IM K–12 California

GRADE 5

Student Edition

UNITS 7-8

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 4

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GRADE 5

UNIT

Shapes on the Coordinate Grid

Content Connections

In this unit you will plot coordinate pairs on a coordinate grid, classify two-dimensional shapes using length and angle measurements, and represent and interpret real-world and mathematical problems on a coordinate grid. You will make connections by:

- **Reasoning with Data** while plotting patterns on a line plot or coordinate grid and use a graph to tell a story about data.
- **Exploring Changing Quantities** while generating and analyzing numerical patterns based on pairs of rules and graph pairs of numbers on a coordinate grid.
- **Discovering Shape and Space** while classifying two-dimensional shapes using side lengths and angle measurements.
- **Taking Wholes Apart, Putting Parts Together** while using the structure of line plots and coordinate grids to precisely communicate the location of points and shapes.

Addressing the Standards

As you work your way through **Unit 7 Shapes on the Coordinate Grid,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	
MP2 Reason abstractly and quantitatively.	Lessons 6, 10, 12
MP3 Construct viable arguments and critique the reasoning of others.	Lesson 8
MP4 Model with mathematics.	Lesson 14
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 1, 2, 4, 5
MP7 Look for and make use of structure.	Lessons 3, 4, 5, 7, 9, 12, 13
MP8 Look for and express regularity in repeated reasoning.	Lessons 9, 11, 13

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Plotting Patterns • Telling a Data Story • Shapes on a Plane	**5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	Lessons 1, 2, 3, and 11

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Plotting Patterns • Telling a Data Story • Shapes on a Plane	**5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Lessons 12, 13, and 14
• Shapes on a Plane	**5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	Lessons 4, 6, 7, and 8
• Shapes on a Plane	**5.G.4** Classify two-dimensional figures in a hierarchy based on properties.	Lessons 4, 5, 6, 7, and 8
• Factors and Groups • Powers and Place Value	**5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Lesson 12
• Plotting Patterns • Telling a Data Story • Modeling • Shapes on a Plane	**5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	Lessons 9, 10, and 11

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections	**5.NF.3** Interpret a fraction as division of the numerator by the denominator *(a/b = a ÷ b).* Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Lesson 10
• Modeling • Seeing Division	**5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Lesson 13

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 7, Lesson 1

Addressing CA CCSSM 5.G.1; building towards 5.G.1; practicing MP6

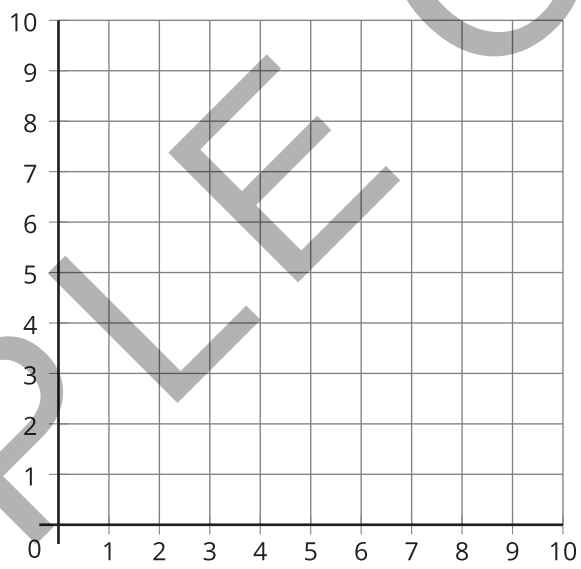
Explore the Coordinate Grid

Let's explore the coordinate grid.

Warm-up

Notice and Wonder: The Grid

What do you notice? What do you wonder?



Activity 1

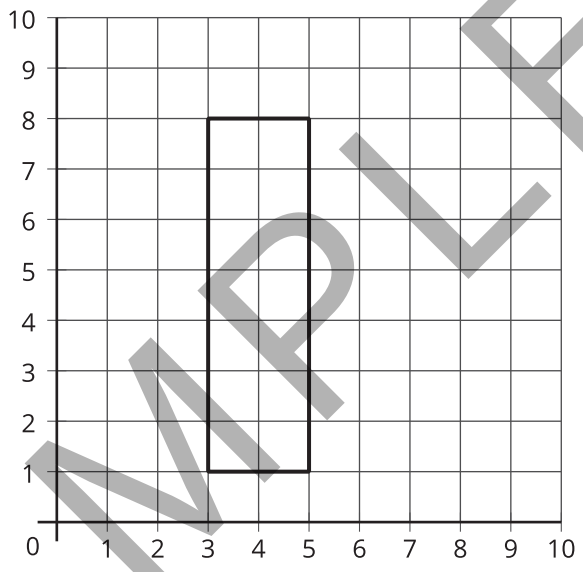
Introduce Can You Draw It—Shapes on the Coordinate Grid

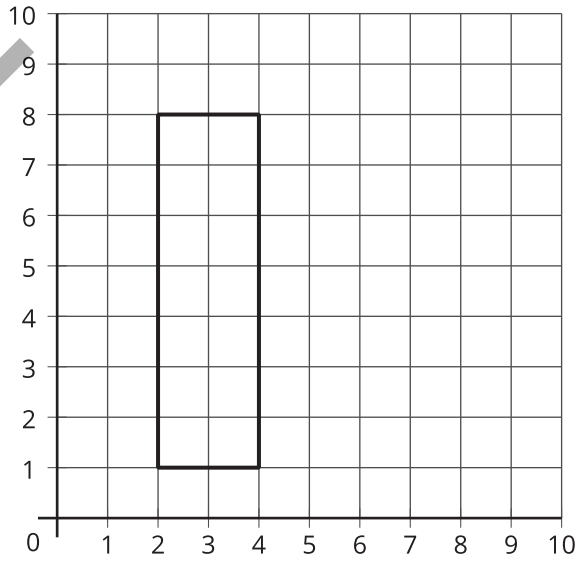
1. Play 3 rounds of *Can You Draw It?* using the 3 sets of cards and 2 Recording Sheets from your teacher.
   a. Use a different set of cards for each round.
      - Round 1: Shapes not on a grid
      - Round 2: Shapes on a grid without numbers
      - Round 3: Shapes on a numbered grid
   b. For each round:
      - Partner A chooses a card and describes the shape without showing it.
      - Partner B draws the shape as described.
      - Partner A reveals the card, and Partner B reveals the drawing.
      - Compare the shapes and discuss: What's the same? What's different?
   c. Score the round for Partner B.
      - Earn 2 points if your shape exactly matches the shape on Partner A's card.
      - Earn 1 point if it doesn't match exactly but matches the given description.
   d. Switch roles and repeat to finish the round.
   e. The partner with the highest score after 3 rounds wins the game.
2. Look at Partner B's drawings for each round. When does Partner B's drawing look most like the shape on the card? Explain why you think that is so.

Activity 2

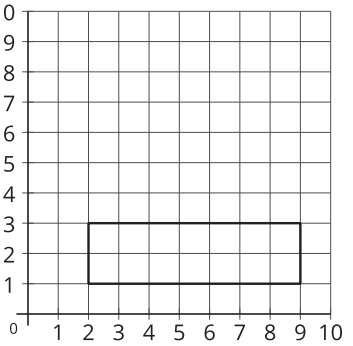
Which One: Shapes on the Coordinate Grid

1. Play a round of *Which One?*
   - Sit next to your partner.
   - Partner A:
     Choose one of the shapes A–L on the next page. Do not tell or show which shape you chose.
   - Partner B:
     Ask *yes* or *no* questions to determine which shape Partner A chose.
2. Switch roles and play another round of *Which One?*
3. Diego and Kiran play a round of *Which One?* Diego chooses one of these two shapes. What question can Kiran ask to determine which shape Diego picked?





A



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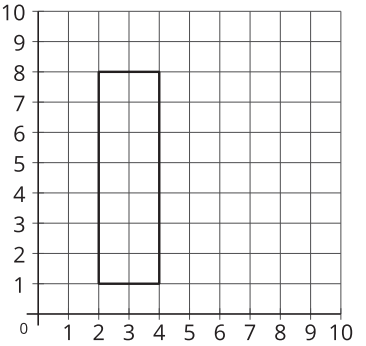

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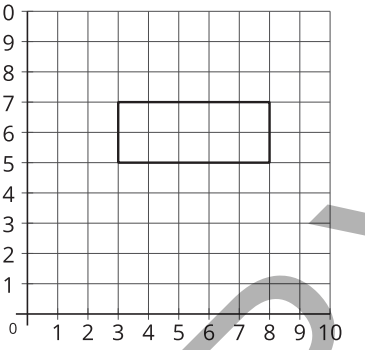

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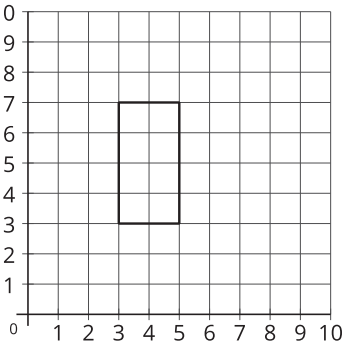

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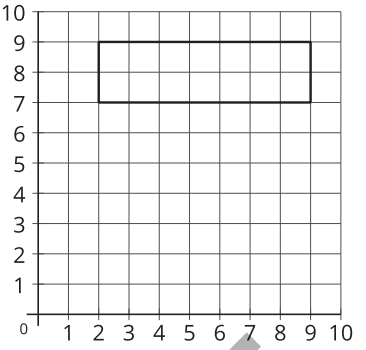

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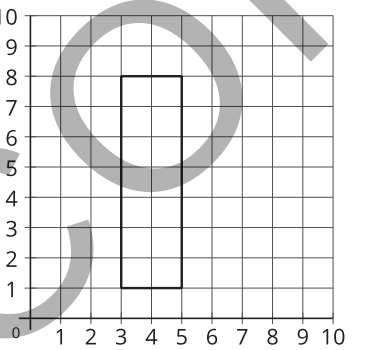

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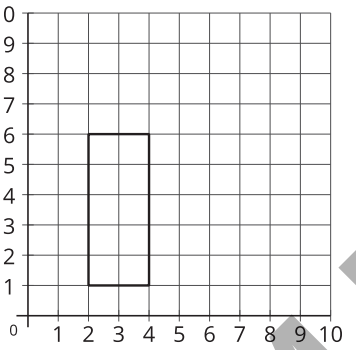

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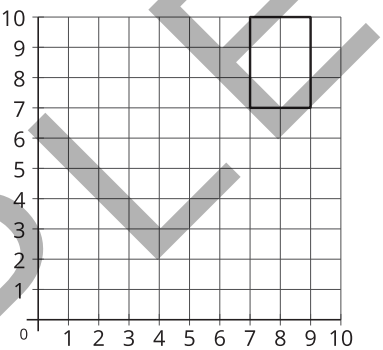

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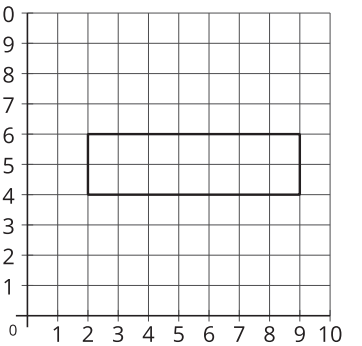

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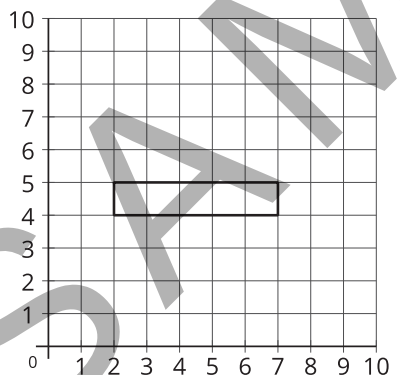

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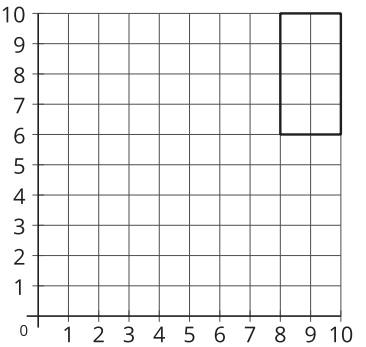

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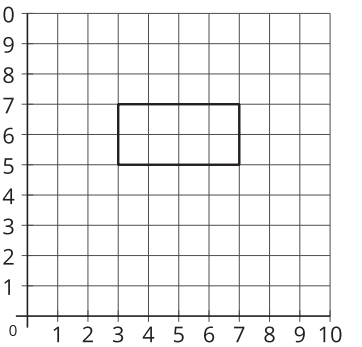


Unit 7, Lesson 2

Addressing CA CCSSM 5.G.1; building towards 5.G.1; practicing MP6

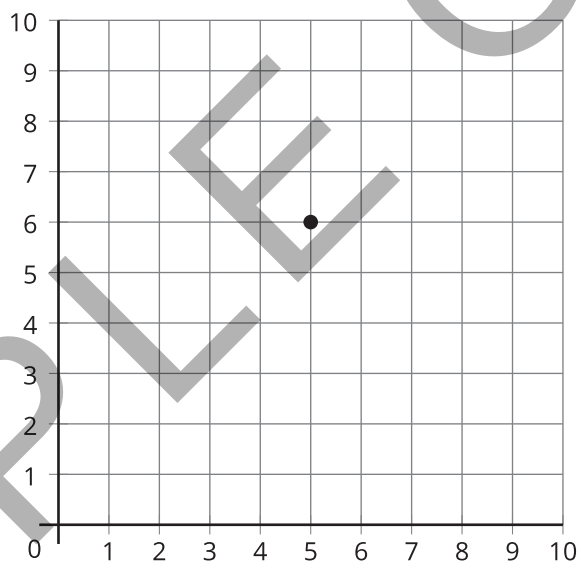
Points on the Coordinate Grid

Let's plot points on the coordinate grid.

Warm-up

Notice and Wonder: A Point

What do you notice? What do you wonder?

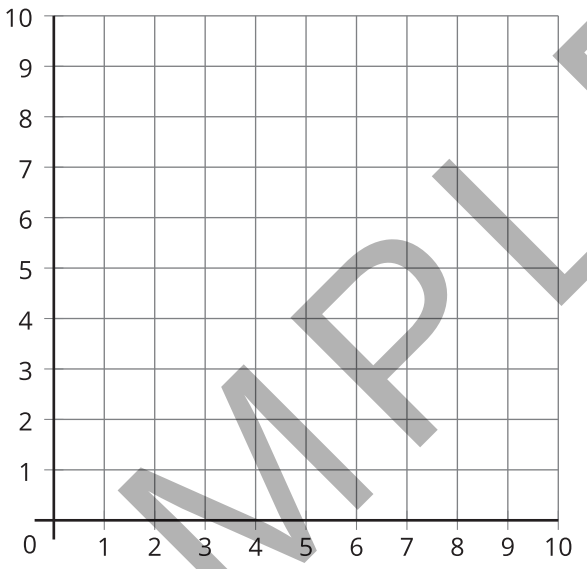


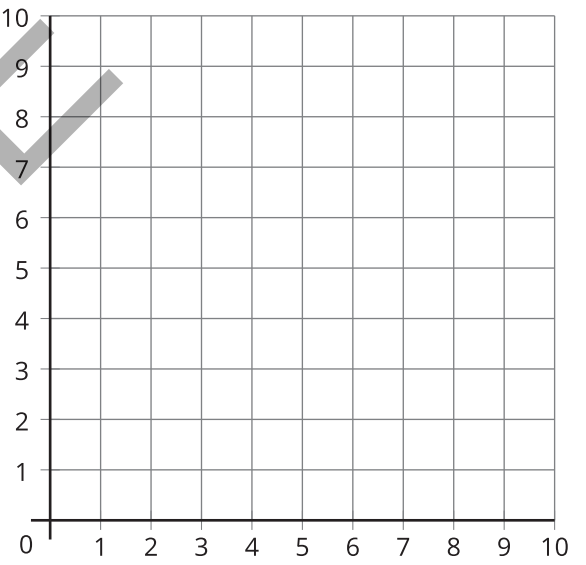
Activity 1

What's the Point?

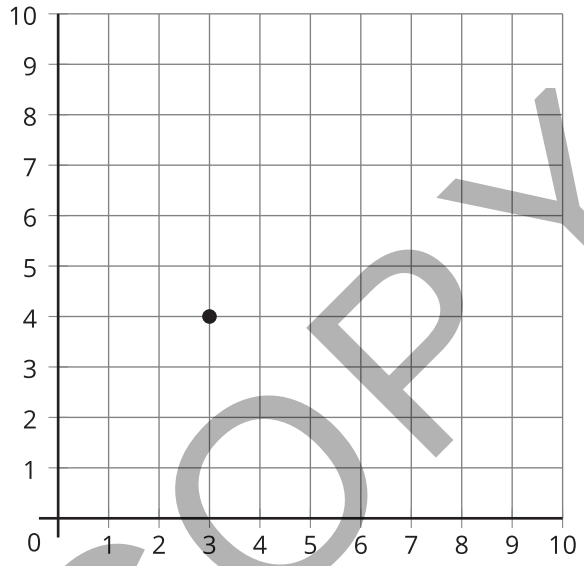
Your teacher will give you a set of cards that show points plotted on coordinate grids.

1. Play 2 rounds of What's the Point? so each partner gets a chance to draw.
   - Sit back to back with your partner.
   - Partner A: Choose a card. Describe the location of the point to your partner.
   - Partner B: Plot the point on the blank coordinate grid.
   - Compare Partner A's card with Partner B's plotted point.
   - Discuss: What's the same? What's different?
   - Switch roles to play a second round.





2. Use words to explain the location of the point on this grid.

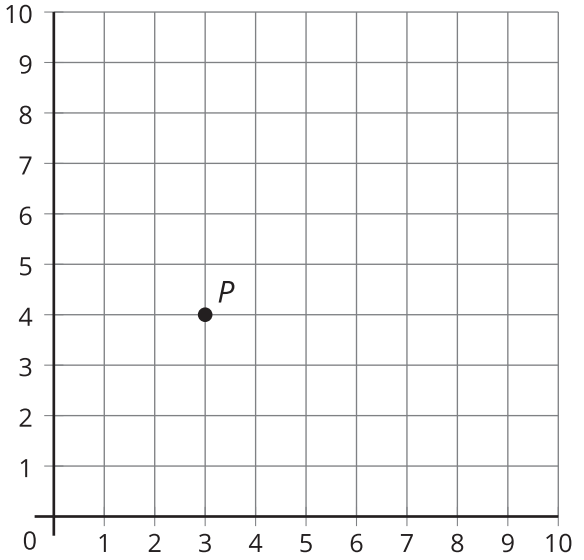


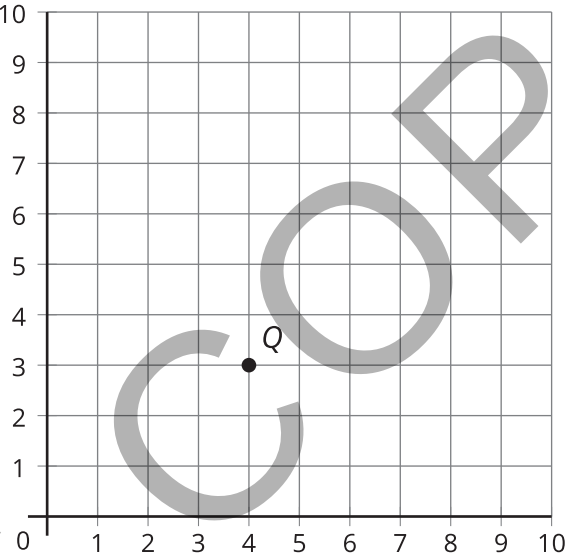
Activity 2

Plot and Label Points

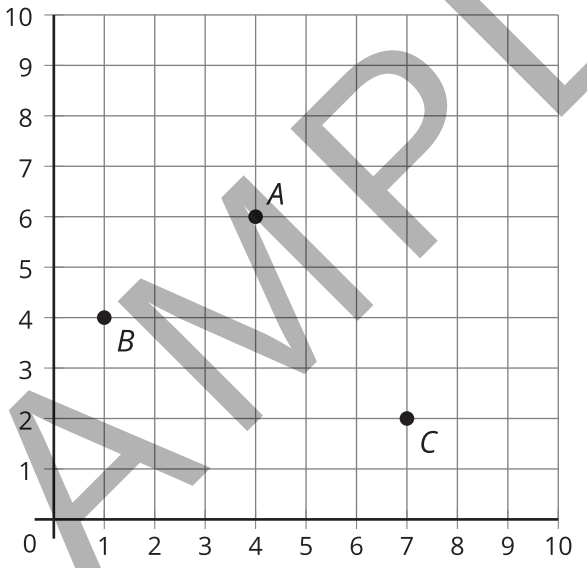
Sec A

1. What do you notice about point P and point Q?





2. List the coordinates for each point.



	coordinates
A	(___ , ___)
B	(___ , ___)
C	(___ , ___)
D	$(6, 4)$
E	$(2, 5)$
F	$(8, 3)$

3. Plot and label points D, E, F on the same grid as points A, B, and C.

Addressing CA CCSSM 5.G.1; building towards 5.G.1; practicing MP7

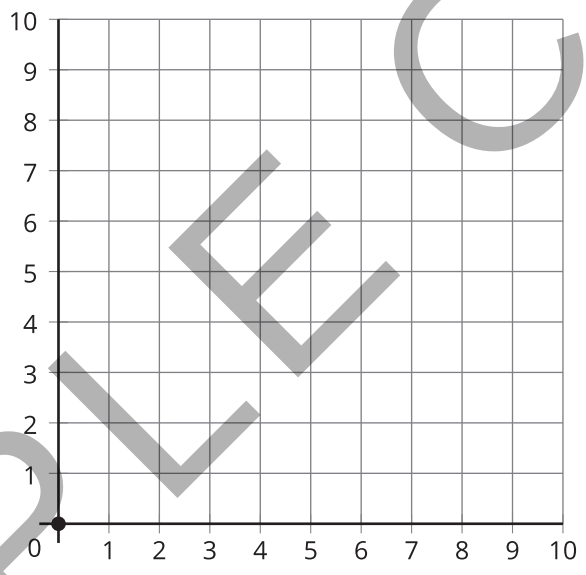
Plot More Points

Let's locate and name points on the coordinate grid.

Warm-up

Notice and Wonder: Points with 0

What do you notice? What do you wonder?



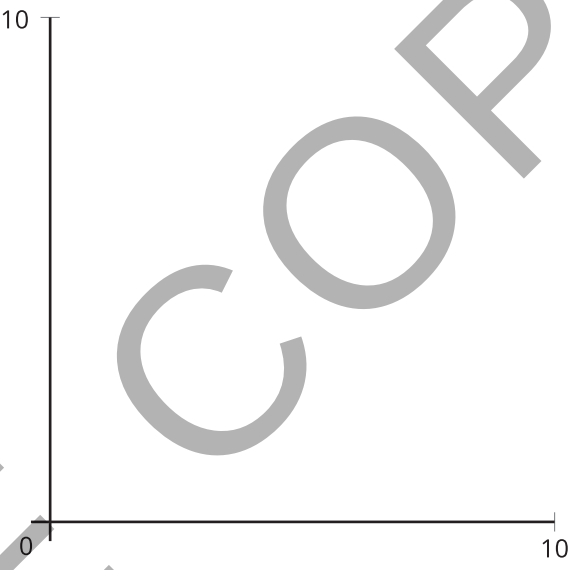
Activity 1

What's the Point?

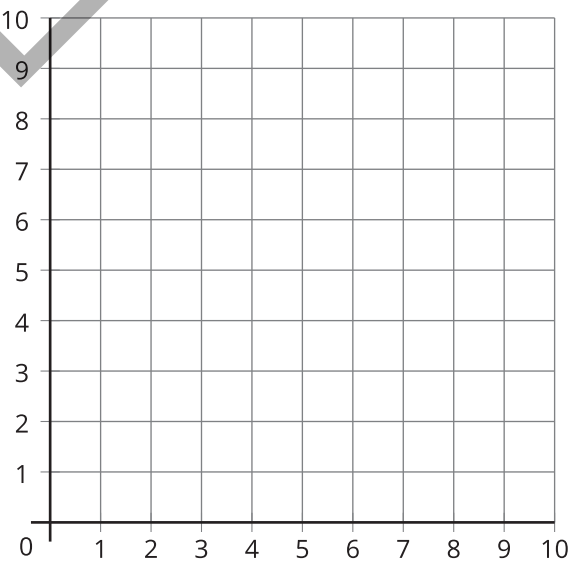
Partner A

1. Estimate to plot and label the location of each point.

point	coordinates
A	$(5, 1)$
B	$(5, 2)$
C	$(5, 3)$
D	$(5, 4)$



2. Plot and label the same points on the coordinate grid.



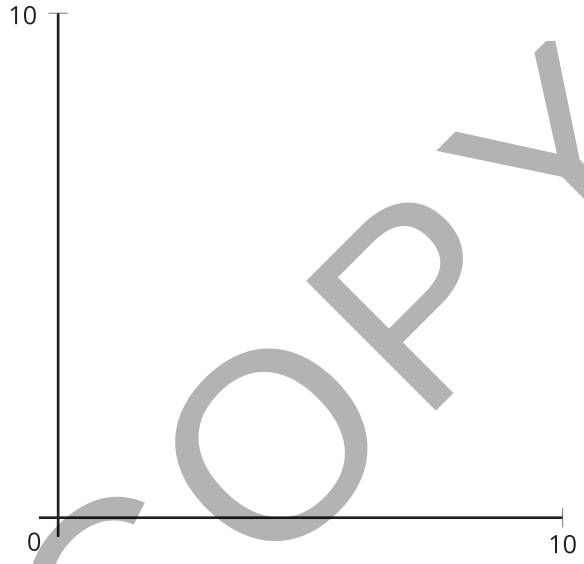
3. What do the points have in common?

4. Plot the point with coordinates $(5, 0)$ on the coordinate grid.

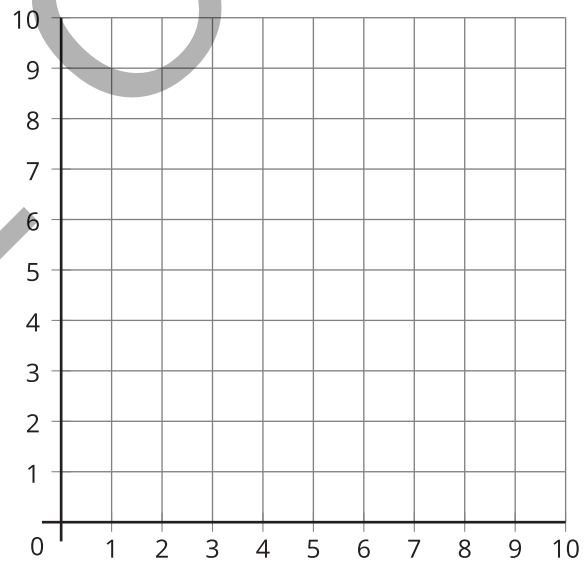
Partner B

1. Estimate to plot and label the location of each point.

point	coordinates
A	$(4, 3)$
B	$(5, 3)$
C	$(6, 3)$
D	$(7, 3)$



2. Plot and label the same points on the coordinate grid.



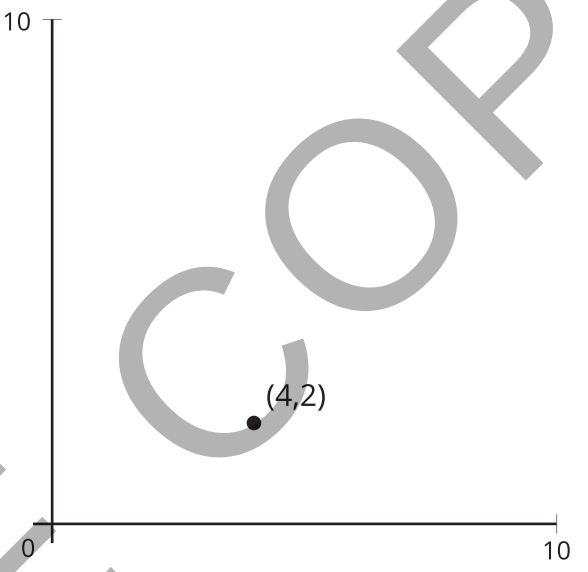
3. What do the points have in common?

4. Plot the point with coordinates $(0, 3)$ on the coordinate grid.

Activity 2

Plotting Points without a Grid

1. The grid lines are removed from this coordinate grid and a point is plotted and labeled. Plot and label some other points. Explain or show your reasoning.



2. Can you plot $(1, 0)$ and $(0, 1)$ accurately? Explain or show your reasoning.

Section A Summary

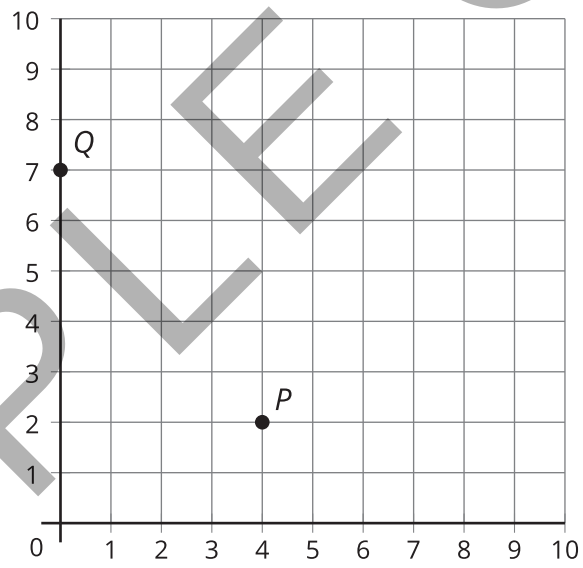
We learned about the **coordinate grid**. We saw that the grid is formed by 2 perpendicular number lines called **axes**. We also saw that the 2 axes intersect at 0.

- The **horizontal axis** runs left to right.
- The **vertical axis** runs up and down.

We used **coordinates** to plot and describe points on the coordinate grid. The coordinates are 2 numbers that tell a point's exact location on a coordinate grid.

- The first coordinate tells the point's horizontal position—how far it is from 0 along the horizontal axis.
- The second coordinate tells the point's vertical position—how far it is from 0 along the vertical axis.

coordinates = (horizontal position, vertical position)



Examples:

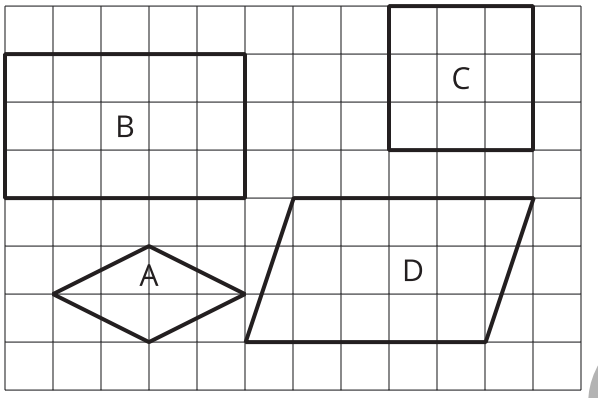
The coordinates of point *P* are 4,2. This means point *P* is located 4 units from 0 along the horizontal axis and 2 units from 0 along the vertical axis. So, to get to point *P*, start at 0 and go 4 units right and 2 units up.

The coordinates of point *Q* are 0,7. This means point *Q* is located 0 units from 0 along the horizontal axis and 7 units from 0 along the vertical axis. So, to get to point *Q*, start at 0 and go 0 units right and 7 units up

Practice Problems

9 Problems

1 Pre-unit

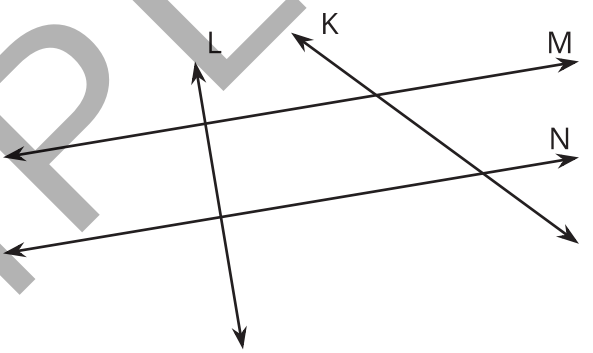


a. Which shapes are rectangles? _____

b. Which shapes are rhombuses? _____

c. Which shapes are squares? _____

2 Pre-unit



a. Name 2 lines in the diagram that are parallel. _____

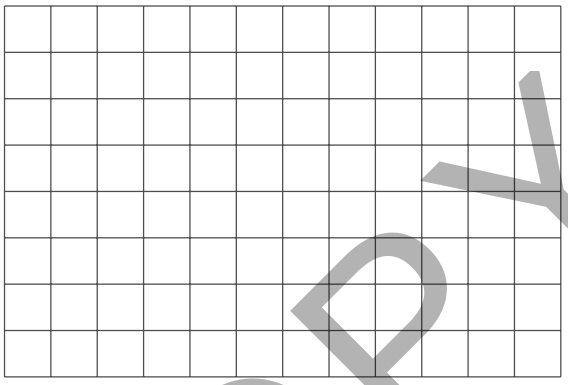
b. Name 2 lines in the diagram that are perpendicular. _____

3

Pre-unit

a. Draw a rectangle on the grid.

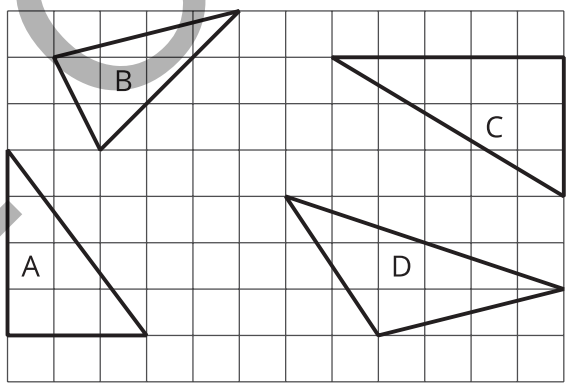
b. What is the perimeter of the rectangle?



4

Pre-unit

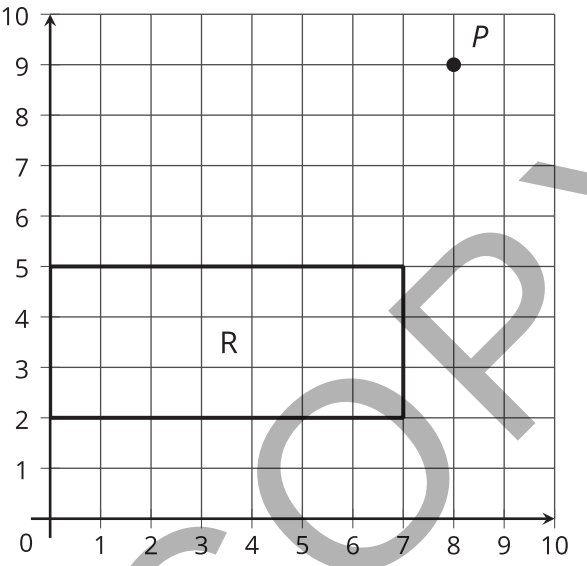
Which triangles are right triangles?



5 from Unit 7, Lesson 1

a. How would you describe point P?

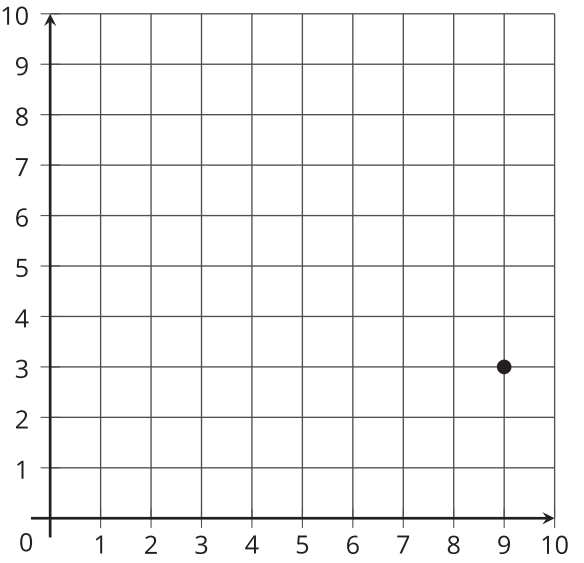
b. How would you describe Rectangle R?



6 from Unit 7, Lesson 2

a. What are the coordinates of the point plotted on this coordinate grid?

b. Plot and label point A with coordinates $(7, 1)$, point B with coordinates $(2, 8)$, and point C with coordinates $(6, 6)$.



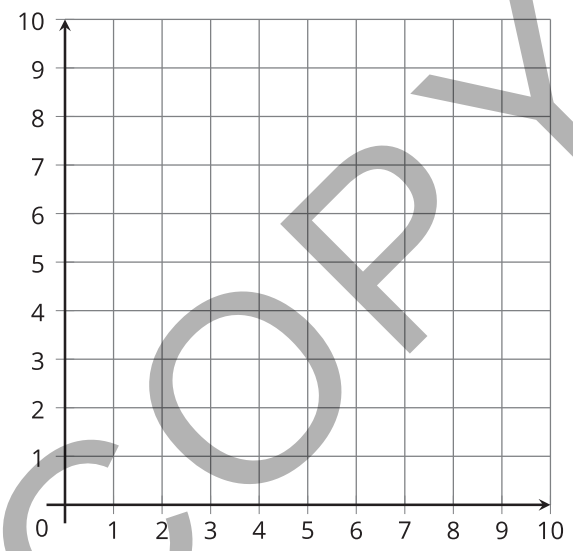
7 from Unit 7, Lesson 3

a. Fill in the blanks with 4 different numbers between 0 and 10 to complete the coordinates for 4 points. Then plot the points on the coordinate grid.

(_____, 0) (_____, 0)

(_____, 0) (_____, 0)

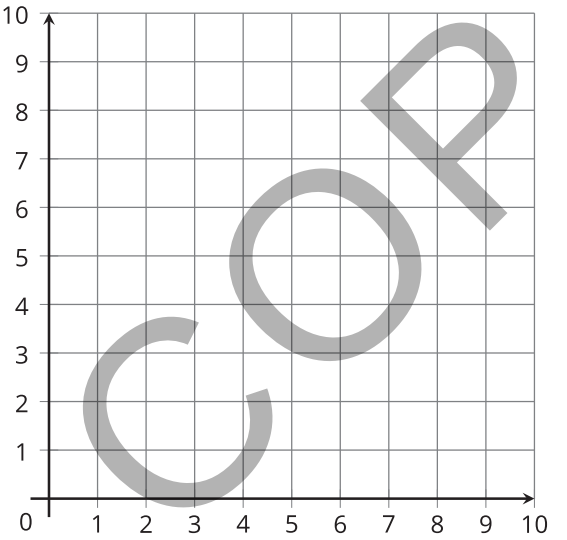
What do you notice about the points?



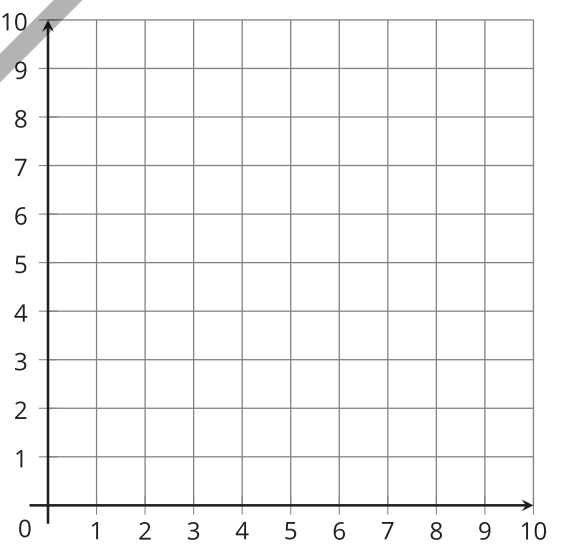
b. Plot points at $(1, 1)$, $(2, 2)$, $(3, 3)$, and $(4, 4)$ on the coordinate grid. What do you notice about the points?

8 a. A student describes the first letter of their name like this: "On a coordinate grid, start at $(2, 4)$ and go to $(2, 8)$, then go to $(3, 6)$. Then go to $(4, 8)$, and then to $(4, 4)$."

Which student is it: Andre, Clare, Diego, Elena, Han, Jada, Kiran, Lin, Mai, Noah, Priya, or Tyler?



b. Describe, using coordinates, how to trace a letter from your name.

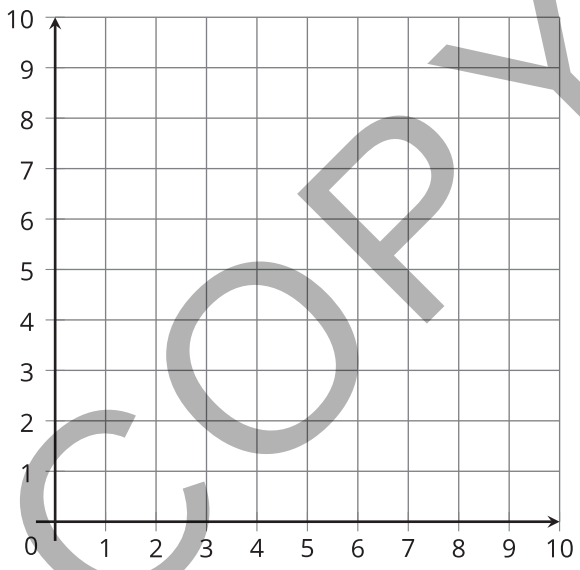


9 Exploration

Work with a partner.

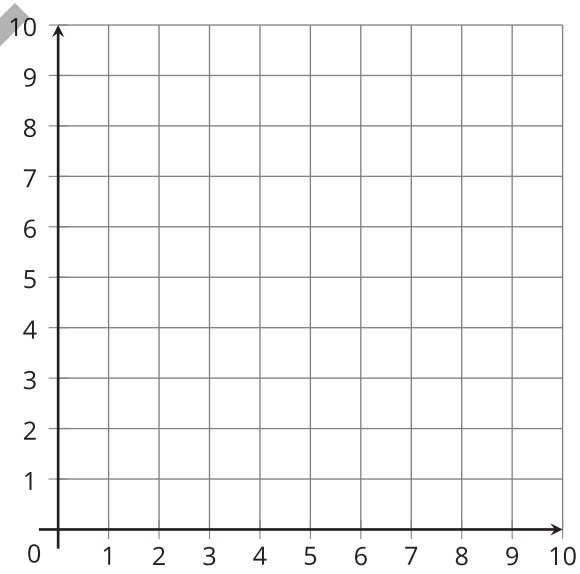
Partner A:

Draw a rectangle on the coordinate grid. Make sure the area of the rectangle is at least 20 square units. Don't show the rectangle to your partner.



Partner B:

Your goal is to figure out what rectangle your partner drew using the fewest points. Use coordinates to name points in the coordinate grid. Your partner will tell you whether the point is or is not on their rectangle.



Unit 7, Lesson 4

Addressing CA CCSSM 5.G.3, 5.G.4; building towards 5.G.3; practicing MP6 and MP7

Sort Quadrilaterals

Let's sort quadrilaterals.

Warm-up

Sec B

What Do You Know about Quadrilaterals?

What do you know about quadrilaterals?

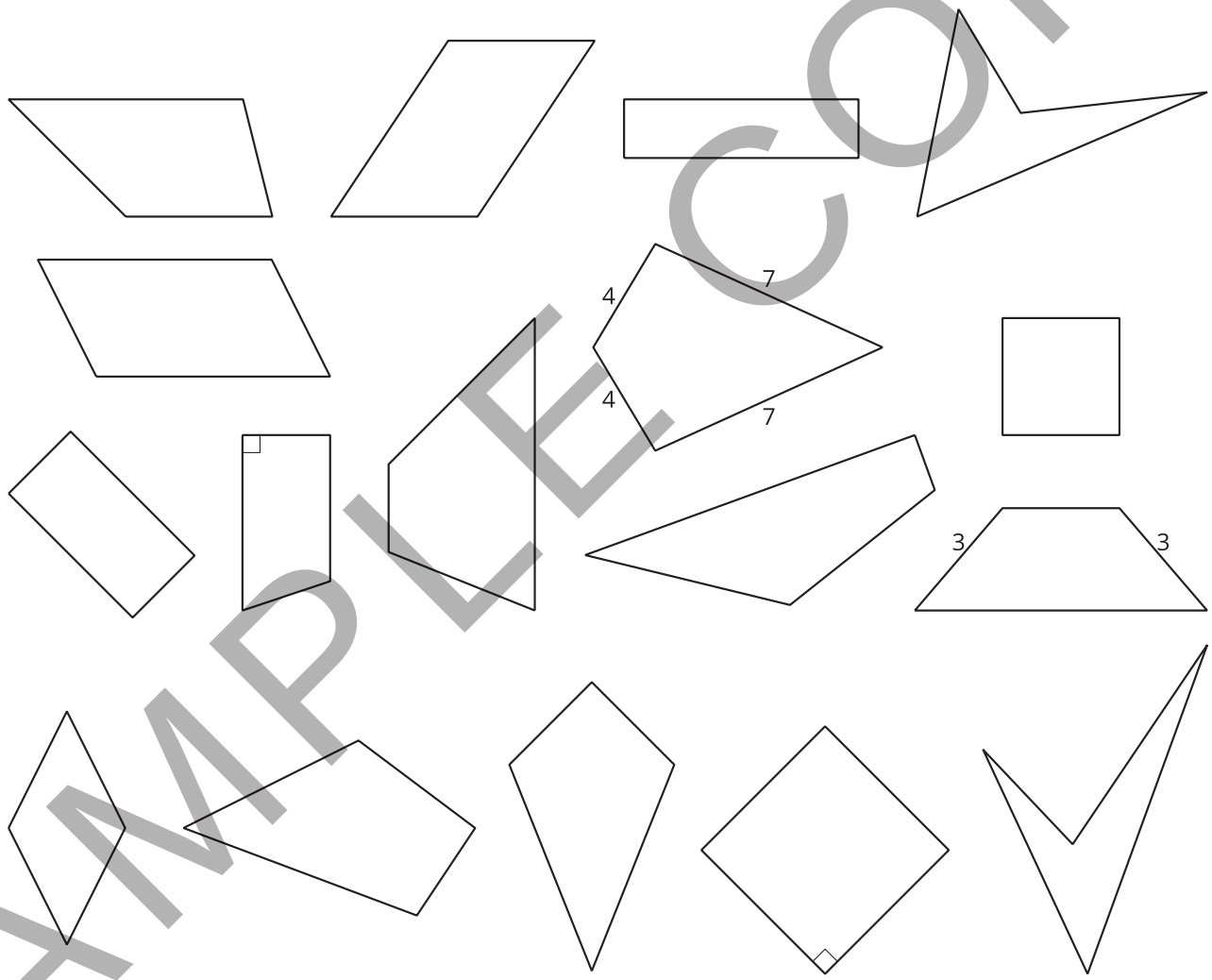
Activity 1

Which One?

1. Play a round of *Which One?*

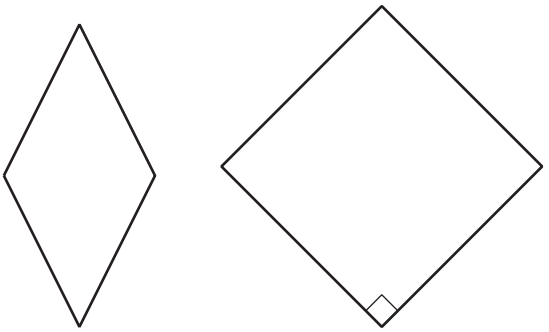
   Partner A: Choose one of these quadrilaterals. Do not tell or show your choice.

   Partner B: Ask *yes* or *no* questions to determine which shape your partner chose. After each question, cross off or cover quadrilaterals based on your partner's answer.



   Use this space to record your questions for this round.

2. Han and Mai play *Which One?* Han chooses one of these two shapes. What question can Mai ask to determine which shape Han chose?



If you have time: Switch roles and play another round of *Which One?*

Activity 2

Card Sort: Quadrilaterals

Your teacher will give you a set of cards that show quadrilaterals.

1. Sort the cards into at least 2 categories of your choosing. Be ready to explain the meaning of your categories.

   (Pause for teacher directions.)

2. Sort the cards into at least 2 categories using parallel lines. Be ready to explain the meaning of your categories.

Unit 7, Lesson 5

Addressing CA CCSSM 5.G.4; building towards 5.G.3; practicing MP6 and MP7

Trapezoids

Let's explore trapezoids.

Warm-up

Sec B

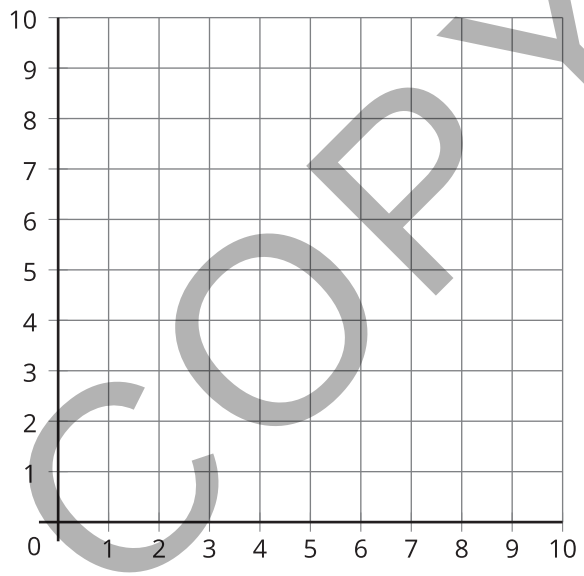
What Do You Know about Trapezoids?

What do you know about trapezoids?

Activity 1

What's a Trapezoid?

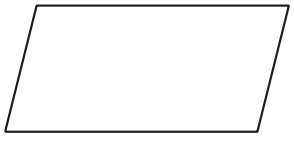
1. Draw a trapezoid on this coordinate grid. Label the coordinates of the 4 points you used for the 4 corners of the trapezoid.



2. Is it a square? Rectangle? Rhombus? Parallelogram? Explain your reasoning

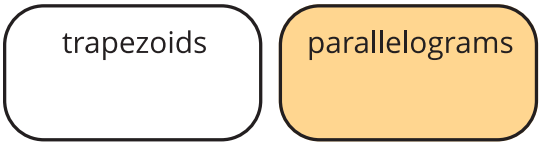
3. Describe a **trapezoid** in your own words. Compare your definition with a partner.

4. Is this shape a trapezoid according to your definition? Explain your reasoning.



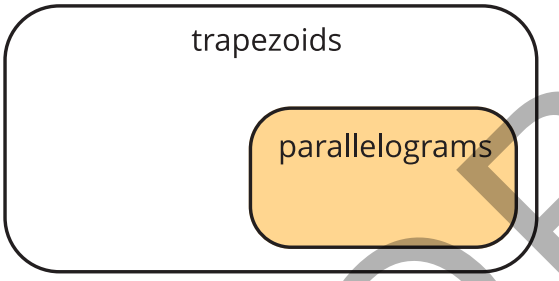
Activity 2

Two Definitions of a Trapezoid



Definition 1

A trapezoid is a quadrilateral that has exactly 1 pair of opposite sides that are parallel.



Definition 2

A trapezoid is a quadrilateral that has at least 1 pair of opposite sides that are parallel.

Which statements go with the first definition? Which statements go with the second definition? Explain or show your reasoning.

1. All parallelograms are trapezoids.

2. No parallelograms are trapezoids.

3. All trapezoids are parallelograms.

4. Some trapezoids are parallelograms.

5. No trapezoids are parallelograms.

Addressing CA CCSSM 5.G.3, 5.G.4; building towards 5.G.3; practicing MP2

Hierarchy of Quadrilaterals

Let's explore the hierarchy of quadrilaterals.

Warm-up

Notice and Wonder: Squares and Rhombuses

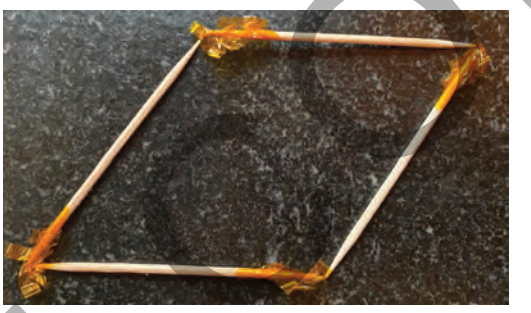
What do you notice? What do you wonder?

Activity 1

Shapes with Toothpicks

1. Build a square with 4 toothpicks. How do you know it is a square?

2. Use the same 4 toothpicks to build this shape. What stayed the same? What changed?



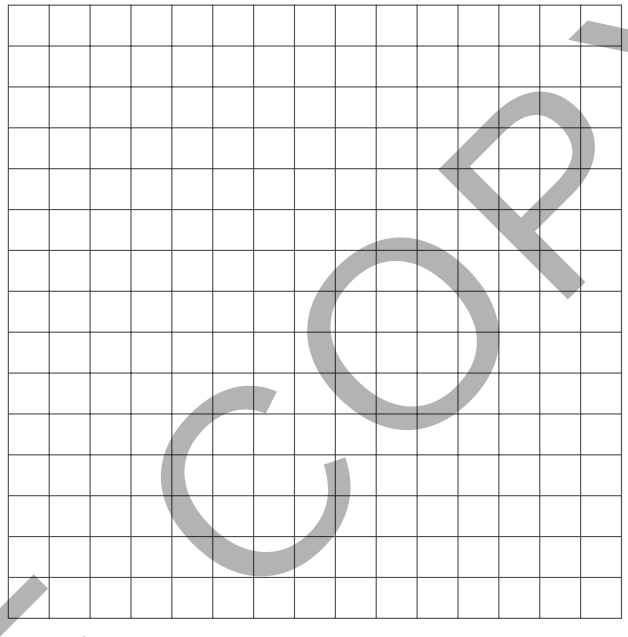
3. Build a rectangle with 6 toothpicks. How do you know it is a rectangle?

4. Use the same 6 toothpicks to build this shape. What stayed the same? What changed?

Activity 2

Three Quadrilaterals

1. Draw 3 different quadrilaterals on the grid. At least 1 of them must be a parallelogram.



2. For each of your quadrilaterals determine if it is a:
   - Square
   - Rhombus
   - Rectangle
   - Parallelogram

   Explain or show your reasoning.

3. Draw a rhombus that is *not* a square. Explain or show how you know it is a rhombus but *not* a square.

4. Draw a rhombus that is a square. Explain or show how you know it is a rhombus and a square.

5. Diego says that it is impossible to draw a square that is *not* a rhombus. Do you agree with him? Explain or show your reasoning.

Addressing CA CCSSM 5.G.3, 5.G.4; building towards 5.G.3; practicing MP7

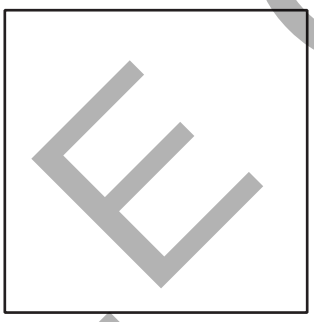
Rectangles and Squares

Let's learn more about rectangles and squares.

Warm-up

What Do You Know about This Shape?

What do you know about this shape?



Activity 1

Quadrilateral Clues

Your teacher will give you a set of cards that show quadrilaterals.

Spread out your cards so you and your partner can see all of them.

Work together to find a quadrilateral that fits each clue. If you don't think it's possible to find that quadrilateral, explain why. Each quadrilateral can only be used for one clue.

1. Find a rhombus that is also a square.

2. Find a quadrilateral that is *not* a parallelogram.

3. Find a rectangle that is *not* a square.

4. Find a rhombus that is *not* a square.

5. Find a square that is *not* a rectangle.

6. Find a trapezoid that is *not* a rectangle.

7. Find a parallelogram that is *not* a rectangle.

Activity 2

Always, Sometimes, Never

Write "always," "sometimes," or "never" in each blank to make the statements true.

For each statement that is completed with “sometimes,” draw a figure for which the statement is true and another figure for which the statement is false.

1. A rhombus is _____ a square.
2. A square is _____ a rhombus.
3. A triangle is _____ a quadrilateral.
4. A square is _____ a rectangle.
5. A rectangle is _____ a parallelogram.
6. A parallelogram is _____ a rhombus.
7. A trapezoid is _____ a parallelogram.

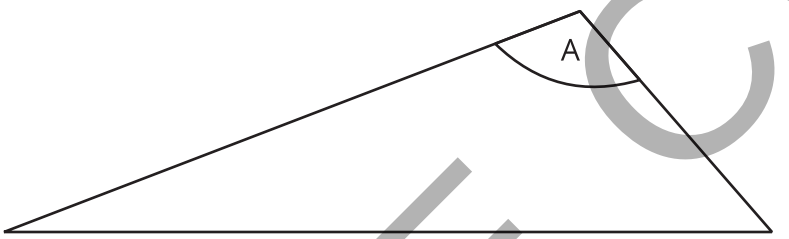
Sort Triangles

Let's sort triangles.

Warm-up

Estimation Exploration: Angle Measure

What is the measure of Angle A?



Record an estimate that is:

too low	about right	too high

Activity 1

The Right Fit

Your teacher will give you a set of cards that show triangles.

1. Find a card that fits the description for each space in the chart. Record the letter of each triangle in the correct space.
2. If you don't think it's possible to find a triangle that fits a space's descriptions, explain why not.

	All 3 side lengths are different.	Exactly 2 of the side lengths are equal.	All 3 side lengths are equal.
Has a 90-degree angle.			
Has an angle that is greater than 90 degrees.			
All 3 angles are less than 90 degrees.			

Explanations:

Activity 2

Card Sort: All, Some, None

Your teacher will give you a set of cards that show triangles.

1. Sort the cards into 2 categories of your choosing. Explain the meaning of your categories.

2. Now find all the triangles with a 90-degree angle. Complete the following statements about those triangles.
   - All of the triangles with a 90-degree angle...
   - Some of the triangles with a 90-degree angle...
   - None of the triangles with a 90-degree angle...

Section B Summary

We sorted and analyzed different kinds of *quadrilaterals* and triangles. We described their properties.

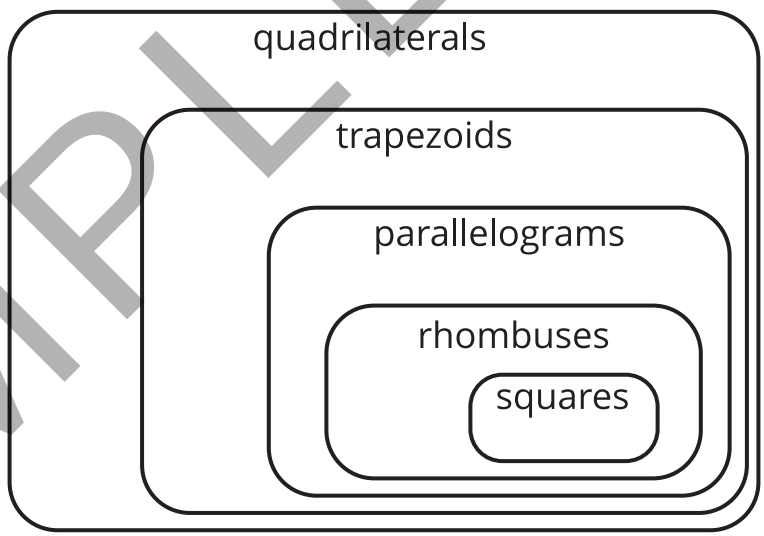
Examples:

- A *rectangle* is a quadrilateral with 4 right angles.
- A *rhombus* is a quadrilateral with 4 equal sides.
- A *square* is a quadrilateral with 4 right angles and 4 equal sides.
- A **trapezoid** is a quadrilateral with at least 1 pair of opposite sides that are parallel.
- A *right triangle* is a triangle with a 90-degree angle.

We also described how the shapes are related to each other.

Examples:

- A square is always a rhombus because all squares have 4 equal sides. But a rhombus is not always a square because some rhombuses do not have 4 right angles.
- A square is always a rectangle because it has 4 right angles. But a rectangle is not always a square because some rectangles do not have 4 equal sides.



Practice Problems

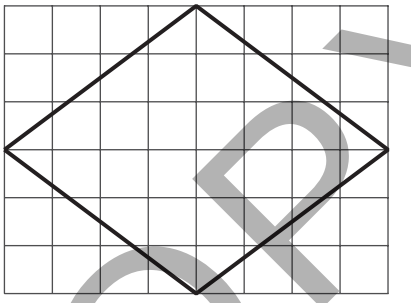
7 Problems

1

from Unit 7, Lesson 4

Determine whether the statement is true or false. Explain or show your reasoning.

a. The shape is a rectangle.



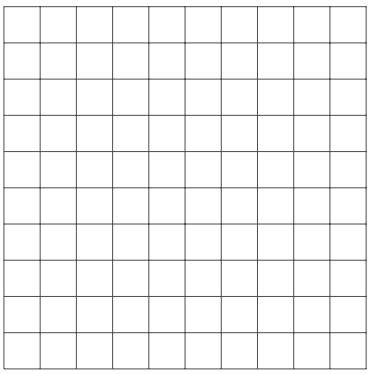
b. The shape is a square.

c. The shape is a rhombus.

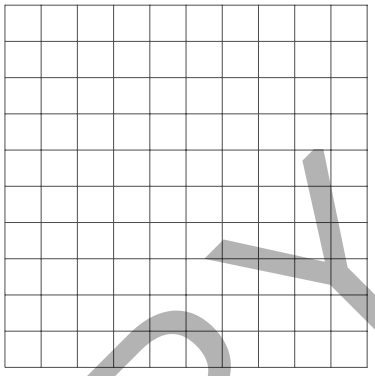
2

from Unit 7, Lesson 5

a. Draw a trapezoid that is also a parallelogram. Explain how you know it is a trapezoid and a parallelogram.

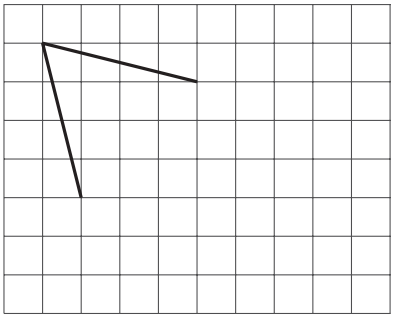


b. Draw a trapezoid that is *not* a parallelogram. Explain how you know it is a trapezoid but not a parallelogram.



3 from Unit 7, Lesson 6

Determine if it's possible to draw each shape so it contains the 2 sides drawn on the grid. Explain your reasoning.



a. a square

b. a rectangle

c. a rhombus

4 from Unit 7, Lesson 7

Decide if each statement is true or false. Explain or show your reasoning.

a. A parallelogram is sometimes a rhombus.

b. A rhombus is always a parallelogram.

c. A trapezoid is never a rectangle.

d. A rectangle is never a square.

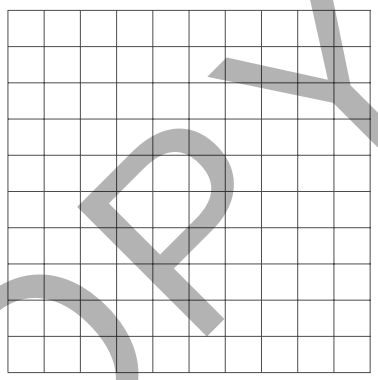
e. A parallelogram is always a trapezoid.

5

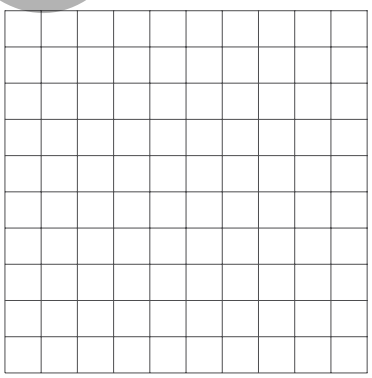
from Unit 7, Lesson 8

Draw a right triangle on the grid to match each description. If you think there is no such triangle, explain why.

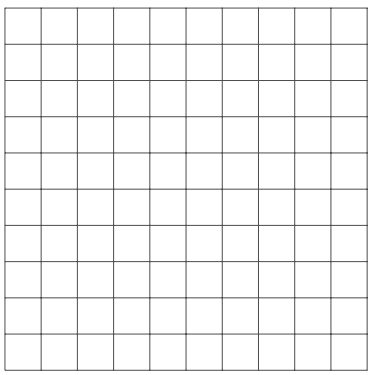
a. It has 2 equal side lengths.



b. It has 3 equal side lengths.



c. It has 3 different side lengths.

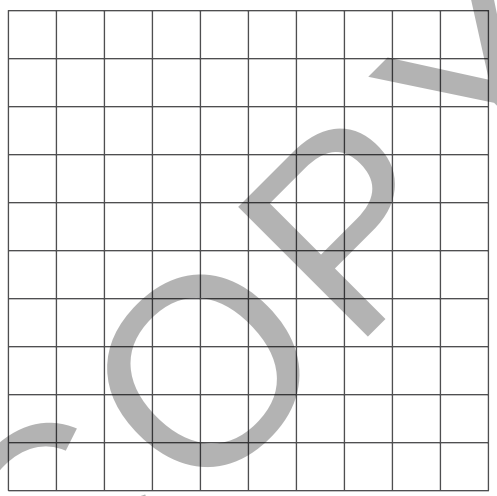


a. Jada cut a quadrilateral in half, from 1 vertex to the opposite vertex, and she got 2 triangles that each have exactly 2 equal side lengths. What kind of quadrilateral could Jada have cut in half? Explain or show your reasoning.

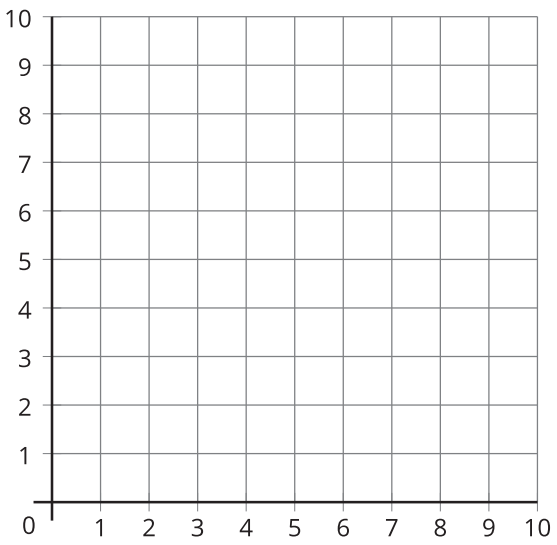
b. Elena put together 2 right triangles to make a quadrilateral. What kind of quadrilateral could Elena have made? Explain or show your reasoning.

7 Exploration

a. Can you draw a square on the grid that does not have a vertical or horizontal side? Explain or show your reasoning.



b. Draw the line segment from $(4, 4)$ to $(6, 5)$. Can you draw a square that contains this segment as one of its sides? Explain or show your reasoning.



Addressing CA CCSSM 5.OA.3; building on 4.OA.5; building towards 5.OA.3; practicing MP7 and MP8

Generate Patterns

Let's explore rules and patterns.

Activity 1

What's the Pattern?

1. Jada and Priya create patterns using rules. Complete the table with the first 10 numbers of their patterns.

   Jada's pattern: Start with 0 and use the rule "Keep adding 4."

   Priya's pattern: Start with 0 and use the rule "Keep adding 8."

2. Jada and Priya continue their patterns. What number will be in Priya's pattern when Jada's pattern has:

   a. 40

   b. 60

3. Jada and Priya continue their patterns. What number will be in Jada's pattern when Priya's pattern has 192?

4. What relationships do you notice between the numbers in Priya's pattern and the numbers in Jada's pattern?

Activity 2

More Patterns

Partner A

1. Complete each table with the first 10 numbers of the pattern.

   Jada's pattern: Start with 0. Use the rule "Keep adding 2."

   Priya's pattern: Start with 0. Use the rule "Keep adding 6."

2. If the patterns continue:

   a. What number will be in Priya's pattern when Jada's pattern has 34? Explain or show your reasoning.

   b. What number will be in Jada's pattern when Priya's pattern has 120? Explain or show your reasoning.

3. What relationships do you notice between the numbers in Priya's pattern and the numbers in Jada's pattern?

Partner B

4. Complete each table with the first 10 numbers of the pattern.

   Jada's pattern: Start with 0. Use the rule "Keep adding 3."

   Priya's pattern: Start with 0. Use the rule "Keep adding 9."

5. If the patterns continue:

   a. What number will be in Priya's pattern when Jada's pattern has 54? Explain or show your reasoning.

   b. What number will be in Jada's pattern when Priya's pattern has 198? Explain or show your reasoning.

6. What relationships do you notice between the numbers in Priya's pattern and the numbers in Jada's pattern?

Unit 7, Lesson 10

Addressing CA CCSSM 5.NF.3, 5.OA.3; building towards 5.OA.3; practicing MP2

Interpret Relationships

Let's find relationships between patterns.

Warm-up

True or False: Multiply and Divide

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $276 \div 3 = \frac{1}{3} \times 276$

- $276 \div 3 = \frac{276}{6}$

- $(276 \div 3) \times 2 = \frac{2}{3} \times 276$

Activity 1

Mix and Match: 3 Patterns

1. Complete each table with the first 5 numbers of the pattern.
2. What are some relationships between the patterns for each set of rules and starting numbers? Be prepared to explain your thinking.

Set A

Pattern 1: Start with 0. Use the rule "Keep adding 3."

Pattern 2: Start with 0. Use the rule "Keep adding 6."

Set B

Pattern 1: Start with 4. Use the rule "Keep adding 3."

Pattern 2: Start with 9. Use the rule "Keep adding 6."

Set C

Pattern 1: Start with 0. Use the rule "Keep adding 5."

Pattern 2: Start with 0. Use the rule "Keep adding 3."

Activity 2

Generate Patterns

Partner A

1. Complete each table with the first 10 numbers of the pattern.

   Pattern 1: Start with 0. Use the rule "Keep adding 4."

   Pattern 2: Start with 0. Use the rule "Keep adding 6."

2. Compare the patterns. What relationships do you notice?

3. What number will be in Pattern 2 when the number in Pattern 1 is 40?

4. What number will be in Pattern 1 when the number in Pattern 2 is 120?

Partner B

1. Complete each table with the first 10 numbers of the pattern.

   Pattern 1: Start with 0. Use the rule "Keep adding 2."

   Pattern 2: Start with 0. Use the rule "Keep adding 3."

2. Compare the patterns. What relationships do you notice?

3. What number will be in Pattern 2 when the number in Pattern 1 is 30?

4. What number will be in Pattern 1 when the number in Pattern 2 is 60?

Unit 7, Lesson 11

Addressing CA CCSSM 5.G.1, 5.OA.3; practicing MP8

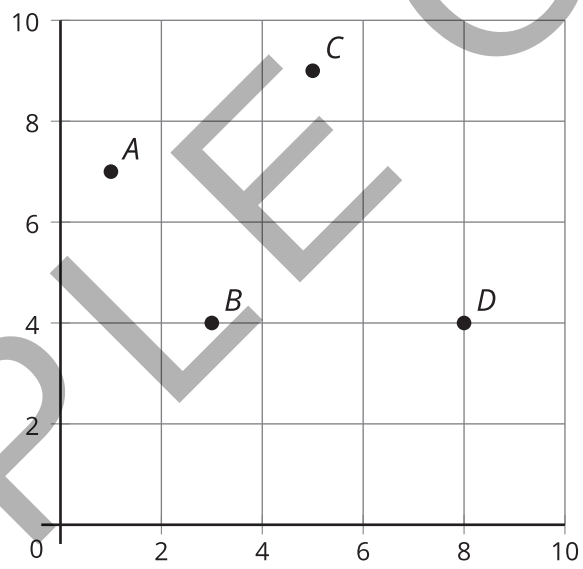
Patterns and Ordered Pairs

Let's represent patterns on the coordinate grid.

Warm-up

Notice and Wonder: The Coordinate Grid

What do you notice? What do you wonder?



Activity 1

Patterns on the Coordinate Grid (Part 1)

Partner A

Both patterns start with 0.

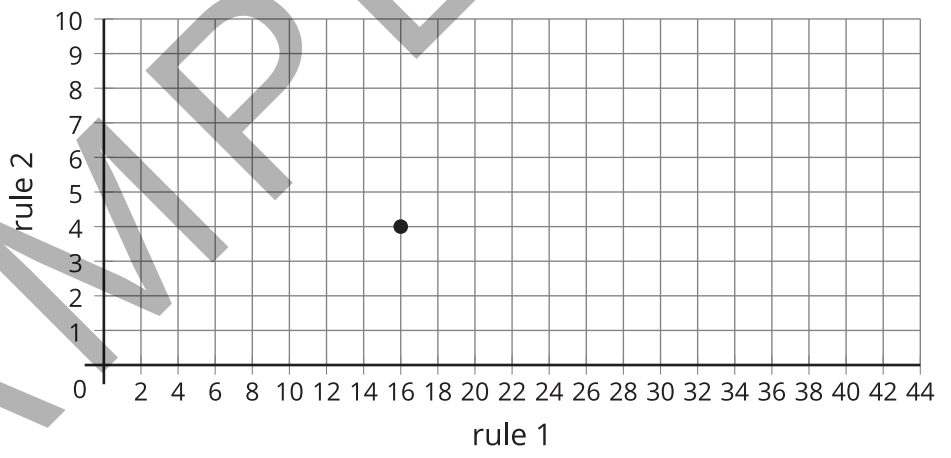
Rule 1: Keep adding 8.

Rule 2: Keep adding 2.

1. Use the rules to complete the table.

	A	*B*	*C*	*D*	*E*	*F*
rule 1						
rule 2						

2. Which column in the table represents the point plotted on this coordinate grid? Label the point with that column's letter.



3. Plot and label the rest of the points.

Partner B

Both patterns start with 0.

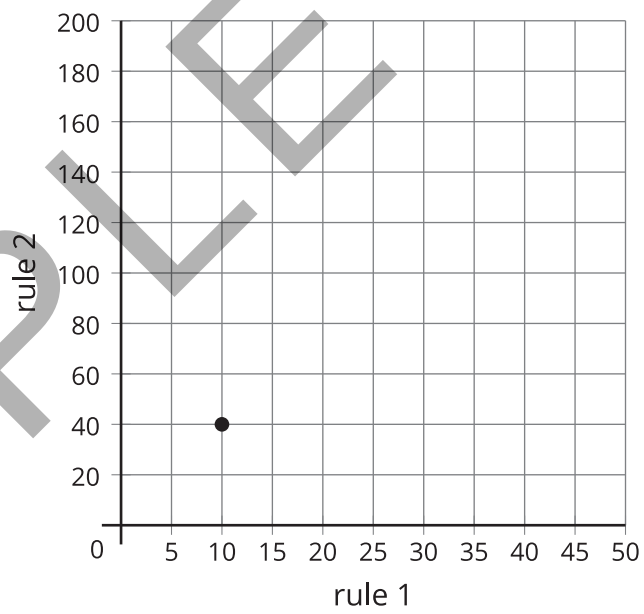
Rule 1: Keep adding 10.

Rule 2: Keep adding 40.

1. Use the rules to complete the table.

	A	*B*	*C*	*D*	*E*	*F*
rule 1						
rule 2						

2. Which column in the table represents the point plotted on this coordinate grid? Label the point with that column's letter.



3. Plot and label the rest of the points.

Activity 2

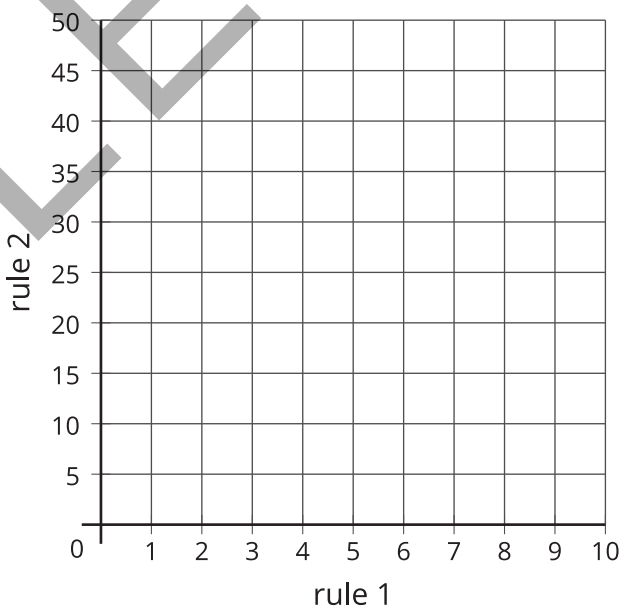
Patterns on the Coordinate Grid (Part 2)

1. Use the rules to complete the table. Both patterns start with 0.
   - Rule 1: Add 2.
   - Rule 2: Add 5.

	A	B	C	D	E	F
rule 1						
rule 2						

2. What relationships do you notice between the corresponding terms of the two patterns?

3. Plot and label the points from the table.



4. What does the Point C tell you about Rule 1 and Rule 2?

Unit 7, Lesson 12

Addressing CA CCSSM 5.G.2, 5.OA.2; practicing MP2 and MP7

Represent Problems on the Coordinate Grid

Let's represent problems on the coordinate grid.

Warm-up

True or False: Addition and Multiplication

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $(2 \times 10) + (3 \times 5) = (3 \times 10) + (1 \times 5)$

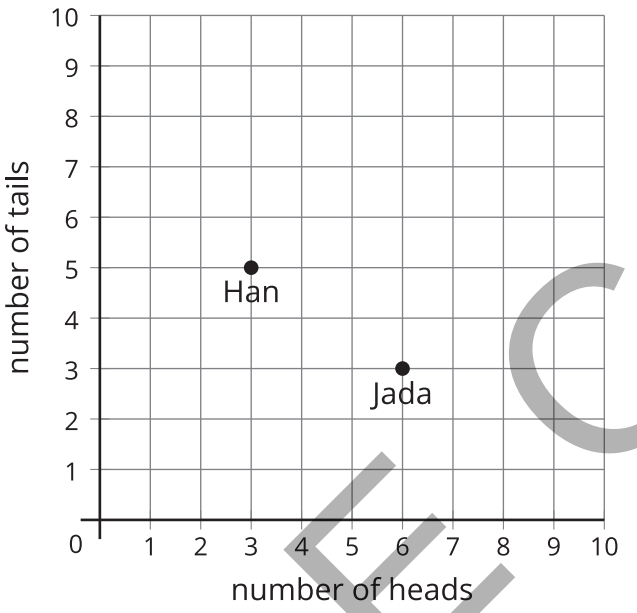
- $(3 \times 25) + (5 \times 5) = 8 \times 25$

- $(4 \times 25) + (10 \times 5) = (2 \times 25) + (10 \times 10)$

Activity 1

Heads or Tails

Han and Jada each flip a penny several times. They each count how many times they get heads and how many times they get tails. Their results are plotted on the coordinate grid.



1. How many heads did Jada get? How many tails did Jada get? Explain how you know.

2. How many heads did Han get? How many tails did Han get? Explain how you know.

3. Flip a coin 10 times and record how many heads and tails you get. Plot the point that represents the results of your coin flips on the coordinate grid.

4. Show your partner the point you plotted on the coordinate grid. Look at the point your partner plotted. How many heads did your partner get? How many tails did your partner get? Explain or show your reasoning.

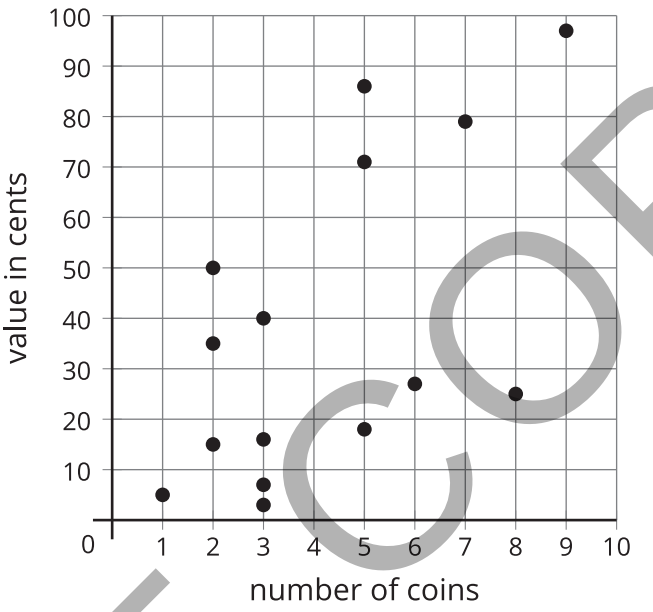
5. Do either of the points you or your partner plotted lie on the horizontal axis? What would a point on the horizontal axis mean in this situation?

6. If time allows, flip the coin 10 more times and record your results on the coordinate grid.

Activity 2

Coin Values

The points plotted on the coordinate grid represent the number and value of coins some students had with them.



1. Tyler has 1 dime, 3 nickels, and 2 pennies. Which point represents Tyler's coins? Label the point.
2. Lin has 3 quarters, 1 dime, and 1 penny. Which point represents Lin's coins? Label the point.
3. Diego has 1 quarter and 1 dime. Write the coordinates of the plotted point that represents Diego's coins. Explain or show your reasoning.

4. Clare has 5 coins and does not have a quarter. Write the coordinates of the point that represents Clare's coins.
5. Which coins might Clare have? Explain or show your reasoning.

Addressing CA CCSSM 5.G.2, 5.NBT.7; building on 5.OA.3; building towards 5.G.2; practicing MP7 and MP8

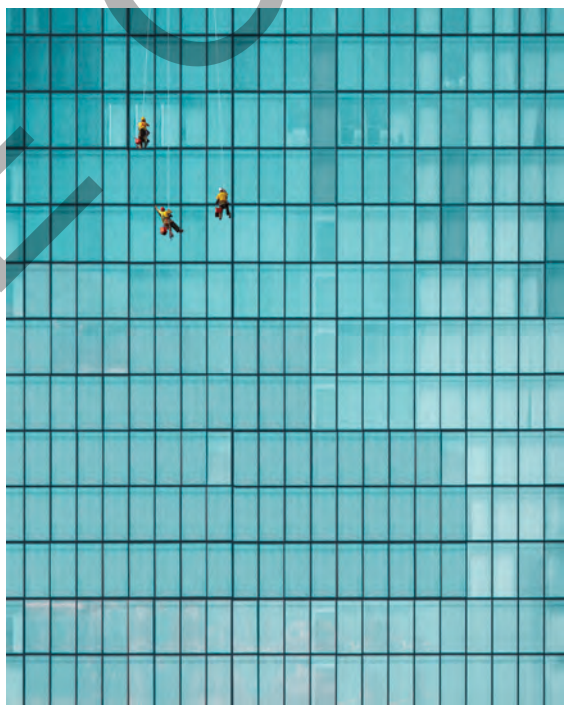
Perimeter and Area of Rectangles

Let's explore the perimeter and area of rectangles on the coordinate grid.

Warm-up

Estimation Exploration: Window Washing

What is the area of 1 window?



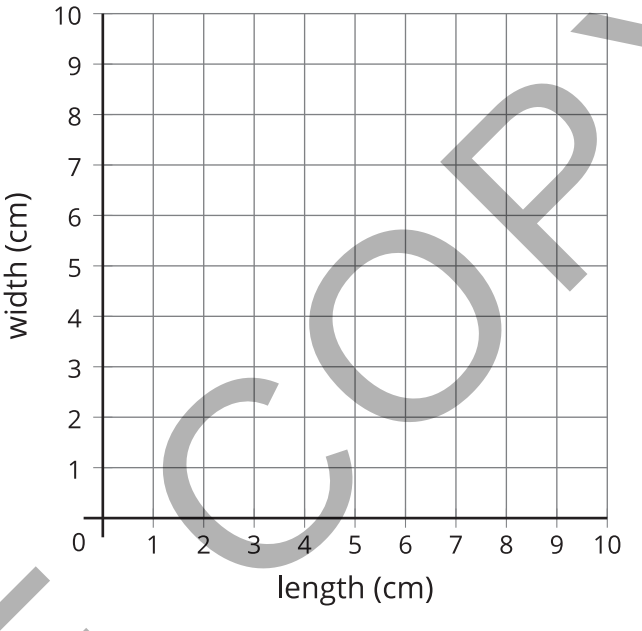
Record an estimate that is:

too low	about right	too high

Activity 1

Rectangle Perimeters

length (cm)	width (cm)



1. Jada draws a rectangle with a perimeter of 12 centimeters. What could be the length and width of the rectangle if all the side lengths are whole numbers? Use the table to record 5 possible answers.

2. Represent the length and width of each rectangle as a point on the coordinate grid.

3. If Jada draws a square, how long and wide will it be?

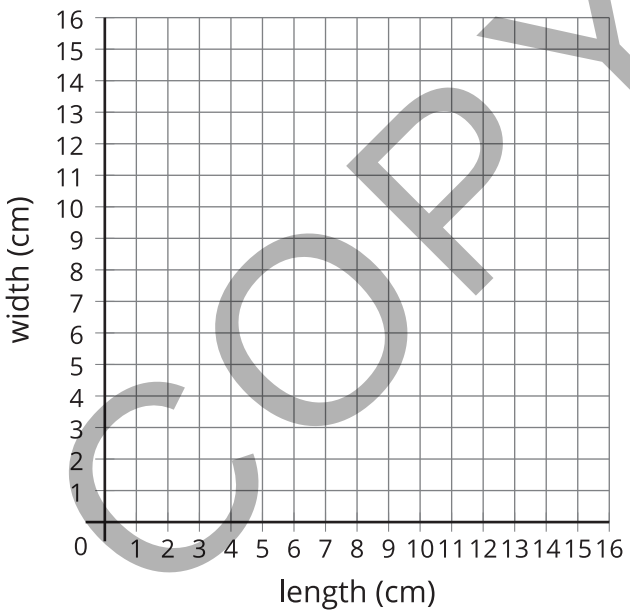
4. If Jada's rectangle is 2.5 cm long, how wide is it? Represent this rectangle as a point on the coordinate grid.

5. If Jada's rectangle is 3.25 cm long, how wide is it? Represent this rectangle as a point on the coordinate grid.

Activity 2

Rectangle Areas

length (cm)	width (cm)



1. Jada draws a rectangle with an area of 16 square centimeters. What could be the length and width of the rectangle if all the side lengths are whole numbers? Use the table to record 5 possible answers.

2. Represent the length and width of each rectangle as a point on the coordinate grid.

3. If Jada's rectangle is 5 cm long, how wide is it? Represent this rectangle as a point on the coordinate grid.

4. If Jada's rectangle is 3 cm long, how wide is it? Represent this rectangle as a point on the coordinate grid.

5. If Jada draws a square, how long and wide is it? Explain how you know.

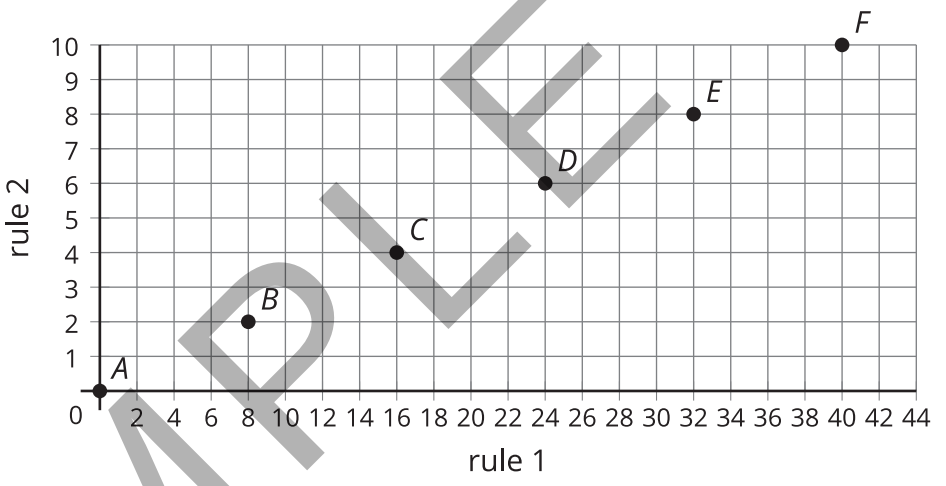
Section C Summary

We generated patterns and analyzed relationships between two different patterns.

Example: Both patterns start with 0.

	A	B	C	D	E	F
rule 1: Add 8.	0	8	16	24	32	40
rule 2: Add 2.	0	2	4	6	8	10

Each number in Rule 1's pattern is 4 times the value of the corresponding number in Rule 2's pattern. Each number in Rule 2's pattern is $\frac{1}{4}$ times the value of the corresponding number in Rule 1's pattern. We represented 2 patterns together as points on a coordinate grid.



We also used points on a coordinate grid to represent other situations, such as the length and width of rectangles with a given area or perimeter.

Addressing CA CCSSM 5.G.2; building towards 5.G.2; practicing MP4

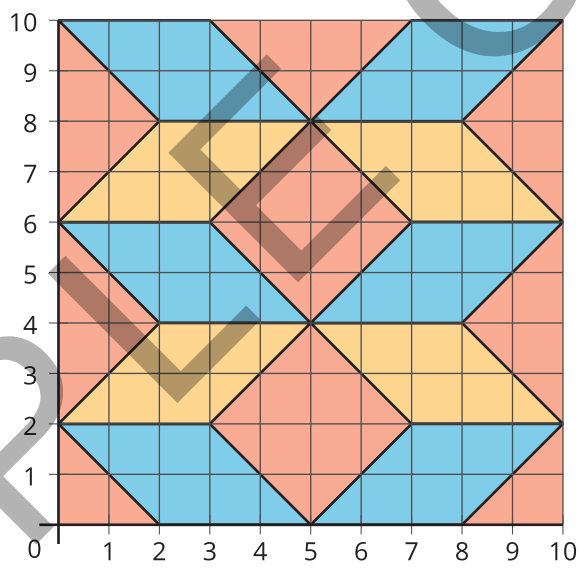
Copies of Figures

Let's use coordinate grids to make a copy of a figure.

Warm-up

Notice and Wonder: Figure with a Grid

What do you notice? What do you wonder?



Activity 1

Plan a Copy

Choose the figure that you will copy using a coordinate grid.

1. Create a coordinate grid on your chosen figure.
2. Decide which points you'll use to copy the figure and give the coordinates for each point. Explain your reasoning.

Activity 2

Make a Copy

Create a coordinate grid on grid paper. Use the points you chose in the previous activity to copy your figure.

1. How are the original figure and the copy of the figure alike?

2. How are they different?

Practice Problems

7 Problems

1 from Unit 7, Lesson 9

a. Complete each table with the first 10 numbers of the pattern.

i. Pattern 1: Start with 0. Use the rule "Keep adding 5."

ii. Pattern 2: Start with 0. Use the rule "Keep adding 10."

b. What relationships do you notice between the numbers in the 2 patterns?

Sec C

2 from Unit 7, Lesson 10

a. Complete each table with the first 10 numbers of the pattern.

i. Pattern 1: Start with 0. Use the rule "Keep adding 6."

ii. Pattern 2: Start with 4. Use the rule "Keep adding 6."

b. When Pattern 1 has the number 222, what number will be in Pattern 2? Explain or show your reasoning.

3

from Unit 7, Lesson 11

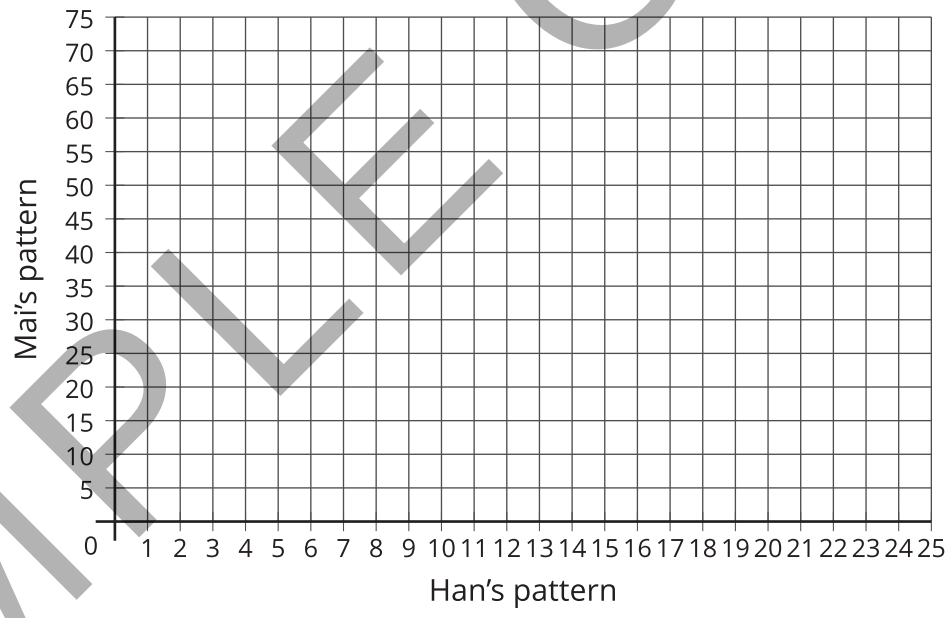
Han and Mai created different patterns using these rules and starting numbers.

Han's pattern: Start with 0. Use the rule "Keep adding 3."
Mai's pattern: Start with 0. Use the rule "Keep adding 10."

a. Complete the table with the first 8 numbers in each pattern.

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Han's rule								
Mai's rule								

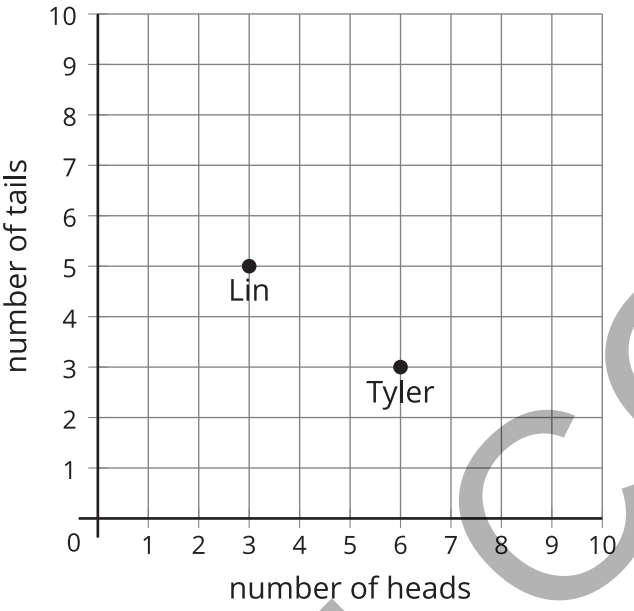
b. Locate and label the points on the coordinate grid.



c. What relationships do you notice between the corresponding terms of these 2 patterns?

4 from Unit 7, Lesson 12

The points on the coordinate grid show the results Lin and Tyler got when they each flipped a coin several times.

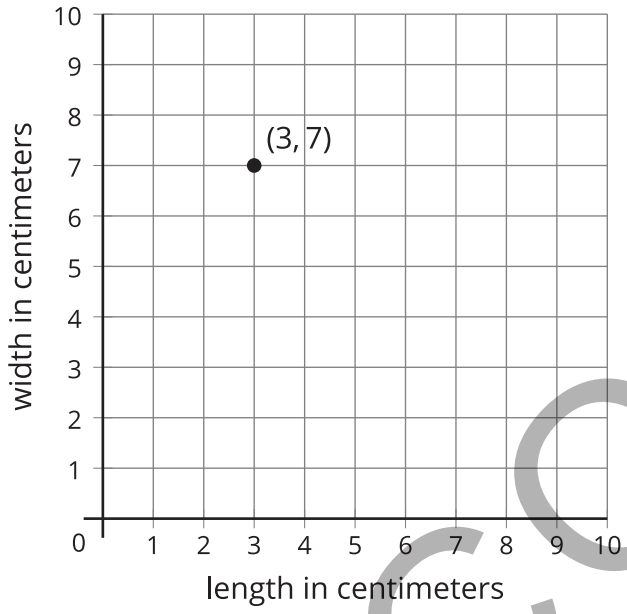


a. Who flipped the coin more times, Lin or Tyler? Explain or show your reasoning.

b. Who got more tails, Lin or Tyler? Explain or show your reasoning.

c. Flip a coin 7 times and plot the point to represent your results on the coordinate grid. Explain or show your reasoning.

5 from Unit 7, Lesson 13

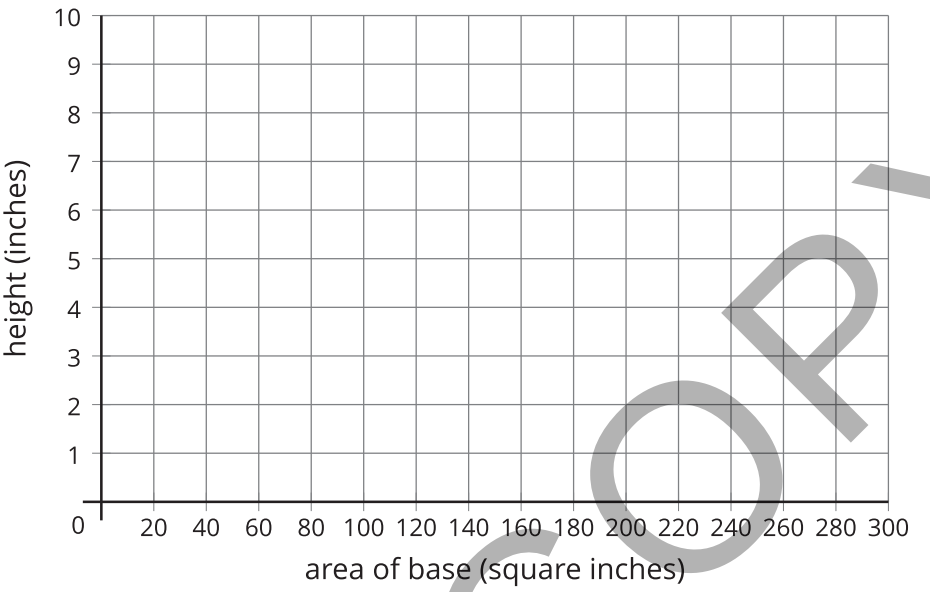


a. The point on the coordinate grid represents the length and width of a rectangle. What is the perimeter of the rectangle?

b. Plot 4 more points to represent 4 different rectangles with the same perimeter as the given rectangle.

c. What point would represent a square with the same perimeter as the given rectangle?

area of base (square inches)	height (inches)



a. A box is shaped like a rectangular prism. The volume of the box is 240 cubic inches. List some possible values for the area of the base of the box and its height in the table.

b. Plot the listed base area and height pairs as points on the coordinate.

c. What do you notice about the plotted points?

d. Which point do you think represents the most reasonable measurements for the box? Explain your reasoning.

7 Andre and Clare create patterns.

- Andre's pattern: Start with 2. Use the rule "Keep adding 6."
- Clare's pattern: Start with 1,000. Use the rule "Keep subtracting 7."

a. List the first 6 numbers in Andre and Clare's patterns.

b. Will Andre and Clare ever have the same number in the same spot in their patterns? Explain or show your reasoning.

GRADE 5

UNIT

8

Putting It All Together

Content Connections

In this unit you will use the standard algorithm to find the value of products, solve real-world problems about volume, and continue to work on operations with decimals and fractions. You will make connections by:

- **Exploring Changing Quantities** while using the standard algorithm to solve multiplication and division problems.
- **Taking Wholes Apart, Putting Parts Together** while practicing operations with decimals and fractions.
- **Discovering Shape and Space** while solving real world problems about volume and create models within mathematics.

Addressing the Standards

As you work your way through **Unit 8 Putting It All Together,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 6, 8, 10, 12, and 17
MP2 Reason abstractly and quantitatively.	Lessons 6, 8, 14, and 16
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 1, 2, 5, and 15
MP4 Model with mathematics.	Lessons 7 and 9
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lesson 18
MP7 Look for and make use of structure.	Lessons 1, 2, 3, 4, 10, 11, and 13
MP8 Look for and express regularity in repeated reasoning.	Lessons 2 and 13

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling	**5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm.	Lessons 1, 2, 3, 7, 8, and 15
• Seeing Division	**5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Lessons 4, 5, 8, and 16
• Modeling • Seeing Division	**5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Lesson 12

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes on a Plane	**5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	Lesson 4
• Shapes on a Plane	**5.G.4** Classify two-dimensional figures in a hierarchy based on properties.	Lesson 4
• Seeing Division • Layers of Cubes	**5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	Lessons 6 and 18
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Lesson 6
• Factors and Groups • Modeling • Seeing Division • Layers of Cubes	**5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ *and* $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	Lessons 6, 7, 8, and 9

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Modeling • Fraction Connections	**5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	Lessons 10, 11, and 17
• Modeling • Fraction Connections	**5.NF.3** Interpret a fraction as division of the numerator by the denominator *(a/b = a ÷ b).* Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Lesson 14
• Modeling • Fraction Connections • Shapes on a Plane	**5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product *(a/b) × q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Lesson 13

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 8, Lesson 1

Addressing CA CCSSM 5.NBT.5; practicing MP3 and MP7

Find the Greatest Product

Let's look for patterns when we multiply multi-digit numbers.

Warm-up

Notice and Wonder: Digits

What do you notice? What do you wonder?

$$\begin{array}{r} 8\ 4\ 1 \\ \times\quad\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 6\ 4\ 1 \\ \times\quad\ 8 \\ \hline \end{array}$$

Activity 1

Talk about It

1. Consider the statement below. Decide whether you agree, disagree, or are unsure.

	agree	disagree	unsure
Round 1: The greatest product using the digits 7, 5, and 2 is 75×2 because 75 is the greatest number you can make.			
Round 2: The greatest product using the digits 7, 5, and 2 is 75×2 because 75 is the greatest number you can make.			

Write about something new that you learned from your group or something you still wonder about:

2. Use the digits 6, 3, and 1 to make the greatest product. (Use each digit only once.) Explain your reasoning.

Activity 2

More Digits

1. Use the digits 7, 3, 2, and 5 to make the greatest product. Use each digit only once.

2. Explain your reasoning.

Addressing CA CCSSM 5.NBT.5; practicing MP3, MP7, MP8

More Multiplication

Let's practice using the multiplication algorithm.

Warm-up

Estimation Exploration: Large Product

$9{,}999 \times 896$

Record an estimate that is:

too low	about right	too high

Activity 1

Kiran's Work

1. Find the value of the product. Explain or show your reasoning.

$$\begin{array}{r} 650 \\ \times \quad 27 \\ \hline \end{array}$$

2. Here is how Kiran found the product. Is his answer reasonable? Explain your reasoning.

$$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 650 \\ \times \quad 27 \\ \hline 4,550 \\ +\ 1,300 \\ \hline 5,850 \end{array}$$

3. What parts of the work do you agree with?

4. What parts of the work do you disagree with?

5. Look at the value you found for 650×27. Is there anything you want to revise? Explain or show your reasoning.

Activity 2

Zero the Hero

Find the value of each product.

1.

$$\begin{array}{r} 260 \\ \times\ 35 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 2,600 \\ \times\ 35 \\ \hline \end{array}$$

3.

$$\begin{array}{r} 670 \\ \times\ 89 \\ \hline \end{array}$$

4.

$$\begin{array}{r} 6,700 \\ \times\ 89 \\ \hline \end{array}$$

Unit 8, Lesson 3

Addressing CA CCSSM 5.NBT.5; practicing MP7

Factors as a Factor in Our Strategy Choice

Let's reason about strategies we use to multiply.

Warm-up

Number Talk: Increasing Factors

Find the value of each expression mentally.

- 230×10
- 230×12
- 230×15
- 232×15

Activity 1

Choose a Multiplication Strategy

Find the value of each expression. Explain or show your reasoning.

1. 14×3

2. 14×101

3. 14×25

4. 14×9

5. 14×136

Activity 2

Compare Strategies

Find the value of each expression.

1. 29×7

2. 12×45

3. 15×199

4. 24×154

Unit 8, Lesson 4

Addressing CA CCSSM 5.G.3, 5.G.4, 5.NBT.6; practicing MP7

Dive Back into Division

Let's estimate and divide.

Warm-up

Notice and Wonder: Blank Spaces

What do you notice? What do you wonder?

$\_\_\_\_ \div \_\_\_\_ = 136$

Activity 1

Reasonable Estimates

1. Circle the most reasonable estimate. Explain or show your reasoning.

   a. $364 \div 13$

   20

   30

   40

   b. $540 \div 12$

   40

   50

   60

   c. $1{,}008 \div 14$

   70

   80

   90

2. Find the value of each quotient.

a.

$13\overline{)364}$

b.

$12\overline{)540}$

c.

$14\overline{)1,008}$

Activity 2

Unknown Dividends and Divisors

1. Find numbers that make each equation true. Use a different number in each blank.

_____ ÷ _____ = 700	_____ ÷ _____ = 78
_____ ÷ _____ = 700	_____ ÷ _____ = 78
_____ ÷ _____ = 700	_____ ÷ _____ = 78
_____ ÷ _____ = 700	_____ ÷ _____ = 78
_____ ÷ _____ = 700	_____ ÷ _____ = 78

2. How did you choose the numbers to write in the blanks?

More Division

Let's divide.

Warm-up

Estimation Exploration: Large Quotient

$9{,}953 \div 37$

Record an estimate that is:

too low	about right	too high

Activity 1

Elena's Work

1. Find the value of the quotient.

$13\overline{)6,773}$

2. Here is how Elena found the quotient. Is her answer reasonable?

   Explain or show your reasoning.

$$
\begin{array}{rl}
 & \boxed{53} \\
 & 1 \\
 & 2 \\
 & 50 \\
13\overline{)6773} & \\
-\ 650 & (50 \times 13) \\
\hline
 & 273 \\
-\ 26 & (2 \times 13) \\
\hline
 & 13 \\
-\ 13 & (1 \times 13) \\
\hline
 & 0
\end{array}
$$

3. What parts of the work do you agree with?

4. What parts of the work do you disagree with?

5. Look at your solution to problem 1. Is there anything you want to revise? Explain or show your reasoning.

Activity 2

Partial Quotients Practice

1. Use partial quotients to find the value of one of the quotients.

   Partner A

   $37\overline{)2,516}$

   Partner B

   $32\overline{)2,272}$

2. Explain to your partner how you found the quotient in your problem.

Addressing CA CCSSM 5.MD.3-5; practicing MP1 and MP2

Revisit Volume

Let's solve problems about volume.

Warm-up

Estimation Exploration: Sugar Cubes

How many cubes are in the bowl?

Record an estimate that is:

too low	about right	too high

Activity 1

126 Cubes

A company packages 126 sugar cubes in each box. The box is a rectangular prism.

1. What are some possible ways to pack the cubes?

2. How would you pack the cubes? Explain or show your reasoning

Activity 2

Colossal Structures Old and New

1. The Great Pyramid of Giza was built in Egypt more than 4,000 years ago. Today, it is 137 meters tall. The base of the pyramid is a square. Each side of the base is 230 meters long. If the pyramid was shaped like a rectangular prism, what would the volume of the prism be?

2. The Empire State Building is in New York City. The base is 129 meters by 57 meters. The building is 381 meters tall. Estimate the volume of the Empire State Building.

3. Which do you think is larger, the Great Pyramid or the Empire State Building? Explain or show your reasoning.

Unit 8, Lesson 7

Addressing CA CCSSM 5.MD.5, 5.NBT.5; building towards 5.MD.5; practicing MP4

Estimate the Volume of the World's Largest Wagon

Let's solve problems about volume.

Sec B

Warm-up

Notice and Wonder: The Big Red Wagon

What do you notice? What do you wonder?

Activity 1

Anatomy of an Estimate

1. What measurements do you need to accurately estimate the volume of the wagon?

2. What units would you use to measure the wagon? Explain your reasoning.

3. Record an estimate for the volume of the wagon that is:

too low	about right	too high

4. What can you use in the picture to refine your estimate?

Activity 2

Estimate the Size of the Big Red Wagon

1. Use the picture of the wagon to make a better estimate of the length, width, and height of the wagon bed.

   Make sure to:

   - Explain how you estimated each measurement.
   - Include how accurate you think each estimate is.

2. Then improve your estimate for the volume of the wagon bed.

Addressing CA CCSSM 5.MD.5, 5.NBT.5, 5.NBT.6; building towards 5.MD.5, 5.NBT.5, 5.NBT.6; practicing MP1 and MP2

Fill the World's Largest Wagon

Let's solve more problems about volume.

Warm-up

Notice and Wonder: Toy Boxes

What do you notice? What do you wonder?

Activity 1

Sand Wagon

The wagon bed is approximately 27 feet long, 13 feet wide, and 2 feet deep.

1. A 150-pound bag of sand fills about 9 cubic feet. How many bags of sand does it take to fill the wagon with sand?

2. A 150-pound bag of sand costs about $12. About how much does it cost to fill the wagon with sand? Explain or show your reasoning.

3. How many pounds of sand does the wagon hold when it is full? Explain or show your reasoning.

Activity 2

More Boxes

The bed of the big red wagon is approximately 27 feet long, 13 feet wide, and 2 feet deep.

The wagon is being used to deliver 4,000 boxes. Each box has side lengths of 2 feet by 2 feet by 2 feet. How many trips does the wagon have to make to deliver all the boxes? Explain or show your reasoning.

Unit 8, Lesson 9

Addressing CA CCSSM 5.MD.5; building towards 5.MD.5; practicing MP4

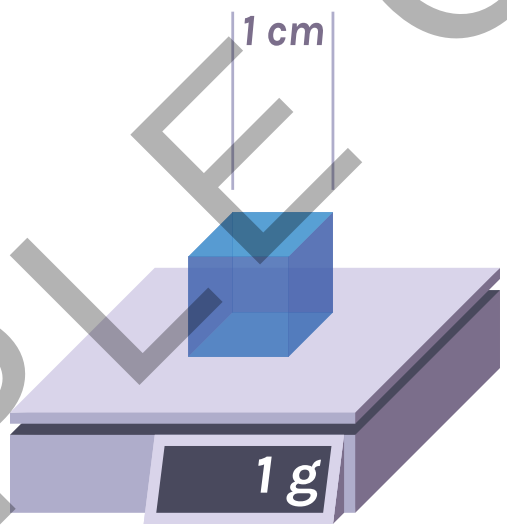
Problem Solving with Volume: Water

Let's solve problems about volume of water.

Warm-up

Notice and Wonder: Cubic Centimeters and Grams

What do you notice? What do you wonder?

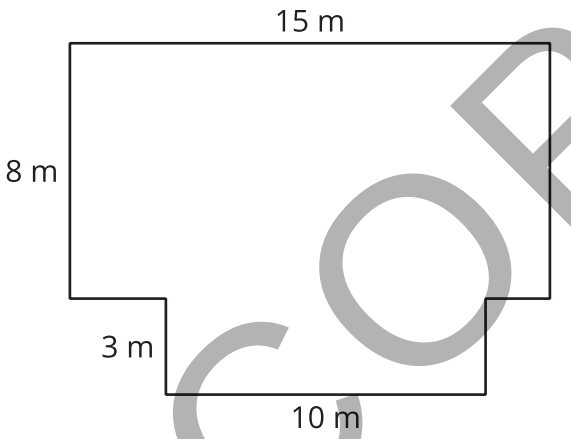


Activity 1

Catching Rainfall

Here is a diagram showing the roof of a house.

1. What is the area of the roof?



2. Each month an average of 5 cm of rain falls on the house. How many cubic cm of rain is that?

3. There are 1,000 cubic cm in 1 liter. How many liters of water fall on the house each month?

4. You decide to build a reservoir to catch the rain that falls on the roof. It will be shaped like a rectangular prism. What edge lengths would you use for the reservoir? Explain or show your reasoning.

Activity 2

How Much Water?

1. What are some of the ways you use water at home?

2. Estimate how much water, in liters, you use at your home in a month.

3. How many centimeters of rain need to fall on your home each month to supply your home?

4. What are some challenges with using rainwater that falls on the roof of your home? Explain your reasoning.

Unit 8, Lesson 10

Addressing CA CCSSM 5.NF.1; practicing MP7 and MP1

Here Comes the Sum

Let's play some games to practice adding fractions.

Warm-up

Number Talk: Adding Fractions

Find the value of each expression mentally.

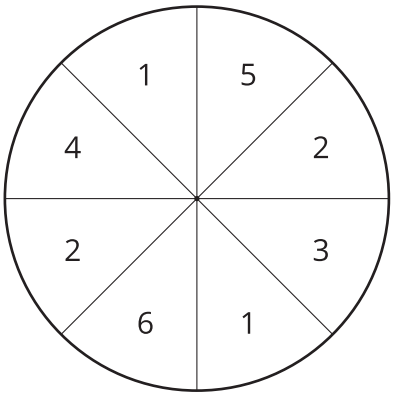
- $\frac{2}{12} + \frac{1}{6}$
- $\frac{2}{6} + \frac{1}{2}$
- $\frac{1}{3} + \frac{1}{2}$
- $\frac{1}{3} + \frac{3}{2}$

Activity 1

Greatest Sum

Use the directions to play Greatest Sum with a partner.

1. Spin the spinner.
2. Write the number in an empty box for Round 1. Be sure your partner can't see your paper. Once a number is written down, it can't be changed.
3. Take turns. Spin and write numbers in the empty boxes until all 4 boxes have been filled.
4. Find the sum.
5. The partner with the greater sum wins the round.
6. The partner who won the most rounds wins the game. If there is a tie, players add the sums from all 4 rounds. The greater total sum wins the game.



Round 1

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Round 2

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Round 3

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Round 4

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

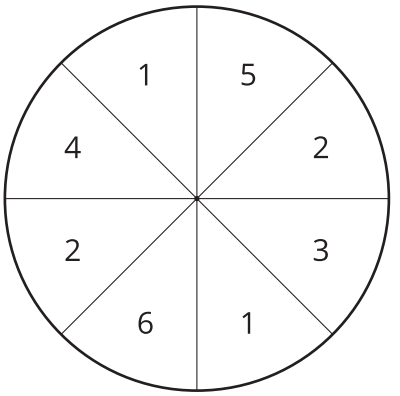
Total sum of all 4 rounds:

Activity 2

Least Sum

Use the directions to play Least Sum with a partner.

1. Spin the spinner.
2. Write the number in an empty box for Round 1. Be sure your partner can't see your paper. Once a number is written down, it can't be changed.
3. Take turns. Spin and write numbers in the empty boxes until all 4 boxes have been filled.
4. Find the sum.
5. The partner with the lesser sum wins the round.
6. The partner who won the most rounds wins the game. If there is a tie, partners add the sums from all 4 rounds. The lesser total sum wins the game.



Round 1

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Round 2

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Round 3

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Round 4

$\frac{\square}{\square} + \frac{\square}{\square} = \square$

Total sum of all 4 rounds:

Unit 8, Lesson 11

Addressing CA CCSSM 5.NF.1; practicing MP7

What's the Difference?

Let's subtract fractions.

Warm-up

Number Talk: Subtracting Fractions

Find the value of each difference mentally.

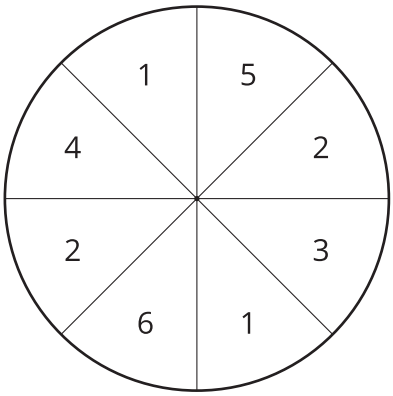
- $\frac{2}{3} - \frac{1}{6}$
- $\frac{2}{3} - \frac{2}{6}$
- $\frac{2}{3} - \frac{4}{6}$
- $\frac{2}{3} - \frac{1}{2}$

Activity 1

Greatest Difference

Use the directions to play Greatest Difference with a partner.

1. Spin the spinner.
2. Write the number in an empty box for Round 1. Be sure your partner can't see your paper. Once a number is written down, it can't be changed.
3. Take turns. Spin and write numbers in the empty boxes until all 4 boxes have been filled.
4. Find the difference.
5. The partner with the greater difference wins the round.
6. The partner who won the most rounds wins the game. If there is a tie, partners add the differences from all 4 rounds. The greater total sum wins the game.



Round 1

$$\frac{\square}{\square} - \frac{\square\square}{\square\square} =$$

Round 2

$$\frac{\square\square}{\square\square} - \frac{\square\square}{\square\square} =$$

Round 3

$$\frac{\square}{\square} - \frac{\square\square}{\square\square} =$$

Round 4

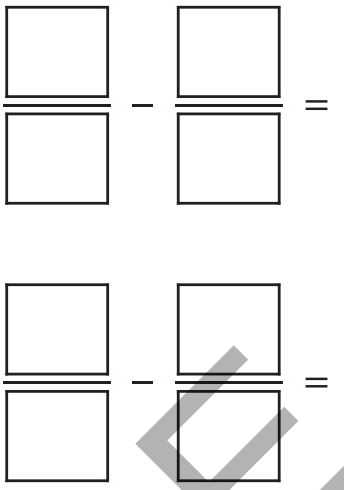
$$\frac{\square}{\square} - \frac{\square\square}{\square\square} =$$

Activity 2

What is the Least Difference?

Use the numbers below to fill in the squares. Use each number only once. Find each difference. Add the 2 differences together.

1 1 2 2 3 4 5 6



Unit 8, Lesson 12

Addressing CA CCSSM 5.NBT.7; practicing MP1

Decimal Game Day

Let's play some games and practice adding decimals.

Warm-up

True or False: Adding Decimals

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $0.99 + 0.1 = 0.9 + 0.1 + 0.09$

- $0.99 + 0.01 = 0.9 + 0.1$

- $0.99 + 0.1 = 1.99$

Activity 1

Race to One or One-Tenth

Use the directions to play Race to One or One-Tenth with your partner. If there is time, play both versions of the game.

Race to One

1. Roll the number cube.
2. Decide if you want the number to represent tenths or hundredths.
3. Add the number to the last sum on your score sheet. If it is your first turn, add the number you roll to 0.
4. Take turns. Roll the number cube, decide the value, and add the number to your previous sum.
5. The first partner to reach exactly 1 wins the game.
6. If you go over 1, you lose your turn. For example, if your last sum was .95 and you roll a 6, you lose your turn.

	number rolled	0.1	0.01	equation to represent the total
1				
2				
3				
4				
5				
6				

Race to One-Tenth

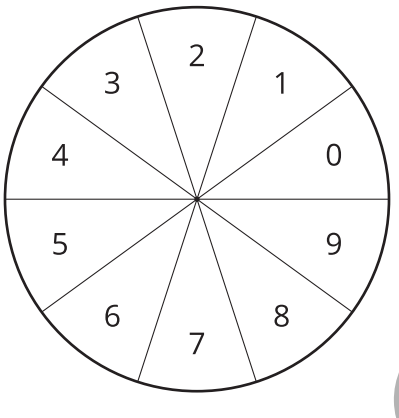
1. Roll the number cube.
2. Decide if you want the number to represent hundredths or thousandths.
3. Add the number to the last sum on your score sheet. If it is your first turn, add the number you roll to 0.
4. Take turns. Roll the number cube, decide the value, and add the number to your previous sum.
5. The first partner to reach exactly 0.1 wins the game.
6. If you go over 0.1, you lose your turn. For example, if your last sum was .095 and you roll a 6, you lose your turn.

	number rolled	0.01	0.001	equation to represent the total
1				
2				
3				
4				
5				
6				

Activity 2

Decimal Race to 500

Use the directions to play Decimal Race to 500 with a partner.



1. Spin the spinner 3 times.
2. Arrange the digits to make a decimal number that follows this rule:
   - Odd numbers can only be used in the tenths, hundredths, and thousandths place.
   - Even numbers can only be used in the ones, tens, and hundreds place.

   Example: You spin the numbers 2, 3, and 9. Some of the possible numbers you can make are 2.39 and 2.93.
3. Add your number to your previous sum. If it is your first turn, add your number to 0.
4. Take turns until one partner reaches 500 or more.

Unit 8, Lesson 13

Addressing CA CCSSM 5.NF.4; practicing MP7 and MP8

Multiply Fractions Game Day

Let's multiply a fraction or whole number by a fraction.

Warm-up

Number Talk: Multiply One-Third

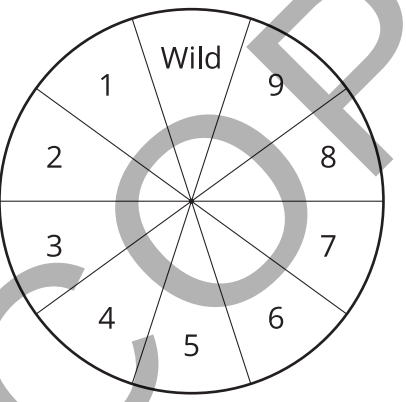
Find the value of each expression mentally.

- $\frac{1}{3} \times 3$
- $\frac{1}{3} \times 4$
- $\frac{1}{3} \times \frac{6}{3}$
- $\frac{1}{3} \times \frac{1}{4}$

Activity 1

Fraction Multiplication Compare

1. Use the directions to play Fraction Multiplication Compare with your partner.
   - Spin the spinner.
   - Write the number in one of the 4 blank boxes on your gameboard. Once you write a number, you can't change it.
   - Take turns until all 4 blank boxes are filled.
   - Multiply your fractions.
   - The partner with the greater product wins.
   - Play again.



Round 1 $\frac{\square}{\square} \times \frac{\square}{\square} = \_\_\_\_$

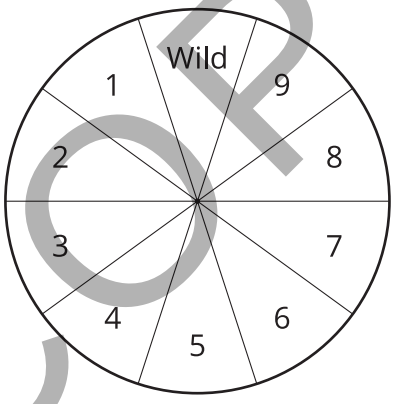
Round 2 $\frac{\square}{\square} \times \frac{\square}{\square} = \_\_\_\_$

2. What strategy did you use to decide where to write the numbers?

Activity 2

Fraction Multiplication Compare Round 2

1. Use the directions to play Fraction Multiplication Compare with your partner.
   - Spin the spinner.
   - Write the number in one of the 4 blank boxes on your gameboard. Once you write a number, you can't change it.
   - Take turns until all 4 blank boxes are filled.
   - Multiply your fractions.
   - The partner with the greater product wins.
   - Play again.



Round 1 $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\quad}{\quad}$

Round 2 $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\quad}{\quad}$

2. What strategy did you use to decide where to write the numbers?

Addressing CA CCSSM 5.NF.3; building towards 5.NF.3; practicing MP2

Notice and Wonder

Let's create a *Notice and Wonder*.

Warm-up

Notice and Wonder: Sharing Bread

What do you notice? What do you wonder?

Activity 1

Design Your Notice and Wonder (Part 1)

1. Find an image that encourages your classmates to notice and wonder about sharing and interpreting the result as a fraction.
2. Write some things students might notice and wonder about your image.

Students may notice:

- _____
- _____
- _____

Students may wonder:

- _____
- _____
- _____

Activity 2

Facilitate Your Notice and Wonder

1. Display your image for your classmates.
2. Ask them, "What do you notice? What do you wonder?"
3. Give them a minute to discuss together.
4. Have them share what they notice and wonder.
5. Record their ideas.

Activity 3

Design Your Notice and Wonder (Part 2)

1. Find an image that you find interesting. The image should encourage your classmates to notice and wonder about a mathematical topic you have learned this year.
2. Fill in some things students might notice and wonder about your image.

Students may notice:

- _____
- _____
- _____

Students may wonder:

- _____
- _____
- _____

Unit 8, Lesson 15

Addressing CA CCSSM 5.NBT.5; practicing MP3

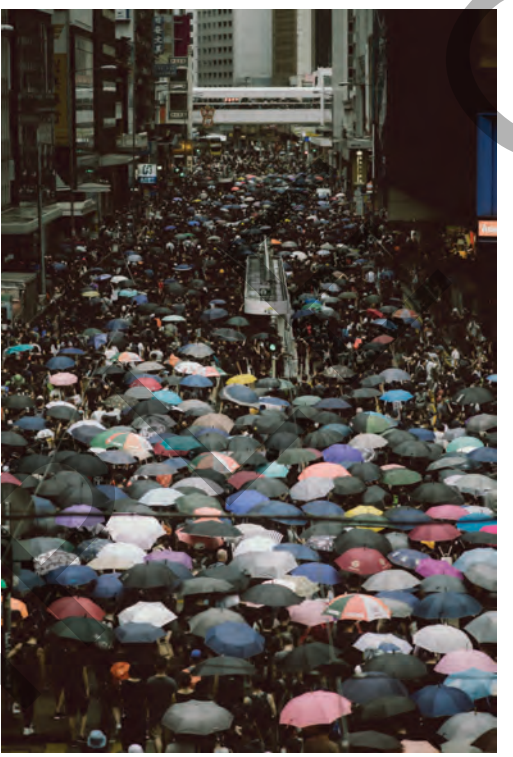
Estimation Exploration

Let's create an *Estimation Exploration*.

Warm-up

Estimation Exploration: Umbrellas

How many umbrellas?



Record an estimate that is:

too low	about right	too high

Activity 1

Design Your Estimation Exploration

1. Find an image that encourages your classmates to estimate the total number of an item using strategies for multi-digit multiplication.
2. Fill in the possible estimates students might make.

   Record an estimate that is:

too low	about right	too high

Activity 2

Facilitate Your Estimation Exploration

1. Display your image for your classmates.
2. Ask them, "What is an estimate that's too high?" "Too low?" "About right?"
3. Give them a minute of quiet think time.
4. Give them a minute to discuss together.
5. Have them share estimates.
6. Record their ideas.

Unit 8, Lesson 16

Addressing CA CCSSM 5.NBT.6; practicing MP2

Number Talk

Let's create a *Number Talk*.

Warm-up

Number Talk: Division

Find the value of each expression mentally.

- $28 \div 14$
- $70 \div 14$
- $98 \div 14$
- $350 \div 14$

Activity 1

Number Talk: Design 1

Write an expression to complete the *Number Talk*. Be prepared to explain how you chose the last expression.

Find the value of each expression mentally.

- $30 \div 15$
- $45 \div 15$
- $300 \div 15$
- _____

Activity 2

Number Talk: Design 2

Choose one of the *Number Talk* activities to complete. Be prepared to share your reasoning for the expressions you chose.

Option 1:

Find the value of each expression mentally.

- $220 \div 22$
- $66 \div 22$
- _____
- _____

Option 2:

Find the value of each expression mentally.

- $260 \div 26$
- $260 \div 13$
- _____
- _____

Activity 3

Number Talk: Design 3

Write expressions to complete the *Number Talk*. Be prepared to share your reasoning for the expressions.

Find the value of each expression mentally.

- $430 \div 43$

- _____

- _____

- _____

Unit 8, Lesson 17

Addressing CA CCSSM 5.NF.1; practicing MP1

True or False?

Let's create a *True or False?* activity.

Warm-up

True or False: Fraction Addition

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{3}{4} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8}$

- $\frac{7}{5} + \frac{2}{3} = \frac{21}{15} + \frac{8}{15}$

- $\frac{8}{9} + \frac{5}{12} = \frac{32}{36} + \frac{15}{36}$

Activity 1

True or False: Design 1

Write an equation to complete the *True and False?* task. Be prepared to share your reasoning for the equation you chose.

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{5}{6} - \frac{4}{9} = \frac{45}{54} - \frac{24}{54}$

- $\frac{5}{6} - \frac{4}{9} = \frac{15}{36}$

- _____

Activity 2

True or False: Design 2

Write two equations to complete the *True or False?* task. Be prepared to share your reasoning for the equations you chose.

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{8}{14} + \frac{3}{7} = \frac{4}{7} + \frac{3}{7}$

- _____

- _____

Activity 3

True or False: Design 3

Write three equations to complete the *True or False?* task. Be prepared to share your reasoning for the equations.

Decide if each statement is true or false. Be prepared to explain your reasoning.

- _____
- _____
- _____

Unit 8, Lesson 18

Addressing CA CCSSM 5.MD.3; practicing MP6

Which Three Go Together?

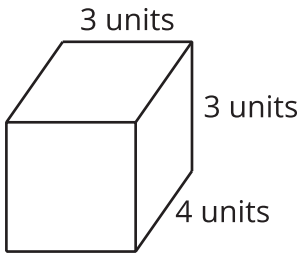
Let's create a *Which Three Go Together?* activity.

Warm-up

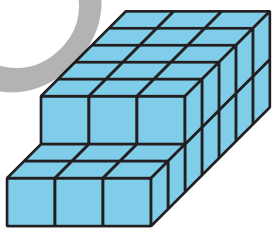
Which Three Go Together: Volume

Which 3 go together?

A



B



C

D

Activity 1

Which Three Go Together: Design 1

1. Choose 3 shapes from the set of cards.
2. Draw a fourth shape to complete the *Which Three Go Together?* activity.
3. For each group of 3 shapes, discuss one reason why they go together.

Which 3 go together?

A

B

C

D

Activity 2

Which Three Go Together: Design 2

1. Choose 2 shapes from the set of cards.
2. Draw a third and fourth shape to complete the *Which Three Go Together?* activity.
3. For each group of 3 shapes, discuss one reason why they go together.

Which 3 go together?

A

B

C

D

Activity 3

Which Three Go Together: Design 3

Create your own *Which Three Go Together?* about any mathematical idea you want others to notice.

Which 3 go together?

A

B

C

D

Glossary

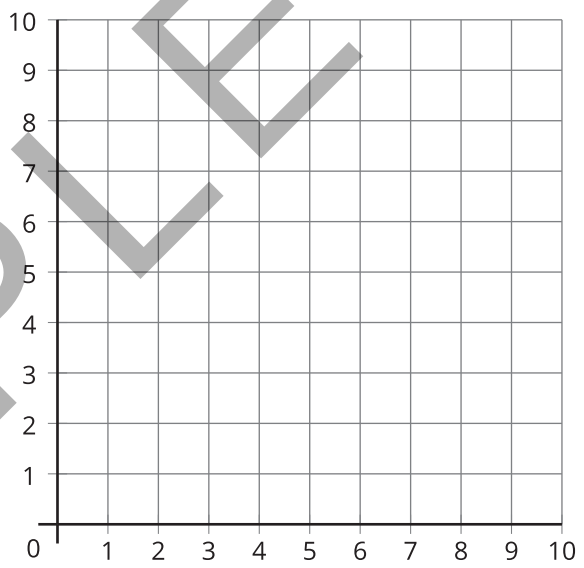
- algorithm
  A set of steps that works every time as long as the steps are carried out correctly.

- area
  The number of square units that cover a flat figure without gaps or overlaps.

- axes
  The two perpendicular number lines that intersect to form a coordinate grid. The numbering on both lines starts where they intersect at 0.

- coordinate grid
  A grid, or pattern of evenly spaced intersecting lines, formed by the horizontal and vertical axes.

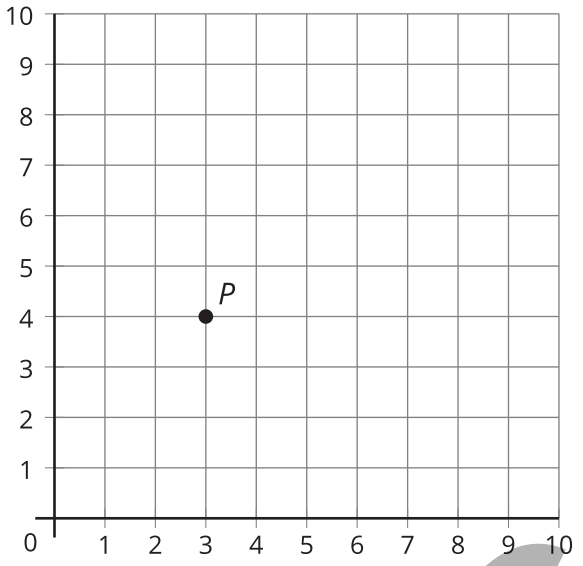
  Example:



- coordinates
  An ordered pair of numbers that tells an exact location on a coordinate grid. The first number represents a location on the horizontal axis. The second number represents a location on the vertical axis.

  Example: The coordinates of point *P* are $(3, 4)$ because *P* is located 3 units from 0 on

the horizontal axis and 4 units from 0 on the vertical axis.



- cubic unit
  The volume of a cube whose sides are each 1 unit long.

  Example: A cube whose sides are each 1 inch long has a volume of 1 cubic inch.

- exponent
  A number that tells how many times something is multiplied by itself.

  Example: In 10^3, the number 3 is an exponent. It means to multiply 10 by itself 3 times.
  $10^3 = 10 \times 10 \times 10 = 1,000$

- horizontal axis
  The number line that runs from left to right in a pair of axes.

- parallelogram
  A quadrilateral that has 2 pairs of parallel sides.

- power of 10
  The result of multiplying 10 by itself a given number of times.

Example: 1,000 is a power of 10 because $10 \times 10 \times 10 = 1,000$.

- quadrilateral
  A flat shape with 4 straight sides and 4 angles.

- rectangle
  A quadrilateral with 2 pairs of parallel sides and 4 right angles.

- rectangular prism
  A solid figure with 6 faces that are all rectangles.

- rhombus
  A quadrilateral with 4 equal sides.

- right triangle
  A triangle with a 90-degree angle.

- square
  A quadrilateral with 4 equal sides and 4 right angles.

- standard algorithm (for multiplication)
  A set of steps used to multiply numbers by place value. Write the numbers vertically, with the digits lined up by place value. Starting with the least place value, multiply the digits in each place of the bottom number by the digits in each place of the top number. Compose units in each place, as needed, representing that unit with a number over the place value to the left of the digit being multiplied. Add the partial products, including composed units.

- trapezoid
  A quadrilateral with at least 2 parallel opposite sides.

- unit cube
  A cube whose sides are each 1 unit long. Unit cubes are used to measure volume.

- vertical axis
  The number line that runs up and down in a pair of axes.

- volume
  The number of unit cubes that fill a solid figure without gaps or overlap.

Attributions

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California Common Core State Standards for Mathematics (CA CCSSM) References

5.G: Grade 5 – Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1
Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,x-axis and x-coordinate, y-axis and y-coordinate).

5.G.2
Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5.G.4
Classify two-dimensional figures in a hierarchy based on properties.

5.MD: Grade 5 – Measurement and Data

Convert like measurement units within a given measurement system.

5.MD.1
Convert among different-sized standard measurements units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

Represent and interpret data.

5.MD.2
Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3
Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.3a
A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

5.MD.3b
A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4
Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5
Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

5.MD.5a
Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

5.MD.5b
Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

5.MD.5c
Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

5.NBT: Grade 5 – Number and Operations in Base Ten

Understand the place value system.

5.NBT.1
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.3
Read, write, and compare decimals to thousandths.

5.NBT.3a
Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

5.NBT.3b
Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4
Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5
Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.6
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF: Grade 5 – Number and Operations—Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

5.NF.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3
Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.4a
Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

5.NF.4b
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5
Interpret multiplication as scaling (resizing), by:

5.NF.5a
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

5.NF.5b
Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

5.NF.6
Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

5.NF.7a
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.

5.NF.7b
Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.

5.NF.7c
Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

5.OA: Grade 5 – Operations and Algebraic Thinking

Write and interpret numerical expressions.

5.OA.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

5.OA.2.1
Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$.

Analyze patterns and relationships.

5.OA.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.