IMKH California

GRADE 2

Student Edition

UNITS

3 | 4

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

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UNIT 4

ADDITION AND SUBTRACTION ON THE NUMBER LINE

GRADE 2

UNIT

Measuring Length

Content Connections

In this unit you will measure and estimate lengths in standard units and solve measurement story problems. You will make connections by:

- **Reasoning with Data** while measuring objects, using appropriate tools to classify objects and create and interpret line graphs.
- **Exploring Changing Quantities** while solving one- and two-step story problems involving addition and subtraction of lengths.

- **Discovering Shape and Space** while solving problems involving length measurements using addition and subtraction.
- **Taking Wholes Apart, Putting Parts Together** while adding and subtracting two-digit numbers using different strategies such as models, symbols and drawings.

Addressing the Standards

As you work your way through **Unit 3 Measuring Length,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use these MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 6, 7, 11, 12, and 13
MP2 Reason abstractly and quantitatively.	Lesson 3, 6, 7, 10, 11, and 14
MP3 Construct viable arguments and critique the reasoning of others.	Lesson 4, 8, 9, and 16
MP4 Model with mathematics.	Lesson 9 and 18
MP5 Use appropriate tools strategically.	Lesson 2, 5, 8, 9, and 17

MP6 Attend to precision.	Lesson 1, 4, 8, 14, and 15
MP7 Look for and make use of structure.	Lesson 9, 10, and 16
MP8 Look for and express regularity in repeated reasoning.	Lesson 3, 6, and 10

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measure and Compare Objects • Problem Solving with Measure	**2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Lesson 2, 3, 4, 5, 8, 9, 13, 14, 15, and 18
• Measure and Compare Objects • Problem Solving with Measure	**2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Lesson 9

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measure and Compare Objects • Problem Solving with Measure	**2.MD.3** Estimate lengths using units of inches, feet, centimeters, and meters.	Lesson 4, 5, 8, 9, and 13
• Measure and Compare Objects • Problem Solving with Measure	**2.MD.4** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	Lesson 2, 3, 13, and 18
• Dollars and Cents • Problem Solving with Measure • Number Strategies	**2.MD.5** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	Lesson 6, 10, 11, 12, and 18

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measure and Compare Objects • Measure and Compare Objects • Squares in an Array	**2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . , and represent whole-number sums and differences within 100 on a number line diagram.	Lesson 3 and 18
• Measure and Compare Objects • Represent Data • Problem Solving with Measure	**2.MD.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	Lesson 14, 15, 16, and 17

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Problem Solving with Measure • Number Strategies	**2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Lesson 6
• Number Strategies	**2.OA.2** Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.	Lesson 6, 10, and 18
• Represent Data • Dollars and Cents	**2.NBT.2** Count within 1000; skip-count by 2s, 5s, 10s, and 100s.	Lesson 5

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Dollars and Cents • Number Strategies	**2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relation-ship between addition and subtraction.	Lesson 3, 6, 7, 11, 12, 13, 15, 16, and 17

Unit 3, Lesson 1

Building on CA CCSSM 1.MD.2; building towards 2.MD.1, 2.MD.2; practicing MP6

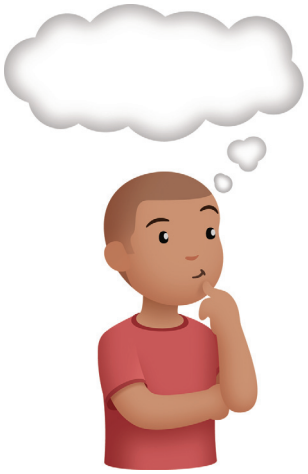
Standard Units of Measure

Let's measure length.

Warm-up

What Do You Know about Measuring?

What do you know about measuring?



Activity 1

Priya's Pet

Use straws to measure your string. The string shows the length of Priya's iguana.

Priya's iguana is _____ straws long.

Activity 2

Use a Standard Unit

Use the cubes to measure Priya's string.

1. Priya's iguana is _____ cubes long.
2. Compare your measurement with another group.

Addressing CA CCSSM 2.MD.1, 2.MD.4; building towards 2.MD.1; practicing MP5

Measure in Centimeters

Let's measure in centimeters.

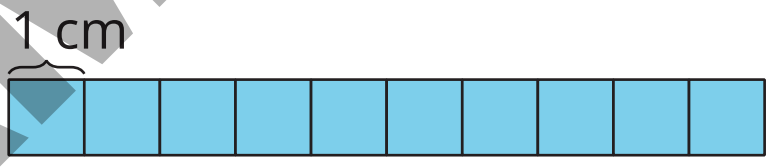
Warm-up

Notice and Wonder: Centimeters

What do you notice? What do you wonder?

A

B



Activity 1

Length in Centimeters

1. Measure the length of the bearded dragon using the centimeter cubes. A centimeter is a metric unit of length.

   The bearded dragon is _____ centimeter cubes long.

2. Measure the length of the bearded dragon using a 10-centimeter tool.

   The bearded dragon is _____ 10-centimeter tools long.

3. How many centimeters long is the bearded dragon?

   The bearded dragon is _____ centimeters long.

Activity 2

Measure with 10-Centimeter Tools

1. Measure the length of each reptile in centimeters.

   a. Jaragua dwarf gecko: _____ cm

   b. blue-tongued skink: _____ cm

   c. musk turtle: _____ cm

   d. ringneck snake: _____ cm

2. Compare measurements with your partner.

3. How much longer is the day gecko than the threadsnake?

Addressing CA CCSSM 2.MD.1, 2.MD.4, 2.MD.6, 2.NBT.5; practicing MP2 and MP8

Create and Use a Ruler

Let's make rulers to measure and compare lengths in centimeters.

Warm-up

Number Talk: Subtract 2 Digits

Find the value of each expression mentally.

- 63 – 3
- 63 – 20
- 63 – 23
- 63 – 24

Activity 1

Create a Ruler

1. Use your tools. Make a ruler that shows lengths in centimeters.

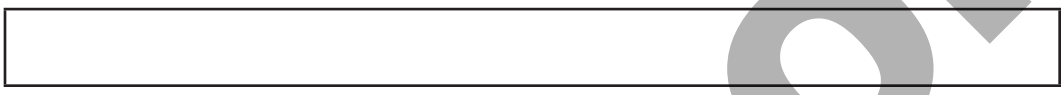
2. Compare your ruler with your partner's.

Activity 2

Measure and Compare Lengths

1. Use your ruler. Measure the length of each rectangle. Label your measurements.

A



B

C

D

E

F

2. How many centimeters longer is Rectangle A than Rectangle B?

3. How many centimeters longer is Rectangle F than Rectangle D?

4. Which 2 rectangles are the longest? How long would the rectangle be if you put them together?

Addressing CA CCSSM 2.MD.1, 2.MD.3; building towards 2.MD.1; practicing MP3 and MP6

Measure and Estimate in Centimeters

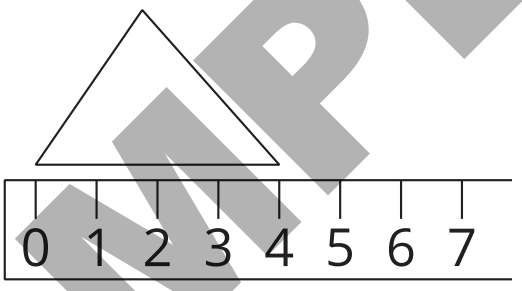
Let's estimate and measure in centimeters.

Warm-up

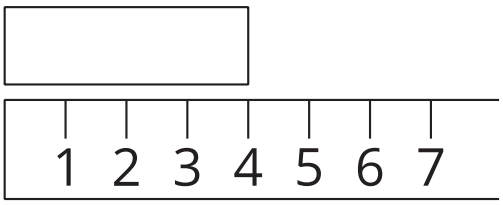
Which Three Go Together: Length

Which 3 go together?

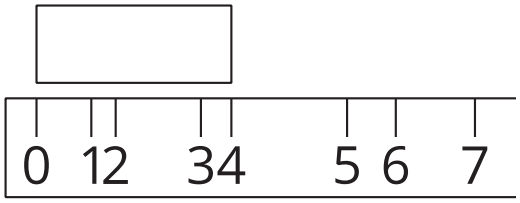
A



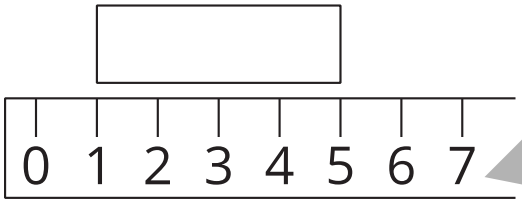
B



C



D



Activity 1

Estimate Length in Centimeters

1. Record an estimate that is:

too low	about right	too high

2. Record an estimate that is:

too low	about right	too high

3. Record an estimate for each object.

4. Tell your partner why your estimates are “about right.”

object	estimate	measurement
Choose an object:		

Activity 2

Measure and Compare

1. Measure each object on your recording sheet. Record each length in centimeters.

2. Compare the measurements to your estimates.

Addressing CA CCSSM 2.MD.1, 2.MD.3, 2.NBT.2; building towards 2.MD.2; practicing MP5

Measure in Meters

Let's measure lengths in meters.

Activity 1

Reptiles to Measure

The tape pieces on the floor represent the lengths of each reptile.

1. Measure each reptile. Write the unit.

   a. What is the length of the Gila monster?

A: Gila monster

b. What is the length of the baby alligator?

B: baby alligator

c. What is the length of the baby cobra?

C: baby cobra

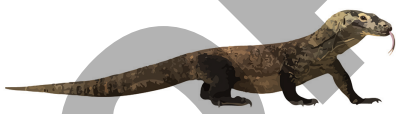
d. What is the length of the Komodo dragon?

D: Komodo dragon

Measure with a Meter Stick

1. Measure the length of the Komodo dragon in meters.

D: Komodo dragon



2. Compare your Komodo dragon measurements in the first activity to the measurements in the second activity. How are they alike? How are they different?

3. Measure each reptile in centimeters or meters. Write the units.

a. How long is the adult alligator?

E: adult alligator

b. How long is the adult cobra?

F: adult cobra

c. How long is the ribbon snake?

G: ribbon snake

Addressing CA CCSSM 2.MD.5, 2.NBT.5, 2.OA.1, 2.OA.2; practicing MP1, MP2, MP8

Compare Reptile Lengths in Story Problems

Let's solve story problems about reptile lengths.

Warm-up

Number Talk: Fives and Tens

Find the value of each expression mentally.

- 5 + 5
- 15 + 5
- 15 + 15
- 15 + 25

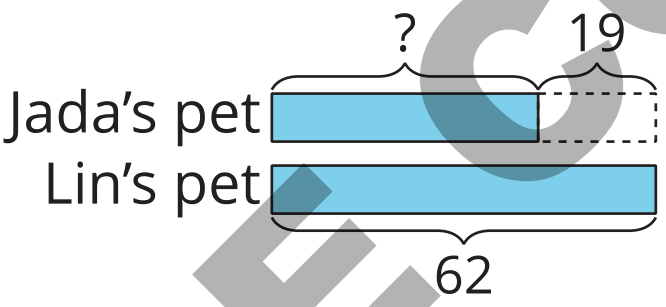
Activity 1

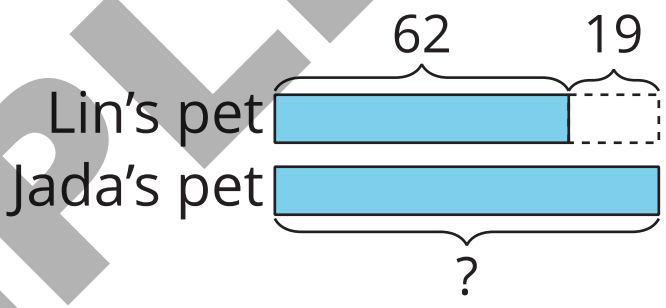
Whose Pet is Longer?

1. Lin's pet lizard is 62 cm long. It is 19 cm shorter than Jada's. How long is Jada's pet lizard?

   a. Whose pet is longer? _____

   b. Circle the diagram that matches the story.





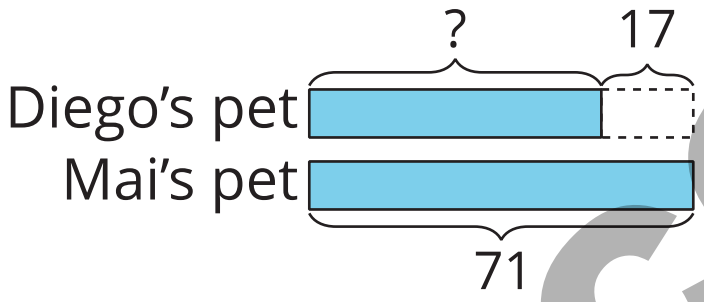
   c. Solve. Show your thinking using drawings, numbers, or words.

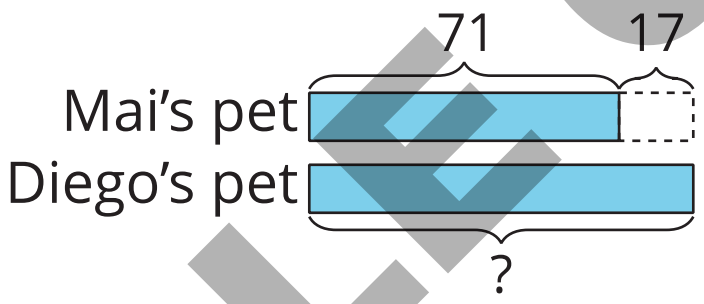
   Jada's pet lizard is _____ cm long.

2. Diego and Mai have pet snakes. Mai's pet snake is 17 cm longer than Diego's. Her snake is 71 cm. How long is his pet snake?

   a. Whose pet is shorter? _____

   b. Circle the diagram that matches the story.





   c. Solve. Show your thinking using drawings, numbers, or words.

   Diego's pet snake is _____ cm long.

Activity 2

Guess My Reptiles

Partner A's reptiles

1. day gecko, 28 cm

2. Komodo dragon, 98 cm

3. baby cobra, 46 cm

4. iguana, 65 cm

Partner B's reptiles

1. ribbon snake, 83 cm

2. Gila monster, 55 cm

3. baby alligator, 71 cm

4. ringneck snake, 38 cm

1. Choose 1 reptile from your list and 1 reptile from your partner's list.

2. Create a story problem using the lengths of the 2 reptiles you chose.

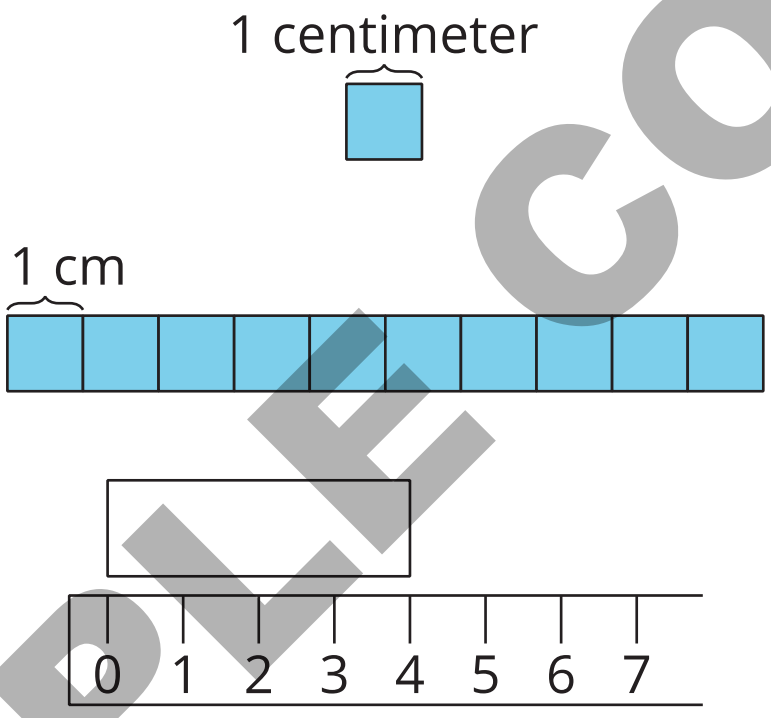
   My reptile is _____ cm long.

   It is _____ cm _____ (shorter/longer) than one of your reptiles.

3. Which reptiles did your partner choose? Show your thinking using drawings, numbers, or words.

Section A Summary

We measured the length of objects with different length units. We learned that the **centimeter** is a metric unit of length. We measured objects in centimeters using base-ten blocks, rulers, and meter sticks. A **meter** is a metric unit of length. There are 100 centimeters in a meter. We learned that rulers represent length units. They use tick marks to show a length away from 0.



Addressing CA CCSSM 2.NBT.5; practicing MP1 and MP2

Center Day 1

Let's practice addition and subtraction.

Warm-up

Number Talk: Subtract from 37

Find the value of each expression mentally.

- 37 − 20
- 37 − 21
- 37 − 17
- 37 − 16

Activity 2

Centers: Choice Time

Choose a center.

Number Puzzles

$14 = 8 + \square$

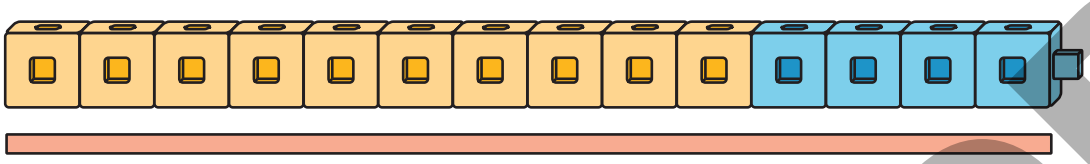
Math Stories

Practice Problems

12 Problems

1 Pre-unit

How many connecting cubes long is the rectangle?



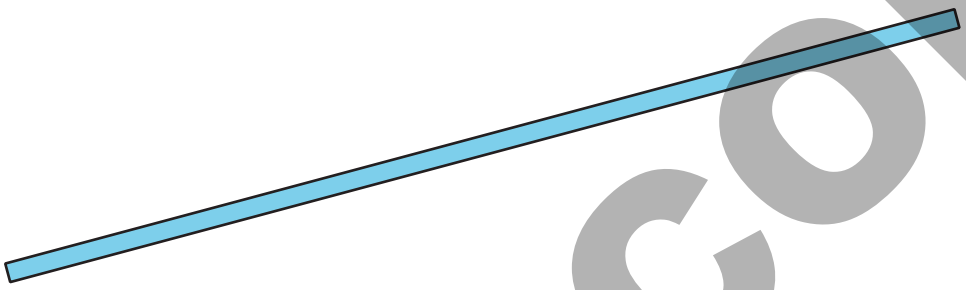
2 Pre-unit

Jada has 16 red counters. She has 7 fewer yellow counters than red counters. How many yellow counters does she have? Show your thinking using drawings, numbers, or words.

3 Pre-unit

A

B



C

a. Which rectangle is the longest?

b. Which rectangle is the shortest?

4 Pre-unit

Find the value of each expression.

a. $52 + 30$

b. $38 + 9$

c. $35 + 16$

d. $85 - 7$

5 from Unit 3, Lesson 1

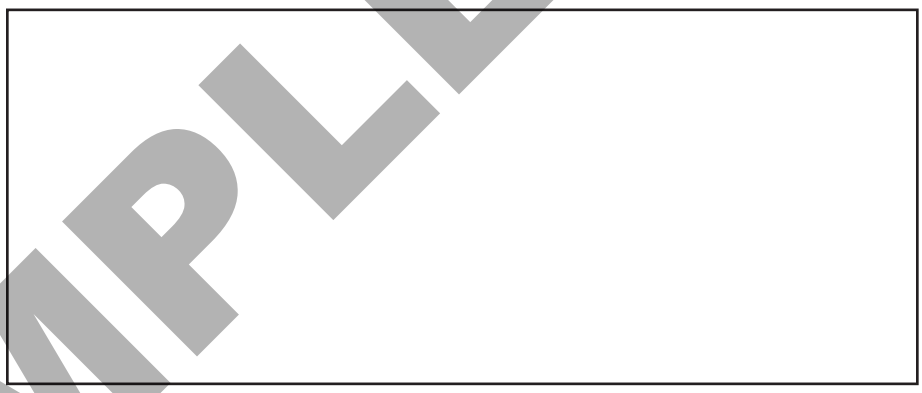
How many centimeters long is each rectangle?

a. 

b. 

6 from Unit 3, Lesson 2

Here is a rectangle.

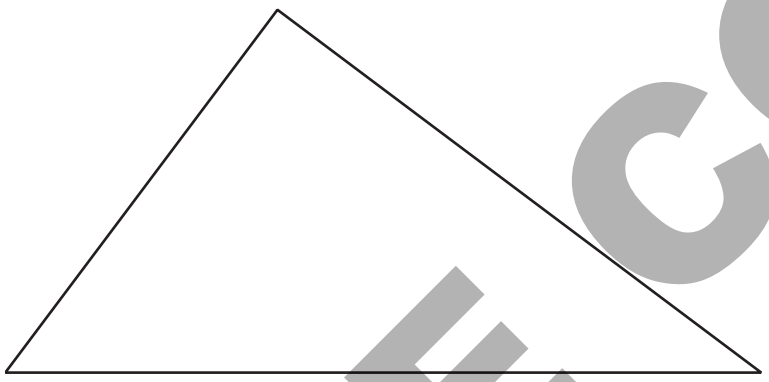


a. How many centimeters long is the rectangle?

b. How many centimeters wide is the rectangle?

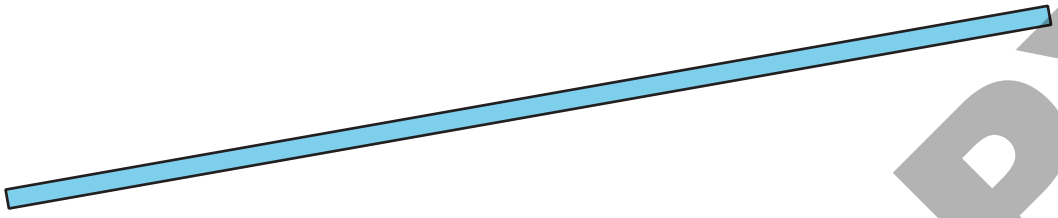
7 from Unit 3, Lesson 3

How many centimeters long is each side of the triangle? Label each side.



8 from Unit 3, Lesson 4

Here is a rectangle.



a. Estimate how many centimeters long the rectangle is.
_____ cm

b. Measure the rectangle. Record the measurement.
_____cm

c. How does your estimate compare to the actual length?

_____

_____

_____

9 from Unit 3, Lesson 5

A python is a snake that grows up to 9 meters long.

a. What tool would you use to measure the length of a python? Show your thinking using drawings, numbers, or words.

b. Is the python longer or shorter than the other reptiles you measured?

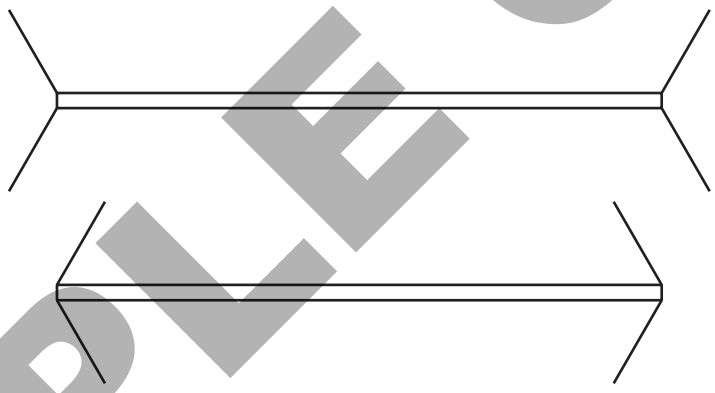
c. Is the python longer or shorter than the adult alligator and the adult cobra together?

10 from Unit 3, Lesson 6

Andre's snake is 85 centimeters long. His lizard is 28 centimeters in length. How many centimeters longer is Andre's snake than his lizard? Show your thinking using drawings, numbers, or words.

11 Exploration

Here are 2 rectangles.



a. Which rectangle do you think is longer?

b. Check your estimate with a centimeter ruler.

12 Exploration

a. Look for objects in the classroom that are about 1 centimeter long. Measure them. How close are they to 1 centimeter?

b. Look for objects in the classroom that look about 1 meter long.
Measure them. How close are they to 1 meter?

Addressing CA CCSSM 2.MD.1, 2.MD.3; building towards 2.MD.1; practicing MP3, MP5, MP6

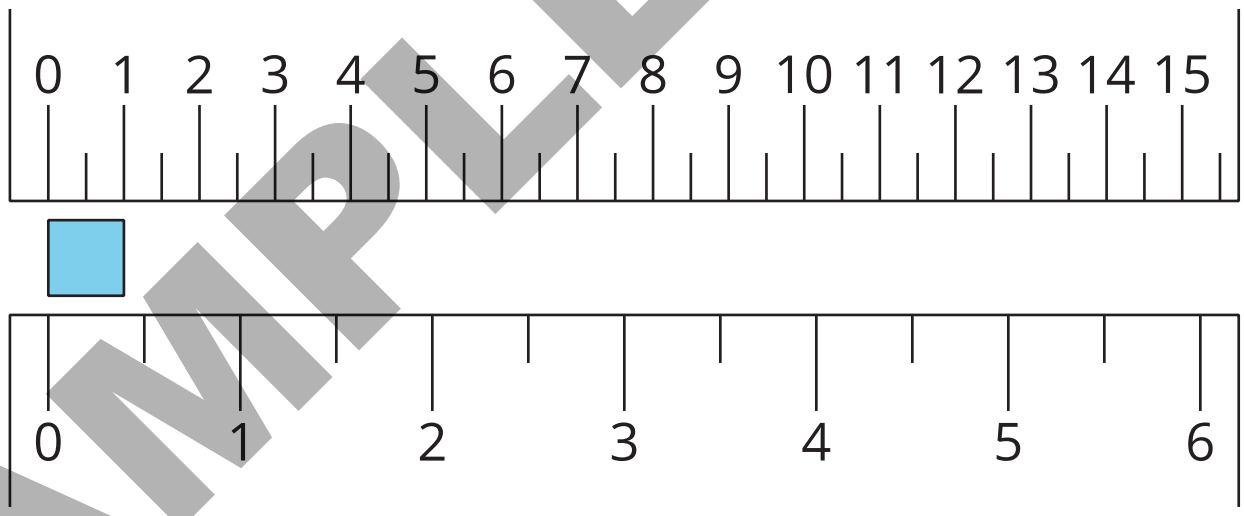
What Is an Inch?

Let's measure in inches.

Warm-up

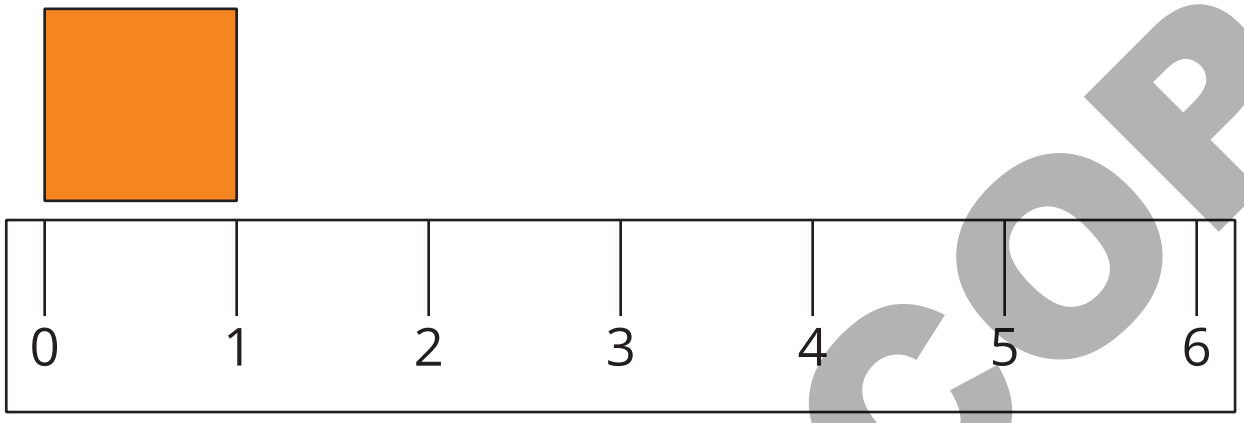
Notice and Wonder: Inches and Centimeters

What do you notice? What do you wonder?



Activity 1

What Is an Inch?



1. Find 2 items that are about an inch long.

   a. _____

   b. _____

2. Measure the length of each object.

object to measure	length in inches
marker	
colored pencil	
11 connecting cubes	
a book	
your choice objects:	

Activity 2

Measure the Sides of Shapes

1. Here is a rectangle.

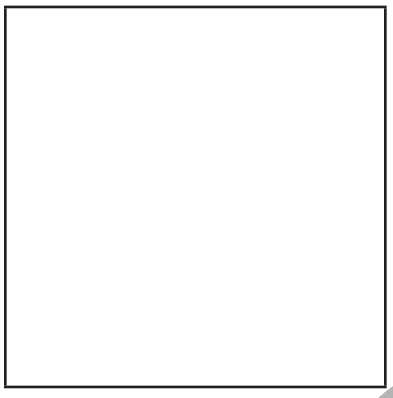
Estimate the length of the long side of the rectangle. Use inches.

Estimate: _____ in

Measure the long side of the rectangle. Use inches.

Actual length: _____ in

2. Here is a square.



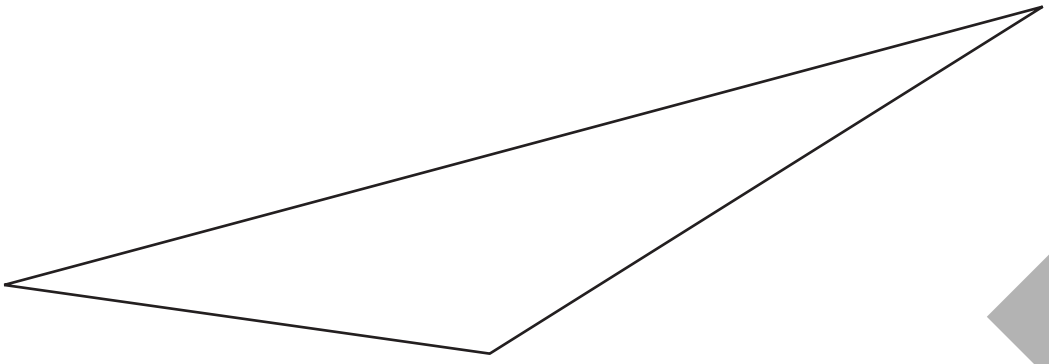
Estimate the length of 1 side of the square. Use inches.

Estimate: _____ in

Measure 1 side of the square. Use inches.

Actual length: _____ in

3. Here is a triangle.



Estimate the length of the longest side of the triangle. Use inches.

Estimate: _____ in

Measure the longest side of the triangle. Use inches.

Actual length: _____ in

Addressing CA CCSSM 2.MD.1, 2.MD.2, 2.MD.3; practicing MP3, MP4, MP5, MP7

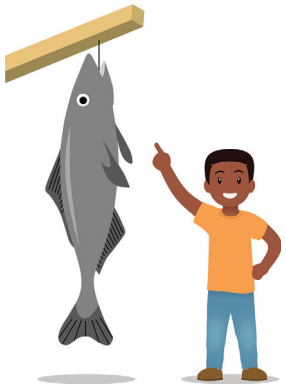
From Feet to Inches

Let's measure longer lengths with U.S. units.

Warm-up

Estimation Exploration: Small Fry, Big Fish

How long is this cobia fish?
Use inches.



1. Record an estimate that is:

too low	about right	too high

2. Record an estimate that is:

too low	about right	too high

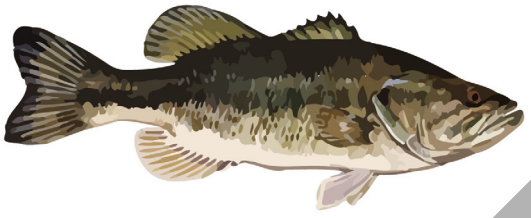
Activity 1

Measure the Length of Fish

1. Work with your group. Measure the tape strips around the classroom. Use feet. Then measure the length in inches.

   Tape A

   largemouth bass



   Length in feet: _____ ft
   Length in inches: _____ in

Tape B

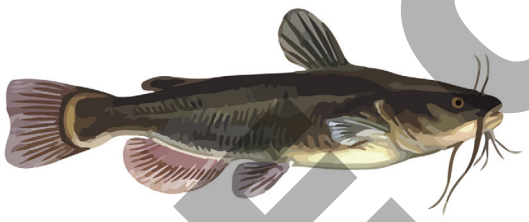
spiny dogfish shark

Length in feet: _____ ft

Length in inches: _____ in

Tape C

catfish



Length in feet: _____ ft

Length in inches: _____ in

Tape D

koi

Length in feet: _____ ft

Length in inches: _____ in

2. What do you notice about the number of feet compared to the number of inches?

_____

_____

_____

Activity 2

Inches or Feet?

1. Estimate the length of objects around the room. Decide if you will measure in inches or feet. Circle the unit.

object to measure	my estimate	circle inches or feet
		inches feet
		inches feet
		inches feet
		inches feet

2. Choose the best tool to measure each object. Record your measurements.

object to measure	measurement tool	length (include unit)

Addressing CA CCSSM 2.MD.5, 2.OA.2; building on 1.OA.7; building towards 2.MD.6; practicing MP2, MP7, MP8

Measure with a Torn Tape

Let's measure without starting at 0.

Warm-up

True or False: Constant Difference

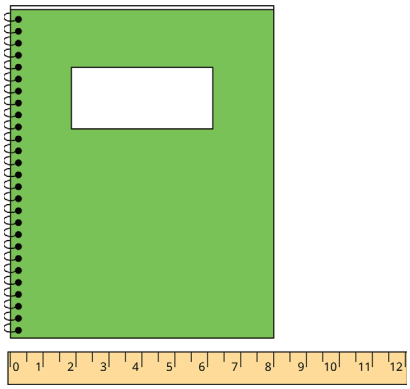
Decide if each statement is true or false. Be prepared to explain your reasoning.

- $10 - 0 = 12 - 2$
- $8 - 4 = 10 - 6$
- $12 - 4 = 10 - 3$
- $15 - 2 = 13 - 0$

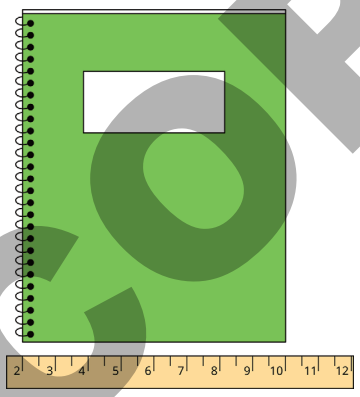
Activity 1

The Notebook Problem

Jada and Han use an inch ruler. They measure the short side of the notebook.



Han says 8 inches.



Jada says 8 inches.

1. How did Han and Jada get the same measurement?

2. Write an equation that shows Jada's thinking.

3. Measure an object using Jada's method.

   - I measured a _____.
   - I started with the number _____.
   - I ended with the number _____.
   - Equation: _____.
   - The length of my object is _____.

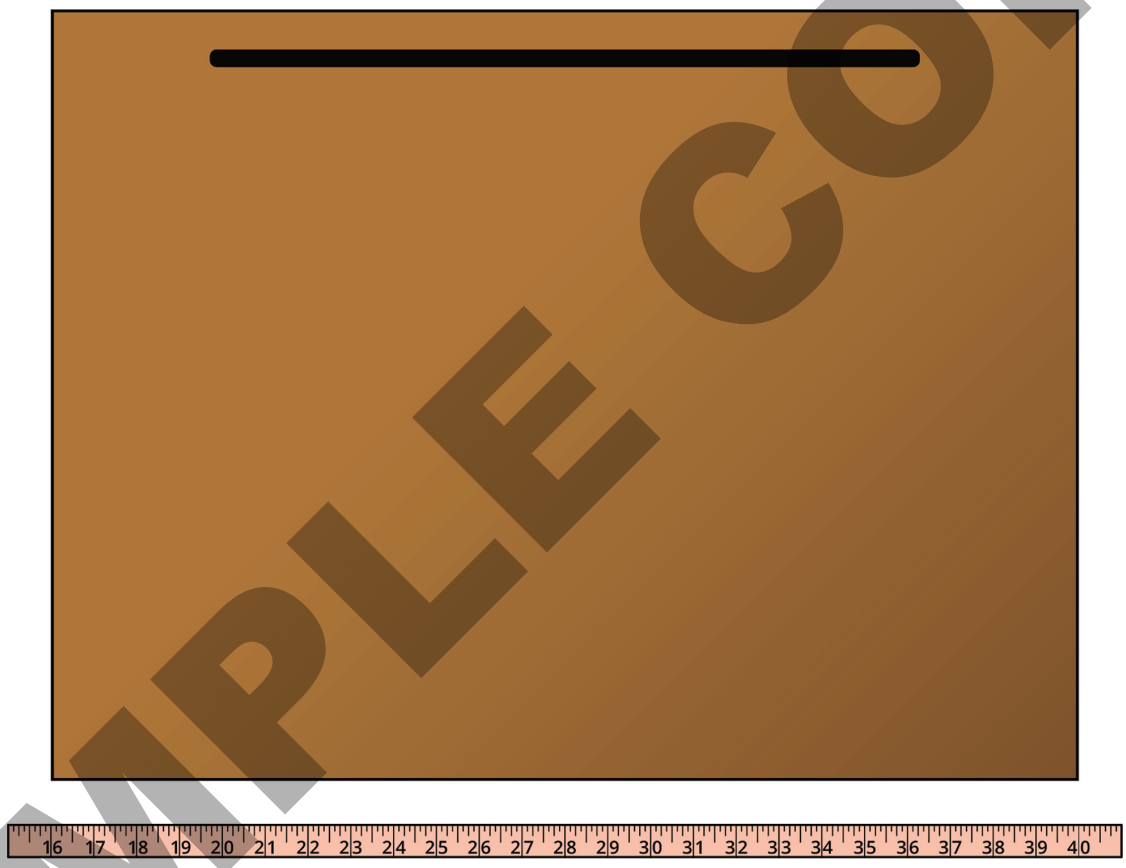
4. Compare measurements with a partner.

Activity 2

A Desktop to Measure

Lin measures her desktop in inches.

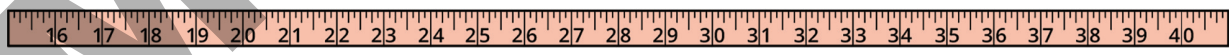
1. What is the length of the long side of the desktop? Show your thinking using drawings, numbers, or words.



Equation: _____

The long side of the desktop is _____.

2. What is the length of the short side of the desktop? Show your thinking using drawings, numbers, or words.



Equation: _____

The short side of the desktop is _____.

Addressing CA CCSSM 2.MD.5, 2.NBT.5; building towards 2.MD.5; practicing MP1 and MP2

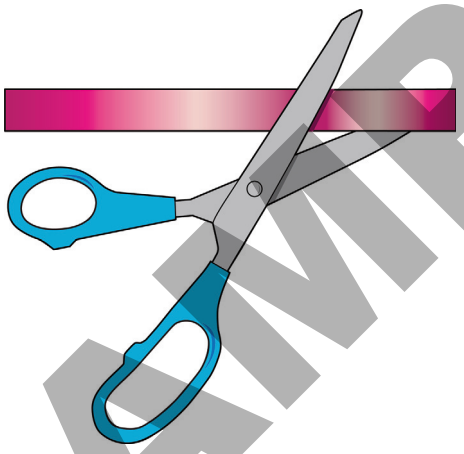
Saree Silk Stories: Necklaces and Bracelets

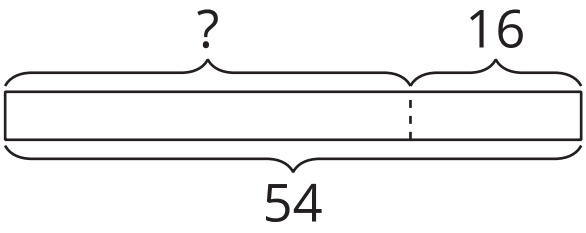
Let's solve story problems about length.

Warm-up

Notice and Wonder: Take From

What do you notice? What do you wonder?





54 − 16 =?

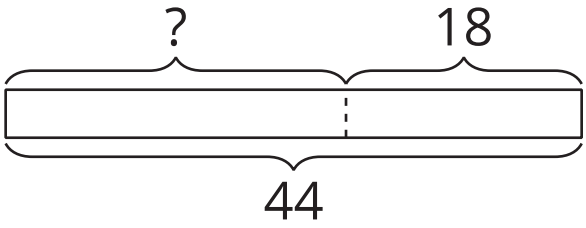
Activity 1

Saree Silk Ribbon Necklaces

What do you notice? What do you wonder?

Priya has a ribbon that is 44 inches long. She cuts off 18 inches. How long is it now?

Andre draws this diagram to help him think about the problem.



1. What does the "?" represent in the story?

2. Why do you think there is a dotted line between the parts?

3. Find the unknown value. Show your thinking using drawings, numbers, or words.

4. Priya's ribbon is _____ long.

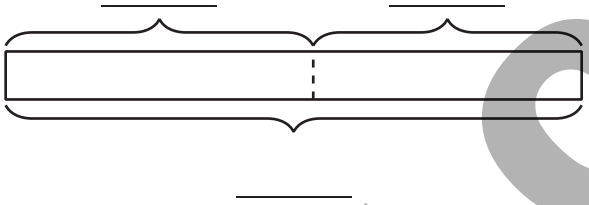
Activity 2

Saree Silk Ribbon Projects

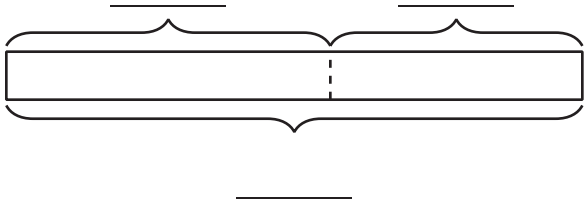
Label the diagram. Find the unknown value. Show your thinking using drawings, numbers, or words. Write the units.

Sec B

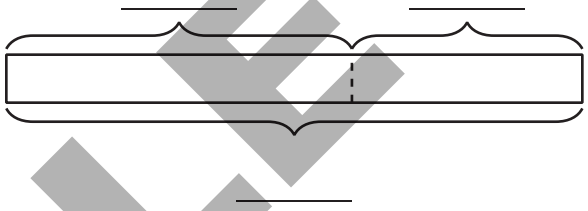
1. Elena starts with 58 inches of ribbon. She gives Clare 27 inches of ribbon. How much did Elena keep?

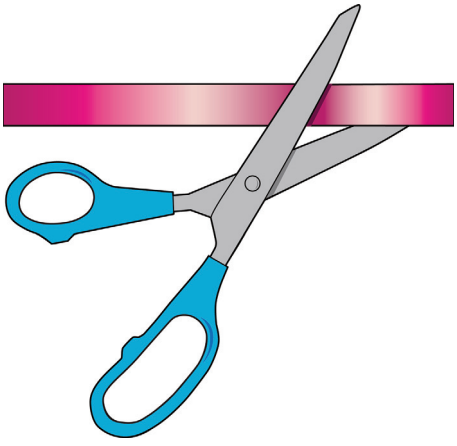


2. Han has a piece of ribbon that is 64 inches long. He cuts off 28 inches to make a necklace for his sister. How much ribbon is left?



3. Priya cuts off 25 inches of ribbon. She has 38 inches of ribbon left. How much ribbon did Priya start with?





Addressing CA CCSSM 2.MD.5, 2.NBT.5; building on 1.NBT.3; practicing MP1

Saree Silk Stories: Friendship Bracelets

Let's solve 2-step story problems about length.

Warm-up

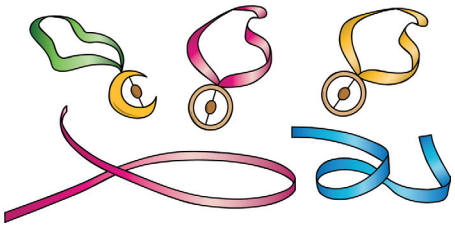
True or False: Place Value Comparisons

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $24 = 10 + 14$
- $15 + 12 = 27$
- $26 = 10 + 6 + 10$
- $58 = 20 + 20 + 8$

Activity 1

Share Ribbon with Friends

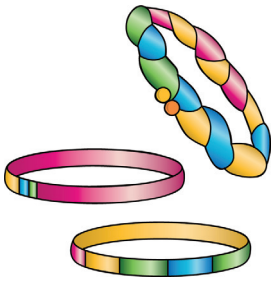


Lin finds a piece of ribbon that is 92 cm long. She cuts a piece for Noah that is 35 cm.

Then Lin cuts off 28 cm of her ribbon. She gives it to Jada. How much ribbon does Lin have left?

Activity 2

Friendship Bracelets and Gifts



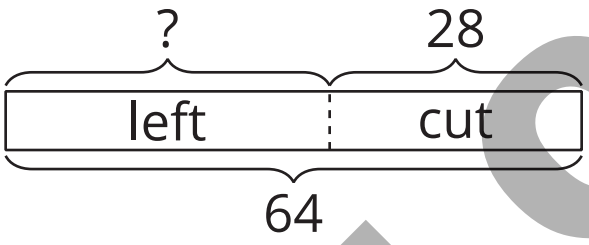
Solve. Show your thinking using drawings, numbers, or words. Write the units.

1. Andre's ribbon is 50 inches long. He cuts off 26 inches of ribbon. Mai's ribbon is now 8 inches longer than his ribbon. How long is her ribbon?

2. Han has 62 inches of ribbon. He cuts off 28 inches of ribbon. He keeps the rest. Clare has a ribbon that is 27 inches long. How much longer is Han's ribbon than Clare's ribbon?

Section B Summary

We learned more about standard length units. We measured using **inches** and feet—two U.S. length units. An **inch** is easier to use when measuring smaller things. A **foot** is 12 inches. We solved 2-step story problems about length. We used diagrams to represent taking a part away. This diagram shows that we know the length of the ribbon and how much was cut. The ? represents the length of ribbon that is left.



Han has a piece of ribbon that is 64 inches long. He cuts off 28 inches to make a necklace for his sister. How much ribbon does he have left?

Addressing CA CCSSM 2.MD.1, 2.MD.3, 2.MD.4, 2.NBT.5; practicing MP1

Center Day 2

Let's estimate and measure objects.

Warm-up

Number Talk: Use Ten to Add

Find the value of each expression mentally.

- 58 + 10
- 58 + 12
- 58 + 13
- 67 + 14

Practice Problems

7 Problems

1 from Unit 3, Lesson 8

a. What do you know about inches?

b. What is the length of this rectangle in inches?

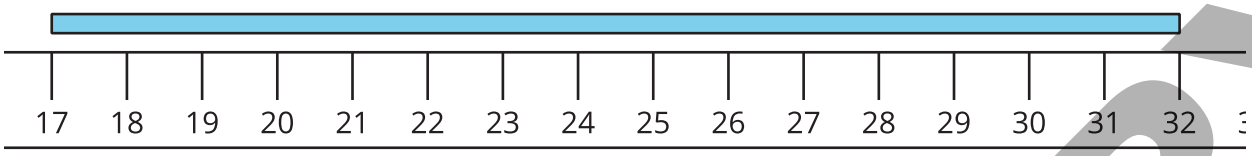
2 from Unit 3, Lesson 9

a. An adult hippopotamus is 13 units long. Are the units inches or feet? Explain your reasoning.

b. A piece of chalk is 2 units long. Are the units inches or feet? Explain your reasoning.

3 from Unit 3, Lesson 10

a. What is the length of this rectangle? Use centimeters.



b. Write an equation.

4 from Unit 3, Lesson 11

Mai makes a headband. She uses a piece of fabric that is 71 centimeters long. She cuts off 18 centimeters. How many centimeters long is the fabric that is left?

5 from Unit 3, Lesson 12

Diego has a piece of ribbon that is 65 inches long. He cuts off a piece that is 20 inches long. He keeps the rest. Then he cuts off another piece that is 28 inches long. How much ribbon does he have left now?

Find objects in your classroom or at home that match the given measurements. Complete the table.

object I measured	estimated measurement	actual measurement
	7 centimeters	
	7 inches	
	7 feet	
	7 meters	

7 Exploration

About how long of a line do you think you can make if your classmates stand side to side with your arms stretched out? What unit of measure would you choose? Explain your reasoning.

Unit 3, Lesson 14

Addressing CA CCSSM 2.MD.1, 2.MD.9; building on 2.MD.10; building towards 2.MD.9; practicing MP2 and MP6

What Is a Line Plot?

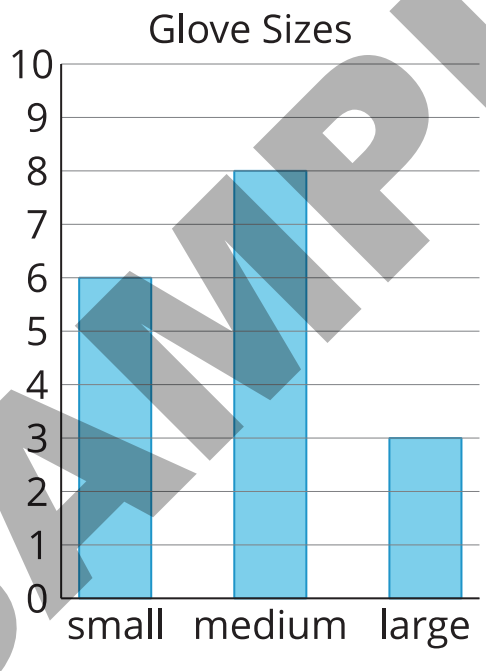
Let's learn a new way to represent data.

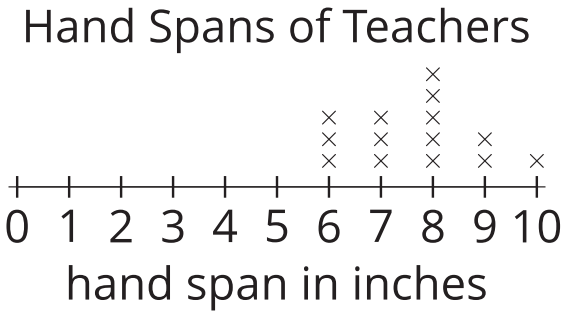
Warm-up

Sec C

Notice and Wonder: "Handy" Graphs

What do you notice? What do you wonder?



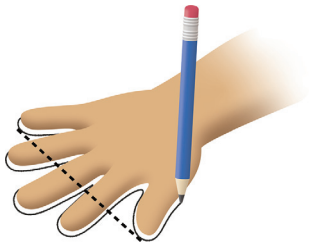


Activity 1

Measure Our Hand Spans

1. Trace your hand. (Spread your fingers wide.)
2. Draw a line from your thumb to your pinky. This line represents your hand span. Measure the length of your hand span in inches.

My hand span is _____ inches.



Activity 2

Interpret Our Numerical Data

1. What is the longest hand span in your class?

2. What is the shortest hand span?

3. Write another statement about the hand spans using the line plot.

Unit 3, Lesson 15

Addressing CA CCSSM 2.MD.1, 2.MD.9, 2.NBT.5; practicing MP6

Create Line Plots

Let's represent data in line plots.

Warm-up

Number Talk: Subtraction within 50

Find the value of each expression mentally.

- 47 − 20
- 47 − 24
- 36 − 10
- 36 − 15

Activity 1

Measure and Plot Pencil Lengths

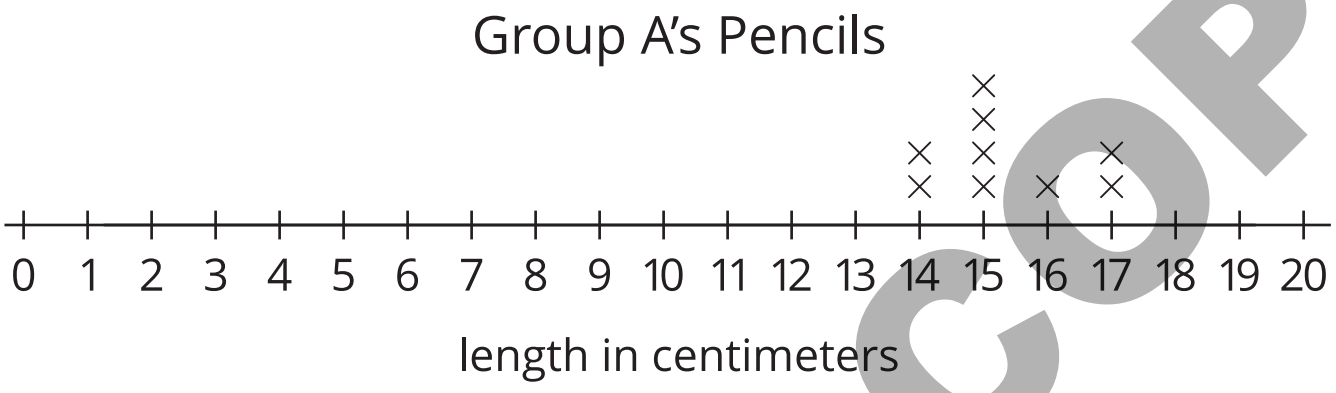
1. Measure the pencils. Use centimeters. Work with a partner. Check each other's measurements. Record each measurement in the table.

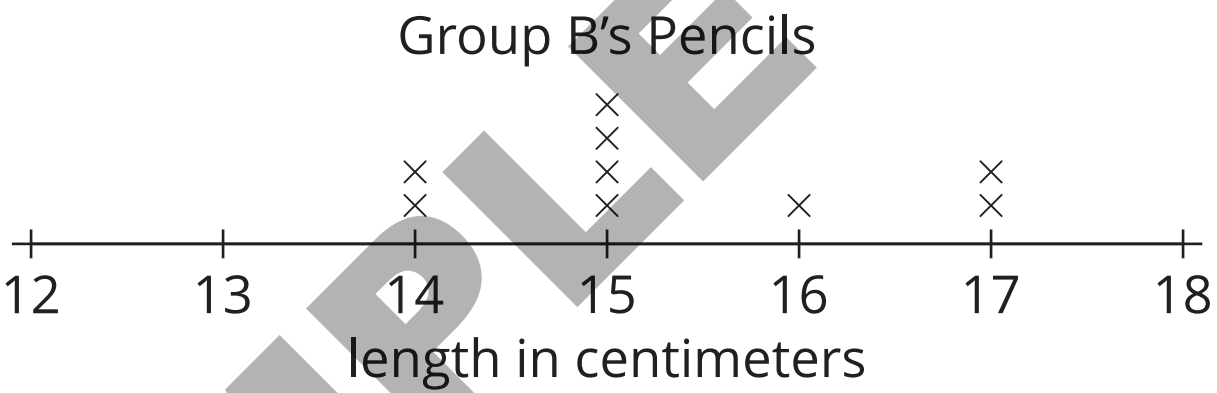
pencil length (cm)

2. Create a line plot to represent the lengths of all the pencils in your group.

Activity 2

Plot Pencil Lengths





Use this data to create a line plot.

Group	C pencil length (centimeters)
Andre	12
Clare	10
Diego	10
Elena	10
Han	13
Jada	12
Kiran	14
Noah	16
Priya	14
Tyler	13

Unit 3, Lesson 16

Addressing CA CCSSM 2.MD.9, 2.NBT.5; practicing MP3 and MP7

Interpret Measurement Data

Let's represent and make sense of data in line plots.

Warm-up

Sec C

Number Talk: Addition within 50

Find the value of each expression mentally.

- 15 + 5 + 1
- 25 + 6
- 16 + 7
- 37 + 6

Activity 1

The Plant Project

Use the data in this table. Create a line plot.

Group B	plant heights (centimeters)
Andre	33
Clare	25
Diego	27
Elena	25
Han	35
Jada	33
Kiran	26
Noah	30
Priya	26
Tyler	33

Activity 2

Interpret Measurement Data on a Line Plot

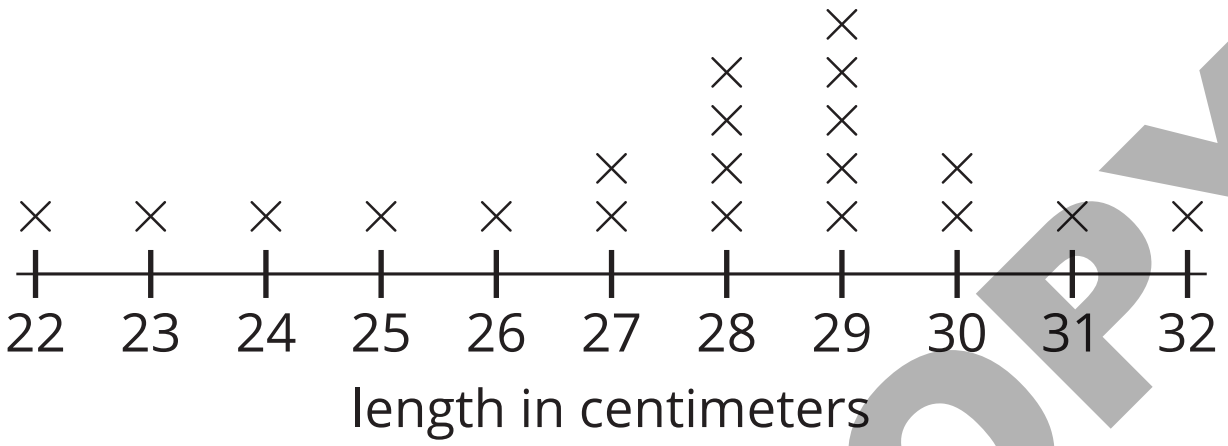
The Plant Project

Answer the questions using your line plot.

1. What is the shortest plant height?

2. What is the tallest plant height?

3. What is the height difference between the tallest and shortest plants? Write an equation.



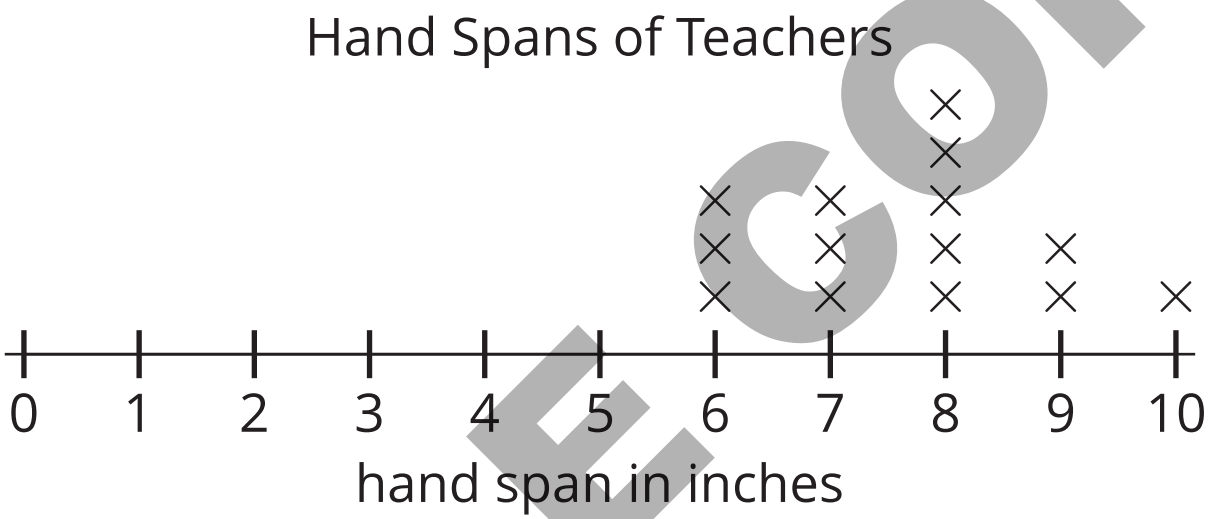
4. Han looks at this line plot. He says the tallest plant is 29 centimeters. Do you agree? Explain your reasoning.

5. How many plants were measured in all?

6. Write a statement about Han's line plot.

Section C Summary

We learned about a new kind of graph. A **line plot** shows data with Xs or other marks above a line with numbers. Each mark represents 1 number or 1 measurement. Line plots look like a ruler or a tape measure. We made our own line plots. Then we used them to answer questions about data.



This line plot shows data about hand spans of teachers. The line with numbers shows inches, like a ruler. Each X represents the hand span of 1 teacher.

We know that 5 teachers have a hand span of 8 inches because there are 5 Xs above the 8.

Addressing CA CCSSM 2.MD.9, 2.NBT.5; practicing MP5

Center Day 3

Let's measure to create line plots and add and subtract within 100.

Warm-up

Number Talk: Subtract from a Multiple of 10

Find the value of each expression mentally.

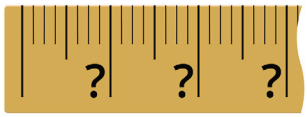
- 60 − 10
- 60 − 11
- 60 − 21
- 70 − 32

Activity 2

Centers: Choice Time

Choose a center.

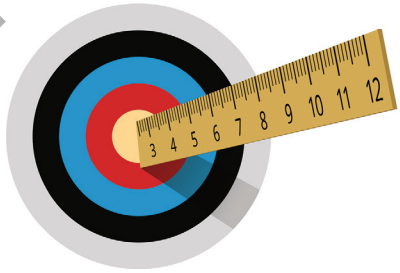
Estimate and Measure



Number Puzzles

14 = 8 + ☐

Target Measurements



Unit 3, Lesson 18

Addressing CA CCSSM 2.MD.1, 2.MD.4, 2.MD.5, 2.MD.6, 2.OA.2; building on 2.MD.1; practicing MP4

Make a Yardstick

Let's make a yardstick.

Warm-up

Sec C

Number Talk: Threes and Fives

Find the value of each expression mentally.

- $3 + 5$
- $5 - 3$
- $5 - 3 + 5$
- $3 + 5 + 3 + 3$

Activity 1

Make a Yardstick

Use the card. Make a yardstick that shows all inch marks from 1 to 36 inches.

The long side of the card measures 5 inches. The short side measures 3 inches.

Activity 2

Compare Yardsticks

1. Compare your yardstick with a yardstick from another group. How are they alike? How are they different?

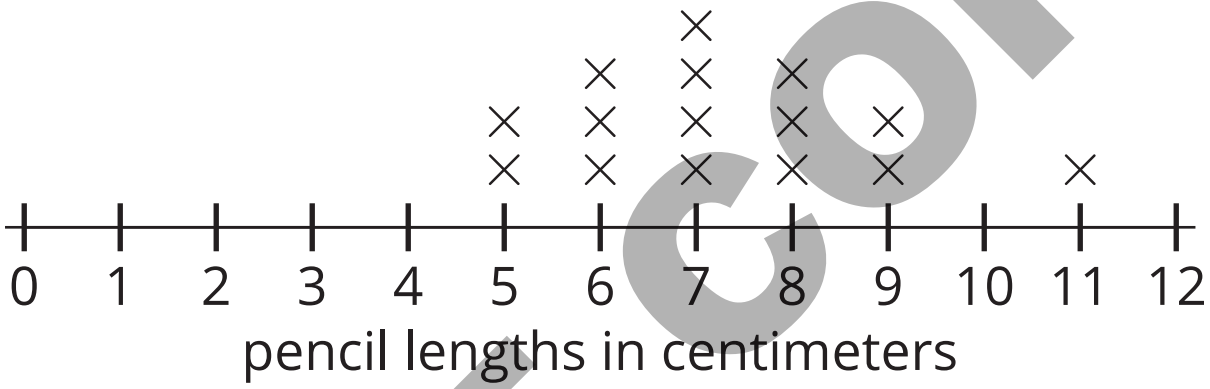
2. Choose an object in the room. Measure it with both yardsticks. Do you get the same measurement? Explain or show your reasoning.

Practice Problems

5 Problems

1 from Unit 3, Lesson 14

The line plot shows the lengths of some pencils in centimeters.



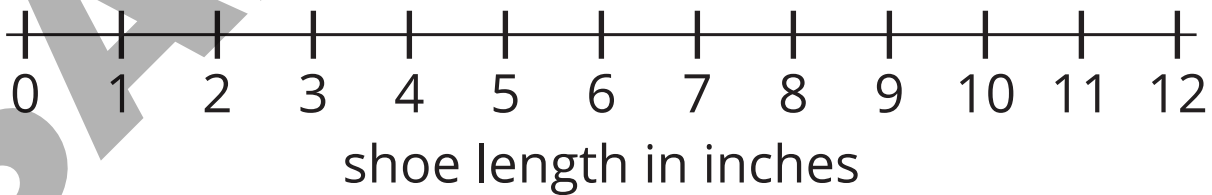
a. How long is the longest pencil? How long is the shortest?

b. What is the most common pencil length?

2 from Unit 3, Lesson 15

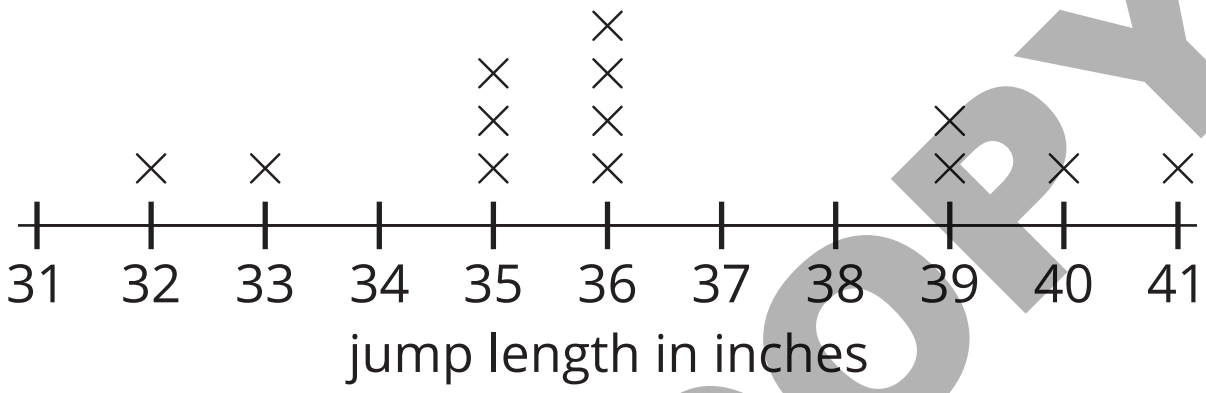
The table shows the lengths of shoes. Complete the line plot with the shoe length data.

student	shoe length in inches
Andre	7
Clare	7
Diego	8
Elena	8
Lin	6
Mai	7
Tyler	9



3 from Unit 3, Lesson 16

The line plot shows the lengths some students jumped.



a. What is the longest jump? What is the shortest?

b. How much longer is the longest jump than the shortest jump?

c. How many measurements are in the line plot?

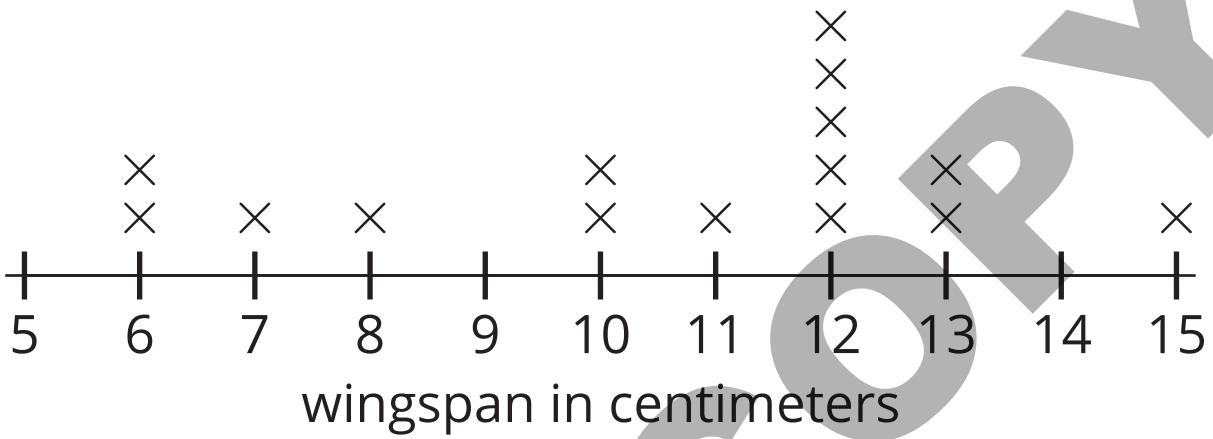
4 Exploration

Create and label a line plot for foot lengths so these statements are true.

a. There are 7 total foot lengths.
b. The most common foot length measured is 6 inches.
c. The difference between the longest foot and the shortest foot is 4 inches.

5 Exploration

Here are wingspans of some butterflies in Diego's collection.



Diego needs to record 2 more lengths. Which of these could change when Diego records the lengths? Explain your reasoning.

a. The most common measurement
b. The total number of measurements
c. The longest measurement
d. The shortest measurement

GRADE 2

UNIT

Addition and Subtraction on the Number Line

Content Connections

In this unit you will use a number line to represent numbers within 100 and relate addition and subtraction to length. You will make connections by:

- **Reasoning with Data** while interpreting and comparing linear measurements on a number line.
- **Exploring Changing Quantities** while solving one- and two-step story problems involving addition and subtraction of lengths.

- **Discovering Shape and Space** while solving problems involving length measurements using addition and subtraction.
- **Taking Wholes Apart, Putting Parts Together** while adding and subtracting two-digit numbers using different strategies such as models, symbols and drawings.

Addressing the Standards

As you work your way through **Unit 4 Addition and Subtraction on the Number Line,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use these MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 5 and 13
MP2 Reason abstractly and quantitatively.	Lesson 7, 9, 10, 11, and 12
MP3 Construct viable arguments and critique the reasoning of others.	Lesson 2, 3, 5, and 8
MP4 Model with mathematics.	Lesson 15
MP5 Use appropriate tools strategically.	Lesson 1, 6, and 14

Mathematical Practices	Where You Use these MPs
MP6 Attend to precision.	Lesson 2, 4, and 11
MP7 Look for and make use of structure.	Lesson 2, 4, 7, 8, and 10
MP8 Look for and express regularity in repeated reasoning.	Lesson 2 and 8

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Measure and Compare Objects • Problem Solving with Measure	**2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Lesson 15

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Dollars and Cents • Problem Solving with Measure • Number Strategies	**2.MD.5** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	Lesson 9 and 13
• Measure and Compare Objects • Measure and Compare Objects • Squares in an Array	**2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . , and represent whole-number sums and differences within 100 on a number line diagram.	Lesson 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15
• Represent Data • Dollars and Cents	**2.NBT.2** Count within 1000; skip-count by 2s, 5s, 10s, and 100s.	Lesson 2, 3, 6, and 8

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Strategies	**2.NBT.4** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	Lesson 15
• Dollars and Cents • Number Strategies	**2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relation-ship between addition and subtraction.	Lesson 4, 6, 9, 10, 11, 12, 13, 14, and 15
• Problem Solving with Measure • Number Strategies	**2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Lesson 12 and 13

Addressing CA CCSSM 2.MD.6; building on 2.MD.1; building towards 2.MD.6; practicing MP5

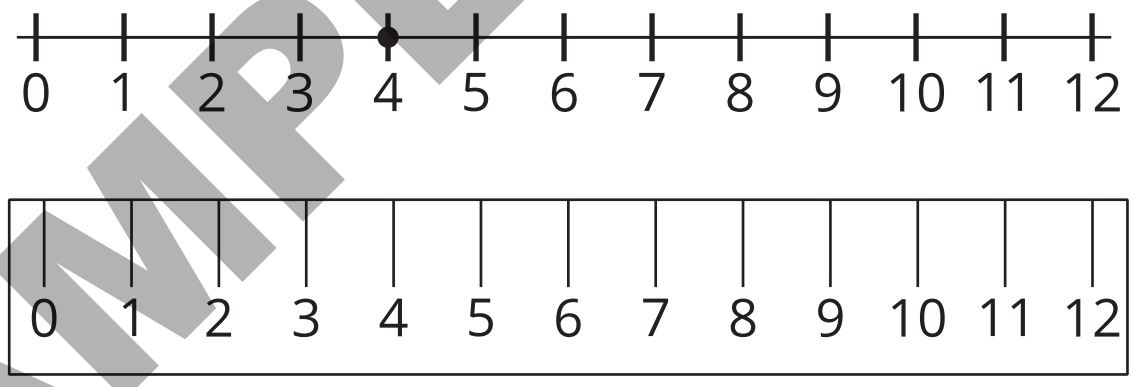
Whole Numbers on the Number Line

Let's represent numbers on a number line.

Warm-up

Notice and Wonder: Rulers and Number Lines

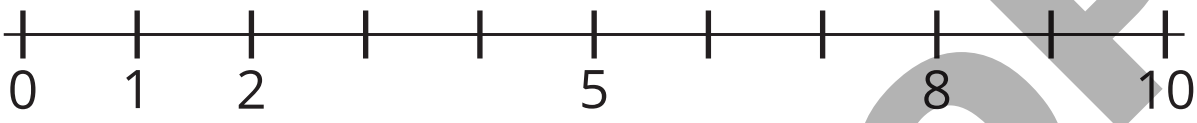
What do you notice? What do you wonder?



Activity 1

What Is a Number Line?

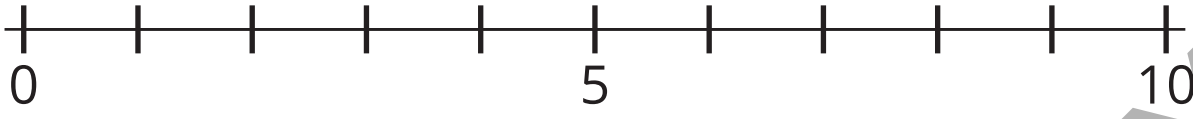
1. Label each tick mark with the number it represents.



2. Locate 6. Mark it with a point.

3. Locate a number that is greater than 6. Mark it with a point.

4. Label each tick mark with the number it represents.



5. Locate 9. Mark it with a point.

6. Locate a number that is less than 9. Mark it with a point.

Activity 2

Make Your Own Number Line

1. Make a number line that goes from 0 to 20.
2. Locate 13. Mark it with a point.
3. Locate 3. Mark it with a point.
4. Compare number lines with your partner.

Addressing CA CCSSM 2.MD.6, 2.NBT.2; practicing MP3, MP6, MP7, MP8

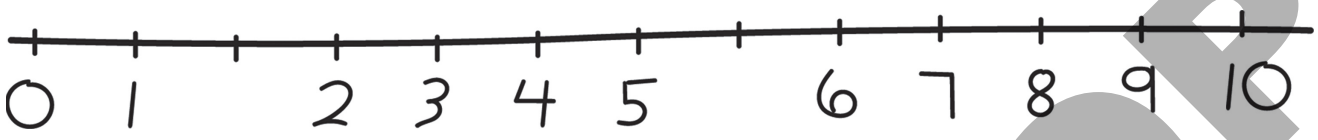
Features of a Number Line

Let's explore the features of a number line.

Activity 2

Analyze Number Lines

1. Jada's number line



How should Jada revise her number line?

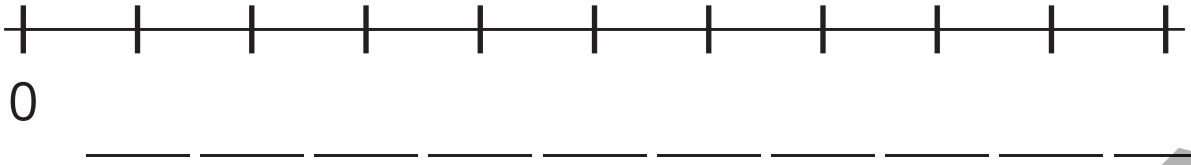
2. Andre's number line

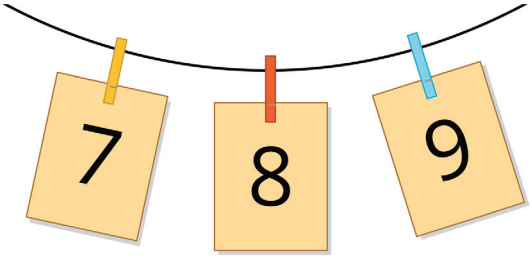
How should Andre revise his number line?

3. Elena's number line

How should Elena revise her number line?

4. Fill in the numbers to create your own number line.





Addressing CA CCSSM 2.MD.6, 2.NBT.2; building towards 2.MD.6; practicing MP3

Unlabeled Tick Marks

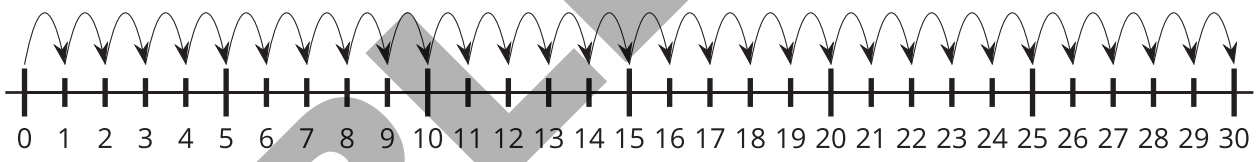
Let's locate numbers on the number line.

Warm-up

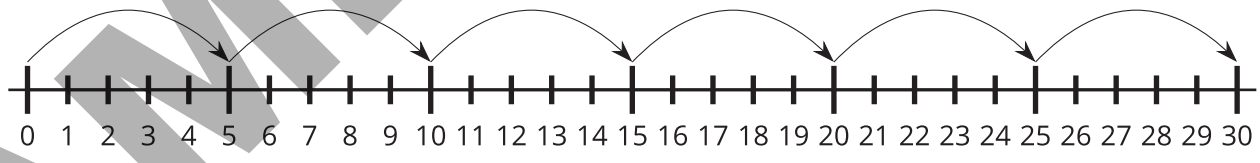
Notice and Wonder: From 0 to 30

What do you notice? What do you wonder?

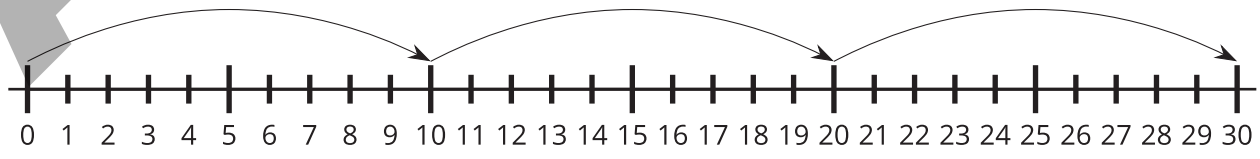
A



B



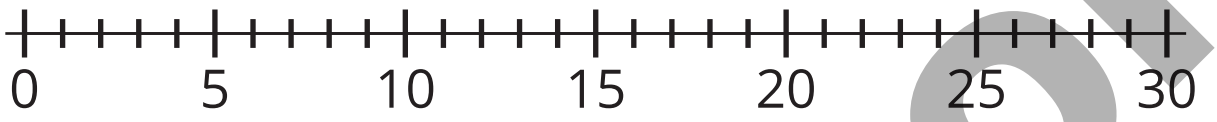
C



Activity 1

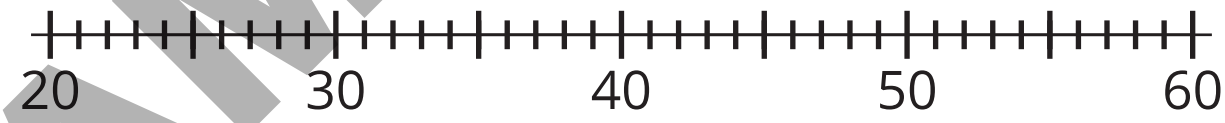
Locate the Numbers

1. Locate 24. Mark it with a point.

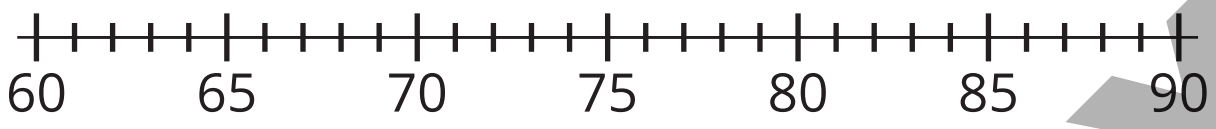


2. Locate 37. Mark it with a point.

3. Locate 48. Mark it with a point.



4. Locate 83. Mark it with a point.



5. Are the numbers you marked on the number lines in the correct locations? How do you know?

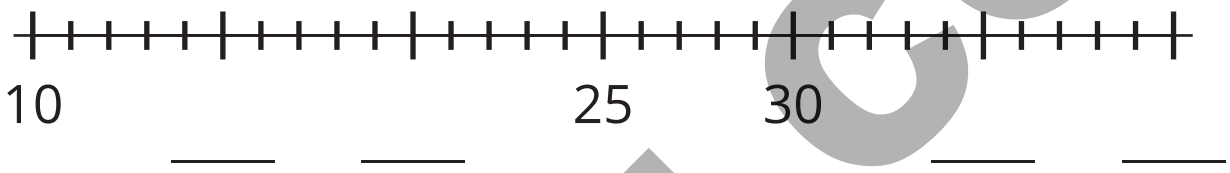
Activity 2

Are You Missing Something?

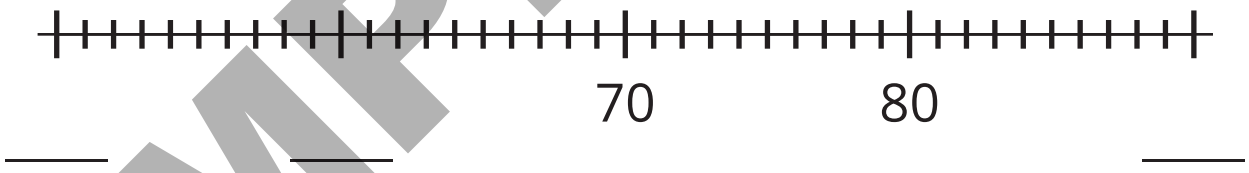
Sec A

Complete each number line. Fill in the labels with the number the tick mark represents. Locate each number. Mark it with a point. Label it with the number it represents.

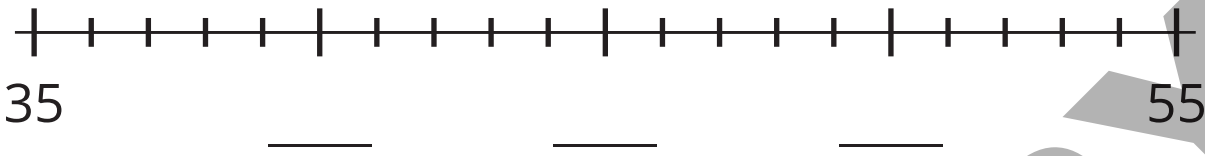
1. Locate and label 17.



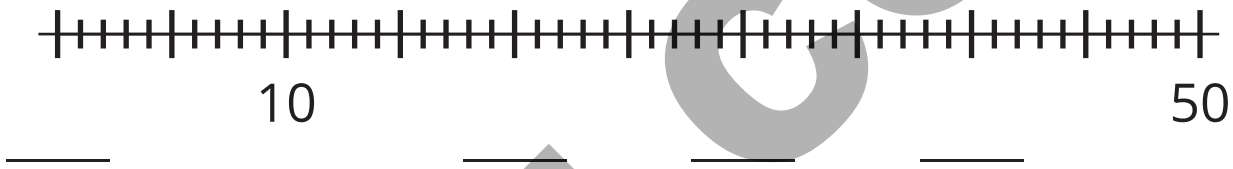
2. Locate and label 59.



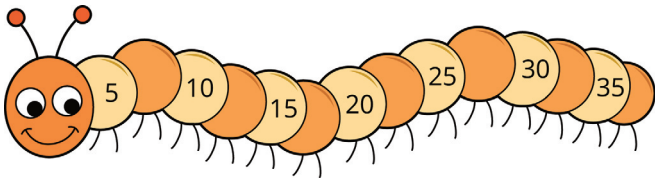
3. Locate and label 43.



4. Locate and label 35.



5. Share your number lines with your partner.



Addressing CA CCSSM 2.MD.6, 2.NBT.5; practicing MP6 and MP7

Compare Numbers on a Number Line

Let's use the number line to compare numbers.

Warm-up

Number Talk: Subtract Fives

Find the value of each expression mentally.

- 35 – 5
- 35 – 10
- 35 – 15
- 35 – 25

Activity 1

Compare the Numbers

- Partner A:
  - Roll 3 number cubes. Find the sum.
  - Put a counter on the location of the sum.
- Partner B:
  - Roll 3 number cubes. Find the sum.
  - Put a counter on the location of the sum on the same number line.
- Decide which number is greater. Explain your reasoning.
- Use <, >, or = to compare the 2 numbers represented on your number line.

Partner A	<, >, or =	Partner B

Activity 2

Compare Larger Numbers

- Each partner rolls 2 number cubes and makes a 2-digit number.
- Locate and label your numbers on the number line.
- Use <, >, or = to compare the numbers.
- Explain why your comparison is true.

Partner A	<, >, or =	Partner B

Addressing CA CCSSM 2.MD.6; practicing MP1 and MP3

Estimate on a Number Line

Let's estimate numbers on a number line.

Warm-up

Estimation Exploration: What Number?

What number could this be?

1. Record an estimate that is:

too low	about right	too high

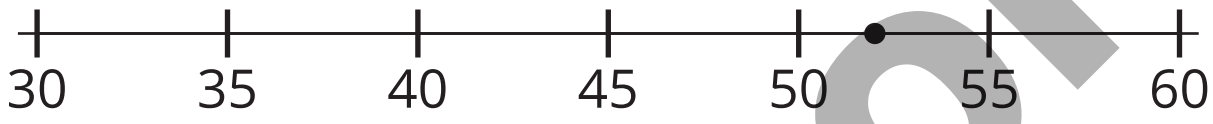
2. Record an estimate that is:

too low	about right	too high

Activity 1

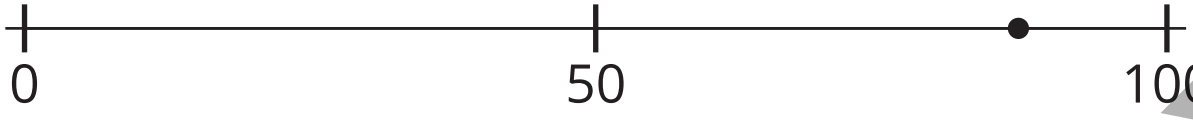
Estimate the Numbers

1. What number could this be? _____



2. What number could this be? _____

3. What number could this be? _____



4. What number could this be? _____

Activity 2

Order the Numbers

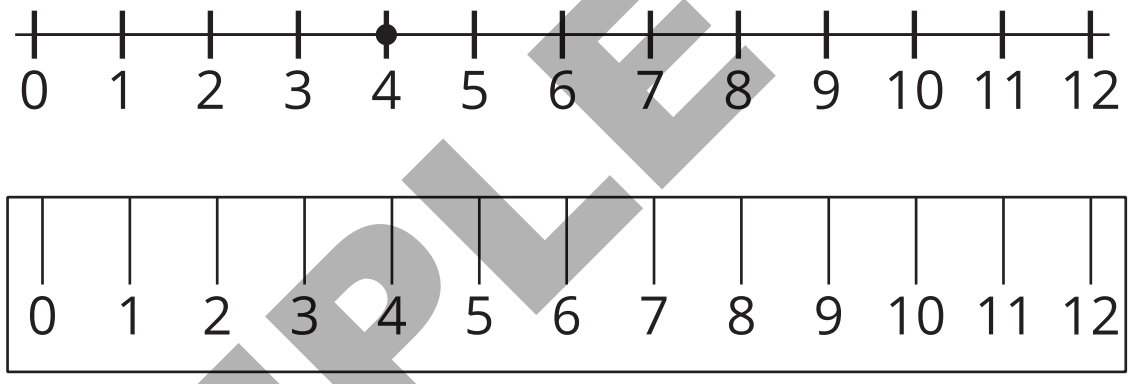
- Pick a card. Place it on the number line.
- Explain your reasoning.
- As a group, revise the position of any cards.
- Repeat until all cards are placed.
- Draw and label points to represent each number on the number line.

Section A Summary

We learned that **number lines** are diagrams that represent numbers as lengths away from 0 along a straight line.

- There are equal-size spaces between dots or tick marks.
- Each dot or tick mark represents a number.
- Number lines show how close or how far numbers are from 0 and each other.

We represented numbers with tick marks and points on number lines. We learned that values go up when moving to the right. We counted by 5 and 10 on number lines to locate and label numbers. We also estimated numbers by thinking about how close they were to 0 and other numbers.



Addressing CA CCSSM 2.MD.6, 2.NBT.2, 2.NBT.5; practicing MP5

Center Day 1

Let's put numbers on a number line.

Warm-up

Number Talk: Decompose and Subtract

Find the value of each expression mentally.

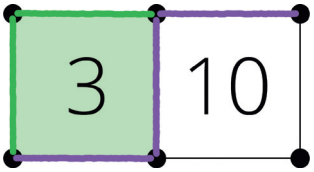
- 65 – 25
- 65 – 27
- 55 – 17
- 46 – 18

Activity 2

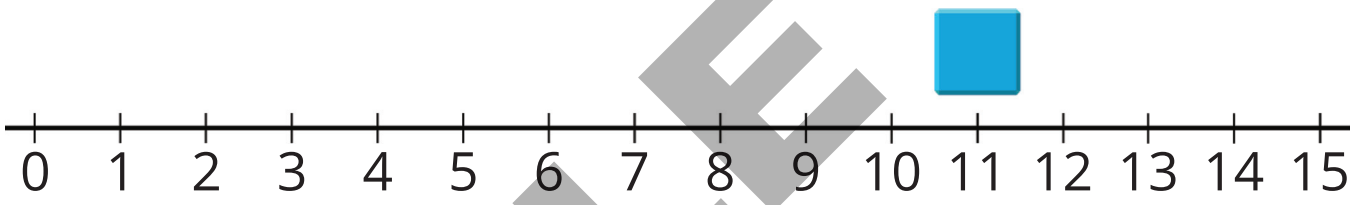
Centers: Choice Time

Choose a center.

Capture Squares



Number Line Scoot



Number Puzzles

$14 = 8 + \square$

Practice Problems

11 Problems

1 Pre-unit

Write the number that makes each statement true.

a. $9 + 7 = \square$

b. $15 - \square = 8$

c. $\square - 11 = 8$

2 Pre-unit

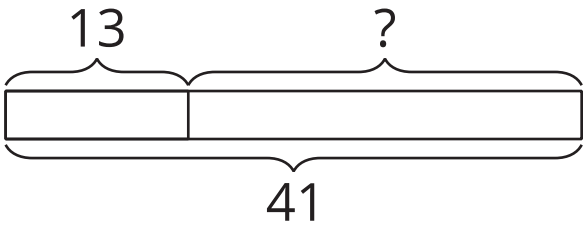
Write < or > in each box to make each statement true.

a. 91 ☐ 19

b. 84 ☐ 87

c. 52 ☐ 36

a. Write an equation that matches the diagram.



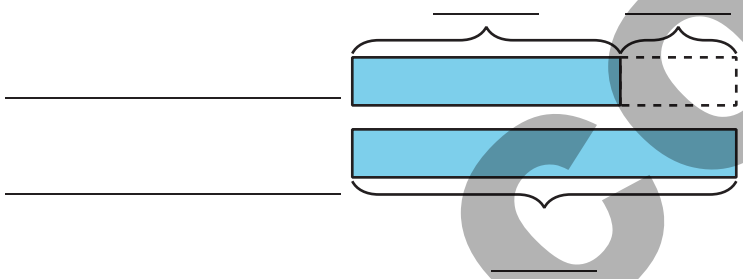
b. Find the unknown value.

4 Pre-unit

There are 37 frogs in the pond. There are 16 more goldfish than frogs.

How many goldfish are there in the pond?

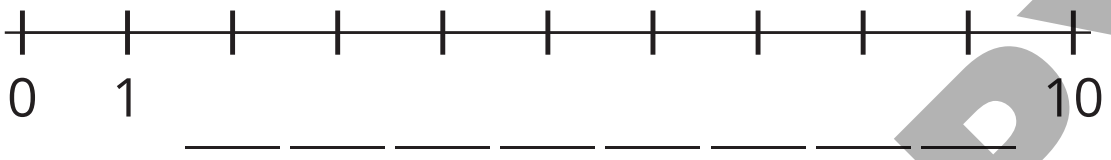
a. Complete the tape diagram to match the story problem.



b. Solve the problem. Explain your reasoning.

5 from Unit 4, Lesson 1

a. Label each tick mark with the number it represents.



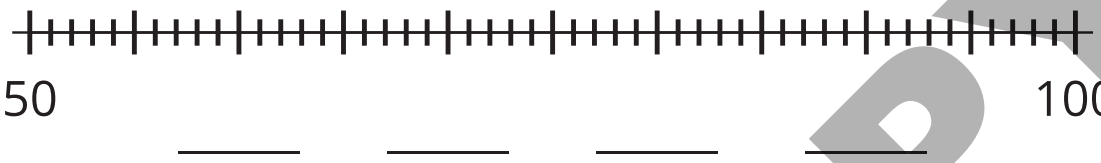
b. Locate 7. Mark it with a point.

6 from Unit 4, Lesson 2

Here is Mai's number line. How should she revise her number line?

7 from Unit 4, Lesson 3

a. Count by 10. Start at 50. End at 100. Label each number.



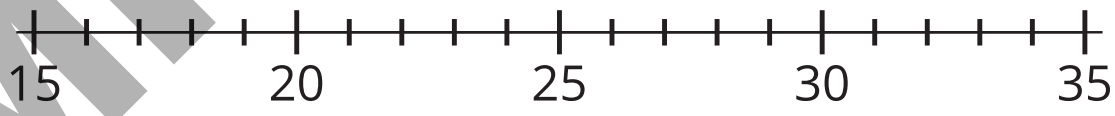
_____ _____ _____ _____

b. Locate and label 78.

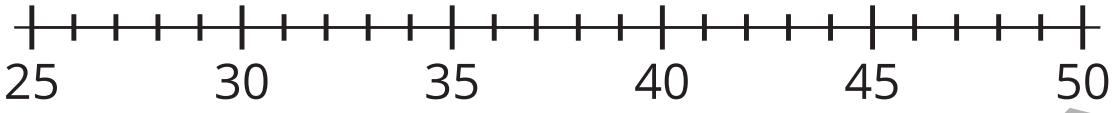
8 from Unit 4, Lesson 4

Locate and label each number pair. Use < or > to compare the numbers.

a. 23 and 27



b. 28 and 46



9 from Unit 4, Lesson 5

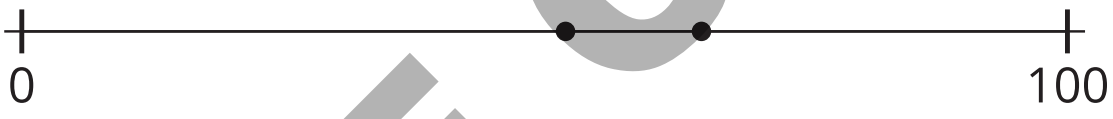
What number could this be? Explain your reasoning.

10 Exploration

a. Locate and label where you think 35 is on the number line.

Explain your reasoning.

b. Elena and Han mark where they think 83 is.

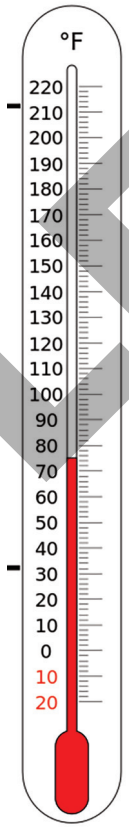


Why do you think they put their points at different locations?

Where do you think 83 is on the number line?

11 Exploration

a. Here is a picture of a thermometer.



How are the thermometer and a number line alike? How are they different?

b. Here is a picture of a rain gauge.

How are the rain gauge and a number line alike? How are they different?

Addressing CA CCSSM 2.MD.6; building towards 2.MD.6; practicing MP2 and MP7

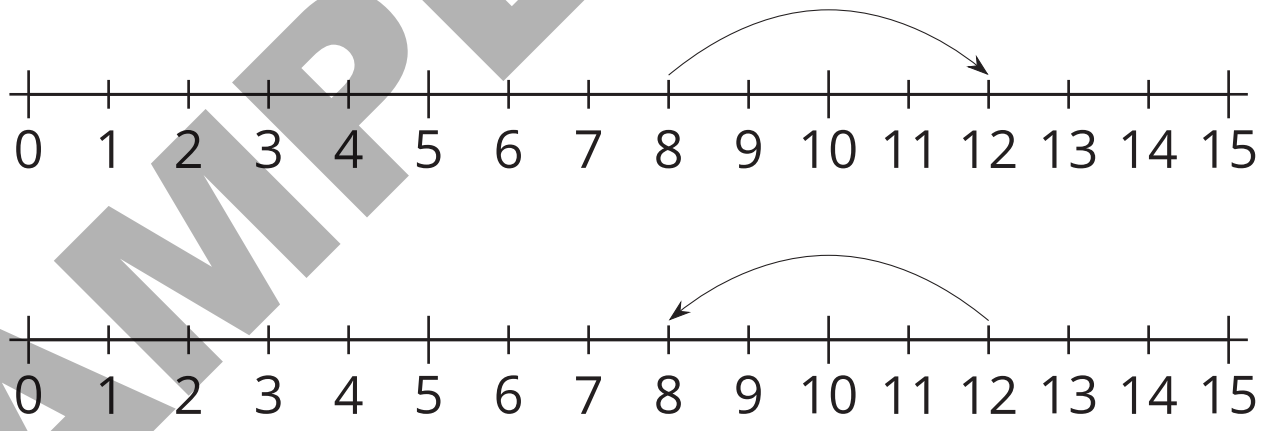
Addition and Subtraction on the Number Line

Let's match equations to number lines.

Warm-up

Notice and Wonder: Jumps on the Number Line

What do you notice? What do you wonder?

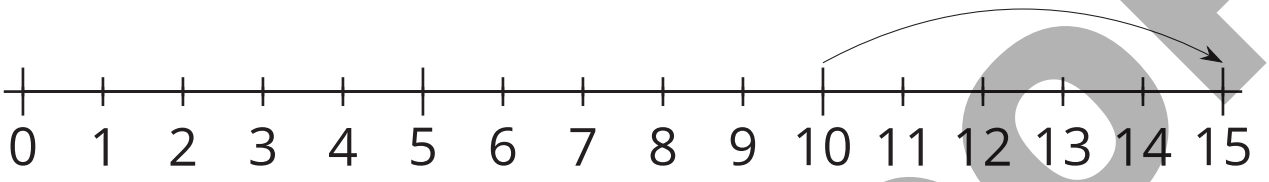


Activity 1

Add and Subtract

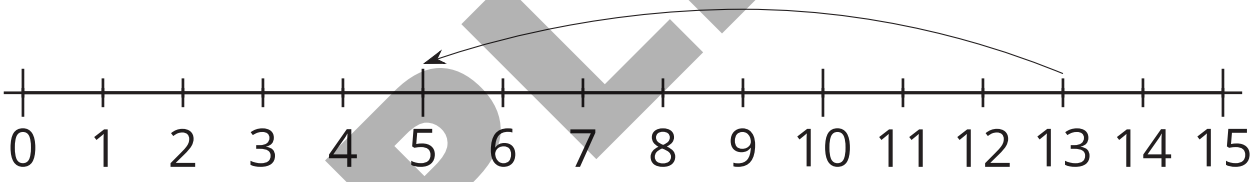
Circle the equation each number line represents.

1.



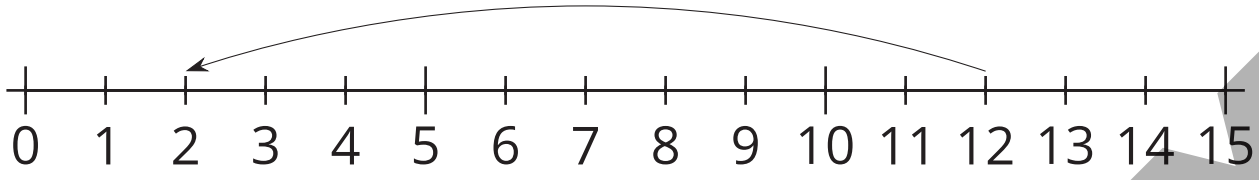
$10 + 5 = 15$ $15 - 5 = 10$

2.



$8 + 5 = 13$ $13 - 8 = 5$

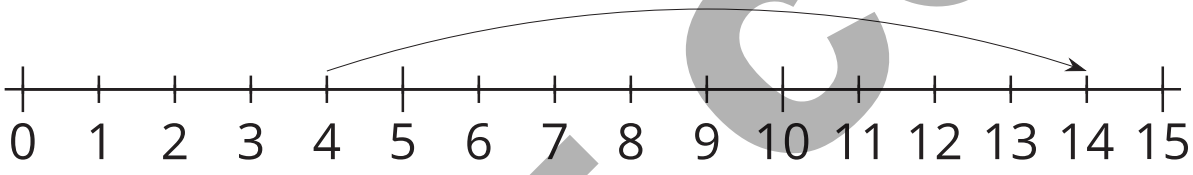
3.



$12 - 2 = 10$

$12 - 10 = 2$

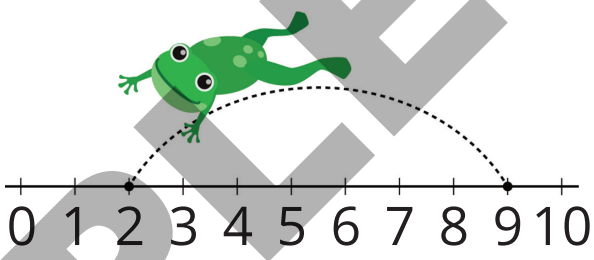
4. a.



$4 + 10 = 14$

$10 + 4 = 14$

b. Why doesn't the other equation match the number line? Explain your reasoning.

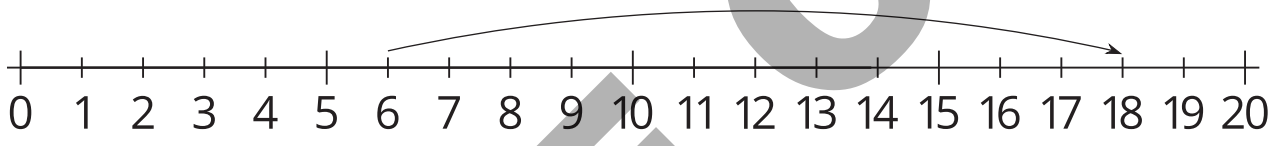


Activity 2

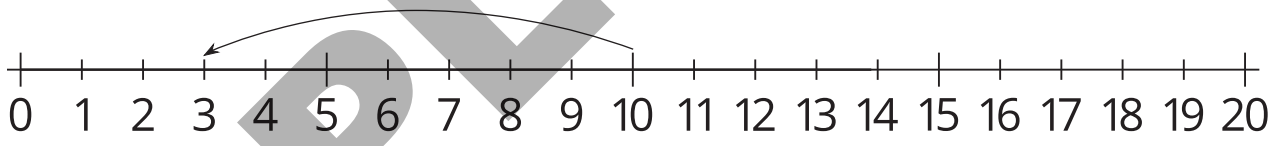
Number Lines and Equations

1. Cut out the equations.
2. Paste each equation next to the number line that represents it.
3. Paste the equation that didn't have a match. Represent it on the blank number line.

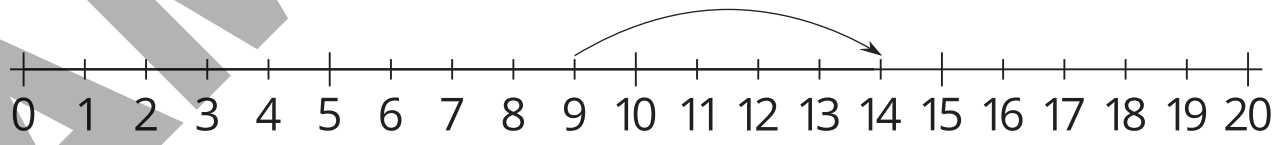
a.



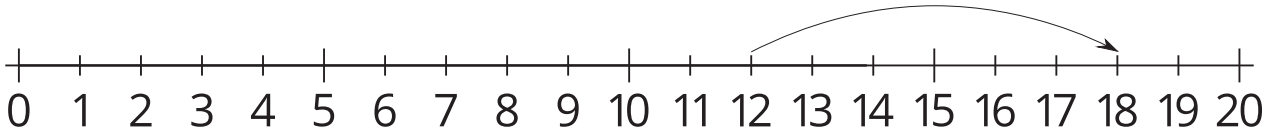
b.



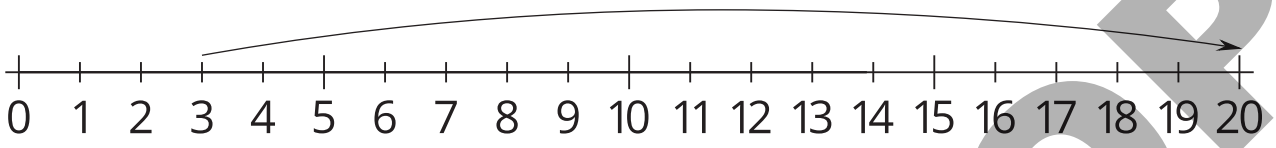
c.



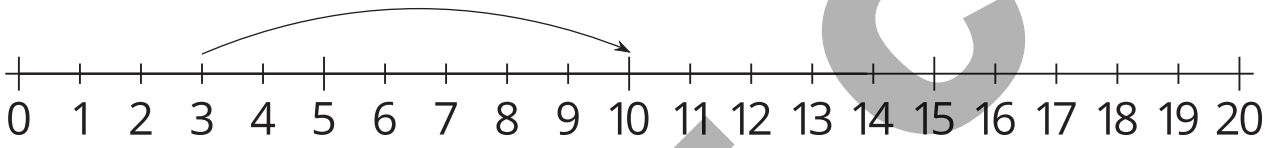
d.



e.

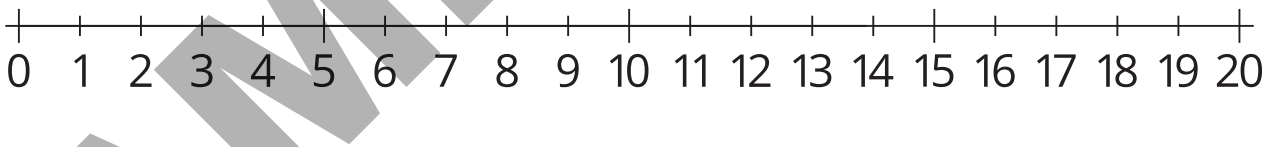


f.



g.

h.



$3 + 7 = 10$	$10 - 7 = 3$	$12 + 6 = 18$	$6 + 12 = 18$
$14 - 5 = 9$	$9 + 5 = 14$	$20 - 3 = 17$	$3 + 17 = 20$

Unit 4, Lesson 8

Addressing CA CCSSM 2.MD.6, 2.NBT.2; practicing MP3, MP7, MP8

Equations on a Number Line

Let's write equations and represent them on a number line.

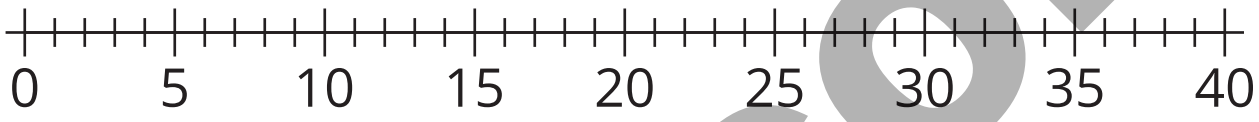
Sec B

Activity 1

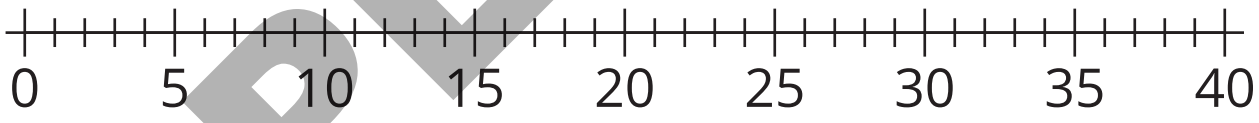
Represent Equations

Represent each equation on the number line.

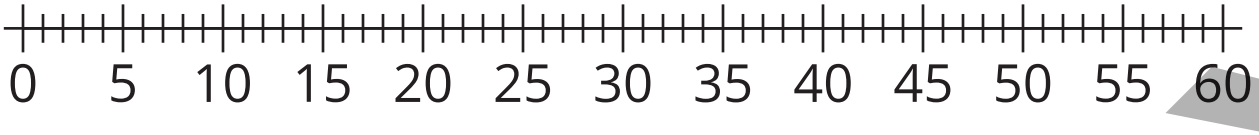
1. $15 + 7 = 22$



2. $18 - 6 = 12$

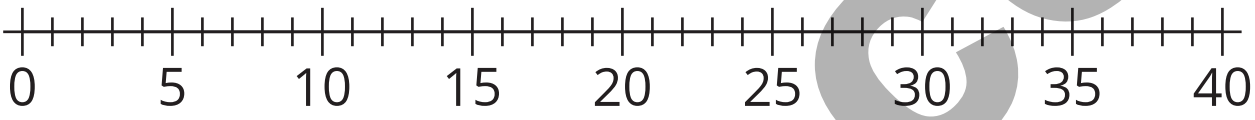


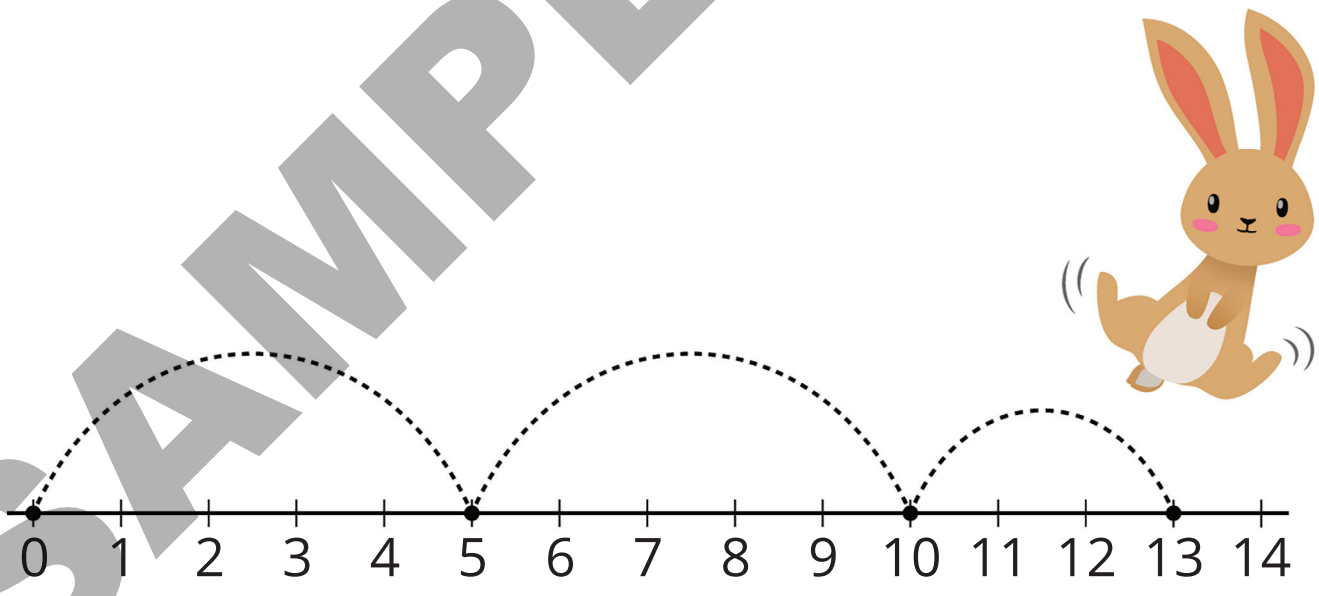
3. $46 + 7 = 53$



Sec B

4. $33 - 9 = 24$



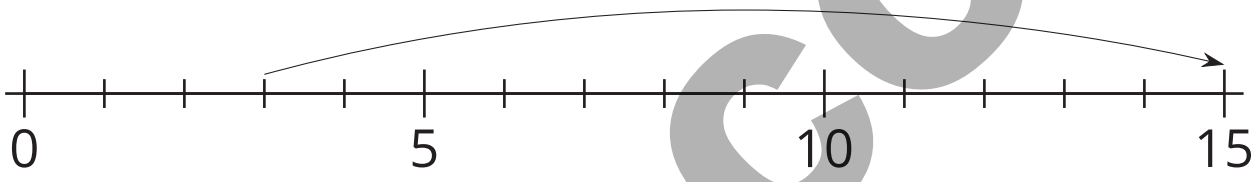


Activity 2

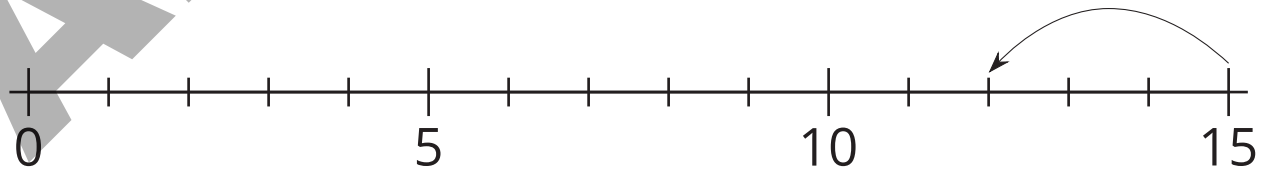
Write Equations

Write an equation that matches the diagram.

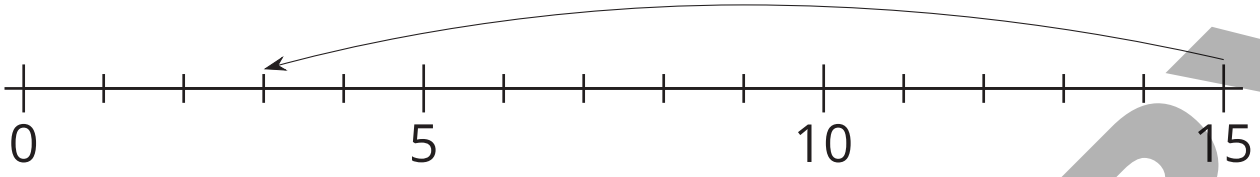
1. Equation: _____



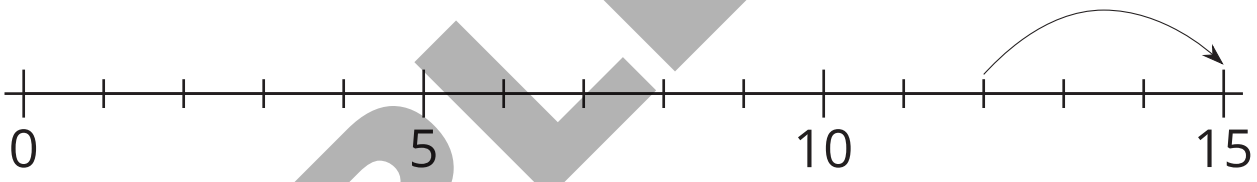
2. Equation: _____



3. Equation: _____



4. Equation: _____



5. Compare your equations with your partner.

6. Pick the 2 number lines you think are most alike. Explain your choice to your partner.

Addressing CA CCSSM 2.MD.5, 2.MD.6, 2.NBT.5; practicing MP2

The Difference between Numbers

Let's represent ways to subtract on the number line.

Warm-up

Number Talk: Add To

Find the value of each expression mentally.

- 20 – 2
- 20 – 17
- 49 – 3
- 67 – 64

Activity 1

Add or Subtract

1. What number makes this equation true? _____
   $38 - 4 = ?$
   Show your thinking using the number line.

2. What number makes this equation true? _____
   $75 - 68 = ?$
   Show your thinking using the number line.

3. What number makes this equation true? _____
   $57 - 24 = ?$
   Show your thinking using the number line.

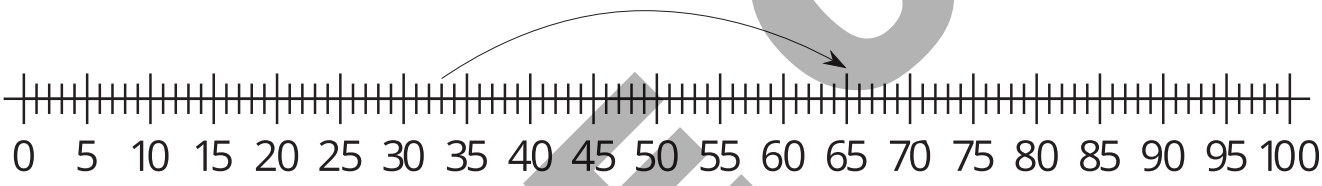
Activity 2

Different Ways to Find the Difference

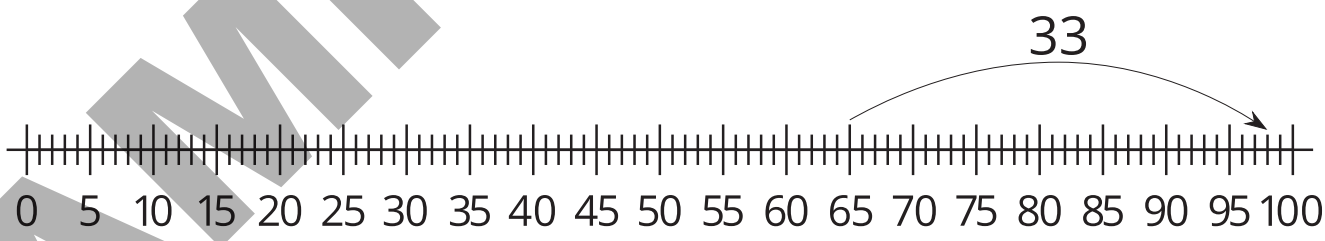
1. Elena's string is much too long. The string is 65 inches long. Elena cuts 33 inches off the length. How long is the string now?

   Choose 2 number lines that show a way to find the length of Elena's string.

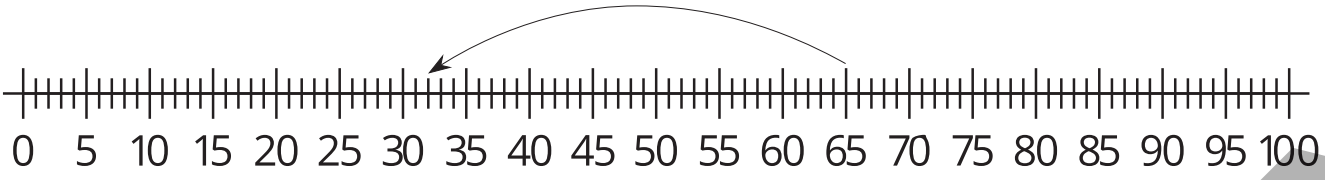
   a.



   b.



c.



2. Han has 87 inches of string. He cuts off 85 inches of it. How much string does he have left?

   a. Write an equation to represent the problem. Use a ? for the unknown.

   b. Find the number that makes the equation true.

   c. Show your thinking using the number line.

3. Find someone who used a different method.
   Show their method on the number line.

Unit 4, Lesson 10

Addressing CA CCSSM 2.MD.6, 2.NBT.5; building on 2.MD.6; building towards 2.NBT.5; practicing MP2 and MP7

Place Value and the Number Line

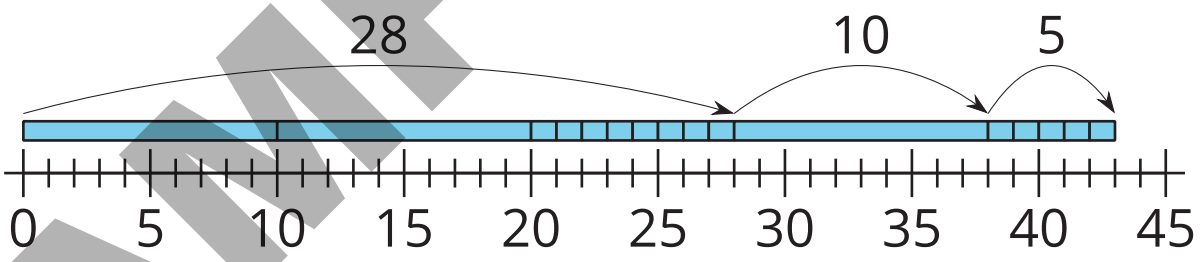
Sec B

Let's compare methods using the number line.

Warm-up

Notice and Wonder: Base-Ten and the Number Line

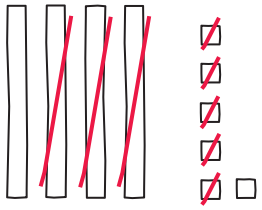
What do you notice? What do you wonder?



Activity 1

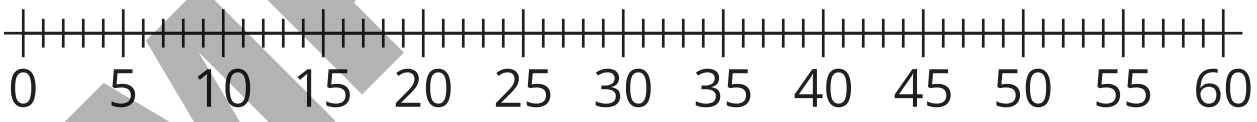
Compare Representations

Clare subtracts and represents her work with a base-ten diagram.



1. Write an equation to represent Clare's work.

2. Represent Clare's method on the number line.



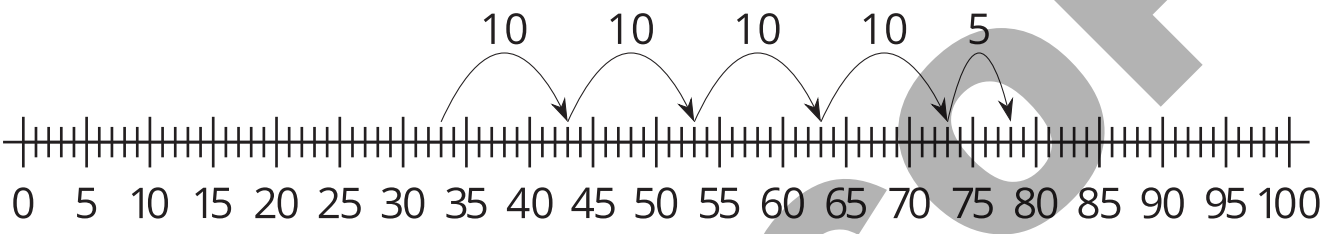
3. Find the value of 58 – 24.
   Show your thinking using a base-ten diagram.

4. Represent how you found the value of 58 – 24 on the number line.

Activity 2

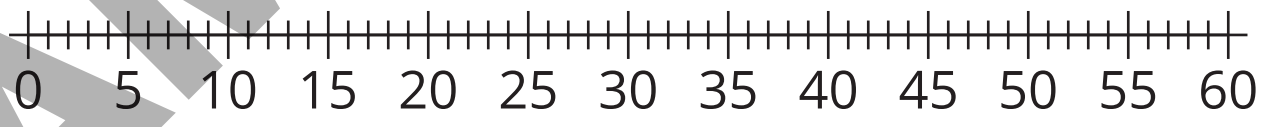
On the Number Line

Diego finds the value of 33 + 45. He says he can count on by tens, then by ones. He uses a number line to show what he means.

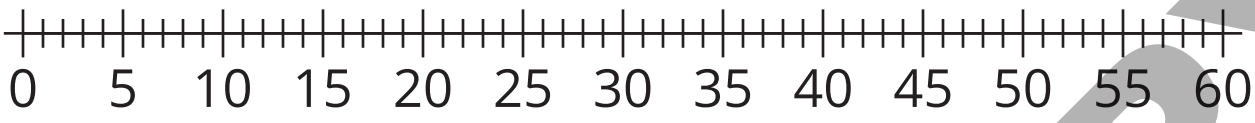


1. Write an equation to show the sum for Diego's work.

2. Find the value of 23 + 24.
   Show your thinking using the number line.



3. Find the value of 50 – 32.
   Show your thinking using the number line.



4. Find the value of 40 – 26.
   Show your thinking using the number line.

Addressing CA CCSSM 2.MD.6, 2.NBT.5; practicing MP2 and MP6

Different Ways to Add and Subtract

Let's add and subtract by using a ten.

Warm-up

Number Talk: Use Tens

Find the value of each expression mentally.

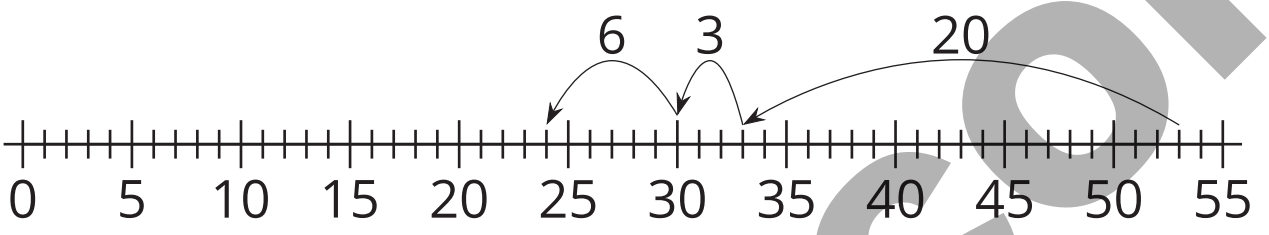
- 32 – 6
- 43 – 6
- 51 – 5
- 52 – 7

Activity 1

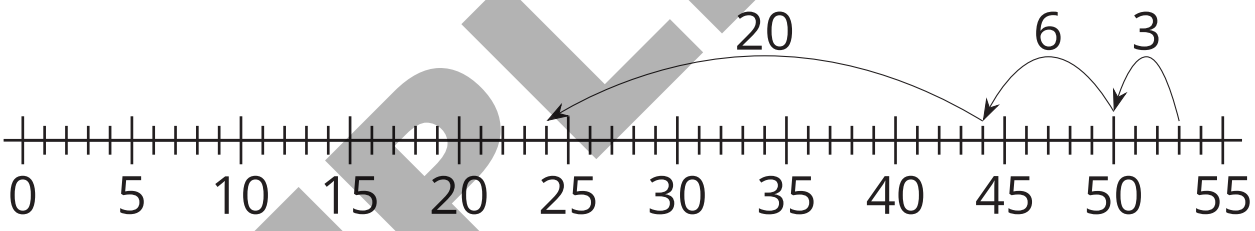
Compare Methods

Diego and Tyler find the value of 53 – 29. Their work is shown.

Diego's method:



Tyler's method:



1. Use Diego's method to find the value of 82 – 35.

2. Use Tyler's method to find the value of 66 – 28.

Activity 2

More Methods

Partner A

1. Find the value of 59 + 27.

Sec B

2. Find the value of 65 – 18.

Partner B

1. Find the value of 68 – 39.

2. Find the value of 22 + 49.

Addressing CA CCSSM 2.MD.6, 2.NBT.5, 2.OA.1; practicing MP2

Equations with Unknowns

Let's represent addition and subtraction with number lines and with equations using a ? for the unknown.

Warm-up

True or False: Making Tens

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $40 = 10 + 27 + 3$

- $47 = 20 + 7 + 3 + 10$

- $60 = 3 + 47 + 10$

Activity 1

Number Line Riddles

Solve each riddle to find the mystery number.
For each riddle:

- Write an equation that represents the riddle. Write a ? for the unknown.
- Represent the equation on the number line. Write the mystery number.

1. I started at 15 and jumped 17 to the right. Where did I end?

   Equation: _____

   Mystery number: _____

2. I started at a number and jumped 20 to the left. I ended at 33. Where did I start?

   Equation: _____

   Mystery number: _____

3. I started at 42 and ended at 80. How far did I jump?

   Equation: _____

   Mystery number: _____

4. I started at 76 and jumped 27 to the left. Where did I end?

   Equation: _____

   Mystery number: _____

5. I started at a number and jumped 19 to the right. I ended at 67. Where did I start?

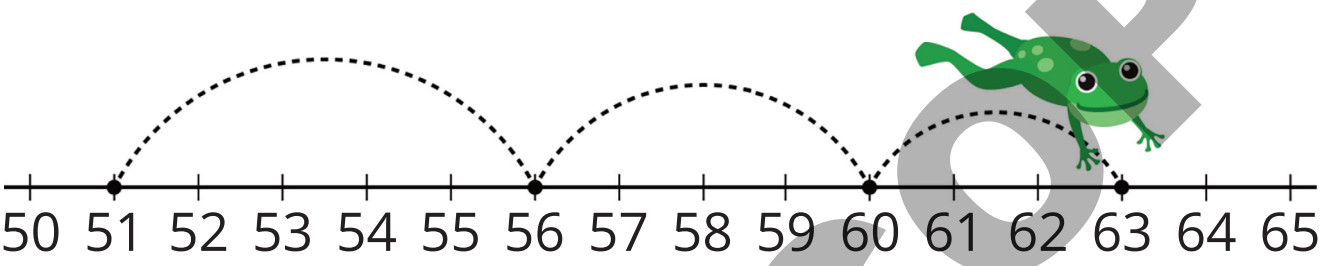
   Equation: _____

   Mystery number: _____

6. I started at 92 and ended at 33. How far did I jump?

Equation: _____

Mystery number: _____



Activity 2

Make the Equations True

Find the number that makes each equation true.
Show your thinking using a number line.

1. ? – 48 = 19

2. 86 – ? = 39

3. ? + 57 = 72

4. 73 + ? = 91

Addressing CA CCSSM 2.MD.5, 2.MD.6, 2.NBT.5, 2.OA.1; building on 2.MD.6, Building towards 2.OA.1; practicing MP1

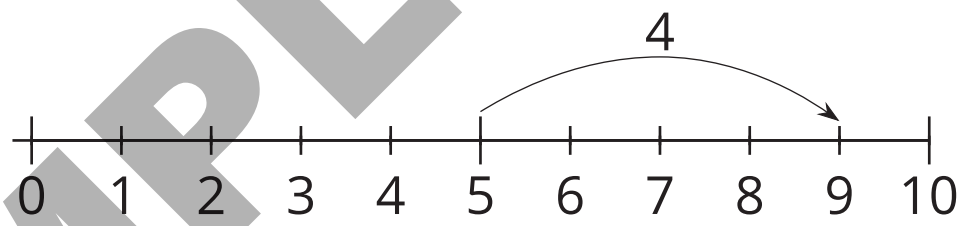
Represent Story Problems

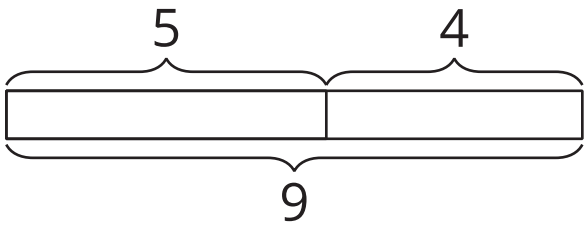
Let's compare representations and solve problems.

Warm-up

Notice and Wonder: Compare Representations

What do you notice? What do you wonder?





Activity 1

Card Sort: Represent Stories

Your teacher will give you a set of cards that show story problems, diagrams, number lines, and equations.

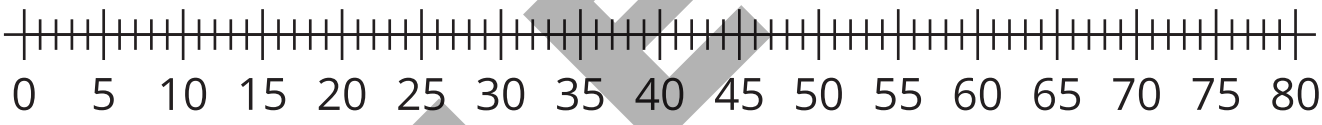
Match each story with a diagram, a number line, and an equation. Be ready to explain your reasoning.

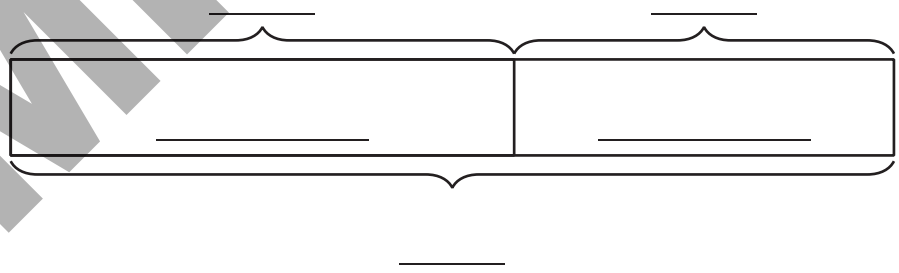
Activity 2

All Kinds of Representations

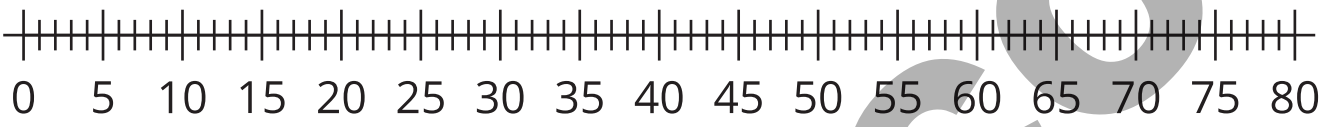
Solve each problem. Use a number line or diagram if it helps.

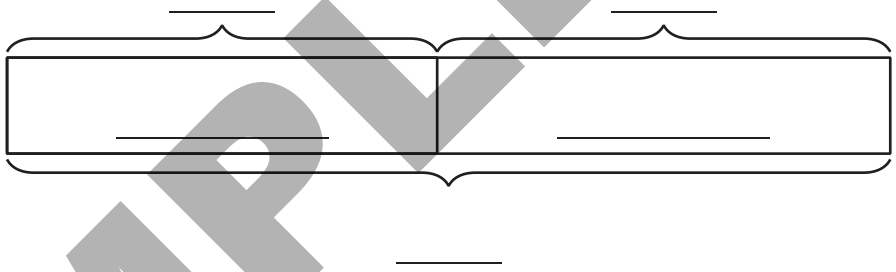
1. Clare starts with 24 cubes and adds some more. She makes a train with 42 cubes. How many cubes does she add on?



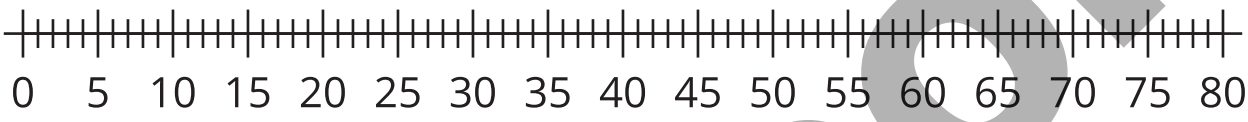


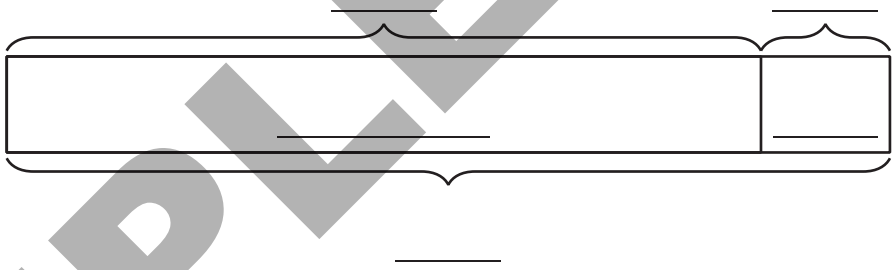
2. Andre has 37 cubes. He adds 39 more to make the train longer. How many cubes does he use?





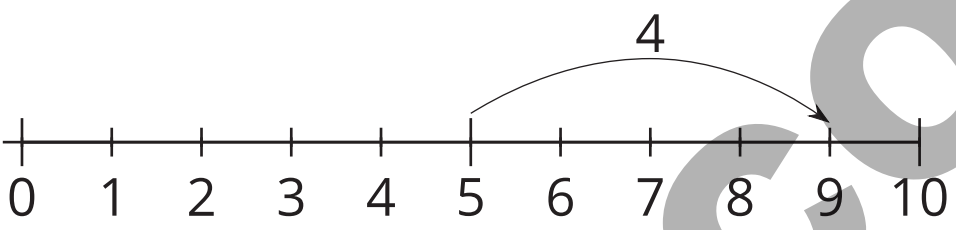
3. Mai adds 8 cubes to her train. Now it is 55 cubes long. How long was her train before she added cubes?

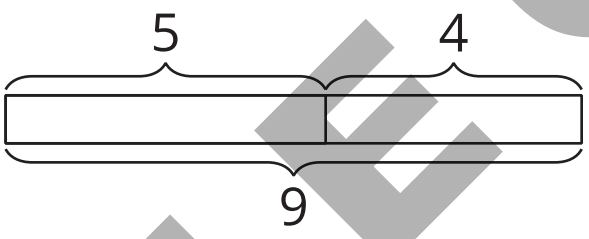


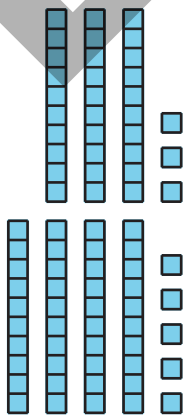


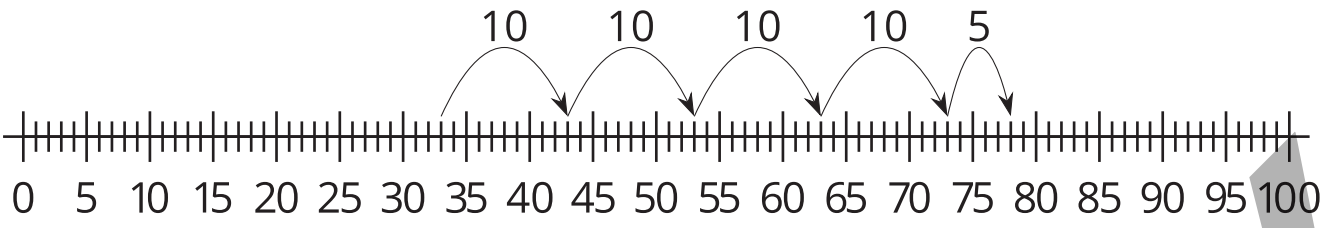
Section B Summary

We solved all types of problems and represented them in different ways. We represented addition and subtraction on number lines. We made connections to equations and strategies. We added and subtracted within 100. We used diagrams, base-ten blocks, number lines, and equations to show our thinking and make sense of stories and situations.








10
10
10
10
5
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

Addressing CA CCSSM 2.MD.6, 2.NBT.5; practicing MP5

Center Day 2

Let's practice adding and subtracting using the number line.

Warm-up

True or False: Is This 50?

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $28 + 2 + 20 = 50$

- $16 + 3 + 30 = 50$

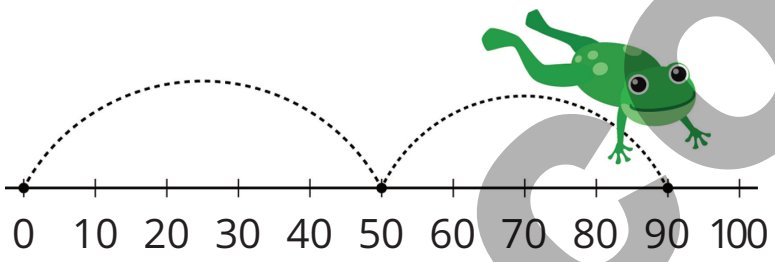
- $36 + 4 + 20 = 50$

Activity 2

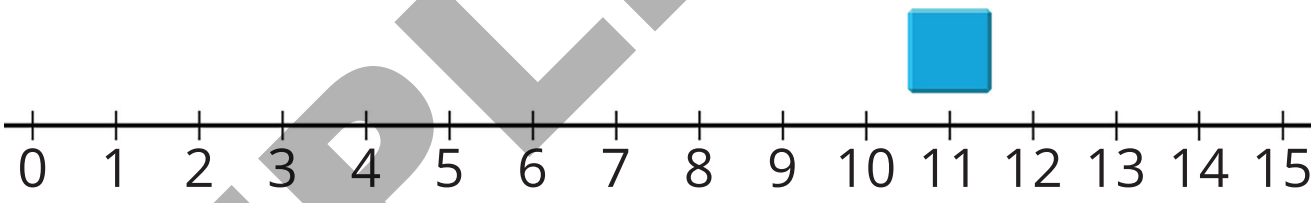
Centers: Choice Time

Choose a center.

Jump the Line



Number Line Scoot



How Close?

☐☐ + ☐☐ = _____

Addressing CA CCSSM 2.MD.1, 2.MD.6, 2.NBT.4, 2.NBT.5; building towards 2.MD.1; practicing MP4

Tall Enough to Ride

Let's find out what amusement park rides people are tall enough to ride.

Warm-up

Notice and Wonder: How Tall Are You?

What do you notice? What do you wonder?

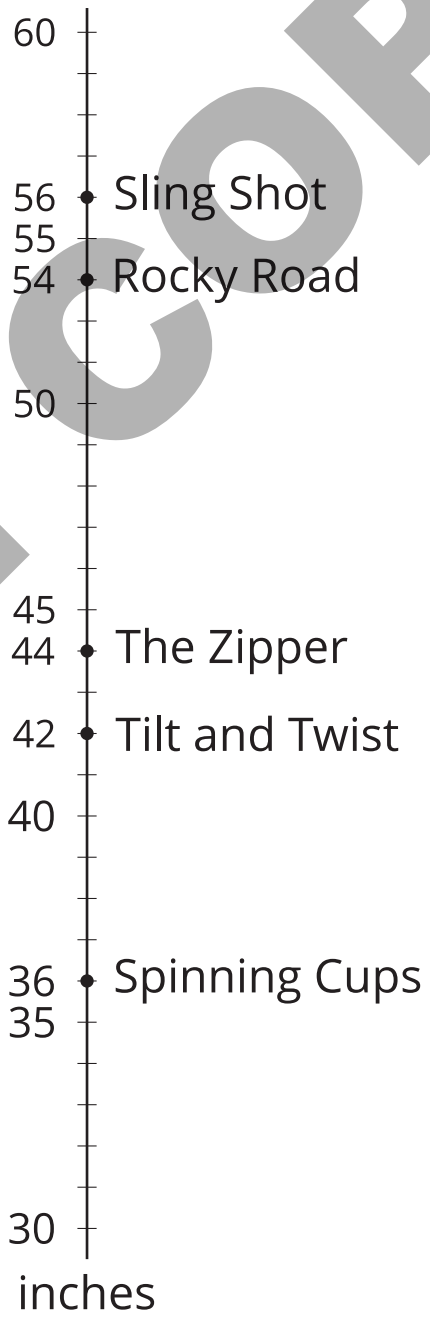
Activity 1

Must Be This Tall to Ride

The number line shows how tall you must be to go on some rides at an amusement park.

What do you notice?

What do you wonder?



1. How many of the rides could a person go on if they are:

   a. 40 inches tall?

   b. 43 inches tall?

   c. 30 inches tall?

   d. 62 inches tall?

2. Show each person's height on the number line.
   - Jada is 48 inches tall.
   - Jada's brother is 12 inches taller than her.
   - Jada's sister is 22 inches shorter than her brother.

Activity 2

A Day at the Park

Help Jada's family plan their day at the amusement park. Use the following information.

- There is time for 2 activities before lunch and 1 activity after it.
- The family wants to go on at least 1 ride together.
- Jada and her sister want time to play games in the arcade.
- Jada's sister hopes there is time to get ice cream.
- Jada's brother wants to go on rides all day.

Practice Problems

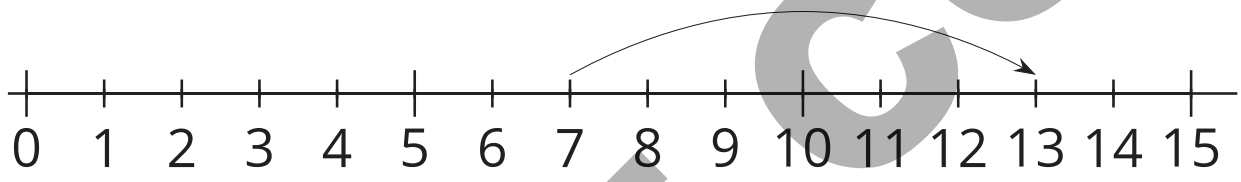
9 Problems

1 from Unit 4, Lesson 7

Which equation does the number line represent? Explain your reasoning.

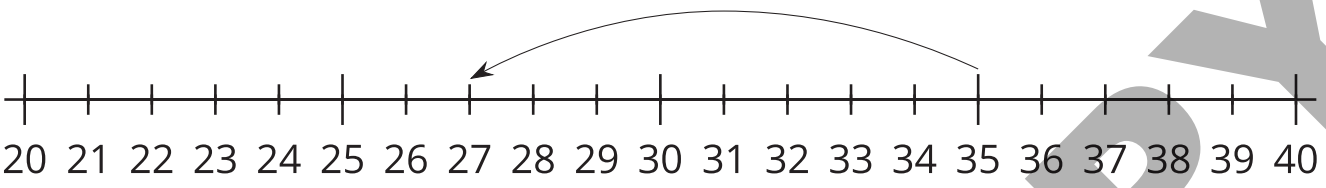
A. $7 + 6 = 13$

B. $13 - 6 = 7$



2 from Unit 4, Lesson 8

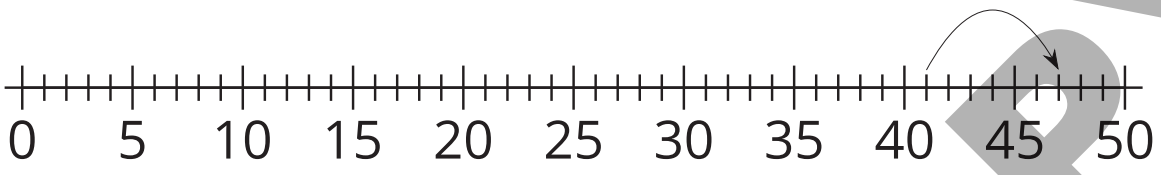
Here is a number line.

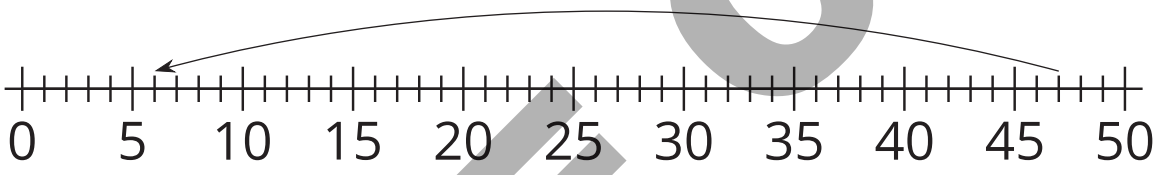


a. Write an equation that the number line represents.

b. How does your equation match the number line? Explain your reasoning.

a. How does each number line represent the value of 47 – 41? Explain your reasoning.

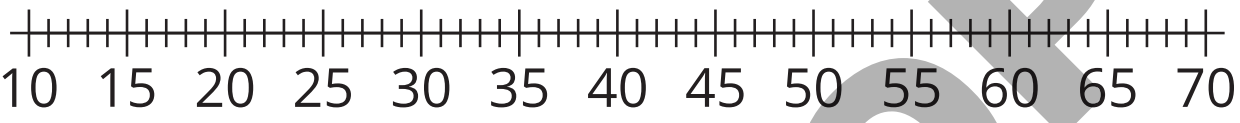




b. Which method do you prefer for finding the value of 47 – 41? Explain your reasoning.

4 from Unit 4, Lesson 10

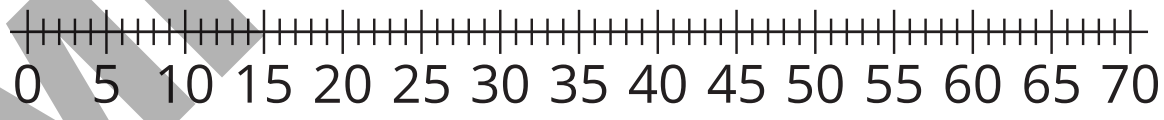
Find the value of 32 + 26. Represent your thinking on the number line.



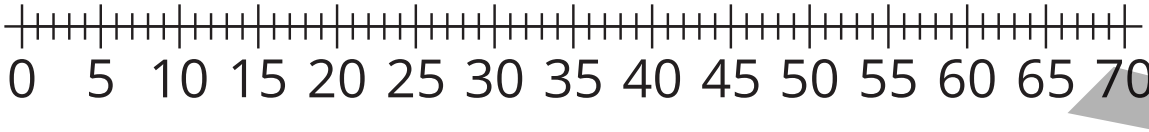
5 from Unit 4, Lesson 11

Find the value of 65 – 58 two different ways. Use the number lines.

a. Method 1:



b. Method 2:



Sec B

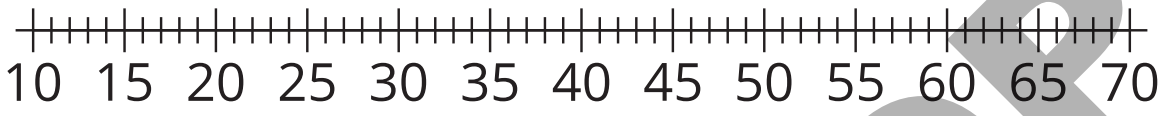
6 from Unit 4, Lesson 12

I started at a number on the number line. I jumped 37 to the left. I ended at 26. Where did I start?

a. Write an equation with a ? for the unknown.

b. Find the number that makes the equation true.

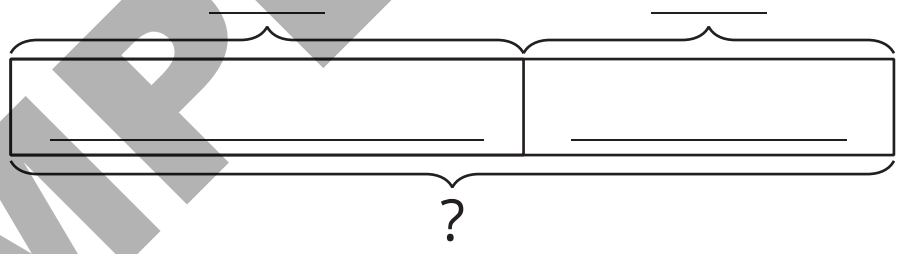
c. Show your thinking using the number line.



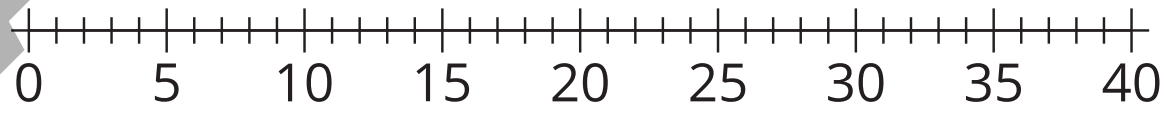
7 from Unit 4, Lesson 13

There are 18 students in the classroom. Then 13 more join them.

a. Label the tape diagram to match the story.



b. Label the number line to match the story.



c. How are the diagram and number line alike? How are they different?

_____

_____

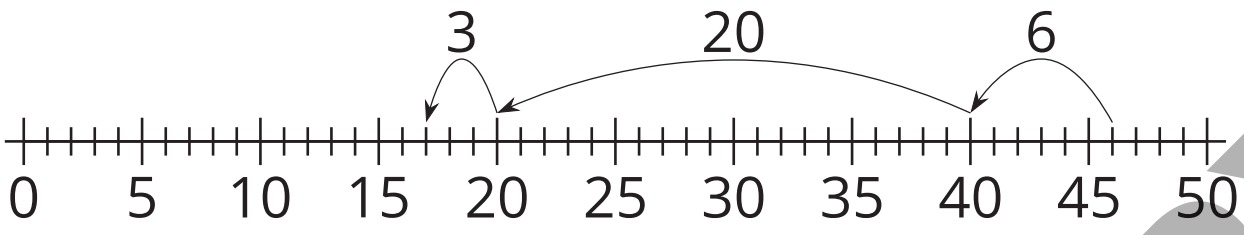
d. How many students are in the classroom now?

8 Exploration

a. Use addition or subtraction. How many equations can you make with these three numbers: 20, 13, and 7?

b. Draw number lines to match each of your equations.

c. How are the number lines alike? How are they different?



a. Write a story problem that can be solved with this number line.

b. Explain how you know the number line solves your story.

Glossary

- bar graph
  A way to show data using the height or length of rectangles to represent how many in each group or category.

- centimeter
  A metric unit of length.

  Examples:
  A paper clip is about 1 centimeter wide.
  A staple is about 1 centimeter long.
  A pinkie finger is about 1 centimeter wide.

- compose
  To put a number or shape together using its parts.

  Examples:
  Compose 1 ten from 10 ones.
  Compose 14 from 1 ten and 4 ones.
  Compose 1 rectangle from two triangles.

- data
  A collection of facts, such as numbers, measurements, or observations.

  Examples:
  the color of each pencil in a box
  the number of pencils sold each day
  the length of each pencil in a box

- decompose
  To break apart a number or shape into its parts.

  Examples:
  Decompose 1 ten into 10 ones.
  Decompose 14 into 1 ten and 4 ones.
  Decompose 1 rectangle into 2 triangles.

- foot
  A U.S. unit of length.

  Examples:
  A ruler is 1 foot long.
  A notebook is about 1 foot long.
  A football is about 1 foot long.

- inch
  A U.S. unit of length.

  Examples:
  A paper clip is about 1 inch long.
  A quarter is about 1 inch wide.
  The side of a connecting cube is about 1 inch long.

- line plot
  A way to show data with Xs or other marks above a number line. Each mark represents 1 number or 1 measurement.

- meter
  A metric unit of length.

  Examples:
  A baseball bat is about 1 meter long.
  A yardstick is about 1 meter long.
  A door is about 1 meter wide.

- number line

  A diagram that represents numbers as lengths away from 0 along a straight line.

- picture graph

  A way to show data using pictures or symbols to represent how many in each group or category.

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Notes

Notes

California Common Core State Standards for Mathematics (CA CCSSM) Reference

2.G: Grade 2 – Geometry

Reason with shapes and their attributes.

2.G.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Sizes are compared directly or visually, not compared by measuring. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

2.G.2

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

2.G.3

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

2.MD: Grade 2 – Measurement and Data

Measure and estimate lengths in standard units.

2.MD.1

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2

Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3

Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4

Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5

Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same

units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

2.MD.7

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. Know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year). CA

2.MD.8

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Represent and interpret data.

2.MD.10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve

simple put-together, take-apart, and compare problems See Glossary, Table 1. using information presented in a bar graph.

2.MD.9

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked offinwhole-numberunits.

2.NBT: Grade 2 – Number and Operations in Base Ten

Understand place value.

2.NBT.1

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

2.NBT.1a

100 can be thought of as a bundle of ten tens—called a "hundred."

2.NBT.1b

The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.2

Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.3

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.NBT.4

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

2.NBT.5

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6

Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition

and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.7.1

Use estimation strategies to make reasonable estimates in problem solving. CA

2.NBT.8

Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.

2.NBT.9

Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.

2.OA: Grade 2 – Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of

adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1.

Add and subtract within 20.

2.OA.2

Fluently add and subtract within 20 using mental strategies. See standard 1.OA.6 for a list of mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3

Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous

problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the

representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is

flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these

may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and

compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a

collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$.

Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.