IMKH California

GRADE 7

Student Edition

UNITS 1-3

Book 1

Certified by Illustrative Mathematics®

ISBN 979-8-3851-6670-1

MS_vIV

20240207

Table of Contents

UNIT 3 MEASURING CIRCLES

GRADE 7

UNIT

1

Scale Drawings

Content Connections

In this unit you will study scaled copies of plane figures and scale drawings of real-world objects. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while exploring the relationships between different angles and recognizing the relationships as the measures change.
- **Discovering Shape and Space** while solving problems involving scale drawings and construct geometric figures using unit rates to accurately represent real-world figures.
- **Exploring Changing Quantities** while using scale drawings to calculate actual lengths and areas as you create scale drawings.

Addressing the Standards

As you work your way through **Unit 1 Scale Drawings,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 1 and 13
MP2 Reason abstractly and quantitatively.	Lessons 7 and 8
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 1, 3, 4, and 12
MP4 Model with mathematics.	Lesson 13
MP5 Use appropriate tools strategically.	Lessons 2, 3, 5, and 7
MP6 Attend to precision.	Lessons 1, 4, 11, and 12
MP7 Look for and make use of structure.	Lessons 2, 3, 4, 5, 6, 9, 10, 11, and 12
MP8 Look for and express regularity in repeated reasoning.	Lessons 2, 5, 6, and 10

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• 2-D and 3-D Connections • Scale Drawings • Shapes in the World	**7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13
• Angle Relationships • Shapes in the World	**7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Lesson 6

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 1, Lesson 1

Addressing CA CCSSM 7.G.1; practicing MP1, MP3, MP6

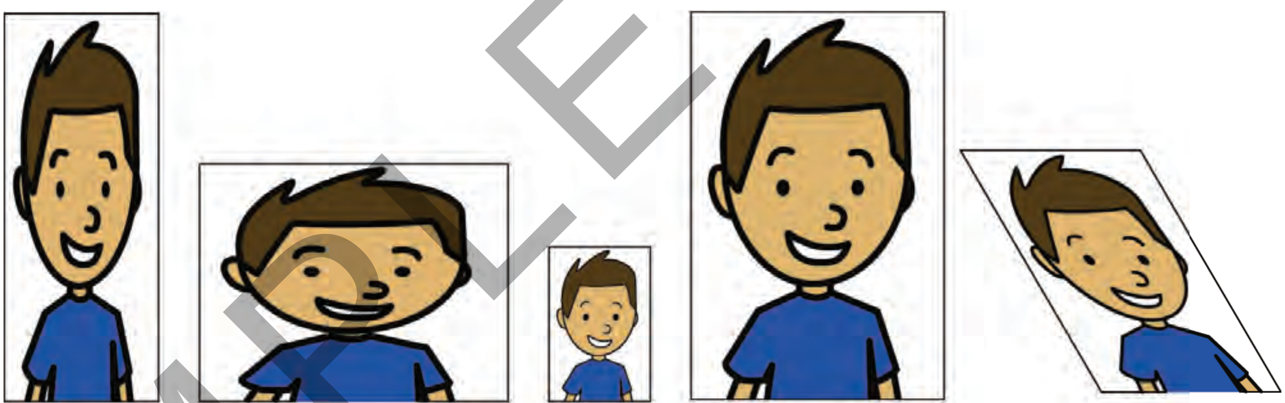
What Are Scaled Copies?

Let’s explore scaled copies.

1.1 Printing Portraits

Here is a portrait of a student.

1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?



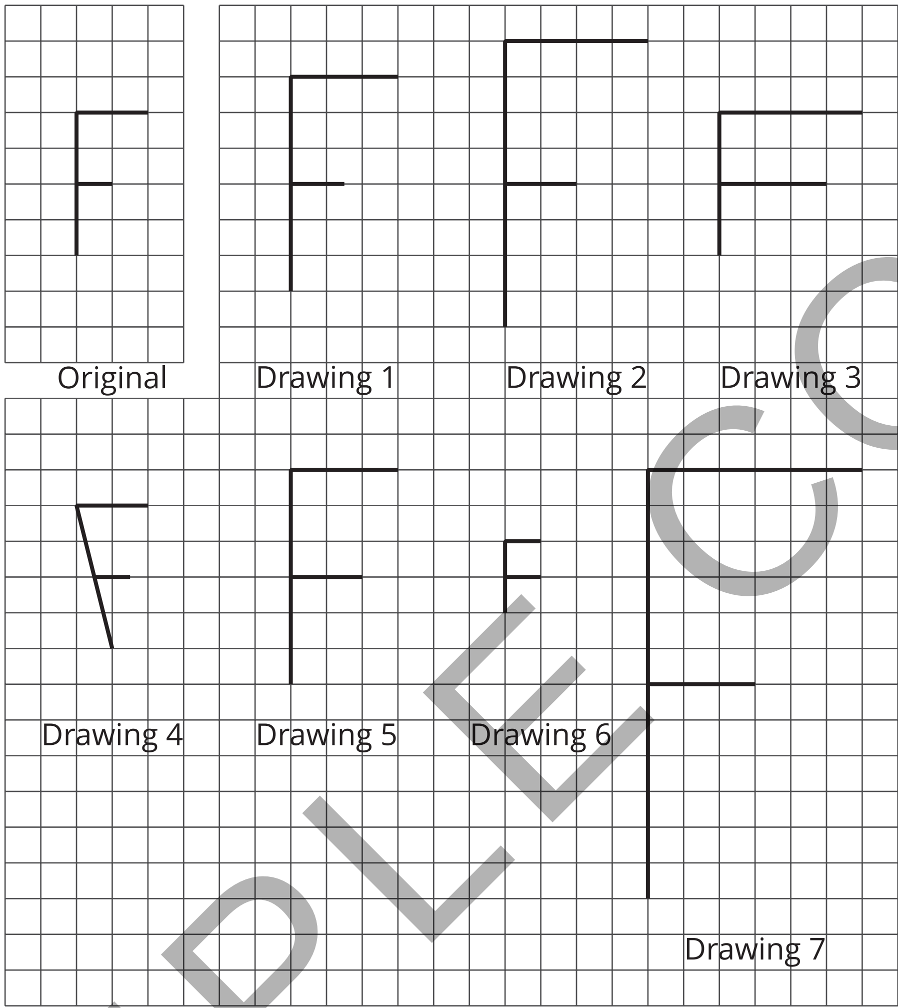
A B C D E

2. Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.

3. What do you think “scaled copy” means?

1.2 Scaling F

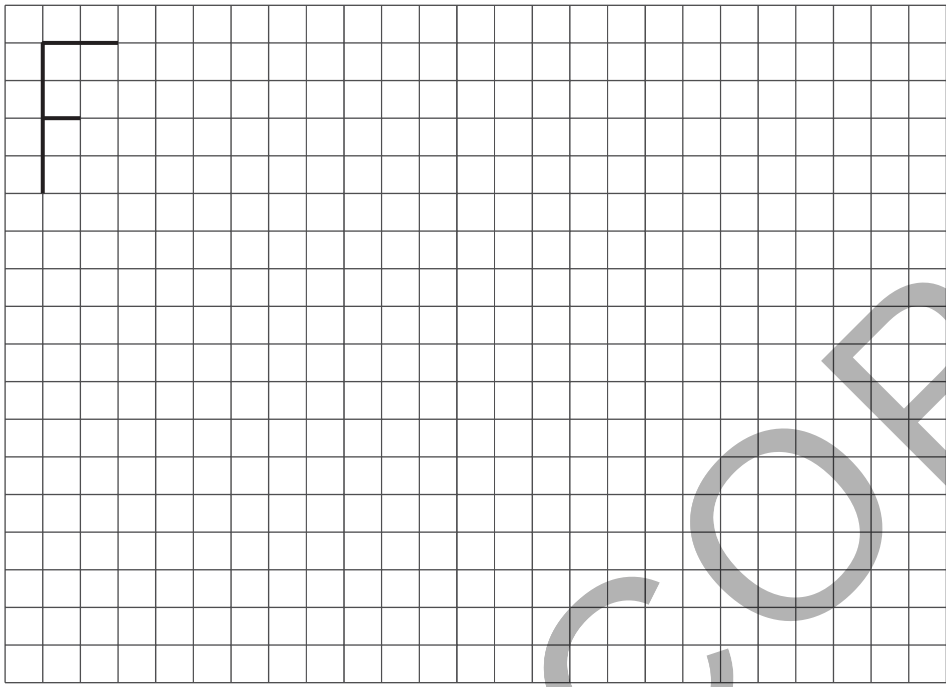
Here is an original drawing of the letter F and some other drawings.



1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.

2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?

3. On the grid, draw a different scaled copy of the original letter F.



1.3 Pairs of Scaled Polygons

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a polygon with another polygon that is a scaled copy.
   a. For each match you find, explain to your partner how you know it's a match.
   b. For each match your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

3. Select one pair of polygons to examine further. Explain or show how you know that one polygon is a scaled copy of the other. You can draw both polygons on the grid if it helps.

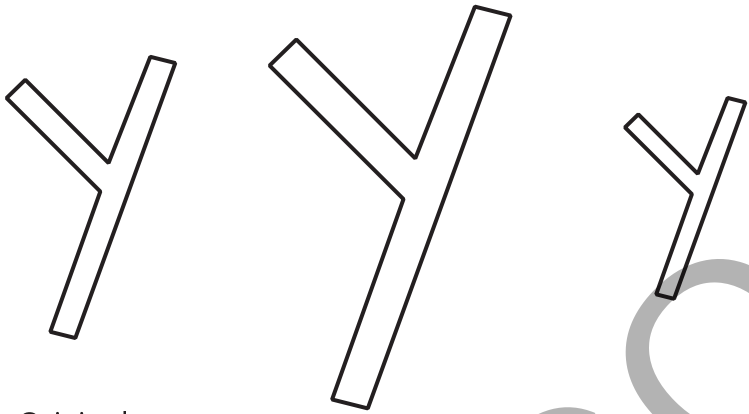
Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.

Lesson 1 Summary

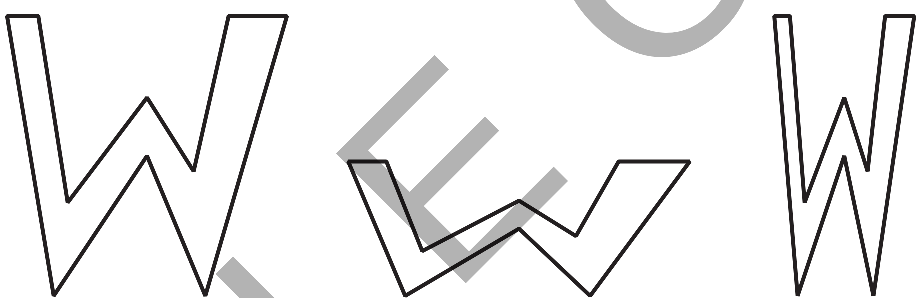
What is a **scaled copy** of a figure? Let’s look at some examples.

The second and third drawings are both scaled copies of the original Y.



Original

However, here, the second and third drawings are *not* scaled copies of the original W.



Original

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

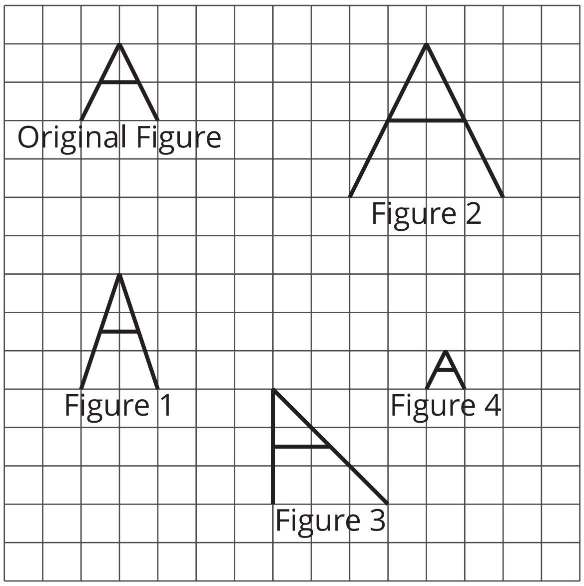
We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

Glossary

- scaled copy

Practice Problems

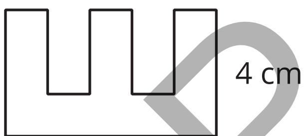
1 Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.



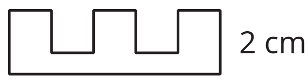
2 Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

Do you agree with Tyler? Explain your reasoning.

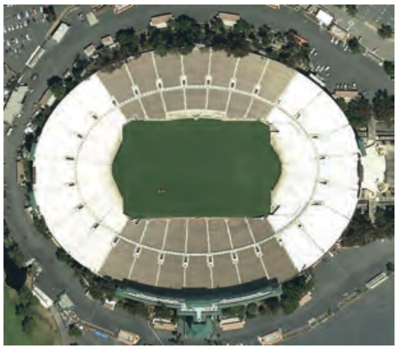
A



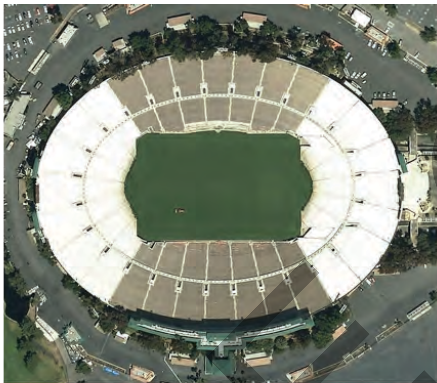
B

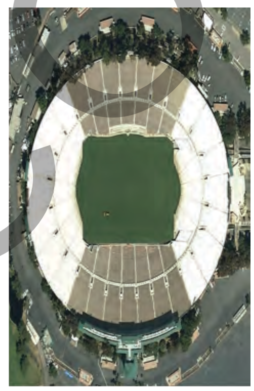


3 Here is a picture of the Rose Bowl Stadium in Pasadena, CA.



Here are some copies of the picture. Select **all** the pictures that are scaled copies of the original picture.

A


B


C


D


4 from an earlier course

Complete each equation with a number that makes it true.

a. $5 \cdot$ _______ $= 15$

b. $4 \cdot$ _______ $= 32$

c. $6 \cdot$ _______ $= 9$

d. $12 \cdot$ _______ $= 3$

Unit 1, Lesson 2

Addressing CA CCSSM 7.G.1; building on 5.NBT.7, 5.NF.4; building towards 7.RP.2; practicing MP5, MP7, and MP8

Corresponding Parts and Scale Factors

Let's describe features of scaled copies.

Sec A

2.1 Math Talk: Multiplying by a Unit Fraction

Find the value of each expression mentally.

- $\frac{1}{4} \cdot 20$
- $44 \cdot \frac{1}{4}$
- $\frac{1}{3} \cdot 63$

Addressing CA CCSSM 7.G.1; building on 5.NBT.7, 5.NF.4; building towards 7.RP.2; practicing MP5, MP7, and MP8

- $90 \cdot \frac{1}{6}$

2.2 Corresponding Parts

Here is a figure and two copies, each with some points labeled.

ORIGINAL COPY 1 COPY 2

1. Complete this table to show **corresponding parts** in the three figures.

original	copy 1	copy 2
point P		
segment LM		
	segment EF	
		point W
angle KLM		
		angle XYZ

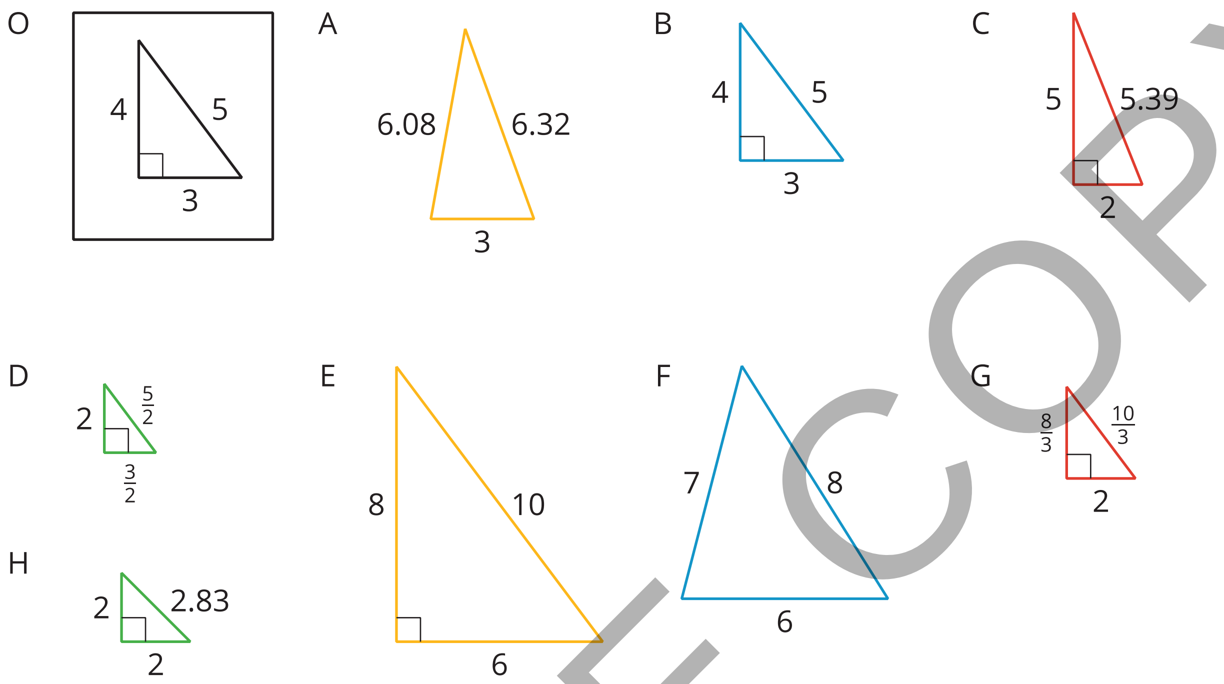
2. Is either copy a scaled copy of the original figure? Explain your reasoning.

3. Use tracing paper to compare angle KLM with its corresponding angles in Copy 1 and Copy 2. What do you notice?

4. Use tracing paper to compare angle NOP with its corresponding angles in Copy 1 and Copy 2. What do you notice?

2.3 Scaled Triangles

Here is Triangle O, followed by a number of other triangles.



Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of Triangle O? Be prepared to explain your reasoning.
2. As a group, identify *all* the scaled copies of Triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.
3. List all the triangles that are scaled copies in the table. Record the side lengths that correspond to the side lengths of Triangle O listed in each column.

Triangle O	3	4	5

4. Explain or show how each copy has been scaled from the original (Triangle O).

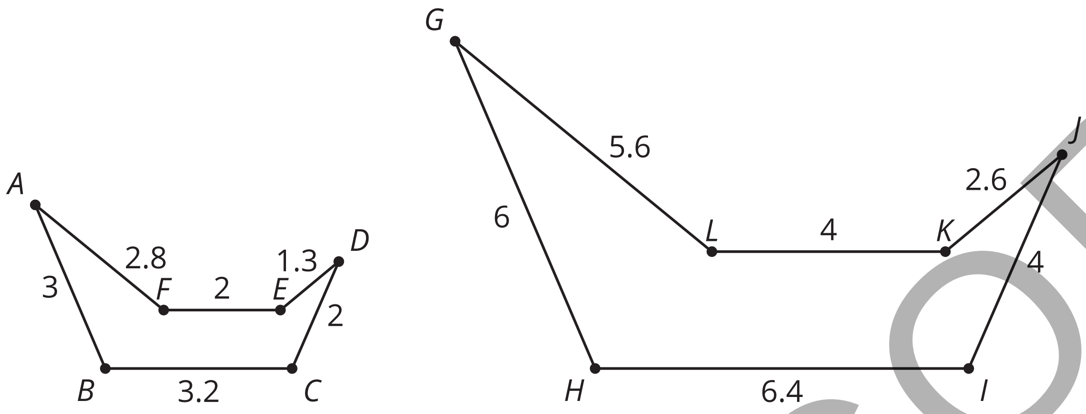
Sec A

Are you ready for more?

Choose one of the triangles that is not a scaled copy of Triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.

Lesson 2 Summary

A figure and its scaled copy have **corresponding parts**, or parts that are in the same position in relation to the rest of each figure. These parts could be points, segments, or angles. For example, Polygon 2 is a scaled copy of Polygon 1.



Polygon 1

Polygon 2

- Each point in Polygon 1 has a *corresponding point* in Polygon 2.
  For example, point B corresponds to point H and point C corresponds to point I.
- Each segment in Polygon 1 has a *corresponding segment* in Polygon 2.
  For example, segment AF corresponds to segment GL.
- Each angle in Polygon 1 also has a *corresponding angle* in Polygon 2.
  For example, angle DEF corresponds to angle JKL.

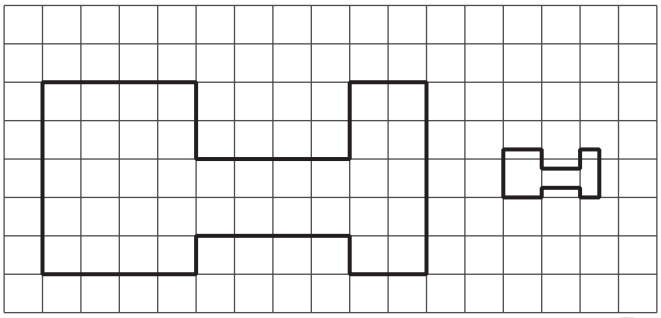
The **scale factor** between Polygon 1 and Polygon 2 is 2, because all of the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1. For example, the measure of angle JKL is the same as the measure of angle DEF.

Glossary

- corresponding
- scale factor

Practice Problems

1 The second H-shaped polygon is a scaled copy of the first.



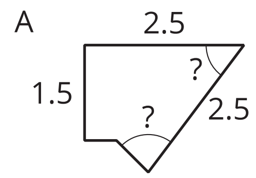
a. Show one pair of corresponding points and two pairs of corresponding sides in the original polygon and its copy. Consider using colored pencils to highlight corresponding parts or labeling some of the vertices.

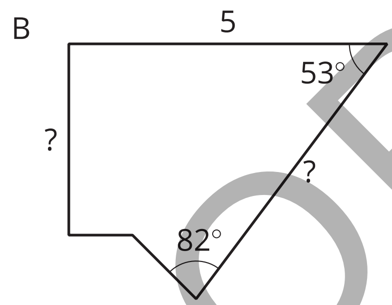
b. What scale factor takes the original polygon to its smaller copy? Explain or show your reasoning.

2 Zapotec people in southern Mexico make woven rugs like this one. Find a pair of figures in the design that are scaled copies. Outline the two figures, and give the scale factor.

3 Polygon B is a scaled copy of Polygon A.

a. What is the scale factor from Polygon A to Polygon B? Explain your reasoning.





b. Find the missing length of each side marked with a "?" in Polygon B.

c. Determine the measure of each angle marked with a "?" in Polygon A.

4 Figure 2 is a scaled copy of Figure 1. Select **all** of the statements that must be true:

A. Figure 2 is larger than Figure 1.

B. Figure 2 has the same number of edges as Figure 1.

C. Figure 2 has the same perimeter as Figure 1.

D. Figure 2 has the same number of angles as Figure 1.

E. Figure 2 has angles with the same measures as Figure 1.

5 from an earlier course

Complete each equation with a number that makes it true.

a. $8 \cdot$ ______ $= 40$

b. $8 +$ ______ $= 40$

c. $21 \div$ ______ $= 7$

d. $21 -$ ______ $= 7$

e. $21 \cdot$ ______ $= 7$

Addressing CA CCSSM 7.G.1; building on 6.NS.3; building towards 7.RP.2; practicing MP3, MP5, and MP7

Making Scaled Copies

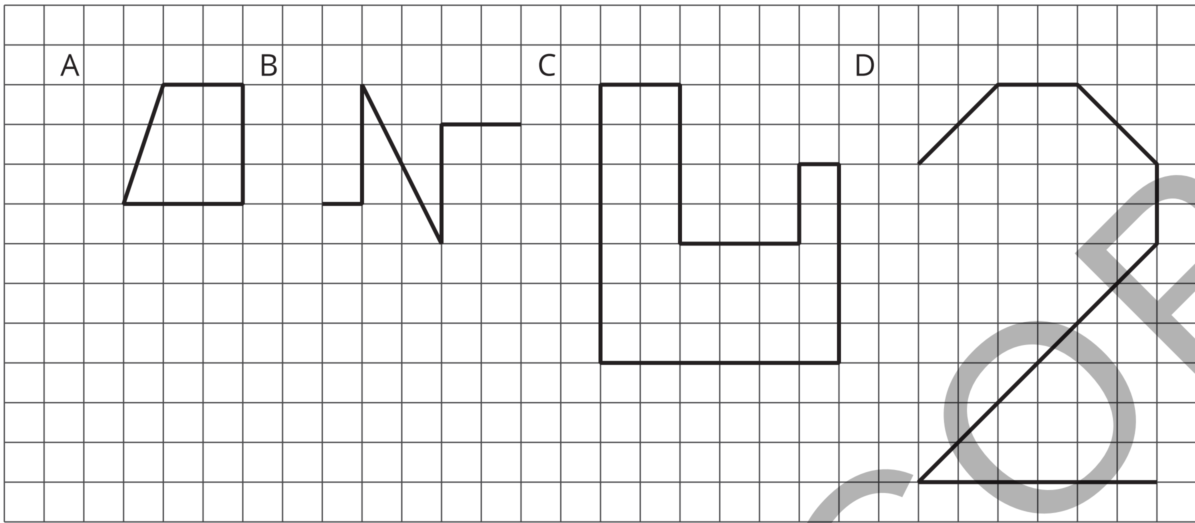
Let's draw scaled copies.

3.1 Math Talk: Missing Operation

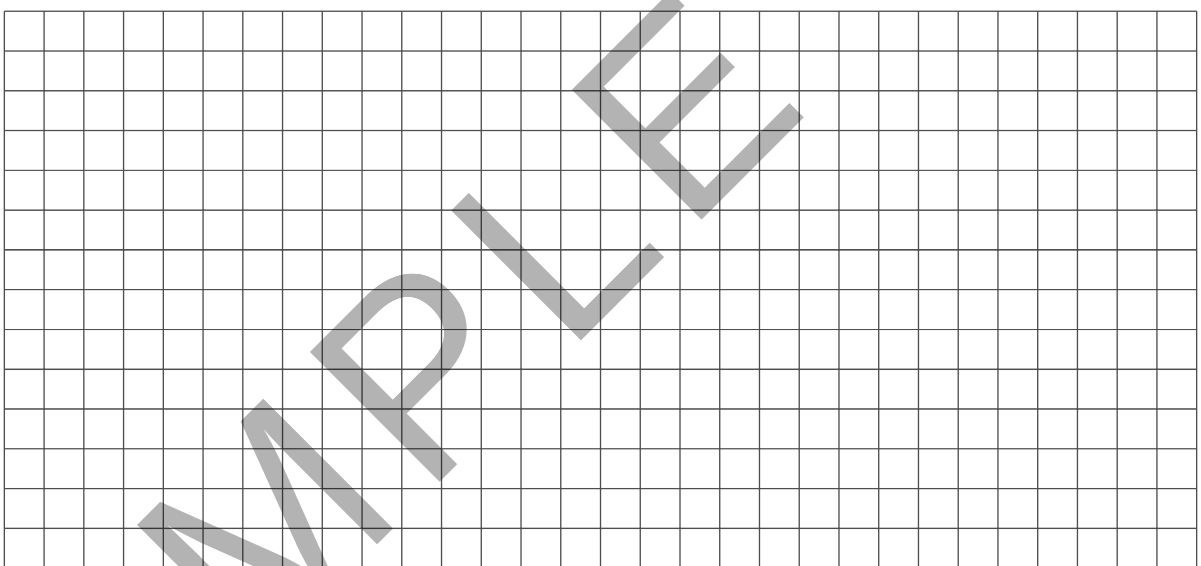
Complete each equation to make it true.

- 5 _______ = 10
- 3 _______ = 15
- 14 _______ = 21
- 30 _______ = 6

3.2 Drawing Scaled Copies

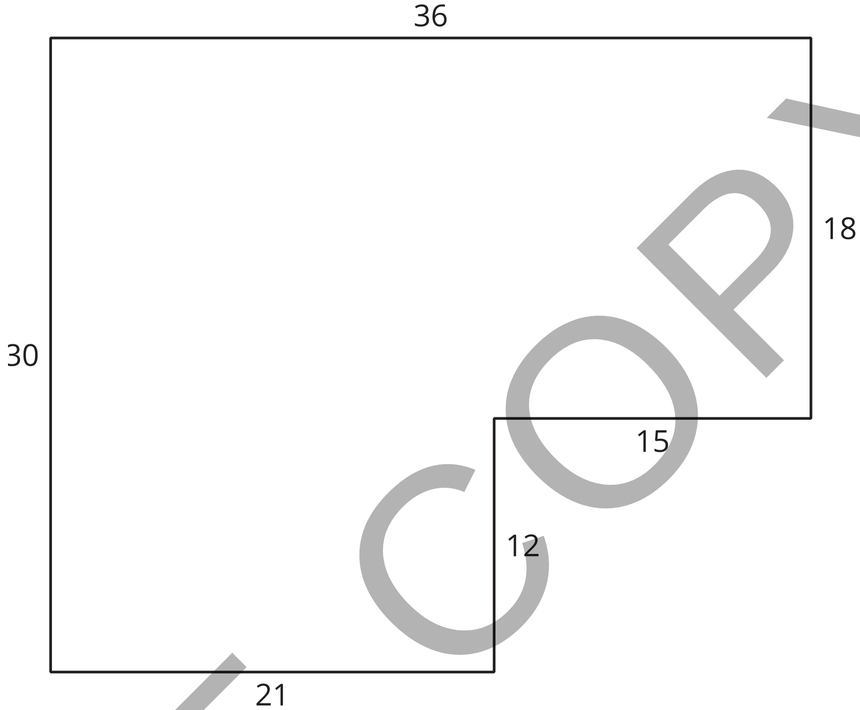


1. Draw a scaled copy of either Figure A or B using a scale factor of 3.
2. Draw a scaled copy of either Figure C or D using a scale factor of $\frac{1}{2}$.



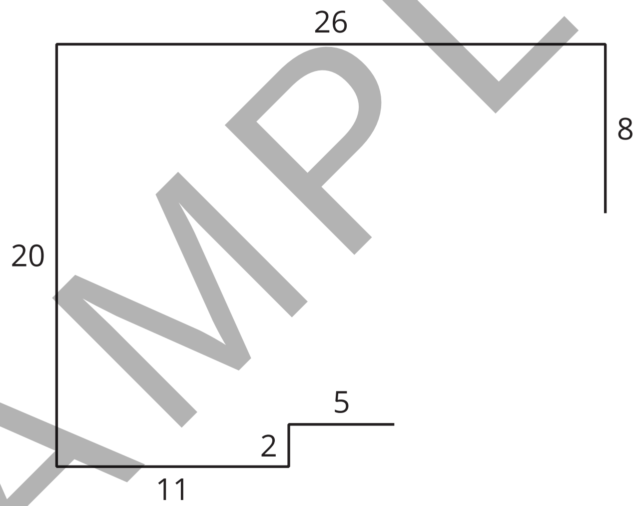
3.3 Which Operations? (Part 1)

Diego and Jada want to scale this polygon so the side that corresponds to 15 units in the original is 5 units in the scaled copy.

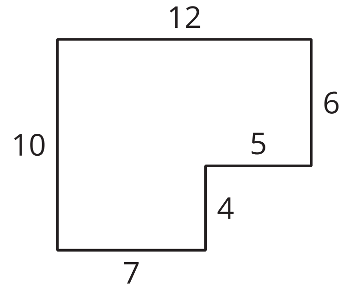


Diego and Jada each use a different operation to find the new side lengths. Here are their finished drawings.

Diego's drawing



Jada's drawing



1. What operation do you think Diego used to calculate the lengths for his drawing?

2. What operation do you think Jada used to calculate the lengths for her drawing?

3. Did each method produce a scaled copy of the polygon? Explain your reasoning.

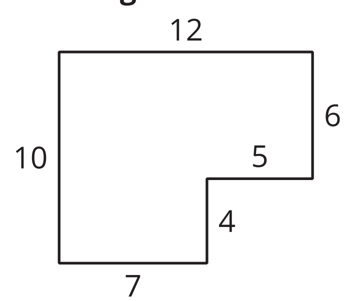
3.4 Which Operations? (Part 2)

Andre wants to make a scaled copy of Jada's drawing so that the side that corresponds to 4 units in Jada's polygon is 8 units in his scaled copy.

1. Andre says "I wonder if I should add 4 units to the lengths of all of the segments?" What would you say in response to Andre? Explain or show your reasoning.

2. Create the scaled copy that Andre wants. If you get stuck, consider using the edge of an index card or paper to measure the lengths needed to draw the copy.

Jada's drawing



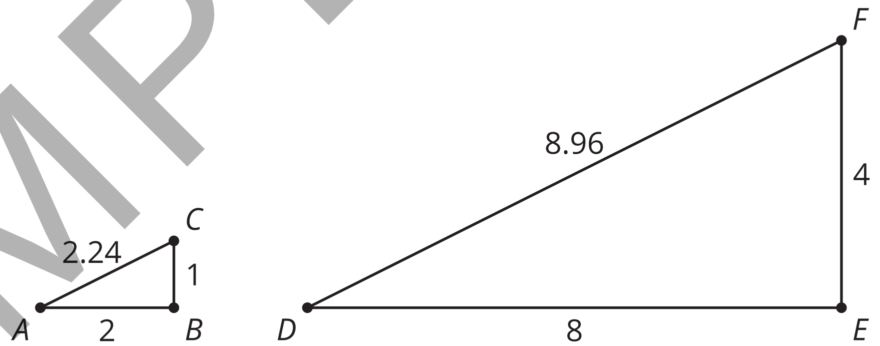
Are you ready for more?

The side lengths of Triangle B are all 5 more than the side lengths of Triangle A. Can Triangle B be a scaled copy of Triangle A? Explain your reasoning.

Lesson 3 Summary

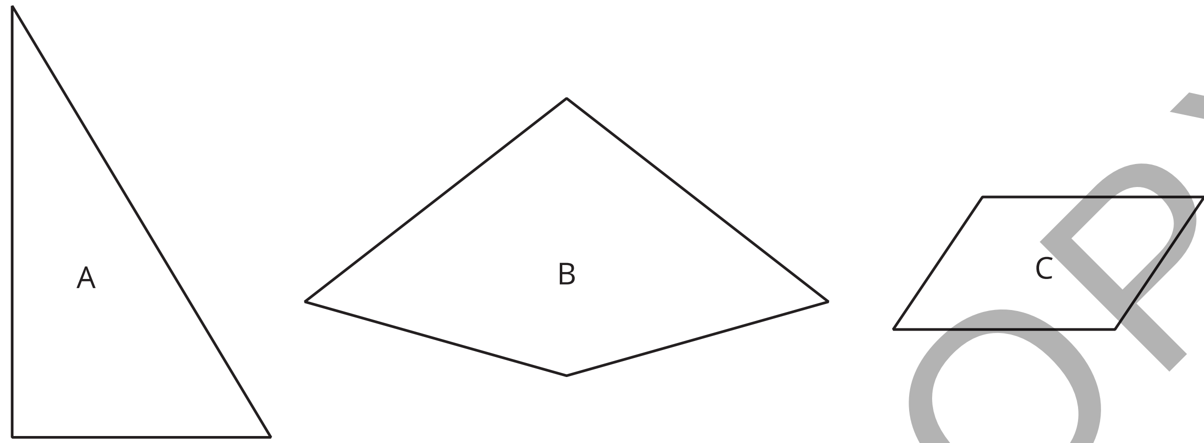
Creating a scaled copy involves *multiplying* the lengths in the original figure by a scale factor.

For example, to make a scaled copy of triangle ABC where the base is 8 units, we would use a scale factor of 4. This means multiplying all the side lengths by 4, so in triangle DEF, each side is 4 times as long as the corresponding side in triangle ABC.



Practice Problems

1 Here are 3 polygons.



Draw a scaled copy of Polygon A using a scale factor of 2.

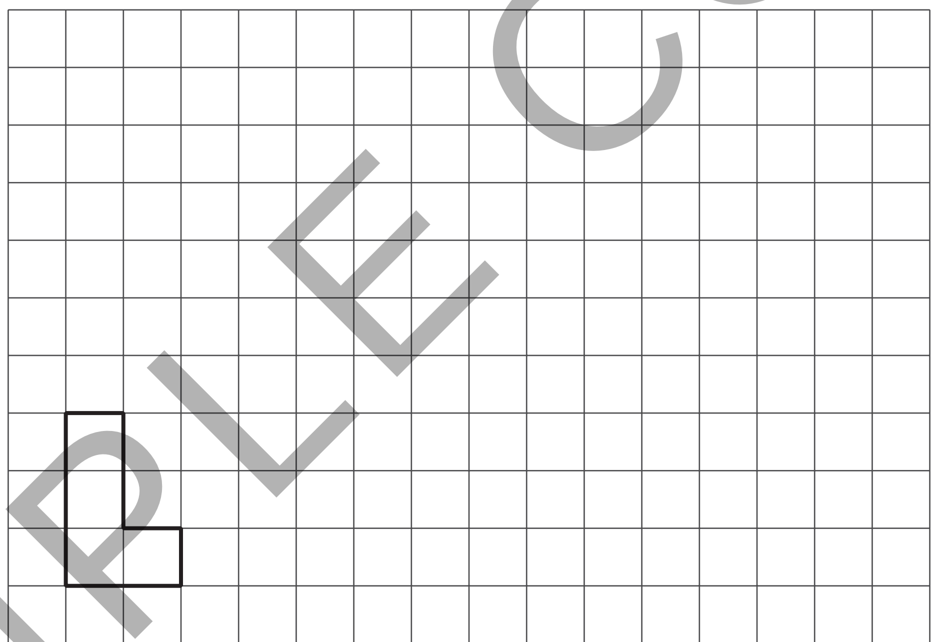
Draw a scaled copy of Polygon B using a scale factor of $\frac{1}{2}$.

Draw a scaled copy of Polygon C using a scale factor of $\frac{3}{2}$.

2 Quadrilateral A has side lengths 6, 9, 9, and 12. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side of length 2. What is the perimeter of Quadrilateral B?

3

a. Draw a scaled copy of this polygon so that the scaled copy has a perimeter of 30 units.



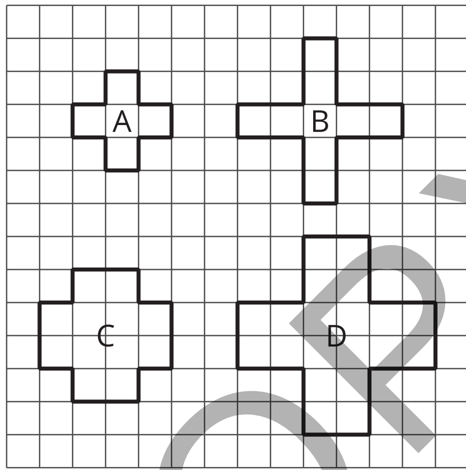
b. What is the scale factor? Explain how you know.

4 from Unit 1, Lesson 1

Priya and Tyler are discussing the figures.

- Priya says B, C, and D are scaled copies of A.
- Tyler says B and D are scaled copies of A.

Do you agree with either of them? Explain your reasoning.



5 from Unit 1, Lesson 2

Persian people made khatam by gluing many small pieces of wood together. Find a pair of figures in the design that are scaled copies. Outline the two figures, and give the scale factor.

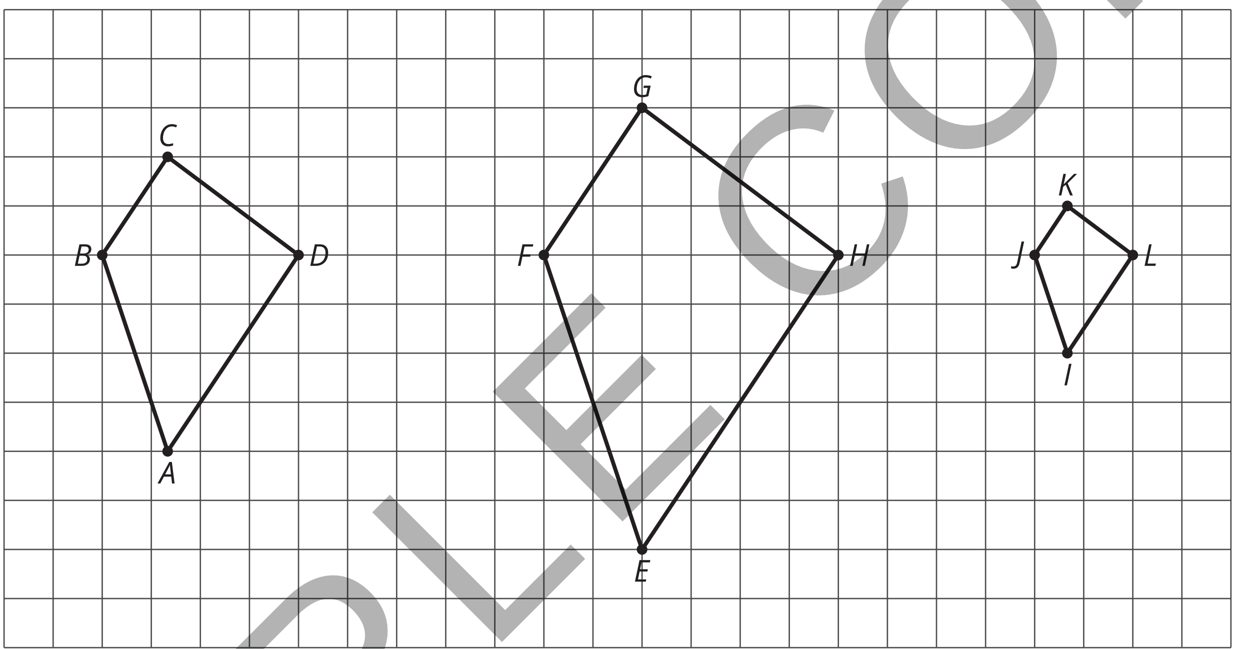
Addressing CA CCSSM 7.G.1; practicing MP3, MP6, and MP7

Scaled Relationships

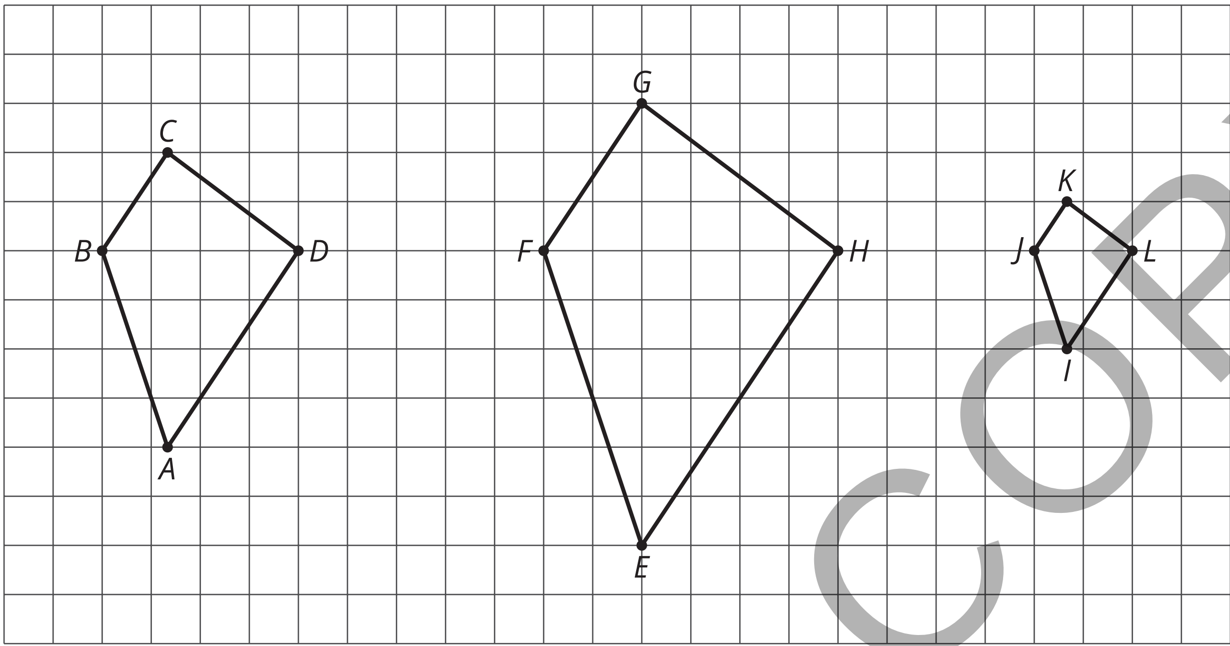
Let’s find relationships between scaled copies.

4.1 Notice and Wonder: Three Quadrilaterals

What do you notice? What do you wonder?



4.2 Measuring the Three Quadrilaterals



1. Measure at least one set of corresponding angles using a protractor. Record your measurements to the nearest 5°.
2. What do you notice about the angle measures?

   Pause here so your teacher can review your work.

3. The side lengths of the polygons are hard to tell from the grid, but there are other corresponding distances that are easier to compare. Identify the distances in the other two polygons that correspond to DB and AC, and record them in the table.

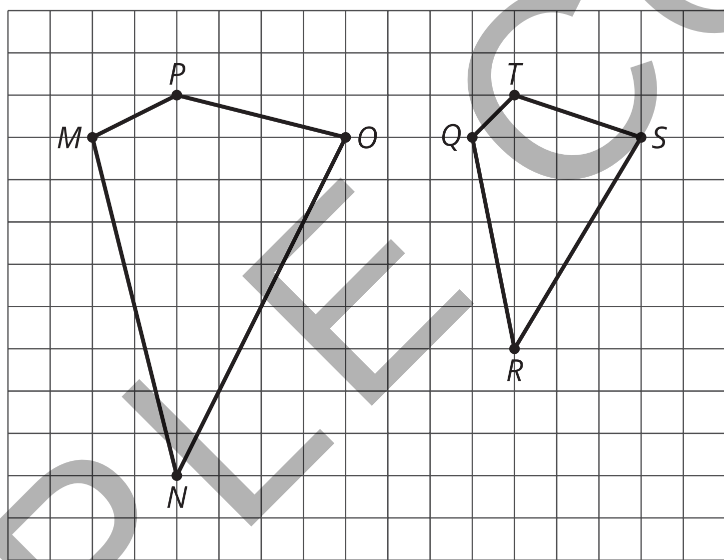
quadrilateral	distance that corresponds to DB	distance that corresponds to AC
$ABCD$	$DB = 4$	$AC = 6$
$EFGH$		
$IJKL$		

4. Look at the values in the table. What do you notice?

5. Are these three quadrilaterals scaled copies? Explain your reasoning.

4.3 Scaled or Not Scaled?

Here are two quadrilaterals.



1. Mai says that polygon $QRST$ is a scaled copy of polygon $MNOP$, but Noah disagrees. Do you agree with either of them? Explain or show your reasoning.

2. Record the corresponding distances in the table. What do you notice?

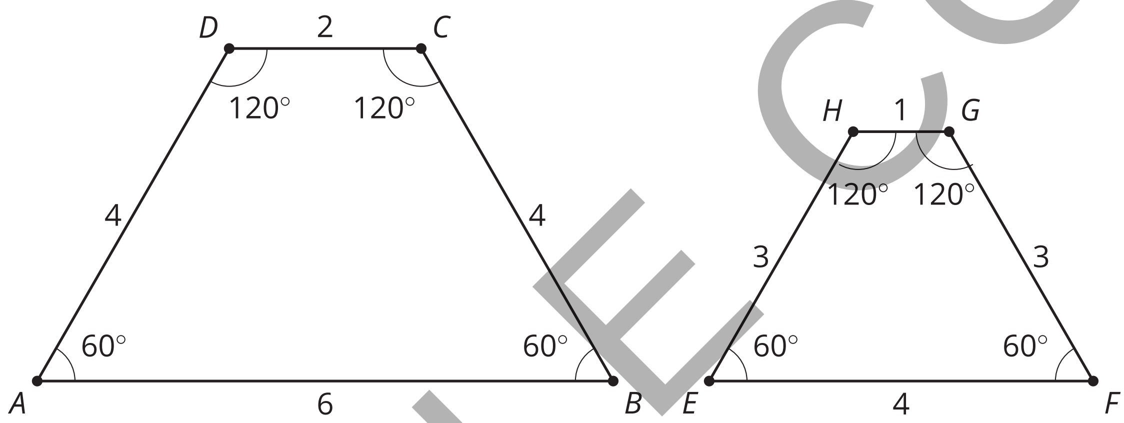
quadrilateral	horizontal distance	vertical distance
$MNOP$	$MO =$	$NP =$
$QRST$	$QS =$	$RT =$

3. Measure at least three pairs of corresponding angles in $MNOP$ and $QRST$ using a protractor. Record your measurements to the nearest 5°. What do you notice?

Sec A

4. Do these results change your answer to the first question? Explain.

5. Here are two more quadrilaterals.



Kiran says that polygon $EFGH$ is a scaled copy of $ABCD$, but Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

Are you ready for more?

All side lengths of Quadrilateral Y are 2, and all side lengths of Quadrilateral Z are 3. Does Quadrilateral Y have to be a scaled copy of Quadrilateral Z? Explain your reasoning.

4.4 Comparing Pictures of Birds

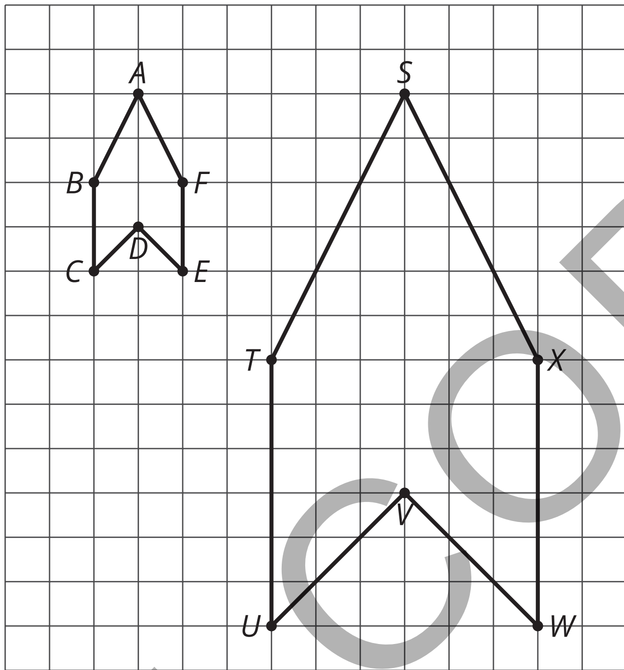
Here are two pictures of a bird. Find evidence that one picture is not a scaled copy of the other. Be prepared to explain your reasoning.

Lesson 4 Summary

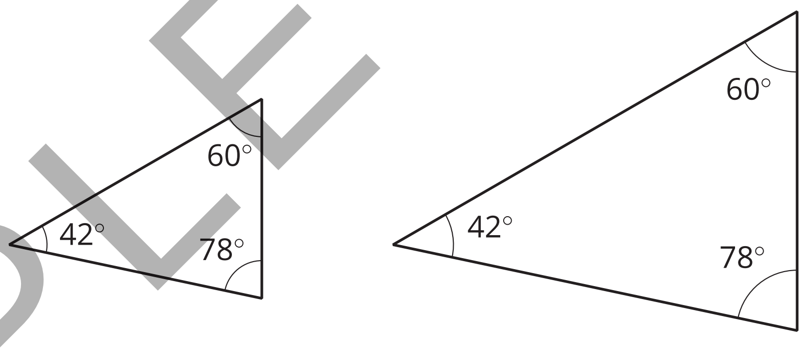
When a figure is a scaled copy of another figure, we know that:

- All distances in the copy can be found by multiplying the *corresponding distances* in the original figure by the same scale factor, whether or not the endpoints are connected by a segment.

  For example, Polygon $STUVWX$ is a scaled copy of Polygon $ABCDEF$. The scale factor is 3. The distance from T to X is 6, which is three times the distance from B to F.

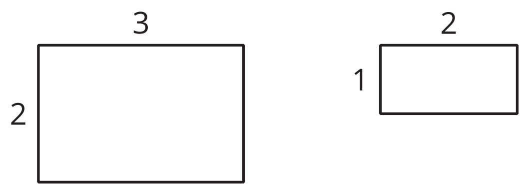


- All angles in the copy have the same measure as the corresponding angles in the original figure, as in these triangles.



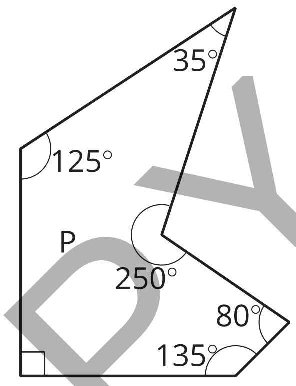
These observations can help explain why one figure is *not* a scaled copy of another.

For example, the second rectangle is not a scaled copy of the first rectangle, even though their corresponding angles have the same measure. Different pairs of corresponding lengths have different scale factors, $2 \cdot \frac{1}{2} = 1$ but $3 \cdot \frac{2}{3} = 2$.



Practice Problems

1 Select **all** the statements that must be true for *any* scaled copy Q of Polygon P.



A. The side lengths are all whole numbers.

B. The angle measures are all whole numbers.

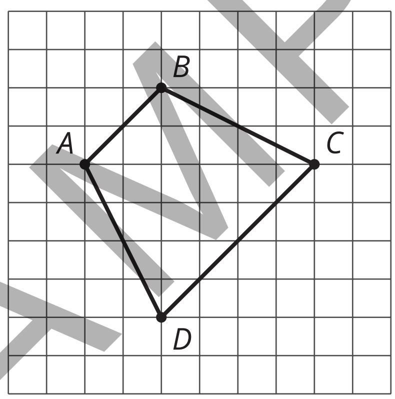
C. Q has exactly 1 right angle.

D. If the scale factor between P and Q is $\frac{1}{5}$, then each side length of P is multiplied by $\frac{1}{5}$ to get the corresponding side length of Q.

E. If the scale factor is 2, each angle in P is multiplied by 2 to get the corresponding angle in Q.

F. Q has 2 acute angles and 3 obtuse angles.

2 Here is quadrilateral $ABCD$.

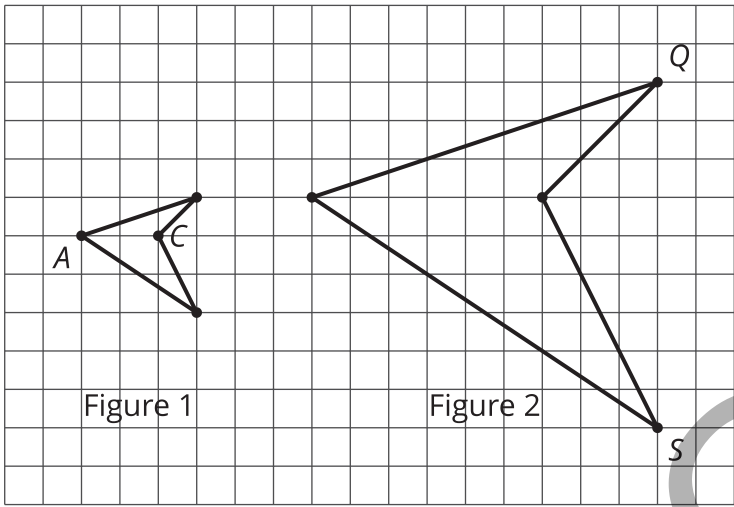


Quadrilateral $PQRS$ is a scaled copy of quadrilateral $ABCD$. Point P corresponds to A, Q to B, R to C, and S to D.

If the distance from P to R is 3 units, what is the distance from Q to S? Explain your reasoning.

Sec A

3 Figure 2 is a scaled copy of Figure 1.



a. Identify the points in Figure 2 that correspond to the points A and C in Figure 1. Label them P and R. What is the distance between P and R?

b. Identify the points in Figure 1 that correspond to the points Q and S in Figure 2. Label them B and D. What is the distance between B and D?

c. What is the scale factor that takes Figure 1 to Figure 2?

d. G and H are two points on Figure 1, but they are not shown. The distance between G and H is 1. What is the distance between the corresponding points on Figure 2?

4 from an earlier course

To make 1 batch of lavender paint, the ratio of cups of pink paint to cups of blue paint is 6 to 5. Find two more ratios of cups of pink paint to cups of blue paint that are equivalent to this ratio.

Addressing CA CCSSM 7.G.1; building on 5.NBT.6, 5.NF.4, 5.NF.5, 6.NS.1; building towards 7.RP.2; practicing MP5, MP7, and MP8

The Size of the Scale Factor

Let's look at the effects of different scale factors.

Sec A

5.1 Math Talk: Missing Factor

Solve each equation mentally.

- $8x = 4$
- $8x = 1$
- $\frac{1}{5}x = 1$
- $\frac{2}{5}x = 1$

Sec A

5.2 Card Sort: Scaled Copies

Your teacher will give you a set of cards. On each card, Figure A is the original and Figure B is a scaled copy.

1. Sort the cards based on their scale factors. Be prepared to explain your reasoning.
2. Examine cards 10 and 13 more closely. What do you notice about the shapes and sizes of the figures? What do you notice about the scale factors?
3. Examine cards 8 and 12 more closely. What do you notice about the figures? What do you notice about the scale factors?

Are you ready for more?

Triangle B is a scaled copy of Triangle A with scale factor $\frac{1}{2}$.

1. How many times bigger are the side lengths of Triangle B when compared with Triangle A?
2. Imagine you scale Triangle B by a scale factor of $\frac{1}{2}$ to get Triangle C. How many times bigger will the side lengths of Triangle C be when compared with Triangle A?
3. Triangle B has been scaled once. Triangle C has been scaled twice. Imagine you scale triangle A n times to get Triangle N, always using a scale factor of $\frac{1}{2}$. How many times bigger will the side lengths of Triangle N be when compared with Triangle A?

5.3 Scaling A Puzzle

Your teacher will give you one of the six pieces of a puzzle.

1. If you drew scaled copies of your puzzle pieces using a scale factor of $\frac{1}{2}$, would they be larger or smaller than the original pieces? How do you know?

2. Create a scaled copy of each puzzle piece on a blank square with a scale factor of $\frac{1}{2}$.

3. When everyone in your group is finished, put all 6 of the original puzzle pieces together like this:

1	2	3
4	5	6

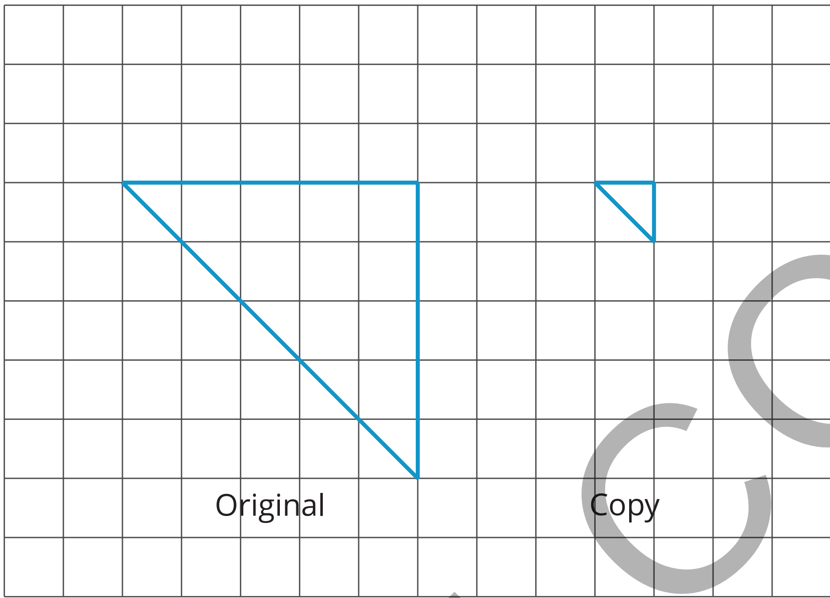
   Next, put all 6 of your scaled copies together. Compare your scaled puzzle with the original puzzle. Which parts seem to be scaled correctly and which seem off? What might have caused those parts to be off?

4. Revise any of the scaled copies that may have been drawn incorrectly.

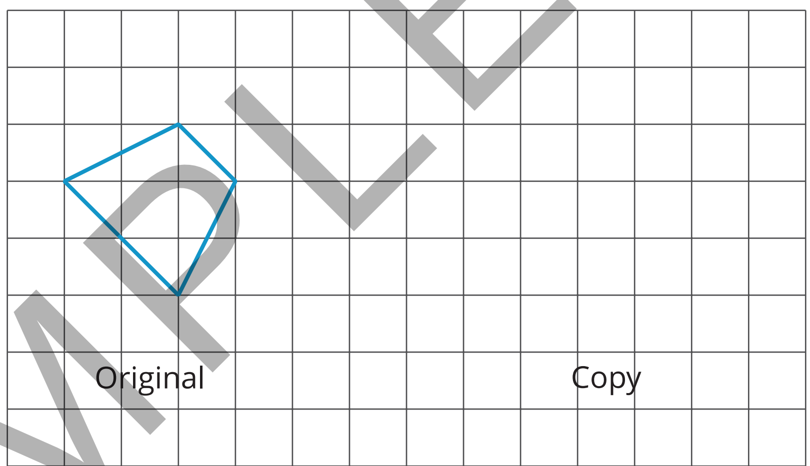
5. If you were to lose one of the pieces of the original puzzle, but still had the scaled copy, how could you recreate the lost piece?

5.4 Missing Figure, Factor, or Copy

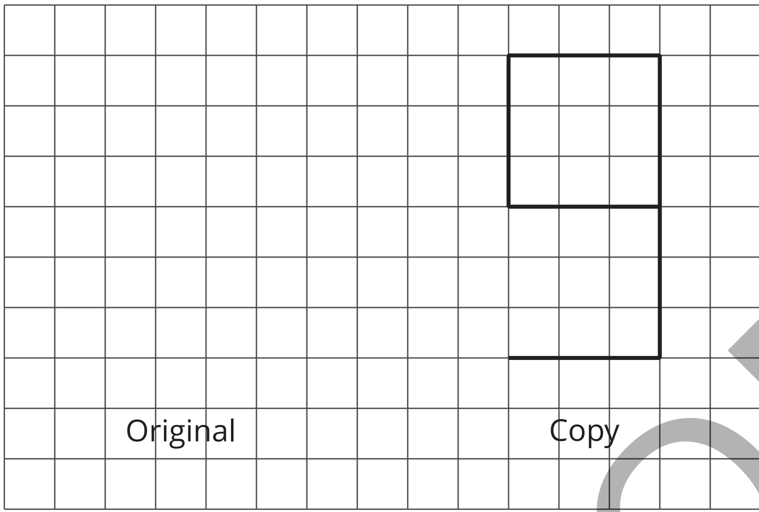
1. What is the scale factor from the original triangle to its copy? Explain or show your reasoning.



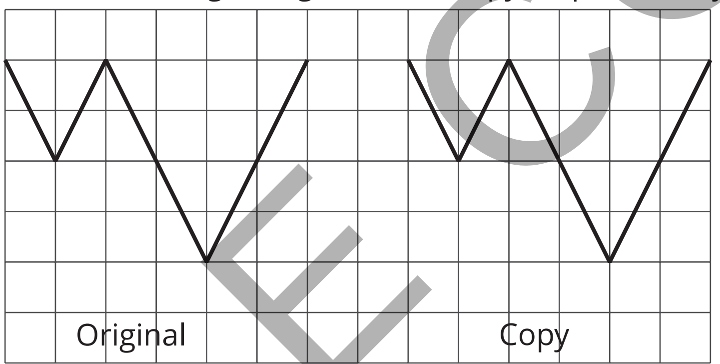
2. The scale factor from the original trapezoid to its copy is 2. Draw the scaled copy.



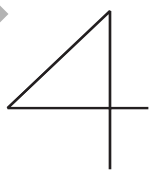
3. The scale factor from the original figure to its copy is $\frac{3}{2}$. Draw the original figure.



4. What is the scale factor from the original figure to the copy? Explain how you know.



5. The scale factor from the original figure to its scaled copy is 3. Draw the scaled copy.



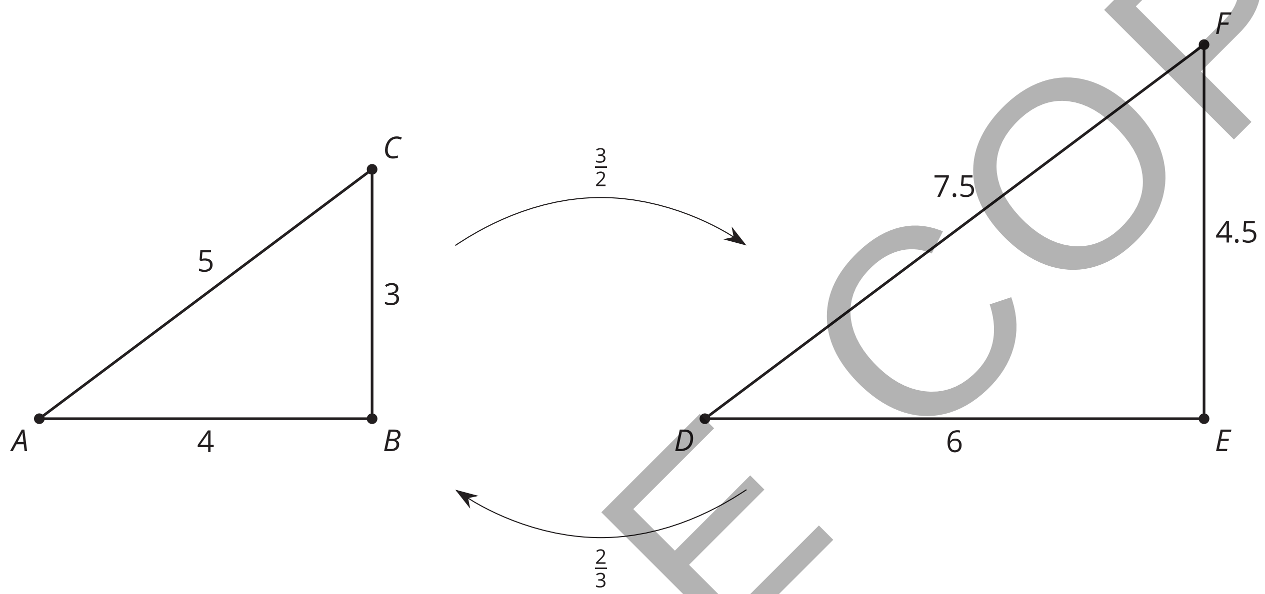
Original

Copy

Lesson 5 Summary

The size of the scale factor affects the size of the copy. When a figure is scaled by a scale factor greater than 1, the copy is larger than the original. When the scale factor is less than 1, the copy is smaller. When the scale factor is exactly 1, the copy is the same size as the original.

Triangle DEF is a larger scaled copy of triangle ABC, because the scale factor from ABC to DEF is $\frac{3}{2}$. Triangle ABC is a smaller scaled copy of triangle DEF, because the scale factor from DEF to ABC is $\frac{2}{3}$.



This means that triangles ABC and DEF are scaled copies of each other. It also shows that scaling can be reversed using **reciprocal** scale factors, such as $\frac{2}{3}$ and $\frac{3}{2}$.

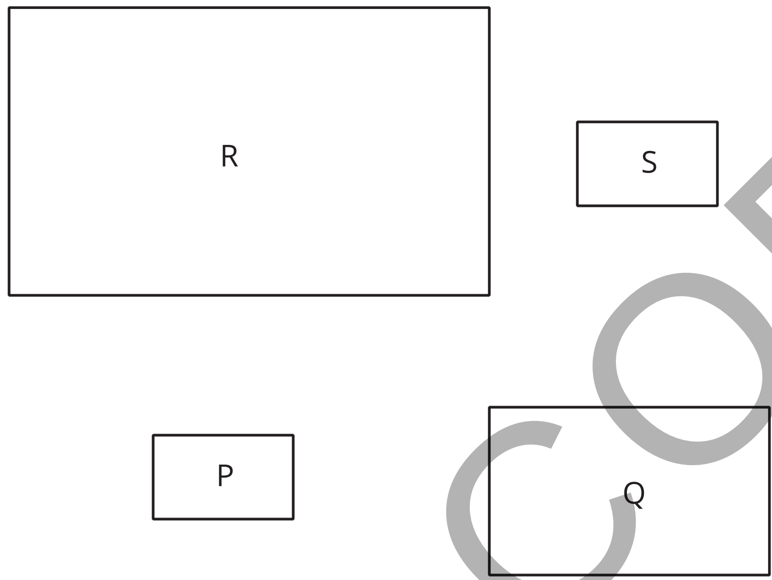
In other words, if we scale Figure A using a scale factor of 4 to create Figure B, we can scale Figure B using the reciprocal scale factor, $\frac{1}{4}$, to create Figure A.

Glossary

- reciprocal

Practice Problems

1 Rectangles P, Q, R, and S are scaled copies of one another. For each pair, decide if the scale factor from one to the other is greater than 1, equal to 1, or less than 1.



a. from P to Q

b. from P to R

c. from Q to S

d. from Q to R

e. from S to P

f. from R to P

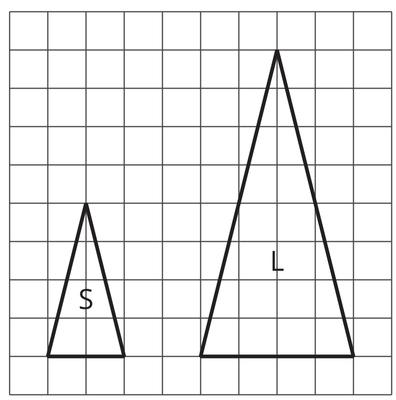
g. from P to S

2 Triangle S and Triangle L are scaled copies of one another.

a. What is the scale factor from S to L?

b. What is the scale factor from L to S?

c. Triangle M is also a scaled copy of S. The scale factor from S to M is $\frac{3}{2}$. What is the scale factor from M to S?



3 Are two squares with the same side lengths scaled copies of one another? Explain your reasoning.

4 from Unit 1, Lesson 2

Quadrilateral A has side lengths 2, 3, 5, and 6. Quadrilateral B has side lengths 4, 5, 8, and 10. Could one of the quadrilaterals be a scaled copy of the other? Explain.

5 from an earlier course

Select **all** the ratios that are equivalent to the ratio $12 : 3$.

A. $6 : 1$

B. $1 : 4$

C. $4 : 1$

D. $24 : 6$

E. $15 : 6$

F. $1{,}200 : 300$

G. $112 : 13$

Addressing CA CCSSM 7.G.1, 7.G.6; building on 6.G.1; building towards 7.G.4, 7.G.6, 7.RP.2a; practicing MP7 and MP8

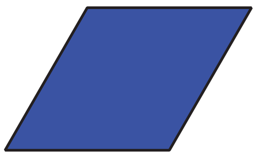
Scaling and Area

Let's build scaled shapes and investigate their areas.

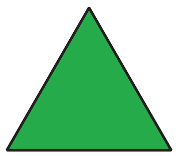
6.1 Scaling a Pattern Block

Your teacher will give you some pattern blocks. Work with your group to build the scaled copies described in each question.

A



B



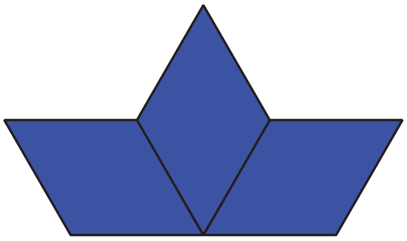
C

1. How many blue rhombus blocks does it take to build a scaled copy of Figure A:
   a. Where each side is twice as long?
   b. Where each side is 3 times as long?
   c. Where each side is 4 times as long?
2. How many green triangle blocks does it take to build a scaled copy of Figure B:
   a. Where each side is twice as long?
   b. Where each side is 3 times as long?
   c. Using a scale factor of 4?
3. How many red trapezoid blocks does it take to build a scaled copy of Figure C:
   a. Using a scale factor of 2?
   b. Using a scale factor of 3?
   c. Using a scale factor of 4?

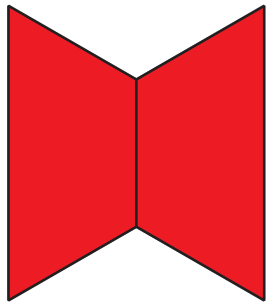
6.2 Scaling More Pattern Blocks

Your teacher will assign your group one of these figures.

D



E



F

1. Build a scaled copy of your assigned shape using a scale factor of 2. Use the same shape blocks as in the original figure. How many blocks did it take?
2. Your classmate thinks that the scaled copies in the previous problem will each take 4 blocks to build. Do you agree or disagree? Explain your reasoning.
3. Start building a scaled copy of your assigned figure using a scale factor of 3. Stop when you can tell for sure how many blocks it would take. Record your answer.
4. Predict: How many blocks would it take to build scaled copies using scale factors 4, 5, and 6? Explain or show your reasoning.
5. How is the pattern in this activity the same as the pattern you saw in the previous activity? How is it different?

Are you ready for more?

1. How many blocks do you think it would take to build a scaled copy of one yellow hexagon where each side is twice as long? Three times as long?

2. Figure out a way to build these scaled copies.

3. Do you see a pattern for the number of blocks used to build these scaled copies? Explain your reasoning.

Sec A

6.3 Area of Scaled Parallelograms and Triangles

1. Your teacher will give you a figure with measurements in centimeters. What is the area of your figure? How do you know?

2. Work with your partner to draw scaled copies of your figure, using each scale factor in the table. Complete the table with the measurements of your scaled copies.

scale factor	base (cm)	height (cm)	area (cm^2)
1			
2			
3			
$\frac{1}{2}$			
$\frac{1}{3}$			

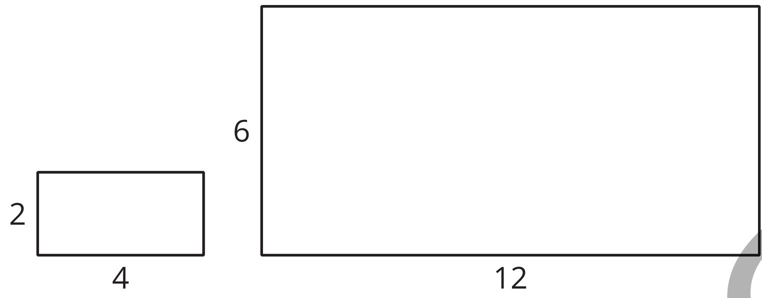
3. Compare your results with a group that worked with a different figure. What is the same about your answers? What is different?

4. If you drew scaled copies of your figure with the following scale factors, what would their areas be? Discuss your thinking. If you disagree, work to reach an agreement. Be prepared to explain your reasoning.

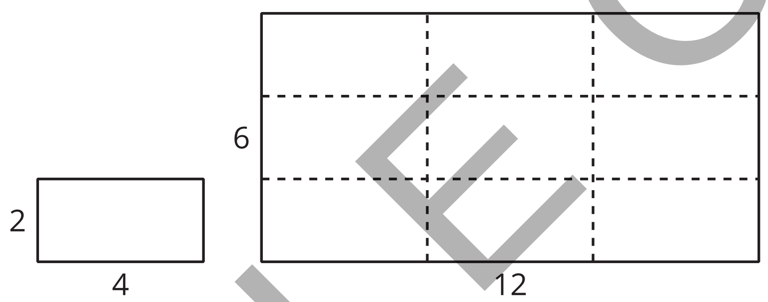
scale factor	area (cm^2)
5	
$\frac{3}{5}$	

Lesson 6 Summary

Scaling affects lengths and areas differently. When we make a scaled copy, all original lengths are multiplied by the scale factor. If we make a copy of a rectangle with side lengths 2 units and 4 units using a scale factor of 3, the side lengths of the copy will be 6 units and 12 units, because $2 \cdot 3 = 6$ and $4 \cdot 3 = 12$.



The area of the copy, however, changes by a factor of (scale factor)^2. If each side length of the copy is 3 times longer than the original side length, then the area of the copy will be 9 times the area of the original, because $3 \cdot 3$, or 3^2, equals 9.

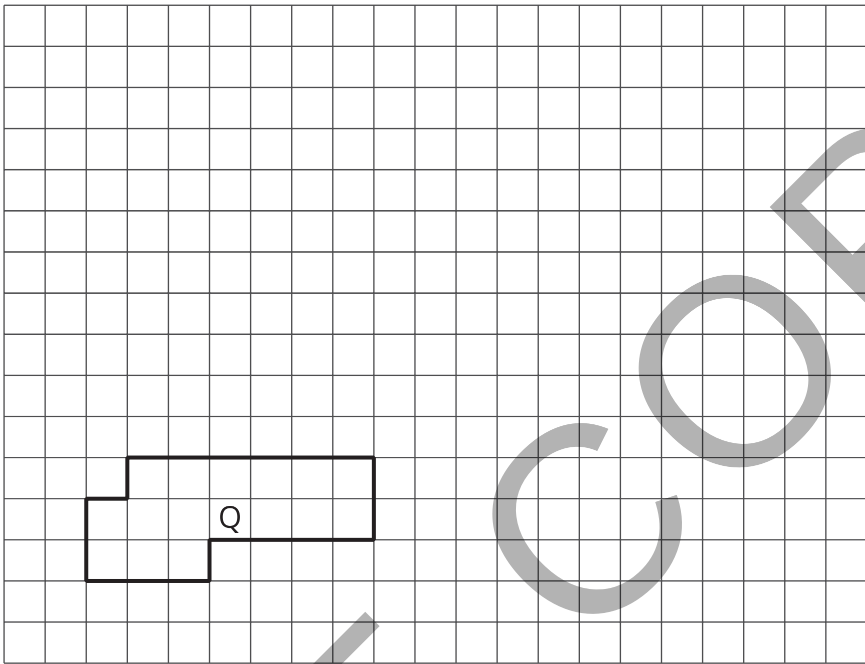


In this example, the area of the original rectangle is 8 units^2 and the area of the scaled copy is 72 units^2, because $9 \cdot 8 = 72$. We can see that the large rectangle is covered by 9 copies of the small rectangle, without gaps or overlaps. We can also verify this by multiplying the side lengths of the large rectangle: $6 \cdot 12 = 72$.

Lengths are one-dimensional, so in a scaled copy, they change by the scale factor. Area is two-dimensional, so it changes by the *square* of the scale factor. We can see this is true for a rectangle with length l and width w. If we scale the rectangle by a scale factor of s, we get a rectangle with length $s \cdot l$ and width $s \cdot w$. The area of the scaled rectangle is $A = (s \cdot l) \cdot (s \cdot w)$, so $A = (s^2) \cdot (l \cdot w)$. The fact that the area is multiplied by the square of the scale factor is true for scaled copies of other two-dimensional figures too, not just for rectangles.

Practice Problems

1 On the grid, draw a scaled copy of Polygon Q using a scale factor of 2. Compare the perimeter and area of the new polygon to those of Q.

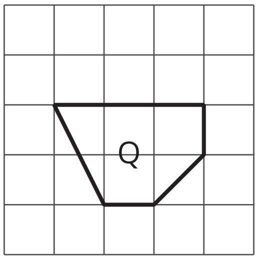


2 A right triangle has an area of 36 square units.

If you draw scaled copies of this triangle using the scale factors in the table, what will the areas of these scaled copies be? Explain or show your reasoning.

scale factor	area (units^2)
1	36
2	
3	
5	
$\frac{1}{2}$	
$\frac{2}{3}$	

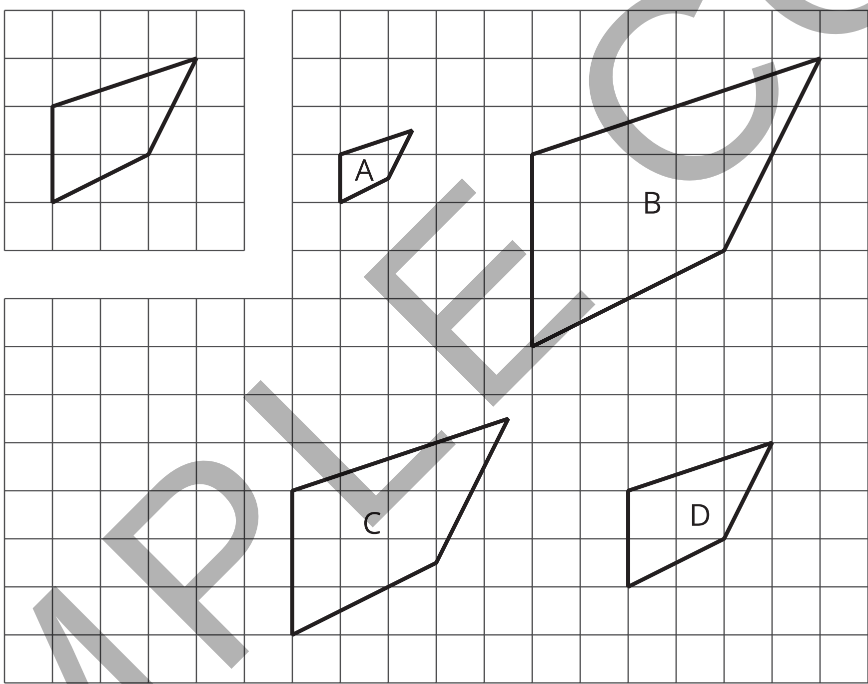
3 Diego drew a scaled version of a Polygon P and labeled it Q.



If the area of Polygon P is 72 square units, what scale factor did Diego use to go from P to Q? Explain your reasoning.

4 from Unit 1, Lesson 2

Here is an unlabeled polygon, along with its scaled copies Polygons A–D. For each copy, determine the scale factor. Explain how you know.



5 from Unit 1, Lesson 5

Solve each equation mentally.

a. $\frac{1}{7} \cdot x = 1$

b. $x \cdot \frac{1}{11} = 1$

c. $1 \div \frac{1}{5} = x$

Addressing CA CCSSM 7.G.1; practicing MP2 and MP5

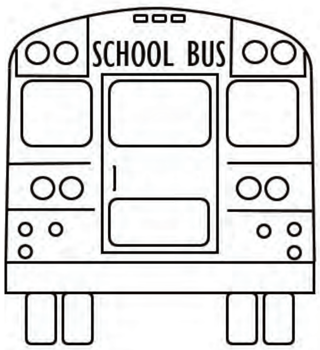
Scale Drawings

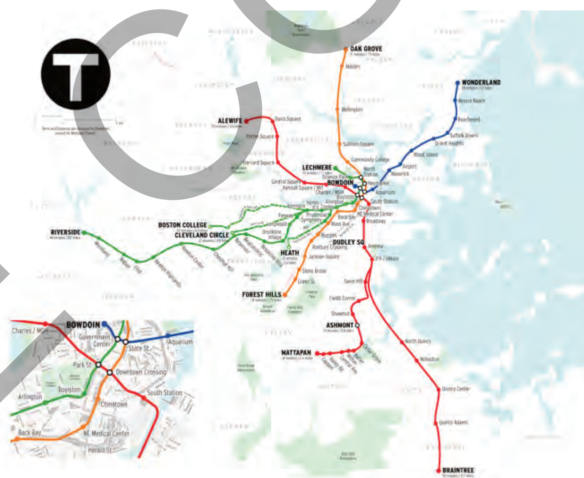
Let's explore scale drawings.

7.1 What is a Scale Drawing?

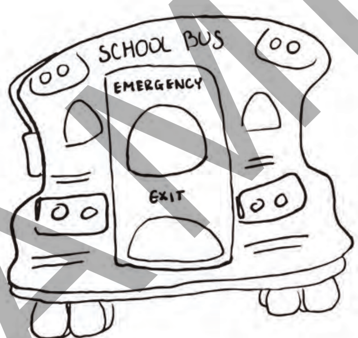
Here are some drawings of a school bus, a quarter, and the subway lines around Boston, Massachusetts.

The first three drawings are **scale drawings** of these objects.





The next three drawings are *not* scale drawings of these objects.



Discuss with your partner what a scale drawing is.

7.2 Sizing Up a Basketball Court

Your teacher will give you a scale drawing of a basketball court. The drawing does not have any measurements labeled, but it says that 1 centimeter represents 2 meters.

1. To the nearest tenth of a centimeter, measure the distances on the scale drawing that are labeled a–d. Record your results in the first row of the table.
2. The statement "1 cm represents 2 m" is the **scale** of the drawing. It can also be expressed as "1 cm to 2 m," or "1 cm for every 2 m." What do you think the scale tells us?

Sec B

3. How long would each measurement from the first question be on an actual basketball court? Explain or show your reasoning.

measurement	(a) length of court	(b) width of court	(c) hoop to hoop	(d) 3 point line to sideline
scale drawing				
actual court				

4. On an actual basketball court, the bench area is typically 9 meters long.
   a. Without measuring, determine how long the bench area should be on the scale drawing.
   b. Check your answer by measuring the bench area on the scale drawing. Did your prediction match your measurement?

7.3 Tall Structures

Here is a scale drawing of some of the world's tallest structures.

1. About how tall is the actual Willis Tower? About how tall is the actual Great Pyramid? Be prepared to explain your reasoning.

2. About how much taller is the Burj Khalifa than the Eiffel Tower? Explain or show your reasoning.

3. Measure the line segment that shows the scale to the nearest tenth of a centimeter. Express the scale of the drawing using numbers and words.

Are you ready for more?

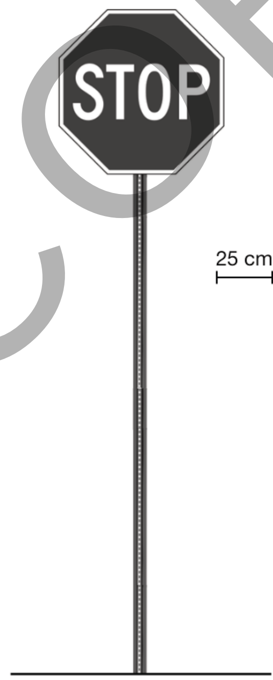
The tallest mountain in the United States, Mount Denali in Alaska, is about 6,190 m tall. If this mountain were shown on the scale drawing, how would its height compare to the heights of the structures? Explain or show your reasoning.

Lesson 7 Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings. On a scale drawing:

- Every part corresponds to something in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A **scale** tells us how actual measurements are represented on the drawing. For example, if a map has a scale of "1 inch to 5 miles" then a $\frac{1}{2}$-inch line segment on that map would represent an actual distance of 2.5 miles.

Sometimes the scale is shown as a segment on the drawing itself. For example, here is a scale drawing of a stop sign with a line segment that represents 25 cm of actual length.



The width of the octagon in the drawing is about three times the length of this segment, so the actual width of the sign is about $3 \cdot 25$, or 75 cm.

Because a scale drawing is two-dimensional, some aspects of the three-dimensional object are not represented. For example, this scale drawing does not show the thickness of the stop sign.

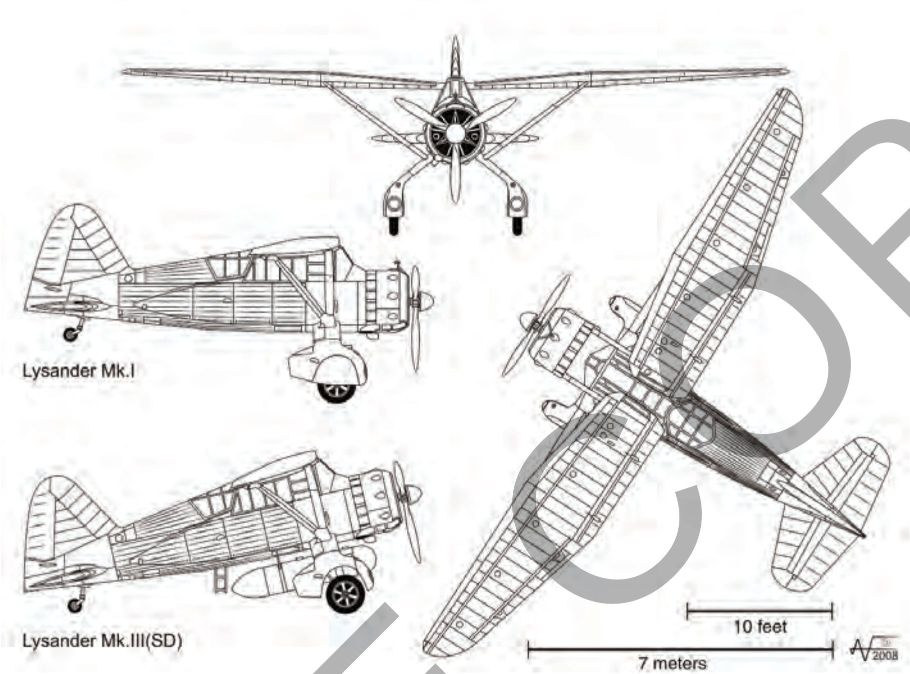
A scale drawing may not show every detail of the actual object; however, the features that are shown correspond to the actual object and follow the specified scale.

Glossary

- scale
- scale drawing

Practice Problems

1 The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.



Use the scales and scale drawings to approximate the actual lengths of:

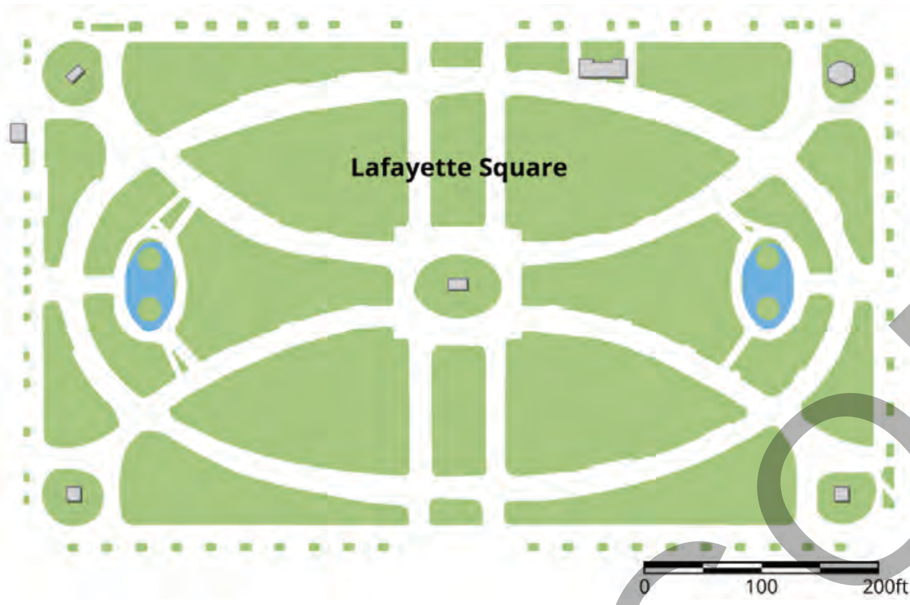
a. The wingspan of the plane, to the nearest foot.

b. The height of the plane, to the nearest foot.

c. The length of the Lysander Mk. I, to the nearest meter.

2 A scale drawing of a rectangular room measures 3 inches long and 5.5 inches wide. The scale says that 1 inch on the drawing represents 10 feet in the actual building. What are the length and width of the actual room?

3 Here is a scale map of Lafayette Square, a rectangular garden north of the White House.

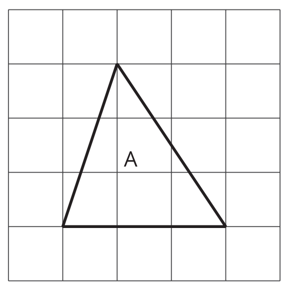


a. The scale is shown in the lower right corner. Find the actual side lengths of Lafayette Square in feet.

b. Use an inch ruler to measure the line segment of the graphic scale. About how many feet does one inch represent on this map?

4 from Unit 1, Lesson 6

Here is Triangle A. Lin created a scaled copy of Triangle A with an area of 72 square units.



a. How many times larger is the area of the scaled copy compared to that of Triangle A?

b. What scale factor did Lin apply to the Triangle A to create the copy?

c. What is the length of the bottom side of the scaled copy?

Addressing CA CCSSM 7.G.1; building on 6.NS.2, 6.RP.3b; building towards 7.RP.2b; practicing MP2

Scale Drawings and Maps

Let’s use scale drawings to solve problems.

8.1 A Train and a Car

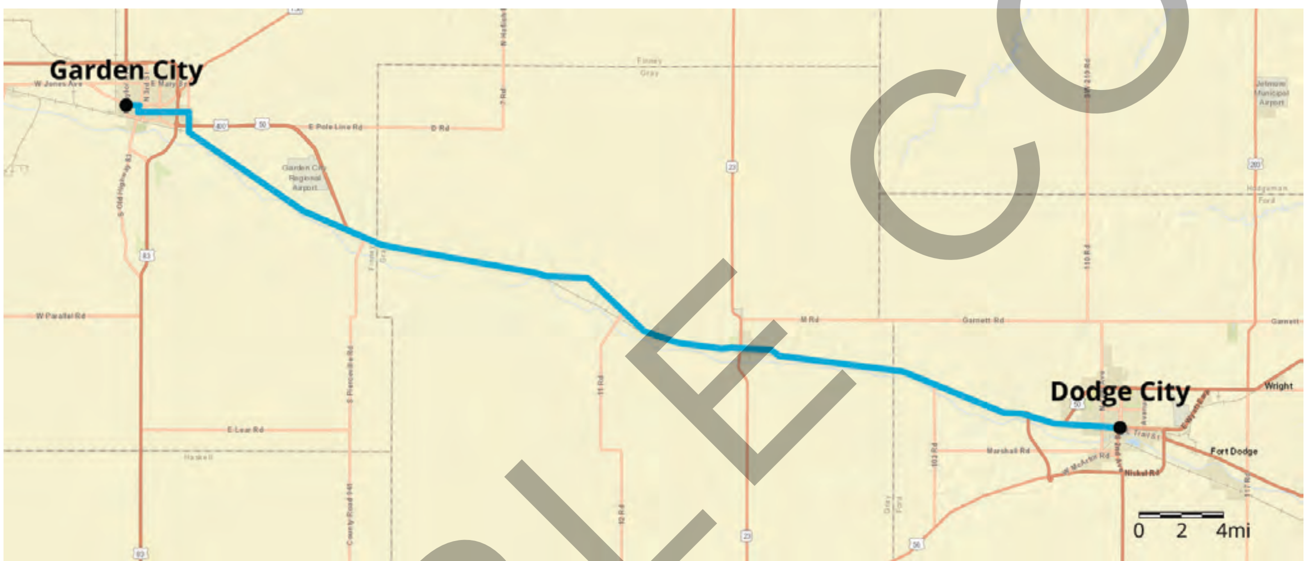
Two cities are 243 miles apart.

- It takes a train 4 hours to travel between the two cities at a constant speed.
- A car travels between the two cities at a constant speed of 65 miles per hour.

Which is traveling faster, the car or the train? Be prepared to explain your reasoning.

8.2 Biking through Kansas

A cyclist rides at a constant speed of 15 miles per hour. At this speed, about how long would it take the cyclist to ride from Garden City to Dodge City, Kansas?

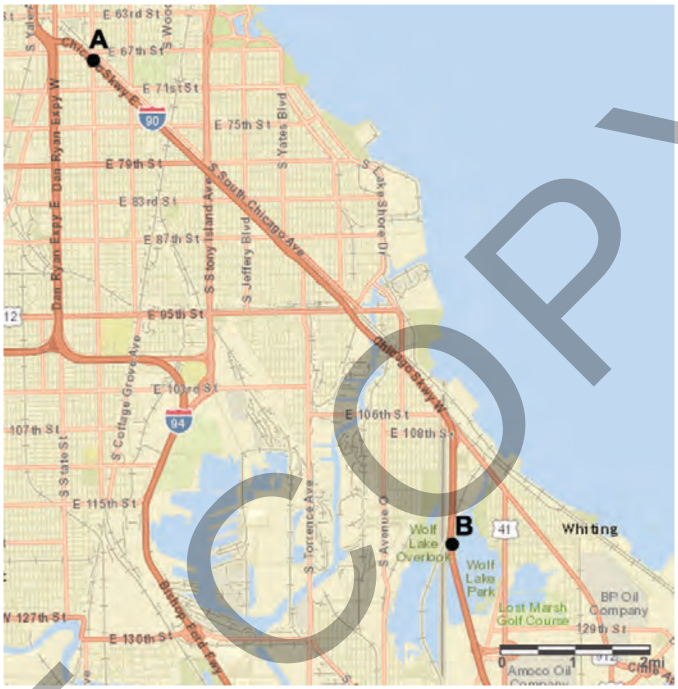


Are you ready for more?

Jada finds a map that says, “Note: This map is not to scale.” What do you think this means? Why is this information important?

8.3 Driving on I-90

1. A driver is traveling at a constant speed on Interstate 90 outside Chicago. If she traveled from Point A to Point B in 10 minutes, at what speed was she driving? Explain your reasoning.

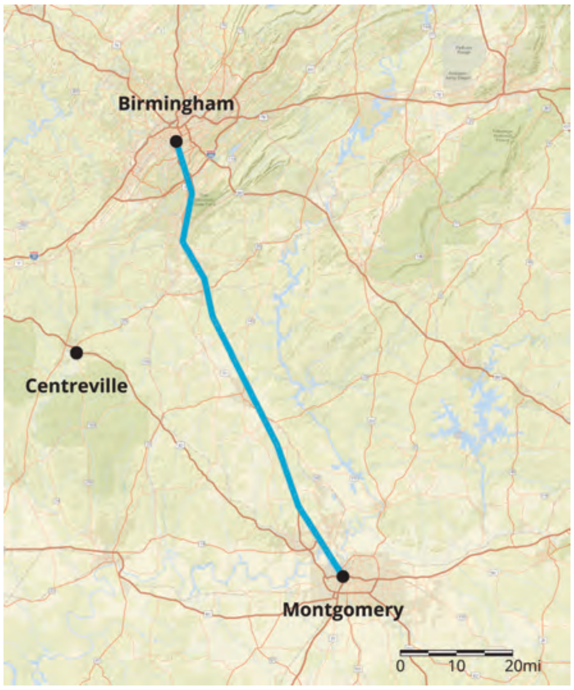


2. A helicopter flew directly from Point A to Point B in 9 minutes. Did the helicopter travel faster or slower than the driver? Explain or show your reasoning.

Sec B

Lesson 8 Summary

Maps with scales are useful for making calculations involving speed, time, and distance. Here is a map of part of Alabama.



Suppose it takes a car 1 hour and 30 minutes to travel at constant speed from Birmingham to Montgomery. How fast is the car traveling?

To make an estimate, we need to know about how far it is from Birmingham to Montgomery. The scale of the map represents 20 mi, so we can estimate that the distance between these cities is about 90 mi.

Since 90 miles in 1.5 hours is the same speed as 180 mi in 3 hours, the car is traveling about 60 mi per hour.

time (hours)	distance (miles)
1.5	90
3	180
1	60

Suppose a car is traveling at a constant speed of 60 miles per hour from Montgomery to Centreville. How long will it take the car to make the trip? Using the scale, we can estimate that it is about 70 mi. Since 60 miles per hour is the same as 1 mile per minute, it will take the car about 70 minutes (or 1 hour and 10 minutes) to make this trip.

Practice Problems

1

a. A whale swims at a constant speed of 4 meters per second. How far does it travel in 40 seconds?

b. A horse runs at a constant speed of 5 meters per second. How much time does it take for the horse to travel 50 meters?

c. A goose flies at a constant speed, traveling 201 meters in 15 seconds. What is its speed in meters per second?

2 Here is a map that shows parts of Texas and Oklahoma.

a. About how far is it from Amarillo to Oklahoma City? Explain your reasoning.

b. Driving at a constant speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain how you know.

Sec B

3 A map of a park is made with the scale 1 inch to 200 feet.

a. On the map, the north side of the park is 1 foot long.

i. How long is the north side of the actual park?

ii. A person walks along the north side of the park, at a constant speed of 5 feet per second. How long does this take them?

b. A straight path in the park is 900 feet long.

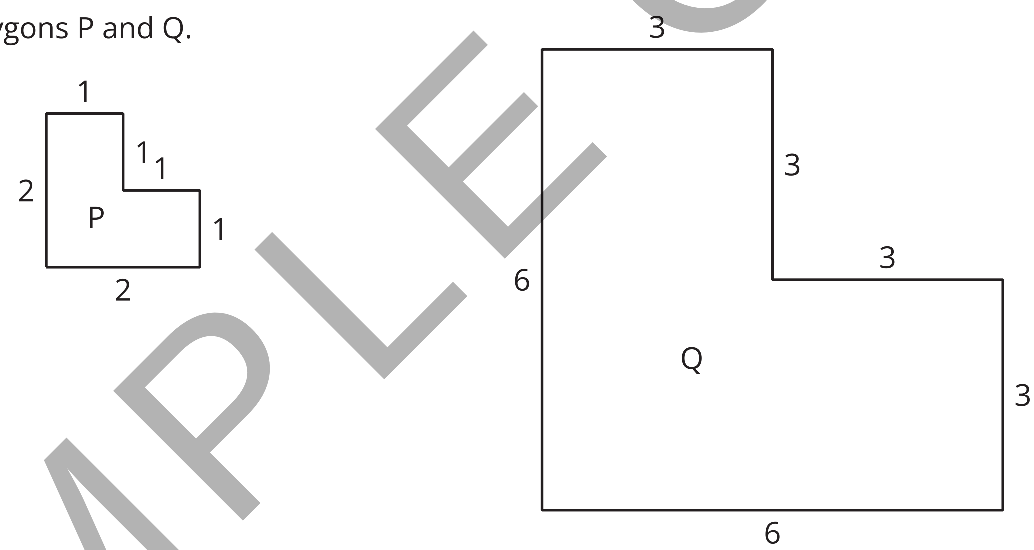
i. How long is this path on the map?

ii. A person skateboards along the path, traveling the entire length of the path in 90 seconds. What is their speed in feet per second?

Sec B

4 from Unit 1, Lesson 5

Here are Polygons P and Q.



a. If Polygon P is the original figure and Polygon Q is the scaled copy, what is the scale factor? Explain your reasoning.

b. If Polygon Q is the original figure and Polygon P is the scaled copy, what is the scale factor? Explain your reasoning.

Unit 1, Lesson 9

Addressing CA CCSSM 7.G.1; building on 3.NF.3, 5.NBT.3; practicing MP7

Creating Scale Drawings

Let's create our own scale drawings.

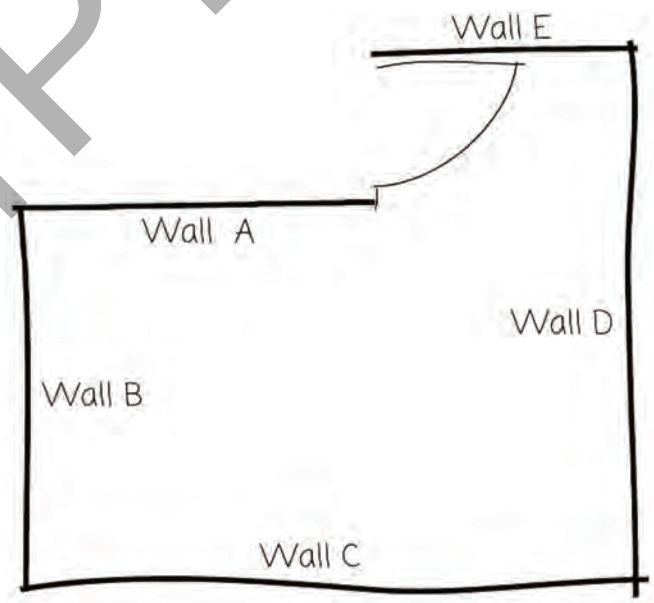
9.1 Math Talk: More Unit Fractions

Find the value of each expression mentally.

- $\frac{1}{3} \cdot 15$
- $15 \cdot \frac{1}{4}$
- $(2.6) \cdot \frac{1}{2}$
- $\frac{1}{5} \cdot (8.5)$

9.2 Bedroom Floor Plan

Here is a rough sketch of Noah's bedroom (not a scale drawing).



Noah wants to create a floor plan that is a scale drawing.

1. The actual length of Wall C is 4 m. To represent Wall C, Noah draws a segment 16 cm long. What scale is he using? Explain or show your reasoning.

2. Find another way to express the scale.

3. Discuss your thinking with your partner. How do your scales compare?

4. The actual lengths of Wall A and Wall D are 2.5 m and 3.75 m. Determine how long these walls will be on Noah's scale floor plan. Explain or show your reasoning.

Are you ready for more?

If Noah wanted to draw another floor plan on which Wall C was 20 cm, would 1 cm to 5 m be the right scale to use? Explain your reasoning.

9.3 Two Maps of Utah

A rectangle around Utah is about 270 mi wide and about 350 mi tall. The upper right corner that is missing is about 110 mi wide and about 70 mi tall.

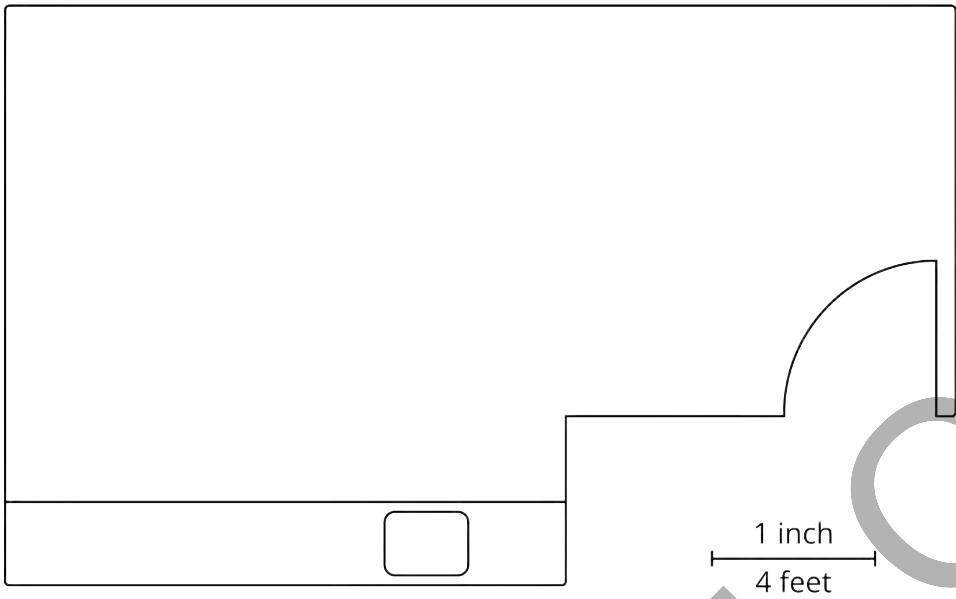
1. Make a scale drawing of Utah where 1 centimeter represents 50 mi.

   Make a scale drawing of Utah where 1 centimeter represents 75 mi.

2. How do the two drawings compare? How does the choice of scale influence the drawing?

Lesson 9 Summary

If we want to create a scale drawing of a room's floor plan that has the scale “1 inch to 4 feet,” we can divide the actual lengths in the room (in feet) by 4 to find the corresponding lengths (in inches) for our drawing.



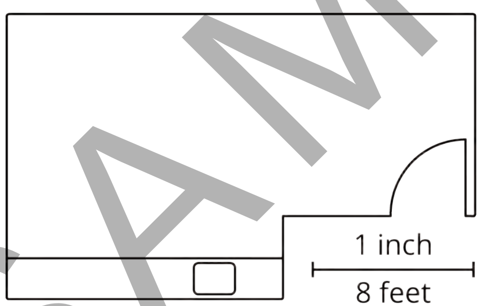
Suppose the longest wall is 23 feet long. We should draw a line 5.75 inches long to represent this wall, because $23 \div 4 = 5.75$.

There is more than one way to express this scale. These three scales are all equivalent, because they represent the same relationship between lengths on a drawing and actual lengths:

- 1 inch to 4 feet
- $\frac{1}{2}$ inch to 2 feet
- $\frac{1}{4}$ inch to 1 foot

Any of these scales can be used to find actual lengths and scaled lengths (lengths on a drawing). For instance, we can tell that, at this scale, an 8-foot long wall should be 2 inches long on the drawing because $\frac{1}{4} \cdot 8 = 2$.

The size of a scale drawing is influenced by the choice of scale. For example, here is another scale drawing of the same room using the scale 1 inch to 8 feet.



Notice that this drawing is smaller than the previous one. Since one inch on this drawing represents twice as much actual distance, each side length needs to be only half as long as it was in the first scale drawing.

Practice Problems

1 The flag of Colombia is a rectangle that is 6 ft long with three horizontal stripes.

- The top stripe is 2 ft tall and is yellow.
- The middle stripe is 1 ft tall and is blue.
- The bottom stripe is also 1 ft tall and is red.

a. Create a scale drawing of the Colombian flag with a scale of 1 cm to 2 ft.

b. Create a scale drawing of the Colombian flag with a scale of 2 cm to 1 ft.

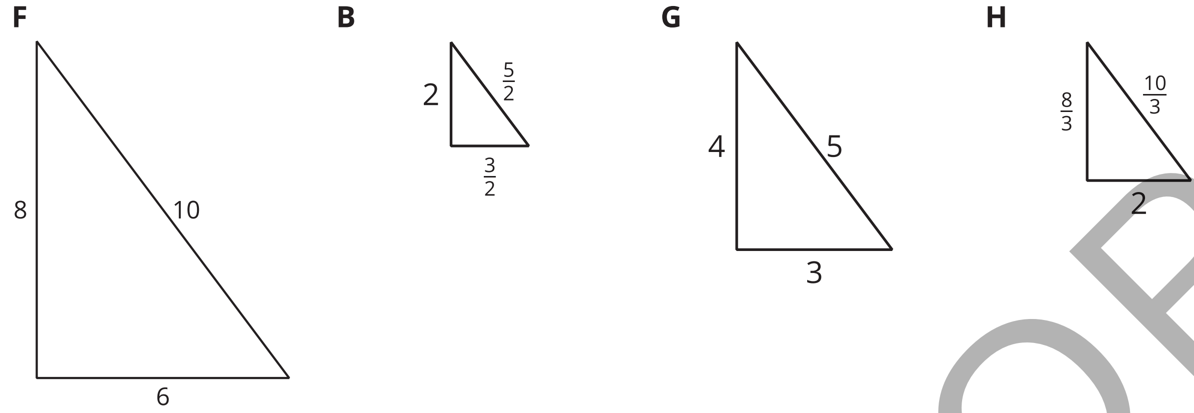
2 An image of a book shown on a website is 1.5 inches wide and 3 inches tall on a computer monitor. The actual book is 9 inches wide.

a. What scale is being used for the image?

b. How tall is the actual book?

3 from Unit 1, Lesson 6

These triangles are scaled copies of each other.



For each pair of triangles listed, the area of the second triangle is how many times larger than the area of the first?

a. Triangle G and Triangle F

b. Triangle G and Triangle B

c. Triangle B and Triangle F

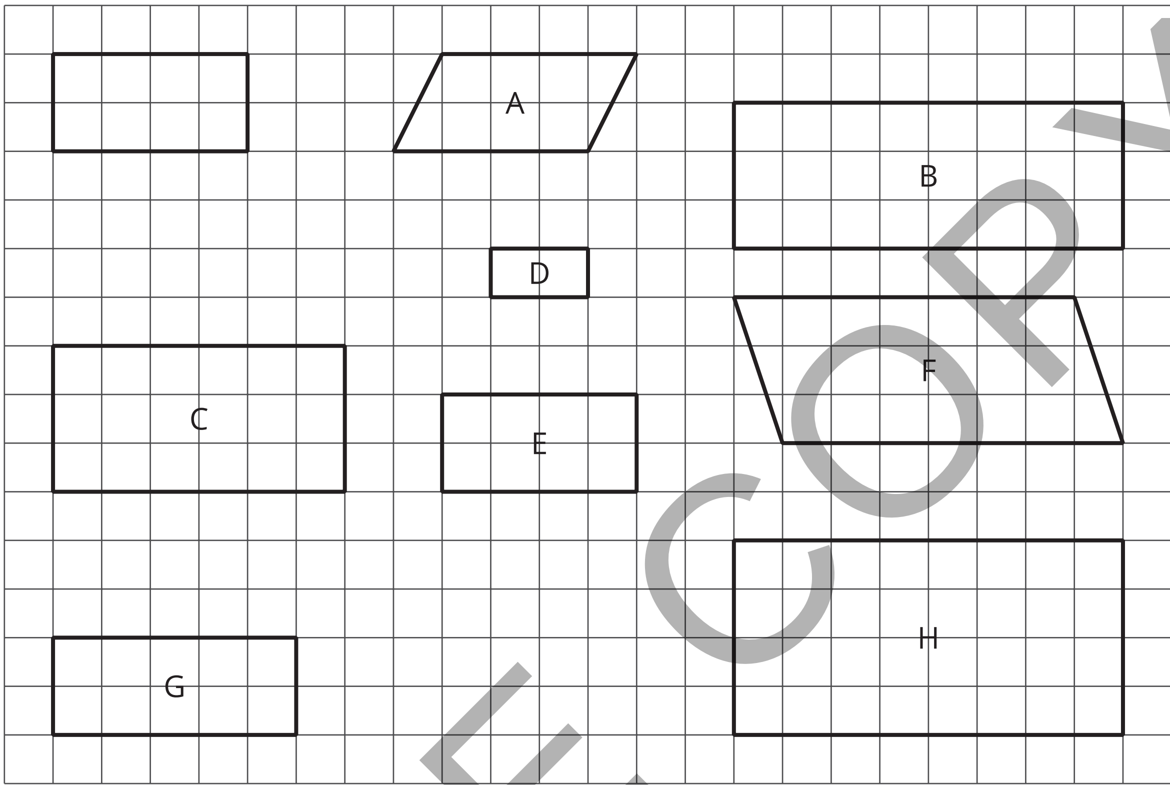
d. Triangle F and Triangle H

e. Triangle G and Triangle H

f. Triangle H and Triangle B

4 from Unit 1, Lesson 3

Here is an unlabeled rectangle, followed by other quadrilaterals that are labeled.



a. Select **all** quadrilaterals that are scaled copies of the unlabeled rectangle. Explain how you know.

b. On graph paper, draw a different scaled version of the original rectangle.

Addressing CA CCSSM 7.G.1; building on 2.MD.1-4, 6.G.1; building towards 7.G.6, 7.RP.3; practicing MP7 and MP8

Changing Scales in Scale Drawings

Let’s explore different scale drawings of the same actual thing.

10.1 Appropriate Measurements

1. If a student uses a ruler like this to measure the length of their foot, which choices would be appropriate measurements? Select **all** that apply. Be prepared to explain your reasoning.

   A. $9\frac{1}{4}$ inches

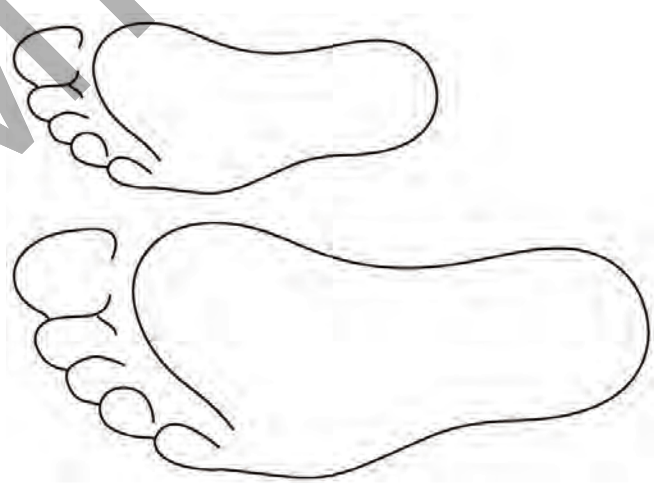
   B. $9\frac{5}{64}$ inches

   C. 23.47659 centimeters

   D. 23.5 centimeters

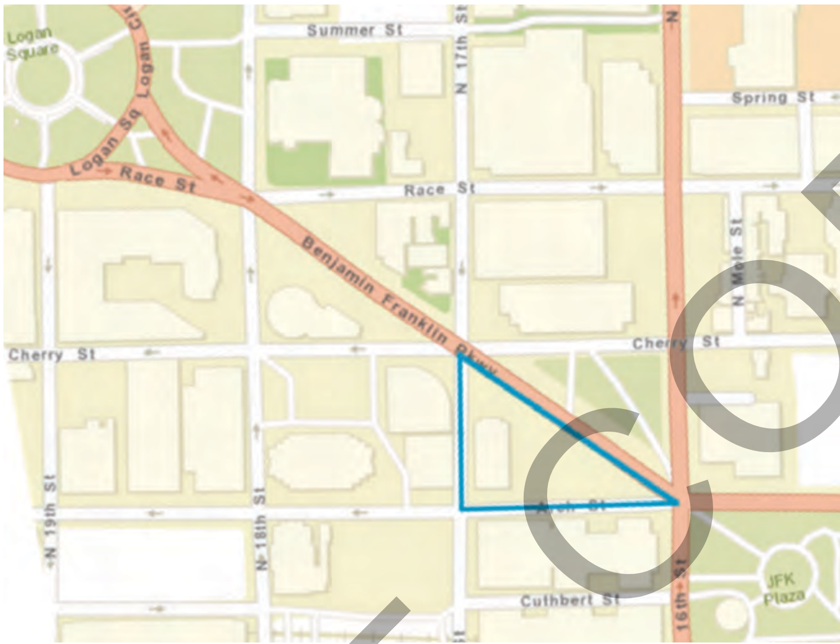
   E. 23.48 centimeters

2. Here is a scale drawing of an average seventh-grade student&required;s foot next to a scale drawing of a foot belonging to the person with the largest feet in the world. Estimate the length of the larger foot.



10.2 Same Plot, Different Drawings

Here is a map showing a plot of land in the shape of a right triangle.



1. Your teacher will assign you a scale to use. On centimeter graph paper, make a scale drawing of the plot of land. Make sure to write your scale on your drawing.
2. What is the area of the triangle you drew? Explain or show your reasoning.
3. How many square meters are represented by 1 square centimeter in your drawing?
4. After everyone in your group is finished, order the scale drawings from largest to smallest. What do you notice about the scales when your drawings are placed in this order?

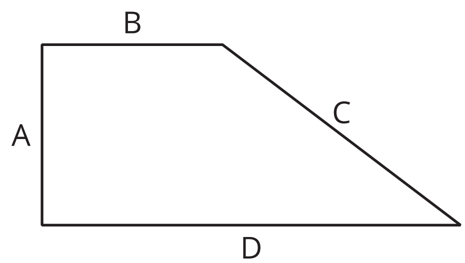
Are you ready for more?

Noah and Elena each make a scale drawing of the same triangular plot of land, using the following scales. Make a prediction about the size of each drawing. How would they compare to the scale drawings made by your group?

1. Noah uses the scale 1 cm to 200 m.
2. Elena uses the scale 2 cm to 25 m.

10.3 A New Drawing of the Playground

Here is a scale drawing of a playground.



The scale is 1 centimeter to 30 meters.

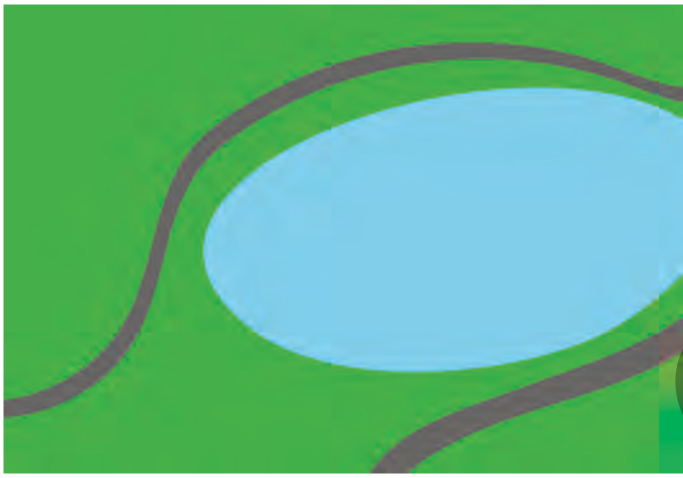
1. Make another scale drawing of the same playground at a scale of 1 centimeter to 20 meters.

2. How do the two scale drawings compare?

Lesson 10 Summary

Sometimes we have a scale drawing of something, and we want to create another scale drawing of it that uses a different scale. We can use the original scale drawing to find the size of the actual object. Then we can use the size of the actual object to figure out the size of our new scale drawing.

For example, here is a scale drawing of a park where the scale is 1 cm to 90 m.



The rectangle is 10 cm by 4 cm, so the actual dimensions of the park are 900 m by 360 m, because $10 \cdot 90 = 900$ and $4 \cdot 90 = 360$.

Suppose we want to make another scale drawing of the park where the scale is 1 cm to 30 meters. This new scale drawing should be 30 cm by 12 cm, because $900 \div 30 = 30$ and $360 \div 30 = 12$.

Another way to find this answer is to think about how the two different scales are related to each other. In the first scale drawing, 1 cm represented 90 m. In the new drawing, we would need 3 cm to represent 90 m. That means each length in the new scale drawing should be 3 times as long as it was in the original drawing. The new scale drawing should be 30 cm by 12 cm, because $3 \cdot 10 = 30$ and $3 \cdot 4 = 12$.

Since the length and width are 3 times as long, the area of the new scale drawing will be 9 times as large as the area of the original scale drawing because $3^2 = 9$.

Practice Problems

1 Here is a scale drawing of a swimming pool where 1 cm represents 1 m.

a. How long and how wide is the actual swimming pool?

b. Will a scale drawing where 1 cm represents 2 m be larger or smaller than this drawing?

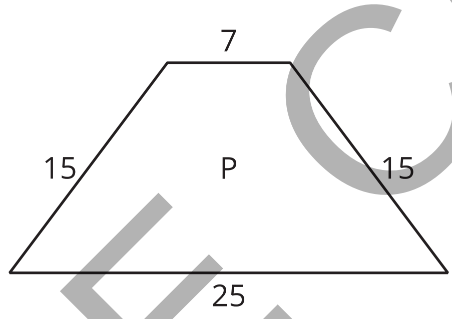
c. Make a scale drawing of the swimming pool where 1 cm represents 2 m.

2 A map of a park has a scale of 1 inch to 1,000 feet. Another map of the same park has a scale of 1 inch to 500 feet. Which map is larger? Explain or show your reasoning.

3 On a map with a scale of 1 inch to 12 feet, the area of a restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your reasoning.

4 from Unit 1, Lesson 3

If Quadrilateral Q is a scaled copy of Quadrilateral P created with a scale factor of 3, what is the perimeter of Q?



5 from Unit 1, Lesson 2

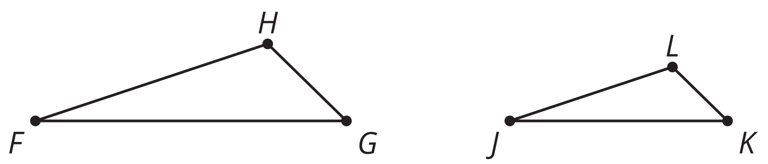
Triangle JKL is a scaled copy of triangle FGH. For each of the following parts of triangle FGH, identify the corresponding part of triangle JKL.

a. angle FGH

b. angle GHF

c. segment FH

d. segment GF



Sec B

Unit 1, Lesson 11

Addressing CA CCSSM 7.G.1; practicing MP6 and MP7

Scales without Units

Let's explore a different way to express scales.

11.1 One to One Hundred

A map of a park says its scale is 1 to 100.

1. What do you think that means?

2. Give an example of how this scale could tell us about measurements in the park.

11.2 Apollo Lunar Module

Your teacher will give you a scale drawing of the Apollo Lunar Module. It is drawn at a scale of 1 to 50.

1. Estimate the actual length of one leg of the spacecraft to the nearest 10 centimeters. Explain or show your reasoning.

2. Estimate the actual height of the spacecraft to the nearest meter. Explain or show your reasoning.

3. Neil Armstrong was 71 inches tall when he went to the Moon. How tall would he be in this scale drawing? Show your reasoning.

4. Sketch a stick figure to represent yourself standing next to the Apollo Lunar Module. Make sure the height of your stick figure is to scale. Show how you determined your height on the drawing.

Are you ready for more?

The table shows the distance between the Sun and 8 planets in our solar system.

1. If you wanted to create a scale model of the solar system that could fit somewhere in your school, what scale would you use?

2. The diameter of Earth is approximately 8,000 mi. What would the diameter of Earth be in your scale model?

planet	average distance (millions of miles)
Mercury	35
Venus	67
Earth	93
Mars	142
Jupiter	484
Saturn	887
Uranus	1,784
Neptune	2,795

11.3 Same Drawing, Different Scales

A rectangular parking lot is 120 feet long and 75 feet wide.

- Lin made a scale drawing of the parking lot at a scale of 1 inch to 15 feet. The drawing she produced is 8 inches by 5 inches.
- Diego made another scale drawing of the parking lot at a scale of 1 to 180. The drawing he produced is also 8 inches by 5 inches.

1. Explain or show how each scale would produce an 8 inch by 5 inch drawing.

2. Make another scale drawing of the same parking lot at a scale of 1 inch to 20 feet. Be prepared to explain your reasoning.

3. Express the scale of 1 inch to 20 feet as a scale without units. Explain your reasoning.

Lesson 11 Summary

In some scale drawings, the scale specifies one unit for the distances on the drawing and a different unit for the actual distances represented. For example, a drawing could have a scale of 1 cm to 10 km.

In other scale drawings, the scale does not specify any units at all. For example, a map may simply say that the scale is 1 to 1,000. In this case, the units for the scaled measurements and actual measurements can be any unit, as long as the same unit is being used for both. If a map of a park has a scale 1 to 1,000, then 1 inch on the map represents 1,000 inches in the park, and 12 centimeters on the map represent 12,000 centimeters in the park. In other words, 1,000 is the scale factor that relates distances on the drawing to actual distances, and $\frac{1}{1{,}000}$ is the scale factor that relates an actual distance to its corresponding distance on the drawing.

A scale with units can be expressed as a scale without units by converting one measurement in the scale into the same unit as the other (usually the unit used in the drawing). For example, these scales are equivalent:

- 1 inch to 200 feet
- 1 inch to 2,400 inches (because there are 12 inches in 1 foot, and $200 \cdot 12 = 2{,}400$)
- 1 to 2,400

This scale tells us that all actual distances are 2,400 times their corresponding distances on the drawing, and distances on the drawing are $\frac{1}{2{,}400}$ times the actual distances that they represent.

Sec B

Practice Problems

1 A scale drawing of a car is presented in the following three scales. Order the scale drawings from smallest to largest. Explain your reasoning. (There are about 1.1 yards in 1 meter, and 2.54 centimeters in 1 inch.)

a. 1 inch to 1 foot

b. 1 inch to 1 meter

c. 1 inch to 1 yard

Sec B

2 Which scales are equivalent to 1 inch to 1 foot? Select **all** that apply.

A. 1 to 12

B. $\frac{1}{12}$ to 1

C. 100 to 0.12

D. 5 to 60

E. 36 to 3

F. 9 to 108

3 A model airplane is built at a scale of 1 to 72. If the model plane is 8 inches long, how many feet long is the actual airplane?

4 from Unit 1, Lesson 3

Quadrilateral A has side lengths 3, 6, 6, and 9. Quadrilateral B is a scaled copy of A with a shortest side length equal to 2. Jada says, "Since the side lengths go down by 1 in this scaling, the perimeter goes down by 4 in total." Do you agree with Jada? Explain your reasoning.

5 from Unit 1, Lesson 6

Polygon B is a scaled copy of Polygon A using a scale factor of 5. Polygon A's area is what fraction of Polygon B's area?

6 from Unit 1, Lesson 5

Figures R, S, and T are all scaled copies of one another. Figure S is a scaled copy of R using a scale factor of 3. Figure T is a scaled copy of S using a scale factor of 2. Find the scale factors for each of the following:

a. From T to S
b. From S to R
c. From R to T
d. From T to R

Unit 1, Lesson 12

Addressing CA CCSSM 7.G.1; building on 6.RP.3d; practicing MP3, MP6, and MP7

Units in Scale Drawings

Let's use different scales to describe the same drawing.

12.1 Equal Measures

Use the numbers and units from the list to find as many equivalent measurements as you can. For example, you might write "30 minutes is $\frac{1}{2}$ hour."

You can use the numbers and units more than once.

1	$\frac{1}{2}$	0.3	centimeter (cm)
12	40	24	meter (m)
0.4	100	$\frac{1}{10}$	kilometer (km)
8	$3\frac{1}{3}$	6	inch (in)
50	30	2	foot (ft)
		$\frac{2}{3}$	yard (yd)

KH | iM Illustrative Mathematics® LEARN MATH FOR LIFE

12.2 Card Sort: Scales

Your teacher will give you some cards with a scale on each card.

1. Take turns with your partner to sort the cards into sets of equivalent scales. Each set should have at least two cards.

   a. For each match that you find, explain to your partner how you know it's a match.

   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

2. Trade places with another group and check each other's work. If you disagree about how the scales should be sorted, work to reach an agreement.

   Pause here so your teacher can review your work.

3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.

12.3 The World's Largest Flag

As of 2016, Tunisia holds the world record for the largest version of a national flag. It was almost as long as four soccer fields. The flag has a circle in the center, a crescent moon inside the circle, and a star inside the crescent moon.

1. Complete the table. Explain or show your reasoning.

	flag length	flag height	height of crescent moon
actual	396 m		99 m
at 1 to 2,000 scale		13.2 cm	

2. Complete each scale with the value that makes it equivalent to the scale of 1 to 2,000. Explain or show your reasoning.

   a. 1 cm to __________ cm

   b. 1 cm to __________ m

   c. 1 cm to __________ km

   d. 2 m to __________ m

   e. 5 cm to __________ m

   f. __________ cm to 1,000 m

   g. __________ mm to 20 m

3. a. What is the area of the large flag?

   b. What is the area of the smaller flag?

   c. The area of the large flag is how many times the area of the smaller flag?

12.4 Pondering Pools

Your teacher will give you a floor plan of a recreation center.

1. What is the scale of the floor plan if the actual side length of the square pool is 15 m? Express your answer both as a scale with units and without units.

2. Find the actual area of the large rectangular pool. Show your reasoning.

3. The kidney-shaped pool has an area of 3.2 cm^2 on the drawing. What is its actual area? Explain or show your reasoning.

Are you ready for more?

1. Square A is a scaled copy of Square B with scale factor 2. If the area of Square A is 10 $units^2$, what is the area of Square B?
2. Cube A is a scaled copy of Cube B with scale factor 2. If the volume of Cube A is 10 $units^3$, what is the volume of Cube B?
3. The four-dimensional Hypercube A is a scaled copy of Hypercube B with scale factor 2. If the "volume" of Hypercube A is 10 $units^4$, what do you think the "volume" of Hypercube B is?

Lesson 12 Summary

Sometimes scales come with units, and sometimes they don’t. For example, a map of Nebraska may have a scale of 1 mm to 1 km. This means that each millimeter of distance on the map represents 1 kilometer of distance in Nebraska. Notice that there are 1,000 millimeters in 1 meter and 1,000 meters in 1 kilometer. This means there are $1{,}000 \cdot 1{,}000$ or 1,000,000 millimeters in 1 kilometer. So, the same scale without units is 1 to 1,000,000, which means that each unit of distance on the map represents 1,000,000 units of distance in Nebraska. This is true for *any* choice of unit to express the scale of this map.

Sometimes when a scale comes with units, it is useful to rewrite it without units. For example, let's say we have a different map of Rhode Island, and we want to use the two maps to compare the size of Nebraska and Rhode Island. It is important to know if the maps are at the same scale. The scale of the map of Rhode Island is 1 inch to 10 miles. There are 5,280 feet in 1 mile, and 12 inches in 1 foot, so there are 63,360 inches in 1 mile (because $5{,}280 \cdot 12 = 63{,}360$). Therefore, there are 633,600 inches in 10 miles. The scale of the map of Rhode Island without units is 1 to 633,600. The two maps are not at the same scale, so we should not use these maps to compare the size of Nebraska to the size of Rhode Island.

Here is some information about equal lengths that you may find useful.

Customary Units

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

Metric Units

1 centimeter (cm) = 10 millimeters (mm)
1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters

Equal Lengths in Different Systems

1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers

1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

Practice Problems

1 The Empire State Building in New York City is about 1,450 feet high (including the antenna at the top) and 400 feet wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an $8\frac{1}{2}$-inch-by-11-inch piece of paper. Select a scale that you think is the most appropriate for the scale drawing. Explain your reasoning.

A. 1 inch to 1 foot

B. 1 inch to 100 feet

C. 1 inch to 1 mile

D. 1 centimeter to 1 meter

E. 1 centimeter to 50 meters

F. 1 centimeter to 1 kilometer

2 Elena finds that the area of a house on a scale drawing is 25 square inches. The actual area of the house is 2,025 square feet. What is the scale of the drawing?

3 Which of these scales are equivalent to 3 cm to 4 km? Select **all** that apply. Recall that 1 inch is 2.54 centimeters.

A. 0.75 cm to 1 km

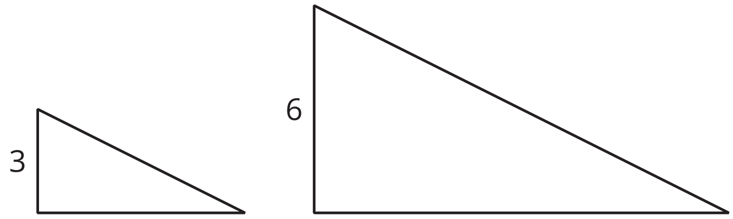
B. 1 cm to 12 km

C. 6 mm to 2 km

D. 0.3 mm to 40 m

E. 1 in to 7.62 km

4 These two triangles are scaled copies of one another. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Explain or show how you know.



5 from an earlier course

Water costs $1.25 per bottle. At this rate, what is the cost of:

a. 10 bottles?

b. 20 bottles?

c. 50 bottles?

6 from an earlier course

The first row of the table shows the amount of dish detergent and water needed to make a soap solution.

a. Complete the table for 2, 3, and 4 batches.

b. How much water and detergent is needed for 8 batches? Explain your reasoning.

number of batches	cups of water	cups of detergent
1	6	1
2		
3		
4		

Addressing CA CCSSM 7.G.1; practicing MP1 and MP4

Draw It to Scale

Let's draw a floor plan.

13.1 Which Measurements Matter?

Which measurements would you need in order to draw a scale floor plan of your classroom? List which parts of the classroom you would measure and include in the drawing. Be as specific as possible.

13.2 Creating a Floor Plan (Part 1)

1. On a blank sheet of paper, make a *rough sketch* of a floor plan of the classroom. Include parts of the room that the class has decided to include or that you would like to include. Accuracy is not important for this rough sketch, but be careful not to leave out important features like a door.
2. Trade sketches with a partner and check each other's work. Specifically, check if any parts are missing or incorrectly placed. Return their work and revise your sketch as needed.
3. Discuss with your group a plan for measuring. Work to reach an agreement on:
    - Which classroom features must be measured and which are optional.
    - The units to be used.
    - How to record and organize the measurements (on the sketch, in a list, in a table, etc.).
    - How to share the measuring and recording work (or the role each group member will play).
4. Gather your tools, take your measurements, and record them as planned. Be sure to double-check your measurements.
5. Make your own copy of all the measurements that your group has gathered, if you haven't already done so. You will need them for the next activity.

13.3 Creating a Floor Plan (Part 2)

Your teacher will give you several paper options for your scale floor plan.

1. Determine an appropriate scale for your drawing based on your measurements and your paper choice. Your floor plan should fit on the paper and not end up too small.
2. Use the scale and the measurements your group has taken to draw a scale floor plan of the classroom. Make sure to:
    - Show the scale of your drawing.
    - Label the key parts of your drawing (the walls, main openings, etc.) with their actual measurements.
    - Show your thinking and organize it so it can be followed by others.

Are you ready for more?

1. If the flooring material in your classroom is to be replaced with 10-inch by 10-inch tiles, how many tiles would it take to cover the entire room? Use your scale drawing to approximate the number of tiles needed.
2. How would using 20-inch by 20-inch tiles (instead of 10-inch by 10-inch tiles) change the number of tiles needed? Explain your reasoning.

13.4 Creating a Floor Plan (Part 3)

1. Trade floor plans with another student who used the same paper size that you used. Discuss your observations and thinking.
2. Trade floor plans with another student who used a different paper size than you used. Discuss your observations and thinking.
3. Based on your discussions, record ideas for how your floor plan could be improved.

Learning Targets

Lesson 1 What Are Scaled Copies?

- I can describe some characteristics of a scaled copy.
- I can tell whether or not a figure is a scaled copy of another figure.

Lesson 2 Corresponding Parts and Scale Factors

- I can describe what the scale factor has to do with a figure and its scaled copy.
- In a pair of figures, I can identify corresponding points, corresponding segments, and corresponding angles.

Lesson 3 Making Scaled Copies

- I can draw a scaled copy of a figure using a given scale factor.
- I know what operation to use on the side lengths of a figure to produce a scaled copy.

Lesson 4 Scaled Relationships

- I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another.
- When I see a figure and its scaled copy, I can explain what is true about corresponding angles.
- When I see a figure and its scaled copy, I can explain what is true about corresponding distances.

Lesson 5 The Size of the Scale Factor

- I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1.
- I can explain how the scale factor that takes Figure A to its copy Figure B is related to the scale factor that takes Figure B to Figure A.

Lesson 6 Scaling and Area

- I can describe how the area of a scaled copy is related to the area of the original figure and the scale factor that was used.

Lesson 7 Scale Drawings

- I can explain what a scale drawing is, and I can explain what its scale means.
- I can use actual distances and a scale to find scaled distances.
- I can use a scale drawing and its scale to find actual distances.

Lesson 8 Scale Drawings and Maps

- I can use a map and its scale to solve problems about traveling.

Lesson 9 Creating Scale Drawings

- I can determine the scale of a scale drawing when I know lengths on the drawing and corresponding actual lengths.
- I know how different scales affect the lengths in the scale drawing.
- When I know the actual measurements, I can create a scale drawing at a given scale.

Lesson 10 Changing Scales in Scale Drawings

- Given a scale drawing, I can create another scale drawing that shows the same thing at a different scale.
- I can use a scale drawing to find actual areas.

Lesson 11 Scales without Units

- I can explain the meaning of scales expressed without units.
- I can use scales without units to find scaled distances or actual distances.

Lesson 12 Units in Scale Drawings

- I can tell whether two scales are equivalent.
- I can write scales with units as scales without units.

Lesson 13 Draw It to Scale

- I can create a scale drawing of my classroom.
- When given requirements on drawing size, I can choose an appropriate scale to represent an actual object.

GRADE 7

UNIT

2

Introducing Proportional Relationships

Content Connections

In this unit you will work with proportional relationships that are represented in tables, as equations, and on graphs. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while working with common contexts associated with proportional relationships such as constant speed, unit pricing, and measurement conversions including decimals and fractions.
- **Discovering Shape and Space** while investigating graphs of proportional relationships and the connections between the graphs, tables and equations that represent the same proportional relationship.
- **Exploring Changing Quantities** while exploring and understanding proportional relationships using fractions, graphs, and tables.
- **Reasoning with Data** while justifying whether or not a relationship is proportional and representing proportional and nonproportional relationships in multiple ways.

Addressing the Standards

As you work your way through **Unit 2 Introducing Proportional Relationships,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 7, 9, and 15
MP2 Reason abstractly and quantitatively.	Lessons 3, 4, 5, 6, 10, 11, and 12
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 10 and 14
MP4 Model with mathematics.	Lessons 1, 7, 11, 14, and 15
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 1, 4, 5, 7, 8, 9, 11, and 13
MP7 Look for and make use of structure.	Lessons 2, 3, 6, 8, 10, 12, and 13
MP8 Look for and express regularity in repeated reasoning.	Lessons 4, 5, 6, and 12

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• 2-D and 3-D Connections • Scale Drawings • Shapes in the World	**7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Lesson 1
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawing	**7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*	Lesson 8

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawing	**7.RP.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.	Lessons 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, and 15
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawing	**7.RP.2a** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	Lessons 3, 7, and 10
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawing	**7.RP.2b** Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	Lessons 2, 3, and 11

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawing	**7.RP.2c** Recognize and represent proportional relationships between quantities. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*	Lessons 4, 6, and 7
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawing	**7.RP.2d** Recognize and represent proportional relationships between quantities. d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.	Lesson 11

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

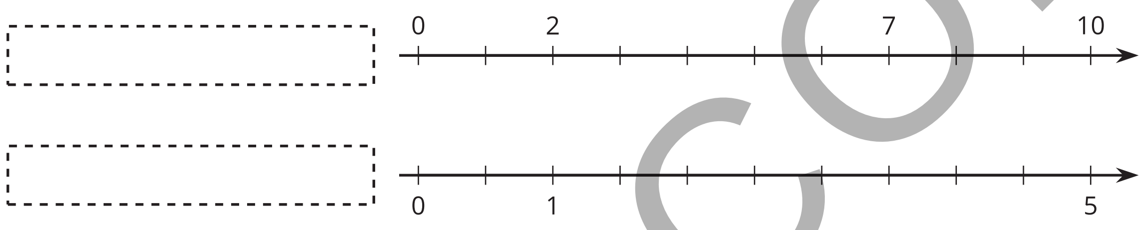
Addressing CA CCSSM 7.G.1; building on 6.RP.1-3; building towards 7.RP.1; practicing MP4 and MP6

One of These Things Is Not Like the Others

Let's remember what equivalent ratios are.

1.1 Remembering Double Number Lines

1. Complete the double number line diagram with the missing numbers.



2. What could each of the number lines represent? Invent a situation and label the diagram.

   Make sure your labels include appropriate units of measure.

1.2 Mystery Mixtures

Your teacher will show you three mixtures. Two taste the same, and one is different.

1. Which mixture would taste different? Why?

2. Here are the recipes that were used to make the three mixtures:

1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix

2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix

Which of these recipes is for the stronger tasting mixture? Explain how you know.

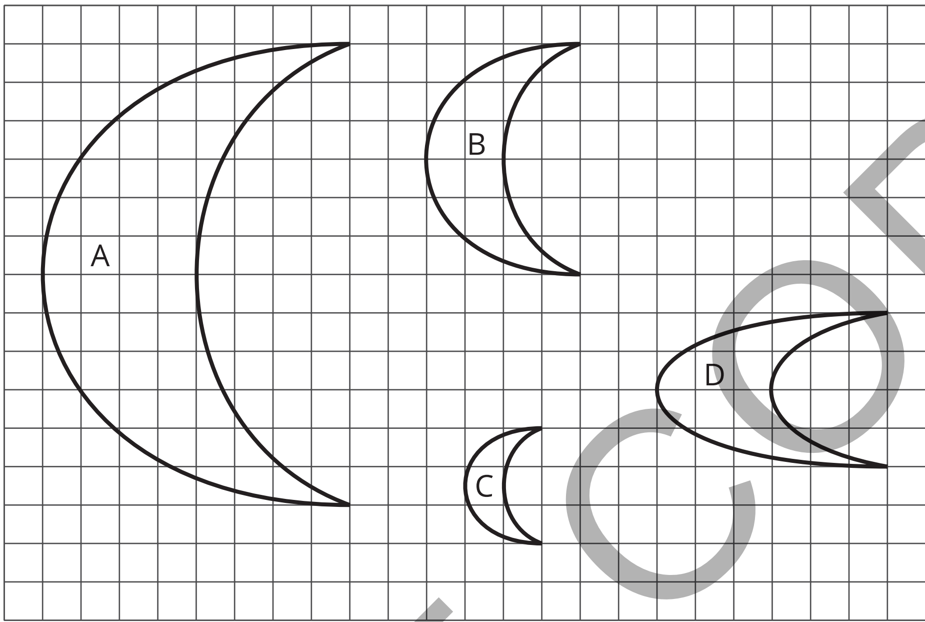
Are you ready for more?

Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same?

- Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar
- Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar
- Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar

1.3 Crescent Moons

Here are four different crescent moon shapes.



1. What do Moons A, B, and C all have in common that Moon D doesn't?

2. Use numbers to describe how Moons A, B, and C are different from Moon D.

Pause here so your teacher can review your work.

3. Use a table or a double number line to show how Moons A, B, and C are different from Moon D.

Lesson 1 Summary

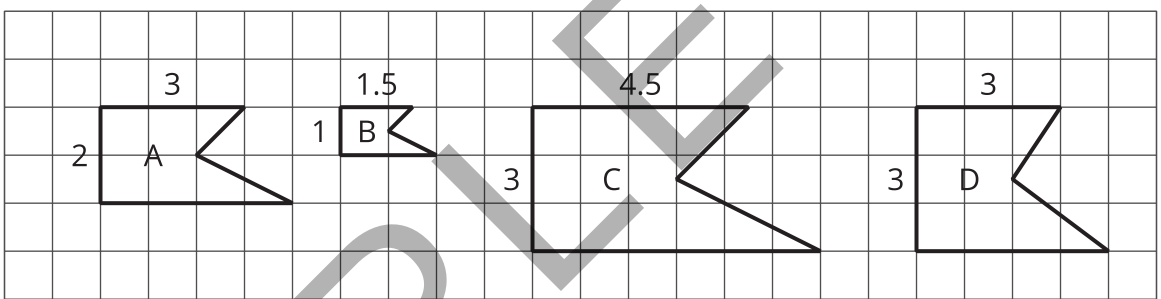
When two different situations can be described by **equivalent ratios**, that means they are alike in some important way.

An example is a recipe. If two people make something to eat or drink, the taste will only be the same as long as the ratios of the ingredients are equivalent. For example, all of the mixtures of water and drink mix in this table taste the same, because the ratios of cups of water to scoops of drink mix are all equivalent ratios.

water (cups)	drink mix (scoops)
3	1
12	4
1.5	0.5

If a mixture were not equivalent to these, for example, if the ratio of cups of water to scoops of drink mix were $6 : 4$, then the mixture would taste different.

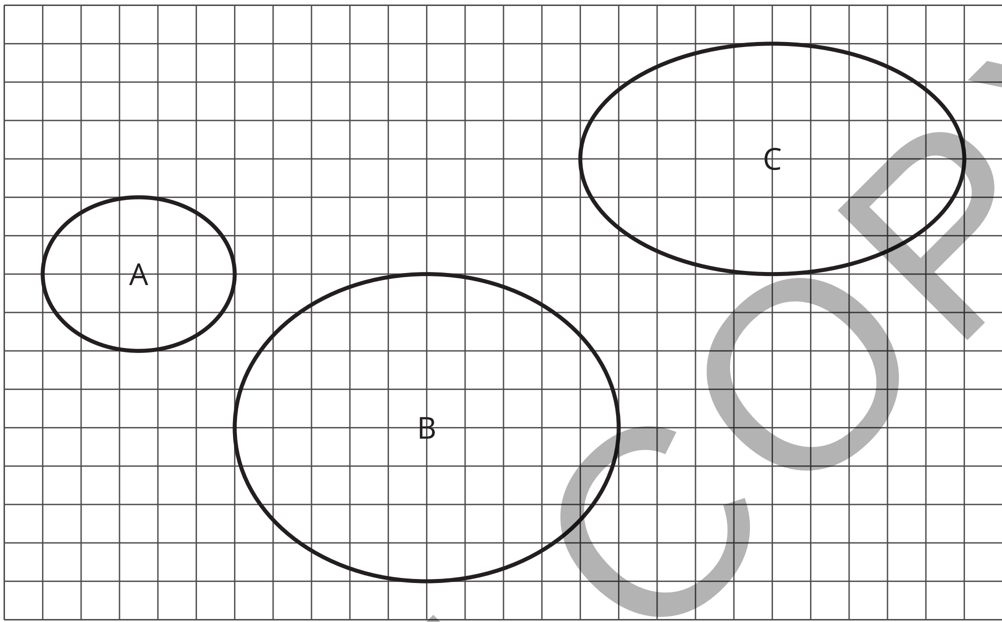
Notice that the ratios of pairs of corresponding side lengths are equivalent in Figures A, B, and C. For example, the ratios of the length of the top side to the length of the left side for Figures A, B, and C are equivalent ratios. Figures A, B, and C are *scaled copies* of each other.
This is the important way in which they are alike.



If a figure has corresponding sides that are not in a ratio equivalent to these, like Figure D, then it's not a scaled copy. In this unit, you will study relationships like these that can be described by a set of equivalent ratios.

Practice Problems

1 Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.



2 In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

3 from Unit 1, Lesson 12

For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

Objects

a. A person

b. A football field (120 yd by $53\frac{1}{3}$ yd)

c. The state of Washington (about 240 mi by 360 mi)

d. The floor plan of a house

e. A rectangular farm (6 mi by 2 mi)

Scales

- 1 in : 1 ft
- 1 cm : 1 m
- 1 : 1,000
- 1 ft : 1 mile
- 1 : 100,000
- 1 mm : 1 km
- 1 : 10,000,000

4 from Unit 1, Lesson 11

Which scale is equivalent to 1 cm to 1 km?

A. 1 to 1000

B. 10,000 to 1

C. 1 to 100,000

D. 100,000 to 1

E. 1 to 1,000,000

5 from an earlier course

a. Find 3 different ratios that are equivalent to $7 : 3$.

b. Explain why these ratios are equivalent.

Unit 2, Lesson 2

Addressing CA CCSSM 7.RP.2, 7.RP.2b; building on 6.RP.3; building towards 7.RP.2b; practicing MP7

Introducing Proportional Relationships with Tables

Let's solve problems involving proportional relationships using tables.

Sec A

2.1 Notice and Wonder: Paper Towels by the Case

Here is a table that shows how many rolls of paper towels a store receives when they order different numbers of cases.

number of cases they order	number of rolls of paper towels
1	12
3	36
5	60
10	120

$\cdot 2$ $\cdot 2$

What do you notice? What do you wonder?

2.2 Feeding a Crowd

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

   a. How many people will 10 cups of rice serve?

   b. How many cups of rice are needed to serve 45 people?

cups of rice	number of people
2	6
3	9
10	
	45

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table.

number of spring rolls	number of people
6	3
30	
40	
	28

2.3 Making Coco Bread

Coco bread is a popular food in Jamaica. To make coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

Complete the table as you answer the questions. Be prepared to explain your reasoning.

1. How many grams of flour do they use with 500 milliliters of coconut milk?

2. How many grams of flour do they use with 235 milliliters of coconut milk?

3. How many milliliters of coconut milk do they use with 450 grams of flour?

4. What is the proportional relationship represented by this table?

coconut milk (millimeters)	flour (grams)
200	360
500	
375	
	450

Sec A

2.4 Quarters and Dimes

4 quarters are equal in value to 10 dimes.

number of quarters	number of dimes
1	
4	10
6	
14	

1. How many dimes equal the value of 6 quarters?

2. How many dimes equal the value of 14 quarters?

3. What value belongs next to the 1 in the table? What does it mean in this context?

Are you ready for more?

Pennies made before 1982 are 95% copper and weigh about 3.11 grams each. (Pennies made after that date are primarily made of zinc). Some people claim that the value of the copper in one of these pennies is greater than the face value of the penny. Find out how much copper is worth right now, and decide if this claim is true.

Lesson 2 Summary

If the ratios between two corresponding quantities are always equivalent, the relationship between the quantities is called a **proportional relationship**.

This table shows different amounts of milk and chocolate syrup. The ingredients in each row, when mixed together, would make a different total amount of chocolate milk, but these mixtures would all taste the same.

Notice that each row in the table shows a ratio of tablespoons of chocolate syrup to cups of milk that is equivalent to $4 : 1$.

About the relationship between these quantities, we could say:

tablespoons of chocolate syrup	cups of milk
4	1
6	$1\frac{1}{2}$
8	2
$\frac{1}{2}$	$\frac{1}{8}$
12	3
1	$\frac{1}{4}$

- The relationship between the amount of chocolate syrup and the amount of milk is proportional.
- The table represents a proportional relationship between the amount of chocolate syrup and amount of milk.
- The amount of milk is proportional to the amount of chocolate syrup.

We could multiply any value in the chocolate syrup column by $\frac{1}{4}$ to get the value in the milk column. We might call $\frac{1}{4}$ a *unit rate*, because $\frac{1}{4}$ cup of milk is needed for 1 tablespoon of chocolate syrup. We also say that $\frac{1}{4}$ is the **constant of proportionality** for this relationship. It tells us how many cups of milk we would need to mix with 1 tablespoon of chocolate syrup.

Glossary

- constant of proportionality
- proportional relationship

Practice Problems

1 When Han makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. Here is a table that shows how to make batches of different sizes. Use the information in the table to complete the statements. Some terms are used more than once.

cups of milk	tablespoons of chocolate syrup
2	3
8	12
1	$\frac{3}{2}$
10	15

a. The table shows a proportional relationship between ____________ and ____________.

b. The scale factor shown is ____________.

c. The constant of proportionality for this relationship is____________.

d. The units for the constant of proportionality are ____________ per ____________.

Bank of Terms: tablespoons of chocolate syrup, 4, cups of milk, cup of milk, $\frac{3}{2}$

2 A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

a. How many cups of red paint should be added to 1 cup of white paint?

cups of white paint	cups of red paint
1	
7	3

b. What is the constant of proportionality?

3 from Unit 1, Lesson 12

A map of a rectangular park has a length of 4 inches and a width of 6 inches. It uses a scale of 1 inch for every 30 miles.

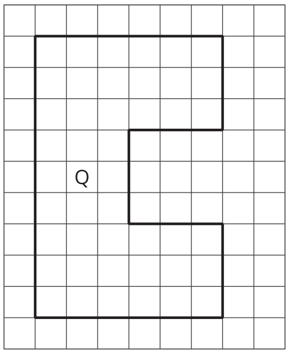
a. What is the actual area of the park? Show how you know.

b. The map needs to be reproduced at a different scale so that it has an area of 6 square inches and can fit in a brochure. At what scale should the map be reproduced so that it fits on the brochure? Show your reasoning.

4 from Unit 1, Lesson 6

Noah drew a scaled copy of Polygon P and labeled it Polygon Q.

If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain or show how you know.



5 from an earlier course

Select **all** the ratios that are equivalent to each other.

A. $4 : 7$

B. $8 : 15$

C. $16 : 28$

D. $2 : 3$

E. $20 : 35$

Unit 2, Lesson 3

Addressing CA CCSSM 7.RP.2a, 7.RP.2b; building on 5.MD.1; practicing MP2 and MP7

More about Constant of Proportionality

Let's solve more problems involving proportional relationships using tables.

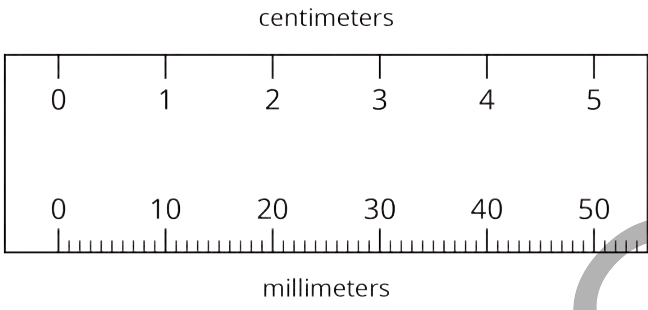
3.1 Math Talk: Division

Find the value of each expression mentally.

- $645 \div 10$
- $645 \div 100$
- $645 \div 50$
- $64.5 \div 50$

3.2 Centimeters and Millimeters

There is a proportional relationship between any length measured in centimeters and the same length measured in millimeters.



There are two ways of thinking about this proportional relationship.

1. If the length of something in centimeters is known, its length in millimeters can be calculated.

   a. Complete the table.

   b. What is the constant of proportionality?

length (cm)	length (mm)
9	
12.5	
50	
88.49	

2. If the length of something in millimeters is known, its length in centimeters can be calculated.

   a. Complete the table.

   b. What is the constant of proportionality?

length (mm)	length (cm)
70	
245	
4	
699.1	

Sec A

3. How are these two constants of proportionality related to each other?

4. Complete each sentence:

   a. To convert from centimeters to millimeters, the value in centimeters is multiplied by ________.

   b. To convert from millimeters to centimeters, the value in millimeters is divided by ________, or multiplied by ________.

Are you ready for more?

1. How many square millimeters are there in a square centimeter?

2. How do you convert square centimeters to square millimeters? How do you convert the other way?

3.3 Pittsburgh to Phoenix

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.

segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330 miles	

1. What is the distance between Saint Louis and Albuquerque?

2. How many minutes did it take to fly between Albuquerque and Phoenix?

3. What is the proportional relationship represented by this table?

4. Diego says the constant of proportionality is 550. Andre says the constant of proportionality is $9\frac{1}{6}$. Do you agree with either of them? Explain your reasoning.

Lesson 3 Summary

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled.

The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

We can multiply any number in the first column by $\frac{2}{3}$ to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is $\frac{2}{3}$. This means that the bug's *pace* is $\frac{2}{3}$ seconds per centimeter.

distance traveled (cm)	elapsed time (sec)
$\frac{3}{2}$	1
1	$\frac{2}{3}$
3	2
10	$\frac{20}{3}$

$\cdot \frac{2}{3}$

This table represents the same situation, except the columns are switched.

We can multiply any number in the first column by $\frac{3}{2}$ to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is $\frac{3}{2}$. This means that the bug's *speed* is $\frac{3}{2}$ centimeters per second.

elapsed time (sec)	distance traveled (cm)
1	$\frac{3}{2}$
$\frac{2}{3}$	1
2	3
$\frac{20}{3}$	10

$\cdot \frac{3}{2}$

Notice that $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$. When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column by to get the values in the second.

Practice Problems

1 Noah is running a portion of a marathon at a constant speed of 6 miles per hour.

Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.

time in hours	miles traveled at 6 miles per hour
1	
$\frac{1}{2}$	
$1\frac{1}{3}$	
	$1\frac{1}{2}$
	9
	$4\frac{1}{2}$

2 One kilometer is 1000 meters.

a. Complete the tables. What is the interpretation of the constant of proportionality in each case?

meters	kilometers
1,000	1
250	
12	
1	

The constant of proportionality tells us that:

kilometers	meters
1	1,000
5	
20	
0.3	

The constant of proportionality tells us that:

b. What is the relationship between the two constants of proportionality?

Sec A

3 Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12. Lin says it is $\frac{1}{12}$. Do you agree with either of them? Explain your reasoning.

4 from Unit 1, Lesson 12

The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

5 from Unit 1, Lesson 11

Which of these scales is equivalent to the scale 1 cm to 5 km? Select **all** that apply.

A. 3 cm to 15 km

B. 1 mm to 150 km

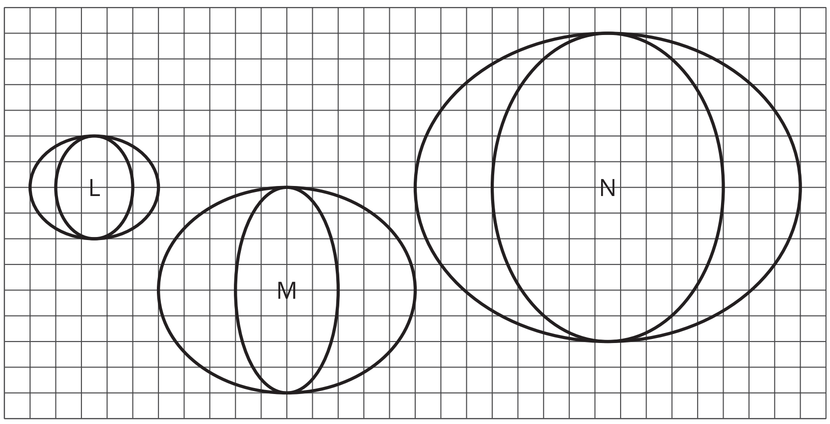
C. 5 cm to 1 km

D. 5 mm to 2.5 km

E. 1 mm to 500 m

6 from Unit 2, Lesson 1

Which one of these pictures is not like the others? Explain what makes it different using ratios.



Addressing CA CCSSM 7.RP.2, 7.RP.2c; building on 5.NBT.7; practicing MP2, MP6, and MP8

Proportional Relationships and Equations

Let's write equations describing proportional relationships.

4.1 Which Three Go Together: Expressions

Which three go together? Why do they go together?

A

$$5 \cdot 2$$

B

$$4 + ? = 20$$

C

$$x + 5$$

D

$$5x$$

4.2 Feeding a Crowd, Revisited

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

   a. How many people will 1 cup of rice serve?

   b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

   c. How many people will x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

   a. How many people will 1 spring roll serve?

   b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

   c. How many people will n spring rolls serve?

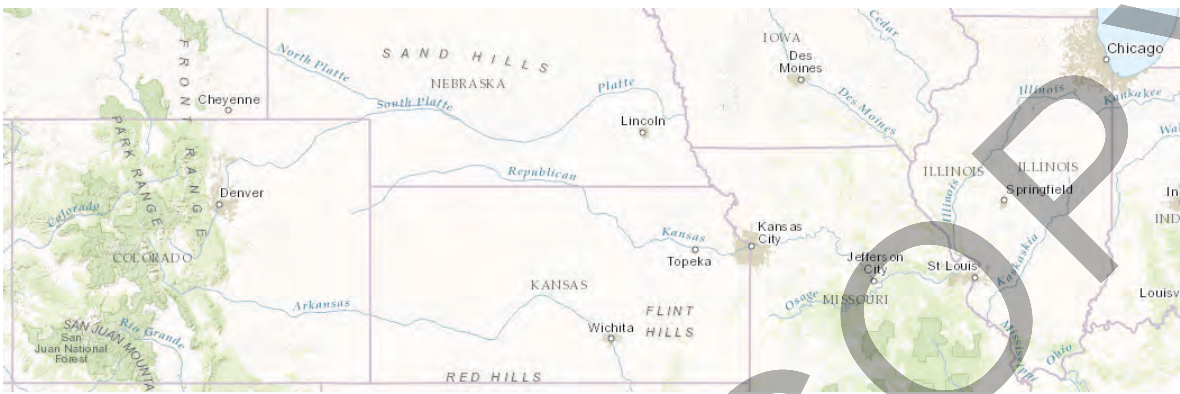
number of spring rolls	number of people
1	
6	3
10	
16	
25	
n	

3. How was completing the table about spring rolls different from completing the table about rice? How was it the same?

Sec B

4.3 Denver to Chicago

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.



1. Complete the table.

time (hours)	distance (miles)
1	
1.5	915
2	
2.5	
t	

2. How far does the plane fly in 1 hour?
3. How far would the plane fly in t hours at this speed?
4. If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d.
5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

Sec B

Sec B

Are you ready for more?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)

2. Imagine there are two twins. One twin leaves Earth on a spaceship and travels to a planet near Proxima Centauri. The spaceship travels at 90% of the speed of light. The other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the twin on the spaceship? Consider researching "The Twin Paradox" to learn more.)

4.4 Revisiting Coco Bread

To bake coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

1. Complete the table.
2. Use f to represent the grams of flour needed for c milliliters of coconut milk. Write an equation that relates f and c.
3. How much flour is needed for 680 milliliters of coconut milk? 945 milliliters? Explain or show your reasoning.

coconut milk (milliliters)	flour (grams)
100	
200	360
450	
c	

Lesson 4 Summary

In this lesson, we wrote equations to represent proportional relationships described in words and shown in tables.

This table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

Note that "parts" can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

The last row in the table shows that if we know the amount of red paint, r, we can always multiply it by 4 to find the amount of blue paint needed to make Venusian Sunset. If b is the amount of blue paint, we can say this more succinctly with the equation $b = 4r$. So, the amount of blue paint is proportional to the amount of red paint, and the constant of proportionality is 4.

We can also look at this relationship the other way around.

If we know the amount of blue paint, b, we can always multiply it by $\frac{1}{4}$ to find the amount of red paint, r, needed to make Venusian Sunset. So, the equation $r = \frac{1}{4}b$ also represents the relationship. The amount of red paint is proportional to the amount of blue paint, and the constant of proportionality $\frac{1}{4}$.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

In general, when y is proportional to x, we can always multiply x by the same number k—the constant of proportionality—to get y. We can write this much more succinctly with the equation $y = kx$.

Practice Problems

1 A ceiling is made up of tiles. Every square meter of the ceiling requires 10.75 tiles. Fill in the table with the missing values.

square meters of ceiling	number of tiles
1	
10	
	100
a	

2 On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d, to the number of hours flying, t, is $t = \frac{1}{500}d$. How long will it take the airplane to travel 800 miles?

3 Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality:

Equation: $P =$

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality:

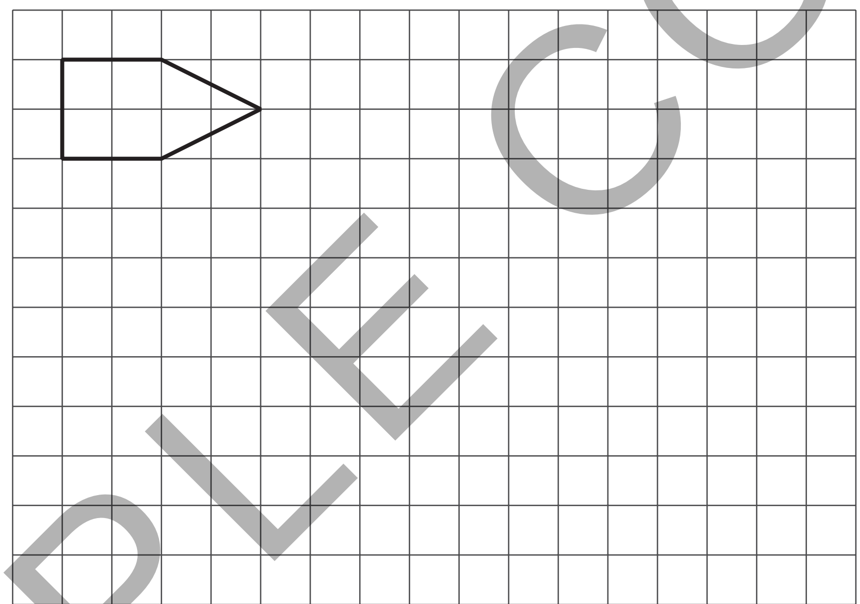
Equation: $C =$

4 from Unit 1, Lesson 11

A map of Colorado says that the scale is 1 inch to 20 miles or 1 to 1,267,200. Are these two ways of reporting the scale the same? Explain your reasoning.

5 from Unit 1, Lesson 3

Here is a polygon on a grid.



a. Draw a scaled copy of the polygon using a scale factor 3. Label the copy A.

b. Draw a scaled copy of the polygon with a scale factor $\frac{1}{2}$. Label it B.

c. Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

Addressing CA CCSSM 7.RP.2; building on 5.OA.3; practicing MP2, MP6, and MP8

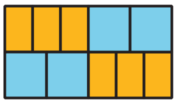
Two Equations for Each Relationship

Let's investigate equations that represent proportional relationships.

5.1 Which Three Go Together: Tiles

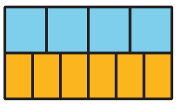
Which three go together? Why do they go together?

A

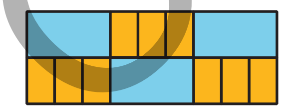


B

C



D



5.2 Meters and Centimeters

There are 100 centimeters (cm) in every meter (m).

length (m)	length (cm)
1	100
0.94	
1.67	
57.24	
x	

length (cm)	length (m)
100	1
250	
78.2	
123.9	
y	

1. Complete the tables.
2. For each table, find the constant of proportionality.
3. Describe the relationship between these two constants of proportionality.
4. For each table, write an equation for the proportional relationship. Let x represent a length measured in meters and y represent the same length measured in centimeters.

Are you ready for more?

1. How many cubic centimeters are there in 1 cubic meter?
2. How do you convert cubic centimeters to cubic meters?
3. How do you convert the other way?

5.3 Filling a Water Cooler

It took Priya 5 minutes to fill a cooler with 8 gallons of water from a faucet that was flowing at a steady rate. Let w be the number of gallons of water in the cooler after t minutes.

1. Which of the following equations represent the relationship between w and t? Select **all** that apply.

   A. $w = 1.6t$

   B. $w = 0.625t$

   C. $t = 1.6w$

   D. $t = 0.625w$

2. What does 1.6 tell you about the situation?

3. What does 0.625 tell you about the situation?

4. Priya changed the rate at which water flowed through the faucet. Write an equation that represents the relationship of w and t when it takes 3 minutes to fill the cooler with 1 gallon of water.

5. Was the cooler filling faster before or after Priya changed the rate of water flow? Explain how you know.

5.4 Feeding Shrimp

At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day.

1. How much food does a shrimp get fed in 1 day?

2. Complete the table to show how many grams of food the shrimp is fed over different numbers of days.

number of days	grams of food
1	
7	
30	

3. What is the constant of proportionality? What does it tell us about the situation?

4. If the columns in the table were switched, what would be the constant of proportionality? Explain your reasoning

5. Use d for number of days and f for amount of food in grams that a shrimp is fed to write *two* equations that represent the relationship between d and f.

6. At this rate, how much food does a shrimp get fed in 75 days?

7. At this rate, how many days would 75 grams of shrimp food last? Explain or show your reasoning.

Lesson 5 Summary

If Kiran rode his bike at a constant 10 miles per hour, his distance in miles, d, is proportional to the number of hours, t, that he rode. We can write the equation

$$d = 10t$$

to represent the proportional relationship. With this equation, it is easy to find the distance Kiran rode when we know how long it took, because we can just multiply the time by 10.

We can rewrite the equation:

$$d = 10t$$
$$\left(\frac{1}{10}\right)d = t$$
$$t = \left(\frac{1}{10}\right)d$$

This version of the equation tells us that the amount of time Kiran rode is proportional to the distance he traveled, and the constant of proportionality is $\frac{1}{10}$. That form of the equation is easier to use when we know his distance and want to find how long it took, because we can just multiply the distance by $\frac{1}{10}$.

When two quantities x and y are in a proportional relationship, we can write the equation

$$y = kx$$

and say, "y is proportional to x." In this case, the number k is the corresponding constant of proportionality. We can also write the equation

$$x = \frac{1}{k}y$$

and say, "x is proportional to y." In this case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each equation can be useful, depending on the information we have and the quantity we are trying to figure out.

Practice Problems

1 The table represents the relationship between a length measured in meters and the same length measured in kilometers.

a. Complete the table.

b. Write an equation for converting the number of meters to kilometers. Use x for the number of meters and y for the number of kilometers.

meters	kilometers
1,000	1
3,500	
500	
75	
1	
x	

2 Concrete building blocks weigh 28 pounds each. Using b for the number of concrete blocks and w for the weight, write two equations that relate the two variables. One equation should begin with $w =$ and the other should begin with $b =$.

3 A store sells rope by the meter. The equation $p = 0.8L$ represents the price, p, in dollars of a piece of nylon rope that is L meters long.

a. How much does the nylon rope cost per meter?

b. How long is a piece of nylon rope that costs $1.00?

4 from Unit 2, Lesson 4

The table represents a proportional relationship. Find the constant of proportionality and write an equation to represent the relationship.

a	y
2	$\frac{2}{3}$
3	1
10	$\frac{10}{3}$
12	4

Constant of proportionality: ________

Equation: $y =$

5 from Unit 1, Lesson 8

On a map of Chicago, 1 cm represents 100 m. Select **all** statements that express the same scale.

A. 5 cm on the map represents 50 m in Chicago.

B. 1 mm on the map represents 10 m in Chicago.

C. 1 km in Chicago is represented by 10 cm on the map.

D. 100 cm in Chicago is represented by 1 m on the map.

Unit 2, Lesson 6

Addressing CA CCSSM 7.RP.2, 7.RP.2c; building on 5.NBT.7, 6.RP.2; practicing MP2, MP7, and MP8

Writing Equations to Represent Relationships

Let’s use equations to solve problems involving proportional relationships.

6.1 Math Talk: Products with Decimal Points

Find the value of each expression mentally.

- $32 \cdot (1.5)$
- $32 \cdot (0.15)$
- $3{,}200 \cdot (0.15)$
- $3{,}200 \cdot (0.03)$

6.2 Bottle Deposits

Answer the following questions. Be prepared to explain your reasoning.

In Iowa, collection centers pay 5¢ per bottle that is returned.

1.
   a. How much would 30 bottles be worth?

   b. How much would 250 bottles be worth?

   c. How much would 860 bottles be worth?

2.
   a. How many bottles would it take to earn $100?

   b. How many bottles would it take to earn $2,750?

3. Write an equation that relates the number of bottles to the amount of money received when the bottles are returned. What do your variables represent?

6.3 Recycling

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of $0.14.

1. A family threw away 2.4 kg of aluminum cans in a month.
   a. How many cans did they throw away? Explain or show your reasoning.

   b. What would be the dollar value if they recycled those same cans? Explain or show your reasoning.

2. Write an equation to represent the relationship between each pair of quantities:
   a. the number of cans c and their weight w, in kilograms

   b. the number of cans c and their recycled value r, in dollars

   c. the weight of cans w and their recycled value r

Are you ready for more?

The U.S. Environmental Protection Agency (EPA) estimates that in 2018, the average amount of garbage produced in the United States was 4.9 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.

Lesson 6 Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20,310 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

$$f = 5{,}280m$$

where f represents a distance measured in feet and m represents the same distance measured in miles. Since we know Denali is 20,310 feet above sea level, we can write

$$20{,}310 = 5{,}280m$$

Solving this equation for m gives $m = \frac{20{,}310}{5{,}280} \approx 3.85$, so we can say that Denali is approximately 3.85 miles above sea level.

Practice Problems

1 A car is traveling on a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.

a. What does the 65 tell us in this situation?

b. How many miles does the car travel in 1.5 hours?

c. How long does it take the car to travel 26 miles at this speed?

2 Elena has some bottles of water that each holds 17 fluid ounces.

a. Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.

b. How much water is in 51 bottles?

c. How many bottles does it take to hold 51 fluid ounces of water?

3 from Unit 2, Lesson 5

There are about 1.61 kilometers in 1 mile. Use x to represent a distance measured in kilometers and y to represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

4 from Unit 2, Lesson 2

In Canadian coins, 16 quarters is equal in value to 2 toonies.

a. Complete the table.

b. What does the value in the right column that is next to 1 in the left column mean in this situation?

number of quarters	number of toonies
1	
16	2
20	
24	

Sec B

5 from Unit 2, Lesson 2

Each table represents a proportional relationship. For each table:

a. Fill in the unknown values.

b. Draw a circle around the constant of proportionality.

x	y
2	10
	15
7	
1	

a	b
12	3
20	
	10
1	

m	n
5	3
10	
	18
1	

6 from Unit 1, Lesson 4

Describe some things you could observe about two polygons that would help you decide that they were not scaled copies.

Unit 2, Lesson 7

Addressing CA CCSSM 7.RP.2, 7.RP.2a, 7.RP.2c; building on 6.RP.3; building towards 7.RP.1; practicing MP1, MP4, and MP6

Comparing Relationships with Tables

Let's explore how proportional relationships are different from other relationships.

7.1 Adjusting a Recipe

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

Invent four new versions of this lemonade recipe:

1. One that would make more lemonade but taste the same as the original recipe.

2. One that would make less lemonade but taste the same as the original recipe.

3. One that would have a stronger lemon taste than the original recipe.

4. One that would have a weaker lemon taste than the original recipe.

7.2 Visiting the State Park

Entrance to a state park costs \$6 per vehicle, plus \$2 per person in the vehicle.

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle	total entrance cost in dollars
2	
4	
10	

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?

3. How might you determine the entrance cost for a bus with 50 people?

4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

Are you ready for more?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

7.3 Running Laps

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	4	
4	9	
6	15	
8	23	

Clare's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	5	
4	10	
6	15	
8	20	

1. Is Han running at a constant pace? Is Clare? How do you know?

2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

Sec C

Lesson 7 Summary

Here are the prices for some smoothies at two different smoothie shops:

Smoothie Shop A

smoothie size (fl oz)	price ($)	dollars per ounce
8	6	0.75
12	9	0.75
16	12	0.75
s	$0.75s$	0.75

Smoothie Shop B

smoothie size (fl oz)	price ($)	dollars per ounce
8	6	0.75
12	8	0.67
16	10	0.625
s	???	???

Sec C

For Smoothie Shop A, smoothies cost $0.75 per ounce no matter which size we buy. There could be a proportional relationship between smoothie size and the price of the smoothie. An equation representing this relationship is

$$p = 0.75s$$

, where s represents size in ounces and p represents price in dollars. (The relationship could still not be proportional, if there were a different size on the menu that did not have the same price per ounce.)

For Smoothie Shop B, the cost per ounce is different for each size. Here the relationship between smoothie size and price is definitely *not* proportional.

In general, two quantities in a proportional relationship will always have the same quotient. When we see some values for two related quantities in a table and we get the same quotient when we divide them, that means they might be in a proportional relationship—but if we can't see all of the possible pairs, we can't be completely sure. However, if we know the relationship can be represented by an equation of the form $y = kx$, then we are sure it is proportional.

Practice Problems

1 Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

a. How loud a sound is

distance from listener (ft)	sound level (dB)
5	85
25	79
35	73
40	67

b. The cost of fountain drinks at Hot Dog Hut.

volume (fl oz)	cost ($)
16	1.49
20	1.59
30	1.89

2 A taxi service charges \$1.00 for the first $\frac{1}{10}$ mile then \$0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance traveled and price of the trip is a proportional relationship.

distance traveled (mi)	price (\$)
$\frac{9}{10}$	
2	
$3\frac{1}{10}$	
10	

Sec C

3 A rabbit and turtle are in a race. Is the relationship between distance traveled and time proportional for either one? If so, write an equation that represents the relationship.

Turtle's run:

distance (meters)	time (minutes)
108	2
405	7.5
540	10
1,768.5	32.75

Rabbit's run:

distance (meters)	time (minutes)
800	1
900	5
1,107.5	20
1,524	32.5

4 from Unit 2, Lesson 2

For each table, answer: What is the constant of proportionality?

a	b
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

a	b
3	360
5	600
8	960
12	1440

a	b
75	3
200	8
1525	61
10	0.4

a	b
4	10
6	15
22	55
3	$7\frac{1}{2}$

5 from Unit 1, Lesson 4

Kiran and Mai are standing at one corner of a rectangular field of grass looking at the diagonally opposite corner. Kiran says that if the field were twice as long and twice as wide, then it would be twice the distance to the far corner. Mai says that it would be more than twice as far, since the diagonal is even longer than the side lengths. Do you agree with either of them?

Unit 2, Lesson 8

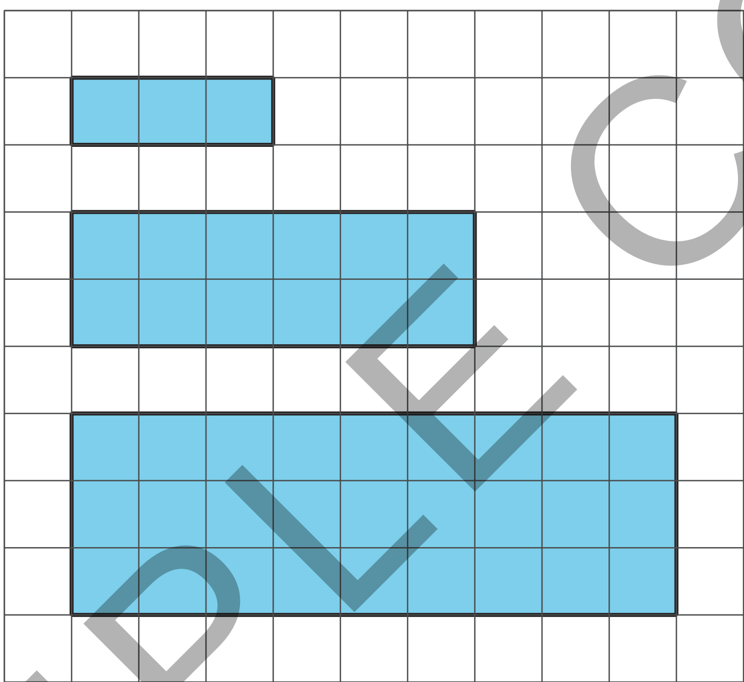
Addressing CA CCSSM 7.RP.1, 7.RP.2; building on 4.OA.5, 6.EE.2; building towards 7.G.6; practicing MP6 and MP7

Comparing Relationships with Equations

Let's develop methods for deciding if a relationship is proportional.

8.1 Notice and Wonder: Patterns with Rectangles

What do you notice? What do you wonder?



8.2 More Conversions

The other day you worked with converting meters, centimeters, and millimeters. Here are some more unit conversions.

1. Use the equation $F = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius, to complete the table.

temperature $(^\circ\text{C})$	temperature $(^\circ\text{F})$
20	
4	
175	

2. Use the equation $c = 2.54n$, where c represents the length in centimeters and n represents the length in inches, to complete the table.

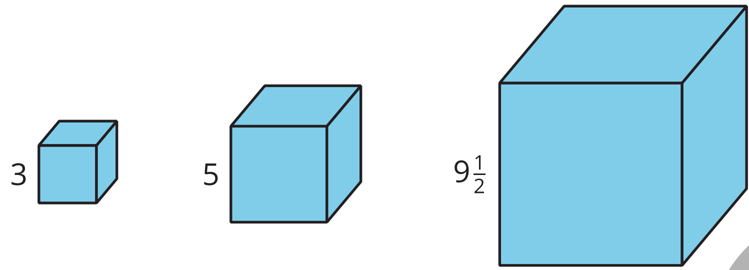
length (in)	length (cm)
10	
8	
$3\frac{1}{2}$	

3. Are these proportional relationships? Explain why or why not.

Sec C

8.3 Total Edge Length, Surface Area, and Volume

Here are some cubes with different side lengths. Complete each table. Be prepared to explain your reasoning.



1. How long is the total edge length of each cube?

side length	total edge length
3	
5	
$9\frac{1}{2}$	
s	

2. What is the surface area of each cube?

side length	surface area
3	
5	
$9\frac{1}{2}$	
s	

3. What is the volume of each cube?

side length	volume
3	
5	
$9\frac{1}{2}$	
s	

4. Which of these relationships is proportional? Explain how you know.

5. Write equations for the total edge length E, total surface area A, and volume V of a cube with side length s.

Are you ready for more?

1. A rectangular solid has a square base with side length ℓ, height 8, and volume V. Is the relationship between ℓ and V a proportional relationship?

2. A different rectangular solid has length ℓ, width 10, height 5, and volume V. Is the relationship between ℓ and V a proportional relationship?

3. Why is the relationship between the side length and the volume proportional in one situation and not the other?

8.4 All Kinds of Equations

Here are six different equations.

$y = 4 + x$

$y = 4x$

$y = \frac{4}{x}$

$y = \frac{x}{4}$

$y = 4^x$

$y = x^4$

$y = 4 + x$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = 4x$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = \frac{4}{x}$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = \frac{x}{4}$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = 4^x$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$y = x^4$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

1. Predict which of these equations represent a proportional relationship.

2. Complete each table using the equation that represents the relationship.
3. Do these results change your answer to the first question? Explain your reasoning.

4. What do the equations of the proportional relationships have in common?

Lesson 8 Summary

If two quantities are in a proportional relationship, then their quotient is always the same. This table represents different values of a and b, two quantities that are in a proportional relationship.

a	b	$\frac{b}{a}$
20	100	5
3	15	5
11	55	5
1	5	5

Notice that the quotient of b and a is always 5. To write this as an equation, we could say $\frac{b}{a} = 5$. If this is true, then $b = 5a$. (This doesn't work if $a = 0$, but it works otherwise.)

If quantity y is proportional to quantity x, we will always see that $\frac{y}{x}$ has a constant value. This value is the constant of proportionality, which we often refer to as k. We can represent this relationship with the equation $\frac{y}{x} = k$ (as long as x is not 0) or $y = kx$.

Note that if an equation cannot be written in this form, then it does not represent a proportional relationship.

Practice Problems

1 The relationship between a distance in yards (y) and the same distance in miles (m) is described by the equation $y = 1{,}760m$.

a. Find measurements in yards and miles for distances by completing the table.

distance measured in miles	distance measured in yards
1	
5	
	3,520
	17,600

b. Is there a proportional relationship between a measurement in yards and a measurement in miles for the same distance? Explain why or why not.

2 Decide whether or not each equation represents a proportional relationship.

a. The remaining length (L) of 120-inch rope after x inches have been cut off: $120 - x = L$

b. The total cost (t) after 8% sales tax is added to an item's price (p): $1.08p = t$

c. The number of marbles each sister gets (x) when m marbles are shared equally among four sisters: $x = \frac{m}{4}$

d. The volume (V) of a rectangular prism whose height is 12 cm and base is a square with side lengths s cm: $V = 12s^2$

3

a. Use the equation $y = \frac{5}{2}x$ to complete the table.
Is y proportional to x? Explain why or why not.

x	y
2	
3	
6	

b. Use the equation $y = 3.2x + 5$ to complete the table.
Is y proportional to x? Explain why or why not.

x	y
1	
2	
4	

Sec C

4

from Unit 2, Lesson 6

To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has 1,500 bytes of information. An equation relating packets to bytes of information is given by $b = 1{,}500p$, where p represents the number of packets and b represents the number of bytes of information.

a. How many packets would be needed to transmit 30,000 bytes of information?

b. How much information could be transmitted in 30,000 packets?

c. Each byte contains 8 bits of information. Write an equation to represent the relationship between the number of packets and the number of bits.

Unit 2, Lesson 9

Addressing CA CCSSM 7.RP.2; practicing MP1 and MP6

Solving Problems about Proportional Relationships

Let’s solve problems about proportional relationships.

9.1 What Do You Want to Know?

A person is running a distance race at a constant rate. What time will they finish the race?

What specific information do you need to be able to solve the problem?

Sec C

9.2 Info Gap: Biking and Rain

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me _____?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know _____ because . . . .” Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know _____?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

9.3 Moderating Comments

A company is hiring people to read through all the comments posted on their website to make sure they are appropriate. Four people applied for the job and were given one day to show how quickly they could check comments.

- Person 1 worked for 210 minutes and checked a total of 50,000 comments.
- Person 2 worked for 200 minutes and checked 1,325 comments every 5 minutes.
- Person 3 worked for 120 minutes, at a rate represented by $c = 331t$, where c is the number of comments checked and t is the time in minutes.
- Person 4 worked for 150 minutes, at a rate represented by $t = \left(\frac{3}{800}\right) c$.

1. Order the people from greatest to least in terms of total number of comments checked.

2. Order the people from greatest to least in terms of how fast they checked the comments.

Are you ready for more?

1. Write equations for each job applicant that allow you to easily decide who is working the fastest.

2. Make a table that allows you to easily compare how many comments the four job applicants can check.

Sec C

Lesson 9 Summary

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between quantities of interest.

- When a bird is flying at a constant speed, then there is a proportional relationship between the flying time and distance flown.
- If water is filling a tub at a constant rate, then there is a proportional relationship between the amount of water in the tub and the time the tub has been filling up.
- If an aardvark is eating termites at a constant rate, then there is a proportional relationship between the number of termites the aardvark has eaten and the time since it started eating.

Sometimes we are presented with a situation, and it is not so clear whether a proportional relationship is a good model. How can we decide if a proportional relationship is a good representation of a particular situation?

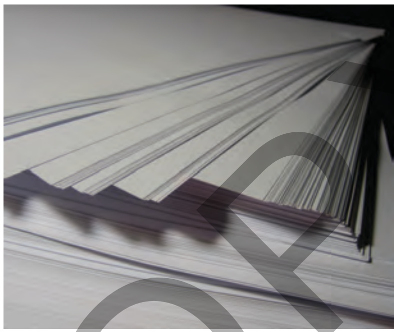
- If you aren't sure where to start, look at the quotients of corresponding values. If they are not always the same, then the relationship is definitely not a proportional relationship.
- If you can see that there is a single value that we always multiply one quantity by to get the other quantity, it is definitely a proportional relationship.

After establishing that it is a proportional relationship, setting up an equation is often the most efficient way to solve problems related to the situation.

Practice Problems

1 For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

a. The weight of a stack of standard 8.5x11 copy paper vs. the number of sheets of paper.



b. The weight of a stack of different-sized books vs. the number of books in the stack.

Sec C

2 Every package of a certain toy also includes 2 batteries.

a. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.

b. Use t for the number of toys and b for the number of batteries to write two equations relating the two variables.

$b =$ $t =$

3 Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

a. Find their ages in different years by filling in the table.

b. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

Lin's age	her brother's age
5	2
6	
15	
	25

Sec C

4 from Unit 2, Lesson 8

A student argues that $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because we need to *multiply* one variable by a constant to get the other one, not *divide* by a constant. Do you agree or disagree with this student?

5 from Unit 1, Lesson 3

Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select **all** of the following that are side lengths of Quadrilateral B.

A. 5

B. 6

C. 7

D. 8

E. 9

Unit 2, Lesson 10

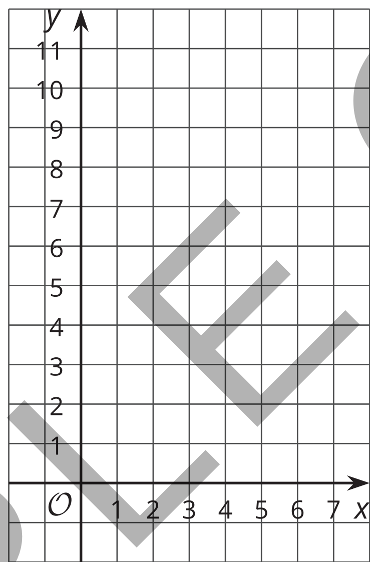
Addressing CA CCSSM 7.RP.2, 7.RP.2a; building on 5.G.1, 6.NS.8; building towards 7.RP.2a; practicing MP2, MP3, and MP7

Introducing Graphs of Proportional Relationships

Let's see how graphs of proportional relationships differ from graphs of other relationships.

10.1 Notice These Points

1. Plot the points $(0, 10), (1, 8), (2, 6), (3, 4), (4, 2)$.



2. What do you notice about the graph?

Sec D

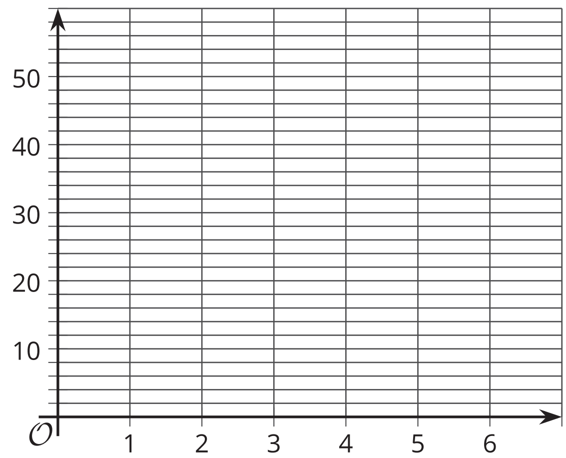
10.2 T-shirts for Sale

Some T-shirts cost $8 each.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.
   a. What does x represent?
   b. What does y represent?
   c. Is there a proportional relationship between x and y?

2. Plot the pairs in the table on the coordinate plane.



3. What do you notice about the graph?

10.3 Card Sort: Tables and Graphs

Your teacher will give you a set of cards that show representations of relationships.

1. Sort the cards into categories of your choosing. Be prepared to describe your categories.

   Pause for a whole-class discussion.

2. Take turns with your partner to match a table with a graph.

   a. For each match that you find, explain to your partner how you know it's a match.

   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

3. Which of the relationships are proportional?

4. What do you notice about the graphs of proportional relationships? Do you think this will hold true for all graphs of proportional relationships?

Sec D

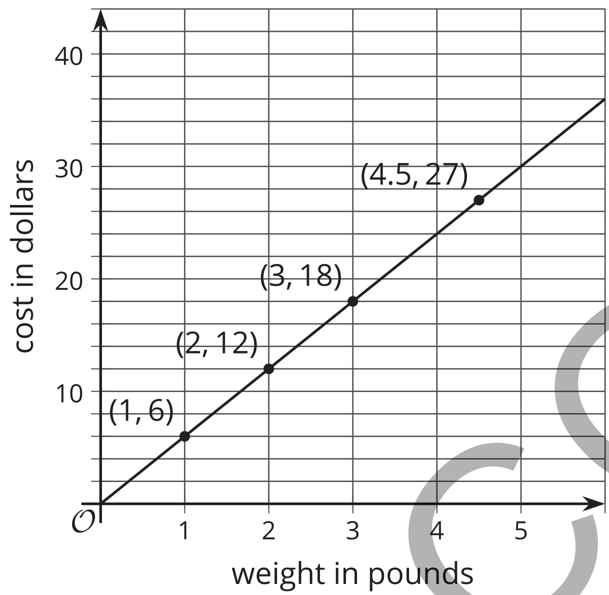
Are you ready for more?

1. All the graphs in this activity show points where both coordinates are positive. Would it make sense for any of them to have one or more coordinates that are negative?

2. The equation of a proportional relationship is of the form $y = kx$, where k is a positive number, and the graph is a line through $(0, 0)$. What would the graph look like if k were a negative number?

Lesson 10 Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, “Blueberries cost \$6 per pound.”



Different points on the graph tell us, for example, that 2 pounds of blueberries cost \$12, and 4.5 pounds of blueberries cost \$27.

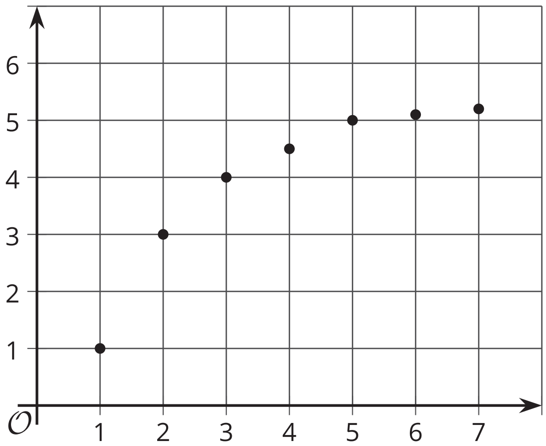
Sometimes it makes sense to connect the points with a line, and sometimes it doesn’t. We could buy, for example, 4.5 pounds of blueberries or 1.875 pounds of blueberries, so all the points in between the whole numbers make sense in the situation, so any point on the line is meaningful.

If the graph represented the cost for different *numbers of sandwiches* (instead of pounds of blueberries), it might not make sense to connect the points with a line, because it is often not possible to buy 4.5 sandwiches or 1.875 sandwiches. Even if only points make sense in the situation, though, sometimes we connect them with a line anyway to make the relationship easier to see.

Graphs that represent proportional relationships all have a few things in common:

- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the **origin**, $(0, 0)$.

Here are some graphs that do *not* represent proportional relationships:



These points do not lie on a line.

This is a line, but it doesn't go through the origin.

Glossary

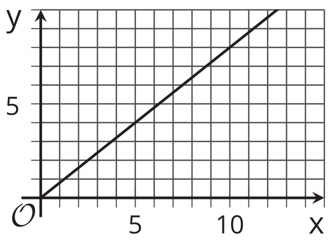
- origin

Sec D

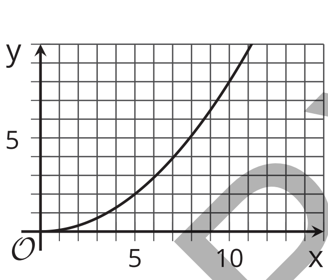
Practice Problems

1 Which graphs could represent a proportional relationship?

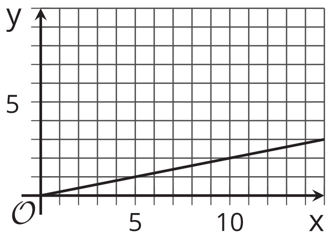
A



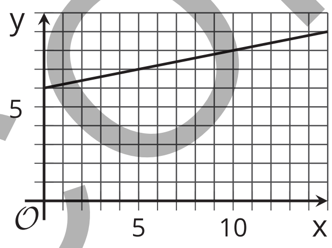
B



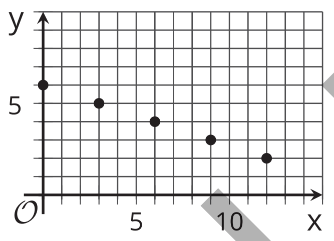
C



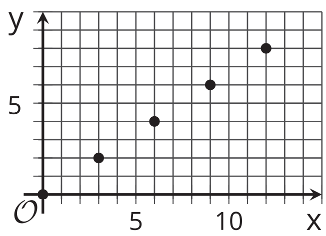
D



E



F



A. A

B. B

C. C

D. D

E. E

F. F

Sec D

2 A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every 1 cup of water.

a. Use the table to answer these questions.

i. What does x represent?

ii. What does y represent?

iii. Is there a proportional relationship between x and y?

x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

b. Plot the pairs in the table in a coordinate plane.

3 from Unit 2, Lesson 9

Select **all** the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in meters between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

A. $y = 3x$

B. After 4 minutes, the turtle has walked 12 feet away from the rock.

C. The turtle walks for a bit, then stops for a minute before walking again.

D. The turtle walks away from the rock at a constant rate.

E. The turtle starts out walking slowly and speeds up as it gets farther away from the rock.

4 from Unit 2, Lesson 7

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The sizes a photo can be printed

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

b. The distance from which a lighthouse is visible

height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

Sec D

Unit 2, Lesson 11

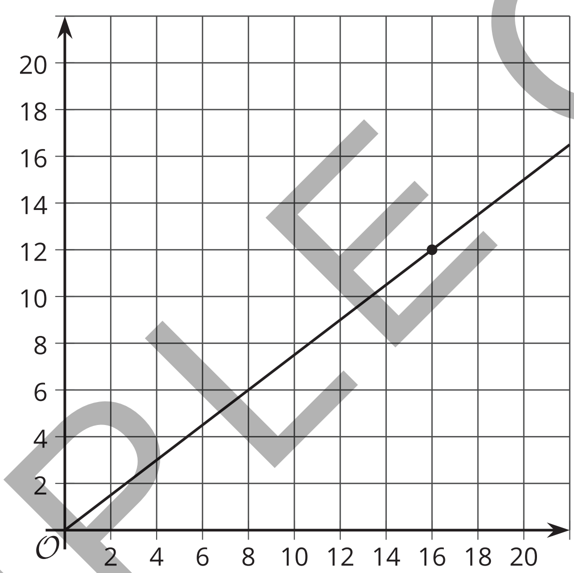
Addressing CA CCSSM 7.RP.2b, 7.RP.2d; practicing MP2, MP4, and MP6

Interpreting Graphs of Proportional Relationships

Let’s read stories from the graphs of proportional relationships.

11.1 What Could the Graph Represent?

Here is a graph that represents a proportional relationship.



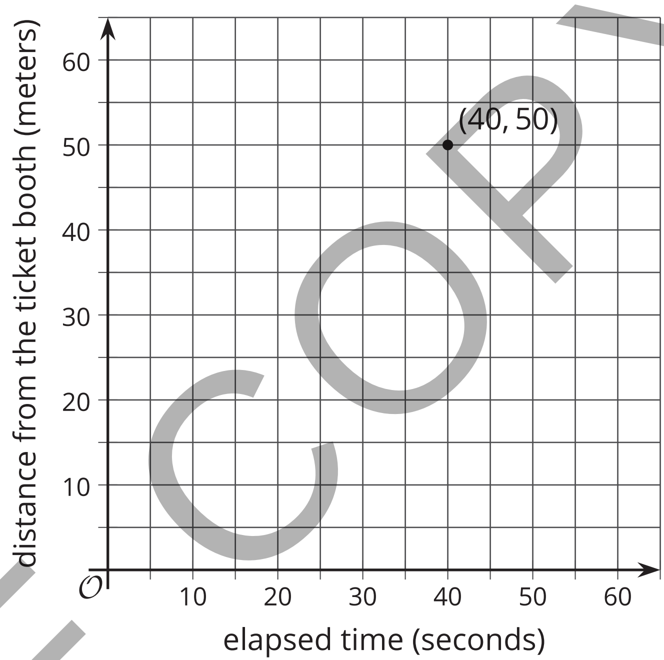
Invent a situation that could be represented by this graph.

1. Label the axes with the quantities in your situation.
2. Give the graph a title.
3. There is a point on the graph. What does it represent in your situation?

11.2 Tyler's Walk

Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.

1. The point on the graph shows his arrival at the bumper cars. What do the coordinates of the point tell us about the situation?



2. The table representing Tyler's walk shows other values of time and distance. Complete the table. Next, plot the pairs of values on the grid.

time (seconds)	distance (meters)
0	0
20	25
30	37.5
40	50
1	

3. What does the point $(0, 0)$ mean in this situation?

4. How far away from the ticket booth was Tyler after 1 second? Label the point on the graph that shows this information with its coordinates.

5. What is the constant of proportionality for the relationship between time and distance? What does it tell you about Tyler's walk? Where do you see it in the graph?

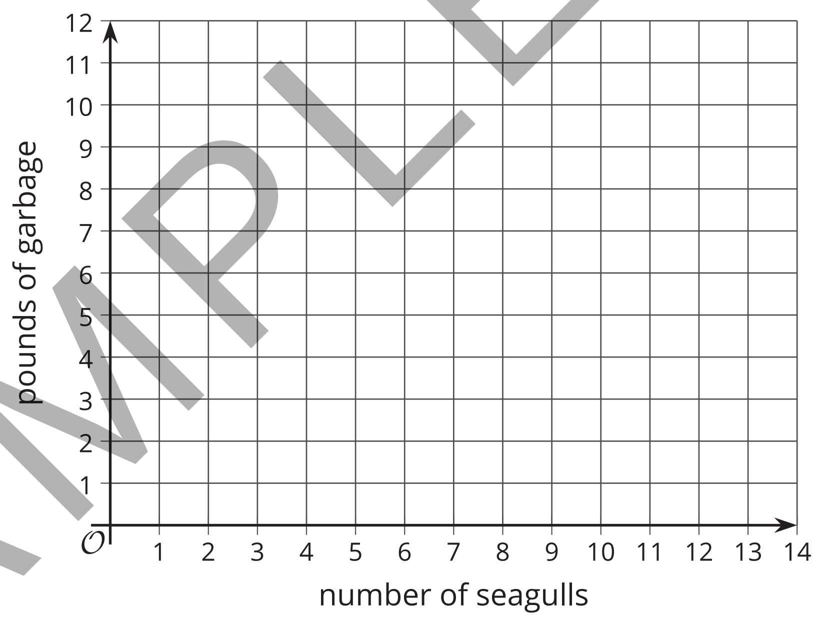
Are you ready for more?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

11.3 Seagulls Eat What?

4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.
2. Use a straightedge to draw a line through this point and $(0, 0)$.
3. Plot the point $(1, k)$ on the line. What is the value of k? What does the value of k tell you about this context?



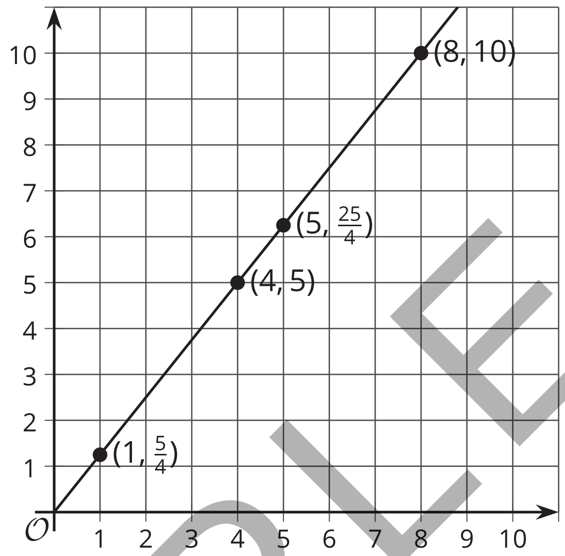
Lesson 11 Summary

For the relationship represented in this table, y is proportional to x. We can see in this table that $\frac{5}{4}$ is the constant of proportionality because it's the y value when x is 1.

The equation $y = \frac{5}{4}x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Here is the graph of this relationship.



If y represents the distance in feet that a snail crawls in x minutes, then the point $(4, 5)$ tells us that the snail can crawl 5 feet in 4 minutes.

If y represents the cups of yogurt and x represents the teaspoons of cinnamon in a recipe for fruit dip, then the point $(4, 5)$ tells us that you can mix 4 teaspoons of cinnamon with 5 cups of yogurt to make this fruit dip.

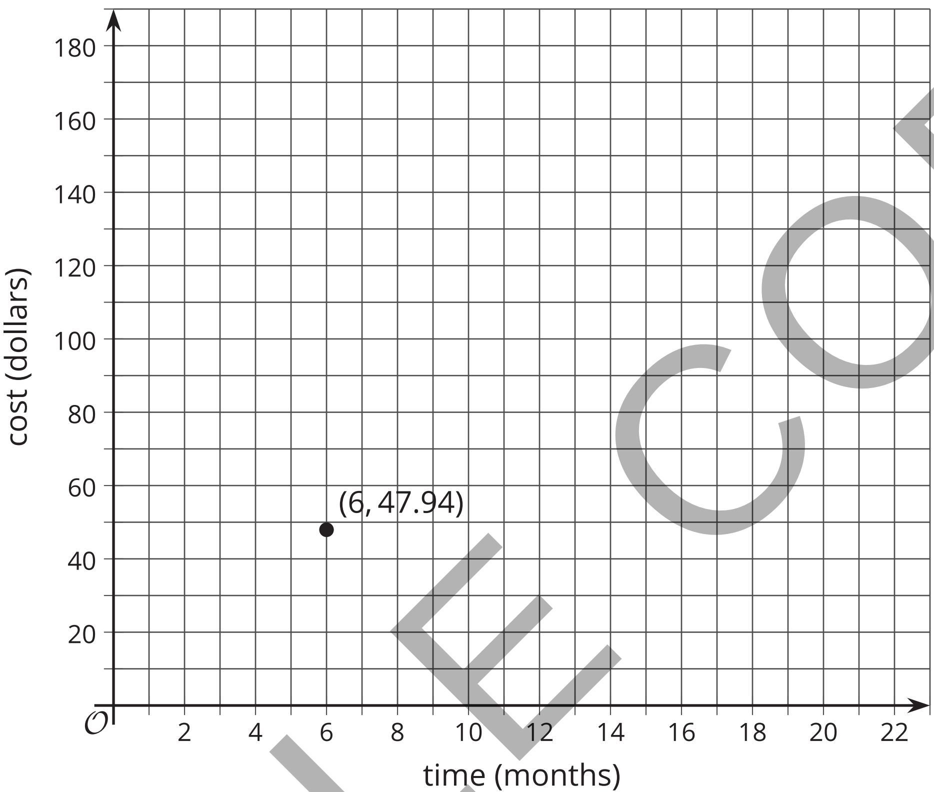
We can find the constant of proportionality by looking at the graph: $\frac{5}{4}$ is the y-coordinate of the point on the graph where the x-coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

In general, when y is proportional to x, the corresponding constant of proportionality is the y-value when $x = 1$.

Sec D

Practice Problems

1 There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is \$47.94. The point $(6, 47.94)$ is shown on this graph:



a. What is the constant of proportionality in this relationship?

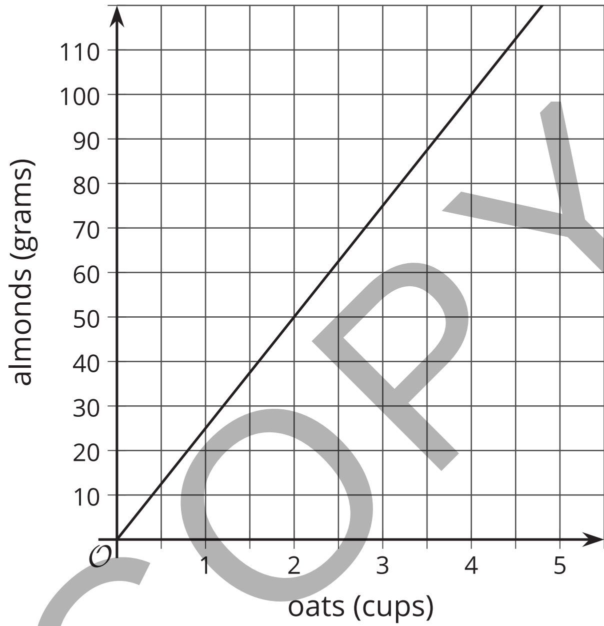
b. What does the constant of proportionality tell us about the situation?

c. Add at least three more points to the graph and label them with their coordinates.

d. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

Sec D

2 The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point $(1, k)$ on the graph, find the value of k, and explain its meaning.



3 from Unit 2, Lesson 9

To make a friendship bracelet, some long strings are lined up. One string is taken and tied in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around a friend's wrist.

Are the number of knots proportional to the number of rows? Explain your reasoning.

4 from Unit 2, Lesson 9

What information do you need to know to write an equation relating two quantities that have a proportional relationship?

Sec D

Addressing CA CCSSM 7.RP.2; building towards 7.RP.1; practicing MP2, MP7, and MP8

Using Graphs to Compare Relationships

Let's graph more than one relationship on the same grid.

12.1 Math Talk: More Division

Find the value of each expression mentally.

- $3 \div 6$
- $4 \div 5$
- $5 \div 4$
- $10 \div 6$

12.2 Race to the Bumper Cars

Diego, Lin, and Mai went from the ticket booth to the bumper cars.

1. Use each description to complete the table representing that person's journey.

   a. Diego left the ticket booth at the same time as Tyler. Diego jogged ahead at a steady pace and reached the bumper cars in 30 seconds.

   b. Lin left the ticket booth at the same time as Tyler. She ran at a steady pace and arrived at the bumper cars in 20 seconds.

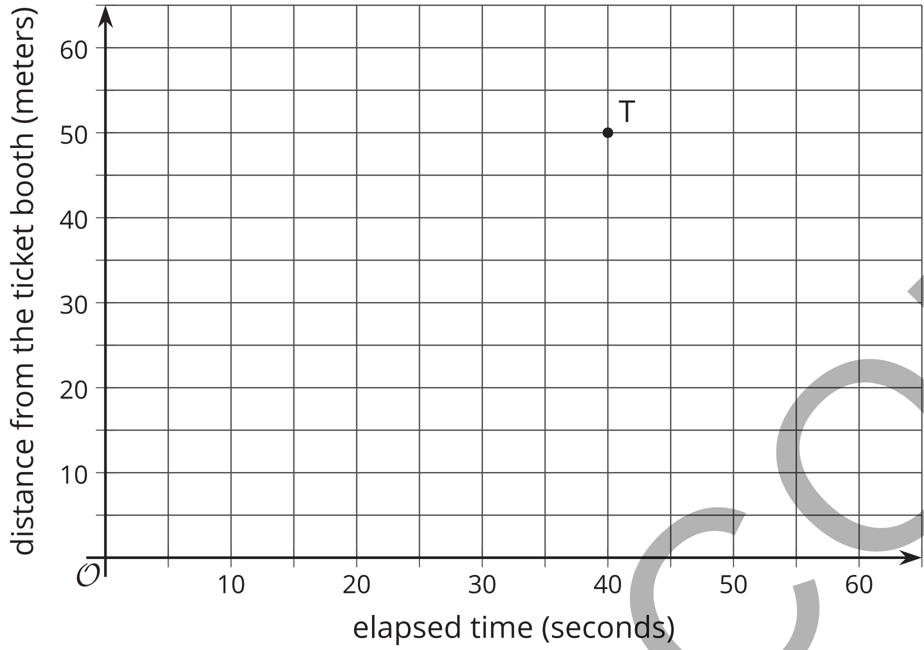
   c. Mai left the booth 10 seconds later than Tyler. Her steady jog enabled her to catch up with Tyler just as he arrived at the bumper cars.

Diego's time (seconds)	Diego's distance (meters)
0	
15	
30	50
1	

Lin's time (seconds)	Lin's distance (meters)
	0
	25
20	50
1	

Mai's time (seconds)	Mai's distance (meters)
10	
	25
40	50
1	

2. Using a different color for each person, draw a graph of all four people's journeys (including Tyler's from the other day).



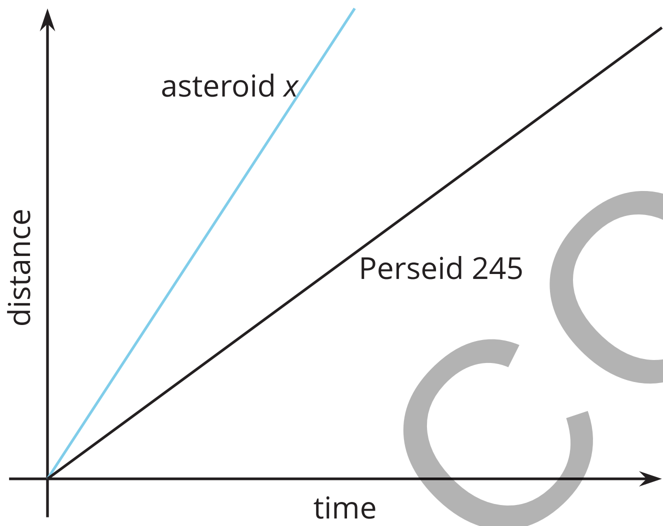
3. Which person is moving the most quickly? How is that reflected in the graph?

Are you ready for more?

Write equations to represent the relationship between time and distance for each person.

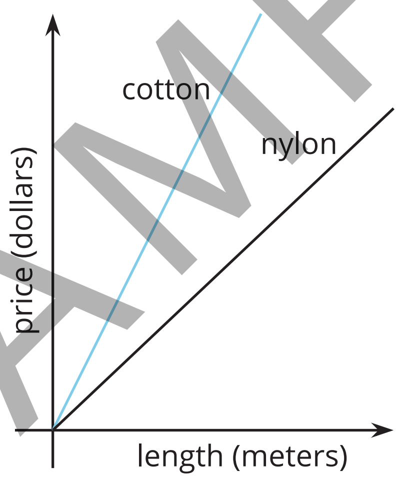
12.3 Space Rocks and the Price of Rope

1. Meteoroid Perseid 245 and Asteroid x travel through the solar system. The graph shows the distance each traveled after a given point in time.



Is Asteroid x traveling faster or slower than Perseid 245? Explain how you know.

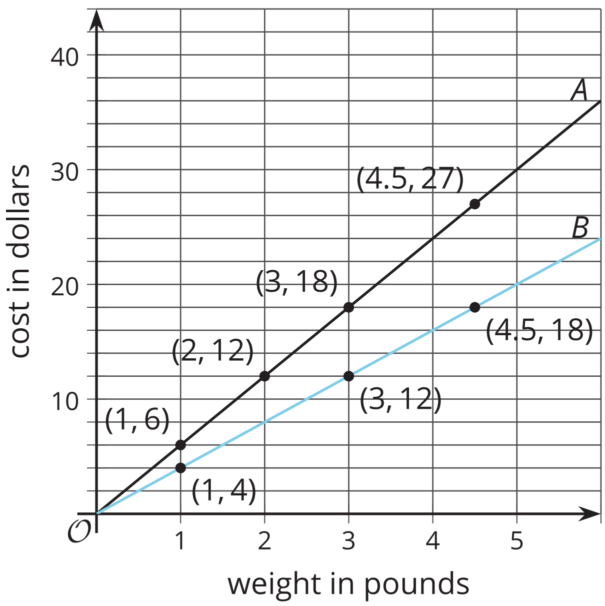
2. The graph shows the price of different lengths of two types of rope.



If you buy $1.00 of each kind of rope, which one will be longer? Explain how you know.

Lesson 12 Summary

Here is a graph that shows the price of blueberries at two different stores. Which store has a better price?



We can compare points that have the same x value or the same y value. For example, the points $(2, 12)$ and $(3, 12)$ tell us that at Store B you can get more pounds of blueberries for the same price.

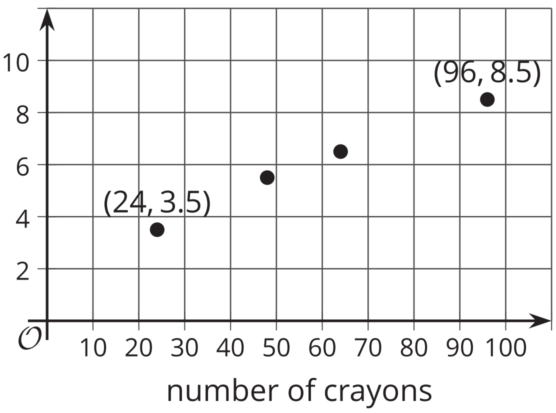
The points $(3, 12)$ and $(3, 18)$ tell us that at Store A you have to pay more for the same quantity of blueberries. This means Store B has the better price.

We can also use the graphs to compare the constants of proportionality. The line representing Store B goes through the point $(1, 4)$, so the constant of proportionality is 4. This tells us that at Store B the blueberries cost \$4 per pound. This is cheaper than the \$6 per pound unit price at Store A.

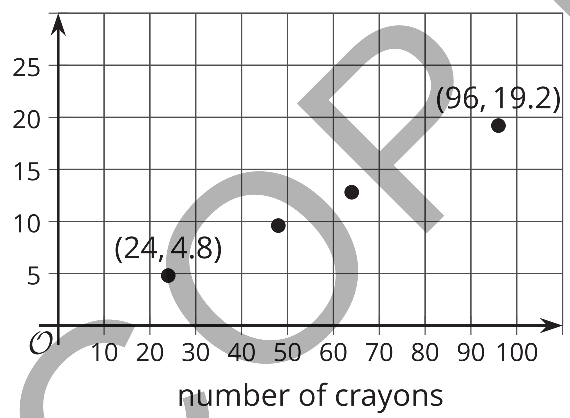
Practice Problems

1 The following graphs show some information about boxes of crayons at a store. One of the graphs shows cost in dollars vs. number of crayons in the box, and one of the graphs shows weight in ounces vs. number of crayons in the box.

_________________ vs. number of crayons



_________________ vs. number of crayons



a. Which graph is which? Give them the correct titles.

b. Which quantities appear to be in a proportional relationship? Explain how you know.

c. For the proportional relationship, find the constant of proportionality. What does that number mean?

2 Lin and Andre biked home from school at a steady pace. Lin biked 1.5 kilometers and it took her 5 minutes. Andre biked 2 kilometers and it took him 8 minutes.

a. Draw a graph with two lines that represent the bike rides of Lin and Andre.

b. For each line, highlight the point with coordinates $(1, k)$ and find k.

c. Who was biking faster?

Sec D

3 Match each equation to its graph.

a. $y = 2x$

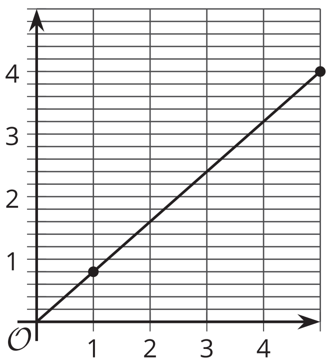
b. $y = \frac{4}{5}x$

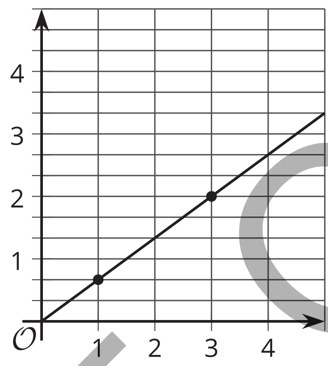
c. $y = \frac{1}{4}x$

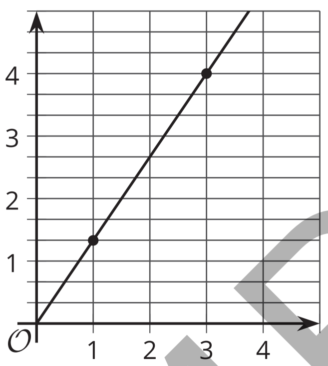
d. $y = \frac{2}{3}x$

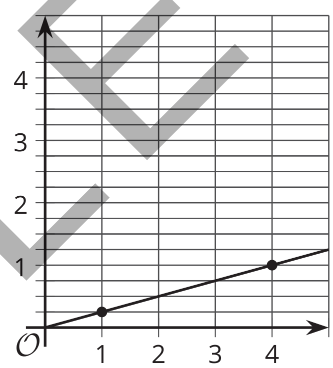
e. $y = \frac{4}{3}x$

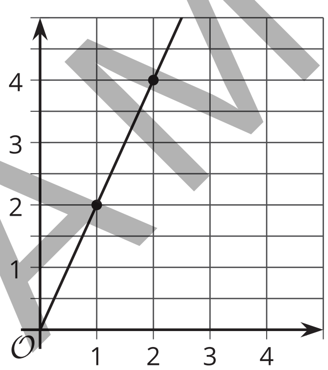
f. $y = \frac{3}{2}x$

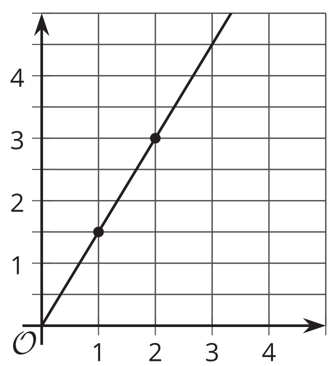








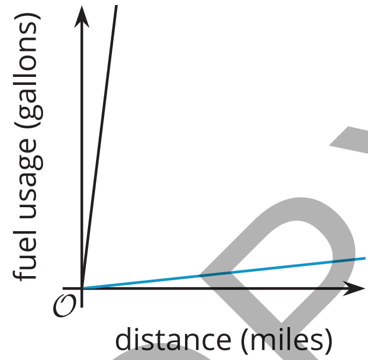




4 A semi truck can travel 1,300 miles on 200 gallons of fuel. A train can travel 1,000 miles on 2 gallons of fuel. This graph shows the fuel usage in miles per gallon for the two vehicles.

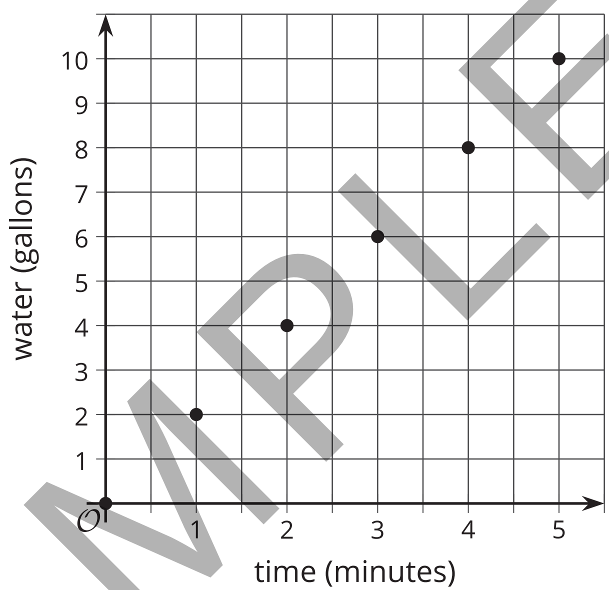
a. Which line represents the semi truck and which represents the train?

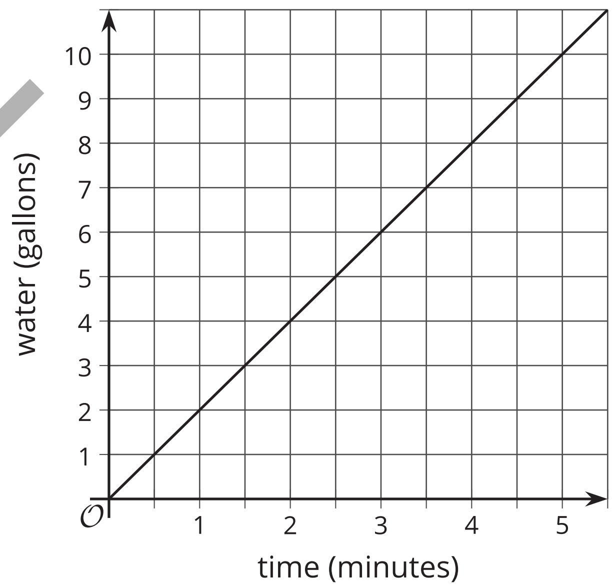
b. Which vehicle uses less fuel to travel the same distance? How can you tell from the graph?



5 from Unit 2, Lesson 10

Here are two graphs for the relationship between the time that a water faucet has been on and the amount of water in a bucket.





a. What is the same about the two graphs?

b. What is different about the two graphs?

c. Which graph makes more sense for representing this situation?

Addressing CA CCSSM 7.RP.2; building towards 7.EE.1-2; practicing MP6 and MP7

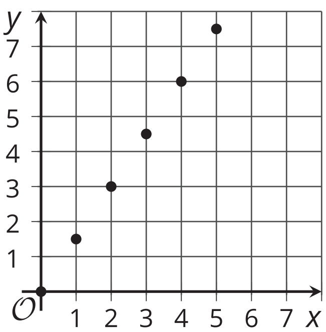
Two Graphs for Each Relationship

Let's use tables, equations, and graphs to answer questions about proportional relationships.

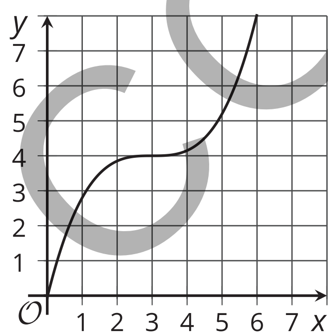
13.1 Which Three Go Together: Graphs

Which three go together? Why do they go together?

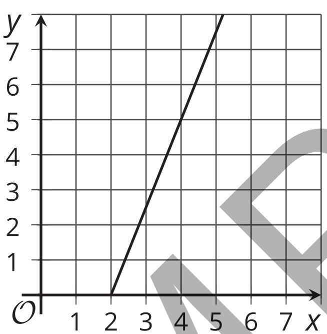
A



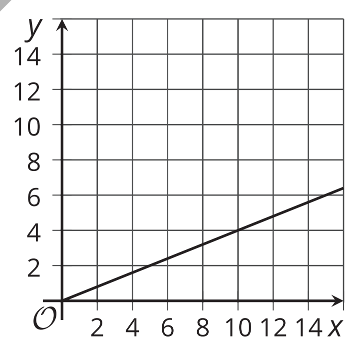
B



C



D

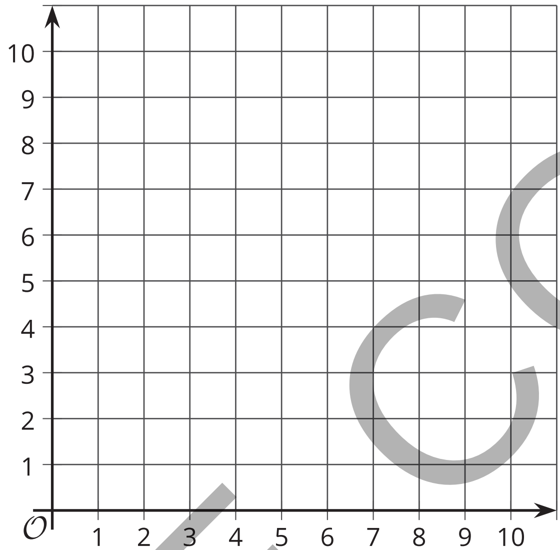


13.2 Tables, Graphs, and Equations

Your teacher will assign you *one* of these three points:

$$A(5,2),\ B(4,5),\ C(8,5).$$

1. On the coordinate plane, plot and label only your assigned point.



2. Graph the proportional relationship that is defined by your point. That is, use a ruler to draw a line that starts at the origin, goes through your point, and continues to the edge of the grid.
3. Use your graph to find the y-value that goes with each of these x-values.

x	y
2	
6	

Your teacher will give you a completed table. Use it to check your values.

Sec D

4. Choose three rows, other than the row that represents the origin, from the completed table. Record the values and compute $\frac{y}{x}$ for each row. What do you notice about these values?

x	y	$\frac{y}{x}$

5. Write an equation that represents the relationship between x and y .

6. What is the y-coordinate of your graph when the x-coordinate is 1? Plot and label this point on your graph.

7. Based on your observations, describe any connections you see between the graph, the table, and the equation.

8. Compare your representations with the rest of your group. Discuss what is the same and what is different about:

   a. Your graphs.

   b. Your tables.

   c. Your equations.

Are you ready for more?

The graph of an equation of the form $y = kx$, where k is a positive number, is a line through $(0, 0)$ and the point $(1, k)$.

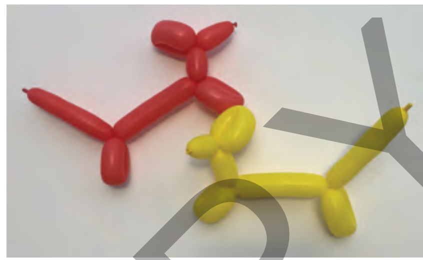
1. Name at least one line through $(0, 0)$ that cannot be represented by an equation like this.

2. If you could draw the graphs of *all* of the equations of this form in the same coordinate plane, what would it look like?

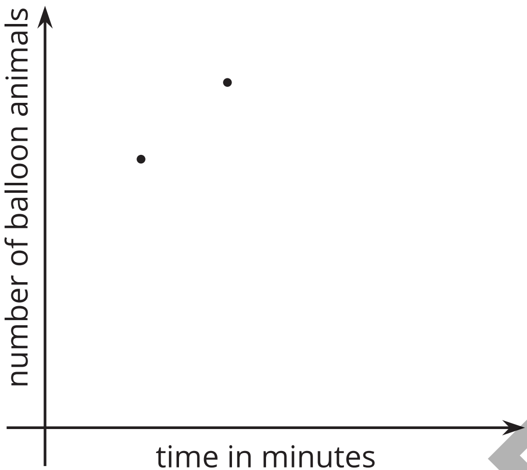
13.3 Balloon Animal Contest

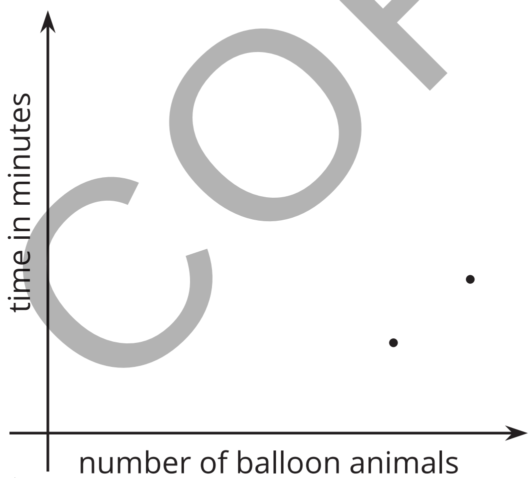
Andre and Jada had a contest making balloon animals.

- Andre made 10 balloon animals in 3 minutes.
- Jada made 12 balloon animals in 5 minutes.



Here are two different graphs that both represent this situation.





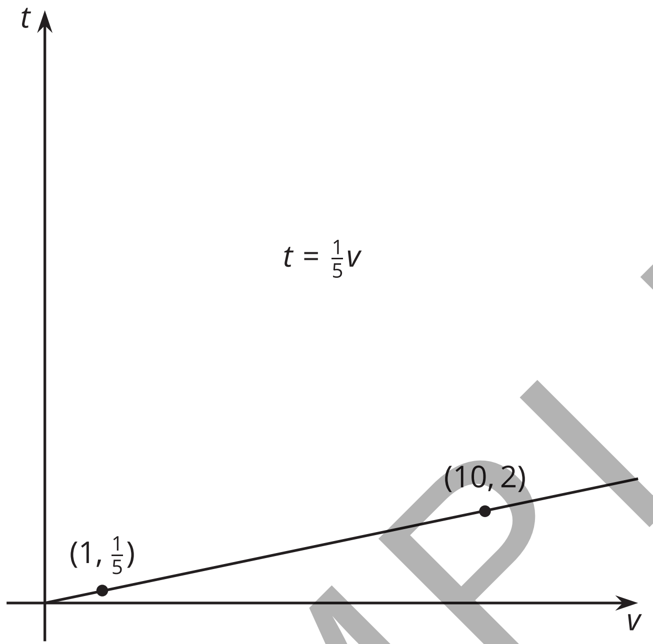
1. On the first graph, which point shows Andre's work and which shows Jada's work? Label them.
2. Draw two lines: one through the origin and Andre's point, and one through the origin and Jada's point.
3. Write an equation for each line. Use t to represent time in minutes and b to represent the number of balloon animals.
   a. Andre:
   b. Jada:
4. For each equation, what does the constant of proportionality tell you?
5. Repeat the previous steps for the second graph.
   a. Andre:
   b. Jada:

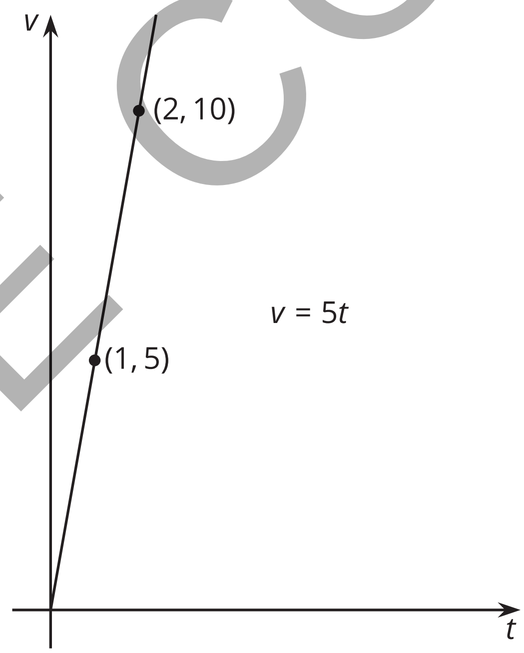
Lesson 13 Summary

Imagine that a faucet is leaking at a constant rate and that every 2 minutes, 10 milliliters of water leaks from the faucet. There is a proportional relationship between the volume of water and elapsed time.

- We could say that the elapsed time is proportional to the volume of water. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of $\frac{1}{5}$ of a minute per milliliter.
- We could say that the volume of water is proportional to the elapsed time. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of 5 milliliters per minute.

Let's use v to represent volume in milliliters and t to represent time in minutes. Here are graphs and equations that represent both ways of thinking about this relationship:





Even though the relationship between time and volume is the same, we are making a different choice in each case about which variable to view as the independent variable. The graph on the left has v as the independent variable, and the graph on the right has t as the independent variable.

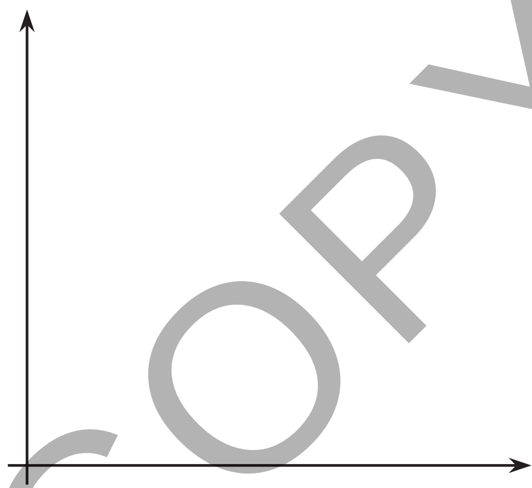
Practice Problems

1 At the supermarket you can fill your own honey bear container. A customer buys 12 ounce of honey for \$5.40.

a. How much does honey cost per ounce?

b. How much honey can you buy per dollar?

c. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.



d. Choose one of your equations, and sketch its graph. Be sure to label the axes.

2 The point $(3, \frac{6}{5})$ lies on the graph representing a proportional relationship. Which of the following points also lie on the same graph? Select **all** that apply.

A. $(1, 0.4)$

B. $(1.5, \frac{6}{10})$

C. $(\frac{6}{5}, 3)$

D. $(4, \frac{11}{5})$

E. $(15, 6)$

3 A trail mix recipe asks for 4 cups of raisins for every 6 cups of peanuts. There is proportional relationship between the amount of raisins, r (cups), and the amount of peanuts, p (cups), in this recipe.

a. Write the equation for the relationship that has a constant of proportionality greater than 1. Graph the relationship.

b. Write the equation for the relationship that has a constant of proportionality less than 1. Graph the relationship.

4 from Unit 2, Lesson 11

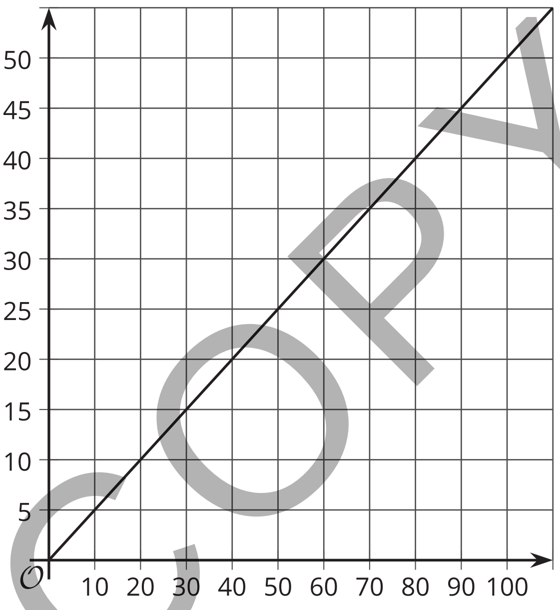
Here is a graph that represents a proportional relationship.

a. Come up with a situation that could be represented by this graph.

b. Label the axes with the quantities in your situation.

c. Give the graph a title.

d. Choose a point on the graph. What do the coordinates represent in your situation?



Sec D

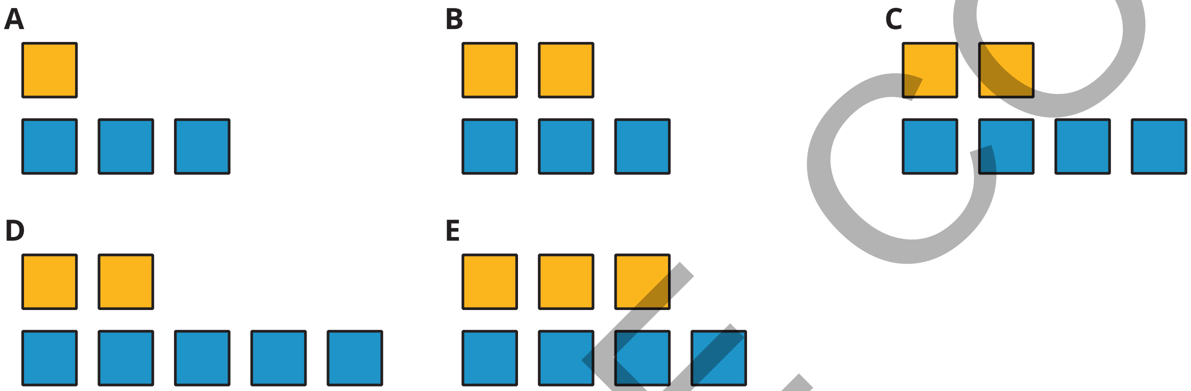
Addressing CA CCSSM 7.RP.2; building on 6.RP.3; practicing MP3 and MP4

Four Representations

Let's contrast relationships that are and are not proportional in four different ways.

14.1 Which Group Is the Bluest?

1. Which group of blocks is the bluest?



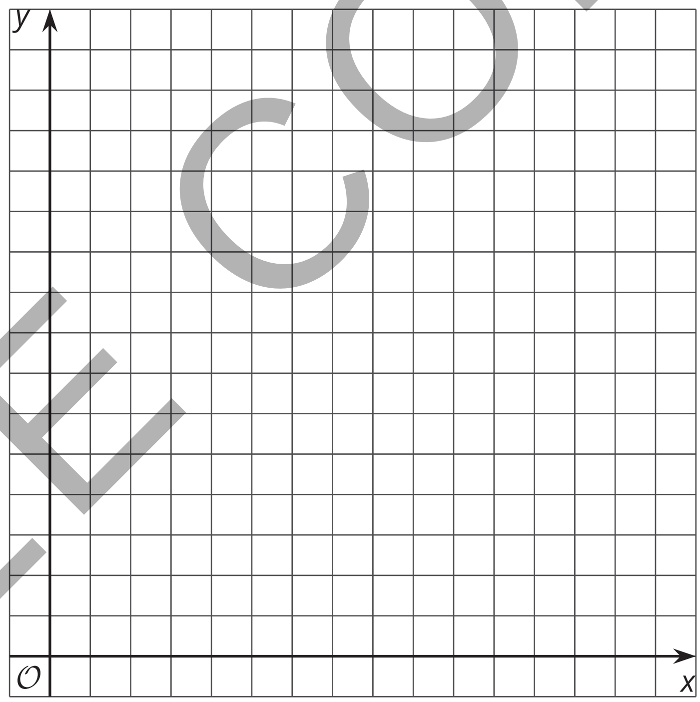
2. Order the groups of blocks from least blue to bluest.

14.2 Creating and Representing Situations

1. Make up a situation where there is a proportional relationship between two quantities.

   a. Write one or more sentences describing the relationship.

   b. Make a table with at least 5 pairs of numbers relating the two quantities.



   c. Graph the relationship and label the axes.

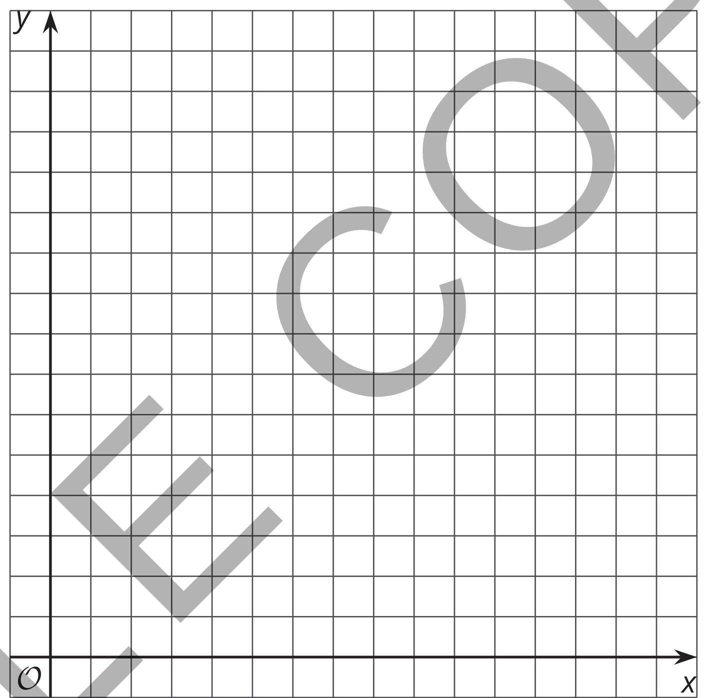
   d. Write an equation showing the relationship. Explain in your own words what each number and letter in your equation represents.

Sec E

2. If you have time, make up another situation where there is a relationship between two quantities, but the relationship is not proportional.

   a. Write one or more sentences describing the relationship.

   b. Make a table with at least 5 pairs of numbers relating the two quantities.



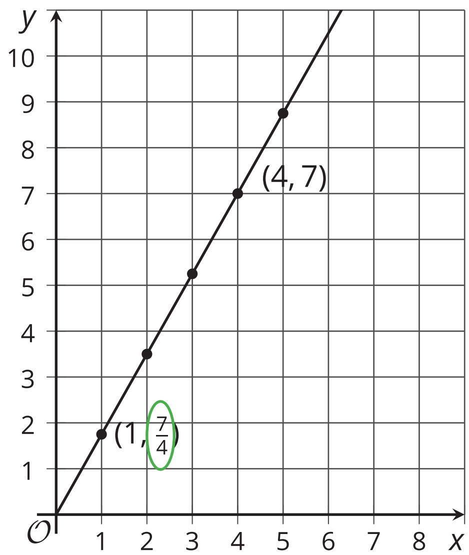
   c. Graph the relationship and label the axes.

   d. If possible, write an equation showing the relationship and explain in your own words what each number and letter in your equation represents.

3. For each of your situations, explain how you know whether the relationship is proportional or not. Give as many reasons as you can.

Lesson 14 Summary

The constant of proportionality for a proportional relationship can often be easily identified in a graph, a table, and an equation that represents it. Here is an example of all three representations for the same relationship. The constant of proportionality is circled:



$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

On the other hand, some relationships are not proportional. If the graph of a relationship is not a straight line through the origin, if the equation cannot be expressed in the form $y = kx$, or if the table does not have a constant of proportionality that can be multiplied by any number in the first column to get the corresponding number in the second column, then the relationship between the quantities is not a proportional relationship.

Sec E

Practice Problems

1 The equation $c = 2.95g$ shows how much it costs to buy gas at a gas station on a certain day. In the equation, c represents the cost in dollars, and g represents how many gallons of gas were purchased.

a. Write down at least four (gallons of gas, cost) pairs that fit this relationship.

b. Create a graph of the relationship.

c. What does 2.95 represent in this situation?

d. Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? How did she come up with that?

2 There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.

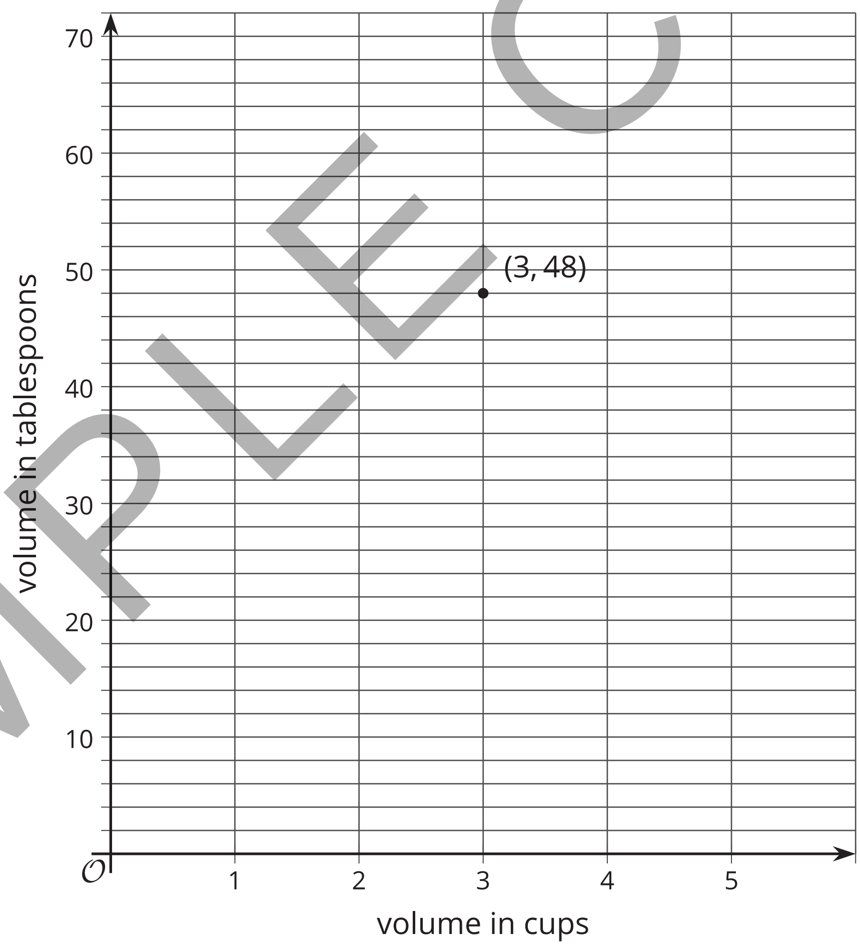
a. Plot and label at least two more points that represent the relationship.

b. Use a straightedge to draw a line that represents this proportional relationship.

c. For which value y is $(1, y)$ on the line you just drew?

d. What is the constant of proportionality for this relationship?

e. Write an equation representing this relationship. Use c for cups and t for tablespoons.



Unit 2, Lesson 15

Addressing CA CCSSM 7.RP.2; practicing MP1 and MP4

Using Water Efficiently

Let's investigate saving water.

15.1 Notice and Wonder: Water

What do you notice? What do you wonder?

15.2 Comparing Baths and Showers

Some people say that it uses more water to take a bath than a shower. Others disagree.

1. What information would you need in order to answer the question?

2. Describe how you could get the information and how you would use the information to find the answer.

3. Find out values for the measurements you need to use the method you described. You may ask your teacher or research them yourself.

4. Under what conditions does a bath use more water? Under what conditions does a shower use more water? Explain or show your reasoning.

15.3 Representing Water Usage

1. Continue considering the problem from the previous activity. Name two quantities that are in a proportional relationship. Explain how you know they are in a proportional relationship.

2. What are two constants of proportionality for the proportional relationship? What do they tell us about the situation?

3. On graph paper, create a graph that shows how the two quantities are related. Make sure to label the axes.

4. Write two equations that relate the quantities in your graph. Make sure to record what each variable represents.

Learning Targets

Lesson 1 One of These Things Is Not Like the Others

- I can use equivalent ratios to describe scaled copies of shapes.
- I know that two recipes will taste the same if the ingredients are in equivalent ratios.

Lesson 2 Introducing Proportional Relationships with Tables

- I can use a table to reason about two quantities that are in a proportional relationship.
- I understand the terms proportional relationship and constant of proportionality.

Lesson 3 More about Constant of Proportionality

- I can find missing information in a proportional relationship using a table.
- I can find the constant of proportionality from information given in a table.

Lesson 4 Proportional Relationships and Equations

- I can write an equation of the form $y = kx$ to represent a proportional relationship shown in a table or described in a story.
- I can write the constant of proportionality as an entry in a table.

Lesson 5 Two Equations for Each Relationship

- I can find two constants of proportionality for a proportional relationship.
- I can write two equations representing a proportional relationship described by a table or story.

Lesson 6 Writing Equations to Represent Relationships

- I can find missing information in a proportional relationship using the constant of proportionality.
- I can relate all parts of an equation like $y = kx$ to the situation it represents.

Lesson 7 Comparing Relationships with Tables

- I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional.

Lesson 8 Comparing Relationships with Equations

- I can decide if a relationship represented by an equation is proportional or not.

Lesson 9 Solving Problems about Proportional Relationships

- I can ask questions about a situation to determine whether two quantities are in a proportional relationship.

- I can solve all kinds of problems involving proportional relationships.

Lesson 10 Introducing Graphs of Proportional Relationships

- I know that the graph of a proportional relationship lies on a line through $(0, 0)$.

Lesson 11 Interpreting Graphs of Proportional Relationships

- I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
- I can find the constant of proportionality from a graph.
- I understand the information given by graphs of proportional relationships that are made up of points or a line.

Lesson 12 Using Graphs to Compare Relationships

- I can compare two, related proportional relationships based on their graphs.
- I know that the steeper graph of two proportional relationships has a larger constant of proportionality.

Lesson 13 Two Graphs for Each Relationship

- I can interpret a graph of a proportional relationship using the situation.
- I can write an equation representing a proportional relationship from a graph.

Lesson 14 Four Representations

- I can make connections between the graphs, tables, and equations of a proportional relationship.
- I can use units to help me understand information about proportional relationships.

Lesson 15 Using Water Efficiently

- I can answer a question by representing a situation using proportional relationships.

GRADE 7

UNIT

3

Measuring Circles

Content Connections

In this unit you will apply your knowledge of proportional relationships to measuring circles. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while describing features of deconstructed circles and their measurements.
- **Discovering Shape and Space** while using the relationship between radius, diameter, circumference and area to solve problems involving circles.
- **Exploring Changing Quantities** while using the formula A=πr^2 when solving problems involving circles.
- **Reasoning with Data** while critiquing reasoning about circles and circle measurements and describe features of graphs.

Addressing the Standards

As you work your way through **Unit 3 Measuring Circles,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 4, 5, and 11
MP2 Reason abstractly and quantitatively.	Lessons 4, 5, 9, and 10
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 2, 5, 8, and 10
MP4 Model with mathematics.	Lessons 1, 6, and 11
MP5 Use appropriate tools strategically.	Lessons 2 and 7
MP6 Attend to precision.	Lessons 2, 4, 6, 7, 9, and 10
MP7 Look for and make use of structure.	Lessons 3, 8, and 9
MP8 Look for and express regularity in repeated reasoning.	Lessons 1, 3, 5, 7, and 8

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• 2-D and 3-D Connections • Scale Drawings • Shapes in the World	**7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Lessons 6, 7, and 11
• 2-D and 3-D Connections • Shapes in the World	**7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Lesson 2

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes in the World	**7.G.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Lessons 3, 4, 5, 7, 8, 9, 10, and 11
• Angle Relationships • Shapes in the World	**7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Lesson 6
• Populations and Samples • Probability Models • Proportional Relationships • Graphing Relationships • Scale Drawings	**7.RP.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.	Lesson 3
• Populations and Samples • Probability Models • Proportional Relationships • Graphing Relationships • Scale Drawings	**7.RP.2a** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	Lessons 1, 3, 5, and 7

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Populations and Samples • Probability Models • Proportional Relationships • Graphing Relationships • Scale Drawings	**7.RP.2c** Recognize and represent proportional relationships between quantities. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*	Lesson 5
• Populations and Samples • Probability Models • Proportional Relationships • Unit Rates in the World • Graphing Relationships • Scale Drawings	**7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	Lesson 5
• Populations and Samples • Visualize Populations • Probability Models • Proportional Relationships • Unit Rates in the World • Scale Drawings	**7.EE.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	Lesson 11

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

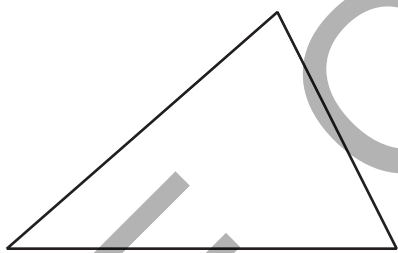
Addressing CA CCSSM 7.RP.2a; building on 6.RP.3c; building towards 7.G.4, 7.RP.3; practicing MP4 and MP8

How Well Can You Measure?

Let's see how accurately we can measure.

1.1 Perimeter of a Triangle

Measure the perimeter of the triangle to the nearest tenth of a centimeter.

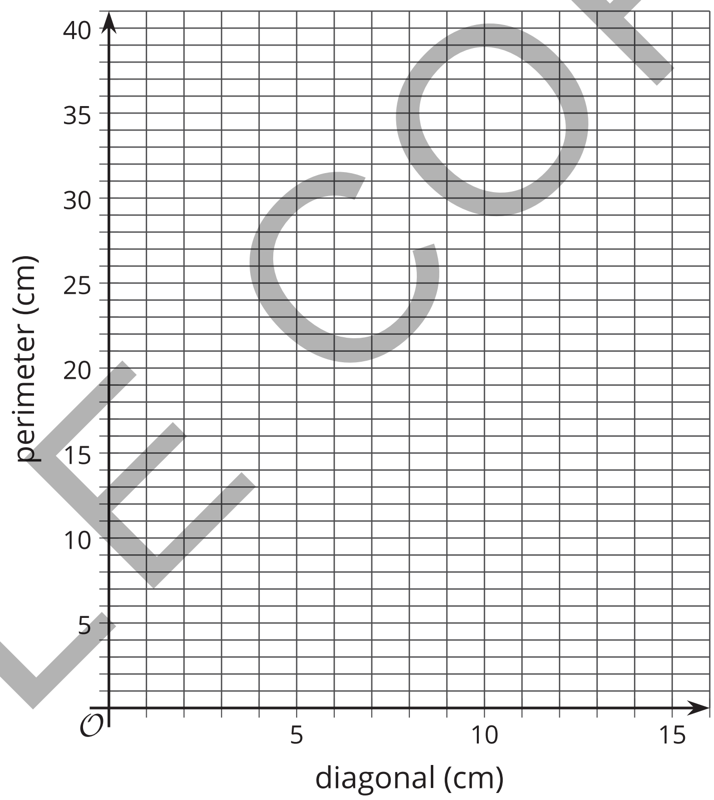


1.2 Perimeter of a Square

Your teacher will give you a picture of 9 different squares and will assign your group 3 of these squares to examine more closely.

1. For each of your assigned squares, measure the length of the diagonal and the perimeter of the square in centimeters. Check your measurements with your group. After you come to an agreement, record your measurements in the table.

	diagonal (cm)	perimeter (cm)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		



2. Plot the diagonal and perimeter values from the table on the coordinate plane.

3. What do you notice about the points on the graph?

Pause here so your teacher can review your work.

4. Record measurements of the other squares to complete your table.

1.3 Area of a Square

1. In the table, record the length of the diagonal for each of your assigned squares from the previous activity. Next, determine the area of each of your squares.

	diagonal (cm)	area (cm^2)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

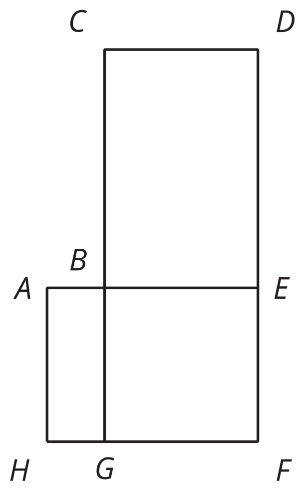
Pause here so your teacher can review your work. Be prepared to share your values with the class.

2. Examine the class graph of these values. What do you notice?

3. How is the relationship between the diagonal and area of a square the same as the relationship between the diagonal and perimeter of a square from the previous activity? How is it different?

Are you ready for more?

Here is a rough map of a neighborhood.



There are 4 mail routes during the week.

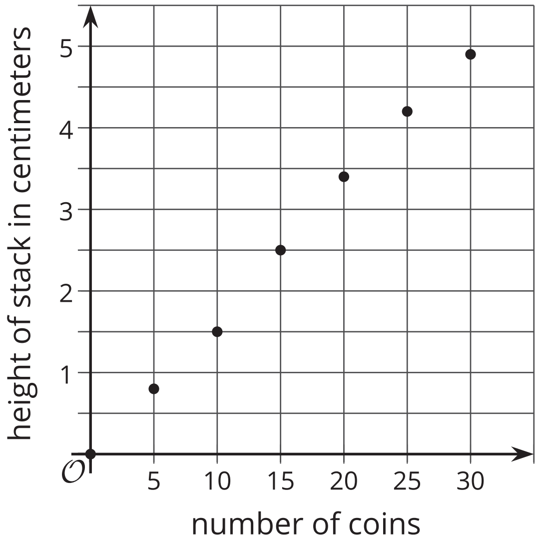
- On Monday, the mail truck follows the route A-B-E-F-G-H-A, which is 14 miles long.
- On Tuesday, the mail truck follows the route B-C-D-E-F-G-B, which is 22 miles long.
- On Wednesday, the truck follows the route A-B-C-D-E-F-G-H-A, which is 24 miles long.
- On Thursday, the mail truck follows the route B-E-F-G-B.

How long is the route on Thursdays?

Lesson 1 Summary

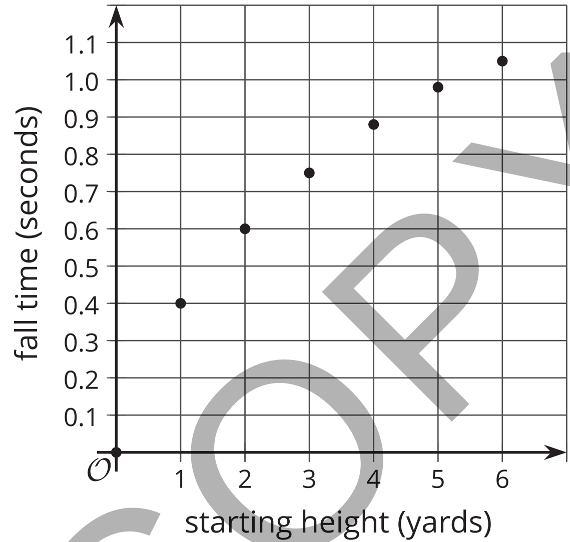
When we measure the values for two related quantities, plotting the measurements in the coordinate plane can help us decide if it makes sense to model them with a proportional relationship. If the points are close to a line through $(0, 0)$, then a proportional relationship is a good model.

This graph shows the height of the stack for different numbers of stacked coins.



These points are close to a straight line through $(0, 0)$, so the relationship may be proportional.

This graph shows the time it takes for a tennis ball to fall from different starting heights.



These points are not close to a straight line through $(0, 0)$, so the relationship is not proportional.

Another way to investigate whether or not a relationship is proportional is by making a table and dividing the values on each row. Here are tables that represent the same relationships as the previous graphs.

number of coins	height in centimeters	centimeters per coin
5	0.8	0.16
10	1.5	0.15
15	2.5	0.167
20	3.4	0.17
25	4.2	0.168
30	4.9	0.163

The centimeters of height per coin are close to the same value, so this relationship appears to be proportional.

starting height (yards)	fall time (seconds)	seconds per yard
1	0.40	0.40
2	0.60	0.30
3	0.75	0.25
4	0.88	0.22
5	0.98	0.196
6	1.05	0.175

The seconds of fall time per yard of starting height are not close to the same value, so this relationship is not proportional.

Practice Problems

1 Mai measured the height, perimeter, and area of some equilateral triangles. Her measurements are shown in the tables.

a. Could the relationship between the triangles' heights and their perimeters be proportional? Explain your reasoning.

height (cm)	perimeter (cm)
1.1	3.9
2.4	8.1
4.1	14.4
5.5	19.2
7.9	27.3

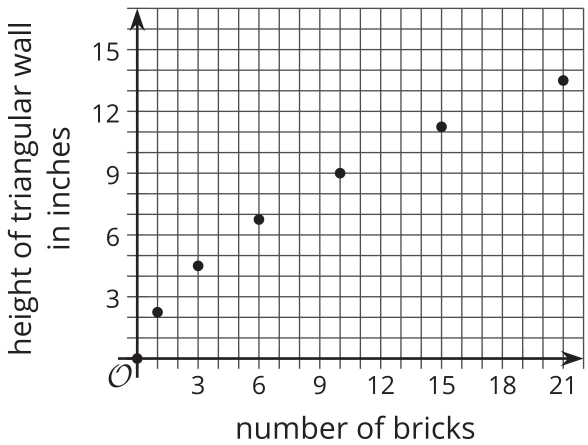
b. Could the relationship between the triangles' heights and their areas be proportional? Explain your reasoning.

height (cm)	area (cm^2)
1.1	0.715
2.4	3.24
4.1	9.84
5.5	17.6
7.9	35.945

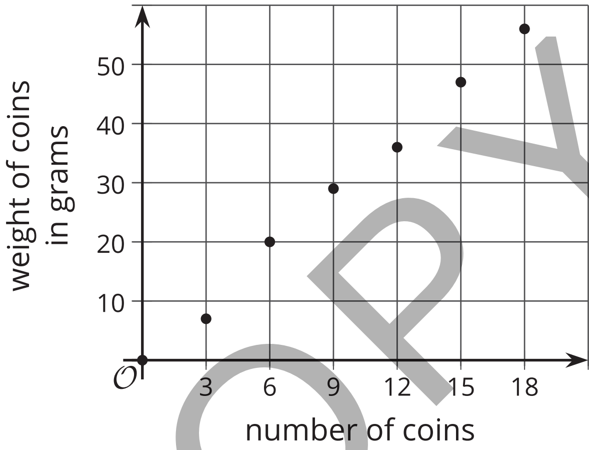
2 Diego made a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities can't be proportional." Do you agree with Diego? Explain.

3 For each graph, explain whether the relationship could be proportional.

A



B

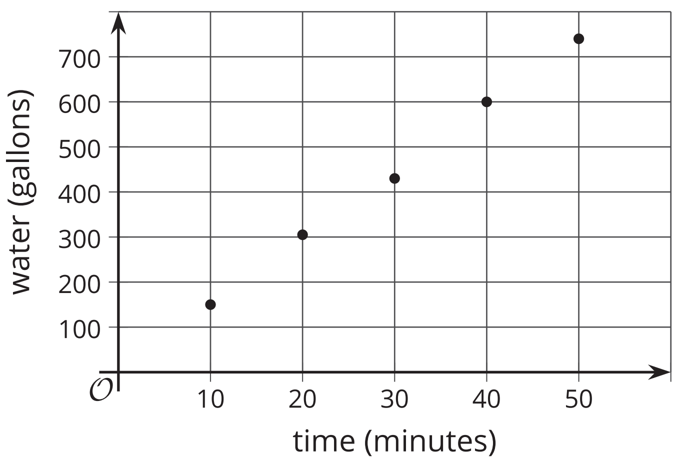


4 The graph shows that while it was being filled, the amount of water in gallons in a swimming pool was approximately proportional to the time that has passed in minutes.

a. About how much water was in the pool after 25 minutes?

b. Approximately when were there 500 gallons of water in the pool?

c. Estimate the constant of proportionality for the gallons of water per minute going into the pool.



Addressing CA CCSSM 7.G.2; building towards 7.G.4; practicing MP3, MP5, and MP6

Exploring Circles

Let's explore circles.

2.1 How Do You Figure?

Here are two figures.

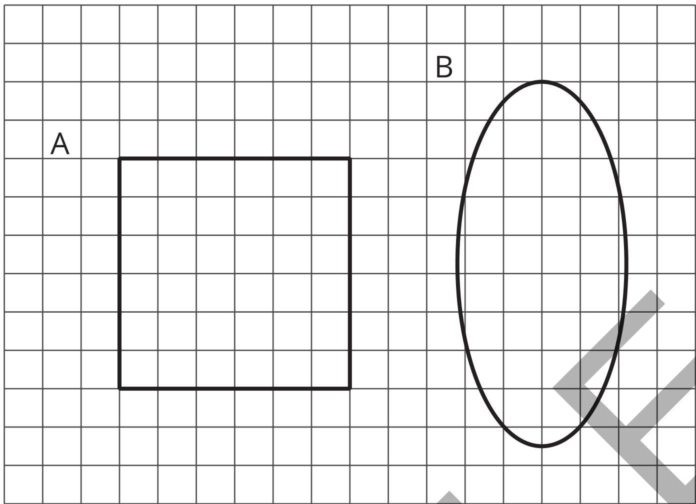


Figure C looks more like Figure A than like Figure B. Sketch what Figure C might look like. Explain your reasoning.

2.2 Card Sort: Sorting Round Objects

Your teacher will give you some pictures of different objects.

1. How could you sort these pictures into two groups? Be prepared to share your reasoning.

2. Take turns with your partner to sort the pictures into the categories that your class has agreed on.
    a. For each match that you find, explain to your partner how you know it's a match.
    b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

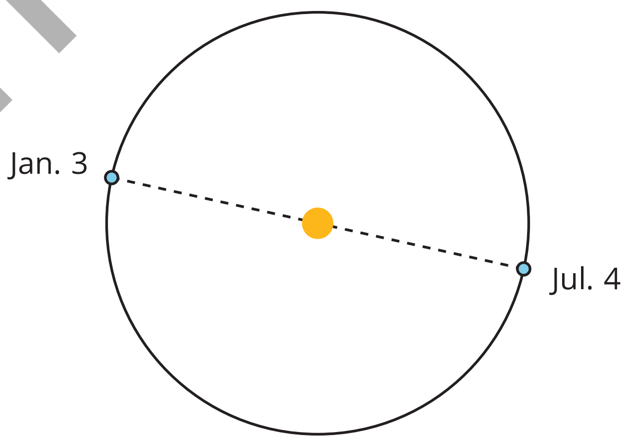
    Pause here so your teacher can review your work.
3. What are some characteristics that all **circles** have in common?

4. Put the circular objects in order from smallest to largest.
5. Select one of the pictures of a circular object. What are some ways that you could measure the actual size of your circle?

Are you ready for more?

On January 3rd, Earth is 147,500,000 kilometers away from the Sun. On July 4th, Earth is 152,500,000 kilometers away from the Sun. The Sun has a radius of about 865,000 kilometers.

Could Earth's orbit be a circle with some point in the Sun as its center? Explain your reasoning.



2.3 Measuring Circles

Priya, Han, and Mai each measured one of the circular objects from earlier.

- Priya says that the bike wheel is 24 inches.
- Han says that the yo-yo trick is 24 inches.
- Mai says that the glow necklace is 24 inches.

1. Do you think that all these circles are the same size?

2. What part of the circle did each person measure? Explain your reasoning.

2.4 Drawing Circles

Draw and label each circle.

1. Circle A, with a diameter of 6 cm.

KH | iM Illustrative® Mathematics LEARN MATH FOR LIFE

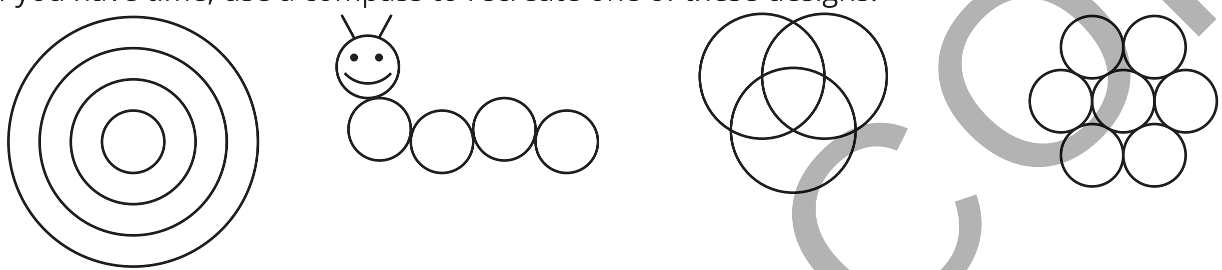
2. Circle B, with a radius of 5 cm.

Pause here so your teacher can review your work.

3. Circle C, with a radius that is equal to Circle A’s diameter.

4. Circle D, with a diameter that is equal to Circle B's radius.

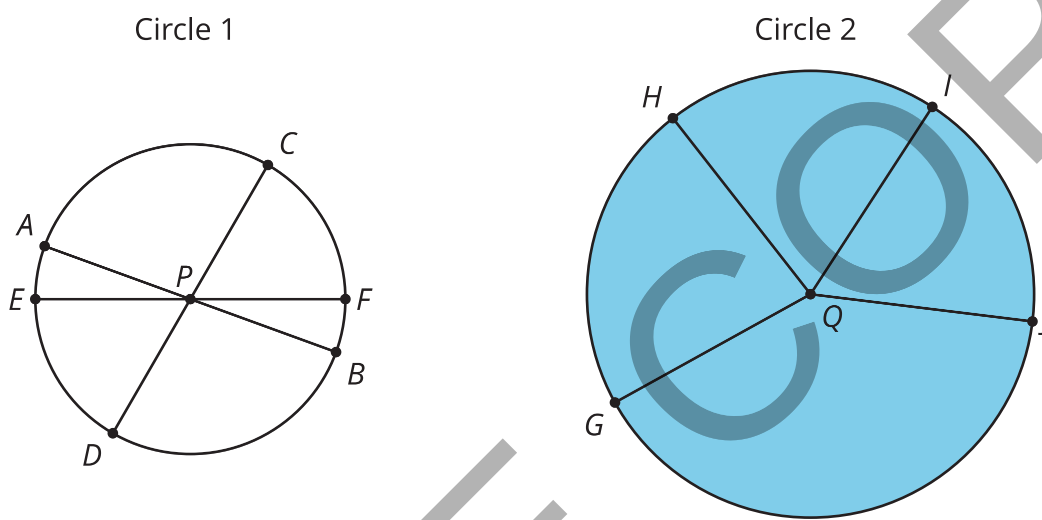
5. If you have time, use a compass to recreate one of these designs.



Lesson 2 Summary

A **circle** consists of all of the points that are the same distance away from a particular point called the *center* of the circle.

A segment that connects the center with any point on the circle is called a **radius**. For example, segments QG, QH, QI, and QJ are all radii of Circle 2. (We say one radius and two radii.) The length of any radius is always the same for a given circle. For this reason, people also refer to this distance as the *radius* of the circle.



A segment that connects two opposite points on a circle (passing through the circle's center) is called a **diameter**. For example, segments AB, CD, and EF are all diameters of Circle 1. All diameters in a given circle have the same length because they are composed of two radii. For this reason, people also refer to the length of such a segment as the *diameter* of the circle.

The **circumference** of a circle is the distance around it. If a circle was made of a piece of string and we cut it and straightened it out, the circumference would be the length of that string. A circle always encloses a circular region. The region enclosed by Circle 2 is shaded, but the region enclosed by Circle 1 is not. When we refer to the area of a circle, we mean the area of the enclosed circular region.

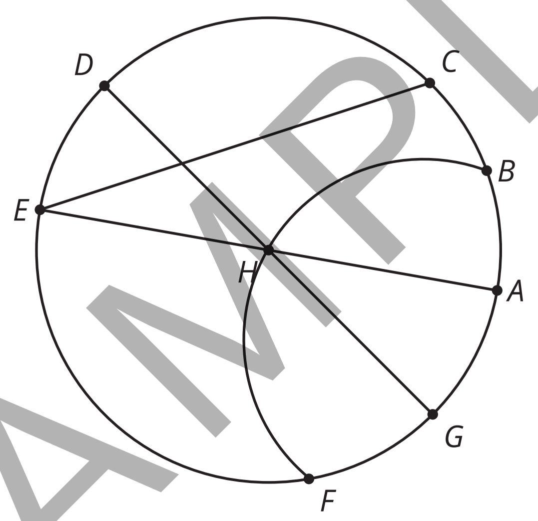
Glossary

- circle
- circumference
- diameter
- radius

Practice Problems

1 Use a geometric tool to draw a circle. Draw and measure a radius and a diameter of the circle.

2 Here is a circle with center H and some line segments and curves joining points on the circle.



Identify examples of the following. Explain your reasoning.

a. Diameter

b. Radius

3 Lin measured the diameter of a circle in two different directions.

- Measuring vertically, she got 3.5 cm.
- Measuring horizontally, she got 3.6 cm.

Explain some possible reasons why these measurements differ.

Sec A

4 from Unit 2, Lesson 1

A small batch of lemonade used $\frac{1}{4}$ cup of sugar added to 1 cup of water and $\frac{1}{4}$ cup of lemon juice. A larger batch is going to be made using 10 cups of water. How much sugar should be added so that the large batch tastes the same as the small batch?

5 from Unit 2, Lesson 13

The graph of a proportional relationship contains the point $(3, 12)$. What is the constant of proportionality of the relationship?

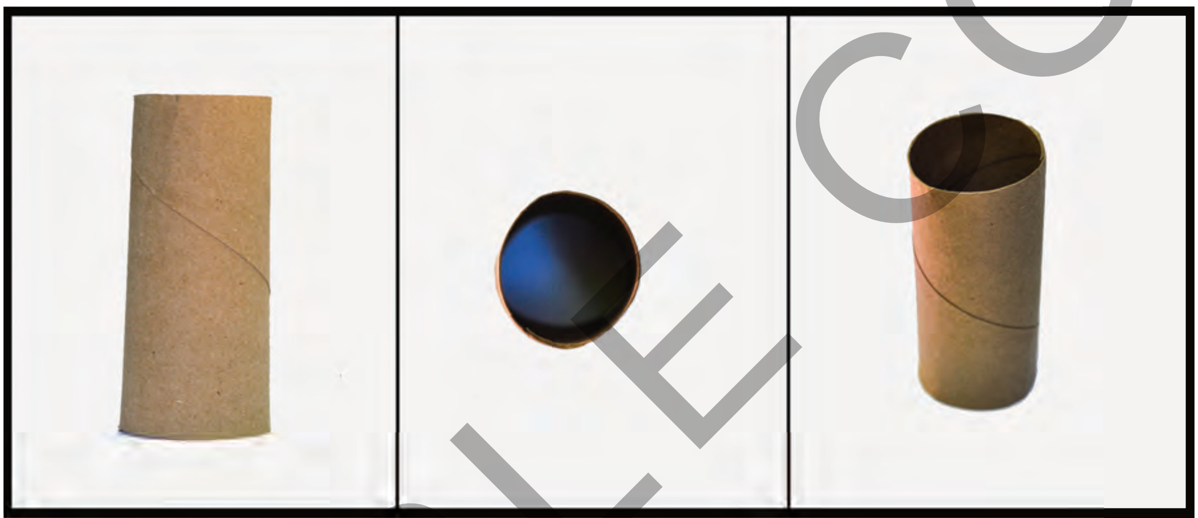
Addressing CA CCSSM 7.G.4, 7.RP.2, 7.RP.2a; building on 2.MD.1-4, 6.SP.5c; building towards 7.G.4; practicing MP7 and MP8

Exploring Circumference

Let's explore the circumference of circles.

3.1 Which Is Greater?

Clare wonders if the height of the toilet paper tube or the distance around the tube is greater. What information would she need in order to solve the problem? How could she find this out?



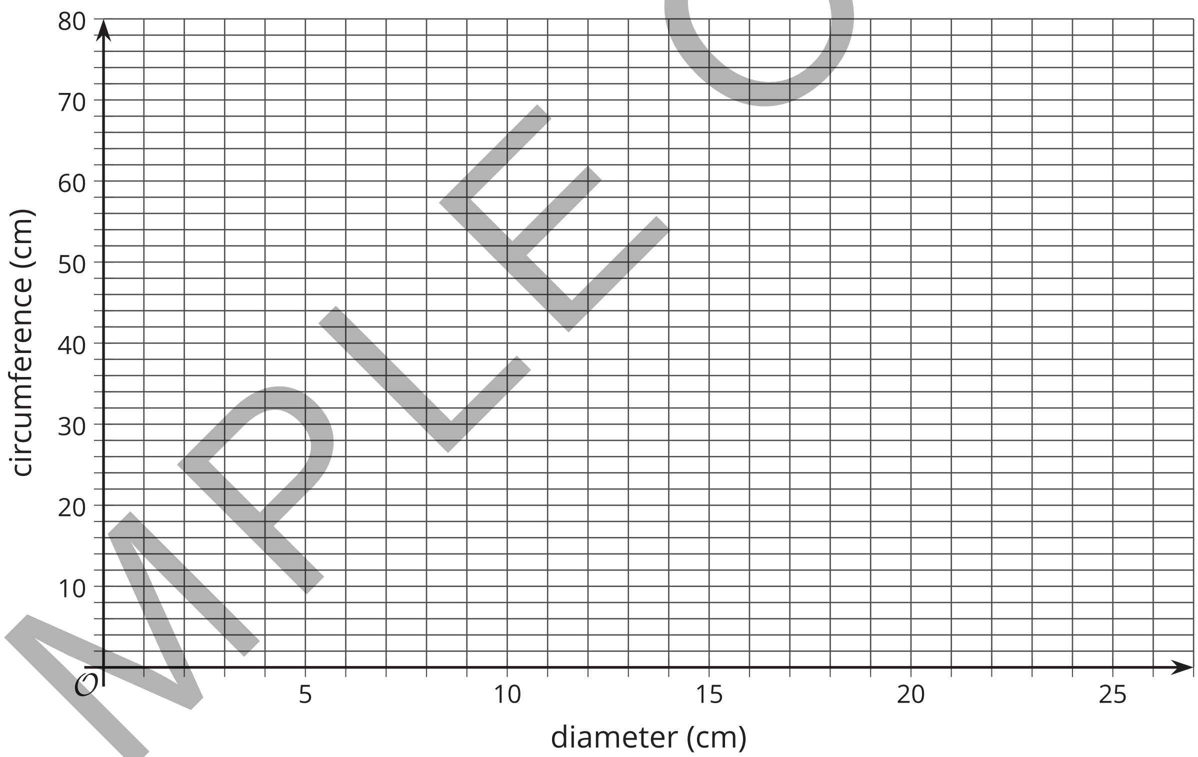
3.2 Measuring Circumference and Diameter

Your teacher will give you two circular objects.

1. Measure the diameter and the circumference of each circle to the nearest tenth of a centimeter. Record your measurements in the first two rows of the table.

object	diameter (cm)	circumference (cm)

2. Plot your diameter and circumference values on the coordinate plane. What do you notice?

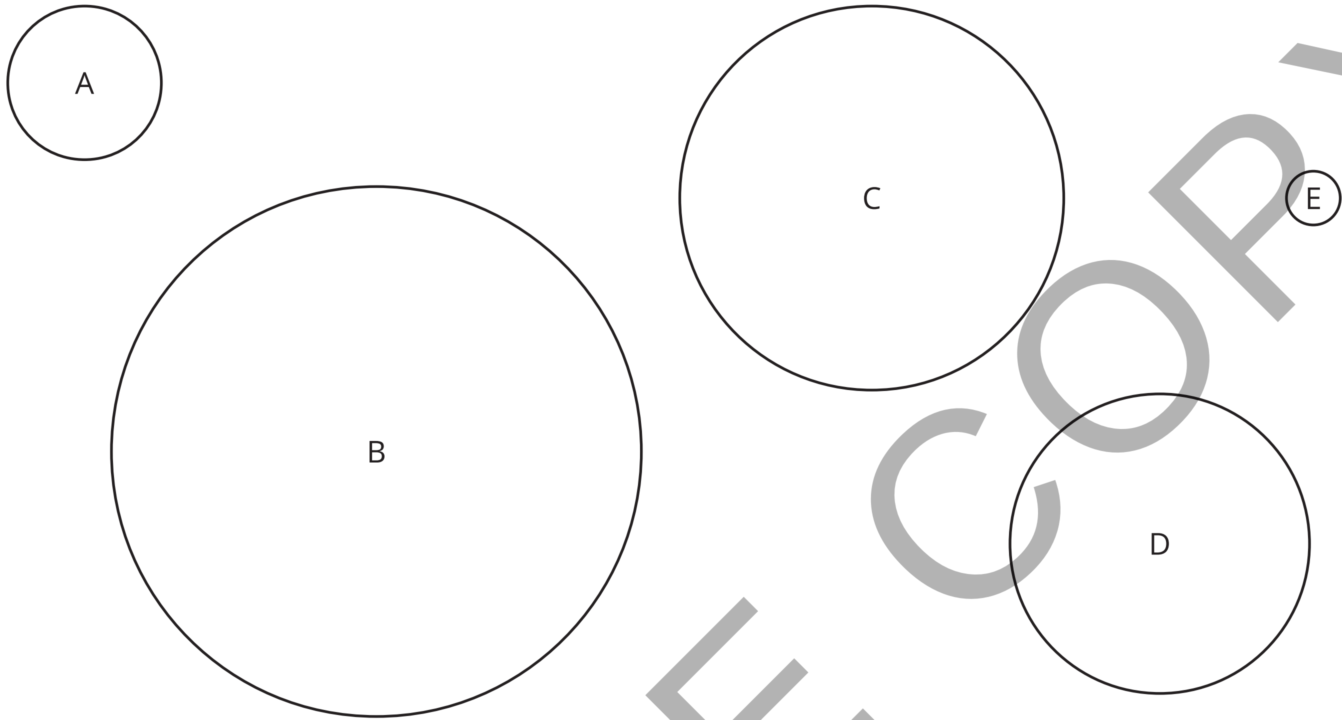


3. Find out the measurements from another group that measured different objects. Record their values in your table and plot them on your same coordinate plane.

4. What do you notice about the diameter and circumference values for these four circles?

3.3 Calculating Circumference and Diameter

Here are five circles. One measurement for each circle is given in the table.



Use the constant of proportionality estimated in the previous activity to complete the table.

	diameter (cm)	circumference (cm)
circle A	3	
circle B	10	
circle C		24
circle D		18
circle E	1	

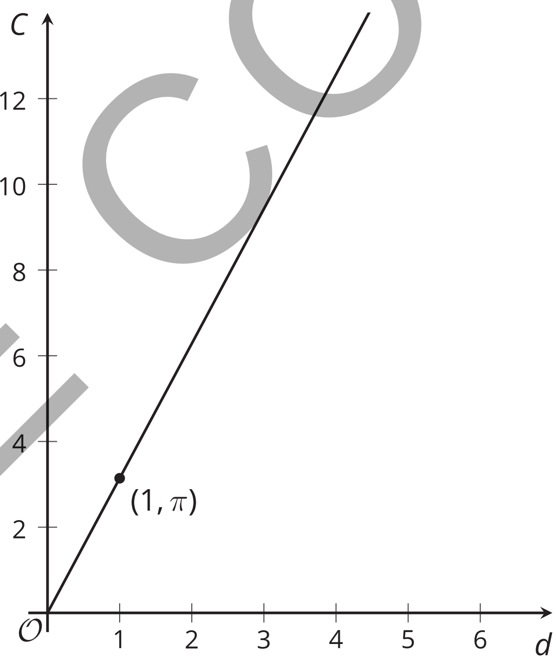
Are you ready for more?

The circumference of Earth is approximately 40,000 km. If you made a circle of wire around the globe, and that wire is only 10 meters (0.01 km) longer than the circumference of the globe, could a flea, a mouse, or even a person creep under it?

Lesson 3 Summary

There is a proportional relationship between the diameter and circumference of any circle. That means that if we write C for circumference and d for diameter, we know that $C = kd$, where k is the constant of proportionality.

The exact value for the constant of proportionality is called pi, and its symbol is π. Some frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these is exactly π.



We can use this to estimate the circumference if we know the diameter, and vice versa. For example, using 3.1 as an approximation for π, if a circle has a diameter of 4 cm, then the circumference is about $(3.1) \cdot 4 = 12.4$, or 12.4 cm.

The relationship between the circumference and the diameter can be written as

$$C = \pi d$$

Glossary

- pi (π)

Practice Problems

1 Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table.

object	diameter (cm)	circumference (cm)
half dollar coin	3	10
flying disc	23	28
jar lid	8	25
flower pot	15	48

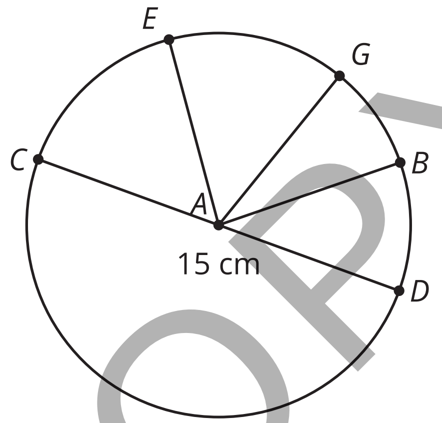
One of his measurements is inaccurate. Which measurement is it? Explain how you know.

2 Complete the table. Use one of the approximate values for π discussed in class (for example 3.14, $\frac{22}{7}$, 3.1416). Explain or show your reasoning.

object	diameter	circumference
hula hoop	35 in	
circular pond		556 ft
magnifying glass	5.2 cm	
car tire		71.6 in

3 from Unit 3, Lesson 2

A is the center of the circle, and the length of CD is 15 centimeters.

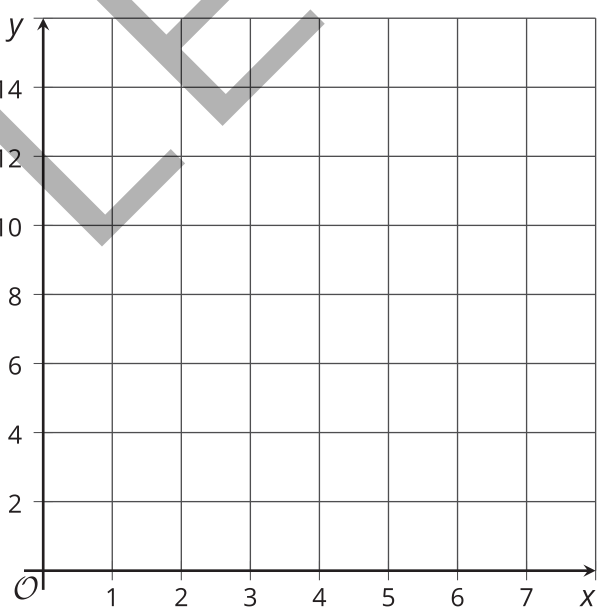


a. Name a segment that is a radius. How long is it?

b. Name a segment that is a diameter. How long is it?

4 from Unit 2, Lesson 10

a. Consider the equation $y = 1.5x + 2$. Find four pairs of x and y values that make the equation true. Plot the points (x, y) on the coordinate plane.



b. Based on the graph, can this be a proportional relationship? Why or why not?

Sec A

Unit 3, Lesson 4

Addressing CA CCSSM 7.G.4; practicing MP1, MP2, and MP6

Applying Circumference

Sec A

Let's use π to solve problems.

4.1 What Do We Know?

For each picture, which measurement is shown? Be prepared to explain your reasoning.

- Wagon wheel: 3 feet

- Plane propeller: 24 inches

- Sliced orange: 20 centimeters

KH | iM Illustrative® Mathematics LEARN MATH FOR LIFE

4.2 Using π

Earlier, we looked at pictures of circular objects. One measurement for each object is listed in the table.

Your teacher will assign you an approximation of π to use for this activity.

1. Complete the table. Be prepared to explain your reasoning.

object	radius	diameter	circumference
wagon wheel		3 ft	
airplane propeller	24 in		
orange slice			20 cm

2. A bug was sitting on the tip of the propeller blade when the propeller started to rotate. The bug held on for 5 rotations before flying away. How far did the bug travel before it flew off?

4.3 Hopi Basket Weaving

Hopi (HOH-pee) weavers make baskets by weaving thin strips of yucca onto a circular willow frame.

Sec A

Sifter Basket

Tray with Handles

1. To make a basket with a radius of $6\frac{1}{2}$ inches, how long does the piece of willow for the circular frame need to be?

2. If a weaver uses a piece of willow that is 33 inches long, what will the radius of the basket be?

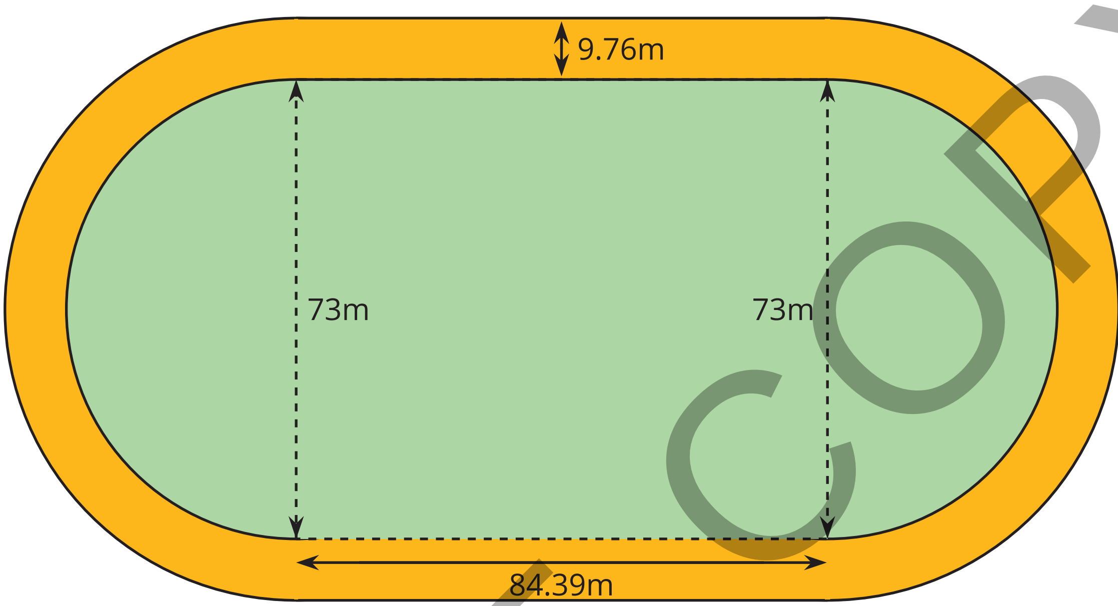
Are you ready for more?

Hopi weavers also make coil plaques that have thin strips of yucca wrapped around a spiraling core.

Describe a method you could use to approximate the length of the spiral core for a plaque made of 19 coils that are 1 cm thick.

4.4 Around the Running Track

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.



1. What is the distance around the inside of the track? Explain or show your reasoning.

2. The track is 9.76 m wide all the way around. What is the distance around the outside of the track? Explain or show your reasoning.

Are you ready for more?

This size running track is usually called a 400-meter track. However, if a person ran as close to the “inside” as possible on the track, they would run less than 400 meters in one lap. How far away from the inside border would someone have to run to make one lap equal exactly 400 meters?

Lesson 4 Summary

The circumference of a circle, C, is π times the diameter, d. The diameter is twice the radius, r. So if we know any one of these measurements for a particular circle, we can find the others. We can write the relationships between these different measures using equations:

$$d = 2r$$
$$C = \pi d$$
$$C = 2\pi r$$

If the diameter of a car tire is 60 cm, that means the radius is 30 cm, and the circumference is $60 \cdot \pi$, or about 188 cm.

If the radius of a clock is 5 in, that means the diameter is 10 in, and the circumference is $10 \cdot \pi$, or about 31 in.

If a ring has a circumference of 44 mm, that means the diameter is $44 \div \pi$, which is about 14 mm, and the radius is about 7 mm.

Practice Problems

1 Here is a picture of a Ferris wheel. It has a diameter of 80 meters.

a. On the picture, draw and label a diameter.

b. How far does a rider travel in one complete rotation around the Ferris wheel?

2 Identify each measurement as the diameter, radius, or circumference of the circular object. Then, estimate the other two measurements for the circle.

a. The length of the minute hand on a clock is 5 inches.

b. The fence around a circular pool is 75 feet long.

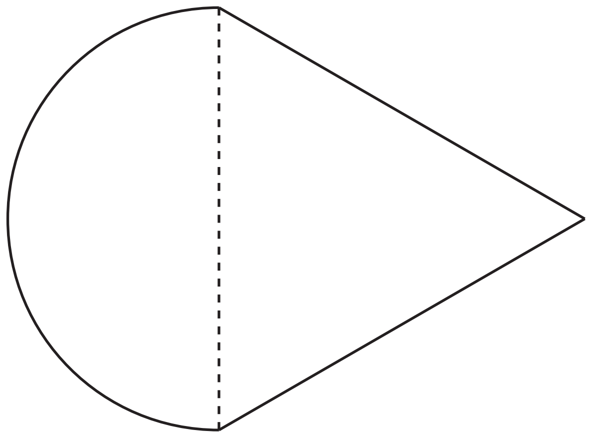
c. The tires on a mining truck are 14 feet tall.

d. The distance from the tip of a slice of pizza to the crust is 7 inches.

e. Folding a tortilla in half creates a straight side 16 centimeters long.

f. The length of the metal rim around a glass lens is 190 millimeters.

3 A half circle is joined to an equilateral triangle with side lengths of 12 units. What is the perimeter of the resulting shape?



4 Circle A has a diameter of 1 foot. Circle B has a circumference of 1 meter. Which circle is bigger? Explain your reasoning. (1 inch = 2.54 centimeters)

5 from Unit 3, Lesson 3

The circumference of Tyler's bike tire is 72 inches. What is the diameter of the tire?

Unit 3, Lesson 5

Addressing CA CCSSM 7.G.4, 7.RP.2a, 7.RP.2c, 7.RP.3; practicing MP1, MP2, MP3, and MP8

Circumference and Wheels

Let's explore how far different wheels roll.

Sec A

5.1 A Rope and a Wheel

Han says that you can wrap a 5-foot rope around a wheel with a 2-foot diameter because $\frac{5}{2}$ is less than pi. Do you agree with Han? Explain your reasoning.

5.2 Rolling, Rolling, Rolling

Your teacher will give you a circular object.

1. Follow these instructions to create the drawing:
   a. On a separate piece of paper, use a ruler to draw a diagonal line all the way across the page.
   b. Roll your object along the line and mark where it completes one rotation.
   c. Use your object to draw tick marks along the line that are spaced as far apart as the diameter of your object.
2. What do you notice?
3. Use your ruler to measure these lengths to the nearest tenth of a centimeter:
   a. the diameter of your object
   b. how far your object rolled in one complete rotation
4. Find the quotient of how far your object rolled divided by its diameter. What do you notice?
5. If you wanted to mark where your object completes 2 rotations, how long a line would you need?
6. Compare your measurements and calculations with another group's that used a different object.
   a. What do you notice?
   b. If both groups rolled their object along the entire length of the classroom, which object would complete the most rotations? Explain or show your reasoning.

5.3 Rotations and Distance

1. A car wheel has a diameter of 20.8 inches.

   a. About how far does the car wheel travel in 1 rotation? 5 rotations? 30 rotations?

   b. Write an equation relating the distance that the car travels in inches, c, to the number of wheel rotations, x.

   c. About how many rotations does the car wheel make when the car travels 1 mile? Explain or show your reasoning.

2. A bike wheel has a radius of 13 inches.

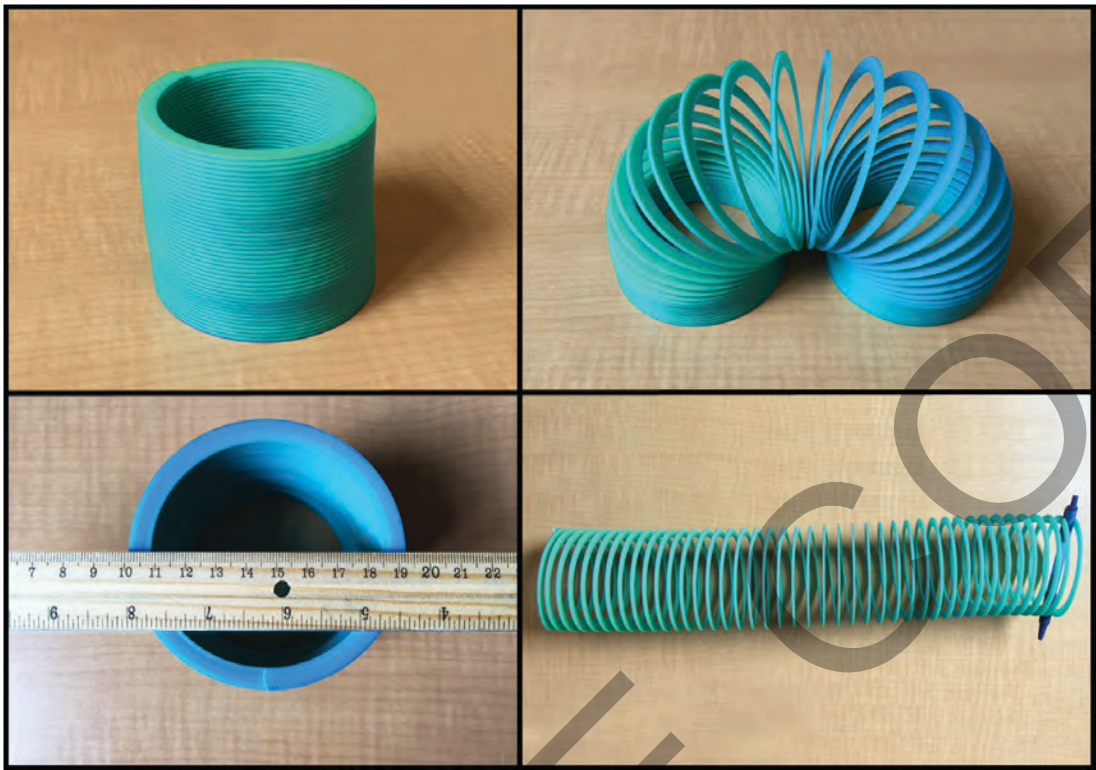
   a. About how far does the bike wheel travel in 1 rotation? 5 rotations? 30 rotations?

   b. Write an equation relating the distance that the bike travels in inches, b, to the number of wheel rotations, x.

   c. About how many rotations does the bike wheel make when the bike travels 1 mile? Explain or show your reasoning.

Are you ready for more?

Here are some photos of a spring toy.



If you could stretch out the spring completely straight, how long would it be? Explain or show your reasoning.

5.4 Rotations and Speed

The circumference of a car wheel is about 65 inches.

1. If the car wheel rotates once per second, how far does the car travel in one minute?

2. If the car wheel rotates once per second, about how many miles does the car travel in one hour?

3. If the car wheel rotates 5 times per second, about how many miles does the car travel in one hour?

4. If the car is traveling 65 miles per hour, about how many times per second does the wheel rotate?

Lesson 5 Summary

The circumference of a circle is the distance around the circle. This is also how far the circle rolls on flat ground in one rotation. For example, a bicycle wheel with a diameter of 2 feet has a circumference of 2π feet, which is about 6.3 feet. This means that the wheel will travel about 6.3 feet in one complete rotation.

Sec A

We can use this relationship to calculate the distance traveled for any number of rotations. Here is a table showing approximately how far the bike travels when the wheel makes different numbers of rotations.

number of rotations	distance traveled in feet
1	6.3
2	12.6
3	18.9
10	63
50	315
x	$6.3x$

In the table, we see that the relationship between the distance traveled and the number of wheel rotations is a proportional relationship. The constant of proportionality is equal to the circumference of the wheel, 2π, or about 6.3.

For this wheel, the equation $d = 6.3x$ gives the distance traveled, d, when the wheel makes x rotations.

For a wheel of any size, the equation is $d = Cx$, where C is the circumference of the wheel.

Practice Problems

1 Find the distance each wheel travels.

a. The circumference of a wagon wheel is 25 inches. The wheel makes 4 complete rotations.

b. The diameter of a bike wheel is 27 inches. The wheel makes 15 complete rotations.

c. The radius of a skateboard wheel is 2.6 centimeters. The wheel makes 100 complete rotations.

2 The wheels on Kiran's bike are 64 inches in circumference. How many times do the wheels rotate if Kiran rides 300 yards?

Sec A

3 from Unit 3, Lesson 4

The numbers are measurements of diameter, radius, and circumference of Circles A and B. Circle A is smaller than Circle B. Which number belongs to which quantity?

2.5 5 7.6 15.2 15.7 47.7

diameter of Circle A: _________

radius of Circle A: _________

circumference of Circle A: _________

diameter of Circle B: _________

radius of Circle B: _________

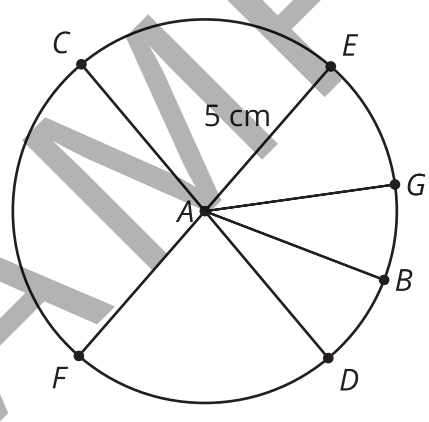
circumference of Circle B: _________

4 from Unit 3, Lesson 3

Circle A has circumference $2\frac{2}{3}$ m. Circle B has a diameter that is $1\frac{1}{2}$ times as long as Circle A's diameter. What is the circumference of Circle B?

5 from Unit 3, Lesson 2

A is the center of the circle, and the length of AE is 5 centimeters.



a. What is the length of segment CD?

b. What is the length of segment AB?

c. Name a segment that has the same length as segment AB.

Addressing CA CCSSM 7.G.1, 7.G.6; building on 5.OA.1-2; practicing MP4 and MP6

Estimating Areas

Let's estimate the areas of weird shapes.

6.1 Math Talk: Mental Calculations

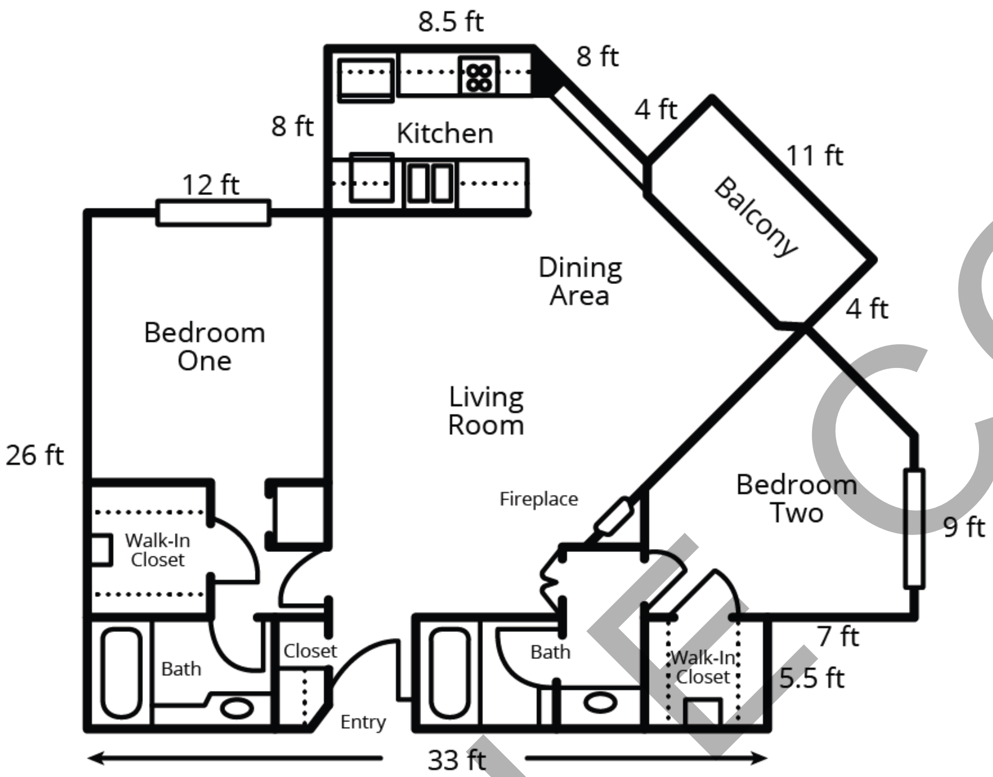
Find the value of each expression mentally.

- $599 + 87$
- $48 + 313$
- $440 - 29$
- $254 - 88$

Sec B

6.2 House Floor Plan

Here is a floor plan of a house. Approximate lengths of the walls are given. What is the approximate area of the home, including the balcony? Explain or show your reasoning.

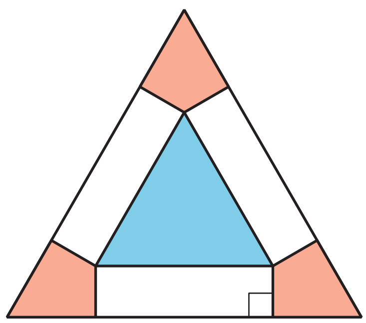


6.3 Area of Nevada

Estimate the area of Nevada in square miles. Explain or show your reasoning.

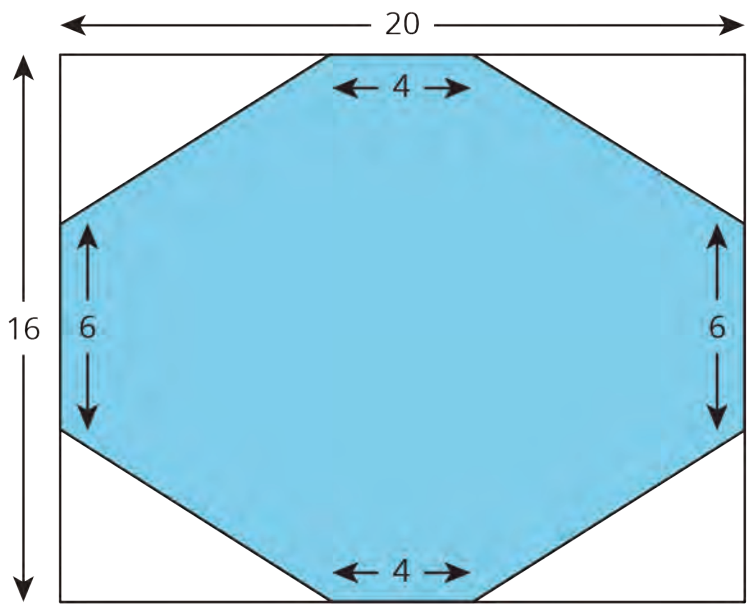
Are you ready for more?

The two triangles are equilateral, and the three pink regions are identical. The blue equilateral triangle has the same area as the three pink regions taken together. What is the ratio of the sides of the two equilateral triangles?



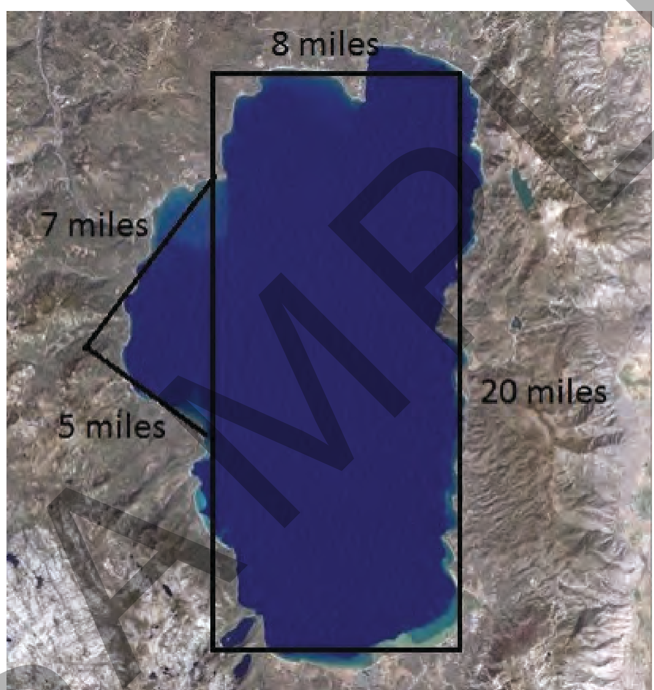
Lesson 6 Summary

We can find the area of some complex polygons by surrounding them with a simple polygon like a rectangle. For example, this octagon is contained in a rectangle.



The rectangle is 20 units long and 16 units wide, so its area is 320 square units. To get the area of the octagon, we need to subtract the areas of the four right triangles in the corners. These triangles are each 8 units long and 5 units wide, so they each have an area of 20 square units. The area of the octagon is $320 - (4 \cdot 20)$, or 240 square units.

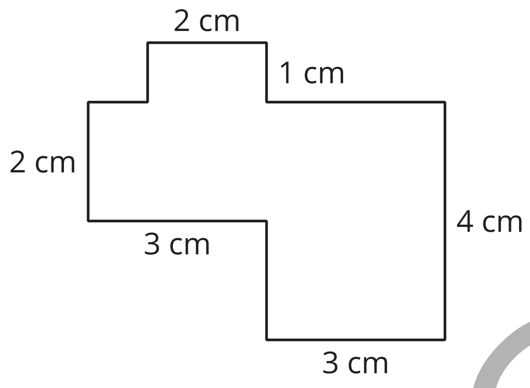
We can estimate the area of irregular shapes by approximating them with a polygon and finding the area of the polygon. For example, here is a satellite picture of Lake Tahoe with some one-dimensional measurements around the lake.



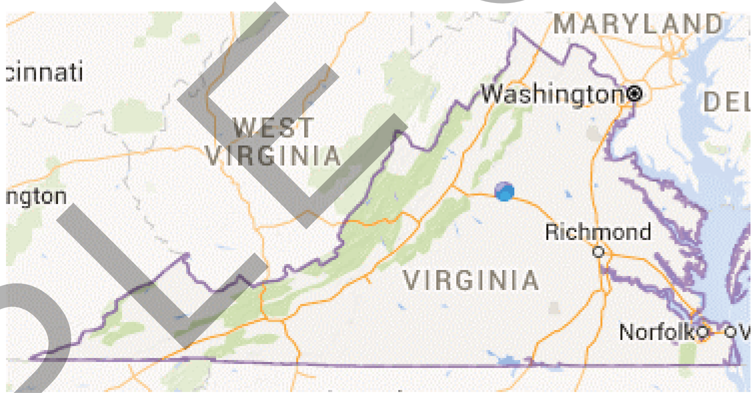
The area of the rectangle is 160 square miles, and the area of the triangle is 17.5 square miles for a total of 177.5 square miles. We recognize that this is an approximation, and not likely the exact area of the lake.

Practice Problems

1 Find the area of the polygon.



2 a. Draw polygons on the map that could be used to approximate the area of Virginia.



b. Which measurements would you need to know in order to calculate an approximation of the area of Virginia? Label the sides of the polygons whose measurements you would need. (Note: You aren't being asked to calculate anything.)

Sec B

3 Estimate the area of Idaho. Explain your reasoning.

4 from Unit 3, Lesson 4

The radius of Earth is approximately 6,400 km. The equator is the circle around Earth dividing it into the northern and southern hemispheres. (The center of the earth is also the center of the equator.) What is the length of the equator?

5 from Unit 3, Lesson 5

Jada's bike wheels have a diameter of 20 inches. How far does she travel if the wheels rotate 37 times?

Unit 3, Lesson 7

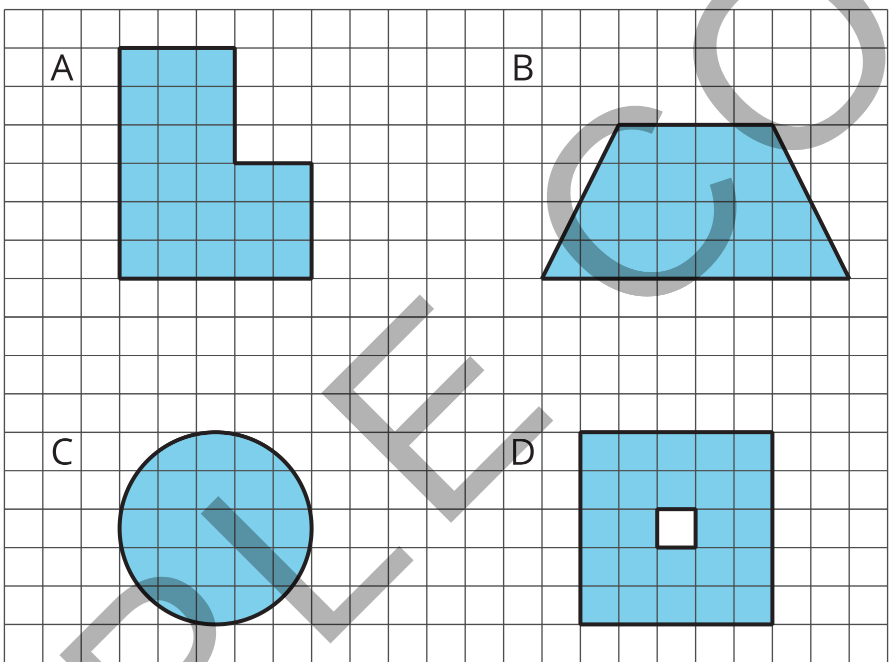
Addressing CA CCSSM 7.G.1, 7.G.4, 7.RP.2a; building towards 7.G.4; practicing MP5, MP6, and MP8

Exploring the Area of a Circle

Let’s investigate the areas of circles.

7.1 Which Three Go Together: Figures on a Grid

Which three go together? Why do they go together?



Sec B

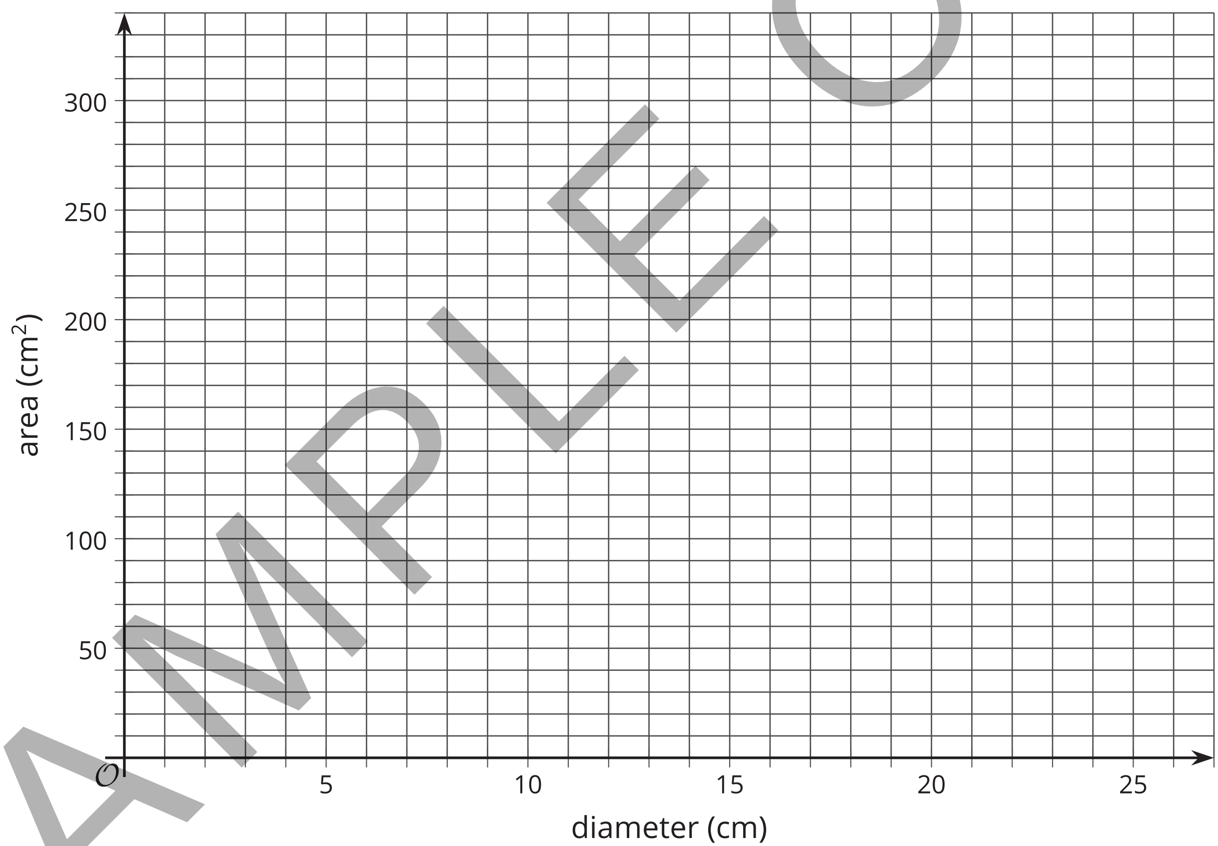
7.2 Estimating Areas of Circles

Your teacher will give your group two circles of different sizes.

1. For each circle,
   a. Use the grid squares to determine the diameter of the circle.
   b. Estimate the **area of the circle**.
   c. Record your measurements in the first two rows of the table.

diameter (cm)	estimated area (cm^2)

2. Plot your diameter and area values on the coordinate plane. What do you notice?



3. Find out the measurements from another group that measured different circles. Record their values in your table, and plot them on your same coordinate plane.

4. Earlier, you graphed the relationship between the diameter and circumference of a circle. How is this graph the same? How is it different?

Are you ready for more?

How many circles of radius 1 unit can you fit inside each of the following so that they do not overlap?

1. a circle of radius 2 units?

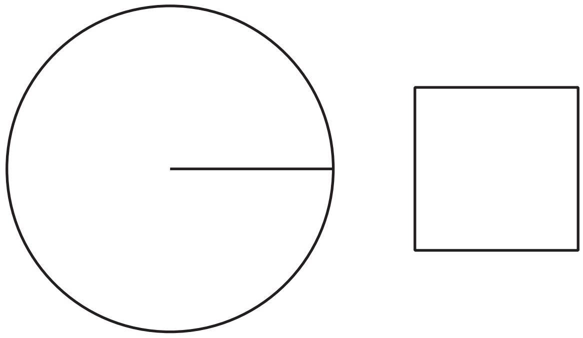
2. a circle of radius 3 units?

3. a circle of radius 4 units?

If you get stuck, consider using coins or other circular objects.

7.3 Covering a Circle

Here is a square whose side length is the same as the radius of the circle.



How many of these squares do you think it would take to cover the circle exactly?

Sec B

Lesson 7 Summary

The circumference C of a circle is proportional to the diameter d, and we can write this relationship as $C = \pi d$. The circumference is also proportional to the radius of the circle, and the constant of proportionality is $2 \cdot \pi$ because the diameter is twice as long as the radius. However, the **area of a circle** is *not* proportional to the diameter (or the radius).

The area of a circle with radius r is a little more than 3 times the area of a square with side r so the area of a circle of radius r is approximately $3r^2$. We saw earlier that the circumference of a circle of radius r is $2\pi r$. If we write C for the circumference of a circle, this proportional relationship can be written $C = 2\pi r$.

The area A of a circle with radius r is approximately $3r^2$. Unlike the circumference, the area is not proportional to the radius because $3r^2$ cannot be written in the form kr for a number k. We will investigate and refine the relationship between the area and the radius of a circle in future lessons.

Glossary

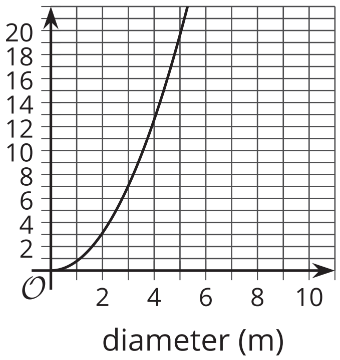
- area of a circle

Practice Problems

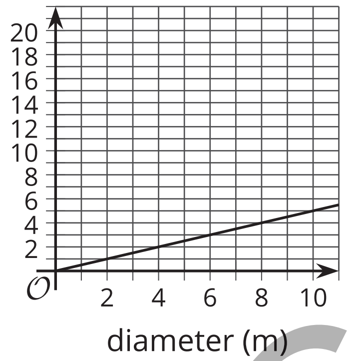
1 The x-axis of each graph has the diameter of a circle in meters. Label the y-axis on each graph with the appropriate measurement of a circle:

radius (m), circumference (m), or area (m^2).

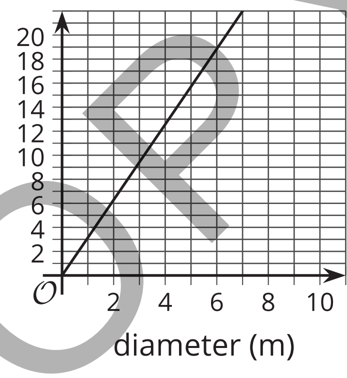
A



B



C



2

a. Priya drew a circle with a circumference of 25 cm. Clare drew a circle with a diameter that is 3 times the diameter of Priya's circle.

What is the circumference of Clare's circle?

b. Noah drew a circle with an area of 500 in^2. Han drew a circle with a diameter that is 3 times the diameter of Noah's circle.

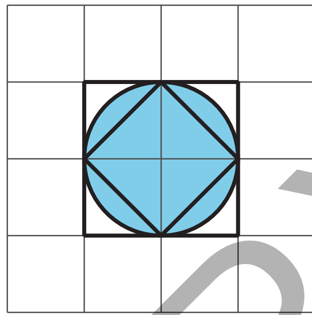
Estimate the area of Han's circle. Explain your reasoning.

Sec B

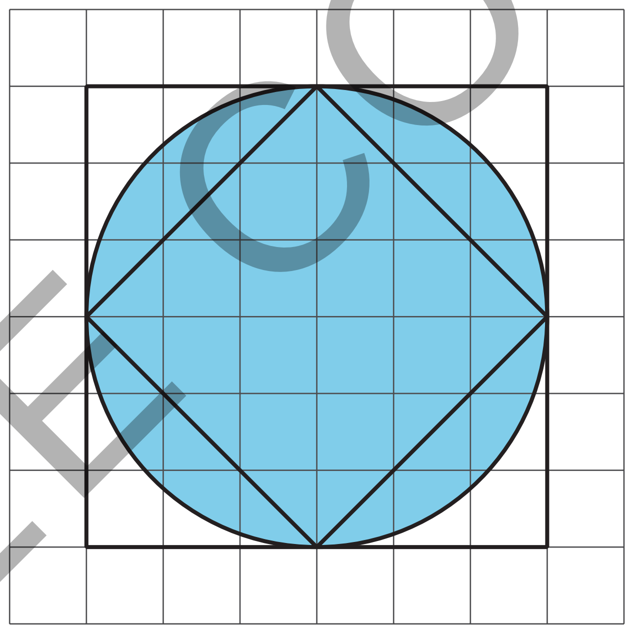
Sec B

3 Each picture shows two squares and a circle.

a. Explain why the area of this circle is more than 2 square units but less than 4 square units.

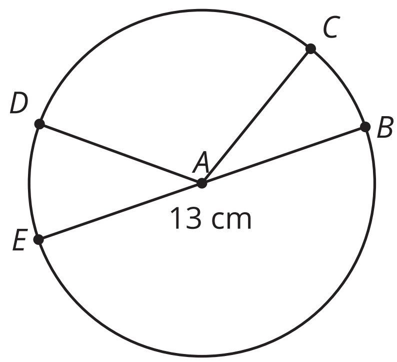


b. Explain why the area of this circle is more than 18 square units and less than 36 square units.



4 from Unit 3, Lesson 3

Point A is the center of the circle, and the length of EB is 13 centimeters.



a. What is the radius of this circle?

b. What is the circumference of this circle?

5 from Unit 3, Lesson 4

The Carousel on the National Mall has 4 rings of horses. Kiran is riding on the inner ring, which has a radius of 9 feet. Mai is riding on the outer ring, which is 8 feet farther out from the center than the inner ring is.

a. In one rotation of the carousel, how much farther does Mai travel than Kiran?

b. One rotation of the carousel takes 12 seconds. How much faster does Mai travel than Kiran?

6 from Unit 3, Lesson 5

Lin's bike travels 100 meters when her wheels rotate 55 times. What is the circumference of her wheels?

7 from an earlier course

Each student has a goal to read for 40 minutes. How many minutes has each student read so far?

a. Elena has read for 25% of the goal.

b. Tyler has read for 75% of the goal.

c. Jada has read for 150% of the goal.

Unit 3, Lesson 8

Addressing CA CCSSM 7.G.4; practicing MP3, MP7, and MP8

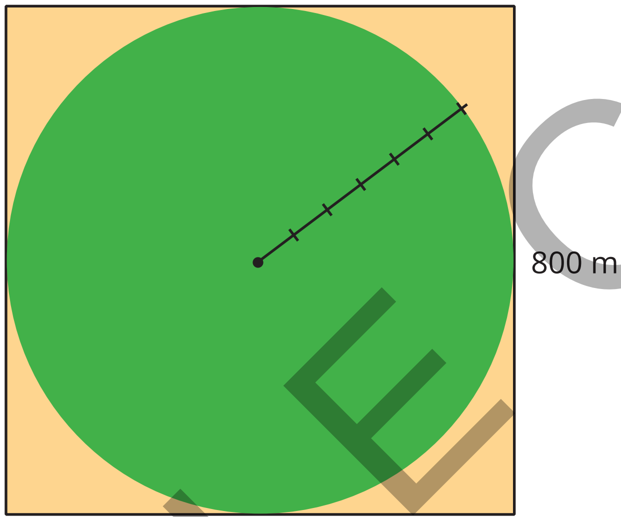
Relating Area to Circumference

Let's rearrange circles to calculate their areas.

8.1 Irrigating a Field

Sec B

A circular field is set into a square with an 800-m side length.



What is the field's area? Record an estimate that is:

too low	about right	too high

8.2 Making a Polygon out of a Circle

Your teacher will give you a page with two circles on it and a piece of paper that is a different color.

Follow these instructions to create a visual display:

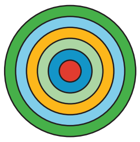
1. Cut out both circles, cutting around the thick outline.
2. Fold and cut *one* of the circles into fourths.
3. Arrange the fourths so that straight sides are next to each other, but the curved edges are alternately on top and on bottom.
   Pause here so your teacher can review your work.
4. Fold and cut the fourths in half to make eighths. Arrange the eighths next to each other, like you did with the fourths.
   Pause here so your teacher can review your work.
5. Glue the remaining circle and the new shape onto a piece of paper that is a different color.

After you finish gluing your shapes, answer the following questions.

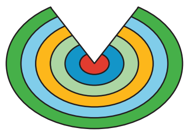
1. How do the areas of the two shapes compare?
2. What polygon does the shape made of the circle pieces most resemble?
3. How could you find the area of this polygon?

8.3 Making Another Polygon out of a Circle

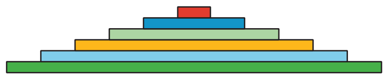
Imagine a circle made of rings that can bend, but not stretch.



A circle is made of rings.



The rings are cut and unrolled.



The circle has been made into a new shape.

Sec B

1. What polygon does the new shape resemble?

2. How does the area of the polygon compare to the area of the circle?

3. How can you find the area of the polygon?

4. Show, in detailed steps, how you could find the polygon's area in terms of the circle's measurements. Show your thinking. Organize it so it can be followed by others.

8.4 Objects for a Powwow

Here are some special objects that might be seen at a Lakota powwow, or *wacipi* (wah-CHEE-pee).

1. A hoop drum has a radius of 7 inches. What is the area of the drum?

2. A beaded medallion has a diameter of 6 centimeters. What is the area of the medallion?

Sec B

Are you ready for more?

If each bead covers about 3.5 mm^2, how many beads are there on the medallion?

Lesson 8 Summary

If C is a circle's circumference and r is its radius, then $C = 2\pi r$. The area of a circle can be found by taking the product of half the circumference and the radius.

If A is the area of the circle, this gives the equation:

$$A = \frac{1}{2}(2\pi r) \cdot r$$

This equation can be rewritten as:

$$A = \pi r^2$$

Remember that when we have $r \cdot r$ we can write r^2, and we can say "r **squared**."

This means that if we know the radius, we can find the area. For example, if a circle has a radius of 10 cm, then its area is about $(3.14) \cdot 100$ which is 314 cm^2.

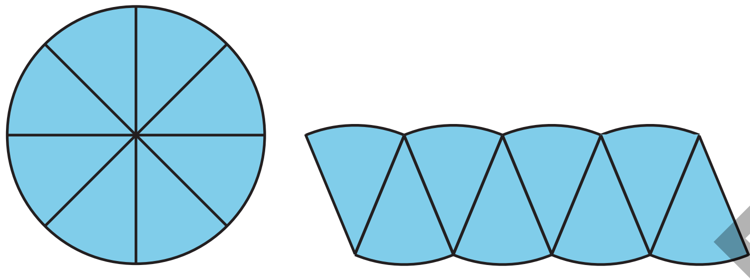
If we know the diameter, we can figure out the radius, and then we can find the area. For example, if a circle has a diameter of 30 ft, then the radius is 15 ft, and the area is about $(3.14) \cdot 225$ which is approximately 707 ft^2.

Glossary

- squared

Practice Problems

1 The picture shows a circle divided into 8 equal wedges, which are rearranged.



The radius of the circle is r, and its circumference is $2\pi r$. How does the picture help to explain why the area of the circle is πr^2?

2 A paper plate has a radius of 4.5 inches. What is the area of the plate?

3 Jada paints a circular table that has a diameter of 37 inches. What is the area of the table?

4 A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle.

Sec B

5 from Unit 3, Lesson 7

Which of these pairs of quantities are proportional to each other? For the quantities that are proportional, what is the constant of proportionality?

a. Radius and diameter of a circle

b. Radius and circumference of a circle

c. Radius and area of a circle

d. Diameter and circumference of a circle

e. Diameter and area of a circle

6 from Unit 3, Lesson 5

Here are the diameters of four coins:

coin	penny	nickel	dime	quarter
diameter	1.9 cm	2.1 cm	1.8 cm	2.4 cm

a. A coin rolls a distance of 33 cm in 5 rotations. Which coin is it?

b. A quarter makes 8 rotations. How far did it roll?

c. A dime rolls 41.8 cm. How many rotations did it make?

7 from an earlier course

Andre has a goal to exercise for 60 minutes this week. How many minutes has he exercised by each day?

a. By Monday he has exercised for 10% of his goal.

b. By Wednesday he has exercised for 60% of his goal.

c. By Saturday he has exercised for 130% of his goal.

Unit 3, Lesson 9

Addressing CA CCSSM 7.G.4; practicing MP2, MP6, and MP7

Applying Area of Circles

Let's find the areas of shapes made up of circles.

Sec B

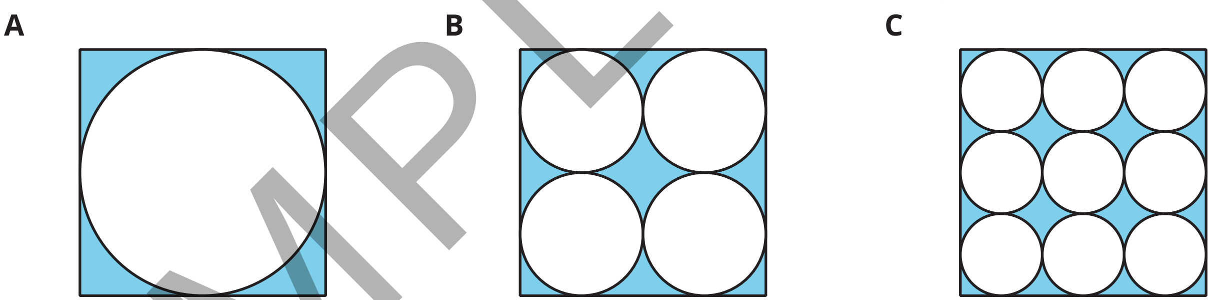
9.1 Math Talk: Expressions with Variables

For each expression, find an equivalent expression with fewer terms.

- $a + a + a + a$
- $a + a + a + b + b$
- $9x - x$
- $5 + 6x + 7$

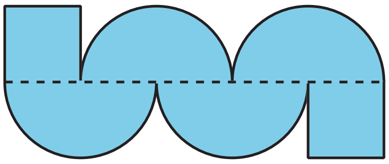
9.2 Comparing Areas Made of Circles

1. Each square has a side length of 12 units. Compare the areas of the shaded regions in the 3 figures. Which figure has the largest shaded region? Explain or show your reasoning.

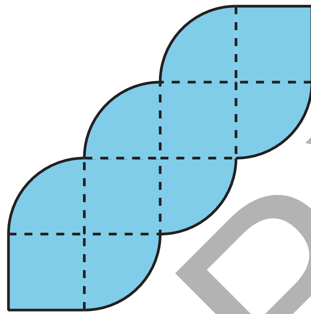


2. Each square in Figures D and E has a side length of 1 unit. Compare the area of the two figures. Which figure has more area? How much more? Explain or show your reasoning.

D



E

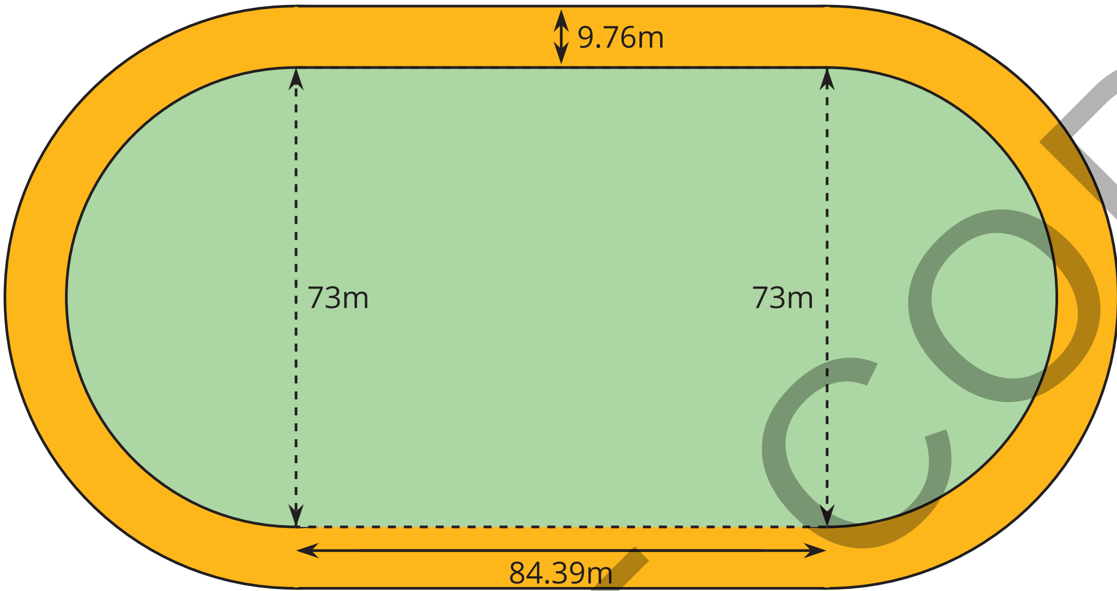


Are you ready for more?

Which figure has a longer perimeter, Figure D or Figure E? How much longer?

9.3 The Running Track Revisited

The field inside a running track is made up of a rectangle 84.39 m long and 73 m wide, together with a half-circle at each end. The running lanes are 9.76 m wide all the way around.



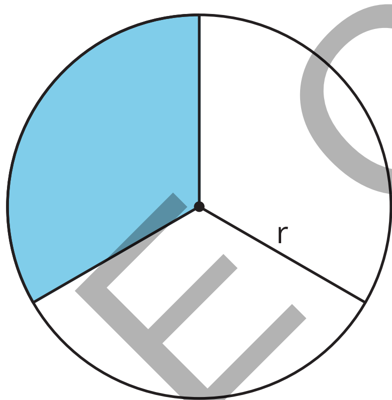
What is the area of the running track that goes around the field? Explain or show your reasoning.

Lesson 9 Summary

The relationship between A, the area of a circle, and r, its radius, is $A = \pi r^2$. We can use this to find the area of a circle if we know the radius. For example, if a circle has a radius of 10 cm, then the area is $\pi \boldsymbol{\cdot} 10^2$, or 100π cm^2. We can also use the formula to find the radius of a circle if we know the area. For example, if a circle has an area of 49π m^2 then its radius is 7 m and its diameter is 14 m.

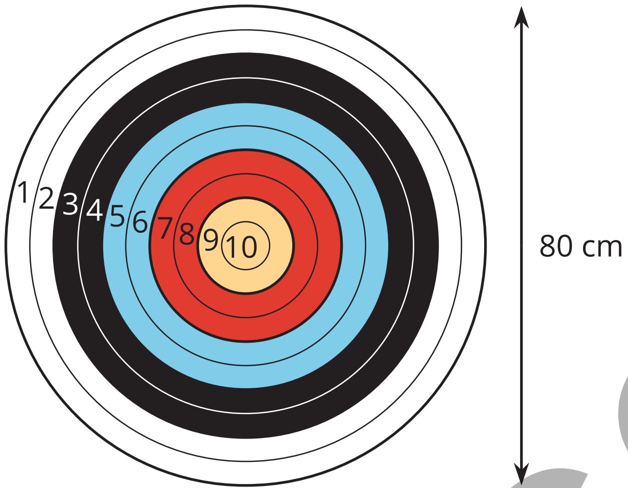
Sometimes instead of leaving π in expressions for the area, a numerical approximation can be helpful. For the examples above, a circle of radius 10 cm has an area of about 314 cm^2. In a similar way, a circle with an area of 154 m^2 has a radius of about 7 m.

We can also figure out the area of a fraction of a circle. For example, the figure shows a circle divided into 3 pieces of equal area. The shaded part has an area of $\frac{1}{3}\pi r^2$.



Practice Problems

1 There are 10 rings on an archery target. Each ring is 4 cm wide.



Find the area of:

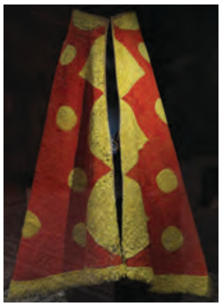
a. the yellow region (rings 9 and 10)

b. the red region (rings 7 and 8)

c. the black region (rings 3 and 4)

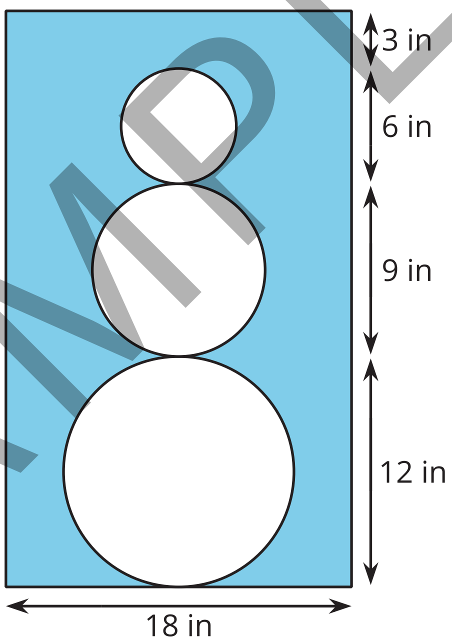
2 A circle with a 12-inch diameter is folded in half and then folded in half again. What is the area of the resulting shape?

3 Some Hawaiian chiefs wore capes that were covered with red, yellow, and black feathers.



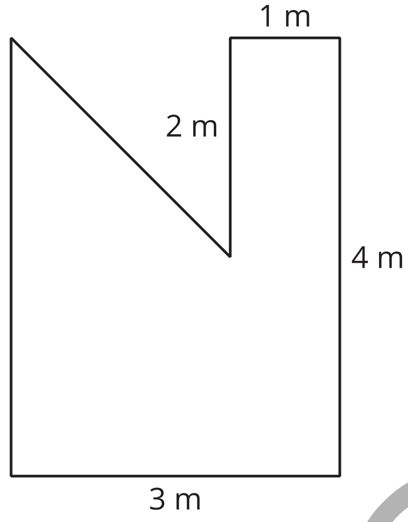
Describe how you could estimate the area of this cape.

4 Find the area of the shaded region. Express your answer in terms of π.



5 from Unit 3, Lesson 6

Find the area of this shape in two different ways.



6 from Unit 2, Lesson 5

Elena and Jada both read at a constant rate, but Elena reads more slowly. For every 4 pages that Elena can read, Jada can read 5.

a. Complete the table.

pages read by Elena	pages read by Jada
4	5
1	
9	
e	
	15
	j

b. Here is an equation for the table: $j = 1.25e$. What does the 1.25 mean?

c. Write an equation for this relationship that starts $e = ...$

Unit 3, Lesson 10

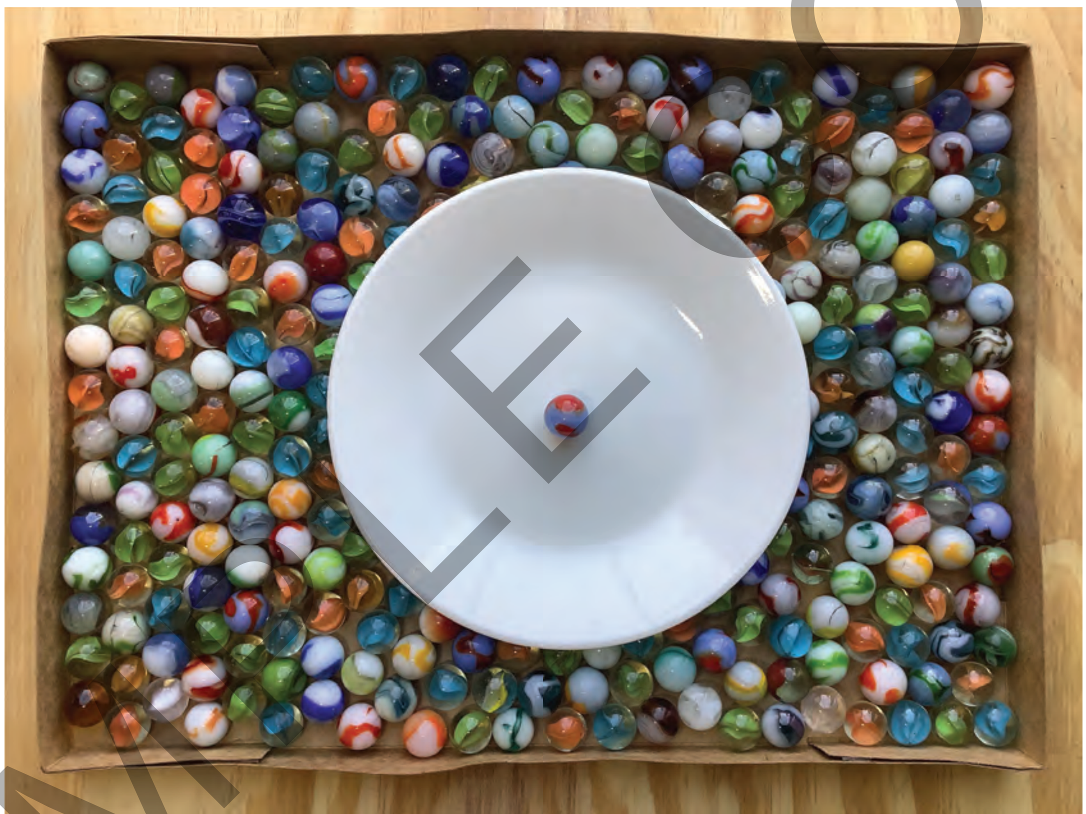
Addressing CA CCSSM 7.G.4; practicing MP2, MP3, and MP6

Distinguishing Circumference and Area

Let's contrast circumference and area.

10.1 A Plate of Marbles

About how many marbles can fit on the plate in a single layer? Be prepared to explain your reasoning.



10.2 Card Sort: Circle Problems

Your teacher will give you a set of cards with questions about circles.

1. Take turns with your partner to sort the cards into two groups based on whether you would use the circumference or the area of the circle to answer the question
   a. For each card that your partner sorts, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
   b. For each card that you sort, explain to your partner how you know which group it goes in.
2. Your teacher will assign you one card to examine more closely. What additional information would you need in order to answer the question on your card?

3. Estimate measurements for the circle that is described on your card.

4. Use your estimates to calculate the answer to the question.

10.3 Visual Display of Circle Problem

In the previous activity you estimated the answer to a question about circles.

Create a visual display that includes:

- The question you were answering.
- A diagram of a circle labeled with your estimated measurements.
- Your thinking, organized so that others can follow it.
- Your answer, expressed in terms of π and also expressed as a decimal approximation.

10.4 Analyzing Circle Claims

Here are two students' answers for each question. Do you agree with either of them? Explain or show your reasoning.

1. How many feet are traveled by a person riding once around the merry-go-round?

   - Clare says, "The radius of the merry-go-round is about 4 feet, so the distance around the edge is about 8π feet."
   - Andre says, "The diameter of the merry-go-round is about 4 feet, so the distance around the edge is about 4π feet."

2. How much room is there to put glue on the back of a paper circle?

   - Clare says "The radius of the circle is about 3 centimeters, so the space for glue is about 6π cm^2."
   - Andre says "The diameter of the circle is about 3 inches, so the space for glue is about 2.25π in^2."

Sec C

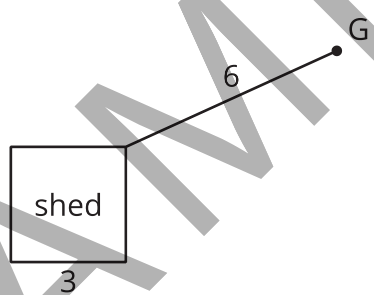
3. How far does the unicycle move when the wheel makes 5 full rotations?

- Clare says, "The diameter of the unicycle wheel is about 0.5 meters. In 5 complete rotations it will go about $\frac{5}{2}\pi$ m^2."
- Andre says, "I agree with Clare's estimate of the diameter, but that means the unicycle will go about $\frac{5}{4}\pi$ m."

Sec C

Are you ready for more?

A goat (point G) is tied with a 6-foot rope to the corner of a shed. The floor of the shed is a square whose sides are each 3 feet long. The shed is closed and the goat can't go inside. The space all around the shed is flat and grassy, and the goat can't reach any other structures or objects. What is the area over which the goat can roam?



10.5 Info Gap: Merry-go-round and Unicycle

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Lesson 10 Summary

Sometimes we need to find the circumference of a circle, and sometimes we need to find the area.

Here are some examples of quantities related to the circumference of a circle:

- The length of a circular path.
- The distance a wheel will travel after one complete rotation.
- The length of a piece of rope coiled in a circle.

Here are some examples of quantities related to the area of a circle:

- The amount of land that is cultivated on a circular field.
- The amount of frosting needed to cover the top of a round cake.
- The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to make the calculation. The circumference of a circle with radius r is $2\pi r$ while its area is πr^2. The circumference is measured in linear units (such as cm, in, km) while the area is measured in square units (such as cm^2, in^2, km^2).

Practice Problems

1 For each problem, decide whether the circumference of the circle or the area of the circle is most useful for finding a solution. Explain your reasoning.

a. A car's wheels spin at 1000 revolutions per minute. You want to know how fast the car is traveling.

b. A circular kitchen table has a diameter of 60 inches. You want to know how much fabric is needed to cover the table top.

c. A circular puzzle is 20 inches in diameter. All of the pieces are about the same size. You want to know about how many pieces there are in the puzzle.

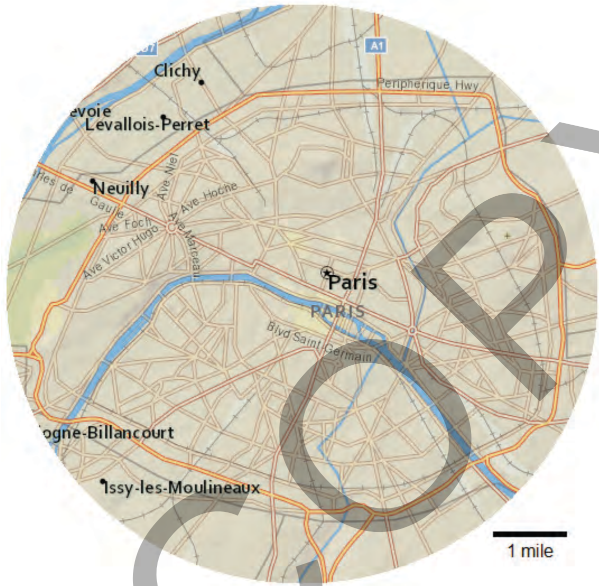
d. You want to know about how long it takes to walk around a circular pond.

2 from Unit 3, Lesson 8

The face of a clock has a circumference of 63 in. What is the area of the face of the clock?

3 The city of Paris, France, is completely contained within an almost circular road that goes around the edge. Use the map with its scale to:

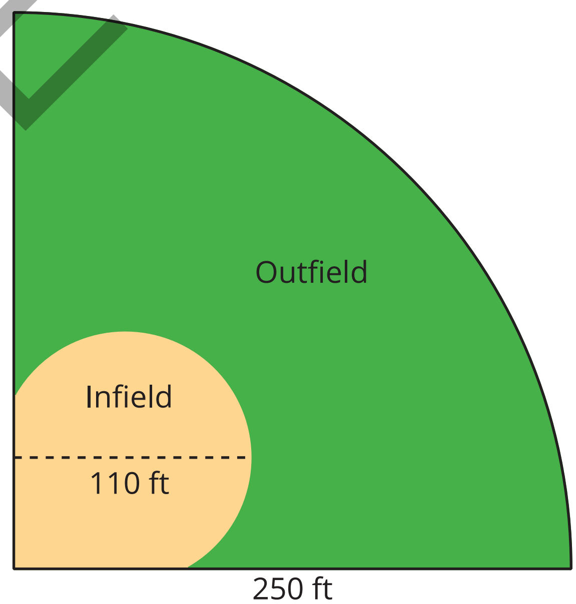
a. Estimate the circumference of Paris.



b. Estimate the area of Paris.

4 Here is a diagram of a softball field:

a. About how long is the fence around the field?



b. About how big is the outfield?

5 from Unit 2, Lesson 5

While in math class, Priya and Kiran come up with two ways of thinking about the equation $5k = 1750$.

- Priya says, "I can solve this equation by dividing 1,750 by 5."
- Kiran says, "I can solve this equation by multiplying 1,750 by $\frac{1}{5}$."

a. What value of k would each student get using their own method?

b. How are Priya and Kiran's approaches related?

c. Explain how each student might approach solving the equation $\frac{2}{3}k = 50$.

Sec C

6 from an earlier course

A moving company needs to load 350 boxes onto a truck.

a. So far, they have loaded 64% of all the boxes. How many boxes have they loaded?

b. There are labels that say "heavy" on 70 of the boxes. What percentage of all the boxes are labeled "heavy"?

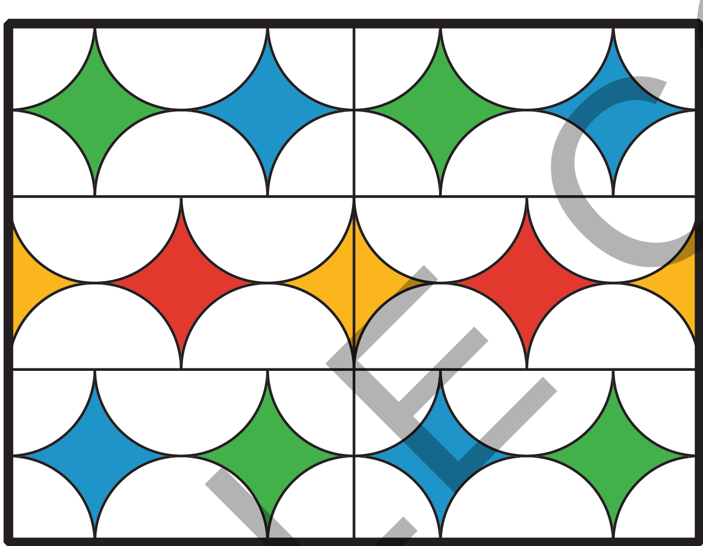
Addressing CA CCSSM 7.EE.3, 7.G.1, 7.G.4; practicing MP1 and MP4

Stained-Glass Windows

Let's use circumference and area to design stained-glass windows.

11.1 Cost of a Stained-Glass Window

The students in art class are designing a stained-glass window to hang in the school entryway. The window will be 3 feet tall and 4 feet wide. Here is their design.



- They have raised \$100 for the project.
- The colored glass costs \$5 per square foot.
- The clear glass costs \$2 per square foot.
- The material they need to join the pieces of glass together costs 10 cents per foot.
- The frame around the window costs \$4 per foot.

Do they have enough money to cover the cost of making the window?

11.2 A Bigger Window

A local community member sees the school's stained-glass window and really likes the design. They ask the students to create a larger copy of the window using a scale factor of 3.

Would \$450 be enough to buy the materials for the larger window? Explain or show your reasoning.

Sec C

11.3 Invent Your Own Design

Draw a stained-glass window design that could be made for less than \$450. Show your thinking. Organize your work so it can be followed by others.

Learning Targets

Lesson 1 How Well Can You Measure?

- I can examine quotients and use a graph to decide whether two associated quantities are in a proportional relationship.
- I understand that it can be difficult to measure the quantities in a proportional relationship accurately.

Lesson 2 Exploring Circles

- I can describe the characteristics that make a shape a circle.
- I can identify the diameter, center, radius, and circumference of a circle.

Lesson 3 Exploring Circumference

- I can describe the relationship between circumference and diameter of any circle.
- I can explain what π means.

Lesson 4 Applying Circumference

- I can choose an approximation for π based on the situation or problem.
- If I know the radius, diameter, or circumference of a circle, I can find the other two.

Lesson 5 Circumference and Wheels

- If I know the radius or diameter of a wheel, I can find the distance the wheel travels in some number of revolutions.

Lesson 6 Estimating Areas

- I can calculate the area of a complicated shape by breaking it into shapes whose area I know how to calculate.

Lesson 7 Exploring the Area of a Circle

- If I know a circle's radius or diameter, I can find an approximation for its area.
- I know whether or not the relationship between the diameter and area of a circle is proportional and can explain how I know.

Lesson 8 Relating Area to Circumference

- I can explain how the area of a circle and its circumference are related to each other.
- I know the formula for area of a circle.

Lesson 9 Applying Area of Circles

- I can calculate the area of more complicated shapes that include fractions of circles.
- I can write exact answers in terms of π.

Lesson 10 Distinguishing Circumference and Area

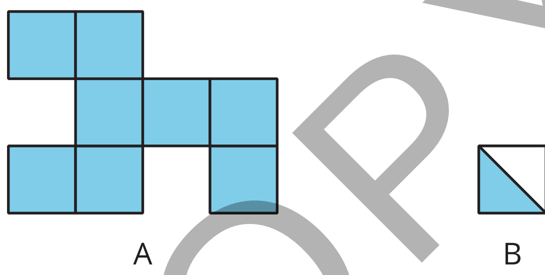
- I can decide whether a situation about a circle has to do with area or circumference.
- I can use formulas for circumference and area of a circle to solve problems.

Lesson 11 Stained-Glass Windows

- I can apply my understanding of area and circumference of circles to solve more complicated problems.

Glossary

- area
  Area is the number of square units that cover a two-dimensional region with no any gaps or overlaps.
  - The area of region A is 8 square units.
  - The area of the shaded region of B is $\frac{1}{2}$ square unit.

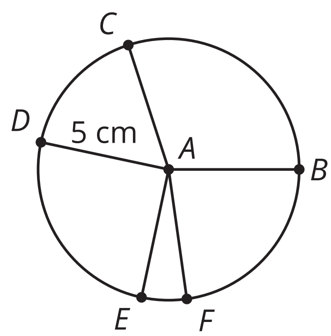


- area of a circle
  The area of a circle is the number of square units it covers. If a circle has radius r units, its area is πr^2 square units.

  For example, a circle has radius of 3 inches. Its area is $\pi 3^2$, or 9π, square inches. This is about 28.3 square inches.

- circle
  A circle is made of all the points that are the same distance from a given point. That given point is the center of the circle.

  Every point on this circle is 5 cm away from point A.



- circumference
  The circumference of a circle is the distance around the circle. If a circle has radius r units, its circumference is $2\pi r$ units.

  For example, a circle has a radius of 3 inches. Its circumference is $2 \cdot \pi \cdot 3$, or 6π inches. This is about 18.85 inches.

- constant of proportionality

In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity. This number is called the *constant of proportionality*.

In this example, the constant of proportionality is 3.

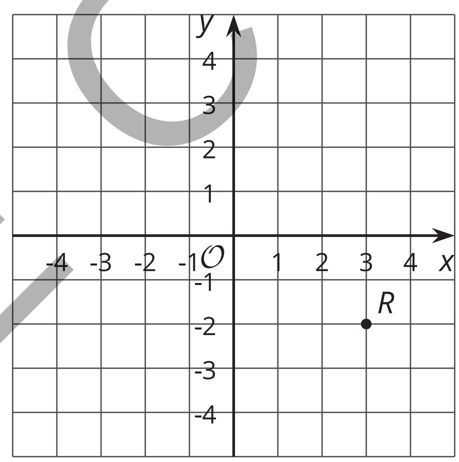
number of oranges	number of apples
2	6
3	9
5	15

$\cdot 3$ $\cdot 3$ $\cdot 3$

- coordinate plane
  The coordinate plane is one way to represent pairs of numbers. The plane is made of a horizontal number line and a vertical number line that cross at 0.

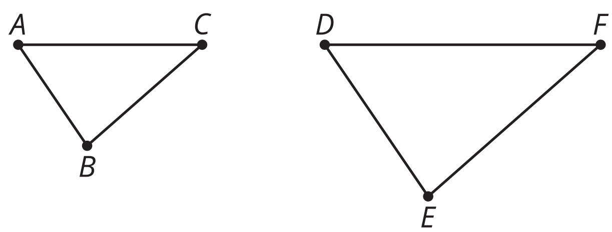
  Pairs of numbers can be used to describe the location of a point in the coordinate plane.

  Point R is located at $(3, -2)$. This means R is 3 units to the right and 2 units down from $(0, 0)$.



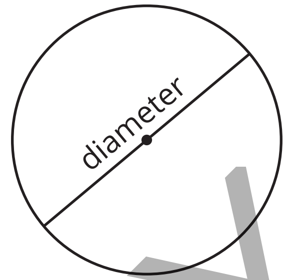
- corresponding
  Corresponding parts are the parts that match up between a figure and its scaled copy. They have the same relative position. Points, segments, angles, or distances can be corresponding.

  Point B in the first triangle corresponds to point E in the second triangle. Segment AC corresponds to segment DF.



- diameter

A diameter is a line segment that goes from one point on a circle to another and passes through the center. The length of this segment is also called the *diameter*. Every diameter of a circle is the same length.

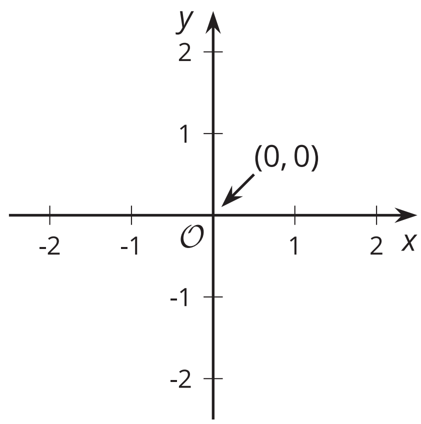


- equivalent ratios
  Two ratios are equivalent if each of the numbers in the first ratio can be multiplied by the same factor to get the numbers in the second ratio. For example, $8 : 6$ is equivalent to $4 : 3$, because $8 \cdot \frac{1}{2} = 4$ and $6 \cdot \frac{1}{2} = 3$.

  A recipe for lemonade says to use 8 cups of water and 6 lemons. If 4 cups of water and 3 lemons are used, it will make half as much lemonade. Both recipes taste the same, because $8 : 6$ and $4 : 3$ are equivalent ratios.

cups of water	number of lemons
8	6
4	3

- origin
  The origin is the point $(0, 0)$ in the coordinate plane. This is where the horizontal axis and the vertical axis cross. The origin is sometimes marked with the symbol $\mathcal{O}$.

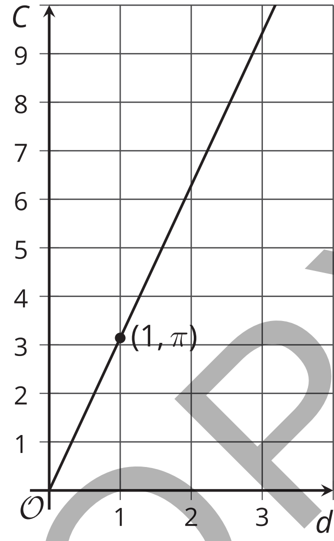


- pi (π)

There is a proportional relationship between the diameter and circumference of any circle. The constant of proportionality is pi. The symbol for pi is π.

This relationship can be represented with the equation $C = \pi d$, where C represents the circumference and d represents the diameter. In the graph, pi can be seen as the value of C when the value of d is 1.

Some approximations for π are $\frac{22}{7}$, 3.14, and 3.14159.



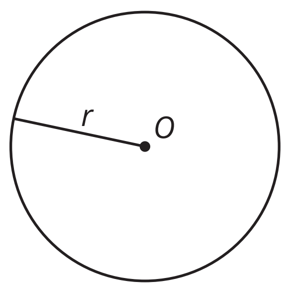
- proportional relationship
  In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity.

  This table shows a proportional relationship between s and p. Each value of p is 4 times a value of s. This relationship can be written as $p = 4s$.

s	p
2	8
3	12
5	20
10	40

- radius
  A radius is a line segment that goes from the center of a circle to any point on the circle. The length of this segment is also called the *radius*. Every radius of a circle is the same length.

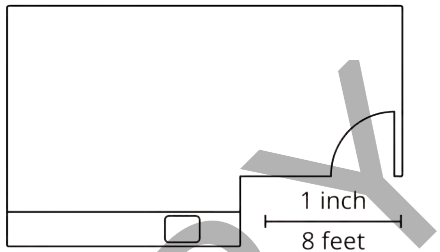
  For example, r is the radius of this circle with center O.



- reciprocal
  Two numbers that multiply to equal 1 are reciprocals.
  - 8 and $\frac{1}{8}$ are reciprocals because $8 \cdot \frac{1}{8} = 1$.
  - $\frac{2}{5}$ is a reciprocal of $\frac{5}{2}$ because $\frac{2}{5} \cdot \frac{5}{2} = 1$.

- scale
  A scale tells how the measurements in a scale drawing represent the actual measurements.

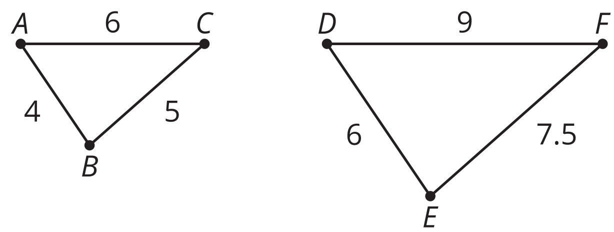
  The scale on this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room. This means that 2 inches represent 16 feet, and $\frac{1}{2}$ inch represents 4 feet.



- scale drawing
  A scale drawing represents an actual place or object. All the measurements in the drawing correspond to the measurements of the actual object by the same scale.

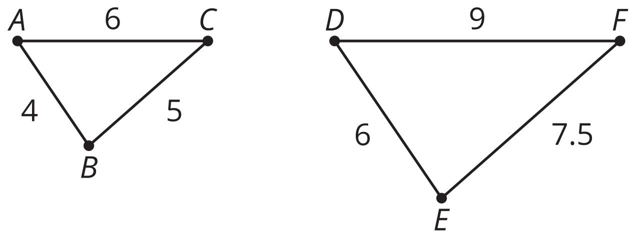
- scale factor
  To create a scaled copy of a figure, all the side lengths in the original figure are multiplied by the same number. This number is called the *scale factor*.

  In this example, the scale factor is 1.5, because $4 \cdot (1.5) = 6$, $5 \cdot (1.5) = 7.5$, and $6 \cdot (1.5) = 9$.



- scaled copy
  A scaled copy is a copy of a figure where every side length in the original figure is multiplied by the same number.

  Triangle DEF is a scaled copy of triangle ABC. Each side length on triangle ABC is multiplied by 1.5 to get the corresponding side length on triangle DEF.



* squared
  The word *squared* means “to the second power.” This is because a square with side length s has an area of $s \cdot s$, or s^2.

Attributions

The *5 Practices* are from Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. National Council of Teachers of Mathematics.

Notice and Wonder and *I Notice/I Wonder* are trademarks of the National Council of Teachers of Mathematics, reflecting approaches developed by the Math Forum (http://www.nctm.org/mathforum/), and used here with permission.

Images that are not the original work of Illustrative Mathematics are in the public domain or released under a Creative Commons Attribution (CC-BY) license, and include an appropriate citation. Images that are the original work of Illustrative Mathematics do not include such a citation.

Reference links are provided as a convenience for informational purposes only and were active and accurate at the time of publication. They do not constitute an endorsement or approval by Illustrative Mathematics of any products, services, or opinions of the corporation, organization, or individual. Illustrative Mathematics is not responsible for the content of external sites.

Image Attributions

Rose Bowl aerial, by NASA (http://worldwind.arc.nasa.gov/). Public Domain. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Rose_Bowl_aerial.jpg.

Zapotec rug, by Gina Pankowski. CC BY 4.0. https://www.etsy.com/listing/293123307/hand-woven-zapotec-rug-softest-hand-spun.

Khatam-bandi, by Telementor. CC BY-SA 4.0. https://en.wikipedia.org/wiki/File:Khatam-bandi.jpg.

Blue Tit Clipart, by Tanya Hall. Public Domain. PublicDomainPictures.net. http://www.publicdomainpictures.net/view-image.php?image=129730..

“MBTA Boston subway map”, by Citynoise. CC BY-SA. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:MBTA_Boston_subway_map.png.

“2013 unofficial MBTA subway map by Michael Kvrivishvili” , by Michael Kvrivishvili. CC BY 2.0.

Wikimedia Commons. https://commons.wikimedia.org/wiki/
File:2013_unofficial_MBTA_subway_map_by_Michael_Kvrivishvili.png.

Westland Lysander, by Emoscopes 23:36, 28 April 2008 (UTC) (self made using XaraXtreme). CC BY 3.0. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Westland_Lysander.png.

By American Fact Finder by the United States Census Bureau. Public Domain. American Fact Finder. http://factfinder.census.gov/faces/nav/jsf/pages/
searchresults.xhtml?ref=geo&refresh=t&tab=map&src=bkmk.

By skeeze. Public Domain. Pixabay. https://pixabay.com/en/bicycling-riding-bike-riding-1814454/.

By American Fact Finder by the United States Census Bureau. Public Domain. American Fact Finder. https://factfinder.census.gov/faces/nav/jsf/pages/index.xhtml.

By American Fact Finder by the United States Census Bureau. CC BY 2.0. American Fact Finder. http://factfinder.census.gov/faces/nav/jsf/pages/
searchresults.xhtml?ref=geo&refresh=t&tab=map&src=bkmk.

By United States Census Bureau. Public Domain. American Fact Finder. https://factfinder.census.gov/faces/nav/jsf/pages/.

By United States Census Bureau. Public Domain. American Fact Finder. http://factfinder.census.gov/faces/nav/jsf/pages/
searchresults.xhtml?ref=geo&refresh=t&tab=map&src=bkmk.

Flag of Colombia, by SKopp. Public Domain. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Flag_of_Colombia.svg.

By American Fact Finder. Public Domain. American Fact Finder. http://factfinder.census.gov.

By United States Census Bureau. Public Domain. American Fact Finder. http://factfinder.census.gov/faces/nav/jsf/pages/
searchresults.xhtml?ref=geo&refresh=t&tab=map&src=bkmk.

By United States Census Bureau. Public Domain. American Fact Finder . http://factfinder.census.gov/faces/nav/jsf/pages/

searchresults.xhtml?ref=geo&refresh=t&tab=map&src=bkmk.

Shrimp in aquarium, by uzilday. Public Domain. Pixabay. https://pixabay.com/en/shrimp-cherry-shrimp-aquarium-99521/.

Paper ream , by Bluesnap. Public Domain. Pixabay. https://pixabay.com/en/paper-ream-stack-tiered-white-224223/.

Stack of books, by Hermann. Public Domain. Pixabay. https://pixabay.com/en/books-education-school-literature-441866/.

U.S. Navy Chief Master-at-Arms Marlon Thomas, foreground, practices hose handling techniques during firefighting training, by MC2 Matthew Cole. Public Domain. https://commons.wikimedia.org/. http://www.defenseimagery.mil/imagery.html?&guid=8133610052a8848d5155972049f86fac68b70f2c.

Natural Rim Medium Sifter Basket, by Marvene Dawahoya. CC BY 4.0. Museum of Northern Arizona.

Yucca Tray Basket with Handles, by Marvene Dawahoya. CC BY 4.0. Museum of Northern Arizona.

Large Hopi Coil Maiden Basket, by Bryjane Kewenvoyouma. CC BY 4.0. Museum of Northern Arizona.

“Steiger Ferris Wheel 1102009 1”, by Zonk43. Public Domain. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:Steiger_Ferris_Wheel_11102009_1.JPG.

By US Geological Survey. Public Domain. US Geological Survey. https://calval.cr.usgs.gov/rst-resources/sites_catalog/radiometric-sites/lake-tahoe/.

By United States Census Bureau. Public Domain. American Fact Finder. https://factfinder.census.gov/faces/nav/jsf/pages/.

Map of Idaho NA, by National Atlas of the United States. Public Domain. https://commons.wikimedia.org/wiki/File:Map_of_Idaho_NA.png.

Frame Drum, by The Metropolitan Museum of Art. Public Domain. https://www.metmuseum.org/

art/collection/search/502895.

'Ahu'ula 11094, by Hiart. CC0. https://commons.wikimedia.org/wiki/File:%27Ahu%27ula,_Hawaiian_feather_cloak,_Bishop_Museum,_11094.JPG.

'Ahu'ula 1969.181, by Hiart. CC0. https://commons.wikimedia.org/wiki/File:%27Ahu%27ula,_Hawaiian_feather_cloak,_Bishop_Museum,_1969.181.JPG.

Haalelea's Feather Cape, by nagualdesign. CC0. https://commons.wikimedia.org/wiki/File:Haalelea%27s_Feather_Cape.png.

By United States Census Bureau. Public Domain. American Fact Finder. http://factfinder.census.gov/faces/nav/jsf/pages/searchresults.xhtml?ref=geo&refresh=t&tab=map&src=bkmk.

California Common Core State Standards for Mathematics (CA CCSSM) References

7.EE: Grade 7 – Expressions and Equations

Use properties of operations to generate equivalent expressions.

7.EE.1
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

7.EE.4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.4a
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

7.EE.4b
Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. $

7.G: Grade 7 – Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.2
Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.3
Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.5
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.6
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

7.NS: Grade 7 – The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.1a
Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

7.NS.1b
Understand $p+q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

7.NS.1c
Understand subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

7.NS.1d
Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.2a
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.2b
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

7.NS.2c
Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.2d
Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3
Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

7.RP: Grade 7 – Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1
Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.

7.RP.2
Recognize and represent proportional relationships between quantities.

7.RP.2a
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.2b
Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.2c
Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

7.RP.2d
Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

7.RP.3
Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

7.SP: Grade 7 – Statistics and Probability

Use random sampling to draw inferences about a population.

7.SP.1
Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2
Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Draw informal comparative inferences about two populations.

7.SP.3
Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

7.SP.4
Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.7
Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.7a
Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

7.SP.7b
Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

7.SP.8
Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

7.SP.8a
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

7.SP.8b
Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

7.SP.8c
Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.