IM K–12 California

GRADE 5

Student Edition

UNITS 1-2

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 1
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GRADE 5

UNIT

Finding Volume

Content Connections

In this unit you will find the volumes of right rectangular prisms and solid figures composed of two right rectangular prisms. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while solving real-world problems involving the volume of a solid figure.
- **Discovering Shape and Space** while finding the volume of rectangular prisms using unit cubes.
- **Exploring Changing Quantities** while using multiplication to find the volume of rectangular prisms.

Addressing the Standards

As you work your way through **Unit 1 Finding Volume,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 5, 6, 7, and 11
MP2 Reason abstractly and quantitatively.	Lessons 3, 4, 5, 6, 9, and 10
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 1, 6, 7, and 10
MP4 Model with mathematics.	Lesson 12
MP5 Use appropriate tools strategically.	Lesson 1
MP6 Attend to precision.	Lessons 1, 2, and 7
MP7 Look for and make use of structure.	Lessons 3, 6, 8, and 10
MP8 Look for and express regularity in repeated reasoning.	Lesson 8

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Seeing Division • Layers of Cubes	**5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Lessons 1, 11, and 12
• Seeing Division • Layers of Cubes	**5.MD.3a** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.	Lesson 2

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Seeing Division • Layers of Cubes	**5.MD.3b** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Lesson 2
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Lessons 2, 3, and 7
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	Lessons 11 and 12

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.5a** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	Lessons 4 and 6
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.5b** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. b. Apply the formulas $V = l \times w \times h$ *and* $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.	Lessons 5 and 6
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.5c** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	Lessons 8, 9, 10, and 12
• Factors and Groups • Powers and Place Value • Layers of Cubes	**5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Lessons 6 and 10

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Factors and Groups • Powers and Place Value	**5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Lessons 4, 6, 9, and 10
• Factors and Groups • Powers and Place Value	**5.OA.2.1** *Express a whole number in the range 2 -50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as 2 × 2 × 2 × 3.*	Lesson 6

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Addressing CA CCSSM 5.MD.3; building towards 5.MD.3; practicing MP3, MP5, MP6

What Is Volume?

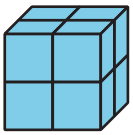
Let's build and compare objects made of cubes.

Warm-up

Which Three Go Together: Objects Made of Cubes

Which 3 go together?

A



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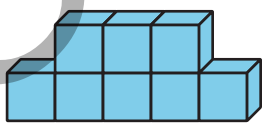

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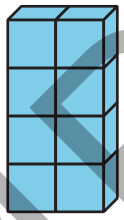

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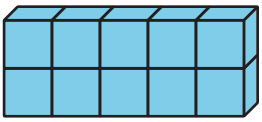
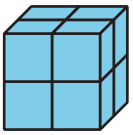


Activity 1

Build Objects with Cubes

1. Which is bigger? Explain or show your reasoning.

A



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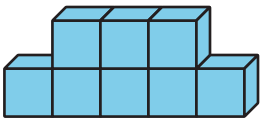
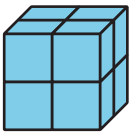


2. Which is bigger? Explain or show your reasoning.

A



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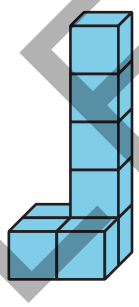


3. What does it mean for an object to be "bigger"?

Activity 2

Build and Order

1. Each group member:
   a. Take a handful of connecting cubes.
   b. Build an object.
2. Order the objects by volume. Discuss your reasoning with your group.
3. Repeat.
4. Each group member:
   a. Take 9 connecting cubes.
   b. Build an object.
5. Order the objects by volume. Discuss your reasoning with your group.

Addressing CA CCSSM 5.MD.3a, 5.MD.3b, 5.MD.4; practicing MP6

Measure Volume

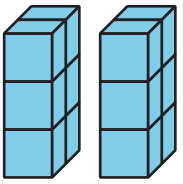
Let's count cubes.

Warm-up

Which Three Go Together: Cubes

Which 3 go together?

A



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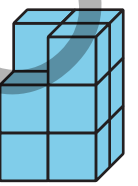

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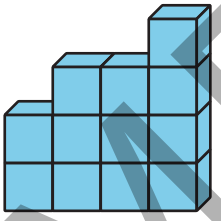

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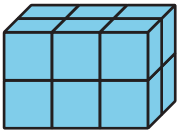


Activity 1

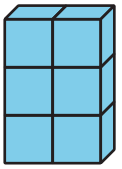
Finding Volume

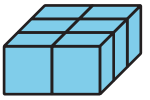
1. Partner A: Build an object, using 8–12 unit cubes. Give the object to Partner B.
2. Partner B: Count the unit cubes. Then explain to your partner how you counted them.
3. Switch roles and repeat. Continue to take turns with your partner.
4. Which objects were easiest to count? Explain your reasoning.

Activity 2

Guess My Prism

What is the same? What is different?





1. The goal of the game is to get your partner to build the same prism.
   - Partner A: Use at least 10 unit cubes to build a prism. Describe it to your partner.
   - Partner B: Build the prism your partner describes to you.

2. Place the 2 prisms next to each other. How are the prisms alike? How are they different?

3. Switch roles and repeat.

Addressing CA CCSSM 5.MD.4; building on 3.OA.7; building towards 5.MD.5; practicing MP2 and MP7

Volume of a Prism Drawings

Let's use layers to find volume.

Warm-up

Number Talk: Multiplication

Find the value of each expression mentally.

- 6×4
- $3 \times 2 \times 4$
- $3 \times 2 \times 5$
- $3 \times 2 \times 6$

Activity 1

Build Rectangular Prisms

The prisms on the cards are completely packed with unit cubes.

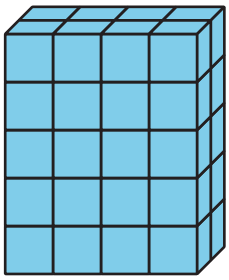
1. Pick a card.
2. Build the rectangular prism.
3. Find the volume. Explain how you found the volume to your partner.
4. Repeat.

Activity 2

Layers, Layers, and More Layers

The prisms are completely packed with unit cubes. Determine the volume of each prism. Explain or show your reasoning.

1.



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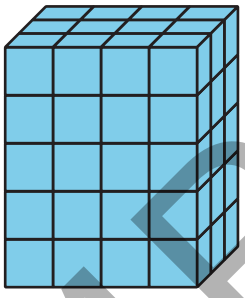

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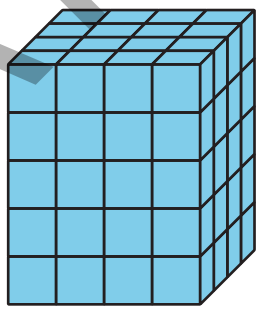


Addressing CA CCSSM 5.MD.5a, 5.OA.2; building towards 5.MD.5a; practicing MP2

Use Layers to Determine Volume

Let's relate multiplication to how we use layers to find volume.

Warm-up

Estimation Exploration: How Many Cubes?

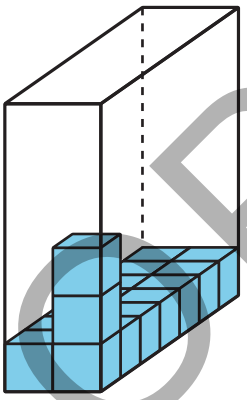
Estimate the number of cubes used to build this prism.

Record an estimate that is:

too low	about right	too high

Activity 1

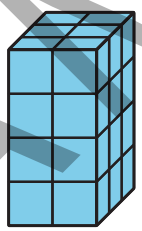
Layers in Rectangular Prisms



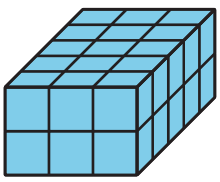
1. Complete the table.

prism	number of cubes in one layer	number of layers	volume
A			
B			
C			
D			

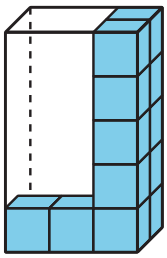
Prism A



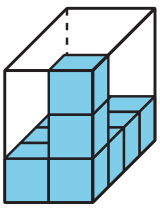
Prism B



Prism C

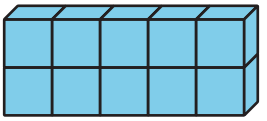


Prism D

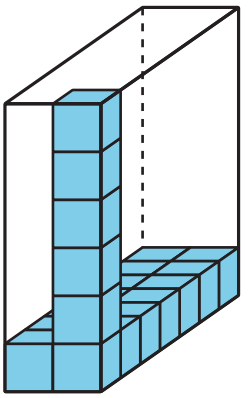


2. Find the volume of each prism. Explain or show your reasoning.

Prism E



Prism F

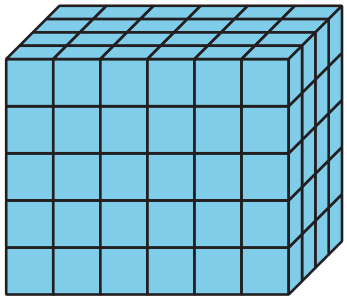


3. How can you find the volume of any rectangular prism?

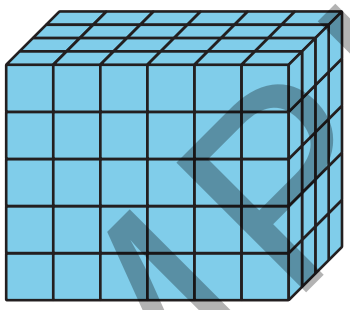
Activity 2

Finding Volume in Different Ways

1. How does the expression 5×24 represent the volume of this rectangular prism? Explain or show your reasoning.



2. How does the expression 6×20 represent the volume of this rectangular prism? Explain or show your reasoning.

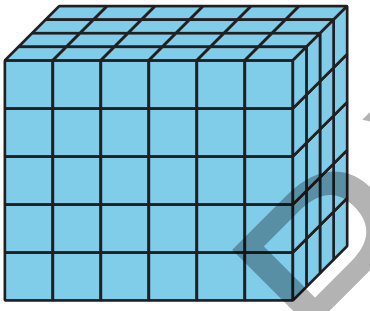


3. Find a different way to calculate the volume of this rectangular prism. Write an expression to represent the way that you calculated the volume. Explain or show your reasoning.

Section A Summary

We learned the amount of space an object takes up is **volume**.

This prism has a volume of 120 **unit cubes**.



We learned to calculate the volume of any prism by finding the number of cubes in one layer and multiplying that number by the number of layers. We can describe this **rectangular prism** as having 6 layers of 20 cubes, 4 layers of 30 cubes, or 5 layers of 24 cubes. All of these expressions represent the volume of the prism:

5×24 or $5 \times (6 \times 4)$

6×20 or $6 \times (5 \times 4)$

4×30 or $4 \times (5 \times 6)$

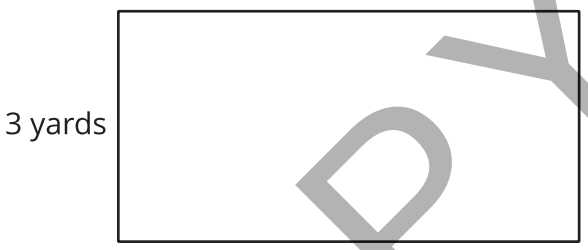
Practice Problems

10 Problems

1 Pre-unit

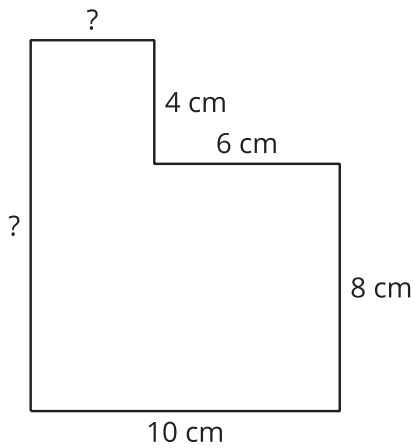
Here is a diagram of the floor in a room.

What is the area of the floor? Explain or show your reasoning.



2 Pre-unit

What are the unknown side lengths? Explain or show your reasoning.



3 Pre-unit

Which of these units would you use to measure the length of a pencil? Select **all** that apply.

A. centimeter

B. meter

C. kilometer

D. inch

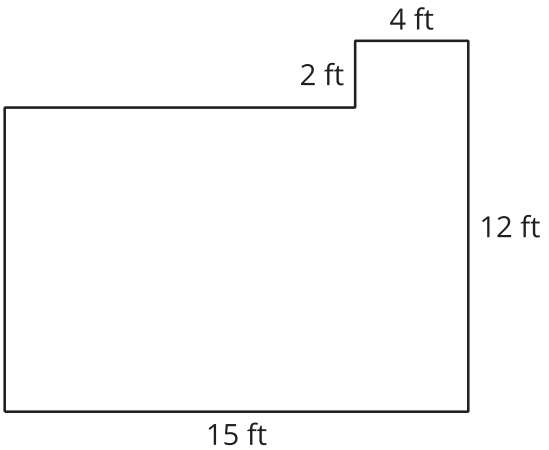
E. foot

F. yard

G. mile

4 Pre-unit

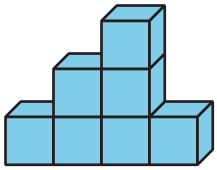
Find the area of the figure shown here. Explain or show your reasoning.



5 from Unit 1, Lesson 1

Which has the greater volume? Explain or show your reasoning.

A



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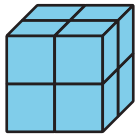
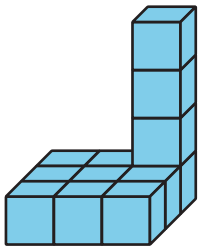


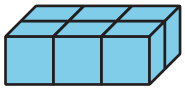
6 from Unit 1, Lesson 2

What is the volume of the figure? Explain or show your reasoning.

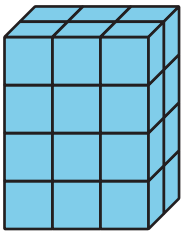


7 from Unit 1, Lesson 3

a. What is the volume of this rectangular prism? Explain or show your reasoning.



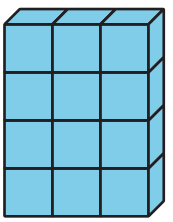
b. What is the volume of this rectangular prism? Explain or show your reasoning.



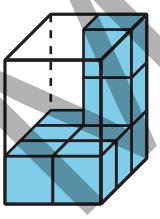
8 from Unit 1, Lesson 4

Find the volume of each rectangular prism. Explain or show your reasoning.

a.



b.



9 Exploration

Find the volume of some objects around you.

a. List the objects.

b. Which has the least volume? Which has the greatest?

c. Pick 2 of your objects and compare their volumes.

a. How many different rectangular prisms can you make with 18 cubes? Explain or show your reasoning.

b. How many different rectangular prisms can you make with 24 cubes? Explain or show your reasoning.

c. How do the side lengths of the prisms compare to each other? What patterns do you notice? Is this pattern true for the rectangular prisms you can make with 36 cubes?

Addressing CA CCSSM 5.MD.5b; building towards 5.MD.5b; practicing MP1 and MP2

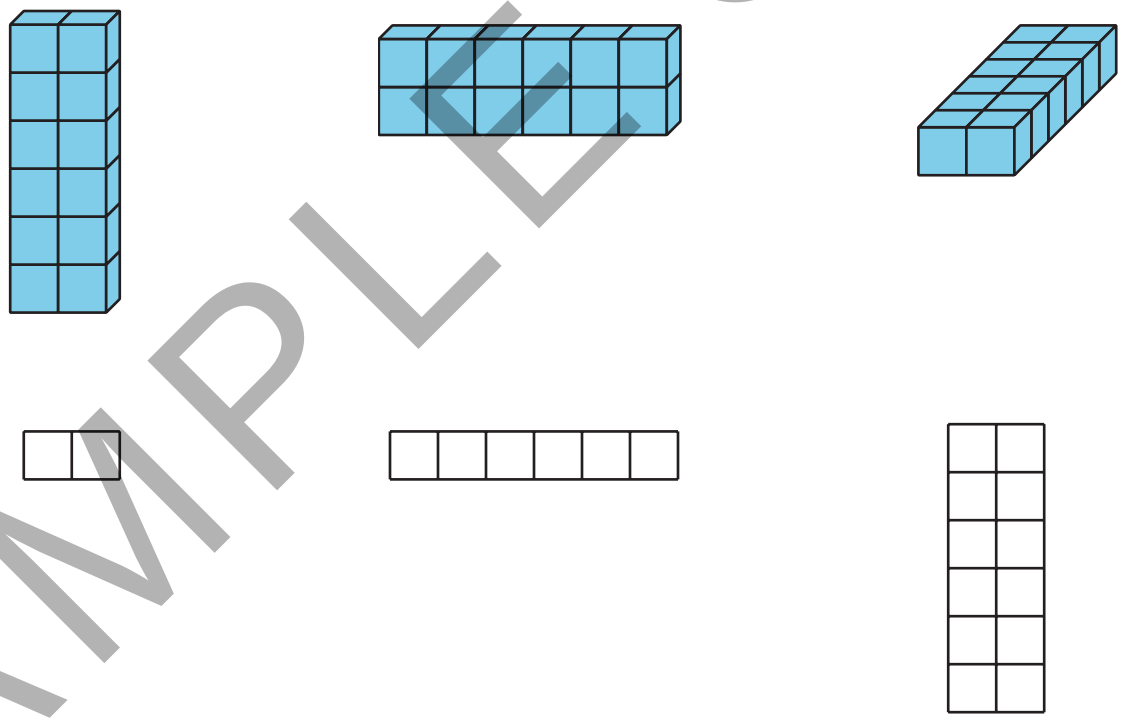
Side Lengths of Rectangular Prisms

Let's describe the side lengths of a prism and find the volume.

Warm-up

Notice and Wonder: Prism Print

What do you notice? What do you wonder?

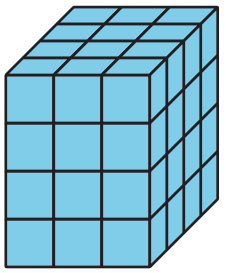


Activity 1

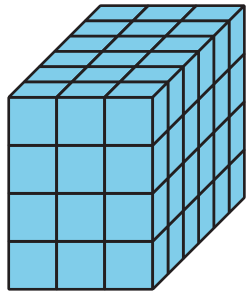
All about That Base

Here are 3 rectangular prisms.

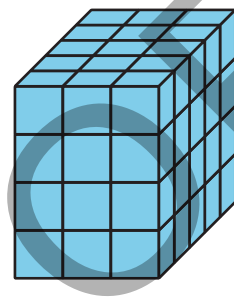
1



2

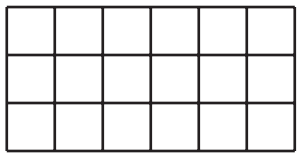


3

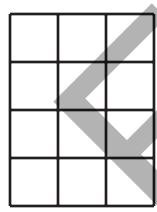


These rectangles represent bases of the prisms.

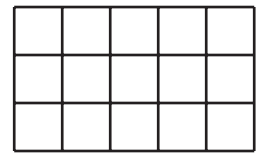
A



B



C

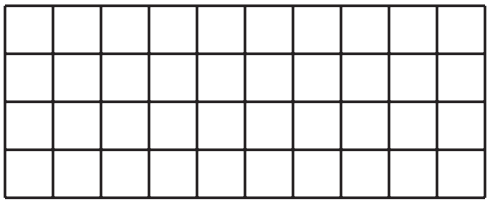


1. Match each prism with a rectangle that represents its base. Some prisms may match more than 1 rectangular base.
2. Find the volume of each prism. Explain or show your reasoning.

Activity 2

Growing Prism

Here is a base of a rectangular prism.



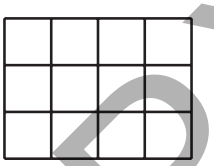
Complete the table for the volumes of rectangular prisms. Use this base and each of these heights.

height (units)	multiplication expression to represent volume	volume (unit cubes)
1		
2		
3		
10		
25		

Activity 3

What Is the Question?

This is the base of a rectangular prism that has a height of 5 units.



Sec B

These are answers to questions about the prism. Determine the question for each answer.

1. 3 is the answer. What is the question?

2. 5 is the answer. What is the question?

3. $3 \times 4 = 12$. The answer is 12. What is the question?

4. $12 \times 5 = 60$. The answer is 60 unit cubes. What is the question?

5. 3 units by 4 units by 5 units is the answer. What is the question?

Addressing CA CCSSM 5.MD.5a, 5.MD.5b, 5.OA.1, 5.OA.2; building towards 5.MD.5b; practicing MP7

Expressions for Volume—Part 1

Let's write expressions for the volume of rectangular prisms.

Warm-up

True or False: Parentheses or No Parentheses?

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $(4 \times 2) \times 5 = 4 \times (2 \times 5)$

- $(2 \times 5) \times 4 = 2 \times 20$

- $5 \times 4 \times 2 = 10 \times 40$

Activity 1

Card Sort: Match the Expression

Your teacher will give you a set of cards.

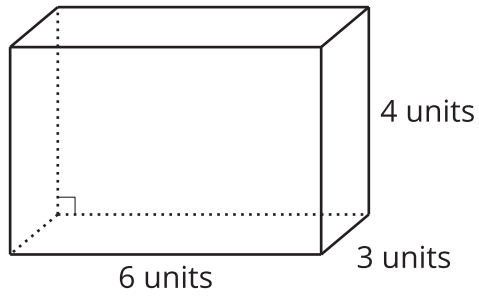
1. Sort the cards into categories in a way that makes sense to you. Be ready to explain the meaning of your categories.
2. Match each rectangular prism with the expression(s) that represents its volume in cubic units. Be ready to explain your reasoning.
3. Write one additional expression for each prism. Represent the volume in cubic units.

Activity 2

A Tale of Two Tables

1. Work with your partner to complete the tables. One partner completes Table 1 and the other completes Table 2.

Prism A



Prism B

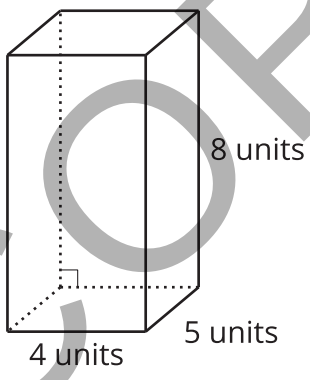


Table 1

	length (units)	width (units)	height (units)	volume (cubic units)
Prism A				
Prism B				

Table 2

	area of the base (square units)	height (units)	volume (cubic units)
Prism A			
Prism B			

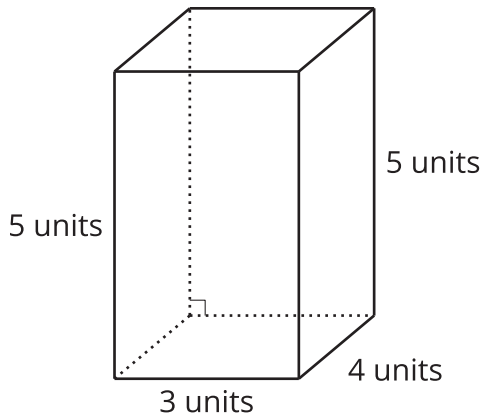
2. Compare the tables and discuss:
   a. What do the tables have in common?
   b. What is different about the tables?

Activity 3

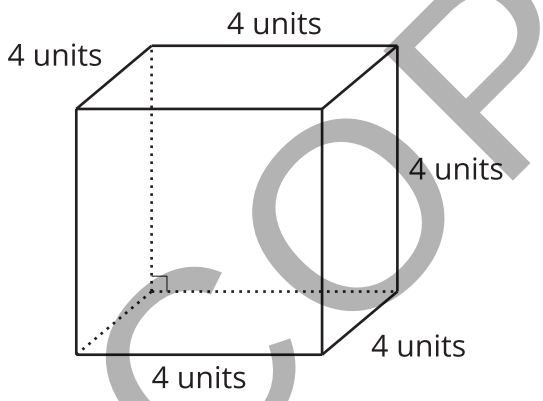
Two Truths and a Lie

Your teacher will assign you and your partner 2 prisms.

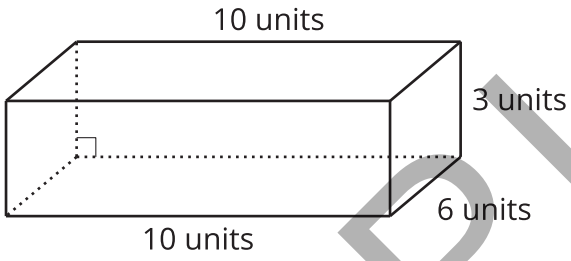
A



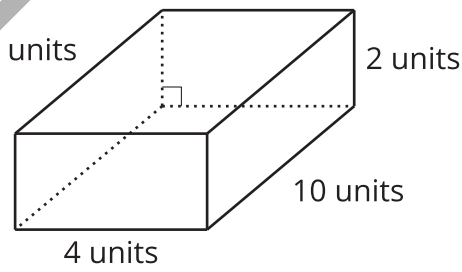
B



C



D



For each of your assigned prisms:

- Write 2 expressions to represent the volume in cubic units.
- Write 1 expression that does not represent the volume in cubic units.

Give your expressions to your partner.

1. Which expression *does not* represent the volume of the prism in cubic units? How do you know?

2. What other expressions represent the volume of the prism in cubic units?

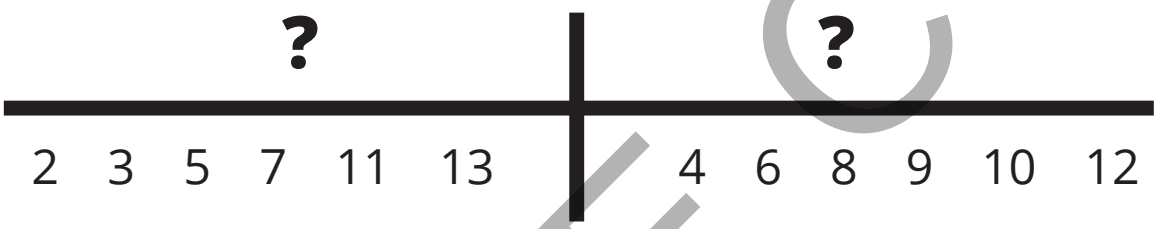
Unit 1, Lesson 6 (Part 2)

Addressing 5.MD.5.b, 5.OA.2.1; Building Towards 5.OA.2.1; Practicing MP1, MP2, MP3

Expressions for Volume—Part 2

Warmup

What do you notice? What do you wonder?



Activity 1

Same Volume, Different Prisms

1. Fill in the table to show **all** the different side lengths for a rectangular prism that has a volume of 18 cubic units. Use only whole number side lengths and use each combination of 3 numbers only one time.

   a. Write an equation to represent the volume of each prism.

volume (cubic units)	side length (units)	side length (units)	side length (units)	equation
18				
18				
18				
18				

   b. Which equation uses only prime number factors?

2. Fill in the table to show **all** the different side lengths for a rectangular prism that has a volume of 30 cubic units. Use only whole number side lengths and use each combination of 3 numbers only one time.

   a. Write an equation to represent the volume of each prism.

volume (cubic units)	side length (units)	side length (units)	side length (units)	equation
30				
30				
30				
30				
30				

   b. Which equation uses only prime number factors?

Activity 2

Volume Challenge

1. Which volumes of rectangular prisms can be created with side lengths that are only prime numbers? Be prepared to explain your reasoning.

   a. 16 cubic units

   b. 27 cubic units

   c. 28 cubic units

   d. 32 cubic units

   e. 35 cubic units

   f. 42 cubic units

   g. 50 cubic units

2. For each rectangular prism that can be created, write an expression to represent the volume using only prime factors.

Activity 3

Finding Prime Factors

Express each number as a product using only factors that are prime numbers.

a. 8

b. 14

c. 20

d. 32

e. 36

f. 40

g. 45

Addressing CA CCSSM 5.MD.4; practicing MP1, MP3, MP6

Cubic Units of Measure

Let's use different cubic units to measure volume.

Warm-up

Notice and Wonder: Two Prisms

What do you notice? What do you wonder?

Activity 1

What Are the Units?

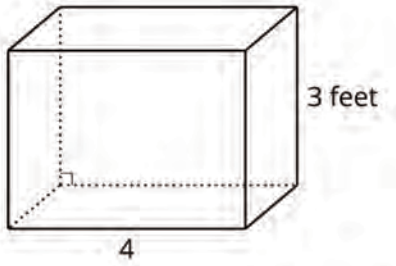
For each object, choose the cubic unit you would use to measure the volume: cubic centimeter, cubic inch, or cubic foot.

volume of	unit
a moving truck	
a freezer	
a juice box	
a classroom	
a dumpster	
a lunch box	

Activity 2

Info Gap: Sizing Up Cubic Units (Part 1)

Problem Card

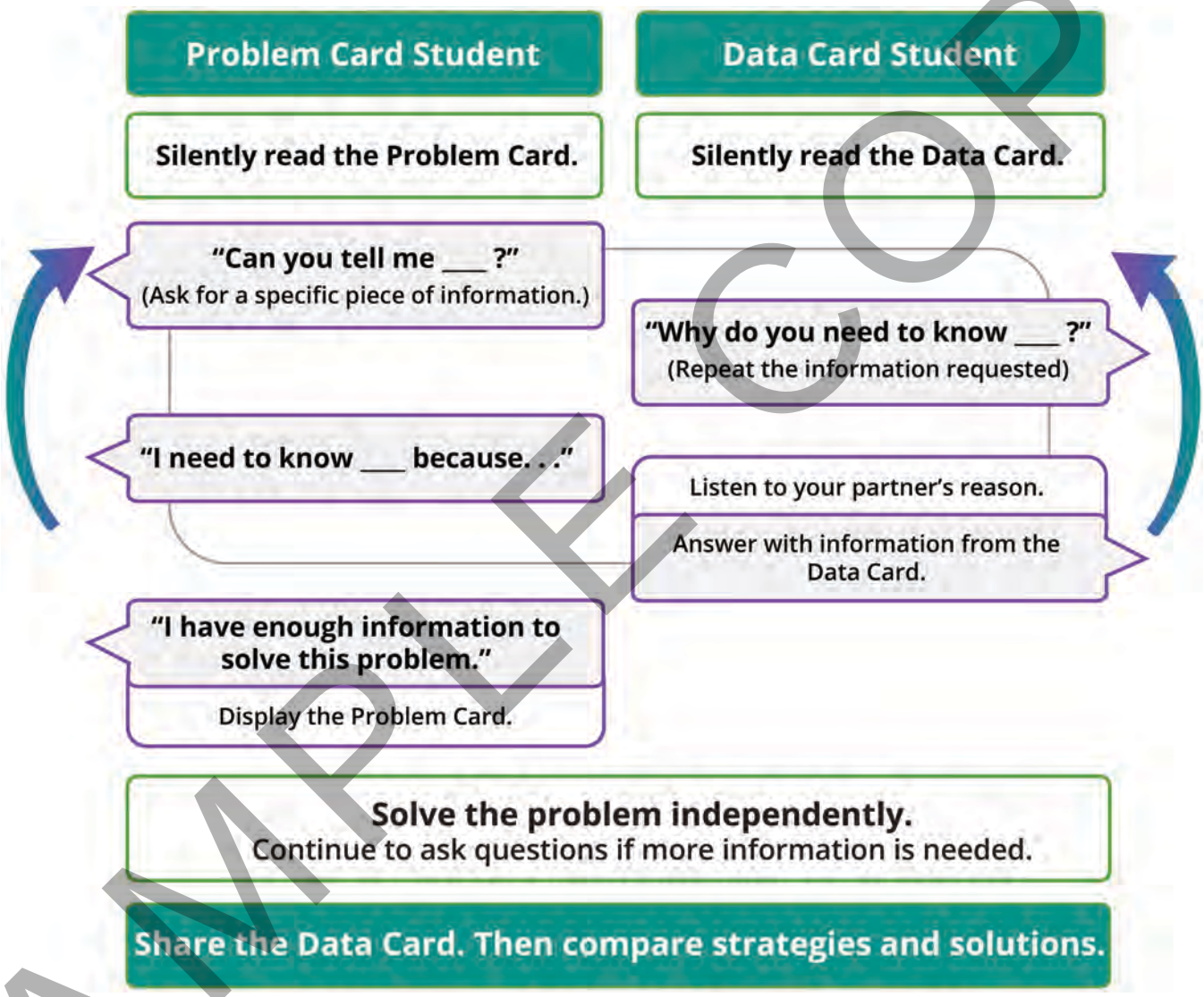


What is the volume of this freezer?

Activity 3

Info Gap: Sizing Up Cubic Units (Part 2)

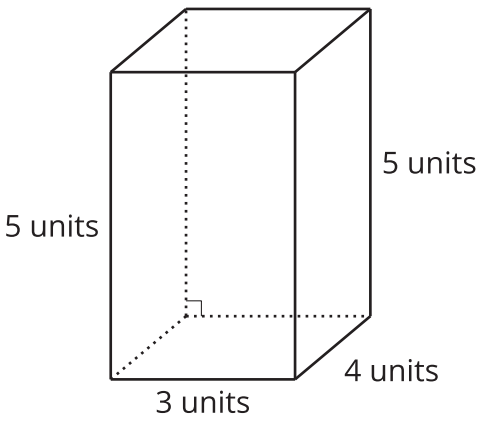
Your teacher will give you either a Problem Card or a Data Card. Do not show or read your card to your partner.



Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Section B Summary

We learned to find the volume of a right rectangular prism by multiplying the side lengths or by multiplying the **area** of the base by the height.



$$4 \times (5 \times 3)$$

$$(4 \times 5) \times 3$$

$$15 \times 4$$

Each of these expressions represents the volume of this prism. The volume of this rectangular prism is 60 cubic units.

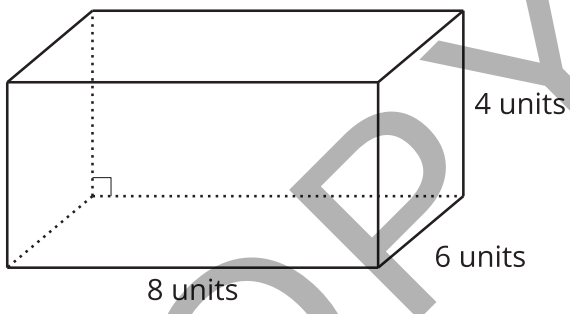
We learned to use different cubic units to measure the volume of objects of different sizes. We used cubic inches, cubic feet, cubic yards, and cubic centimeters to measure volume.

Practice Problems

7 Problems

1 from Unit 1, Lesson 5

Andre and Clare used different strategies to find the volume of this rectangular prism.



a. Andre says the volume of this rectangular prism is 8×24 cubic units. Is Andre correct? Explain or show your reasoning.

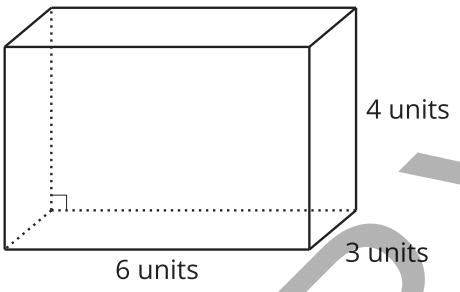
b. Clare says the volume of the rectangular prism is 6×32 cubic units. Is this correct? Explain or show your reasoning.

2

from Unit 1, Lesson 6

Which expressions represent the volume of this rectangular prism in cubic units?

Select **all** that apply.



A. $3 \times 4 \times 6$

B. 24×12

C. $12 + 12 + 12$

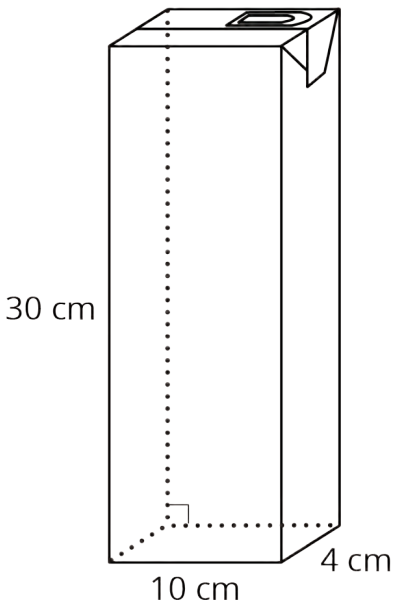
D. $24 + 24 + 24$

E. 18×4

3

from Unit 1, Lesson 7

A box of milk measures 4 centimeters by 10 centimeters by 30 centimeters. What is its volume in cubic centimeters? Explain or show your reasoning.



4 Exploration

A sugar cube has a volume of about 1 cubic centimeter. Estimate the size of a box you would need to hold 1,000,000 sugar cubes.

5 Exploration

Find some objects around the school or your home. What units would you use to measure their volumes? Choose one of your objects and estimate its volume.

6 Exploration

An object has a volume of 36 cubic inches. A box has side lengths 1 foot by 3 inches by 4 inches.

a. What is the least number of these objects that can fit within the box? Explain your reasoning.

b. What is the greatest number of these objects that can fit within the box? Explain your reasoning.

7 Exploration

A container has a volume of 120 cubic inches.

a. What could be the length, the width, and the height of the container?

b. Can one of the side lengths be 9 inches? Explain or show your reasoning.

Addressing CA CCSSM 5.MD.5c; building on 5.MD.5a; practicing MP7 and MP8

Figures Made of Prisms

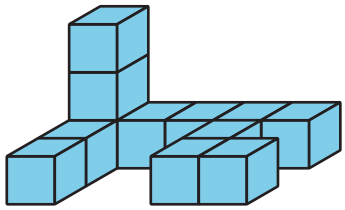
Let's find the volumes of figures made of prisms.

Warm-up

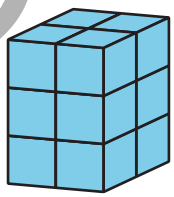
Which Three Go Together: Different Figures

Which 3 go together?

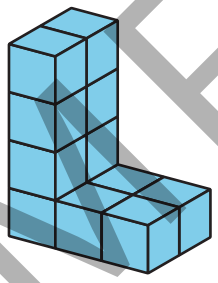
A



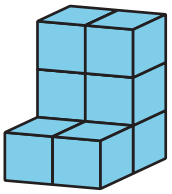
B



C



D

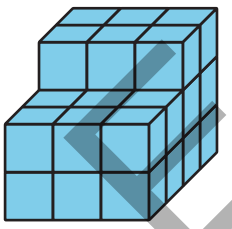


Activity 1

Put It Together

1. Partner A: Build a rectangular prism with 12 cubes.
2. Partner B: Build a rectangular prism with 10 cubes.
3. Put your 2 rectangular prisms together to make one figure. What is the volume of the new figure? Explain or show your reasoning.

4. Diego and Jada put together 2 rectangular prisms to make this figure.



   a. What is the volume of the figure Diego and Jada made?

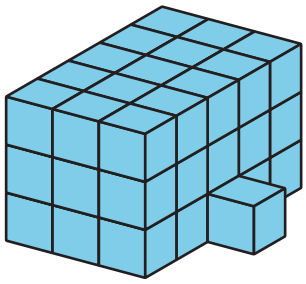
   b. Which rectangular prisms could Diego and Jada each have built? Show your thinking. Organize your work so it can be followed by others.

Activity 2

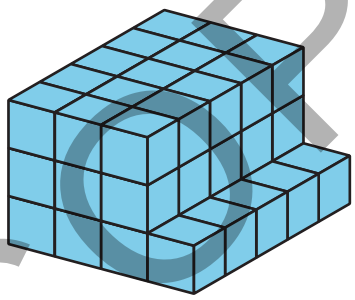
I See Two Prisms

Find the volume of each figure. Explain or show your reasoning.

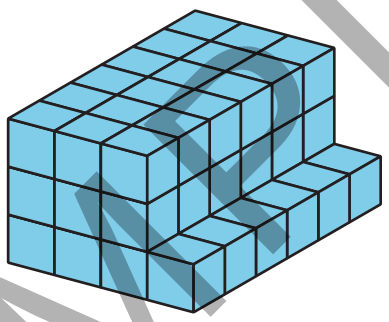
A



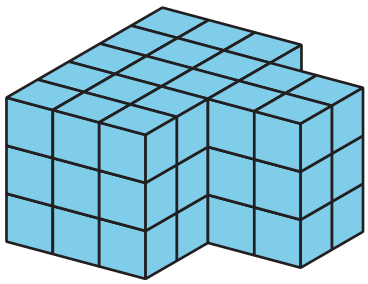
B



C



D



Addressing CA CCSSM 5.MD.5c, 5.OA.2; building towards 5.NBT.2; practicing MP2

Measure Figures Made from Prisms

Let's find the volumes of more figures.

Warm-up

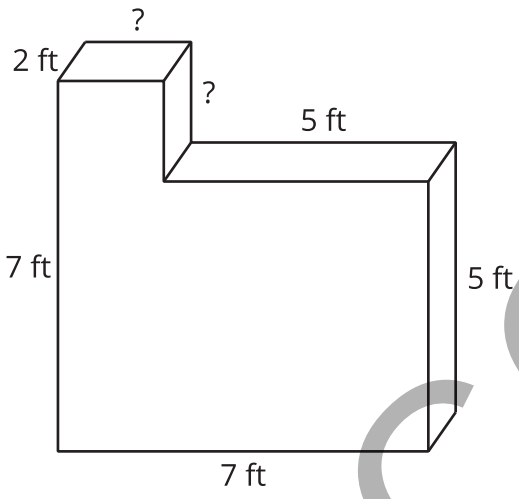
Number Talk: Times Ten

Find the value of each expression mentally.

- 6×2
- $6 \times 2 \times 10$
- $6 \times 20 \times 10$
- $60 \times 20 \times 10$

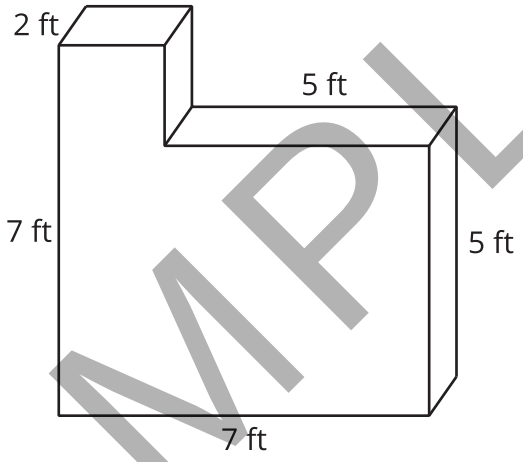
Activity 1

Find the Volumes of Figures



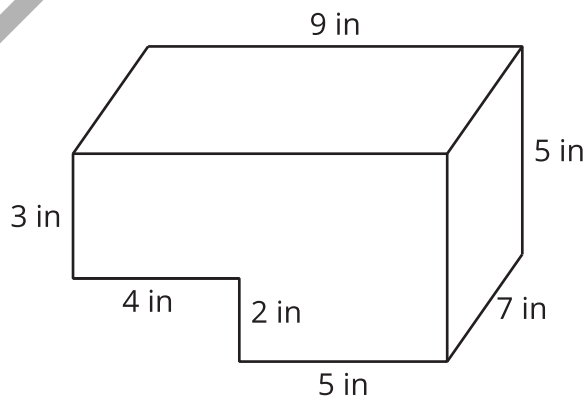
Partner A: Find the volume of Figure 1.

Figure 1



Partner B: Find the volume of Figure 2.

Figure 2

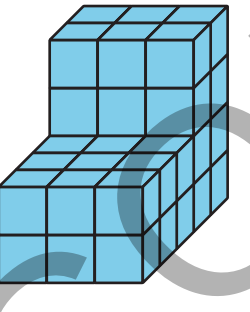


Activity 2

Expressions for the Volume of a Figure

1. Explain how each expression represents the volume of the figure. Show your reasoning. Organize your work so it can be followed by others.

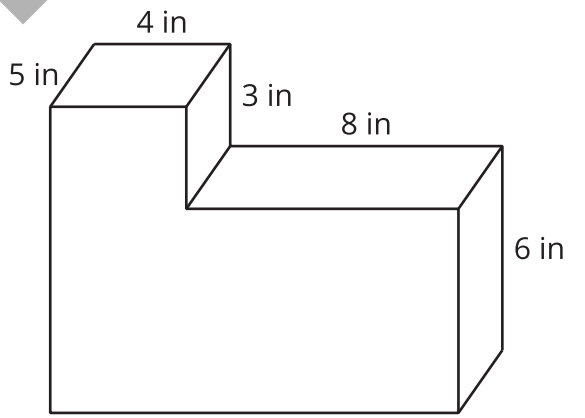
   a. $((2 \times 3) \times 4) + ((3 \times 3) \times 2)$



   b. $(5 \times 6) + (3 \times 4)$

2. How does each expression represent the volume of the prism? Explain or show your reasoning. Organize your work so it can be followed by others.

   a. $(5 \times 8 \times 6) + (5 \times 4 \times 9)$ cubic inches



   b. $(5 \times 4 \times 3) + (5 \times 12 \times 6)$ cubic inches

Addressing CA CCSSM 5.MD.5c, 5.OA.1, 5.OA.2; building towards 5.MD.5c; practicing MP2, MP3, MP7

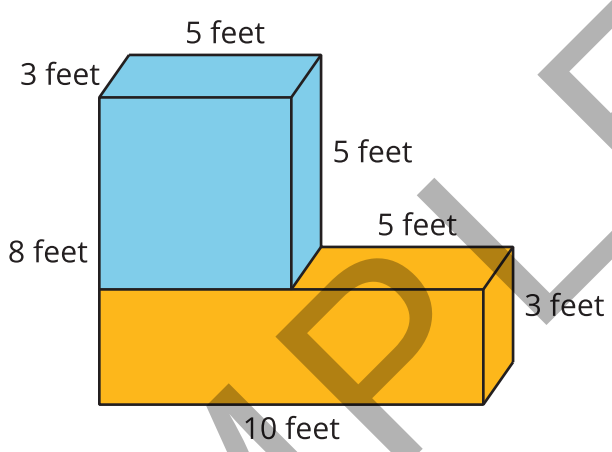
Represent Volume with Expressions

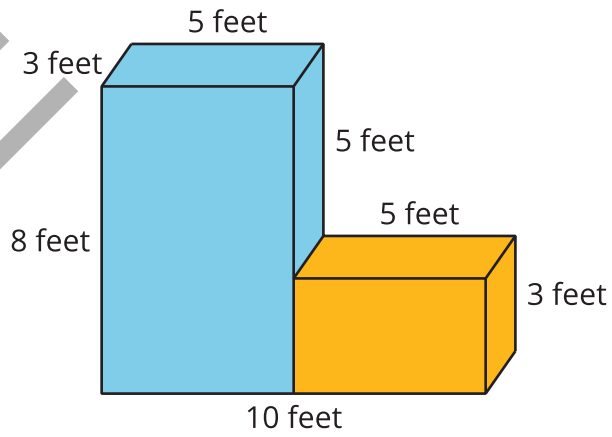
Let's write expressions for the volumes of figures.

Warm-up

Notice and Wonder: Prism Pieces

What do you notice? What do you wonder?

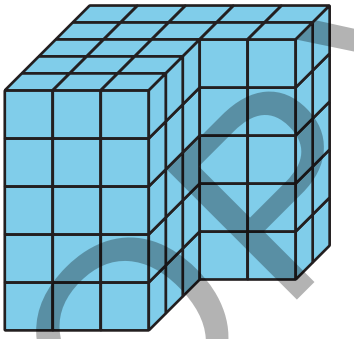




Activity 1

Compare Expressions

1. Write an expression to represent the volume of the figure in unit cubes.



2. Compare expressions with your partner.
   a. How are they alike?

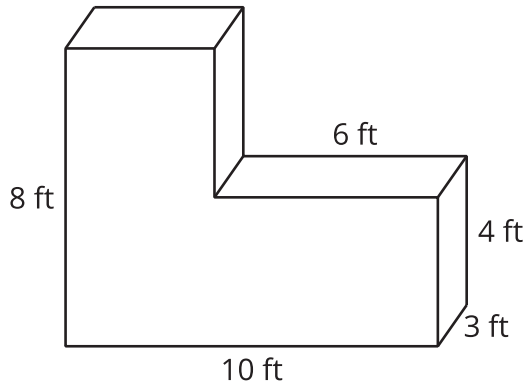
   b. How are they different?

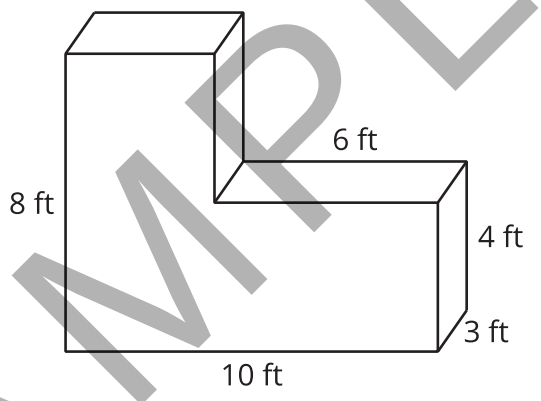
3. If they are the same, try to find another way to represent the volume.

Activity 2

Find the Volume in Different Ways

1. Find the volume by decomposing the figure in as many ways as you can. Show your thinking. Organize your work so it can be followed by others.



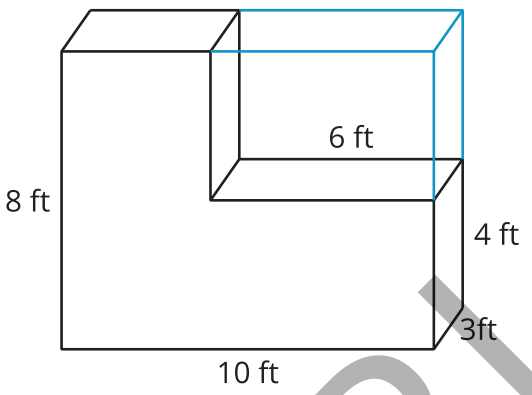


2. Write expressions to represent each way that you decompose the figure.

3. Mai used this expression to find the volume of the figure:

$$(10 \times 8 \times 3) - (6 \times 4 \times 3).$$

Use the diagram to interpret Mai's expression. Show your thinking. Organize your work so it can be followed by others.



Addressing CA CCSSM 5.MD.3, 5.MD.5; practicing MP1

All Kinds of Prisms

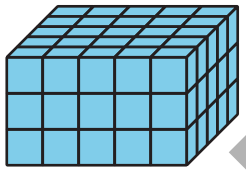
Let's find the volume of all different kinds of prisms.

Warm-up

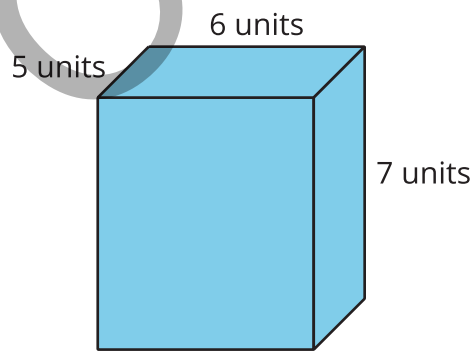
Which Three Go Together: Many Prisms

Which 3 go together?

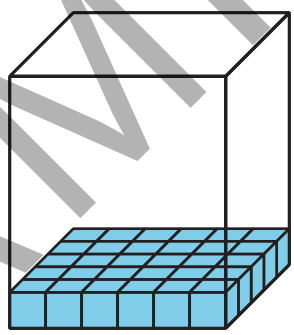
A



B



C



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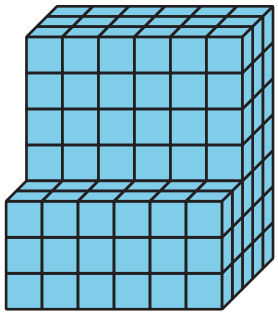
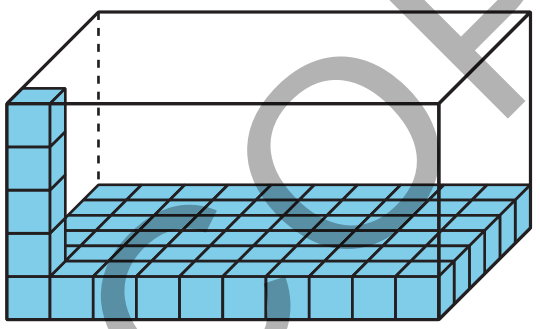


Activity 1

Prism-palooza

For each problem, explain or show your reasoning.

1. Han fills a box with cubes. Below is a diagram of the box. How many cubes can fit in the box if Han completely packs it, without gaps between cubes?



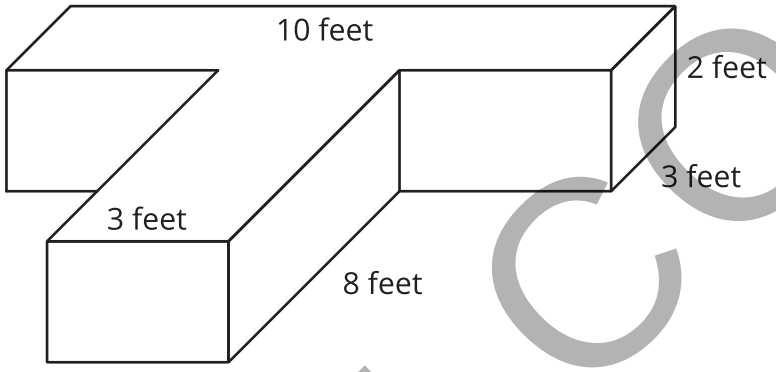
2. Clare buys a storage container for her art supplies. The storage container is 4 feet wide, 9 feet long, and 5 feet high. What is the volume of her container?

3. Mai's new bedroom has a walk-in closet with a floor that measures 30 square feet. Her closet ceiling is 9 feet from the floor. What is the volume of her closet?

Activity 2

Problem Solving with Figures

The elementary school builds a raised bed garden. A raised-bed garden is a box with soil that is higher than the ground around it. Here is a diagram that shows the side lengths of the garden.



1. What is the volume of the garden? Explain or show your reasoning.

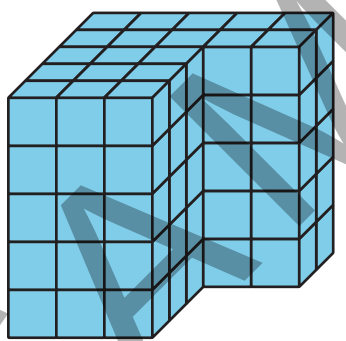
2. Write an expression to represent the volume of the garden.

3. Noah designs a garden with the same volume but different side lengths. What could be the side lengths of his garden?

4. Which garden design do you like better? Explain or show your reasoning.

Section C Summary

We learned that some figures are made from two rectangular prisms. We can decompose these figures and find the volume of each prism. Then we add the volumes of the two prisms to find the total volume of the figure.



There is often more than one way to decompose figures made from two rectangular prisms. These expressions can be used to find the volume of the figure.

$$(3 \times 3 \times 5) + (5 \times 2 \times 5)$$

$$(3 \times 5 \times 5) + (2 \times 2 \times 5)$$

Unit 1, Lesson 12

Addressing CA CCSSM 5.MD.3, 5.MD.5, 5.MD.5c; practicing MP4

Tons and Tons of Garbage

Let's investigate what happens to garbage.

Warm-up

Notice and Wonder: Garbage Truck

What do you notice? What do you wonder?

Activity 1

Sixty Containers

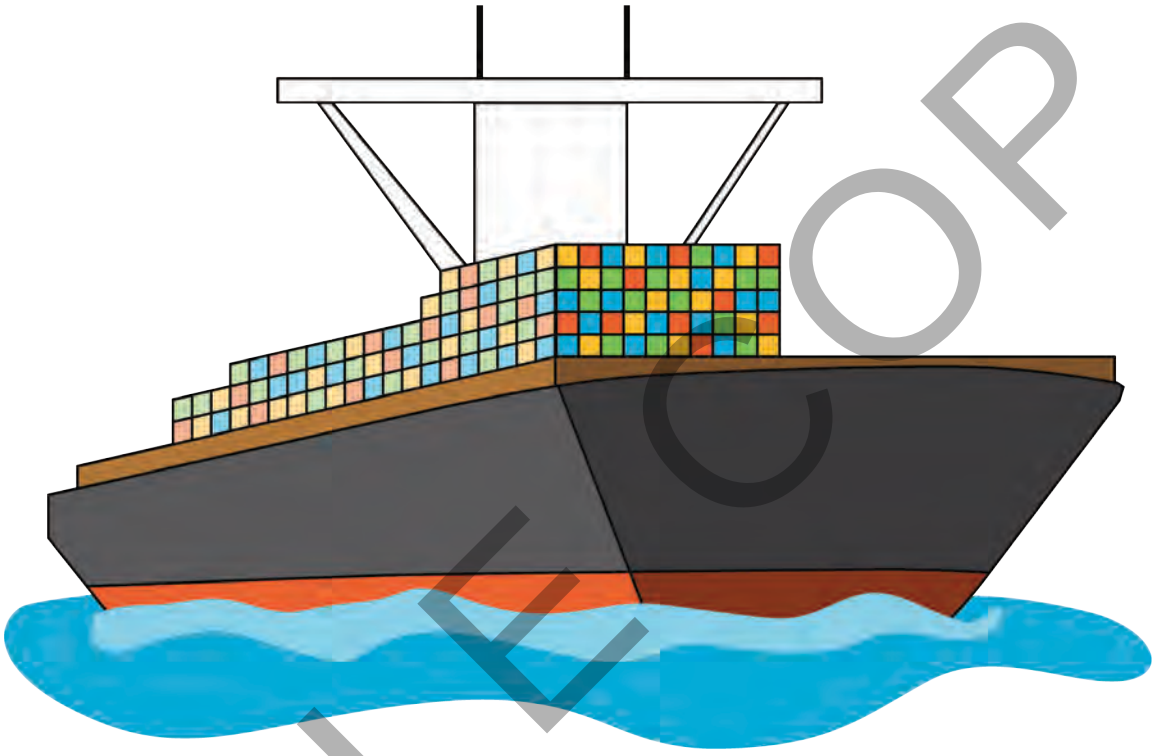
1. Find at least 5 different ways to arrange 60 containers. Represent each arrangement with an expression.

2. Create a display to show which is the best arrangement for shipping the 3,300 tons of garbage.

Activity 2

How Many Containers on the Ship?

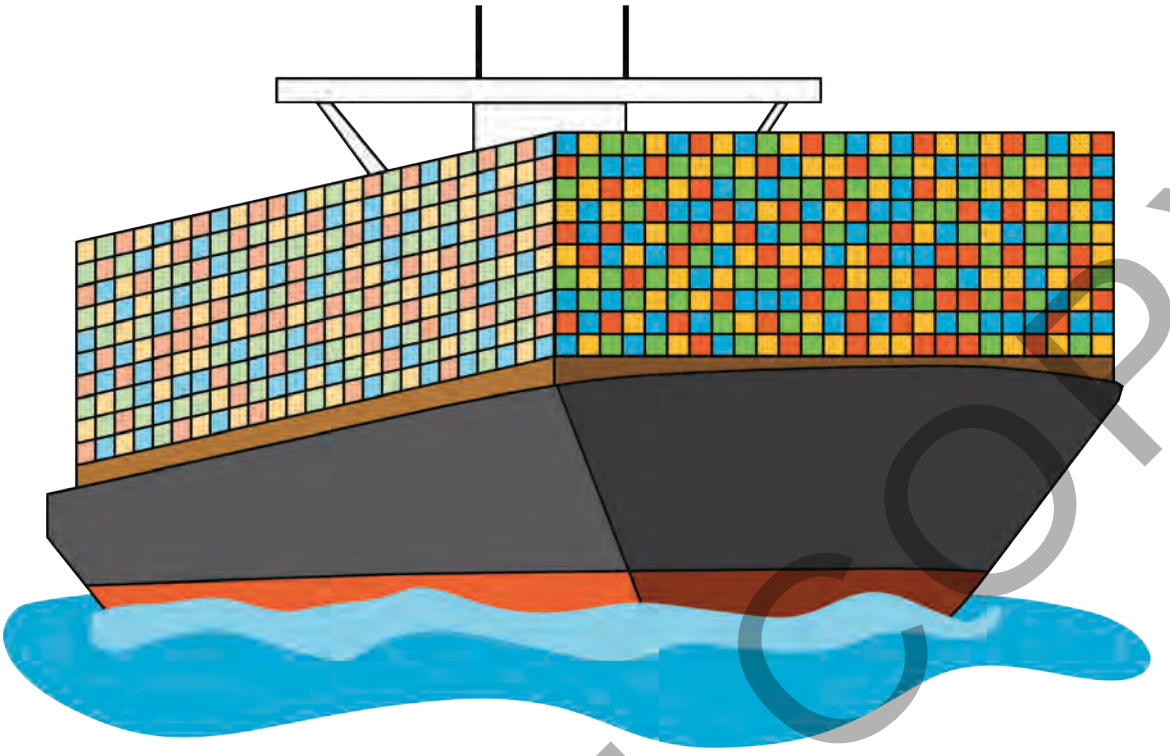
1. How many containers are on the cargo ship?



Record an estimate that is:

too low	about right	too high

2. How many containers are on the cargo ship?



Record an estimate that is:

too low	about right	too high

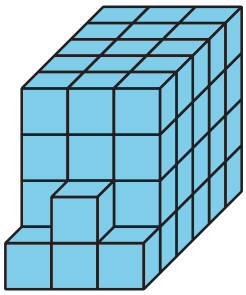
3. What assumptions did you make when you came up with your estimates?

Practice Problems

6 Problems

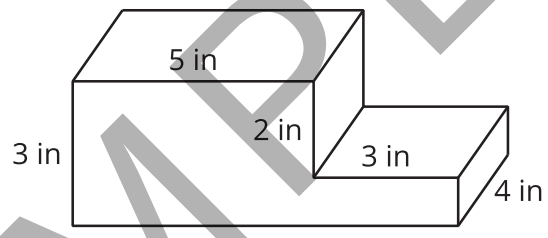
1 from Unit 1, Lesson 8

What is the volume of this figure? Explain or show your reasoning.



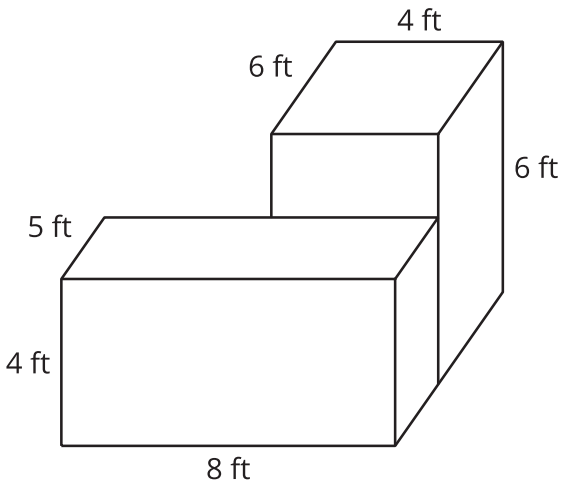
2 from Unit 1, Lesson 9

Find the volume of the figure. Explain or show your reasoning.



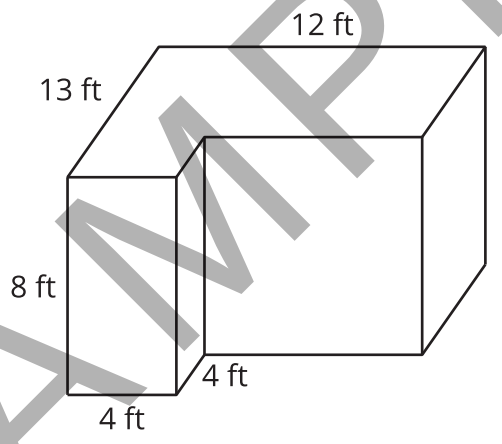
3 from Unit 1, Lesson 10

Find the volume of the figure. Explain or show your reasoning.



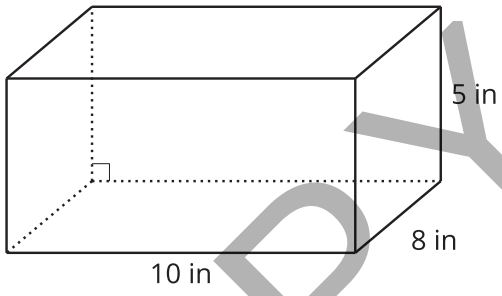
4 from Unit 1, Lesson 11

This is a diagram of a bedroom. What is the volume of the bedroom? Explain or show your reasoning.



5 Exploration

a. Han says that the volume of this rectangular prism is 50 times as great as a 2-inch cube. Do you agree with Han? Explain or show your reasoning.



b. Han says that he can fit fifty 2-inch cubes in this rectangular prism. Do you agree with Han? Explain or show your reasoning.

6 Exploration

There are 2 common sizes of shipping boxes: 10 inches by 6 inches by 16 inches and 12 inches by 7 inches by 12 inches. Which size box would you choose to ship the books for your math class? Explain or show your reasoning.

GRADE 5

UNIT

Fractions as Quotients and Fraction Multiplication

Content Connections

In this unit you will interpret fractions as quotients, use division to understand fractions and multiply a whole number and a fraction. You will make connections by:

- **Exploring Changing Quantities** while multiplying a whole number and a fraction, including fractions greater than 1.
- **Taking Wholes Apart, Putting Parts Together** while dividing the numerator and denominator of a fraction to create groups.
- **Discovering Shape and Space** while interpreting the product of a whole number and a fraction in terms of the side lengths of a rectangle.

Addressing the Standards

As you work your way through **Unit 2 Fractions as Quotients and Fraction Multiplication,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 1, 3, 14, 15, and 16
MP2 Reason abstractly and quantitatively.	Lessons 2, 4, 7, and 11
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3, 5, 7, and 12
MP4 Model with mathematics.	Lesson 17
MP5 Use appropriate tools strategically.	Lesson 9
MP6 Attend to precision.	Lessons 2, 5, 10, and 14
MP7 Look for and make use of structure.	Lessons 2, 3, 6, 9, and 10
MP8 Look for and express regularity in repeated reasoning.	Lessons 8 and 13

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections	**5.NF.3** Interpret a fraction as division of the numerator by the denominator *(a/b = a ÷ b)*. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Lessons 1, 2, 3, 4, 5, 6, 7, 11, and 15

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Lessons 7, 8, 12, 13, 14, 15, 16, and 17
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.4a** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*	Lessons 7, 8, 10, and 15
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.4b** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Lessons 9, 10, 11, 12, 13, 15, and 16

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Factors and Groups • Powers and Place Value • Layers of Cubes	**5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Lesson 13
• Factors and Groups • Powers and Place Value	**5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Lessons 6 and 8

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Addressing CA CCSSM 5.NF.3; building on 3.NF.1, 3.OA.2; building towards 5.NF.3; practicing MP1

Share Sandwiches

Let's share sandwiches.

Warm-up

Which Three Go Together: Sandwiches

Which 3 go together?

A

B

C

D

Activity 1

Share Sandwiches

_____ sandwiches are shared equally by _____ people.

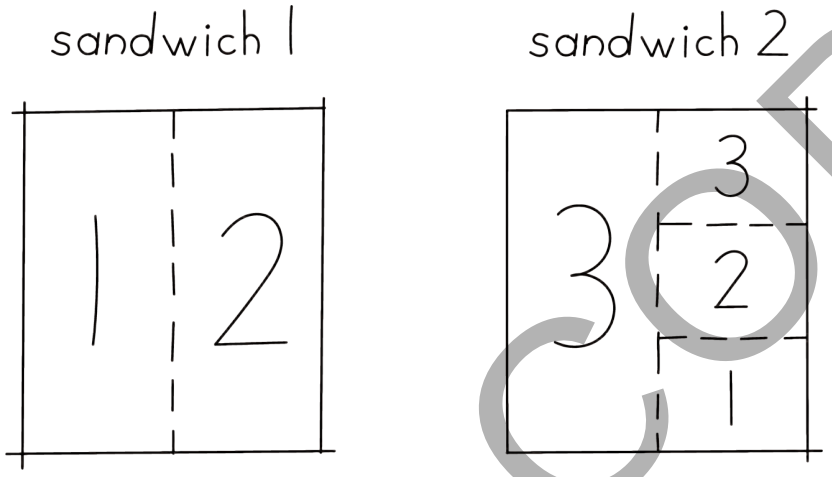
1. Choose numbers 2, 3, or 5 to fill in the blanks. You can use each number only once.
2. Represent the situation with a diagram or drawing.

3. Explain or show how you know that each person will get the same amount of sandwich.

Activity 2

The Same Amount

1. Han's work shows how 3 people can equally share 2 sandwiches.



How do you know that each person gets the same amount of sandwich? Explain or show your thinking.

2. Draw a diagram to show a different way that 3 people can share 2 sandwiches so that each person gets the same amount.

Addressing CA CCSSM 5.NF.3, building on 3.NF.1, 3.OA.2; building towards 5.NF.3; practicing MP2, MP6, MP7

Share More Sandwiches

Let's use diagrams and expressions to represent division situations.

Warm-up

Estimation Exploration: Name That Fraction

The whole rectangle represents 1. What fraction of the rectangle is shaded?

Record an estimate that is:

too low	about right	too high

Activity 1

One Sandwich

Jada's family made sandwiches to share equally at a picnic. Complete the table to show how much sandwich each person gets.

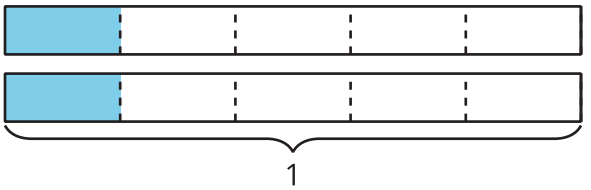
sandwiches being shared	number of people sharing sandwiches equally	amount of sandwich each person gets	division expression
1	2		
1	3		
1	4		
1	5		

1. Choose one row from the table and represent your thinking with a diagram.

2. What patterns do you notice in the table?

Activity 2

Card Sort: Sandwich Match



Your teacher will give you a set of cards. Match each diagram with a situation and expression. Some situations and expressions will have more than 1 matching diagram.

Choose 1 set of matched cards.

1. Explain or show how the diagram(s) and expression represent the number of sandwiches being shared.

2. Explain or show how the diagram(s) and expression represent the number of people sharing the sandwiches.

3. How much sandwich does each person get in the situation?

Addressing CA CCSSM 5.NF.3; building towards 5.NF.3; practicing MP1, MP3, MP7

Interpret Equations

Let's use equations to show the relationship between division and fractions.

Warm-up

What Do You Know about $\frac{3}{2}$?

What do you know about $\frac{3}{2}$?

Activity 1

Dehydrated Dancers

1. There are 3 dancers. They share 2 liters of water equally. How much water does each dancer get? Write a division equation to represent the situation.

2. Mai says each dancer gets $\frac{3}{2}$ liters of water because 3 divided into 2 equal groups is $\frac{3}{2}$. Do you agree with Mai? Explain or show your reasoning.

Activity 2

Interpret Expressions

1. Complete the table. Draw a diagram if it is helpful.

number of dancers	liters of water shared equally	division expression	amount of water each dancer gets in liters
4	2		
		$3 \div 4$	
		$3 \div 5$	
4			
	5		

2. What patterns do you notice in the table?

Addressing CA CCSSM 5.NF.3; practicing MP2

Division Situations

Let's solve and represent division problems.

Warm-up

Number Talk: Division

Find the value of each expression mentally.

- $35 \div 7$
- $1 \div 7$
- $36 \div 7$
- $37 \div 7$

Activity 1

Pounds of Blueberries

1. Complete the 2 missing parts of your assigned table. Explain your reasoning.
2. Discuss the equations, situations, and diagrams in your group's 3 tables. What is similar? What is different?

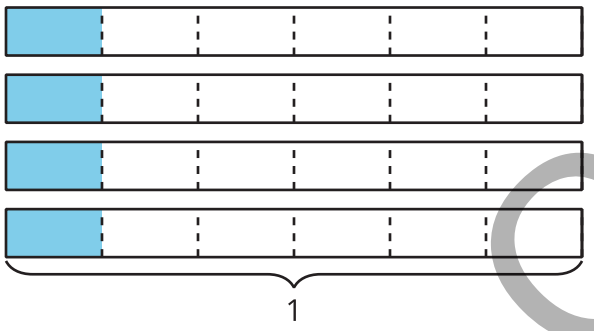
Partner A

equation	situation
$4 \div 6 = \frac{4}{6}$	
diagram	

Partner B

equation	situation

diagram



Partner C

equation	situation
	Six students share 4 pounds of blueberries equally. How many pounds of blueberries does each student get?

diagram

Activity 2

Grams of Gold

1. A group of 3 friends go panning for gold. They share the gold equally. Each friend gets $\frac{4}{3}$ grams of gold. How much gold did they collect together? Explain or show your reasoning.

2. Another group of friends goes panning for gold. They collect 5 grams of gold together and share it equally. Each friend gets $\frac{5}{6}$ gram. How many friends shared the gold? Explain or show your reasoning.

Addressing CA CCSSM 5.NF.3; practicing MP3 and MP6

Relate Division and Fractions

Let's explain the relationship between division and fractions.

Warm-up

True or False: Interpret Fractions

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5 \div 2 = \frac{5}{2}$
- $\frac{5}{2} = 5\frac{1}{2}$
- $\frac{6}{2} = 3$

Activity 1

Relate Pounds to People

	Each person gets _____ pound(s) of blueberries.			
	more than 1	exactly 1	less than 1	$\frac{1}{2}$
_____ people share 7 pounds of blueberries equally.	☑			
_____ people share _____ pounds of blueberries equally.		☑		
Three people share _____ pounds of blueberries equally.			☑	
_____ people share _____ pounds of blueberries equally.				☑

1. Fill in the blanks to match the rules in the table.
2. How many pounds of blueberries does each person get when they get more than 1 pound?

3. How many pounds of blueberries does each person get when they get less than 1 pound of blueberries?

(Pause your work after you answer the 2 questions.)

4. Work with your group to make a poster that explains or shows your thinking about the following questions.
   - What is true about all the pairs of numbers that are used when each person gets less than 1 pound of blueberries?
   - What is true about all the pairs of numbers that are used when each person gets more than 1 pound of blueberries?
   - What is true about all the pairs of numbers that are used when each person gets exactly $\frac{1}{2}$ pound of blueberries?

Activity 2

Why Does It Work?

1. What numbers can replace the question marks in each equation? Explain your reasoning.

$$? \div 2 = \frac{?}{2} \qquad 2 \div ? = \frac{2}{?}$$

(Pause for teacher directions.)

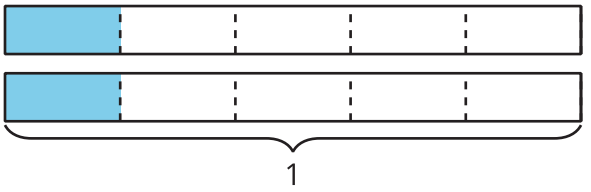
2. Work with your partner to explain why any division expression can be interpreted as a fraction. You can use diagrams, expressions, equations, and words.

Section A Summary

We learned that there is a relationship between division and fractions.

We can see this relationship in diagrams, situations, and equations.

Example: This diagram represents 2 sandwiches shared equally by 5 people. Each person gets $\frac{2}{5}$ of a sandwich. The equation $2 \div 5 = \frac{2}{5}$ also represents the situation.

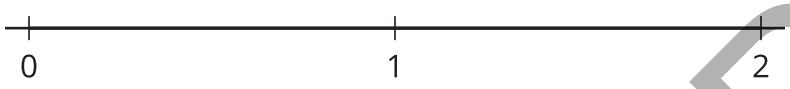


Practice Problems

15 Problems

1 Pre-unit

a. Locate and label $\frac{6}{4}$ on the number line.



b. Explain or show why your point represents $\frac{6}{4}$.

2 Pre-unit

Shade $\frac{3}{4}$ of the rectangle. Explain or show your reasoning.

3 Pre-unit

Explain or show why $\frac{4}{3} = 4 \times \frac{1}{3}$.

4 Pre-unit

Each book is $\frac{3}{8}$ inch thick. How many inches thick is a stack of 5 books? Explain or show your reasoning.

5 Pre-unit

a. There are 36 fish in 4 aquariums. There are the same number of fish in each aquarium. How many fish are in each aquarium? Explain or show your reasoning.

b. There are 24 dogs at a shelter. There are 4 times as many dogs as cats at the shelter. How many cats are at the shelter? Explain or show your reasoning.

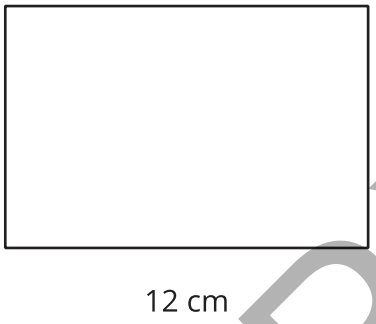
6 Pre-unit

A bottle holds $\frac{7}{10}$ liter of water. How much water do 6 bottles hold? Explain or show your reasoning.

7

Pre-unit

What is the area of the rectangle? Explain or show your reasoning.



8

from Unit 2, Lesson 1

a. There are 3 students. They equally share 18 sheets of construction paper for an art project. How much paper does each student get? Explain or show your reasoning.

b. There are 3 students. They equally share 1 tube of clay for an art project. How much clay does each student get? Explain or show your reasoning.

9 from Unit 2, Lesson 2

a. There are 4 hikers. They equally share 3 liters of water. How much water does each hiker get? Explain or show your reasoning.

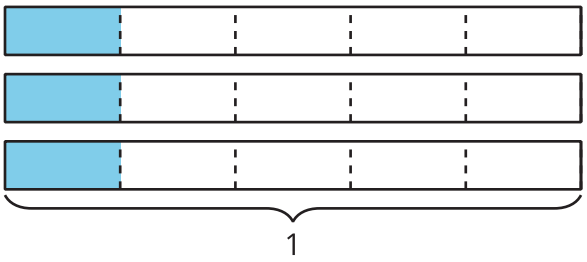
b. There are 4 hikers. They equally share 5 liters of water. How much water does each hiker get? Explain or show your reasoning.

10 from Unit 2, Lesson 3

a. Jada cuts an 11-inch strip of paper into 5 equal parts. How many inches long is each part?

b. Jada cuts a strip of paper into 5 equal parts. Each part is $\frac{7}{5}$ inches long. How long was the strip of paper?

11 from Unit 2, Lesson 4



a. Describe a division situation that the diagram could represent.

b. Write an equation that represents the diagram and the situation.

12 from Unit 2, Lesson 5

Decide whether each equation is true or false. Explain or show your reasoning.

a. $3 \div 7 = \frac{3}{7}$

b. $18 \div 5 = \frac{5}{18}$

c. $15 \div 6 = 2\frac{1}{2}$

13 Exploration

a. Describe a situation in the classroom or at home where you share something equally that results in fractional size parts.

b. Draw a picture or diagram to represent the situation.

c. Write a division equation to represent the situation.

14 Exploration

Elena travels to visit her grandparents. Their house is 125 miles from Elena's.

a. Elena stops for lunch $\frac{2}{3}$ of the way to her grandparents' house. How far has Elena traveled? Explain or show your reasoning.

b. Elena reaches the city where her grandparents live after 110 miles. Is she more or less than $\frac{9}{10}$ of the way to their house? Explain or show your reasoning.

15 Exploration

a. Describe a situation that represents the equation $4 \div 6 = \frac{4}{6}$.

b. Draw a diagram to represent the situation.

Addressing CA CCSSM 5.NF.3, 5.OA.2; building on 4.NF.4; building towards 5.NF.4; practicing MP7

Relate Division and Multiplication

Let's explore the relationship between multiplication and division.

Warm-up

Number Talk: Multiply and Divide

Find the value of each expression mentally.

- $3 \times \frac{1}{2}$
- $3 \times \frac{2}{2}$
- $3 \times \frac{3}{2}$
- $5 \times \frac{3}{2}$

Activity 1

The Race

1. Lin and Han run a 3-mile relay race as a team. They each run the same distance. Draw a diagram to represent the situation.

2. Take turns describing to your partner how your diagrams represent the situation.
3. How far does each person run?

Activity 2

Where Do You See It?

Diagram A

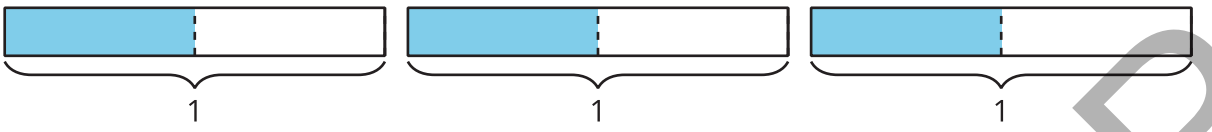
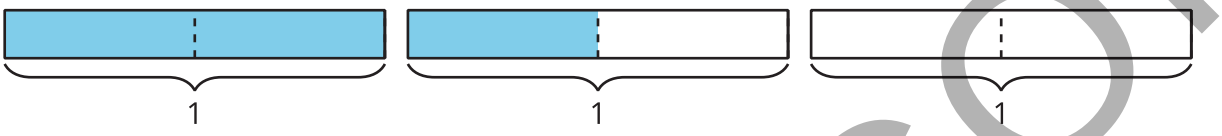


Diagram B



Choose 1 diagram that you think best represents each expression. Describe how your chosen diagram represents that expression. Explain your reasoning.

1. $3 \times \frac{1}{2}$

2. $3 \div 2$

3. $\frac{1}{2} \times 3$

Addressing CA CCSSM 5.NF.3, 5.NF.4, 5.NF.4a; building towards 5.NF.4; practicing MP2 and MP3

Divide to Multiply Unit Fractions

Let's solve problems about multiplying whole numbers by unit fractions.

Warm-up

Estimation Exploration: Number Line

What number is marked on the number line?

Record an estimate that is:

too low	about right	too high

Activity 1

How Far Did They Run?

Solve each problem. Draw a diagram if it is helpful.

1. Mai runs $\frac{1}{4}$ the length of her road. The road is 9 miles long. How far does Mai run?

2. Han runs $\frac{1}{4}$ the length of his road. The road is 7 miles long. How far does Han run?

Activity 2

Card Sort: Match the Situation

Your teacher will give you a set of cards that show expressions and diagrams.

1. Find the cards that match this situation. Be ready to explain your reasoning.

   Han, Lin, Kiran, and Jada run a 3-mile relay race as a team. They each run the same distance.

2. How far does each person run?

Addressing CA CCSSM 5.NF.4, 5.NF.4a, 5.OA.2; practicing MP8

Divide to Multiply Non-Unit Fractions

Let's solve problems about multiplying whole numbers by fractions.

Warm-up

True or False: A Fraction by a Whole Number

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $2 \times \left(\frac{1}{3} \times 6\right) = \frac{2}{3} \times 6$

- $2 \times \left(\frac{1}{3} \times 6\right) = 2 \times (6 \div 3)$

- $\frac{2}{3} \times 6 = 2 \times \left(\frac{1}{4} \times 6\right)$

Activity 1

Multiply a Whole Number by a Fraction

Find the value of each expression. Explain or show your reasoning. Draw a diagram if it is helpful.

1. $\frac{1}{5} \times 3$

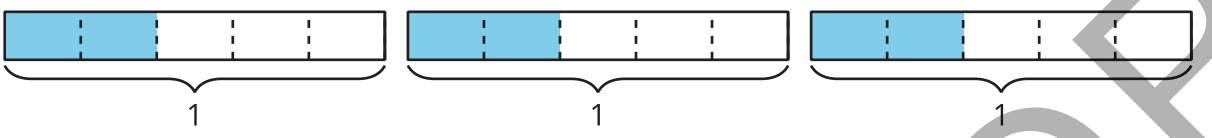
2. $\frac{2}{5} \times 3$

3. $\frac{3}{5} \times 3$

Activity 2

Match Expressions to Diagrams

Explain how each expression represents the shaded region of this diagram.



Sec B

1. $2 \times (3 \div 5)$

2. $\frac{6}{5}$

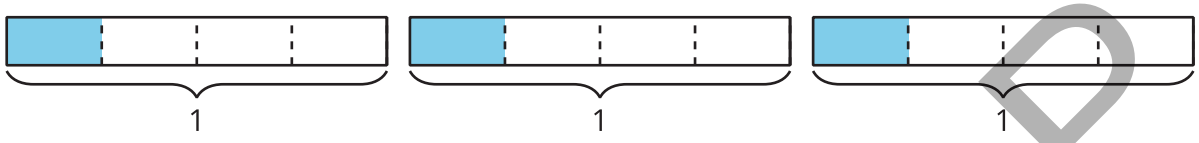
3. $3 \times \frac{2}{5}$

4. $3 \times 2 \times \frac{1}{5}$

Section B Summary

We explored the relationship between multiplication and division. We learned that a diagram can represent different multiplication and division expressions.

Example: We can represent this diagram with 4 different expressions:



- $\frac{3}{4}$

  Each rectangle is divided into 4 equal parts, and 3 parts are shaded.

- $3 \times \frac{1}{4}$

  There are 3 parts shaded, and each part is $\frac{1}{4}$ of the rectangle.

- $3 \div 4$

  There are 3 rectangles, and each rectangle is divided into 4 equal parts.

- $\frac{1}{4} \times 3$

  There are 3 rectangles, and $\frac{1}{4}$ of each rectangle is shaded.

We know that all of these expressions have the same value because they all represent the same diagram. We can use any of these expressions to represent and solve this problem:

- Mai eats $\frac{1}{4}$ of a 3-pound bag of blueberries. How many pounds of blueberries does Mai eat?

Practice Problems

5 Problems

1 from Unit 2, Lesson 6

Han cuts a 15-foot piece of rope into 4 equal parts. Decide whether each expression represents the length of each part of the rope in feet. Explain or show your reasoning.

a. $15 \div 4$

b. 4×15

c. $3\frac{3}{4}$

2 from Unit 2, Lesson 7

Find the value of each expression.

a. $\frac{1}{2} \times 6$

b. $\frac{1}{7} \times 6$

c. $\frac{1}{8} \times 11$

d. $\frac{1}{3} \times 34$

3 from Unit 2, Lesson 8

Kiran runs $\frac{2}{5}$ the length of his road. The road is 9 miles long. How far does Kiran run? Explain or show your reasoning.

4 Exploration

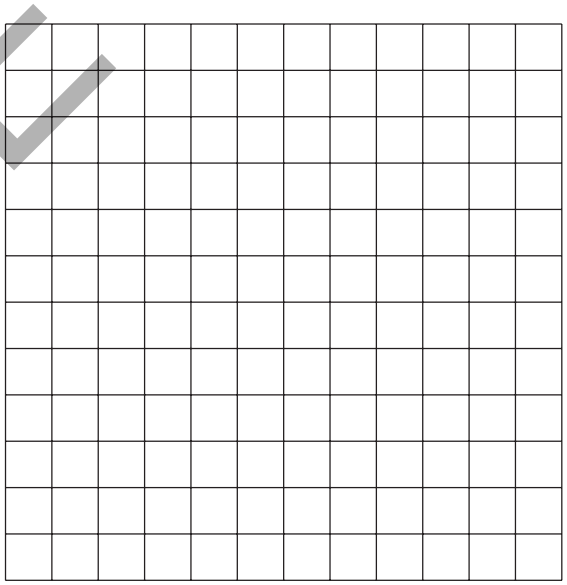
a. A map divides a state park into about 400 equal-size squares. Each square represents 2,178 square feet. Make an estimate for the size of the park in square feet. Explain or show your reasoning.

b. Each square on the map represents $\frac{1}{20}$ acre. How many square feet are in 1 acre? Explain or show your reasoning.

5 Exploration

A standard rectangular sheet of paper is $8\frac{1}{2}$ inches wide and 11 inches long. How many square inches does the sheet of paper cover?

If you get stuck, consider using the grid.



Unit 2, Lesson 9

Addressing CA CCSSM 5.NF.4b; building towards 5.NF.4b; practicing MP5 and MP7

Relate Area to Multiplication

Let's explore the area of rectangles with one side length that is a unit fraction.

Warm-up

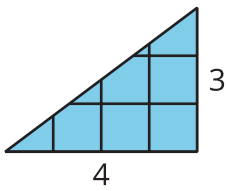
Which Three Go Together: Area

Which 3 go together?

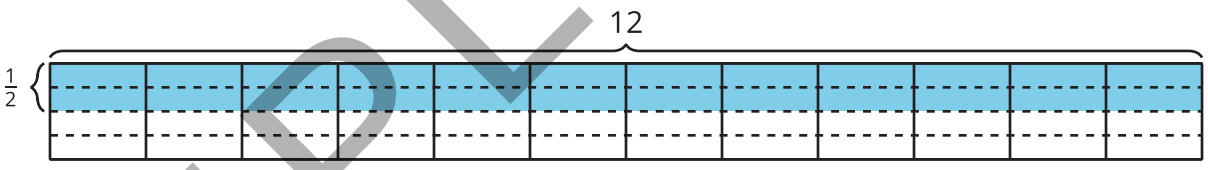
A

B

C



D

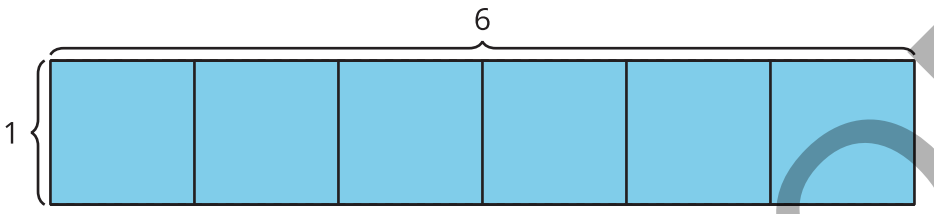


Activity 1

Find the Area

Find the area of the shaded region in square units. Explain or show your reasoning.

1.



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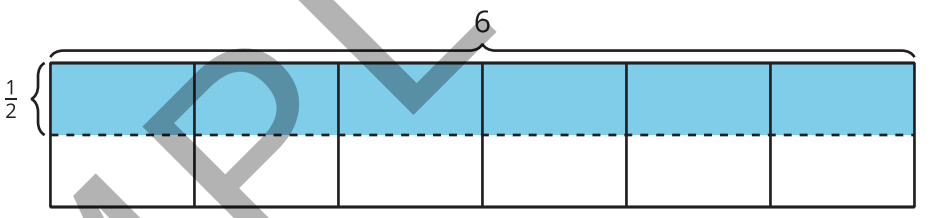
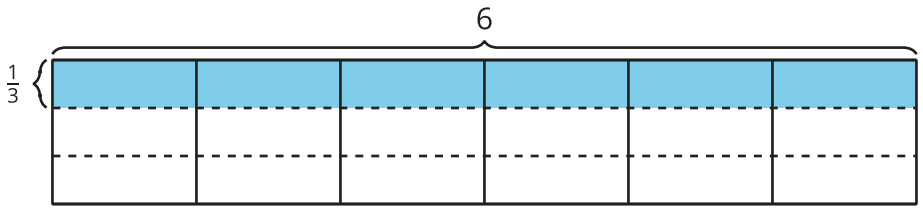


3.



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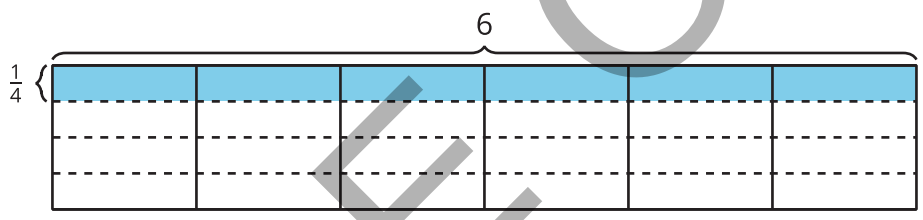


Activity 2

Draw Rectangles

1. Represent each rectangle on grid paper:
   - $\frac{1}{2}$ unit by 1 unit
   - $\frac{1}{2}$ unit by 2 units
   - $\frac{1}{2}$ unit by 3 units
   - $\frac{1}{2}$ unit by 4 units

2. Find the area of each rectangle that you drew.

3. What information do you need in order to find the area of the shaded region?

4. What might the area of the shaded region be? Explain or show your reasoning.

Addressing CA CCSSM 5.NF.4a, 5.NF.4b; building towards 5.NF.4; practicing MP6 and MP7

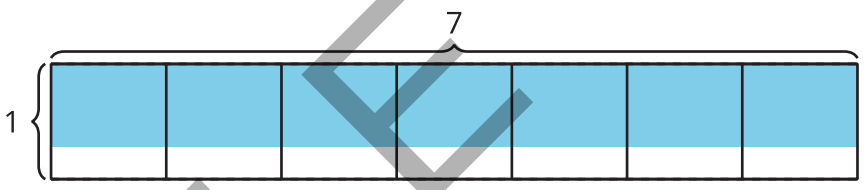
Fractional Side Lengths Less than 1

Let's find the area of rectangles with a fractional side length.

Warm-up

Estimation Exploration: What Is the Area?

What is the area of the shaded region?



Record an estimate that is:

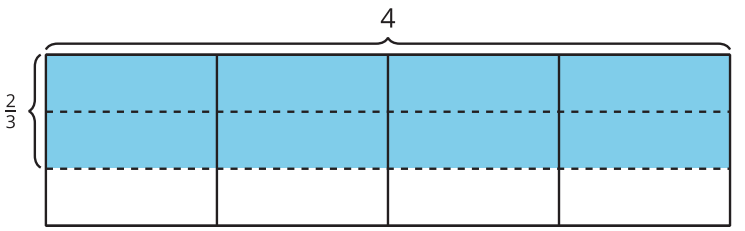
too low	about right	too high

Activity 1

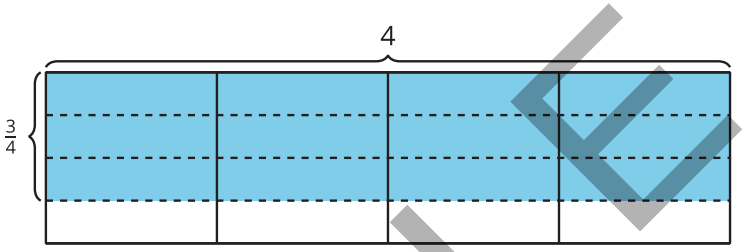
Rectangle with a Fractional Side Length

Write a multiplication expression to represent the area of each shaded region in square units. Then find the area.

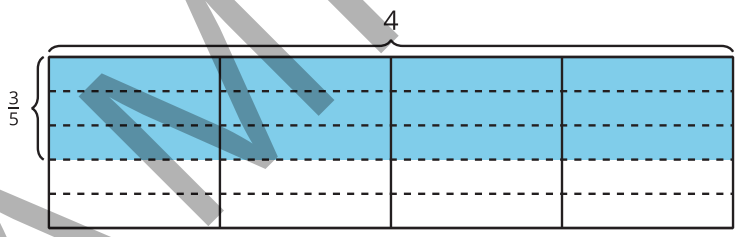
1.



2.



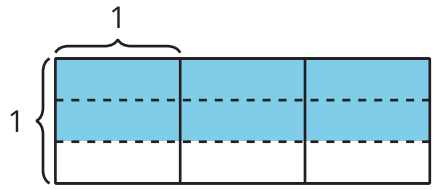
3.



Activity 2

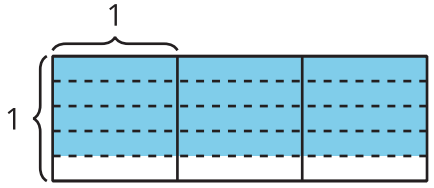
What Are the Side Lengths?

1. Write a multiplication expression to represent the area of the shaded region in square units. What is the area?

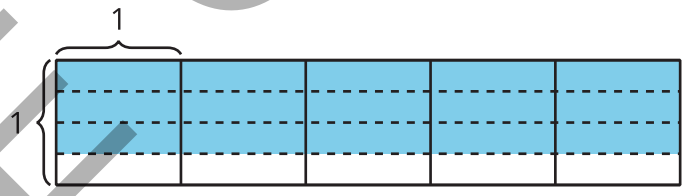


2. Here are two diagrams. Decide whether each expression represents the shaded region in one of the diagrams.

X



Y



a. $\frac{3}{4} \times 5$

b. $3 \times \frac{3}{5}$

c. $3 \times 4 \times \frac{1}{5}$

d. $4 \times \frac{3}{4}$

e. $3 \times 3 \times \frac{1}{4}$

3. For each diagram, what is the area of the shaded region?

Unit 2, Lesson 11

Addressing CA CCSSM 5.NF.3, 5.NF.4b; practicing MP2

Fractional Side Lengths Greater than 1

Let's find the area of more rectangles.

Warm-up

True or False: Thirds

Sec C

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $10 \div 3 = 10 \times \frac{1}{3}$

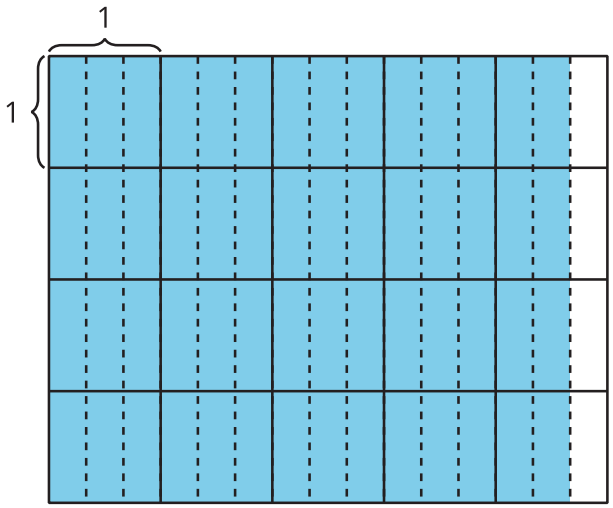
- $10 \div 3 = 10\frac{1}{3}$

- $\frac{10}{3} = 5 \times \frac{2}{3}$

Activity 1

Greater than 1

1. Find the area of the shaded region in square units. Explain or show your reasoning.



2. Select **all** the expressions that represent the area of the shaded region in square units. For each correct expression, explain your reasoning.

   A. $4\frac{2}{3} \times 4$

   B. $16 \times \frac{8}{3}$

   C. $\frac{14}{3} \times 4$

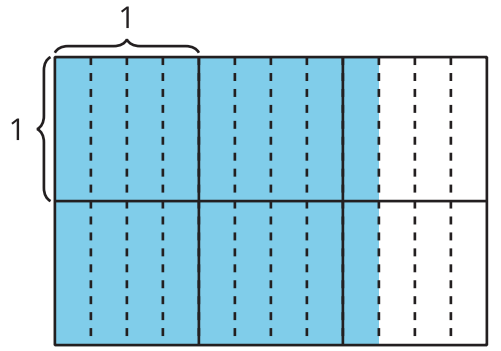
   D. $\frac{56}{3}$

   E. $4 \times \frac{5}{3}$

Activity 2

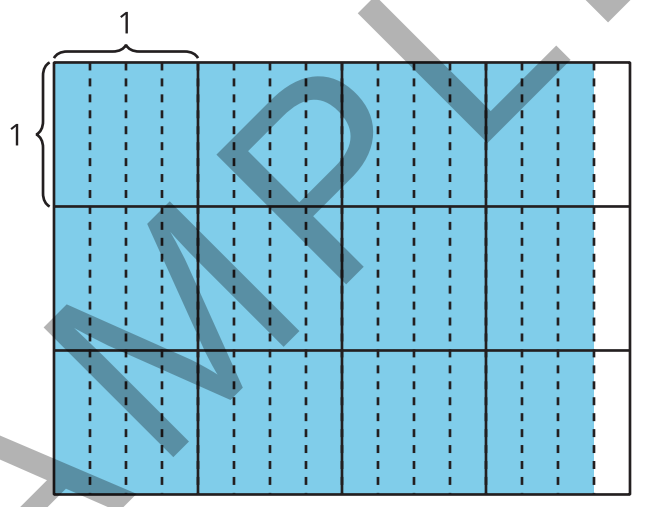
Diagrams and Expressions for Area

1. a. Write a multiplication expression to represent the area of the shaded region in square units.



b. What is the area of the shaded region?

2. a. Write a multiplication expression to represent the area of the shaded region in square units.



b. What is the area of the shaded region?

Unit 2, Lesson 12

Addressing CA CCSSM 5.NF.4, 5.NF.4b; practicing MP3

Decompose Area

Let's decompose rectangles to find their area.

Warm-up

Number Talk: Partial Products

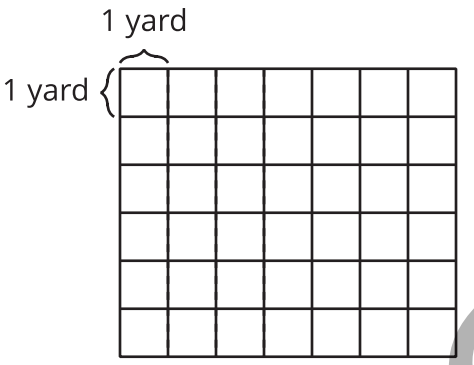
Find the value of each expression mentally.

- 3×20
- 3×24
- 5×2
- $5 \times 2\frac{1}{2}$

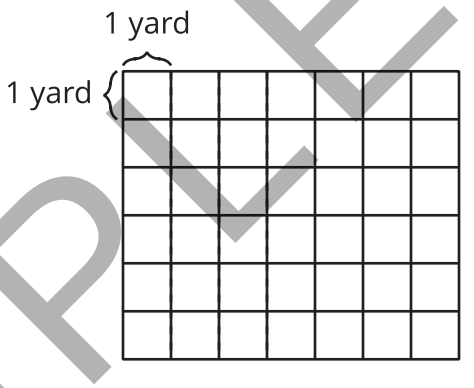
Activity 1

Which Garden Is Larger?

1. Noah's rectangular garden is 5 yards by $6\frac{1}{4}$ yards. Draw a diagram of Noah's garden on the grid.



2. Priya's rectangular garden is 6 yards by $5\frac{1}{4}$ yards. Draw a diagram of Priya's garden on the grid.

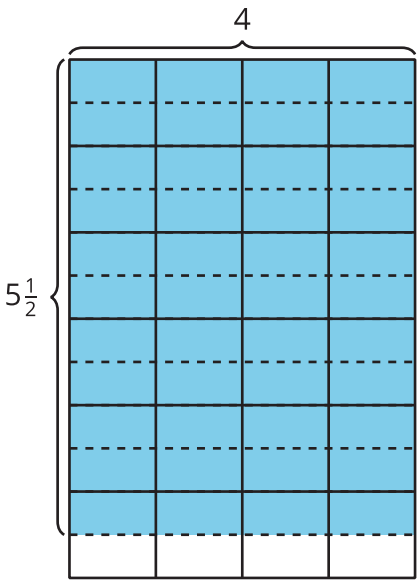


3. Whose garden covers a larger area? Explain your reasoning.

Activity 2

Different Ways to Find the Area

Partner A

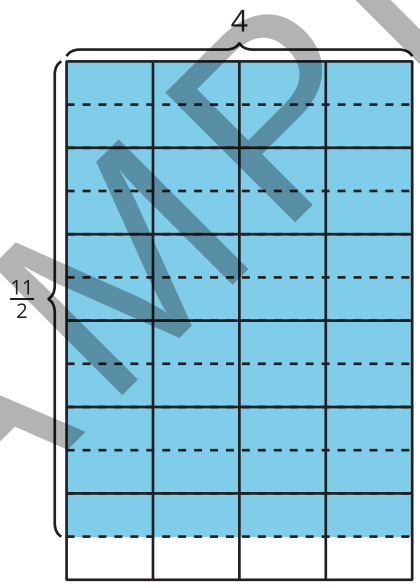


Jada: 4×5

Priya: $4 \times \frac{1}{2}$

Tyler: 6×4

Partner B



Clare: $\frac{10}{2} \times 4$

Diego: 4×6

Elena: 4×11

1. Each expression shows the first step a student uses to find the area of the shaded region. Explain how each student could finish their work to find the area. Show your thinking on the diagram.

2. Share your response with your partner. How are your responses alike? How are they different?

Unit 2, Lesson 13

Addressing CA CCSSM 5.NF.4, 5.NF.4b, 5.OA.1; practicing MP8

Area and Properties of Operations

Let's write expressions to represent the area of rectangles.

Warm-up

Number Talk: Parentheses

Find the value of each expression mentally.

- $5 \times (7 + 4)$

- $(5 \times 7) + (5 \times 4)$

- $(5 \times 7) + (5 \times \frac{1}{4})$

- $(5 \times 7) - (5 \times \frac{1}{4})$

Activity 1

Card Sort: Diagrams and Expressions

Your teacher will give you a set of cards.

1. Sort the cards into 2 categories of your choosing. Be ready to explain the meaning of your categories.

2. Match each diagram to an expression. Some diagrams match more than 1 expression.

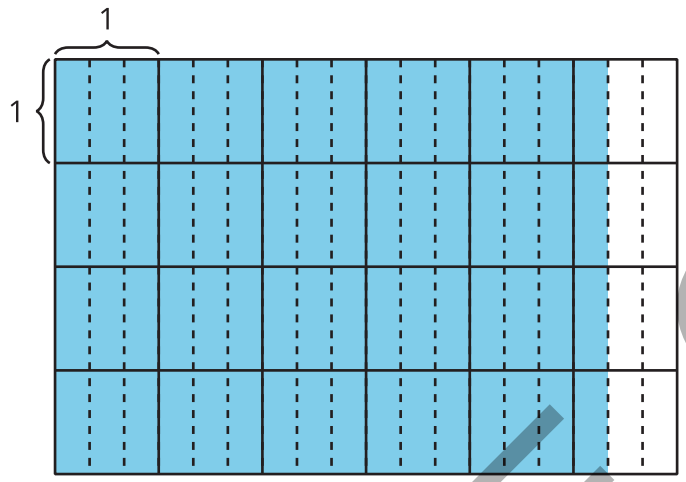
3. Work with your partner to find the area of each shaded region. Explain or show your reasoning.

Activity 2

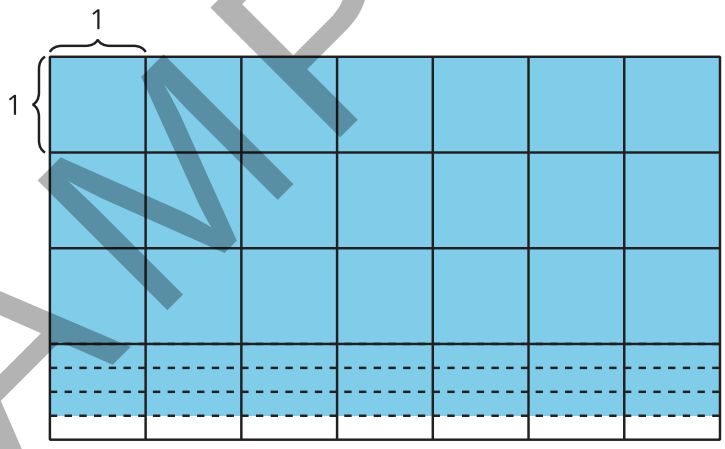
Write Expressions

For each diagram, write as many expressions as you can to represent the area of the shaded region in square units.

1.



2.



Unit 2, Lesson 14

Addressing CA CCSSM 5.NF.4; practicing MP1 and MP6

Area Situations

Let's apply what we've learned about fraction multiplication.

Warm-up

Number Talk: Multiply Fractions

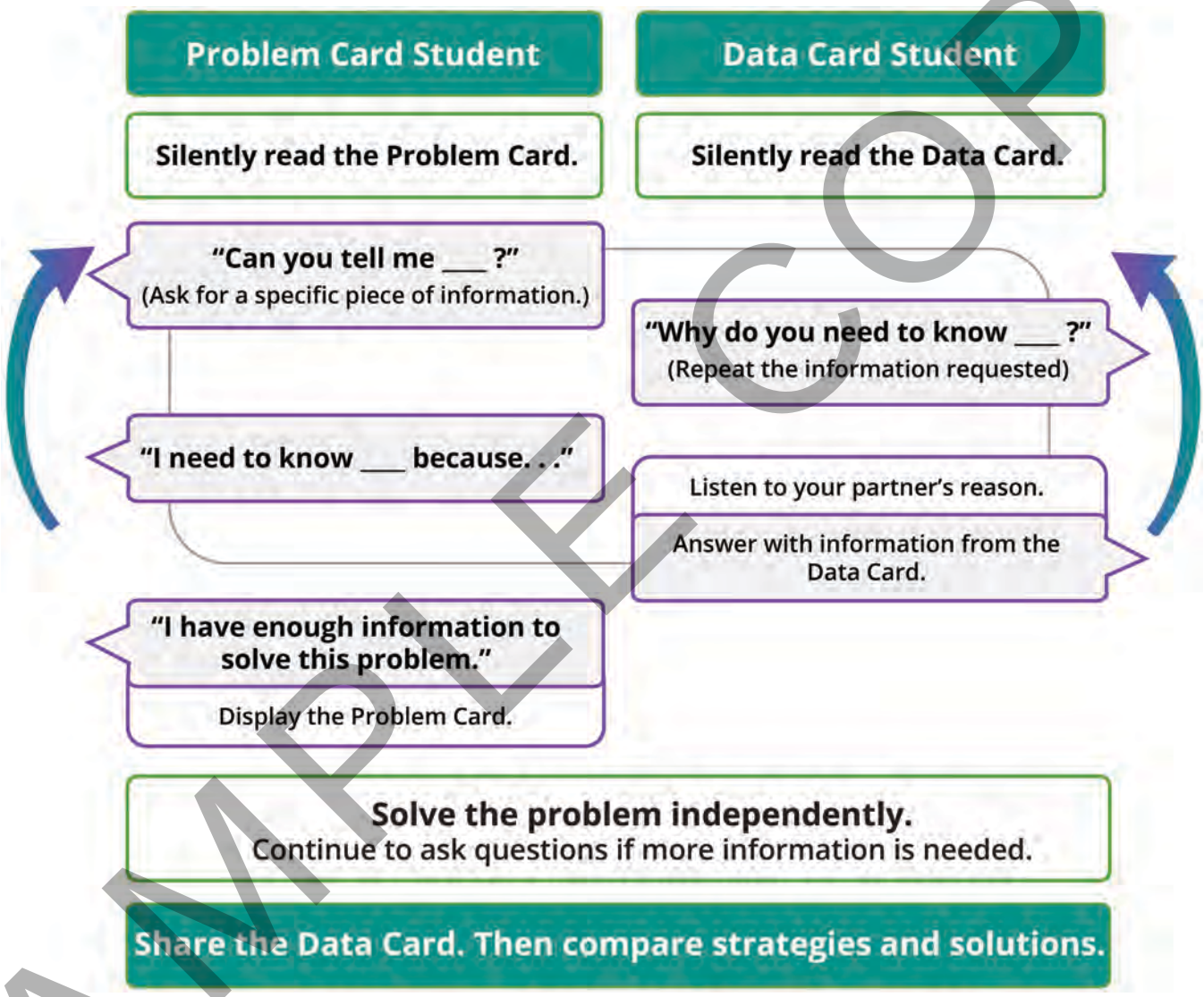
Find the value of each expression mentally.

- $3 \times (10 \div 2)$
- $\frac{3}{2} \times 10$
- $\left(\frac{14}{7}\right) \times 10$
- $14 \times \frac{10}{7}$

Activity 1

Info Gap: Area

Your teacher will give you either a Problem Card or a Data Card. Do not show or read your card to your partner.



Pause here so your teacher can review your work. Ask your teacher for a new set of cards. Repeat the activity, trading roles with your partner.

Activity 2

Fill in the Blank

Fill in the blanks to make each equation true.

1. $\frac{1}{3} \times 18 = \_\_\_\_$

2. $\frac{7}{9} \times \_\_\_\_ = \frac{21}{9}$

3. $\frac{1}{15} \times \_\_\_\_ = 2$

4. $9 \times 6\frac{2}{3} = \_\_\_\_$

5. $14\frac{99}{100} \times 10 = \_\_\_\_$

6. $7\frac{3}{5} \times 6 = \_\_\_\_$

7. $4 \times 6\frac{9}{10} = \_\_\_\_$

Unit 2, Lesson 15

Addressing CA CCSSM 5.NF.3, 5.NF.4, 5.NF.4a, 5.NF.4b; practicing MP1

Multiply More Fractions

Let's multiply mixed numbers.

Warm-up

Number Talk: Multiply Mixed Numbers

Find the value of each expression mentally.

- $6 \times \frac{3}{8}$
- $6 \times 2\frac{3}{8}$
- $7 \times \frac{9}{10}$
- $7 \times 3\frac{9}{10}$

Activity 1

Multiply Your Way

Write 1 number from the list in each blank so the situations make sense. Each number can be used only once.

4	5	$5\frac{1}{2}$	3	$5\frac{3}{4}$

1. The area of a rectangular rug is $16\frac{1}{2}$ square feet. The length of the rug is _____ feet.

   The width of the rug is _____ feet.

2. A rectangular puzzle is $2\frac{1}{2}$ feet wide. It is _____ feet long. It has an area of _____ square feet.

3. The area of a rectangular whiteboard is 23 square feet. The length of the whiteboard is

   _____ feet. The width of the whiteboard is _____ feet.

Share your work with your partner. Explain what choices you made and why.

Activity 2

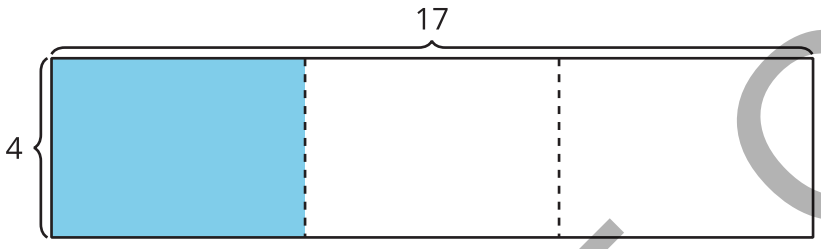
Equivalent Expressions

Each diagram represents a way to calculate $4 \times 5\frac{2}{3}$.
Each expression is equivalent to $4 \times 5\frac{2}{3}$.

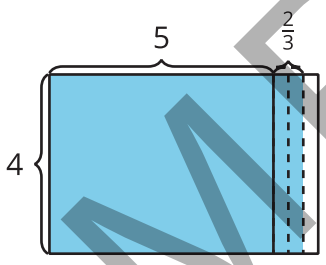
Match each diagram to an expression. Explain or show your reasoning.

1. $(4 \times 5) + \left(4 \times \frac{2}{3}\right)$
2. $(4 \times 6) - \left(4 \times \frac{1}{3}\right)$
3. $4 \times \frac{17}{3}$
4. $(4 \times 17) \div 3$

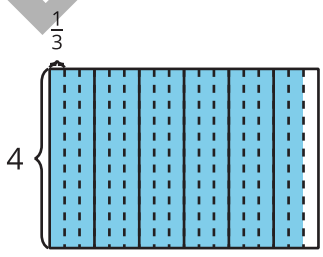
A



B



C



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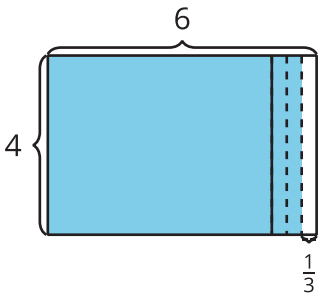
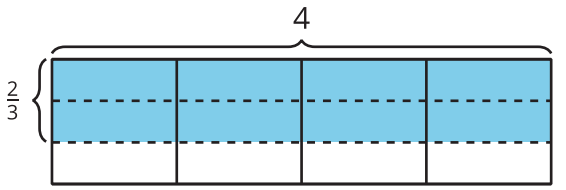


Choose your favorite diagram and expression to find the value of $4 \times 5\frac{2}{3}$. Be prepared to explain why it is your favorite.

Section C Summary

We learned how to find the area of a rectangle with a fractional side length.

Example: The shaded region has an area of $4 \times \frac{2}{3}$ because there are 4 groups of $\frac{2}{3}$ of a square unit shaded. The area is $\frac{8}{3}$ or $2\frac{2}{3}$ because there are 8 shaded parts and each one is $\frac{1}{3}$ of a square unit.

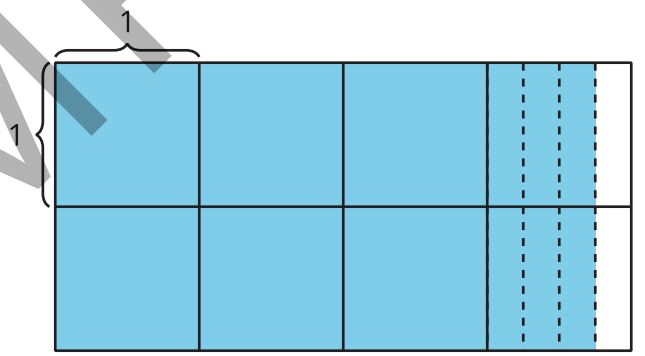


We also learned to multiply a mixed number by a whole number. We used diagrams and expressions to see why our strategies work.

Example: To multiply $3\frac{3}{4} \times 2$, we can use the expression $(3 \times 2) + (\frac{3}{4} \times 2)$. We can see both expressions represented by the shaded region in the diagram.

- The 2 rows of 3 and $\frac{3}{4}$ squares shaded show $3\frac{3}{4} \times 2$.
- The 2 rows of 3 squares shaded show $3 \times 2 = 6$.
- The 2 rows of $\frac{3}{4}$ of a square shaded show $\frac{3}{4} \times 2 = \frac{6}{4}$.
- The shaded region in the diagram represents the area of a rectangle with the dimensions of $3\frac{3}{4}$ units by 2 units.

So, the area of the shaded region is $6 + \frac{6}{4}$ or $7\frac{2}{4}$ square units.



Addressing CA CCSSM 5.NF.4, 5.NF.4b; building towards 5.NF.4; practicing MP1

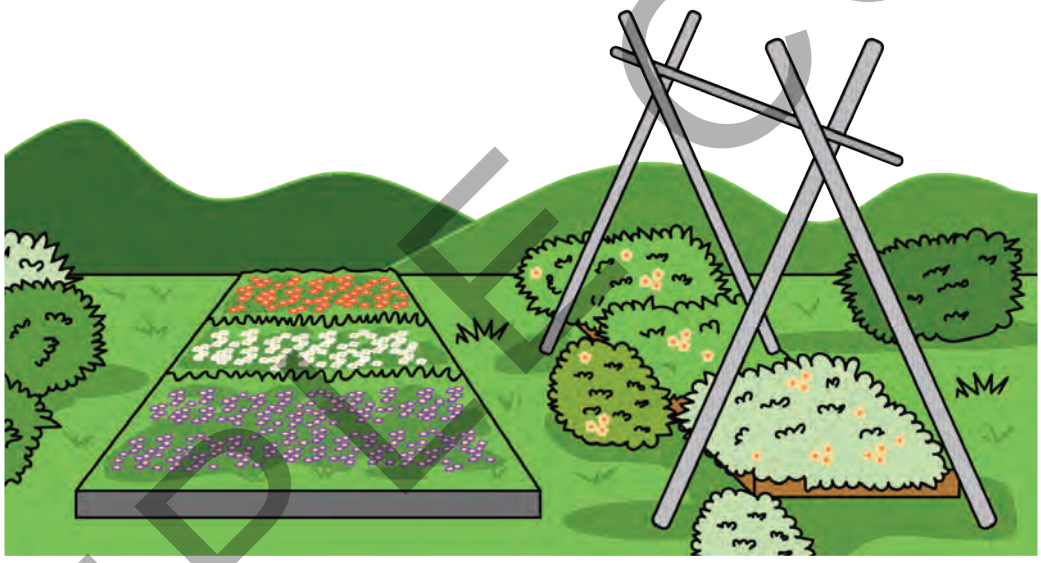
Estimate Products

Let's estimate products of a whole number and a fraction.

Warm-up

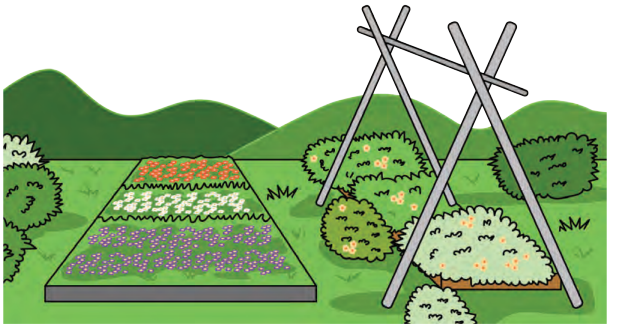
Notice and Wonder: Garden Size

What do you notice? What do you wonder?



Activity 1

Priya's Garden



Priya has enough materials to build a rectangular garden that covers 36 square feet.

Choose **all** the side lengths that are reasonable for her garden. Explain your reasoning.

1. 9 feet by $4\frac{2}{3}$ feet

2. 9 feet by $3\frac{8}{9}$ feet

3. 12 feet by $2\frac{11}{12}$ feet

4. 9 feet by $2\frac{2}{3}$ feet

Activity 2

Too High, Too Low, Just about Right

1. Write a multiplication expression with whole numbers whose value is slightly less than, slightly greater than, or about equal to the value of $7 \times 12\frac{8}{9}$.

   a. slightly less:

   b. slightly greater:

   c. about equal to:

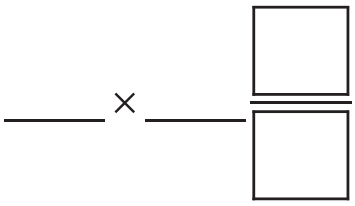
2. Write a multiplication expression with whole numbers whose value is slightly less than, slightly greater than, or about equal to the value of $9 \times 4\frac{2}{29}$.

   a. slightly less:

   b. slightly greater:

   c. about equal to:

3. Without calculating, use the numbers 2, 3, 5, 6, and 7, to complete the expression so it has a value close to 20.



4. Explain how you know your expression represents a value close to 20.

Unit 2, Lesson 17

Addressing CA CCSSM 5.NF.4; building towards 5.NF.4; practicing MP4

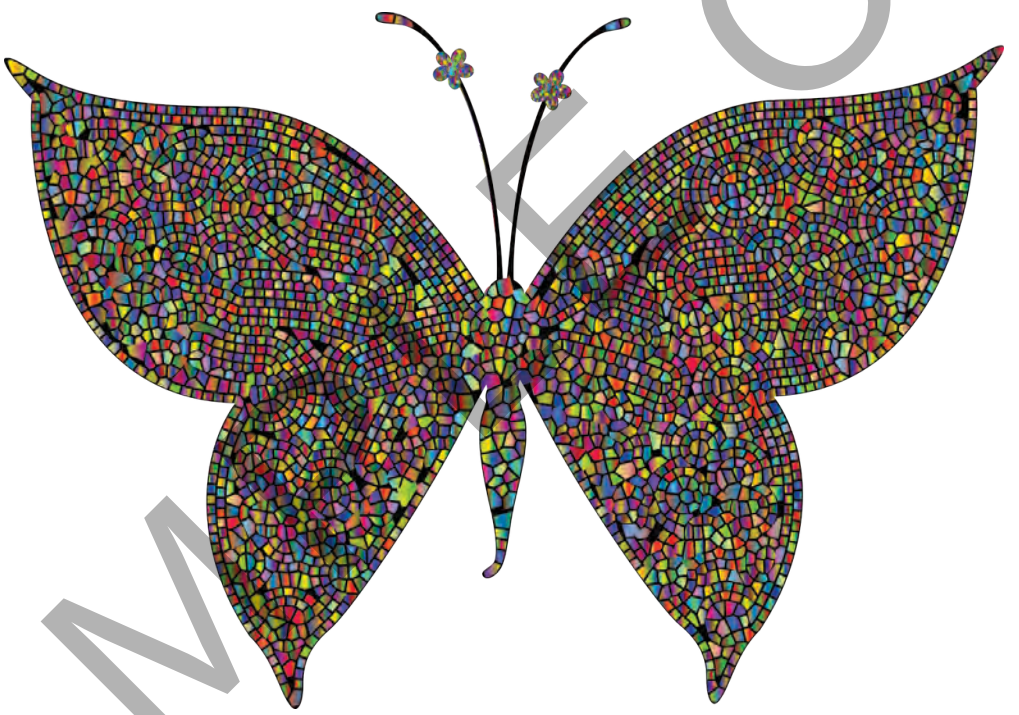
Mosaic Pictures

Let's make a design for a mosaic.

Warm-up

Notice and Wonder: Mosaic

What do you notice? What do you wonder?



Activity 1

Create a Mosaic Design

1. Use colored paper and scissors to cut at least 2 identical rectangles. Make sure the measurement of one side of the rectangle is a whole number of inches and the other is a fraction greater than 1 inch.

2. What is the area of one of your rectangles? Explain or show your reasoning.

3. Arrange some or all of the rectangles from your group on a piece of chart paper to show a design for a mosaic.

Activity 2

Cost of Mosaic

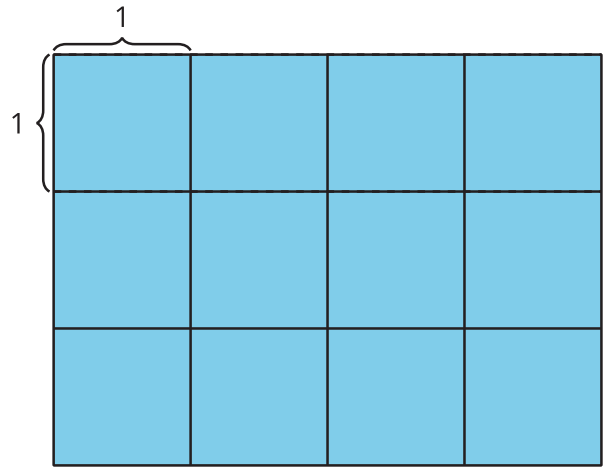
Choose one material in the table. About how much would it cost to create your mosaic with that material? Explain or show your reasoning.

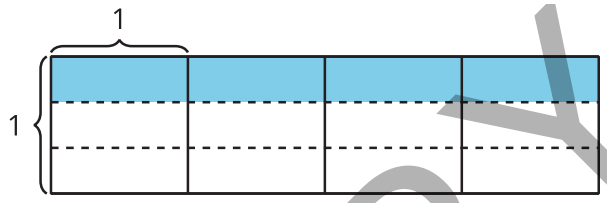
material	cost per square inch
Stone	$5
Tile	$3
Glass	$2

Practice Problems

11 Problems

1 from Unit 2, Lesson 9



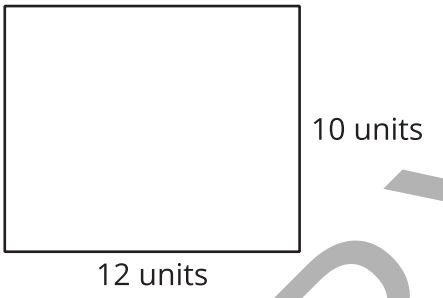


a. How are the diagrams alike? How are they different?

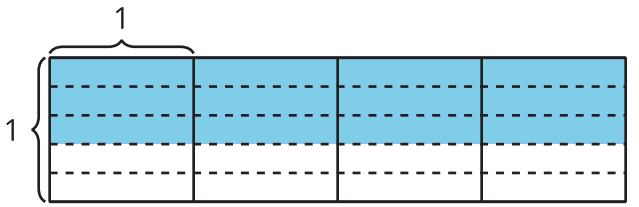
b. How is finding the area of the shaded region in each diagram alike? How is it different?

2 from Unit 2, Lesson 10

a. What is the area of this rectangle? Explain or show your reasoning.



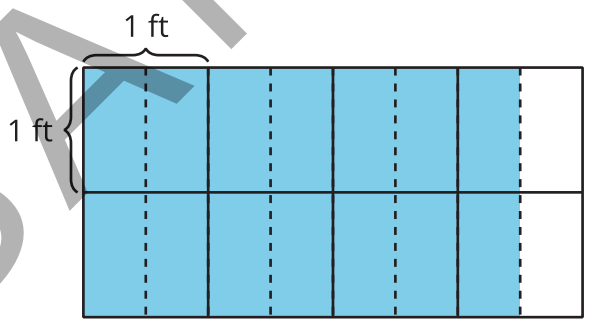
b. What is the area of the shaded region? Explain or show your reasoning.



c. How are these two area calculations alike? How are they different?

3 from Unit 2, Lesson 11

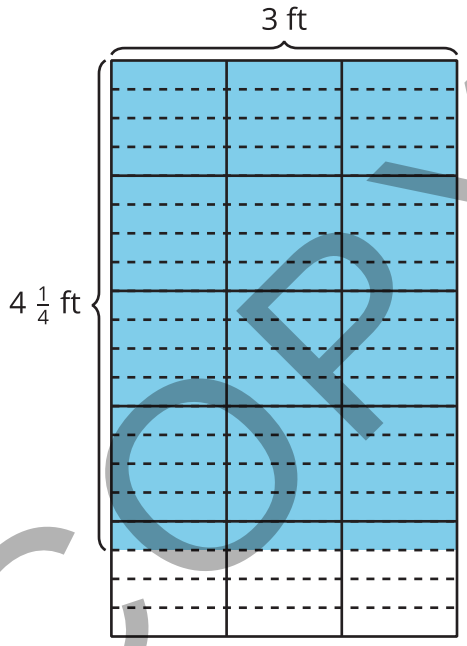
The shaded part of this diagram represents the top of a stove. What is the area of the stovetop? Explain or show your reasoning.



4

from Unit 2, Lesson 12

Find the area of the shaded region. Explain or show your reasoning.



5

from Unit 2, Lesson 13

Select **all** of the expressions that represent the area of the shaded region in square feet.

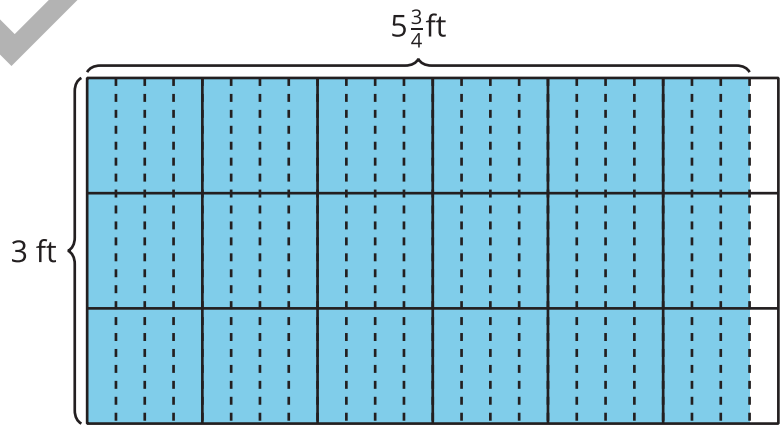
A. $3 + 5\frac{3}{4}$

B. $3 \times 5\frac{3}{4}$

C. $3 \times \left(5 + \frac{3}{4}\right)$

D. $(3 \times 5) + \frac{3}{4}$

E. $3 \times 6 - \left(3 \times \frac{1}{4}\right)$



Write one more expression that represents the area of the shaded region in square feet.

6 from Unit 2, Lesson 16

Tyler says that $9\frac{11}{12} \times 5$ is a little less than 50.

a. Do you agree with Tyler? Explain or show your reasoning.

b. What is the value of $9\frac{11}{12} \times 5$?

7 from Unit 2, Lesson 14

A banner at a sporting event is 8 feet long and $2\frac{1}{3}$ feet wide.

a. Sketch and label a diagram of the banner.

b. Find the area of the banner.

8 from Unit 2, Lesson 15

Find the value of each expression. Explain or show your reasoning.

a. $3\frac{2}{5} \times 10$

b. $8 \times \frac{14}{3}$

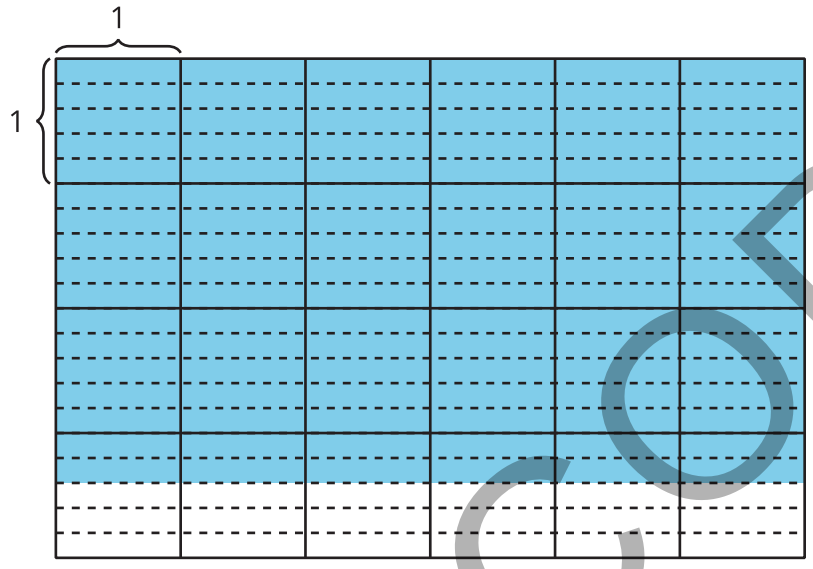
c. $3\frac{41}{100} \times 5$

a. A standard rectangular sheet of paper is $8\frac{1}{2}$ inches wide and 11 inches long. How many times would you need to fold the sheet of paper in half before the area is less than 1 square inch? Explain or show your reasoning.

b. A rectangular piece of chart paper is 23 inches wide by 33 inches long. How many times would you need to fold it in half before its area is less than 1 square inch?

10 Exploration

Part of the rectangle is shaded.



a. Write a multiplication expression that represents the shaded area.

b. Write a division expression that represents the shaded area.

c. Write some other expressions that represent the shaded area.

Here is an image of the Empire State Building in New York City.

The base of the Empire State Building is shaped like a rectangle. What do you think the area of the rectangle is in square meters?

(Hint: A typical bathtub covers about 1 square meter. A typical car parking space is about 10 square meters.)

a. Make an estimate that is too low.

b. Make an estimate that is too high.

c. The length of the rectangle is about $129\frac{1}{5}$ meters. The width is about 57 meters. What is the area of the base of the Empire State Building?

Glossary

- area
  The number of square units that cover a flat figure without gaps or overlaps.

- cubic unit
  The volume of a cube whose sides are each 1 unit long.

  Example: A cube whose sides are each 1 inch long has a volume of 1 cubic inch.

- rectangular prism
  A solid figure with 6 faces that are all rectangles.

- unit cube
  A cube whose sides are each 1 unit long. Unit cubes are used to measure volume.

- volume
  The number of unit cubes that fill a solid figure without gaps or overlap.

Attributions

Image Attributions

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Empire State Building (aerial view), by Sam Valadi. CC BY 2.0. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Empire_State_Building_(aerial_view).jpg.

Citations

Unit 1: Finding Volume

Lesson Grade5.1.C12

Smith-Schoenwalder, C. (2020, May 28). Malaysia to Ship Plastic Trash Back to the U.S., Other Origin Countries. *U.S. News and World Report*. https://www.usnews.com/news/world-report/articles/2019-05-28/malaysia-to-ship-plastic-trash-back-to-the-us-other-origin-countries

Unit 2: Fractions as Quotients and Fraction Multiplication

Lesson Grade5.2.C17

Mosaic. *Wikipedia, The Free Encyclopedia*. Retrieved from https://en.wikipedia.org/wiki/Mosaic

Practice Problem Grade5.2.C.11

Empire State Building. *Wikipedia, The Free Encyclopedia*. Retrieved from https://en.wikipedia.org/wiki/Empire_State_Building

California Common Core State Standards for Mathematics (CA CCSSM) References

5.G: Grade 5 – Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1
Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2
Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4
Classify two-dimensional figures in a hierarchy based on properties.

5.MD: Grade 5 – Measurement and Data

Convert like measurement units within a given measurement system.

5.MD.1
Convert among different-sized standard measurements units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

Represent and interpret data.

5.MD.2
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3
Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.3a
A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

5.MD.3b
A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4
Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5
Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

5.MD.5a
Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

5.MD.5b
Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

5.MD.5c
Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

5.NBT: Grade 5 – Number and Operations in Base Ten

Understand the place value system.

5.NBT.1
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.3
Read, write, and compare decimals to thousandths.

5.NBT.3a
Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

5.NBT.3b
Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4
Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5
Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.6
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF: Grade 5 – Number and Operations—Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

5.NF.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3
Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.4a
Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

5.NF.4b
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5
Interpret multiplication as scaling (resizing), by:

5.NF.5a
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

5.NF.5b
Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

5.NF.6
Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

5.NF.7a
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.

5.NF.7b
Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.

5.NF.7c
Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

5.OA: Grade 5 – Operations and Algebraic Thinking

Write and interpret numerical expressions.

5.OA.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

5.OA.2.1
Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$.

Analyze patterns and relationships.

5.OA.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.