IMKH California

GRADE 4

Student Edition

UNIT 5-6

Illustrative Mathematics — LEARN MATH FOR LIFE | Kendall Hunt

Book 3
Certified by Illustrative Mathematics®

Table of Contents

GRADE 4

UNIT

5

Multiplicative Comparison and Measurement

Content Connections

In this unit you will use multiplication to compare numbers and solve problems about measurement. You will make connections by:

- **Reasoning with Data** while exploring rectangles, measuring, and plotting.
- **Exploring Changing Quantities** while working with number and shape patterns, multi-digit numbers, factors, and area models.
- **Taking Wholes Apart and Putting Parts Together** when you investigate circles, fractions, and decimals.
- **Discovering Shapes and Space** when you solve measurement problems related to circles, fractions, and decimals.

Addressing the Standards

As you work your way through **Unit 5 Multiplicative Comparison and Measurement,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 13
MP2 Reason abstractly and quantitatively.	Lessons 2, 4, 5, 9, 11, and 15
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3, 7, 10, and 11
MP4 Model with mathematics.	Lesson 18
MP5 Use appropriate tools strategically.	Lesson 1
MP6 Attend to precision.	Lessons 5, 6, 13, 14, and 17
MP7 Look for and make use of structure.	Lessons 6, 7, 8, 10, 12, and 16
MP8 Look for and express regularity in repeated reasoning.	Lesson 6

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number and Shape Patterns • Factors and Area Models • Multi-Digit Numbers • Fraction Flexibility • Circles, Fractions, and Decimals	**4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	Lessons 1, 2, 3, 4, 5, and 6
• Number and Shape Patterns • Factors and Area Models • Connected Problem Solving	**4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	Lessons 1, 2, 3, 4, 5, 6, 10, 15, 16, 17, and 18

5B

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
Connected Problem Solving	**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Lessons 6, 10, 11, 12, and 13
• Measuring and Plotting • Rectangle Investigations • Connected Problem Solving	**4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 21), (3, 36), ...	Lessons 7, 8, 9, 11, 12, 14, and 18
• Rectangle Investigations • Circles, Fractions, and Decimals • Connected Problem Solving	**4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Lessons 7, 8, 9, 10, 11, 12, 13, 14, 15, and 17
• Rectangle Investigations • Connected Problem Solving	**4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	Lesson 16 and 17

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number and Shape Patterns • Multi-Digit Numbers • Shapes and Symmetries • Connected Problem Solving	**4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Lesson 14
• Factors and Area Models • Connected Problem Solving	**4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lessons 6, 8, and 14
• Factors and Area Models • Connected Problem Solving	**4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lesson 14
Fraction Flexibility	**4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	Lesson 16
Fraction Flexibility	**4.NF.4.c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	Lesson 15

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

6B

Times as Many

Let's represent "times as many" situations.

Warm-up

Notice and Wonder: Compare Cubes

What do you notice? What do you wonder?

Andre

Han

Andre

Mai
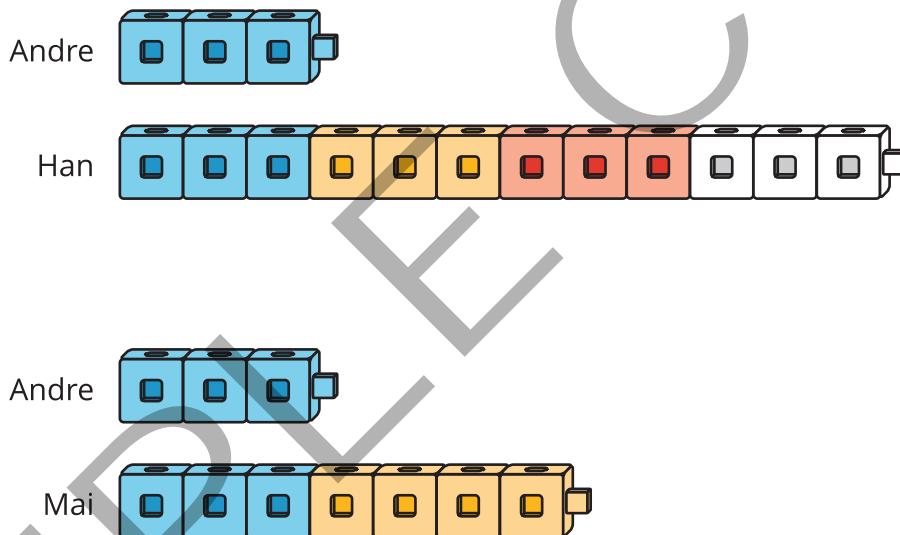
Activity 1

Twice as Many

Andre has some cubes. Han has twice as many cubes as Andre.

Use cubes, pictures, or other diagrams to show how many cubes Andre could have and how many cubes Han could have.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Times as Many

Draw a picture to show the number of cubes the students have in each situation.

1. Andre has the following cubes and Han has 4 times as many.

    Andre

    Han
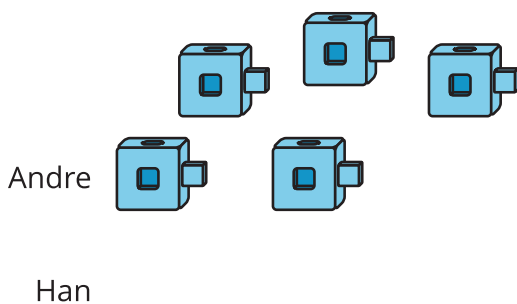
2. Priya has the following cubes and Jada has 6 times as many.

    Priya

    Jada
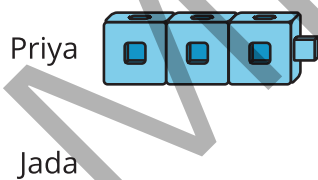
3. Tyler has the following cubes and Mai has 8 times as many.

Tyler

Mai

4. What number represents "8 times as many as 2"? Show your reasoning.

Activity 3

Make n Times as Many

Work with a partner on this activity.

1. Partner A: Create a set, using 2–6 connecting cubes, and show it to Partner B.

    Partner B: Roll a number cube. Let's call the number rolled n.

2. Partner A: Use connecting cubes or a diagram to show n times as many as the original set of cubes. Show your reasoning.

    Partner B: Check your partner's work and discuss any disagreements.

3. Switch roles and repeat.

Interpret Representations of Multiplicative Comparison

Let's make sense of representations of problems with "times as many."

Warm-up

How Many Do You See: Times as Many

How many do you see? How do you see them?

Activity 1

Represent "Times as Many"

Mai's cubes

Kiran's cubes

1. Jada has 4 times as many cubes as Kiran. Draw a diagram to represent the situation.

2. Diego has 5 times as many cubes as Kiran. Draw a diagram to represent the situation.

3. Lin has 6 times as many cubes as Kiran. How many cubes does Lin have? Explain or show your reasoning.

Diagrams to Solve Multiplicative Comparison Problems

Here are 4 sets of descriptions, diagrams, and equations that compare pairs of quantities.

Match each description to a diagram and an equation that represent the same situation.

A Lin has 3 cubes. Jada has 2 times as many.	**B** Han has 3 cubes. Elena has 3 times as many.
C $2 \times 3 = 6$	**D** Priya had 5 cubes. Noah had 3 times as many.
E $3 \times 5 = 15$	**F** $4 \times 4 = 16$
G quantity 1 quantity 2	**H** quantity 1 quantity 2
I $3 \times 3 = 9$	**J** Mai has 4 cubes. Diego has 4 times as many.
K quantity 1 quantity 2	**L** quantity 1 quantity 2

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Record your matches here:

Set 1: _____, _____, _____

Set 2: _____, _____, _____

Set 3: _____, _____, _____

Set 4: _____, _____, _____

Addressing CA CCSSM 4.OA.1, 4.OA.2; practicing MP3; Environmental Principle 4, Concepts A and C

Solve Multiplicative Comparison Problems

Sec A

Let's solve multiplicative comparison problems.

Warm-up

Number Talk: Find the Unknown Factor

Find the value of each unknown mentally.

- $8 \times ? = 16$

- $3 \times ? = 24$

- $? \times 8 = 32$

- $40 \div 8 = ?$

KH | iM Illustrative Mathematics® LEARN MATH FOR LIFE

Activity 1

A Book Drive

This diagram shows the books Lin and Diego donated for the school book drive.

Lin's books ⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚

Diego's books ⬚⬚⬚⬚
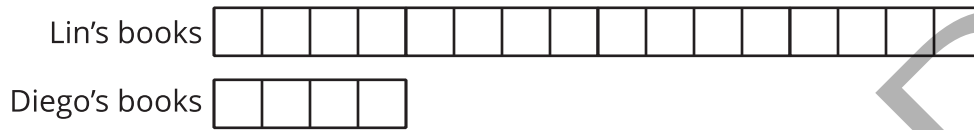
1. Lin donates 16 books. Diego donates 4 books. How many times the number of books as Diego does Lin donate? Explain or show your reasoning. Use the diagram if it is helpful.

2. Priya donates 3 books. Noah donates 21 books. How many times the number of books as Priya does Noah donate? Explain or show your reasoning. Draw a diagram if it is helpful.

3. Mai makes a pile of 27 donated books. Tyler makes his own pile of 3 books. How many times the number of books as Tyler does Mai stack? Explain or show your reasoning.

4. Is a book drive an example of reduce, reuse or recycle? Explain how this helps our planet.

Activity 2

Represent an Unknown Amount

1. Clare donates 48 books. Clare donates 6 times as many books as Andre.

   a. Draw a diagram to represent the situation.

   b. How many books does Andre donate? Explain your reasoning.

   _____

   _____

   _____

   _____

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

2. Han says he can figure out the number of books Andre donates, using division.

Tyler says we have to use multiplication because it says "times as many."

   a. Do you agree with Han or Tyler? Explain your reasoning.

_____

_____

_____

_____

   b. Write an equation to represent Tyler's thinking.

   c. Write an equation to represent Han's thinking.

3. Elena donates 9 times as many books as Diego. Elena donates 81 books.

Use multiplication or division to find the number of books Diego donates.

Solve Multiplicative Comparison Problems with Large Numbers

Let's represent and solve multiplicative comparison problems, with greater numbers.

Sec A

Warm-up

Notice and Wonder: Too Many Times More?

What do you notice? What do you wonder?
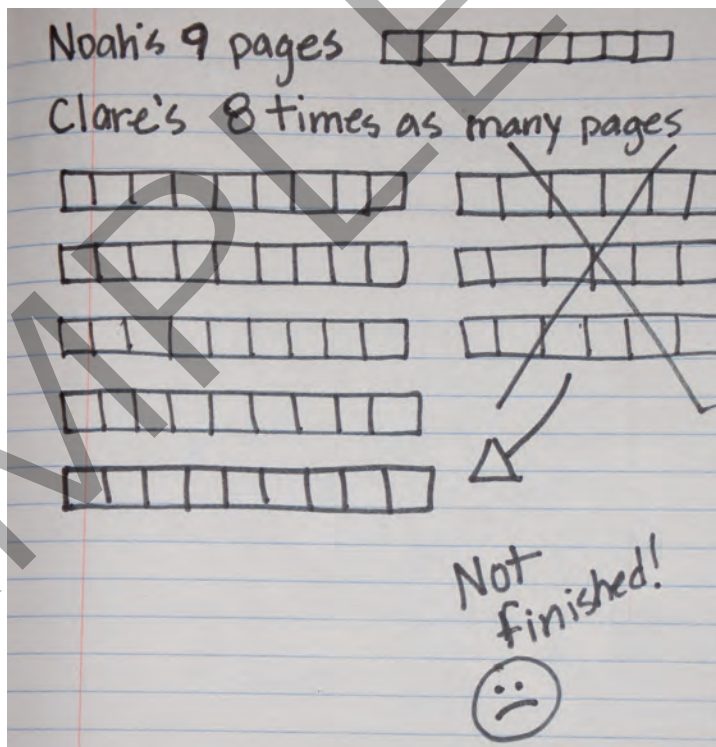
KH | iM Illustrative Mathematics®
LEARN MATH FOR LIFE

Activity 1

A New Kind of Diagram

1. Mai and Andre compare the numbers of pages they read on the first day of the reading competition.

   Mai's pages | 15 |

   Andre's pages [nine segments bar]

   ?

   What would be a good estimate for the number of pages Andre read?

2. The diagram shows the pages Lin and Kiran each read on 1 day of the reading competition.

   Lin's pages | 9 |

   Kiran's pages | 9 | 9 | 9 | 9 | 9 | 9 |

   ?

   a. Complete the statement and explain how you know.

      Kiran read _____ times as many pages as Lin.

      _____

      _____

   b. Write a multiplication equation that compares the pages read by Lin and by Kiran.

   c. How many total pages did Kiran read?

Activity 1

A New Kind of Diagram

1. Mai and Andre compare the numbers of pages they read on the first day of the reading competition.

   Mai's pages | 15 |

   Andre's pages [bar divided into nine segments]

   ?

   What would be a good estimate for the number of pages Andre read?

2. The diagram shows the pages Lin and Kiran each read on 1 day of the reading competition.

   Lin's pages | 9 |

   Kiran's pages | 9 | 9 | 9 | 9 | 9 | 9 |

   ?

   a. Complete the statement and explain how you know.

      Kiran read _____ times as many pages as Lin.

      _____

      _____
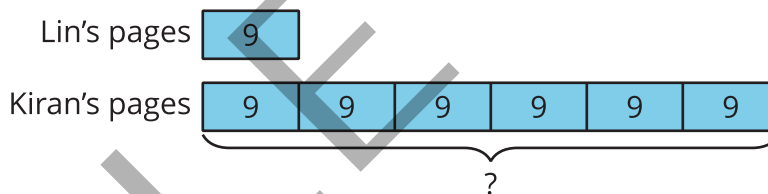
   b. Write a multiplication equation that compares the pages read by Lin and by Kiran.

   c. How many total pages did Kiran read?

3. Jada read some pages. Han read 60 pages altogether.

Jada's pages | ? |

Han's pages | ? | ? | ? |

60
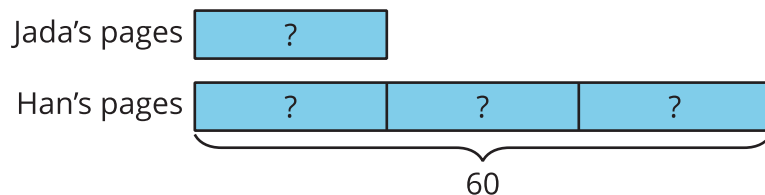
a. How many times as many pages as Jada did Han read? Explain how you know.

_____

_____

b. Write a multiplication equation to compare the pages Han read with the number of pages Jada read. Use a symbol to represent the unknown.

c. How many pages did Jada read? Explain how you know.

_____

_____

_____

_____

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

4. Elena reads 72 pages. Clare reads 9 pages.

Clare's pages `9`

Elena's pages `9` ⌐---------------------------------------⌐

72

a. How is this diagram different from the earlier diagrams?

_____

_____

_____

_____

b. Write a multiplication equation to compare the pages read by Elena and by Clare. Use a symbol to represent the unknown.

c. How many times the number of pages as Clare does Elena read?

Who Read More?

For each situation:

- Write an equation to represent the situation. Use a symbol to represent the unknown.
- Draw a diagram to show the comparison.
- Answer the question about the situation.

1. Lin read 7 books. Diego read 8 times as many books as Lin.

    a. Equation:

    b. Diagram:

    c. How many books did Diego read?

2. Tyler has some books. Clare has 72 books, which is 12 times as many books as Tyler.

    a. Equation:

    b. Diagram:

    c. How many books does Tyler have?

3. Noah read 13 books. Elena read 130 books.

    a. Equation:

    b. Diagram:

    c. Complete the statement:

    _____ read _____ times as many books as _____.

One- and Two-Step Comparison Problems

Let's solve multiplicative comparison word problems.

Sec A

Warm-up

Which Three Go Together: Something's Missing

Which 3 go together?

A

| quantity 1 | 7 |
| quantity 2 | 7 | 7 | 7 | 7 | 7 | 7 |

B

$6 \times 7 = 42$

C

| quantity 1 | 7 |
| quantity 2 | 7 |

42

D

42 is 6 times as many as _____.

KH | iM Illustrative Mathematics®
LEARN MATH FOR LIFE
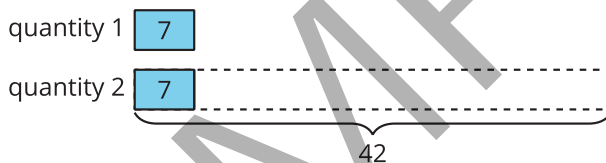
Activity 1

The Book Fair

1. On Thursday, Mai purchased a biography for $9 and a comic book for $4. The amount of total sales that day at the book fair was 8 times as much money as Mai spent.

   What was the amount of total sales for the book fair on Thursday?

2. On Friday, the sales at the book fair were $13 in the morning and 8 times as much as that in the afternoon.

   What was the amount of total sales for the book fair on Friday?

Activity 2

More Book Fair Purchases

For this year's book fair, a school ordered 15 science experiment books and 6 times as many picture books.

Last year, the school ordered 4 times as many picture books and 4 times as many science experiment books than they did this year.

1. How many picture books were ordered last year? Show or explain your reasoning.

2. How many total books were ordered last year? Show or explain your reasoning.

Ten Times as Many

Let's represent "10 times as many."

Activity 1

Ten Times as Many

Here is a diagram that represents 2 quantities, A and B.

A □

B □□□□□□□□□□

1. What are some possible values of A and B?

2. Select the equations that could be represented by the diagram.

   A. $15 \times 10 = 150$

   B. $16 \times 100 = 1,600$

   C. $30 \div 3 = 10$

   D. $5,000 \div 5 = 1,000$

   E. $80 \times 10 = 800$

   F. $12,000 \div 10 = 1,200$

3. For the equations that can't be represented by the diagram:

   a. Explain why the diagram does not represent these equations.

   _____

   _____

   b. How would you change the equations so the diagram could represent them?

   c. Compare your equations with your partner's. Make at least 2 observations about the equations you and your partner wrote.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

What Remains the Same?

1. Use the diagram to complete the table.

A □

B ▢▢▢▢▢▢▢▢▢▢

value of A	value of B
14	
	1,000
160	
	850
1,000	
	2,070
	3,900

2. Select some values from your table to explain or show:

   a. How you found the value of B when the value of A is known.

   b. How you found the value of A when the value of B is known.

Section A Summary

We learned to use multiplication and the phrase "____ times as many" or "____ times as much" for comparing two quantities.

At first, we used cubes and drawings to represent the quantities. For example: Andre has 3 cubes and Han has 12 cubes. We compared the number of cubes by:

- Saying "Han has 4 times as many cubes as Andre."

- Drawing diagrams that show 3 pieces for Andre and 4 times as many pieces for Han.
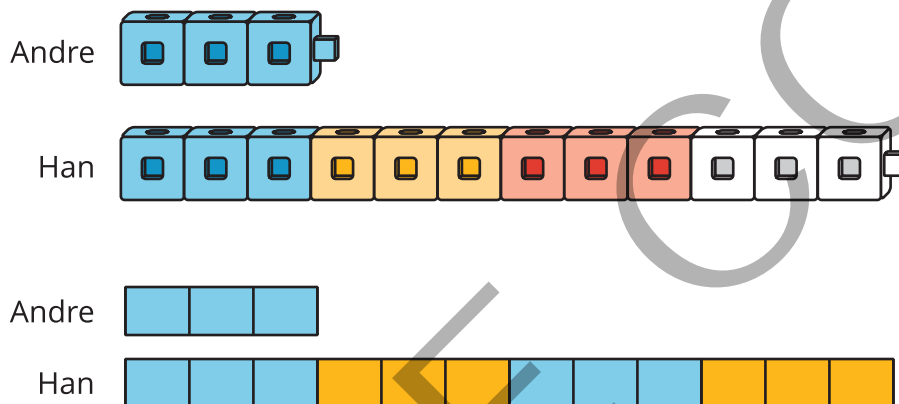
- Writing an equation, such as $4 \times 3 = 12$.

Drawing every unit became less convenient as the numbers became greater, so we used simpler diagrams, with numbers to represent the sizes of these quantities.

If Andre has 30 cubes and Han has 4 times as many, we can represent the comparison, with a diagram like this:
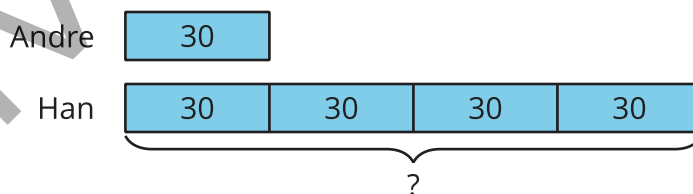
Finally, we compared quantities in which one quantity is 10 times as much as another. We used our understanding of patterns in numbers to multiply a number by 10.

Practice Problems

1 (Pre-unit)

Write a multiplication expression represented by each diagram.

a.

| 5 | 5 | 5 | 5 |

b.
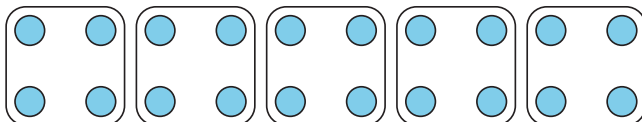
2 (Pre-unit)

Each bottle contains 2 liters of water. How many liters of water are there in 6 bottles?

3 (Pre-unit)

Find the value of each product. Explain or show your reasoning.

a. $5 \times \frac{1}{8}$

b. $5 \times \frac{3}{8}$

c. $4 \times \frac{7}{12}$

4 (Pre-unit)

Find the value of each expression.

a. $\frac{2}{5} + \frac{4}{5}$

b. $1\frac{5}{8} - \frac{4}{8}$

c. $2\frac{1}{6} + 1\frac{5}{6}$

5 (from Unit 5, Lesson 1)

Here are some connecting cubes.

Select **all** pictures that show 4 times as many connecting cubes.

A.

B.

C.

D.

E.

6 from Unit 5, Lesson 2

Han read 4 books during the summer. Priya read 3 times as many books as Han.

a. Draw a diagram that represents the situation.

b. Write an equation that represents the situation.

7 from Unit 5, Lesson 3

Mai exercised 3 times as many hours as Tyler last week. Mai exercised for 15 hours.

a. Write an equation to represent the situation.

b. How many hours did Tyler exercise last week?

8

Diego has 8 songs on his playlist. Mai has 3 times as many songs as Diego.

a. Write an equation that represents the situation.

b. Draw a diagram that represents the situation.

c. Explain how the diagram and the equation are related.

_____

_____

d. How many songs are on Mai's playlist?

9

Lin scored 6 points in her basketball game on Saturday. She scored 3 times as many points in her basketball game on Sunday. How many points did Lin score altogether in the two games? Explain or show your reasoning.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Sec A

10 from Unit 5, Lesson 6

a. If Diagram A represents 15, what does Diagram B represent? Explain your reasoning.

A [ ]

B [| | | |]

_____

_____

b. If Diagram B represents 100, what does Diagram A represent? Explain your reasoning.

_____

_____

11 (Exploration)

For each question, explain or show your reasoning.

Lin's books [| | | |]

Diego's books [| | | | | |]

a. If Lin brought 10 books, how many books did Diego bring?

b. If Diego brought 21 books to the fair, how many books did Lin bring?

c. If Lin and Diego brought 60 books altogether, how many did they each bring?

12

Write a comparison story problem.

_____

_____

_____

_____

Trade problems with a partner.

  a. Draw a diagram and write an equation to represent the situation.

  b. Solve the story problem.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.1; building on 2.MD.1; practicing MP3 and MP7

Meters and Centimeters

Let's explore lengths in meters and in centimeters.

Warm-up

Notice and Wonder: Big Bug, Little Bug

What do you notice? What do you wonder?

Sec B
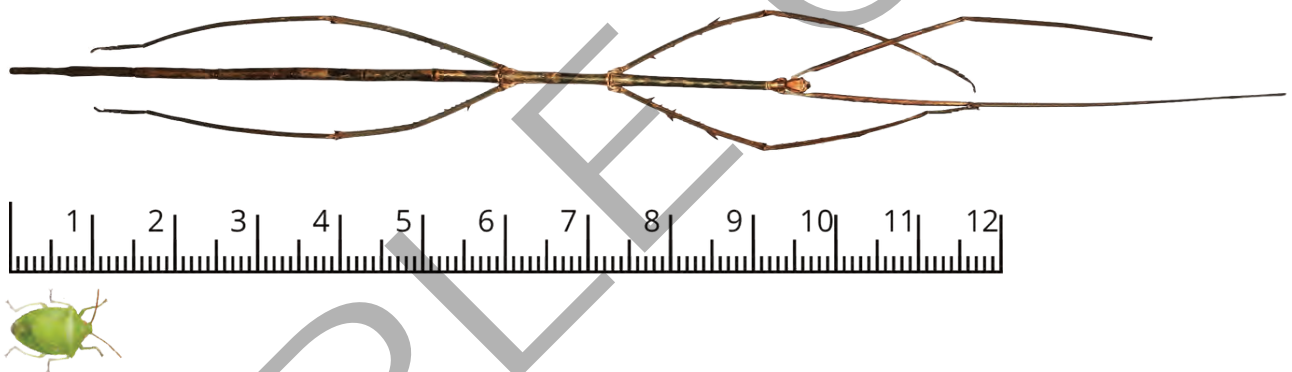
Activity 1

How Long Is 1 Meter?

1. Use the centimeter paper to build a strip that is 100 centimeters long. You will need scissors and tape.

   If you do it precisely, your paper strip will be 1 meter long.

2. List 5 items in the classroom that you think are about 1 meter long.

   Then use your paper strip to check how close your prediction is to 1 meter.

3. Decide whether each of the following is more than 1 meter, less than 1 meter, or about 1 meter.

   a. The stick insect in the *Warm-up* activity

   b. The step you make when walking

   c. The step you make when running

   d. Your arm span

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

In and around the School

Priya took some measurements in meters and recorded them in the table, but she made some errors when converting them to centimeters. She also left out 1 measurement.

	measurement (meters)	measurement (centimeters)
a. height of door	2	200
b. height of hallway	3	30
c. width of hallway	5	500
d. length of gym	18	180
e. length of hallway	27	2,700
f. length of playground	50	

1. Find and correct Priya's conversion errors. Explain how you know.

_____

_____

_____

_____

2. Record the length of the playground in centimeters. Write an equation to represent your thinking.

Meters and Kilometers

Let's explore measurements in meters and in kilometers.

Sec B

Warm-up

Number Talk: Times Hundreds and Thousands

Find the value of each expression mentally.

- 3×100

- 40×100

- 43×100

- $43 \times 1,000$

KH | iM | Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 1

How Long Is 1 Kilometer?

There are 1,000 meters in 1 kilometer.

1. The shaded section of the track is the length of a 100-meter race. How many 100-meter races does it take to travel 1 kilometer?

100 meters
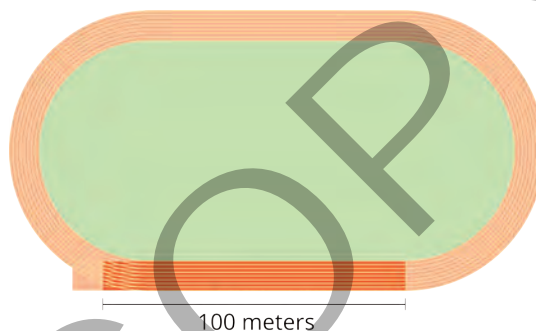
2. Your teacher will give you images of something with a length or a height measured in meters.

   About how many of the items in the handout are needed to make 1 kilometer? Explain or show how you know.

3. Work with your group to write a number in the blank so that each statement is true. Decide if your length is "about" or "exactly" 1 kilometer, and circle the correct word.

   a. One kilometer is the length of (about, exactly) _____ soccer fields.

   b. One kilometer is the length of (about, exactly) _____ Statues of Liberty.

   c. One kilometer is the length of (about, exactly) _____ Olympic-size swimming pools.

   d. One kilometer is the length of (about, exactly) _____ basketball courts.

4. Estimate where you might end up if you travel 1 kilometer from your school's front door.

Activity 2

Meters and Kilometers

1. Complete the table with the unknown lengths in meters or in kilometers.

kilometers (km)	meters (m)
$\frac{1}{2}$	
1	1,000
5	
	6,000
$8\frac{1}{2}$	
10	
	12,000
27	

2. Andre says 100 meters is longer than 10 kilometers. Do you agree or disagree? Explain or show your reasoning.

3. Which is greater?

   a. 2,000 meters or 3 kilometers

   b. 500 meters or 1 kilometer

   c. 14 kilometers or 14,000 meters

   d. 8 kilometers or 80,000 meters

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.1, 4.MD.2; practicing MP2

Grams and Kilograms, Liters, and Milliliters

Let's explore measurements in grams, kilograms, liters, and milliliters.

Warm-up

Which Three Go Together: Meter, Meter on the Page

Which 3 go together?

A	B	C
2 m	2,000 meters	200 centimeters

D

1 meter	1 meter
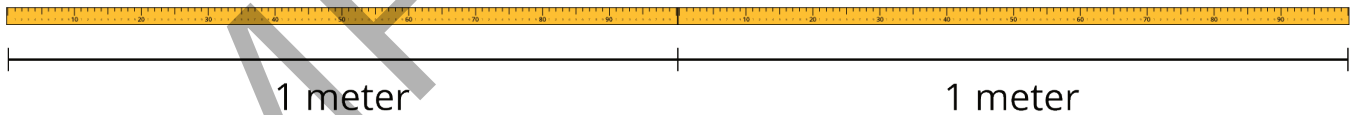
Activity 1

A Whole Lot of Paper Clips

One paper clip weighs 1 gram.

1. How many grams are the paper clips in each image?

a.

b.

office supplies
100 paper clips
100 paper clips

c.

office supplies
250 paper clips
250 paper clips

2. The paper clips in this image weigh 1 kilogram.

   What is the relationship between kilograms and grams?

office supplies
100 paper clips
office supplies
100 paper clips
office supplies
100 paper clips
office supplies
100 paper clips
office supplies
100 paper clips
100 paper clips

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Complete the table, with the unknown amounts in grams.

kilograms (kg)	grams (g)
2	
7	
15	
$\frac{1}{2}$	
$9\frac{1}{2}$	

4. Which weighs more?

a. 8 kilograms or 8 boxes, with 100 paper clips in each box

b. 1,250 paper clips or 1 kilogram

c. 500 grams or 2 boxes of 250 paper clips each

d. $\frac{1}{2}$ kilogram or 500 paper clips

Liters and Milliliters

1. a. Estimate: How many times do we fill the 100-milliliter glass to get 1 liter of liquid? (Assume that each time the liquid is filled to the 100-milliliter line.)

   b. Write a sentence to describe the relationship between milliliters and liters.

   _____

   _____

2. How many times do we need to fill each of these containers to get 1 liter?

   a. A 1-milliliter medicine dropper

   b. A 250-milliliter measuring cup

   c. A 20-milliliter cup

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Complete the table, with the unknown amounts in liters (L) or milliliters (mL).

liters (L)	milliliters (mL)
1	
4	
7	
16	
	500
	20,000

Multi-Step Measurement Problems

Let's solve multi-step measurement problems.

Warm-up

Notice and Wonder: Distances Traveled

What do you notice? What do you wonder?
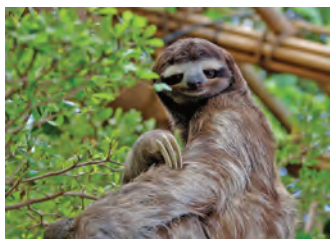
animal	distance traveled in a day
three-toed sloth	30 meters
snail	2,500 centimeters
dromedary	40 kilometers
giant tortoise	300 meters

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 1

Long Hikes, Short Hikes

Here are estimates of the farthest distances some animals travel in one day.

animal	distance traveled in a day
three-toed sloth	30 meters
snail	2,500 centimeters
dromedary	40 kilometers
giant tortoise	300 meters

1. Put the animals and their travel distances in order, from shortest to longest. Explain or show your reasoning.

2. Do you agree with each statement? Explain your reasoning.

   a. A giant tortoise can travel 10 times as far as a three-toed sloth can travel in a day.

   b. A dromedary can travel 80 times as far as a giant tortoise can travel in a day.

Sec B

Big Bottles, Little Bottles

Here are 6 water bottles arranged from largest to smallest size.

A   B   C   D   E   F
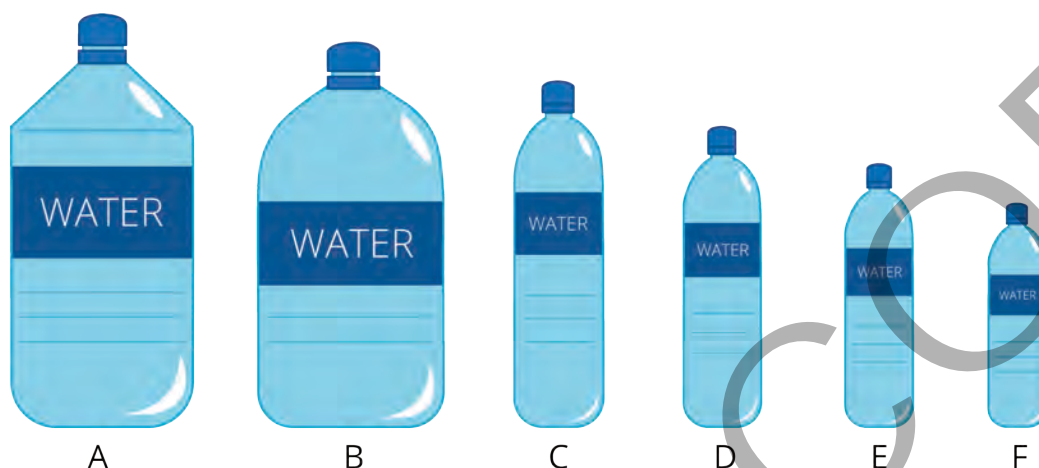
Here are four clues about the amount of water each bottle holds.

- One bottle holds 350 mL.
- A bottle in size B holds 5 times as much water as the bottle that holds 1 L.
- The largest bottle holds 20 times the amount of water in the smallest bottle.
- One bottle holds 1,500 mL, which is 3 times as much water as a bottle in size E.

Use the clues to find out the amount of water, in milliliters, that each bottle size holds.

A: _____ mL

B: _____ mL

C: _____ mL

D: _____ mL

E: _____ mL

F: _____ mL

Addressing CA CCSSM 4.MD.1, 4.MD.2, 4.OA.3; practicing MP2 and MP3

Pounds and Ounces

Let's explore measurements in pounds and ounces.

Warm-up

Notice and Wonder: A Recipe

What do you notice? What do you wonder?

❏ 14 ounces of rice

❏ ½ pound of ground beef

❏ 2 cans (2 pounds) of kidney beans, drained and rinsed

❏ 1 jar (24 ounces) of picante sauce

❏ 12 ounces of shredded Cheddar cheese

Sec B

Pounds and Ounces

Your teacher will show images of some packaged food items.

1. Use the information on the images to find out how pounds (lb) and ounces (oz) are related.

   a. Write a sentence to describe the relationship between the 2 units.

   _____

   _____

   b. Use 2 pieces of information to show that your conclusion is correct.

2. Complete this table, with amounts in ounces.

pounds (lb)	ounces (oz)
1	
2	
3	
4	
5	
8	
10	

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Party Prep

1. A family is cooking for a party and needs to buy some ingredients for a main dish:

   - 5 pounds of rice
   - $\frac{1}{2}$ pound of almonds
   - 2 pounds 8 ounces of pinto beans

   How many ounces of each ingredient does the family need to buy? Show your reasoning.

2. To make the dessert, they need 6 pounds of flour. If they already have 40 ounces of flour in the pantry, how many more ounces of flour do they need to buy?

Hours, Minutes, and Seconds

Let's explore time in hours, minutes, and seconds.

Warm-up

What Do You Know about 1 Hour?

What do you know about 1 hour?

Activity 1

Mai's School Day

The table shows how Mai spends the time she is awake on a school day.

activity	hours	minutes
morning routine	1	
getting to school	$\frac{1}{2}$	
time at school	8	
getting home from school	$\frac{3}{4}$	
homework and reading	$1\frac{1}{2}$	
playing and family time	2	
bedtime routine	$\frac{1}{4}$	

1. Complete the table to show how many minutes Mai spends on each activity.

2. How many hours does Mai spend at school? What is the time in minutes?

3. How many minutes does Mai sleep on a school night? Explain or show your reasoning.

Activity 2

Precious Minutes and Seconds

Diego sets a timer to make sure his daily activities do not take too long or too short an amount of time.
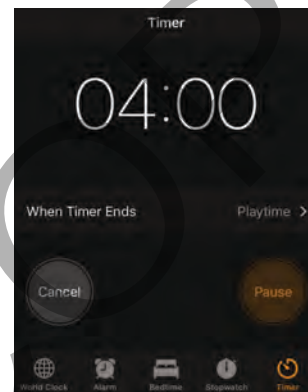
activity	minutes	seconds
brushing teeth	2	
showering	3	
heating a cup of milk in the microwave	$\frac{1}{2}$	
break during homework time	5	
quick workout	10	
daily reading	30	

1. Complete the table, with the number of seconds for each activity.

2. Diego notices on a television channel that commercial breaks often are between $1\frac{1}{2}$ minutes and $2\frac{1}{2}$ minutes long. How long are they in seconds? Explain or show your reasoning.

3. Diego's workout starts with 4 minutes of warming-up and stretching, followed by 100 seconds of jumping jacks.

   If he works out for 10 minutes exactly, how many more seconds are left in his workout?

KH | iM Illustrative Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.2, 4.OA.3; building on 4.NF.4.b; practicing MP1 and MP6

Multi-Step Measurement Problems with Fractions

Let's solve multi-step measurement problems.

Warm-up

True or False: Some Number Times a Fraction

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $16 \times \frac{1}{4} = 4$

- $8 \times \frac{3}{4} = 12$

- $32 \times \frac{2}{8} = 8$

- $60 \times \frac{1}{12} = 10$

Info Gap: Noah's School Day (Part 1)

Problem Card

On a school day, Noah usually spends 40 minutes on his morning routine and 75 minutes on his sports practice.

Which one takes more time:

1. Noah's morning routine or his bedtime routine?

2. Noah's sports practice or his homework and reading time?

Activity 2

Info Gap: Noah's School Day (Part 2)

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Problem Card Student	Data Card Student
Silently read the Problem Card.	Silently read the Data Card.

"Can you tell me ___ ?"
(Ask for a specific piece of information.)

"Why do you need to know ___ ?"
(Repeat the information requested)

"I need to know ___ because. . ."

Listen to your partner's reason.

Answer with information from the Data Card.

"I have enough information to solve this problem."

Display the Problem Card.

Solve the problem independently.
Continue to ask questions if more information is needed.

Share the Data Card. Then compare strategies and solutions.

Pause here so your teacher can review your work.

Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Shopping List

Here are 6 ingredients that a shopper bought and some clues about each quantity.

The items are listed in order of weight, from least to greatest.

ingredient	pounds	ounces
rice noodles		
shrimp		
tapioca flour		
tofu		
carrots		
brown rice		

- The heaviest item weighs 4 times the weight of tofu.
- One ingredient weighs $\frac{1}{2}$ pound.
- The item that weighs 10 pounds is 10 times the weight of shrimp.
- The carrots are 3 times as heavy as the shrimp.
- The carrots are 2 times as heavy as the tapioca flour.
- Brown rice weighs 20 times as much as the weight of noodles.

Use the clues to find out the weight of each ingredient in both pounds and ounces.

👥 Section B Summary

We learned about various units for measuring length, distance, weight, capacity, and time. We saw how different units that measure the same property are related.

Here are the relationships we saw:

- One meter (m) is 100 times as long as 1 centimeter (cm).
- One kilometer (km) is 1,000 times as long as 1 meter (m).
- One kilogram (kg) is 1,000 times as heavy as 1 gram (g).
- One liter (L) is 1,000 times as much as 1 milliliter (mL).
- One pound (lb) is 16 times as heavy as 1 ounce (oz).
- One hour is 60 times as long as 1 minute.
- One minute is 60 times as long as 1 second.

When given a measurement in one unit, we can find the value in another unit by reasoning and writing equations. For example, to express 5 kilograms in grams, we can write $5 \times 1,000 = 5,000$. To express 4 pounds in ounces, we can write $4 \times 16 = 64$.

Throughout the section, we used these relationships to convert measurements from one unit to another, to compare and order measurements, and to solve problems in different situations.

Practice Problems

1 from Unit 5, Lesson 7

Select **all** measurements greater than 1 meter.

A. the width of a paper clip

B. the length of your math book

C. the width of the classroom

D. the height of the door in the classroom

E. the length of a finger

2 from Unit 5, Lesson 8

a. The Burj Khalifa (BURJZH kha-LEE-fah) is a skyscraper in Dubai (doo-BAI). It is 828 meters tall. Is the height more than or less than 1 kilometer? Explain your reasoning.

_____

_____

b. An airplane flies 9,500 meters above the ground. Is the distance more or less than 1 kilometer? Explain your reasoning.

_____

_____

KH | Illustrative Mathematics
LEARN MATH FOR LIFE

Sec B

3

a. A bottle of water holds 250 milliliters.

- How many of those bottles does it take to make 1 liter?

- How many liters of water does the bottle hold?

b. Jada's backpack weighs 3.5 kilograms. How many grams is Jada's backpack? Explain or show your reasoning.

4

a. A dromedary travels 40 kilometers in a day. A tuna fish swims 200 kilometers in a day. How many times farther does the tuna fish travel than the dromedary?

b. A three-toed sloth travels 30 meters in a day. What is the distance in centimeters?

c. A giant tortoise travels 300 meters in a day. How many times farther than the three-toed sloth does the giant tortoise travel?

5 from Unit 5, Lesson 11

The average weight of a newborn baby is 7 pounds 8 ounces. What is the weight in ounces?

6 from Unit 5, Lesson 12

Diego said it took him 1,800 seconds to get home from school yesterday. Lin thinks he is exaggerating.

a. It took Diego $\frac{1}{2}$ hour to get home from school yesterday. What is the time in minutes? Explain your reasoning.

_____

_____

b. What is the time in seconds? Explain or show your reasoning.

c. Was Diego's statement correct or is he exaggerating?

KH | Illustrative® Mathematics LEARN MATH FOR LIFE

7

Han makes sparkling juice by mixing 1.5 liters of juice and 500 milliliters of sparkling water.

   a. How many milliliters of sparkling juice did Han make?

   b. How many times more juice did Han use than sparkling water?

8 (Exploration)

Jada is trying to decide which option will take less time to make a trip. Here are the details.

Option 1 (Fly)

- 30 minutes to get to the airport
- $1\frac{1}{2}$ hours waiting at the airport
- 45-minute flight
- $1\frac{1}{4}$ hours more to get to the destination

Option 2 (Drive)

- $3\frac{3}{4}$-hour drive
- $\frac{1}{2}$-hour stop for lunch

Which option takes less total time? Explain your reasoning.

Three different trails start and end at the same location. Here are the round-trip distances:

Trail A: 800 m   Trail B: 2.5 km   Trail C: 1,300 m   Trail D: 3 km

   a. Andre hiked a total of more than 3 kilometers but less than 4 kilometers. Which trails might Andre have taken?

   b. Jada hiked more than 2 kilometers but less than 3 kilometers. Which trails might Jada have taken?

Addressing CA CCSSM 4.MD.1, 4.MD.2, 4.NBT.4-6, 4.OA.2; practicing MP6

Weight and Capacity Measurements

Let's solve problems about weight and about capacity.

Warm-up

Number Talk: A Lot of Thousands

Find the value of each expression mentally.

- $1,200 + 900$

- $12,500 + 9,000$

- $13,000 + 9,900$

- $130,000 + 99,000$

Activity 1

Milk and Mango Lassi

1. Here are 3 true statements about this jug of milk.

   ◦ This jug contains 1 gallon of milk.

   ◦ This jug contains 4 quarts of milk.

   ◦ This jug contains 16 cups of milk.

   Complete each statement so that it is true:

   a. One gallon is _____ times as much as 1 quart.

   b. One gallon is _____ times as much as 1 cup.

   c. One quart is _____ times as much as 1 cup.

2. For a potluck party, Priya and 3 other relatives are bringing *mango lassi* (MAYNG-goh LAH-see).

   a. Who prepared the most mango lassi?
      Explain or show your reasoning.

guest	amount of lassi
Priya	10 cups
uncle	3 quarts
cousin	8 cups
grandma	2 gallons

   b. How many cups of lassi did all the guests bring?

   c. Complete this sentence: Priya's grandma made _____ times as much lassi as Priya's cousin. Show how you know.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Clay for Art Class

At a craft store, clay is sold in packs of different sizes: 1 pound, 24 ounces, 3 pounds, and 5 pounds.

An art teacher needs 120 ounces of clay for her class.

1. Would she have enough clay if she bought each of the following combinations? Explain or show your reasoning.

	1-pound pack	24-ounce pack	3-pound pack	5-pound pack
A	1		1	
B	1	1	1	1
C		1	2	

2. Decide if each statement is true or false. Explain or show your reasoning.

   a. A 3-pound pack weighs 2 times as much as a 24-ounce pack.

   b. If we combine a 1-pound pack, a 3-pound pack, and a 5-pound pack, we'd have 6 times as much clay as what's in a 24-ounce pack.

Length Measurements

Let's solve problems about distances and lengths.

Warm-up

Which Three Go Together: Measurements

Which 3 go together?

A

3 feet

B

(3×1) yards

C

(2×18) inches

D

$(\frac{1}{3} + \frac{1}{3} + \frac{1}{3})$ yard

Activity 1

Disc Throws

There are 6 students throwing discs on field day. Here is some information about each person's first throw.

student	distance
Han	17 yards
Lin	$51\frac{1}{2}$ feet
Clare	$21\frac{1}{3}$ feet
Andre	22 yards 2 feet
Elena	
Tyler	

- Elena's disc went 3 times as far as Clare's disc.

- Andre's disc went 4 times as far as Tyler's disc.

1. Complete the table, with Elena and Tyler's distances. Explain or show your reasoning.

2. Who are the top 3 throwers for that round?

   Find out by ordering the students and their distances from longest to shortest.

rank	student	distance (feet)
1		
2		
3		
4		
5		
6		

Stone Towers

While on an outing, a group of friends had a stone-stacking contest to see who could build the tallest stone tower.

- Andre's tower is 3 times as tall as Diego's, but Diego didn't build the shortest tower.
- Tyler built the tallest tower at 4 feet and 2 inches.
- One person built a tower that is 39 inches tall.
- Tyler's tower is 5 times as tall as the shortest tower.

1. How tall is each person's stone tower?

person	tower height (inches)
Andre	
Tyler	
Clare	
Diego	

2. Elena came along and built a tower that is 5 times as tall as Diego's tower. Is Elena's tower more than 6 feet? Show your reasoning.

KH | Illustrative Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.3, 4.NF.4, 4.OA.2; practicing MP7

Compare Perimeters of Rectangles

Let's solve problems about rectangles of different sizes.

Warm-up

Number Talk: Two and Four Times a Fraction

Find the value of each expression mentally.

- $2 \times \frac{3}{2}$

- $4 \times \frac{3}{4}$

- $4 \times \frac{9}{4}$

- $\left(2 \times \frac{3}{4}\right) + \left(2 \times \frac{9}{4}\right)$

Activity 1

Chenille Stem Perimeters

How many different rectangles can be made, using the entire length of one 12-inch chenille stem?

1. Record as many pairs of side lengths as you can think of.

length (inches)	width (inches)

2. Which pair of measurements represents the side lengths of a square?

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Perimeter Predictions

1. Your teacher will assign a pair of side lengths to you. Use a chenille stem to build a rectangle with those side lengths.

   What is the perimeter of the rectangle?

2. Two 12-inch chenille stems are joined end to end (with no overlaps) and used to build a square.

   a. What is the side length of this square? What is its perimeter?

   b. How do the side length and the perimeter of this square compare to those of the first square?

3. Several chenille stems are joined end to end (with no overlaps) to build a square with a perimeter of 60 inches.

   a. How many chenille stems are used? Explain or show how you know.

   b. What is the side length of the square?

   c. How do the side length and the perimeter compare to those of the first square?

Sec C

Activity 3

Gridded Rectangles

1. Draw the following rectangles on centimeter grid paper. Label each rectangle. Record the side lengths and the perimeter of each.

   ◦ Rectangle A has a perimeter of 16 centimeters.

   ◦ Rectangle B has side lengths that are 3 times the side lengths of Rectangle A.

   ◦ Rectangle C has side lengths that are $\frac{1}{2}$ of the side lengths of B.

rectangle	length (cm)	width (cm)	perimeter (cm)
A			16
B			
C			

2. Rectangle D has a perimeter of 96 centimeters.

   The perimeter of D is:

   ◦ _____ times the perimeter of A

   ◦ _____ times the perimeter of B

   ◦ _____ times the perimeter of C

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.2, 4.MD.3, 4.OA.2; building on 4.NF.3.b; practicing MP6

More Perimeter Problems

Let's solve problems about the perimeter of various shapes.

Warm-up

True or False: Fractions and Whole Numbers

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{8}{12} + \frac{3}{12} + \frac{9}{12} + \frac{4}{12} = 2$

- $\frac{20}{4} + \frac{10}{4} + \frac{6}{4} = 8$

- $2 = \frac{59}{100} + \frac{41}{100} + \frac{89}{100} + \frac{11}{100}$

- $2 = \frac{3}{8} + \frac{3}{8} + \frac{12}{8}$

Sec C

Along the Walls in Tiny Steps

A rectangular room has a perimeter of 39 feet and a length of $10\frac{1}{2}$ feet.

1. What is the width of the room in feet? Explain or show your reasoning.

2. An ant walked along two walls of the room, always in a straight line. It started in one corner and ended up in a corner opposite of where it started. How many inches did it travel? Explain or show your reasoning.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Unknown Measurements

Your teacher has posted six quadrilaterals around the room. Each one has an unknown side length or an unknown perimeter.

1. Choose two diagrams—one with an unknown side length and another with an unknown perimeter. Make sure all six shapes will be visited by at least one person in your group

   Find the unknown values. Show your reasoning and remember to include the units.

2. Discuss your responses with your group until everyone agrees on the unknown measurements for all six figures.

3. Answer one of the following questions. Explain or show your reasoning.

   a. The perimeter of B is how many times the perimeter of D?

   b. The perimeter of one figure is 1,000 times that of another figure. Which are the two figures?

   c. The perimeter of F is how many times the perimeter of B?

👥 Section C Summary

We solved problems and puzzles by converting units of measurement—gallons, quarts, cups, pounds, ounces, yards, feet, and inches—and by comparing measurements in different units. We saw that the problems could be solved in different ways.

For example, if Priya threw a disc 16 yards and this is 4 times as far as the distance Jada threw in feet, how far did Jada throw the disc?

- One way to solve this problem is by finding $16 \div 4$, which is Jada's throw distance in yards ($16 \div 4 = 4$), and then by multiplying the result to convert the yards to feet ($4 \times 3 = 12$, so 4 yards is 12 feet).

- Another way is first to convert the 16 yards to feet ($16 \times 3 = 48$, so 16 yards is 48 feet), and then to divide the result by 4 to find Jada's throw distance ($48 \div 4 = 12$).

In the last two lessons, we solved multiplication and comparison problems that involve the perimeter of rectangles and some other quadrilaterals.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.1, 4.OA.2; practicing MP4

Two Truths and a Lie

Let's compare facts about animals.

Warm-up

Notice and Wonder: Animal Facts

What do you notice? What do you wonder?

30 feet
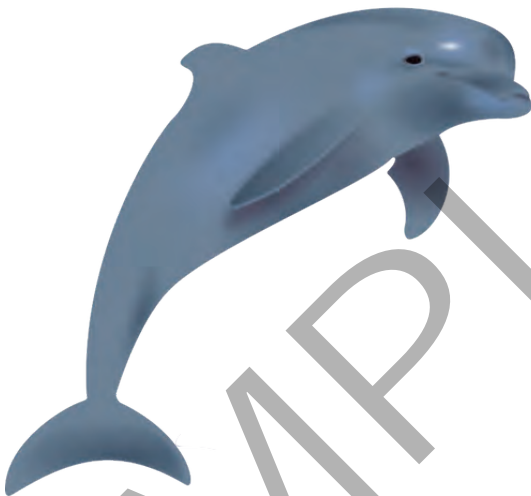
130 inches

3 feet

Two Truths and a Lie

Your teacher will give you a fact sheet about some animals or ask you to perform some research.

1. Choose 2 animals that you would like to compare.

2. Use the information from your teacher or your research to write 2 true comparison statements and 1 false comparison statement about these animals.

   a. Write 1 statement on the front side of each index card.

   b. On the back side of each card, write whether the statement is true or false.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Gallery Walk: Two Truths and a Lie

1. Read at least 5 statements that are not yours and decide if they are true.

   - Find at least one statement that is false. Write down the statement here. Explain or show how you know.

   _____

   _____

   _____

   _____

   - If a statement you read is unclear, use a sticky note to leave a comment or question for the writer.

2. Collect your cards and any notes that your classmates might have left for you.

   - Think about how to make revisions to make your statements stronger and clearer.

   - Record your revised statements here.

   _____

   _____

   _____

   _____

   _____

   _____

   _____

Sec C

Practice Problems

1 from Unit 5, Lesson 14

a. Jada's class drinks 2 gallons of milk at lunch. What is the amount of milk in cups?

b. The third graders at Jada's school drink 3 times as much milk as Jada's class. What is the amount in quarts?

2 from Unit 5, Lesson 15

Diego sees a snake that is 9 inches long at the pet store. The world's longest snake, a python, is 10 yards long. How many times longer is the python than the snake Diego sees at the pet store? Explain or show your reasoning.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

3 from Unit 5, Lesson 16

   a. Andre draws a rectangle that is 6 centimeters long and 5 centimeters wide. What is the perimeter of Andre's rectangle?

   b. Jada draws a rectangle with side lengths that are 4 times as long as Andre's rectangle. What is the perimeter of Jada's rectangle?

4 from Unit 5, Lesson 18

   a. A rectangle has a perimeter of 19 centimeters. One side is 4 centimeters. How long is the other side of the rectangle?

   b. A square has a perimeter of 19 centimeters. How long is each side of the square? Explain or show your reasoning.

5 (Exploration)

A furlong is 220 yards. It is a unit usually used to measure the distance of horse races. A mile is 5,280 feet. Mai reads that a furlong is $\frac{1}{8}$ mile. Is this true? Explain or show your reasoning.

6 (Exploration)

Many of the length measurements we use today have interesting histories. Choose one of them, such as a foot, a yard, or a marathon, and investigate its origin.

GRADE 4

UNIT

6

Multiplying and Dividing Multi-digit Numbers

Content Connections

In this unit you will use multiplication to compare numbers and solve problems about measurement. You will make connections by:

- **Reasoning with Data** while exploring rectangles, measuring, and plotting.
- **Exploring Changing Quantities** while working with number and shape patterns, multi-digit numbers, factors, and area models.
- **Taking Wholes Apart and Putting Parts Together** when you investigate circles, fractions, and decimals.
- **Discovering Shapes and Space** when you solve measurement problems related to circles, fractions, and decimals.

Addressing the Standards

As you work your way through **Unit 6 Multiplying and Dividing Multi-digit Numbers,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 12, 23
MP2 Reason abstractly and quantitatively.	Lessons 2, 12, 20, 22, 24, and 25
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 7, 9, 11, 14, 17, and 18
MP5 Use appropriate tools strategically.	Lessons 14, 15
MP4 Model with mathematics.	Lesson 26
MP6 Attend to precision.	Lessons 5, 10, 19, and 21
MP7 Look for and make use of structure.	Lessons 1, 3, 4, 6, 8, 10, 13, and 16
MP8 Look for and express regularity in repeated reasoning.	Lesson 3

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number and Shape Patterns • Factors and Area Models • Multi-Digit Numbers • Fraction Flexibility • Circles, Fractions, and Decimals	**4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	Lesson 22
• Number and Shape Patterns • Factors and Area Models • Connected Problem Solving	**4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	Lessons 22 and 25

89B

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
Connected Problem Solving	**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Lessons 14, 18, 19, 20, 21, 22, 23, 25, and 26
Factors and Area Models	**4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	Lesson 19
Number and Shape Patterns	**4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	Lessons 1, 2, 3, and 4
• Measuring and Plotting • Rectangle Investigations • Connected Problem Solving	**4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 21), (3, 36), ...	Lesson 26

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Rectangle Investigations • Circles, Fractions, and Decimals • Connected Problem Solving	**4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Lessons 12, 23, 24, and 26
• Rectangle Investigations • Connected Problem Solving	**4.MD.3** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	Lessons 14 and 23
• Number and Shape Patterns • Multi-Digit Numbers • Shapes and Symmetries • Connected Problem Solving	**4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Lessons 19, 24 and 25
• Factors and Area Models • Connected Problem Solving	**4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lessons 5, 6, 7, 8, 9, 10, 11, 12, 19, 22, 23, and 24
• Factors and Area Models • Connected Problem Solving	**4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lessons 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, and 24

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Patterns that Grow

Let's create and describe growing patterns that follow a given rule.

Warm-up

Notice and Wonder: Sets of Circles

What do you notice? What do you wonder?
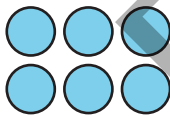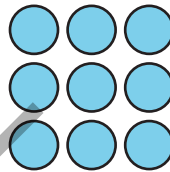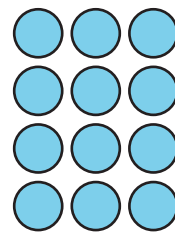
step 1   step 2   step 3   step 4

Activity 1

Bottle Cap Patterns

Han and Lin create patterns with bottle caps. Here are the first 2 steps.

1. Han starts with 1 row of 5 bottle caps. He adds a new row of 5 bottle caps in each step.

   a. Describe or draw how Han's pattern will continue.

   b. What do you notice about Han's pattern? Why do you think this happens?

   _____

   _____

   _____

   _____

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

2. Lin creates a different pattern. She starts with 1 row of 5 bottle caps. Then she doubles the number of rows of bottle caps.

    a. Describe or draw how Lin's pattern will continue.

    b. What do you notice about Lin's pattern? Why do you think this happens?

_____

_____

_____

_____

Sec A

Activity 2

Taller and Taller

Jada uses pattern blocks to make a giraffe. She wants to create a pattern with this giraffe as the first step. For each step that follows, her rule is to add 2 square blocks to make the giraffe's neck longer.
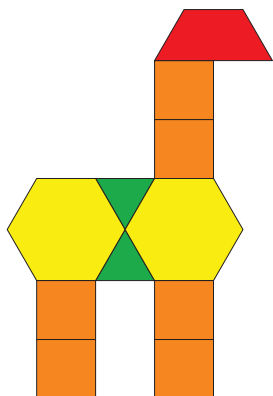
Partner A:

1. Create the next 3 steps of Jada's pattern. Use pattern blocks or draw to continue Jada's pattern.

2. List the number of square blocks in each of the first 5 steps. Write 2 observations about the numbers in the pattern.

_____

_____

_____

_____

3. Will a step ever have 25 square blocks? Explain or show your reasoning

KH | iM Illustrative Mathematics® LEARN MATH FOR LIFE

Jada uses pattern blocks to make a giraffe. She wants to create a pattern with this giraffe as the first step. For each step that follows, her rule is to add 2 square blocks to make the giraffe's neck longer.
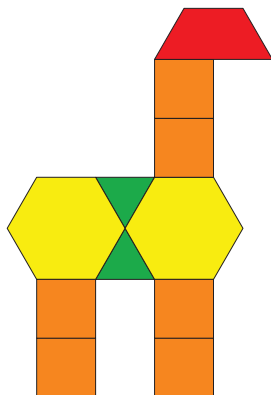
Partner B:

1. Create the next 3 steps of Jada's pattern. Use pattern blocks or draw to continue Jada's pattern.

2. List the total number of blocks in each of the first 5 steps. Write 2 observations about the numbers in the pattern.

_____

_____

_____

_____

3. Will a step ever have a total of 25 blocks? Explain or show your reasoning.

Patterns that Repeat

Let's create and describe repeating patterns that follow a given rule.

Warm-up

How Many Do You See: Colorful Tiles

How many tiles do you see? How do you see them?

| * | ? | ! | # | * | ? | ! | # | * | ? | ! | # | * | ? | ! | # | * | ? | ! | # | * | ? |

Activity 1

Clare's Pattern

Clare creates a pattern using 3 shapes—a triangle, a circle, and a square—that repeat in that order.

1. Draw the first 10 shapes in Clare's pattern.

2. What do you notice about the first 10 shapes in Clare's pattern?

_____

_____

_____

_____

Sec A

3. Clare numbers the shapes 1–10 to show their position in her pattern.

    a. What numbers represent the position of the first 5 squares?

    b. What else do you notice about the shapes and their numbers?

_____

_____

_____

_____

4. What is the 31st shape in Clare's pattern? Explain or show your reasoning.

Activity 2

Generate More Patterns

1. Draw the first 12 shapes for each of these patterns.

    a. Pattern A: Repeat circle, square.

    b. Pattern B: Repeat triangle, circle, square.

    c. Pattern C: Repeat circle, triangle, circle, square.

2. What do you notice about these patterns? Why do you think this happens? Number the shapes if it helps.

3. What do you predict is the 25th shape in each pattern? Explain how you know.

4. Kiran creates a pattern with a rule that repeats these 5 shapes.

Repeat triangle, circle, triangle, circle, square.

Kiran draws the first 12 shapes in the pattern.

a. How many times does his pattern repeat?

b. What is the 12th shape?

c. Then he draws the first 24 shapes in the pattern. What would the 24th shape be?

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.OA.5; building on 3.OA.9; building towards 4.NBT.5; practicing MP7 and MP8
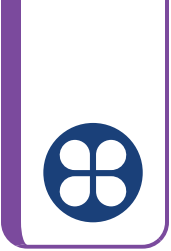
Numerical Patterns

Let's explore numerical patterns.

Activity 1

Keep Adding 9

Andre's rule for a pattern is "start with 9, keep adding 9."

Use what you've noticed about Andre's pattern to make some predictions.

1. Complete the table with the first 10 numbers in Andre's pattern.

   What do you notice about the numbers in Andre's pattern? Make at least 2 observations to share with your partner.

keep adding 9
9

2. Choose one observation you or your partner made. Explain or show why you think it happens.

a. What is the 12th number in Andre's pattern?

b. What is the 15th number?

c. What is the 25th number?

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Add 99

Elena's rule for a pattern is "start with 99, keep adding 99."

1. Complete the table with the first 5 numbers in Elena's pattern.

   Look closely at the list of numbers. Make at least 3 observations about the numbers in the pattern.

   _____

   _____

   _____

   _____

   _____

   _____

keep adding 99
99

2. Complete the table with the next 5 numbers in Elena's pattern.

Addressing CA CCSSM 4.OA.5; building on 4.NBT.1; building towards 4.NBT.5

More Numerical Patterns

Let's create and explore more numerical patterns that follow a rule.

Warm-up

Number Talk: Patterns in Multiplication

Find the value of each expression mentally.

- 20×3

- 21×3

- 40×3

- 42×3

KH | iM Illustrative Mathematics®
LEARN MATH FOR LIFE

Activity 1

A Rule with Two Digits

Mai creates a pattern that follows the rule "start with 15, keep adding 15."

keep adding 15										

1. Complete the table with the first 10 numbers in Mai's pattern.

2. What do you notice about the numbers in Mai's pattern? Describe as many observations as you can.

_____

_____

_____

_____

3. Choose one of your observations and explain why you think it happens.

_____

_____

_____

_____

4. Could 250 be a number in Mai's pattern? Explain or show your reasoning.

Activity 2

Double It

1. Andre has an idea for a pattern. His rule is "start with 1, double it."

double it	1							

   a. Complete the table with the first 8 numbers in Andre's pattern.

   b. What do you notice about the numbers in Andre's pattern?

   _____

   _____

2. What happens to the pattern if Andre starts with 10 instead of 1? His new rule is "start with 10, double it."

double it	10							

   a. Complete the table with the first 8 numbers in Andre's new pattern.

   b. What do you notice about the numbers in Andre's pattern? Describe as many observations as you can.

   _____

   _____

   _____

   _____

   c. Choose one of your observations and explain why you think it happens.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Section A Summary

We created and described shape patterns and number patterns.

We saw shapes that grew or repeated by certain rules. Then we used numbers to help us notice and explain different ways the patterns changed.

We also saw numbers that increased by certain rules and used what we know about even and odd numbers, place value, factors, multiples, and properties of operations to describe and explain the patterns.

Here are some examples of the patterns:

- Shapes that grow by a rule: start with 5, keep adding 5

  5   10   15   20
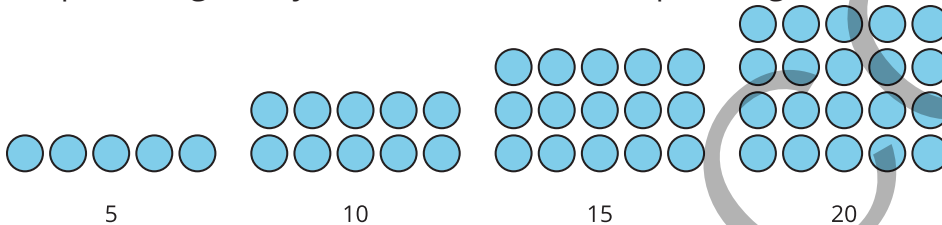
- Shapes that repeat by a rule: triangle, circle, triangle, square, repeat

  ▲ ○ ▲ ■ ▲ ○ ▲ ■ ▲ ○ ▲ ■
  1 2 3 4 5 6 7 8 9 10 11 12

  ▲ : 1, 3, 5, 7, . .

  ○ : 2, 6, 10, . . .

  ▨ : 4, 8, 12, . . .

- Numbers that change by a rule

start with 9, keep adding 9	start with 10, keep adding 10	start with 15, keep adding 15	start with 10, double it
9	10	15	10
18	20	30	20
27	30	45	40
36	40	60	80
45	50	75	160

Sec A

We learned we can use what we notice about a pattern to predict other shapes or numbers in the sequence.

Sometimes, we can use addition and multiplication to predict other shapes or numbers. Other times we can see how the digits in the numbers change to make predictions or determine if a shape or number is in the pattern.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Practice Problems

1 (Pre-unit)

Here is a list of the first ten multiples of 5:

$$5, 10, 15, 20, 25, 30, 35, 40, 45, 50$$

a. Circle the multiples of 10 in the list.

b. What do you notice about where the multiples of 10 are on the list?

_____

_____

c. Why do you think that is?

_____

_____

2 (Pre-unit)

Find the value of each expression.

a. 14×7

b. 13×6

c. 23×4

d. $85 \div 5$

3 (Pre-unit)

There are 418 students at Jada's school. There are 135 fewer students at Noah's school. How many students are there at Jada's and Noah's schools together? Explain or show your reasoning.

4 (Pre-unit)

a. What is the value of the digit 6 in each of the numbers?

- 165

- 18,622

- 675,219

b. Complete this statement so that it is true:

The value of the 6 in 675,219 is _____ times that of the 6 in 165.

5 (Pre-unit)

Find the value of each sum and difference.

$$\begin{array}{r} 1\ 5,\ 8\ 2\ 6 \\ +\ \ 4\ 4,\ 3\ 7\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 3\ 3,\ 5\ 1\ 7 \\ -\ \ 1\ 6,\ 3\ 2\ 2 \\ \hline \end{array}$$

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

6 from Unit 6, Lesson 1

a. Mai follows a rule to create a pattern of square blocks. Her rule is to keep adding 1 square to the top of her L design and 1 square to the right. Sketch or describe the next 2 shapes in Mai's pattern.
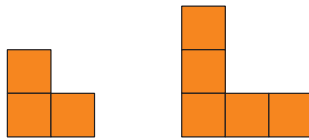
b. Will Mai's pattern ever use 20 squares? Explain your reasoning.

_____

_____

7 from Unit 6, Lesson 2

Diego types the letters a, s, d, f and then repeats them in that order, over and over.

a. What is the 5th letter Diego will type? What about the 10th? The 20th?

b. Diego numbers each letter he types, starting with 1 for the first a. What are the numbers given to the first 6 f's in his pattern?

c. What do you notice about the numbers for the f's?

_____

_____

8 from Unit 6, Lesson 3

The rule for a pattern is "start with 8, keep adding 8."

a. Complete the table with the first 10 numbers in the pattern.

b. What do you notice about the digits in the ones place? How do the digits change?

_____

_____

_____

_____

c. Why do you think it changes that way?

_____

_____

_____

_____

start with 8, keep adding 8

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

9

The rule for a pattern is "start with 25, keep adding 25."

keep adding 25	25							

a. Complete the table with the first 8 numbers of the pattern.

b. What do you notice about the numbers in the pattern? Explain or show why you think it happens.

c. Could 475 be a number in this pattern? Explain or show your reasoning.

a. Make a list of the multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10. Stop when you get a multiple of 10. For example, for 2, the list is 2, 4, 6, 8, 10.

b. What do you notice about your lists? Make some observations.

_____

_____

_____

_____

KH | **i M** Illustrative® Mathematics
LEARN MATH FOR LIFE

11 Exploration

Tyler draws this picture and writes the equation $1 + 3 + 5 = 9$.
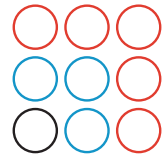
a. How do you think the equation relates to the picture?

_____

_____

b. Tyler keeps drawing circles to make larger squares. How many new circles does he need to draw to make a 4-by-4 square, and then a 5-by-5 square?

c. What pattern do you notice in the number of circles Tyler adds in each step?

_____

_____

d. Why do you think the number of circles is increasing that way?

_____

_____

Here is a growing pattern of squares that makes rectangles. The pattern follows the rule "keep adding 1 square to the row."

step 1   step 2   step 3

a. Find the area and perimeter of the rectangles in steps 2 and 3.

step	number of squares	area of rectangle (square units)	perimeter of rectangle (units)
1	2	2	6
2			
3			

b. What do you notice when you look at the numbers in the chart? Use what you notice to complete the chart for steps 4 and 5.

c. Draw the next two diagrams (for steps 4 and 5). Were your predictions for the area and perimeter of each rectangle correct?

d. How would you describe what you noticed about this pattern to a classmate?

_____

_____

_____

_____

KH | **i Illustrative® Mathematics** LEARN MATH FOR LIFE

13 (Exploration)

Mai and Tyler make their own pattern. Mai's pattern repeats @, #, and $. Tyler's pattern repeats ~ and @.

Some of their pattern symbols are the same, some are different. The table shows the first 6 symbols in Mai's pattern and the first 4 in Tyler's pattern.

Mai's pattern	@	#	$	@	#	$			
Tyler's pattern	~	@	~	@					

a. Complete the table with the next symbols in each pattern.

b. At what step do you think Mai and Tyler will draw the same symbol at the same time? Explain how you know.

Products Beyond 100

Let's find products beyond 100.

> **Warm-up**

Number Talk: A Number Times Some Multiple of 10

Find the value of each expression mentally.

- 8×30

- 5×30

- 10×30

- 15×30

Activity 1

Elena's Sticky Gift

Elena receives a sheet of fancy stickers as a gift.

How could you find the number of stickers Elena receives without counting every sticker? Show your thinking. Organize it so it can be followed by others.

More and More Stickers

1. Elena has another sheet of stickers that has 9 rows with 21 stickers in each row. How many stickers does Elena have? Explain or show your reasoning.

2. Noah's sticker sheet has 3 rows with 48 stickers in each row. Andre's sticker sheet has 7 rows with 23 stickers in each row.

   Who has more stickers? Explain or show your reasoning.

Addressing CA CCSSM 4.NBT.5; practicing MP7

Multiply Two-Digit Numbers and One-Digit Numbers

Let's multiply two-digit and one-digit numbers.

Warm-up

Notice and Wonder: With and Without a Grid

What do you notice? What do you wonder?
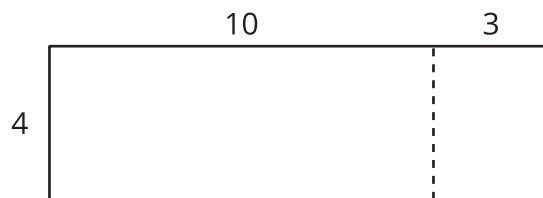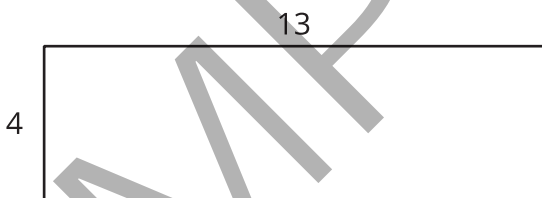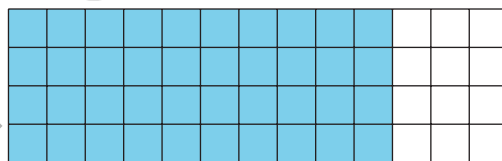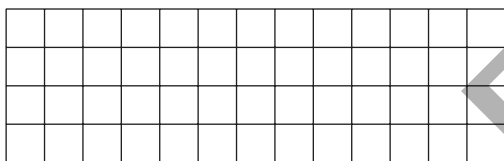
Activity 1

Tyler's Diagrams

1. Tyler uses this base-ten diagram to find the value of 4×36.
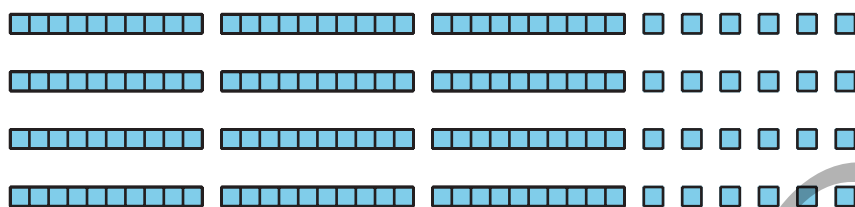
   a. Where is the 36 in Tyler's diagram?

   b. Where is the 4 in his diagram?

   c. What is the value of 4×36?

2. Tyler makes a diagram to find the value of 9×18.

   Explain or show how his diagram helps him find the value of 9×18.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE
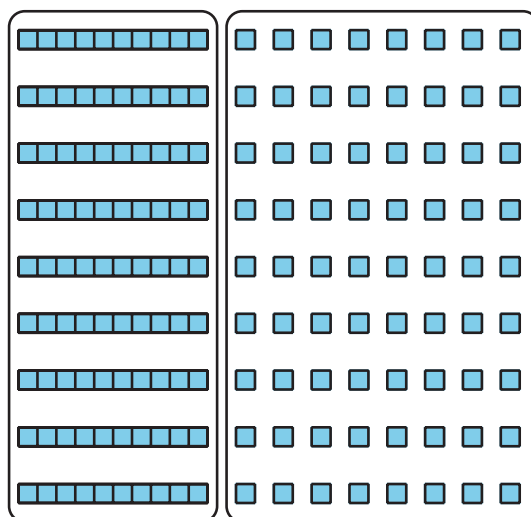
Activity 2

Two Kinds of Diagrams

1. Priya draws a base-ten diagram to multiply 6×53. She says it shows that the product can be found by adding 300 and 18.
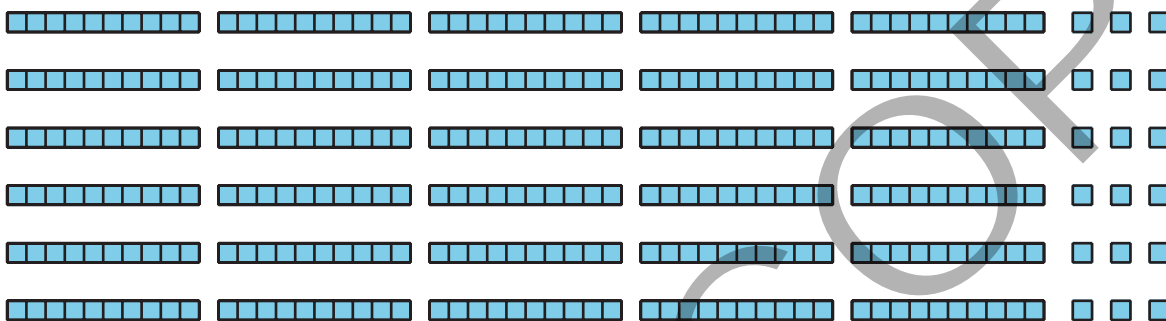
   a. Where do you see 6 and 53 in Priya's diagram?

   b. Where do you see 300 and 18 in her diagram? What do they represent?

2. Han draws this diagram to multiply 6×53:
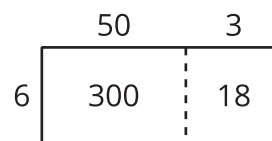
   Where do you see 300 and 18 in his diagram? What do they represent?

3. Which diagram do you prefer for multiplying 6×53: Han's way or Priya's way? Explain your reasoning.

_____

_____

_____

_____

4. Find the value of 6×53.

5. Draw a diagram to represent each multiplication expression. Then find the value of each product.

   a. 6×48

   b. 9×67

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.NBT.5; practicing MP3
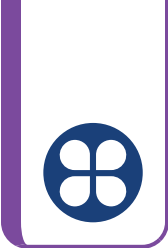
Multiply Three- and Four-Digit Numbers by One-Digit Numbers

Let's multiply three- and four-digit numbers by one-digit numbers.

Warm-up

Estimation Exploration: Mysterious Area

What is the area of the rectangle represented by the diagram?

```
            395
      ┌──────────────────┐
  6   │         ?        │
      └──────────────────┘
```
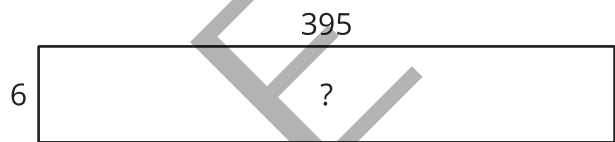
Record an estimate that is:

too low	about right	too high

Larger Numbers to Multiply

1. Clare draws this diagram.
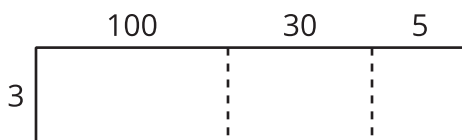
a. What multiplication expression can be represented by the diagram?

b. Find the value of the expression. Show your thinking using diagrams, symbols, or other representations.

2. Consider the expression 6×252.

a. Draw a diagram to represent the expression.

b. Find the value of the expression. Show your thinking using diagrams, symbols, or other representations.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Lin draws a diagram to represent $3 \times 2{,}135$.

100		5
3		90	
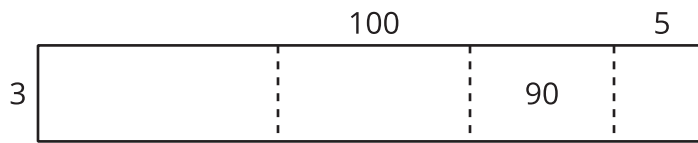
a. Complete Lin's diagram.

b. Write an expression to represent the value of each part of the diagram.

c. Find the value of $3 \times 2{,}135$. Show your thinking using diagrams, symbols, or other representations.

Activity 2

Jada's Errors

1. Jada uses this diagram to multiply $3 \times 6{,}489$. She makes a few errors in the diagram.

	6,000	400	80	9
3	18	12	24	27

a. Explain the errors Jada made.

_____

_____

_____

_____

b. Find the value of $3 \times 6{,}489$. Show your thinking using diagrams, symbols, or other representations.

KH | i⋀ Illustrative® Mathematics
LEARN MATH FOR LIFE

2. Find the value of 5×699. Show your thinking using diagrams, symbols, or other representations.

3. Find the value of $8 \times 4,973$. Show your thinking using diagrams, symbols, or other representations.

Multiply 2 Two-Digit Numbers

Let's multiply 2 two-digit numbers.

Warm-up

Sec B

Number Talk: Extra Groups

Find the value of each expression mentally.

- 20×60

- 21×60

- 20×62

- 19×60

Activity 1

Two by Two

1. For each diagram, write a multiplication expression that the diagram can represent. Then find the value of the expression. Use equations to explain or show your reasoning.

   a.

	10	8

   (6 on the left side)

   b.

	10	8

   (10 on the left side)

   c.

	10	8

   (10 and 6 on the left side)
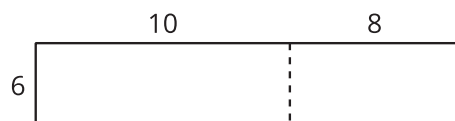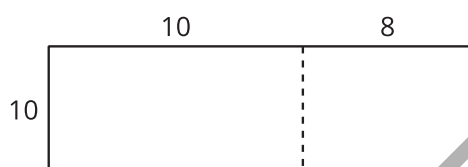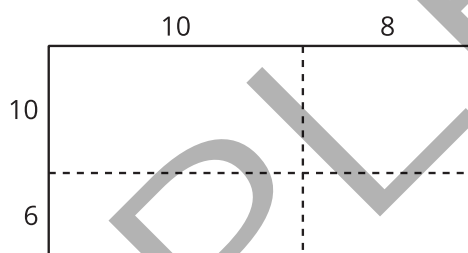
2. How are these diagrams alike? How are they different? Discuss with your partner.

3. Use a diagram to find each product.

   ◦ 13×21 ◦ 25×46

Decompose by Place Value

These diagrams could be used to find the value of 49×57.

Diagram A

	50	7
40		
9		

Diagram B

	50	7
37		
12		
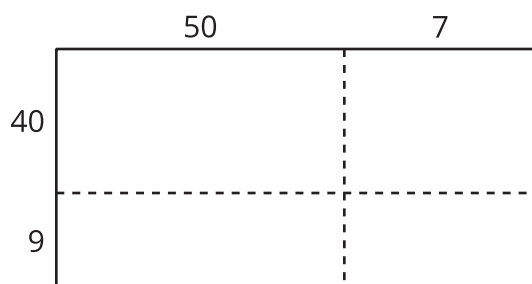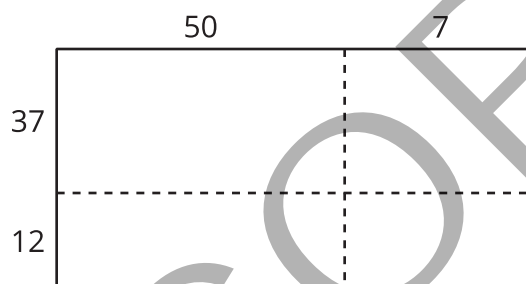
1. Which diagram is more helpful when finding the value of 49×57? Why?

_____

_____

_____

_____

2. Use a diagram to find each product.

    ◦ 49×57 ◦ 29×55

Recording Partial Products: One-Digit and Three- or Four-Digit Factors

Let's analyze and try an algorithm that uses partial products.

Warm-up

Which Three Go Together: Expressions Galore

Which 3 go together?

A

$$7 \times 50$$

B

$$(3 \times 50) + (4 \times 50)$$

C

$$(5 \times 10) \times 7$$

D

$$50 + 50 + 50 + 50 + 50 + 50 + 50$$

Activity 1

An Algorithm for Noah

1. Noah draws a diagram and writes expressions to multiply 2 numbers.
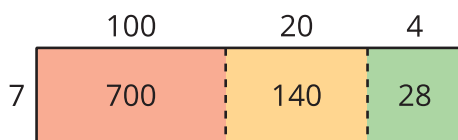
	100	20	4
7	700	140	28

700 + 140 + 28 = 868

7×124
$7 \times (100 + 20 + 4)$
$(7 \times 100) + (7 \times 20) + (7 \times 4)$
$700 + 140 + 28$

How does each expression represent Noah's diagram?

_____

_____

2. Noah learns another way to record the multiplication.

Step 1

```
    1  2  4
  ×       7
  ─────────
       2  8     7 × 4
```

Step 2

```
    1  2  4
  ×       7
  ─────────
       2  8     7 × 4
    1  4  0     7 × 20
```

Step 3

```
    1  2  4
  ×       7
  ─────────
       2  8     7 × 4
    1  4  0     7 × 20
  + 7  0  0     7 × 100
  ─────────
    8  6  8
```

Make sense of each step of the calculations and record your thoughts.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Complete the diagram to find the value of 384×5. Use Noah's recording method to check your work.


```
    3  8  4
×         5
```

5×4

5×80

5×300

```
+ _____
```
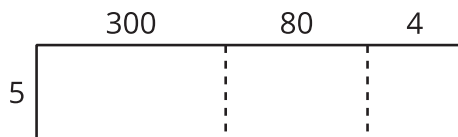
Activity 2

Try an Algorithm with Partial Products

Noah and Mai want to find the value of $6 \times 2{,}947$. They record their steps in different ways.

Noah

```
      2, 9  4  7
  ×          6
  ───────────────
            4  2
         2  4  0
      5, 4  0  0
  + 1 2, 0  0  0
```

Mai

```
      2, 9  4  7
  ×          6
  ───────────────
  1 2, 0  0  0
   5, 4  0  0
      2  4  0
  +        4  2
```

1. How are these notations alike? How are they different?

    _____

    _____

    _____

    _____

2. Use a diagram to show what each of the partial products 42, 240, 5,400 and 12,000 represent. Then find the value of $6 \times 2{,}947$.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Find the value of each expression. Use the algorithm that Noah used for at least one expression. Show your thinking using diagrams, symbols, or other representations.

a. $4 \times 5{,}342$

b. 7×983

Using Algorithms with Partial Products: 2 Two-Digit Numbers

Let's try to multiply two-digit numbers with an algorithm that uses partial products.

Warm-up

Notice and Wonder: Ways to Keep Track

What do you notice? What do you wonder?

$$
\begin{array}{r}
14000 \\
700 \\
350 \\
56 \\
\hline
15,106
\end{array}
\qquad
\begin{array}{r}
2158 \\
\times \quad 7 \\
\hline
56 \\
350 \\
700 \\
14000 \\
\hline
15,106
\end{array}
$$

KH | iM Illustrative Mathematics®
LEARN MATH FOR LIFE

Activity 1

Partial Products, Recorded

1. Tyler uses an algorithm to find the value of 64×87.

$$
\begin{array}{r}
64 \\
\times\ 87 \\
\hline
28 \\
420 \\
320 \\
4800 \\
\hline
5,568
\end{array}
$$

Describe each step in Tyler's method. How do you think he arrived at the last five numbers? Record your thinking.

2. Use Tyler's method to find the value of 31×15. Then draw a diagram to check your answer.

Activity 2

Han's Multiplication Mishap

1. Decide with your partner who will find each product. Show your thinking. Organize it so it can be followed by others.

$$
\begin{array}{r}
1\ \ 9 \\
\times\ \ 3\ \ 2 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
3\ \ 2 \\
\times\ \ 1\ \ 9 \\
\hline
\end{array}
$$

2. Here is Han's computation of 81×47.

$$
\begin{array}{r}
8\ 1 \\
\times\ \ 4\ 7 \\
\hline
7 \\
5\ 6 \\
4\ 0 \\
3\ 2\ 0 \\
\hline
4\ 2\ 3
\end{array}
$$

a. What error or errors did Han make?

b. Show the correct computation for finding the value of 81×47.

$$
\begin{array}{r}
8\ \ 1 \\
\times\ \ 4\ \ 7 \\
\hline
\end{array}
$$

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Partial Products and the Standard Algorithm

Let's compare multiplication algorithms.

Warm-up

Number Talk: The Value of the Digits

Find the value of each expression mentally.

- 5×101

- 5×102

- 5×203

- 5×404

Activity 1

Two Algorithms to Multiply

1. Here are two algorithms for finding the value of 3×713.

    Kiran

    ```
        7  1  3
      ×       3
      2, 1  3  9
    ```

    Diego

    ```
          7  1  3
      ×         3
                9
             3  0
      +  2, 1  0  0
      2, 1  3  9
    ```

    Discuss with your partner:

    a. How are these algorithms alike? How are they different?

    b. How do you think Kiran found the product 2,139?

2. Find the value of each product.

    ◦ 212×4

    ◦ $3 \times 4{,}132$

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Algorithm Comparison

1. Analyze the two algorithms used to find the value of 4×223.

Kiran

$$
\begin{array}{r}
\overset{1}{} \\
2\ 2\ 3 \\
\times \qquad 4 \\
\hline
8\ 9\ 2
\end{array}
$$

Diego

$$
\begin{array}{r}
2\ 2\ 3 \\
\times \qquad 4 \\
\hline
1\ 2 \\
8\ 0 \\
+\ 8\ 0\ 0 \\
\hline
8\ 9\ 2
\end{array}
$$

a. How are these algorithms alike? How are they different?

_____

_____

b. Where is the 12 in Kiran's algorithm?

2. a. Try using Kiran's algorithm to find the value of 512×3.

b. Check your work using a different method.

Sec B

Addressing CA CCSSM 4.NBT.5, 4.MD.2; practicing MP1 and MP2

Solve Problems Involving Multiplication

Let's solve problems using what we learned about multiplication of whole numbers.

Sec B

Warm-up

What Do You Know about 1 Year?

What do you know about 1 year?

KH | iM Illustrative Mathematics®
LEARN MATH FOR LIFE

Activity 1

Time Flies When We Leap Years

1. A baby elephant was born exactly 48 weeks ago. How many days old is the elephant?

2. A leap year has 366 days. A non-leap year has 365 days. How many days are in 3 leap years?

3. In our calendar system, some months are 31 days long, some are 30 days long, and one month (February) is either 28 or 29 days long.

   What if the calendar system changed so that each month has 31 days? How many more days would there be in a year?
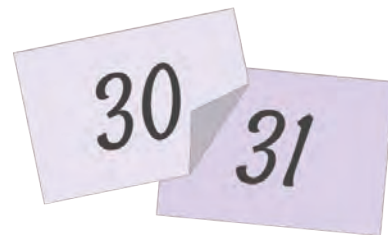
Activity 2

Coin Collection

1. Lin's family collects 2,074 nickels. How many pennies are worth the same amount?

2. If Lin's family saves 2,074 nickels each year for 4 years, how many nickels will her family have?

3. Create a situation that involves a problem that can be solved by finding the value of $8 \times 1,049$. Solve the problem. Explain or show your reasoning.

_____

_____

_____

_____

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Section B Summary

We learned to multiply factors whose products are greater than 100, using different representations and strategies.

When working with multi-digit factors, it helps to decompose them by place value before multiplying. For example, to find the value of $4 \times 5{,}342$, we can decompose the 5,342 into its expanded form, $5{,}000 + 300 + 40 + 2$, and then use a diagram or an algorithm to help us multiply.

	5,000	300	40	2
4	20,000	1,200	160	8

20,000 + 1,200 + 160 + 8 = 21,368

$$\begin{array}{r} 5{,}342 \\ \times \quad 4 \\ \hline 8 \\ 160 \\ 1{,}200 \\ + \quad 20{,}000 \\ \hline 21{,}368 \end{array}$$

In both the diagram and the algorithm, the 20,000, 1,200, 160, and 8 are called the partial products. They are the result of multiplying each decomposed part of 5,342 by 4.

We can do the same to multiply a two-digit number by another two-digit number.

For example, here are two ways to find the value of 31×15. The 31 is decomposed into $30 + 1$ and 15 is decomposed into $10 + 5$.

$$\begin{array}{r} 31 \\ \times \quad 15 \\ \hline 5 \\ 150 \\ 10 \\ + \quad 300 \\ \hline 465 \end{array}$$

	30	1
10	10 × 30 = 300	10 × 1 = 10
5	5 × 30 = 150	5 × 1 = 5

Practice Problems

1 from Unit 6, Lesson 5

Mai has a sheet of stickers with 23 rows and 8 stickers in each row.

a. Does Mai have more or less than 100 stickers? Explain your reasoning.

_____

_____

b. How many stickers does Mai have? Explain or show your reasoning.

2 from Unit 6, Lesson 6

Find the value of 7×64. Use a diagram if it is helpful.

3 from Unit 6, Lesson 7

a. Use the diagram to find the value of 8×573.

	500	70	3
8			
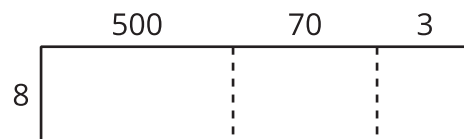
b. Find the value of $4 \times 3{,}516$.

4 from Unit 6, Lesson 8

a. Use the diagram to find the value of 47×62.

	60	2
40		
7		
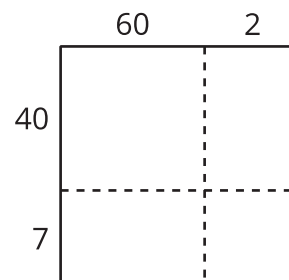
b. Is this diagram helpful to finding the value of 47×62? Explain your reasoning.

	48	14
34		
13		

_____

_____

_____

_____

Sec B

5
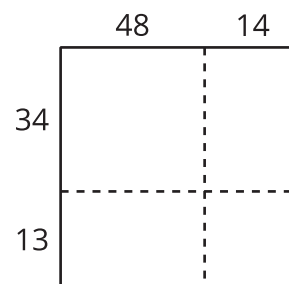
The diagram and calculations show two ways for finding the value of $2{,}518 \times 6$.

	2,000	500	10	8
6				
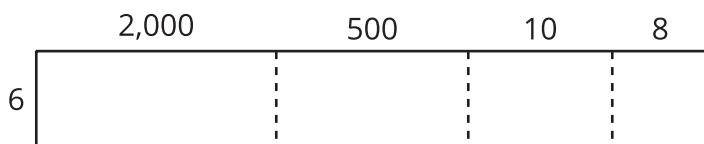
```
       2,  5  1  8
    ×            6
             1
             4  8
             6  0
       3,  0  0  0
 +  1  2,  0  0  0
    1  5,  1  0  8
```

a. How does each part of the vertical calculation relate to the diagram?

_____

_____

b. Find the value of $3{,}172 \times 5$ using a method of your choice.

6 from Unit 6, Lesson 10

Here is an incomplete calculation that uses partial products of 65×43.

a. Write multiplication expressions that the numbers 15, 180, 200, and 2,400 each represent. Then find the value of 65×43.

$$
\begin{array}{r}
6\ 5 \\
\times\ \ 4\ 3 \\
\hline
1\ 5 \\
1\ 8\ 0 \\
2\ 0\ 0 \\
+\ \ 2,\ 4\ 0\ 0 \\
\hline
\end{array}
$$

b. Find the value of the product 45×38.

7 from Unit 6, Lesson 11

Here is how Elena calculates the value of 723×3.

$$
\begin{array}{r}
7\ 2\ 3 \\
\times\ \ \ \ \ \ \ 3 \\
\hline
2,\ 1\ 6\ 9 \\
\end{array}
$$

a. Where does the 9 in Elena's calculation come from? What about the 6?

b. Where do the 2 and the 1 in calculation come from?

c. Use Elena's method to find the value of 534×2.

8 from Unit 6, Lesson 12

There are 4,218 students in school district A. School district B has 3 times as many students as school district A. How many students are in school district B? Explain or show your reasoning.

9 (Exploration)

Clare checks her answers for some products. Without doing the computation again, she knows that these answers are incorrect. How might Clare have known?

a. $5 \times 5{,}783 = 27{,}914$

b. $7 \times 8{,}419 = 54{,}253$

c. $9 \times 9{,}999 = 99{,}999$

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

10 (Exploration)

Here is Mai's strategy to find the value of $9 \times 8{,}235$.

$$\begin{array}{r} 8\,2{,}3\,5\,0 \\ -\quad 8{,}2\,3\,5 \\ \hline 7\,4{,}1\,1\,5 \end{array}$$

a. Explain why Mai's method works.

_____

_____

_____

_____

b. Use Mai's method to find the value of $9 \times 6{,}789$.

c. Find the value of $9 \times 6{,}789$ using a strategy you learned. How is Mai's method like yours? How is it different?

_____

_____

_____

_____
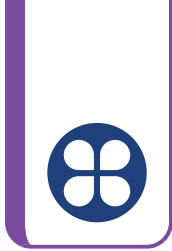
Addressing CA CCSSM 4.NBT.6; building on 3.OA.2, 3.OA.3; practicing MP7

Situations Involving Equal-Size Groups

Let's solve division problems.

Warm-up

Estimation Exploration: Lots of Paletas

How many paletas (pah-LAY-tuhs) are in the case?

Record an estimate that is:

too low	about right	too high

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 1

Paletas for a Class Party

Diego's aunt buys paletas, which are ice pops, for a class party. At the local market, paletas come in different flavors. She buys the same number of paletas of each flavor.

1. What mathematical questions can we ask about this situation?

_____

_____

_____

_____

2. Here is an equation: $84 \div 7 = ?$

   In the situation about the class party, what questions could the equation represent?

_____

_____

_____

_____

3. Find the answer to one of the questions you wrote. Explain or show your reasoning.

Activity 2

More Snacks for a Class Party

1. Priya's mom makes 85 gulab jamuns for the class to share. Priya gives 5 gulab jamuns to each student in the class.

   How many students are in Priya's class? Explain or show your reasoning.

2. Han's uncle sends 108 chocolate-covered breadsticks for a snack. The students in Han's class are seated at 6 tables. Han plans to give the same number of breadsticks to each table.

   How many breadsticks does each table get? Explain or show your reasoning.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.MD.3, 4.NBT.6, 4.OA.3; building on 3.MD.7, 4.NBT.5; practicing MP3 and MP5
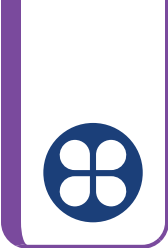
Situations Involving Area

Let's divide to find the side length of a rectangle.

Warm-up

Estimation Exploration: Area of a Soccer Field

Estimate: What is the length of the soccer field in meters?

? meters

40 meters

area: 2,280 square meters
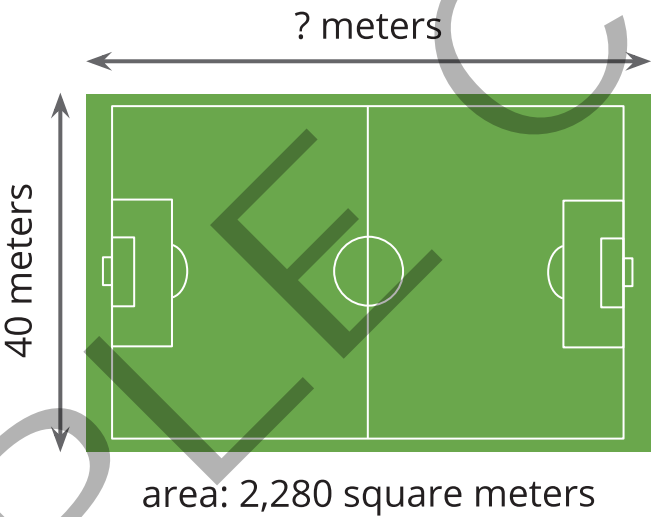
Record an estimate that is:

too low	about right	too high

Sec C

Activity 1

Elena's Mural

Elena uses 189 square tiles to create a rectangular mural for the art club. The mural is 7 tiles wide.
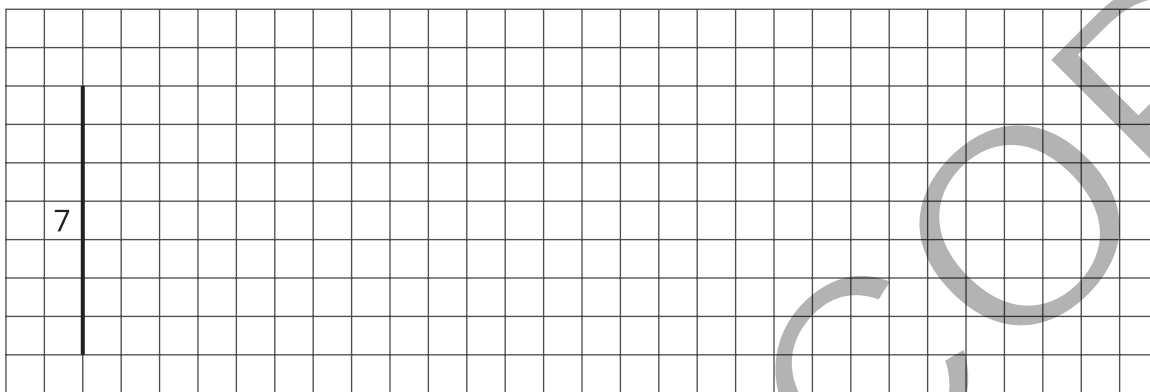
1. How many tiles long is Elena's mural? Explain or show your reasoning.

2. Write one or more equations that show how you solved this problem.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

Tyler's Mural

Tyler is also creating a rectangular mural for the art club. He has 197 tiles for his mural. He wants the mural to be 6 tiles wide with no gaps or overlaps between the tiles.

1. Will Tyler use all of his tiles in the mural? Explain your reasoning.

_____

_____

_____

_____

2. How many tiles long will Tyler's mural be? Show your thinking using diagrams, symbols, or other other representations.

Sec C

Base-Ten Blocks to Divide

Let's use base-ten blocks to divide.

Warm-up

What Do You Know about Base-Ten Blocks?

What do you know about base-ten blocks?

KH | iM Illustrative Mathematics®
LEARN MATH FOR LIFE
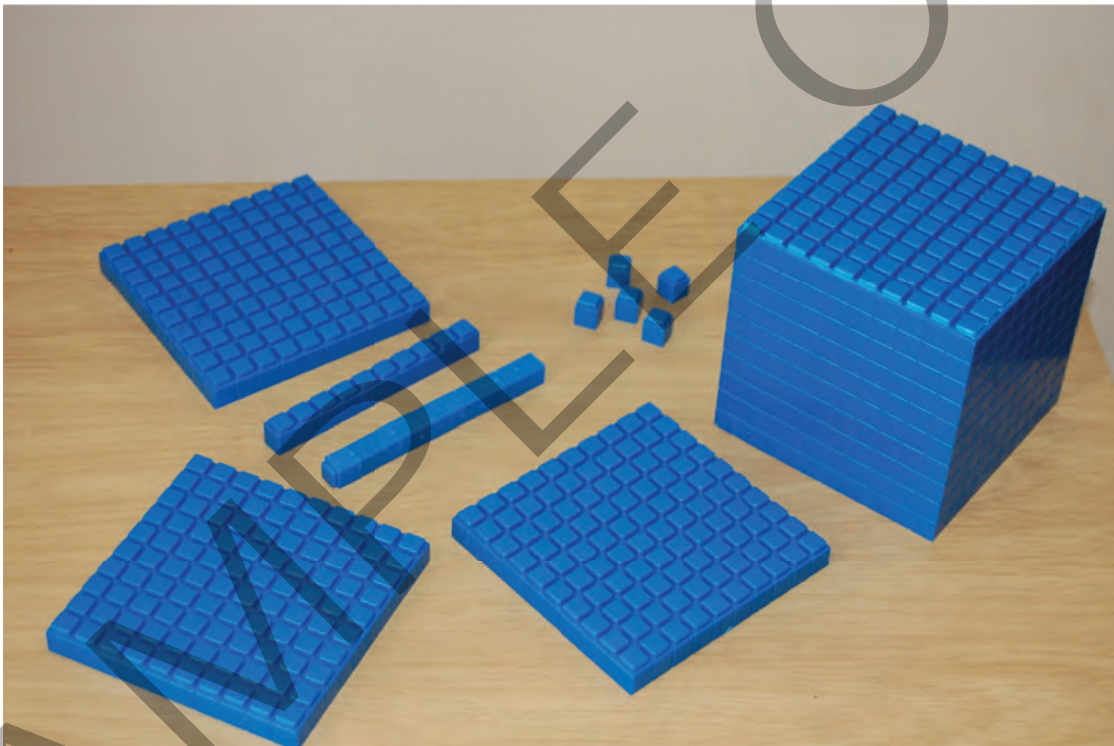
Activity 1

Blocks to Divide

Use the base-ten blocks to represent each expression. Then find the value of each expression.

1. $488 \div 4$

2. $104 \div 8$

Sec C

Show Us Your Blocks

Find the value of each expression. Explain or show your reasoning. Use base-ten blocks if it helps.

1. $96 \div 4$

2. $86 \div 2$

3. $108 \div 9$

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Base-Ten Diagrams to Represent Division

Let's make sense of base-ten diagrams that represent division.

Warm-up

Which Three Go Together: Base-Ten Diagrams

Which 3 go together?

A
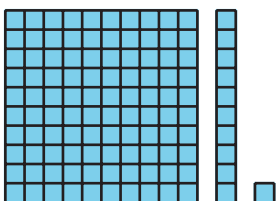
B
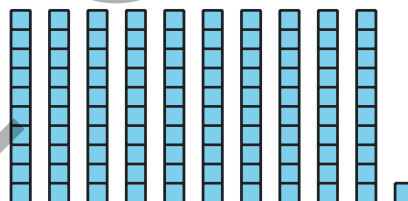
C
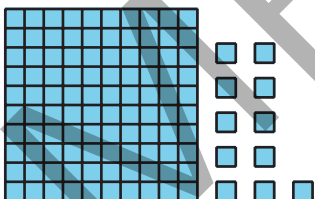
D
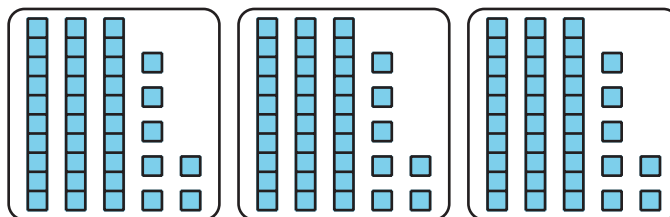
Activity 1

Divide with Diagrams or Blocks

1. Priya draws a base-ten diagram to find the value of $64 \div 4$. A rectangle represents 10. A small square represents 1.

   Use the diagram (or actual blocks) to help Priya complete the division. Explain or show your reasoning.
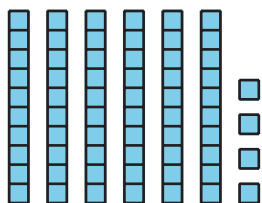

2. Use this base-ten diagram (or actual blocks) to find the value of $117 \div 3$.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE
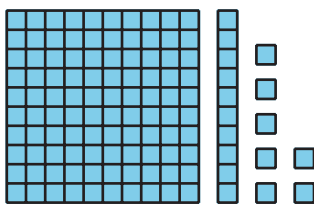
Activity 2

Help Noah Get Unstuck

1. This diagram represents 235.
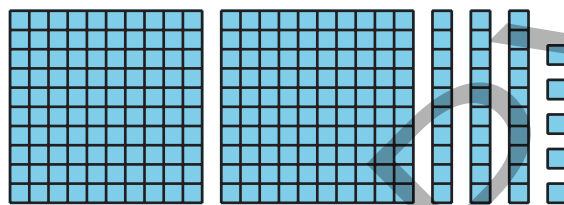
Noah draws this diagram to find 235 ÷ 5 and then gets stuck.
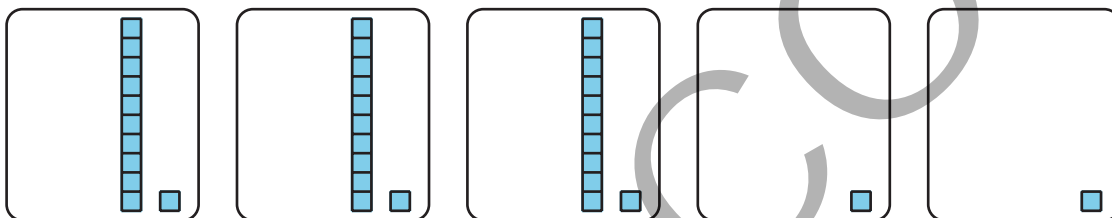
He says, "There are not enough of the hundreds or the tens pieces to put into 5 groups."

Explain or show how Noah could find 235 ÷ 5 with his diagram.

2. Find the value of 432 ÷ 6. Show your thinking using diagrams, symbols, or other representations. Use base-ten diagrams or blocks if you find them helpful.

An Algorithm with Partial Quotients

Let's make sense of and use an algorithm that uses partial quotients.

Warm-up

Number Talk: Divide by 3

Find the value of each expression mentally.

- $90 \div 3$

- $96 \div 3$

- $960 \div 3$

- $954 \div 3$

Activity 1

Decompose Dividends

1. Find the value of $465 \div 5$. Explain or show your reasoning. You may use base-blocks if you find them helpful.

2. Here is how Priya finds the value of $465 \div 5$.

$$400 \div 5 = 80$$
$$60 \div 5 = 12$$
$$5 \div 5 = 1$$
$$\overline{465 \div 5 = 93}$$

a. Describe the steps in Priya's method.

_____

_____

b. How is Priya's method similar to your method?

_____

_____

c. Use Priya's method to find the value of $428 \div 4$.

Activity 2

Tyler's Method

Tyler uses a different method to find the value of $465 \div 5$. Let's compare Priya's and Tyler's work.

Priya's method

$$400 \div 5 = 80$$
$$60 \div 5 = 12$$
$$5 \div 5 = 1$$
$$\overline{465 \div 5 = 93}$$

Tyler's method

```
        93
         1
        12
        80
    5)465
   −  400      5 × 80
       65
   −   60      5 × 12
        5
   −    5      5 × 1
        0
```

1. How are these methods alike? How are they different? List as many similarities and differences as you can find.

   _____

   _____

   _____

   _____

2. Why do you think Tyler uses subtraction in his method?

   _____

   _____

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Show how Tyler might record the process of finding the value of 428 ÷ 4.

Addressing CA CCSSM 4.NBT.6, 4.OA.3; practicing MP3

Use an Algorithm with Partial Quotients

Let's find quotients using an algorithm that uses partial quotients.

Warm-up

Number Talk: Partial Quotients

Find the value of each expression mentally.

- $500 \div 5$

- $60 \div 5$

- $5 \div 5$

- $565 \div 5$

KH | iM Illustrative Mathematics® LEARN MATH FOR LIFE

Activity 1

A Stack of Partial Quotients

Jada uses partial quotients to find out how many groups of 7 are in 392.

1. Analyze Jada's steps in the partial-quotients algorithm.

$$
\begin{array}{r}
\boxed{56} \\
9 \\
7 \\
40 \\
7\,\overline{)392} \\
-\ 280 \\
\hline
112 \\
-\ \ 49 \\
\hline
63 \\
-\ \ 63 \\
\hline
0
\end{array}
$$

a. Look at the 3 numbers above 392. What do they represent?

_____

_____

b. Look at the 3 subtractions below 392. What do they represent?

_____

_____

2. Show another way you can decompose 392 to divide by 7.

3. Use a partial-quotients algorithm to find the value of $702 \div 3$.

Activity 2

Andre and Elena's Work

Andre and Elena divide 2,315 by 5. Before they begin, Andre says, "I can already tell that the quotient is going to be less than 500."

1. Decide if you agree with Andre without doing any calculations. Explain your reasoning.

_____

_____

_____

_____

2. Here is Andre and Elena's work. Each student made one or more errors. Identify the errors each student made. Then show a correct computation.

Andre's Work

$$
\begin{array}{r}
\boxed{103} \\
3 \\
60 \\
40 \\
5\,\overline{)\,2{,}315} \\
-2{,}000 \\
\hline
315 \\
-300 \\
\hline
15 \\
-15 \\
\hline
0
\end{array}
$$

Elena's Work

$$
\begin{array}{r}
\boxed{400} \\
60 \\
100 \\
300 \\
5\,\overline{)\,2{,}315} \\
-1{,}500 \\
\hline
815 \\
-500 \\
\hline
315 \\
-300 \\
\hline
15
\end{array}
$$

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Incomplete Calculations

Here are 4 calculations to find the value of $3,294 \div 3$, but each one is unfinished.

Complete at least 2 of the unfinished calculations.

A

$$
\begin{array}{r}
90 \\
1,000 \\
3\overline{)3,294} \\
-3,000 \quad 3 \times 1,000 \\
\hline
294 \\
-270 \quad 3 \times 90 \\
\hline
\end{array}
$$

B

$$
\begin{array}{r}
80 \\
200 \\
400 \\
400 \\
3\overline{)3,294} \\
-1,200 \quad 3 \times 400 \\
\hline
2,094 \\
-1,200 \quad 3 \times 400 \\
\hline
894 \\
-600 \quad 3 \times 200 \\
\hline
294 \\
-240 \quad 3 \times 80 \\
\hline
\end{array}
$$

C

$$600 \div 3 =$$
$$600 \div 3 =$$
$$600 \div 3 =$$
$$600 \div 3 =$$
$$600 \div 3 =$$
$$270 \div 3 =$$

D

$$
\begin{array}{r}
3,300 \div 3 = 1,100 \\
-6 \div 3 = 2 \\
\hline
\end{array}
$$

Sec C

Divide with Remainders

Let's solve problems involving division including interpreting remainders.

Warm-up

Notice and Wonder: Expressions and Equations

What do you notice? What do you wonder?

division expression	multiplication equations
$100 \div 5$	$100 = 20 \times 5$
$101 \div 5$	$101 = 20 \times 5 + 1$
$102 \div 5$	$102 = 20 \times 5 + 2$
$103 \div 5$	$103 = 20 \times 5 + 3$
$104 \div 5$	$104 = 20 \times 5 + 4$
$105 \div 5$	$105 = 20 \times 5 + 5$

Sec C

Activity 1

$182 \div 6$

$182 \div 6$

Write division situations that the expression can represent.

_____

_____

_____

_____

Sec C

Activity 2

Is There a Remainder?

1. Decide if each expression will result in a remainder. Explain how you know.

    a. $753 \div 6$

    _____

    _____

    b. $918 \div 9$

    _____

    _____

    c. $1,263 \div 2$

    _____

    _____

    d. $2,630 \div 5$

    _____

    _____

2. Find the value of 2 expressions. Choose one with a remainder and one without. Explain or show your reasoning.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Interpret Remainders in Division Situations

Let's solve problems involving division and interpret remainders.

Activity 1

Hearts and Seats

1. Two artists paint 378 wooden hearts. They sell the hearts in boxes of 4.

   The first artist says they will need 94 boxes for all the hearts.

   The second artist says 95 boxes are needed.

   Who do you agree with? Explain or show your reasoning.

2. An auditorium seats 258 people. The seats are arranged in rows of 9, but there is one short row with fewer than 9 seats.

   How many rows of 9 seats are there? How many seats are in the shorter row?

Activity 2

Save for a Garden

1. A school needs $1,270 to build a garden. After saving the same amount each month for 8 months, the school still needs $6.

    How much did they save each month? Explain or show your reasoning.

2. Choose one of the following division expressions.

    $$711 \div 3 \qquad\qquad 3{,}128 \div 8$$

    a. Write a situation to represent the expression.

    _____

    _____

    b. Find the value of the quotient. Explain or show your reasoning.

    c. What does the value of the quotient represent in your situation?

    _____

    _____

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.NBT.6, 4.OA.3; building on 4.NBT.5, 4.OA.4; practicing MP6

Problems with Remainders

Let's think about remainders to solve problems.

Warm-up

Number Talk: Dividing by 7

Find the value of each expression mentally.

- $21 \div 7$

- $35 \div 7$

- $140 \div 7$

- $196 \div 7$

Activity 1

Write Multiples

1. Han starts writing multiples of a number. When he reaches 104, he has written 8 numbers.

   For each of the following questions, explain or show your reasoning.

130
117
104

   a. What number is Han writing multiples of?

   b. What is the 15th multiple of this number?

   c. Han gets to 286. How many numbers has he written at that point?

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

2. Kiran wants to know how many multiples of 7 are between 0 and 150.

   a. He thinks he can use division to find out. Do you agree? Explain your reasoning.

   _____

   _____

   _____

   _____

   b. How many multiples will he find? Show your reasoning.

   c. Is 150 a multiple of 7? Show how you know.

3. Han writes the number 295. He is wondering if he made a mistake. Determine if 295 is a multiple of 8 and explain your reasoning.

   _____

   _____

Sec C

Jada's Mystery Number

Jada writes multiples of a mystery number. After writing some numbers, she writes the number 126.

- Mai says 6 is the mystery number.
- Priya says 8 is the mystery number.
- Andre says 9 could be the mystery number.

1. Which student do you agree with? Show how you know using equations.

2. Jada gives one more clue: "If I keep writing multiples, I'll get to 153."

   What is the mystery number? Explain or show your reasoning.

Activity 3

Watch Your Remainder!

Directions:

- Spin the spinner to get your one-digit divisor.

- Each partner:

  ◦ Use 3–4 cards to create a dividend.

  ◦ Write a multiplication equation to represent the quotient. (For example, $109 \div 9$ is written as $(9 \times 12) + 1$ and your score is 1.)

  ◦ Check your partner's work to make sure you agree.

  ◦ Your score for each round is the remainder.

- Take new cards so that you have 4 cards to start the next round.

- The partner who has the fewest points once the recording sheet is full wins the game.

Sec C

👥 Section C Summary

We solved different problems that involved dividing whole numbers.

We recalled two ways of thinking about division.

For example, if $274 \div 8$ represents a situation where 274 markers are put into equal groups. The value of $274 \div 8$ can tell us:

- How many markers are in each group if there were 8 groups.

- How many groups can be made if there were 8 markers in each group.

We learned that in $274 \div 8$, the 274 is called the **dividend**, and the 8 is called the **divisor.** We then identified many ways to find the value of a **quotient** —or the result of the division. For $274 \div 8$, we can:

- Think about whether one number is a multiple or factor of another number. For example, "Is 274 a multiple of 8?" or "Is 8 a factor of 274?"

- Divide by place value and think about putting 2 hundred, 7 tens, and 4 ones into 8 equal groups.

- Divide in parts and find partial quotients. For example, we can first find $160 \div 8$ (which is 20), and then $80 \div 8$ (which is 10), and then $32 \div 8$ (which is 4).

- Think in terms of multiplication. For example, we can think of $8 \times 20 = 160$, $8 \times 10 = 80$, and so on.

Here is one way to record division using partial quotients.

$$
\begin{array}{r}
\boxed{34} \\
4 \\
10 \\
20 \\
8\overline{)274} \\
-\ 160 \qquad 8 \times 20 \\
\hline
114 \\
-\ \ 80 \qquad 8 \times 10 \\
\hline
34 \\
-\ \ 32 \qquad 8 \times 4 \\
\hline
2
\end{array}
$$

Sometimes a division results in a leftover that can't be put into equal groups or is not enough to make a new group. We call the leftover a **remainder**. Dividing 274 by 8 gives 34 and a remainder of 2.

KH | i̇M Illustrative Mathematics
LEARN MATH FOR LIFE

Practice Problems

1 from Unit 6, Lesson 13

a. If 5 pencils cost 95 cents, how much does each pencil cost? Explain or show your reasoning.

b. If 68 colored pencils are split evenly between 4 students, how many pencils does each student get? Explain or show your reasoning.

2 from Unit 6, Lesson 14

Clare has 194 square tiles. Can Clare put all of her tiles in 6 rows with the same number of tiles in each row? Explain or show your reasoning.

Sec C

3 from Unit 6, Lesson 15

A long, rectangular hallway is 8 feet wide and has an area of 368 square feet. How long is the hallway?

a. Write a multiplication equation and a division equation that represent the situation.

b. Find the length of the hallway. Explain or show your reasoning.

4 from Unit 6, Lesson 16

Here is 378 represented with base-ten blocks.
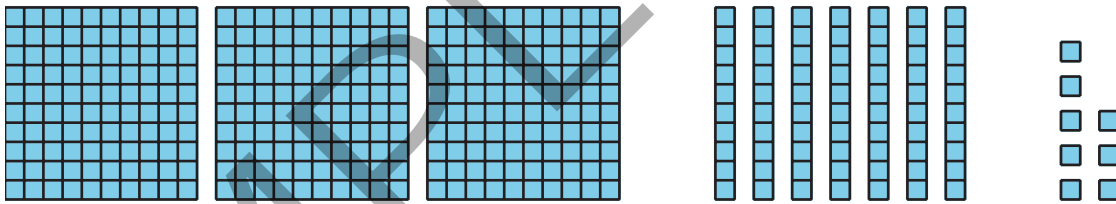
Use words, diagrams, or equations to show how to use the base-ten blocks to find the value of $378 \div 6$.

5

Here are 2 incomplete calculations of $864 \div 4$. Complete each calculation to find the value of the quotient.

A

$$800 \div 4 = 200$$
$$40 \div 4 =$$
$$20 \div 4 =$$
$$4 \div 4 =$$
$$\overline{864 \div 4 =}$$

B

$$
\begin{array}{r}
100 \\
100 \\
4\overline{)864} \\
-400 \quad 4 \times 100 \\
\hline
464 \\
-400 \quad 4 \times 100 \\
\hline
\end{array}
$$

6

What is the value of $3{,}420 \div 5$?

Use partial quotients to show your reasoning.

7 from Unit 6, Lesson 19

   a. Use partial quotients to find the value of $637 \div 4$.

   b. There are 637 toothpicks and 4 people. What could $637 \div 4$ mean in this situation? What could each step you took in the algorithm mean?

   c. What does the value of the quotient represent in the situation?

8 from Unit 6, Lesson 20

There are 875 peaches at the orchard. Each box contains 9 peaches. How many boxes are needed for the peaches? Explain your reasoning.

KH | iM Illustrative Mathematics
LEARN MATH FOR LIFE

9 from Unit 6, Lesson 21

Priya writes the multiples of a number and 63 is on her list. Priya's number is not 1.

a. What could Priya's number be? Explain your reasoning.

_____

_____

_____

_____

b. 112 is the last number on Priya's list. What is Priya's number? How many numbers are on Priya's list?

10 (Exploration)

Consider the expression $286 \div 5$.

a. Write a division story with a question that can be answered by finding the value of $286 \div 5$. Then answer the question.

_____

_____

_____

_____

Sec C

b. Write a different story with a question that can be answered by finding the value of $286 \div 5$ but with a different answer than your first story. Answer the question.

_____

_____

_____

_____

11 (Exploration)

Mai has a special way of knowing that 531 is a multiple of 9. She says, "Each hundred is 11 nines and 1 more and each ten is one nine and 1 more, so 531 is 58 nines and 9 more."

a. Make sense of and explain Mai's reasoning. Is 531 a multiple of 9?

_____

_____

_____

_____

b. Use Mai's reasoning to decide if 648 is a multiple of 9.

KH | **iM** Illustrative® Mathematics
LEARN MATH FOR LIFE

Addressing CA CCSSM 4.NBT.5, 4.NBT.6, 4.OA.3; practicing MP2

Different Ways to Solve Problems

Let's reason about and solve multi-step problems.

Warm-up

Which Three Go Together: Expressions with 5 or 90

Which 3 go together?

A

$$5 \times 90$$

B

$$90 + 90 + 90 + 90 + 90$$

C

$$(4 \times 90) + (1 \times 90)$$

D

$$3 \times 3 \times 10 \times 5$$

Going on a Field Trip

1. There are 45 students going on a field trip to a museum. Tickets for the museum are $18 each. Teachers have $900 to buy tickets for the trip. Will this be enough money to buy a ticket for every student?

   If yes, will there be any leftover money? How much?

   If no, how much more money is needed?

2. Your teacher will show 5 strategies for answering the first question. Analyze the strategies.

   a. Which strategy is closest to yours? With a partner, take turns explaining how your strategy is close to the poster you chose.

   b. Discuss a different strategy with your partner. Try using this strategy to find the value of 14×35.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Activity 2

A Trip to the Movies

Movie tickets are $9 each. The theater sells the same number of tickets 2 days in a row.

The theater made $3,132 from ticket sales on the first day.

1. Record and answer one question of your choice from the list the class generated. Discuss your strategy with your partner.

   _____

   _____

2. Use the given information about movie tickets to complete the following statement:

   _____ tickets were sold on the first and second days.

3. A medium drink is $7 and small popcorn is $5. If each ticket holder purchases popcorn and a drink, how much money will the theater collect from the sales of popcorn and drinks?

Sec D

Problems about Perimeter and Area

Let's solve situations involving perimeter and area.

Sec D

Warm-up

How Many Do You See: Shaded Squares

How many shaded squares do you see? How do you see them?

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE
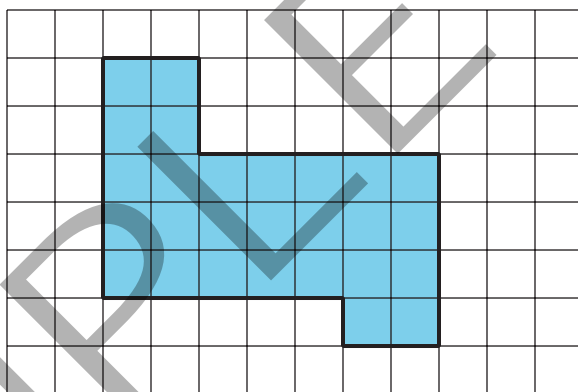
Activity 1

Create a Class Banner

Jada's teacher buys a poster paper that measures 36 inches by 24 inches. Her plan is to cut it into pieces, rearrange them, and tape them back together to create a welcome banner that is 8 inches tall and 8 feet long.

36 inches

24 inches
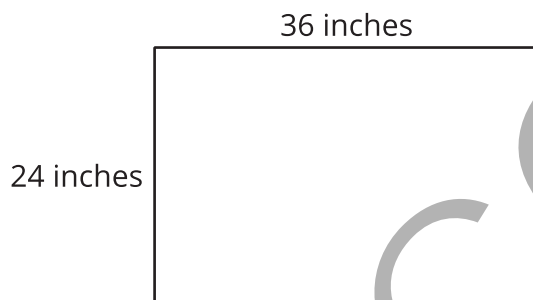
1. Does she have enough paper to make the banner? Explain or show your reasoning.

2. What is the area of the poster paper?

Replace the Classroom Carpet

A classroom orders new carpet and baseboards. Tyler and a couple of friends are helping to take measurements.

Here is a sketch of the classroom and the measurements they record.

For each question, show your reasoning.

8 feet

3 feet

3 yards

180 inches

4 yards

204 inches
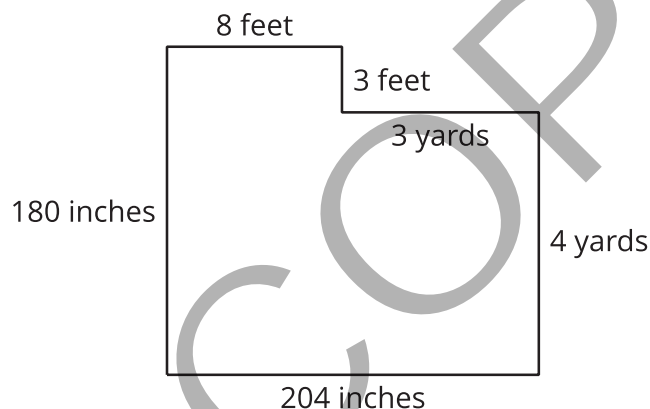
1. How many feet of baseboard do they need to replace in the classroom? How many inches is that?

2. 1,200 inches of baseboard material was delivered. Is that enough?

3. How many square feet of carpet is needed to cover the floor area?

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Solve Problems with Many Operations

Let's solve multi-step problems involving the four operations.

Warm-up

True or False: Differences

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $50{,}000 - 999 = 49{,}001$

- $4{,}799 = 5{,}000 - 311$

- $3{,}005 = 4{,}000 - 1{,}995$

- $2{,}000 - 1{,}234 = 1{,}876$

Back and Forth

Mai's cousin is in middle school. She travels from her homeroom to math, then English, history, and science. When she finishes her science class, she takes the same path back to her homeroom.

Mai's cousin makes the same trip 5 times each week. The distances between the classes are shown.

		157 ft	134 ft	162 ft	275 ft
music	homeroom	math	English	history	science
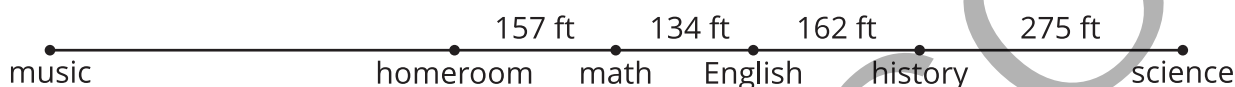
1. How far does Mai's cousin travel each round trip—from her homeroom to the 4 classes and back? Write one or more expressions or equations to show your reasoning.

2. Each week, Mai's cousin makes 3 round trips from her homeroom to her music class. The total distance traveled on those 3 round trips is 2,364 feet.

   How far away is the music room from her homeroom? Explain or show your reasoning.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. Mai thinks her cousin travels 2 miles each week just going between classes. Do you agree? Explain or show your reasoning.

Activity 2

Fitness Challenge

Han's school holds a fitness challenge with prizes to motivate students to exercise.

Fitness Challenge!

4,000 steps a day
120,000 steps total } **4 weeks**

Sign up & get your free step tracker today!

1. Han walks 32,550 steps in the first week. He walks the same number of steps every day. How many steps does Han walk each day? Explain or show your reasoning.

2. The table shows the number of steps Han takes each week for the first 3 weeks. How much does the number of steps drop from the first week to the second week?

week 1	week 2	week 3	week 4
32,550	28,098	36,249	

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

3. If Han wants to meet the challenge, what is the fewest number of steps that he needs to take in week 4? Explain or show your reasoning.

4. How do you know your answer to the third problem is reasonable?

_____

_____

Addressing CA CCSSM 4.NBT.4, 4.OA.2, 4.OA.3; practicing MP2

Assess the Reasonableness of Solutions

Let's solve problems and assess the reasonableness of solutions.

Warm-up

Notice and Wonder: Native American Languages

What do you notice? What do you wonder?

Activity 1

Do You Speak Navajo?

1. In the list of the ten most widely spoken Native American languages in the U.S., Navajo and Yupik are the most widely spoken.

    a. How many more Navajo speakers are there than Yupik speakers? Explain or show your reasoning.

language	number of speakers
Navajo	166,826
Yupik	19,750
Dakota	17,855
Apache	13,445
Keres	13,190
Cherokee	11,465
Ojibwa	9,735
Choctaw	9,635
Zuni	9,615
Pima	6,990

    b. About how many times as many Navajo speakers are there as Yupik speakers? Explain or show your reasoning.

2. Navajo, Apache, and Cherokee languages have been used during wartime to help the U.S. military keep its communications secure.

   a. Based on the data here, how many people might have been able to understand the communications? Explain or show your reasoning.

   b. How do you know that your answer is reasonable?

_____

_____

3. Are there more Navajo speakers than the speakers of all the other nine languages combined? Show how you know.

Languages in Philadelphia and Chicago

Philadelphia is a diverse city and home to people of different backgrounds.

The table shows 2020 data on some languages spoken in Philadelphia and the numbers of people who speak them.

language	number of speakers in Philadelphia
English only	1,131,303
Spanish	159,343
Other Indo-European	81,924
Asian	77,706

1. Use the data to determine if there are more people in Philadelphia who only speak English or more people who speak a language other than English. Show how you know.

2. What is the difference between the number of people who speak only English and those who speak another language? Show how you know.

3. Chicago is a city with a similar population to Philadelphia.

   This table shows data on some types of languages spoken by people in Chicago.

language	number of speakers in Chicago
English only	1,634,103
Spanish	600,655
Other Indo-European	150,354
Asian	105,790

   a. How many more speakers of Spanish and other Indo-European languages are in Chicago than in Philadelphia? Explain or show your reasoning.

   b. How do you know your answer is reasonable?

   _____

   _____

Sec D

👥 Section D Summary

We solved problems that involve large numbers from different contexts using different strategies.

In the beginning, we saw at least five ways to find the product of 45 and 18: by multiplying and dividing in parts, using a series of equations, drawing diagrams, and more.

Later, we solved problems about measurements, with numbers up to four digits. We found that, often, the same problem could be solved using different operations.

For example, in the fitness challenge activity, Han took 32,550 steps in 7 days. We can find the number of steps he took each day by using multiplication (what number times 7 is 32,550?) or division (what is 32,550 divided by 7?).

We can also write different equations.

$$7 \times n = 32{,}550 \qquad\qquad 32{,}550 \div 7 = n$$

To find out how many steps Han had to take to reach a goal of 120,000 steps if he had 96,897 steps, we can use addition (what number must be added to 96,897 to make 120,000?) or subtraction (what is the difference between 120,000 and 96,897?).

$$96{,}897 + n = 120{,}000 \qquad\qquad 120{,}000 - 96{,}897 = n$$

Paper Flower Decorations

Let's make patterns with paper flowers.

Warm-up

How Many Do You See: Paper Flowers

How many do you see? How do you see them?

Sec D

KH | iM Illustrative Mathematics® LEARN MATH FOR LIFE

Paper Flower Construction

Follow these steps to make paper flowers:

- Place 6 pieces of tissue paper on top of each other.

- Starting at one side, fold over about 1 inch, then fold in the opposite direction. Repeat with this accordion fold (like a paper fan) until you have a strip that is 1 inch wide and the length of the original paper.

- Tie a rubber band around the middle of the folded paper strip. Then open up the folds.

- Carefully, one layer at a time, fold the layers up into the middle to make the petals.

Sec D

Plan Quinceañera Decorations
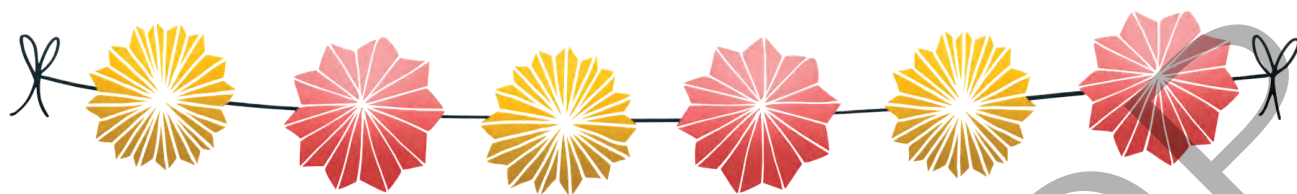
Create a plan for creating paper flower garlands for a quinceañera. In your plan include:

- A pattern for the paper flower garlands you will create.

- How many paper flower garlands you can make in 3 hours.

- How much tissue paper and string you will need to complete the paper flower garlands.

Explain or show your reasoning.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

Sec D

Practice Problems

1 from Unit 6, Lesson 22

a. An author takes a 4-hour airplane flight for a work trip. The plane flies 478 miles each hour. How many miles does she travel?

b. A photographer takes a 4-hour drive for his work trip. The car travels 57 miles each hour. How many miles does he travel?

c. How many miles further did the author travel than the photographer?

2 from Unit 6, Lesson 23

The diagram shows the side lengths of a sports field.

a. What is the perimeter of the field? Explain or show your reasoning.

85 meters

43 meters
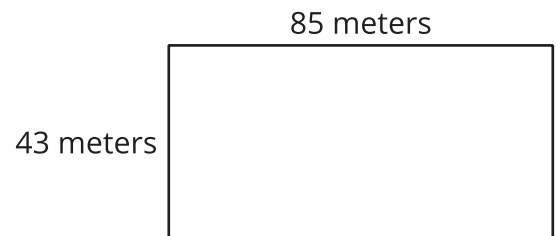
b. What is the area of the field? Explain or show your reasoning.

Sec D

3 from Unit 6, Lesson 24

The table shows the number of students who have different numbers of pets at a school.

How many pets do all the students have together? Explain or show your reasoning.

number of pets	number of students
1	218
2	167
3	287
4	138

4 from Unit 6, Lesson 25

A builder covers the floor of a rectangular room that is 23 feet by 25 feet with tiles that are 1 foot by 1 foot. The tiles are sold in boxes of 12.

Diego says 59 boxes are needed to cover the floor, and there will be a few leftover tiles.

a. Is Diego's answer reasonable? Explain or show your reasoning.

b. How many boxes of tiles would you buy to cover this floor? Explain or show your reasoning.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

5 (Exploration)

Find a rectangular room at home or in the school.

    a. Choose inches, feet, or yards to measure the length and width of the room. Explain your reasoning.

        _____

        _____

    b. Measure the length and width to the nearest whole number.

    c. Find the perimeter and area of the room.

    d. Can you find a length and width pair for a room that has the same perimeter but a different area?

    e. Can you find a length and width pair for a room that has the same area but a different perimeter?

6

The area of a rectangle is 720 square centimeters. One side is 6 centimeters longer than the other. What is the perimeter of the rectangle? Explain or show your reasoning.

Glossary

- common denominator
  The same denominator in two or more fractions. Example, $\frac{1}{4}$ and $\frac{5}{4}$ have the common denominator 4.

- composite number
  A whole number with more than one factor pair.

- decimal notation
  A way to write tenths, hundredths, and other decimal fractions as numerals with digits and a decimal point. The digits to the left of the decimal point show the whole-number part of the number. The digits to the right of the decimal point show the fractional part less than 1.

  Examples:

  $\frac{3}{10}$ written in decimal notation is 0.3.

  $\frac{25}{100}$ written in decimal notation is 0.25.

  $\frac{17}{10}$ written in decimal notation is 1.7.

  $2\frac{7}{100}$ written in decimal notation is 2.07.

- denominator
  The bottom part of a fraction that tells how many equal parts the whole was partitioned into.

- dividend
  The number being divided. Example: When 37 is divided by 5, we call 37 the dividend.

- divisor
  The number we are dividing by, which can represent the size of the groups or the number of groups. Example: When 37 is divided by 5, we call 5 the divisor.

- equivalent fractions
  Fractions that have the same size and describe the same point on the number line. Example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

- expanded form
  A way of writing a number as a sum of the values of the digits.

  Example: 482 written in expanded form is $400 + 80 + 2$.

- factor pair of a whole number
  Two whole numbers that multiply to result in that number. Example: 5 and 4 are a factor pair of 20.

- mixed number
  A number expressed as a whole number and a fraction less than 1.

- multiple of a number
  The result of multiplying that number by a whole number. Example: 18 is a multiple of 3, because it is a result of multiplying 3 by 6.

- numerator
  The top part of a fraction that tells how many of the equal parts are being described.

- prime number
  A whole number that is greater than 1 and has exactly one factor pair: the number itself and 1.

- quotient
  The result of dividing one number by another.

- remainder
  The amount left over when we divide a number. The amount is not enough to put into equal groups or to make a new group.

KH | iM Illustrative® Mathematics
LEARN MATH FOR LIFE

- standard algorithm (for addition or subtraction)
  A set of steps used to add or subtract numbers by place value. Write the numbers vertically with digits lined up by place value. Add or subtract the digits in each place value, starting with the least place value. Compose or decompose units, as needed in each place value.

Attributions

The *Common Core State Standards* are trademarks of the Common Core State Standards Initiative. © Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved. http://www.corestandards.org/

The *5 Practices* are from Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. National Council of Teachers of Mathematics.

Notice and Wonder and *I Notice/I Wonder* are trademarks of the National Council of Teachers of Mathematics, reflecting approaches developed by the Math Forum (http://www.nctm.org/mathforum/), and used here with permission.

Images that are not the original work of Illustrative Mathematics are in the public domain or released under a Creative Commons Attribution (CC-BY) license, and include an appropriate citation. Images that are the original work of Illustrative Mathematics do not include such a citation.

Reference links are provided as a convenience for informational purposes only and were active and accurate at the time of publication. They do not constitute an endorsement or approval by Illustrative Mathematics of any products, services, or opinions of the corporation, organization, or individual. Illustrative Mathematics is not responsible for the content of external sites.

Image Attributions

By P.E. Bragg. CC-BY-SA 3.0. Wikimedia. https://commons.wikimedia.org/wiki/File:Phobaeticus_chani_Bragg,_2008;_Holotype_Female_dorsal_view.jpg and.

By Sander van der Molen. CC-BY-SA 3.0. Wikimedia. https://commons.wikimedia.org/wiki/File:Paperclip-01_(xndr).jpg.

Paper Clips on White Background, by Rainier Wong. CC0. PublicDomainPictures.net. https://www.publicdomainpictures.net/en/view-image.php?image=37375&picture=paper-clips-on-white-background.

By Sergio Delgado. CC-BY-SA 3.0. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:SlothDWA.jpg.

By Hanne. CC0. pxhere. https://pxhere.com/en/photo/1610126.

KH | iM Illustrative Mathematics
LEARN MATH FOR LIFE

By Judith Scharnowski. Pixabay. Pixabay. https://pixabay.com/photos/camel-wadi-rum-desert-travel-1120371/.

By Elli60. CC0. Needpix.com. https://www.needpix.com/photo/943767/turtle-giant-tortoise-panzer.

By Florian Prischl/Wikimedia Commons. CC-BY-SA 3.0. Wikimedia. https://sco.m.wikipedia.org/wiki/File:Camelus_dromedarius_on_Sinai.jpg.

By Luisa2020. Pixabay. Pixabay. https://pixabay.com/photos/carrots-carrot-healthy-vegetables-3933837/.

By Fireworks. CC0. Wikimedia. https://commons.wikimedia.org/wiki/File:Tofu_4.jpg.

By Rlsheehan. CC BY-SA 4.0. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Bottle_of_milk.jpg.

By Imo Flow. Pixabay. Pixabay. https://pixabay.com/photos/mango-drink-fruit-diet-healthy-3380630/.

Blocks of polymer clay, by Dan Bollinger. CC-BY-SA 3.0. Wikipedia. https://simple.wikipedia.org/wiki/Polymer_clay#/media/File:Polymer_clay_examples.jpg.

By A_Different_Perspective. Pixabay. Pixabay. https://pixabay.com/photos/stone-tower-stones-cairn-stone-pile-3280616/.

By Essie. CC BY 2.0. Flickr. https://www.flickr.com/photos/64148767@N00/14468746263.

By Keistutis. CC0. GoodFreePhotos. Adapted from https://www.goodfreephotos.com/vector-images/dolphins-vector-clipart.png.php.

By Jose R. Cabello. Pixabay. Pixabay. Adapted from https://pixabay.com/illustrations/frog-brown-amphibian-animal-nature-2457344/.

By Gorkhs. Pixabay. Pixabay. Adapted from https://pixabay.com/vectors/rabbit-silhouette-black-2262101/.

By AnnaliseArt. Pixabay. Pixabay. https://pixabay.com/illustrations/kawaii-animals-animal-stickers-sheep-3995577/.

Paletas, by Arnold Gatilao. CC BY 2.0. Wikipedia. https://commons.wikimedia.org/wiki/File:Paletas_for_days..._(16492073013).jpg.

Artists Palette And Brush, by Free*SVG OpenClipart. Public Domain. Free SVG. https://freesvg.org/artists-palette-and-brush.

By חפנון. CC BY-SA 4.0. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Flowers_in_school_garden,_botanical_garden,_Jerusalem,_Israel.jpg.

By Victoria Borodinova. Pixabay. Pixabay. https://pixabay.com/illustrations/tickets-cinema-entrance-theatre-4267025/.

By United States Census Bureau. Public Domain. census.gov. https://www.census.gov/content/dam/Census/newsroom/facts-for-features/2015/cb15-ff22_graphic.jpg.

By AnnaliseArt. Pixabay. Pixabay. https://pixabay.com/illustrations/bunting-banners-decoration-banner-4707979/.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

Citations

Unit 5: Multiplicative Comparison and Measurement

Lesson Grade4.5.B7
Image of two insects and a ruler by Illustrative Mathematics is licensed under CC BY-SA 4.0, https://creativecommons.org/licenses/by-sa/4.0/. The image of the stick insect Phobaeticus chani by P.E. Bragg, https://commons.wikimedia.org/wiki/File:Phobaeticus_chani_Bragg,_2008;_Holotype_Female_dorsal_view.jpg, is licensed under CC BY-SA 3.0, https://creativecommons.org/licenses/by-sa/3.0/. The image of the green potato bug (Cuspicona simplex) by Jacob Littlejohn, https://commons.wikimedia.org/wiki/File:Original_cuspicona_simplex_New_Zealand.jpg, is licensed under CC BY-SA 4.0, https://creativecommons.org/licenses/by-sa/4.0/

Unit 6: Multiplying and Dividing Multi-digit Numbers

Lesson Grade4.6.D25
Chicago, Illinois Population 2020. *United States Census Bureau.* Retrieved from https://data.census.gov/cedsci/table?tid=ACSST5Y2020.S1601&g=1600000US1714000

Lesson Grade4.6.D25
Native North American Languages. *Encyclopedia.com.* Retrieved from https://www.encyclopedia.com/literature-and-arts/language-linguistics-and-literary-terms/language-and-linguistics/native

Lesson Grade4.6.D25
Philadelphia, Pennsylvania Population 2020. *United States Census Bureau.* Retrieved from https://data.census.gov/cedsci/table?tid=ACSST5Y2020.S1601&g=1600000US4260000

Lesson Grade4.6.D26
Quinceañera. *Wikipedia, The Free Encyclopedia.* Retrieved from https://en.wikipedia.org/wiki/Quincea%C3%B1era

Notes

California Common Core State Standards for Mathematics (CA CCSSM) Reference

4.G: Grade 4 – Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (Two-dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA

4.G.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

4.MD: Grade 4 – Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1, 12)$, $(2, 24)$, $(3, 36)$, ...

4.MD.2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3
Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Represent and interpret data.

4.MD.4
Make a line plot to display a data set of measurements in fractions of a unit $(1/2, 1/4, 1/8)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

4.MD.5.a
An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

4.MD.5.b
An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.NBT: Grade 4 – Number and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

4.NBT.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.

4.NBT.3
Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

KH | Illustrative® Mathematics
LEARN MATH FOR LIFE

4.NF: Grade 4 – Number and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

4.NF.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3
Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

4.NF.3.a
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.3.b
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

4.NF.3.c
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3.d
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4.a
Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

4.NF.4.b
Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

4.NF.4.c
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. For example, express 3/10 as 30/100, and add $3/10 + 4/100 = 34/100$.

4.NF.6
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using the number line or another visual model. CA

4.OA: Grade 4 – Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1
Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4.OA.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

KH | **i V** Illustrative Mathematics
LEARN MATH FOR LIFE

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize— to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument— explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

KH | iM Illustrative Mathematics® LEARN MATH FOR LIFE

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.