IMKH California

GRADE 5

Student Edition

UNITS 5-6

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 3

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GRADE 5

UNIT

5

Place Value Patterns and Decimal Operations

Content Connections

In this unit you will use place value knowledge to round, compare, order, add, subtract, multiply, and divide decimals. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while solving problems involving decimals with the use of rounding and estimation to check accuracy and justify results.
- **Exploring Changing Quantities** while multiplying, dividing, adding and subtracting decimals using place value and visual models.
- **Discovering Shape and Space** while using hundredths grids and number lines to represent and compare decimals to the tenths and hundredths.
- **Reasoning with Data** while using number lines and line plots to compare, analyze patterns, and solve problems involving decimals.

Addressing the Standards

As you work your way through **Unit 5 Place Value Patterns and Decimal Operations,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 9 and 24
MP2 Reason abstractly and quantitatively.	Lessons 3, 4, 8, 10, and 18
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 2, 8, 12, 13, 15, and 22
MP4 Model with mathematics.	Lesson 26
MP5 Use appropriate tools strategically.	Lessons 5 and 14
MP6 Attend to precision.	Lessons 2, 3, 6, 7, 9, 13, 16, and 25
MP7 Look for and make use of structure.	Lessons 1, 4, 6, 11, 17, 18, 19, and 20
MP8 Look for and express regularity in repeated reasoning.	Lessons 14, 17, 21, and 23

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Powers and Place Value	**5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Lessons 1, 2, 3, 4, and 21
• Modeling • Fraction Connections • Powers and Place Value	**5.NBT.3** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Lessons 1, 2, 4, 5, 6, 7, 9, and 10

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections • Powers and Place Value	**5.NBT.3a** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).	Lessons 3 and 4
• Modeling • Fraction Connections • Powers and Place Value	**5.NBT.3b** Read, write, and compare decimals to thousandths. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Lessons 5, 6, 8, and 9
• Seeing Division	**5.NBT.4** Use place value understanding to round decimals to any place.	Lessons 7, 8, and 10
• Modeling • Seeing Division	**5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Lessons 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, and 26
• Factors and Groups • Powers and Place Value • Layers of Cubes	**5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Lessons 3, 18, and 19
• Factors and Groups • Powers and Place Value	**5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Lessons 17, 18, 19, and 23

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Lesson 20
• Plotting Patterns • Modeling • Fraction Connections	**5.NF.7b** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*	Lesson 22

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 5, Lesson 1

Addressing CA CCSSM 5.NBT.1, 5.NBT.3; practicing MP7

What Is a Thousandth?

Let's make sense of thousandths.

Warm-up

Estimation Exploration: 1 Tiny Piece

What fraction of the whole image is 1 single square tile?

Record an estimate that is:

too low	about right	too high

Activity 1

What Do You Know about Thousandths?

1. What do you know about 1 tenth?

2. What do you know about 1 hundredth?

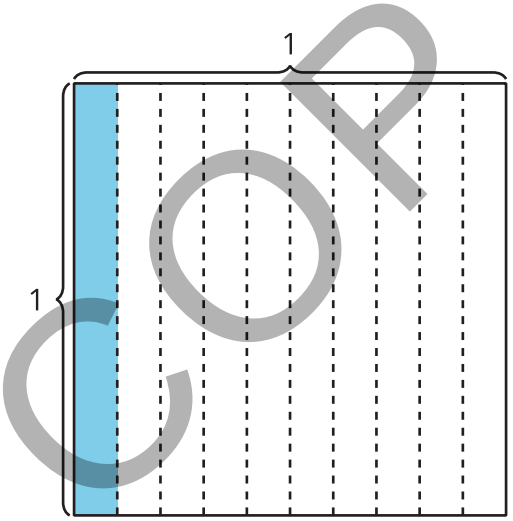
3. What do you know about 1 thousandth?

Activity 2

Represent Numbers on a Hundredths Diagram

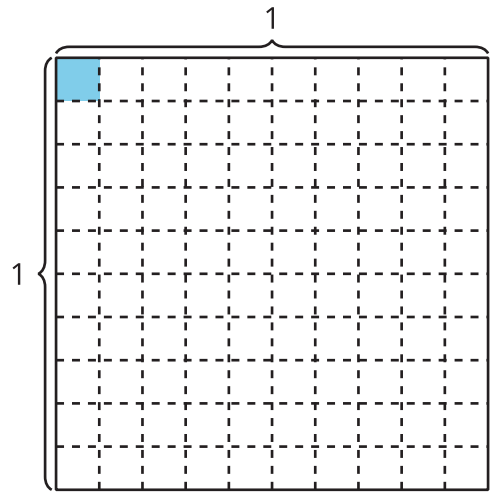
1. The large square represents 1. It is divided into 10 equal-size rectangles. What number does the shaded rectangle represent?

   Explain or show your reasoning.



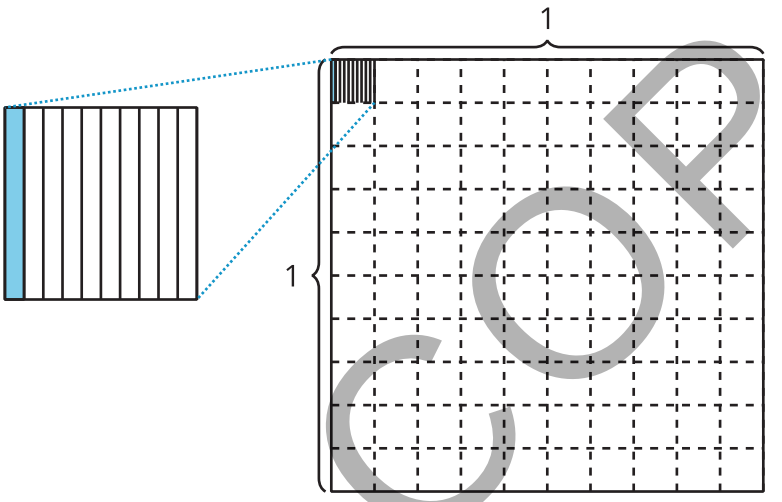
2. Each rectangle part from the previous diagram is now divided into 10 equal-size small squares. What number does the shaded small square represent?

   Explain or show your reasoning.



3. The shaded small square from the previous diagram is now divided into 10 equal-size small rectangles. One of those small rectangles is shaded. What number does the shaded small rectangle represent?

   Explain or show your reasoning.



4. How do you think $\frac{1}{1,000}$ is written as a decimal? Explain or show your reasoning.

fraction	decimal
$\frac{1}{10}$	0.1
$\frac{1}{100}$	0.01
$\frac{1}{1,000}$	?

Unit 5, Lesson 2

Addressing CA CCSSM 5.NBT.1, 5.NBT.3; practicing MP3 and MP6

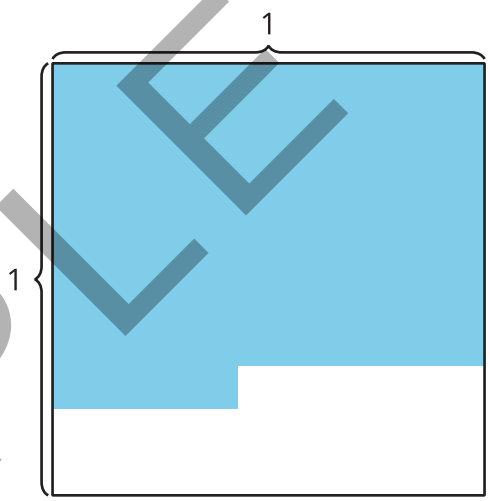
Thousandths on Diagrams and in Words

Let's represent numbers on diagrams as decimals, fractions, and words.

Warm-up

Estimation Exploration: What Part of the Square Is Shaded?

How much of the square is shaded?



Record an estimate that is:

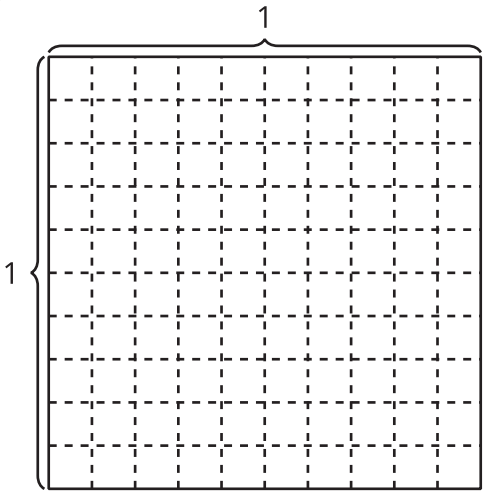
too low	about right	too high

Activity 1

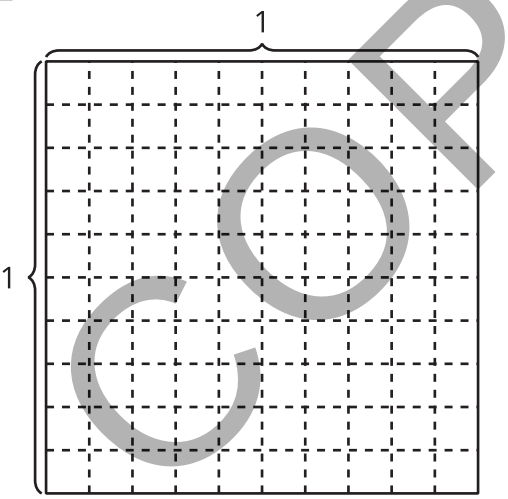
Represent Thousandths on a Diagram

1. Shade each diagram to represent the given number.

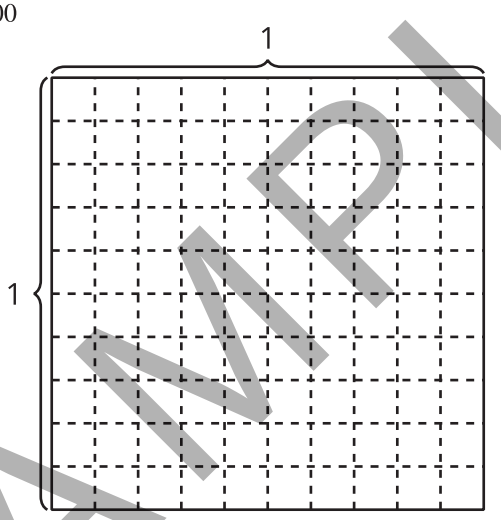
a. $\frac{2}{10}$



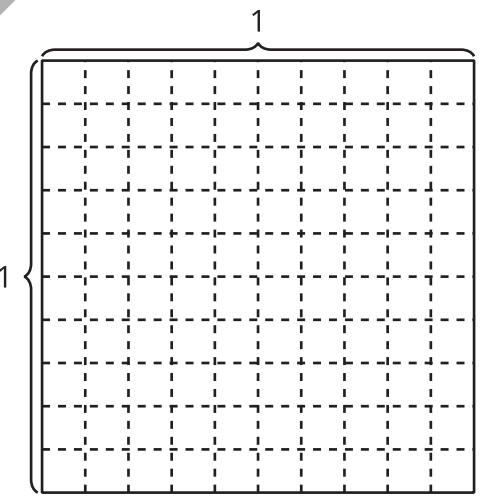
b. 0.2



c. $\frac{15}{100}$

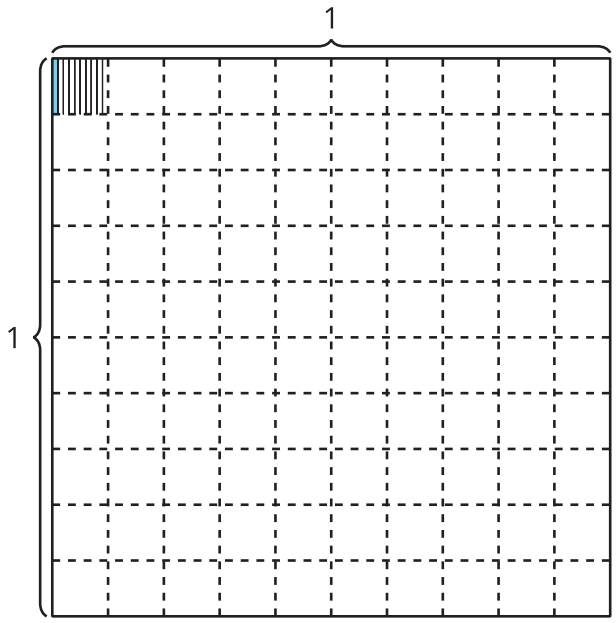


d. 0.34

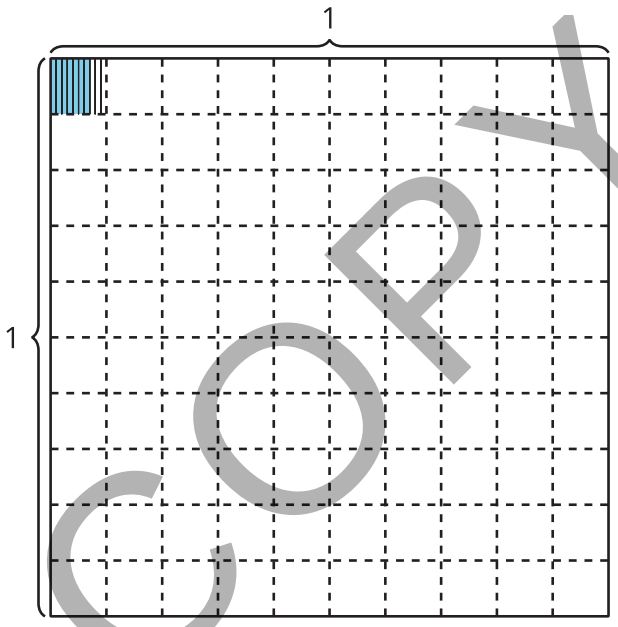


2. Write a decimal number to represent the shaded part of each diagram. Explain or show your reasoning.

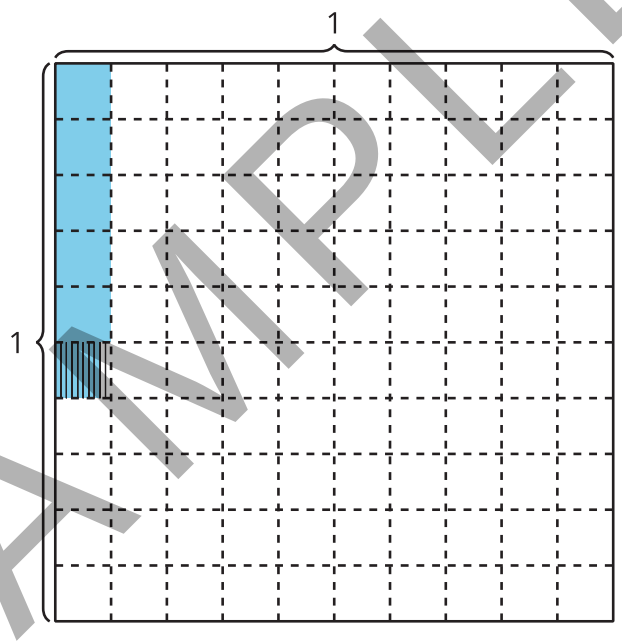
a.



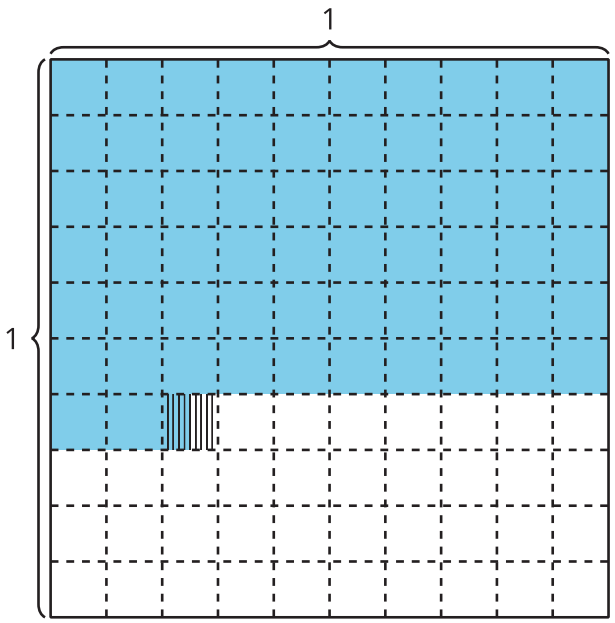
b.



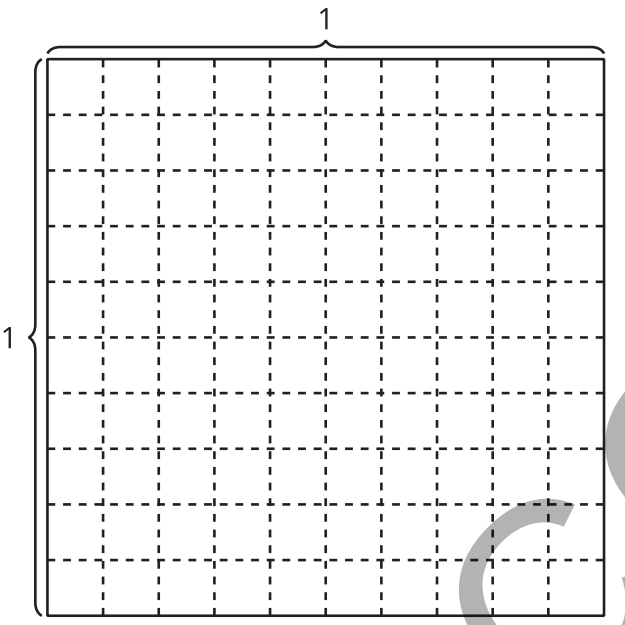
c.



d.



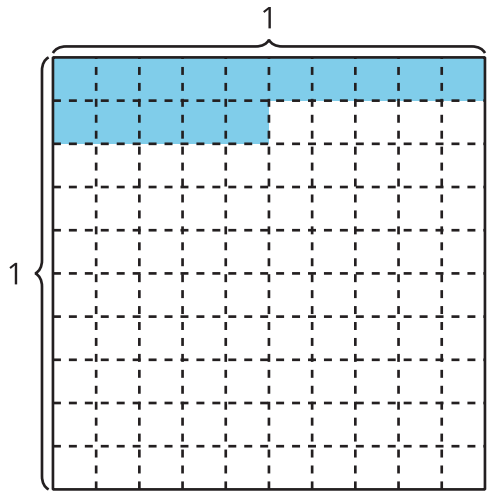
3. Shade the diagram to represent 0.328. Explain or show your reasoning.



Activity 2

Say What?

Several students look at the diagram. They describe the shaded region in different ways. Who do you agree with? Explain or show your reasoning.



A. Jada says, "15 hundredths."
B. Priya says, "150 thousandths."
C. Tyler says, "15 thousandths."
D. Diego says, "1 tenth and 5 hundredths."
E. Mai says, "1 tenth and half of a tenth."

Addressing CA CCSSM 5.NBT.1, 5.NBT.3a, 5.OA.1; building towards 5.NBT.3a; practicing MP2 and MP6

Thousandths in Expanded Form

Let's represent thousandths.

Warm-up

Which Three Go Together: Different Ways to Express a Decimal Number

Which 3 go together?

A

$26 \div 100$

B

0.26

C

26×0.001

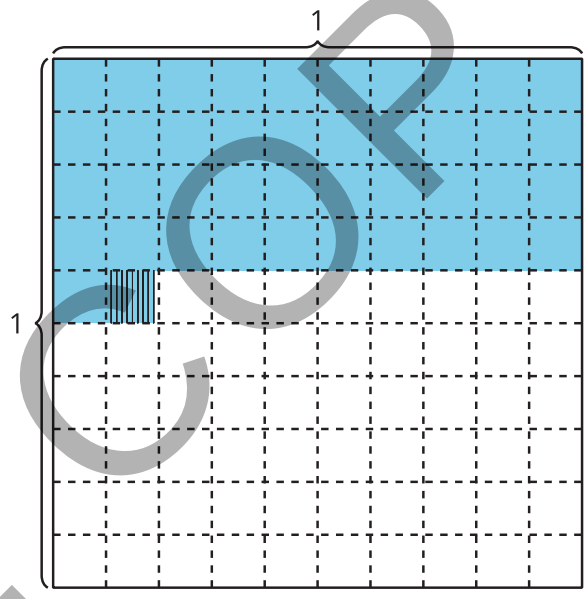
D

$(2 \times 0.1) + (6 \times 0.01)$

Activity 1

Expanded Form

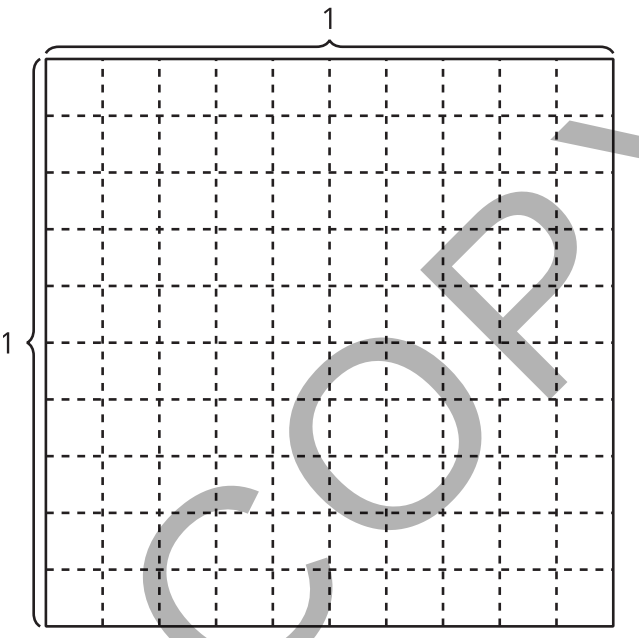
1. a. Explain or show why the shaded region represents $(4 \times 0.1) + (1 \times 0.01) + (9 \times 0.001)$.



b. What decimal number represents the shaded region?

2. a. Shade the diagram to represent $(8 \times 0.1) + (3 \times 0.01) + (5 \times 0.001)$.

   b. Write the number $(8 \times 0.1) + (3 \times 0.01) + (5 \times 0.001)$ in decimal notation.



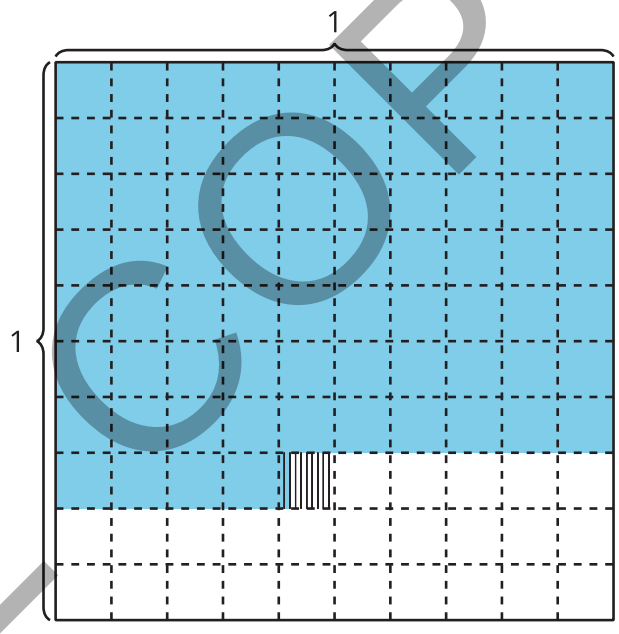
3. Mai says the decimal 0.105 represents $(1 \times 0.1) + (5 \times 0.01)$. Do you agree?

Activity 2

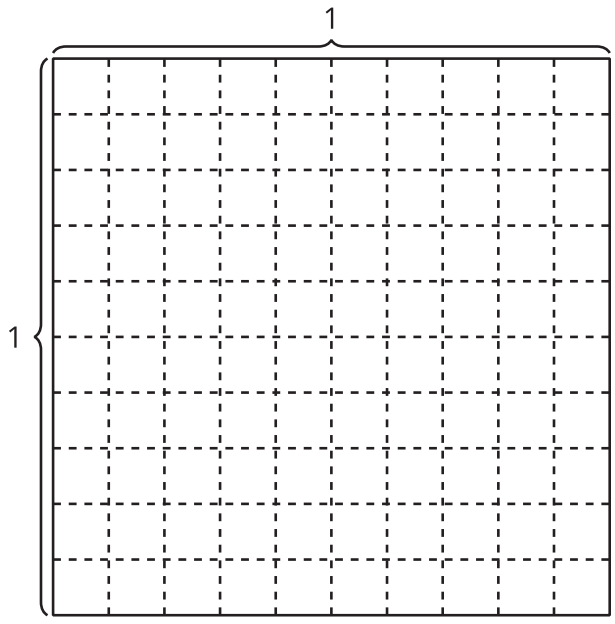
Decimal Numbers in Numerous Ways

Represent each number in as many ways as you can.

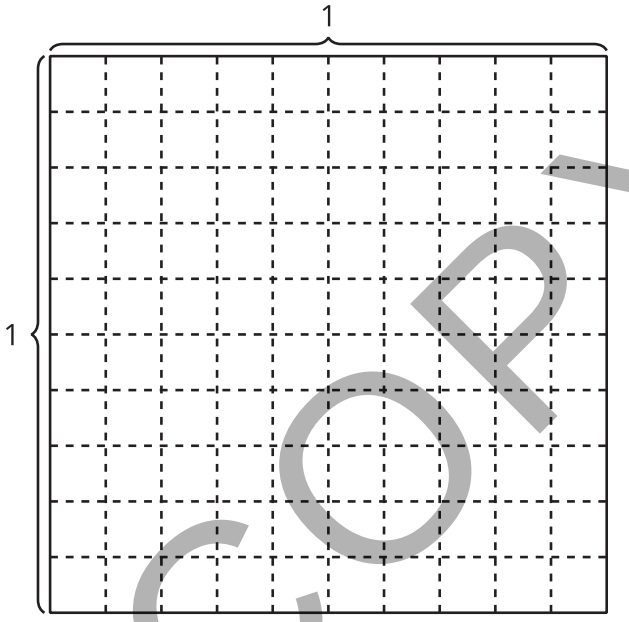
1.



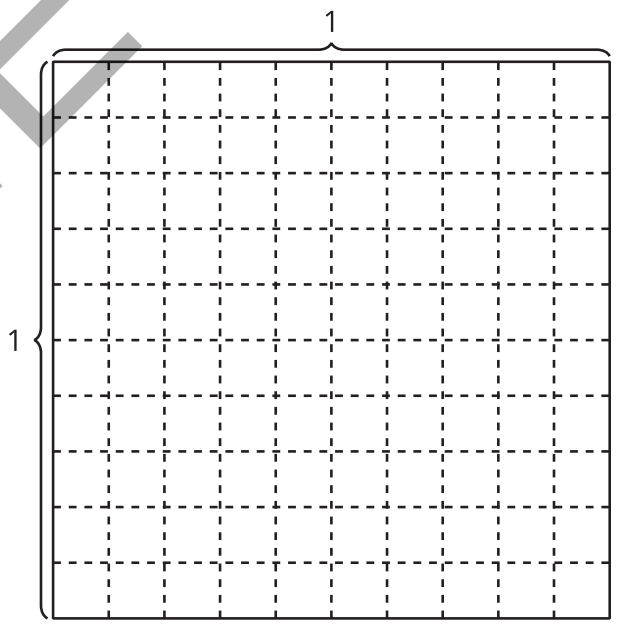
2. $\frac{477}{1{,}000}$



3. one hundred thirty-six thousandths



4. $(3 \times 0.1) + (6 \times 0.01) + (8 \times 0.001)$



Unit 5, Lesson 4

Addressing CA CCSSM 5.NBT.1, 5.NBT.3, 5.NBT.3a; practicing MP2 and MP7

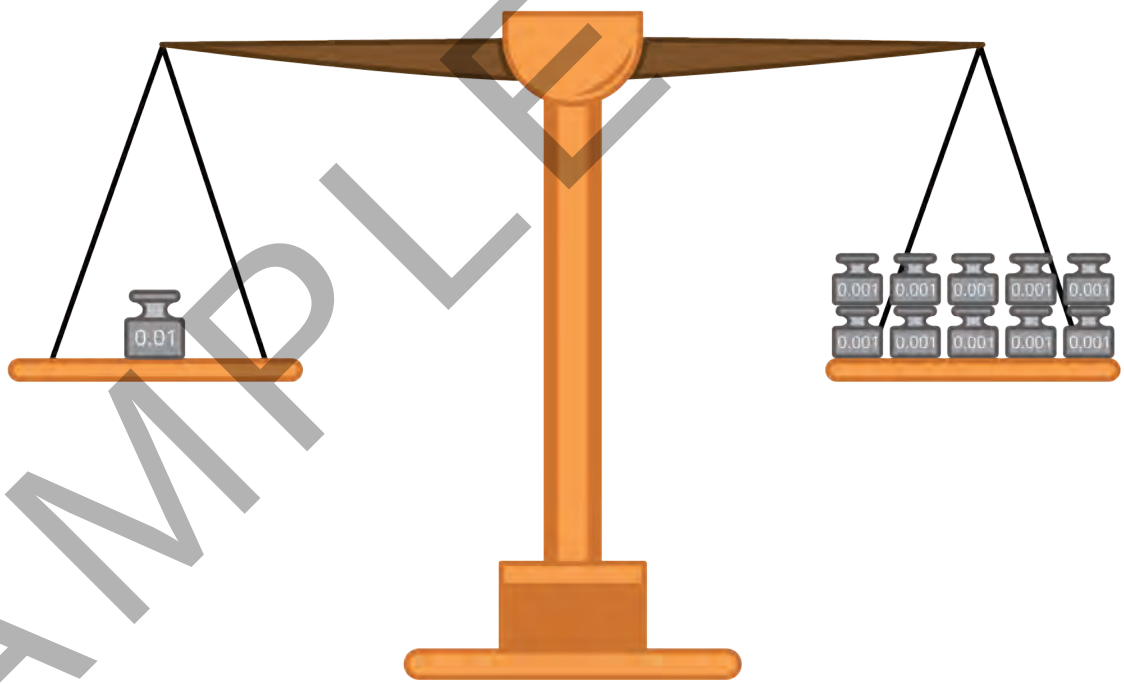
Explore Place Value Relationships

Let's explore place value relationships.

Warm-up

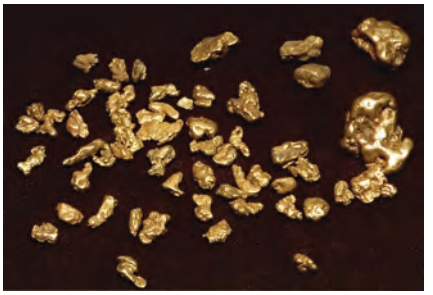
Notice and Wonder: Maintain Your Balance

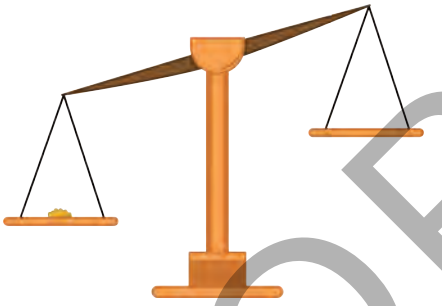
What do you notice? What do you wonder?



Activity 1

Balance the Weight





You have a balance and 0.1-ounce, 0.01-ounce, and 0.001-ounce weights.

1. A gold nugget weighs 0.2 ounce.
   a. What is one set of weights you can use to balance the nugget? Explain or show your reasoning.

   b. What is another set of weights you can use to balance the nugget? Explain or show your reasoning.

   c. How many 0.01-ounce weights do you need to balance the nugget? How many 0.001-ounce weights?

2. Another nugget weighs 0.385 ounce.

   a. What is one set of weights you can use to balance the nugget? Explain or show your reasoning.

   b. What is the fewest number of weights you can use to balance the nugget? Explain or show your reasoning.

   c. What is the greatest number of weights you can use to balance the nugget? Explain or show your reasoning.

3. Write a decimal number for the weight of each gold nugget that is balanced with:

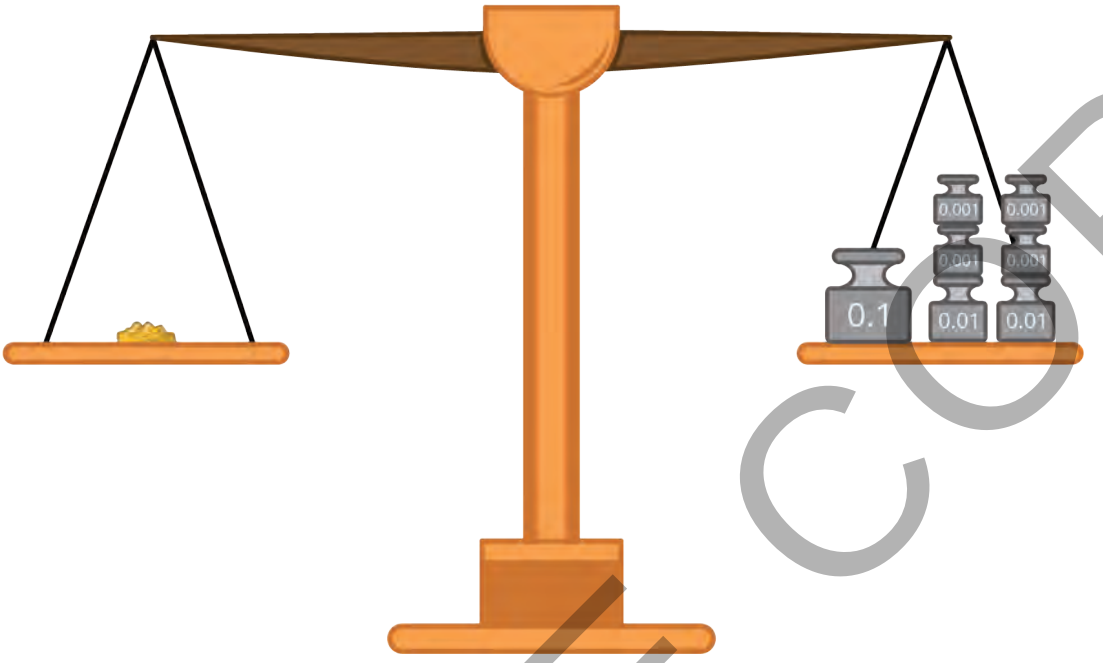
   a. 266 of the 0.001-ounce weights

   b. 150 of the 0.01-ounce weights

   c. 27 of the 0.1-ounce weights

Activity 2

Weights and Place Values



1. Each of these sets of weights is used to balance a different gold nugget. Write the weight of each gold nugget in ounces in expanded form.

   a. three 0.1-ounce weights, five 0.01-ounce weights, and eight 0.001-ounce weights

   b. six 0.1-ounce weights and two 0.001-ounce weights

   c. two 0.01-ounce weights and six 0.1-ounce weights

2. Here are the weights of 2 gold nuggets represented in word form. Write each weight in expanded form.

   a. two hundred eighty-three thousandths ounce

   b. four hundred nine thousandths ounce

3. A gold nugget weighs 0.527 ounce.

   a. What is the value of each digit in the decimal 0.527?

   b. How does the expanded form of 0.527 show the value of each digit in the decimal?

Activity 3

Comparing Place Values with Weights

1. How many 0.01-ounce weights will balance one 0.1-ounce weight? Explain or show your reasoning.

2. How many 0.001-ounce weights will balance a 0.1-ounce weight? Explain or show your reasoning.

3. The table shows the weights of 3 gold nuggets. Fill in the blanks. Explain or show your reasoning.

gold	weight (grams)
Nugget A	0.6
Nugget B	0.06
Nugget C	0.006

   a. Nugget A weighs _____ times as much as Nugget B.

   b. Nugget A weighs _____ times as much as Nugget C.

   c. Nugget C weighs _____ times as much as Nugget B.

   d. Nugget C weighs _____ times as much as Nugget A.

Unit 5, Lesson 5

Addressing CA CCSSM 5.NBT.3, 5.NBT.3b; practicing MP5

Compare Decimals

Let's compare decimals.

Warm-up

True or False: Decimals

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $7.06 = 7.006$

- $7.06 = 7.060$

- $7.06 = 7.600$

Activity 1

Farther and Faster

1. Diego and Jada compete to see who can throw a disc farther. Diego throws the disc 5.10 meters. Jada throws it 5.01 meters.

   Who throws the disc farther? Explain your reasoning.

2. Tyler and Han compete to see who can swim the length of a pool faster. Tyler swims the length of the pool in 35.15 seconds. Han swims the length in 35.30 seconds.

   Who swims faster? Explain your reasoning.

Activity 2

Farthest Flight

Recall that Diego threw a disc 5.1 meters, and Jada threw it 5.01 meters. For each question, find 2 possible answers.

1. Han threw the disc farther than Diego. How far might Han have thrown the disc?

2. Tyler threw the disc farther than Diego but less than 6 meters. How far might Tyler have thrown the disc?

3. Mai threw the disc a shorter distance than Jada. How far might Mai have thrown the disc?

4. Priya threw the disc a shorter distance than Jada but more than 5 meters. How far might Priya have thrown the disc?

Unit 5, Lesson 6

Addressing CA CCSSM 5.NBT.3, 5.NBT.3b; practicing MP6 and MP7

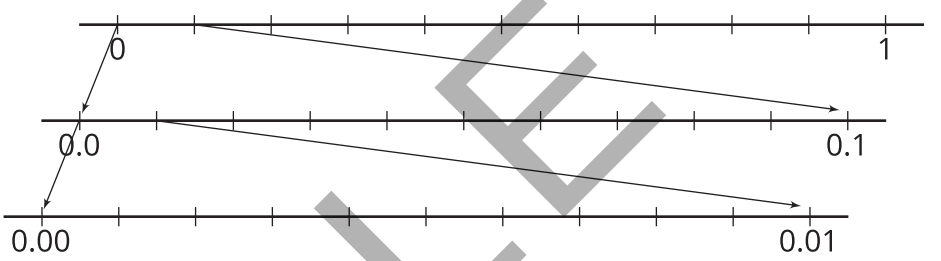
Compare Decimals on the Number Line

Let's locate and label decimals on number lines.

Warm-up

Notice and Wonder: Nested Lines

What do you notice? What do you wonder?



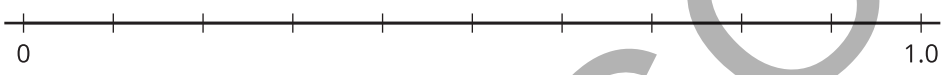
Activity 1

Locate 1 Thousandth

On each number line:

- Label all of the tick marks.
- Locate and label the number 0.001.

1.



2.

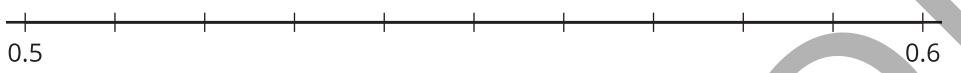
3.

Activity 2

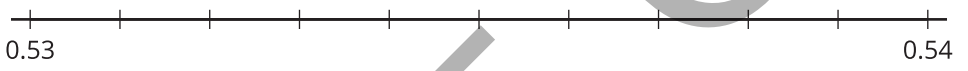
Label and Compare Decimals

1. Label the tick marks on each number line.

a.



b.



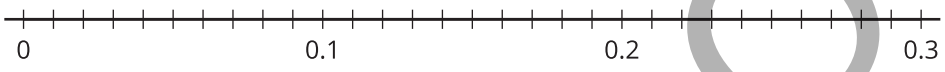
c.

2. Which of the number lines would you use to compare 0.534 and 0.537? Explain or show your reasoning.

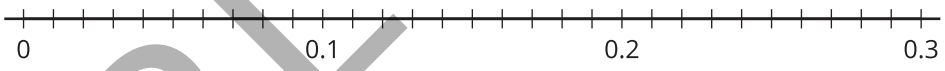
Activity 3

Locate and Compare With Symbols

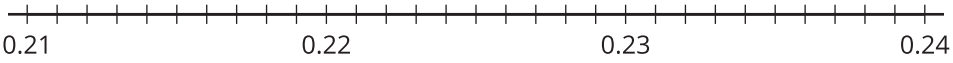
1. Use the symbol > or < to compare the decimals 0.2 and 0.02. Use the number line to explain or show your reasoning.



2. Use the symbol > or < to compare the decimals 0.3 and 0.14. Use the number line to explain or show your reasoning.



3. Use the symbol > or < to compare the decimals 0.23 and 0.216. Use the number line to explain or show your reasoning.



Addressing CA CCSSM 5.NBT.3, 5.NBT.4; building towards 5.NBT.4; practicing MP6

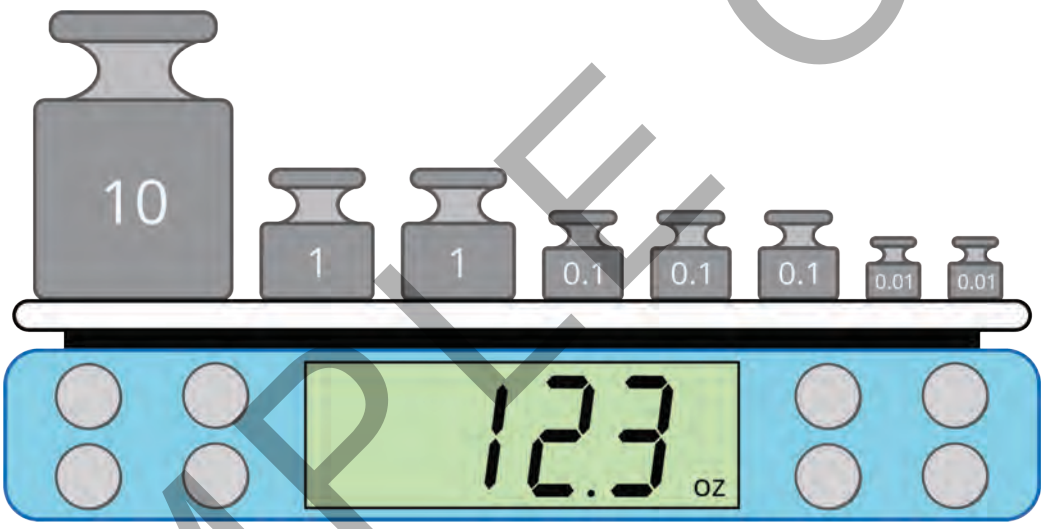
Round Doubloons

Let's explore rounding with decimals.

Warm-up

Notice and Wonder: A Digital Scale

What do you notice? What do you wonder?



Activity 1

Gold Doubloons

- Doubloons made before 1728 weigh 6.867 grams.
- Doubloons made after 1728 weigh 6.766 grams.

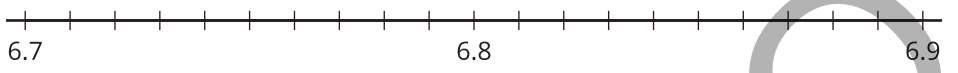
1. The scale measures weight to the nearest tenth of a gram.

   Was the doubloon on the scale made before or after 1728?

2. The same doubloon is put on a different scale that measures to the nearest gram. Could you use that measurement to know if the coin was made before or after 1728? Explain or show your reasoning.

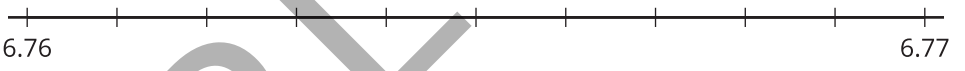
3. a. Which doubloons weigh more, the ones made before 1728 or the ones made after 1728? Explain or show your reasoning.

   b. Show the 2 doubloon weights in grams on the number line.

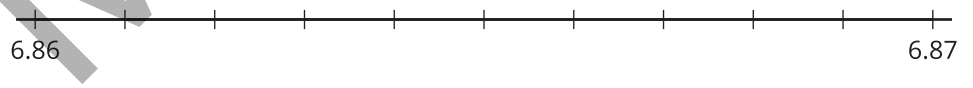


4. Show each doubloon weight on the number line. Which hundredth of a gram is each weight closer to?

   a. 6.766 grams



   b. 6.867 grams



Activity 2

Exact or Approximate?

Decide if you think each quantity is exact or estimated. Explain your reasoning.

1. There are 14 pencils on the desk.

2. The population of Los Angeles is 12,400,000.

3. It's 2.4 miles from the school to the park.

4. The runner finishes the race in 19.78 seconds.

Addressing CA CCSSM 5.NBT.3b, 5.NBT.4; building on 5.NBT.4; building towards 5.NBT.3b; practicing MP2 and MP3

Round Decimals

Let's round decimals to the nearest whole, tenth, and hundredth.

Warm-up

Estimation Exploration: Number Line

What number might be represented on the point on the number line?

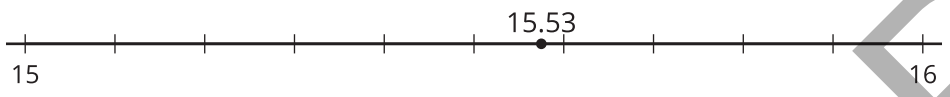
Record an estimate that is:

too low	about right	too high

Activity 1

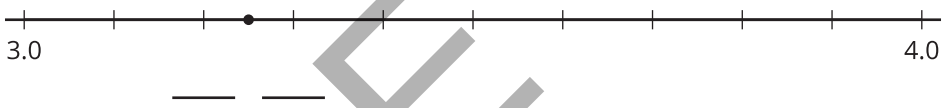
Name that Number

Jada tries to locate 15.53 on the number line. Do you think she accurately located the number?

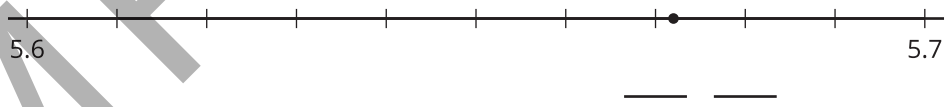


A number is located between 2 tick marks on each number line. Label those tick marks. Then estimate the number.

1.



2.

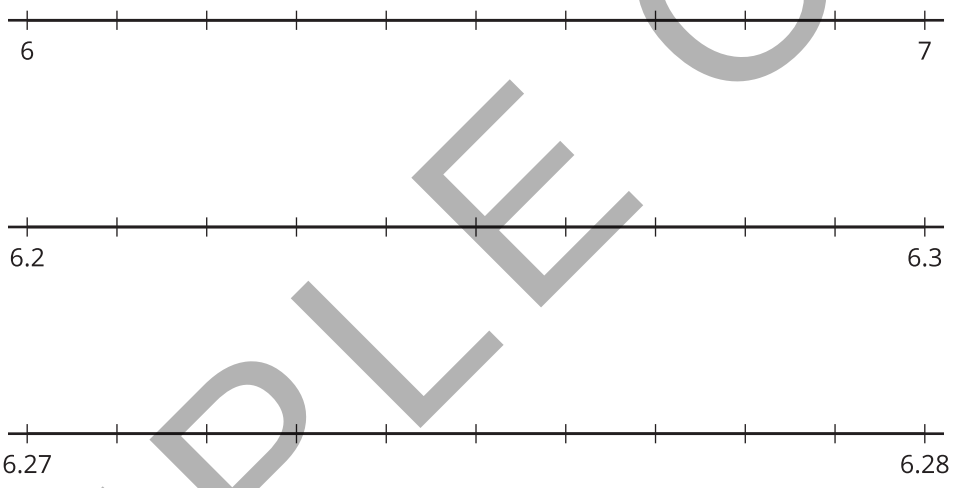


3.

Activity 2

Which Number Is Closest?

1. Round 6.273 to the nearest whole number, tenth, and hundredth. Use the number lines if they are helpful. Explain or show your reasoning.



2. Round 4.158 to the nearest whole number, tenth, and hundredth.

Activity 3

Round the Numbers

Round each number to the nearest whole number, tenth, and hundredth.

	nearest whole number	nearest tenth	nearest hundredth
34.482			
99.909			
5.555			
19.509			

Addressing CA CCSSM 5.NBT.3, 5.NBT.3b; practicing MP1 and MP6

Order Decimals

Let's put decimals in order.

Warm-up

True or False: Decimal Inequalities

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $0.909 > 0.91$

- $4.1 < 4.100$

- $0.99 < 0.999$

Activity 1

Caught in the Middle

1. Fill in the blank to make each statement true. Use the number lines if they are helpful.

   a. $786.2 < \_\_\_\_\_ < 786.3$

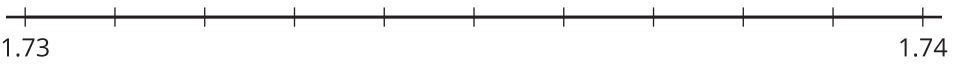
   b. $9.99 < \_\_\_\_\_ < 10$

   c. $0.46 > \_\_\_\_\_ > 0.45$

   d. $0.5 < \_\_\_\_\_ < 0.51$

   e. $0.99 < \_\_\_\_\_ < 0.999$

2. Kiran says that there is no number between 1.731 and 1.732. Do you agree? Use the number line if it is helpful.



Activity 2

Least to Greatest

Write each set of numbers in order from least to greatest.

1. 67.020, 67.200, 67.002

2. 1.101, 1.02, 1.1

3. 0.333, 0.323, 0.3

4. 99.99, 99.09, 99.091

Addressing CA CCSSM 5.NBT.3, 5.NBT.4; practicing MP2

Solve Problems with Decimals

Let's round and order decimals to solve problems.

Warm-up

Notice and Wonder: The Luge

What do you notice? What do you wonder?

A	B
48.532	82.13
48.561	82.75
48.626	82.81
48.634	83.07
48.708	82.80

Activity 1

How Accurate Is It?

The table shows the race times for 5 luge athletes.

athlete	time (seconds)
Athlete 1	48.532
Athlete 2	48.561
Athlete 3	48.626
Athlete 4	48.634
Athlete 5	48.708

1. How would the results of the race change if the times were recorded to the nearest second?

2. How would the results of the race change if the times were recorded to the nearest tenth of a second?

3. How would the results of the race change if the times were recorded to the nearest hundredth of a second?

4. An athlete records a time of 48.85 seconds to the nearest hundredth of a second. What could that time be if it was recorded to the thousandth of a second?

5. An athlete records a time of 48.615 seconds to the nearest thousandth of a second. What could that time be if it was recorded to the nearest hundredth of a second?

Activity 2

Compare Speeds

The table shows the top speeds of 5 luge athletes.

athlete	speed (miles per hour)
Athlete 1	82.13
Athlete 2	82.75
Athlete 3	82.81
Athlete 4	83.07
Athlete 5	82.80

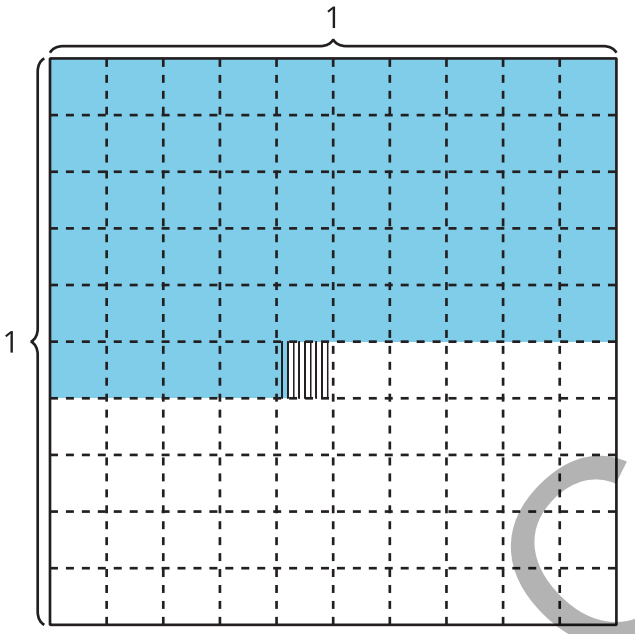
1. List the speeds in decreasing order.

2. Do any of these athletes have the same top speed if rounded to the nearest tenth of a mile per hour? To the nearest mile per hour?

3. A sixth athlete's top speed is faster than Athlete 5's but slower than Athlete 3's. What could the top speeds of these 3 athletes be if they were each measured to the nearest thousandth of a mile per hour?

Section A Summary

We represented decimals to the thousandths place.



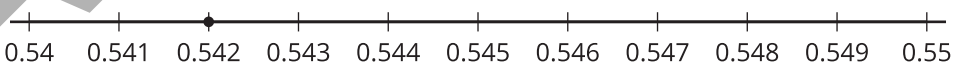
The shaded region of the diagram represents 0.542.

- The 5 shaded rows are each a tenth or 0.1
- The 4 shaded small squares are each a hundredth or 0.01.
- The 2 shaded tiny rectangles are each a thousandth or 0.001.

The decimal 0.542 can be represented in other ways.

- $\frac{542}{1{,}000}$
- five hundred forty-two thousandths
- $(5 \times 0.1) + (4 \times 0.01) + (2 \times 0.001)$

We also learned how to locate 0.542 on a number line.



The number line shows that 0.542 is closer to 0.54 than to 0.55. So, 0.542 rounded to the nearest hundredth is 0.54.

Practice Problems

18 Problems

1 Pre-unit

Find the value of each expression.

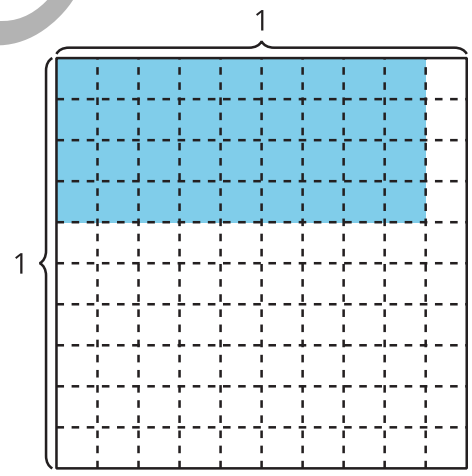
a. $\frac{1}{3} \times \frac{1}{10}$

b. $\frac{1}{10} \times \frac{1}{10}$

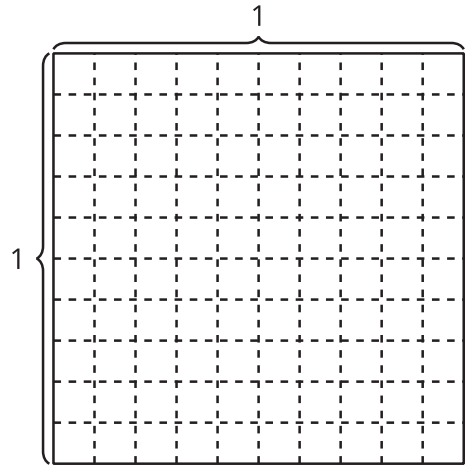
c. $\frac{1}{10} \times \frac{1}{100}$

2 Pre-unit

a. Write a multiplication equation to represent the shaded region of the diagram.

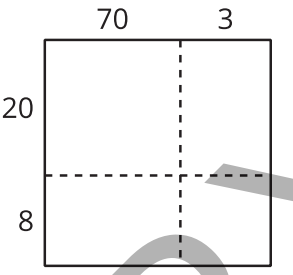


b. What is the value of $\frac{7}{10} \times \frac{5}{10}$? Use the diagram if it is helpful.



3 Pre-unit

Find the value of 73×28. Use the diagram if it is helpful.



4 Pre-unit

a. What is the value of the digit 6 in 618,923?

b. How many times greater is the value of the digit 6 in 618,923 than the value of the digit 6 in 27,652?

5 Pre-unit

Find the value of $3{,}724 \div 7$. Explain or show your reasoning.

6 Pre-unit

Find the value of each sum or difference.

a.

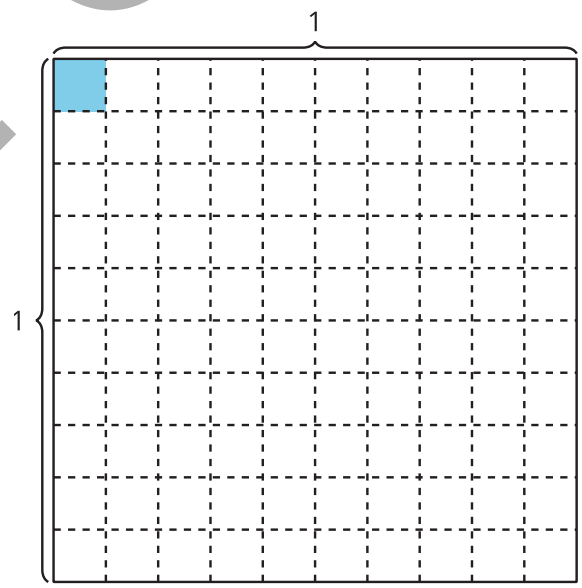
$$\begin{array}{r} 13,817 \\ +\ \ 6,544 \\ \hline \end{array}$$

b.

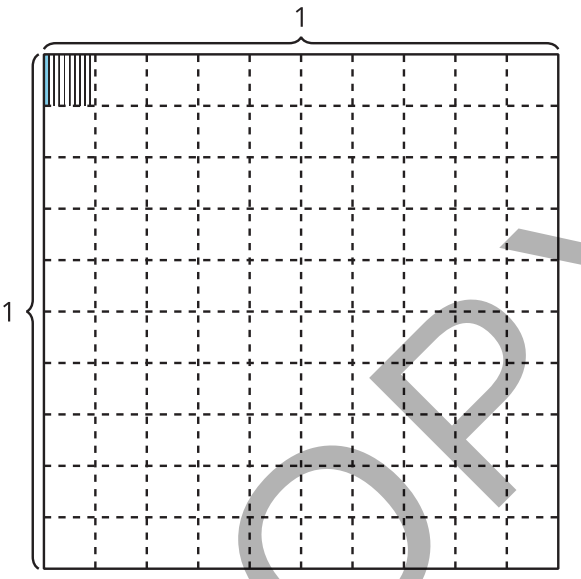
$$\begin{array}{r} 8,793 \\ -\ 4,675 \\ \hline \end{array}$$

7 from Unit 5, Lesson 1

a. What fraction of the whole square is shaded? Explain or show your reasoning.

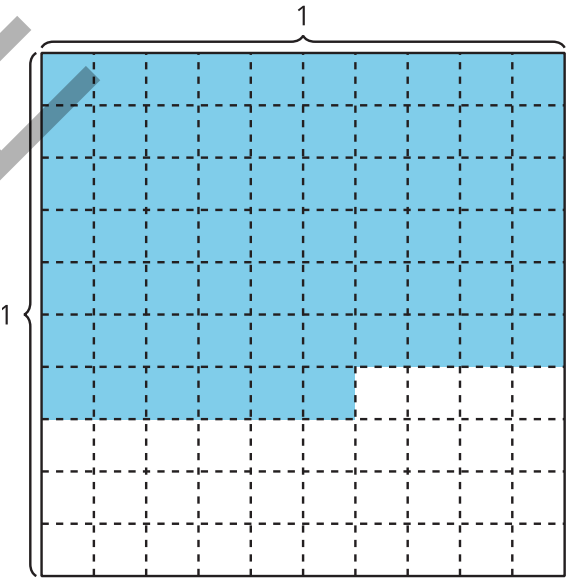


b. What fraction of the whole square is shaded? Explain or show your reasoning.

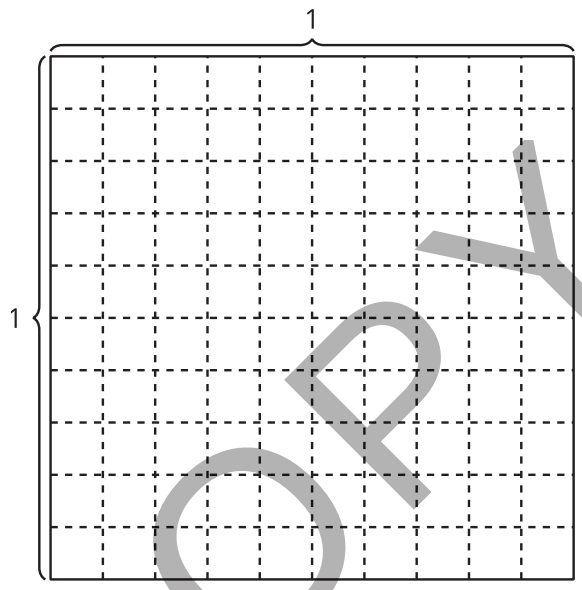


8 from Unit 5, Lesson 2

a. Write a decimal number to represent how much of the square is shaded.



b. Shade the square to represent the decimal one hundred fifteen thousandths.

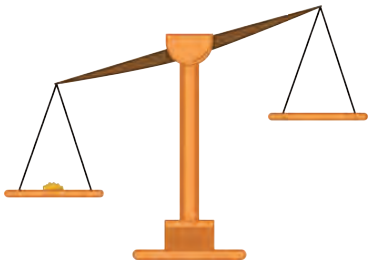


9 from Unit 5, Lesson 3

Write the decimal 0.418 as a fraction, in words, and in expanded form.

10 from Unit 5, Lesson 4

a. The gold nugget on the scale weighs 0.265 ounce. Name 2 different sets of 0.1-ounce, 0.01-ounce, and 0.001-ounce weights you can use to balance the nugget.



b. One gold nugget weighs 0.008 ounce. A second gold nugget weighs 0.8 ounce.

- How many times as much as the first nugget does the second nugget weigh?

- How many times as much as the second nugget does the first nugget weigh?

11 from Unit 5, Lesson 5

Noah threw the ball 4.89 yards.

a. Noah threw the ball farther than Lin. How far could Lin have thrown it?

b. Andre threw the ball farther than Noah but less than 4.9 yards. How far could Andre have thrown it? Explain or show your reasoning.

12 from Unit 5, Lesson 6

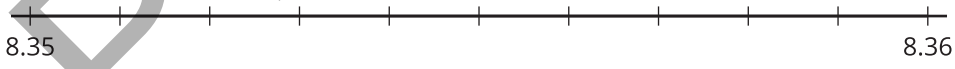
a. Label the tick marks. Use the number line to explain your reasoning.

b. Which is greater, 0.654 or 0.658? Explain or show your reasoning.

13 from Unit 5, Lesson 7

A $5 gold coin weighs 8.359 grams.

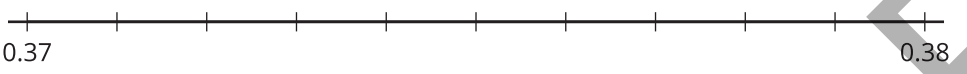
a. Locate 8.359 on the number line.



b. A scale measures to the nearest 0.01 gram. What will the scale show for the weight of the coin? Explain or show your reasoning.

14 from Unit 5, Lesson 8

a. What is 0.374 rounded to the nearest hundredth? Explain or show your reasoning. Use the number line if it's helpful.



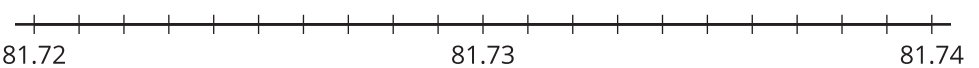
b. What is 9.893 rounded to the nearest tenth? To the nearest hundredth? Explain or show your reasoning.

15 from Unit 5, Lesson 9

List the decimals from least to greatest: 6.95, 6.895, 6.598, 6.985, 5.986

16 from Unit 5, Lesson 10

To the nearest hundredth of a mile per hour, a luge rider's top speed was 81.73 mph. What are some possible speeds to the thousandth of a mile per hour? Use the number line if it is helpful.



17 Exploration

a. Jada has 3 doubloons. She knows that 2 have the same weight and 1 is heavier than the other 2. Jada also has a balance to compare the weights of coins. Explain or show how Jada can use the balance to figure out which doubloon is heavier and which 2 weigh the same.

b. What if Jada has 5 doubloons and knows that 4 have the same weight and 1 is heavier?

18 Exploration

There are 2 packages of ground beef at the store. One package says it has 1 pound of beef. The second package says it has 0.97 pound of beef. Jada says that the 1-pound package has more beef. Do you agree? Explain or show your reasoning.

Unit 5, Lesson 11

Addressing CA CCSSM 5.NBT.7; building towards 5.NBT.7; practicing MP7

Make Sense of Decimal Addition

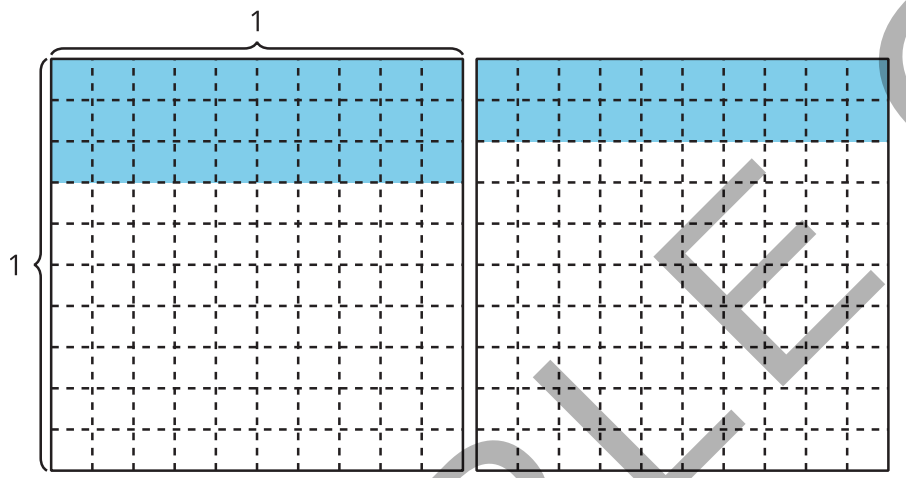
Let's add decimals.

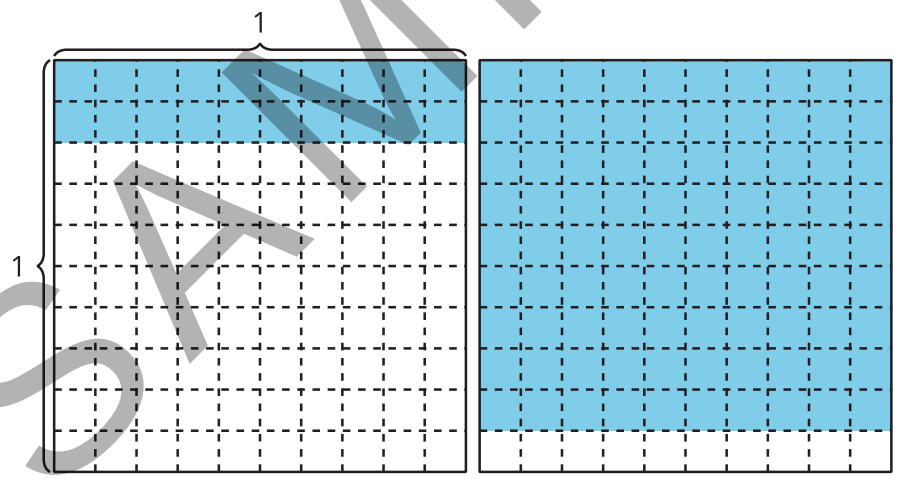
Warm-up

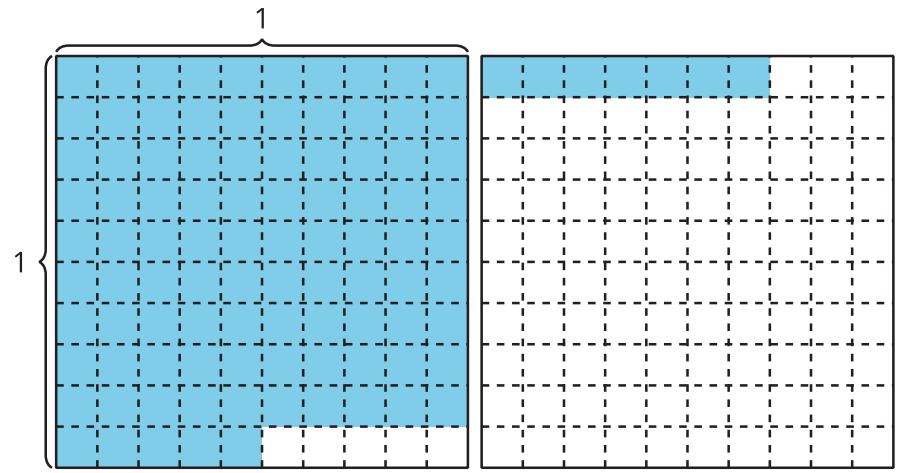
Sec B

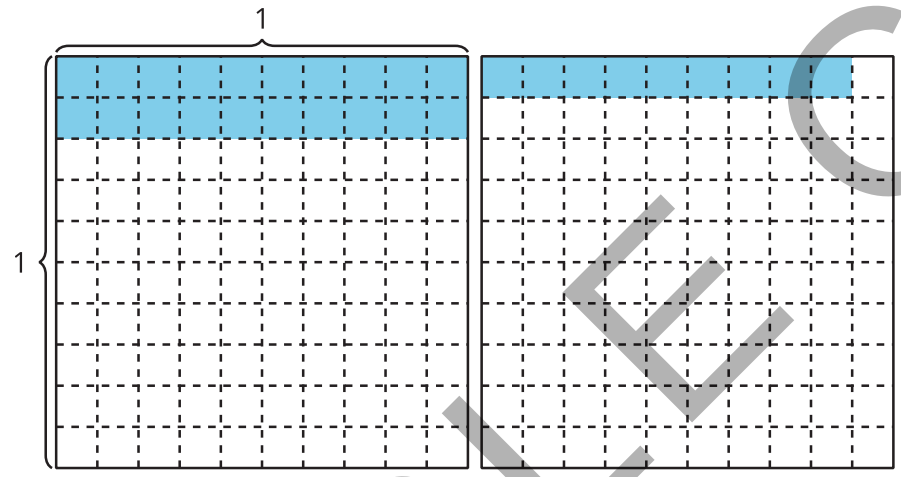
How Many Do You See: Diagrams

How many do you see? How do you see them?









Activity 1

The Sum

1. Find the value of the expression. Show your thinking. Organize your work so it can be followed by others. Use diagrams if they are helpful.

   $2.26 + 1.87$

2. What questions do you have about adding decimals?

Activity 2

Introduce Target Numbers—Add Tenths or Hundredths

Directions:

1. Play 1 game of *Target Numbers*.
   - Take out the cards that show 0 and 10 and set them aside.
   - On your turn.
     - Start at 0. Pick a number card. Choose whether to add that number of tenths or hundredths to your starting number.
     - Write an equation to represent the sum.
   - The sum is the starting number in the next equation.
   - Take turns for 6 rounds.
   - The partner to get a sum closer to 1 without going over wins.

2. Describe a move that you could have made differently to change the outcome of the game.

Unit 5, Lesson 12

Addressing CA CCSSM 5.NBT.7; practicing MP3

Estimate and Add

Let's add decimals and decide if our answers are reasonable.

Warm-up

Number Talk: 99 Hundredths

Find the value of each expression mentally.

- $1.00 + 0.99 + 0.02$
- $1.99 + 0.02$
- $1.99 + 0.03$
- $1.99 + 0.13$

Activity 1

Use the Standard Algorithm to Add Decimals

1. Find the value of $5.61 + 2.53$. Explain or show your reasoning.

2. Han uses the standard algorithm to add decimals. Describe what he does in each step.

Step 1	Step 2	Step 3
5.61	5.61	5.61
+ 2.53	+ 2.53	+ 2.53
. 4	.14	8.14

3. Use the standard algorithm to find the value of $6.62 + 3.74$.

Activity 2

Estimate and Find the Value

1. Which whole number is the sum $2.82 + 5.2$ closest to? Explain or show your reasoning.

2. What is the value of $2.82 + 5.2$? Explain or show your reasoning. Use the standard algorithm if it is helpful.

3. Which whole number is the sum $6.8 + 4.97$ closest to? Explain or show your reasoning.

4. What is the value of $6.8 + 4.97$? Explain or show your reasoning. Use the standard algorithm if it is helpful.

Addressing CA CCSSM 5.NBT.7; practicing MP3 and MP6

Analyze Addition Mistakes

Let's use place value strategies to add decimals.

Warm-up

Estimation Exploration: Many Places

$$1{,}987.89 + 658.54$$

Record an estimate that is:

too low	about right	too high

Activity 1

Compare Calculations

1. Find the value of $621.45 + 72.3$. Explain or show your reasoning.

Sec B

2. Elena and Andre try to find the value of $621.45 + 72.3$. Who do you agree with? Explain or show your reasoning.

Elena

$$\begin{array}{r} 621.45 \\ +\ \ 72.3 \\ \hline 628.68 \end{array}$$

my answer makes sense because it is more than 621.

Andre

$$\begin{array}{r} 621.45 \\ +\ 72.30 \\ \hline 693.75 \end{array}$$

My answer makes sense because 620 + 70 = 690 and then I still have to add a little bit more than 3 to 690.

Activity 2

Same Digits, Different Sums

Find the value of each expression. Explain or show your reasoning.

1. $2.63 + 7.74$

2. $26.3 + 774$

3. $46.3 + 31.42$

4. $463 + 3.14$

Unit 5, Lesson 14

Addressing CA CCSSM 5.NBT.7; practicing MP5 and MP8

Make Sense of Decimal Subtraction

Let's subtract decimals.

Warm-up

True or False: Decimal Differences

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $0.5 - 0.01 = 0.4$

- $0.61 - 0.02 = 0.59$

- $1 - 0.07 = 0.93$

Activity 1

The Difference

1. Find the value of $2.26 - 1.32$. Explain or show your reasoning.

2. What questions do you have about subtracting decimals?

Activity 2

Introduce Target Numbers—Subtract Tenths or Hundredths

Directions:

1. Play 1 game of *Target Numbers*.
   - Take out the cards that show 0 and 10 and set them aside.
   - On your turn:
     - Start at 2. Pick a number card. Choose whether to subtract that number of tenths or hundredths from your starting number.
     - Write an equation to represent the difference.
   - The difference is the starting number in the next equation.
   - Take turns for 6 rounds.
   - The partner to get a difference closer to 1 without going under wins.

2. Describe a move that you could have made differently to change the outcome of the game.

Unit 5, Lesson 15

Addressing CA CCSSM 5.NBT.7; practicing MP3

Estimate and Subtract

Let's use place value to subtract decimals.

Warm-up

Number Talk: One and Five Tenths

Find the value of each expression mentally.

- $1.50 - 0.51$
- $1.50 - 0.52$
- $1.50 - 0.60$
- $1.50 - 0.62$

Activity 1

Revisit the Algorithm

1. Find the value of $8.53 - 2.7$. Explain or show your reasoning.

Sec B

2. Han uses the standard algorithm to subtract decimals. Describe what he does in each step.

Step 1	Step 2	Step 3
8.53	8.53	8.53
− 2.7	− 2.7	− 2.7
. 3	.83	5.83

3. Use the standard algorithm to find the value of $6.62 - 3.71$.

Activity 2

Estimate and Subtract

1. Which whole number is the difference $15.27 - 4.3$ closest to? Explain or show your reasoning.

2. What is the value of $15.27 - 4.3$? Explain or show your reasoning.

3. Which whole number is the difference $16.14 - 9.8$ closest to? Explain or show your reasoning.

4. What is the value of $16.14 - 9.8$? Explain or show your reasoning.

Activity 3

Compare Calculations for a Difference

1. Find the value of $622.35 - 71.4$. Explain or show your reasoning.

Sec B

2. Elena and Andre try to find the value of $622.35 - 71.4$. Who do you agree with? Explain or show your reasoning.

Elena

$$\begin{array}{r} 6\overset{1}{2}\overset{1}{2}.35 \\ -\ \ 71.4 \\ \hline 615.21 \end{array}$$

my answer makes sense because 615.21 is less than 622.35.

Andre

$$\begin{array}{r} \overset{5}{6}\overset{12}{2}\overset{1}{2}.\overset{13}{3}5 \\ -\ 71.40 \\ \hline 550.95 \end{array}$$

My answer makes sense because 620 - 70 = 550. So my answer should be right around 550.

Unit 5, Lesson 16

Addressing CA CCSSM 5.NBT.7; practicing MP6

Addition and Subtraction

Let's use place value strategies to add and subtract decimals.

Warm-up

Number Talk: Subtracting Decimals

Find the value of each expression mentally.

- $2.57 - 2.55$
- $2.57 - 2.49$
- $2.57 - 0.99$
- $2.57 - 0.59$

Activity 1

What's the Difference?

Find the value of each expression. Explain or show your reasoning.

1. $7.35 - 2.6$

2. $100.8 - 6.03$

3. $26.5 - 13.62$

4. $465 - 463.14$

Activity 2

Sums and Differences

Find the value of each expression. Explain or show your reasoning.

1. $36.51 - 4.3$

2. $100 + 31.05$

3. $100 - 31.05$

4. $266.43 + 75.9$

Activity 3

Subtraction with Larger Numbers

Find the value of each expression.

1. $43.14 - 18.6$

Sec B

2. $73.3 - 52.99$

3. $128.44 - 62.57$

4. $261.25 - 260.7$

Section B Summary

We learned that we can use the same strategies and algorithms for adding and subtracting whole numbers to add and subtract decimals.

We learned that it is helpful to estimate a sum before calculating it. For example, this sum will be close to $620 + 70$ or 690.

$$\begin{array}{r} 6\ 2\ 1.\ 4\ 5 \\ +\quad 7\ 2.\ 3\ 0 \\ \hline 6\ 9\ 3.\ 7\ 5 \end{array}$$

We learned to align the place values when adding and subtracting.

$$\begin{array}{r} {\scriptstyle 5\ 12\ 1\ 13} \\ \cancel{6}\ \cancel{2}\ \cancel{2}.\ \cancel{3}\ 5 \\ -\quad 7\ 1.\ 4\ 0 \\ \hline 5\ 5\ 0.\ 9\ 5 \end{array}$$

We estimated that the value of the difference will be about $620 - 70$ or 550.

Practice Problems

8 Problems

1 from Unit 5, Lesson 11

Mai and Tyler play a game. They each roll 6 dice. They must make each number they roll either a tenth or a hundredth. Then they find the sum of those 6 numbers

a. Mai rolls 6 sixes. How close can she get to a sum of 1 (without going over)?

b. Tyler rolls 6 fours. How close can he get to a sum of 1 (without going over)?

2 from Unit 5, Lesson 12

a. Which whole number is $3.62 + 1.49$ closest to? Explain or show your reasoning.

b. Find the value of $3.62 + 1.49$.

3 from Unit 5, Lesson 13

Find the value of the expression $215.7 + 64.94$.

4 from Unit 5, Lesson 14

a. Which whole number is $9.36 - 6.52$ closest to? Explain or show your reasoning.

b. Find the value of $9.36 - 6.52$.

5 from Unit 5, Lesson 15

a. Here is how Elena found the value of $15.37 - 8.19$.

$$\begin{array}{r} {\scriptstyle 15\ \ 2\ 17} \\ 1\not{5}.\not{3}\not{7} \\ -\ \ 8.\ 1\ 9 \\ \hline 7.\ 1\ 8 \end{array}$$

Explain her calculations and the meaning of the 15 written above the digit 5 and the 17 written above the digit 7 in 15.37.

b. Use Elena's algorithm to calculate $52.63 - 17.55$.

6 from Unit 5, Lesson 16

Find the value of each expression.

a. $37.06 - 22.57$

b. $555 - 4.44$

7 Exploration

a. Kiran finds the value of $35.16 - 18.79$ with these calculations.

$18.79 + 0.21 = 19$
$19 + 16.16 = 35.16$
$16.16 + 0.21 = 16.37$

Explain why Kiran's strategy works.

b. Find the difference $22.86 - 9.99$ in a way that makes sense to you.

8 Exploration

Lin tries to use the digits 1, 3, 4, 2, 5, and 6 to make 2 two-digit decimals whose sum is 1. Each digit can only be used once.

a. Explain why this can't be done.

b. What is the closest Lin can get to 1? Explain how you know.

Addressing CA CCSSM 5.NBT.7, 5.OA.2; building towards 5.NBT.7; practicing MP7 and MP8

Multiply Decimals and Whole Numbers

Let's multiply whole numbers by tenths and hundredths.

Warm-up

True or False: Place Value Products

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $100 \times 0.01 = 1$

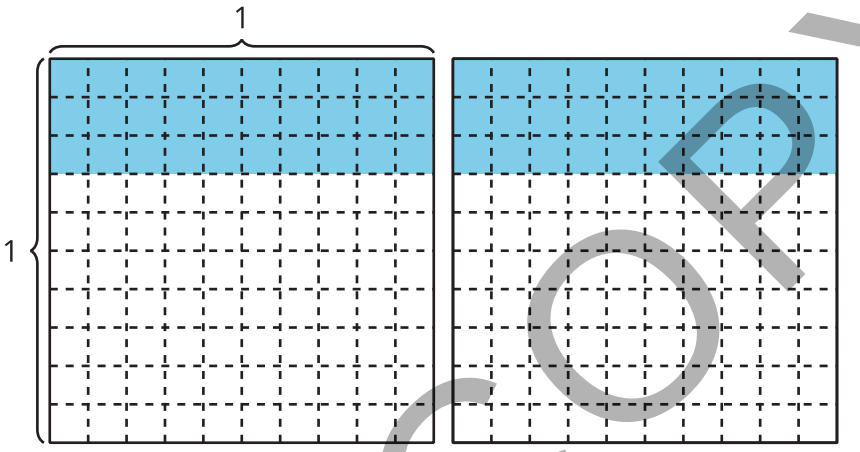
- $10 \times 0.1 = 0.01$

- $10 \times 0.01 = 0.1$

Activity 1

Multiply Decimals by Whole Numbers

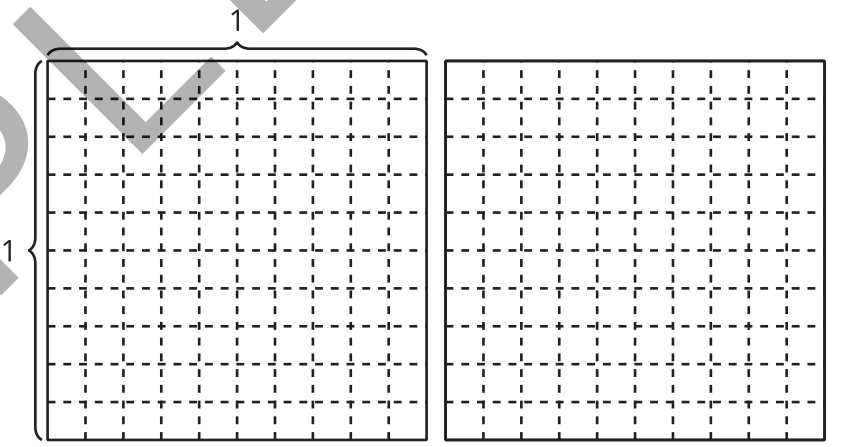
How many do you see?



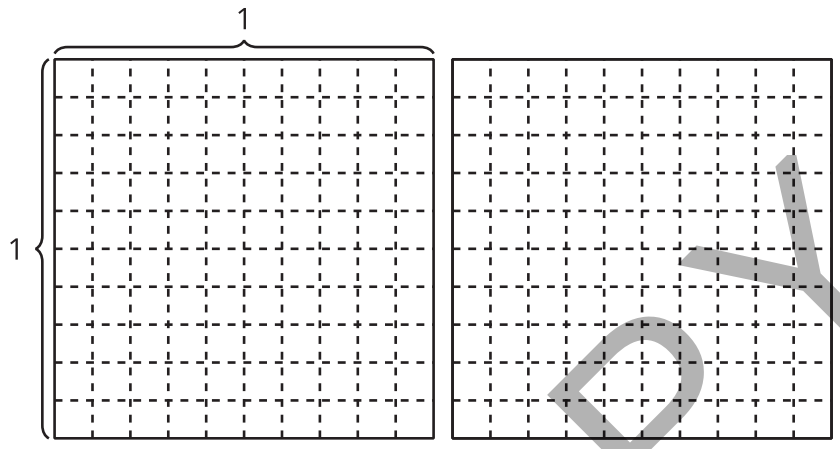
Sec C

Find the value of each expression in a way that makes sense to you. Explain or show your reasoning. Use the diagrams if they are helpful.

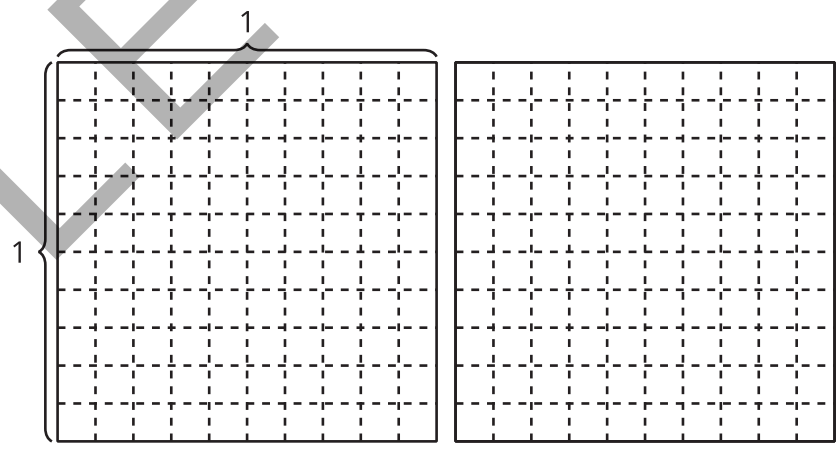
1. 2×0.7



2. 2×0.08



3. 2×0.78



Activity 2

Using Whole Number Products

1. Find the value of each expression. Explain or show your reasoning.

   a. 3×0.5

   b. 5×0.3

   c. 7×0.02

2. Kiran writes this explanation to describe the strategy he used to multiply a whole number by some tenths:

   "I just turn the numbers into whole numbers, multiply them, and call them tenths."

   Fill in the blanks to show how Kiran's strategy works.

   $6 \times 0.7 = 6 \times$ _____ tenths = _____ tenths = 4.2

   $6 \times 0.07 = 6 \times$ _____ hundredths = _____ hundredths = _____

Unit 5, Lesson 18

Addressing CA CCSSM 5.NBT.7, 5.OA.1, 5.OA.2; practicing MP2 and MP7

Use Whole Number Facts

Let's multiply whole numbers and decimals.

Warm-up

True or False: Group Dynamics

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $30 \times 2 \times 10 = 6 \times 10$

- $30 \times 2 \times 10 = 20 \times 3 \times 10$

- $60 \times 10 = 30 \times 20$

Activity 1

Agree or Disagree

1. Decide if each equation is true or false. Explain or show your reasoning.

   a. $4 \times 0.7 = 28$

   b. $5 \times 0.8 = 0.40$

   c. $6 \times 0.03 = (6 \times 3) \times 0.01$

   d. $8 \times 0.07 = (8 \times 7) \times 0.1$

2. Fill in the blank to make each equation true.

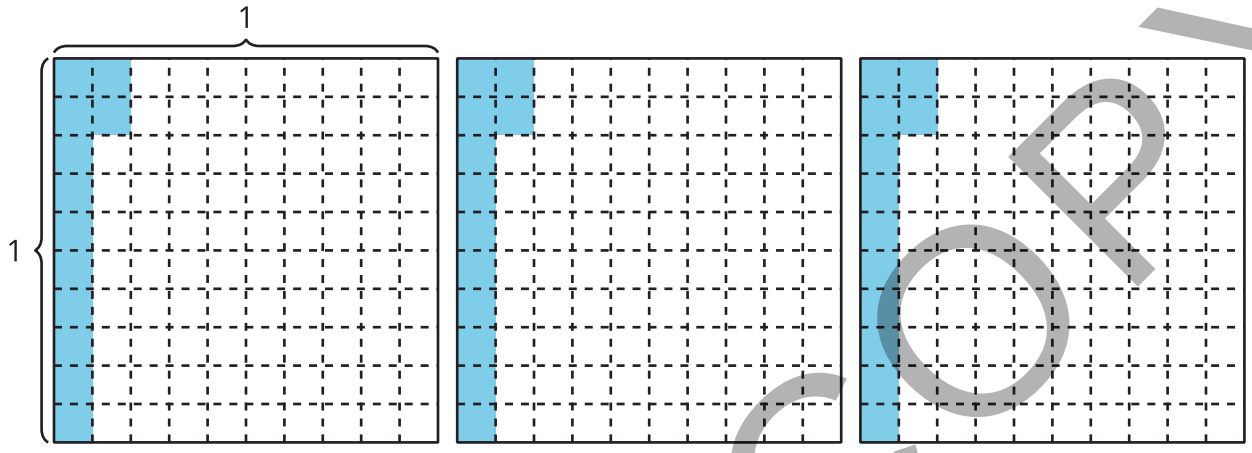
   a. $3 \times 0.7 = 3 \times 7 \times$ _____

   b. $3 \times 0.07 = 3 \times 7 \times$ _____

   c. $5 \times$ _____ $= (5 \times 4) \times 0.1$

Activity 2

Interpret Diagrams and Expressions



1. Explain or show how the diagram represents each expression.

   a. 3×0.12

   b. $(3 \times 12) \times 0.01$

   c. $(3 \times 0.1) + (3 \times 0.02)$

2. Find the value of $(3 \times 12) \times 0.01$. Explain or show your reasoning.

3. Find the value of $(3 \times 0.1) + (3 \times 0.02)$. Explain or show your reasoning.

Unit 5, Lesson 19

Addressing CA CCSSM 5.NBT.7, 5.OA.1-2; practicing MP7

Use Properties to Multiply Decimals

Let's interpret and find the value of multiplication expressions with decimals and whole numbers.

Warm-up

Number Talk: Many Hundredths

Find the value of each expression mentally.

- $40 \times 2 \times 0.1$

- $20 \times 0.1 \times 4$

- 0.1×80

- $0.01 \times 20 \times 40$

Activity 1

Card Sort: Decimal Multiplication Expressions

Your teacher will give you a set of cards that show multiplication expressions.

1. Sort the cards into 2 categories of your choosing. Be ready to explain the meaning of your categories.

   Then sort the cards into 2 categories in a different way. Be ready to explain the meaning of your new categories. (Pause for teacher directions.)

2. Find Cards A, B, and C. Group each expression with the other expressions that have the same value. Be ready to explain your reasoning.

3. Choose 1 expression from each group. Find the value of the expressions on Cards A, B, and C.

4. Write at least 1 more expression that has the same value as each expression on Cards A, B, and C.

Activity 2

Choose a Strategy

Find the value of each expression. Explain or show your reasoning.

1. 6×0.12

2. 4×1.4

3. 5×3.9

4. 25×0.41

Activity 3

More Multiplication Problems

Find the value of each expression.

1. 35×0.08

2. 35×0.7

3. 35×0.78

4. 42×0.66

Unit 5, Lesson 20

Addressing CA CCSSM 5.NBT.7, 5.NF.4; building towards 5.NF.7; practicing MP7

Products in the Hundredths Place

Let's multiply tenths by tenths.

Warm-up

What do you know about 1×0.1 and 0.1×0.1?

What do you know about these expressions?

- 1×0.1

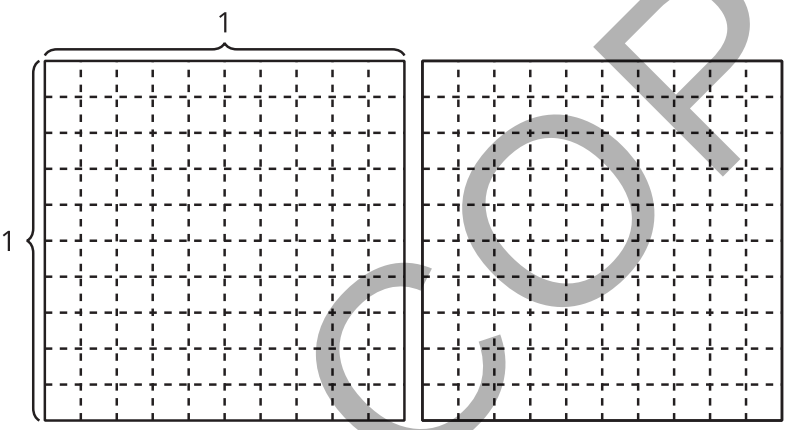
- 0.1×0.1

Activity 1

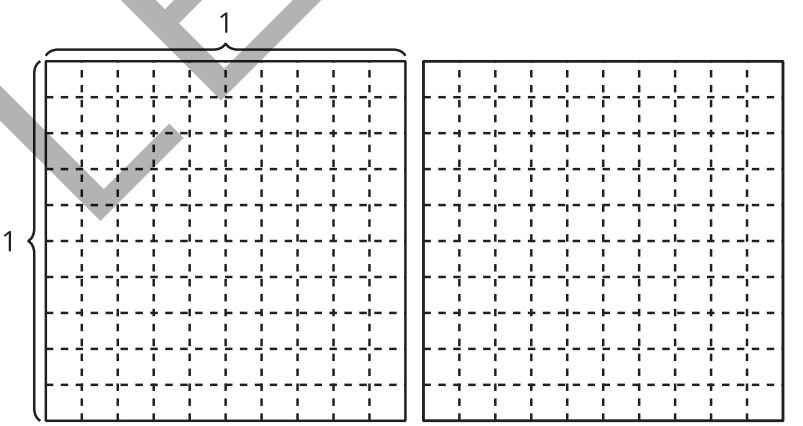
Products of Tenths

1. Find the value of each expression. Explain or show your reasoning. Use the diagrams if they are helpful.

   a. 2×0.3



   b. 0.2×0.3



2. Kiran says $0.2 \times 0.4 = 0.8$. Do you agree? Explain or show your reasoning.

Activity 2

Multiply Tenths

1. Find the value of each expression. Explain or show your reasoning.

   a. 1.8×0.4

   b. 2.5×0.6

   c. 3.8×0.7

2. How are these products alike? How are they different?
   - 74×6
   - 7.4×6
   - 7.4×0.6

Unit 5, Lesson 21

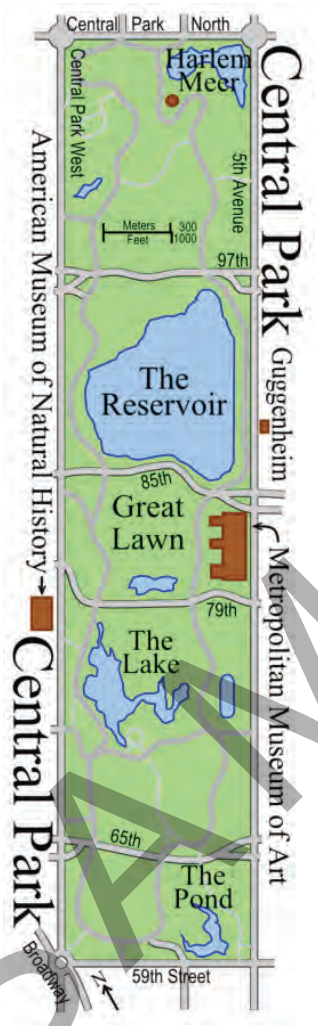
Addressing CA CCSSM 5.NBT.1, 5.NBT.7; practicing MP8

Multiply More Decimals

Let's multiply decimals.

Warm-up

Estimation Exploration: Central Park



Central Park is a large park in Manhattan. It is about 3.85 kilometers long and 0.79 kilometer wide. What is the approximate area of Central Park?

Record an estimate that is:

too low	about right	too high

Activity 1

Multiply More Decimals

1. Explain or show why each pair of expressions have the same value.

   a. 7.2×5.3 and $(72 \times 53) \times 0.01$

   b. 6.5×2.8 and $(65 \times 28) \div 100$

   c. 31×0.44 and $(31 \times 44) \times \frac{1}{100}$

2. Find the value of the products in the previous problem.

Activity 2

Choose Your Strategy

Find the value of each product.

1. 7.3×4.2

2. 38×0.55

3. 285×0.17

Section C Summary

We learned different strategies for multiplying with decimals.

We used place value relationships to reason about the multiplication.

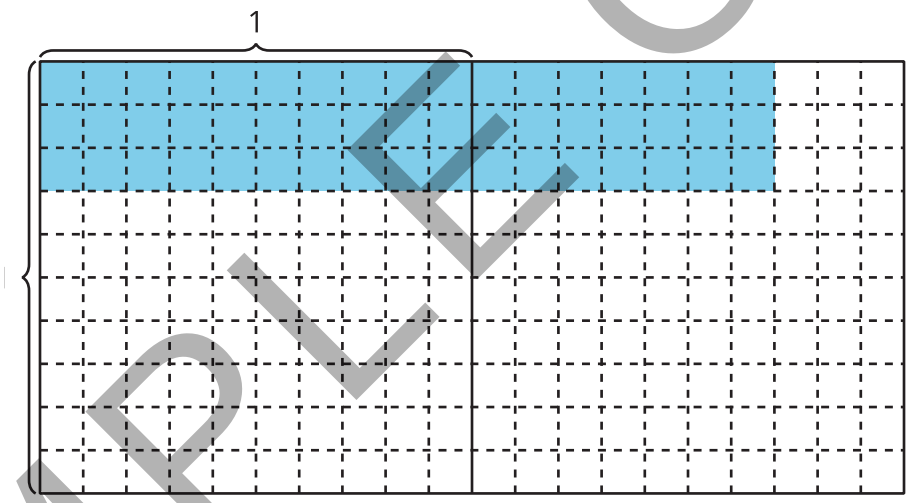
Example: $6 \times 0.14 = 0.84$ because 6 groups of 14 hundredths is 6×14 or 84 hundredths.

We used properties of operations to break up the multiplication.

Example: $0.9 \times 0.3 = (9 \times 3) \times 0.01 = 27 \times 0.01 = 0.27$

We also used diagrams to represent the multiplication.

Example: This diagram shows 17 groups of 3 hundredths is 51 hundredths, so $1.7 \times 0.3 = 0.51$.

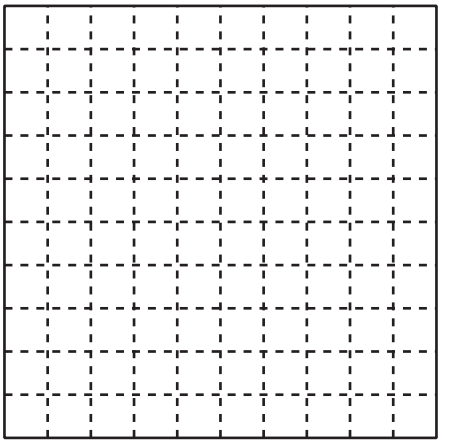


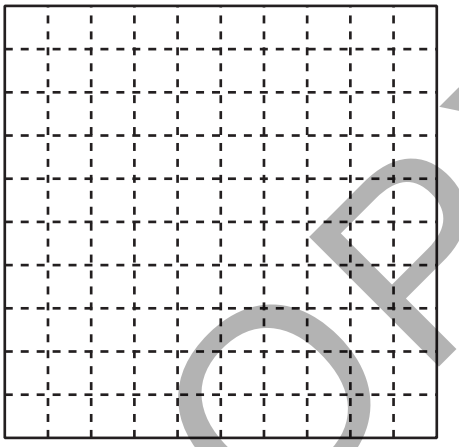
Practice Problems

7 Problems

1

from Unit 5, Lesson 17





Sec C

a. Shade the first diagram to represent 5×0.07.

b. What is the value of 5×0.07? Explain or show your reasoning.

c. What is the value of 5×0.2? Use the second diagram if it is helpful.

2

from Unit 5, Lesson 18

a. Mai says that 7×0.4 and 7×0.04 have the same value of 28. Do you agree? Explain or show your reasoning.

b. Explain why $8 \times 0.03 = (8 \times 3) \times 0.01$.

3

from Unit 5, Lesson 19

a. Explain why each expression has the same value as 9×0.45.

$(9 \times 0.4) + (9 \times 0.05)$

$(9 \times 45) \div 100$

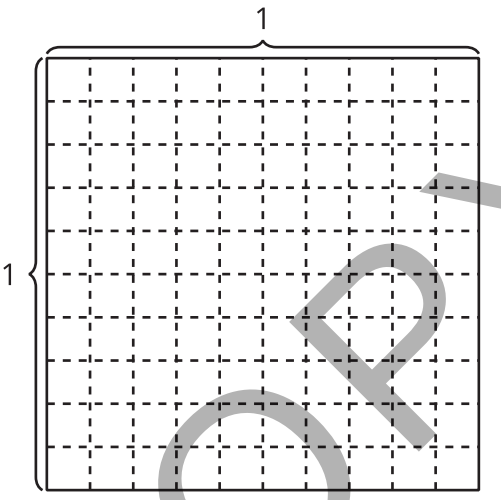
$(10 \times 0.45) - (1 \times 0.45)$

b. Find the value of 9×0.45 using one of the expressions or your own strategy.

4 from Unit 5, Lesson 20

Shade the diagram to represent 0.7×0.4.

What is the value of 0.7×0.4?



5 from Unit 5, Lesson 21

a. Explain or show why $5.6 \times 3.4 = (56 \times 34) \times 0.01$.

b. Use this strategy to calculate 5.6×3.4.

6 Exploration

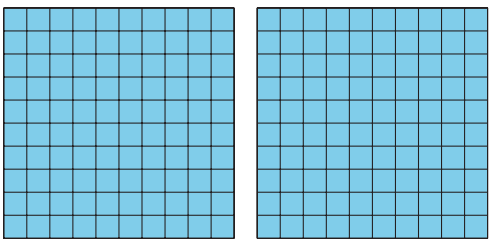
Diego finds the value of 17.5×3.3. He says, "I know $\frac{175}{10} \times \frac{33}{10} = \frac{175 \times 33}{100}$, so I just find 175×33 and then divide by 100."

a. Explain or show why Diego's method works.

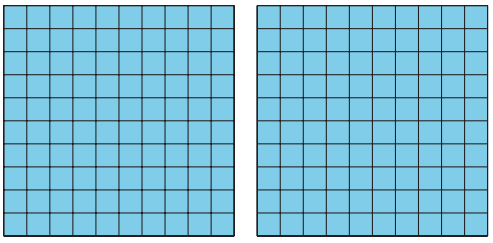
b. Use his method to find the value of 17.5×3.3.

7 Exploration

a. Han says the diagram shows $4 \times 0.5 = 2$. Label the diagram to show Han's reasoning.



b. Mai says it shows $10 \times 0.2 = 2$. Label the diagram to show Mai's reasoning.



c. What other products can the diagram represent? Explain or show your reasoning.

Unit 5, Lesson 22

Addressing CA CCSSM 5.NBT.7, 5.NF.7b; building towards 5.NBT.1; practicing MP3

Divide Whole Numbers by 0.1 and 0.01

Let's divide whole numbers by one tenth and one hundredth.

Warm-up

Number Talk: Remember Division of Unit Fractions

Find the value of each expression mentally.

- $1 \div \frac{1}{10}$
- $2 \div \frac{1}{10}$
- $1 \div 0.01$
- $2 \div 0.01$

Activity 1

Patterns in Dividing by Decimal Units

1. Find the value of each expression. Explain or show your reasoning. Use a diagram if it is helpful.

   a. $3 \div 0.1$

   b. $3 \div 0.01$

   c. $4 \div 0.1$

   d. $4 \div 0.01$

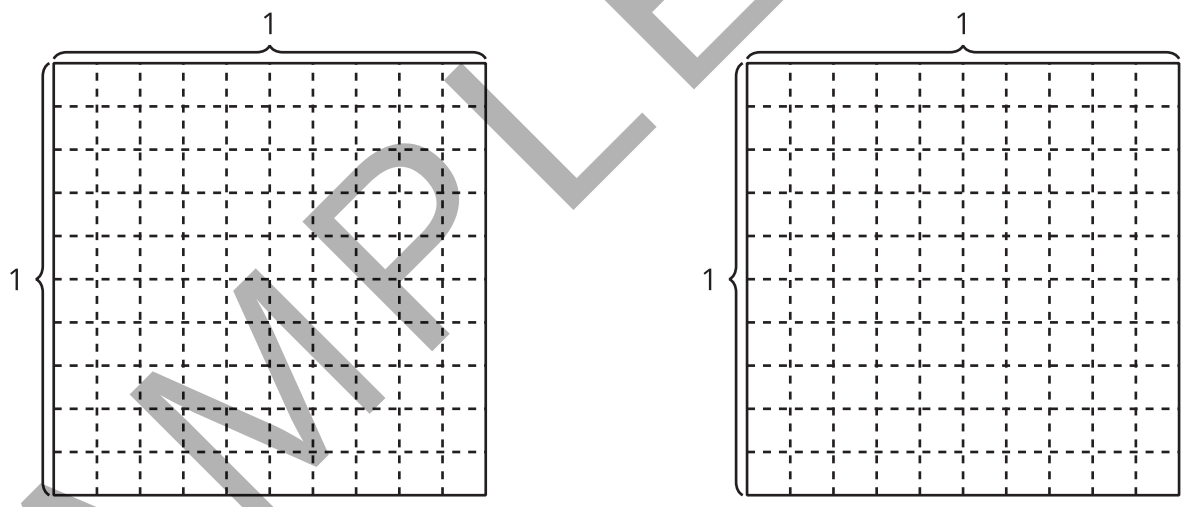
2. What patterns do you notice?

Activity 2

Divide Whole Numbers by Decimals

1. Describe how you can find the value of any whole number divided by 0.1. Use a diagram if it is helpful.

2. Describe how you can find the value of any whole number divided by 0.01. Use a diagram if it is helpful.



Unit 5, Lesson 23

Addressing CA CCSSM 5.NBT.7, 5.OA.2; practicing MP8

Divide Whole Numbers by Decimals

Let's divide whole numbers by decimals.

Warm-up

True or False: Tenths and Hundredths

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $6 \div 0.01 = 60$

- $6 \div 0.1 < 6 \div 0.01$

- $6 \div 0.01 = 60 \div 0.1$

Activity 1

Same Divisor, Different Dividend

1. Find the value of each expression. Explain or show your reasoning.

   a. $1 \div 0.2$

   b. $2 \div 0.2$

   c. $3 \div 0.2$

   d. $4 \div 0.2$

2. Find the value of each expression. Explain or show your reasoning.

   a. $1 \div 0.02$

   b. $2 \div 0.02$

   c. $3 \div 0.02$

   d. $4 \div 0.02$

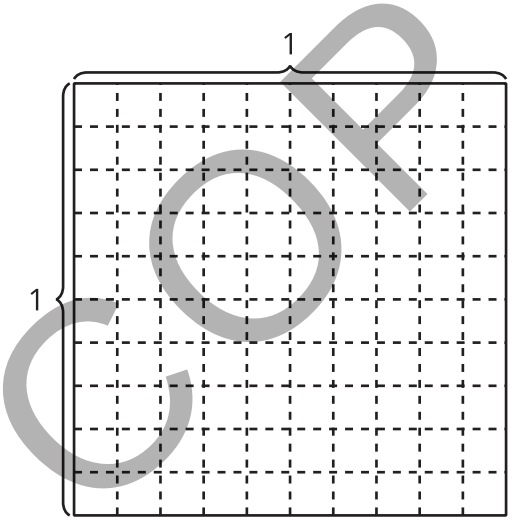
3. What patterns do you notice?

Activity 2

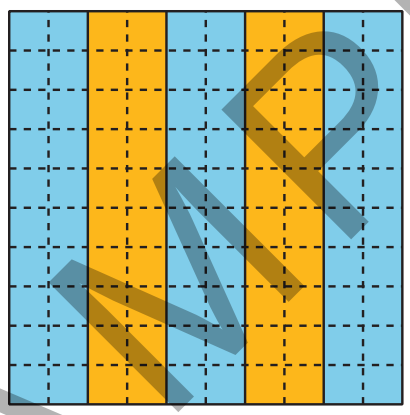
Strategies with Larger Dividends

1. Find the value of the expression. Explain or show your reasoning. Use a diagram if it is helpful.

$12 \div 0.2$



2. Tyler uses this diagram and explanation to justify why $12 \div 0.2 = 60$.



$12 \div 0.2 = 60$

There are 5 groups of 0.2 in 1 and there are 12 so that is 12 groups of 5.

Explain how the expression $12 \times (1 \div 0.2)$ relates to Tyler's reasoning.

3. Find the value of each expression.

   a. $14 \div 0.5$

   b. $5 \div 0.25$

Unit 5, Lesson 24

Addressing CA CCSSM 5.NBT.7; building towards 5.NBT.7; practicing MP1

Divide Decimals by Whole Numbers

Let's divide decimals by whole numbers.

Warm-up

Estimation Exploration: Divide by Whole Numbers

What is the value of $0.42 \div 5$?

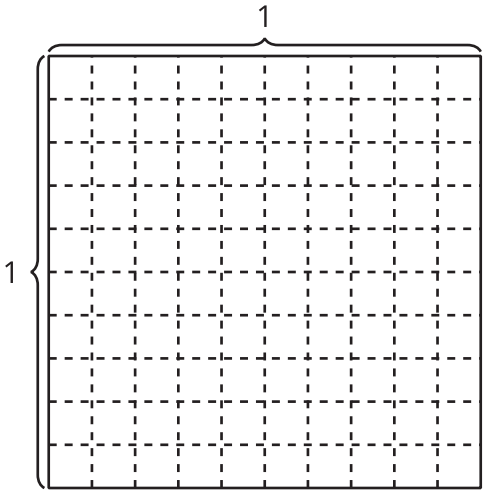
Record an estimate that is:

too low	about right	too high

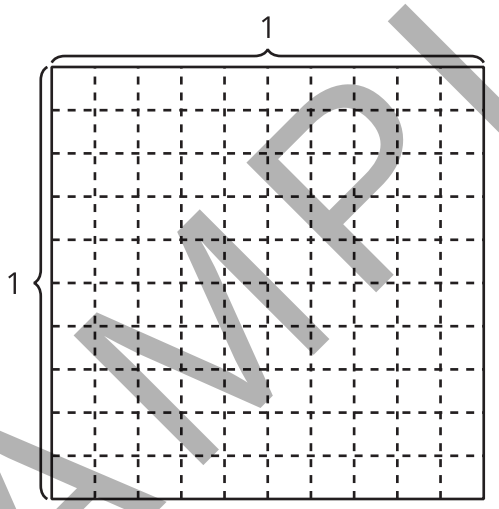
Activity 1

Whole Number Groups

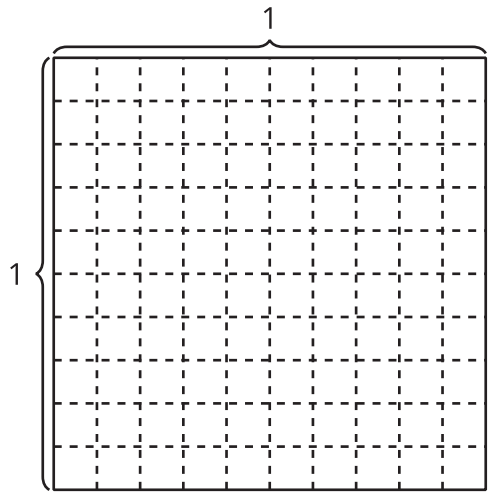
1. Find the value of the expression $0.8 \div 4$. Explain or show your reasoning.



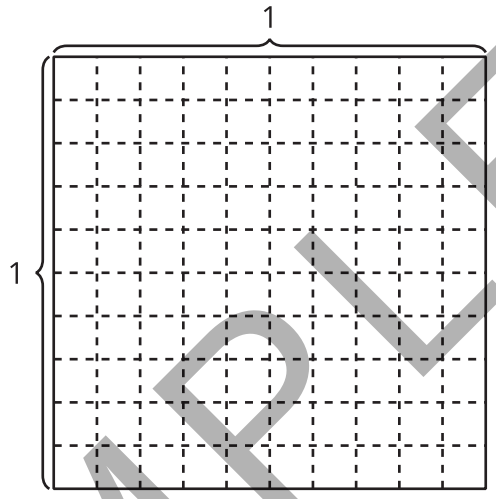
2. Find the value of the expression $0.6 \div 3$. Explain or show your reasoning.



3. Find the value of $0.5 \div 2$. Explain or show your reasoning.



4. Find the value of $0.75 \div 5$. Explain or show your reasoning.



Activity 2

Expressions with Decimal Quotients

1. Find the value of each expression. Explain your reasoning.

   a. $60 \div 5$

   b. $6 \div 5$

   c. $0.6 \div 5$

2. Find the value of each expression. Explain your reasoning.

   a. $0.7 \div 2$

   b. $0.45 \div 5$

Unit 5, Lesson 25

Addressing CA CCSSM 5.NBT.7; practicing MP6

Divide Decimals by Decimals

Let's divide decimals by decimals.

Warm-up

Number Talk: Same/Different

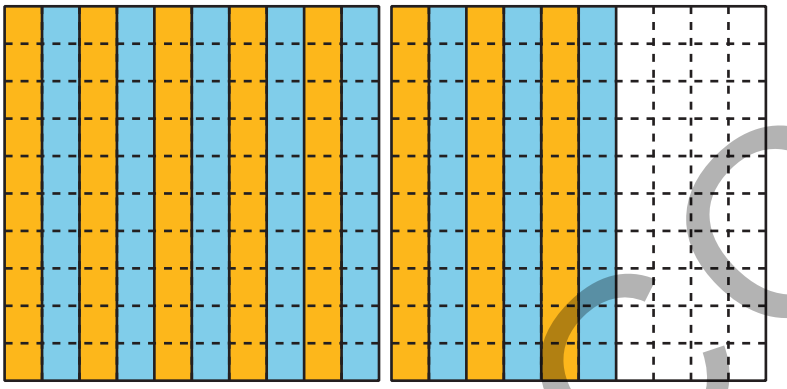
Find the value of each expression mentally.

- $20 \div 2$
- $2 \div 0.2$
- $50 \div 2$
- $5 \div 0.2$

Activity 1

Dividing by a Tenth and a Hundredth

1. Jada draws this diagram to find the value of $1.6 \div 0.1$.



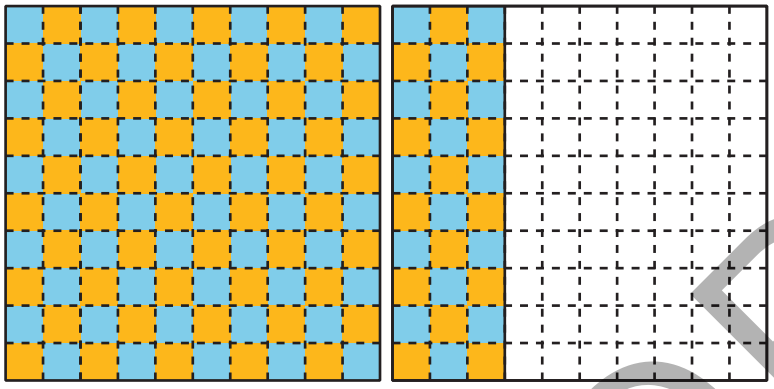
a. Describe how the diagram shows 1.6.

b. Describe how the diagram shows 16 groups of 1 tenth.

c. Describe how the diagram shows the value of $1.6 \div 0.1$.

d. Describe how the diagram also represents the expression $160 \div 10$.

2. a. Describe how this diagram represents $1.3 \div 0.01$.



b. What is the value of $1.3 \div 0.01$? Explain or show your reasoning.

Activity 2

Divide Decimals by Decimals

Find the value of each expression. Explain or show your reasoning.

1. $5 \div 0.1$

2. $5 \div 0.01$

3. $0.5 \div 0.1$

4. $0.5 \div 0.01$

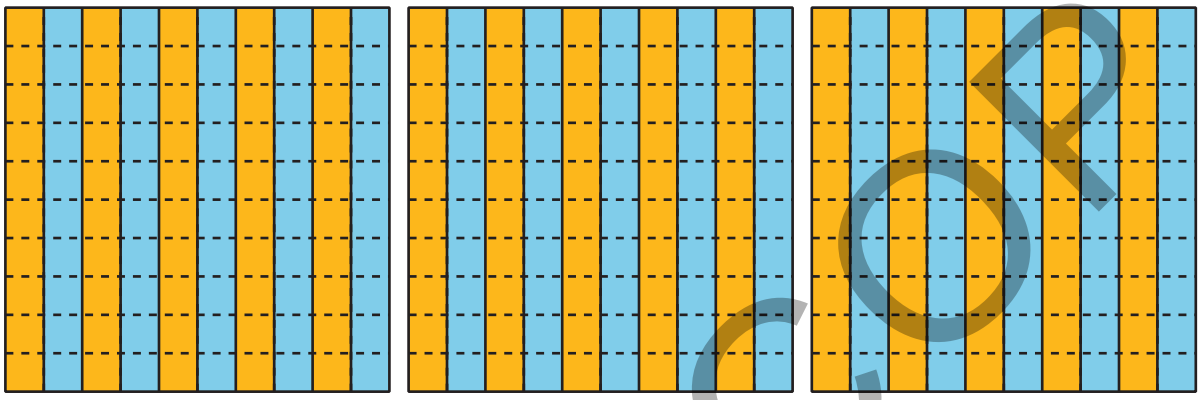
5. $0.02 \div 0.01$

6. $1.53 \div 0.01$

Section D Summary

We learned different strategies for dividing with decimals.

We used diagrams. This diagram shows there are 10 groups of 0.1 in each whole. So, there are 3×10 or 30 groups of 0.1 in 3 wholes.



We thought about place value. Since 3 is 30 tenths and 0.1 is 1 tenth, $3 \div 0.1$ is equivalent to $30 \div 1$. Both have the value 30.

We also used the relationship between multiplication and division. We know that $10 \times 0.1 = 1$. So, $3 \div 0.1 = 30$.

Unit 5, Lesson 26

Addressing CA CCSSM 5.NBT.7; building towards 5.NBT.7; practicing MP4

Book Fair

Let's plan a book fair fundraiser.

Warm-up

Notice and Wonder: Books for Sale

What do you notice? What do you wonder?

Sec D

Activity 1

Book Prices

Two schools buy science books for $8 each from a publisher to sell at their book fairs. School A sells the books for $12. School B sells the books for $12.90

1. Who do you think sells more science books? Explain or show your reasoning.

2. How much profit does each school make if they each sell 35 books?

3. School B sells 10 science books. How many science books does School A have to sell to raise about the same amount of money?

Activity 2

Plan a Book Fair

Price list from the publisher:

type of book	price
boxed sets & collections	$24.95
comic books	$2.60
science books	$8.00
chapter books	$9.99
history books	$14.49
audiobooks	$20.00
activity books	$4.50
reference books	$12.00
Spanish language books	$6.00
biographies	$6.05

Plan a book fair:

1. Choose 3–5 types of books to order.

2. Decide on the mark-up price for each type of book you chose.

3. Estimate how much money your school will raise as a profit with your book fair.

   Record an estimate that is:

too low	about right	too high

4. Explain or show your reasoning for the estimate. Include the assumptions you made.

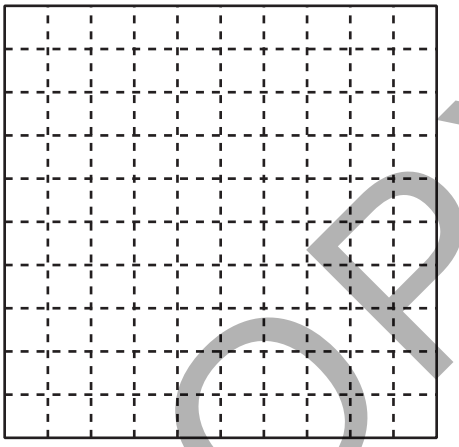
Practice Problems

7 Problems

1

from Unit 5, Lesson 22

a. Find the value of $1 \div 0.01$. Use the diagram if it is helpful.



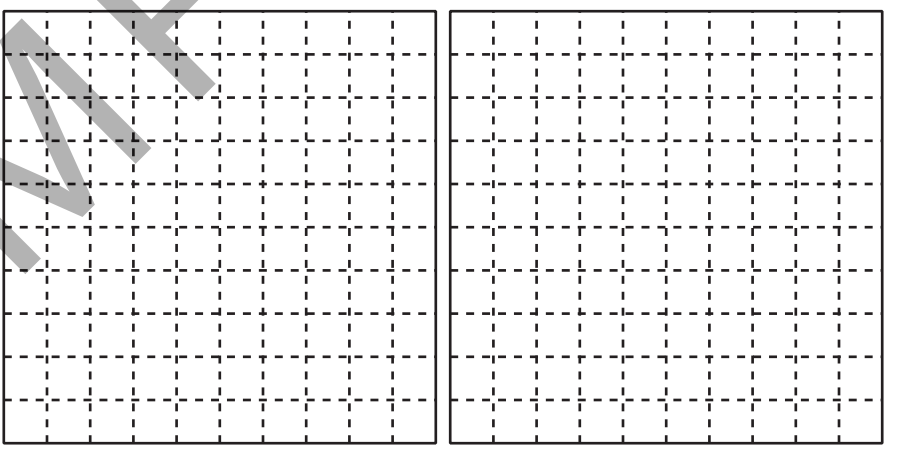
b. Jada says there are 100 hundredths in 1, so $1 \div 0.01$ is 100. Do you agree? Explain or show or explain your reasoning.

Sec D

2

from Unit 5, Lesson 23

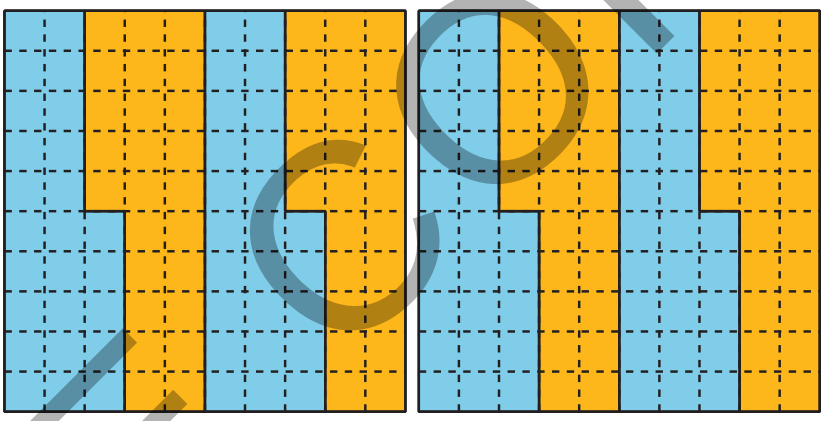
a. Find the value of $2 \div 0.2$. Use the diagram if it is helpful.



b. Find the value of $21 \div 0.2$.

3 from Unit 5, Lesson 23

Here is a diagram.



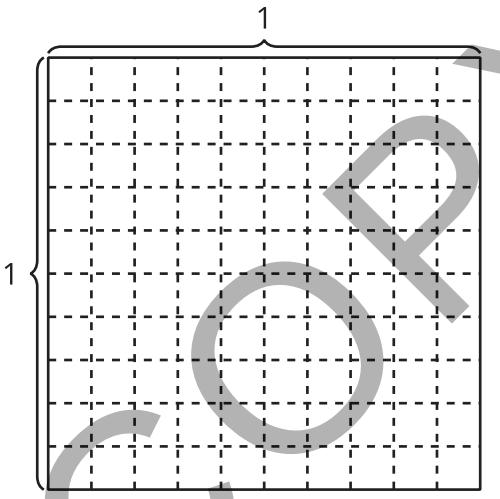
a. Explain or show how the diagram shows $200 \div 25$. What is the value of the expression?

b. Explain or show how the diagram shows $2 \div 0.25$. What is the value of the expression?

4 from Unit 5, Lesson 24

Find the value of each expression. Explain or show your reasoning.

a. $0.2 \div 5$. Use the diagram if it is helpful.



b. $6 \div 3$

c. $6 \div 0.3$

5 from Unit 5, Lesson 25

Find the value of each expression. Explain or show your reasoning.

a. $0.5 \div 0.1$

b. $0.5 \div 0.01$

c. $3.5 \div 0.01$

6 Exploration

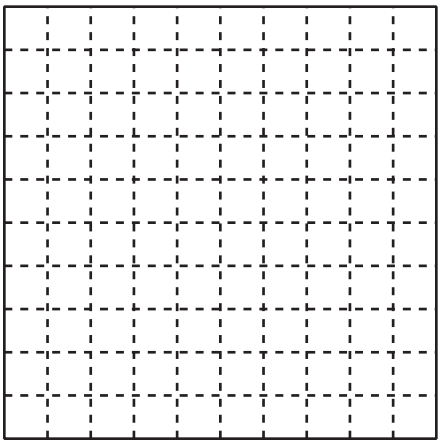
Noah has a scale that weighs to the nearest ounce. The table shows the weights of different numbers of paper clips Noah found using the scale.

How many ounces do you think each paper clip weighs? Explain or show your reasoning.

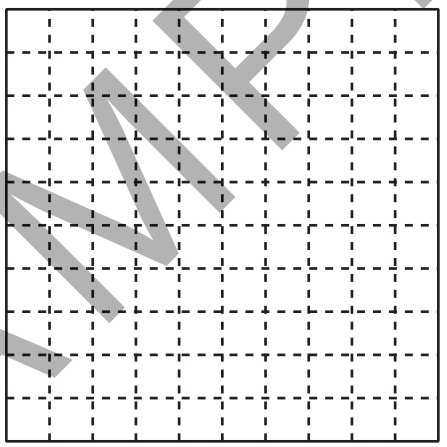
number of paper clips	weight (ounces)
1	0
10	0
20	1
25	1
50	2
100	3

The daily recommended allowance of vitamin C for a 5th grader is 0.045 gram.

a. One vitamin C tablet has 1 gram of vitamin C. Approximately how many times the daily recommended allowance of vitamin C is 1 tablet? Use the diagram if it is helpful.



b. A large orange has 0.18 gram of vitamin C. Approximately how many times the daily recommended allowance of vitamin C is in a large orange? Use the diagram if it is helpful.



GRADE 5

UNIT

More Decimal and Fraction Operations

Content Connections

In this unit you will solve multi-step problems involving measurement conversions, line plots, and fraction operations. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while connecting decimal notation to fraction representations.
- **Discovering Shape and Space** while using diagrams and expressions to show their work.
- **Reasoning with Data** while looking for patterns when multiplying and dividing by powers of ten, interpreting multiplication as scaling and comparing products with factors.
- **Exploring Changing Quantities** while converting decimals from a smaller unit to a larger unit and adding and subtracting fractions with different denominators.

Addressing the Standards

As you work your way through **Unit 6 More Decimal and Fraction Operations,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	
MP2 Reason abstractly and quantitatively.	Lessons 4, 6, 12, 14, 16, and 17
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 7, 10, 13, and 14
MP4 Model with mathematics.	Lesson 21
MP5 Use appropriate tools strategically.	Lesson 8
MP6 Attend to precision.	Lessons 5, 12, 15, and 20
MP7 Look for and make use of structure.	Lessons 1, 2, 3, 4, 9, 11, 17, 18, and 19
MP8 Look for and express regularity in repeated reasoning.	Lessons 4, 10, and 19

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Powers and Place Value	**5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Lessons 1, 2, 5, and 6
• Powers and Place Value	**5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	Lessons 2, 3, 4, and 6
• Layers of Cubes	**5.MD.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.	Lessons 3, 4, 5, 6, and 7

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Plotting Patterns • Fraction Connections	**5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	Lessons 14, 15, and 21
• Modeling • Fraction Connections	**5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	Lessons 8, 9, 10, 11, 12, 13, and 14
• Modeling • Fraction Connections	**5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	Lessons 9, 11, 12, 15, and 21
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Lessons 15 and 21

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.5** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Lesson 20
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.5a** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	Lessons 16, 17, and 18
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.5b** Interpret multiplication as scaling (resizing), by: b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Lessons 17, 19, and 20
• Factors and Groups • Powers and Place Value • Layers of Cubes	**5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Lesson 20

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 6, Lesson 1

Addressing CA CCSSM 5.NBT.1; practicing MP7

Place-Value Patterns

Let's observe place-value patterns.

Warm-up

Notice and Wonder: Same Digits

What do you notice? What do you wonder?

8,200

820

82

8.2

0.82

0.082

Activity 1

Many True Equations

Use these numbers and symbols to write as many true equations as you can. You may use each number and symbol more than once.

600	0.06	100
60	×	10
6	÷	0.1
0.6	=	0.01

Activity 2

Describe Multiplicative Relationships

600
60
6
0.6
0.06

1. Explain or show how the value of the 6 changes in each of the numbers.

2. Which numbers come above 600 if the list continues? Explain your reasoning.

3. Which numbers come below 0.06 if the list continues? Explain your reasoning.

Addressing CA CCSSM 5.NBT.1, 5.NBT.2; building towards 5.NBT.2; practicing MP7

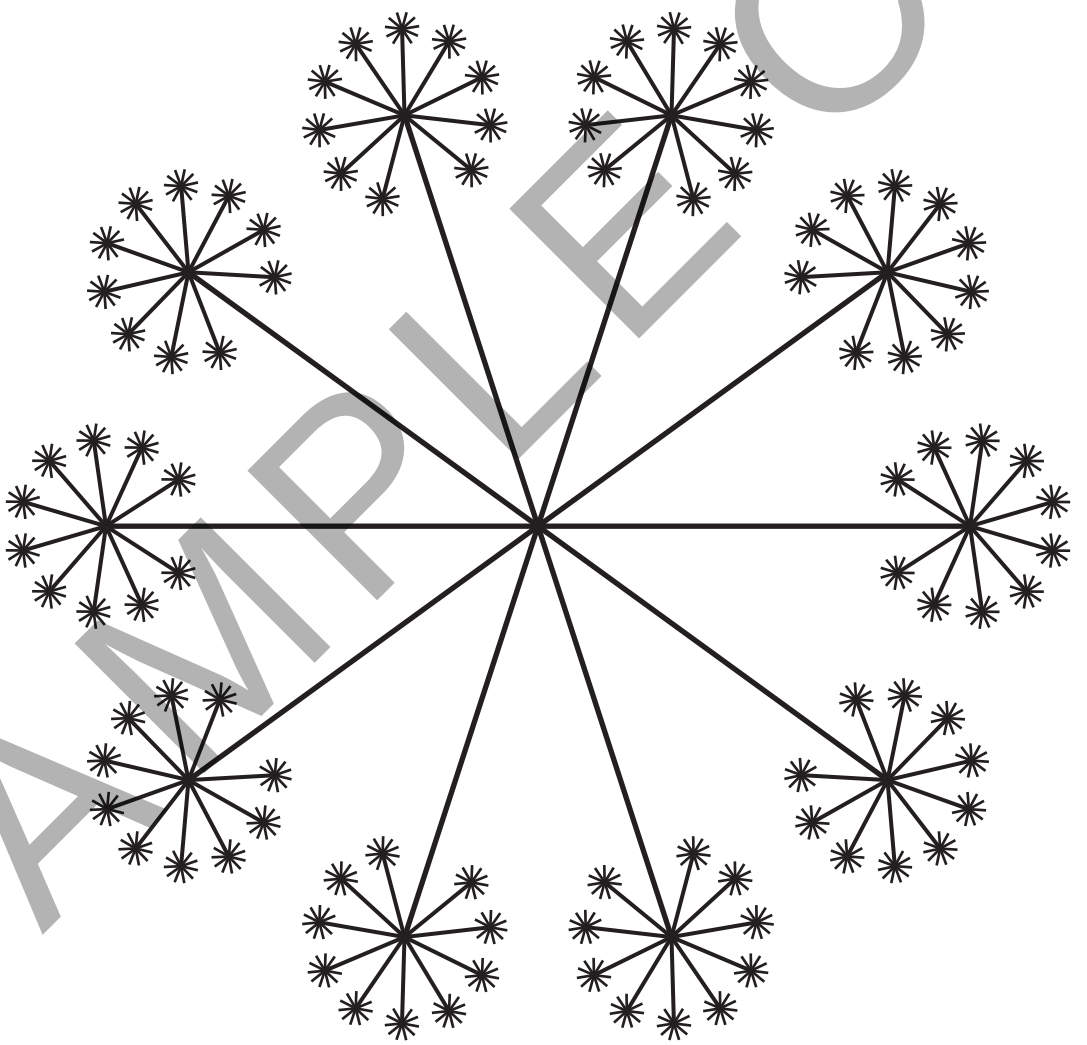
Powers of 10

Let's use exponents to show powers of 10.

Warm-up

How Many Do You See: Snowflake

How many do you see? How do you see them?



Activity 1

Populations of Delaware and the United States

1. About 1,000,000 people live in Delaware.

   a. How do you say this number?

   b. How many thousands is 1,000,000? Explain or show your reasoning.

   c. Write the number, using powers of 10.

   d. Look at the diagram in the *Warm-up*. If each snowflake is made up of smaller snowflakes, how many times do you expand the focus of the diagram to see 1,000,000 tiny segments? Explain or show your reasoning.

2. In 2023, the population of the United States was about one-third of 1,000,000,000.

   a. How would you say 1,000,000,000?

   b. How many millions is 1,000,000,000? How many thousands is it? Explain or show your reasoning.

   c. Write the number, using powers of 10.

   d. Look at the diagram in the *Warm-up*. If each snowflake is made up of smaller snowflakes, how many times do you expand the focus of the diagram to see 1,000,000,000 tiny segments? Explain or show your reasoning.

Activity 2

Powers of 10

1. Find the unknown number that makes each equation true. Explain your reasoning.

   a. $2{,}000 = \_\_\_\_\_ \times 20$

   b. $20 \times 10 \times \_\_\_\_\_ = 20{,}000$

   c. $\_\_\_\_\_ \times 10 = 100{,}000$

   d. $1{,}000 \times 10{,}000 = \_\_\_\_\_$

2. How are products of 10 useful in solving these problems?

3. Write each power of 10 as a number.

   a. 10^3

   b. 10^4

   c. 10^7

Activity 3

Beyond One Billion

1. How would you say the number 1,000,000,000,000?

2. How many billions is 1,000,000,000,000? How many millions? Explain or show your reasoning.

3. Write the number, using powers of 10.

4. Describe an example of something in the world that numbers 1,000,000,000,000.

Unit 6, Lesson 3

Addressing CA CCSSM 5.MD.1, 5.NBT.2; practicing MP7

Metric Conversion and Multiplication by Powers of 10

Let's notice patterns in metric measurements.

Warm-up

Number Talk: Multiply Then Divide

Find the value of each expression mentally.

- 100×1.5
- $1{,}000 \times 1.5$
- $15 \div 10$
- $15 \div 100$

Activity 1

How Tall? How Long? How Far?

1. Complete the table.

meters	centimeters	millimeters
1		
10		
10^2		

2. What patterns do you notice in the table?

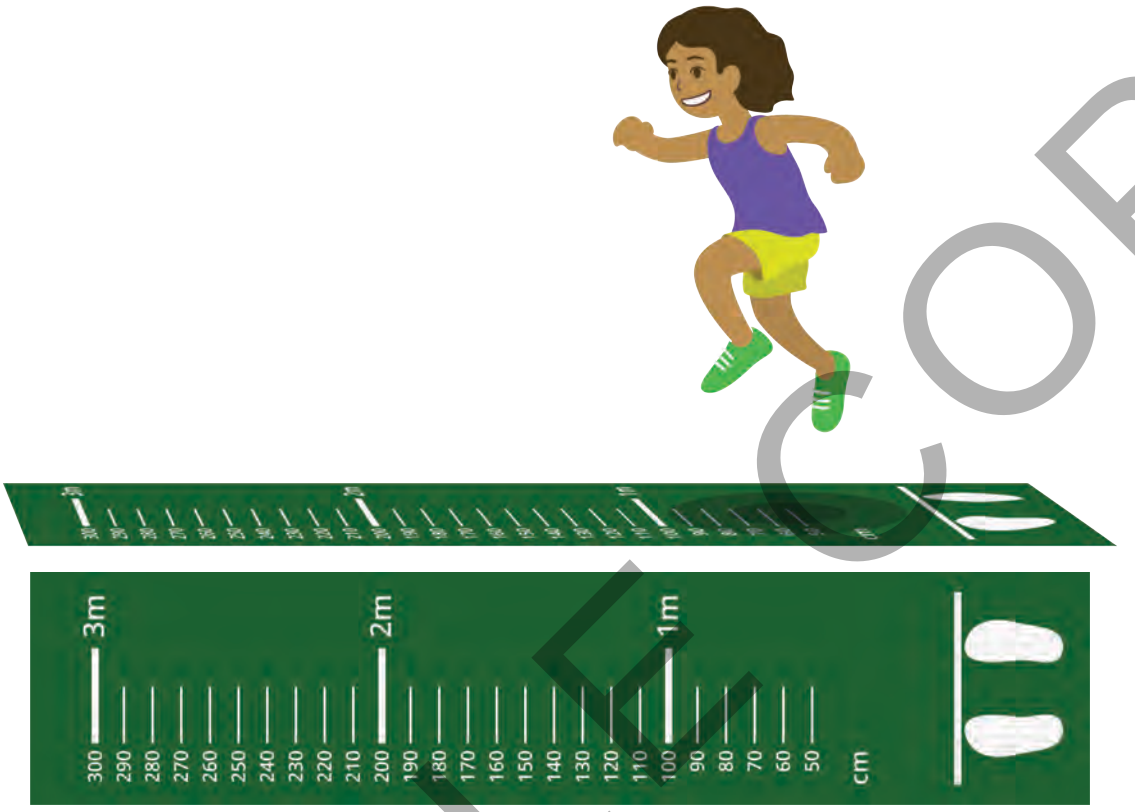
3. There are 3 long-distance races: 10 kilometers, 100 kilometers, and 1,000 kilometers. Complete the table to show these distances in meters.

distance in kilometers	distance in meters
1	1,000
10	
100	
10^3	

4. What patterns do you notice in the table?

Activity 2

Broad Jump



Here are the distances that each student jumped in the standing broad jump.

student	distance
Mai	1.61 meters
Elena	1.43 meters
Clare	1.57 meters

1. The average distance for 5th graders is 148 centimeters. Is each student in the table below, at, or above the average distance? Explain or show your reasoning.

2. Elena says her jump sounds more impressive if she reports it in millimeters.

   a. How far is Elena's jump in millimeters? What about Mai's and Clare's jumps?

   b. Which unit do you think is best for reporting the jumps? Explain your reasoning.

Unit 6, Lesson 4

Addressing CA CCSSM 5.MD.1, 5.NBT.2; practicing MP2, MP7, MP8

Metric Conversion and Division by Powers of 10

Let's convert units.

Warm-up

True or False: Divide by a Hundred and by a Thousand

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5 \div 1{,}000 = 0.05$

- $36 \div 100 = 0.36$

- $1{,}328 \div 1{,}000 = 1.328$

Activity 1

Long Jump, Javelin Throw, and Shot Put

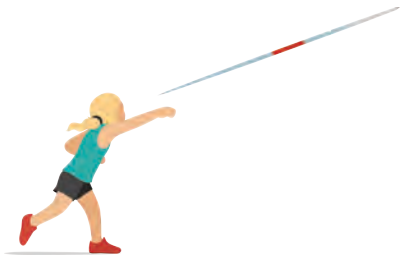
athlete	long jump	javelin throw	shot put
Jackie Joyner-Kersee, USA	727 cm	4,566 cm	1,580 cm
Sabine John, East Germany	671 cm	4,256 cm	1,623 cm
Anke Behmer, East Germany	678 cm	4,454 cm	1,420 cm

1. At the 1988 Summer Olympics, Jackie Joyner-Kersee set the still-standing world record in the heptathlon, which is a combination of 7 track-and-field events. These are the results, in centimeters, of some of the events, showing the first-, second-, and third-place athletes. Complete the table to show Joyner-Kersee's distances in meters.

event	centimeters	meters
long jump	727	
javelin throw	4,566	
shot put	1,580	

2. Which unit of measure, centimeters, or meters, is most helpful when you picture each distance? Explain or show your reasoning.

3. Why do you think distances are measured to the nearest centimeter?



Activity 2

Hurdles

1. The table shows how many meters students ran during a week. Complete the table to show these distances in kilometers.

student	distance (meters)	distance (kilometers)
Diego	9,513	
Clare	11,018	
Priya	8,210	
Andre	10,000	

2. What patterns do you notice in the table?

3. This is Tyler's strategy to divide a whole number by 10, by 100, or by 1,000.

I find the quotient by shifting the digits to the right — once when I divide by 10, twice when I divide by 100, 3 times when I divide by 1,000.

5,632 / 10 = 563.2
5,632 / 100 = 56.32
5,632 / 1,000 = 5.632

Describe Tyler's strategy to your partner.

(Pause for teacher direction.)

4. Why does Tyler's strategy work? Does Tyler's strategy always work? Explain or show your reasoning.

Unit 6, Lesson 5

Addressing CA CCSSM 5.MD.1, 5.NBT.1; practicing MP6

Multi-step Conversion Problems: Metric Lengths

Let's solve multi-step problems about metric length.

Warm-up

True or False: Powers of 10

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5{,}423 \times 10 = 50{,}423$

- $5{,}423 \div 10 = 542.3$

- $5{,}423 \div 100 = 54.23$

Activity 1

Walk All Day

Lin has a watch that counts the number of steps she takes during the day. The watch displays her steps in centimeters, meters, or kilometers.

1. Here is a list of activities Lin did on Monday. Next to each activity, write whether it would make sense to display the distance in centimeters, meters, or kilometers.
   - walked to her friend's desk
   - walked to the front of the classroom
   - walked from her classroom to the bus
   - ran twice around the playground

2. The table shows the amount of steps Lin's watch displayed for each activity. If each of Lin's steps is 50 centimeters, how many centimeters and how many meters did she walk for each activity?

activity	number of steps	distance (cm)	distance (m)
walked to her friend's desk	5		
walked to the front of the classroom	12		
walked from her classroom to the bus	250		
ran twice around the playground	1,000		

3. At the end of the day, Lin's watch displayed 8,500 steps. Should her watch record the distance in centimeters, meters, or kilometers? Explain your reasoning.

4. How many kilometers did Lin walk on Monday?

Activity 2

Who Ran Farther?

1. Use the table to find the total distance, in kilometers, Tyler ran during the week. Explain or show your reasoning.

day	distance (km)
Monday	8.5
Tuesday	6.25
Wednesday	10.3
Thursday	5.75
Friday	9.25

2. Use the table to find the total distance, in meters, Clare ran during the week. Explain or show your reasoning.

day	distance (m)
Monday	5,400
Tuesday	7,500
Wednesday	8,250
Thursday	6,750
Friday	7,250

3. Who ran farther, Clare or Tyler? How much farther? Explain or show your reasoning.

Unit 6, Lesson 6

Addressing CA CCSSM 5.MD.1, 5.NBT.1, 5.NBT.2; building on 5.MD.1, 5.NBT.2; practicing MP2

Multi-step Conversion Problems: Metric Liquid Volumes

Let's solve multi-step problems about metric liquid volume.

Warm-up

Number Talk: Divide by Powers of 10

Find the value of each expression mentally.

- $1{,}400 \div 10$
- $1{,}400 \div 100$
- $1{,}400 \div 1{,}000$
- $1{,}401 \div 1{,}000$

Activity 1

Liquid-Volume Conversions

330 mL 500 mL 750 mL 1 L 1.5 L

1. Complete the table.

L	mL
5	
6.3	
0.95	
10^2	
	800,000
	10^6
	65

2. Decide if the two measurements are equal. If not, choose the measurement that is greater. Explain or show your reasoning.

   a. 15 mL and 0.15 L

   b. 2,500 mL and 2.5 L

   c. 200 mL and $\frac{1}{4}$ L

   d. 1 mL and $\frac{1}{1{,}000}$ L

   e. 15,600 mL and 15.5 L

Activity 2

Hydrating Dancers

There are 25 dancers in the performance group. During practice, each dancer drinks $1\frac{1}{2}$ bottles of water.

1. Each bottle holds 500 mL of water. How many liters of water do the dancers drink in total? Explain or show your reasoning.

2. Each cooler holds 15 L of water. How many coolers does the group need? How many liters of water are left after practice if all of the coolers are full at the start of practice? Explain or show your reasoning.

3. The dancers make a sports drink by dissolving 30 mL of drink mix into each 500 mL of water. How many liters of drink mix does the team need for their practice? Explain or show your reasoning.

Unit 6, Lesson 7

Addressing CA CCSSM 5.MD.1; building towards 5.MD.1; practicing MP3

Multi-step Conversion Problems: Customary Lengths

Let's solve multi-step problems about customary length.

Warm-up

Number Talk: Multiples of 12

Find the value of each expression mentally.

- 45×2
- 45×10
- 45×12
- 46×12

Activity 1

Card Sort: Customary Measurements

Your teacher will give you a set of cards that show different measurements.

1. Sort the cards into categories in a way that makes sense to you. Be ready to explain the meaning of each category.

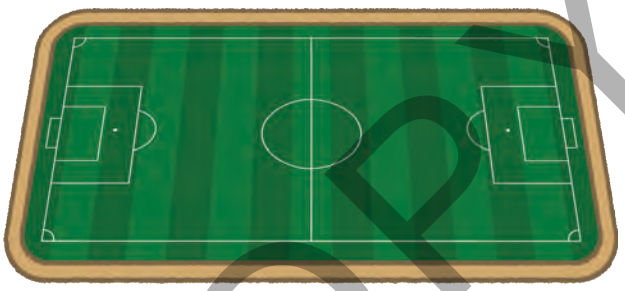
   (Pause for teacher directions.)

2. Sort the cards into groups that represent equal lengths.
3. For each unit of measure, order the lengths from shortest to longest.

Activity 2

Run a Mile or Two

1. A rectangular field is 90 yards long and $42\frac{1}{4}$ yards wide. Priya says that 6 laps around the field is a greater distance than a mile. Do you agree? Explain or show your reasoning.



2. A different rectangular field is $408\frac{1}{2}$ feet long and $240\frac{1}{4}$ feet wide. How many laps around this field does Priya run if she runs at least 2 miles?

Section A Summary

We studied **powers of 10** and conversions between units. We learned that we can write a product of 10s, such as $10 \times 10 \times 10 \times 10$, as a power of 10, such as 10^4. The number 4 is an **exponent** and it means that there are 4 factors of 10.

We also converted between different units of measure. There are 1,000 millimeters in a meter and 1,000 meters in a kilometer. This means that there are $1{,}000 \times 1{,}000$ or $1{,}000{,}000$ millimeters in a kilometer. We also can say there are 10^6 millimeters in a kilometer.

We used our understanding of decimals to make conversions. There are 1,000 meters in a kilometer. Each meter is $\frac{1}{1{,}000}$ or 0.001 kilometer. So, 853 meters can also be written as 0.853 kilometer.

Practice Problems

16 Problems

1 Pre-unit

Find the number that makes each equation true. Explain or show your reasoning.

a. $\frac{\square}{12} = \frac{2}{3}$

b. $\frac{5}{6} = \frac{\square}{12}$

2 Pre-unit

a. The road around a lake is 15 kilometers long. What is the distance in meters?

b. The length of an alligator is 4 meters. What is the length in centimeters?

3 Pre-unit

The value of the 6 in 618,204 is how many times the value of the 6 in 563? Explain or show your reasoning.

4 Pre-unit

Find the value of each sum.

a. $\frac{3}{8} + \frac{9}{8}$

b. $3\frac{1}{5} + \frac{3}{5}$

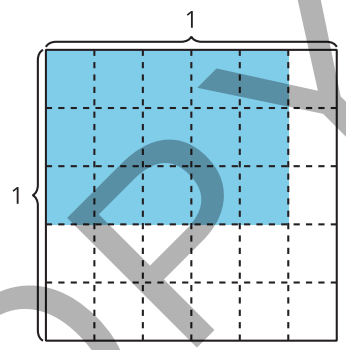
c. $2\frac{4}{10} + 1\frac{7}{10}$

5 Pre-unit

Lin spends 5 minutes reading a story. Noah spends 3 times as long as Lin. How long does Noah spend reading the story? Explain or show your reasoning.

6 Pre-unit

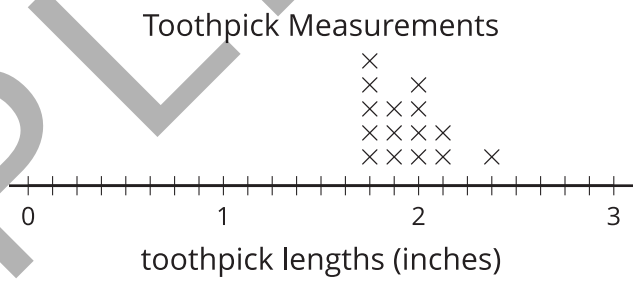
a. Write a multiplication expression for the shaded area, and find the value of the expression. Explain or show your reasoning.



b. Find the value of $\frac{5}{7} \times \frac{10}{3}$.

7 Pre-unit

The line plot shows the lengths of some toothpicks, in inches.



a. How many measurements are there?

b. What is the difference in length between the longest toothpick and the shortest toothpick?

8 from Unit 6, Lesson 1

a. Write a multiplication equation relating the values of 0.5 and 0.05.

b. Write a division equation relating the values of 0.5 and 0.05.

9 from Unit 6, Lesson 2

Write each number, using exponential form.

a. $10 \times 10 \times 10$

b. 100×100

c. 100,000

d. 1,000,000,000

10 from Unit 6, Lesson 3

a. How many centimeters are in each measurement?

0.12 m	3.5 m	19 m

b. How many millimeters are in each measurement?

3 m	37 m	1,915 m

c. How does a whole number of meters change when it is represented in millimeters? Explain or show your reasoning.

11 from Unit 6, Lesson 4

a. How many meters are in each measurement?

16 millimeters 1,375 millimeters 57 millimeters

b. How does a whole number of millimeters change when you represent the measurement in meters? Explain or show your reasoning.

12 from Unit 6, Lesson 5

A track is 366 meters around. An athlete runs 15 laps. What is the distance in kilometers? Explain or show your reasoning.

13 from Unit 6, Lesson 6

Clare drinks 8 glasses of water each day. There are 235 milliliters in each glass. How many liters of water does Clare drink each day? Explain or show your reasoning.

14 from Unit 6, Lesson 7

A track is 400 yards around. How many full laps does Tyler run if he runs at least 2 miles?

15 Exploration

a. Write each number, using exponential form.

i. 1,000,000,000 (the approximate combined population of North America and South America, in 2023)

ii. 100,000,000,000 (the estimated number of stars in the Milky Way)

iii. 100,000,000,000,000 (the denomination of a bill in Zimbabwean (zim-BAHB-way-yen) dollars, from 2008)

iv. 10,000,000,000,000,000,000 (the estimated number of grains of sand on Earth)

v. 1,000,000,000,000,000,000,000,000 (the estimated number of stars in the universe)

vi. 10,000,000,000,000,000,000,000,000,000,000,000,000,000 (the estimated number of living cells that ever existed on Earth)

vii. 100,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000 (the estimated number of atoms in the universe!)

b. How is exponential form helpful for writing these numbers?

16 Exploration

You have a piggy bank with 1 kilogram of coins inside. Which coins do you want to have in the piggy bank? Explain or show your reasoning.

coin	approximate weight (grams)
penny	2.5
nickel	5
dime	2.3
quarter	5.7

Addressing CA CCSSM 5.NF.1; building towards 5.NF.1; practicing MP5

Add and Subtract Fractions

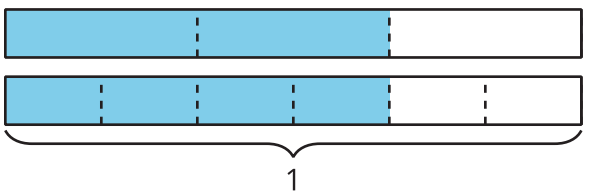
Let's add and subtract fractions.

Warm-up

Which Three Go Together: Fraction Representations

Which 3 go together?

A



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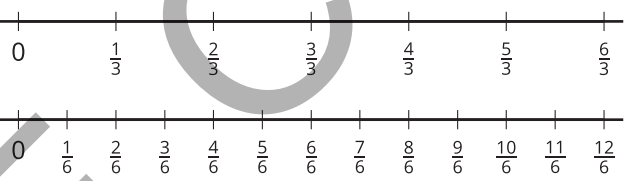

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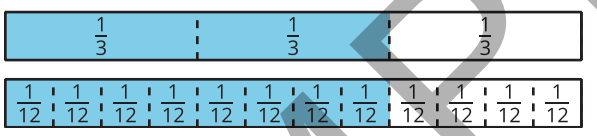

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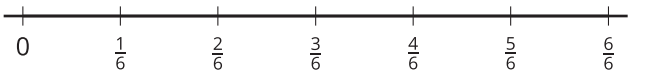


Activity 1

Card Sort: Fraction Sums and Differences

Your teacher will give you a set of cards that show expressions.

1. Sort the cards into 2 categories in a way that makes sense to you. Be ready to explain the meaning of each category.
2. Sort the cards into 2 categories in a different way. Be ready to explain the meaning of each new category.

Activity 2

Add and Subtract

Find the value of each expression. Show your thinking. Organize your work so it can be followed by others.

1. $\frac{2}{3} + \frac{2}{3}$

2. $\frac{2}{3} - \frac{1}{6}$

3. $\frac{2}{3} + \frac{1}{2}$

Unit 6, Lesson 9

Addressing CA CCSSM 5.NF.1, 5.NF.2; practicing MP7

Use Expressions with the Same Value

Let's use expressions with the same value to add and subtract fractions with unlike denominators.

Warm-up

True or False: Fraction Addition and Subtraction

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

- $\frac{1}{2} + \frac{1}{4} = \frac{2}{4}$

- $\frac{3}{4} - \frac{1}{2} = \frac{2}{4}$

Activity 1

Sums with the Same Value

1. Why is each expression equivalent to $\frac{2}{3} + \frac{10}{12}$? Explain or show your reasoning.
   - $\frac{8}{12} + \frac{10}{12}$
   - $\frac{4}{6} + \frac{5}{6}$
2. Find the value of the expression $\frac{2}{3} + \frac{10}{12}$. Explain or show your reasoning.

Activity 2

Find the Value of the Difference

1. Find the value of the expression $\frac{16}{12} - \frac{3}{6}$. Explain or show your reasoning.

2. Compare strategies with a partner. How are they alike? How are they different?

Activity 3

Grow Plants

Jada and Andre compare the growth of their plants. Jada's plant grew $1\frac{3}{4}$ inches since last week. Andre's plant grew $\frac{7}{8}$ inch. How much more did Jada's plant grow? Explain or show your reasoning.

Unit 6, Lesson 10

Addressing CA CCSSM 5.NF.1; practicing MP3 and MP8

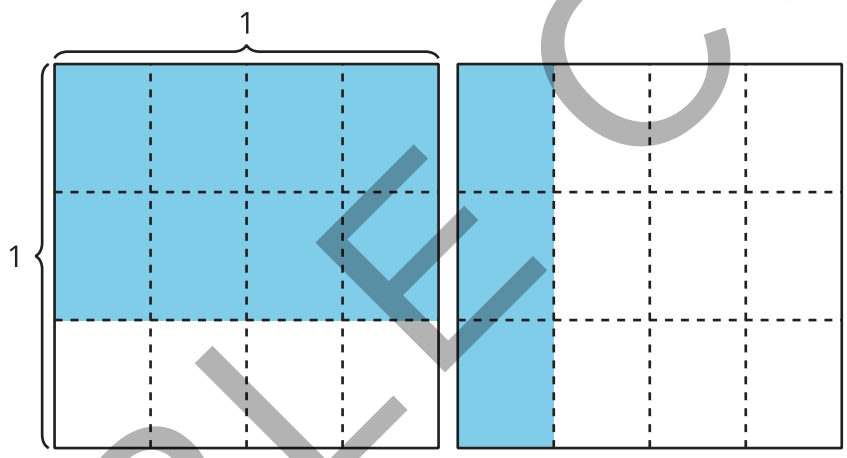
All Sorts of Denominators

Let's find common denominators.

Warm-up

How Many Do You See: Fraction Sum

How many do you see? How do you see them?



Activity 1

Different Denominators

Find the value of each expression. Show your thinking. Organize your work so it can be followed by others.

1. $\frac{3}{4} + \frac{7}{8}$

2. $\frac{3}{4} + \frac{4}{6}$

3. $\frac{3}{4} - \frac{2}{5}$

Activity 2

Multiply Denominators

1. Here is Lin's strategy for finding the value of $\frac{2}{5} + \frac{4}{9}$: "I know 5×9 is a common denominator so I'll use that." Does Lin's strategy for finding a common denominator work? Explain or show your reasoning. Then find the value of $\frac{2}{5} + \frac{4}{9}$.

2. Find the value of each expression.

   a. $\frac{3}{8} + \frac{1}{5}$

   b. $\frac{7}{10} - \frac{2}{3}$

   c. $\frac{7}{20} + \frac{41}{50}$

   d. $\frac{2}{9} - \frac{1}{6}$

Addressing CA CCSSM 5.NF.1, 5.NF.2; practicing MP7

Different Ways to Subtract

Let's subtract fractions and mixed numbers.

Warm-up

Number Talk: Mixed-Number Addition and Subtraction

Find the value of each expression mentally.

- $3 + \frac{7}{8}$
- $3 - \frac{7}{8}$
- $1\frac{5}{8} + \frac{6}{8}$
- $1\frac{5}{8} - \frac{6}{8}$

Activity 1

Challenging Differences

1. Circle all of the expressions that are equivalent to $3\frac{5}{8}$. Explain or show your reasoning.
   - $\frac{20}{8}$
   - $2\frac{13}{8}$
   - $3\frac{10}{16}$
2. Find the value of each expression. Explain or show your reasoning.
   - $3\frac{5}{8} - \frac{3}{16}$
   - $3\frac{5}{8} - 1\frac{15}{16}$
   - $3\frac{5}{8} - 1\frac{12}{16}$

Activity 2

Find the Difference

Find the value of each difference. Explain or show your reasoning.

1. $9\frac{1}{8} - 8\frac{8}{9}$

2. $3\frac{1}{2} - \frac{10}{4}$

3. $4\frac{3}{5} - 1\frac{2}{3}$

Addressing CA CCSSM 5.NF.1, 5.NF.2; practicing MP2 and MP6

Solve Problems

Let's solve more problems by adding and subtracting fractions with unlike denominators.

Warm-up

Estimation Exploration: Large Denominators

What is the value of the sum?

$$\frac{3}{17} + \frac{17}{19}$$

Record an estimate that is:

too low	about right	too high

Activity 1

Priya's Salad Dressing

Priya's Salad Dressing Recipe

- $\frac{3}{4}$ cup olive oil
- $\frac{1}{3}$ cup lemon juice
- $\frac{1}{2}$ cup mustard
- Pinch of salt and pepper

1. Priya has $\frac{2}{3}$ cup of olive oil. She is going to borrow some more from her neighbor. How much olive oil does she need to borrow to have enough to make the dressing?

2. 1 tablespoon is equal to $\frac{1}{16}$ of a cup. Priya decides that 1 tablespoon of olive oil is close enough to what she needs to borrow. Do you agree? Explain or show your reasoning.

3. Priya says her recipe makes about $1\frac{1}{2}$ cups of dressing. Do you agree? Explain or show your reasoning.

Activity 2

More Problems to Solve

1. Choose a problem to solve.

   Problem A:

   Jada is baking protein bars for a hike. She adds $\frac{1}{2}$ cup of walnuts and then decides to add another $\frac{1}{3}$ cup. How many cups of walnuts has she added altogether?

   The recipe requires $1\frac{1}{3}$ cups of walnuts. How many more cups of walnuts does Jada add? Explain or show your reasoning.

   Problem B:

   Kiran and Jada hiked $1\frac{1}{2}$ miles and took a rest. Then they hiked another $\frac{4}{10}$ mile before stopping for lunch. How many miles have they hiked so far?

   The trail is a total of $2\frac{1}{2}$ miles. How much farther do they hike? Explain or show your reasoning.

2. Discuss the problems and solutions with your partner. How are your strategies alike? How are they different?

3. Revise your work if necessary.

Put It All Together: Add and Subtract Fractions

Let's add and subtract fractions with unlike denominators.

Warm-up

Number Talk: Sums with $\frac{1}{8}$

Find the value of each expression mentally.

- $\frac{1}{8} + \frac{5}{8}$
- $\frac{1}{8} + \frac{6}{16}$
- $\frac{1}{8} + \frac{1}{3}$
- $\frac{1}{8} + \frac{5}{12}$

Activity 1

Common Denominators

$$\frac{4}{6} + \frac{5}{8}$$

Tyler says: "I can find the sum, using 18 as a common denominator."

Han says: "I can find the sum, using 24 as a common denominator."

Clare says: "I can find the sum, using 48 as a common denominator."

1. With whom do you agree? Explain or show your reasoning.

2. What is the value of $\frac{4}{6} + \frac{5}{8}$?

3. Are there other common denominators you can use to find the sum? Explain or show your reasoning.

Activity 2

Unlike Denominators

Find the value of each expression. Explain or show your reasoning.

1. $\frac{2}{5} + \frac{13}{15}$

2. $\frac{6}{5} - \frac{1}{3}$

3. $\frac{11}{12} + 3\frac{5}{9}$

4. $\frac{6}{10} - \frac{9}{25}$

Addressing CA CCSSM 5.MD.2, 5.NF.1; building towards 5.MD.2; practicing MP2 and MP3

Representing Fractions on a Line Plot

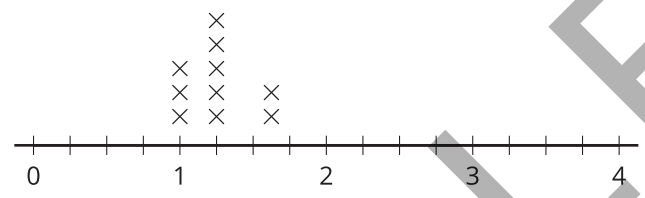
Let's make a line plot and analyze the data we collect.

Warm-up

Which Three Go Together: Line Plot

Which 3 go together?

A



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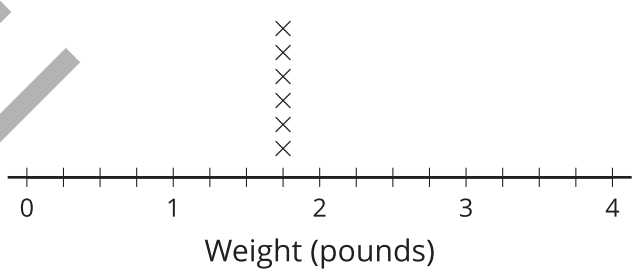

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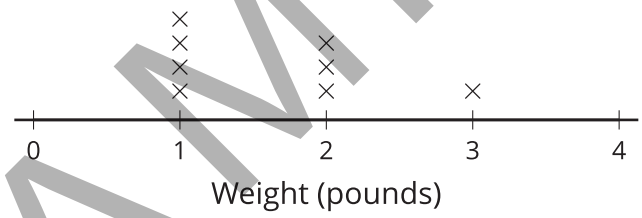

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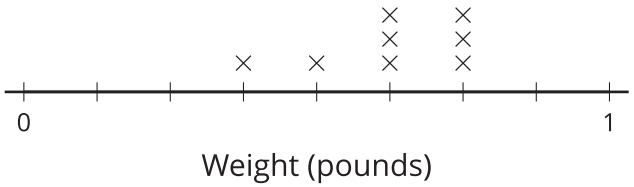
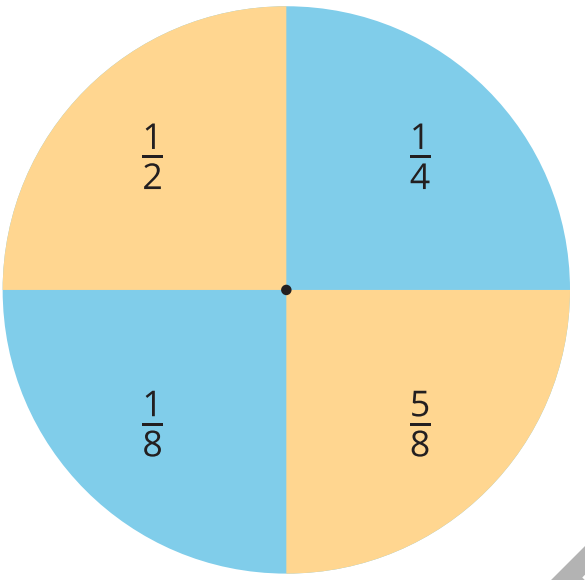


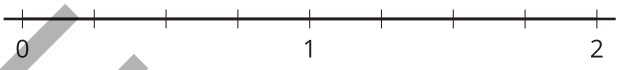
Activity 1

Sums of Fractions

1. Play Sums of Fractions with your partner.



- Spin the spinner twice.
- Add the 2 fractions.
- Record the sum on the line plot.
- Take turns until you and your partner together have 12 data points.



2. How did you know where to plot the sums of eighths?

3. What is the difference between your greatest and least numbers?

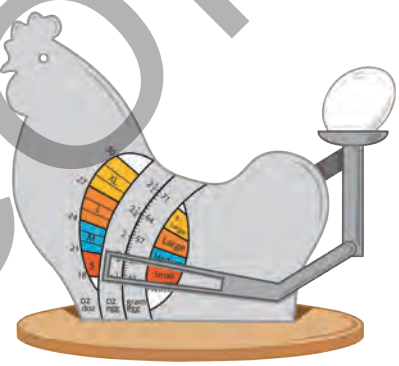
4. What do you notice about the data you collected?

Activity 2

A Lot of Eggs

1. Here are the weights of some eggs, in ounces. Use them to make a line plot.

$1\frac{7}{8}$ $2\frac{1}{2}$ $2\frac{3}{8}$ $1\frac{3}{4}$ $2\frac{1}{4}$ $2\frac{4}{8}$ $2\frac{1}{8}$ $1\frac{7}{8}$ $2\frac{1}{4}$ $1\frac{6}{8}$ $2\frac{1}{8}$ $1\frac{7}{8}$



2. Jada says that $\frac{1}{4}$ of the eggs weigh $1\frac{7}{8}$ ounces. Do you agree? Explain or show your reasoning.

3. How much heavier is the heaviest egg than the lightest egg? Explain or show your reasoning.

Unit 6, Lesson 15

Addressing CA CCSSM 5.MD.2, 5.NF.2, 5.NF.4; practicing MP6

Problem Solving with Line Plots

Let's solve problems, using a line plot.

Warm-up

Number Talk: Multiply by 18

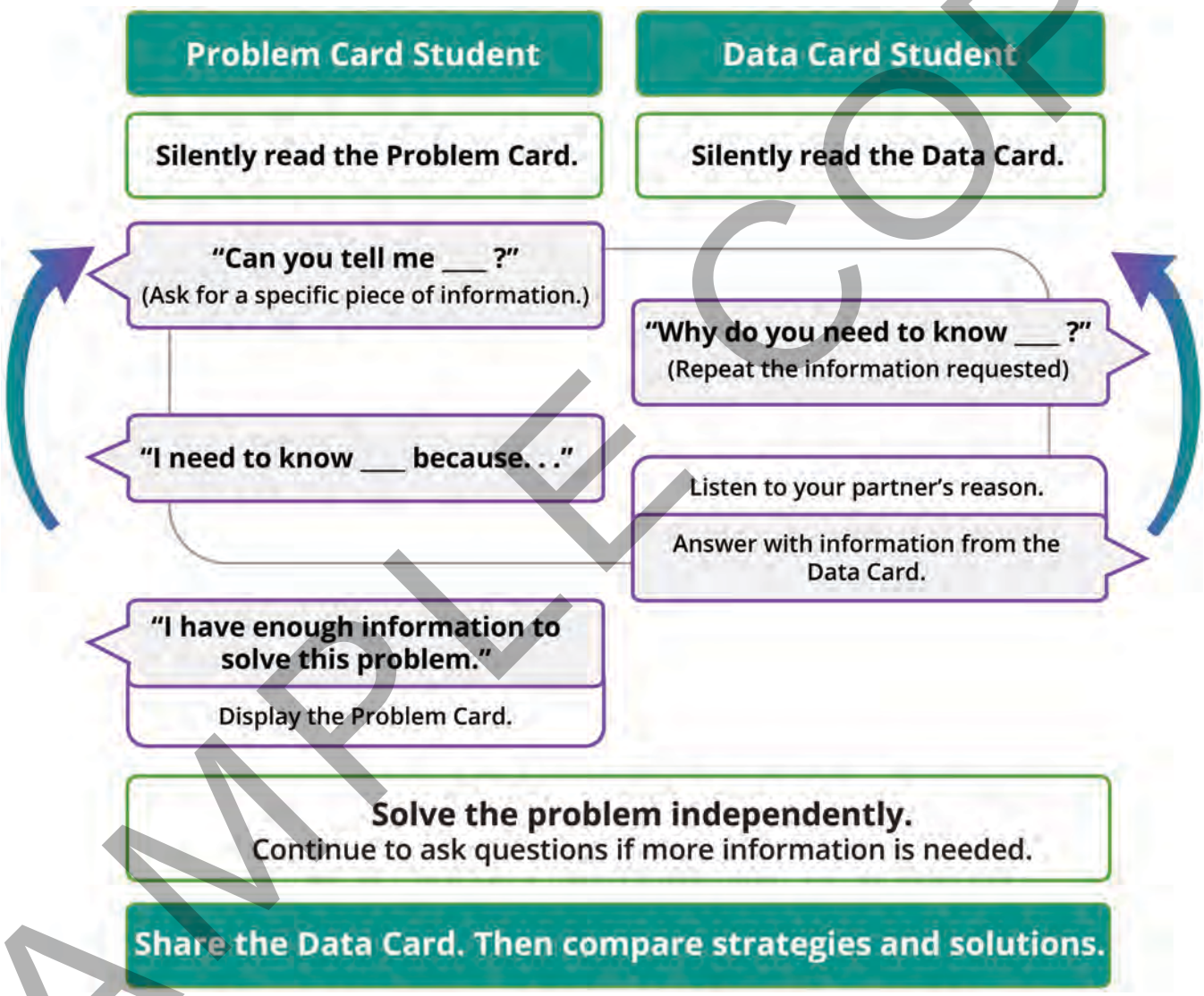
Find the value of each expression mentally.

- $\frac{1}{3} \times 18$
- $\frac{2}{3} \times 18$
- $\frac{4}{3} \times 18$
- $\frac{5}{3} \times 18$

Activity 1

Info Gap: Picking Fruit

Your teacher will give you either a Problem Card or a Data Card. Do not show or read your card to your partner.

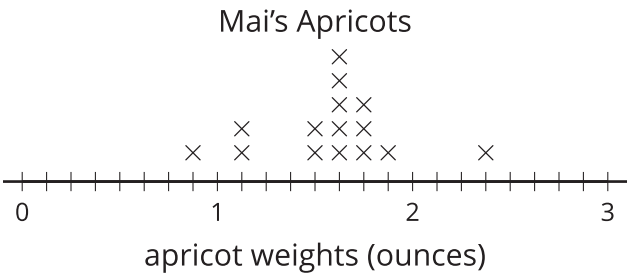


Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Activity 2

Mathematical Questions

This line plot shows the weights of the apricots that Mai picked.



1. What fraction of the apricots weigh less than $1\frac{1}{2}$ ounces each? Explain or show your reasoning.

2. Write a multiplication equation that represents the total weight of the apricots that each weigh $1\frac{5}{8}$ ounces.

3. Do all of Mai's apricots together weigh more than or less than 1 pound? Explain or show your reasoning.

Section B Summary

We learned to add and subtract fractions.

We learned how to add and subtract fractions with denominators that are the same.

Example: $\frac{7}{10} + \frac{4}{10}$

We add the tenths. There are 11 tenths, so $\frac{7}{10} + \frac{4}{10} = \frac{11}{10}$.

We also learned how to add and subtract fractions with denominators that are not the same.

Example: $\frac{1}{6} + \frac{3}{8}$

We look for a common denominator, so we can add parts of the same size.

One way to find a common denominator is to use the product of the two denominators. This is always a common multiple.

Using 48 as a denominator, we find $\frac{1}{6} + \frac{3}{8} = (\frac{1}{6} \times \frac{8}{8}) + (\frac{3}{8} \times \frac{6}{6})$. This means $\frac{1}{6} + \frac{3}{8} = \frac{26}{48}$.

We also can use a smaller common denominator.

Since 24 is a multiple of 6 and 8, we can rewrite $\frac{1}{6} + \frac{3}{8}$ as $\frac{4}{24} + \frac{9}{24}$, which is $\frac{13}{24}$.

Practice Problems

10 Problems

1 from Unit 6, Lesson 8

a. Find the value of each sum. Explain or show your reasoning.

i. $\frac{5}{6} + \frac{2}{6}$

ii. $\frac{5}{6} + \frac{2}{3}$

b. How are the calculations alike? How are they different?

2 from Unit 6, Lesson 9

a. Explain why the expressions $\frac{2}{3} - \frac{7}{12}$ and $\frac{8}{12} - \frac{7}{12}$ are equivalent.

b. How is the expression $\frac{8}{12} - \frac{7}{12}$ helpful to find the value of $\frac{2}{3} - \frac{7}{12}$?

3 from Unit 6, Lesson 10

Find the value of each expression. Explain or show your reasoning.

a. $\frac{1}{4} + \frac{1}{5}$

b. $\frac{10}{9} - \frac{3}{4}$

4 from Unit 6, Lesson 11

a. Find the value of $2\frac{3}{4} - \frac{1}{3}$. Explain or show your reasoning.

b. Find the value of $3\frac{2}{7} - \frac{4}{5}$. Explain or show your reasoning.

5 from Unit 6, Lesson 12

Jada picks $4\frac{2}{3}$ cups of blackberries. Andre picks $3\frac{5}{8}$ cups of blackberries.

a. How many cups of blackberries do Jada and Andre pick together? Explain or show your reasoning.

b. How many more cups of blackberries does Jada pick than Andre? Explain or show your reasoning.

6 from Unit 6, Lesson 13

Find the value of each expression. Explain or show your reasoning.

a. $\frac{7}{8} + \frac{4}{13}$

b. $\frac{7}{8} - \frac{3}{20}$

7 from Unit 6, Lesson 14

Here are the lengths of some pieces of ribbon, measured in inches

$3\frac{1}{4}$ $4\frac{1}{8}$ $3\frac{6}{8}$ $3\frac{1}{8}$ $2\frac{5}{8}$ $3\frac{2}{4}$ $3\frac{1}{4}$ $3\frac{7}{8}$ $4\frac{1}{8}$ $3\frac{1}{2}$ $2\frac{7}{8}$ $4\frac{1}{8}$

$3\frac{3}{4}$ $3\frac{2}{8}$

a. Complete the line plot, with the ribbon lengths.

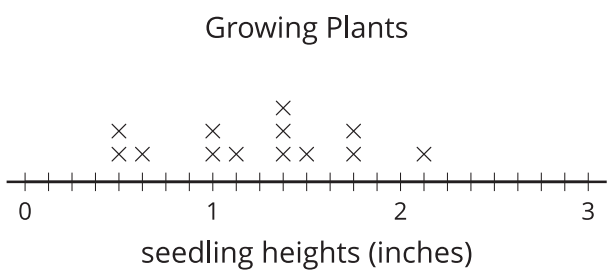
b. What is the sum of the ribbon lengths that measure greater than 4 inches each? Explain or show your reasoning.

8 from Unit 6, Lesson 15

Han is making a line plot of the seedlings his class grew.

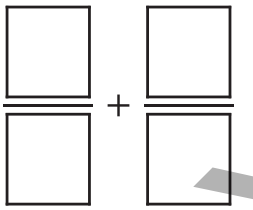
Use this information to complete the line plot. Explain or show your reasoning.

- There are 15 seedlings altogether.
- The tallest seedling is $2\frac{1}{8}$ inches taller than the shortest seedling.
- There are 3 seedlings of the shortest height.

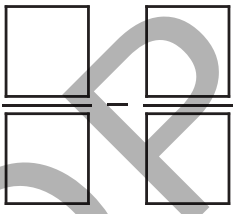


9 Exploration

a. Put the numbers 2, 3, 4, and 5 in the four boxes so that the expression is as close to 1 as possible.



b. Put the numbers 2, 3, 4, and 5 in the four boxes so that the expression is as close to 1 as possible.



10 Exploration

Make a line plot of seedling heights so that each of these statements is true.

- There are 12 measurements.
- The tallest measurement is $2\frac{3}{8}$ inches taller than the shortest measurement.
- The sum of the measurements is $18\frac{3}{8}$ inches.

Explain how you made the line plot.

Unit 6, Lesson 16

Addressing CA CCSSM 5.NF.5a; building towards 5.NF.5a; practicing MP2

Compare Products

Let's compare products.

Warm-up

True or False: Compare Products

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{4}{5} \times 100 = 120$

- $\frac{4}{5} \times 100 < 100$

- $\frac{4}{5} \times 100 = 80$

Activity 1

Go the Distance

Kiran, Noah, and Elena each ran as far as they could in 1 hour.

- Elena ran $\frac{3}{4}$ of a 5-mile trail.
- Noah ran $\frac{1}{2}$ of a 5-mile trail.
- Kiran ran $1\frac{1}{4}$ of a 5-mile trail.

1. List the distances the students ran, in increasing order. Explain your reasoning.

2. Find the number that makes each statement true. Explain your reasoning.
   a. Diego ran farther than Noah, but not as far as Kiran.

      Diego ran _____ of a 5-mile trail.

   b. Lin ran farther than Kiran, but not twice as far as Kiran.

      Lin ran _____ of a 5-mile trail.

   c. Tyler ran farther than Noah, but not as far as Elena.

      Tyler ran _____ of a 5-mile trail.

Activity 2

Compare Expressions

1. Write $<$ or $>$ in each blank to make the statement true. Explain or show your reasoning.

   a. $\frac{5}{4} \times 100 \_\_\_\_ 100$

   b. $\frac{5}{7} \times 2 \_\_\_\_ 2$

   c. $\frac{1}{3} \times 25 \_\_\_\_ 25$

2. Write a number in each box to make the statement true. Explain or show your reasoning.

   a. $\frac{\square}{9} \times 50 < 50$

   b. $\frac{\square}{9} \times 50 = 50$

   c. $\frac{\square}{9} \times 50 > 50$

3. Write a number in each box to make the statement true. Explain or show your reasoning.

a. $\frac{9}{\square} \times 50 < 50$

b. $\frac{9}{\square} \times 50 = 50$

c. $\frac{9}{\square} \times 50 > 50$

Unit 6, Lesson 17

Addressing CA CCSSM 5.NF.5a, 5.NF.5b; practicing MP2 and MP7

Interpret Diagrams

Let's compare products without multiplying.

Warm-up

Estimation Exploration: Fraction of a Whole Number

$$\frac{5}{3} \times 9{,}625$$

Record an estimate that is:

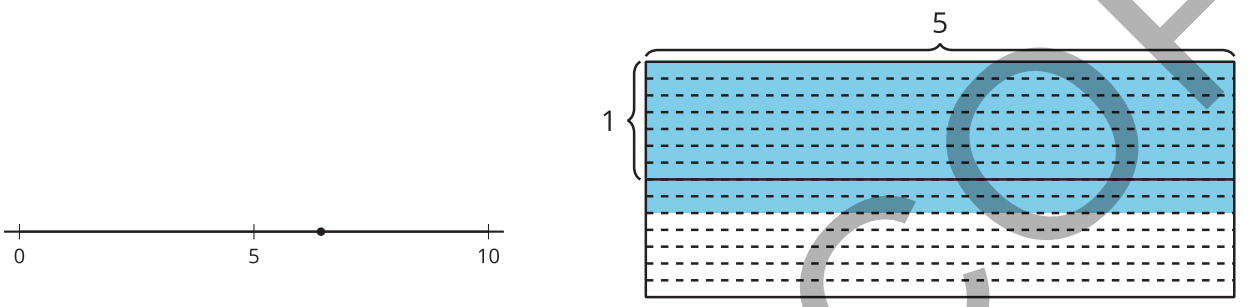
too low	about right	too high

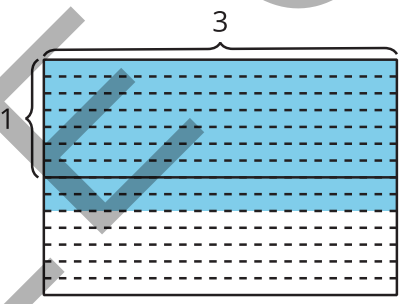
Activity 1

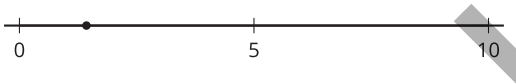
Match the Diagram

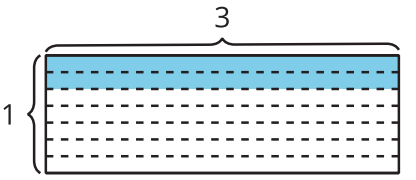
1. Match each expression to a number line and a diagram.

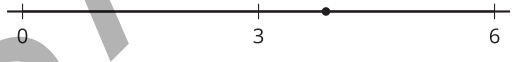
$\frac{2}{7} \times 3$ $\frac{9}{7} \times 3$ $\frac{2}{7} \times 5$ $\frac{9}{7} \times 5$

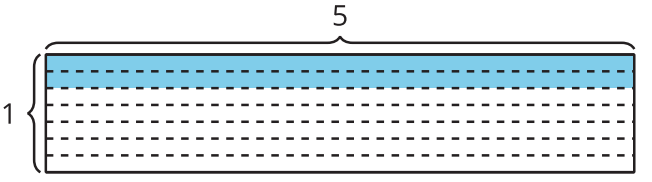












2. Write $<$ or $>$ in each blank to make the inequality true.

   a. $\frac{2}{7} \times 3 \_\_\_\_ 3$

   b. $\frac{9}{7} \times 3 \_\_\_\_ 3$

   c. $\frac{2}{7} \times 5 \_\_\_\_ 5$

   d. $\frac{9}{7} \times 5 \_\_\_\_ 5$

Activity 2

Who Ran Farther?

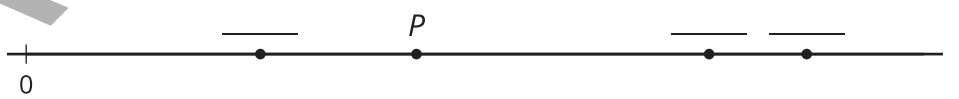
- Priya ran to her grandmother's house.
- Jada ran twice as far as Priya.
- Han ran $\frac{6}{7}$ as far as Priya.
- Clare ran $\frac{14}{8}$ as far as Priya.
- Mai ran $\frac{3}{5}$ times as far as Priya.

1. Which students ran farther than Priya? _____

2. Which students did not run as far as Priya? _____

3. List the runners in order, from the shortest distance to the longest distance. Explain or show your reasoning.

4. Point *P* represents how far Priya ran. Find the distance of each runner on the number line. Write the runner's initial in the blank. One runner does not have a point on the number line.



5. Label the distance for the missing runner on the number line.

Unit 6, Lesson 18

Addressing CA CCSSM 5.NF.5a; practicing MP7

Compare without Multiplying

Let's compare expressions without evaluating them.

Warm-up

Notice and Wonder: Expressions and Number Lines

What do you notice? What do you wonder?

$\frac{2}{3} \times 5$

Activity 1

Approximate Location

1. Label each expression at its approximate location on the number line.

Partner A

a. $\frac{2}{5} \times 12$

b. $\frac{5}{3} \times 12$

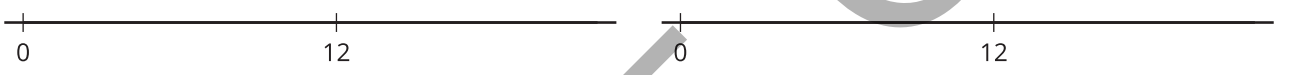
c. $\frac{7}{7} \times 12$

Partner B

a. $\frac{4}{7} \times 12$

b. $\frac{8}{5} \times 12$

c. $\frac{9}{9} \times 12$



2. Write a number in each box to make the statement true.

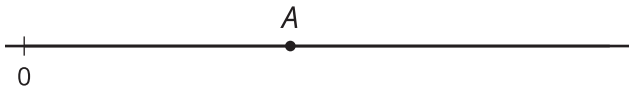
a. $\frac{\square}{11} \times 12 > 12$

b. $\frac{\square}{15} \times 12 = 12$

c. $\frac{13}{\square} \times 12 < 12$

Activity 2

An Unknown Number



1. The number A is shown on the number line. Label the approximate location of the value of each expression. Explain or show your reasoning.
   - $\frac{1}{4} \times A$
   - $2 \times A$
   - $\frac{13}{8} \times A$
   - $\frac{2}{3} \times A$

2. Is $\frac{13}{8} \times \frac{11}{39}$ less than, greater than, or equal to $\frac{11}{39}$? Explain or show your reasoning.

3. Is $\frac{2}{3} \times \frac{17}{53}$ less than, greater than, or equal to $\frac{17}{53}$? Explain or show your reasoning.

Unit 6, Lesson 19

Addressing CA CCSSM 5.NF.5b; building towards 5.NF.5b; practicing MP7 and MP8

Compare to 1

Let's explain what happens when we multiply a fraction by a fraction greater than 1, less than 1, or equal to 1.

Warm-up

What Do You Know about $\frac{15}{14} \times \frac{23}{30}$?

What do you know about $\frac{15}{14} \times \frac{23}{30}$?

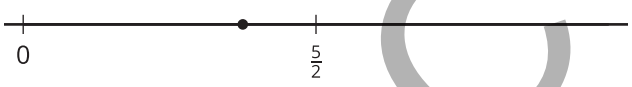
Activity 1

Compare Fraction Products on the Number Line

1. Match each expression to the number line that shows the same value.

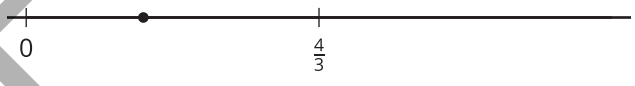
   - $\frac{2}{5} \times \frac{4}{3}$
   - $\frac{3}{4} \times \frac{5}{2}$
   - $\frac{4}{3} \times \frac{5}{2}$
   - $\left(1 + \frac{1}{3}\right) \times \frac{5}{2}$
   - $\left(1 - \frac{3}{5}\right) \times \frac{4}{3}$
   - $\left(1 - \frac{1}{4}\right) \times \frac{5}{2}$

A



B

C



2. Choose one expression from each set. Explain whether the value is greater than or less than the second factor.

Activity 2

True Statement

1. Rewrite each expression as a sum or difference of 2 products.

   a. $\left(1 - \frac{2}{5}\right) \times \frac{4}{7}$

   b. $\left(1 + \frac{1}{5}\right) \times \frac{4}{7}$

   c. $\left(1 - \frac{3}{8}\right) \times \frac{4}{7}$

   d. $\left(1 + \frac{1}{8}\right) \times \frac{4}{7}$

2. Write $<$ or $>$ to make the inequality true.

   a. $\left(1 - \frac{2}{5}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$

   b. $\left(1 + \frac{1}{5}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$

   c. $\left(1 - \frac{3}{8}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$

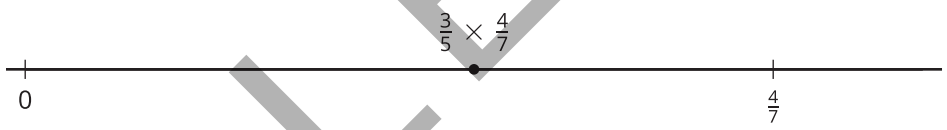
   d. $\left(1 + \frac{1}{8}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$

3. Describe the value of the product when $\frac{4}{7}$ is multiplied by a fraction greater than 1. Explain your reasoning.

4. Describe the value of the product when $\frac{4}{7}$ is multiplied by a fraction less than 1. Explain your reasoning.

Section C Summary

We learned how to compare the size of a product to the sizes of its factors.



To compare $\frac{3}{5} \times \frac{4}{7}$ with $\frac{4}{7}$, we can put them on a number line.
Since $\frac{3}{5}$ is 3 equal parts with 5 parts in the whole, it is to the left of $\frac{4}{7}$. We also can write $\frac{3}{5}$ as $1 - \frac{2}{5}$.

$$\left(1 - \frac{2}{5}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{2}{5} \times \frac{4}{7}\right)$$

The product is less than $\frac{4}{7}$, because it is $\frac{4}{7}$ minus a fraction.

Unit 6, Lesson 20

Addressing CA CCSSM 5.NF.5, 5.NF.5b, 5.OA.1; practicing MP6

Will It Always Work?

Let's make generalizations about multiplying a whole number by a fraction.

Warm-up

True or False: Distributing

Sec C

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{3}{4} = 1 - \frac{1}{4}$

- $\left(1 - \frac{1}{4}\right) \times 9 = 9 - \left(\frac{1}{4} \times 9\right)$

- $\left(1 + \frac{1}{4}\right) \times 7 = (1 \times 7) + \frac{1}{4}$

Activity 1

True Statements

Write >, <, or = in each blank to make true statements.

Choose one problem to explain or show your reasoning.

1. $567 \_\_\_\_ 345 \times 567$

2. $\frac{4}{5} \times 851 \_\_\_\_ 851$

3. $\frac{1}{4} \_\_\_\_ \frac{5}{5} \times \frac{1}{4}$

4. $\frac{103}{104} \_\_\_\_ \frac{103}{104} \times \frac{103}{104}$

5. $\frac{99}{8} \times \frac{23}{22} \_\_\_\_ \frac{99}{8}$

6. $\frac{10}{10} \times \frac{1}{2} \_\_\_\_ \frac{1}{2}$

7. $\frac{100}{7} \times \frac{9}{13} \_\_\_\_ \frac{9}{13}$

Activity 2

Andre's Rules

Andre says:

- When you multiply any fraction by a number less than 1, the product will be less than the fraction.
- When you multiply any fraction by a number greater than 1, the product will be greater than the fraction.

Each partner chooses a different statement and describes why it is true. Show your thinking, using diagrams, symbols, or other representations.

Unit 6, Lesson 21

Addressing CA CCSSM 5.MD.2, 5.NF.2, 5.NF.4; practicing MP4

Weekend Investigation

Let's find out how students spend free time on the weekend.

Warm-up

Number Talk: Fractions of 60

Find the value of each expression mentally.

- $\frac{1}{4} \times 60$
- $\frac{3}{4} \times 60$
- $\frac{5}{4} \times 60$
- $\frac{9}{4} \times 60$

Activity 1

Data Collection

Imagine you have 2 hours of free time this weekend that you can spend any way you like.

1. How would you spend it? Record your answer in halves, fourths, or eighths of an hour. Explain or show your reasoning.

2. Record the time for each activity from your list on the appropriate poster.

Activity 2

Data Analysis

Your teacher will assign a poster with a data set for one of the categories from the previous activity.

1. Create a line plot that represents the data. Make sure to label the line plot.

2. Analyze the data. Tell the story of your data. Choose at least 3 things you learn about the data. Use the following questions if they are helpful.
   - What is the total number of hours the class spends on this activity?
   - What is the difference between the greatest time and the least time?
   - Is there anything surprising?
   - How many data points are there? What does that tell you?
   - What fraction of your classmates spend less than 1 hour on this activity? More than 1 hour?

Practice Problems

7 Problems

1

from Unit 6, Lesson 16

a. Andre ran $\frac{4}{5}$ of a 7-mile trail. Did Andre run a distance greater than or less than 7 miles? Explain or show your reasoning.

b. Clare ran $\frac{\square}{10}$ of a 7-mile trail. She ran a distance greater than 7 miles. Find a number that makes this statement true. Explain or show your reasoning.

Sec C

2

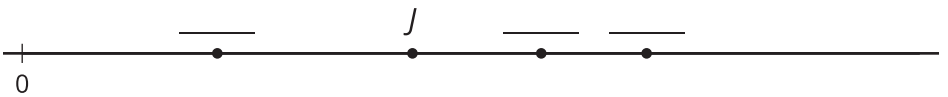
from Unit 6, Lesson 17

Point *J* on the number line shows how many miles Jada ran. Label the points on the number line to show each runner's distance, in miles.

a. Clare ran $\frac{8}{5}$ as far as Jada.

b. Tyler ran $\frac{4}{3}$ as far as Jada.

c. Lin ran $\frac{1}{2}$ as far as Jada.



3 from Unit 6, Lesson 18

Point A is labeled on the number line.

Label each of these points on the number line.

- $\frac{2}{5} \times A$
- $\frac{13}{10} \times A$
- $\frac{7}{4} \times A$

4 from Unit 6, Lesson 19

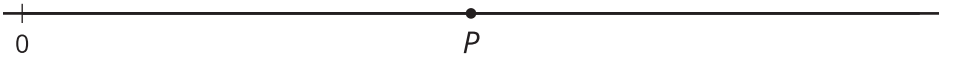
Use the equation $\frac{5}{7} = \left(1 - \frac{2}{7}\right)$ to explain why $\frac{5}{7} \times \frac{11}{3} < \frac{11}{3}$.

5 from Unit 6, Lesson 20

Explain why multiplying a fraction by a number less than 1 makes the fraction smaller.

6 Exploration

Point P is labeled on the number line.



a. Point P is $\frac{3}{4}$ of a number A. Plot A on the number line. Explain or show your reasoning.

b. Point P is $\frac{5}{9}$ of a number B. Plot B on the number line. Explain or show your reasoning.

7 Exploration

a. About 10^6 people live in Michigan. About 10^4 of the people in Michigan live in Flint.

   i. How many times as many people live in Michigan as in Flint?

   ii. How many times as many people live in Flint as in Michigan?

b. There are about 10^{11} stars in the Milky Way. There are about 10^{24} stars in the universe.

i. How many times as many stars are there in the universe than in the Milky Way?

ii. How many times as many stars are there in the Milky Way than in the universe?

Glossary

- algorithm
  A set of steps that works every time as long as the steps are carried out correctly.

- area
  The number of square units that cover a flat figure without gaps or overlaps.

- cubic unit
  The volume of a cube whose sides are each 1 unit long.

  Example: A cube whose sides are each 1 inch long has a volume of 1 cubic inch.

- exponent
  A number that tells how many times something is multiplied by itself.

  Example: In 10^3, the number 3 is an exponent. It means to multiply 10 by itself 3 times.
  $10^3 = 10 \times 10 \times 10 = 1,000$

- power of 10
  The result of multiplying 10 by itself a given number of times.

  Example: 1,000 is a power of 10 because $10 \times 10 \times 10 = 1,000$.

- rectangular prism
  A solid figure with 6 faces that are all rectangles.

- standard algorithm (for multiplication)
  A set of steps used to multiply numbers by place value. Write the numbers vertically, with the digits lined up by place value. Starting with the least place value, multiply the digits in each place of the bottom number by the digits in each place of the top number. Compose units in each place, as needed, representing that unit with a number over the place value to the left of the digit being multiplied. Add the partial products, including composed units.

- unit cube
  A cube whose sides are each 1 unit long. Unit cubes are used to measure volume.

- volume
  The number of unit cubes that fill a solid figure without gaps or overlap.

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California Common Core State Standards for Mathematics (CA CCSSM) References

5.G: Grade 5 – Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1
Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2
Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4
Classify two-dimensional figures in a hierarchy based on properties.

5.MD: Grade 5 – Measurement and Data

Convert like measurement units within a given measurement system.

5.MD.1
Convert among different-sized standard measurements units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

Represent and interpret data.

5.MD.2
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3
Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.3a
A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

5.MD.3b
A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4
Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5
Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

5.MD.5a
Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

5.MD.5b
Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

5.MD.5c
Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

5.NBT: Grade 5 – Number and Operations in Base Ten

Understand the place value system.

5.NBT.1
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.3
Read, write, and compare decimals to thousandths.

5.NBT.3a
Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

5.NBT.3b
Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4
Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5
Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.6
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF: Grade 5 – Number and Operations—Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

5.NF.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3
Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.4a
Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

5.NF.4b
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5
Interpret multiplication as scaling (resizing), by:

5.NF.5a
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

5.NF.5b
Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

5.NF.6
Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

5.NF.7a
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.

5.NF.7b
Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.

5.NF.7c
Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

5.OA: Grade 5 – Operations and Algebraic Thinking

Write and interpret numerical expressions.

5.OA.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

5.OA.2.1
Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$.

Analyze patterns and relationships.

5.OA.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.