IMKH California

iM v.360 An IM K–12 Math Curriculum

GRADE 4

Student Edition

UNITS 3-4

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 2

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UNIT 4 FROM HUNDREDTHS TO HUNDRED-THOUSANDS

GRADE 4

UNIT

3

Extending Operations to Fractions Overview

In this unit you will learn more about measurements and analyzing data, and extend your mathematical knowledge to apply to fractions. Throughout the unit you will make connections by:

- **Reasoning with Data** when you create line plots, analyze measurement data, then apply your math knowledge to adding fractions.
- **Putting Parts Together** while creating fraction models and studying the flexibility of fractions.
- **Discovering Shape and Space** through your study of shapes, fractions, and decimals.

Addressing the Standards

As you work your way through **Unit 3 Extending Operations to Fractions**, you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 14, 15, and 18
MP2 Reason abstractly and quantitatively.	Lessons 1, 2, 6, 14, 16, and 19
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3, 6, 9, 11, and 12
MP4 Model with mathematics.	Lessons 15 and 20
MP5 Use appropriate tools strategically.	Lesson 8
MP6 Attend to precision.	Lessons 12, 13, and 15
MP7 Look for and make use of structure.	Lessons 5, 10, 11, 12, and 17
MP8 Look for and express regularity in repeated reasoning.	Lessons 3, 4, 7, and 17

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measuring and Plotting	**4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	Lessons 13 and 14
• Measuring and Plotting • Fraction Flexibility • Visual Fraction Models	**4.NF.1** Explain why a fraction *a/b* is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	Lessons 15 and 16

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measuring and Plotting • Visual Fraction Models	**4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lesson 15
• Fraction Flexibility • Visual Fraction Models	**4.NF.3** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*	Lesson 7
• Fraction Flexibility • Visual Fraction Models	**4.NF.3.a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	Lessons 8, 9
• Fraction Flexibility • Visual Fraction Models	**4.NF.3.b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.	Lessons 7, 8
• Fraction Flexibility • Visual Fraction Models	**4.NF.3.c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Lessons 9, 10, 11, 12, 14, and 19
• Fraction Flexibility • Visual Fraction Models	**4.NF.3.d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Lessons 10, 11, 12, 13, 15, 19 and 20
Fraction Flexibility	**4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	Lessons 1, 2, 3, 4, 19, and 20

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
Fraction Flexibility	**4.NF.4.a** Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).	Lessons 1, 2, 3, 5, 6
Fraction Flexibility	**4.NF.4.b** Understand a multiple of a/b as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, *n* × (*a*/*b*) = (*n* × *a*)/*b*.)	Lessons 4, 5, and 6
Fraction Flexibility	**4.NF.4.c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	Lessons 2, 5, 6, 10, and 18
• Fraction Flexibility • Visual Fraction Models • Circles, Fractions, and Decimals	**4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	Lessons 16, 17, 18 and 19

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 3, Lesson 1

Addressing 4.NF.4, 4.NF.4.a; practicing MP2

Equal Groups of Unit Fractions

Let's look at equal groups of fractions.

Sec A

Warm-up

How Many Do You See: Oranges

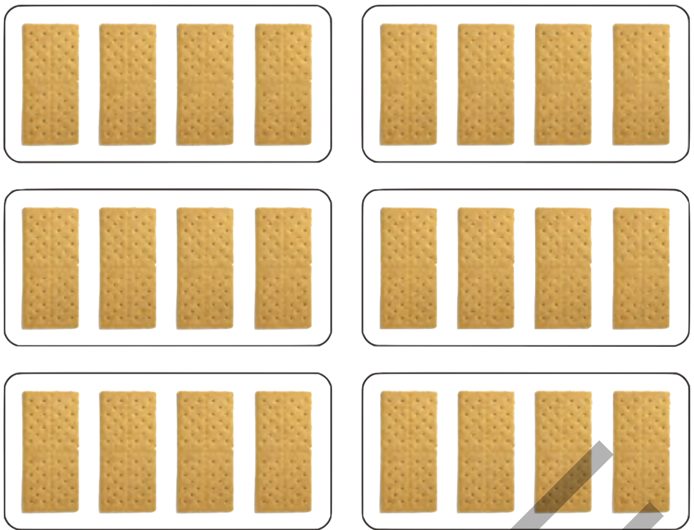
How many do you see? How do you see them?

Activity 1

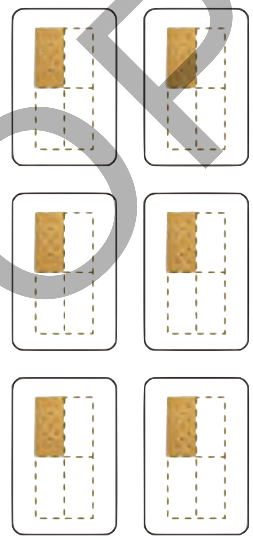
Crackers, Kiwis, and More

1. Here are some images of crackers:

A



B



   a. How are the crackers in Image A and Image B alike?

   b. How are they different?

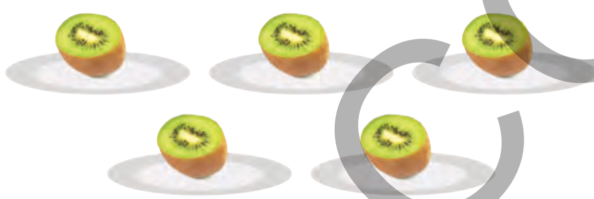
   c. How many crackers are in each image?

   d. Write an expression to represent the crackers in each image.

2. Here are more images and descriptions of food items. For each question, write a multiplication expression to represent the quantity. Then answer the question.

   a. Clare has 3 baskets. She put 4 eggs into each basket. How many eggs did she put in baskets?

   b. Diego has 5 plates. He put $\frac{1}{2}$ of a kiwi fruit on each plate. How many kiwis did he put on plates?



   c. Priya prepares 7 plates with $\frac{1}{8}$ of a pie on each. How much pie does she put on plates?

   d. Noah scoops $\frac{1}{3}$ cup of brown rice 8 times. How many cups of brown rice does he scoop?

Activity 2

What Could It Mean?

For each expression:

- Write a story that the expression could represent. The story should be about a situation with equal groups.
- Create a drawing to represent the situation.
- Find the value of the expression. What does this number mean in your story?

1. $8 \times \frac{1}{2}$

2. $7 \times \frac{1}{5}$

Unit 3, Lesson 2

Addressing 4.NF.4, 4.NF.4.a, 4.NF.4.c; building towards 4.NBT.5; practicing MP2

Representations of Equal Groups of Fractions

Sec A

Let's look at diagrams and expressions that can help us multiply a whole number and a fraction.

Warm-up

Number Talk: Three, Six, Nine, Twelve

Find the value of each expression mentally.

- 3×6
- 3×9
- 6×9
- 12×9

Activity 1

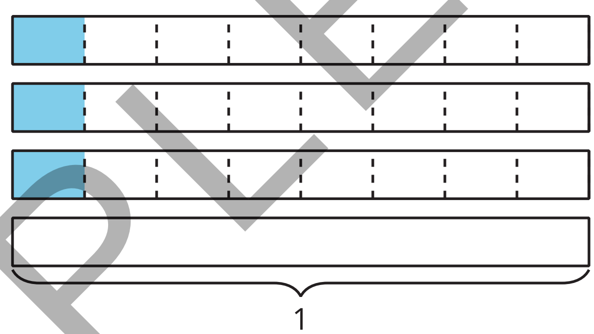
Card Sort: Expressions and Diagrams

Sec A

Your teacher will give you a set of cards.

1. Match each expression to a diagram that represents the same quantity.
2. Record each expression without a match.

3. Han starts drawing a diagram to represent $7 \times \frac{1}{8}$ and does not finish. Complete his diagram. Be prepared to explain your reasoning.



4. Choose one expression that you recorded earlier that didn't have a match.

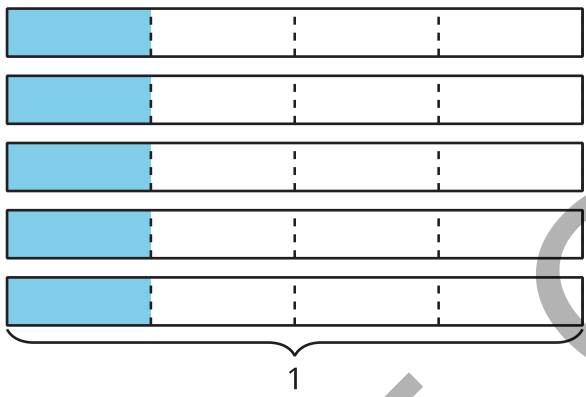
   Draw a diagram that can be represented by the expression. What value do the shaded parts of your diagram represent?

Activity 2

Different Representations

1. a. Write a multiplication expression that represents the shaded parts of the diagram. Then find the value of the expression.

Diagram:



Expression:

Value:

b. Draw a diagram that the expression $6 \times \frac{1}{3}$ could represent. Then find the value of the expression.

Diagram:

Expression: $6 \times \frac{1}{3}$

Value:

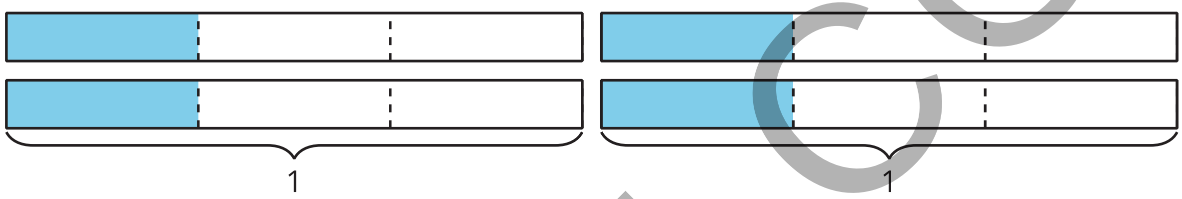
c. Draw a diagram and write an expression that show the value $\frac{7}{2}$.

Diagram:

Expression:

Value: $\frac{7}{2}$

2. To represent $4 \times \frac{1}{3}$, Diego drew this diagram:



Elena drew this diagram:

Are they representing the same expression and the same value? Explain or show how you know.

Unit 3, Lesson 3

Addressing 4.NF.4, 4.NF.4.a; practicing MP3 and MP8

Patterns in Multiplication

Let's look at patterns in multiplication of a fraction by a whole number.

Sec A

Activity 1

Describe the Pattern

1. Here are two tables with expressions. Find the value of each expression. Use a diagram if you find it helpful.

   Leave the last two rows of each table blank for now.

   Set A

expression	value
$1 \times \frac{1}{8}$	
$2 \times \frac{1}{8}$	
$3 \times \frac{1}{8}$	
$4 \times \frac{1}{8}$	
$5 \times \frac{1}{8}$	
$6 \times \frac{1}{8}$	

   Set B

expression	value
$2 \times \frac{1}{3}$	
$2 \times \frac{1}{4}$	
$2 \times \frac{1}{5}$	
$2 \times \frac{1}{6}$	
$2 \times \frac{1}{7}$	
$2 \times \frac{1}{8}$	

2. Look at your completed tables. What patterns do you see in how the expressions and the values are related?

3. In the last two rows of the table of Set A, write $\frac{11}{8}$ in one row and $\frac{13}{8}$ in the other, in the "value" column. Write the expressions with those values.

4. In the last two rows of the table of Set B, write $\frac{2}{12}$ in one row and $\frac{2}{15}$ in the other, in the "value" column. Write the expressions with those values.

Activity 2

What's Missing?

1. Use the patterns you observed earlier to complete each equation so that it's true.

   a. $5 \times \frac{1}{10} =$ ______

   b. $8 \times \frac{1}{6} =$ ______

   c. $4 \times$ ______ $= \frac{4}{5}$

   d. $6 \times$ ______ $= \frac{6}{10}$

   e. ______ $\times \frac{1}{4} = \frac{3}{4}$

   f. ______ $\times \frac{1}{12} = \frac{7}{12}$

2. Your teacher will give you a sheet of paper. Work with your group to complete these steps on the paper. After each step, pass your paper to your right.

   - Step 1: Write a fraction with a numerator other than 1 and a denominator no greater than 12.
   - Step 2: Write the fraction you received as a product of a whole number and a unit fraction.
   - Step 3: Draw a diagram to represent the equation you just received.
   - Step 4: Collect your original paper. If you think the work is correct, explain why the expression and the diagram both represent the fraction that you wrote. If not, discuss what revisions are needed.

Unit 3, Lesson 4

Addressing 4.NF.4, 4.NF.4.b; practicing MP8

Equal Groups of Non-unit Fractions

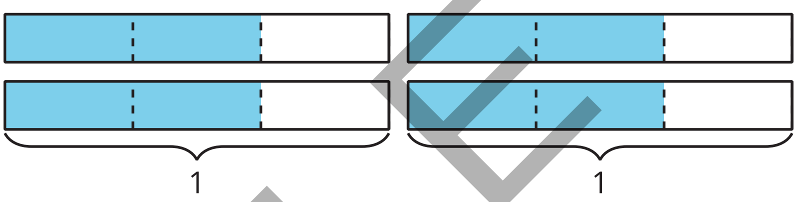
Sec A

Let's multiply any fraction by a whole number.

Warm-up

Notice and Wonder: Thirds

What do you notice? What do you wonder?



Activity 1

Jars of Slime

Elena's science club makes red and blue slime. She fills 5 small jars with slime to share with her friends. Each jar can fit $\frac{3}{4}$ cup of slime. How many cups of slime are in the jars?

If you have time: Elena still has some slime left. She takes 2 large jars and puts $\frac{5}{4}$ cups of slime in each jar. How many cups of slime are in the jars?

Activity 2

How Do We Multiply?

1. This diagram represents $\frac{2}{5}$.

   a. Show how you would change the diagram to represent $4 \times \frac{2}{5}$.

   b. What is the value of the shaded parts in your diagram?

2. This diagram represents $\frac{5}{8}$.

   a. Show how you would change the diagram to represent $3 \times \frac{5}{8}$.

   b. What is the value of the shaded parts in your diagram?

3. Find the value of each expression. Draw a diagram if you find it helpful.

   a. $2 \times \frac{1}{6}$

   b. $2 \times \frac{4}{6}$

   c. $2 \times \frac{5}{6}$

   d. $4 \times \frac{5}{6}$

4. Mai says that to multiply any fraction by a whole number, she multiplies the whole number and the numerator of the fraction and keeps the same denominator. Do you agree with Mai? Explain your reasoning.

Unit 3, Lesson 5

Addressing 4.NF.4.a, 4.NF.4.b, 4.NF.4.c; practicing MP7

Equivalent Multiplication Expressions

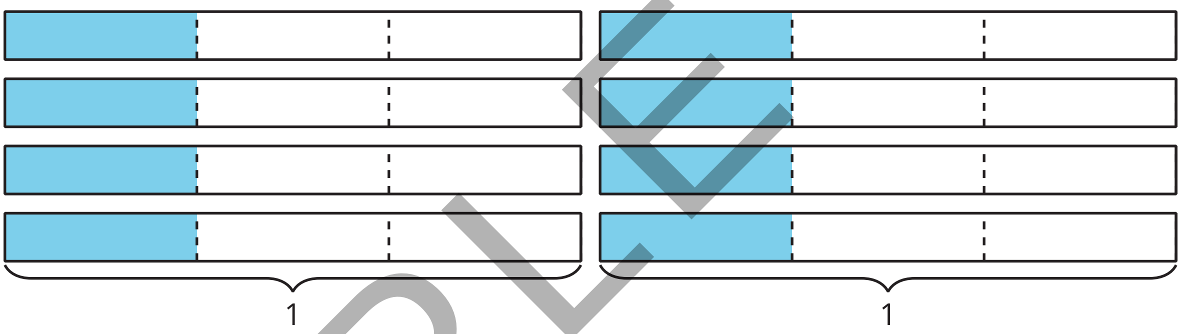
Sec A

Let's write multiplication expressions in different ways.

Warm-up

How Many Do You See: Thirds

How many thirds do you see? How do you see them?



Activity 1

Complete the Equations

1. Find the number that makes each equation true. Draw a diagram if it is helpful.

$\frac{12}{5} = 12 \times$ ________

$\frac{12}{5} = 3 \times$ ________

$\frac{12}{5} = 6 \times$ ________

$\frac{12}{5} = 2 \times$ ________

$\frac{12}{5} = 4 \times$ ________

$\frac{12}{5} = 1 \times$ ________

2. Here are two sets of numbers:

Set A:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Set B:

$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$

a. Choose a number from Set A and a number from Set B to complete this equation and make it true:

$$\frac{6}{7} = \_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_$$

b. Choose a different number from Set A and from Set B to complete the equation to make it true.

$$\frac{6}{7} = \_\_\_\_\_\_\_ \times \_\_\_\_\_\_\_$$

3. Explain or show how you know that the two equations you wrote are both true.

Activity 2

Fractions and Matching Expressions

Sec A

Here is a set of expressions:

A. $6 \times \frac{1}{10}$

B. $2 \times 4 \times \frac{1}{9}$

C. $4 \times \frac{1}{5}$

D. $3 \times 2 \times \frac{1}{10}$

E. $5 \times 2 \times \frac{1}{12}$

F. $2 \times 2 \times \frac{1}{5}$

G. $4 \times 4 \times \frac{1}{9}$

H. $10 \times \frac{1}{12}$

I. $4 \times \frac{1}{12}$

1. Match each expression to one of the following fractions, if possible. Record your matches.

   $\frac{4}{5}$ $\frac{10}{12}$ $\frac{6}{10}$ $\frac{8}{9}$

2. Complete each equation to make it true. Try to complete the equation, without using unit fractions.

   a. $\frac{4}{5} =$ _______ $\times$ _______ $\frac{4}{5} =$ _______ $\times$ _______

   b. $\frac{10}{12} =$ _______ $\times$ _______ $\frac{10}{12} =$ _______ $\times$ _______

   c. $\frac{6}{10} =$ _______ $\times$ _______ $\frac{6}{10} =$ _______ $\times$ _______

   d. $\frac{8}{9} =$ _______ $\times$ _______ $\frac{8}{9} =$ _______ $\times$ _______

Unit 3, Lesson 6

Addressing 4.NF.4.a, 4.NF.4.b, 4.NF.4.c; practicing MP2 and MP3

Problems with Equal Groups of Fractions

Sec A

Let’s solve problems with fractions.

Warm-up

True or False: Two and Three Factors

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $\frac{10}{12} = 5 \times \frac{2}{12}$
- $1 \times \frac{10}{12} = 5 \times \frac{2}{12}$
- $\frac{24}{4} = 6 \times 3 \times \frac{1}{4}$
- $12 \times 2 \times \frac{1}{4} = 8 \times 3 \times \frac{1}{4}$

Activity 1

Banana Bread Recipe

A bakery makes banana bread. Here is the list of ingredients for 1 batch:

Ingredients:

- 1 banana
- $\frac{2}{3}$ cup butter
- $\frac{3}{2}$ teaspoons baking soda
- $\frac{5}{8}$ cup sugar
- 2 large eggs
- $\frac{5}{2}$ cups all-purpose flour

1. The bakery makes 2 batches of banana bread on Monday. Complete the table to show how much of each ingredient is used.

Monday's banana bread

ingredient	expression	amount of ingredient
bananas		______
butter		______ cup(s)
baking soda		______ teaspoon(s)
sugar		______ cup(s)
eggs		______
flour		______ cup(s)

2. On Tuesday, the bakery needs $\frac{8}{3}$ cups of butter to make enough banana bread for the day. How many batches are made on Tuesday? Explain or show your reasoning.

Recipe:

- 1 banana
- $\frac{2}{3}$ cup butter
- $\frac{3}{2}$ teaspoons baking soda
- $\frac{5}{8}$ cup sugar
- 2 large eggs
- $\frac{5}{2}$ cups all-purpose flour

3. Based on the number of the batches made on Tuesday, complete the table for each ingredient.

Tuesday's banana bread

ingredient	expression	amount of ingredient
bananas		_______
butter		$\frac{8}{3}$ cups
baking soda		_______ teaspoon(s)
sugar		_______ cup(s)
eggs		_______
flour		_______ cup(s)

Activity 2

How Much Milk Was Used?

The bakery that sells banana bread also sells fresh milkshakes. Each milkshake uses $\frac{1}{10}$ gallon of milk.

Here are 5 descriptions of the milkshakes sold in a week and 5 expressions that represent the gallons of milk used.

Match each description to an expression that represents it.

1. On Monday, the bakery sold 8 milkshakes. How much milk was used?
2. On Tuesday, 2 customers bought 4 milkshakes each. How much milk was used?
3. On Wednesday, 4 customers bought 2 milkshakes each. How much milk was used?
4. On Thursday, 2 customers each bought a milkshake. They placed the same order 3 more times for their friends that day. How much milk was used?
5. On Saturday, 4 friends each bought a milkshake for breakfast. They placed the same order after dinner. How much milk was used?

$4 \times (2 \times \frac{1}{10})$

$4 \times \frac{2}{10}$

$8 \times \frac{1}{10}$

$2 \times (4 \times \frac{1}{10})$

$2 \times \frac{4}{10}$

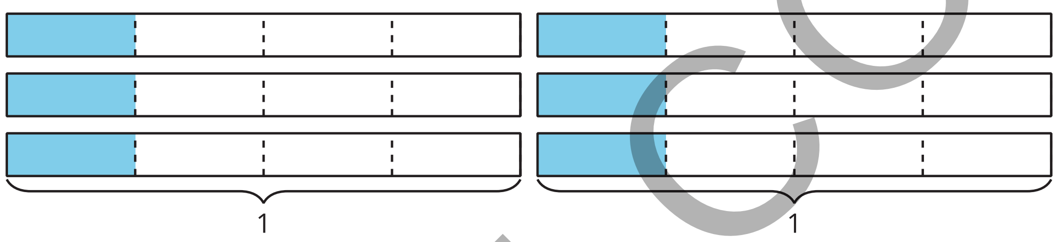
Section A Summary

We learned to multiply a whole number and a fraction by thinking about equal-size groups, just as we did when multiplying two whole numbers.

We can think of 6×4 as 6 groups of 4. A diagram like this can help to show that the product is 24:

4	4	4	4	4	4

We also can think of $6 \times \frac{1}{4}$ as 6 groups of $\frac{1}{4}$. Diagrams can help us see that the product is $\frac{6}{4}$:



$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

After looking at patterns closely, we noticed that when we multiply a whole number and a fraction, the whole number is multiplied only by the numerator of the fraction and the denominator stays the same.

Example:

$$6 \times \frac{1}{2} = \frac{6}{2}$$

$$2 \times \frac{4}{5} = \frac{8}{5}$$

We also learned that:

- Every fraction can be written as the multiplication of a whole number and a unit fraction. For example, $\frac{5}{4}$ can be written as $5 \times \frac{1}{4}$.
- We can write different multiplication expressions for the same fraction. For example, $\frac{8}{3}$ can be written as:

$$8 \times \frac{1}{3} \qquad 4 \times 2 \times \frac{1}{3} \qquad 4 \times \frac{2}{3} \qquad 2 \times \frac{4}{3}$$

Practice Problems

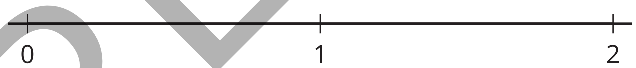
12 Problems

1 Pre-unit

What fraction of the rectangle is shaded? Explain how you know.

2 Pre-unit

a. Locate and label $\frac{3}{4}$ and $\frac{6}{4}$ on the number line.

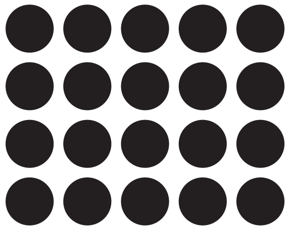


b. Explain why your points represent $\frac{3}{4}$ and $\frac{6}{4}$.

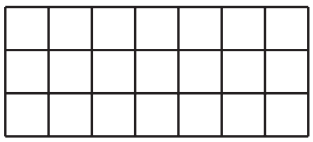
3 Pre-unit

Write a multiplication expression for each image. Explain your reasoning.

a.



b.



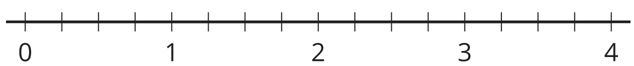
4 Pre-unit

Here are the lengths of some lizards in inches. Use the lengths to complete the line plot.

$2\frac{1}{4}$ $1\frac{1}{2}$ $2\frac{2}{4}$ 3 $3\frac{2}{4}$ 2

$2\frac{1}{4}$ $2\frac{1}{4}$ $2\frac{3}{4}$ 2 $2\frac{1}{4}$ 3

Length of Lizards



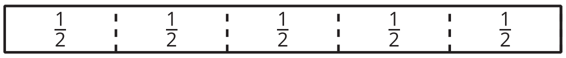
lizard length in inches

Sec A

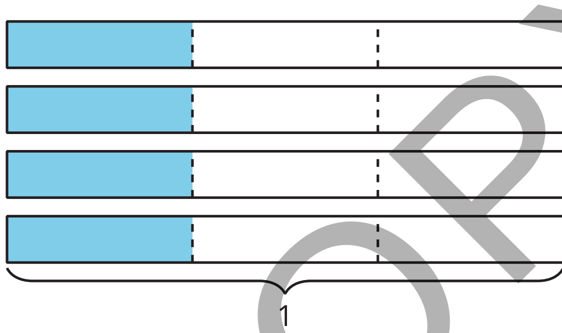
5 from Unit 3, Lesson 1

Write an expression that matches each diagram. Then find the value of each expression.

a.



b.

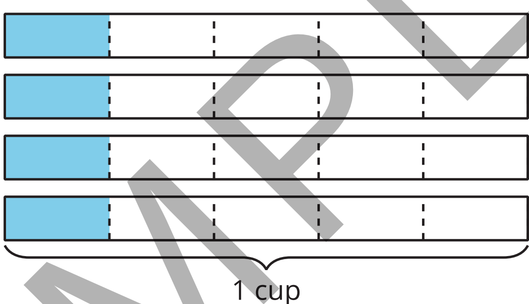


6 from Unit 3, Lesson 2

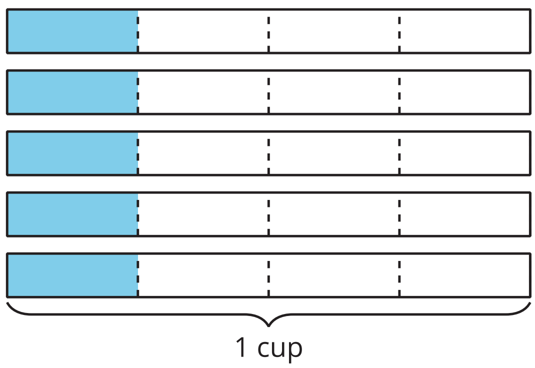
There are 5 friends that go on a hike. They each bring $\frac{1}{4}$ cup of nuts.

a. If the shaded parts represent the amount of nuts the friends bring on their hike, which diagram matches the story? Explain your reasoning.

A



B



b. How many cups of nuts do the friends bring on the hike?

7 from Unit 3, Lesson 3

Kiran's cat eats $\frac{1}{2}$ cup of food each day.

a. How much food does Kiran's cat eat in a week?

b. Draw a diagram to represent the situation.

8 from Unit 3, Lesson 4

a. Draw a diagram to show $3 \times \frac{7}{8}$.

b. How does the diagram help you find the value of the expression $3 \times \frac{7}{8}$?

Sec A

9 from Unit 3, Lesson 5

Find the number that makes each equation true. Draw a diagram if it is helpful.

a. $\frac{10}{3} = \_\_\_\_ \times \frac{1}{3}$

b. $\frac{10}{3} = \_\_\_\_ \times \frac{2}{3}$

c. $\frac{10}{3} = \_\_\_\_ \times \frac{5}{3}$

10 from Unit 3, Lesson 6

Each bead weighs $\frac{5}{8}$ gram. How much do 7 beads weigh? Explain or show your reasoning.

11 Exploration

a. Measure how thick your book is to the nearest $\frac{1}{8}$ inch.

b. If your classmates stacked their books together, how tall would the stack be? Explain or show your reasoning.

c. Check your answer by measuring the stack, if possible.

12 Exploration

Diego walks the same number of miles to school each day. He says he walks $\frac{48}{5}$ miles in total, but he does not say how many days that distance includes.

What are some possible numbers of days Diego walks to school? What is the distance he walks each of those days?

Sec A

Unit 3, Lesson 7

Addressing 4.NF.3, 4.NF.3.b; building on: 3.NF.1; practicing MP8

Fractions as Sums

Let's write fractions as sums.

Sec B

Activity 1

Barley Soup

Lin is learning to make barley soup, using a family recipe. Here are some ingredients in the recipe:

- $\frac{3}{4}$ cup barley
- $\frac{5}{4}$ cups chopped celery
- $\frac{6}{4}$ cups chopped carrots
- 1 cup chopped onions
- $2\frac{1}{4}$ cups vegetable broth

1. Lin only has one measuring cup that measures $\frac{1}{4}$ cup. Show how Lin could use the cup to measure the ingredients in the recipe.
   - Barley:
   - Celery:
   - Carrots:
   - Onions:
   - Vegetable broth:

2. Lin finds a $\frac{3}{4}$-cup. She can now measure the ingredients, using both a $\frac{3}{4}$-cup and a $\frac{1}{4}$-cup. Show how she could use the cups to measure the ingredients in the recipe.

    - Barley:
    - Onions:
    - Celery:
    - Vegetable broth:
    - Carrots:

Activity 2

Sums in Fifths and Thirds

1. Use different combinations of fifths to make a sum of $\frac{9}{5}$.

   a. $\frac{9}{5}$ = _______ + _______ + _______ + _______ + _______

   b. $\frac{9}{5}$ = _______ + _______ + _______ + _______

   c. $\frac{9}{5}$ = _______ + _______ + _______

   d. $\frac{9}{5}$ = _______ + _______

2. Write different ways to use thirds to make a sum of $\frac{4}{3}$. How many can you find? Write an equation for each combination.

3. Is it possible to write any fraction with a denominator of 5 as a sum of other fifths? Explain or show your reasoning.

Addressing 4.NF.3.a, 4.NF.3.b; building towards 4.NF.3.c; practicing MP3

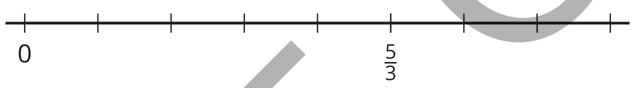
Addition of Fractions

Let's explore sums of fractions on a number line.

Warm-up

Notice and Wonder: A Fraction on a Number Line

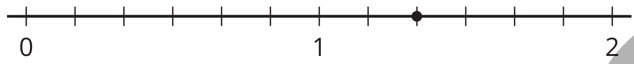
What do you notice? What do you wonder?

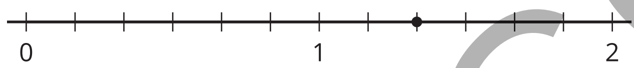


Activity 1

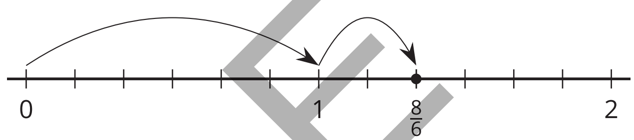
Sum of Jumps

1. a. On each number line, draw two jumps to show how to use sixths to make a sum of $\frac{8}{6}$. Then write an equation to represent each combination of jumps.





b. Noah draws the following diagram and writes: $\frac{8}{6} = \frac{6}{6} + \frac{2}{6}$ and $\frac{8}{6} = 1 + \frac{2}{6}$. Which equation is correct? Explain your reasoning.



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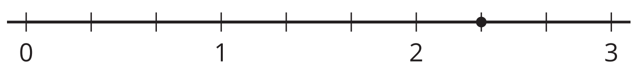
2. a. On each number line, draw jumps to show how to use thirds to make a sum of $\frac{7}{3}$. Then write an equation to represent each combination of jumps.



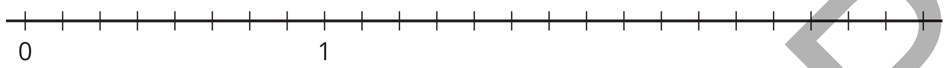
b. Write $\frac{7}{3}$ as a sum of a whole number and a fraction.

Activity 2

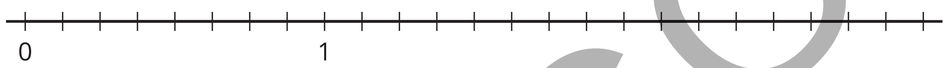
What Is the Sum?

1. Use a number line to represent each addition expression and find its value.

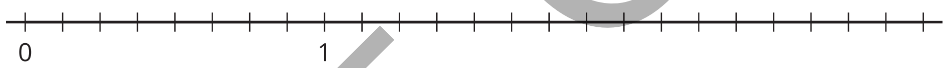
   a. $\frac{5}{8} + \frac{2}{8}$



   b. $\frac{1}{8} + \frac{9}{8}$

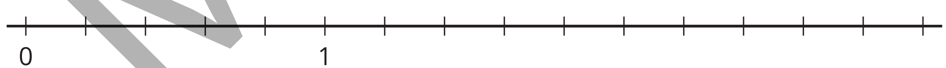


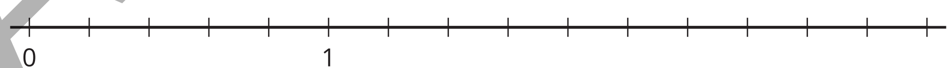
   c. $\frac{11}{8} + \frac{9}{8}$

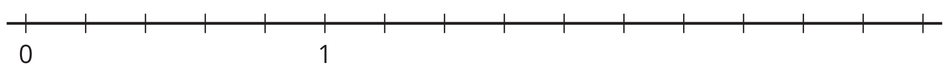


   d. $2\frac{1}{8} + \frac{4}{8}$

2. Priya says the sum of $1\frac{2}{5}$ and $\frac{4}{5}$ is $1\frac{6}{5}$. Kiran says the sum is $\frac{11}{5}$. Tyler says it is $2\frac{1}{5}$. Do you agree with any of them? Explain or show your reasoning. Use 1 or more number lines if you find them helpful.





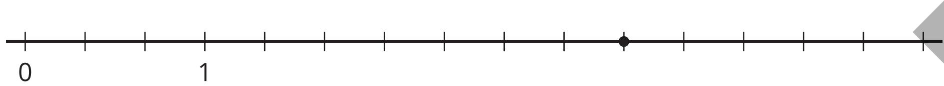


Activity 3

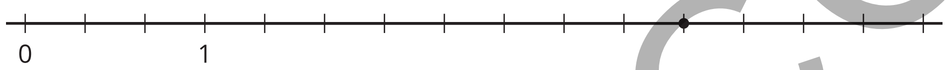
Jump Forward

Here are four number lines. There is a point on each number line.

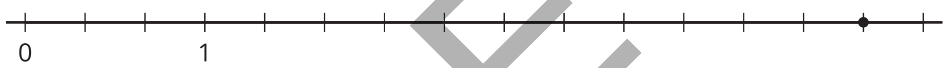
1.



2.



3.



4.

For each number line, label the point with the fraction it represents. This is your target. Make 2 forward jumps to get from 0 to the target.

- Pick a card from the set given to you. Use the fraction on it for your first jump. Draw the jump and label it with the fraction.
- From that point, draw the second jump to reach the target. What fraction do you need to add? Label the jump with the fraction.
- Write an equation to represent the sum of your two fractions.

Unit 3, Lesson 9

Addressing 4.NF.3.a, 4.NF.3.c; practicing MP3

Differences of Fractions

Let's explore differences of fractions on a number line.

Sec B

Warm-up

True or False: Sums of Tenths

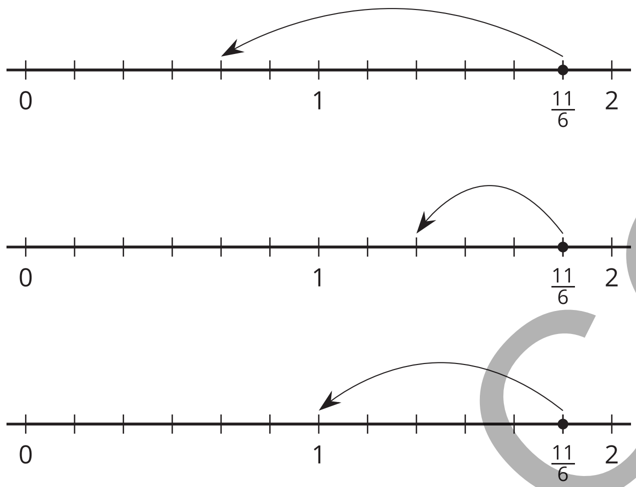
Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} = 1$
- $1 + \frac{7}{10} = \frac{3}{10} + \frac{4}{10} + \frac{10}{10}$
- $\frac{5}{10} + 1 = \frac{6}{10}$
- $\frac{2}{10} + \frac{10}{10} = 1 + \frac{1}{5}$

Activity 1

Jump to Subtract

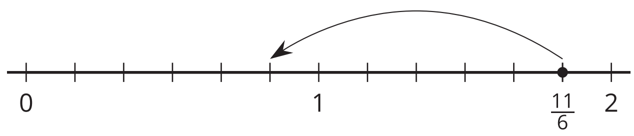
1. To subtract different fractions from $\frac{11}{6}$, Noah draws jumps on number lines.



a. The first diagram shows how he finds $\frac{11}{6} - \frac{7}{6}$. What is the value of $\frac{11}{6} - \frac{7}{6}$?

b. Write an equation to show the difference represented by each of Noah's diagrams.

2. Here is another diagram Noah draws:



Which equations could the diagram represent? Explain your reasoning.

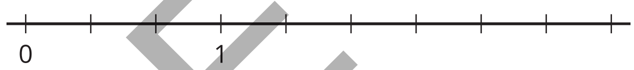
$\frac{11}{6} - \frac{6}{6} = \frac{5}{6}$

$\frac{11}{6} - 1 = \frac{5}{6}$

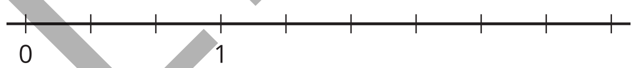
$1\frac{5}{6} - 1 = \frac{5}{6}$

3. Use a number line to represent each difference and find its value.

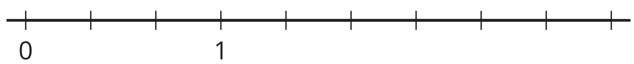
a. $\frac{8}{3} - \frac{2}{3}$



b. $\frac{8}{3} - \frac{4}{3}$



c. $\frac{8}{3} - 1$



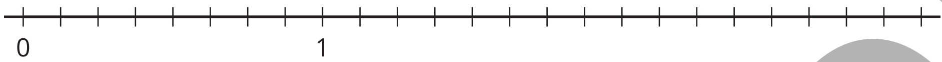
Sec B

Activity 2

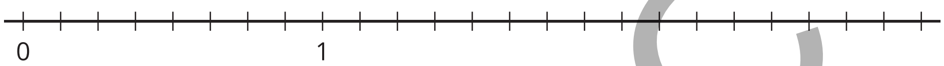
What's the Difference?

Use a number line to represent each difference and find its value.

1. $\frac{13}{8} - \frac{2}{8}$



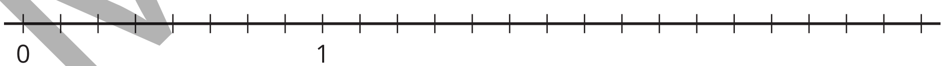
2. $\frac{13}{8} - \frac{6}{8}$



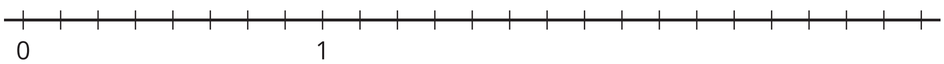
3. $\frac{13}{8} - 1$

4. $1\frac{5}{8} - \frac{7}{8}$

5. $1\frac{5}{8} - 1$



6. $1\frac{5}{8} - 1\frac{4}{8}$



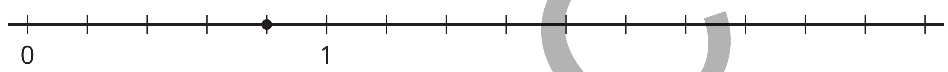
Activity 3

Jump Back

Here are 4 number lines, each with a point on it. Label each point with the fraction it represents.

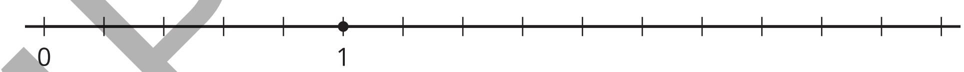
1.

2.



3.

4.



The points you labeled are your targets. Follow these directions for each number line:

- Pick a card from the set given to you. Locate and label the fraction on the number line.
- From that point, draw one or more jumps to reach the target. What do you need to subtract? Label each jump you draw.
- Write an equation to represent the difference of your two fractions.

Unit 3, Lesson 10

Addressing 4.NF.3.c, 4.NF.3.d, 4.NF.4.c; building on 4.NF.3.a, 4.NF.4.b; practicing MP7

The Numbers in Subtraction

Let’s subtract fractions from whole numbers.

Warm-up

Sec B

Number Talk: Groups of Twelfths

Find the value of each expression mentally.

- $2 \times \frac{3}{12}$
- $6 \times \frac{3}{12}$
- $12 \times \frac{3}{12}$
- $12 \times \frac{30}{12}$

Activity 1

What's Left?

1. A pitcher contains 3 cups of watermelon juice. If you pour each of these amounts from the full pitcher, how many cups are left after each pour?

   a. $\frac{1}{4}$ cup

   b. $\frac{5}{4}$ cup

   c. $1\frac{1}{4}$ cups

   d. $2\frac{2}{4}$ cups

2. A second pitcher contains 4 cups of water. If you pour each of these amounts from the full pitcher, how many cups are left after each pour?
   Explain or show your reasoning. Use diagrams or equations, if they are helpful.

   a. $\frac{1}{3}$ cup

   b. $\frac{5}{3}$ cups

   c. $2\frac{2}{3}$ cups

Activity 2

Card Sort: Twelfths

Your teacher will give you a set of cards that show fractions and expressions with fractions.

1. Sort the cards into two categories in a way that makes sense to you. Be ready to explain the meaning of your categories.

Sec B

2. Find the value of each difference. Show your reasoning.

   a. $1 - \frac{5}{8}$

   b. $2 - \frac{7}{8}$

   c. $3 - \frac{9}{8}$

Unit 3, Lesson 11

Addressing 4.NF.3.c, 4.NF.3.d; practicing MP3 and MP7

Subtract Fractions Flexibly

Let's find all kinds of differences.

Sec B

Warm-up

Which Three Go Together: Fractional Values

Which 3 go together?

A. $2 - \frac{3}{5}$

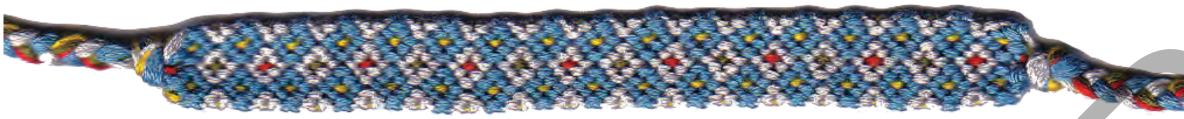
B. $\frac{10}{5} - \frac{3}{5}$

C. $1\frac{3}{5} - \frac{1}{5}$

D. $\frac{10}{5} - 1$

Activity 1

Friendship Bracelets



Clare, Elena, and Andre make *macramé* (MAA-kruh-may) friendship bracelets. They'd like their bracelets to be $9\frac{4}{8}$ inches long. For each question, explain or show your reasoning.

1. Clare starts her bracelet first and has only $\frac{7}{8}$ inch left until she finishes it. How long is her bracelet so far?

2. So far, Elena's bracelet is $5\frac{1}{8}$ inches long and Andre's is $3\frac{5}{8}$ inches long. How many more inches do they each need to reach $9\frac{4}{8}$ inches?

3. How much longer is Elena's bracelet than Andre's bracelet?

Activity 2

Multiple Ways to Subtract

Here are 4 expressions that you may have written about the friendship bracelets.

$9\frac{4}{8} - \frac{7}{8}$ $\qquad$ $9\frac{4}{8} - 5\frac{1}{8}$ $\qquad$ $9\frac{4}{8} - 3\frac{5}{8}$ $\qquad$ $5\frac{1}{8} - 3\frac{5}{8}$

1. Here is one way to find the value of the first expression. Look closely at the calculation. Talk to your partner about why $9\frac{4}{8}$ is written as different sums.

<table>
<tr><td colspan="2">$9\frac{4}{8} \quad - \quad \frac{7}{8}$</td></tr>
<tr><td>first number</td><td>second number</td></tr>
<tr><td>$9\frac{4}{8}$
$8 + 1 + \frac{4}{8}$
$8 + \frac{8}{8} + \frac{4}{8}$
$8 + \frac{12}{8}$</td><td>$\frac{7}{8}$</td></tr>
<tr><td colspan="2">$8 + \frac{12}{8} - \frac{7}{8}$
$8 + \frac{5}{8}$
$8\frac{5}{8}$</td></tr>
</table>

2. Here are some unfinished calculations. Complete them to find the value of each difference.

a

$9\frac{4}{8} - 5\frac{1}{8}$	
first number	second number
$9\frac{4}{8}$	$5\frac{1}{8}$
$9 + \frac{4}{8}$	$5 + \frac{1}{8}$

b

$9\frac{4}{8} - 3\frac{5}{8}$	
first number	second number
$9\frac{4}{8}$	$3\frac{5}{8}$
$8 + 1 + \frac{4}{8}$	$3 + \frac{5}{8}$
$8 + \frac{8}{8} + \frac{4}{8}$	
$8 + \frac{12}{8}$	

c

$5\frac{1}{8} - 3\frac{5}{8}$	
first number	second number
$5\frac{1}{8}$	$3\frac{5}{8}$
$5 + \frac{1}{8}$	$3 + \frac{5}{8}$

Unit 3, Lesson 12

Addressing 4.NF.3.c, 4.NF.3.d; practicing MP3, MP6, and MP7

Sums and Differences of Fractions

Let's add and subtract fractions and analyze our strategies.

Warm-up

Number Talk: Subtract Some Eighths

Find the value of each expression mentally.

- $2\frac{3}{8} - \frac{3}{8}$
- $2\frac{3}{8} - \frac{5}{8}$
- $2\frac{3}{8} - 2$
- $2\frac{3}{8} - 1\frac{7}{8}$

Sec B

Activity 1

Make It True

1. Find the number that makes each equation true. Show your reasoning.

   a. $\_\_\_\_\_\_\_ + \frac{2}{6} = 1\frac{1}{6}$

   b. $2\frac{4}{5} + \_\_\_\_\_\_\_ = 7\frac{1}{5}$

   c. $3 - 2\frac{1}{3} = \_\_\_\_\_\_\_$

   d. $4\frac{1}{12} - 2\frac{5}{12} = \_\_\_\_\_\_\_$

Sec B

2. Write a sentence to describe your first step for finding the unknown value in each equation in the first problem.

   a. First step:

   b. First step:

   c. First step:

   d. First step:

3. Compare and reflect on your first steps with your group. Did you make the same moves?

   Discuss why you might have chosen the same way or a different way to start finding the missing values.

Activity 2

To Decompose or Not to Decompose

1. Here are some addition and subtraction expressions. Sort them into two groups, based on whether you think it would be helpful to decompose a number to find the value of the expression.

   A. $\frac{18}{5} - \frac{7}{5}$

   B. $\frac{1}{6} + \frac{9}{6}$

   C. $7 - 1\frac{3}{8}$

   D. $\frac{102}{100} + 5\frac{27}{100}$

   E. $2\frac{5}{12} + \frac{6}{12}$

   F. $6\frac{1}{10} - \frac{6}{10}$

   G. $3\frac{8}{100} + 4\frac{93}{100}$

   H. $5 - \frac{17}{12}$

   I. $1\frac{3}{10} + \frac{6}{10}$

   J. $\frac{17}{8} - 1\frac{7}{8}$

   - Not necessary or not helpful to decompose any number:
   - Necessary or helpful to decompose one or more numbers:

2. Choose at least one expression from each group and find their values. Show your reasoning.

Unit 3, Lesson 13

Addressing 4.MD.4, 4.NF.3.d; building on 3.MD.4; practicing MP6

Fractional Measurements on Line Plots

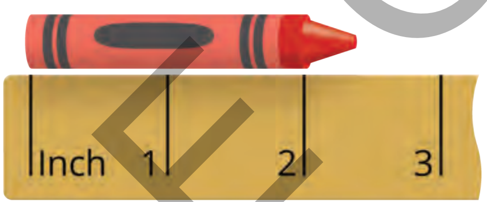
Let’s create line plots and analyze the data.

Warm-up

Notice and Wonder: Which Ruler?

What do you notice? What do you wonder?

A



B

C

D

Activity 1

Measure to the Nearest $\frac{1}{4}$ Inch and $\frac{1}{8}$ Inch

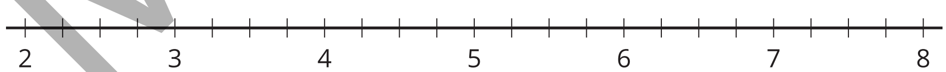
Your teacher will give your group a set of colored pencils.

1. Work with your group to measure each colored pencil to the nearest $\frac{1}{4}$ inch. Check each other's measurements. Record each measurement in the table.

group members	colored pencil length (inches)

2. Create a line plot to represent the data your group collected.

Colored Pencil Data



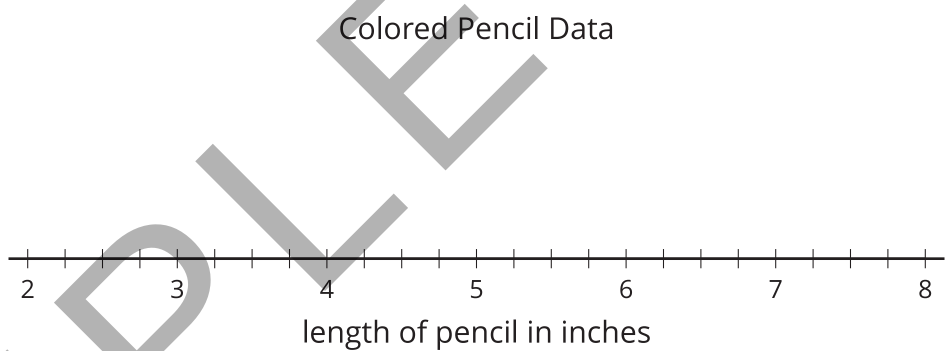
length of pencil in inches

3. Work with your group to measure each colored pencil to the nearest $\frac{1}{8}$ inch.

   Check all measurements. Record each measurement in the table.

group members	colored pencil length (inches)

4. Create a line plot to represent your new data.



5. How is measuring to the nearest $\frac{1}{4}$ inch different from measuring to the nearest $\frac{1}{8}$ inch?

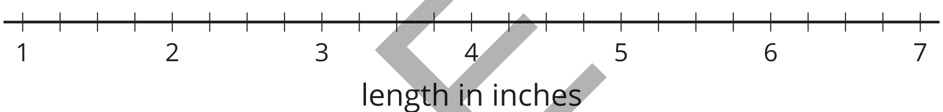
Activity 2

Colored Pencil Measurements

1. Andre's class measures the length of some colored pencils to the nearest $\frac{1}{4}$ inch. Here is the class data:

   $1\frac{3}{4}$ $2\frac{1}{4}$ $5\frac{1}{4}$ $5\frac{1}{4}$ $4\frac{2}{4}$ $4\frac{2}{4}$ $6\frac{1}{4}$ $6\frac{3}{4}$ $6\frac{3}{4}$ $6\frac{3}{4}$

   a. Plot the colored-pencil data on the line plot.



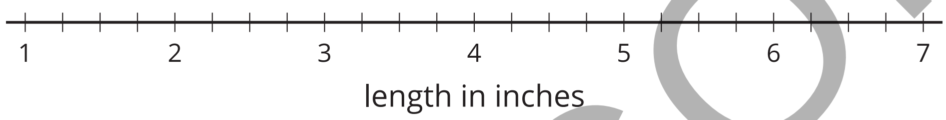
   b. Which colored-pencil length is the most common in the data set?

   c. Write 2 new questions that could be answered using the line plot data.

2. Next, Andre's class measures their colored pencils to the nearest $\frac{1}{8}$ inch. Here is the class data:

$$1\tfrac{6}{8} \quad 2\tfrac{2}{8} \quad 5\tfrac{2}{8} \quad 5\tfrac{3}{8} \quad 4\tfrac{4}{8} \quad 4\tfrac{4}{8} \quad 6\tfrac{3}{8} \quad 6\tfrac{6}{8} \quad 6\tfrac{6}{8} \quad 6\tfrac{6}{8}$$

a. Plot the colored-pencil data on the line plot.



b. Which colored-pencil length is the most common in the line plot?

c. Why did some colored-pencil lengths change on this line plot?

____

____

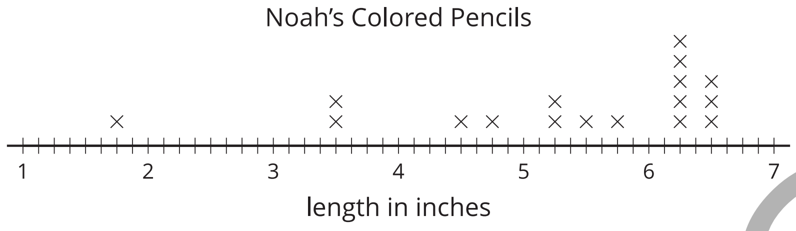
____

____

d. What is the difference between the lengths of the longest colored pencil and the shortest colored pencil? Show your reasoning.

Noah's Colored Pencils

The line plot shows the data Noah collected on a set of colored pencils.



Use the line plot to tell if each of the following statements is true or false. Explain or show your reasoning. For each false statement, correct it so that it is true.

1. Noah measures the colored pencils to the nearest $\frac{1}{2}$ inch.

2. There are 5 pencils that are $6\frac{1}{4}$ inches long.

3. The shortest pencil is $1\frac{3}{4}$ inches long.

4. The 3 longest pencils are exactly 5 inches longer than the shortest pencil.

5. If Noah removes the shortest pencil from the collection, the difference between the longest and shortest pencils is 3 inches.

If You Have Time

Noah wants to create a collection of at least 10 pencils where the difference between the longest and shortest colored pencils is no more than $1\frac{1}{2}$ inches.

Is that possible? If so, which pencils should he remove from his collection?

Unit 3, Lesson 14

Addressing 4.MD.4, 4.NF.3.c; practicing MP1 and MP2

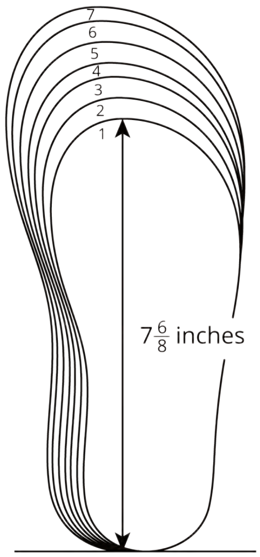
Problems about Fractional Measurement Data

Let’s solve problems involving measurement data on line plots.

Warm-up

Notice and Wonder: Shoe Sizes

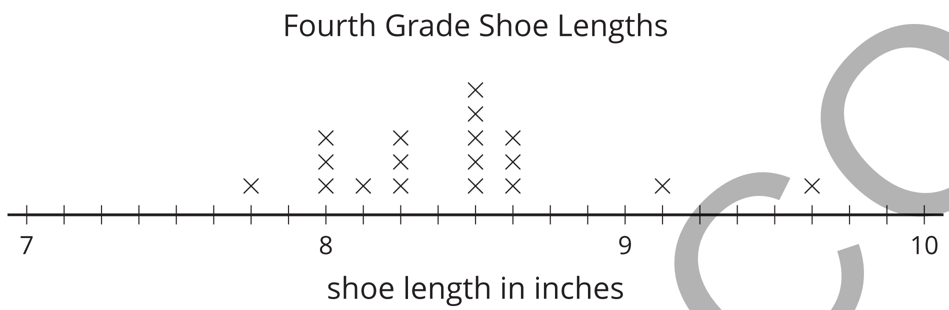
U.S. youth shoe size	insole length (inches)
1	$7\frac{6}{8}$
1.5	8
2	$8\frac{1}{8}$
2.5	$8\frac{2}{8}$
3	$8\frac{4}{8}$
3.5	
4	$8\frac{6}{8}$
4.5	9
5	$9\frac{1}{8}$
5.5	
6	$9\frac{4}{8}$
6.5	$9\frac{5}{8}$
7	$9\frac{6}{8}$



Activity 1

Shoe Lengths

Students in a fourth-grade class collect data on their shoe sizes and lengths. They plot the shoe lengths on a line plot.



The line plot is missing the shoe lengths of 6 students.

$9 \quad 9\frac{1}{8} \quad 8\frac{6}{8} \quad 7\frac{6}{8} \quad 9\frac{2}{8} \quad 8\frac{1}{8}$

1. Complete the line plot with the missing data.
2. Use the completed line plot to answer the following questions:
   a. What is the longest shoe length, in inches?

   b. What is the shortest shoe length, in inches?

   c. What is the difference between the longest and shortest shoe lengths, in inches? Explain or show your reasoning.

   d. The student who records 9 inches for her shoe length makes a mistake when reading the shoe chart. Her actual shoe length is $\frac{7}{8}$ inches shorter.

   What is her shoe length, in inches? Plot her corrected data point on the line plot.

Activity 2

Larger Shoes, Anyone?

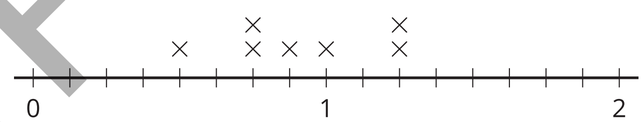
There are 10 students that record their shoe lengths in third grade and then again in fourth grade.

They found how much their feet have grown over a year and organize the data in a table and on a line plot.

student	change in shoe length (inches)
Jada	$\frac{5}{4}$
Priya	$\frac{7}{8}$
Andre	$\frac{3}{4}$
Elena	$\frac{1}{2}$
Han	$1\frac{2}{8}$

student	change in shoe length (inches)
Clare	1
Tyler	$1\frac{1}{8}$
Kiran	$\frac{6}{8}$
Diego	$1\frac{1}{4}$
Lin	$\frac{5}{8}$

How Much Have Our Feet Grown?



change in shoe length in inches

1. The line plot shows only 7 points. Whose information is missing? Add the 3 missing points to the line plot.

2. If Han's shoe length now is $9\frac{1}{8}$ inches, what was his shoe length in third grade?

3. If Priya's shoe length was $7\frac{6}{8}$ inches last year, what's her shoe length this year?

Sec B

4. Tyler makes a calculation error. What he records, $1\frac{1}{8}$ inches, was $\frac{3}{8}$ inches off from the actual change in shoe length.

   a. What could be the actual change in his shoe length? Explain or show your reasoning.

   b. How does his error change the line plot? Explain your reasoning.

Section B Summary

We added and subtracted fractions with the same denominator, using number lines to help with our reasoning.

First, we learned that a fraction can be decomposed into a sum of smaller fractions. For example, here are a few ways to write $\frac{6}{10}$:

$$\frac{6}{10} = \frac{4}{10} + \frac{2}{10} \qquad \frac{6}{10} = \frac{5}{10} + \frac{1}{10}$$

$$\frac{6}{10} = \frac{2}{10} + \frac{2}{10} + \frac{2}{10}$$

If the fraction is greater than 1, it can be decomposed into a whole number and a fraction less than 1. For instance, we can decompose $\frac{17}{10}$ and rewrite it as $1\frac{7}{10}$. A number such as $1\frac{7}{10}$ is called a **mixed number**.

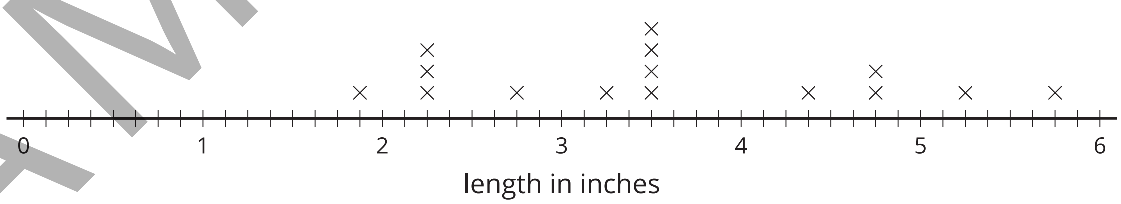
$$\frac{10}{10} + \frac{7}{10}$$
$$1 + \frac{7}{10}$$
$$1\frac{7}{10}$$

Later, we decomposed fractions into sums and wrote equivalent fractions to help us add and subtract fractions. For example, to find the value of $1\frac{2}{5} - \frac{3}{5}$, we can:

- Decompose $1\frac{2}{5}$ into $1 + \frac{2}{5}$ or $\frac{5}{5} + \frac{2}{5}$, which is $\frac{7}{5}$.
- Find the value of $\frac{7}{5} - \frac{3}{5}$, which is $\frac{4}{5}$.

Finally, we organized and analyzed measurement data, using line plots. The data were lengths measured to the nearest inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, and $\frac{1}{8}$ inch.

Colored Pencil Data



We used equivalent fractions to plot the measurements because the fractions have different denominators. Then we used the line plots and what we know about addition and subtraction of fractions to solve problems about the data.

Practice Problems

9 Problems

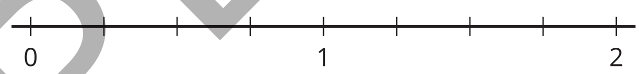
1 from Unit 3, Lesson 7

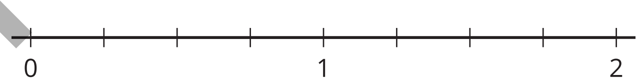
a. Write $\frac{4}{3}$ in as many ways as you can as a sum of fractions.

b. Write $\frac{9}{8}$ in at least 3 different ways as a sum of fractions.

2 from Unit 3, Lesson 8

a. Draw jumps on the number lines to show two ways to use fourths to make a sum of $\frac{7}{4}$.

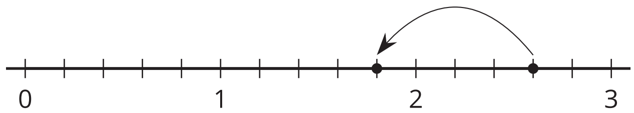




b. Represent each combination of jumps as an equation.

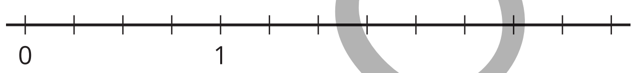
3 from Unit 3, Lesson 9

a. Explain how the diagram represents $\frac{13}{5} - \frac{4}{5}$.



Use the diagram to find the value of $\frac{13}{5} - \frac{4}{5}$.

b. Use a number line to represent and find the difference $\frac{9}{4} - \frac{3}{4}$.



4 from Unit 3, Lesson 10

Show two different ways to find the difference: $2 - \frac{3}{4}$

5 from Unit 3, Lesson 11

Elena is making friendship necklaces and wants the chain and clasp to be a total of $18\frac{1}{4}$ inches long. She is going to use a clasp that is $2\frac{3}{4}$ inches long. How long does her chain need to be? Explain or show your reasoning.

6 from Unit 3, Lesson 12

For each of the expressions, explain whether you think it would be helpful to decompose one or more numbers to find the value of the expression.

a. $\frac{4}{3} + \frac{5}{3}$

b. $5\frac{1}{5} - 2\frac{2}{5}$

c. $9\frac{5}{6} - 6\frac{1}{6}$

7 from Unit 3, Lesson 12

This shows the shoe lengths for a dad and his two daughters.

For each question, show your reasoning.

a. How much longer is the older daughter's shoes than her sister's shoes?

b. Which is longer, the dad's shoes or the combined lengths of his daughters' shoes?

8 Exploration

A chocolate chip cookie recipe calls for $2\frac{3}{4}$ cups of flour. You have only a $\frac{1}{4}$-cup measure and a $\frac{3}{4}$-cup measure that you can use.

a. What are different combinations of the cup measures that you can use to get a total of $2\frac{3}{4}$ cups of flour?

b. Write each of the combinations as an addition equation.

9 Exploration

The table shows some lengths of different shoe sizes in inches.

U.S. shoe size	insole length (inches)
1	$7\frac{6}{8}$
1.5	8
2	$8\frac{1}{8}$
2.5	$8\frac{2}{8}$
3	$8\frac{4}{8}$
3.5	$8\frac{5}{8}$
4	$8\frac{6}{8}$
4.5	9
5	$9\frac{1}{8}$
5.5	$9\frac{2}{8}$
6	$9\frac{4}{8}$
6.5	$9\frac{5}{8}$
7	$9\frac{6}{8}$

a. What do you notice about the insole lengths as the size increases?

b. Based on the data in the table, what is the insole length increase from size 7 to size 7.5? What is the insole length of a size 7.5 shoe?

c. Predict the insole length for sizes 9, 10, and 12. Explain your prediction. Then solve to find out if your prediction is true.

Unit 3, Lesson 15

Addressing 4.NF.1, 4.NF.2, 4.NF.3.d; building on 4.NF.3.c, 4.NF.4; building towards 4.NF.5; practicing MP1 and MP4

An Assortment of Fractions

Let's find the heights of some stacked objects.

Warm-up

Which Three Go Together: Halves, Fourths, Sixths, and Eighths

Which 3 go together?

A

$$1\frac{1}{2}$$

B

$$\frac{4}{4} + \frac{2}{4}$$

C

$$\frac{12}{8}$$

D

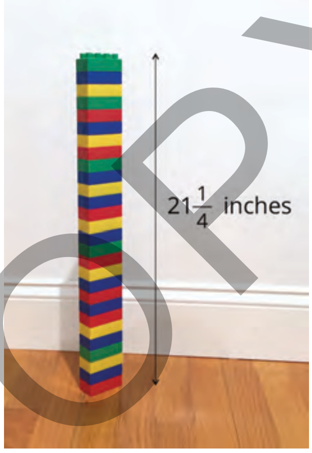
$$\frac{4}{6}$$

Sec C

Activity 1

All the Way to the Top

Priya, Kiran, and Lin use large playing bricks to make towers. Here are the heights of their towers.



- Priya: $21\frac{1}{4}$ inches
- Kiran: $32\frac{3}{8}$ inches
- Lin : $55\frac{1}{2}$ inches

Show your reasoning for each question.

Sec C

1. How much taller is Lin's tower compared to:
   a. Priya's tower?

   b. Kiran's tower?

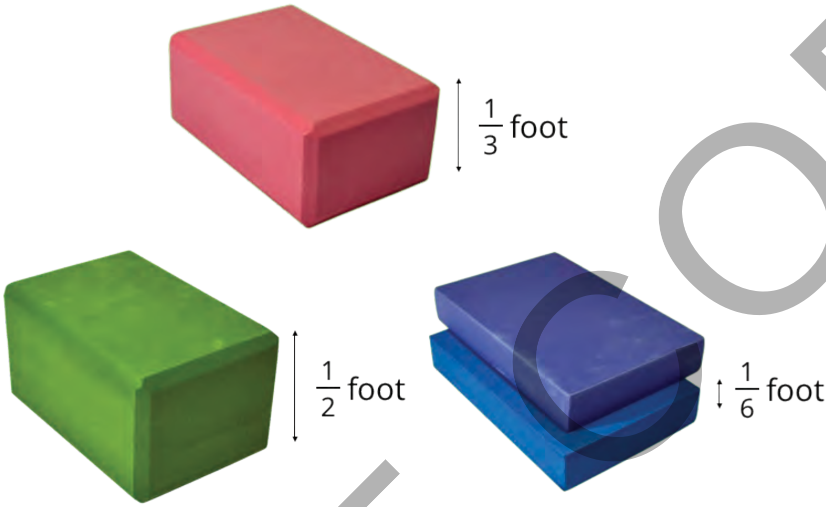
2. They are playing in a room that is 109 inches tall. Priya says that if they combine their towers to make a super tall tower, it would be too tall for the room. She says they must remove 1 brick.

   Do you agree with Priya? Explain your reasoning.

Activity 2

Stacks of Blocks

Andre is building a tower out of foam blocks. The blocks come in three different thicknesses: $\frac{1}{2}$ foot, $\frac{1}{3}$ foot, and $\frac{1}{6}$ foot.



1. Andre stacks 1 block of each size. Is the stack more than 1 foot tall? Explain or show how you know.

2. Can Andre use only the $\frac{1}{6}$ -foot and $\frac{1}{3}$-foot blocks to make a stack that is $1\frac{1}{2}$ feet tall? If you think so, show one or more ways. If not, explain why not.

3. Can Andre use only the $\frac{1}{6}$-foot and $\frac{1}{2}$-foot blocks to make a stack that is $1\frac{1}{3}$ feet tall? If so, show one or more ways. If not, explain why not.

Addressing 4.NF.1, 4.NF.5; building on 4.NF.1, 4.NF.2; practicing MP2

Add Tenths and Hundredths Together

Let's add some tenths and hundredths.

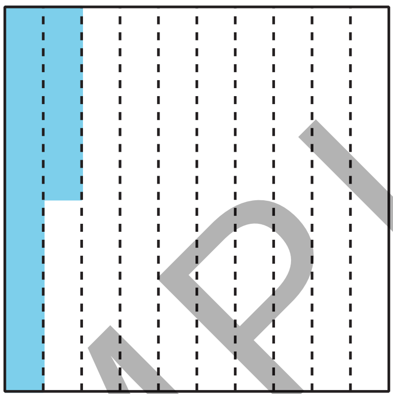
Warm-up

Notice and Wonder: Shaded Rectangles and Squares

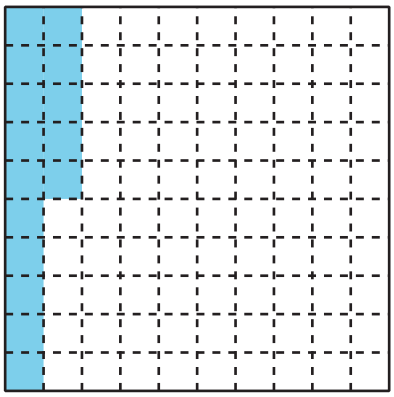
Each large square represents 1.

What do you notice? What do you wonder?

A



B



Activity 1

Tenths and Hundredths

1. Complete the table with equivalent fractions in tenths or hundredths. Write a new pair of equivalent fractions in the last row.

	tenths	hundredths
a.	$\frac{1}{10}$	
b.	$\frac{4}{10}$	
c.	$\frac{6}{10}$	
d.		$\frac{50}{100}$
e.		$\frac{90}{100}$
f.	$\frac{12}{10}$	
g.		$\frac{200}{100}$
h.	$2\frac{3}{10}$	
i.		$\frac{125}{100}$
j.		

2. Name some fractions that are:

   a. between $\frac{50}{100}$ and $\frac{60}{100}$

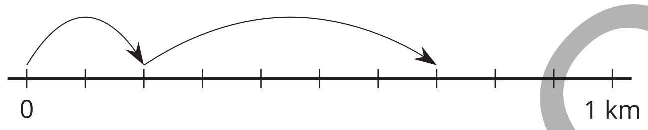
   b. between $\frac{3}{10}$ and $\frac{4}{10}$

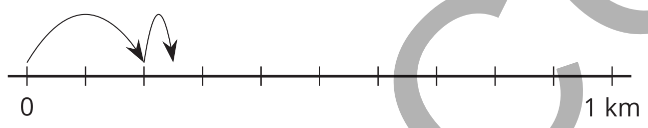
Activity 2

Walk, Stop, and Sip

Noah walks $\frac{2}{10}$ kilometer (km), stops for a drink of water, walks $\frac{5}{100}$ kilometer, and stops for another drink.

1. Which number line diagram represents the distance Noah has walked? Explain how you know.





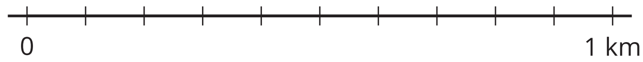
2. The diagram that you didn't choose represents Jada's walk. Write an equation to represent:

   a. the total distance Jada walked

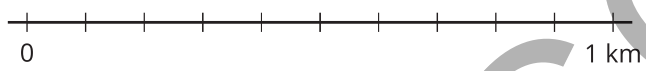
   b. the total distance Noah walked

3. Find the value of each of the following sums. Show your reasoning. Use number lines if you find them helpful.

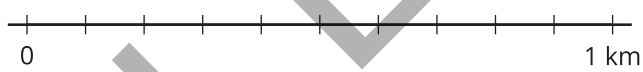
a. $\frac{5}{10} + \frac{1}{10}$



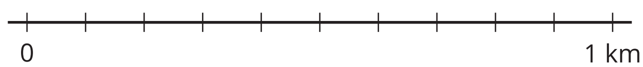
b. $\frac{50}{100} + \frac{10}{100}$



c. $\frac{5}{10} + \frac{30}{100}$



d. $\frac{15}{100} + \frac{4}{10}$



Sec C

Unit 3, Lesson 17

Addressing 4.NF.5; building on 4.NF.1, 4.NF.2; practicing MP7 and MP8

Sums of Tenths and Hundredths

Let's add more tenths and hundredths.

Warm-up

Which Three Go Together: Tenths and Hundredths

Which 3 go together?

A. $\frac{48}{100}$

B. $\frac{8}{10}$

C. $\frac{120}{100}$

D. $\frac{70}{100}$

Sec C

Activity 1

Card Sort: Less than, Equal to, or Greater than 1?

Your teacher will give you a set of cards.

1. Sort the cards, based on whether the value of each expression is less than 1, equal to 1, or greater than 1.

   After you sort the cards, make a quick list of which expressions you have in each group.

2. Visit the sorted collection of another group.
   - Did they sort the cards the same way?
   - Select 1–2 cards that you have a question about or whose placement you disagree with.
   - Leave a note for the group members to discuss.

3. Return to your collection.
   - Discuss any notes that are left for your group, or revise your sorting decision, based on what you learned from another group.
   - Record the expressions here.

less than 1	equal to 1	greater than 1

Activity 2

What's Missing?

1. Each equation has an unknown fraction in hundredths. Find the fraction that makes each equation true.

   a. $\frac{10}{100} + \_\_\_\_\_\_\_\_ = \frac{30}{100}$

   b. $\_\_\_\_\_\_\_\_ + \frac{2}{10} = \frac{80}{100}$

   c. $\frac{7}{10} + \_\_\_\_\_\_\_\_ = \frac{94}{100}$

   d. $\frac{9}{100} + \_\_\_\_\_\_\_\_ = \frac{8}{10}$

   e. $\frac{16}{100} + \frac{4}{10} = \_\_\_\_\_\_\_\_$

   f. $\_\_\_\_\_\_\_\_ + \frac{14}{10} = \frac{172}{100}$

2. Each equation has an unknown fraction in tenths or hundredths. Find the fraction that makes each equation true.

   a. $\frac{20}{100} + \_\_\_\_\_\_\_\_ = \frac{28}{100}$

   b. $\frac{110}{100} + \_\_\_\_\_\_\_\_ = \frac{15}{10}$

   c. $\frac{61}{100} + \frac{3}{10} = \_\_\_\_\_\_\_\_$

   d. $\frac{9}{10} + \_\_\_\_\_\_\_\_ = \frac{170}{100}$

   e. $\_\_\_\_\_\_\_\_ + \frac{72}{100} = \frac{102}{100}$

   f. $\frac{15}{100} + \_\_\_\_\_\_\_\_ = 1\frac{55}{100}$

Activity 3

Fraction Action: Tenths, Hundredths

Play Fraction Action with 2 players:

- Shuffle the cards that your teacher gives you. Place the cards facedown in a stack.
- Each player turns over 2 cards and adds the fractions on the cards.
- Compare the sums. The player with the greater sum wins that round and keeps all 4 cards.
- If the sums are equivalent, each player turns over 1 more card and adds the value to their sum. The player with the greater new sum keeps all cards.
- The player with the most cards wins the game.

Play Fraction Action with 3 or 4 players:

- The player with the greatest sum of fractions wins the round.
- If 2 or more players have the greatest sum, each of those players turns over 2 more cards and finds their sum. The player with the greatest sum keeps all the cards.

Record any pair of fractions whose sum is challenging to find here.

_______ and _______   _______ and _______

_______ and _______   _______ and _______

Addressing 4.NF.4.c, 4.NF.5; building on 3.NBT.2; practicing MP1

A Lot of Fractions to Add

Let's add more than two fractions.

Warm-up

Number Talk: A Bunch of Numbers

Find the value of each expression mentally.

- $54 + 2 + 18$
- $61 + 104 + 39$
- $25 + 63 + 75 + 7$
- $50 + 106 + 19 + 101$

Activity 1

Stack Centavos and Pesos

Diego and Lin each have a small collection of Mexican coins.

The table shows the thickness of different coins in centimeters, cm, and how many of each Diego and Lin have.

coin value	thickness in cm	Diego	Lin
1 centavo	$\frac{12}{100}$	3	1
10 centavos	$\frac{22}{100}$	0	1
1 peso	$\frac{16}{100}$	0	1
2 pesos	$\frac{14}{100}$	0	1
5 pesos	$\frac{2}{10}$	1	1
20 pesos	$\frac{25}{100}$	2	1

1. If Diego and Lin each stack their *centavo* (sen-TAH-voh) coins, whose stack would be taller? Show your reasoning.

2. If they each stack their *peso* (PAY-soh) coins, whose stack would be taller? Show your reasoning.

3. If they each stack all their coins, whose stack would be taller? Show your reasoning.

4. If they combine their coins to make one stack, would it be more than 2 centimeters tall? Show your reasoning.

Activity 2

More than Two Fractions

Find the value of at least 3 of the expressions. Show your reasoning.

1. $\frac{2}{100} + \frac{13}{10} + \frac{1}{10} + \frac{8}{100}$

Sec C

2. $\frac{50}{10} + \frac{16}{100} + \frac{2}{10}$

3. $\frac{3}{10} + \frac{4}{100} + \frac{7}{10} + \frac{26}{100}$

4. $\frac{4}{100} + 3\frac{2}{10} + 1\frac{5}{10}$

5. $1\frac{1}{10} + 5\frac{2}{100} + \frac{78}{100}$

6. $2\frac{7}{10} + \frac{2}{100} + \frac{8}{10}$

Sec C

Section C Summary

We learned more ways to add fractions and to solve problems that involve adding, subtracting, and multiplying fractions.

We started by adding tenths and hundredths, using what we know about equivalent fractions. For example, to find the sum of $\frac{4}{10}$ and $\frac{30}{100}$, we can:

- Write $\frac{4}{10}$ as $\frac{40}{100}$, and then find $\frac{40}{100} + \frac{30}{100}$, or
- Write $\frac{30}{100}$ as $\frac{3}{10}$, and then find $\frac{4}{10} + \frac{3}{10}$.

We learned that when adding a few fractions, it may help to rearrange or group them. Example:

- $\frac{6}{100} + \frac{2}{10} + \frac{74}{100}$ can be rearranged as $\frac{6}{100} + \frac{74}{100} + \frac{2}{10}$.
- Next, the hundredths can be added first, giving $\frac{80}{100} + \frac{2}{10}$.
- Then we can write an equivalent fraction for $\frac{80}{100}$ and find $\frac{8}{10} + \frac{2}{10}$, or write an equivalent fraction for $\frac{2}{10}$ and find $\frac{80}{100} + \frac{20}{100}$.

Sec C

Unit 3, Lesson 19

Addressing 4.NF.3.c, 4.NF.3.d, 4.NF.4, 4.NF.5; practicing MP2

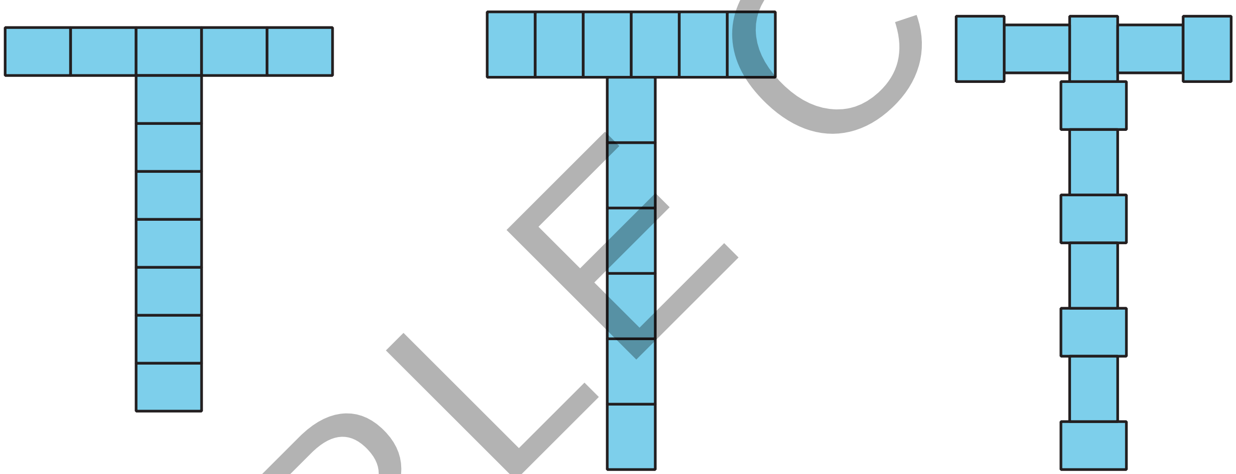
Flexible with Fractions

Let’s solve all kinds of problems involving fractions.

Warm-up

Notice and Wonder: Sticky Notes

What do you notice? What do you wonder?



Sec C

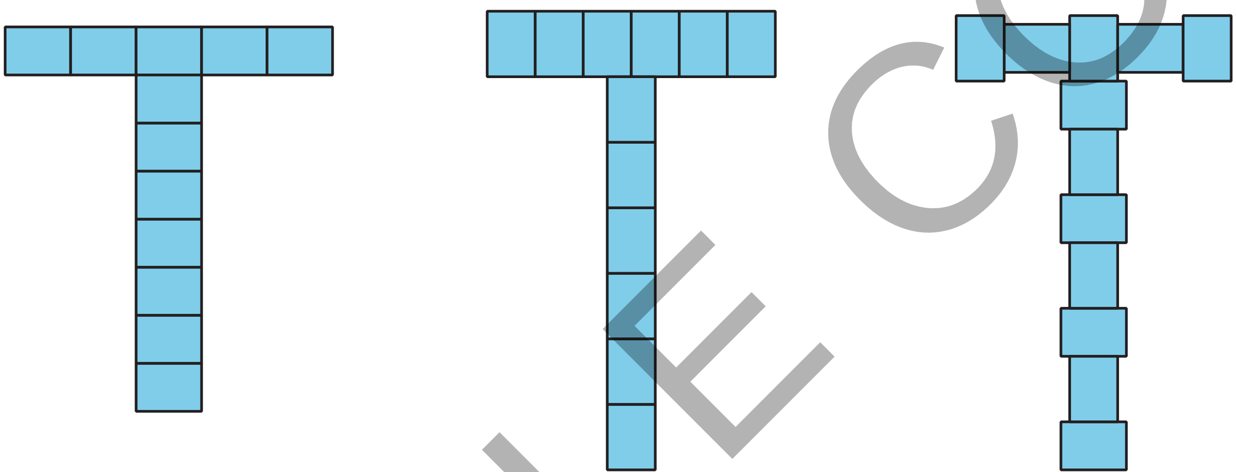
Activity 1

Sticky-Note Designs

Tyler is using small sticky notes to make a T shape to decorate a folder.

The longer side of the sticky note is $\frac{15}{8}$ inches. The shorter side is $\frac{11}{8}$ inches. The folder is 9 inches wide and 12 inches tall.

Here are 3 ways he could arrange the sticky notes:



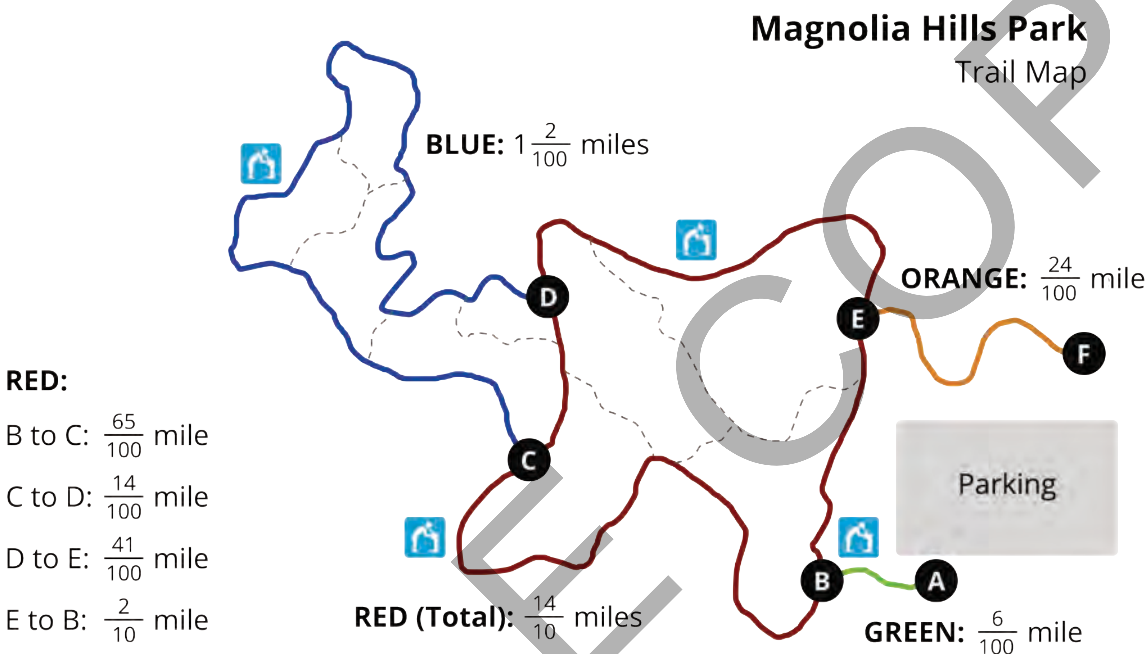
Is the folder tall enough and wide enough for his designs? If so, which design(s) would fit? Show your reasoning.

Sec C

Activity 2

Hiking Trails

Jada and Noah are hiking at a park. Here is a map of the trails. The length of each trail is shown.



1. Jada and Noah hike the orange trail from Point F to Point E, make one full loop on the red trail back to Point E, and then hike from E back to F.

   How many miles do they hike? Show your reasoning.

2. Here are two expressions that represent some hiking situations. What question might each expression help to answer? Write the question and the answer.

   a. $\frac{6}{100} + \frac{65}{100} + 1\frac{2}{100} + \frac{41}{100} + \frac{24}{100}$

   b. $\left(2 \times \frac{14}{10}\right) + \left(2 \times \frac{6}{100}\right)$

3. Use the distances on the map to write a new question and find its answer. Then trade questions with a partner and answer one another's question.

Activity 3

Find a Match

Your teacher will give you 1 card with an expression on it.

1. Find the value of the expression.
2. Find a classmate whose card also has the same value. Prove to each other that you're a match.
3. Work with your partner to find at least 2 features that your expressions share—other than the fact that they have the same value.
4. Write one more expression that has the same value but uses a different operation.

Unit 3, Lesson 20

Addressing 4.NF.3.d, 4.NF.4; practicing MP4

Sticky Notes

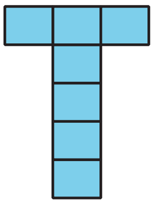
Let’s make a design using sticky notes.

Warm-up

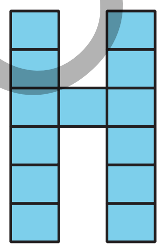
Which Three Go Together: Sticky Notes

Which 3 go together?

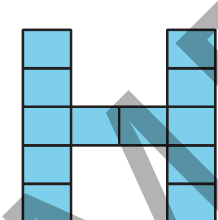
A



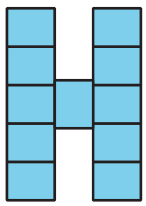
B



C



D



Activity 1

Estimation Exploration: Sticky Notes

1. How many sticky notes will fit across the top or along the side of the page?

   Record an estimate that is:

too low	about right	too high

2. What information do you need to help you make a better estimate?

3. With the new information you have now, make a better estimate. Show or explain your reasoning.

4. Write an expression that represents your estimate of how many sticky notes fit across or along the side of the paper.

Sec C

Activity 2

Design Your Initial

Design a letter with sticky notes.

1. Plan your design and determine the number of sticky notes that you need.

2. Write at least two equations that show your design will fit on a piece of paper.

3. Take turns sharing your design with a partner.
4. Get the supplies and make your design.

Practice Problems

6 Problems

1 from Unit 3, Lesson 15

Andre is building a tower out of different foam blocks. These blocks come in three different thicknesses: $\frac{1}{2}$ foot, $\frac{1}{4}$ foot, and $\frac{1}{8}$ foot.

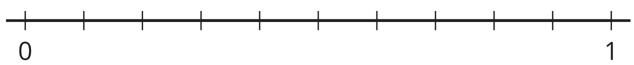
Andre stacks two $\frac{1}{2}$-foot blocks, two $\frac{1}{4}$-foot blocks, and two $\frac{1}{8}$-foot blocks to create a tower. What is the height of the tower in feet? Explain or show how you know.

2 from Unit 3, Lesson 16

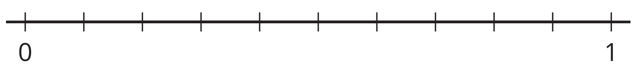
Find the value of each of the following sums. Show your reasoning. Use number lines if you find them helpful.

a. $\frac{1}{10} + \frac{3}{100}$

b. $\frac{24}{100} + \frac{4}{10}$



c. $\frac{7}{10} + \frac{13}{100}$



3 from Unit 3, Lesson 17

Is the value of each expression greater than, less than, or equal to 1? Explain how you know.

a. $\frac{3}{10} + \frac{7}{100}$

b. $\frac{13}{10} + \frac{7}{100}$

c. $\frac{30}{100} + \frac{7}{10}$

Sec C

4 from Unit 3, Lesson 18

Diego and Lin continue to play with their coins.

Diego says he has exactly 3 coins whose thickness adds up to $\frac{50}{100}$ centimeters (cm). What coins does Diego have? Explain or show your reasoning.

coin	thickness (cm)
1 centavo	$\frac{12}{100}$
10 centavos	$\frac{22}{100}$
1 peso	$\frac{16}{100}$
2 pesos	$\frac{14}{100}$
5 pesos	$\frac{2}{10}$
20 pesos	$\frac{25}{100}$

Sec C

5 Exploration

A chocolate cake recipe calls for 2 cups of flour. You gather your measuring cups and notice you have the following sizes: $\frac{1}{2}$ cup, $\frac{1}{3}$ cup, $\frac{1}{4}$ cup, and $\frac{1}{6}$ cup.

a. What are the different ways you could use all 4 measuring cups to measure 2 cups of flour?

b. What are other ways you could use just some of the 4 measuring cups to measure exactly 2 cups of flour?

6 Exploration

A dime is worth $\frac{1}{10}$ of a dollar and a penny is worth $\frac{1}{100}$ of a dollar.

a. What are different combinations of dimes and pennies that represent $\frac{89}{100}$ of a dollar? Use equations to show your reasoning.

Sec C

b. A nickel is worth $\frac{5}{100}$ of a dollar. What are some different combinations of dimes, nickels, and pennies that represent $\frac{89}{100}$ of a dollar? Use equations to show your reasoning.

GRADE 4

UNIT

4

From Hundredths to Hundred-Thousands Overview

In this unit you will continue your exploration of fractions along with other numbers and mathematical operations. Throughout the unit you will make connections by:

- **Exploring Changing Quantities** of multi-digit numbers while looking at shapes and patterns.
- **Taking Wholes Apart and Putting Parts Together** when you create fraction models for adding, subtracting, and comparing numbers.
- **Discovering Shape and Space** by solving problems related to shapes, symmetries, fractions, and decimals.

Addressing the Standards

As you work your way through **Unit 4 From Hundredths to Hundered-Thousands,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 6
MP2 Reason abstractly and quantitatively.	Lessons 1, 4, 5, 15, 17, and 22
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 2, 10, 13, 19, and 20
MP4 Model with mathematics.	Lessons 17 and 23
MP5 Use appropriate tools strategically.	Lessons 14 and 16
MP6 Attend to precision.	Lessons 1, 3, 8, 12, and 13
MP7 Look for and make use of structure.	Lessons 1, 5, 7, and 9
MP8 Look for and express regularity in repeated reasoning.	Lessons 8, 11, 18, and 21

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fraction Flexibility • Visual Fraction Models	**4.NF.3.c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Lesson 19
• Fraction Flexibility • Visual Fraction Models • Circles, Fractions, and Decimals	**4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	Lessons 2, 3, 4, and 5

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Visual Fraction Models • Circles, Fractions, and Decimals	**4.NF.6** Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	Lessons 1, 2, 3, 4, and 5
• Visual Fraction Models • Circles, Fractions, and Decimals	**4.NF.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or < and justify the conclusions, e.g., by using the number line or another visual model.	Lessons 2, 3, 4, and 5
• Multi-Digit Numbers	**4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.	Lessons 6, 8, 9, 10, 11, and 23
• Multi-Digit Numbers	**4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multidigit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Lessons 7, 8, 9, 10, 11, 12, 13, 14, 21, and 23
• Multi-Digit Numbers • Shapes and Symmetries • Connected Problem Solving	**4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place.	Lessons 14, 15, 16, 17, and 23
• Multi-Digit Numbers • Number and Shape Patterns • Shapes and Symmetries • Connected Problem Solving	**4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Lessons 10, 18, 19, 20, 21, 22, and 23

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 4, Lesson 1

Addressing 4.NF.6; practicing MP2, MP6, and MP7

Decimal Numbers

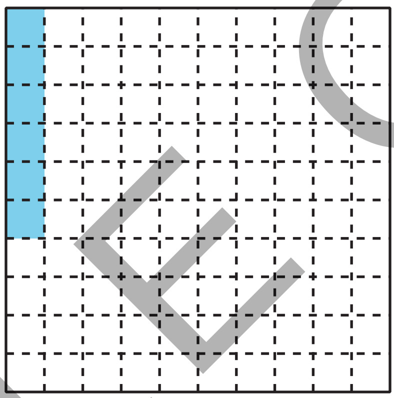
Let's learn about decimals.

Sec A

Warm-up

Notice and Wonder: Shaded Grid

What do you notice? What do you wonder?



Activity 1

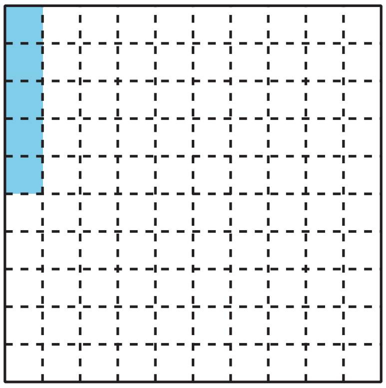
Shady Fractions

Sec A

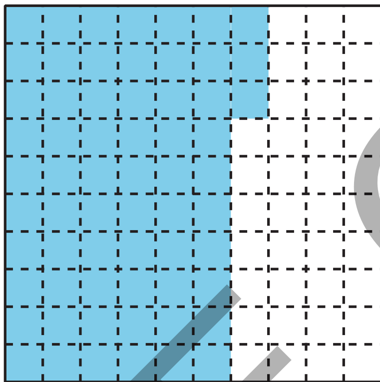
Each large square represents 1.

1. What fraction do the shaded parts of each diagram represent? For the last square, shade in some parts and name the fraction it represents.

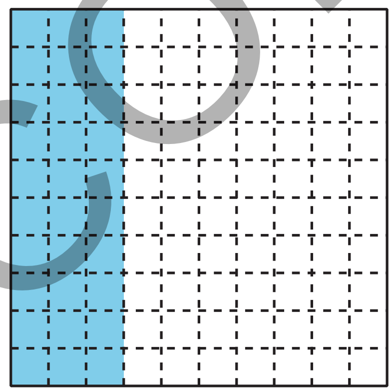
a.



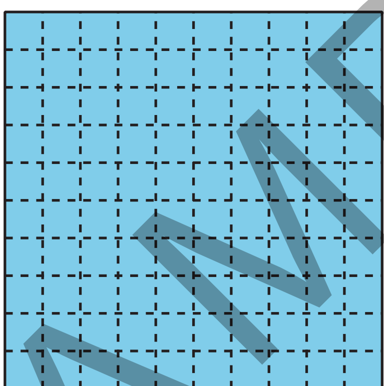
b.



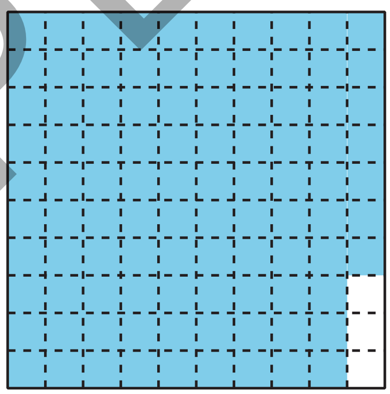
c.



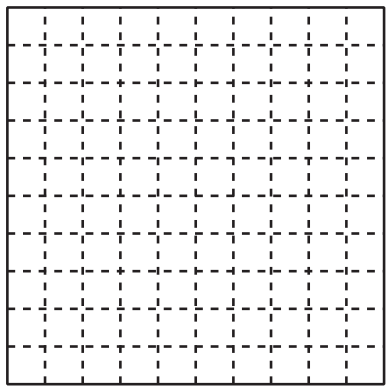
d.



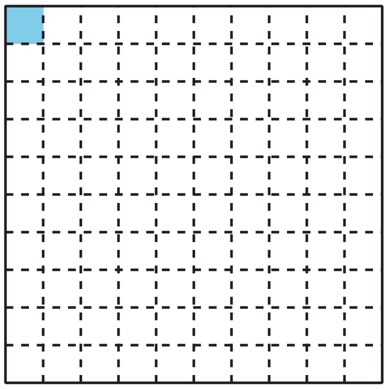
e.



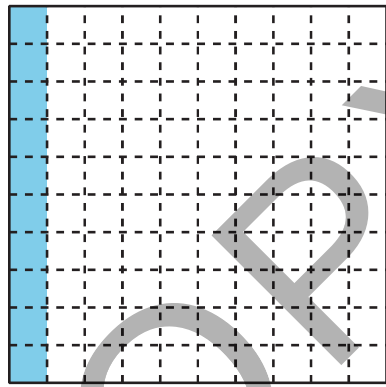
f.



2. The shaded part of this diagram represents 0.01 or “1 hundredth.”



The shaded parts of this diagram represent 0.10 or “10 hundredths.” They also represent 0.1 or “1 tenth.”

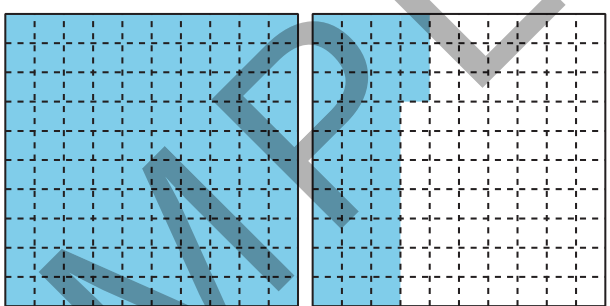


Numbers like 0.01, 0.10, and 0.1 are written in **decimal notation**.

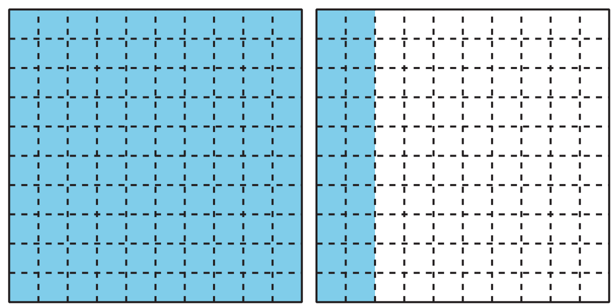
Look at the shaded parts of each diagram in the first problem. Write the numbers they represent in decimal notation.

3. What number does the shaded parts of each diagram represent? Write the number as a fraction and in decimal notation.

a.



b.



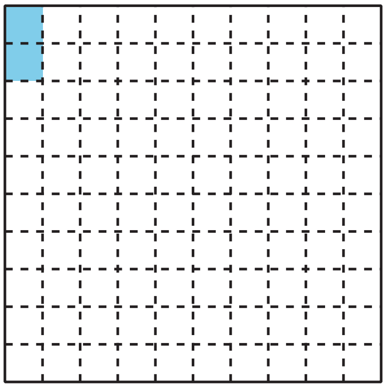
Activity 2

Ways to Express a Number

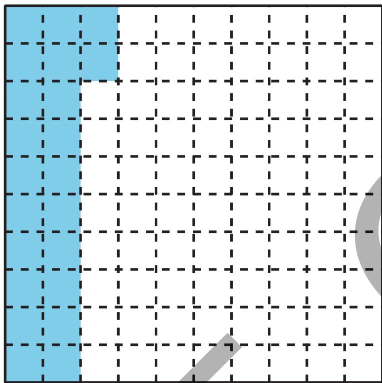
Each large square represents 1.

1. Write a fraction and a decimal that represent the shaded parts of each diagram. Then write each number in words.

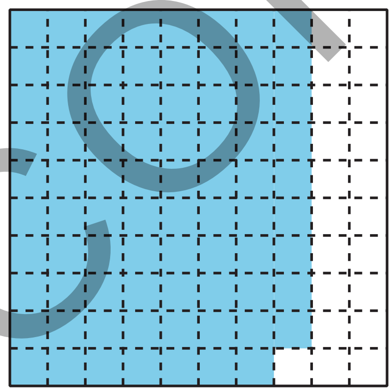
a.



b.

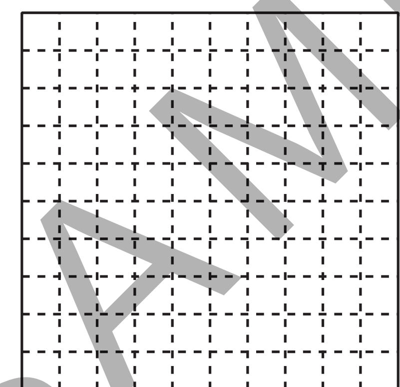


c.



2. Shade each diagram to represent the given number. Then write the number in the form that is not given.

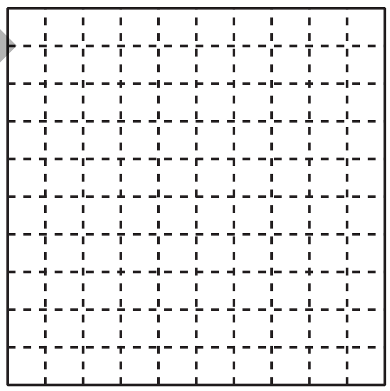
a.



Fraction: ________

Decimal Notation: 0.78

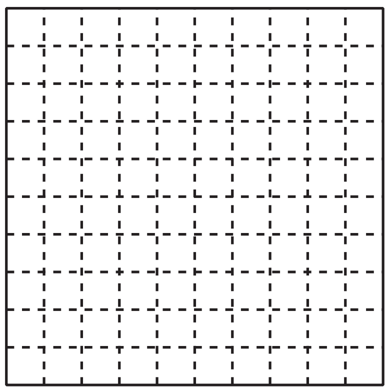
b.



Fraction: $\frac{8}{10}$

Decimal Notation: ________

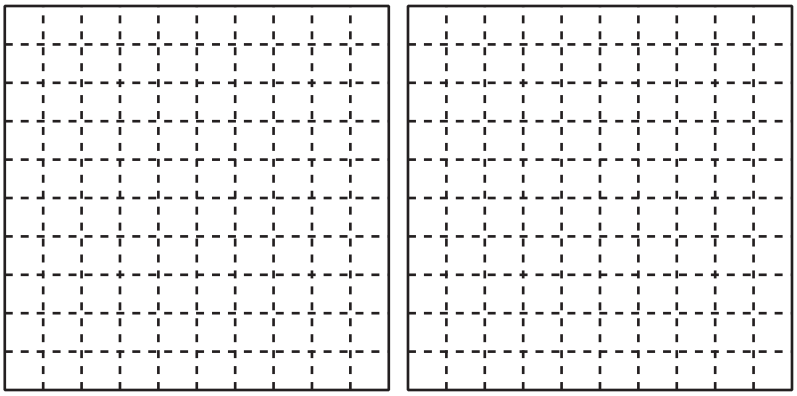
c.



Fraction: $\frac{55}{100}$

Decimal Notation: ________

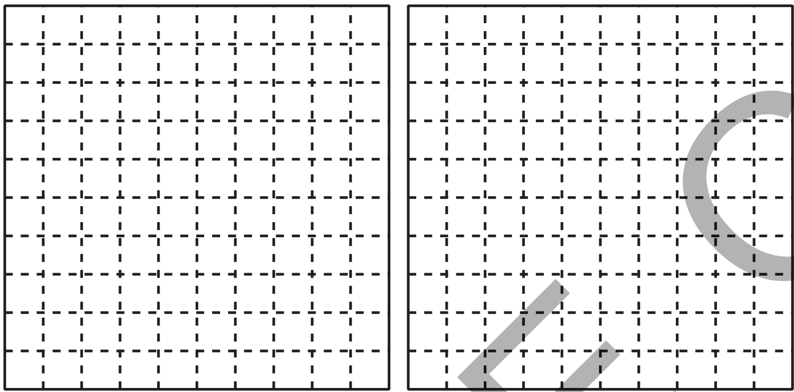
d.



Fraction: $\frac{107}{100}$

Decimal Notation: ________

e.



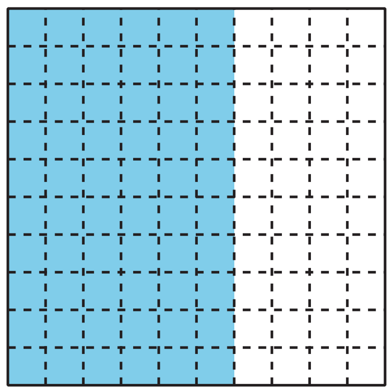
Fraction: ________

Decimal Notation: 1.6

3. Han and Elena disagree about what number the shaded part represents.

   Han says that it represents 0.60 and Elena says it represents 0.6.

   Explain why both Han and Elena are correct.



____

____

____

____

Unit 4, Lesson 2

Addressing 4.NF.5, 4.NF.6, 4.NF7; practicing MP3

Equivalent Decimals

Let's think about equivalent decimals.

Sec A

Warm-up

True or False: Equivalent Fractions

Decide whether each statement is true or false. Explain your reasoning.

- $\frac{50}{100} = \frac{5}{10}$

- $\frac{20}{10} = \frac{20}{100}$

- $2 = 1 + \frac{90}{100}$

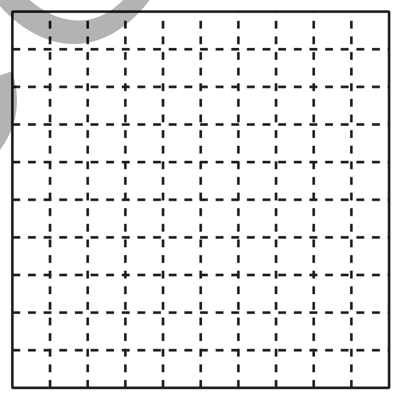
- $3\frac{1}{10} = \frac{31}{10}$

Activity 1

Card Sort: Diagrams of Fractions and Decimals

Your teacher will give you a set of cards. Each large square on the cards represents 1.

1. Sort the cards into groups so that the representations in each group have the same value. Be ready to explain your reasoning.
2. One of the diagrams has no matching number card. What number does it represent? Write that number as a fraction and in decimal notation.
3. Are 0.20 and 0.2 equivalent? Use fractions and a diagram to explain your reasoning.



Activity 2

True or Not True?

1. Decide whether each statement is true or false. For each statement that is false, replace one of the numbers to make it true. A different number should be shown on either side of the equal sign.

   a. $\frac{50}{100} = 0.50$

   b. $0.05 = 0.5$

   c. $0.3 = \frac{3}{10}$

   d. $0.3 = \frac{30}{100}$

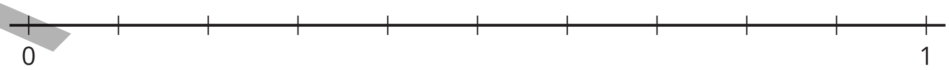
   e. $0.3 = 0.30$

   f. $1.1 = 1.10$

   g. $3.06 = 3.60$

   h. $2.70 = 0.27$

2. Jada says if you locate the numbers 0.05, 0.5, and 0.50 on the number line, you should have only 2 points. Do you agree? Explain your reasoning.



Unit 4, Lesson 3

Addressing 4.NF.7; practicing MP6

Decimals on Number Lines

Let’s compare some decimals.

Warm-up

Which Three Go Together: Decimals

Which 3 go together?

A

8 tenths

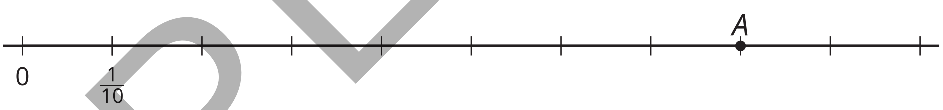
B

$\frac{80}{10}$

C

0.80

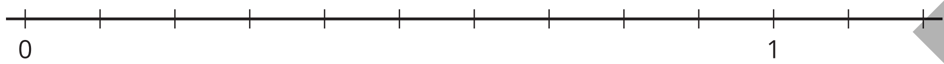
D



Activity 1

Points on Number Lines

1. Label each tick mark on the number line with the number it represents.



2. Here are 8 numbers.

   0.10 0.40 0.80 1.10 0.15 0.45 0.75 1.05

   a. Locate and label each number on the number line.

   b. Which number is greatest? Which is least? Explain how the number line can help determine the greatest and least numbers.

   ____

   ____

3. Locate and label these numbers on the number line.

   0.24 0.96 0.61 1.12 0.08

4. Use 2 numbers from the previous questions to make each comparison statement true.

   a. _______ is greater than ______.

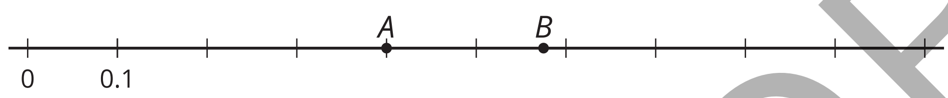
   b. _______ is less than ______.

   c. _______ is the greatest number.

Activity 2

Compare Decimals

1. Here is a number line with 2 points on it.



a. Name the decimal located at point *A*.

b. Is the decimal at point *A* less than or greater than 0.50? Explain or show your reasoning.

c. Is the decimal at point *B* greater or less than 0.06? Explain your reasoning.

d. Estimate the decimal at point *B*.

2. Compare the numbers using >, <, or =. Can you think of a way to make comparisons without using a number line?

   a. 0.51_______0.09

   b. 0.19_______0.91

   c. 0.45_______0.54

   d. 0.62_______0.26

   e. 1.02_______0.95

   f. 0.3_______0.30

   g. 4.01_______4.10

Unit 4, Lesson 4

Addressing 4.NF.7; practicing MP2

Compare and Order Decimals

Let's put some decimals in order.

Warm-up

Estimation Exploration: Eagle Wingspan

The person in the image is 1.7 meters tall.

Estimate the wingspan of the eagle in meters.

Record an estimate that is:

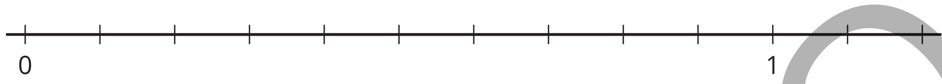
too low	about right	too high

Activity 1

All in Order

1. Order the numbers from least to greatest. Use the number line if it is helpful.

1.08 0.08 0.80 0.9 0.45 0.54



2. Order the numbers from greatest to least. Use the number line if it is helpful.

1.25 0.95 0.4 0.09 12.05 0.25

Activity 2

400-Meter Dash in a Flash

The table shows 8 of the top runners in the Women's 400-Meter event. Here are the running times that put them in the world's top 25 for this event.

48.37	49.3	48.7	49.26
49.07	49.28	48.83	49.05

The names in the table are arranged in order by the fastest running times. The fastest runner is at the top.

runner	time (seconds)	year achieved
Shaunae Miller-Uibo (Bahamas)		2019
Sanya Richards (U.S.A.)		2006
Valerie Brisco-Hooks (U.S.A.)		1984
Chandra Cheesborough (U.S.A.)		1984
Tonique Williams-Darling (Bahamas)		2004
Allyson Felix (U.S.A.)		2015
Pauline Davis (Bahamas)		1996
Lorraine Fenton (Jamaica)		2002

1. Put the running times in order, from least to greatest, to match each time with the runner.
2. How many seconds did it take Sanya Richards to run 400 meters?
3. What is Allyson Felix's running time?

Unit 4, Lesson 5

Addressing 4.NF.5, 4.NF.6; practicing MP2 and MP7

Compare and Order Decimals

Let's put decimals in order.

Sec A

Warm-up

Number Talk: Sums of Fractions

Find the value of each expression mentally.

- $\frac{5}{10} + \frac{50}{100}$
- $\frac{5}{10} + \frac{55}{100}$
- $\frac{6}{10} + \frac{50}{100}$
- $\frac{6}{10} + \frac{65}{100}$

Activity 1

Order Once, Order Twice

Your teacher will give you a set of cards with numbers written as fractions and in decimal notation.

1. Work with your group to order the numbers from least to greatest. Record the numbers in order.

2. Find a group whose cards are different from yours. Combine your cards with their cards. Order the combined set from least to greatest. Record the numbers in order.

3. Use the numbers from your sorted set and >, <, or = symbols to create true comparison statements:

   a. ________ < ________

   b. ________ > ________

   c. ________ < ________

   d. ________ > ________

Activity 2

Long Jumps

American athlete Carl Lewis won 10 Olympic medals and 10 World Championships in track and field— 100-meter dash, 200-meter dash, and long jump.

Here are some long jump records from his career.

year	distance (meters)
1979	8.13
1980	8.35
1982	8.7
1983	8.79
1984	8.24
1987	8.6
1991	8.87

1. On this list, which distance is his shortest jump? Which is his longest jump?

2. Here are the top distances, in meters, of 3 other American long jumpers.
   - Bob Beamon: $8\frac{9}{10}$
   - Jarrion Lawson: $8\frac{58}{100}$
   - Mike Powell: $8\frac{95}{100}$

   Compare their records to Carl Lewis's longest jump. Order the distances from greatest to least.

Section A Summary

We learned to express tenths and hundredths in **decimal notation**, locate them on a number line, and compare them.

We learned $\frac{1}{10}$ written in decimal notation is 0.1, and that this number is read "1 tenth." We also learned $\frac{1}{100}$ written in **decimal notation** is 0.01 and is read "1 hundredth."

The table shows some more examples of tenths and hundredths in decimal notation.

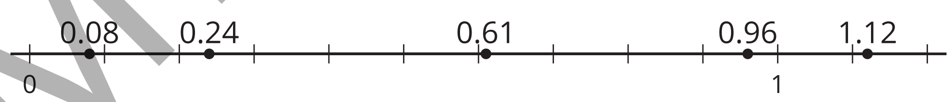
- Because $\frac{5}{10}$ and $\frac{50}{100}$ are equivalent, 0.5 and 0.50 are also equivalent.
- Likewise, $\frac{17}{10}$ and $\frac{170}{100}$ are equivalent, so 1.7 and 1.70 are also equivalent.

fraction	decimal notation
$\frac{4}{100}$	0.04
$\frac{23}{100}$	0.23
$\frac{5}{10}$	0.5
$\frac{50}{100}$	0.50
$\frac{17}{10}$	1.7
$\frac{170}{100}$	1.70

Numbers in decimal notation can be located on a number line to help compare them.

Example:

The decimal 0.24 is equivalent to $\frac{24}{100}$, which is between $\frac{20}{100}$ and $\frac{30}{100}$ (or between $\frac{2}{10}$ and $\frac{3}{10}$) on the number line. We can see 0.24 is greater than 0.08 and less than 0.61.



Practice Problems

11 Problems

1 Pre-unit

Round each number to the nearest 10 and to the nearest 100.

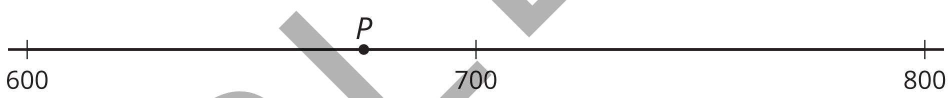
a. 63

b. 350

c. 485

2 Pre-unit

A point *P* is located on the number line and represents a number.



a. Round *P* to the nearest multiple of 100. Explain your reasoning.

b. Can you tell what *P* is if the value is rounded to the nearest multiple of 10? Explain your reasoning.

3 Pre-unit

Find the value of each expression. Show your thinking. Organize it so it can be followed by others.

a. $523 + 278$

b. $418 - 235$

4 Pre-unit

Here are three numbers: 265, 652, and 526. For each question, explain your reasoning.

a. Does the digit 6 have a greater value in 265 or 652?

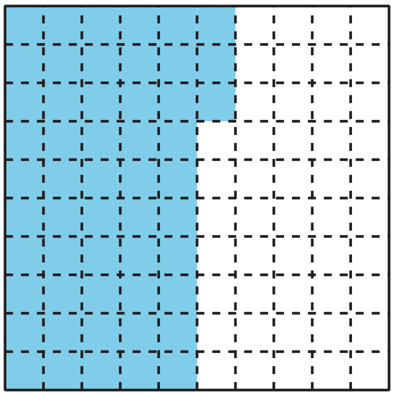
b. Does the digit 5 have a greater value in 265 or 652?

c. In which number does the digit 2 have the greatest value? In which number does the digit 2 have the least value?

5 from Unit 4, Lesson 1

Each large square represents 1.

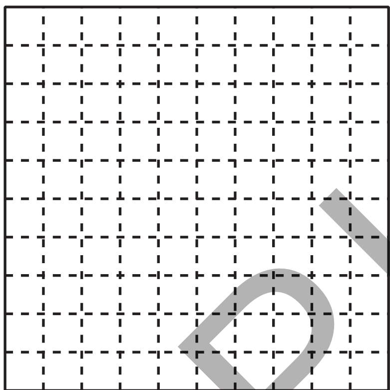
a. Write the number that represents the shaded part of the large square as a fraction and in decimal notation.

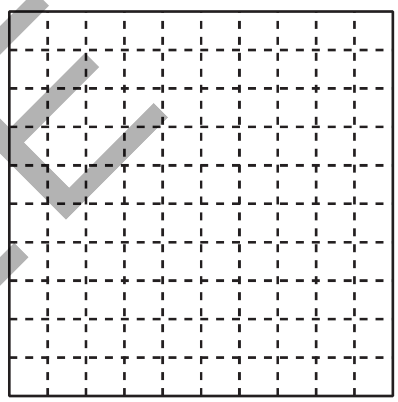


Fraction: _________

Decimal Notation: _________

b. Shade part of each square to represent the given number. Then write the number in the format that is not given.





Fraction: $\frac{13}{100}$

Decimal Notation: _________

Fraction: _________

Decimal Notation: 0.44

6 from Unit 4, Lesson 2

Select **all** the numbers equivalent to $\frac{2}{10}$.

A. 0.5

B. 0.2

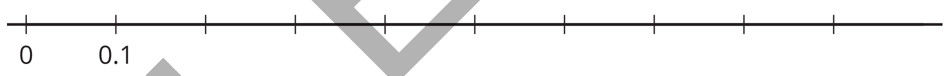
C. $\frac{20}{100}$

D. $\frac{25}{100}$

E. 0.20

7 from Unit 4, Lesson 3

a. Locate and label 0.6 and 0.35 on the number line.



b. Compare 0.6 and 0.35 using < or >.

8 from Unit 4, Lesson 4

Order the numbers from least to greatest.

5.90 9.05 5.95 0.59 5.59

Sec A

9 from Unit 4, Lesson 5

Order the numbers from least to greatest.

$\frac{13}{10}$ 1.25 1.46 $\frac{7}{5}$ $\frac{155}{100}$

10 Exploration

The table shows the distances, in miles, some students walked during the school week.

Order the numbers from least to greatest.

student	distance (miles)
Han	$5\frac{3}{4}$
Tyler	$5\frac{7}{8}$
Mai	5.95
Elena	$5\frac{8}{10}$
Andre	5.79

11 Exploration

In an earlier lesson, you learned about the lengths of the jumps made by Carl Lewis and other athletes.

Create and label a number line to show the distances of all 10 jumps made by the athletes.

Unit 4, Lesson 6

Addressing 4.NBT.1; practicing MP1

How Much is 10,000?

Let’s represent 10,000.

Warm-up

What Do You Know about 1,000?

What do you know about 1,000?

Activity 1

Build Numbers

1. Use 2 cards to make a two-digit number. Name the number and build it with base-ten blocks.

2. Use a third card to make a three-digit number. Name the number and build it with base-ten blocks.

3. Use a fourth card to make a four-digit number. Name the number and build it.

   If you don't have enough blocks, describe what you would need to build the number.

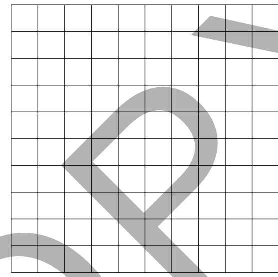
4. Your teacher will give you 1 more digit card. Use the last card from your teacher to make a five-digit number. Make the card the first digit. Name the number and build it.

   If you don't have enough blocks, describe what blocks you would need to build the number.

Activity 2

What is 10,000?

Your teacher will give you a set of 10-by-10 grids.



Sec B

1. Use the grids to represent each of the following numbers. Then describe or draw a sketch of your representation.

   a. 800

   b. 1,000

   c. 1,500

   d. 2,000

2. How many 10-by-10 grids would you need to represent each of the following numbers? Explain or draw a sketch to show your reasoning.

   a. 3,000

   b. 6,400

   c. 9,000

   d. 9,900

3. Draw a sketch to represent 10,000 using 10-by-10 grids. Be sure to clearly label each group of 1,000 in the sketch.

Unit 4, Lesson 7

Addressing 4.NBT.2; practicing MP7

Numbers Within 100,000

Let's read, write, and represent multi-digit numbers.

Sec B

Activity 1

Count and Write Numbers

Record each count in the given spaces. The first number has been recorded for you.

1. Count by 1,000

   5,000 , ____________, ____________, ____________, ____________, ____________

2. Count by 100

   9,500 , ____________, ____________, ____________, ____________, ____________

3. Count by 10

   9,950 , ____________, ____________, ____________, ____________, ____________

4. Count by 1

   9,995, ____________, ____________, ____________, ____________, ____________

5. Complete each statement:

   a. Ten thousand is 1 more than ____________.

   b. Ten thousand is 1,000 more than ____________.

   c. Ten thousand is 10 more than ____________.

   d. Ten thousand is 100 more than ____________.

Activity 2

Many Thousands

1. Complete the table to show how many thousands are in each number. In the last row, write your own five-digit number.

number	number of thousands	name in words	
10,000	10	ten thousand	
20,000			
90,000			
11,000			
27,000			
98,000			

2. With your partner, name each number in words. Leave the last column blank for now.

3. In the top (header) row of the last column, write "number of ten thousands." Complete the table to show how many ten thousands are in each number.

4. Here are 4 numbers.

   20,500   51,300   82,050   5,970

   a. Which number has a 5 in the thousands place?

   b. Which number has a 5 in the ten-thousands place?

Unit 4, Lesson 8

Addressing 4.NBT.1, 4.NBT.2; practicing MP6 and MP8

Beyond 100,000

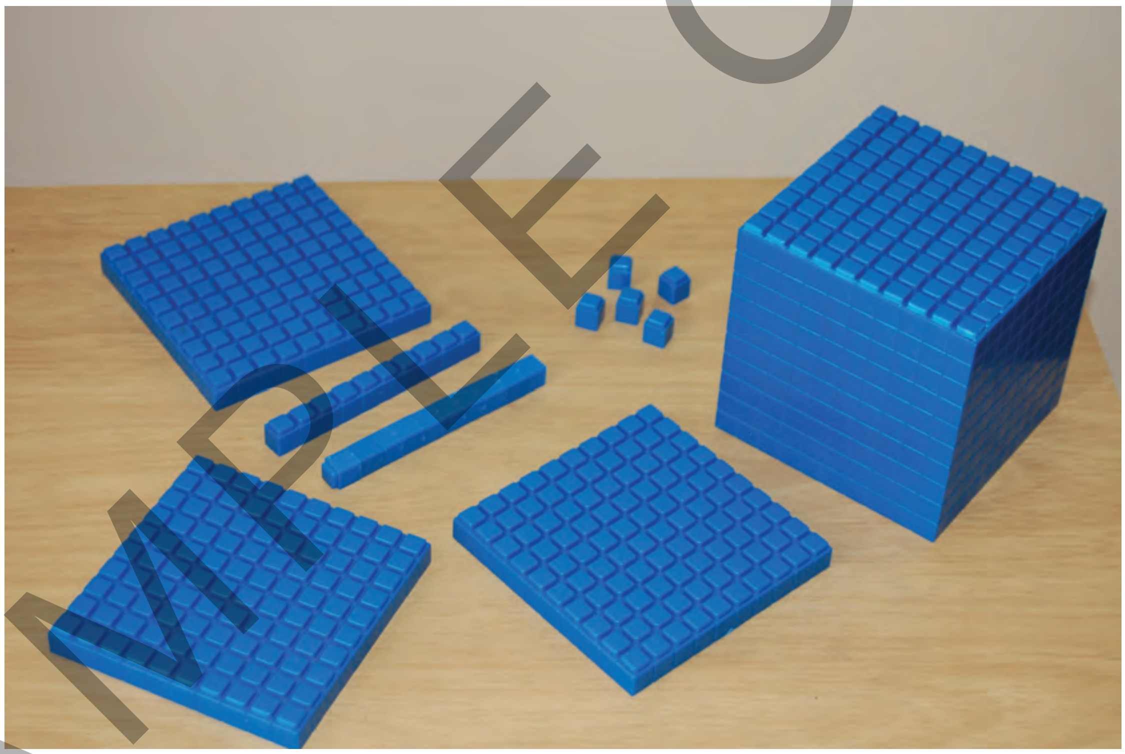
Let's read, write, and represent numbers beyond 100,000.

Sec B

Warm-up

How Many Do You See: Base-ten Blocks

How many do you see? How do you see them?



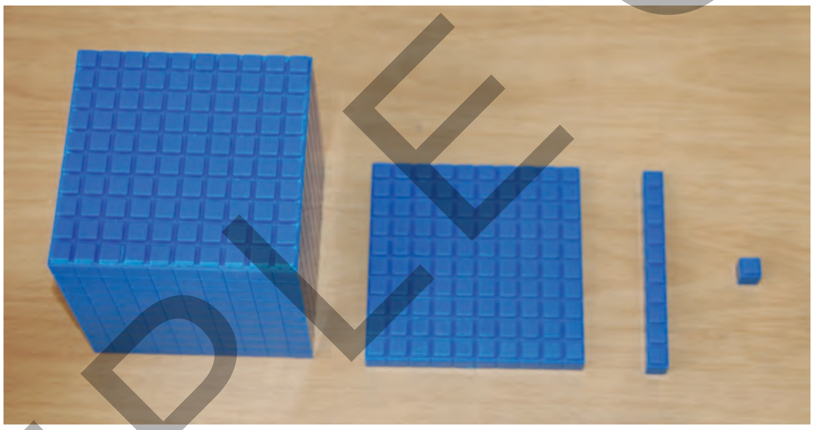
Activity 1

Lin's Representation

1. Use base-ten blocks or draw a base-ten diagram to represent 15,710.

Sec B

2. Lin uses blocks like these to represent 15,710. She decides to change the value of the small cube to represent 10.



What is the value of each block if the value of the small cube is 10?

a. Small cube: 10

b. Long rectangular block: __________

c. Large square block: __________

d. Large cube: __________

3. Use Lin's strategy to represent 15,710.

4. Use Lin's strategy to represent each number.

   a. 23,000

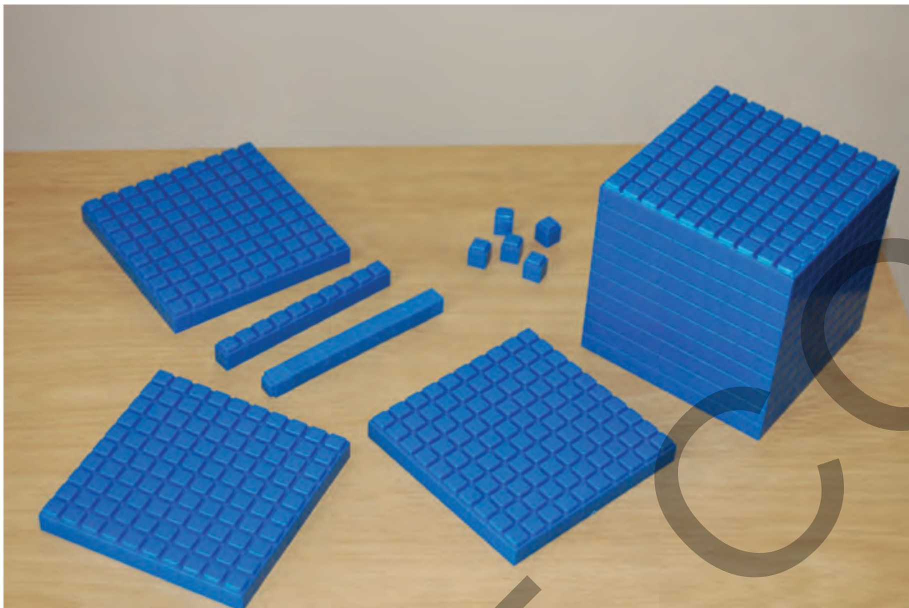
   b. 58,100

   c. 69,470

5. Which base-ten blocks would you use to represent 100,000?

Activity 2

What Number is Represented?



1. A small cube represents 1. What value do the blocks in the picture represent?

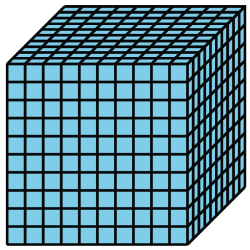
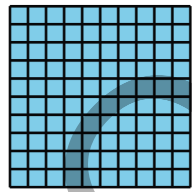
2. A small cube is now worth 10. What is the new value the blocks in the picture represent?

3. Write two statements comparing the numbers used in your base-ten representations.

Activity 3

Build Hundred-thousands

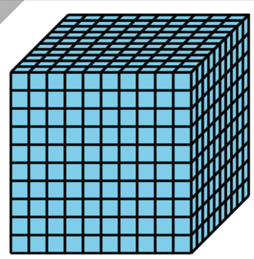
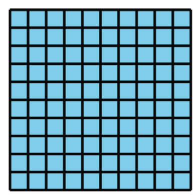
1. Lin changes the value of the small cube to 10 to represent large numbers. She uses these blocks to represent her first number.

type of block				
number of blocks used	4	9	8	3

   a. What number did Lin represent? Explain or show your reasoning.

   b. Write an equation to represent the value of the blocks.

2. She used more blocks to represent another number.

type of block				
number of blocks used	10	20	4	5

   a. What number does Lin represent? Explain or show your reasoning.

   b. Write an equation to represent the value of the blocks.

Unit 4, Lesson 9

Addressing 4.NBT.1, 4.NBT.2; practicing MP7

Same Digit, Different Value

Let's describe the relationship between the digits in multi-digit numbers.

Sec B

Warm-up

True or False: Expanded Expressions

Decide if each statement is true or false. Explain your reasoning.

- $4{,}000 + 600 + 70{,}000 = 70{,}460$

- $900{,}000 + 20{,}000 + 3{,}000 = 920{,}000 + 3{,}000$

- $80{,}000 + 800 + 8{,}000 = 800{,}000 + 80 + 8$

Activity 1

Card Sort: Large Numbers

Your teacher will give you a set of cards that show multi-digit numbers.

1. Sort the cards into categories in a way that makes sense to you. Be ready to explain the meaning of your categories.
2. Join with another group and explain how you sorted your cards.
3. Write each number in expanded form.

   a. 4,620

   b. 46,200

   c. 462,000

4. Write the value of the 4 in each number.

5. Compare the value of the 4 in two of the numbers. Write two statements to describe what you notice about the values.

6. How is the value of the 2 in 46,200 related to the value of the 2 in 462,000?

Activity 2

Expand Large Numbers

1. Express each number in standard form, expanded form, and word form.

number	expanded form	word form
784,003		
	$50{,}000 + 9{,}000 + 300 + 60 + 1$	
		eight hundred three thousand, ninety-nine
310,060		
		nine hundred thirty-four thousand, nine hundred

2. Choose 2 numbers from the table to make this statement true:

   The 3 in ______________ is ten times the value of the 3 in ______________.

3. Explain why you chose those numbers.

____

____

4. Find 2 classmates who chose different numbers than you. Record their numbers. Take turns sharing your completed statements and explaining your reasoning.

    - The 3 in ______________ is ten times the value of the 3 in ______________.

    - The 3 in ______________ is ten times the value of the 3 in ______________.

Unit 4, Lesson 10

Addressing 4.NBT.1, 4.NBT.2, 4.NBT.4; practicing MP3

Ten Times As Much

Let's write equations to show the relationship between the digits in multi-digit numbers.

Sec B

Warm-up

Number Talk: Related Numbers

Find the value of each expression mentally.

- $650 + 75$
- $5{,}650 + 75$
- $50{,}650 + 75$
- $500{,}650 + 75$

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Activity 1

Alike but Not the Same

1. Complete the table with the value of the 8 in each number.

number	value of the 8
180,000	
108,000	
100,800	
100,080	
100,008	

2. Describe the relationship between the value of the 8 in each number.

3. Write a multiplication or division equation to represent the relationship between the values of the 8 in two different numbers in the table.

Sec B

Activity 2

More and More Money

Diego's class counts collections of play money during a math class. There are 4 types of bills: tens, hundreds, thousands, and ten thousands.

Diego finds 9 of each type of bill. He organizes each type into a stack, creating 4 stacks.

Sec B

1. How much money is in each stack of bills?
   a. 9 tens
   b. 9 hundreds
   c. 9 thousands
   d. 9 ten thousands
2. Describe the relationship between the values of each stack of bills.

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3. How is the value of the stack of thousands related to the value of the stack of ten thousands? Write an equation for this relationship.

4. Clare had 21 bills of each type. How much money is in each stack of bills Clare has?

   a. 21 tens

   b. 21 hundreds

   c. 21 thousands

   d. 21 ten thousands

5. What is the value of the 2 in each stack of bills?

6. How is the value of the 2 in the stack of thousands related to the value of 2 in the stack of ten thousands? Write an equation for this relationship.

Unit 4, Lesson 11

Addressing 4.NBT.1, 4.NBT.2; practicing MP8

Large Numbers on a Number Line

Let's locate multi-digit numbers on a number line.

Warm-up

Estimation Exploration: What Number Could This Be?

What number is represented by the point?

Record an estimate that is:

too low	about right	too high

Activity 1

Locate Large Numbers

1. Locate and label each number on the number line.

    a. 347

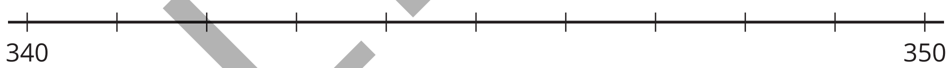
    b. 3,470

    c. 34,700

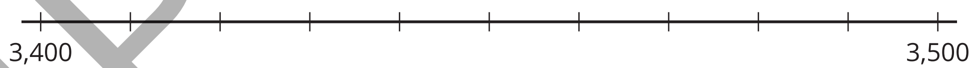
    d. 347,000

2. Locate and label each number on the number line.

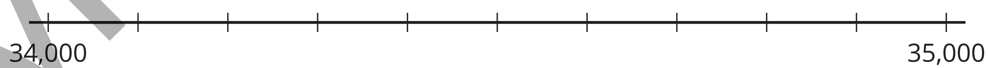
    a. 347



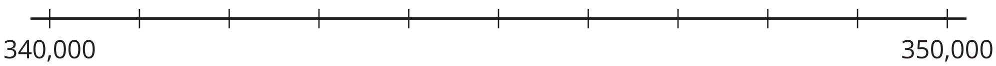
    b. 3,470



    c. 34,700



    d. 347,000



3. What do you notice about the location of these numbers on the number lines? Make 2 observations and discuss them with your partner.

Activity 2

So Many Numbers, So Little Line

Your teacher will assign a number for you to locate on the given number line.

A. 347

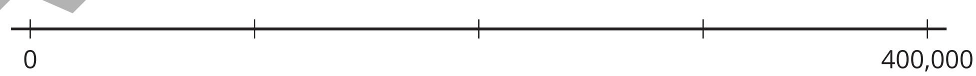
B. 3,470

C. 34,700

D. 347,000

1. Decide where your assigned number falls on this number line. Explain your reasoning.

2. Work with your group to label the tick marks and agree on where each of the numbers should be placed.



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Section B Summary

We worked with numbers to the hundred-thousands.

First, we used base-ten blocks, 10-by-10 grids, and base-ten diagrams to name, write, and represent multi-digit numbers within 1,000,000. We wrote the numbers in **expanded form** so we can see the value of each digit. Example:

$$725{,}400 = 700{,}000 + 20{,}000 + 5{,}000 + 400$$

Next, we learned the value of a digit in a multi-digit number is ten times the value of the same digit in the place to its right. Example:

- Both 14,800 and 148,000 have 4 in them.
- The 4 in 14,800 is in the thousands place. Its value is 4,000.
- The 4 in 148,000 is in the ten thousands place. Its value is 40,000.
- The value of the 4 in 148,000 is ten times the value of the 4 in 14,800.

We used both multiplication and division equations to represent this relationship.

$$10 \times 4{,}000 = 40{,}000$$

$$40{,}000 \div 10 = 4{,}000$$

Finally, we analyzed the “ten times” relationships by locating numbers on number lines.

Practice Problems

8 Problems

1 from Unit 4, Lesson 6

a. Write the name of the number 8,500 in words.

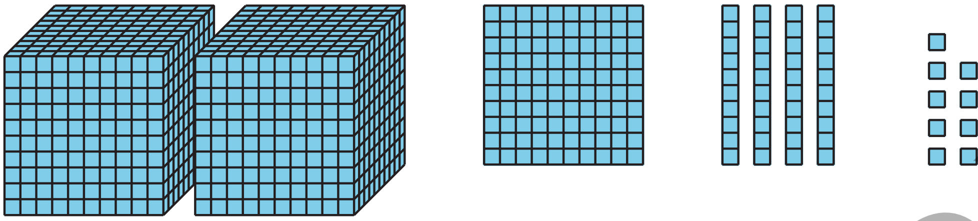
b. How many hundreds are there in 8,500? Explain how you know.

2 from Unit 4, Lesson 7

a. Count by 10,000 starting at 6,500 and stopping at 66,500. Record each number:

b. Pick 2 numbers from your list and write their names in words.

3 from Unit 4, Lesson 8



a. If each small square represents 1, what number does the picture represent?

b. If each small square represents 10, what number does the picture represent?

4 from Unit 4, Lesson 9

a. Write the names of the numbers 702,150, and 73,026 in words.

b. How is the value of the 7 in 702,150 related to the value of the 7 in 73,026?

5 from Unit 4, Lesson 10

a. What is the value of the 6 in 65,247?

b. What is the value of the 6 in 16,803?

c. Write multiplication and division equations to represent the relationship between the value of the 6 in 65,247 and the value of the 6 in 16,803.

6 from Unit 4, Lesson 11

a. Locate and label each number on the number line:

- 100,000
- 10,000
- 1,000

b. Which numbers were easiest to locate? Which were most difficult? Why?

7 Exploration

For each question, use only the digits 1, 0, 5, 9, and 3. You may not use a digit more than once and you do not need to use all the digits.

a. Can you make 3 numbers greater than 3,000 but less than 3,500?

b. Can you make 3 numbers greater than 9,000 but less than 10,000?

c. Which numbers can you make that are greater than 39,500 but less than 40,000?

8 Exploration

Estimate the value of the number labeled *A* on the number line. Explain your reasoning.

Unit 4, Lesson 12

Addressing 4.NBT.2; practicing MP6 and MP7

Compare Multi-Digit Numbers

Let's compare large numbers.

Warm-up

Which Three Go Together: Friendly Numbers

Which 3 go together?

A. 1,095

B. 3,195

C. 9,053

D. 31,095

Activity 1

Which is Greater?

Your teacher will give you a set of cards, each with a single digit, 0–9.

1. Use the cards for 2, 7, and 8 to make two different three-digit numbers. Use > or < to compare them.

2. Now include the digit 1 to make two different four-digit numbers. Compare the numbers.

3. Shuffle the cards. Repeat what you did earlier with new cards.

   a. Four-digit numbers

   b. Five-digit numbers

   c. Six-digit numbers

4. For each pair you compared, how did you decide which number is greater?

Activity 2

Incomplete Numbers

1. Here are 2 numbers. In both numbers, the unknown digit is the same.

$\boxed{}\boxed{1}\boxed{7}$ $\qquad$ $\boxed{}\boxed{6}\boxed{2}$

   - Han says the numbers can't be compared because they are incomplete.
   - Clare says the second number is greater, no matter what the unknown digit is.

   Do you agree with either one of them? Explain your reasoning.

2. Here are some pairs of numbers. The numbers in each pair have the same unknown digit. Can you tell which number is greater? Explain your reasoning.

   a. $\boxed{4}\boxed{9}\boxed{}$
   $\boxed{3}\boxed{}\boxed{9}$

   b. $\boxed{1},\boxed{}\boxed{7}\boxed{2}$
   $\boxed{1},\boxed{}\boxed{8}\boxed{5}$

   c. $\boxed{8},\boxed{}\boxed{1}\boxed{6}$
   $\boxed{5},\boxed{8}\boxed{}\boxed{2}$

Sec C

d. $\boxed{2}\,\boxed{7}\,,\,\boxed{}\,\boxed{9}\,\boxed{5}$
$\boxed{2}\,\boxed{}\,,\,\boxed{7}\,\boxed{4}\,\boxed{5}$

e. $\boxed{}\,\boxed{9}\,\boxed{0}\,,\,\boxed{1}\,\boxed{6}\,\boxed{5}$
$\boxed{9}\,\boxed{}\,\boxed{0}\,,\,\boxed{0}\,\boxed{6}\,\boxed{4}$

Activity 3

Is It Possible?

1. Each of the following pairs of numbers has the same unknown digit but in different places.

   Your teacher will assign a digit to you. Use it as the unknown digit and decide if each comparison statement is true.

   a. $\boxed{}\,,\boxed{9}\boxed{9}\boxed{9} > \boxed{}\,,\boxed{5}\boxed{0}\boxed{0}$

   b. $\boxed{1}\boxed{5}\,,\boxed{2}\boxed{}\boxed{0} > \boxed{1}\boxed{5}\,,\boxed{}\boxed{0}\boxed{2}$

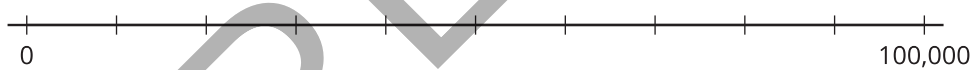
   c. $\boxed{4}\boxed{}\,,\boxed{7}\boxed{0}\boxed{0} < \boxed{7}\boxed{}\,,\boxed{4}\boxed{0}\boxed{0}$

   d. $\boxed{1}\boxed{}\boxed{5}\,,\boxed{0}\boxed{0}\boxed{0} > \boxed{5}\boxed{}\boxed{1}\,,\boxed{0}\boxed{0}\boxed{0}$

2. Here are 2 numbers, each with the same unknown digit.

   $\boxed{4}\boxed{}\,,\boxed{3}\boxed{0}\boxed{0} \qquad \boxed{3}\boxed{}\,,\boxed{4}\boxed{0}\boxed{0}$

   Choose a digit to complete the numbers and show where they would be on the number line.



3. Is it possible to fill in the 2 blanks with the same digit to make each statement true? If you think so, give at least one example of what the digits could be. If not, explain why it is not possible.

   a. $\boxed{4}\boxed{}\,,\boxed{3}\boxed{0}\boxed{0}$ is less than $\boxed{3}\boxed{}\,,\boxed{4}\boxed{0}\boxed{0}$

   b. $\boxed{}\boxed{4}\,,\boxed{3}\boxed{0}\boxed{0}$ is less than $\boxed{}\boxed{3}\,,\boxed{4}\boxed{0}\boxed{0}$.

Unit 4, Lesson 13

Addressing 4.NBT.2; practicing MP3 and MP6

Order Multi-Digit Numbers

Let's put some multi-digit numbers in order.

Warm-up

True or False: Decomposed Numbers

Decide if each statement is true or false. Explain your reasoning.

- $1{,}923 = 1 + 90 + 200 + 3{,}000$
- $1{,}923 = 1{,}000 + 90 + 20 + 3$
- $19{,}203 = 10{,}000 + 9{,}000 + 200 + 3$
- $190{,}023 = 10{,}000 + 90{,}000 + 20 + 3$

Activity 1

Ways to Compare

1. Tyler compares large numbers by looking at the first digit from the left.

   He says, "The greater the first digit, the greater the number. If the first digit is the same, then we compare the second digit."

   In each of these pairs of numbers, is the number with the greater first digit also the greater number?

   a. 985,248 and 320,097

   b. 72,050 and 64,830

   c. 320,097 and 58,978

   d. 54,000 and 587,000

   e. 58,978 and 547,612

   f. 146,001 and 1,483

2. Does Tyler's strategy work for comparing any pair of numbers? Explain your reasoning.

3. How would you compare large numbers? Describe your strategy for comparing 54,000 and 587,000.

4. Use your strategy to order these numbers from least to greatest.

   a. 87,696   847,040   84,381

   b. 63,591   630,951   63,951   631,051

Activity 2

Video Game Scores

Mai and her friends had a video game tournament one weekend.

Here are the scores at the end of the tournament.

player	score
Mai	93,005
Priya	101,012
Kiran	90,298
Noah	90,056
Clare	98,032
Elena	89,100
Andre	--

1. Rank the scores from highest to lowest. Who is in first place?

2. Andre's score was accidentally deleted but everyone agreed he is in second place. Could Andre's score be a six-digit number?

   Describe what Andre's score could be and give a couple of examples.

Addressing 4.NBT.2, 4.NBT.3; practicing MP5

Multiples of 10,000 and 100,000

Let's explore multiples of 1,000, 10,000, and 100,000 and how other numbers relate to them.

Activity 1

On Which Line Do They Belong?

Your teacher will assign a set of numbers to you.

A	140,261	100,025	486,840	676,850
B	450,099	414,500	128,201	379,900
C	158,002	42,326	99,982	428,950
D	194,030	658,340	541,700	621,035
E	215,300	499,600	608,720	644,700

1. Several number lines are posted around the room. Work with your group to decide on which number line each number should go.

   Then estimate the location of the number on the line, put a dot sticker to mark it, and label it with the number.

2. Look at the number line that represents 0 to 100,000 and has 2 points on it.

   a. Name 2 multiples of 10,000 that are closest to each point.

   b. Of the 2 multiples of 10,000 you named, which one is the nearest to each point?

Activity 2

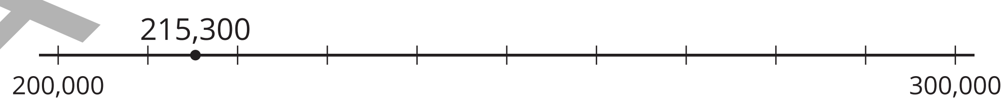
Closer to Some Multiple

Use the number line that represents the numbers between 100,000 and 200,000 for this activity.

1. Name the multiple of 10,000 that is the nearest to each number. (Leave the last column blank for now.)

number	nearest multiple of 10,000	
100,025		
128,201		
140,261		
158,002		
194,030		

2. Here is the number line that shows 215,300. Which multiple of 100,000 is the nearest to 215,300?



3. Label the last column in the table “nearest multiple of 100,000.” Then name the nearest multiple of 100,000 for each number in the table.

Unit 4, Lesson 15

Addressing 4.NBT.3; practicing MP2

The Nearest Multiples of 1,000, 10,000, and 100,000

Let’s find multiples of 1 thousand, 10 thousand, and 100 thousand that are the nearest to a number.

Warm-up

Estimation Exploration: What Could It Be?

What number could this point represent?

Record an estimate that is:

too low	about right	too high

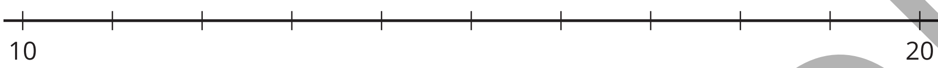
Sec C

Activity 1

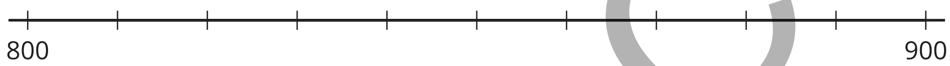
Closer to This or That?

1. Answer each question. Use the number lines if they are helpful.

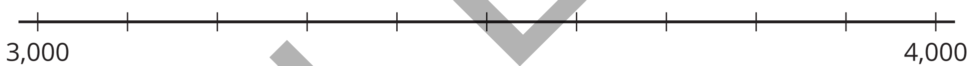
   a. Is 16 closer to 10 or to 20?



   b. Is 816 closer to 800 or to 900?

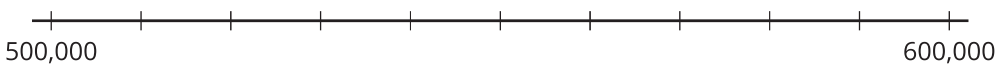


   c. Is 3,816 closer to 3,000 or 4,000?



   d. Is 73,816 closer to 70,000 or 80,000?

   e. Is 573,816 closer to 500,000 or 600,000?



2. For 816:

    - The nearest multiple of 1,000 is 1,000.
    - The nearest multiple of 100 is 800.
    - The nearest multiple of 10 is 820.

    Complete the table with the nearest multiple of 10, 100, 1,000, 10,000, and 100,000 for each number.

nearest multiple of ...	10	100	1,000	10,000	100,000
16		--	--	--	--
816	820	800	1,000	--	--
3,816				--	--
73,816					
573,816					

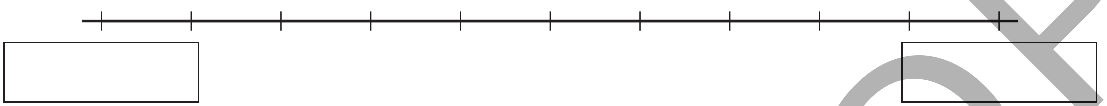
Sec C

Activity 2

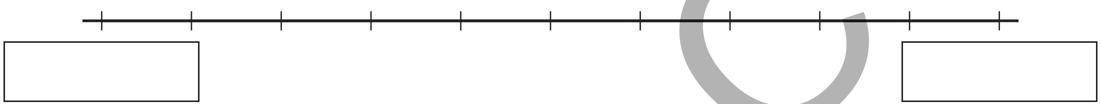
Closer to Which Number?

1. Answer each question. Label and use the number lines if they are helpful.

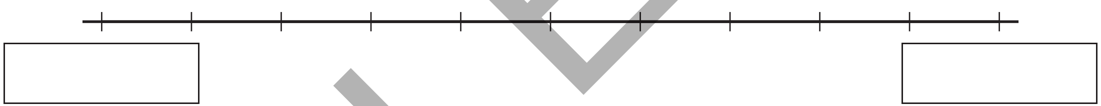
   a. Is 425,193 closer to 400,000 or 500,000?



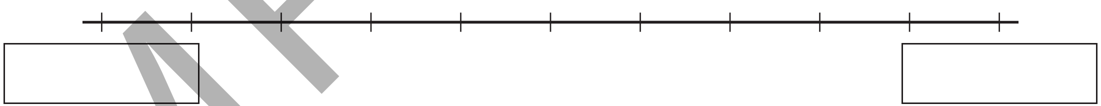
   b. Is 425,193 closer to 420,000 or 430,000?



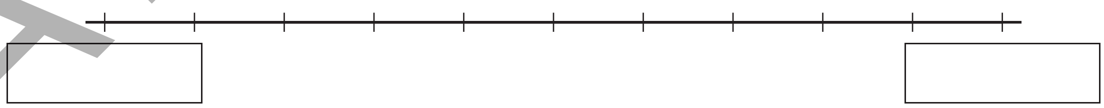
   c. Is 425,193 closer to 425,000 or 426,000?



   d. Is 425,193 closer to 425,100 or to 425,200?



   e. Is 425,193 closer to 425,190 or to 425,200?



2. For the number 425,193:

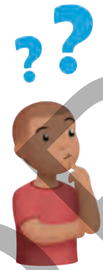
   - The nearest multiple of 100,000 is ______________.
   - The nearest multiple of 10,000 is ______________.
   - The nearest multiple of 1,000 is ______________.
   - The nearest multiple of 100 is ______________.
   - The nearest multiple of 10 is ______________.

Activity 3

What's the Nearest Multiple?

1. For the number 136,850, Han can name the nearest multiple of 100,000, 10,000, and 1,000.

   He is stuck when trying to name the nearest multiple of 100.



nearest multiple of . . .	100,000	10,000	1,000
136,850			

   a. In the table, write the nearest multiples that Han knows for each place value. Use number lines if they are helpful.

   b. Why might it be tricky to name the nearest multiple of 100 for 136,850? What do you think it is?

2. Name the nearest multiples of 100,000, 10,000, 1,000, and 100 for each number.

nearest multiple of . . .	100,000	10,000	1,000	100
191,530				
70,500				

Unit 4, Lesson 16

Addressing 4.NBT.3; practicing MP5

Round Numbers

Let's round some large numbers.

Warm-up

Number Talk: Missing Numbers

Find the value that makes each equation true mentally.

- $421 + \underline{\hspace{3cm}} = 500$
- $421 + \underline{\hspace{3cm}} = 1{,}000$
- $6{,}421 + \underline{\hspace{3cm}} = 7{,}000$
- $6{,}421 + \underline{\hspace{3cm}} = 10{,}000$

Activity 1

Round to What?

Noah says that 489,231 can be rounded to 500,000.

Priya says that it can be rounded to 490,000.

1. Explain or show why both Noah and Priya are correct. Use a number line if it helps.

Sec C

2. Describe all the numbers that round to 500,000 when rounded to the nearest hundred thousand.

3. Describe all the numbers that round to 490,000 when rounded to the nearest ten thousand.

4. Name 2 other numbers that can also be rounded to both 500,000 and 490,000.

Activity 2

Some Numbers to Round

Your teacher will show you 6 numbers. Choose at least 3 numbers and round each to the nearest hundred thousand, ten thousand, thousand, and hundred.

Record your work in the table. Use a number line if it is helpful.

round to the nearest . . .	100,000	10,000	1,000	100
53,487				
4,896				
370,130				
96,500				
985,411				
7,150				

Sec C

Activity 3

Rounded Populations

The table shows the estimated populations of two cities in the States—based on surveys in 2018.

city	population	rounded to the nearest 1,000,000	rounded to the nearest 100,000	rounded to the nearest 10,000
Austin, TX	964,254			960,000
Lincoln, NE	287,401			
		1,000,000		900,000
		1,000,000	900,000	
		0	500,000	

Here are 3 other cities and their estimated populations.

- Charlotte, NC: 872,498
- Jacksonville, FL: 903,889
- Virginia Beach, VA: 450,189

1. Match each of the 3 cities with the rounded population in the table.
2. The table shows 3 ways of rounding large numbers.
   a. Which ways of rounding are more helpful in finding how many people are in each of these cities?
   b. Which ways of rounding are more helpful in comparing or ordering the populations by size? Less helpful?

Unit 4, Lesson 17

Addressing 4.NBT.3; practicing MP2 and MP7

Apply Rounding

Let's round large numbers to learn about situations and solve problems.

Warm-up

Notice and Wonder: Plane Altitudes

What do you notice? What do you wonder?

plane	altitude (feet)
WN11	35,625
SK51	28,999
VT35	15,450
BQ64	36,000
AL16	31,000
AB25	35,175
CL48	16,600
WN90	30,775
NM44	30,245

Activity 1

Apart in the Air

1. Altitude is the vertical distance from sea level. Here are the altitudes of 10 planes.

plane	altitude (feet)	
WN11	35,625	
SK51	28,999	
VT35	15,450	
BQ64	36,000	
AL16	31,000	
AB25	35,175	
CL48	16,600	
WN90	30,775	
NM44	30,245	

Which planes are flying at about 30,000 feet? Explain or show your reasoning.

2. Planes flying over the same area need to stay at least 1,000 feet apart in altitude.

   Mai says that one way to tell if planes are too close is to round each plane's altitude to the nearest thousand. Do you agree that this is a reliable strategy?

   In the last column, round each altitude to the nearest thousand. Use the rounded values to explain why or why not.

Activity 2

Safe or Unsafe?

Use the altitude data table from earlier for the following problems.

1. Look at the column showing exact altitudes.
   a. Find 2 or more numbers that are within 1,000 feet of one another. Mark them with a circle or a color.
   b. Find another pair of numbers that are within 1,000 feet of one another. Mark them with a square or a different color.
   c. Based on what you just did, which planes are too close to one another?

2. Repeat what you just did with the rounded numbers in the last column. Use these numbers to find which planes are too close to one another.

3. Which set of altitude data should air traffic controllers use to keep airplanes safe while in the air? Explain your reasoning.

4. Are there better ways to round these altitudes, or should you not round at all? Explain or show your reasoning.

Activity 3

No Phone Zone?

In some countries, cell phone use is allowed on a flight only when the plane is at a certain altitude, usually around 40,000 feet.

Here are 6 planes and their altitudes.

plane	altitude (feet)
A	40,990
B	39,524
C	36,138
D	40,201
E	35,472
F	30,956

Jada says the passengers in all planes except for Plane F can use their phones.

Elena says only those in B and D can do so.

Do you agree with either of them? Explain your reasoning.

Section C Summary

We learned to compare, order, and **round** numbers up to 1,000,000.

We started by using what we know about place value to compare large whole numbers. For instance, we know 45,892 is less than 407,892 because the 4 in 45,892 represents 4 ten thousands and the 4 in 407,892 represents 4 hundred thousands.

Next, we found multiples of 1,000, 10,000, and 100,000 that are closest to given numbers—at first with the help of number lines, and later without. For example, for 407,892, we know that:

- 408,000 is the nearest multiple of 1,000.
- 410,000 is the nearest multiple of 10,000.
- 400,000 is the nearest multiple of 100,000.

Finally, we used what we know about finding nearest multiples to round large numbers to the nearest thousand, ten thousand, and hundred thousand.

Practice Problems

9 Problems

1 from Unit 4, Lesson 12

Jada writes the same digit in the 2 blanks to make the statement true. Which digits could she write?

$$\boxed{6}\boxed{}, \boxed{4}\boxed{3}\boxed{2} < \boxed{6}\boxed{5}, \boxed{}\boxed{9}\boxed{8}$$

2 from Unit 4, Lesson 13

a. Order these numbers from least to greatest:

98,107 102,356 752,031 88,207 99,653

b. How did you pick the smallest number? Explain your reasoning.

3 from Unit 4, Lesson 14

a. Which multiple of 10,000 is closest to 132,256?

b. Which multiple of 100,000 is closest to 132,256?

c. Which multiple of 100,000 is closest to the number labeled *A*?

4 from Unit 4, Lesson 15

For the number 583,642:

a. What is the nearest multiple of 100,000?

b. What is the nearest multiple of 10,000?

c. What is the nearest multiple of 1,000?

d. What is the nearest multiple of 100?

e. What is the nearest multiple of 10?

5 from Unit 4, Lesson 16

a. Describe the numbers that are 460,000 when rounded to the nearest ten thousand.

b. Where are these numbers located on the number line?

6 from Unit 4, Lesson 17

When rounded to the nearest thousand, Airplane X is flying at 30,000 feet, Airplane Y at 31,000 feet, and Airplane Z at 32,000 feet.

a. Could Airplanes X and Y be within 1,000 feet of each other? If you think so, give some examples. If you don't think so, explain why not.

b. Explain why Airplanes X and Z could not be within 1,000 feet of each other. Use a number line if you find it helpful.

7 Exploration

Rounded to the nearest 10 pounds, 1 bag of sand weighs 50 pounds.

Jada wants at least 1,000 pounds of sand for a sandbox. How many bags of sand does Jada need to buy to be sure that she has enough sand?

8 Exploration

You will need a set of digit cards 0–9 for this exploration.

Place the cards facedown in a stack. Turn over 6 digit cards.

Can you put the 6 digits in the blanks so that all three statements are true?

a. $\boxed{4}, \boxed{}\boxed{2}\boxed{3} > \boxed{}, \boxed{9}\boxed{7}\boxed{8}$

b. $\boxed{}\boxed{2}, \boxed{4}\boxed{0}\boxed{3} > \boxed{4}\boxed{2}, \boxed{}\boxed{0}\boxed{1}$

c. $\boxed{4}\boxed{3}\boxed{}, \boxed{2}\boxed{5}\boxed{7} > \boxed{4}\boxed{}\boxed{5}, \boxed{9}\boxed{3}\boxed{7}$

Sec C

9 Exploration

Round to the nearest ten, hundred, thousand, and ten thousand to answer these riddles. Use a number line if it is helpful.

a. I can be rounded to 100 or to 140. What number could I be?

b. I can be rounded to 7,500 or to 8,000. What number could I be?

c. I can be rounded to 60,000 or to 57,000. What number could I be?

Unit 4, Lesson 18

Addressing 4.NBT.4; practicing MP8

Standard Algorithm to Add and Subtract

Let's find sums and differences of large numbers.

Warm-up

Estimation Exploration: What's the Difference?

Estimate the difference: $42{,}050 - 3{,}790$.

Sec D

Record an estimate that is:

too low	about right	too high

Activity 1

Weekly Steps

A teacher uses an app on her cell phone to track her physical activity. Here is the data on the number of steps over 5 school days.

Monday

Steps
6,285 steps

Tuesday

Steps
9,312 steps

Wednesday

Steps
9,587 steps

Thursday

Steps
7,403 steps

Friday

Steps
8,169 steps

For each question, show your reasoning.

1. On which 2 days did she take the most steps? Over those 2 days, how many steps did she take altogether?

2. What is the difference in the number of steps she took on her most active day and on her least active day?

3. Between Wednesday and Thursday, her activity level dropped. How many fewer steps did she take on Thursday than Wednesday?

Activity 2

Steps During the Weekend

The teacher also keeps track of the number of steps she took during the weekend. The data from Saturday and Sunday of that same week are shown.

Saturday

Steps
17,375 steps

Sunday

Steps
14,024 steps

Here are 2 strategies to compute the total number of steps she took over the weekend.

Strategy A

$$\begin{array}{rrrrrrrrrrrr} & 10{,}000 & + & 7{,}000 & + & 300 & + & 70 & + & 5 & & \\ + & 10{,}000 & + & 4{,}000 & + & 0 & + & 20 & + & 4 & & \\ \hline & 20{,}000 & + & 11{,}000 & + & 300 & + & 90 & + & 9 & = & 31{,}399 \end{array}$$

Strategy B

$$\begin{array}{rccccc} & \overset{1}{1} & 7, & 3 & 7 & 5 \\ + & 1 & 4, & 0 & 2 & 4 \\ \hline & 3 & 1, & 3 & 9 & 9 \end{array}$$

1. Analyze the strategies. Discuss with your partner:
   - What is happening in each strategy?
   - How are they alike? How are they different?

2. Use both strategies to find the difference between the number of steps the teacher took on Saturday and on Sunday.

Sec D

3. The following week, the teacher took 26,815 steps during the weekdays and 11,403 steps during the weekend. Use both strategies to find the total number of steps she took that week.

Unit 4, Lesson 19

Addressing 4.NBT.4, 4.NF.3.c; practicing MP3

Compose and Decompose to Add and Subtract

Let's compose and decompose units to add and subtract.

Warm-up

Number Talk: Subtract Fractions

Find the value of each expression mentally.

- $2\frac{3}{4} - 1\frac{1}{4}$
- $1\frac{1}{4} - \frac{3}{4}$
- $5\frac{1}{8} - 2\frac{3}{8}$
- $3\frac{2}{10} - 2\frac{7}{10}$

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Sec D

Activity 1

Find and Check Sums

1. Find the value of each sum.

a

$$\begin{array}{r} 8{,}299 \\ +\quad 1 \\ \hline \end{array}$$

b

$$\begin{array}{r} 8{,}299 \\ +\quad 11 \\ \hline \end{array}$$

c

$$\begin{array}{r} 8{,}299 \\ +\quad 111 \\ \hline \end{array}$$

d

$$\begin{array}{r} 8{,}299 \\ +\ 1{,}111 \\ \hline \end{array}$$

2. Use the expanded form of both 8,299 and 1,111 to check the value you found for the last sum.

Sec D

3. Each computation has at least one error. Find the errors and show the correct calculation.

a

$$\begin{array}{r} 1 \\ 12{,}309 \\ +\ \ 4{,}567 \\ \hline 16{,}866 \end{array}$$

b

$$\begin{array}{r} 111 \\ 32{,}216 \\ +\ \ \ \ 789 \\ \hline 33{,}995 \end{array}$$

c

$$\begin{array}{r} 1\phantom{0{,}000} \\ 642{,}999 \\ +\ \ 48{,}111 \\ \hline 690{,}000 \end{array}$$

d

$$\begin{array}{r} 111 \\ 18{,}999 \\ +\ \ 9{,}999 \\ \hline 27{,}998 \end{array}$$

Activity 2

Priya's Family Heirlooms

Priya's mom wore an heirloom bracelet at her wedding in 1996. The bracelet was made in 1947.

Priya subtracts to find out how old the bracelet was when her parents were married.

$$\begin{array}{r} \,8\ 16 \\ 1,9\,\cancel{9}\,\cancel{6} \\ -\ 1,9\,4\,7 \\ \hline 4\,9 \end{array}$$

Priya learns that her grandmother also wore the bracelet at her wedding 24 years earlier.

Priya subtracts to find out when her grandparents were married.

$$\begin{array}{r} 1,9\,9\,6 \\ -\ 2\,4 \\ \hline 1,9\,7\,2 \end{array}$$

1. Are both calculations correct? Why does one calculation have some numbers crossed out and some new numbers, but the other one does not? Explain your reasoning.

2. Priya's grandmother wore an heirloom necklace and earring set that was 63 years old when she was married in 1972.

a. Does Priya need to decompose a unit if she uses the standard algorithm to subtract $1972 - 63$? Explain your reasoning.

b. Use the standard algorithm to subtract $1972 - 63$. Find the year the necklace was made.

3. Create a subtraction problem that doesn't require decomposing a unit to subtract. Then solve the problem.

$$\begin{array}{r} \square,\square\square\square \\ \square\square\square \\ \hline \end{array}$$

Sec D

Unit 4, Lesson 20

Addressing 4.NBT.4; practicing MP3

Add and Subtract within 1,000,000

Let’s use the standard algorithm to add and subtract.

Warm-up

Notice and Wonder: Subtracting Tens of Thousands

What do you notice? What do you wonder?

A

$$\begin{array}{rrrrrr} & & & & 13 & \\ & & & 10 & \cancel{3} & 8 \\ & 7 & 5, & \cancel{9} & \cancel{4} & \cancel{0} \\ - & 1 & 2, & 7 & 8 & 6 \\ \hline \end{array}$$

B

$$\begin{array}{rrcrcrcrcrcr} & & & & & & & 130 & & \\ & & & & & 800 & & \cancel{30} & & 10 \\ & 70{,}000 & + & 5{,}000 & + & \cancel{900} & + & \cancel{40} & + & \cancel{0} \\ - & 10{,}000 & + & 2{,}000 & + & 700 & + & 80 & + & 6 \\ \hline & 60{,}000 & + & 3{,}000 & + & 100 & + & 50 & + & 4 \end{array}$$

Sec D

Activity 1

Add and Subtract Large Numbers

1. Use the standard algorithm to find the value of each sum and difference. If you get stuck, try writing the numbers in expanded form.

   a. $7{,}106 + 2{,}835$

   b. $8{,}179 - 3{,}599$

   c. $142{,}571 + 10{,}909$

d. $268{,}322 - 72{,}145$

2. Find the unknown value that would make each computation true.

a

$$\begin{array}{r} 67{,}182 \\ + \\ \hline 129{,}400 \end{array}$$

b

$$\begin{array}{r} 234{,}650 \\ - \\ \hline 193{,}710 \end{array}$$

Sec D

Activity 2

Spot Errors

1. Kiran tries to find the sum of 204,500 and 695. He isn't sure how to set up the calculation, so he writes 2 ideas. Which way is correct? Explain your reasoning.

A

$$
\begin{array}{r}
20{,}450 \\
+\ 69{,}5 \\
\hline
89{,}950
\end{array}
$$

B

$$
\begin{array}{r}
1 \\
204{,}500 \\
+\ 695 \\
\hline
205{,}195
\end{array}
$$

2. Lin makes some errors when subtracting 4,325 from 61,870. Identify as many errors as you can find. Then show the correct way to subtract.

$$
\begin{array}{r}
1010 \\
6\cancel{1}{,}87\cancel{0} \\
-\ 4{,}325 \\
\hline
66{,}555
\end{array}
$$

Sec D

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Unit 4, Lesson 21

Addressing 4.NBT.2, 4.NBT.4; practicing MP8

Zeros in the Standard Algorithm

Let's subtract from numbers with several zeros.

Warm-up

Which Three Go Together: Numbers with 0, 2, and 5

Which 3 go together?

A. 2,050

B. 2,055

C. 205.2

D. 20,005

Sec D

Activity 1

What If There is Nothing to Decompose?

Here are some numbers you saw earlier. Each number has at least one 0. From each number, 1,436 is being subtracted.

1. Make sense of the problems and explain to a partner.

a

$$\begin{array}{rrrrr} & 1 & 10 & 4 & 10 \\ & \cancel{2}, & \cancel{0} & \cancel{5} & \cancel{0} \\ - & 1, & 4 & 3 & 6 \\ \hline & & 6 & 1 & 4 \end{array}$$

b

$$\begin{array}{rrrrr} & 1 & 10 & 4 & 15 \\ & \cancel{2}, & \cancel{0} & \cancel{5} & \cancel{5} \\ - & 1, & 4 & 3 & 6 \\ \hline & & 6 & 1 & 9 \end{array}$$

2. Use the approach in the first problem to find these 2 differences:

a

$$\begin{array}{rrrrr} & 2, & 0 & 0 & 5 \\ - & 1, & 4 & 3 & 6 \\ \hline \end{array}$$

b

$$\begin{array}{rrrrrr} & 2 & 0, & 0 & 0 & 5 \\ - & & 1, & 4 & 3 & 6 \\ \hline \end{array}$$

3. Find the value of each difference. If you get stuck, try subtracting using the expanded form.

a

$$\begin{array}{rrrrr} & 8, & 0 & 3 & 0 \\ - & 2, & 6 & 1 & 5 \\ \hline \end{array}$$

b

$$\begin{array}{rrrrr} & 8, & 0 & 3 & 3 \\ - & 2, & 6 & 1 & 5 \\ \hline \end{array}$$

c

$$\begin{array}{rrrrr} & 8, & 0 & 0 & 3 \\ - & 2, & 6 & 1 & 5 \\ \hline \end{array}$$

d

$$\begin{array}{rrrrrr} & 8 & 0, & 0 & 0 & 3 \\ - & & 2, & 6 & 1 & 5 \\ \hline \end{array}$$

Activity 2

What is Your Age?

Jada records the birth year of some of her maternal grandparents for a family history project.

family member	birth year
grandmother	1952
grandfather	1948
great-grandmother	1930
great-grandfather	1926

As of this year, what is the age of each family member? Explain or show your reasoning. Use the standard algorithm at least once.

Sec D

Unit 4, Lesson 22

Addressing 4.NBT.4; building on 4.NBT.1, 4.NBT.2; practicing MP2

Solve Problems Involving Large Numbers

Let's solve problems by adding and subtracting.

Warm-up

True or False: Sums and Differences

Decide if each statement is true or false. Explain your reasoning.

Sec D

- $7{,}000 + 3{,}000 = 10{,}000$

- $7{,}180 + 3{,}920 = 10{,}100$

- $423{,}450 - 42{,}345 = 105$

- $400{,}000 - 99{,}999 = 311{,}111$

Sec D

Activity 1

The Fundraiser

A school's track teams raised $41,560 from fundraisers and concession sales.

In the fall, the teams paid $3,180 for uniforms, $1,425 in entry fees for track meets, and $18,790 in travel costs.

In the spring, the teams paid $10,475 in equipment replacement, $1,160 for competition expenses, and $912 for awards and trophies.

1. Is the amount collected enough to cover all the payments? Explain how you know.

2. If the amount collected is enough, how much money do the track teams have left after paying all the expenses? If it is not enough, how much money did the track teams overspend? Explain how you know.

Activity 2

The Least and the Greatest of Them All

Your teacher will give you and your partner a set of 10 cards, each with a number between 0 and 9. Place the cards facedown in a stack.

1. Draw 3 cards. Use all 3 cards to form 2 different numbers that would give:

   a. the greatest possible sum

   b. the least possible sum

   c. the greatest possible difference

   d. the least possible difference

Sec D

2. Shuffle the cards and draw 4 cards. Use them to form 2 different numbers that would give:

   a. the greatest possible sum

$$\begin{array}{r} \square,\square\square\square \\ +\ \square,\square\square\square \\ \hline \end{array}$$

   b. the least possible sum

$$\begin{array}{r} \square,\square\square\square \\ +\ \square,\square\square\square \\ \hline \end{array}$$

   c. the greatest possible difference

$$\begin{array}{r} \square,\square\square\square \\ -\ \square,\square\square\square \\ \hline \end{array}$$

   d. the least possible difference

$$\begin{array}{r} \square,\square\square\square \\ -\ \square,\square\square\square \\ \hline \end{array}$$

Section D Summary

We used our understanding of place value and expanded form to add and subtract large numbers using the **standard algorithm.**

We learned how to use the standard algorithm to keep track of addition of digits that result in a number greater than 9.

When there are 10 in a unit, we make a new unit. Then record the new unit at the top of the column of numbers in the next place to the left.

$$\begin{array}{rrrrrr} & & 1 & & & \\ & 2 & 6, & 8 & 1 & 5 \\ + & 1 & 1, & 4 & 0 & 3 \\ \hline & 3 & 8, & 2 & 1 & 8 \end{array}$$

When we subtract numbers, it may be necessary to decompose tens, hundreds, thousands or ten thousands before subtracting.

$$\begin{array}{rrrrr} & & & 8 & 16 \\ & 1, & 9 & \cancel{9} & \cancel{6} \\ - & 1, & 9 & 4 & 7 \\ \hline & & & 4 & 9 \end{array}$$

Finally, we learned if the digit we are subtracting is a zero, we may need to decompose one unit of the digit in the next place to the left.

Sometimes it is necessary to look two or more places to the left to find a unit to decompose. For example, here is one way to decompose a ten and a thousand to find $2,050 - 1,436$.

$$\begin{array}{rrrrr} & 1 & 10 & 4 & 10 \\ & \cancel{2}, & \cancel{0} & \cancel{5} & \cancel{0} \\ - & 1, & 4 & 3 & 6 \\ \hline & & 6 & 1 & 4 \end{array}$$

Sec D

Unit 4, Lesson 23

Addressing 4.NBT.1, 4.NBT.2, 4.NBT.3, 4.NBT.4; practicing MP4

Bees are Buzzing

Let's investigate insect populations.

Warm-up

Estimation Exploration: Bees

How many bees are in the image?

Record an estimate that is:

too low	about right	too high

Activity 1

Termites, Ants, and Bees

Here are some facts about insects.

Termites

- Size of a colony: 100–1,000,000
- A queen lives for 30–50 years.
- There are 3,000–3,500 species of termites.
- The length of a termite is 4 to 15 millimeters.
- In some species, the mature queen may produce around 40,000 eggs a day.

Odorous House Ants

- Size of colony: up to 100,000
- A queen lives for 300–1,800 days.
- The length of an ant is 1.5–3.2 millimeters.
- Foraging ants travel up to 700 feet from their nests.
- There are 12,000–22,000 possible species.

Honey Bees

- Size of a hive: 10,000–60,000
- There are around 500 drones in a hive.
- A queen can lay about 1,500–2,000 eggs each day.
- A hive produces 7–40 liters of honey in a season.
- The length of a bee is 10–20 millimeters.

1. Here are some numbers that could represent facts about termites, house ants, and honey bees. What might each number represent?

number	what it might represent
2.4	
8	
487	
1,794	
6,905	
20,799	
530,097	

2. Add another number to the list. What fact about the insects might this number represent?

3. Discuss your answers with your partner. Explain or show your reasoning.

Activity 2

Bee Population

An entomologist records the number of bees in their beehive over the course of several months. They record:

- The number of bees at the beginning of the month.
- How many bees left, and didn't return, during the month.
- How many new bees were added to the hive during the month.

Unfortunately, some of the entries in the table are missing.

1. Complete the missing information in the table.

month	bees in the hive at the beginning of the month	new bees	bees that left the hive
May	20,000	9,378	342
June		15,870	970
July		14,965	
August	58,107		28,980
September	30,017	No data	No data

2. Discuss your responses with your partner. Explain or show your reasoning.

Sec D

Practice Problems

7 Problems

1 from Unit 4, Lesson 18

Clare walks 11,243 steps on Saturday and 12,485 steps on Sunday.

a. How many steps does Clare take altogether on Saturday and Sunday?

b. How many more steps does Clare take on Sunday than on Saturday?

2 from Unit 4, Lesson 19

Sec D

a. Find the value of the sum. Explain your calculations.

$$\begin{array}{r} 4,\ 5\ 1\ 8 \\ +\ 2,\ 8\ 3\ 5 \\ \hline \end{array}$$

b. Find the value of the difference. Explain your calculations.

$$\begin{array}{r} 5,\ 6\ 2\ 7 \\ -\ 2,\ 1\ 3\ 4 \\ \hline \end{array}$$

3 from Unit 4, Lesson 20

Find the value of each sum and difference using the standard algorithm.

$$\begin{array}{r} 118{,}257 \\ +\ 367{,}095 \\ \hline \end{array} \qquad \begin{array}{r} 122{,}518 \\ -\ 72{,}479 \\ \hline \end{array}$$

4 from Unit 4, Lesson 21

Han finds the value of $300{,}526 - 4{,}472$.

$$\begin{array}{rcccccc} & 2 & & 10 & 4 & 12 & \\ & \cancel{3} & 0 & \cancel{0}, & \cancel{5} & \cancel{2} & 6 \\ - & & & 4, & 4 & 7 & 2 \\ \hline & 2 & 0 & 6, & 0 & 5 & 4 \end{array}$$

a. How can you tell by estimating that Han has made an error?

b. What error did Han make?

c. Find the value of $300{,}526 - 4{,}472$.

Sec D

5 from Unit 4, Lesson 22

In 2018, the population of Boston is estimated as 694,583 and the population of Seattle is estimated as 744,995.

a. Is the population difference between Boston and Seattle more or less than 100,000? Explain how you know.

b. Is the population difference more or less than 50,000? Explain how you know.

c. Find the difference in the populations of the 2 cities.

Sec D

6 Exploration

Han says he uses a method to find the value of $1{,}000{,}000 - 267{,}923$ without decomposing a unit: "I write 1,000,000 as $999{,}999 + 1$."

a. How might rewriting 1,000,000, as Han suggested, help with finding the difference of $1{,}000{,}000 - 267{,}923$?

b. Try Han's method to find $1{,}000{,}000 - 267{,}923$.

Sec D

7 Exploration

Use the information to determine when the telephone, printing press, and automobile were first invented.

- The airplane was invented in 1903.
- The printing press was invented 453 years before the most recent invention.
- The automobile was invented 15 years before 1900.
- It was 426 years after the invention of the printing press that the telephone was invented.
- The automobile and telephone were invented the closest together in time with only 9 years between them.

Glossary

- common denominator
  The same denominator in two or more fractions. Example, $\frac{1}{4}$ and $\frac{5}{4}$ have the common denominator 4.

- composite number
  A whole number with more than one factor pair.

- decimal notation
  A way to write tenths, hundredths, and other decimal fractions as numerals with digits and a decimal point. The digits to the left of the decimal point show the whole-number part of the number. The digits to the right of the decimal point show the fractional part less than 1.

  Examples:

  $\frac{3}{10}$ written in decimal notation is 0.3.

  $\frac{25}{100}$ written in decimal notation is 0.25.

  $\frac{17}{10}$ written in decimal notation is 1.7.

  $2\frac{7}{100}$ written in decimal notation is 2.07.

- denominator
  The bottom part of a fraction that tells how many equal parts the whole was partitioned into.

- equivalent fractions
  Fractions that have the same size and describe the same point on the number line. Example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

- expanded form
  A way of writing a number as a sum of the values of the digits.

Example: 482 written in expanded form is $400 + 80 + 2$.

- factor pair of a whole number
  Two whole numbers that multiply to result in that number. Example: 5 and 4 are a factor pair of 20.

- mixed number
  A number expressed as a whole number and a fraction less than 1.

- multiple of a number
  The result of multiplying that number by a whole number. Example: 18 is a multiple of 3, because it is a result of multiplying 3 by 6.

- numerator
  The top part of a fraction that tells how many of the equal parts are being described.

- prime number
  A whole number that is greater than 1 and has exactly one factor pair: the number itself and 1.

- standard algorithm (for addition or subtraction)
  A set of steps used to add or subtract numbers by place value. Write the numbers vertically with digits lined up by place value. Add or subtract the digits in each place value, starting with the least place value. Compose or decompose units, as needed in each place value.

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4.G: Grade 4 – Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (Two-dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA

4.G.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

4.MD: Grade 4 – Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1, 12)$, $(2, 24)$, $(3, 36)$, ...

4.MD.2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3
Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Represent and interpret data.

4.MD.4
Make a line plot to display a data set of measurements in fractions of a unit $(1/2, 1/4, 1/8)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

4.MD.5.a
An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

4.MD.5.b
An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4.MD.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.NBT: Grade 4 – Number and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

4.NBT.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.

4.NBT.3
Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NF: Grade 4 – Number and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

4.NF.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, =, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3
Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

4.NF.3.a
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.3.b
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

4.NF.3.c
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3.d
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4.a
Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

4.NF.4.b
Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

4.NF.4.c
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.

4.NF.6
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, =, or $<$, and justify the conclusions, e.g., by using the number line or another visual model. CA

4.OA: Grade 4 – Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1
Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4.OA.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.