IM K–12 California

GRADE 4

Student Edition

UNIT 7-9

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 4
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UNIT 9 PUTTING IT ALL TOGETHER

GRADE 4

UNIT

Angles and Angle Measurement

Content Connections

In this unit you will draw and identify points, rays, segments, angels and lines. You will also use a protractor to measure angles and draw angles of given measurements. You will make connections by:

- **Discovering Shape and Space** while drawing, identifying and describing shapes.
- **Taking Wholes Apart, Putting Wholes Together** while solving problems about unknown angle measurements.
- **Reasoning with Data** while investigating shapes by measuring lengths and angles, collecting the data and representing it visually.
- **Exploring Changing Quantities** while using fractions to explore angle measurements and understand that angels are additive.

Addressing the Standards

As you work your way through **Unit 7 Angles and Angle Measurement,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 7,15
MP2 Reason abstractly and quantitatively.	Lessons 9, 10
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3, 6
MP4 Model with mathematics.	Lessons 4, 16
MP5 Use appropriate tools strategically.	Lesson 7
MP6 Attend to precision.	Lessons 1, 2, 4, 5, 11, 12
MP7 Look for and make use of structure.	Lessons 3, 6, 8, 13, 14
MP8 Look for and express regularity in repeated reasoning.	

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes and Symmetries	**4.G.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Lessons 1, 2, 3, 4, 5, 6, 10, 11, 12, and 15
• Shapes and Symmetries • Circles, Fractions and Decimals • Rectangle Investigations	**4.MD.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	Lessons 5, 6, 7, 11, and 16

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes and Symmetries • Circles, Fractions and Decimals • Rectangle Investigations	**4.MD.5a** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.	Lessons 8 and 9
• Shapes and Symmetries • Circles, Fractions and Decimals • Rectangle Investigations	**4.MD.5b** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	Lessons 9 and 10
• Rectangle Investigations • Shapes and Symmetries	**4.MD.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	Lessons 9, 10, 11, 14, and 16
• Circles, Fractions and Decimals • Shapes and Symmetries	**4.MD.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Lessons 8, 9, 11, 13, 14, and 15
• Multi-Digit Numbers • Shapes and Symmetries • Connected Problem Solving	**4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place.	Lesson 2
• Factors and Area Models • Connected Problem Solving	**4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lesson 9

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Factors and Area Models • Connected Problem Solving	**4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lesson 10

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 7, Lesson 1

Addressing CA CCSSM 4.G.1; practicing MP6

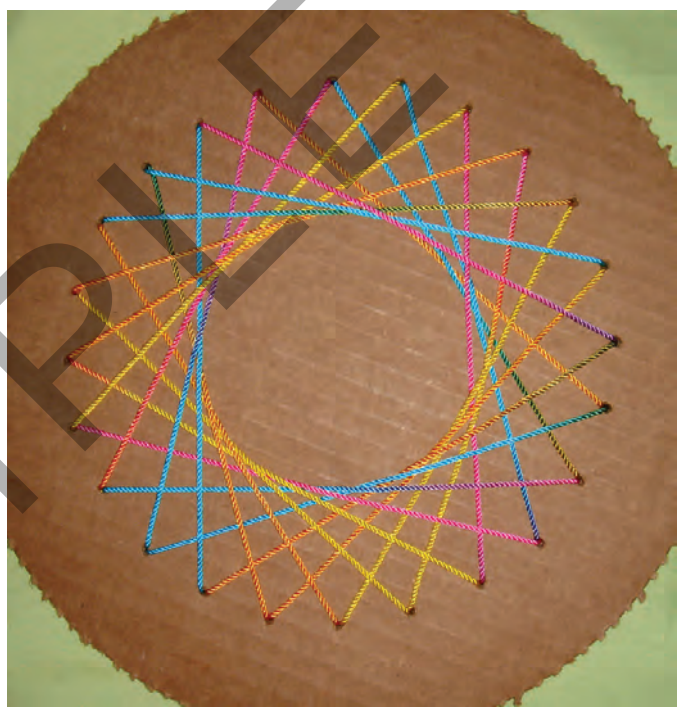
How Would You Describe These Figures?

Let's draw and describe geometric figures.

Warm-up

Notice and Wonder: String Art

What do you notice? What do you wonder?



Activity 1

Do You See What I See?

Work with a partner. Sit back to back, or use a divider to keep one partner from seeing the other partner's work.

Partner A:

- Your teacher will give you a card. Don't show it to your partner.
- Describe the image on the card—as clearly and precisely as possible—so that your partner can draw it on a blank card.

Partner B:

- Your teacher will give you a blank card.
- Your partner will describe an image. Listen carefully to each description and use them to create a drawing on your card.

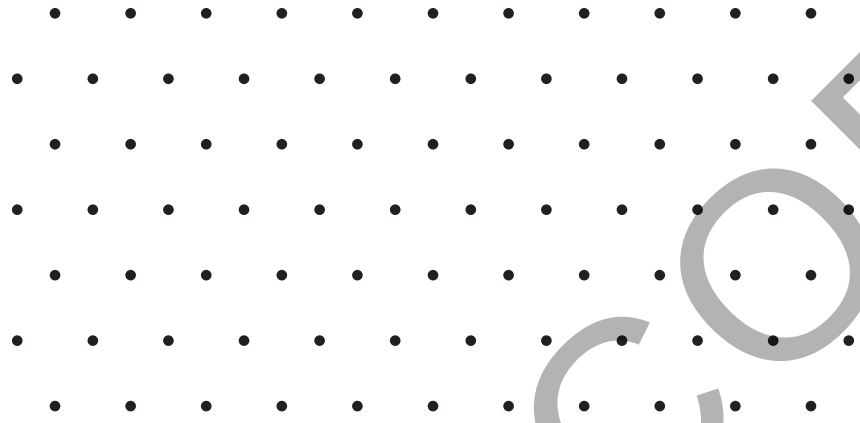
1. Compare the given image and the drawing. Discuss:
    - How are the given image and the drawing alike? How are they different?
    - How would you improve the descriptions?

2. Switch roles. Repeat the directions with a new card.

Activity 2

Lines and Line Segments

Here is a field of dots.



1. Draw 5 lines. Each line should connect at least 2 dots and extend as far as possible.
2. Do your lines make familiar shapes or figures—perhaps a triangle, a quadrilateral, a letter, or a number?

   Identify at least one familiar shape or figure in your drawing. Trace the shape with a heavier mark or use a colored pencil.

3. Share your drawing with your group. Discuss:
   - How are the drawings alike? What do all the shapes have in common?
   - How are the drawings different?

Unit 7, Lesson 2

Addressing CA CCSSM 4.G.1 and 4.NBT.3; practicing MP6

Points, Lines, Rays, and Segments

Let's draw points, lines, line segments, and rays.

Warm-up

Number Talk: Finding Differences

Find the value of each expression mentally.

- $90 - 45$
- $270 - 45$
- $270 - 135$
- $360 - 135$

Activity 1

Card Sort: Who Am I?

Your teacher will give you a set of cards.

Sort the cards into 4 groups. Each group should represent the attributes or characteristics of a **point**, **line**, **ray**, or **line segment**.

Pause for directions from your teacher before completing the graphic organizer.

point	line
ray	line segment

Activity 2

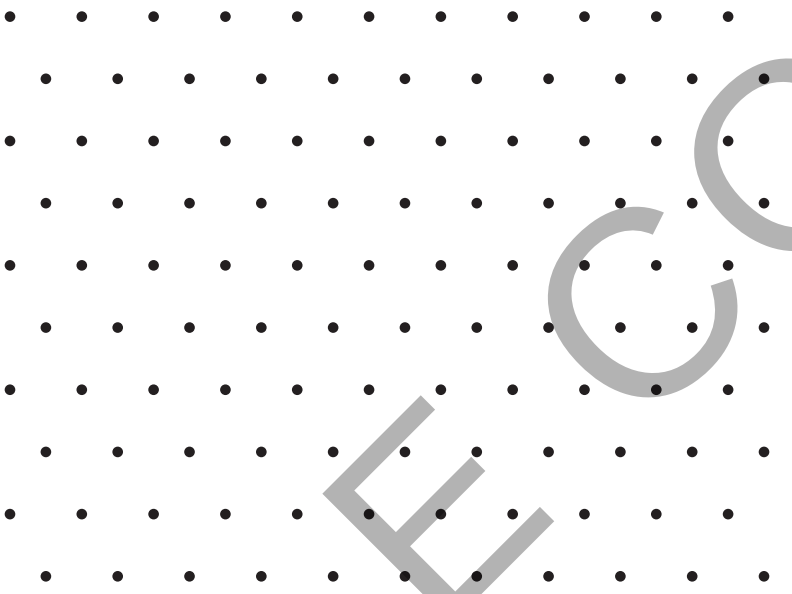
Make Some Shapes

1. Each dot on the grid represents a point. Draw line segments to create:

a triangle
a rhombus

a trapezoid
a hexagon

a pentagon
a rectangle

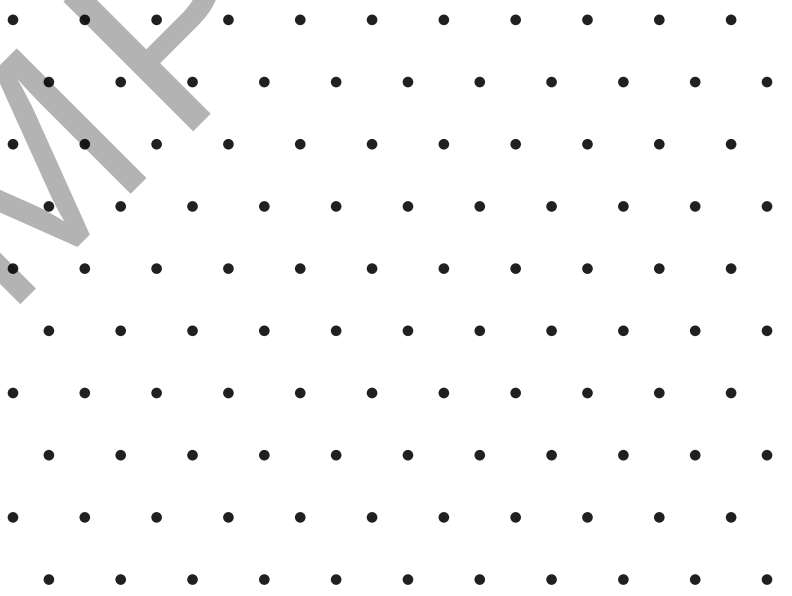


2. Draw a combination of rays and line segments to create:

an uppercase letter

a number

a lowercase letter



Unit 7, Lesson 3

Addressing CA CCSSM 4.G.1; practicing MP3 and MP7

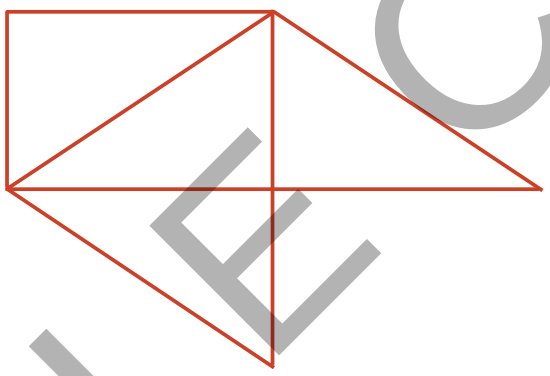
Two or More Lines

Let's look at lines that cross and lines that don't.

Warm-up

How Many Do You See: A Curious Figure

How many line segments do you see? How do you see them?

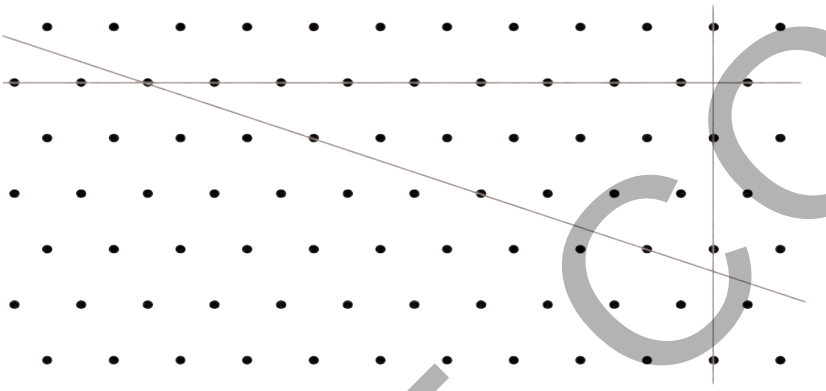


Activity 1

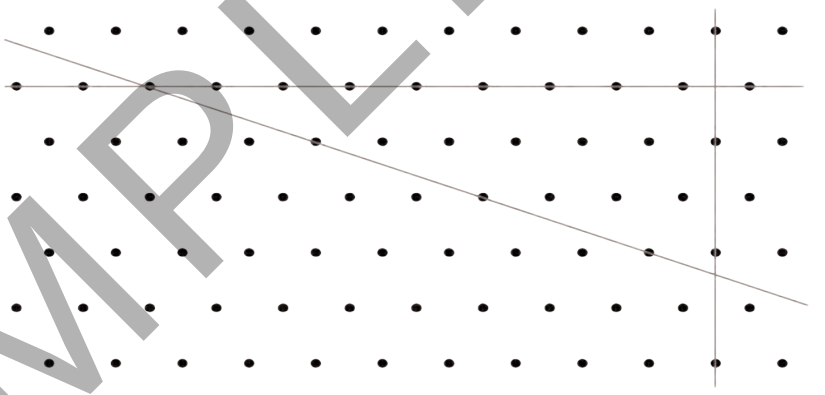
Four Lines

1. Three lines on a field of dots **intersect** (cross) to form a triangle. Can you draw a fourth line so that the four lines form a quadrilateral?

   Use the drawing to explain or show your reasoning.



2. Here is a copy of the same drawing. Can you draw a fourth line to form a rectangle?



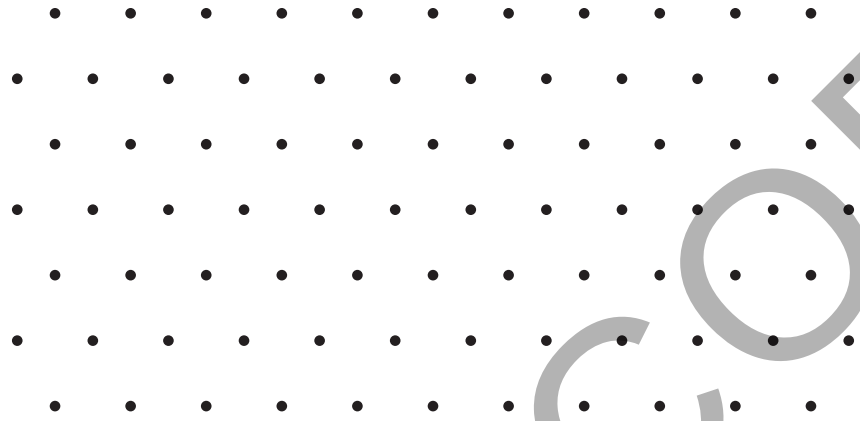
   Use the drawing to explain or show your reasoning.

3. Discuss the drawings with your group. Check if they agree with your conclusions.

Activity 2

To Cross or Not to Cross

Here is another field of dots. Each dot represents a point.



1. Draw a line through at least 2 points. Label it Line H.
2. Draw another line that goes through at least 2 points and intersects your first line. Label it Line G.
3. Can you draw a new line that you think would never intersect:
   a. Line H? If so, draw the line. If not, explain or show why it can't be done.
   b. Line G? If so, draw the line. If not, explain or show why it can't be done.
4. Here is a trapezoid.

   Do you think its top and bottom sides are parallel? What about its left and right sides? Explain or show how you know.

If you have time: Can you draw a new line that would intersect neither Line H nor Line G? If so, draw the line. Explain how you know the lines would never cross. If not, explain why it can't be done.

Addressing CA CCSSM 4.G.1; building towards 4.G.2; practicing MP4 and MP6

Points and Lines All Around

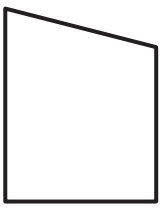
Let's find parallel and intersecting lines all around us.

Warm-up

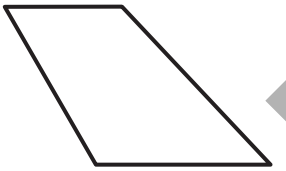
Which Three Go Together: Four-Sided Shapes

Which 3 go together?

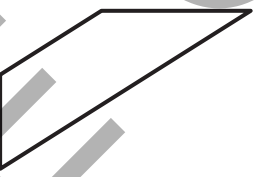
A



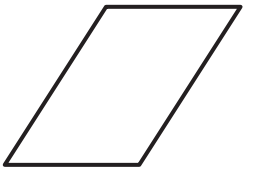
B



C



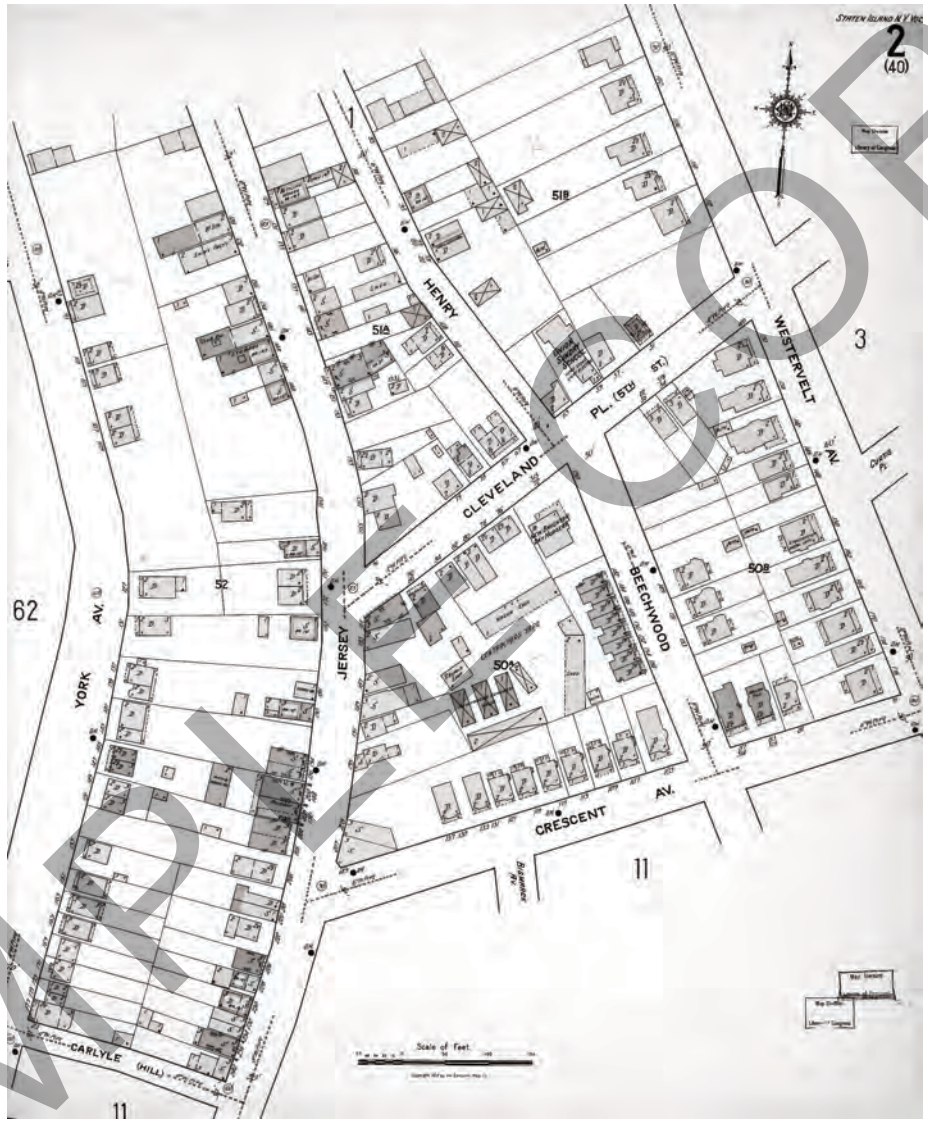
D



Activity 1

Spot Lines and Line Segments

1. Here is a map of a neighborhood in Staten Island, New York.



Find and label each of these items on the map.

- 4 line segments of different lengths
- 3 pairs of parallel lines
- 2 pairs of lines that are not parallel

(Consider using a different color for each type of line.)

2.

WHALE JOY

Use the words WHALE and JOY to find one or more letters that represent each description.

a. No parallel segments _____

b. Exactly one pair of parallel segments _____

c. More than one pair of parallel segments _____

d. Exactly one segment _____

If you have time: Does the uppercase alphabet use more parallel segments or intersecting segments?

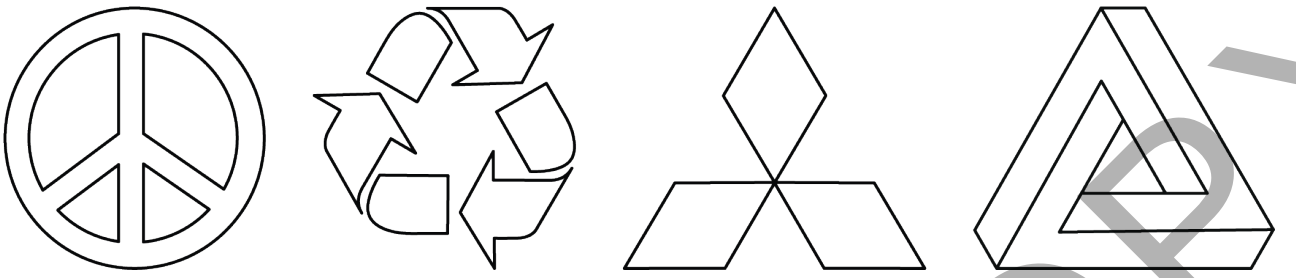
Activity 2

Draw and Design with Lines

1. Use each of these descriptions to draw a sketch of a part of your classroom.
   a. at least 3 pairs of parallel line segments
   b. intersecting line segments that make square corners
   c. intersecting line segments that don't make square corners

   Trade sketches with a partner. Find the line segments represented in each description.

2. Here are some symbols and logos that you may recognize. All of them have intersecting and parallel line segments.



Design a logo with at least 8 parallel segments and 8 intersecting line segments.

Use a ruler for any straight parts of your logo.

Addressing CA CCSSM 4.G.1 and 4.MD.5; building towards 4.MD.5; practicing MP6

What Is an Angle?

Let's look for angles and find out ways to describe them.

Warm-up

Notice and Wonder: A Wall of Clocks

What do you notice? What do you wonder?

Activity 1

Tricky Figures

Work with a partner in this activity. Choose a role: A or B. Sit back to back, or use a divider to keep one partner from seeing the other partner's work.

Partner A:

- Your teacher will give you a card. Don't show it to your partner.
- Describe both images on the card—as clearly and precisely as possible—so that your partner can draw the same images.

Partner B:

- Your partner will describe 2 images. Listen carefully to the descriptions.
- Create the drawings as described. Follow the instructions as closely as possible.

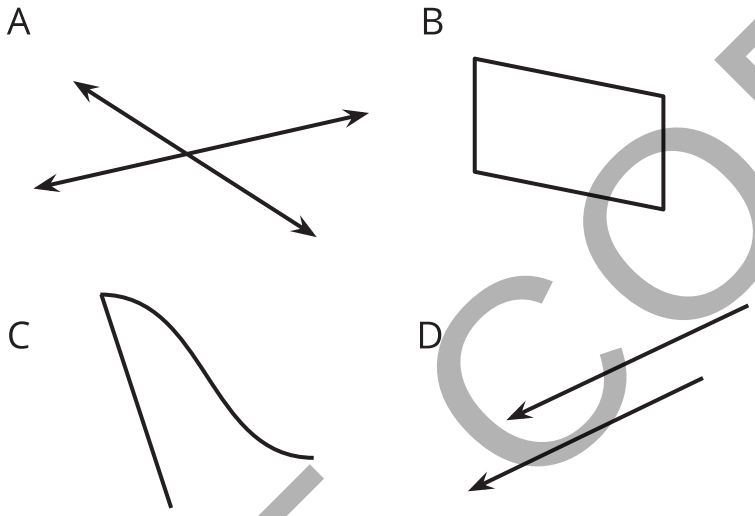
1. Compare your drawings to the original images. Discuss:
    - How are your drawings and the original images alike? How are they different?
    - How would you improve the descriptions?
2. Switch roles. Repeat the directions with a new card.

If you have time: Request two new cards from your teacher (one card at a time). Take turns describing and drawing the geometric figure on each card.

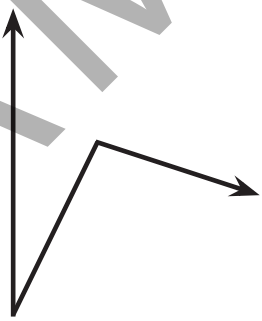
Activity 2

Angles or Not Angles?

1. Decide if each figure shows at least one angle. Explain or show your reasoning.



2. Clare and Kiran look at this diagram. Clare says there are no angles because the rays do not meet at a point. Kiran says he sees 2 angles.



Do you agree with Clare or Kiran? How many angles do you see?

Activity 3

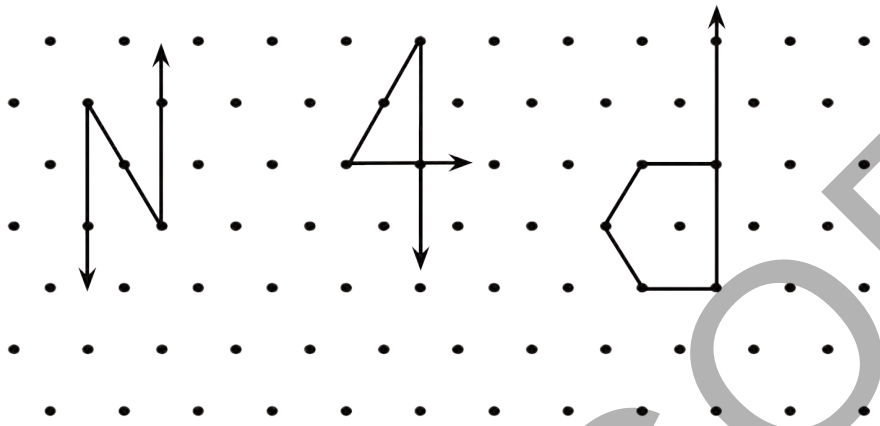
Discover Angles

Here are two figures.

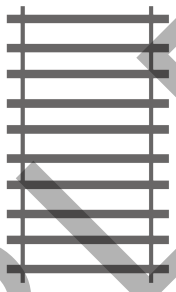
1. Find 2–3 angles in each figure. Draw pairs of rays to show the angles.
2. Sketch a part of your classroom that has 2–3 angles. Draw pairs of rays to show the angles.

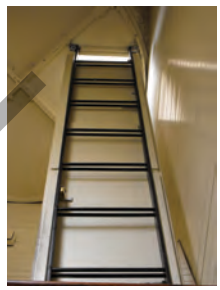
Section A Summary

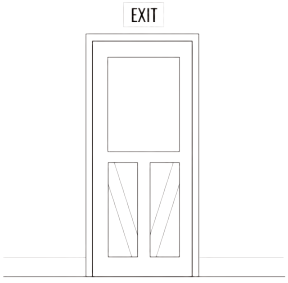
We learned the meaning of **point**, **line**, **line segment**, and **ray**. We used these terms to describe figures and geometric parts to create drawings.



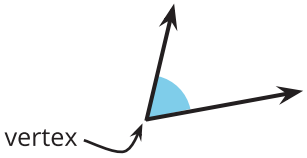
We learned about lines that cross—**intersecting lines**—and lines that never cross—**parallel lines**. Then we looked for examples of intersecting lines, parallel lines, and line segments.







Finally, we learned that an **angle** is a figure made up of two rays that share the same starting point. The shared point is the **vertex** of the angle.



Practice Problems

9 Problems

1

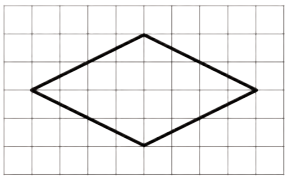
Pre-unit

Draw a rectangle on the grid and label it A. Draw a triangle and label it B. Draw a hexagon and label it C.

2

Pre-unit

a. Is the shape a rhombus? Explain your reasoning.

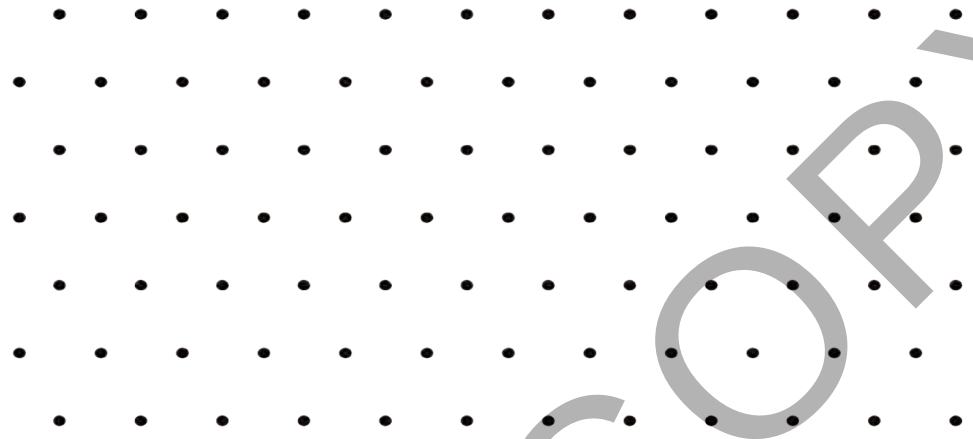


b. Is the shape a rectangle? Explain your reasoning.

c. Is the shape a square? Explain your reasoning.

3 from Unit 7, Lesson 1

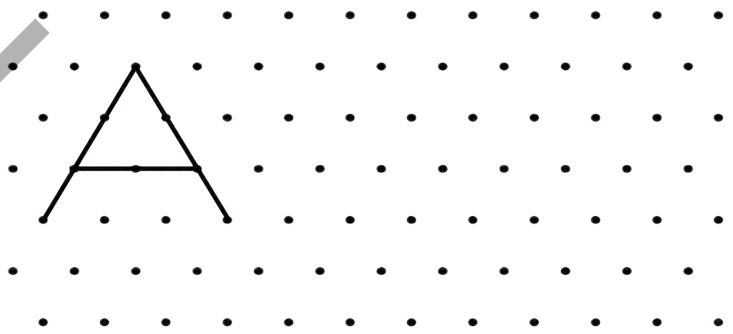
a. Draw 4 different lines through points on the grid. At least 2 of the lines should cross another line.



b. Mark at least 3 different segments in your drawing.

4 from Unit 7, Lesson 2

a. Circle the line segments that make up the letter A.

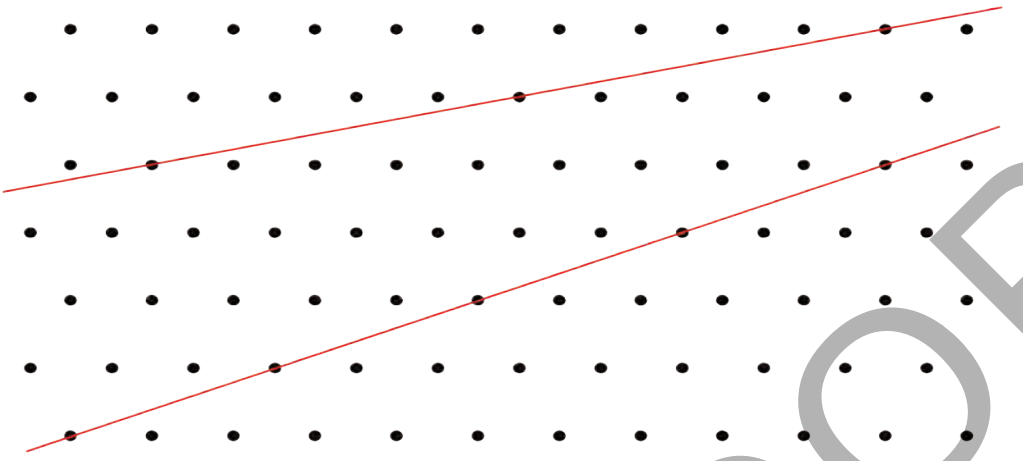


b. Draw 4 rays that surround a rectangle.

c. Can you find 4 different rays that surround the same rectangle?

5 from Unit 7, Lesson 3

Andre says that these 2 lines are parallel because they do not intersect.



a. Explain why Andre is not correct.

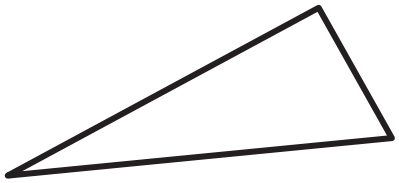
b. Draw a line that is parallel to one of the lines in the image.

6 from Unit 7, Lesson 4

a. Which segments of the letter Z are a pair of parallel lines? Draw the lines.

b. Sketch a line that is parallel to the third segment in the Z.

7 from Unit 7, Lesson 5



a. Find one angle in the figure. Draw a pair of rays to show the angle and extend them as far as you can.

b. Find another angle in the figure. Draw a pair of rays to show the angle. Extend the rays as far as you can. (You can use a different colored pencil for this pair of rays.)

c. Do you see other angles that you didn't see before? If so, label each one with a letter.

8 Here is a riddle. Can you solve it?

"I am a capital letter made of more than 1 segment, with no curved parts. I have no perpendicular segments or parallel segments. What letter could I be?"

a. Name or describe any shapes that you recognize in the painting.

b. Do you see any parallel lines? If so, trace or circle them. (You can use a different colored pencil for each set of parallel lines.)

c. Are there any angles in the painting? If so, mark them or describe where they are in the painting.

Addressing CA CCSSM 4.G.1 and 4.MD.5; building towards 4.MD.5a; practicing MP3 and MP7

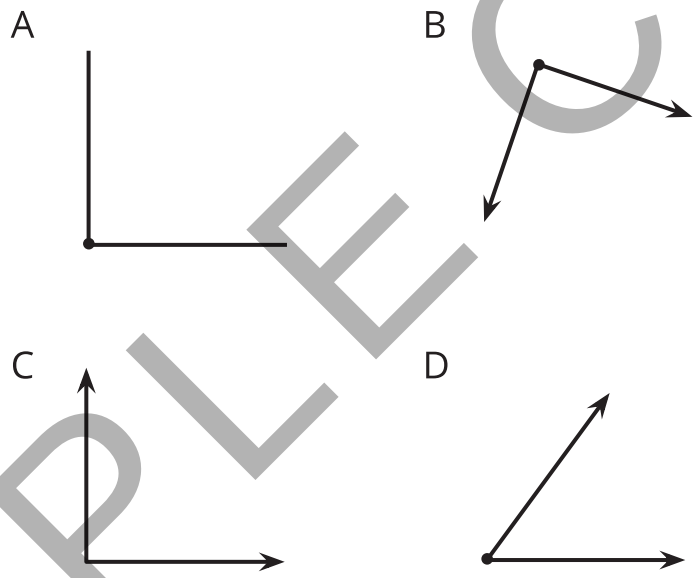
Compare and Describe Angles

Let's think about how to compare and describe angles.

Warm-up

Which Three Go Together: Angles

Which 3 go together?



Activity 1

Card Sort: Angles

Your teacher will give you a set of cards that show angles.

Sort the angles into 3 or more categories in a way that makes sense to you.

Record your sorted angles here. Write words or phrases to describe each category. Be ready to explain the meaning of your categories.

Activity 2

Order Angles

You will need Cards A–P from an earlier activity.

Order the angles on the cards from smallest to largest.

Record your ordered angles. Explain or show how you decided which angle was the smallest and which was the largest.

Unit 7, Lesson 7

Addressing CA CCSSM 4.MD.5; building towards 4.MD.5a; practicing MP1 and MP5

The Size of an Angle on a Clock

Let's describe angles, using the hands of a clock.

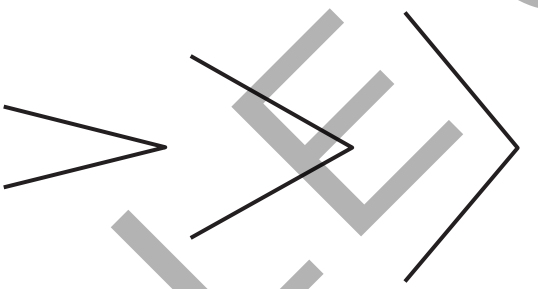
Warm-up

Sec B

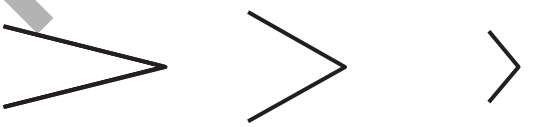
Notice and Wonder: Two Sets of Angles

What do you notice? What do you wonder?

Set 1



Set 2

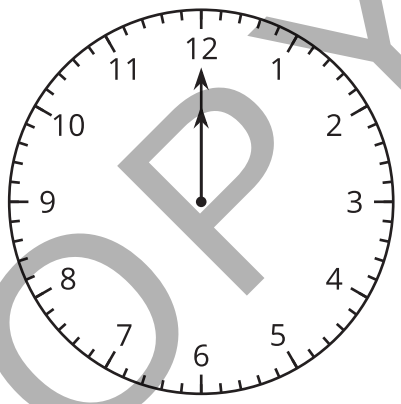


Activity 1

Draw Angles Andre's Way

Andre used the hands of a clock to explain to his partner how to draw an angle.

"Imagine both hands are pointing at the 12. Turn the minute hand so it's pointing at the 3."

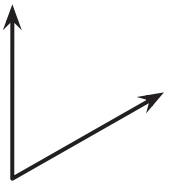


1. Draw Andre's angle.

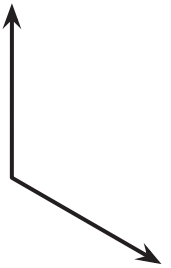
2. What is another way to describe how to draw the same angle, using the clock?

3. Use Andre's way to explain how to draw these angles.

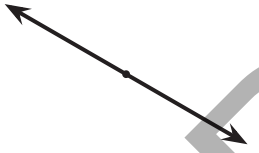
a.



b.



c.



Activity 2

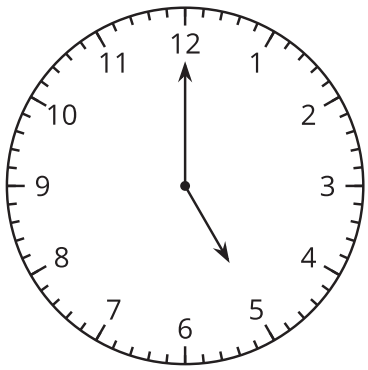
Compare Angles on the Clock

1. Here are some angles formed by the two hands of a clock.

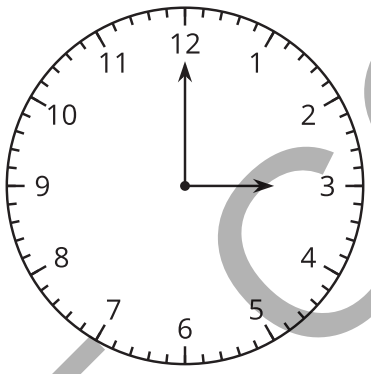
   In each pair of angles, which angle is greater? Explain or show your reasoning.

   a.

   5:00

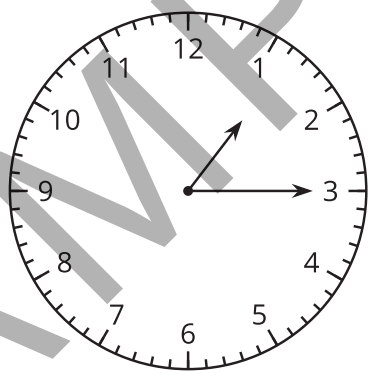


   3:00

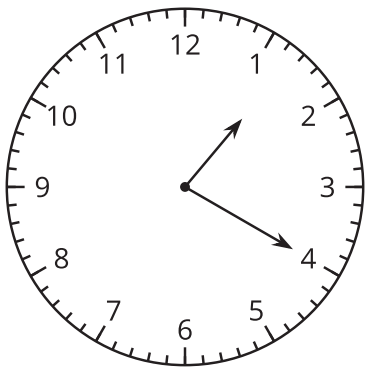


   b.

   1:15

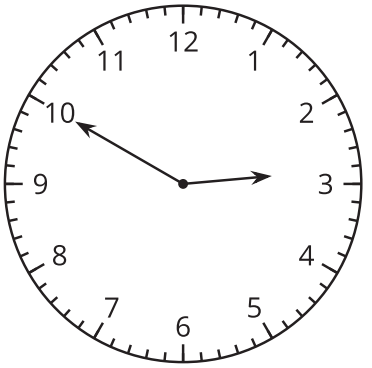


   1:20

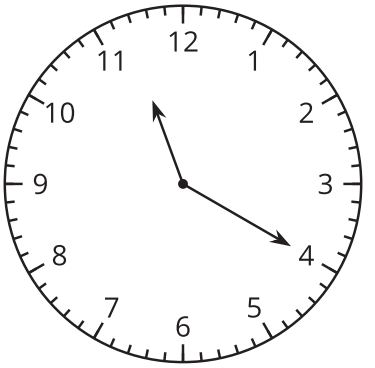


c.

2:50

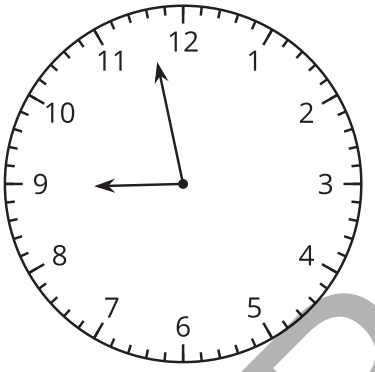


11:20

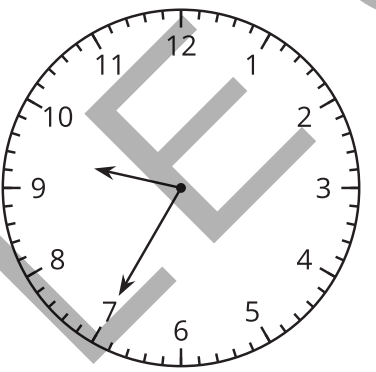


d.

8:58

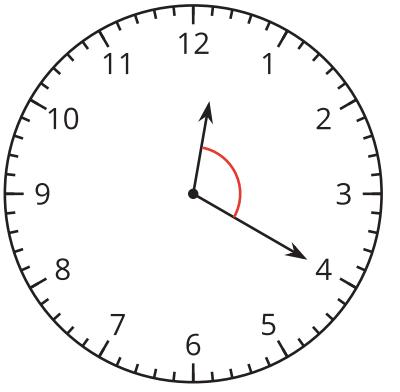


9:35



2. How large is this angle?

   Describe its size in as many ways as you can.



Addressing CA CCSSM 4.MD.5a and 4.MD.7; building on 4.NBT.1; building towards 4.MD.5, 4.MD.6 and 4.MD.7; practicing MP7

The Size of An Angle, in Degrees

Let's describe the size of an angle, using degrees.

Warm-up

What Do You Know about 360?

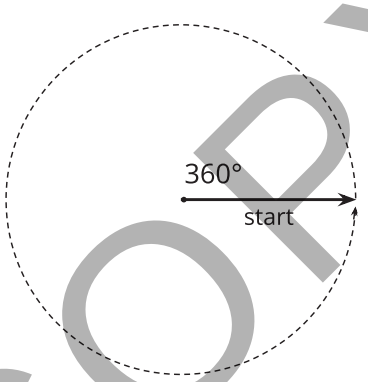
What do you know about 360?

Activity 1

A Full Turn

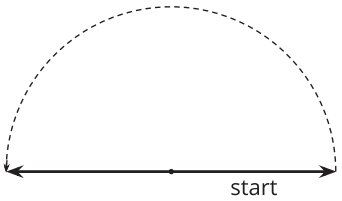
A ray that turns all the way around its starting point has made a full turn.

We say that the ray has turned 360 **degrees**.

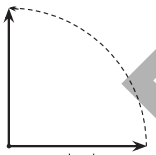


1. How many degrees has the ray turned from where it started?

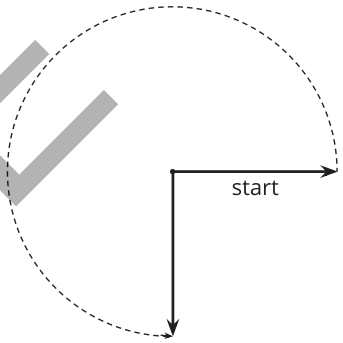
a



b



c

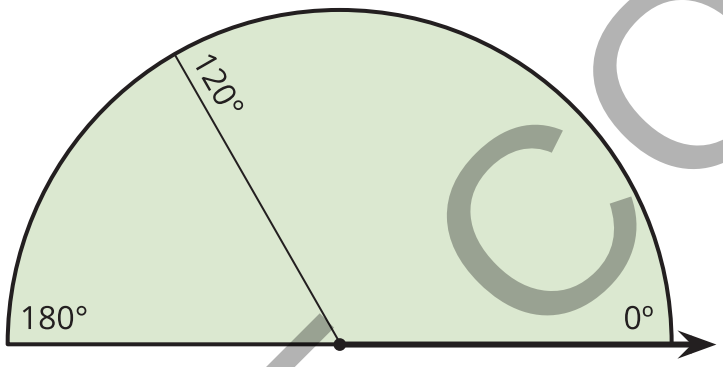


2. Sketch 2 angles:
   a. an angle where a ray has turned 50°
   b. an angle where a ray has turned 130°

Activity 2

Make a Measuring Tool

Your teacher will give you a sheet of paper in the shape of half a circle. It shows a 120° angle and a 180° angle from the ray on the bottom right.



On the half-a-circle paper:

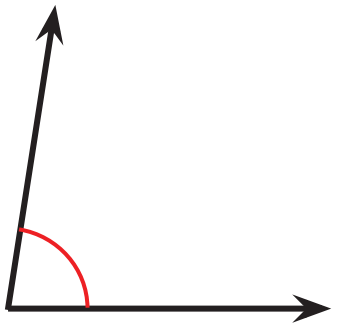
1. Draw a line segment to show a 90° angle from the same ray. Label it with the measurement. Be as precise as possible.
2. Draw line segments to show the following angles (measured from the same ray). Label the measurement on each segment.

   60° 45° 30° 150° 135°
3. Can you find a 1° angle from that same ray? Explain or show how you know.

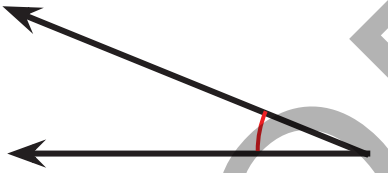
4. You just made a measuring tool!

   How is it used to estimate the size of an angle? Discuss your ideas with your group. Then use your tool to estimate the sizes of at least 2 angles.

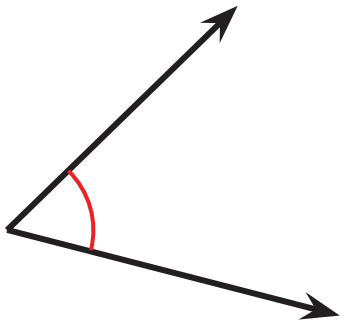
a.



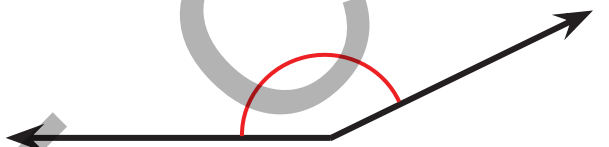
b.



c.



d.



Addressing CA CCSSM 4.MD.5a, 4.MD.5b, 4.MD.6-7, and 4.NBT.5; practicing MP2

Use a Protractor to Measure Angles

Let's use some tools to measure angles.

Warm-up

True or False: There's Something about 45

Decide if each statement is true or false. Explain your reasoning.

- $2 \times 45 = 6 \times 15$

- $4 \times 45 = 2 \times 90$

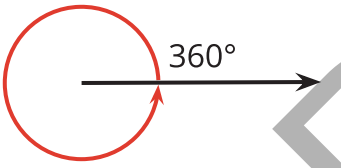
- $3 \times 45 = 180 - 90$

- $6 \times 45 = 45 + 90 + 135$

Activity 1

How Large Is a 1° Angle?

1. A ray that turns all the way around its starting point has made a full turn or has turned 360°.



What fraction of a full turn is each of the following angle measurements?

a. 120°

b. 60°

c. 45°

d. 30°

e. 10°

f. 1°

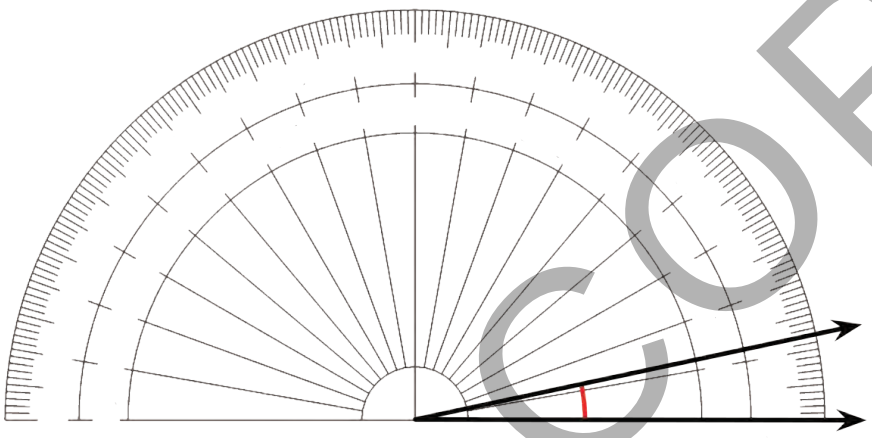
2. Your teacher will give you a **protractor**, a tool for measuring the number of degrees in an angle.

a. How is 1° shown on the protractor?

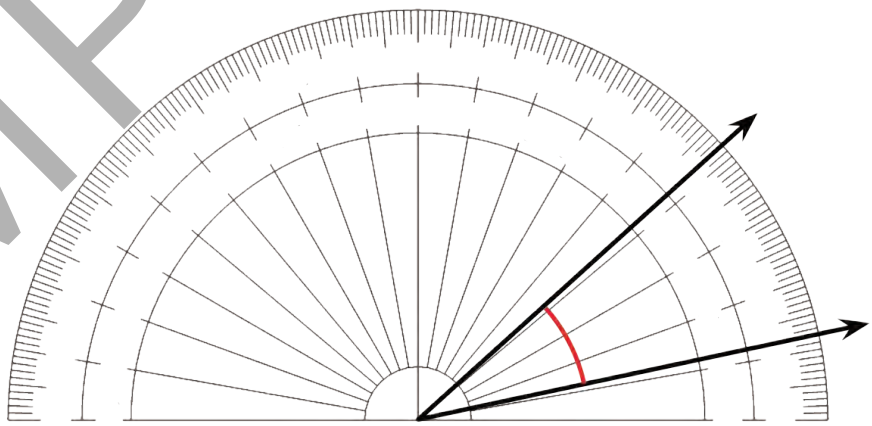
b. How many 1° measurements do you see?

3. A protractor with no numbers has been placed over an angle.
   - The center of the protractor is lined up with the vertex of the angle.
   - The straight edge of the protractor lines up with a ray of the angle.

   How many degrees is this angle? Explain how you know.



4. An angle contains thirty 1° angles. How many degrees is this angle?

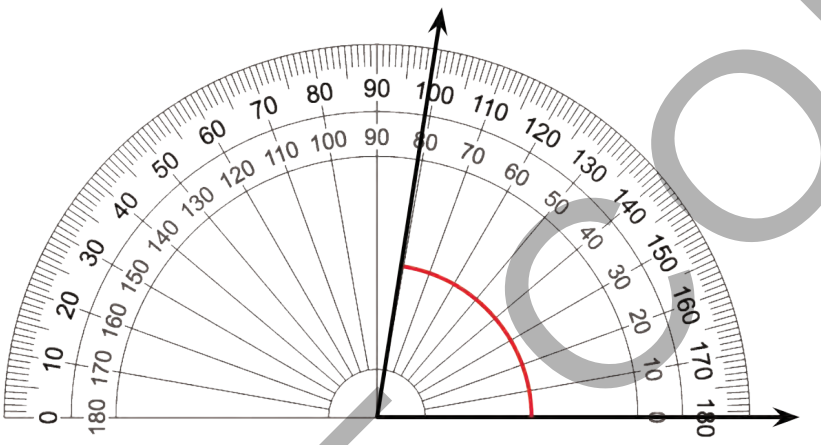


Activity 2

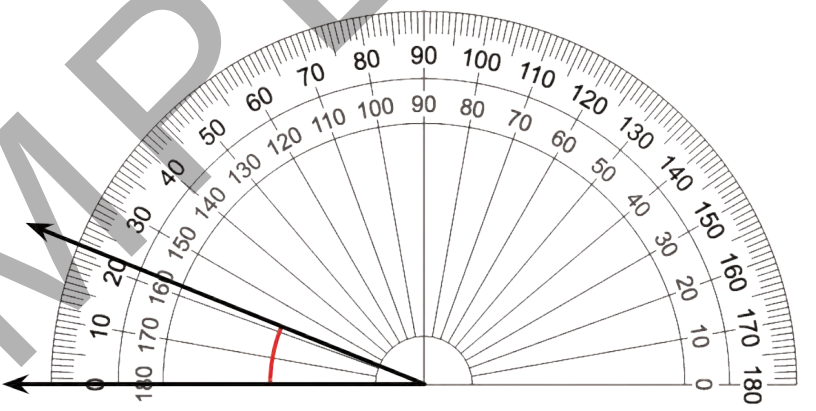
Use a Protractor

1. Here are 4 angles. A protractor has been placed over each angle. Measure the size of each angle in degrees.

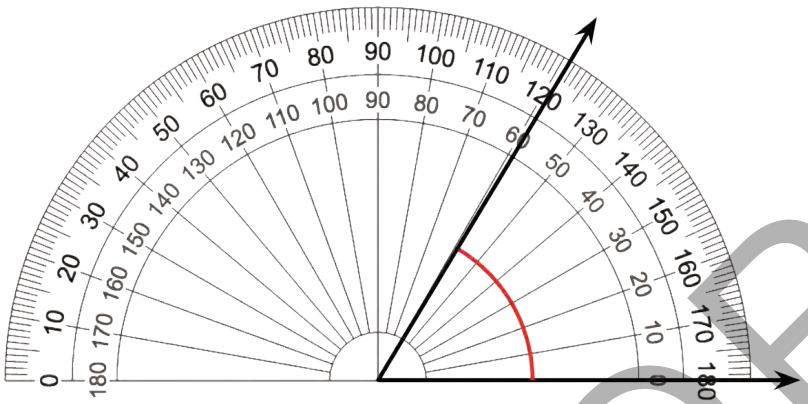
   a.



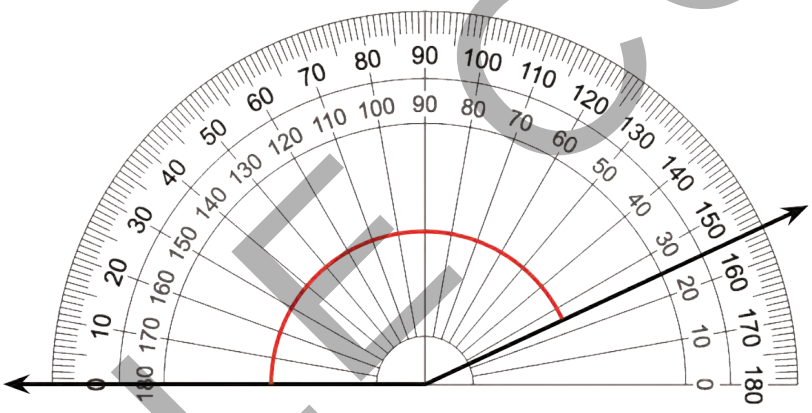
   b.



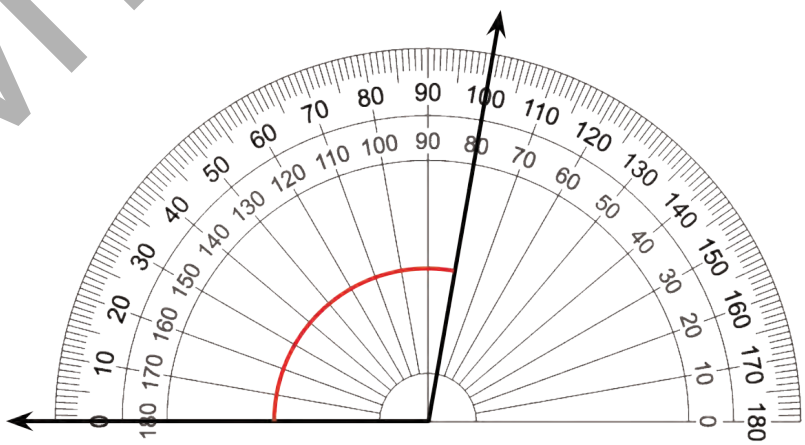
c.



d.



2. Elena and Kiran measure an angle with a protractor. Elena says the angle is 80°. Kiran says the angle is 100°. Why are the measurements different? Which measurement is correct? Explain your reasoning.



Unit 7, Lesson 10

Addressing CA CCSSM 4.G.1, 4.MD.5b, 4.MD.6, and 4.NBT.6; practicing MP2

Angle Measurement and Perpendicular Lines

Let's measure all kinds of angles.

Warm-up

Number Talk: Quotients

Find the value of each expression mentally.

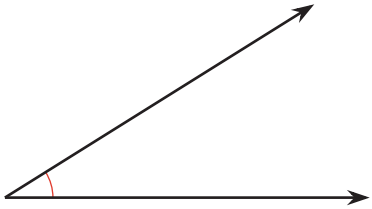
- $180 \div 2$
- $180 \div 4$
- $360 \div 8$
- $360 \div 16$

Activity 1

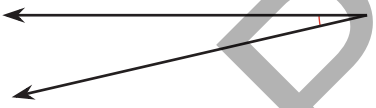
Angles Here, There, Everywhere

Use a protractor to find the value of each angle measurement in degrees.

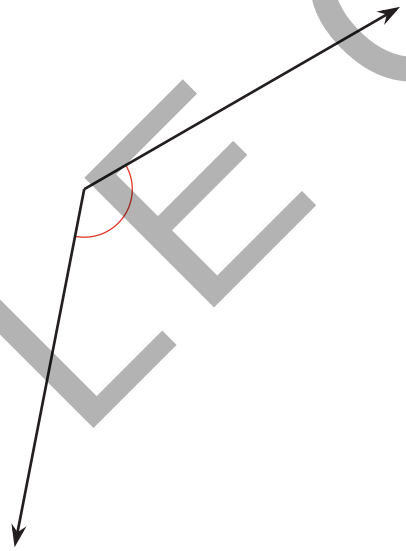
1.



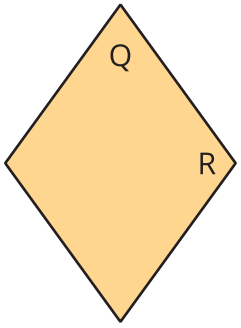
2.



3.



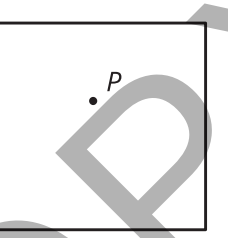
If you have time: Use a protractor to measure the labeled angles in the rhombus.



Activity 2

A Folding Challenge

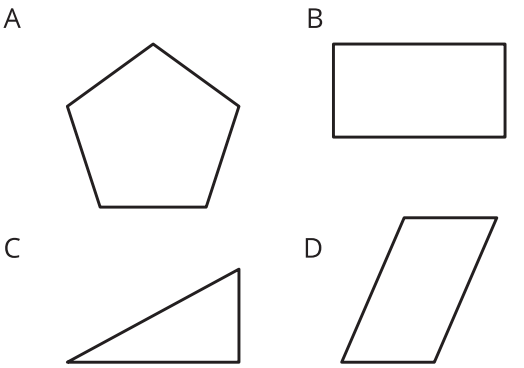
Tyler gives Lin a challenge: "Without using a protractor, draw four 90° angles. All angles have their vertex at point *P*."



Lin folds the paper twice, making sure each fold goes through point *P*. Then she traces the creases.

1. Your teacher will give you a sheet of paper. Draw a point on it. Then show how Lin might have met the challenge.
2. When Lin folds the paper, the creases form a pair of **perpendicular lines**. What do you think "perpendicular lines" mean?

3. Use Lin's method to create a new pair of perpendicular lines through the same point. Trace the creases with a different color.
4. Which shapes have sides that are perpendicular to one another?



Mark the perpendicular sides. Explain how you know the sides are perpendicular.

Addressing CA CCSSM 4.G.1, 4.MD.5, 4.MD.6-7; practicing MP6

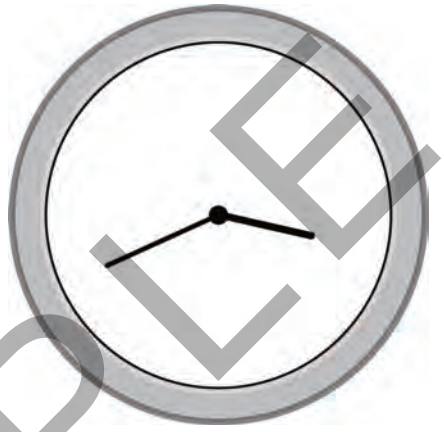
Use a Protractor to Draw Angles

Let's draw some angles.

Warm-up

Estimation Exploration: Long Hand and Short Hand

How many degrees is the angle formed by the long hand and the short hand of the clock?



Make an estimate that is:

too low	about right	too high

Activity 1

Draw These Angles

1. Draw a line that is neither vertical nor horizontal. Put a point somewhere on this line. Use your protractor to draw a perpendicular line through this point. Be as precise as possible. (No folding this time!)

2. Here is a ray that starts at point *M*.

   Use a protractor to draw:

   a. A ray starting at point *M* to create a 40° angle.
   b. Another ray starting at point *M* to create a 20° angle.
   c. One more ray starting at point *M* to create a 95° angle. Label each angle with its measurement.

3. There is one angle that is not labeled with a measurement and is greater than 180°. Label the angle with an arc. How many degrees is this angle? Explain how you know.

Activity 2

Angles Made to Order

Your teacher will give you 4 blank cards. Label each card with a letter A–D.

1. On each labeled card, draw an angle that meets the requirement with the same letter. Use a ruler and a protractor.
   a. an angle that is less than 35°
   b. an angle that is between 35° and 80°
   c. an angle that is greater than 80° but less than 120°
   d. an angle that is greater than 120° but less than 180°
2. Trade cards with your partner.
   a. Record the angle measurement for each angle. Check to make sure each angle meets the requirement.
   b. Have your partner correct the angle if it does not meet the requirement. Save the cards for the next lesson.

If you have time:

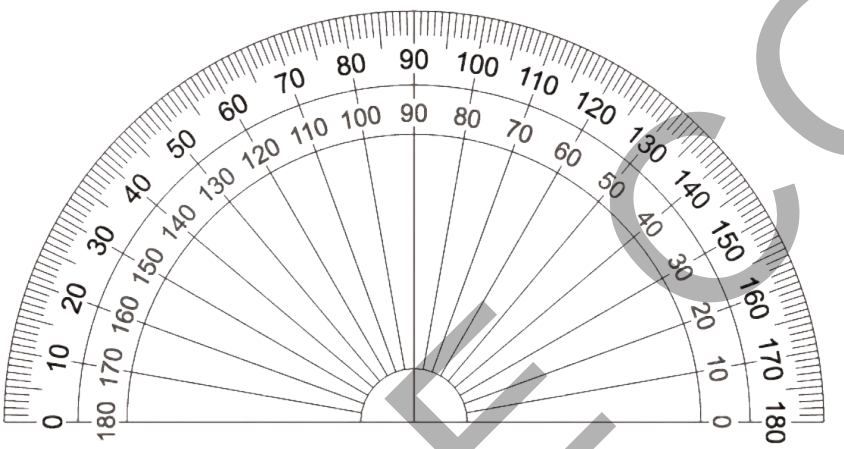
1. Create a drawing that shows several angles. Then write some descriptions about your drawing. Be as specific as possible.
2. Ask a partner to recreate the drawing, based on your descriptions. Does their drawing look like your drawing? If not, adjust your descriptions and ask them to try again.

Section B Summary

We learned ways to describe and measure the size of an angle.

We described angles as a turn of one ray away from the other. We learned that a degree is a measure of the turn around a circle and that 1 degree is $\frac{1}{360}$ of a full turn of a ray through a circle.

Finally, we learned that a protractor is a tool used to measure angles that also can be used to create angles of a certain measure.



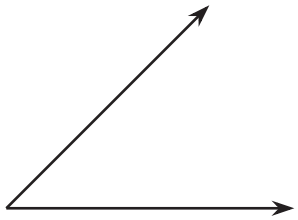
A protractor has two sets of numbers that can be used to measure an angle. We learned to use a protractor to measure and draw different angles.

Practice Problems

9 Problems

1 from Unit 7, Lesson 6

A



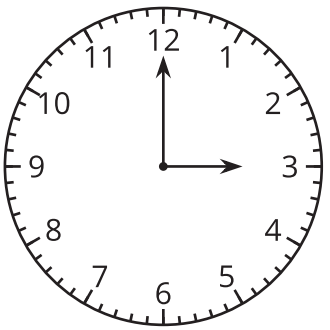
B

a. Write 2 statements that compare the sizes of Angles A and B.

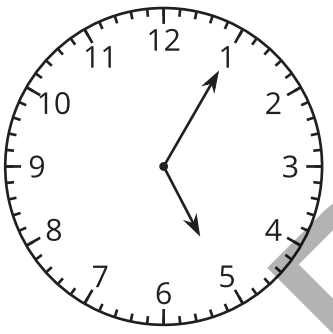
b. Draw an Angle C that is greater than Angle B.

2 from Unit 7, Lesson 7

A



B



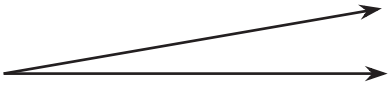
Sec B

a. Which set of clock hands makes a greater angle? Explain how you know.

b. Choose one of the clocks. Describe how to draw the angle represented by the hands on the clock.

3 from Unit 7, Lesson 8

This angle has a measure of 10°.



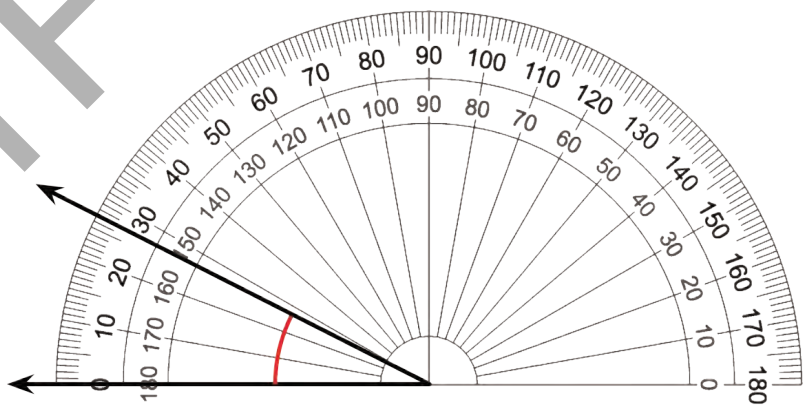
a. How many of these angles can you put together, without overlaps, to make a complete circle? Explain or show how you know.

b. Explain how you can use the given angle to sketch a 5° angle.

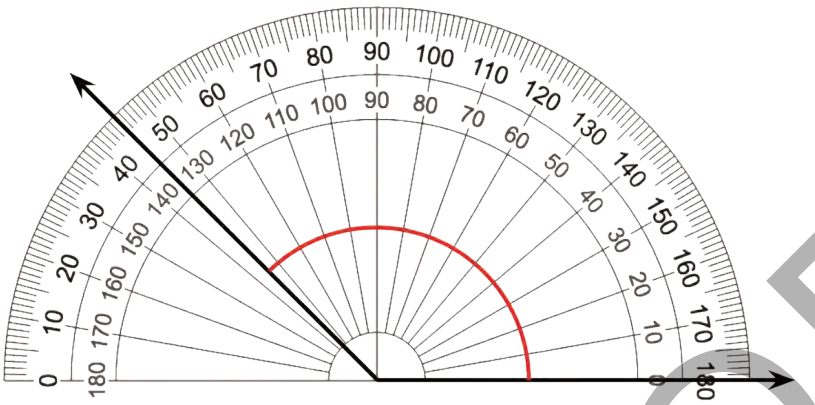
4 from Unit 7, Lesson 9

Use the given protractor to find the measurement of each angle.

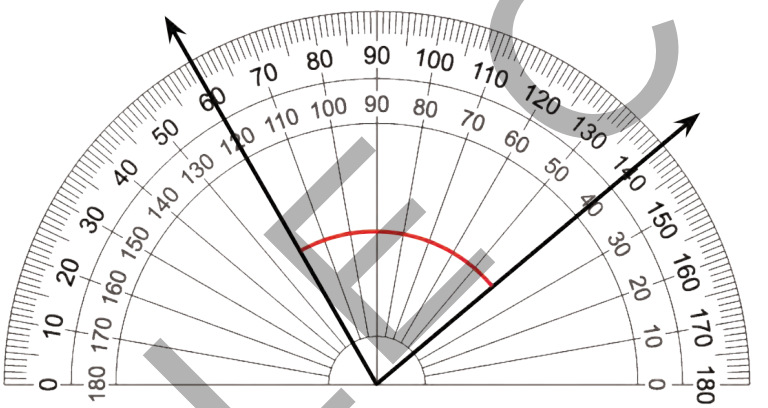
a.



b.



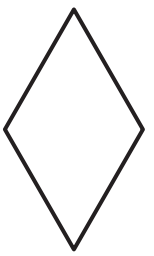
c.



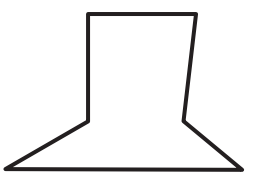
5 from Unit 7, Lesson 10

Which of these shapes have segments that are perpendicular to one another? Trace or circle the perpendicular segments.

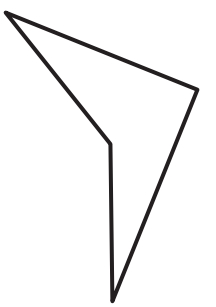
A



B



C



6

from Unit 7, Lesson 11

Draw a ray. How many different 35° angles can you make using your ray and another ray? Explain your reasoning and draw the angles.

7

Exploration

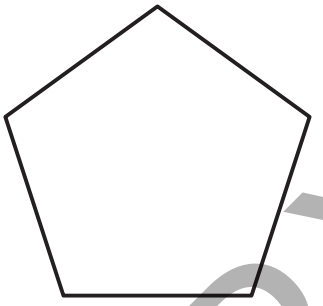
What is the smallest angle you can draw?

a. Can you draw a 10° angle?

b. Can you draw a 5° angle or a 1° angle?

c. What is difficult about drawing a small angle?

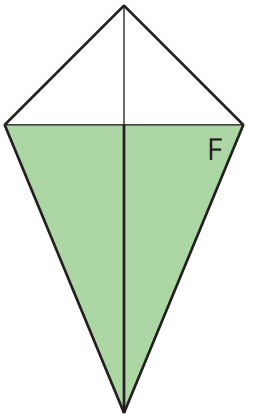
8 a. What is the measurement of each angle on the pentagon?



b. Connect each vertex to every other vertex with line segments that cross the pentagon. What do you notice? What do you wonder?

9 **Exploration**

Can you estimate or find the measurement of Angle F? If so, explain or show how you know.



Addressing CA CCSSM 4.G.1; building on 4.NF.4b; practicing MP6

Types of Angles

Let's look at different types of angles.

Warm-up

Number Talk: Fractions of 120 and 360

Find the value of each expression mentally.

- $12 \times \frac{1}{12}$
- $120 \times \frac{1}{12}$
- $360 \times \frac{1}{12}$
- $360 \times \frac{3}{12}$

Activity 1

Sorting Angles

In an earlier lesson, you and your partner drew some angles on cards.

Put the cards together and sort the angles into 2 groups. Explain your reasoning.

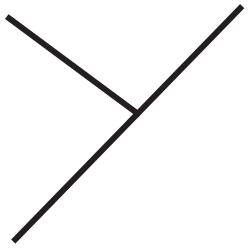
Activity 2

What Is It, Really?

1. Mai and Jada look at this drawing. Jada says it is a line. Mai says it is an angle.

Do you agree with Mai or Jada? Explain your reasoning.

2. Tyler and Andre measure an angle in this letter Y.



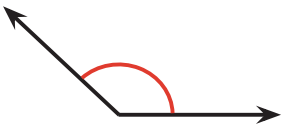
Andre said the angle he measured is obtuse. Tyler said the angle is acute.

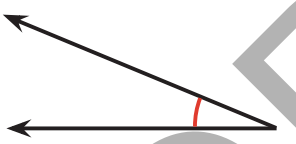
Explain why they could both be correct.

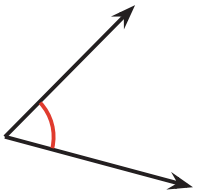
Activity 3

Small Angles, Large Angles

1. Identify each angle as acute, right, obtuse, or straight.

a. 

b. 

c. 

d. An angle formed by two 45° angles

e. A 91° angle

f. An angle that is in a rectangle

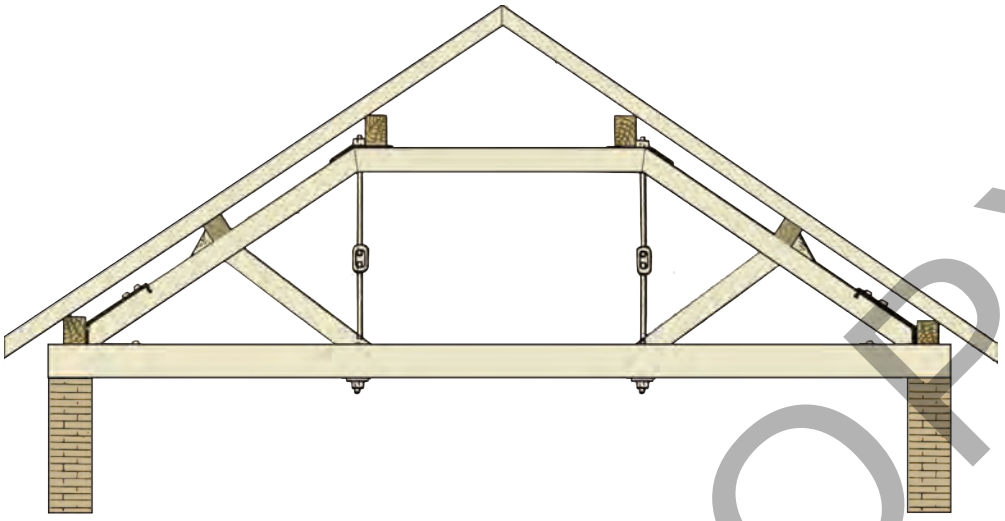
g. 

h. 

i. An angle composed of two right angles

j. An angle composed of five 12° angles

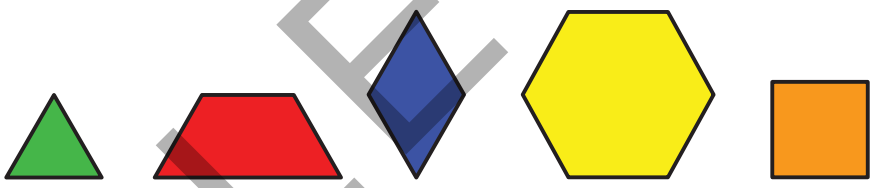
2. Here is a drawing of the structure of a roof.



Find as many acute and obtuse angles as you can in the drawing.

Use an "A" to label acute angles, a square (□) for right angles, and an "O" for obtuse angles.

3.



a. Diego chooses a pattern block that has 2 acute angles and 2 obtuse angles. Which pattern block does Diego choose?

b. He then chooses a pattern block with no obtuse angles. Which pattern block does he choose? Explain your reasoning.

Unit 7, Lesson 13

Addressing CA CCSSM 4.MD.7; practicing MP7

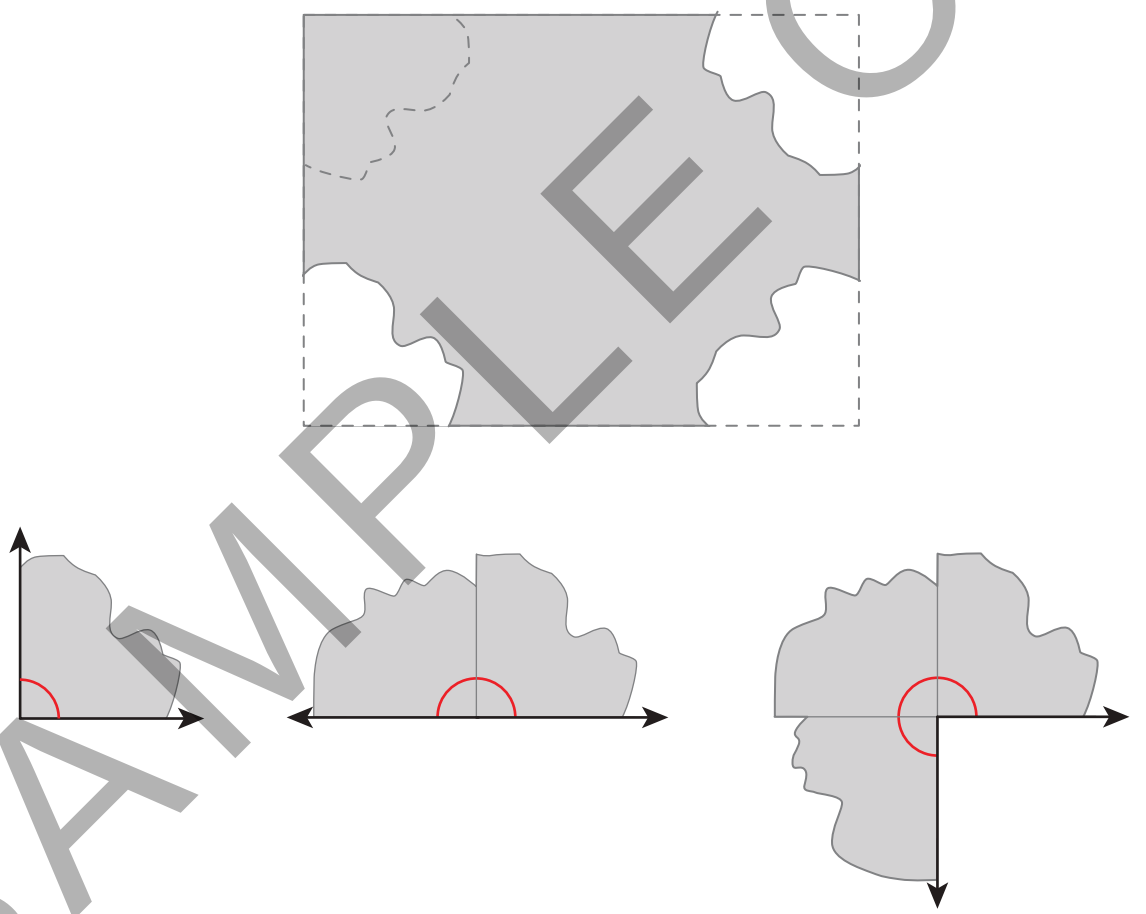
Find Angle Measurements

Let's compose and decompose angles to find their measurements.

Warm-up

Notice and Wonder: Corner Pieces

What do you notice? What do you wonder?

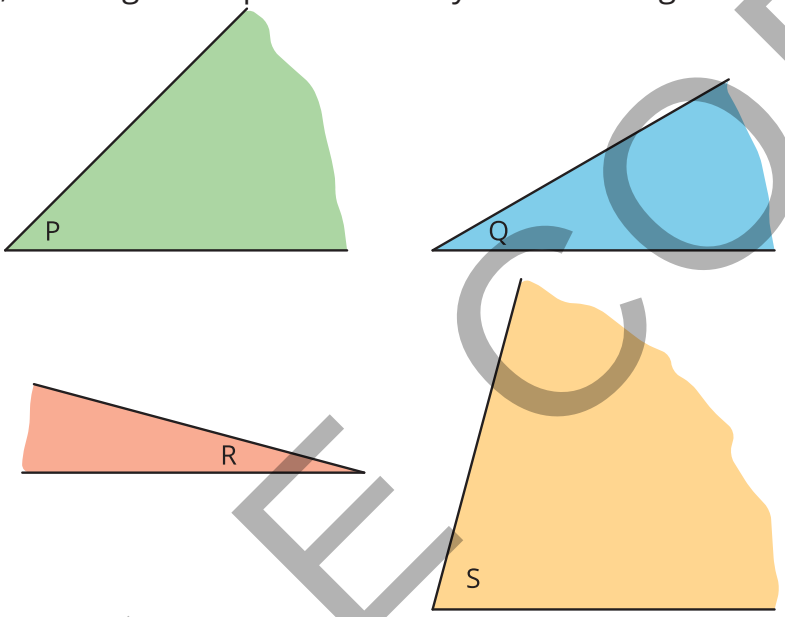


Activity 1

How Great Are These Angles?

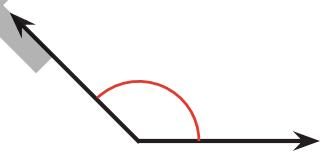
Your teacher will give you materials that can help you find angle measurements.

1. Use the materials and what you know about a right angle to find the sizes of Angle P, Angle Q, Angle R, and Angle S. Explain or show your reasoning.

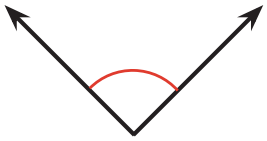


2. Next, use the measurements of Angle P, Angle Q, Angle R, and Angle S to find the measurements of these angles.

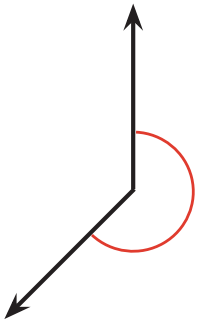
a.



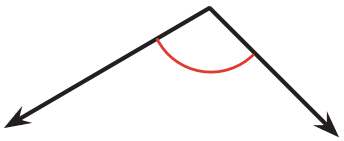
b.



c.



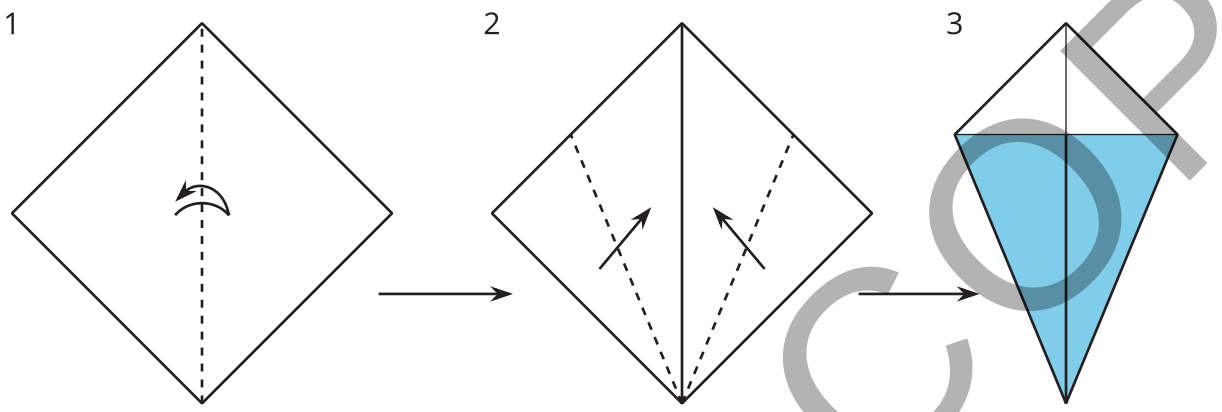
d.



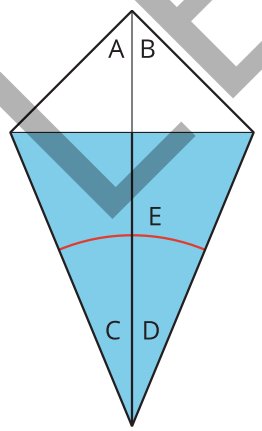
Activity 2

Angles in a Kite

Your teacher will give you a square sheet of paper. Follow the steps to fold the paper into a kite. Fold as precisely as possible.



Can you find the measurement of each labeled angle on the kite? If so, show your reasoning. If not, explain why not.



Unit 7, Lesson 14

Addressing CA CCSSM 4.MD.6-7; building on 4.G.1; practicing MP7

Reasoning about Angles (Part 1)

Let's find the size of an angle on the clock.

Warm-up

Which Three Go Together: Time after Time

Which 3 go together?

A

B

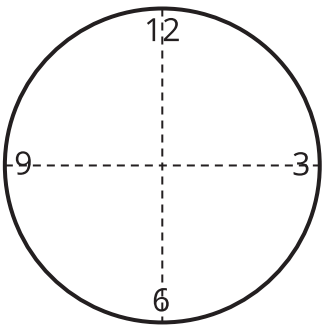
C

D

Activity 1

Draw a Clock

Kiran is drawing a clock. He draws a pair of perpendicular lines to find the placement of the numbers 3, 6, 9, and 12 around the circle.



1. How many degrees is each angle? Explain how you know.

2. Help Kiran find the exact placement of the numbers "1" and "2" on the clock.
   a. How many new lines does he need to draw?

   b. Describe the angles that should be formed between the 2 lines he has already drawn and the new lines.

   c. Draw lines precisely, and place the numbers "1" and "2" on the drawing.

3. Measure and draw as many lines as needed to complete the clock drawing. Make sure each number is placed in the correct location on the clock.

Activity 2

Tick Tock

1. The hour and minute hands form an angle at each of these times. How many degrees is each angle?

   a. 6 o'clock

   b. 8 o'clock

   c. 9 o'clock

   d. 11 o'clock

   e. 12 o'clock

2. How many degrees has the minute hand turned when it moves from 2:00 to 2:05?

   What about from 2:05 to 2:30? Explain how you know.

3. The minute hand of the clock is vertical at 7 p.m. Sometime later, it makes an angle that is 120° from where it was at 7 p.m. What time is it?

4. How many degrees does the minute hand turn in:

   a. 10 minutes?

   b. 1 minute?

   c. 4 minutes?

Unit 7, Lesson 15

Addressing CA CCSSM 4.G.1 and 4.MD.7; practicing MP1

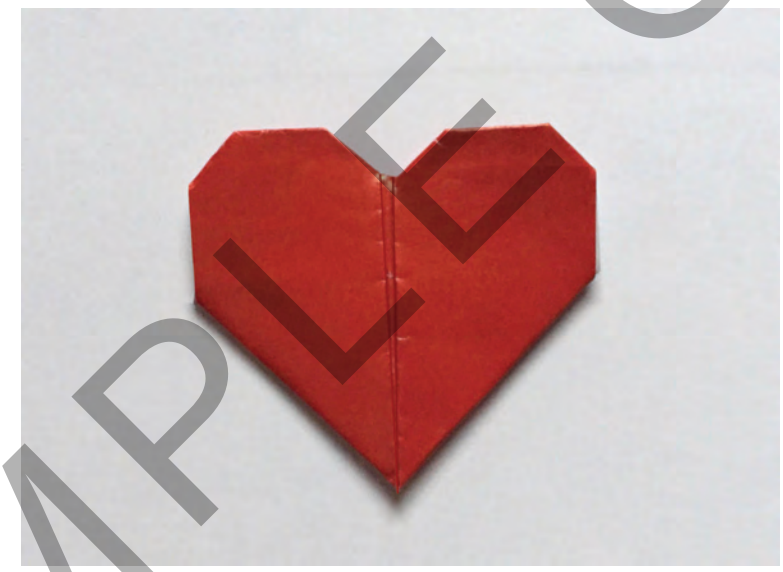
Reasoning about Angles (Part 2)

Let's figure out missing angle measurements.

Warm-up

How Many Do You See: Obtuse Angles

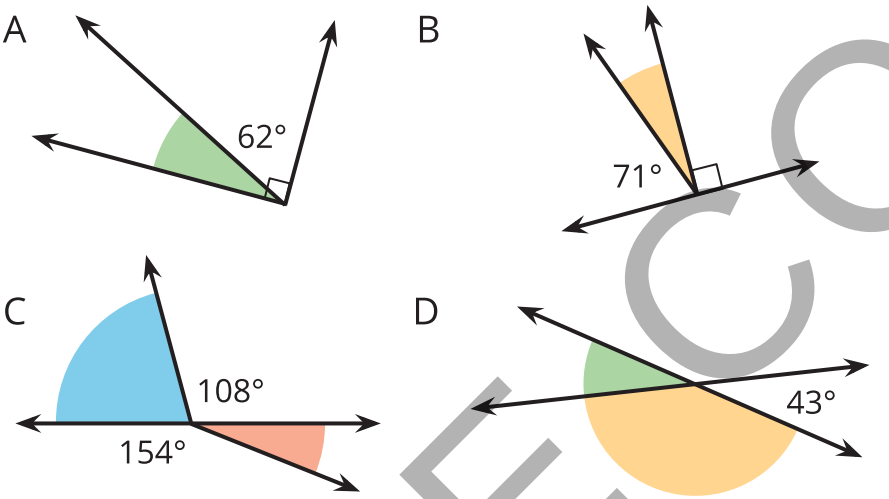
How many angles do you see in the folded paper heart?



Activity 1

Shaded and Unshaded Angles

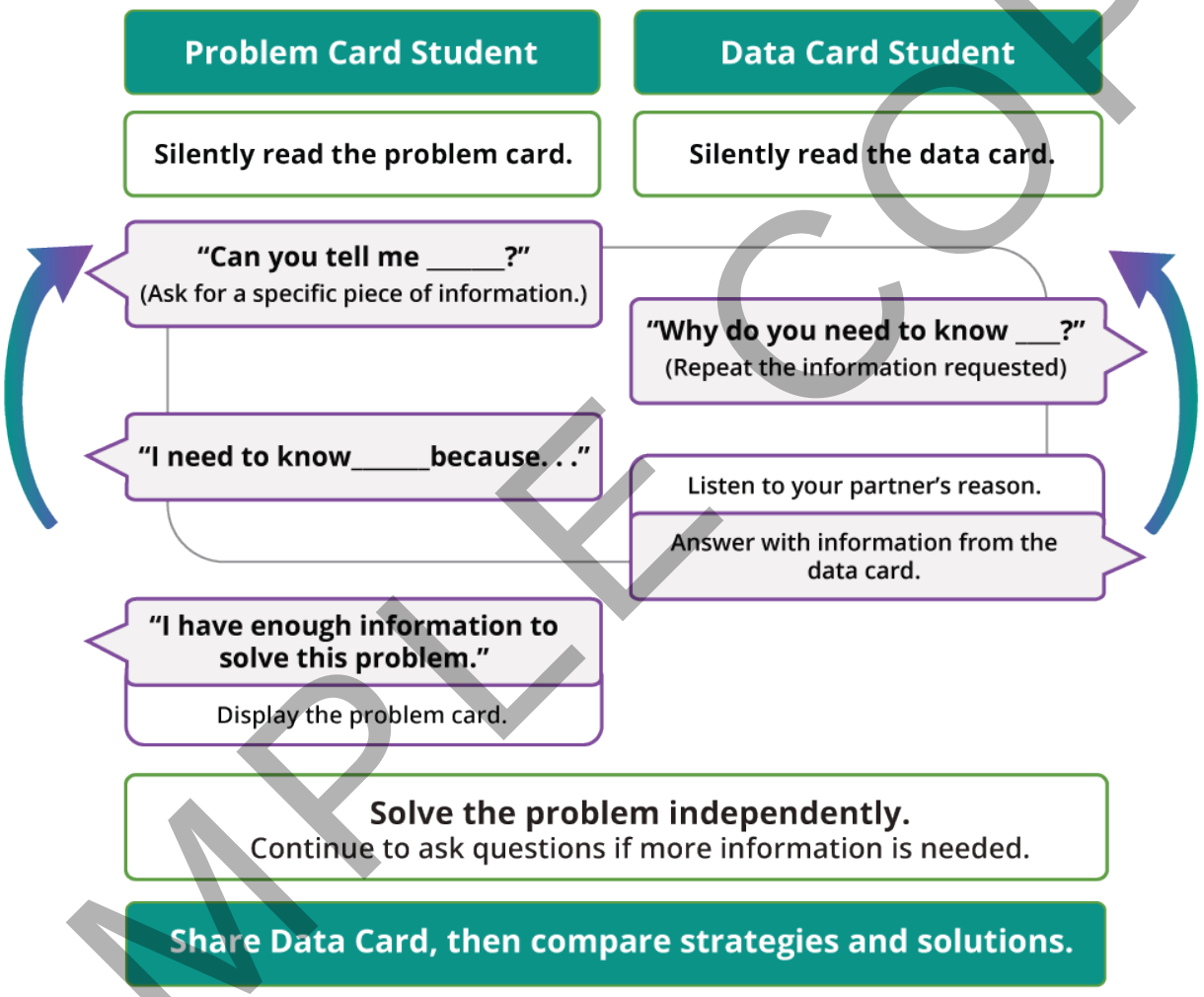
Find the measurement of each shaded angle. Show how you know.



Activity 2

Info Gap: A Whole Bunch of Angles

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

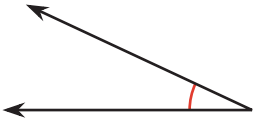


Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity. Switch roles with your partner.

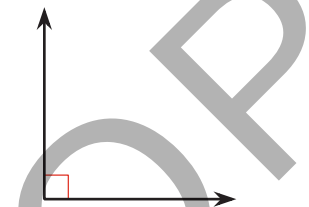
Section C Summary

We learned ways to name angles based on their measurements.

- **Acute angles** are angles that measure less than 90°.



- **Right angles** are angles that measure 90°.



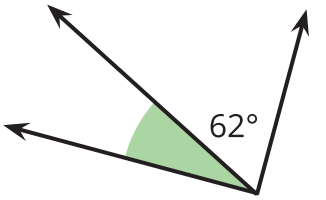
- **Obtuse angles** are angles that measure greater than 90°.
- **Straight angles** are angles that measure 180°.

We also solved problems about angles.

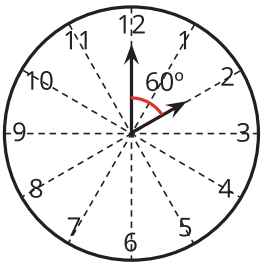
Examples:

If two angles make a right angle or a straight angle, we can use the size of one angle to find the other.

The shaded angle here must be 28° because it makes a right angle when combined with the 62° angle.



We know that the full turn of a clock measures 360°, so we determined that the long hand makes:



- A 360° angle every hour.
- A 180° angle every one-half hour.
- A 90° angle every 15 minutes.
- A 60° angle every 10 minutes.

Unit 7, Lesson 16

Addressing CA CCSSM 4.MD.5-6; practicing MP4

Angles, Streets, and Steps

Let's investigate streets and steps.

Warm-up

Notice and Wonder: Neighborhood Angles

What do you notice? What do you wonder?

Activity 1

How Steep Are These Streets?

1. In each case, draw a horizontal line and measure the angle that the street makes when it meets the horizontal line.

   a.

   b.

c.

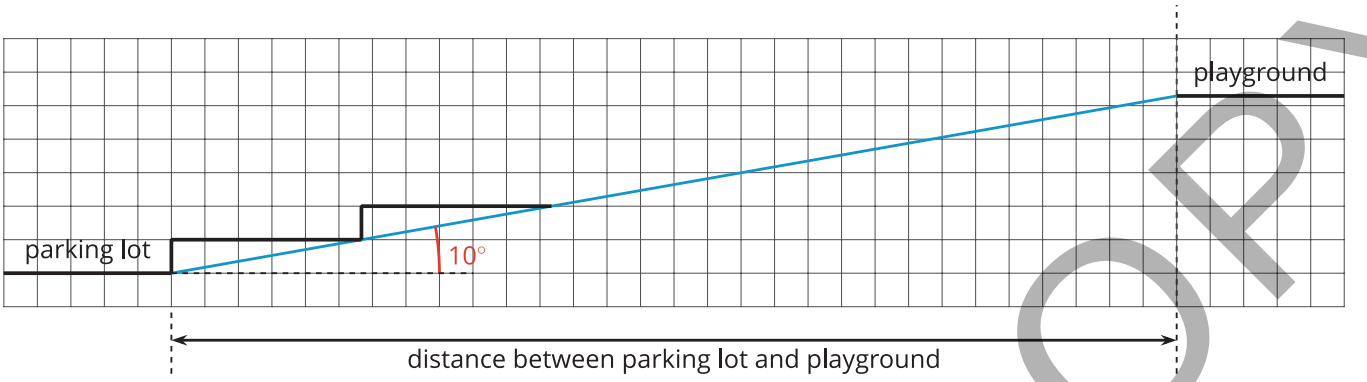
d.

e.

2. How is living on a steep street different from living on a level or flat street? Think of as many differences as you can.

Activity 2

Steep Steps



Sec C

A playground is on higher ground than its parking lot. The angle from the parking lot to the playground is 10°. Here's one way to build steps to get from the parking lot to the playground.

1. Create a drawing to show what the steps look like for the following angles from the parking lot to the playground. Make the height of each step 1 unit tall.
   a. 20°
   b. 25°
   c. an angle of your choice
2. How many steps are needed to get up from the parking lot to the playground for each angle?

3. At about what angle do you think the steps would be too steep or too hard to climb up? Explain or show your reasoning.

Practice Problems

10 Problems

1 from Unit 7, Lesson 12

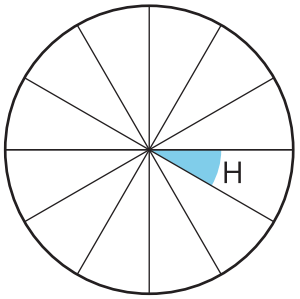
a. Draw an acute angle. Explain how you know the angle is acute.

b. Extend one of the rays of your angle in the opposite direction. Explain why your new angle is obtuse.

2 from Unit 7, Lesson 13

a. The circle is divided into 12 equal parts. What is the measure of Angle H?

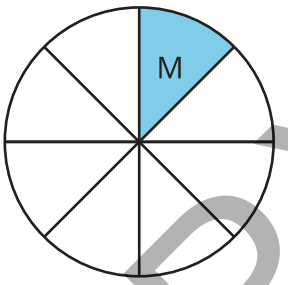
Explain or show how you know.



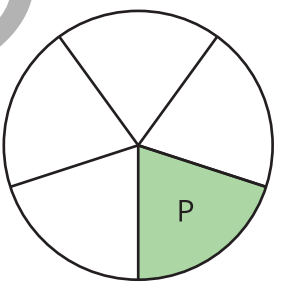
b. Can you put together 20° angles to make a circle? How many of them will it take?

3 from Unit 7, Lesson 13

a. A circle has been cut into eighths. How many degrees is the angle labeled M? Explain or show your reasoning.



b. Another circle has been cut into fifths. How many degrees is the angle labeled P? Explain or show your reasoning.



Sec C

4 from Unit 7, Lesson 14

a. What angles are made by the hour and minute hands on a clock at these times? Explain or show your reasoning.

- 3:00

- 5:00

- 6:00

b. How many degrees does the hour hand move between 3:00 and 7:00? Explain or show how you know.

5

from Unit 7, Lesson 14

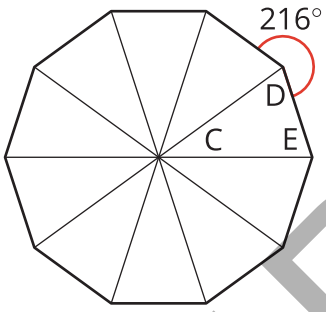
The long hand points at 12 when Jada looks at the clock. Less than an hour later, when she looks up again, the long hand of the clock has turned 210 degrees. How many minutes have passed? Explain or show your reasoning.

6

from Unit 7, Lesson 15

Find the measure of each labeled angle in the drawing. Assume that:

- The angles of the triangles meeting at the point in the middle of the figure have the same measure.
- The other angles of the triangles all have the same measure.



7 Tyler wonders if the hour hand and the minute hand ever point in the same direction at the same time. Can you find some times when the hour hand and the minute hand point in the same direction? Explain or show your reasoning.

8 a. Draw a rhombus with a 50° angle. Explain how you know your shape is a rhombus.

b. Draw another rhombus with a 50° angle. How are your rhombuses the same? How are they different?

9 Exploration

How many degrees does the minute hand turn in each of the following times? Show how you know.

a. 30 seconds

b. 10 seconds

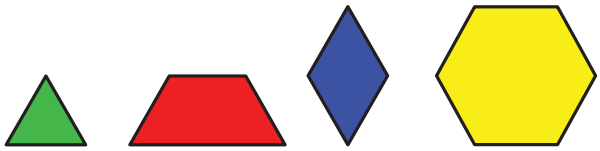
c. 80 minutes

d. 2.5 hours

10 Exploration

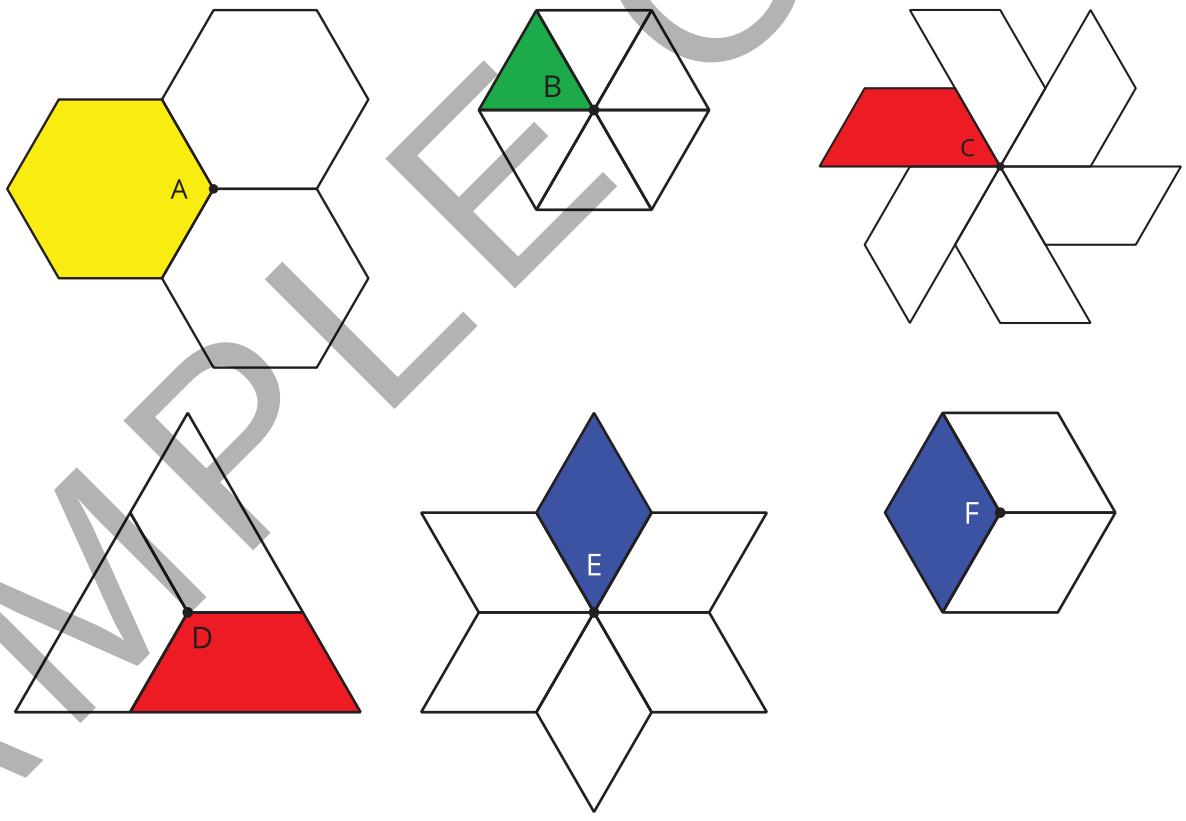
Here are diagrams of some pattern blocks. Each shape has some angles.

a. How many angles do you see inside each shape?



- a triangle
- a trapezoid
- a rhombus
- a hexagon

b. Here are diagrams that show a group of each type of pattern block arranged around a shared point.



Use what you know about angle measurement to find the sizes of Angles A–F. Show your reasoning.

GRADE 4

UNIT

Properties of Two-Dimensional Shapes

Content Connections

In this unit you will classify triangles and quadrilaterals based on the properties of their side lengths and use your understanding of the symmetry of two-dimensional figures to solve problems. You will make connections by:

- **Discovering Shape and Space** while using your knowledge about geometry to describe and reason about characteristics of shapes.
- **Reasoning with Data** while analyzing and categorizing two-dimensional shapes (triangles and quadrilaterals) by their attributes.
- **Taking Wholes Apart, Putting Parts Together** while exploring symmetry, identifying symmetric figures, and drawing lines of symmetry.

Addressing the Standards

As you work your way through **Unit 8 Properties of Two-Dimensional Shapes,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	
MP2 Reason abstractly and quantitatively.	
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 5 and 7
MP5 Use appropriate tools strategically.	Lesson 11
MP4 Model with mathematics.	Lessons 2, 4, 5, 8, 9, and 10
MP6 Attend to precision.	Lessons 1, 3, 4, 6, and 10
MP7 Look for and make use of structure.	Lessons 1, 2, 3, 6, 8, and 9
MP8 Look for and express regularity in repeated reasoning.	Lessons 2 and 7

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes and Symmetries	**4.G.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Lessons 1, 3, 5, 6, and 9
• Shapes and Symmetries	**4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (Two-dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.)	Lessons 1, 2, 3, 4, 6, and 9

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes and Symmetries • Connected Problem Solving	**4.G.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Lessons 4, 5, 6, 7, 8, 9, 10, and 11
• Measuring and Plotting • Rectangle Investigations • Connected Problem Solving	**4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), . . .*	Lessons 7 and 8
• Rectangle Investigations • Connected Problem Solving	**4.MD.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	Lessons 5, 7, and 9
• Rectangle Investigations • Circles, Fractions and Decimals • Shapes and Symmetries	**4.MD.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	Lesson 3
• Rectangle Investigations • Shapes and Symmetries	**4.MD.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	Lesson 3

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Circles, Fractions and Decimals • Shapes and Symmetries	**4.MD.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Lessons 3 and 10
• Fraction Flexibility • Visual Fraction Models	**4.NF.3c** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1*/b.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Lessons 2, 7, and 8
• Fraction Flexibility	**4.NF.4** 4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of 1*/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of 1*/b,* and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	Lessons 2, 7, and 8

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 8, Lesson 1

Addressing CA CCSSM 4.G.1-2; building towards 4.G.2; practicing MP6 and MP7

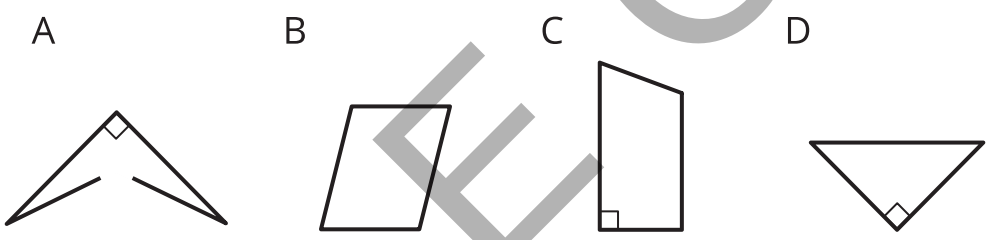
Ways to Look at Figures

Let's sort flat figures.

Warm-up

Which Three Go Together: Geometric Figures

Which 3 go together?



Activity 1

Card Sort: Shapes

Your teacher will give you a set of cards that show flat figures.

1. Sort the cards into 3–5 categories in a way that makes sense to you. For each category, write a title on a sticky note. Be ready to explain the meaning of your categories.

2. Share your categories with another group. Take turns listening to each other's explanations.
   - Do your categories make sense to them?
   - Do their categories make sense to you?
   - Any suggestions or corrections?

3. Cover or hide the titles of your categories. Trade places with another group. Study their sorted cards while they study yours.

   Guess the other group's categories and how they sorted the figures.

Activity 2

Guess the Category

Partner A:

- Record a category from the first activity (or think of a new one). Don't show it to your partner.
- Find 3 figures that fit the category and 3 figures that *do not* fit. Record each figure's letter name in the correct column of the table.

Partner B:

- Study the figures chosen by your partner.
- Pick another figure from the cards. Ask: "Does this shape fit in your category?"
- Find 2 figures that fit the category and 2 figures that *do not* fit.
- Guess the category. If your guess is incorrect, ask more questions before guessing again.

Switch roles after the category is guessed correctly.

- Partner A's category: _____

fit the category	do not fit the category

- Partner B's category: _____

fit the category	do not fit the category

Unit 8, Lesson 2

Addressing CA CCSSM 4.G.2, 4.NF.3c, and 4.NF.4; practicing MP5, MP7 and MP8

Ways to Look at Triangles

Let's sort and analyze triangles.

Warm-up

Number Talk: Sums and Products

Find the value of each expression mentally.

- $12 + 12 + 75$
- $12\frac{1}{2} + 12\frac{1}{2} + 75$
- $(2 \times 12\frac{1}{2}) + (4 \times 12\frac{1}{2})$
- $7 \times 12\frac{1}{2}$

Activity 1

Triangle Hunt

1. From the set of triangle cards, find all the triangles that have each attribute. Record their letter names here.

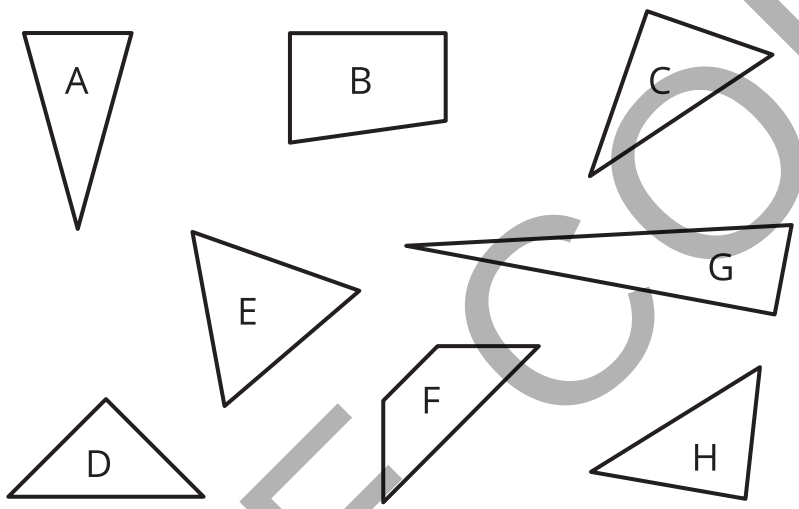
a. No right angles	b. Parallel sides	c. Perpendicular sides
d. The same length for all sides	e. The same size for all angles	f. More than 1 right angle
g. More than 1 obtuse angle	h. More than 1 acute angle	i. Foldable into 2 equal halves

2. Choose 1 sentence to complete based on your work.
   a. I noticed that some triangles . . .
   b. I noticed that all triangles . . .
   c. I noticed that no triangles . . .

Activity 2

The Right Kind of Triangle

1. Identify all shapes that are right triangles. For each right triangle, mark the right angle with a small square.



2. Explain why the other shapes are *not* right triangles.

Unit 8, Lesson 3

Addressing CA CCSSM 4.G.1-2 and 4.MD.5-7; practicing MP6 and MP7

Ways to Look at Quadrilaterals

Let's sort and identify quadrilaterals.

Warm-up

How Many Do You See: Brick Pattern

How many bricks have 2 pairs of parallel sides?

Activity 1

Quadrilateral Hunt

1. Find the quadrilaterals that have each of the following attributes. Record their letter names here.

attribute	quadrilaterals with the attribute
a. no right angles	
b. 1 pair of parallel sides	
c.1 pair of perpendicular sides	
d. same length for all sides	
e. same size for all angles	
f. same length for only 2 sides	
g. no parallel sides	
h. 2 obtuse angles	

2. Choose 1 sentence to complete based on your work.

   a. I noticed some quadrilaterals . . .

   b. I noticed that all quadrilaterals . . .

   c. I noticed that no quadrilaterals . . .

If you have time: Do you think it's possible for a quadrilateral to have:

- More than 2 acute angles?

- More than 2 obtuse angles?

- Exactly 3 right angles?

If you think so, sketch an example. If you don't think so, explain or show why you think it's impossible.

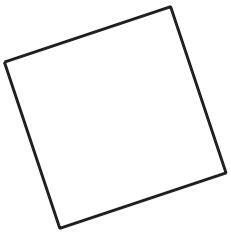
Activity 2

What's True about These Quadrilaterals?

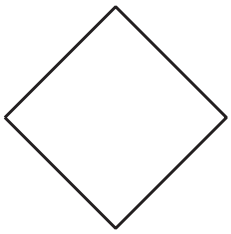
Here are 4 sets of quadrilaterals.

Quadrilaterals D and AA are squares.

D

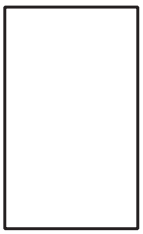


AA

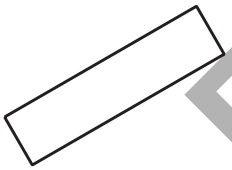


Quadrilaterals K, Z, and AA are rectangles.

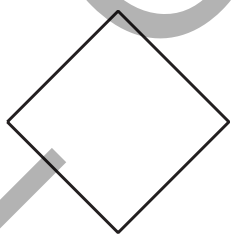
K



Z



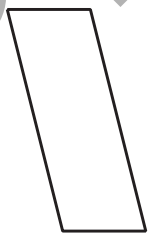
AA



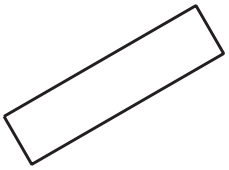
Quadrilaterals N, U, and Z are parallelograms.

N

U

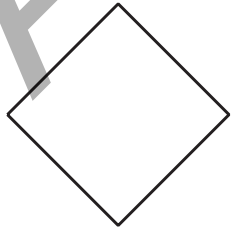


Z

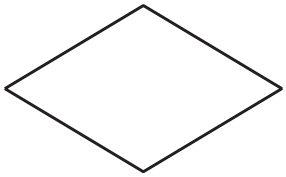


Quadrilaterals AA, EE, and JJ are rhombuses.

AA



EE



JJ

Write 4–5 statements about the sides and angles of the quadrilaterals in each set. Each statement must be true for *all* the shapes in the set.

square	rectangle
rhombus	parallelogram

Activity 3

Guess Again

Partner A:

- Record an attribute that a quadrilateral could have. Don't show it to your partner.
- Find 3 quadrilaterals that have that attribute and 3 that don't. Record each quadrilateral's letter name in the correct column of the table.

Partner B:

- Study the quadrilaterals chosen by your partner.
- Pick another quadrilateral. Ask: "Does this quadrilateral have the attribute?"
- Find at least 1 quadrilateral that has the attribute and 1 that *does not* have it.
- Guess the attribute. If your guess is incorrect, ask more questions before guessing again.

Switch roles after the attribute is guessed correctly.

- Partner A's attribute: _____

have the attribute	do not have the attribute

- Partner B's attribute: _____

have the attribute	do not have the attribute

Unit 8, Lesson 4

Addressing CA CCSSM 4.G.2-3; practicing MP5 and MP6

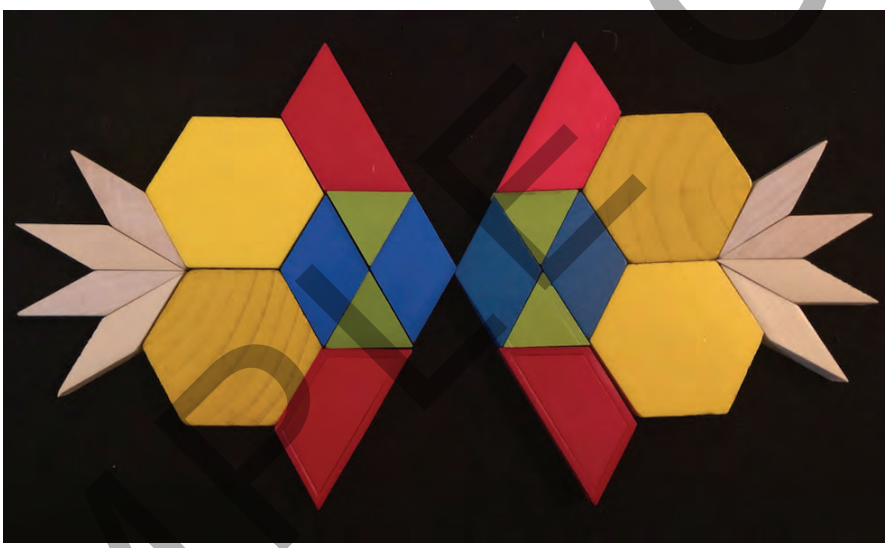
Symmetry in Figures (Part 1)

Let's describe symmetry in flat figures.

Warm-up

Notice and Wonder: Seeing Double

What do you notice? What do you wonder?

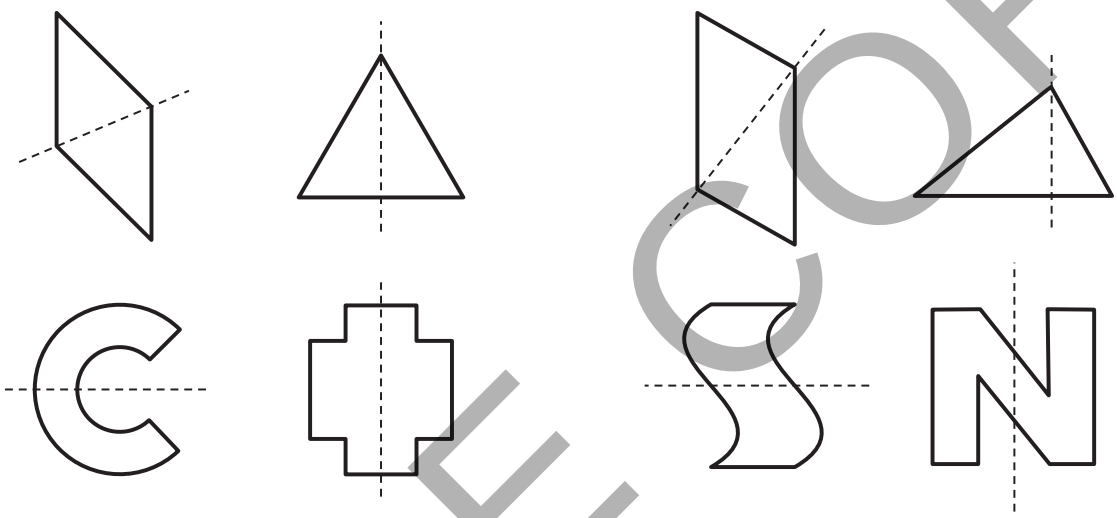


Activity 1

Perfect Matches

1. Lin cuts pieces of paper into different figures. Then she folds each piece of paper once, creating two smaller parts.

   Lin sorts the pieces into 2 categories based on the folding lines.



folding line is a line of symmetry

folding line is not a line of symmetry

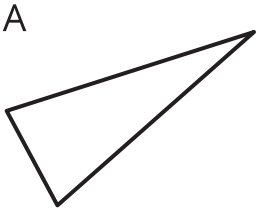
Study the figures in each category. What do you think a **line of symmetry** means?

Complete this sentence:

A line of symmetry is . . .

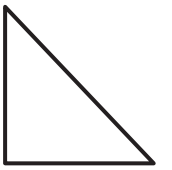
2. Do any of these figures have a line of symmetry? If so, draw the line. If not, explain how you know.

A

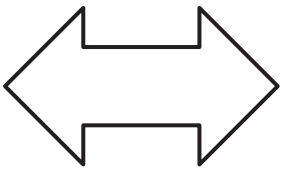


B

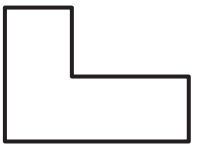
C



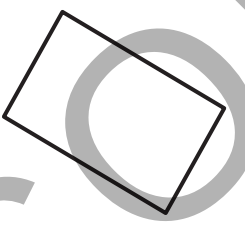
D



E



F



3. Are there any figures with more than 1 line of symmetry? If you think so, draw all the lines of symmetry.

Activity 2

In Search of Symmetry

Your teacher will give your group a set of cards.

1. Sort the figures on the cards by the number of lines of symmetry they have.

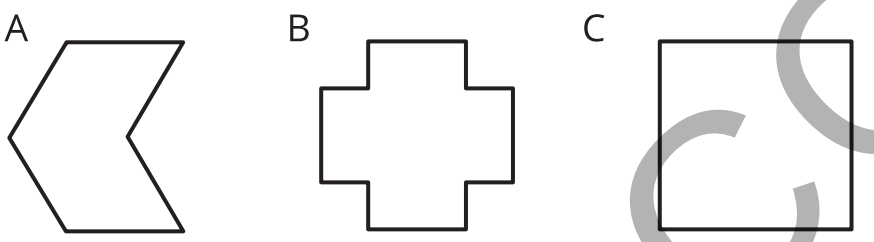
0 lines of symmetry	only 1 line of symmetry	only 2 lines of symmetry	only 3 lines of symmetry

2. Find another group that has the same set of cards. Compare how you sorted the figures. Did you agree with how the other group sorted the figures? If not, discuss any disagreement.

Activity 3

Just Keep Folding

Priya cuts out these 3 paper figures. For each figure, she folds it along a line of symmetry. Then she keeps folding along a line of symmetry until the folded figure has no more lines of symmetry.



1. How many times can she fold each figure before she can no longer continue?

2. What do you notice about each folded figure when it can no longer be folded?

Addressing CA CCSSM 4.G.1, 4.G.3, and 4.NBT.3; practicing MP3 and MP5

Symmetry in Figures (Part 2)

Let's draw some figures that have lines of symmetry.

Warm-up

Number Talk: Keeping Track

Find the value of each expression mentally.

- $43 + 57 + 50 + 7 + 3 + 40$

- $243 + 57 + 43 + 257$

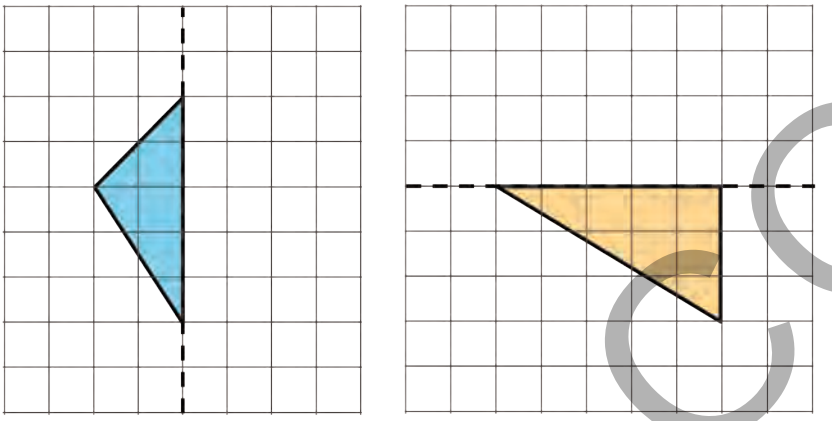
- $1{,}043 + 257 + 57 + 200 + 43 + 1{,}000$

- $1{,}943 + 257 + 1 + 257 + 1{,}000 + 943$

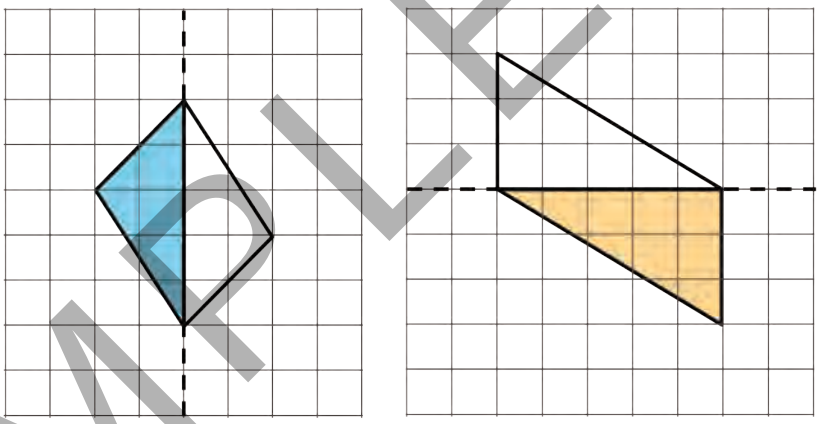
Activity 1

Half-Drawn Figures

Each shaded triangle is half of a whole figure that has a line of symmetry shown by the dashed line.



Clare drew some segments to show the missing half of each figure.

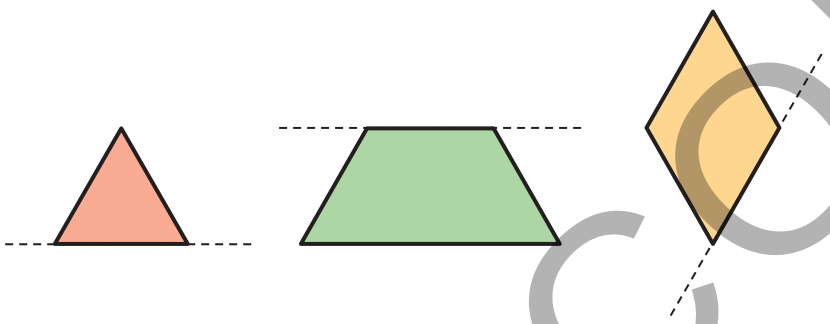


Do you agree that the dashed line is a line of symmetry for each figure Clare completed? Explain your reasoning. If you disagree with Clare's work, show a way to complete the drawing so the dashed line is a line of symmetry.

Activity 2

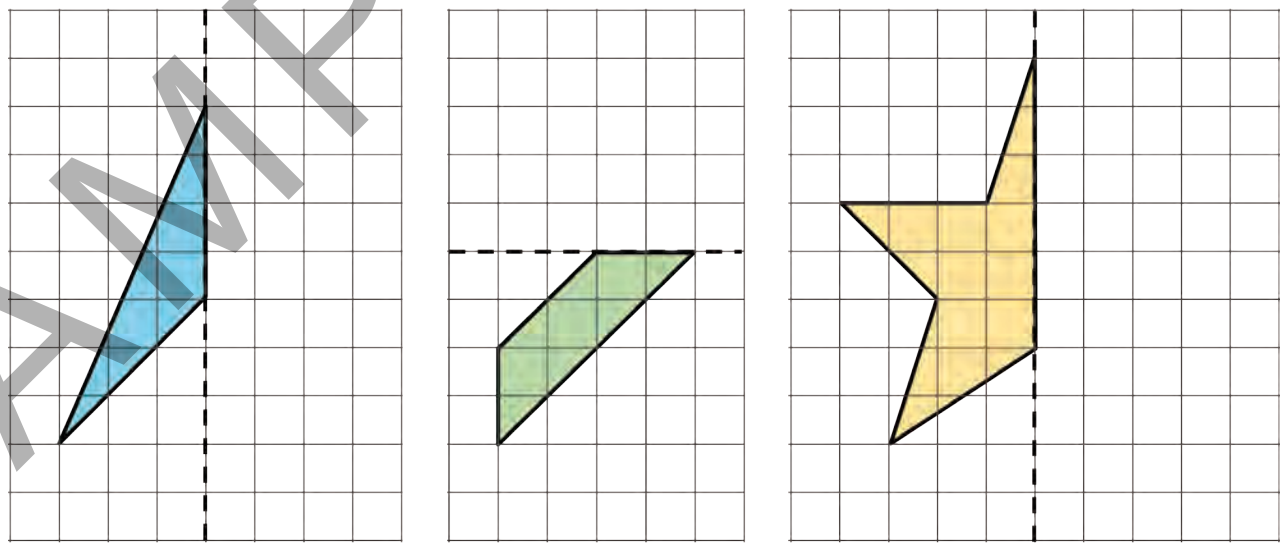
What's the Whole Picture?

1. Here are 3 figures. Each figure is half of a whole figure. The dashed line is a line symmetry of that whole figure.

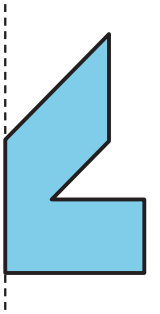


Use patty paper to help you draw the whole figure.

2. Each figure on the grid is half of a whole figure. The dashed line shows the line of symmetry for the whole figure. Use the grid to help you draw the whole figure. Be as precise as possible.



3. Here is another figure that is half of a whole figure with a vertical line of symmetry. Draw the whole figure. Be as precise as possible.



Activity 3

What Could the Whole Figure Be?

Trace a triangle cutout from your teacher.

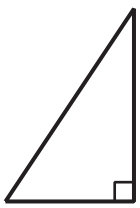
The triangle is half of a whole figure that has a line of symmetry. What could the whole figure look like? Can you show two possibilities? Three possibilities? Show your thinking. Organize it so it can be followed by others.

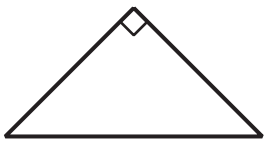
Section A Summary

We looked at different attributes of shapes, such as the number and length of sides, the measurements of sides and angles, and whether the shapes had parallel or perpendicular sides.

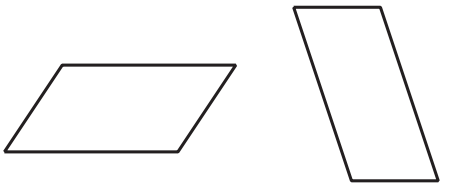
We then used these attributes to classify quadrilaterals and triangles.

Right triangles have 1 right angle.

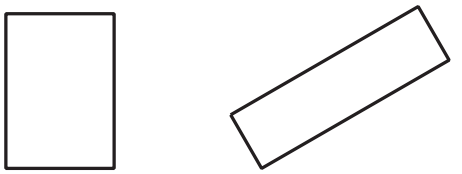




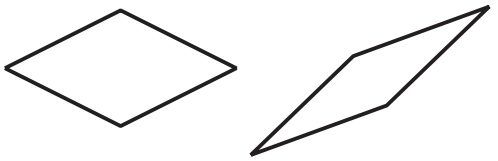
Parallelograms have 2 pairs of parallel sides.



Rectangles have 2 pairs of parallel sides and 4 right angles.

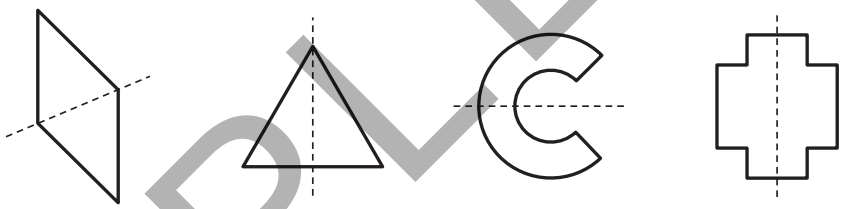


Rhombuses have 4 equal sides.



Squares have 4 equal sides and 4 right angles.

We also learned about symmetry. A figure has **symmetry** if its parts can match up exactly after it is folded or turned. A figure that has a **line of symmetry** can be folded along that line to create 2 halves that match up exactly.



Unit 8, Lesson 6

Addressing CA CCSSM 4.G.1-3; practicing MP6 and MP7

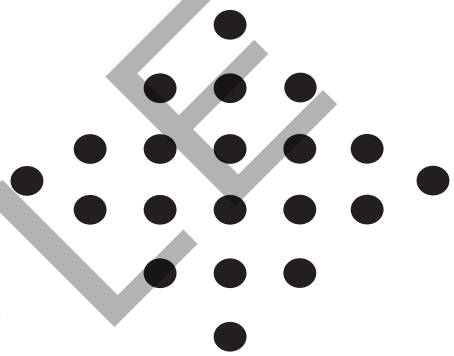
All Kinds of Attributes

Let's use what we know about attributes of figures to create drawings.

Warm-up

How Many Do You See: Dot after Dot

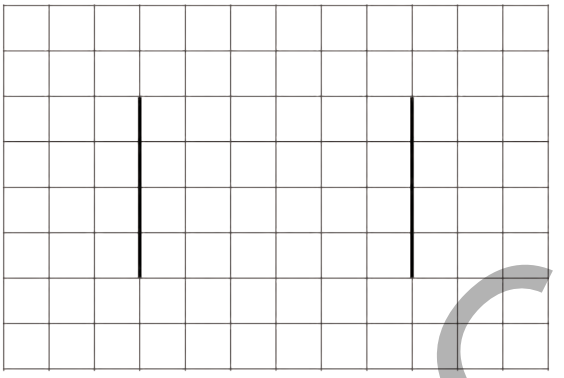
How many do you see? How do you see them?



Activity 1

You're Gonna Draw It: It's Symmetric

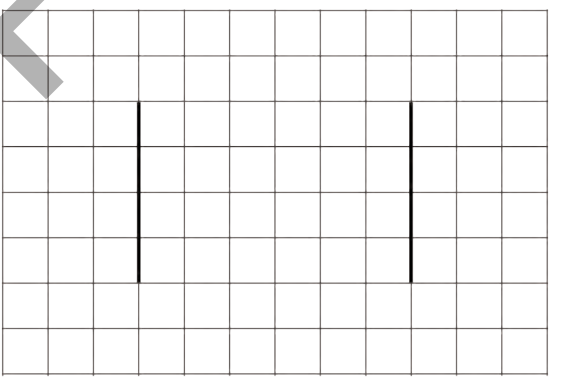
1. Here is a pair of parallel segments that have the same length.



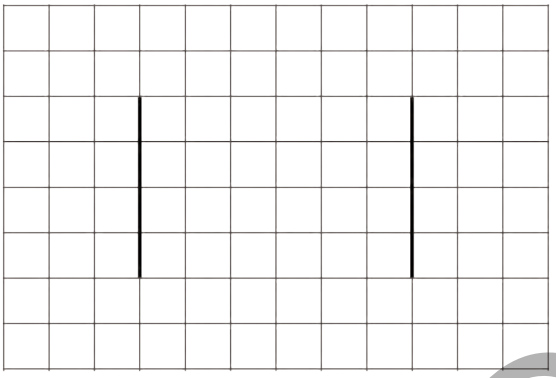
Draw 1 or more segments to create a figure with only 1 line of symmetry.

2. Here are some other pairs of parallel segments. Draw more segments to make:

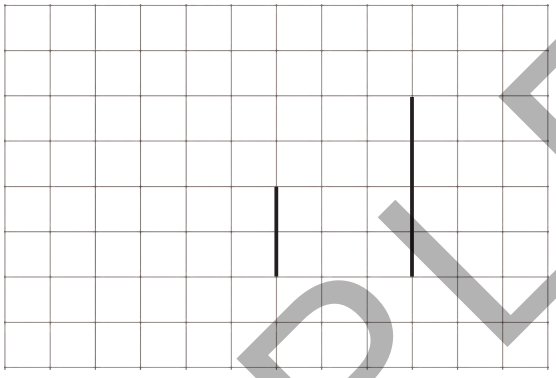
   a. a figure with 2 lines of symmetry

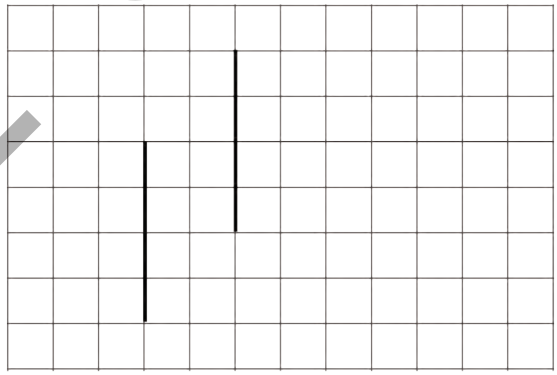


b. a figure with 0 lines of symmetry



If you have time: Here are some other pairs of parallel lines. Draw more segments to create a figure with only 1 line of symmetry.

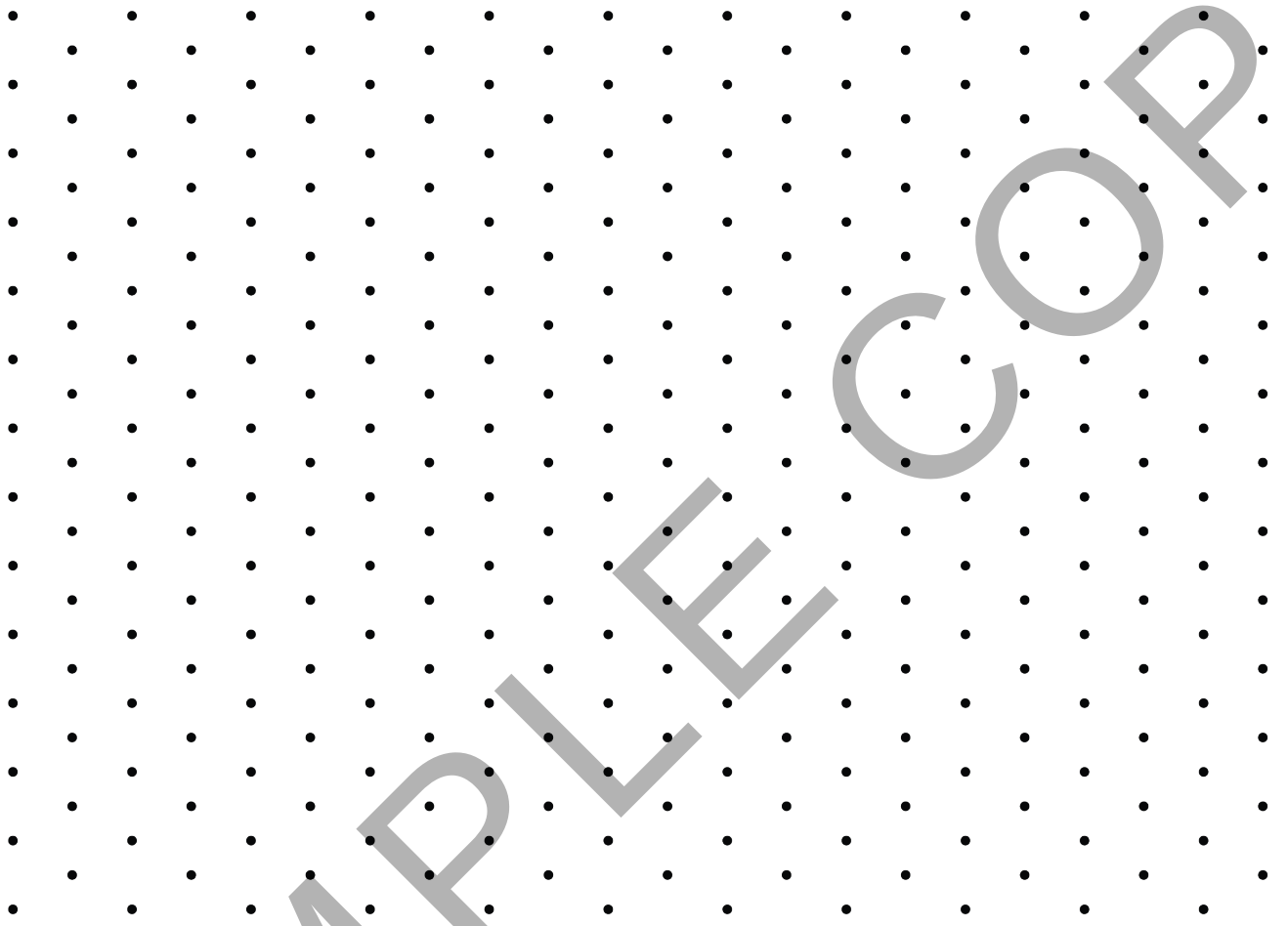




Activity 2

Hidden Shapes

Here is a field of dots.



Can you connect the dots to create each of the following shapes? If so, draw the shapes. If not,Uexplain your reasoning.

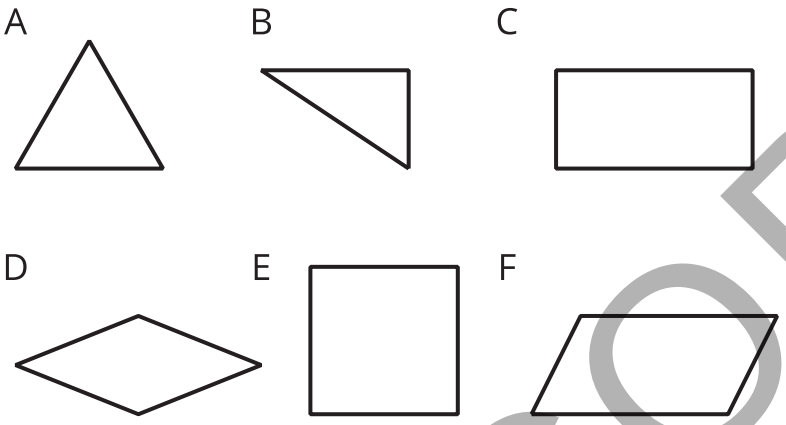
1. a triangle with only 1 line of symmetry
2. a quadrilateral with only 1 line of symmetry
3. a quadrilateral with 2 pairs of parallel sides
4. a quadrilateral with 1 pair of perpendicular sides
5. a rectangle
6. a 6-sided figure with only 1 line of symmetry

Practice Problems

10 Problems

1 Pre-unit

Here are 6 shapes:



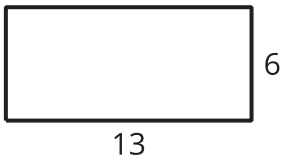
a. Which shapes are quadrilaterals?

b. Which shapes are rhombuses?

c. Which shapes are rectangles?

2 Pre-unit

Find the perimeter and area of the rectangle.
Explain or show your reasoning.

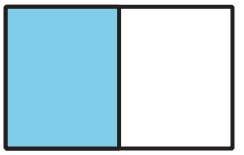


3 Pre-unit

Select **all** images that show half of the rectangle shaded.

A.

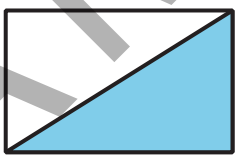
B.



C.

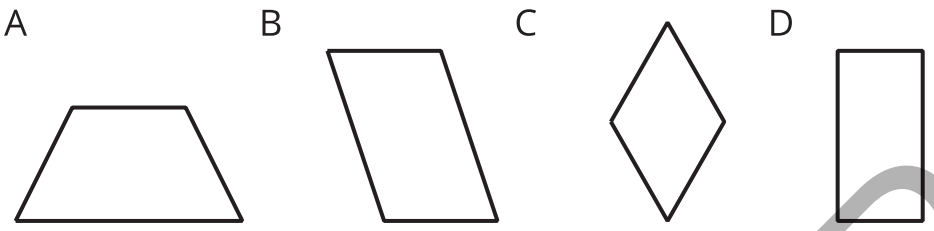
D.

E.



4 from Unit 8, Lesson 1

Here are 4 shapes:

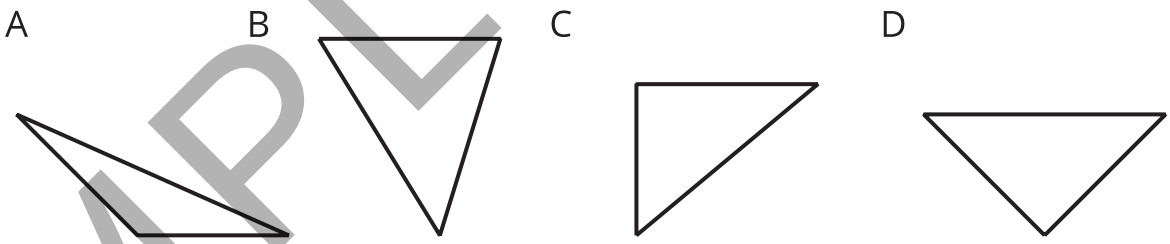


a. Name some attributes that all the shapes share.

b. Name some attributes that the shapes *do not* share.

5 from Unit 8, Lesson 2

Here are 4 triangles:



a. Which triangles are right triangles?

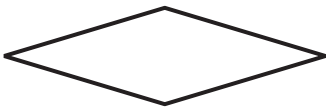
b. Which triangles have an obtuse angle?

c. Which triangles have 3 acute angles?

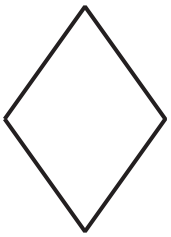
6 from Unit 8, Lesson 3

Here are 3 rhombuses:

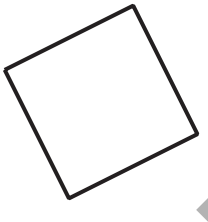
A



B



C



a. What attributes do the 3 rhombuses share?

b. What attributes are different in the 3 rhombuses?

7 from Unit 8, Lesson 4

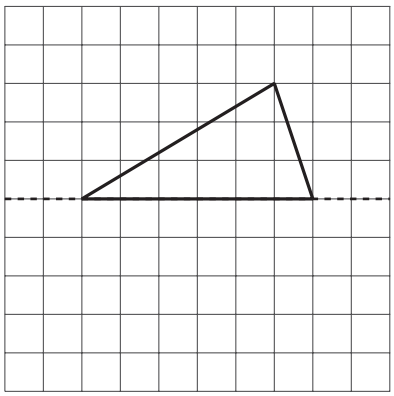
Draw any lines of symmetry for these letters:

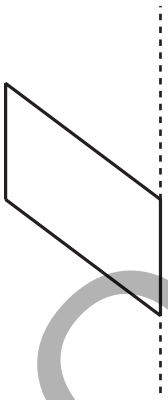
A C G

N O Z

8 from Unit 8, Lesson 5

Complete each figure so that the dashed line is a line of symmetry for the completed figure.





9 Exploration

Draw all the lines of symmetry you can find in this snowflake. How many can you find?

10 Draw each shape and all the lines of symmetry you can find in it.

- Rectangle
- Rhombus
- Square

Addressing CA CCSSM 4.G.3, 4.MD.1, 4.MD.3, 4.NF.3c, 4.NF.4; building on 4.G.3 and 4.MD.3; practicing MP3 and MP8

Ways to Find Unknown Length (Part 1)

Let's find the perimeter of different shapes.

Warm-up

Number Talk: Multiple Thirds

Find the value of each expression mentally.

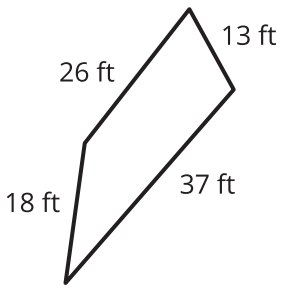
- $6 \times \frac{1}{3}$
- $30 \times \frac{1}{3}$
- $60 \times \frac{2}{3}$
- $90 \times \frac{2}{3}$

Activity 1

All the Way Around

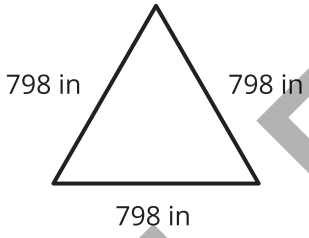
1. Find the perimeter of each shape. Write an expression that shows how you find the perimeter.

a.

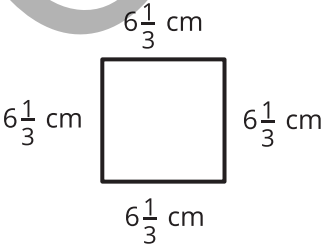


b.

c.



d.



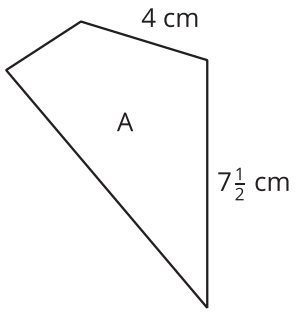
2. Compare your expressions with your partner's expressions. Make 1–2 observations.

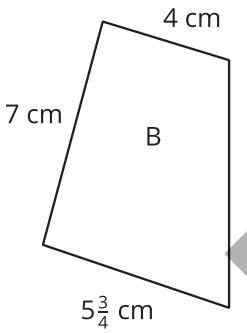
Activity 2

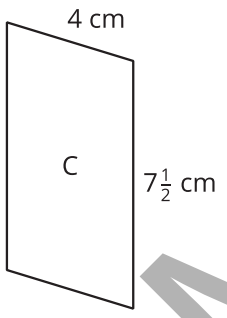
Ponder Perimeter

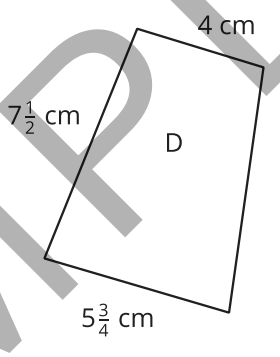
Here are 4 quadrilaterals and what we know about them:

- A, B, and C have no lines of symmetry.
- A has no parallel sides.
- B has 1 pair of parallel sides.
- C has 2 pairs of parallel sides.
- D has 1 pair of parallel sides and 1 line of symmetry.









Mai says, "We can't find the perimeter of any of these shapes because each one is missing labels for one or more side lengths."

Andre disagrees. He says, "We can find the perimeters for C and D but not for A and B."

1. Do you agree with either of them? Explain or show your reasoning.

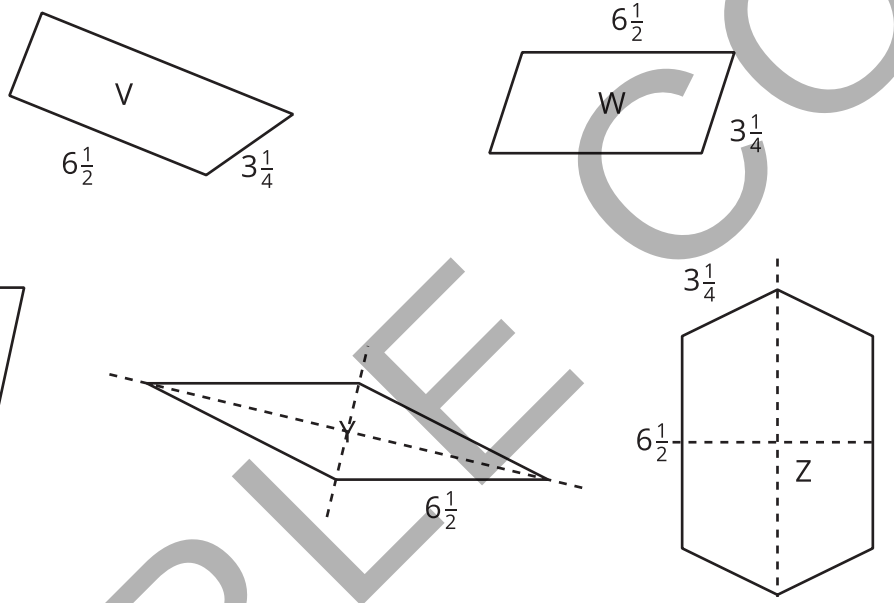
2. Find the perimeters that can be found, if any.

Activity 3

Perimeter Expressions

Here are 5 figures and what we know about them.

- Not all the side lengths are labeled.
- The lines of symmetry are shown.
- Only the triangle has no parallel sides.



1. For which figures is it possible to find the perimeter? For which figures is it not possible? Explain your reasoning.

2. Here are 4 expressions. Each expression represents the perimeter of one of the figures. The $6\frac{1}{2}$ and $3\frac{1}{4}$ in each expression represent side lengths. Which expression represents which figure?

   a. $(2 \times 6\frac{1}{2}) + 3\frac{1}{4}$

   b. $4 \times 6\frac{1}{2}$

   c. $(2 \times 6\frac{1}{2}) + (4 \times 3\frac{1}{4})$

   d. $(2 \times 6\frac{1}{2}) + (2 \times 3\frac{1}{4})$

Unit 8, Lesson 8

Addressing CA CCSSM 4.G.3, 4.MD.1, 4.NF.3c, and 4.NF.4; practicing MP5 and MP7

Ways to Find Unknown Length (Part 2)

Let's find the unknown lengths in figures.

Warm-up

True or False: Equations with Fractions

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $1\frac{1}{5} + 2\frac{2}{5} + 3\frac{3}{5} + 4\frac{4}{5} = 12$

- $10 - \frac{1}{2} - \frac{2}{2} - \frac{3}{2} - \frac{4}{2} = 5$

- $1\frac{1}{6} + 2\frac{2}{6} + 3\frac{3}{6} + 4\frac{4}{6} + 5\frac{5}{6} = 15\frac{3}{6}$

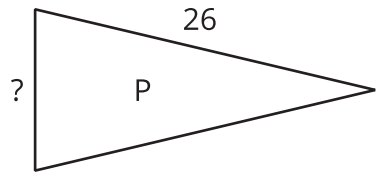
- $\frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 3 \times \frac{2}{3}$

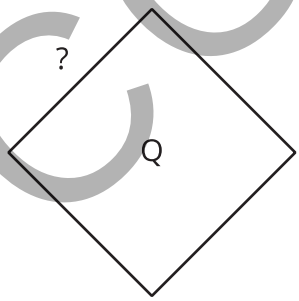
Activity 1

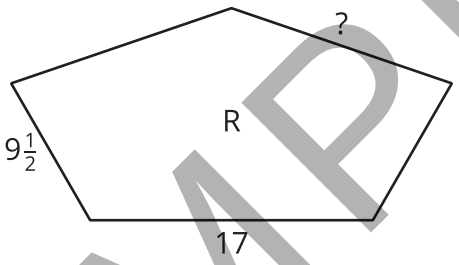
Unknown Lengths

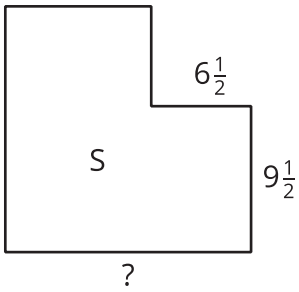
Here are 4 figures and what we know about them.

- Each figure has a perimeter of 64 inches.
- P, R, and S each have 1 line of symmetry.
- Q has 4 lines of symmetry.









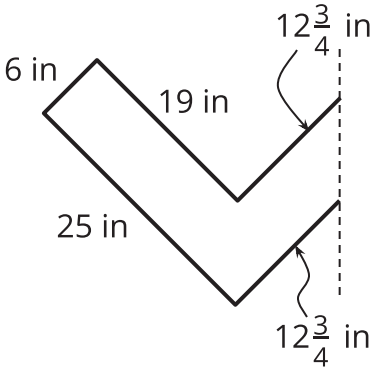
1. Draw the lines of symmetry of each figure.
2. For each figure, find the length of the side labeled "?" Explain or show your reasoning.

Activity 2

Lin's Design

Lin has 145 inches of fancy tape to make the outline for a design.

Here is half of the design. The dashed line is the line of symmetry for the entire design.



1. Sketch Lin's entire design.
2. Does she have enough tape for the entire outline? Explain or show your reasoning.

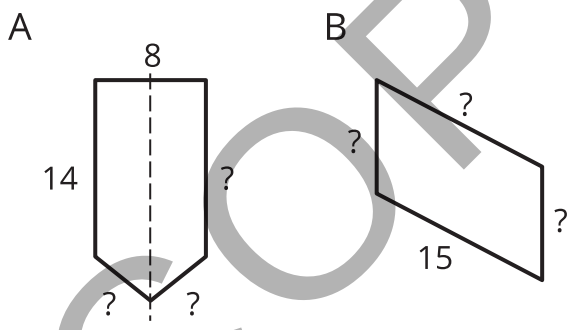
If you have time: Lin has a sheet of fancy paper that she can cut to cover the inside of the design. The paper is a rectangle that is 30 inches by 18 inches. The angles in the design are right angles. Does Lin have enough paper to cover the inside of the design? Explain or show your reasoning.

Section B Summary

We used attributes, such as side lengths, angles, lines of symmetry, and parallel sides, to solve problems about the perimeter of figures.

We learned that, if a figure has certain attributes, we can use them to find its perimeter, even when we don't know all of its side lengths. And if we know the perimeter of a figure and enough information about its attributes, we can find its side lengths.

For example, here are 2 figures:



We know the perimeter of each figure is 48 units.

If we know that the dashed line through Figure A is a line of symmetry, we can find its 3 unknown side lengths.

- Because of symmetry, the side opposite 14 units is also 14 units.
- $48 - 14 - 14 - 8 = 12$, so the other 2 sides have a total length of 12 units.
- $12 \div 2 = 6$, so each of the other 2 sides is 6 units.

If we know that the opposite sides of Figure B have equal lengths, we can find its 3 unknown side lengths.

- The side opposite 15 units is also 15 units.
- $48 - 15 - 15 = 18$, so the other 2 sides have a total length of 18 units.
- $18 \div 2 = 9$, so each of the other 2 sides is 9 units.

Addressing CA CCSSM 4.G.1-3 and 4.MD.3; practicing MP5 and MP7

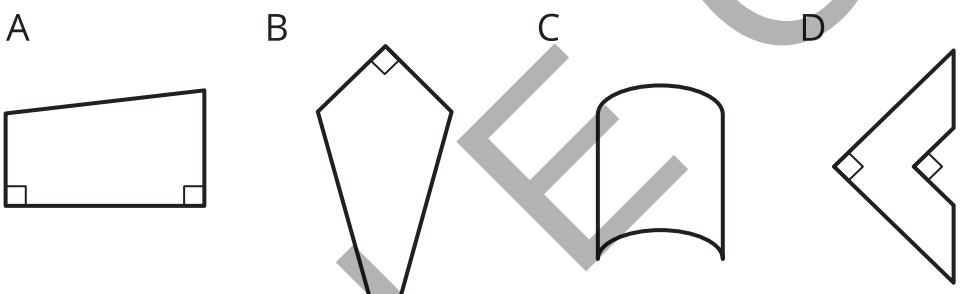
Symmetry in Action

Let's investigate symmetry and perimeter in folded figures.

Warm-up

Which Three Go Together: Figures

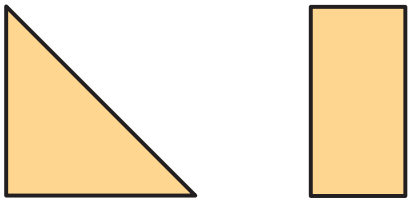
Which 3 go together?



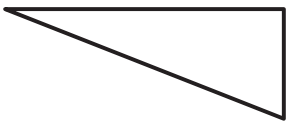
Activity 1

Before and After

1. Mai has a piece of paper. She can get each of these 2 shapes by folding the paper once along a line of symmetry. What is the shape of the unfolded paper?



2. Diego folded a piece of paper once along a line of symmetry and got this right triangle.



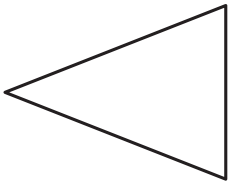
Which shapes could the paper have before it was folded? Explain or show your reasoning.

A

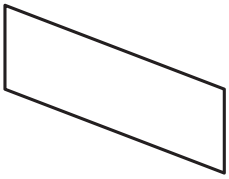
B

C

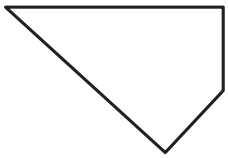
D



E



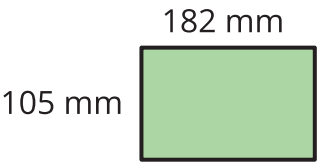
F



Activity 2

Before and After: Perimeter Edition

1. Jada folded a piece of paper along a line of symmetry and got this rectangle.



a. What could the paper look like before being folded? Draw one or more sketches.

b. Write an expression for the perimeter of the unfolded paper.

2. Kiran folded a different piece of paper twice—each time along a line of symmetry. Kiran's folds created the same rectangle as Jada's did.

   Show that each expression could represent the perimeter of Kiran's unfolded paper.

   a. $(4 \times 182) + (4 \times 105)$

   b. $(2 \times 182) + (8 \times 105)$

   c. $(8 \times 182) + (2 \times 105)$

Unit 8, Lesson 10

Addressing CA CCSSM 4.G.3 and 4.MD.7; practicing MP5 and MP6

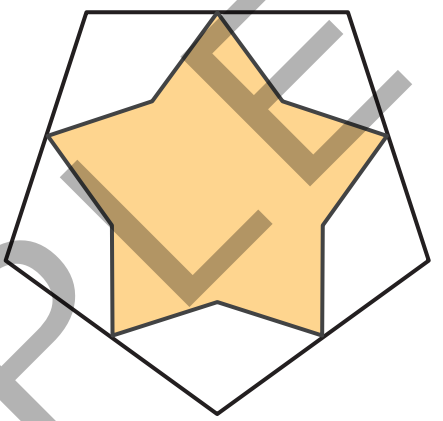
Ways to Find Angle Measurements

Let's find angle measurements in figures with line symmetry.

Warm-up

How Many Do You See: Symmetry in a Star

How many lines of symmetry do you see? How do you see them?

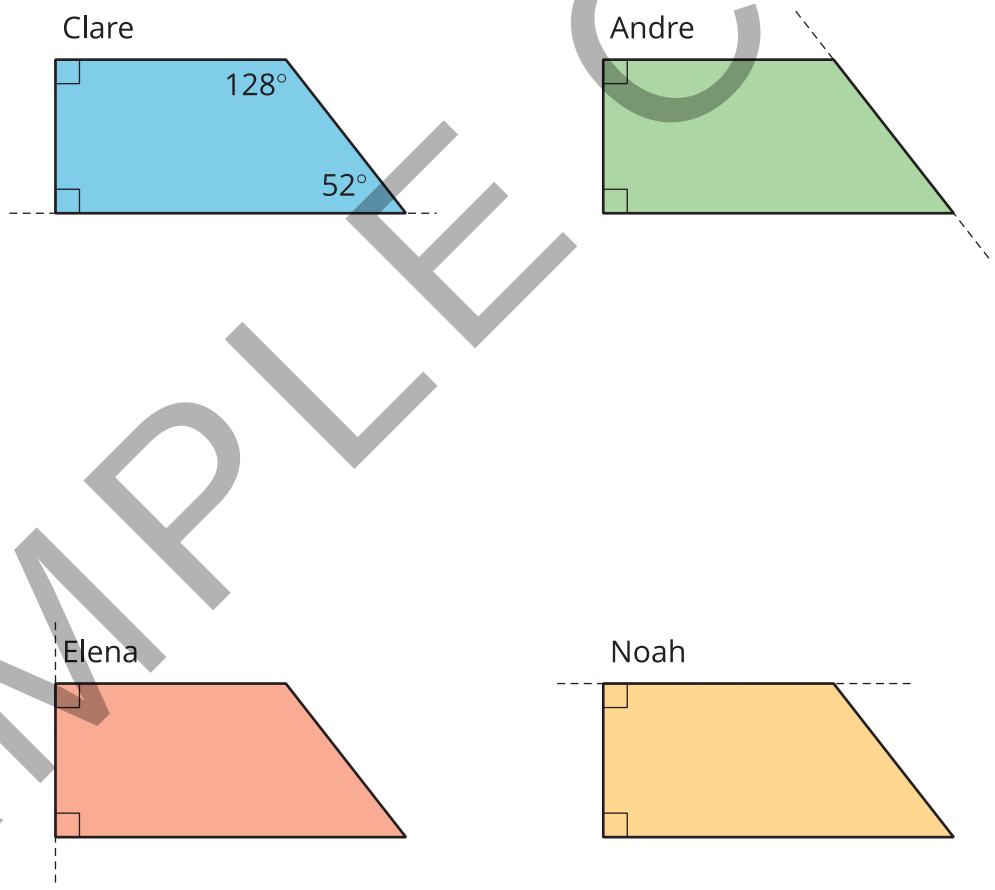


Activity 1

Before and After, Angle Edition

Clare, Andre, Elena, and Noah each have a piece of paper with 1 line of symmetry. When they fold their papers along the line of symmetry, they all produce the same shape. The dashed line represents the folding line.

1. Draw the figure of the unfolded paper that each student had. Be as precise as possible.



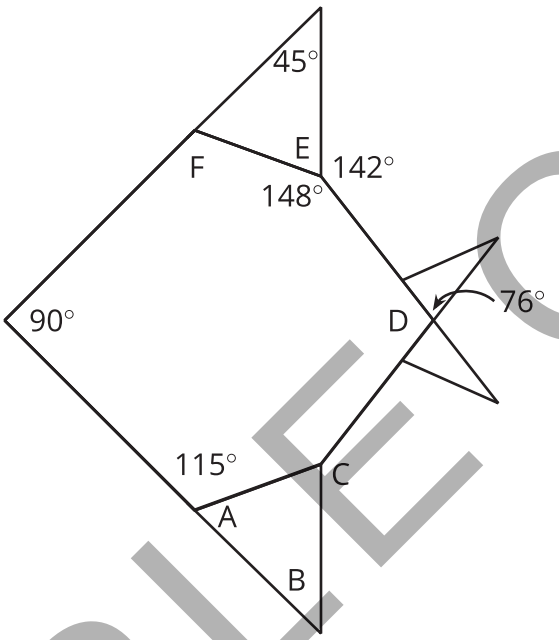
2. Without measuring, find the measurement of all angles within each shape (of the unfolded paper) that you drew.

Activity 2

Angular Fish

Origami is the Japanese art of folding paper.

Here is a diagram of an origami fish that has 1 line of symmetry.



1. Draw the line of symmetry.
2. Without measuring, find the measurement of the angles labeled A–F.

Unit 8, Lesson 11

Addressing CA CCSSM 4.G.3; practicing MP4

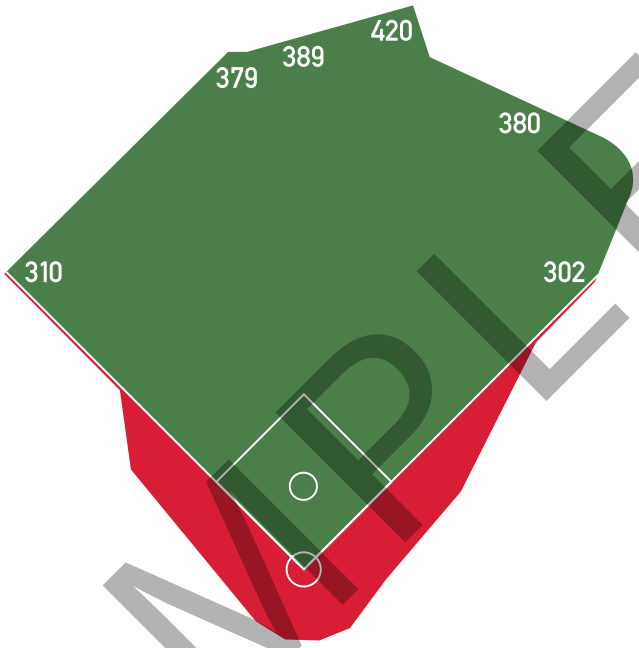
Symmetry in Sports

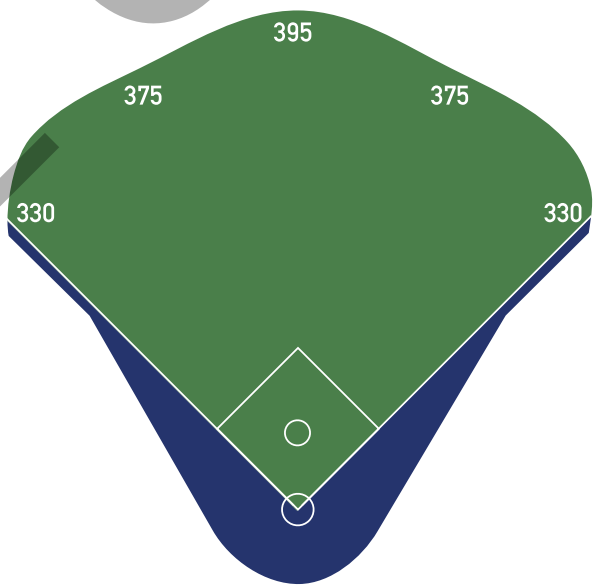
Let's use symmetry to think about sports.

Warm-up

Notice and Wonder: Outfield

What do you notice? What do you wonder?





Activity 1

New Field

Choose a sport that has a symmetrical field or court. Describe how the sport would change if the court or field was *not* symmetrical. Show your thinking. Organize it so it can be followed by others.

Activity 2

New Field, New Rules

1. Choose a sport. It can be the sport from the previous activity, a different sport, or a new sport you create.
   a. Design a field for the sport that has no lines of symmetry. Create a list of rules for playing the sport on this field.
   b. Create a visual display to show your thinking. Organize it so it can be followed by others.

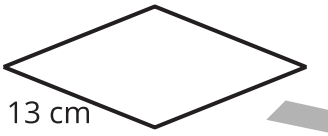
2. Gallery Walk: As you visit the other groups' visual displays, leave a sticky note with:
   a. Something you notice about their design.
   b. Something you wonder about their design.

Practice Problems

5 Problems

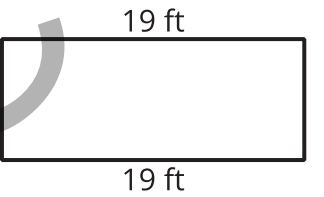
1 from Unit 8, Lesson 7

a. What is the perimeter of this rhombus? Explain or show your reasoning.



b. Diego says he can find the area of this rectangle because he knows 2 side lengths.

Do you agree with Diego? Explain your reasoning.



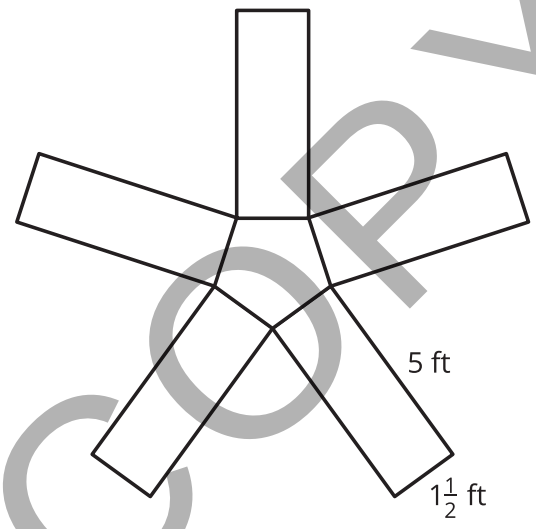
Sec B

2 from Unit 8, Lesson 8

The spinning part of a windmill is called a rotor. This figure represents a rotor with 5 blades.

a. Draw the lines of symmetry for the figure.

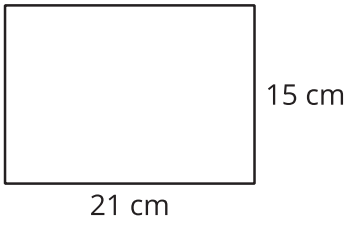
b. Each blade is 5 feet long and $1\frac{1}{2}$ feet wide. What is the perimeter of the rotor, as shown in the diagram? Explain or show your reasoning.



3 from Unit 8, Lesson 9

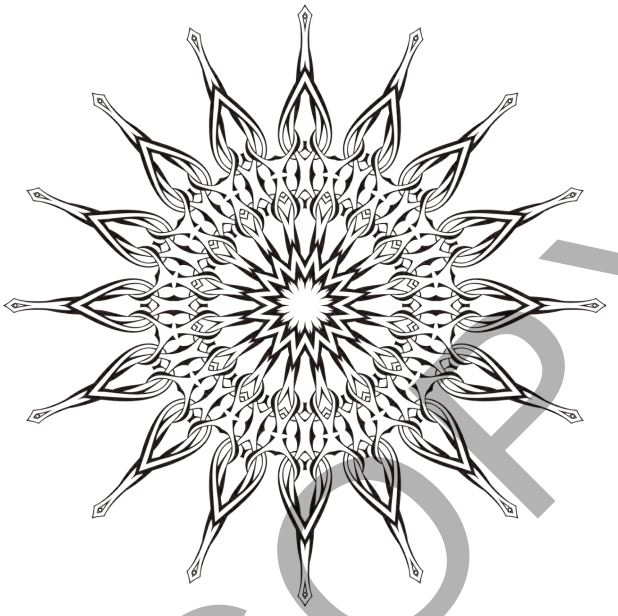
Here is Rectangle R.

a. What shape can be folded once along a line of symmetry to create R? What are the side lengths of that unfolded shape?



b. What shape can be folded twice along lines of symmetry to create R? What are the side lengths of that unfolded shape?

4 How many lines of symmetry are there in this design? Explain or show your reasoning.



5 Make a shape or design with 1 or more lines of symmetry. Trade shapes with a partner and find all the lines of symmetry of your partner's shape. You may find pattern blocks helpful to make your shape or design.

GRADE 4

UNIT

9

Putting It All Together

Content Connections

In this unit you will solidify and apply concepts and skills you have learned this year. You will make connections by:

- **Discovering Shape and Space** while drawing and identifying shapes, looking at the relationships between rays, lines, and angles, and exploring symmetry.
- **Reasoning with Data** while comparing fractions and interpreting data using visual representations.
- **Taking Wholes Apart, Putting Parts Together** while adding and subtracting fractions and multiplying fractions and whole numbers.
- **Exploring Changing Quantities** while adding and subtracting multi-digit numbers fluently, using the standard algorithm and multiply and divide numbers by reasoning about place value.

Addressing the Standards

As you work your way through **Unit 9 Putting It All Together,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 7, 10
MP2 Reason abstractly and quantitatively.	Lessons 1, 2, 3, 7, 8, 9
MP3 Construct viable arguments and critique the reasoning of others.	Lesson 2
MP4 Model with mathematics.	
MP5 Use appropriate tools strategically.	Lesson 9
MP6 Attend to precision.	Lessons 10, 11
MP7 Look for and make use of structure.	Lessons 1, 4, 5, 6, 12
MP8 Look for and express regularity in repeated reasoning.	Lesson 4

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes and Symmetries	**4.G.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	Lesson 11
• Measuring and Plotting • Fraction Flexibility • Visual Fraction Models	**4.NF.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	Lesson 2

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measuring and Plotting • Visual Fraction Models	**4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lesson 2
• Fraction Flexibility • Visual Fraction Models	**4.NF.3** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1*/b.* a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Lessons 1, 2 and 12
• Fraction Flexibility • Visual Fraction Models	**4.NF.3c** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1*/b.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Lesson 3

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fraction Flexibility • Visual Fraction Models	**4.NF.3d** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1*/b.* d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Lesson 3
• Fraction Flexibility	**4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of 1*/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of 1*/b,* and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	Lessons 1 and 12
• Fraction Flexibility • Visual Fraction Models • Circles, Fractions and Decimals	4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.2*	Lessons 1, 3, 11
• Visual Fraction Models • Circles, Fractions and Decimals	**4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*	Lesson 11

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Multi-Digit Numbers	**4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	Lesson 12
• Number and Shape Patterns • Multi-Digit Numbers • Shapes and Symmetries • Connected Problem Solving	**4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Lessons 4, 9, 11 and 12
• Factors and Area Models • Connected Problem Solving	**4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lessons 5, 7, 8, 9, 11, and 12
• Factors and Area Models • Connected Problem Solving	**4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lessons 6, 7, 8, 9, 11, and 12
• Number and Shape Patterns • Factors and Area Models • Multi-Digit Numbers • Fraction Flexibility	**4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	Lesson 10
• Number and Shape Patterns • Factors and Area Models • Connected Problem Solving	**4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	Lesson 7

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Connected Problem Solving	**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Lessons 7, 8, 9, 10
• Factors and Area Models	**4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	Lesson 11
• Number and Shape Patterns	**4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	Lesson 11

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 9, Lesson 1

Addressing CA CCSSM 4.NF.3-4 and 4.NF.5; practicing MP2 and MP7

Add, Subtract, and Multiply Fractions

Let's practice solving problems involving fractions.

Warm-up

Number Talk: Fluency and Fractions

Find the value of each expression mentally.

- $5 \times \frac{10}{5}$
- $9 \times \frac{6}{3}$
- $8 \times \frac{11}{4}$
- $6 \times \frac{12}{10}$

Activity 1

Let's Make Head Wraps!

Jada and Lin see a picture of head wraps made of African wax-print fabric. They decide to make their own head wraps.

1. Jada stitches together 5 pieces of fabric. Each piece has a length of $\frac{2}{6}$ yard. Write an equation to show the total length of fabric Jada uses for her head wrap.

2. Lin stitches together 3 pieces of fabric. Each piece has a length of $\frac{2}{3}$ yard. Write an equation to show the total length of fabric Lin uses for her head wrap.

3. Who uses more fabric? Explain or show your reasoning.

Activity 2

Make Two Yards of Fabric

Jada's and Lin's moms teach the fourth-grade class how to combine and use fabric pieces for head wraps. Here is the length of each piece of fabric.

$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{11}{10}$ yard
$1\frac{2}{5}$ yards	$\frac{9}{10}$ yard	$\frac{2}{6}$ yard	$\frac{6}{12}$ yard
$\frac{3}{6}$ yard	$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{12}{12}$ yard
$\frac{2}{6}$ yard	$\frac{3}{5}$ yard	$\frac{2}{6}$ yard	

Find as many different combinations of fabric that would have a length of 2 yards. Each piece of fabric can be used only one time. Write an equation for each combination.

Activity 3

Play by the Rules

1. Here are 4 fractions: $\frac{15}{12}$ $\frac{7}{12}$ $\frac{21}{12}$ $\frac{18}{12}$

   a. What is the sum of all the fractions?

   b. Select 2 of the fractions with a difference that is less than $\frac{1}{3}$. Explain or show your reasoning.

   c. Select 2 of the fractions with a sum greater than 3. Explain or show your reasoning.

2. Here are 4 new fractions: $\frac{5}{12}$ $\frac{8}{12}$ $\frac{3}{12}$ $\frac{2}{12}$

   Use the fractions to make the value 1. Follow these rules.

   - Use addition, subtraction, or both.
   - Use all four fractions.
   - Use each fraction only one time.

3. Use these fractions and the same rules to make the value 1.

   $\frac{15}{10}$ $\frac{13}{100}$ $\frac{53}{100}$ $\frac{9}{10}$

Addressing CA CCSSM 4.NF.1-2, 4.NF.3; building towards 5.NF.1; practicing MP2 and MP3

Sums and Differences of Fractions

Let's practice solving problems involving fractions.

Warm-up

Number Talk: Wholes and Units

Find the value of each expression mentally.

- $38 + 62$
- $38\frac{2}{6} + 62\frac{3}{6}$
- $38\frac{2}{6} + 62\frac{3}{6} + 17\frac{1}{6}$
- $138\frac{2}{6} + 162\frac{3}{6} + 17\frac{2}{6}$

Activity 1

Straws for a Roller Coaster

In science class, Noah, Tyler, and Jada build a model of a roller coaster out of 1-foot-long paper straws.

- Noah needs a piece that is $\frac{7}{12}$ foot long.
- Tyler needs a piece that is $\frac{1}{4}$ foot long.
- Jada needs a piece that is shorter than the other two.

Jada says, "We can use just 1 straw for all these pieces."

1. Draw a diagram to represent this situation. Explain to your partner how it matches the situation. Then find the length of the piece of straw that could be Jada's piece.

2. Did Noah use more than $\frac{1}{2}$ foot or less than $\frac{1}{2}$ foot of straw? Explain or show your reasoning.

3. Tyler says, "If Jada uses a piece that is $\frac{1}{6}$ foot long, there would be a piece of straw that is $\frac{1}{12}$ foot left."

   Do you agree or disagree with Tyler? Explain your reasoning.

Activity 2

Tall Enough for a Ride?

Lin's class is on a trip to the amusement park. Visitors must be at least a certain height to get on rides. Use the table to answer these questions.

ride	height requirement
tilt and spin	52 inches
roller coaster	54 inches
bumper cars	44 inches

1. Andre is $3\frac{3}{8}$ inches shorter than the height requirement for the roller coaster. How tall is Andre?

2. Lin is $\frac{18}{8}$ inches taller than Andre. How tall is Lin?

3. Elena was $1\frac{3}{4}$ inches too short to ride the bumper cars last year. Since then she grew $4\frac{1}{2}$ inches. How tall was Elena last year? How tall is she now?

4. Mai is tall enough to ride all the rides this year. Mai was $51\frac{7}{8}$ inches tall last year. At least how many inches did Mai grow?

Unit 9, Lesson 3

Addressing CA CCSSM 4.NF.3c-d and 4.NF.5; practicing MP2

Stories with Fractions

Let's add and subtract mixed numbers.

Warm-up

Number Talk: One Whole, Many Names

Find the value of each expression mentally.

- $1 - \frac{8}{10}$
- $1\frac{4}{10} - \frac{8}{10}$
- $2\frac{4}{10} - \frac{8}{10}$
- $10\frac{5}{10} - \frac{8}{10}$

Activity 1

Relay Race at Recess

A fourth-grade class runs a relay race during recess. Each team has 4 runners. Each runner runs the length of the school playground.

Here are the times of the runners for 2 teams.

runner	Diego's team, time (seconds)	Jada's team, time (seconds)
1	$10\frac{25}{100}$	$11\frac{9}{10}$
2	$11\frac{40}{100}$	$9\frac{8}{10}$
3	$9\frac{7}{10}$	$9\frac{84}{100}$
4	$10\frac{5}{100}$	$10\frac{60}{100}$

1. Which team wins the relay race? Show your reasoning.

2. How much faster is the winning team than the other team? Show your reasoning.

3. The record time for the playground relay race was 40.27 seconds. Did the winning team beat this record time? Show your reasoning.

Activity 2

You Be the Author

Write a problem that represents each of these situations. After each problem is written, trade papers with a partner to compare your problems and check your solutions.

1. A problem that can be solved by addition and has $9\frac{2}{5}$ as an answer

2. A problem that can be solved by subtraction and has $\frac{32}{100}$ as an answer

3. A problem that can be solved by writing the equation: $9 - \_\_\_\_\_ = 3\frac{3}{5}$

Addressing CA CCSSM 4.NBT.4; practicing MP7 and MP8

Another Look at the Standard Algorithm

Let's subtract from numbers with zeros.

Warm-up

Number Talk: Differences

Find the value of each difference mentally.

- 87 – 24
- 387 – 124
- 6,387 – 129
- 6,387 – 4,329

Activity 1

A Lot of Zeros

1. Find the value of each difference.

a.

$$\begin{array}{r} 7\,0\,0 \\ -\quad 1\,6 \\ \hline \end{array}$$

b.

$$\begin{array}{r} 7,\!0\,0\,0 \\ -\quad\quad 1\,6 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 7\,0,\!0\,0\,0 \\ -\quad\quad\quad 1\,6 \\ \hline \end{array}$$

d.

$$\begin{array}{r} 7\,0\,0,\!0\,0\,0 \\ -\quad\quad\quad 1\,6 \\ \hline \end{array}$$

2. Find the number that makes each expression true.

a.

$$\begin{array}{r} \quad 4\,3 \\ +\quad\quad \\ \hline 2\,0\,0 \end{array}$$

b.

$$\begin{array}{r} \quad 4\,3 \\ +\quad\quad \\ \hline 2,\!0\,0\,0 \end{array}$$

c.

$$\begin{array}{r} \quad\quad 4\,3 \\ +\quad\quad\quad \\ \hline 2\,0,\!0\,0\,0 \end{array}$$

d.

$$\begin{array}{r} \quad\quad 4\,3 \\ +\quad\quad\quad \\ \hline 2\,0\,0,\!0\,0\,0 \end{array}$$

Activity 2

Ways of Finding Differences

Priya and Han find the value of $20{,}000 - 472$. They use 2 different methods to set up their calculations.

Priya

$$\begin{array}{r} 20{,}000 \\ -\quad\ 472 \\ \hline \end{array}$$

Han

$$\begin{array}{r} 472 \\ +\quad\ \\ \hline 20{,}000 \end{array}$$

1. Use both methods to find the difference of 20,000 and 472.
2. Kiran uses another method. Explain how Kiran finds the value of $20{,}000 - 472$.

$$472 + 8 = 480$$
$$480 + 20 = 500$$
$$500 + 500 = 1{,}000$$
$$1{,}000 + 19{,}000 = 20{,}000$$

$$19{,}000 + 500 + 20 + 8 = 19{,}528$$

3. Which method do you prefer? Or do you prefer another way? Explain your reasoning.

4. Find the value of $50{,}400 - 1{,}389$. Show your reasoning.

Addressing CA CCSSM 4.NBT.5; practicing MP7

Multiplication of Multi-digit Numbers

Let's multiply multi-digit numbers.

Warm-up

Estimation Exploration: A Silly Riddle

- Seven teachers go to the park.
- Each teacher takes 7 students.
- Each student brings 7 fishbowls.
- Each fishbowl has 7 fish.

How many go to the park?

Record an estimate that is:

too low	about right	too high

Activity 1

Two Methods Revisited

1. We used 2 different ways to multiply numbers earlier in the school year.

A

$$\begin{array}{r} 4\ 1\ 6 \\ \times\quad\quad 2 \\ \hline 1\ 2 \\ 2\ 0 \\ +\ 8\ 0\ 0 \\ \hline 8\ 3\ 2 \end{array}$$

B

$$\begin{array}{r} 1 \\ 4\ 1\ 6 \\ \times\quad\quad 2 \\ \hline 8\ 3\ 2 \end{array}$$

a. In Method A, where do the 12, 20, and 800 come from?

b. In Method B, where does the 1 above 416 come from?

2. Diego uses both methods to find the value of 215×3. He has a different result for each method.

$$\begin{array}{r} 2\ 1\ 5 \\ \times\quad\quad 3 \\ \hline 1\ 5 \\ 3\ 0 \\ +\ 6\ 0\ 0 \\ \hline 6\ 4\ 5 \end{array}$$

$$\begin{array}{r} 2\ 1\ 5 \\ \times\quad\quad 3 \\ \hline 6,\ 3\ 1\ 5 \end{array}$$

a. Can you tell which method shows the correct product, without using calculations? How do you know the other product is not correct?

b. For the incorrect result, explain what is correct and what is incorrect in Diego's steps. Then show the correct calculation, using Method B.

3. Use either method to find the value of each product. Show your reasoning.

   a. 521×3

   b. $6{,}121 \times 4$

   c. 305×9

Activity 2

Two by Two

Here are 2 ways to find the value of 34×21.

A

$$\begin{array}{r} 34 \\ \times\ 21 \\ \hline 1 \\ \hline 4 \\ 30 \\ 80 \\ +\ 600 \\ \hline 714 \end{array}$$

B

$$\begin{array}{r} 34 \\ \times\ 21 \\ \hline 1 \\ \hline 34 \\ +\ 680 \\ \hline 714 \end{array}$$

1. In Method A, where do the 4, 30, 80, and 600 come from?

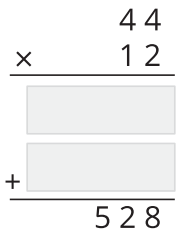
2. In Method B, which 2 numbers are multiplied to get these products.

   a. 34

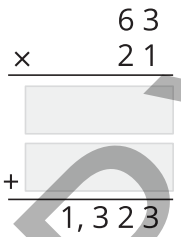
   b. 680

3. Use the 2 methods to show that each equation is true.

a. $44 \times 12 = 528$



b. $63 \times 21 = 1{,}323$



Addressing CA CCSSM 4.NBT.6; practicing MP7

What's the Quotient?

Let's find some quotients of multi-digit numbers.

Warm-up

Number Talk: Divide by 3

Find the value of each expression mentally.

- $48 \div 3$
- $480 \div 3$
- $528 \div 3$
- $5{,}280 \div 3$

Activity 1

Unfinished Divisions

Here are 4 calculations to find the value of $7{,}465 \div 5$, but each calculation is unfinished.

Complete at least 2 of the unfinished calculations.


```
      200
       80
       13
  5 )7,465
 −    65
    7,400
 −   400
    7,000
 −  1,000
```

```
      400
    1,000
  5 )7,465
 −  5,000
    2,465
 −  2,000
```

$5{,}000 \div 5 = 1{,}000$
$60 \div 5 = 12$
$5 \div 5 = 1$

7,465 is a little less than 7,500.

$7{,}500 \div 5 = 1{,}500$
$35 \div 5 = 7$

Activity 2

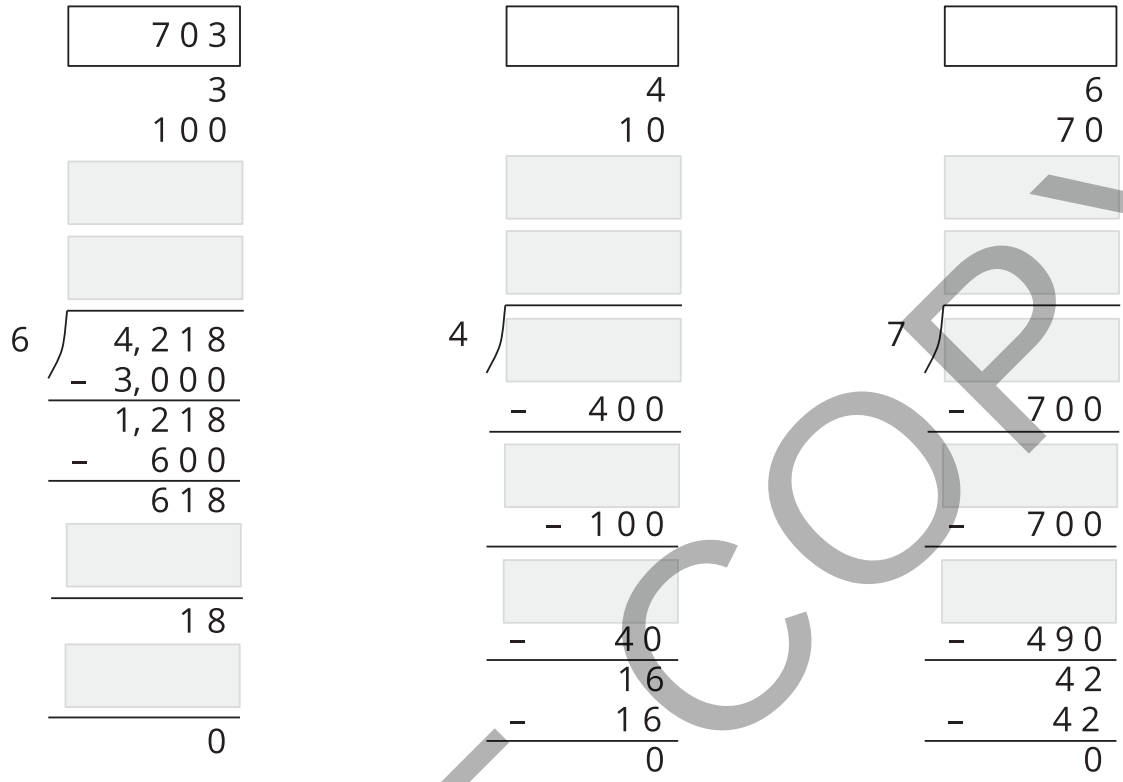
Where Do We Begin?

1. Jada and Noah are finding the value of $3{,}681 \div 9$. Jada says to start by dividing 81 by 9. Noah says to start by dividing 3,600 by 9.

   a. Explain why each suggestion is helpful for finding the quotient.

   b. Find the value of $3{,}681 \div 9$. Show your reasoning.

2. Find the unknown numbers to show a correct division calculation.



3. Consider the expression $5{,}016 \div 8$.

   a. What would you do to start finding the value of the quotient?

   b. Show how you would find the value with as few steps as possible.

Unit 9, Lesson 7

Addressing CA CCSSM 4.NBT.5-6 and 4.OA.2-3; practicing MP1 and MP2

Solve Multiplicative Comparison Problems

Let's solve real-world multiplicative comparison problems.

Warm-up

Notice and Wonder: Two Cities

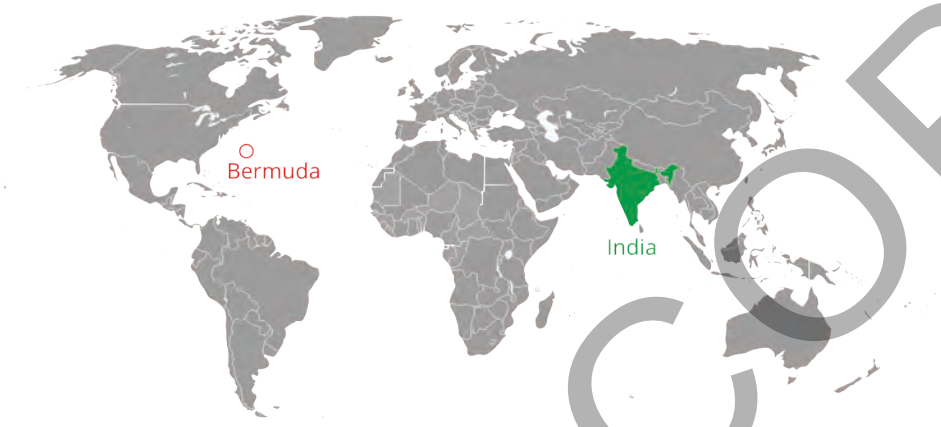
What do you notice? What do you wonder?

2023 prices	San Francisco, CA	Fort Wayne, IN
population	715,717	269,621
milk (1 gallon)	$5.99	$2.79
bread (1 loaf)	$4.79	$3.99
gasoline (1 gallon)	$6.02	$3.56
movie ticket	$17	$10.50
internet connection (1 month)	$89	$60
rent for a 3-bedroom apartment in the city center (1 month)	$5,000	$1,200
cost of a house	$1,380,000	$210,000

Activity 1

The Most and the Least Expensive

Bermuda is one of the most expensive countries to live in the world. India is one of the least expensive.



1. The table shows how prices (in U.S. dollars) in Bermuda and India compared in 2023. Estimate or calculate the missing information in the table.

2023 prices (U.S. dollars)	Bermuda	cost in Bermuda is ____ as in India	India
a meal with drink (1 person)		12.5 times as much	$6
gasoline (1 gallon)	$9	2 times as much	
brand-name jeans		3 times as much	$26
men's leather shoes	$136	4 times as much	
internet connection		13 times as much	$11

2. In 2023, rent for a 1-bedroom apartment in Delhi (DELL-ee), the capital city of India, cost about $182 per month. A similar apartment in Hamilton, Bermuda, costs 18 times as much.

   Utilities (electric, gas, water, and heating) for a small apartment cost about $65 per month in India and 2 times as much in Bermuda.

   If a person earned $3,500 per month, could they afford to pay rent and utilities in Bermuda?

Activity 2

The Cost of Living

The cost of living in the United States is higher than in Ghana.

1. The table shows how prices (in U.S. dollars) in the United States and Ghana compared in 2023.

2023 monthly cost (U.S. dollars)	United States	cost in the US is ____ as in Ghana	Ghana
1-bedroom apartment in city center	$2028	3 times as much	
1-bedroom apartment outside city center		17 times as much	$82
utilities for a 915-square-foot apartment	$191		$47
transportation pass	$76	2 times as much	
private preschool (1 child)	$1218		$58

2. In 2023, if a couple had $3,000 for housing and wanted a 1-bedroom apartment outside of a city center, how many months of rent could they afford in:

   a. the United States? Explain or show your reasoning.

   b. Ghana? Explain or show your reasoning.

Unit 9, Lesson 8

Addressing CA CCSSM 4.NBT.5-6 and 4.OA.3; practicing MP2

Solve Problems with Multiplication and Division

Let's make sense of situations and solve word problems.

Warm-up

Number Talk: Divide by 8

Find the value of each expression mentally.

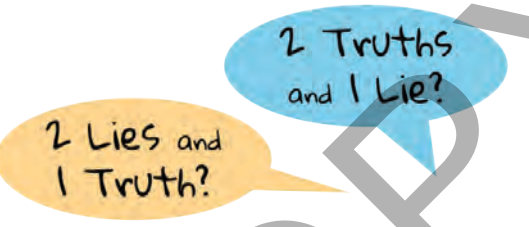
- $848 \div 8$
- $4{,}848 \div 8$
- $4{,}852 \div 8$
- $5{,}848 \div 8$

Activity 1

Two Truths and a Lie, or Two Lies and a Truth?

Here are 3 situations. Which situations are true?
Which situations are not true?

Show how you know.



- Situation A: A high-rise building has 53 stories. The first floor is 17 feet tall. All other stories are each 11 feet tall. The building is 589 feet tall.

- Situation B: A window washer has 600 seconds to wash 17 windows of a building. It takes 54 seconds to wash each window. The washer finishes washing all the windows and has 11 seconds to spare.

- Situation C: There are 11 students who set a goal to raise at least $600 for charity. Each student raises $17 each day. After 3 days of fundraising, the group needs $54 to meet their goal.

Activity 2

Buses for a Field Trip

A school is taking everyone on a field trip. It needs buses to transport 375 people.

Bus Company A has small buses with 27 seats in each.

Bus Company B has large buses with 48 seats in each.

1. What is the least number of buses that will be needed if the school goes with:
   - Bus Company A? Show your reasoning.
   - Bus Company B? Show your reasoning.

2. Which bus company should the school choose? Explain your reasoning.

3. Bus Company C has large buses that can take up to 72 passengers.

   Diego says, "If the school chooses Bus Company C, it will need only 6 buses, but the buses will have more empty seats."

   Do you agree? Explain your reasoning.

Unit 9, Lesson 9

Addressing CA CCSSM 4.NBT.4-6 and 4.OA.3; practicing MP2 and MP4

Create Word Problems

Let's write our own word problems.

Warm-up

Number Talk: Difference and Thousands

Find the value of each expression mentally.

- $5{,}000 - 403$
- $5{,}300 - 473$
- $25{,}300 - 493$
- $26{,}000 - 1{,}493$

Activity 1

What's the Question?

George Meegan walked 19,019 miles between 1977 and 1983. He finished at age 31. He wore out 12 pairs of hiking boots.

Jean Beliveau walked 46,900 miles between 2000 and 2011. He finished at age 56.

Here are the responses Kiran gave to answer some questions about the situation.

In each row, write a question that Kiran could answer, using the given information. In the last row, write a new question about the situation. Show your answer and explain your reasoning.

question	response and reasoning
1.	$1983 - 1977 = 6$ $12 \div 6 = 2$ 2 pairs of hiking boots
2.	$56 - 31 = 25$ 25 years
3.	$2011 - 2000 = 11$ 11 years $11 \times \_\_\_\_\_ = 46{,}900$ $11 \times 4{,}000 = 44{,}000$ $11 \times 200 = 2{,}200$ $11 \times 70 = 770$ $44{,}000 + 2{,}200 + 770 = 46{,}970$ $4{,}000 + 200 + 70 = 4{,}270$ 4,270 miles
4.	

Activity 2

What's the Problem?

Elena, Noah, and Han each create a problem with an answer of 1,564.

- Elena uses multiplication.
- Noah uses multi-digit numbers and addition only.
- Han uses multiplication and subtraction.

Write a problem that each student could have written. Show that the answer to the question is 1,564.

- Elena's problem:

Solution:

- Noah's problem:

Solution:

- Han's problem:

Solution:

Addressing CA CCSSM 4.OA.1 and 4.OA.3; practicing MP1 and MP6

Estimation Exploration

Let's design an Estimation Exploration activity.

Warm-up

Estimation Exploration: No Driver Required

Here are pictures showing the exterior and the interior of a parking tower in Wolfsburg, Germany. The parking is automated—each car goes up on a lift and is then placed in a parking space.

How many cars can fit in the tower?

Record an estimate that is:

too low	about right	too high

Activity 1

Dental Care

Jada brushes for 2 minutes, twice a day. She changes her toothbrush every 3 months.

Use this information to make some estimates. Explain or show your reasoning.

1. In a lifetime, approximately:
   a. How many toothbrushes will Jada use?

   b. How many dollars will she spend on toothbrushes?

2. About how many minutes would she spend brushing her teeth:
   a. in a year?

   b. in a lifetime?

3. Think of another estimation question you could ask about this situation.

Activity 2

Get Your Classmates to Estimate

It's your turn to create an estimation problem.

1. Think of situations or look for images that would make interesting estimation problems. Write down 4–5 ideas or possible topics.

2. Choose your favorite idea.
   - Write an estimation question that would encourage others to use multiplication of multi-digit numbers to answer.
   - Record an estimate that is:

too low	about right	too high

Activity 3

Facilitate Your Estimation Exploration Activity

Follow these steps to facilitate your Estimation Exploration activity for another group:

- Display your image or present your scenario.
- Ask your classmates: "What is an estimate that's too high? Too low? About right?"
- Give them 1 minute of quiet think time.
- Give them 1–2 minutes to discuss together.
- Invite them to share their estimates and how they made them. Record their ideas.
- Reveal the actual number or quantity if you know it.

Unit 9, Lesson 11

Building on CA CCSSM 4.G.1, 4.NBT.4-6, 4.NF.5-6, 4.OA.4, and 4.OA.5; practicing MP6

Which Three Go Together

Let's create a Which Three Go Together.

Warm-up

Which Three Go Together: Strings of Numbers

Which 3 go together?

A. 0, 4, 8, 12, 16

B. 3, 6, 9, 12, 15

C. 5, 105, 205, 305, 405

D. 6, 60, 600, 6,000, 60,000

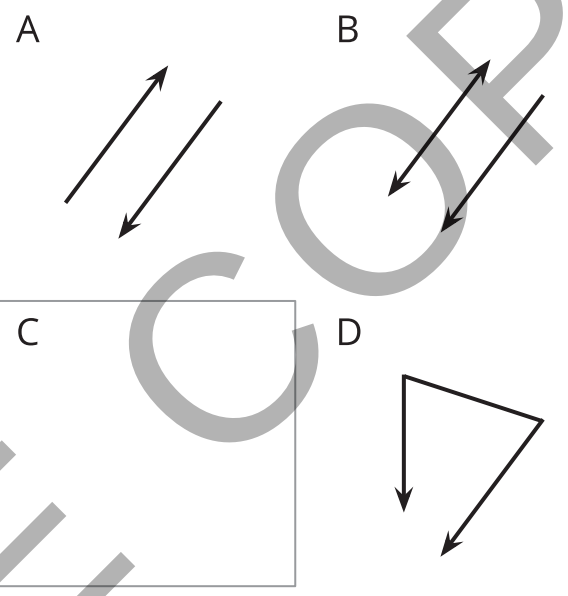
Activity 1

Which Three Go Together: Design 1

Choose either the problem with geometric figures or numbers. Then complete the Which Three Go Together.

Geometric Figures

Which 3 go together?



Decide on a fourth figure to complete the Which Three Go Together.

For each group of 3 figures, discuss one reason why they go together.

- A, B, and C go together because:
- A, B, and D go together because:
- A, C, and D go together because:
- B, C, and D go together because:

Numbers

Which 3 go together?

A	B	C	D
4.8		13.05	5

Decide on a fourth number to complete the Which Three Go Together.

For each group of 3 numbers, discuss one reason why they go together.

- A, B, and C go together because:
- A, B, and D go together because:
- A, C, and D go together because:
- B, C, and D go together because:

Activity 2

Which Three Go Together: Design 2

Here is an incomplete Which Three Go Together. It has two missing expressions.

Which 3 go together?

A

$(4 + 3) \times (2 - 1)$

B

$(4 + 3) + (2 - 1)$

C
D

Write 2 expressions to complete the Which Three Go Together.

For each group of 3 expressions, discuss one reason why they go together.

- A, B, and C go together because:
- A, B, and D go together because:
- A, C, and D go together because:
- B, C, and D go together because:

Activity 3

Which Three Go Together: Design 3

Create your own Which Three Go Together? about any mathematical idea you want others to notice.

Which 3 go together?

A

B

C

D

Addressing CA CCSSM 4.NBT.1, 4.NBT.4-6, and 4.NF.3-4; practicing MP7

Number Talk

Let's create our own Number Talks.

Warm-up

Number Talk: A Whole Number and a Fraction

Find the value of each expression mentally.

- $6 \times \frac{1}{4}$
- $6 \times \frac{3}{4}$
- $18 \times \frac{3}{4}$
- $180 \times \frac{3}{4}$

Activity 1

Related Numbers, Related Expressions

1. Here are 2 addition expressions. Think of at least 2 different ways to find the value of each sum mentally.

   a. $15 + 29$

   b. $30 + 58$

2. Here are 3 subtraction expressions. Think of at least 2 different ways to find the value of each difference mentally.

   a. $91 - 11$

   b. $91 - 16$

   c. $391 - 86$

3. Can you write a fourth subtraction expression that uses the same strategy you used to find the values of the other differences?

Activity 2

Add One New Expression, Then Two

1. Here are 3 division expressions. Find the value of each quotient mentally and think about how they might be related.
   - $35 \div 5$
   - $70 \div 5$
   - $210 \div 5$
   - _____

   Write a new division expression whose value can be found more easily after working through the first three expressions.

2. Here are 2 multiplication expressions. Analyze them and think about how they might be related.
   - 21×7
   - 42×7
   - _____
   - _____

   Write 2 new expressions. Explain your reasoning for each expression.

Activity 3

Add Three New Expressions

Here are 4 expressions you could use to start a Number Talk activity.

$75 + 30$ $160 - 51$ $24 \div 8$ $3 \times \frac{1}{6}$

1. Choose one starting expression. Think of at least 2 different ways to find its value mentally.

2. Write 3 expressions to create a Number Talk activity.

   _____ (starting expression)

   _____

   _____

   _____

3. Create an answer key for your Number Talk. Include at least one way to find the value of each expression mentally.

Glossary

- acute angle
  An angle that measures less than 90 degrees.

- angle
  A figure made up of 2 rays that share the same starting point.

- common denominator
  The same denominator in two or more fractions. Example, $\frac{1}{4}$ and $\frac{5}{4}$ have the common denominator 4.

- composite number
  A whole number with more than one factor pair.

- decimal notation
  A way to write tenths, hundredths, and other decimal fractions as numerals with digits and a decimal point. The digits to the left of the decimal point show the whole-number part of the number. The digits to the right of the decimal point show the fractional part less than 1.

  Examples:

  $\frac{3}{10}$ written in decimal notation is 0.3.

  $\frac{25}{100}$ written in decimal notation is 0.25.

  $\frac{17}{10}$ written in decimal notation is 1.7.

  $2\frac{7}{100}$ written in decimal notation is 2.07.

- degree
  A unit of rotation used to measure angles. Each degree is $\frac{1}{360}$ of a circle.

- denominator
  The bottom part of a fraction that tells how many equal parts the whole was

partitioned into.

- dividend
  The number being divided. Example: When 37 is divided by 5, we call 37 the dividend.

- divisor
  The number we are dividing by, which can represent the size of the groups or the number of groups. Example: When 37 is divided by 5, we call 5 the divisor.

- equivalent fractions
  Fractions that have the same size and describe the same point on the number line. Example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

- expanded form
  A way of writing a number as a sum of the values of the digits.

  Example: 482 written in expanded form is $400 + 80 + 2$.

- factor pair of a whole number
  Two whole numbers that multiply to result in that number. Example: 5 and 4 are a factor pair of 20.

- intersecting lines
  Lines that cross.

- line
  A set of points that are arranged in a straight path and extend infinitely in opposite directions.

- line of symmetry
  A line that divides a figure into 2 halves that match up exactly when the figure is folded along the line.

- line segment or segment
  A part of a line with 2 endpoints and all of the points in between them.

- mixed number
  A number expressed as a whole number and a fraction less than 1.

- multiple of a number
  The result of multiplying that number by a whole number. Example: 18 is a multiple of 3, because it is a result of multiplying 3 by 6.

- numerator
  The top part of a fraction that tells how many of the equal parts are being described.

- obtuse angle
  An angle that measures greater than 90 degrees.

- parallel lines
  Lines that never intersect.

- perpendicular lines
  Intersecting lines that create 4 right angles.

- point
  A location along a line or in space.

- prime number
  A whole number that is greater than 1 and has exactly one factor pair: the number itself and 1.

- quadrilateral
  A flat shape with 4 straight sides and 4 angles.

- quotient
  The result of dividing one number by another.

- ray
  A part of a line that starts at 1 point and goes on infinitely in 1 direction.

- rectangle
  A quadrilateral with 2 pairs of parallel sides and 4 right angles.

- remainder
  The amount left over when we divide a number. The amount is not enough to put into equal groups or to make a new group.

- rhombus
  A quadrilateral with 4 equal sides.

- right angle
  An angle that measures 90 degrees.

- right triangle
  A triangle with a 90-degree angle.

- square
  A quadrilateral with 4 equal sides and 4 right angles.

- standard algorithm (for addition or subtraction)
  A set of steps used to add or subtract numbers by place value. Write the numbers vertically with digits lined up by place value. Add or subtract the digits in each place value, starting with the least place value. Compose or decompose units, as needed in each place value.

- straight angle
  An angle that measures 180 degrees.

- symmetry
  When the parts of a figure can match up exactly after it is folded or turned.

- triangle
  A flat shape with 3 straight sides and 3 angles.

- vertex
  The point that 2 rays share when they form an angle.

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Unit 9: Putting It All Together

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Notes

California Common Core State Standards for Mathematics (CA CCSSM) Reference

4.G: Grade 4 – Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (Two-dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA

4.G.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

4.MD: Grade 4 – Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1, 12)$, $(2, 24)$, $(3, 36)$, ...

4.MD.2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3
Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Represent and interpret data.

4.MD.4
Make a line plot to display a data set of measurements in fractions of a unit $(1/2, 1/4, 1/8)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

4.MD.5.a
An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

4.MD.5.b
An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4.MD.6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7
Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.NBT: Grade 4 – Number and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

4.NBT.2
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.3
Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NBT.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NBT.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.NF: Grade 4 – Number and Operations—Fractions

Extend understanding of fraction equivalence and ordering.

4.NF.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3
Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

4.NF.3.a
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.3.b
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

4.NF.3.c
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3.d
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4.a
Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

4.NF.4.b
Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

4.NF.4.c
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.

4.NF.6
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using the number line or another visual model. CA

4.OA: Grade 4 – Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1
Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.3
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4
Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4.OA.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.