IMKH California

GRADE 4

Student Edition

UNITS 1-2

Illustrative® Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 1

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GRADE 4

UNIT

1

Factors and Multiples

In this unit you will learn even more about multiplication, division, and the area of a rectangle. You should finish the unit with a deeper understanding of factors and multiples. Throughout the unit you will make connections by:

- **Exploring Changing Quantities** by examining number and shape patterns, factors, and area models while using algebraic thinking.
- **Discovering Shapes and Space** while problem solving.

Addressing the Standards

As you work your way through **Unit 1 Factors and Multiples,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	**Where You Use These MPs**
MP1 Make sense of problems and persevere in solving them.	Lessons 4, 5, and 6
MP2 Reason abstractly and quantitatively.	Lesson 5
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 5 and 7
MP4 Model with mathematics.	Lessons 5 and 8
MP5 Use appropriate tools strategically.	--
MP6 Attend to precision.	Lessons 1 and 7
MP7 Look for and make use of structure.	Lessons 1 and 2
MP8 Look for and express regularity in repeated reasoning.	Lessons 2 and 3

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
Connected Problem Solving	**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Lesson 5
Factors and Area Models	**4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	Lessons 1, 2, 3, 4, 5, 6, 7, 8
Number and Shape Patterns	**4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	Lesson 3

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 1, Lesson 1

Addressing CA CCSSM 4.OA.4; building on 3.MD.7.a; practicing MP6 and MP7

Multiples of a Number

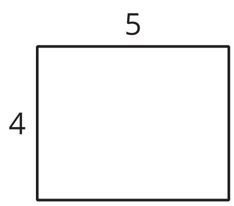
Let’s build some rectangles.

Warm-up

Which Three Go Together: All Kinds of Area

Which 3 go together?

A



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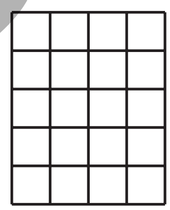

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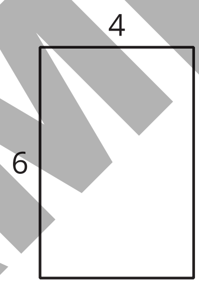

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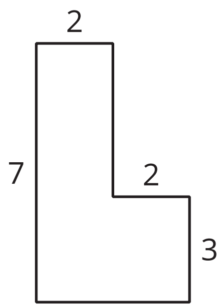
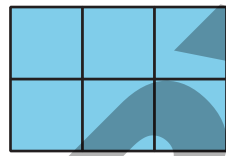


Activity 1

Build Rectangles and Find Area

1. Build 5 different rectangles with each of the given widths. Record the area of each rectangle in the table.



	area of rectangle				
2 tiles wide					
3 tiles wide					
4 tiles wide					

2. Discuss with a partner what you notice about the areas in each row of the table.

3. Predict the area of another rectangle that has each width. Explain your reasoning.

   - 2 tiles:
   - 3 tiles:
   - 4 tiles:

Activity 2

What Areas Can You Build?

1. Elena builds rectangles with a width of 3 units and an area of 30 square units or less.

   a. Build the rectangles Elena could make and draw the rectangles on grid paper. Label the area and the side lengths of each rectangle.

   b. What is the area of each rectangle you built?

   c. What do you notice about the areas?

2. Why is 28 square units not a possible area for a rectangle with a width of 3 units?

3. Elena decides that the area of the rectangle can be more than 30 square units. Find 2 other areas it could have. Explain or show your reasoning.

4. What is an area that is *not* possible for a rectangle with a width of 3 units? Explain or show your reasoning.

Unit 1, Lesson 2

Addressing CA CCSSM 4.OA.4; building on 3.OA.7; practicing MP7 and MP8

Factor Pairs

Let's learn about factor pairs.

Sec A

Warm-up

Number Talk: Multiplication

Find the value of each expression mentally.

- 2×7
- 4×7
- 3×7
- 7×7

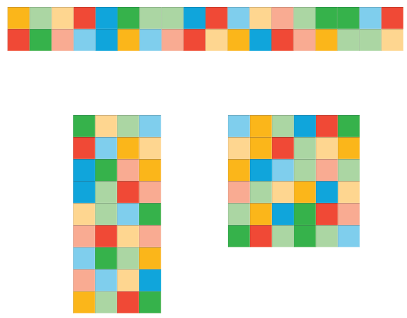
Activity 1

How Many Rectangles?

Your teacher will assign 2 numbers to your group. Each number represents the area of a rectangle.

1. On grid paper:
    - Draw all the possible rectangles that have the given area.
    - Label the area and side lengths of each rectangle.
    - Use only whole numbers for side lengths.
    - Use each pair of side lengths only once.

      (For example, if you draw a rectangle with 4 units across and 6 units down, you don’t need to also draw a rectangle with 6 units across and 4 units down because they have the same pair of side lengths.)
2. When you think you've drawn all the possible rectangles for both areas, cut out your rectangles and put them on a poster. Make 1 poster for each area you were assigned.
3. Display your poster for all to see.



Activity 2

Gallery Walk: How Many Rectangles?

As you visit each poster, discuss with your partner:

1. What do you notice? Use the following sentence frames when you share:
    a. "I notice that some of the posters . . . ."
    b. "I notice the posters for areas _____ and _____ are alike because . . . ."

2. How do you know that all possible rectangles were found for the given area?

Unit 1, Lesson 3

Addressing CA CCSSM 4.OA.4 and 4.OA.5; practicing MP8

Prime and Composite Numbers

Let's identify prime and composite numbers.

Activity 1

Card Sort: Area

Your teacher will give you a set of cards that show rectangles.

1. Sort the cards into 4 categories in a way that makes sense to you. Be ready to explain the meaning of your categories.
2. Group the cards into rectangles that have the same area. Be ready to explain your reasoning.
3. For each group of cards that have the same area, think of at least one more rectangle. Record its length and width. Be prepared to explain your reasoning.

Activity 2

Prime or Composite?

The table shows different areas. How many rectangles with whole-number side lengths can be made for each area?

Complete the table.

Rectangles with the same pair of side lengths should be counted only once. For example, if you count a rectangle with 4 units across and 6 units down, you don't need to also count a rectangle with 6 units across and 4 units down.

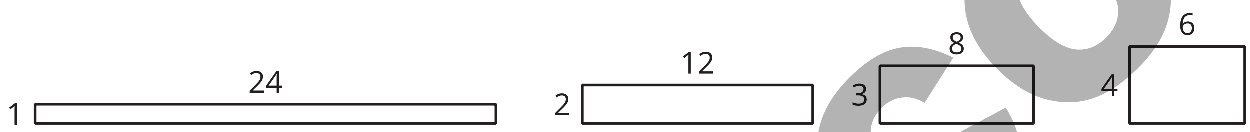
area (square units)	how many rectangles?	prime or composite?
2		
10		
48		
11		
21		
23		
60		
32		
42		
31		
56		

Section A Summary

We used our understanding of the area of rectangles to learn about factors, multiples, factor pairs, prime numbers, and composite numbers.

If we know the side length of a rectangle, we can find the areas that the rectangle could have. For instance, a rectangle with a side length of 3 could have an area of 3, 6, 9, 12, 15, or other numbers that result from multiplying 3 by a whole number. We call these numbers **multiples** of 3.

If we know the area of a rectangle, we can find the side lengths that it could have. For example, a rectangle with an area of 24 square units could have side lengths of 1 and 24, 2 and 12, 3 and 8, or 4 and 6. We call these pairs of side lengths the **factor pairs** of 24.



We also learned that a number that has only one factor pair—1 and the number itself—is called a **prime number**. For instance, 5 is prime because its only factor pair is 1 and 5.

A number that has two or more factor pairs is a **composite number**. For instance, 15 is composite because its factor pairs are 1 and 15, and 3 and 5.

Addressing CA CCSSM 4.OA.4; building on 3.OA.7; practicing MP1

Multiplication Practice

Let's practice multiplication.

Warm-up

Number Talk: Factors Over Ten

Find the value of each expression mentally.

- 10×6
- 3×6
- 13×6
- 12×4

Activity 1

Sort the Multiplication Facts

Sec A

Take turns sorting the multiplication expressions into one of these groups:

- know it right away
- can find it quickly
- don’t know it yet

Multiplication expressions I’m going to practice:

A.

B.

C.

D.

E.

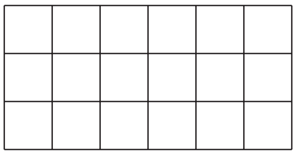
Practice Problems

8 Problems

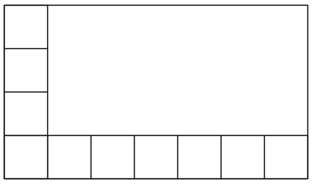
1 Pre-unit

Find the area of each rectangle. Explain your reasoning.

a.



b.

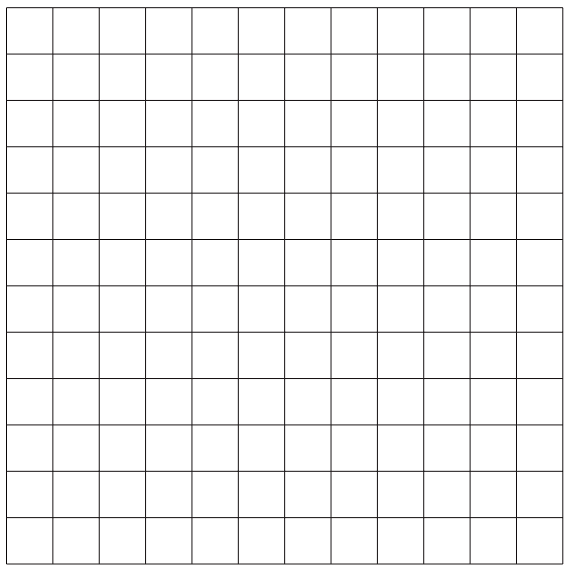


2 Pre-unit

On the grid, draw a rectangle whose area is represented by each expression.

a. 3×5

b. 4×8



3 from Unit 1, Lesson 1

Tyler wants to build a rectangle with an area of 20 square units using square tiles.

a. Can Tyler build a rectangle with a width of 4 units? Explain or show your reasoning.

b. Can Tyler build a rectangle with a width of 6 units? Explain or show your reasoning.

4 from Unit 1, Lesson 2

List the possible whole-number side lengths of rectangles with an area of 32 square units. Explain or show how you know your list is complete.

5 from Unit 1, Lesson 3

List the factor pairs of each number. Is each number prime or composite? Explain or show your reasoning.

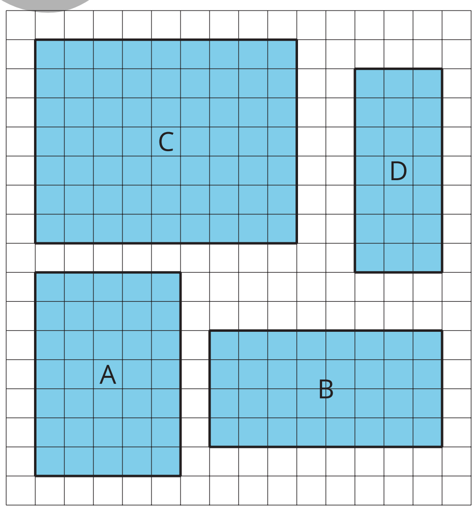
a. 37

b. 27

c. 77

6 from Unit 1, Lesson 4

a. Use multiplication facts to calculate the area of each rectangle.



b. How did you use multiplication facts to calculate the areas?

7 Exploration

a. You want to arrange all of the students in your class in equal rows.

   i. How many rows can you have? How many students would be in each row?

   ii. What if you add the teacher to the arrangement? How would your rows change?

b. Find some objects at home (such as silverware, stuffed animals, cards from a game) and decide how many equal rows you can arrange them in and how many objects are in each row.

8 Exploration

What is the largest prime number you can find? Explain or show why it is a prime number.

Unit 1, Lesson 5

Addressing CA CCSSM 4.OA.3 and 4.OA.4; practicing MP1, MP2, MP3, and MP4

More Multiples

Let's solve problems that involve factors and multiples.

Warm-up

Estimation Exploration: Banquet Seating

About how many chairs are in this room?

Record an estimate that is:

too low	about right	too high

Activity 1

Choose the Right Tables

Students are preparing for a party. The school has tables where 6 people can sit and tables where 8 people can sit.

The students can only choose one type of table and they want to avoid having empty seats.

Sec B

1. Jada's class has 18 students. Which table would you choose for Jada's class? Explain or show your reasoning.

2. Noah's class has 30 students. Which table would you choose for Noah's class? Explain or show your reasoning.

3. Which table would you choose for Noah's and Jada's classes together? Can you find more than one option? Explain or show your reasoning.

4. If you also want places for Noah's teacher and Jada's teacher to sit, which table would you choose? Explain or show your reasoning.

Activity 2

Party Hats and Noisemakers

Lin and Diego are planning school parties.

- Each package of party hats has 10 party hats.
- Each package of noisemakers has 8 noisemakers.

1. Lin needs 50 party hats for his school party.
   a. How many packages of party hats should Lin buy? Explain or show your reasoning.
   b. Can Lin buy exactly 50 noisemakers? How many packages of noisemakers should Lin buy? Explain or show your reasoning.

2. Diego needs 72 party hats for his school party.
   a. How many packages of party hats should Diego buy? Explain or show your reasoning.

Sec B

b. How many packages of noisemakers should Diego buy? Explain or show your reasoning.

Sec B

3. Is it possible to buy exactly the same number of party hats and noisemakers? If so, what would that number be? If not, explain your reasoning.

Unit 1, Lesson 6

Addressing CA CCSSM 4.OA.4, practicing MP1 and MP4

The Locker Problem

Let’s figure out what's happening in a game about lockers.

Activity 1

Questionable Lockers

The picture shows lockers in a school hallway.

The 20 students in Tyler’s fourth-grade class play a game in a hallway that has 20 lockers in a row. The lockers are numbered from 1 to 20.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

- The 1st student starts with the 1st locker, and while going down the hallway, opens all the lockers.
- The 2nd student starts with the 2nd locker, and while going down the hallway, shuts every other locker.
- The 3rd student stops at every 3rd locker and opens the locker if it is closed or shuts the locker if it is open.
- The 4th student stops at every 4th locker and opens the locker if it is closed or shuts the locker if it is open.
- This process continues through the 20th student, so that all 20 students in the class touch the lockers.

Create a representation to show what you understand about this problem. Consider:

- How does your representation show lockers?
- How does your representation keep track of students who touch lockers?
- How does your representation show which lockers are open or closed?

Sec B

Activity 2

An Open-and-Shut Case

Tyler's class plays the same locker game again.

Your goal this time is to find out which lockers are touched as each of the 20 students takes their turns.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. Which locker numbers does the 3rd student touch?

2. Which locker numbers does the 5th student touch?

3. How many students touch locker 17? Explain or show how you know.

4. Which lockers are touched by only 2 students? Explain or show how you know.

Sec B

5. Which lockers are touched by only 3 students? Explain or show how you know.

6. Which lockers are touched the most? Explain or show how you know.

If you have time: Which lockers are still open at the end of the game? Explain or show how you know.

Addressing CA CCSSM 4.OA.4; building on 3.OA.5, 3.OA.7, practicing MP3 and MP6

Find Factors and Multiples

Let's find factors and multiples of whole numbers from 1 to 100.

Warm-up

Number Talk: Division

Find the value of each expression mentally.

- $12 \div 3$
- $30 \div 3$
- $60 \div 3$
- $72 \div 3$

Activity 1

Factor and Multiple Statements

1. For each number, complete a statement using the word "factor" and a statement using the word "multiple."

number	factor	multiple
10	____ is a factor of ____ because . . .	____ is a multiple of ____ because . . .
7	____ is a factor of ____ because . . .	____ is a multiple of ____ because . . .
50	____ is a factor of ____ because . . .	____ is a multiple of ____ because . . .
16	____ is a factor of ____ because . . .	____ is a multiple of ____ because . . .

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number	factor	multiple
35	_____ is a factor of _____ because . . .	_____ is a multiple of _____ because . . .
20	_____ is a factor of _____ because . . .	_____ is a multiple of _____ because . . .
19	_____ is a factor of _____ because . . .	_____ is a multiple of _____ because . . .
6	_____ is a factor of _____ because . . .	_____ is a multiple of _____ because . . .

2. As you compare statements with your partner, discuss one thing you notice and one thing you wonder.

Section B Summary

We used what we learned about factors, multiples, and prime and composite numbers between 1 and 100 to play games and solve problems.

We learned that numbers can share factors and multiples. Example:

- The number 2 is a factor of 6 and and also a factor of 8.
- The number 24 is a multiple 6 and also a multiple of 8.

Knowing about factors and multiples helped us answer questions such as:

- “Can we put 24 chairs in 6 equal rows? What about 7 equal rows or 8 equal rows?”
- “If there are 20 lockers in a row (numbered 1 to 20) and a student touches every fourth locker, how many lockers would they touch? Which locker numbers would they touch?”

Unit 1, Lesson 8

Addressing CA CCSSM 4.OA.4; practicing MP4

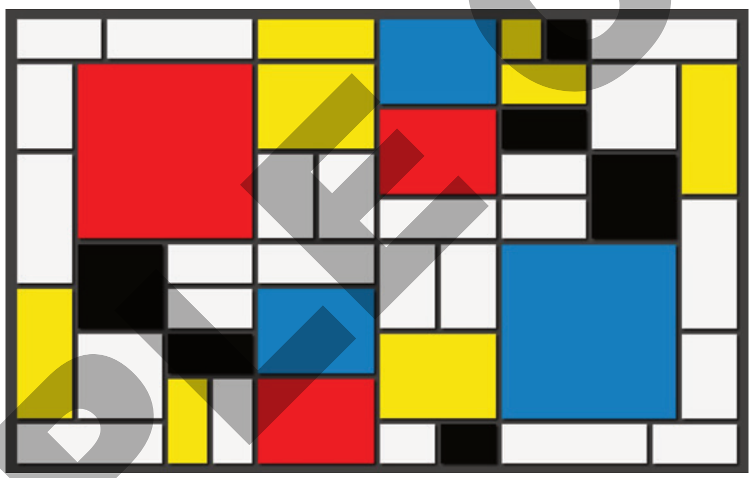
Mondrian's Art

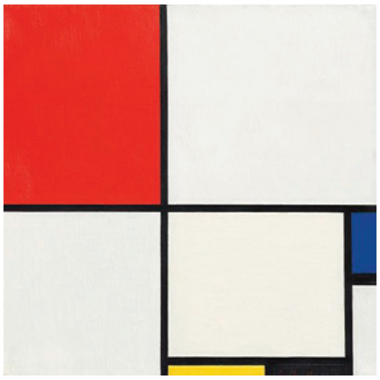
Let's make art with rectangles.

Warm-up

Notice and Wonder: Piet Mondrian's Art

What do you notice? What do you wonder?





Activity 1

My Rectangle Art

1. Use grid paper to create a plan for your own artwork that uses at least 12 rectangles.

   a. The areas of the rectangles should represent at least three of the following:

      _____ all the factors of a number

      _____ at least 6 multiples of a number

      _____ prime numbers

      _____ composite numbers

   b. Explain or show your reasoning.

2. Turn your grid paper over and trace your design with a black marker or crayon. Color the design to emphasize the choices you made and give your artwork a title.

Activity 2

Analyze the Rectangles

Trade artwork with your partner.

Describe which of the following you see in the area of the rectangles in your partner's artwork:

- It shows all the factors of a number.
- It shows at least 6 multiples of a number.
- It shows prime numbers.
- It shows composite numbers.

Sec B

Practice Problems

5 Problems

1 from Unit 1, Lesson 5

Pens are sold in packages of 5 and also in packages of 6.

a. Jada wants to buy 60 pens for her class. Which packages of pens and how many should Jada buy if she doesn't want any extras? Explain or show your reasoning.

b. Han wants to buy 55 pens for his class. Which packages of pens and how many should Han buy? Explain or show your reasoning.

Sec B

2 from Unit 1, Lesson 6

a. Find the factor pairs of 36.

b. How many factors does 36 have?

c. List the factors of 15.

3 from Unit 1, Lesson 7

Select **all** numbers that are multiples of 8.

A. 16

B. 28

C. 40

D. 54

E. 66

F. 72

G. 84

H. 96

4 Exploration

a. List the multiples of 2 from 1 to 30.

b. List the multiples of 3 from 1 to 30.

c. What do you notice about the numbers in the two lists?

5 Exploration

Which number(s) from 1 to 100 have the most factors? Explain or show how you know.

GRADE 4

UNIT

2

Fraction Equivalence and Comparison

In this unit you will learn even more about equivalent fractions and comparing fractions. You will make connections by:

- **Reasoning with Data** as you measure and plot data using fractions.
- **Taking Wholes Apart and Putting Parts Together** when you explore fraction models and the flexibility of fractions.

Addressing the Standards

As you work your way through **Unit 2 Fraction Equivalence and Comparison,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 14
MP2 Reason abstractly and quantitatively.	Lessons 13 and 15
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3, 7, and 9
MP4 Model with mathematics.	Lesson 17
MP5 Use appropriate tools strategically.	Lesson 11
MP6 Attend to precision.	Lessons 2, 7, and 9
MP7 Look for and make use of structure.	Lessons 1, 4, 6, 8, and 16
MP8 Look for and express regularity in repeated reasoning.	Lessons 3, 5, 8, and 10

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measuring and Plotting • Fraction Flexibility • Visual Fraction Models	**4.NF.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	Lessons 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17
• Measuring and Plotting • Visual Fraction Models	**4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lessons 3, 6, 12, 13, 14, 15, 16, and 17

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 2, Lesson 1

Addressing CA CCSSM 4.NF.1; building on 3.NF.1; practicing MP7

Representations of Fractions (Part 1)

Let's name some fractions and represent them visually.

Sec A

Warm-up

What Do You Know about $\frac{1}{2}$?

What do you know about $\frac{1}{2}$?

Sec A

Activity 1

Fraction Strips

Your teacher will give you strips of paper. Each strip represents 1.

1. Use the strips to represent halves, fourths, and eighths.

   Use one strip for each fraction and label the parts.

2. What do you notice about the number of parts or the size of the parts? Make at least 2 observations.

Activity 2

Fractions, Represented

1. Each whole diagram represents 1. What fraction does the shaded part of each diagram represent?

   a. 

   b. 

   c. 

2. Here are four blank diagrams. Each diagram represents 1. Partition each diagram and shade 1 part so that the shaded part represents the given fraction.

   a. $\frac{1}{6}$

   b. $\frac{1}{8}$

   c. $\frac{1}{10}$

   d. $\frac{1}{12}$

3. Suppose you use the same blank diagram to represent $\frac{1}{20}$. Would the shaded part be larger or smaller than the shaded part in the diagram of $\frac{1}{10}$? Explain how you know.

Unit 2, Lesson 2

Addressing CA CCSSM 4.NF.1; building on 3.NF.1; practicing MP6

Representations of Fractions (Part 2)

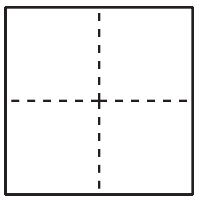
Let’s name some other fractions and represent them with diagrams.

Warm-up

Which Three Go Together: All Cut Up

Which 3 go together?

A



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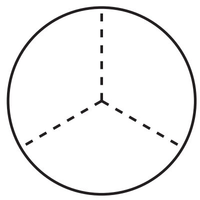

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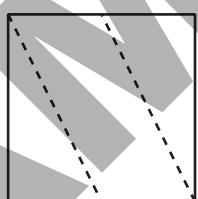

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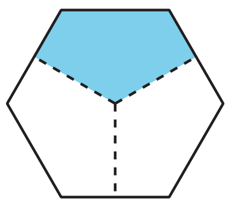

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Activity 1

A Diagram for Each Fraction

Each whole diagram represents 1. Match each fraction to a diagram whose shaded part represents that fraction.

Two of the fractions are *not* represented. Use the blank diagrams to represent each of them.

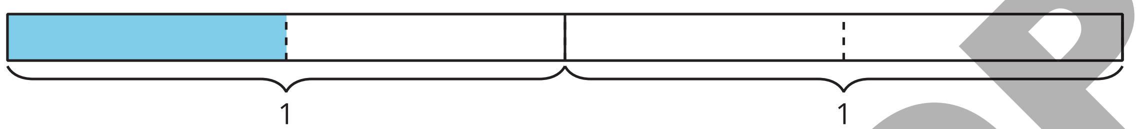
$\frac{2}{3}$: ________	$\frac{3}{8}$: ________	$\frac{4}{10}$: ________	$\frac{4}{6}$: ________	$\frac{6}{6}$: ________
$\frac{3}{5}$: ________	$\frac{4}{8}$: ________	$\frac{6}{12}$: ________	$\frac{6}{10}$: ________	$\frac{3}{4}$: ________
$\frac{5}{6}$: ________	$\frac{2}{5}$: ________	$\frac{5}{12}$: ________	$\frac{7}{10}$: ________	$\frac{7}{8}$: ________

Activity 2

Diagrams for Some Other Fractions

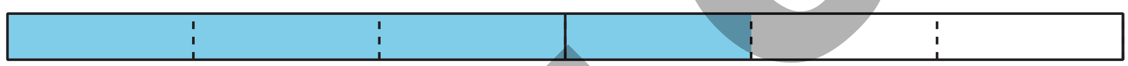
1. What fraction does the shaded part of each diagram represent?

   a.



   b.

   c.

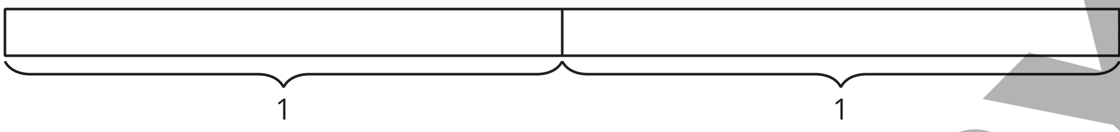


   d.

   e.

2. Here are 4 fractions and 4 blank diagrams. Partition each diagram and shade the parts to represent the fraction.

   a. $\frac{2}{2}$

   b. $\frac{4}{2}$

   c. $\frac{5}{4}$

   d. $\frac{10}{8}$

Unit 2, Lesson 3

Addressing CA CCSSM 4.NF.2; building on 3.NF.ed; building towards 4.NBT.4; practicing MP3 and MP8

Same Denominator or Numerator

Sec A

Let's compare fractions with the same numerator or the same denominator.

Warm-up

Number Talk: Hundreds More

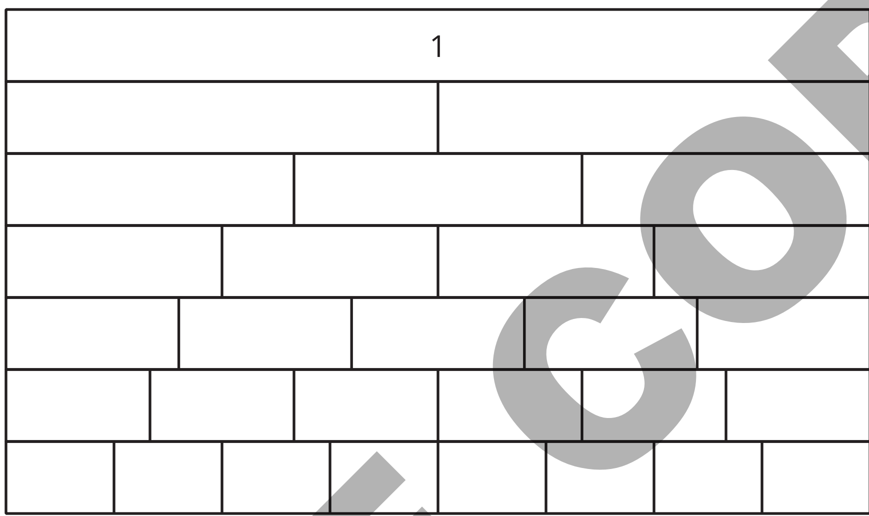
Find the value of each expression mentally.

- $136 + 100$
- $136 + 300$
- $136 + 370$
- $136 + 378$

Activity 1

Fractions with the Same Denominator

1. This diagram shows a set of fraction strips. Label each part of each strip with the fraction it represents.



2. Circle the greater fraction in each pair. If helpful, use the diagram of fraction strips.

   a. $\frac{3}{4}$ or $\frac{5}{4}$

   b. $\frac{3}{5}$ or $\frac{5}{5}$

   c. $\frac{3}{6}$ or $\frac{5}{6}$

   d. $\frac{3}{8}$ or $\frac{5}{8}$

   e. $\frac{3}{10}$ or $\frac{5}{10}$

3. What pattern do you notice about the circled fractions? How can you explain the pattern?

4. Which fraction is greater: $\frac{7}{3}$ or $\frac{10}{3}$? Explain your reasoning.

Activity 2

Fractions with the Same Numerator

1. Circle the greater fraction in each pair. If helpful, use the diagram of fraction strips from Activity 1.

   a. $\frac{1}{3}$ or $\frac{1}{5}$

   b. $\frac{2}{3}$ or $\frac{2}{5}$

   c. $\frac{3}{3}$ or $\frac{3}{5}$

   d. $\frac{4}{3}$ or $\frac{4}{5}$

   e. $\frac{9}{3}$ or $\frac{9}{5}$

2. What pattern do you notice about the circled fractions? How can you explain the pattern?

3. Which fraction is greater: $\frac{7}{12}$ or $\frac{7}{8}$? Explain your reasoning.

4. Tyler is comparing $\frac{4}{10}$ and $\frac{4}{6}$. He says, “I know 10 is greater than 6. So, $\frac{4}{10}$ is greater than $\frac{4}{6}$.” Explain or show why Tyler’s conclusion is incorrect.

Unit 2, Lesson 4

Addressing CA CCSSM 4.NF.1; building on 3.NF.2, 3.NF.2.a, 3NF.3.b; practicing MP7

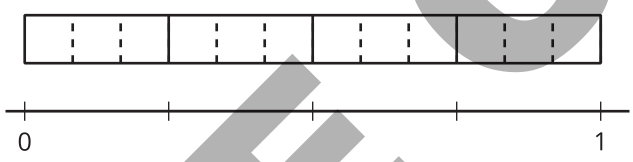
Same Size, Related Sizes

Let’s find some fractions that are the same size.

Warm-up

Notice and Wonder: A Fraction Strip and a Number Line

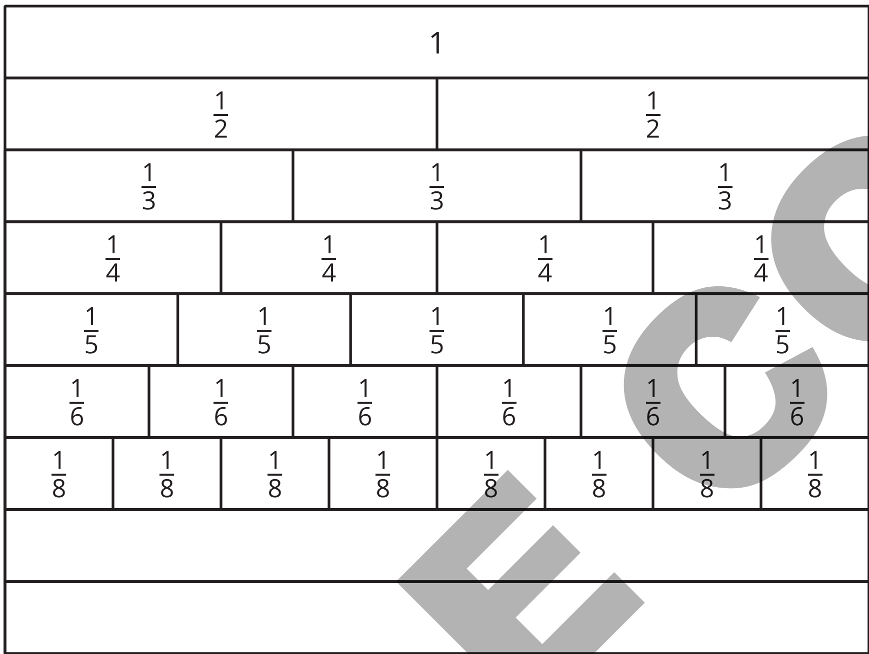
What do you notice? What do you wonder?



Activity 1

Same Size, Different Numbers

Here's a diagram of fraction strips, with two blank strips added.



1. Use one blank strip to show tenths. Label the parts. How did you partition the strip?

2. Use the other blank strip to show twelfths. Label the parts. How did you partition the strip?

3. Jada says, "I notice that 1 of the $\frac{1}{2}$ parts is the same size as 2 of the $\frac{1}{4}$ parts and 3 of the $\frac{1}{6}$ parts. So, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ must be equivalent fractions." Jada's reasoning is correct.

   Find a fraction in the diagram that is equivalent to each of the following fractions. Be prepared to explain your reasoning.

   a. $\frac{1}{6}$

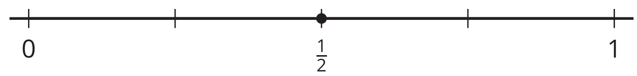
   b. $\frac{2}{10}$

   c. $\frac{3}{3}$

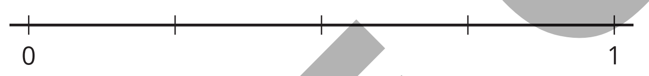
Activity 2

Fractions on Number Lines

1. The point on this number line shows the fraction $\frac{1}{2}$.



Label the tick marks on each number line.



2. You will locate $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$ on one of the number lines.

   a. Which number line would you use for each fraction? Be prepared to explain your reasoning.

   b. Locate and label each fraction ($\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$) on a different number line.

3. Locate and label each fraction on one of the number lines.

$\frac{2}{6}$	$\frac{2}{8}$	$\frac{6}{8}$	$\frac{8}{10}$	$\frac{4}{6}$
$\frac{4}{8}$	$\frac{4}{10}$	$\frac{6}{6}$	$\frac{6}{10}$	$\frac{8}{8}$

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Unit 2, Lesson 5

Addressing CA CCSSM 4.NF.1; building on 3.OA.5; practicing MP8

Fractions on Number Lines

Let's investigate equivalent fractions on a number line.

Sec A

Warm-up

Number Talk: A Number Times 12

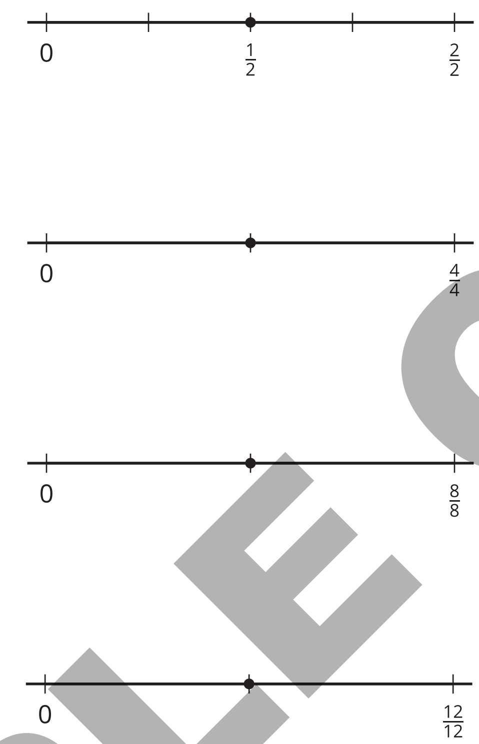
Find the value of each expression mentally.

- 2×12
- 4×12
- 8×12
- 16×12

Activity 1

All Lined Up

1. These number lines have different labels for the tick mark on the far right.



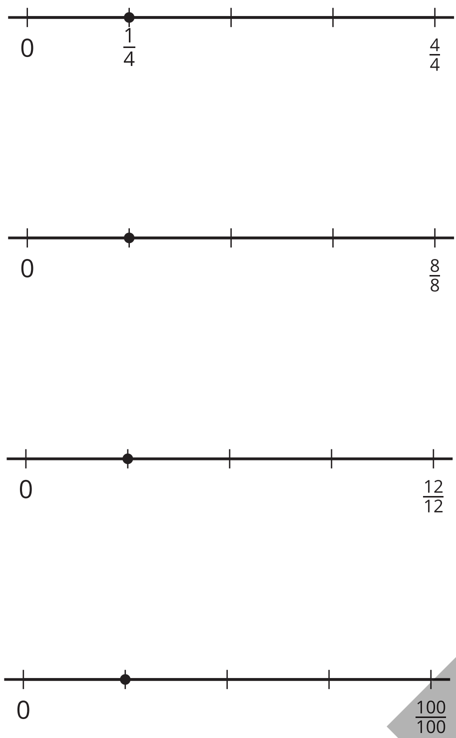
   a. Explain to your partner why the tick marks on the far right can be labeled with different fractions.

   b. Label each point with a fraction it represents (other than $\frac{1}{2}$).

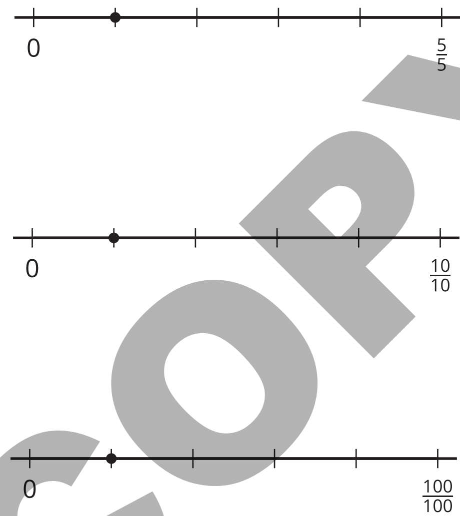
   c. Explain to your partner why the fractions you wrote are equivalent.

2. Label the point on each number line with a fraction it represents. Use a different fraction for each number line. Be prepared to explain your reasoning.

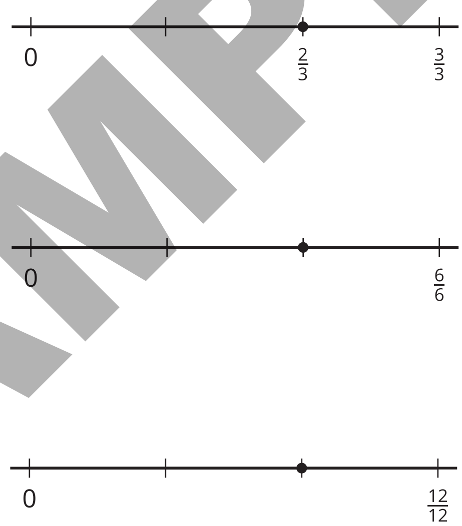
a.



b.



c.



Sec A

Sec A

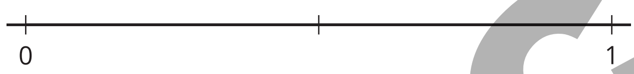
Activity 2

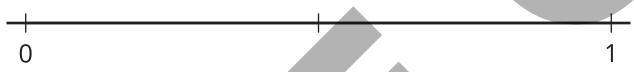
How Far to Run?

1. Han and Kiran plan to go for a run after school.

   - Han says, “Let’s run $\frac{3}{4}$ mile. That’s how far I run to my soccer practice.”
   - Kiran says, “I can only run $\frac{9}{12}$ mile.”

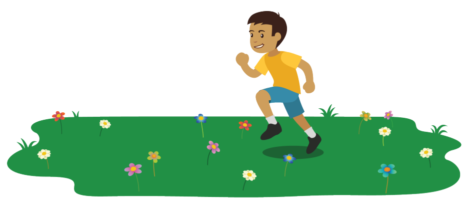
   Which distance should they run? Explain your reasoning. Use one or more number lines to show your reasoning.

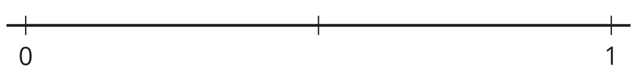




2. Mai wants to join Han and Kiran on their run. She says, “How about we run $\frac{7}{8}$ mile?”

   Is the distance Mai suggests the same as what her friends wanted to run? Explain or show your reasoning.





Unit 2, Lesson 6

Addressing CA CCSSM 4.NF.2; building on 3.NF.2 and 3.NF.3; practicing MP7

Relate Fractions to Benchmarks

Let's compare the size of fractions to $\frac{1}{2}$ and to 1.

Warm-up

Notice and Wonder: A Point on a Number Line

What do you notice? What do you wonder?

Activity 1

Greater than or Less than 1?

Sec A

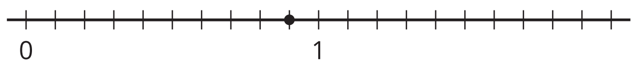
For each number line:

a. Name a fraction that the point represents.

b. Is that fraction greater than or less than 1?

c. How far is it from 1?

1.

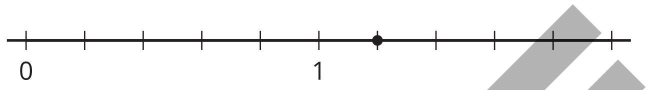


a.

b.

c.

2.



a.

b.

c.

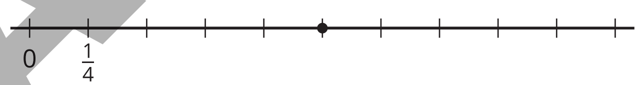
3.

a.

b.

c.

4.



a.

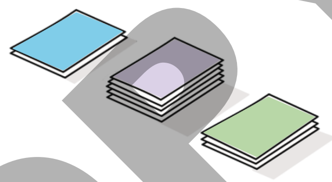
b.

c.

Activity 2

Card Sort: Where Do They Belong?

Your teacher will give you a set of cards that show fractions.



1. Sort the cards into 3 groups: less than $\frac{1}{2}$, equal to $\frac{1}{2}$, and greater than $\frac{1}{2}$. Be ready to explain your reasoning.

   Discuss your sorting with another group. Then record the fractions in the table.

less than $\frac{1}{2}$	equal to $\frac{1}{2}$	greater than $\frac{1}{2}$

2. Discuss your sorting with the class. Then complete the sentences.
   - A fraction is less than $\frac{1}{2}$ when . . .
   - A fraction is greater than $\frac{1}{2}$ when . . .
   - A fraction is between $\frac{1}{2}$ and 1 when . . .

Sec A

Activity 3

Greater than or Less than $\frac{1}{2}$?

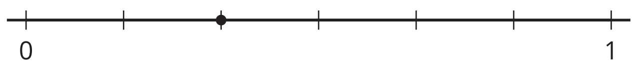
For each number line:

a. Name a fraction that the point represents.

b. Is that fraction greater than or less than $\frac{1}{2}$?

c. What fraction describes how far the point is from $\frac{1}{2}$?

1.

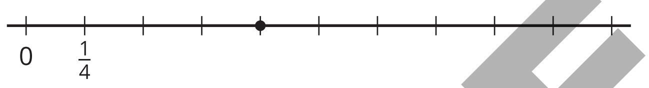


a.

b.

c.

2.



a.

b.

c.

3.

a.

b.

c.

4.

a.

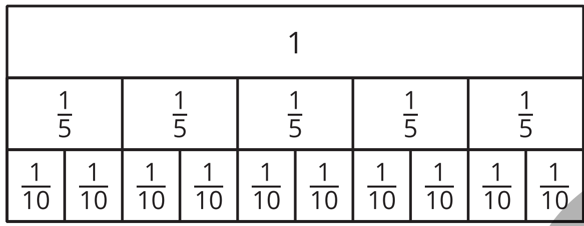
b.

c.

Section A Summary

We used fraction strips to represent fractions with denominators of 2, 3, 4, 5, 6, 8, 10, and 12.

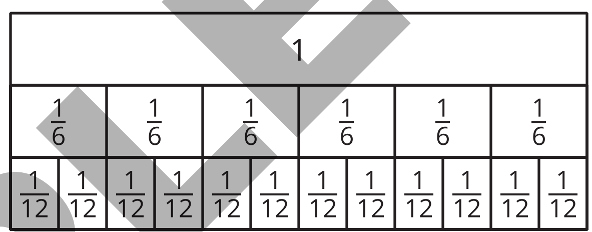
Fraction strips helped us reason about relationships between fractions.



Example:

- One whole split into 5 equal parts makes 5 fifths.
- Each fifth split into 2 equal parts makes 10 equal parts, or 10 tenths.
- When the denominator is larger, there are more parts in a whole.

Fraction strips also helped us reason about the sizes of fractions.



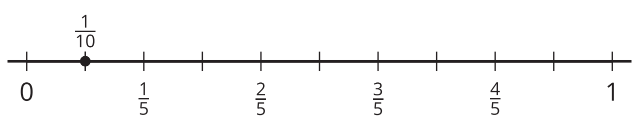
Same denominator: The size of the parts is the same. So, the fraction with more parts is greater.

Example: $\frac{5}{6}$ is greater than $\frac{2}{6}$.

Same numerator: The number of parts is the same. So, the fraction with larger parts is greater.

Example: $\frac{5}{6}$ is greater than $\frac{5}{12}$.

We used what we learned about fraction strips to partition number lines and represent fractions.



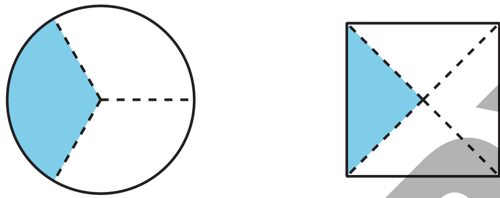
Practice Problems

12 Problems

Sec A

1 Pre-unit

What fraction of each figure is shaded?



2 Pre-unit

Explain why the shaded part represents $\frac{1}{8}$ of the whole rectangle.

3 Pre-unit

Label each tick mark with the number it represents. Explain your reasoning.

4 Pre-unit

Explain or show why $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

5 from Unit 2, Lesson 1

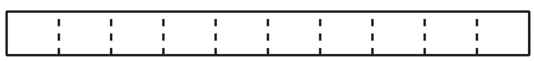
a. The whole diagram represents 1.
Shade the diagram to represent $\frac{1}{4}$.

b. To represent $\frac{1}{6}$ on the diagram, would you shade more or less than what you shaded for $\frac{1}{4}$? Explain your reasoning.

6 from Unit 2, Lesson 2

a. The whole diagram represents 1.
What fraction does the shaded part represent? Explain your reasoning.

b. Shade this diagram to represent $\frac{2}{10}$.



7 from Unit 2, Lesson 3

Circle the greater fraction in each pair. Explain or show your reasoning.

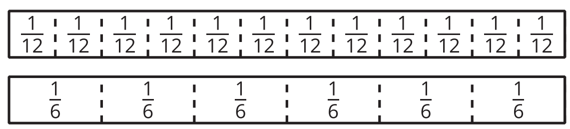
a. $\frac{1}{8}$ or $\frac{1}{10}$

b. $\frac{4}{10}$ or $\frac{7}{10}$

c. $\frac{4}{5}$ or $\frac{5}{4}$

8 from Unit 2, Lesson 4

Use the fraction strips to name 3 pairs of equivalent fractions. Explain how you know the fractions are equivalent.



9 from Unit 2, Lesson 5

a. Explain or show why the point on the number line represents both $\frac{3}{5}$ and $\frac{6}{10}$.

b. Explain why $\frac{6}{10}$ and $\frac{3}{5}$ are equivalent fractions.

10 from Unit 2, Lesson 6

For each question, explain your reasoning. Use a number line if you find it helpful.

a. Is $\frac{4}{5}$ greater than or less than $\frac{1}{2}$?

b. Is $\frac{4}{5}$ greater than or less than 1?

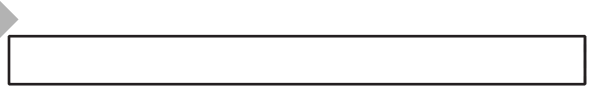
11 Exploration

Fold a strip of paper to represent each fraction. How did you fold the paper to make sure the parts are the correct size for each fraction? Use the blank diagrams to show how you folded.

a. $\frac{1}{3}$s

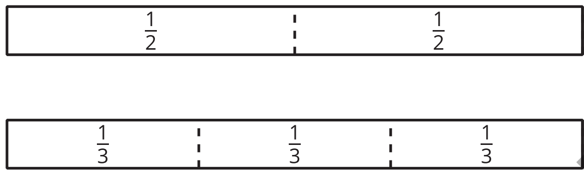
b. $\frac{1}{5}$s

c. $\frac{1}{10}$s

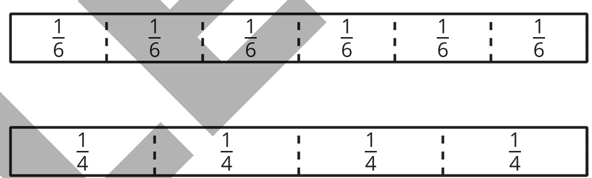


12 Exploration

a. Andre looks at these fraction strips and says, "Each $\frac{1}{2}$ is the same size as $\frac{1}{3}$ and another half of $\frac{1}{3}$." Do you agree with Andre? Explain your reasoning.



b. What relationship do you see between $\frac{1}{6}$ and $\frac{1}{4}$? Explain your reasoning.



c. Can you find a relationship between $\frac{1}{6}$ and $\frac{1}{8}$ using fraction strips? Explain your reasoning.

Addressing CA CCSSM 4.NF.1; building on 3.NF.3.b; practicing MP3 and MP6

Equivalent Fractions

Let's find some equivalent fractions.

Warm-up

True or False: Equivalence

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{4}{8} = \frac{7}{8}$
- $\frac{3}{4} = \frac{6}{8}$
- $\frac{2}{6} = \frac{2}{8}$
- $\frac{6}{3} = \frac{4}{2}$

Activity 1

Two or More Fractions

1. Each whole diagram represents 1. Write 2 or more fractions that the shaded part of each diagram represents. Be prepared to explain your reasoning.

   a. 

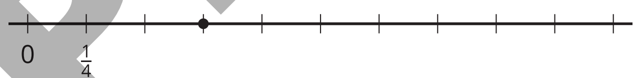
   b. 

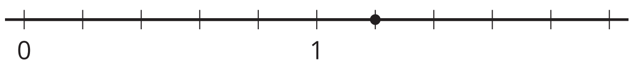
   c. 

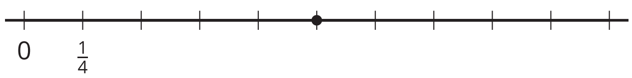
   d. 

2. Write 2 or more fractions that the point on each number line represents. Be prepared to explain your reasoning.

   a. 

   b. 

   c. 

   d. 

3. Draw a new point on a tick mark on one of the last two number lines that you just used. Then write 2 fractions that the point represents.

Activity 2

Equivalent for Sure?

For each fraction, write 2 equivalent fractions.

Partner A

1. $\frac{3}{2}$

2. $\frac{10}{6}$

Partner B

1. $\frac{4}{3}$

2. $\frac{14}{10}$

Next, show or explain to your partner how you know that the fractions you wrote are equivalent to the original. Use any representation that you think is helpful.

Unit 2, Lesson 8

Addressing CA CCSSM 4.NF.1; building on 3.NF.1; practicing MP7 and MP8

Equivalent Fractions on the Number Line

Let’s use number lines to reason about equivalent fractions.

Warm-up

Estimation Exploration: A Shaded Portion

The whole diagram represents 1. What fraction of the diagram is shaded?

Make an estimate that is:

too low	about right	too high

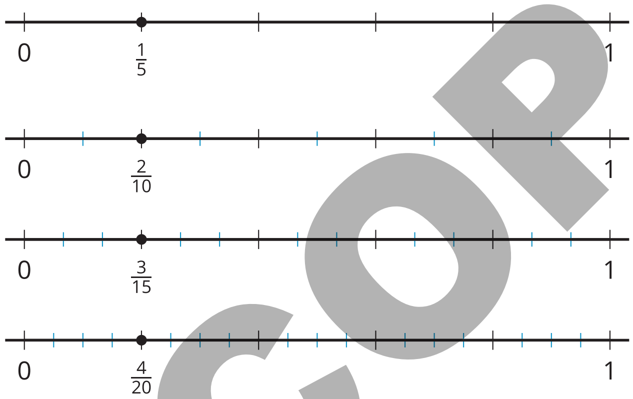
Activity 1

Handy Number Lines

Andre used number lines to find fractions that are equivalent to $\frac{1}{5}$.

He drew this number line:

Then he drew 3 copies of the number line. He wrote a different fraction for the same point on each line:



1. How did Andre use the number lines to find fractions equivalent to $\frac{1}{5}$? Explain your thinking to a partner.
2. How can number lines be used to show whether these pairs of fractions are equivalent?

   a. $\frac{8}{10}$ and $\frac{4}{5}$

   b. $\frac{14}{20}$ and $\frac{4}{5}$

3. Find 3 fractions that are equivalent to $\frac{6}{5}$. Explain or show how Andre's number lines can help.

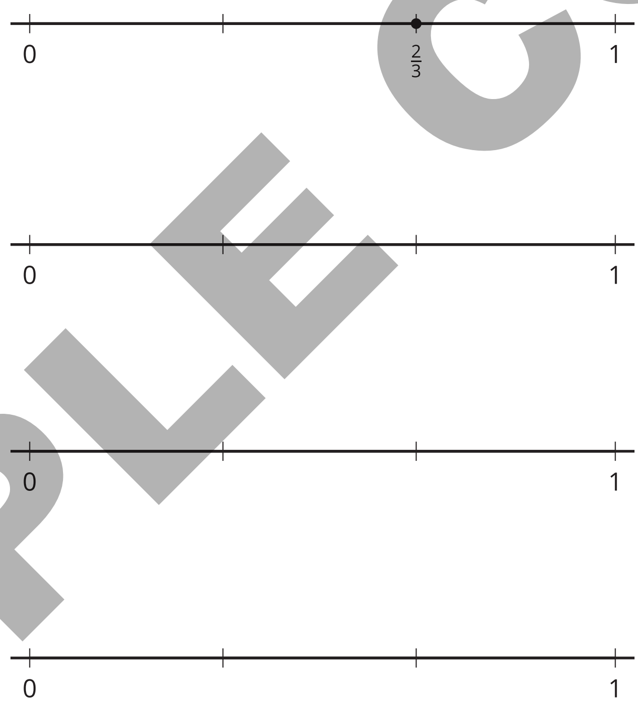
Activity 2

Can It Be Done?

1. Priya wants to find fractions that are equivalent to $\frac{2}{3}$, other than $\frac{4}{6}$. She wonders if she can find equivalent fractions with denominators 9, 10, and 12.

$$\frac{\quad}{9} \qquad \frac{\quad}{10} \qquad \frac{\quad}{12}$$

Can it be done? Use number lines to show your reasoning.



2. Represent $\frac{1}{10}$ on a number line. Then find 2 fractions that are equivalent to $\frac{1}{10}$. How would you use the number lines to show that they are equivalent to $\frac{1}{10}$?

3. Can you find an equivalent fraction for $\frac{1}{10}$ with 100 for the denominator? Explain or show your reasoning.

Addressing CA CCSSM 4.NF.1; building on 3.OA.5; building towards 4.NBT.5; practicing MP3 and MP6

Explain Equivalence

Let's talk about how we know whether two fractions are equivalent.

Warm-up

Number Talk: Familiar Numbers

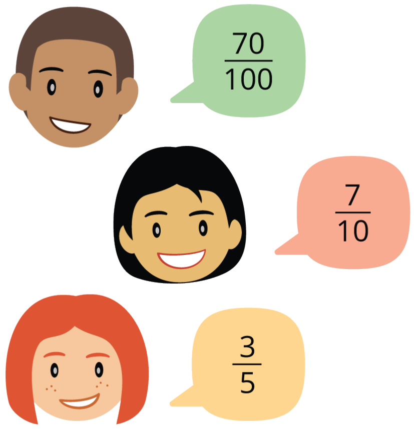
Find the value of each expression mentally.

- 10×6
- 10×12
- 10×24
- 5×24

Activity 1

Pointed Discussion

Andre, Lin, and Clare will represent $\frac{70}{100}$ on a number line.



- Andre says, "Oh, no! We'll need to partition the line into 100 equal parts and count 70 parts just to mark one point!"
- Lin says, "What if we mark $\frac{7}{10}$ instead? We could partition the line into just 10 parts and count 7 parts."
- Clare says, "What if we partition the line into 5 parts and mark $\frac{3}{5}$?"

Do you agree with any of them? Explain or show your reasoning.

Activity 2

How Do You Know?

Around the room you will find 6 posters, each showing either 2 or 3 fractions.

With your group, visit at least 2 posters: one with 2 fractions and one with 3 fractions.

For the poster with 2 fractions:

- Explain or show how you know the fractions are equivalent.

- Write a new equivalent fraction on a sticky note and add it to the poster. Try to find a fraction that hasn't already been written by someone else.

  We visited poster _________, which shows _________ and _________.

  New equivalent fraction: _________

For the poster with 3 fractions:

- Identify 2 fractions that are equivalent. Explain your reasoning.

  We visited poster _________, which shows _________, _________, and _________.

  _________ and _________ are equivalent fractions.

Sec B

Unit 2, Lesson 10

Addressing CA CCSSM 4.NF.1; practicing MP8

Use Multiples to Find Equivalent Fractions

Let's look at a way to find equivalent fractions without using diagrams.

Sec B

Warm-up

Notice and Wonder: Four Equations

What do you notice? What do you wonder?

- $\frac{1}{3} = \frac{2}{6}$
- $\frac{2}{3} = \frac{4}{6}$
- $\frac{3}{3} = \frac{6}{6}$
- $\frac{4}{3} = \frac{8}{6}$

Activity 1

Elena's Way

Elena thought of another way to find equivalent fractions. She wrote:

$$\frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

$$\frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

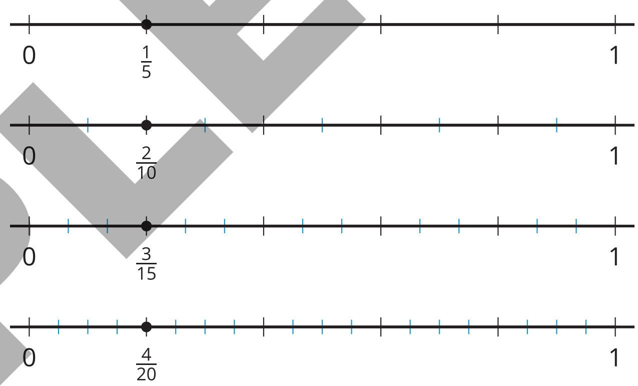
$$\frac{1 \times 4}{5 \times 4} = \frac{4}{20}$$

$$\frac{1 \times 5}{5 \times 5} = \frac{5}{25}$$

$$\frac{1 \times 10}{5 \times 10} = \frac{10}{50}$$

1. Analyze Elena's work. Then discuss these questions with a partner:

   a. How are Elena's equations related to Andre's number lines?



   b. How might Elena find other fractions that are equivalent to $\frac{1}{5}$? Show 2 examples.

2. Use Elena's strategy to find 5 fractions that are equivalent to $\frac{1}{8}$. Use number lines to check your thinking, if they help.

Activity 2

Equivalence Hunting

Look at Elena's strategy from an earlier activity.

1. Could her strategy help you know whether 2 fractions are equivalent? Try using it to check the equivalence of the following pairs of fractions. If they are equivalent, write an equation to show it.

   a. $\frac{5}{2}$ and $\frac{10}{8}$

   b. $\frac{2}{6}$ and $\frac{4}{12}$

2. Find all fractions in the list that are equivalent to $\frac{3}{4}$. Be prepared to explain or show how you know.

$\frac{2}{10}$	$\frac{6}{8}$	$\frac{12}{15}$	$\frac{30}{40}$
$\frac{8}{9}$	$\frac{12}{20}$	$\frac{12}{16}$	$\frac{15}{20}$
$\frac{8}{10}$	$\frac{24}{32}$	$\frac{75}{100}$	$\frac{60}{80}$

Unit 2, Lesson 11

Addressing CA CCSSM 4.NF.1; practicing MP5

Use Factors to Find Equivalent Fractions

Let’s find equivalent fractions by working with numerators and denominators.

Warm-up

Which Three Go Together: Four Representations

Which 3 go together?

A.

B.

C. $\frac{1}{4}$

D.

Activity 1

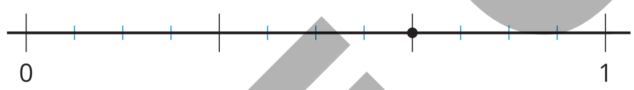
The Other Way Around

1. Andre drew this number line and marked a point on it. Label the point with the fraction it represents.

2. To find other fractions that the point represents, Andre made copies of the number line. He made some of the existing tick marks longer.

   Label the longer tick marks Andre made on each number line.
   Use a different denominator for each number line.

   a.



   b.

3. Kiran wrote the same fractions for the points that Andre did. But Kiran used a different strategy. Analyze his reasoning.

   $$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

   $$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

   How do you think Andre's and Kiran's strategies are related?

4. Try using Kiran's strategy to find 1 or more fractions that are equivalent to $\frac{10}{12}$ and $\frac{18}{12}$.

Activity 2

How Would You Find Them?

Find at least 2 fractions that are equivalent to each fraction. Show your reasoning.

1. $\frac{16}{8}$

2. $\frac{40}{10}$

3. $\frac{7}{6}$

4. $\frac{90}{100}$

5. $\frac{5}{4}$

Sec B

Activity 3

Card Sort: Fractions Galore

Your teacher will give you a set of cards.

Sort the cards by finding as many equivalent fractions as you can. Be ready to explain or show your reasoning.

Sec B

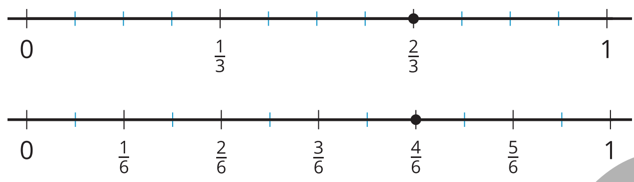
Record the sets of equivalent fractions here.

Record fractions that do *not* have an equivalent fraction in the cards here.

Section B Summary

We learned to identify and write **equivalent fractions**. We represented fractions on number lines. We saw that two fractions that occupy the same spot on a number line are equivalent.

Example:



We also looked at strategies for finding equivalent fractions. We learned that multiplying or dividing the numerator and denominator by the same number will result in an equivalent fraction.

Examples:

$$\frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

$$\frac{1 \times 4}{5 \times 4} = \frac{4}{20}$$

$\frac{1}{5}$ is equivalent to $\frac{2}{10}$ and $\frac{4}{20}$.

$$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

$$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

$\frac{8}{12}$ is equivalent to $\frac{4}{6}$ and $\frac{2}{3}$.

Practice Problems

8 Problems

Sec B

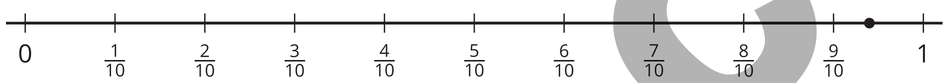
1 from Unit 2, Lesson 7

Name 3 fractions that are equivalent to $\frac{2}{5}$. Explain or show your reasoning.

2 from Unit 2, Lesson 8

Which of these fractions can be represented by the point on the number line? Explain your reasoning.

$\frac{86}{100}$ $\frac{90}{100}$ $\frac{94}{100}$ $\frac{101}{100}$

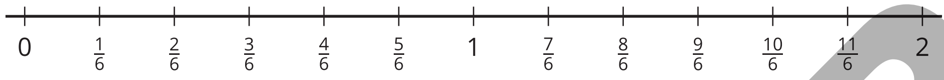


3 from Unit 2, Lesson 9

Explain why the fractions $\frac{10}{3}$ and $\frac{40}{12}$ are equivalent.

4 from Unit 2, Lesson 10

Find 2 fractions equivalent to $\frac{10}{6}$. Explain or show why they are equivalent to $\frac{10}{6}$. Use the number line if you think it is helpful.



5 from Unit 2, Lesson 11

Jada says that $\frac{7}{5}$ is equivalent to $\frac{14}{10}$ because the numerator and denominator of $\frac{14}{10}$ are each 2 times the numerator and denominator of $\frac{7}{5}$.

a. Explain why Jada's reasoning is correct.

____

____

____

____

b. Use Jada's method to find another fraction equivalent to $\frac{7}{5}$.

6 Exploration

Mai is thinking of a fraction. She gives several clues to help you guess her fraction. Try to guess Mai's fraction after each clue.

a. My fraction is equivalent to $\frac{2}{3}$.

b. The numerator of my fraction is greater than 10.

c. A factor of my numerator is 8.

d. The numbers 8 and 5 are a factor pair of my numerator.

7 Exploration

Think of a fraction: __________

Write several clues so a friend or family member can guess your fraction. Then present the clues one at a time and ask them to make a guess after each one.

a. My fraction is equivalent to __________.

b. The numerator of my fraction is less than __________.

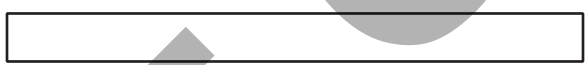
c. One multiple of my numerator is __________.

d. A factor pair of my denominator is __________ and __________.

8 Exploration

a. Diego says he shaded $\frac{10}{20}$ of the diagram. Do you agree with Diego? Explain your reasoning.

b. Shade $\frac{18}{24}$ of the diagram. Explain how you know $\frac{18}{24}$ is shaded.



Unit 2, Lesson 12

Addressing CA CCSSM 4.NF.1 and 4.NF.2; building on 3.NF.1; practicing MP3

Ways to Compare Fractions

Let's compare some fractions.

Sec C

Warm-up

Estimation Exploration: What's That Point?

What number is represented by the point on the number line?

Make an estimate that is:

too low	about right	too high

Activity 1

The Greatest of Them All

Here are 25 fractions in a table.

	A	B	C	D	E
1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{10}$	$\frac{2}{12}$	$\frac{2}{100}$
2	$\frac{4}{3}$	$\frac{4}{5}$	$\frac{4}{10}$	$\frac{4}{12}$	$\frac{4}{100}$
3	$\frac{7}{3}$	$\frac{7}{5}$	$\frac{7}{10}$	$\frac{7}{12}$	$\frac{7}{100}$
4	$\frac{11}{3}$	$\frac{11}{5}$	$\frac{11}{10}$	$\frac{11}{12}$	$\frac{11}{100}$
5	$\frac{26}{3}$	$\frac{26}{5}$	$\frac{26}{10}$	$\frac{26}{12}$	$\frac{26}{100}$

For each question, be prepared to explain your reasoning.

1. Identify the greatest fraction in each column (A, B, C, D, and E).

2. Identify the greatest fraction in each row (1, 2, 3, 4, and 5).

3. Which fraction is the greatest fraction in the entire table?

Activity 2

Relative to $\frac{1}{2}$ and 1

Here is the same table you saw earlier.

	A	B	C	D	E
1	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{10}$	$\frac{2}{12}$	$\frac{2}{100}$
2	$\frac{4}{3}$	$\frac{4}{5}$	$\frac{4}{10}$	$\frac{4}{12}$	$\frac{4}{100}$
3	$\frac{7}{3}$	$\frac{7}{5}$	$\frac{7}{10}$	$\frac{7}{12}$	$\frac{7}{100}$
4	$\frac{11}{3}$	$\frac{11}{5}$	$\frac{11}{10}$	$\frac{11}{12}$	$\frac{11}{100}$
5	$\frac{26}{3}$	$\frac{26}{5}$	$\frac{26}{10}$	$\frac{26}{12}$	$\frac{26}{100}$

1. Which fractions are less than $\frac{1}{2}$? Circle each of them. Then complete this sentence:

   I know a fraction is less than $\frac{1}{2}$ when . . .

2. Which fractions are greater than $\frac{1}{2}$ but less than 1? Circle each of them with a pencil of a different color. (Or draw a triangle around each one.) Then complete this sentence:

   I know a fraction is greater than $\frac{1}{2}$ but less than 1 when . . .

3. Circle the remaining fractions with a pencil of a third color. (Or draw a square around each one.) How would you describe the size of these fractions?

4. Next to the table, create a legend or key to show what each color (or each shape) represents.

5. Here are some pairs of fractions from the table. In each pair, which fraction is greater?

   a. $\frac{2}{5}$ or $\frac{7}{10}$

   b. $\frac{4}{10}$ or $\frac{7}{12}$

   c. $\frac{11}{100}$ or $\frac{4}{3}$

   d. $\frac{26}{10}$ or $\frac{11}{12}$

Unit 2, Lesson 13

Addressing CA CCSSM 4.NF.1 and 4.NF.2; practicing MP2

Use Equivalent Fractions to Compare

Let's compare fractions by writing an equivalent fraction.

Warm-up

Notice and Wonder: Pairs of Numbers

Sec C

What do you notice? What do you wonder?

$5 < 8$ $\qquad \frac{9}{2} > 4\frac{1}{2}$ $\qquad 4 = \frac{3}{2}$ $\qquad \frac{1}{3} < \frac{1}{2}$

Activity 1

Pairs to Compare

Here are some pairs of fractions sorted into 3 groups. Circle the greater fraction in each pair. Explain or show your reasoning.

1. Group 1:

   a. $\frac{2}{10}$ or $\frac{26}{100}$

   b. $\frac{2}{5}$ or $\frac{20}{100}$

2. Group 2:

   a. $\frac{2}{3}$ or $\frac{7}{12}$

   b. $\frac{4}{5}$ or $\frac{7}{10}$

3. Group 3:

   a. $\frac{11}{5}$ or $\frac{26}{10}$

   b. $\frac{11}{3}$ or $\frac{26}{12}$

Sec C

Activity 2

New Pairs to Compare

1. Decide whether each statement is true or false. Be prepared to explain or show how you know.

   a. $\frac{5}{12} = \frac{2}{6}$

   b. $\frac{10}{3} < \frac{44}{12}$

   c. $\frac{1}{4} > \frac{25}{100}$

   d. $\frac{8}{15} < \frac{3}{5}$

2. Compare each pair of fractions. Use the symbols $>$, $<$, or $=$ to make each statement true.

   a. $\frac{6}{12}$ ______ $\frac{4}{6}$

   b. $\frac{4}{3}$ ______ $\frac{7}{6}$

   c. $\frac{8}{5}$ ______ $\frac{400}{100}$

   d. $\frac{12}{10}$ ______ $\frac{35}{5}$

   e. $\frac{11}{4}$ ______ $\frac{17}{8}$

   f. $\frac{7}{12}$ ______ $\frac{4}{3}$

Unit 2, Lesson 14

Addressing CA CCSSM 4.NF.1 and 4.NF.2; building towards 4.NBT.4; practicing MP1

Fraction Comparison Problems

Let's solve different kinds of fraction comparison problems.

Warm-up

Number Talk: Multiples of 10

Find the value of each expression mentally.

- $119 + 119$
- $139 + 139$
- $159 + 159$
- $199 + 199$

Sec C

Activity 1

Mystery Fractions

Six friends were each given a list of 5 fractions. They each chose 1 fraction and wrote clues about their choice. Use their clues to identify the fraction they each chose.

Andre: $\frac{8}{12}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{3}{2}$ $\frac{2}{12}$

- less than 1
- greater than $\frac{1}{3}$
- less than $\frac{2}{3}$

Tyler: $\frac{2}{6}$ $\frac{2}{2}$ $\frac{2}{4}$ $\frac{2}{3}$ $\frac{2}{5}$

- greater than $\frac{1}{3}$
- less than 1
- less than $\frac{1}{2}$

Clare: $\frac{4}{3}$ $\frac{4}{2}$ $\frac{3}{4}$ $\frac{1}{4}$ $\frac{2}{10}$

- greater than $\frac{2}{8}$
- less than $\frac{11}{6}$
- greater than 1

Diego: $\frac{2}{8}$ $\frac{6}{12}$ $\frac{6}{8}$ $\frac{12}{10}$ $\frac{11}{12}$

- greater than $\frac{1}{2}$
- less than 1
- greater than $\frac{3}{4}$

Elena: $\frac{2}{12}$ $\frac{50}{100}$ $\frac{4}{10}$ $\frac{3}{5}$ $\frac{7}{5}$

- greater than $\frac{2}{10}$
- less than 1
- greater than $\frac{3}{6}$

Noah: $\frac{18}{10}$ $\frac{7}{8}$ $\frac{2}{5}$ $\frac{18}{5}$ $\frac{150}{100}$

- greater than $\frac{1}{2}$
- less than $\frac{25}{10}$
- greater than $\frac{8}{5}$

Activity 2

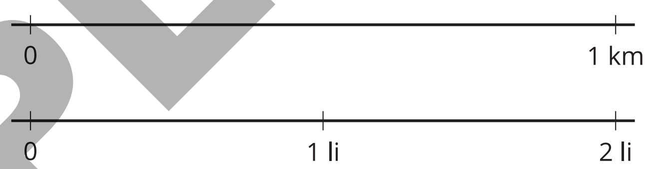
Distances on Foot

The “li” is a traditional unit of length in China and some East Asian countries.

Here are the walking distances between the home of a student in China and the places he visits regularly.

- school: $\frac{7}{5}$ li
- market: $\frac{7}{4}$ li
- library: $\frac{23}{10}$ li
- badminton club: $\frac{23}{12}$ li

1. Which is a shorter distance from the student’s home:
   a. His school or the library?
   b. The market or the badminton club?
   c. The library or the market?
2. A student in the United States walks $\frac{4}{5}$ kilometer (km) to school. These number lines show how 1 kilometer compares to 1 li.



   Which student walks a longer distance to school? Use the number lines to show your reasoning.
3. Explain why you can’t just compare the fractions $\frac{4}{5}$ and $\frac{7}{5}$ to see which student walks a longer distance.

Unit 2, Lesson 15

Addressing CA CCSSM 4.NF.1 and 4.NF.2; building on 4.OA.4; practicing MP2

Use Common Denominators to Compare

Let's compare fractions by writing equivalent fractions with the same denominator.

Warm-up

Sec C

What Do You Know about 15 and 30?

What do you know about 15 and 30?

Activity 1

Tricky Fractions?

1. In each pair of fractions, which fraction is greater? Explain or show your reasoning.

   a. $\frac{4}{3}$ or $\frac{13}{12}$

   b. $\frac{4}{3}$ or $\frac{7}{5}$

2. Han says he can compare $\frac{4}{3}$ and $\frac{13}{12}$ by writing an equivalent fraction for $\frac{4}{3}$. He says he can't use that strategy to compare $\frac{4}{3}$ and $\frac{7}{5}$. Do you agree? Explain your reasoning.

3. Priya and Lin show different ways to compare $\frac{4}{3}$ and $\frac{7}{5}$. Make sense of what they did. How are their strategies alike? How are they different?

Priya		Lin	
$\frac{4 \times 5}{3 \times 5} = \frac{20}{15}$	$\frac{7 \times 3}{5 \times 3} = \frac{21}{15}$	$\frac{4 \times 10}{3 \times 10} = \frac{40}{30}$	$\frac{7 \times 6}{5 \times 6} = \frac{42}{30}$
$\frac{21}{15}$ is greater than $\frac{20}{15}$, so $\frac{7}{5}$ is greater than $\frac{4}{3}$.		$\frac{42}{30}$ is greater than $\frac{40}{30}$, so $\frac{7}{5}$ is greater than $\frac{4}{3}$.	

Sec C

Activity 2

Use a Common Denominator, or Not

1. For each pair of fractions, write a pair of equivalent fractions with a common denominator.

   a. $\frac{5}{6}$ and $\frac{3}{4}$

   b. $\frac{2}{3}$ and $\frac{5}{8}$

   c. $\frac{2}{6}$ and $\frac{4}{10}$

   d. $\frac{7}{4}$ and $\frac{17}{10}$

2. For each pair of fractions, decide which fraction is greater. Be prepared to explain your reasoning.

   a. $\frac{5}{12}$ or $\frac{3}{8}$

   b. $\frac{13}{5}$ or $\frac{11}{6}$

   c. $\frac{71}{10}$ or $\frac{34}{5}$

   d. $\frac{7}{12}$ or $\frac{49}{100}$

Unit 2, Lesson 16

Addressing CA CCSSM 4.NF.1 and 4.NF.2; building on 4.OA.4; practicing MP7

Compare and Order Fractions

Let's put some fractions in order.

Warm-up

Number Talk: Multiples of 6 and 12

Find the value of each expression mentally.

- 5×6
- 5×12
- 6×12
- 11×12

Sec C

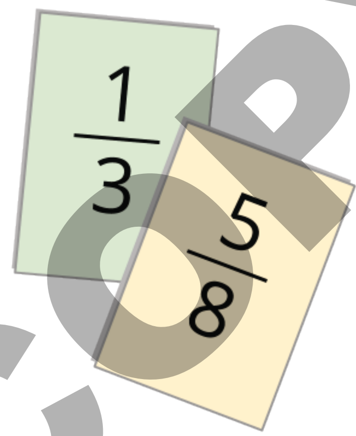
Activity 1

Introduce Compare—Fractions

Play *Compare Fractions* with 2 players:

- Split the deck between the players.
- Each player turns over 1 card.
- Compare the 2 fractions. The player with the greater fraction keeps both cards.
- If the fractions are equivalent, each player turns over 1 more card. The player with the greater fraction keeps all 4 cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

fraction cards



Play *Compare Fractions* with 3 or 4 players:

- The player with the greatest fraction wins the round.
- If 2 or more players have the greatest fraction (equivalent fractions), those players turn 1 more card over. The player with the greatest fraction keeps all the cards.

Record any sets of fractions that are challenging to compare here.

________ and ________   ________ and ________

________ and ________   ________ and ________

Activity 2

Fractions in Order

Put each set of fractions in order, from least to greatest. Be prepared to explain your reasoning.

1. $\frac{3}{12}$ $\frac{2}{4}$ $\frac{2}{3}$ $\frac{1}{8}$

2. $\frac{8}{5}$ $\frac{5}{6}$ $\frac{11}{12}$ $\frac{11}{10}$

3. $\frac{21}{20}$ $\frac{9}{10}$ $\frac{6}{5}$ $\frac{101}{100}$

4. $\frac{5}{8}$ $\frac{2}{5}$ $\frac{3}{7}$ $\frac{3}{6}$

Section C Summary

We compared fractions using:

- What we know about the size of fractions.
- Benchmark fractions, such as $\frac{1}{2}$ and 1.
- Equivalent fractions.

Example: To compare $\frac{3}{8}$ and $\frac{6}{10}$, we can reason that:

- $\frac{4}{8}$ is equivalent to $\frac{1}{2}$, so $\frac{3}{8}$ is less than $\frac{1}{2}$.
- $\frac{5}{10}$ is equivalent to $\frac{1}{2}$, so $\frac{6}{10}$ is greater than $\frac{1}{2}$.
- This means that $\frac{6}{10}$ is greater than $\frac{3}{8}$. (Or $\frac{3}{8}$ is less than $\frac{6}{10}$.)

Sec C

We can also compare by writing equivalent fractions with the same denominator, or a **common denominator**. For example, to compare $\frac{3}{4}$ and $\frac{4}{6}$, we can use 12 as the denominator:

$$\frac{3}{4} = \frac{9}{12} \qquad \frac{4}{6} = \frac{8}{12}$$

Because $\frac{9}{12}$ is greater than $\frac{8}{12}$, we know that $\frac{3}{4}$ is greater than $\frac{4}{6}$.

Addressing CA CCSSM 4.NF.1 and 4.NF.2; practicing MP4

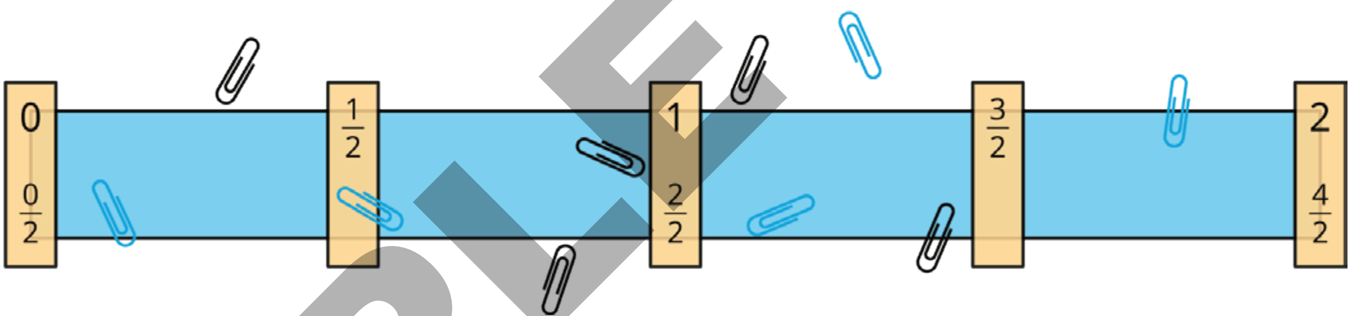
Paper Clip Games

Let’s create a game about locating and comparing fractions on the number line.

Warm-up

Notice and Wonder: Lots of Paper Clips

What do you notice? What do you wonder?



Activity 1

Paper Clip Tossing Game

Let's prepare a gameboard and figure out how to toss paper clips and record the results.

1. Make your gameboard:
   - Fold your paper strip in half, then in half again.
   - Tape the paper strip to your workspace and label the benchmark fractions 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2.
2. Play the game:
   - Decide on which benchmark fraction you'll try to land. This is the target fraction.
   - Take turns tossing the paper clips.
   - Label the fraction where each paper clip lands.
   - If you land on the target fraction, name an equivalent fraction to get a point.
   - The player with the most points at the end of the game wins.

Be prepared to share your strategies for:

- tossing the paper clips
- finding out the fraction for each clip's location on the gameboard
- naming equivalent fractions for the target fractions

Activity 2

A New Game with New Rules

Invent your own game.

1. Make a list with the rules of your game. Your game should include at least one of the following:
    - comparing fractions
    - finding equivalent fractions
    - using common denominators
    - using benchmark fractions like $\frac{1}{2}$ or $\frac{2}{2}$
2. Play your game, paying close attention to the rules.
3. Revise and clarify your game rules, if necessary.

Sec C

Activity 3

Field Test

Let's try out these games!

1. Before playing the game, exchange your game rules with another team. Carefully read the rules. Take turns asking clarifying questions, if you have any.
2. Play each other's games.
3. After playing the game, give feedback to each other about the rules.

   a. What is one thing you liked about the other team's game?

   b. What is one thing you might change?

Practice Problems

8 Problems

1 from Unit 2, Lesson 12

Circle the greater fraction in each pair. Explain or show your reasoning.

a. $\frac{2}{5}$ or $\frac{2}{6}$

b. $\frac{5}{8}$ or $\frac{7}{8}$

c. $\frac{9}{10}$ or $\frac{103}{100}$

2 from Unit 2, Lesson 13

Use $>$, $<$, or $=$ to make each statement true. Explain or show your reasoning.

a. $\frac{2}{3}$ _____ $\frac{10}{15}$

b. $\frac{1}{5}$ _____ $\frac{22}{100}$

c. $\frac{10}{4}$ _____ $\frac{45}{20}$

Sec C

3 from Unit 2, Lesson 14

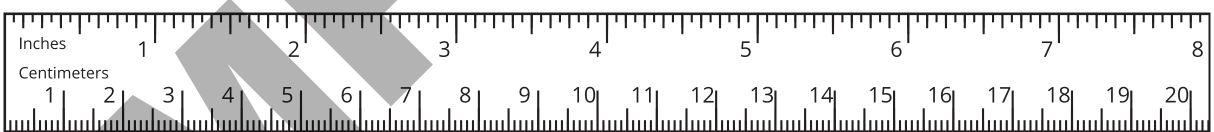
A water fountain is $\frac{7}{10}$ mile from the start of a hiking trail. A pond is $\frac{3}{5}$ mile from the start of the trail. A hiker begins walking at the start of the trail. Which will the hiker pass first, the water fountain or the pond? Explain your reasoning.

4 from Unit 2, Lesson 14

Tyler says he grew $\frac{3}{2}$ centimeters since his height was measured 6 months ago.

Diego says, "Oh, you grew more than I did! My height went up by only $\frac{7}{8}$ inch in the past 6 months."

Explain why Tyler did not grow more than Diego did, even though $\frac{3}{2}$ is greater than $\frac{7}{8}$.



5 from Unit 2, Lesson 15

List these fractions from least to greatest. Explain or show your reasoning.

$\frac{1}{3}$ $\frac{5}{12}$ $\frac{2}{10}$

6 from Unit 2, Lesson 16

List these fractions from least to greatest. Explain or show your reasoning.

$\frac{15}{8}$ $\frac{215}{100}$ $\frac{7}{4}$ $\frac{21}{10}$

7 Exploration

Jada lists these fractions that are all equivalent to $\frac{1}{2}$: $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$

She notices that each time the numerator increases by 1, the denominator increases by 2. Will this pattern continue? Explain your reasoning.

Sec C

8 Exploration

Find a fraction that is between $\frac{2}{5}$ and $\frac{3}{8}$. Explain or show your reasoning.

Sec C

Glossary

- common denominator
  The same denominator in two or more fractions. Example, $\frac{1}{4}$ and $\frac{5}{4}$ have the common denominator 4.

- composite number
  A whole number with more than one factor pair.

- denominator
  The bottom part of a fraction that tells how many equal parts the whole was partitioned into.

- equivalent fractions
  Fractions that have the same size and describe the same point on the number line. Example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

- factor pair of a whole number
  Two whole numbers that multiply to result in that number. Example: 5 and 4 are a factor pair of 20.

- multiple of a number
  The result of multiplying that number by a whole number. Example: 18 is a multiple of 3, because it is a result of multiplying 3 by 6.

- numerator
  The top part of a fraction that tells how many of the equal parts are being described.

- prime number
  A whole number that is greater than 1 and has exactly one factor pair: the number itself and 1.

Attributions

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California Common Core State Standards for Mathematics (CA CCSSM) Reference

4.G: Grade 4 – Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (Two-dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA

4.G.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

4.MD: Grade 4 – Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1, 12)$, $(2, 24)$, $(3, 36)$, ...

4.MD.2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3
Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Represent and interpret data.

4.MD.4
Make a line plot to display a data set of measurements in fractions of a unit $(1/2, 1/4, 1/8)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

4.MD.5.a
An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

4.MD.5.b
An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4.MD.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.NBT: Grade 4 – Number and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

4.NBT.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.

4.NBT.3
Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NF: Grade 4 – Number and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

4.NF.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, =, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3
Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

4.NF.3.a
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.3.b
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

4.NF.3.c
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3.d
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4.a
Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

4.NF.4.b
Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

4.NF.4.c
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.

4.NF.6
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, =, or $<$, and justify the conclusions, e.g., by using the number line or another visual model. CA

4.OA: Grade 4 – Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1
Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4.OA.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.