IMKH California

GRADE 3

Student Edition

UNITS 1-2

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Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 1
Certified by Illustrative Mathematics®

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UNIT 2 AREA AND MULTIPLICATION

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GRADE 3

UNIT

Introducing Multiplication

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Content Connections

In this unit you will represent and solve multiplication problems through the use of scaled picture graphs and equal-group situations. You will make connections by:

- **Reasoning with Data** while using scaled picture graphs and scaled bar graphs to learn about multiplication.
- **Explore Changing Quantities** while multiplying within 100 and using visual representations to show your work.

Addressing the Standards

As you work your way through **Unit 1 Introducing Multiplication,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 4 and 8
MP2 Reason abstractly and quantitatively.	Lessons 1, 10, 11, 13, 18, and 20
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 6, 10, 14, and 16
MP4 Model with mathematics.	Lessons 9, 14, and 21
MP5 Use appropriate tools strategically.	Lesson 4
MP6 Attend to precision.	Lessons 2, 4, 10, 14, 17, and 20
MP7 Look for and make use of structure.	Lessons 3, 7, 9, 10, 11, 12, 14, 16, 18, and 20
MP8 Look for and express regularity in repeated reasoning.	Lessons 5, 11, 14, 15, and 19

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

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Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Represent Multivariable Data	**3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	Lessons 2, 3, 4, 5, 6, 7, 8, and 21
• Represent Multivariable Data	**3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	Lessons 2, 3, 4, and 5
• Number Flexibility to 100 for All Four Operations	**3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	Lessons 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, and 21
• Number Flexibility to 100 for All Four Operations	**3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	Lessons 9, 10, and 21
• Number Flexibility to 100 for All Four Operations	**3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Lessons 9, 10, 12, 13, 14, 15, 19, and 21

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Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations	**3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?.*	Lessons 9, 10, 14, 15, and 21
• Number Flexibility to 100 for All Four Operations	**3.OA.5** Apply properties of operations as strategies to multiply and divide. 2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Lesson 20
• Number Flexibility to 100 for All Four Operations	**3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Lessons 14, 15, and 19
• Patterns in Four Operations	**3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	Lessons 14, 15, and 19

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

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Building on 2.MD.9-10; building towards 3.MD.3; practicing MP2

Make Sense of Data

Let's read and ask questions about data.

Warm-up

Notice and Wonder: Graphs

What do you notice? What do you wonder?

Each  represents 1 student.

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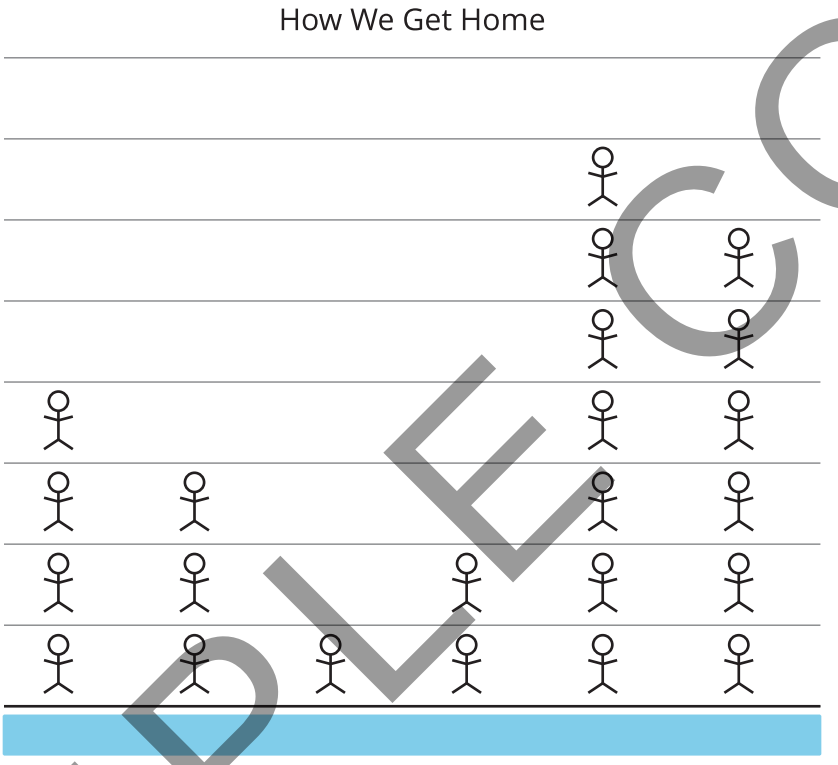
Activity 1

Picture Time

What could the categories be for this picture graph?

Be prepared to explain your reasoning.



Each represents 1 student.

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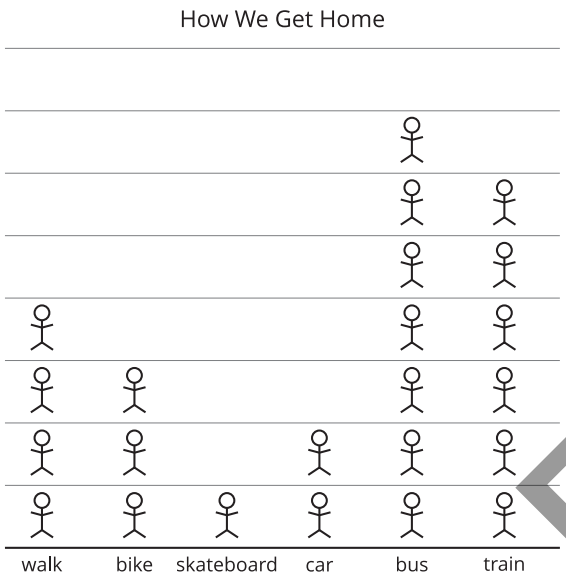
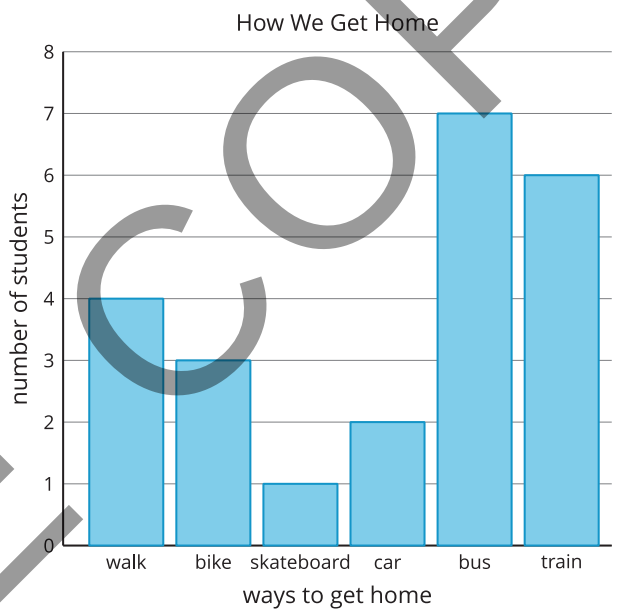
Activity 2

Picture Graphs and Bar Graphs

A group of students were asked, "How do you get home?" Their responses are shown in a picture graph and a bar graph.





Each 🧍 represents 1 student.

1. How are the 2 graphs alike? How are they different?

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2. What can you learn about how students get home based on the graphs?

3. Write 2 questions that can be answered by reading the graphs.

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Addressing CA CCSSM 3.MD.3-4; building on 2.MD.10, 2.OA.3; building towards 3.MD.3; practicing MP6

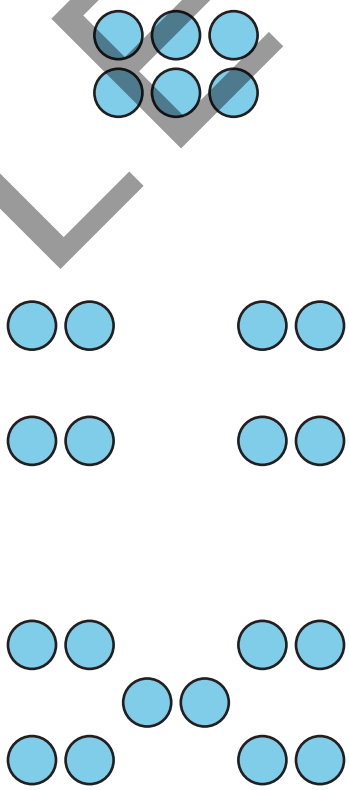
Represent Data and Solve Problems

Let's create graphs and answer questions.

Warm-up

How Many Do You See: Dots in Groups

How many do you see? How do you see them?



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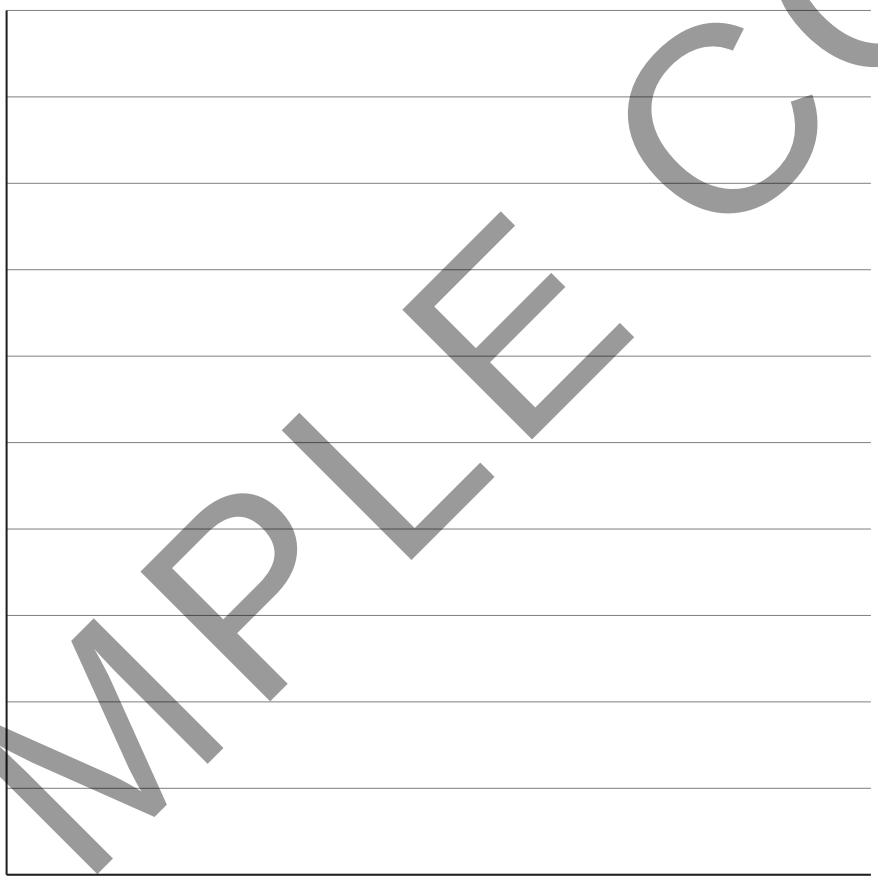
Activity 1

How We Get Home

1. Follow your teacher's instructions to represent the class data in a picture graph.
2. Represent the same data in a bar graph.



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Activity 2

Questions about a Bar Graph

1. Decide if each statement about how our class gets home is true or false. Use the graph to explain your reasoning to your partner.

   a. More students walk than go home any other way.

   _____

   b. More students ride home on a bus than in a car.

   _____

   c. Fewer students walk home than ride their bikes.

   _____

   d. More students walk or ride their bikes than ride in a van.

   _____

2. Fill in the blanks as directed by your teacher. Then answer each question.

   a. "How many more students _____ than _____?"

   b. "How many more students _____ or _____ than _____?"

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Addressing CA CCSSM 3.MD.3-4; building on 2.NBT.5; building towards 3.MD.3; practicing MP7

Scaled Picture Graphs

Let's explore scaled picture graphs.

Warm-up

Number Talk: Addition

Find the value of each expression mentally.

- $50 + 10$
- $50 + 12$
- $60 + 13$
- $65 + 13$

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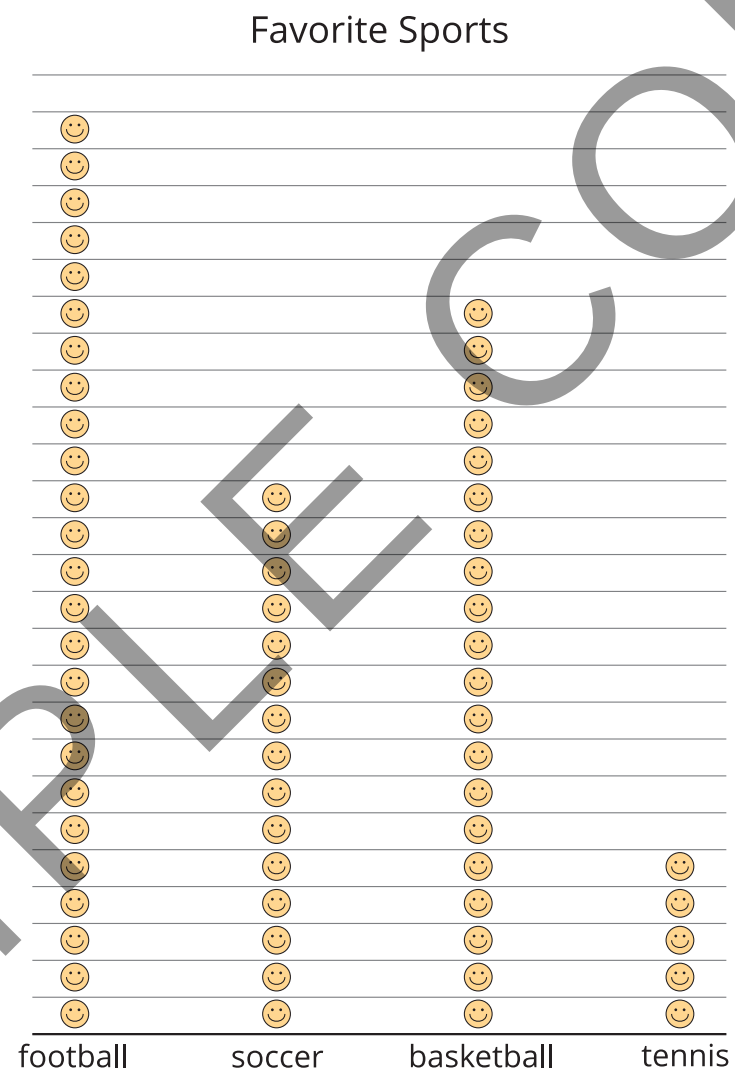
Activity 1

So Many Responses

1. A group of students were asked, "Which of these 4 sports is your favorite?" Their responses are shown in this picture graph:



How many students are represented in the graph? _____

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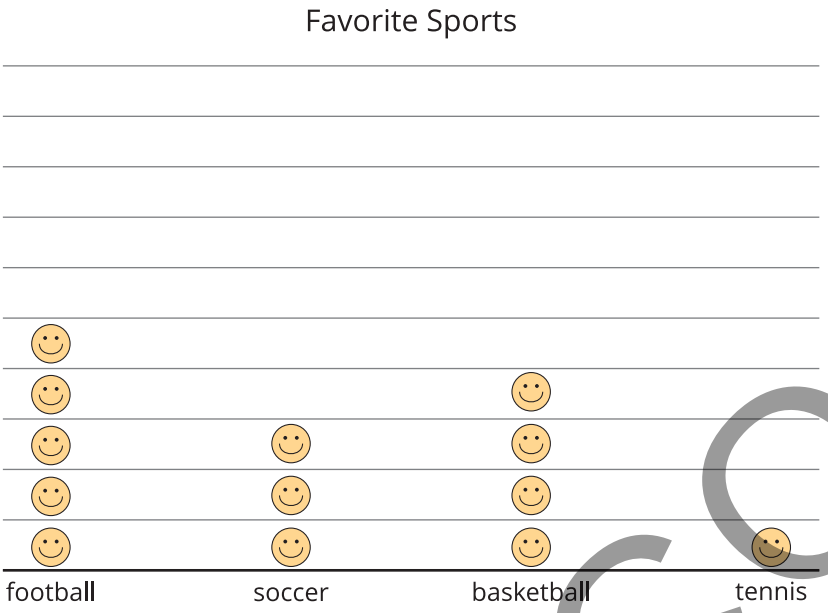
2. The students' responses are also shown in this picture graph:



Each ☺ represents 5 students.

How is counting the total number of students in this graph different from counting the total number of students in the first graph?

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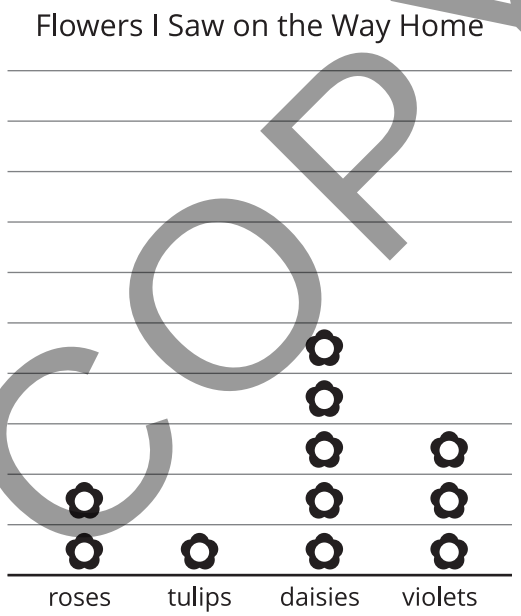
Activity 2

Questions about Scaled Picture Graphs

1. Andre collects data about 4 types of flowers he sees on the way home. The data is shown in this picture graph:



Each ✿ represents 5 flowers.

a. How many of each type of flower did Andre see on the way home?

roses _____ tulips _____ daisies _____ violets _____

b. Write 2 questions that can be answered by reading the graph.

_____

_____

_____

_____

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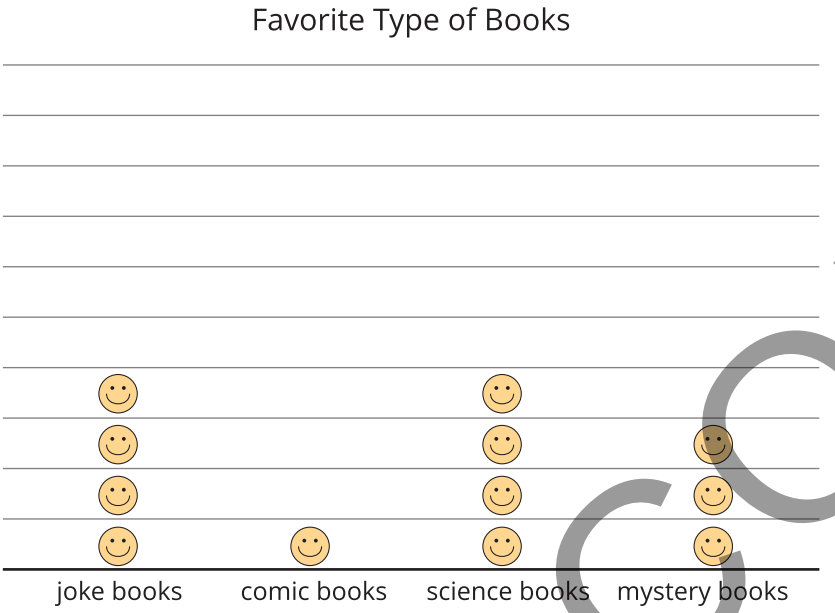
2. A group of students were asked, "Which is your favorite type of book?" Their responses are shown in this picture graph:



Each ☺ represents 2 students.

a. How many students chose each type of book? How do you know?

b. Write 2 questions that can be answered by reading the graph.

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Addressing CA CCSSM 3.MD.3-4; building towards 3.MD.3; practicing MP1, MP5, MP6

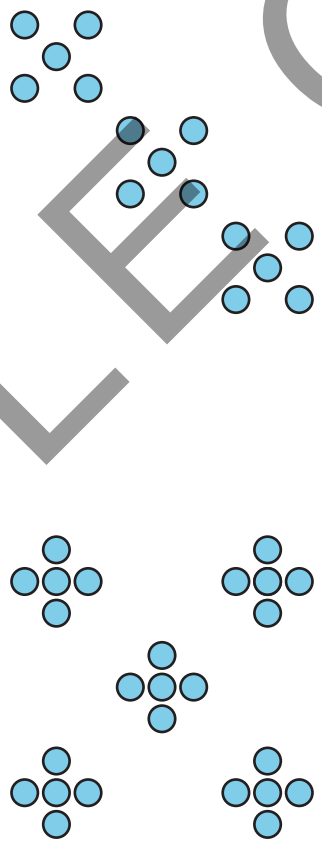
Create Scaled Picture Graphs

Let's make a scaled picture graph.

Warm-up

How Many Do You See: More Groups of Dots

How many do you see? How do you see them?



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Activity 1

Ways to Travel

How would you like to travel?

- car (C)
- train (T)
- boat (B)
- balloon (Bal)
- plane (P)
- helicopter (H)

student's name	way of traveling

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Activity 2

Create a Scaled Picture Graph

Represent the class survey data in a scaled picture graph. Have each picture represent 2 students.

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Addressing CA CCSSM 3.MD.3-4; building on 2.OA.4; building towards 3.MD.3; practicing MP8

Represent Data in Scaled Bar Graphs

Let's make a scaled bar graph.

Warm-up

Number Talk: Twos and Fives

Find the value of each expression mentally.

- $2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$

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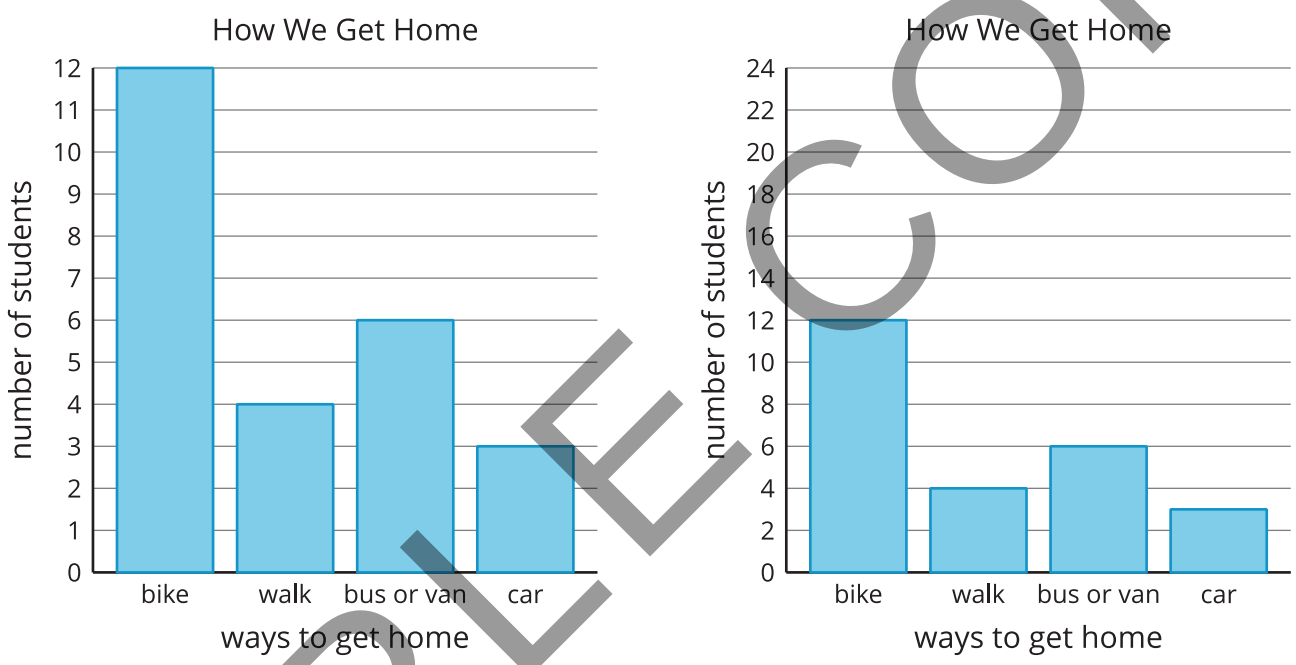
Activity 1

Compare Bar Graphs

All the students in a class were asked, "How do you get home from school?" Their responses are shown in these 2 bar graphs:



Discuss with your partner:

- How are the 2 graphs alike?
- How are they different?

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Activity 2

Create a Scaled Bar Graph

Represent the data we collected earlier about travel choices in a **scaled bar graph.** Use the graph with a scale of 2 or the graph with a scale of 5. If you have time, you can make 2 graphs. Be sure to label your title and categories.

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Addressing CA CCSSM 3.MD.3; building towards 3.MD.3; practicing MP3

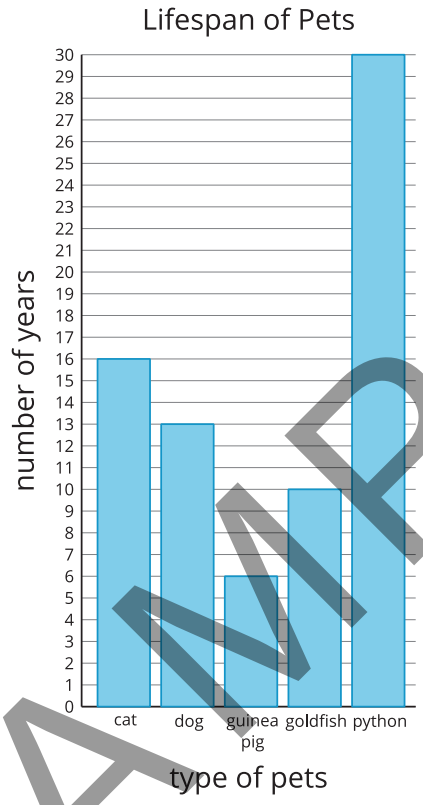
Choose a Scale

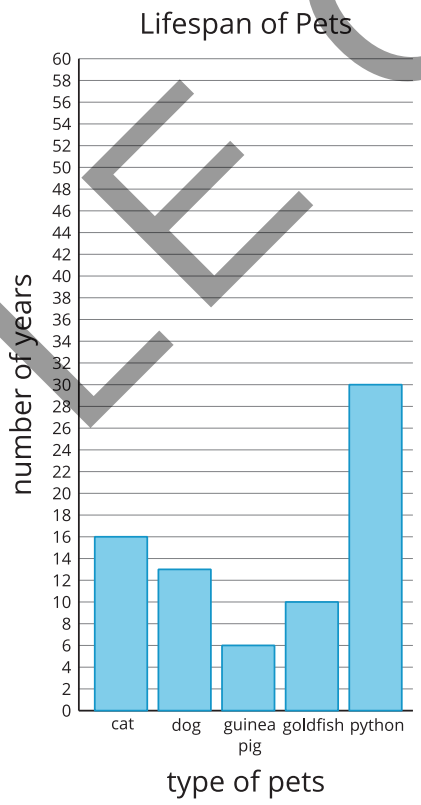
Let's choose a scale for our bar graph.

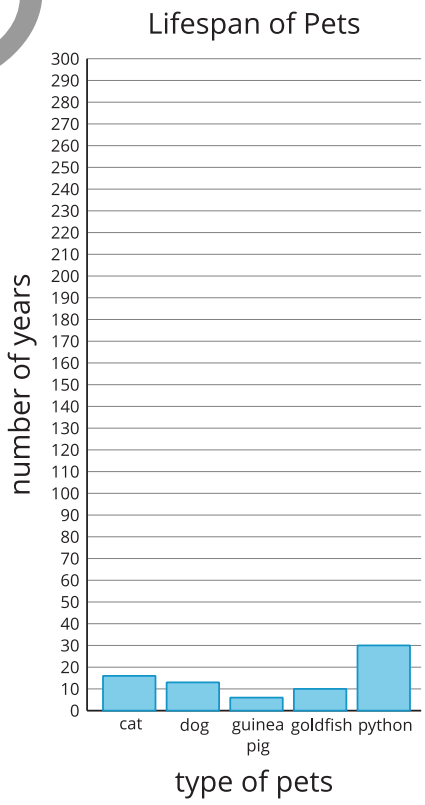
Warm-up

Notice and Wonder: Bar Graph Scales

What do you notice? What do you wonder?







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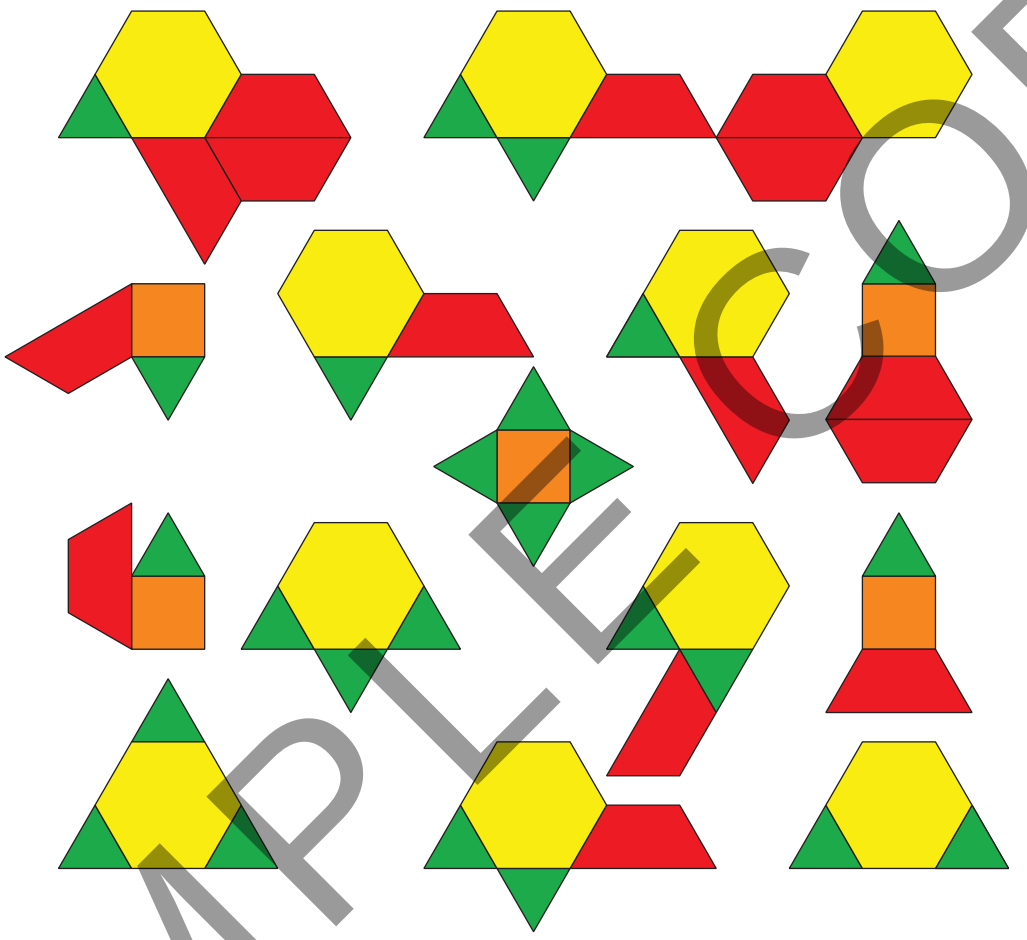
Activity 1

Represent Pattern Blocks

Here is a collection of pattern blocks.



Mai, Noah, and Priya want to make a bar graph to represent the number of triangles, squares, trapezoids, and hexagons in the collection.

- Mai says the scale of the bar graph should be 2.
- Noah says the scale of the bar graph should be 5.
- Priya says the scale of the bar graph should be 10.

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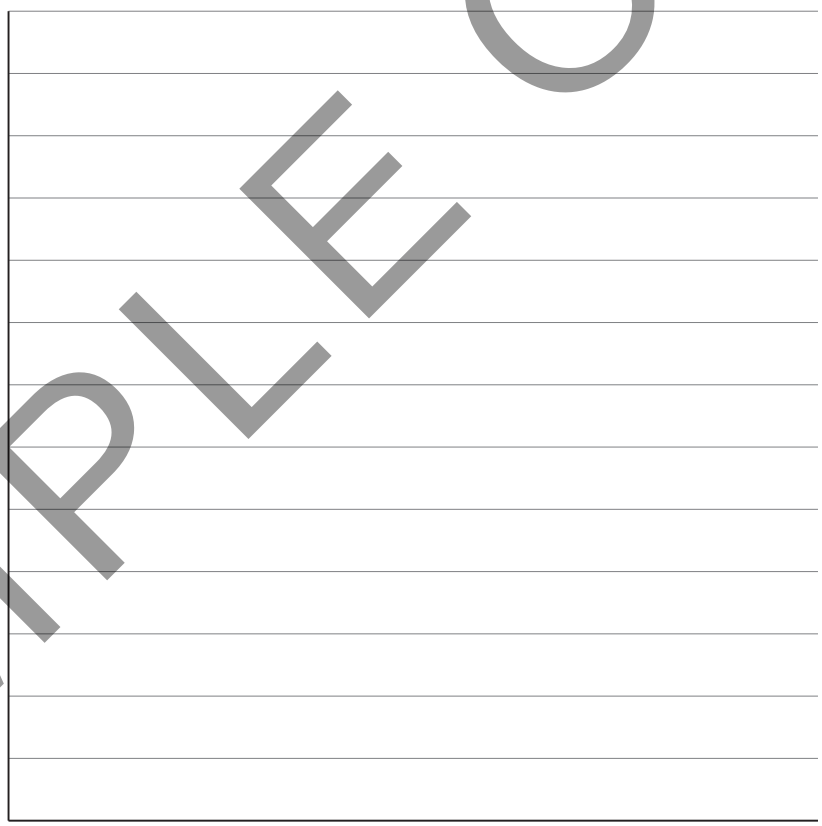
1. Who do you agree with? Explain your reasoning.

2. Use the scale that you chose to create a scaled bar graph to represent the collection.



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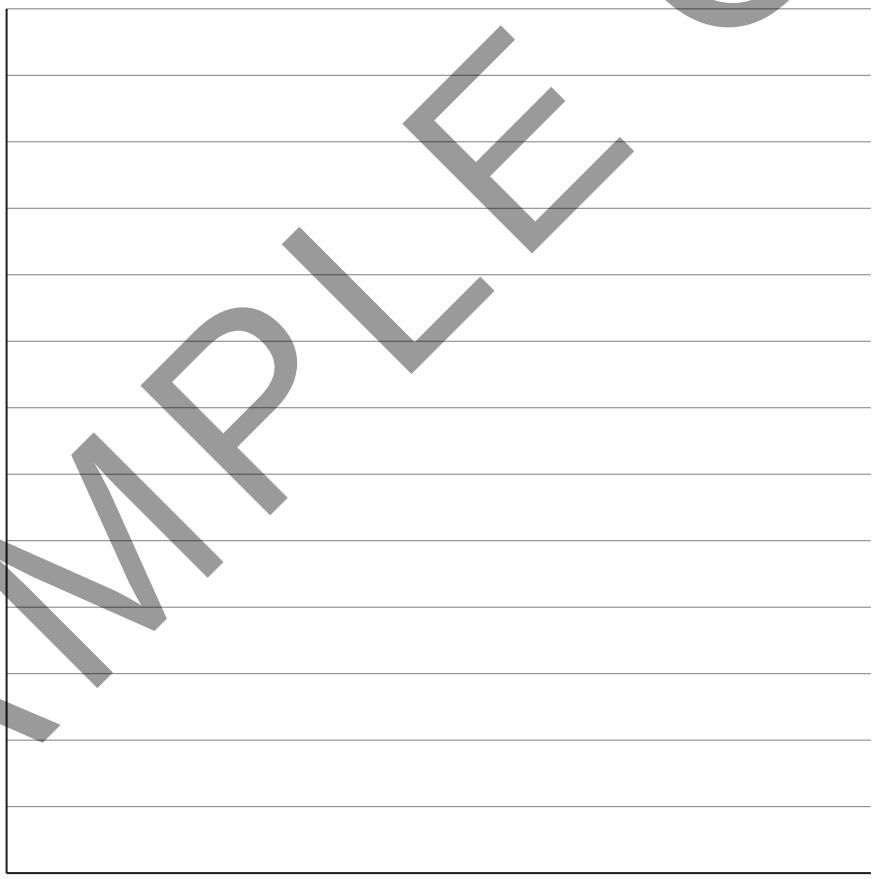
Activity 2

Represent More Data in a Scaled Bar Graph

All the third-grade students at a school were asked, "What is your favorite season?" Their responses are shown in this table.

favorite season of the year	winter	spring	summer	fall
number of students	24	13	40	22

Use the data from the table to create a scaled bar graph.



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Addressing CA CCSSM 3.MD.3; building on 2.OA.3; building towards 3.MD.3; practicing MP7

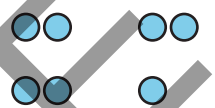
Answer Questions about Scaled Bar Graphs

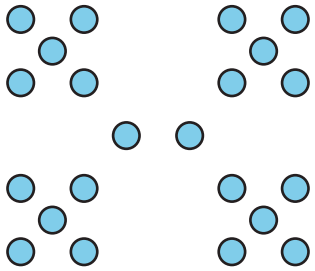
Let's solve problems based on data represented in bar graphs.

Warm-up

How Many Do You See: Groups of Dots

How many do you see? How do you see them?





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Activity 1

Questions about Favorite Season

Use your Favorite Season bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many students are represented in the graph?

2. How many students chose spring or fall as their favorite season?

3. How many more students chose summer than winter?

4. How many fewer students chose spring than fall?

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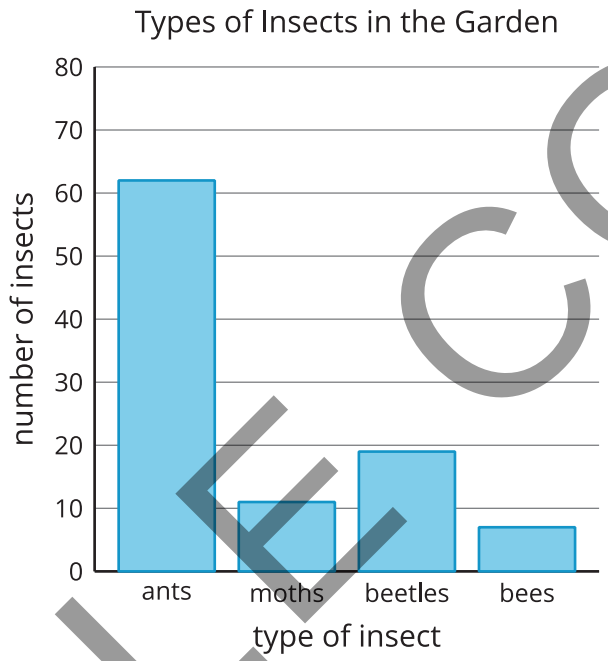
Activity 2

Questions about Insects in the Garden

Data was collected to see how many of the 4 types of insect were in a garden. The data is shown in this bar graph:



Use the bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many insects were in the garden?

2. How many more ants were in the garden than bees?

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3. How many fewer moths were there than ants?

4. Work with your partner to write 2 other questions that can be answered by reading the graph.

5. Trade with another group and answer each other's questions.

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Addressing CA CCSSM 3.MD.3; building on 2.OA.4; building towards 3.MD.3; practicing MP1

More Questions about Scaled Bar Graphs

Let's solve problems using data shown on bar graphs.

Warm-up

Number Talk: Repeated Addition

Find the value of each expression mentally.

- $2 + 2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5$

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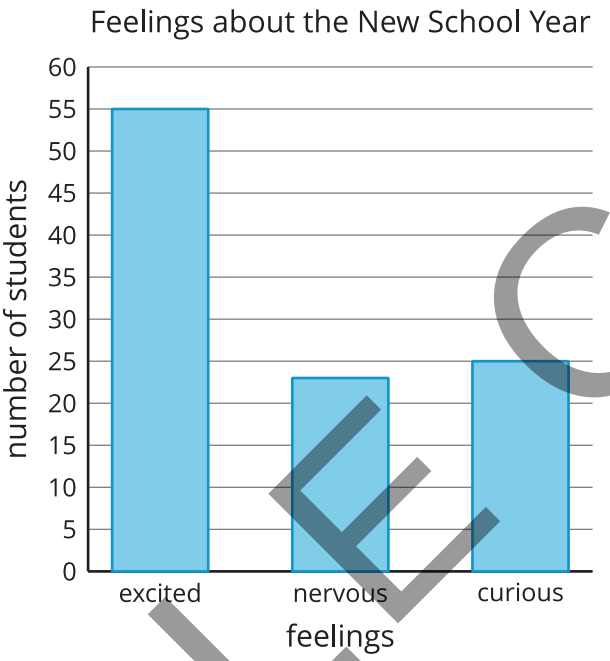
Activity 1

New School Year

A group of students were asked, "Which way do you feel about the new school year?" Their responses are shown in this bar graph:



How many more students are excited about the new school year than are nervous or curious?

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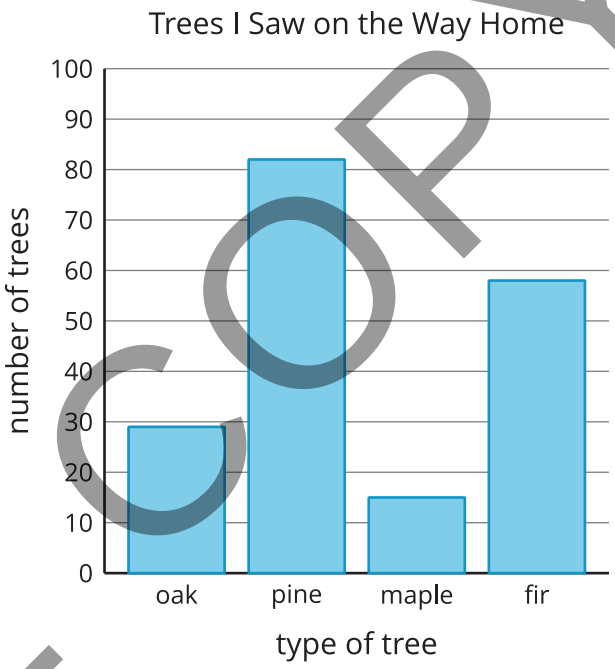
Activity 2

Use Bar Graphs to Solve Problems

The bar graph shows how many of the 4 types of trees Clare saw on the way home. Use the graph to answer the questions. Show your thinking using expressions or equations.



1. How many more pine trees did Clare see than fir trees?

2. How many more pine trees did Clare see than oak or maple trees?

3. How many fewer oak trees did Clare see than pine trees?

4. How many fewer maple or oak trees did Clare see than fir trees?

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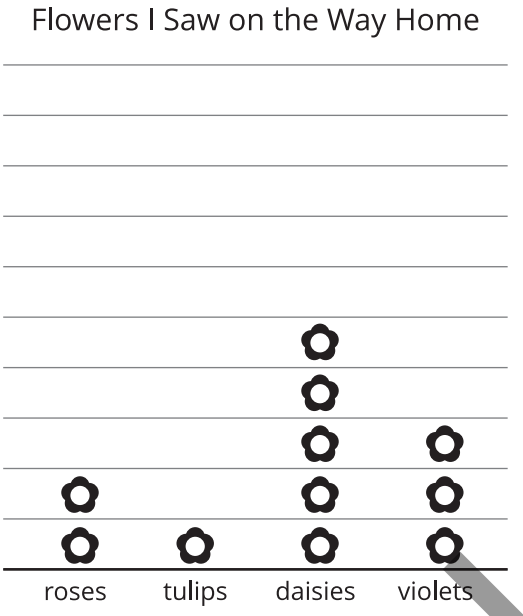
Section A Summary

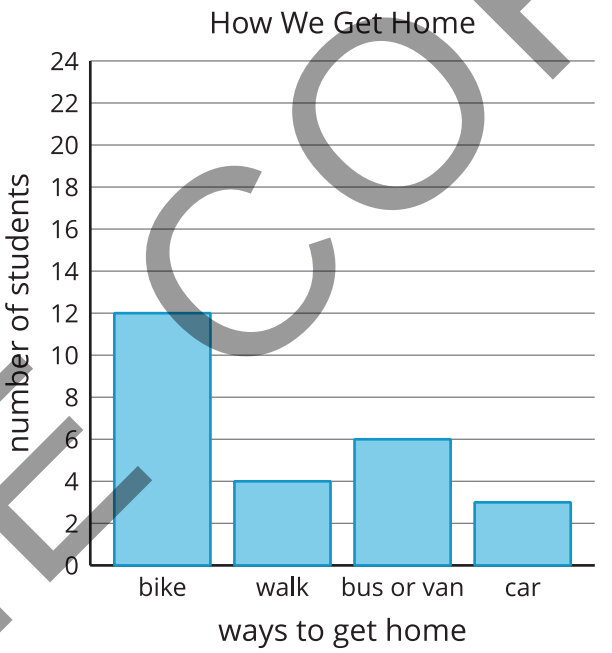
We created **scaled picture graphs** and **scaled bar graphs**.

The **key** tells what each picture represents in a picture graph.

The **scale** tells what number each bar represents in a bar graph.



Each ✿ represents 5 flowers.



We asked and answered questions about data represented in the graphs.

- How many more daisies were seen than violets?
- How many fewer students walk home than bike home?
- How many more students bike home than walk or ride in a car?

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Practice Problems

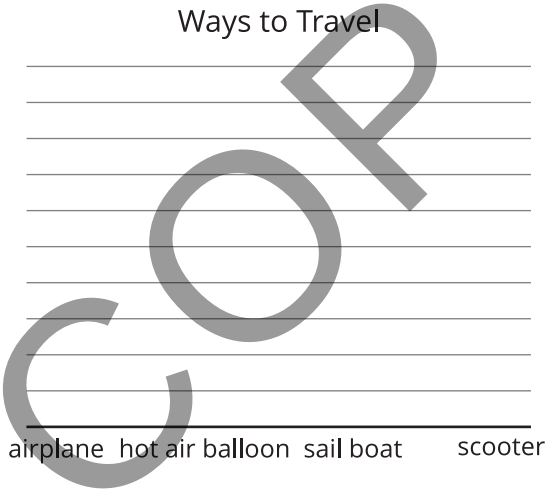
15 Problems

1

Pre-unit

The table shows how a group of students chose between 4 ways they would most like to travel. Use the table to complete the picture graph.

way to travel	number of students
airplane	4
hot air balloon	7
sail boat	6
scooter	3



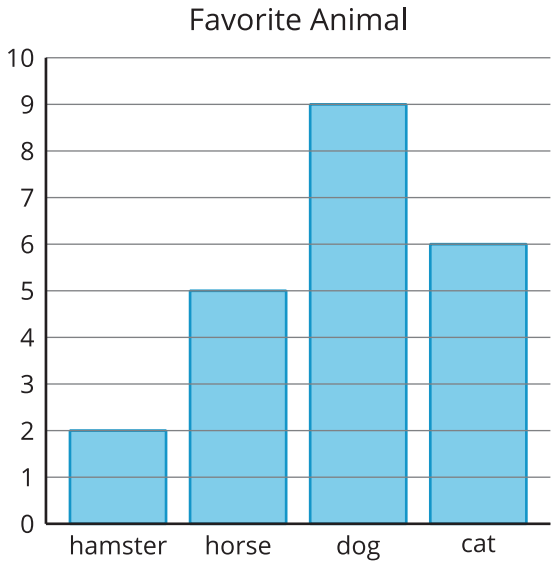
2

Pre-unit

The bar graph shows students' choices for favorite animals.

a. How many students recorded their favorite animal?

b. How many fewer students chose hamsters than dogs?



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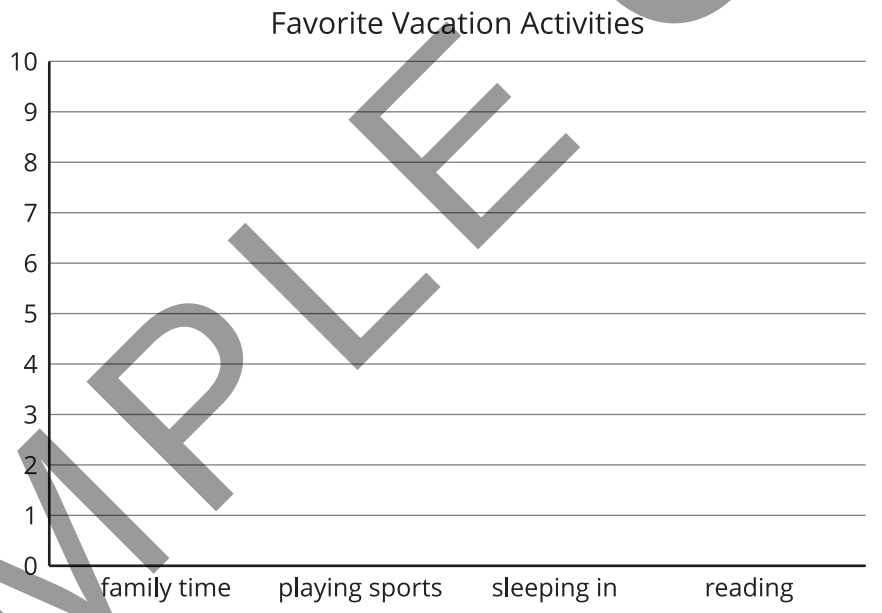
3 Pre-unit

The table shows the favorite summer vacation activity for a group of students.

vacation activities	number of students
family time	6
playing sports	8
sleeping in	5
reading	3

Use the table to complete the bar graph.



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4 Pre-unit

Find each sum or difference. Show your reasoning.

a. $25 + 62$

b. $37 - 9$

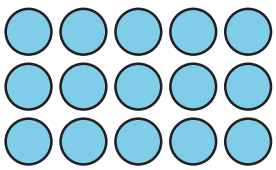
c. $24 + 47$

d. $84 - 59$

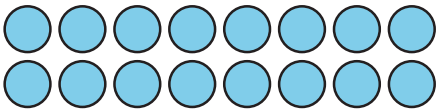
5 Pre-unit

How many objects are in each array? Explain or show your reasoning.

a.



b.



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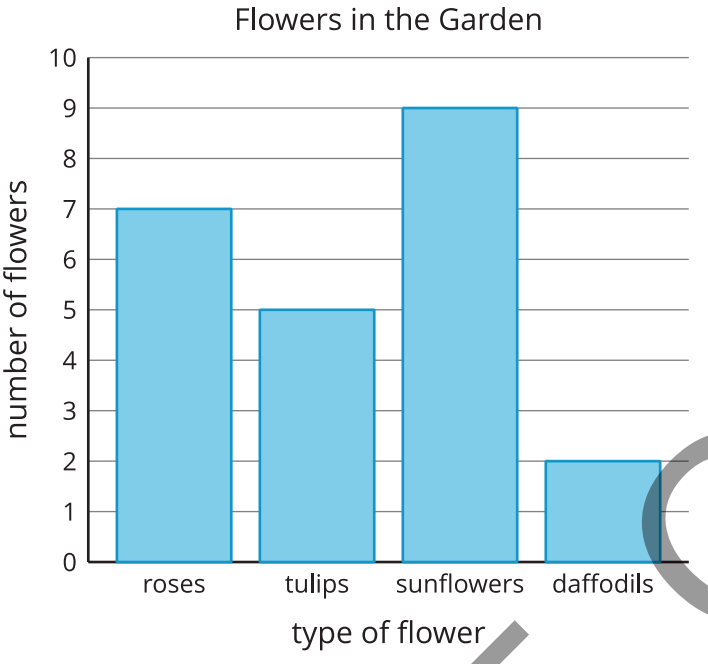
6 from Unit 1, Lesson 1

The graph shows some information about flowers in the garden.



a. Write 1 fact you know based on the data shown in the graph.

b. Write 2 questions that can be answered by reading the graph.

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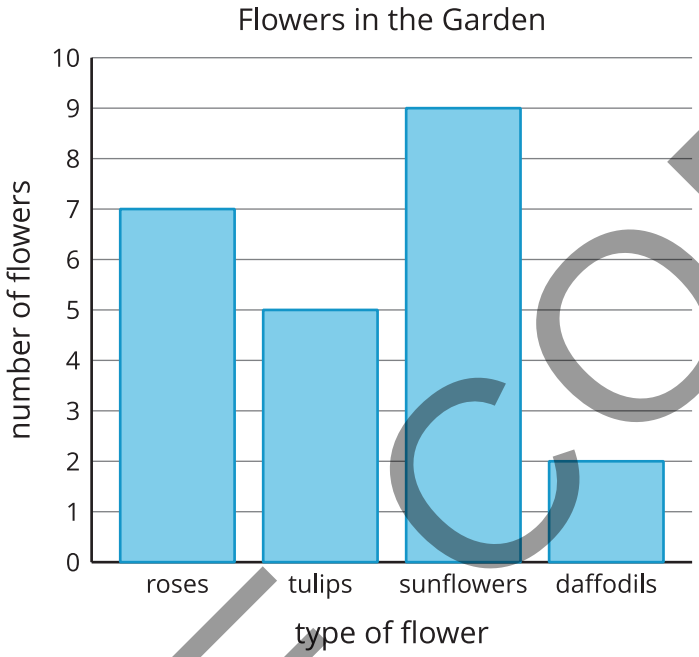
7 from Unit 1, Lesson 2

The bar graph shows the numbers of different types of flowers in the garden. Use the graph to answer the questions.



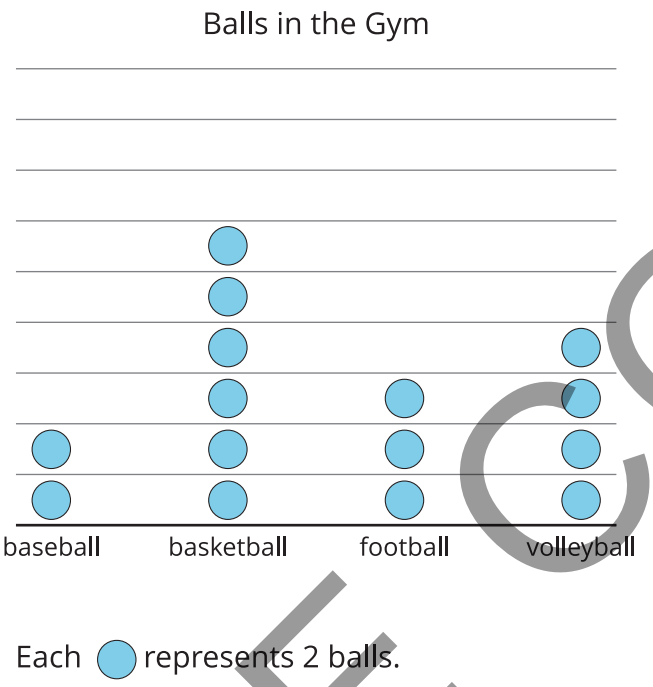
a. How many flowers are represented on the graph?

b. How many tulips, sunflowers, and daffodils are in the garden altogether?

SAMPLE COPY

8 from Unit 1, Lesson 3

This picture graph shows the numbers of different types of balls in the gym. Use the graph to answer the questions.



a. How many basketballs are in the gym?

b. How many more basketballs are there than footballs?

c. Write 1 other question that can be answered by reading the graph.

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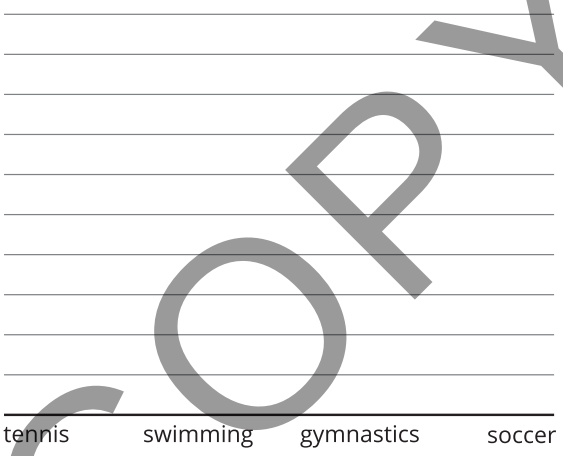
9 from Unit 1, Lesson 4

The table shows the favorite sports of some students. Use the table to complete the scaled picture graph.

sport	number
tennis	6
swimming	6
gymnastics	4
soccer	8

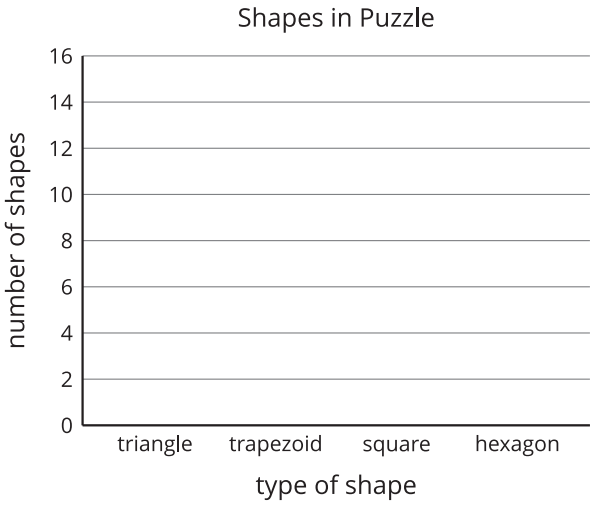


Each ● represents 2 people.

10 from Unit 1, Lesson 5

The table shows the numbers of different shapes in a pattern block puzzle. Use it to complete the scaled bar graph.

shape	number
triangle	13
trapezoid	10
square	9
hexagon	15



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11 from Unit 1, Lesson 6

The table shows the numbers of books some students have at home.

Use the information from the table to create a scaled bar graph.

books	number
Elena	25
Andre	9
Tyler	16
Clare	21

Books at Home

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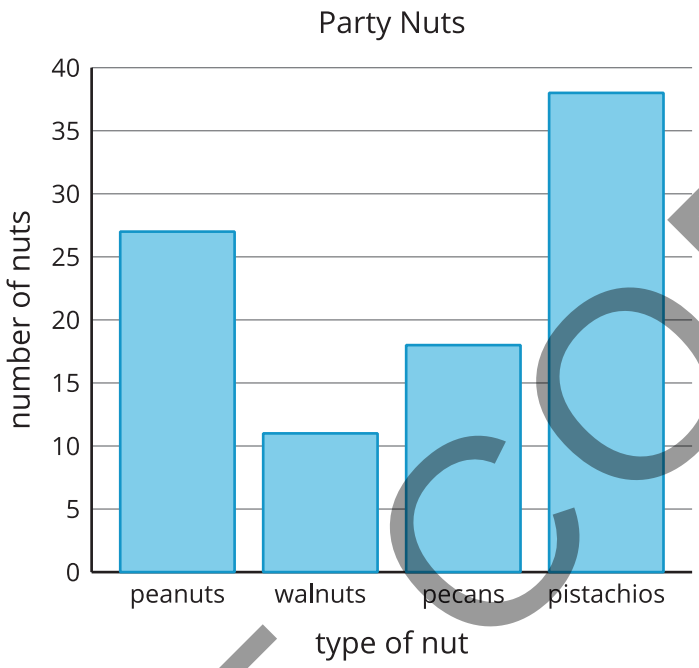
12 from Unit 1, Lesson 7

The bar graph shows the numbers of different kinds of nuts in a bowl.



a. How many more pistachios are there than walnuts? Explain or show your reasoning.

b. How many fewer pecans are there than peanuts? Explain or show your reasoning.

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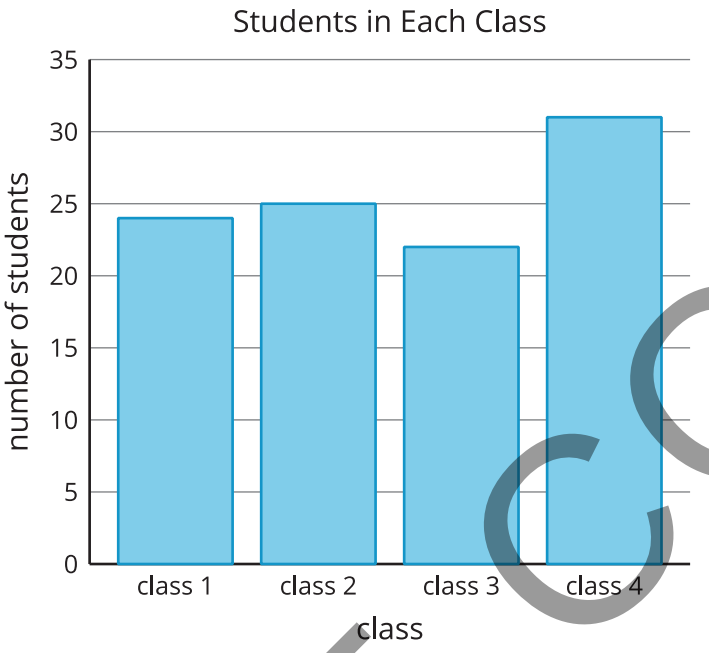
13 from Unit 1, Lesson 8

The bar graph shows how many students are in each class.



a. How many students are in the 4 classes altogether? Explain or show your reasoning.

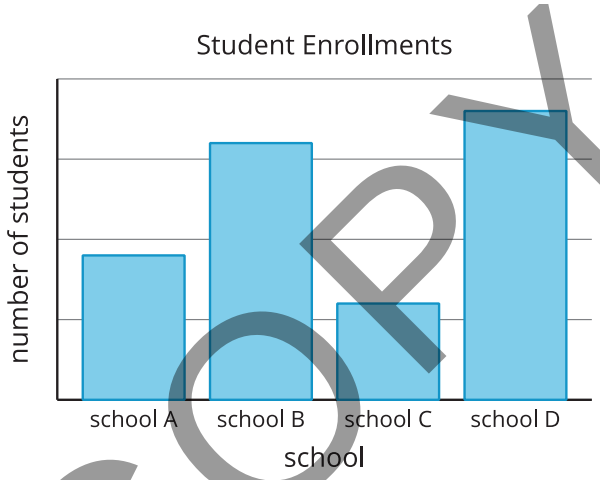
b. How many fewer students are in Class 1 than in Class 4? Explain or show your reasoning.

SAMPLE COPY

14 Exploration

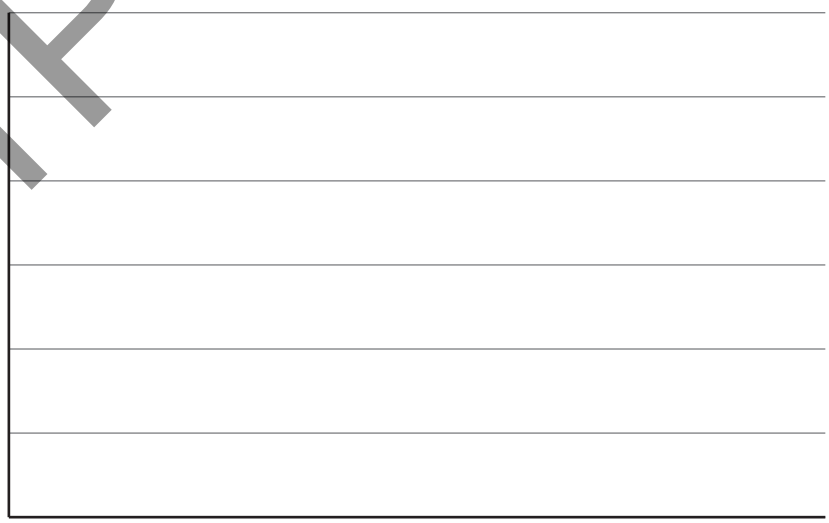
The bar graph shows data about 1,000 students who attend 4 schools.

What is the approximate scale for the bar graph? Explain or show your reasoning.



15 Exploration

Collect data of interest to you and represent the data on a bar graph. You may use the bar graph template if you wish.



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Addressing CA CCSSM 3.OA.1-4; building on 2.NBT.5; building towards 3.OA.1; practicing MP4 and MP7

Multiplication for Equal Groups

Let's work with equal groups of things.

Warm-up

Number Talk: More Addition

Find the value of each expression mentally.

- $40 + 35$
- $45 + 35$
- $45 + 36$
- $34 + 58$

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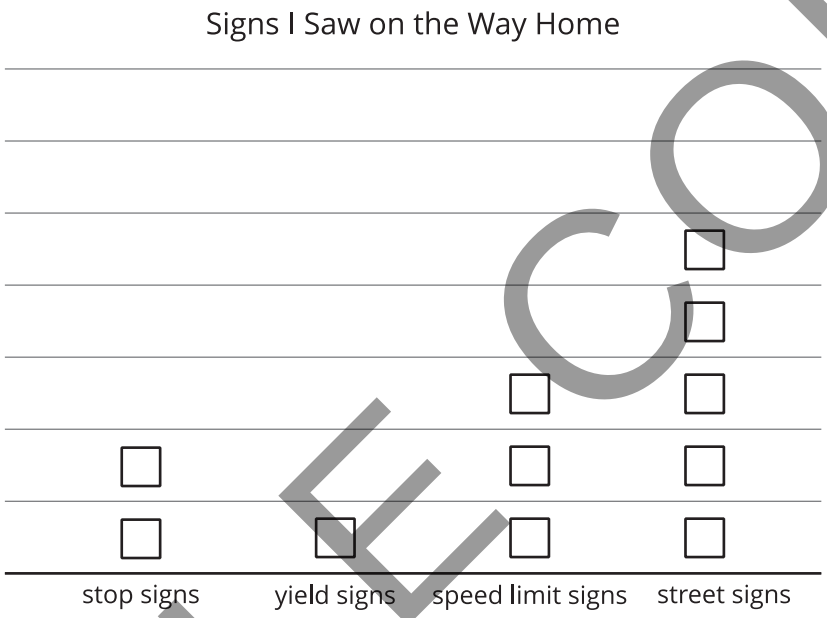
Activity 1

From Scaled Graphs to Equal Groups

Elena collected data about signs she saw on the way home. The data is shown in this picture graph:



Each ☐ represents 2 signs.

1. Represent the number of speed limit signs Elena saw on the way home.

SAMPLE COPY

2. Which statement describes how the graph represents the number of speed limit signs Elena saw? Explain your reasoning.

   A. There are 3 pictures, and each picture represents 1 speed limit sign.

   B. There are 3 pictures, and each picture represents 2 speed limit signs.

   C. There are 2 pictures, and each picture represents 2 speed limit signs.

3. How could this drawing represent the street signs Elena saw on the way home?

SAMPLE COPY

Activity 2

Situations with Equal Groups

Represent each situation.

1. There are 4 people wearing shoes. Each person is wearing 2 shoes.

2. There are 2 boxes of markers. Each box has 10 markers.

3. There are 3 basketball teams. Each team has 5 players.

SAMPLE COPY

Addressing CA CCSSM 3.OA.1-4; building towards 3.OA.1; practicing MP2, MP3, MP6, MP7

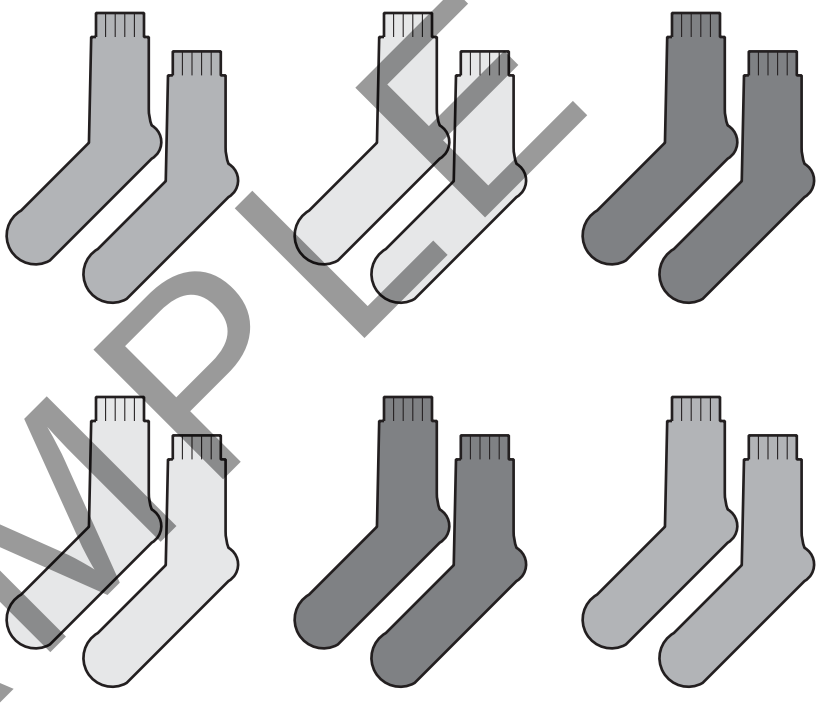
Situations, Drawings, and Diagrams, Oh My!

Let's represent equal groups.

Warm-up

Notice and Wonder: Socks

What do you notice? What do you wonder?



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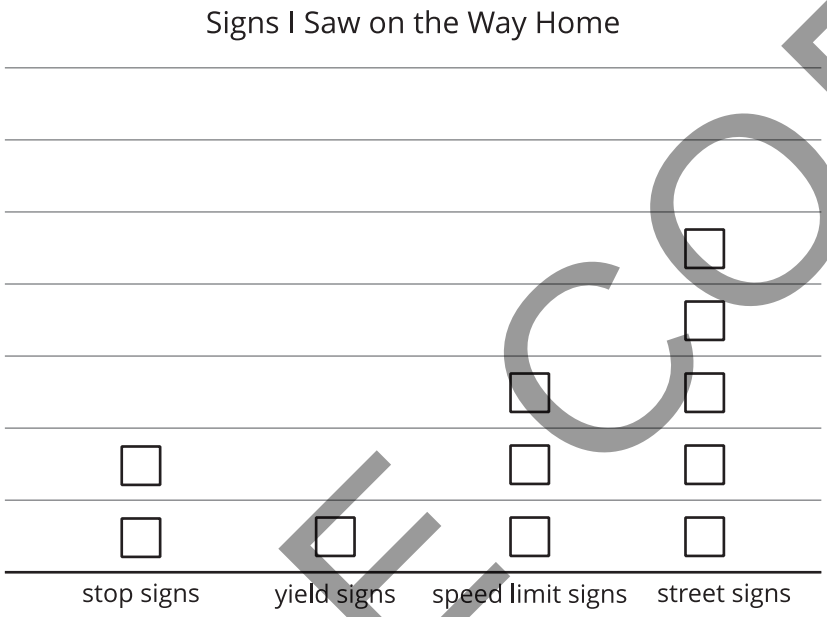
Activity 1

Scaled Picture Graph to Diagram

The graph shows the number of signs Elena saw on the way home.



Each ☐ represents 2 signs.

1. How does the diagram represent the speed limit signs that Elena saw?

2	2	2

SAMPLE COPY

2. Represent the data from another category in the graph with your own drawing or diagram.

SAMPLE COPY

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Activity 2

Card Sort: Equal Groups

Your teacher will give you a set of cards that show situations, drawings, and diagrams.

1. Find the cards that match. Be ready to explain your reasoning.

2. Create a drawing or diagram for each situation.

   a. There are 4 bags. Each bag has 2 strawberries.

   b. There are 4 hands. Each hand has 5 fingers.

SAMPLE COPY

Addressing CA CCSSM 3.OA.1; practicing MP2, MP7, MP8

Multiplication Expressions

Let's write multiplication expressions.

Activity 1

Multiplication Expression Match

Your teacher will give you a card showing a situation, a drawing, or a diagram.

Match it to 1 of the expressions posted around the room. Be prepared to explain your reasoning.

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Activity 2

Expressions to Drawings and Diagrams

1. Create a drawing or diagram for each expression. Explain your reasoning.

   a. 5×2

   b. 3×4

SAMPLE COPY

c. 3×10

2. Write your own expression and matching diagram. Explain your reasoning.

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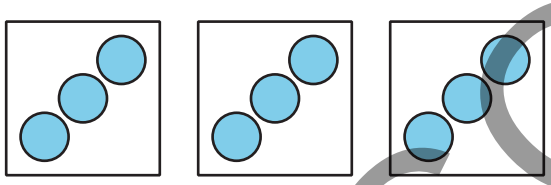
Activity 3

Write Multiplication Expressions

Write a multiplication expression to match each situation, drawing, or diagram. Explain your reasoning.

1. 

2. 

3. There were 2 packs of water. Each pack had 6 bottles of water.

SAMPLE COPY

Addressing CA CCSSM 3.OA.1, 3.OA.3; practicing MP7

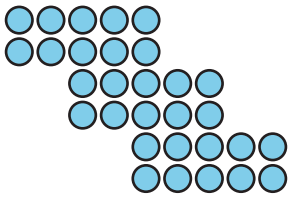
Represent and Solve Multiplication Problems

Let's represent and solve problems involving equal groups.

Warm-up

How Many Do You See: Lots of Dots

How many do you see? How do you see them?



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Activity 1

Tyler's Boxes

Tyler has 3 boxes. He has 5 baseballs in each box. How many baseballs does he have altogether? Show your thinking using diagrams, symbols, or other representations.

SAMPLE COPY

Activity 2

Solve Equal Groups Problems

Solve each problem. Show your thinking using diagrams, symbols, or other representations.

1. There are 4 soccer fields. Two teams are on each field. How many teams are there altogether?

2. There are 7 windows. Each window has 2 pieces of glass. How many pieces of glass are there in the windows?

SAMPLE COPY

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3. Jada has 5 bags. Each bag has 10 earrings. How many earrings does Jada have?

4. Kiran has 4 boxes. Each box has 5 pencils in it. How many pencils does Kiran have?

5. Andre has 3 bags of carrots. Each bag has 10 carrots. How many carrots does Andre have?

SAMPLE COPY

Addressing CA CCSSM 3.OA.1, 3.OA.3; practicing MP2

Multiplication Equations

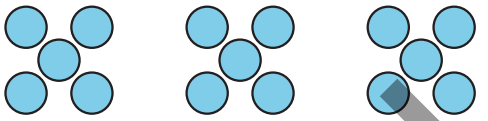
Let's learn about multiplication equations.

Warm-up

Which Three Go Together: Representations

Which 3 go together?

A



B

3×5

C

$2 \times 5 = 10$

D

$7 + 8 = 15$

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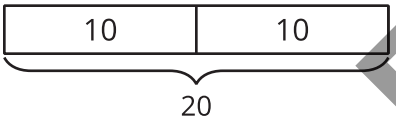
Activity 1

Multiplication Equation Match

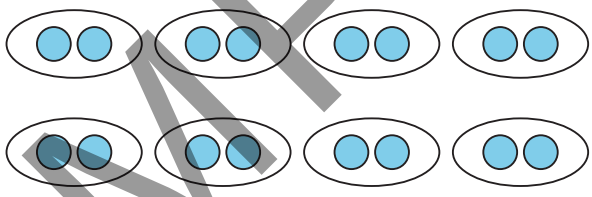
Find an equation from the list that can represent each situation, drawing, or diagram. Record the equation. Be prepared to explain your reasoning.

- $3 \times 5 = 15$
- $4 \times 10 = 40$
- $2 \times 10 = 20$
- $10 = 5 \times 2$
- $30 = 6 \times 5$
- $4 \times 2 = 8$
- $16 = 8 \times 2$
- $4 \times 5 = 20$
- $50 = 5 \times 10$

1. 

_____

2. Andre has 5 pairs of socks. _____

3.  _____

4. There are 6 hands on the table.
Each hand has 5 fingers. _____

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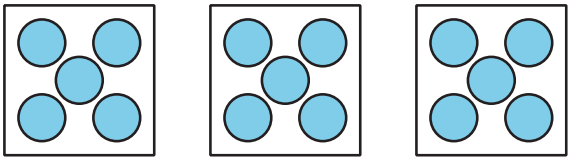
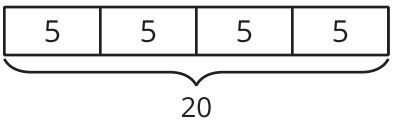
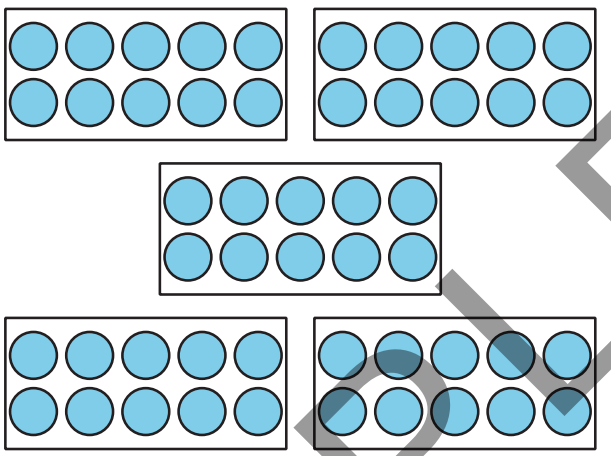
5.



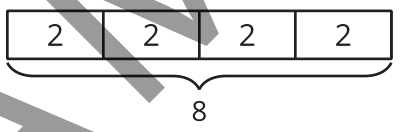
6.



7.



8.



9. There are 4 boxes of markers.
Each box has 10 markers.

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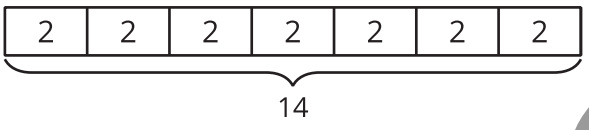
Activity 2

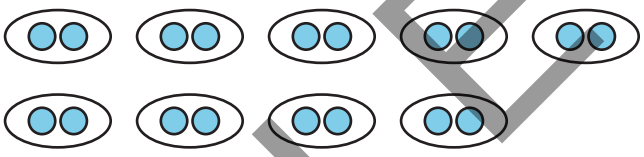
Write Multiplication Equations

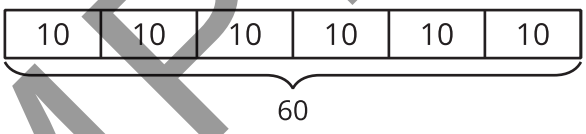
Write a multiplication equation that represents each situation, drawing, or diagram. Be prepared to explain your reasoning.

1. A package has 6 pairs of socks.

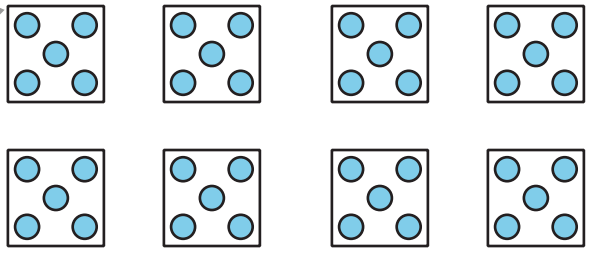
2. 

3. Diego has 7 sections in his notebook. Each section has 10 pages.

4. 

5. 

6. Elena has 4 bags of oranges. Each bag has 5 oranges in it.

7. 

SAMPLE COPY

Addressing CA CCSSM 3.OA.1, 3.OA.3-4, 3.OA.7, 3.OA.9; practicing MP3, MP4, MP6, MP7, MP8

Write and Solve Equations with Unknowns

Let's work with equations with unknown numbers.

Warm-up

Number Talk: Fives

Find the value of each expression mentally.

- 1×5
- 2×5
- 3×5
- 4×5

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Activity 1

Card Sort: Unknown Numbers

Your teacher will give you a set of cards. Match each equation to a situation or diagram. Be ready to explain your reasoning.

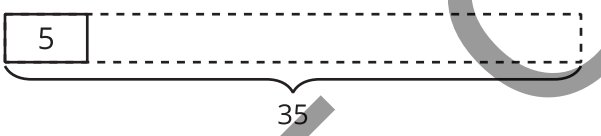
SAMPLE COPY

Activity 2

Write Equations with an Unknown Number

- Write a multiplication equation to represent each diagram or situation. Use a symbol for the unknown. Be prepared to share your reasoning.
- Find the number that makes each equation true. Rewrite the equation with the solution.

1.



   - equation with symbol: _____
   - equation with solution: _____

2. Jada has some packs of sports cards. Each pack has 5 cards. If Jada has 45 cards, how many packs of cards does she have?

   - equation with symbol: _____
   - equation with solution: _____

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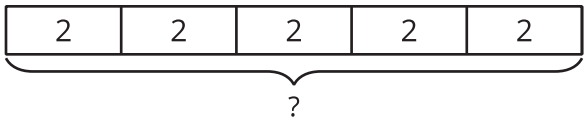
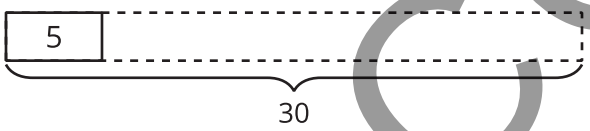
3.



- equation with symbol: _____
- equation with solution: _____

4.



- equation with symbol: _____
- equation with solution: _____

5. The school has 6 bags. Each bag has 10 basketballs in it. How many basketballs does the school have?

- equation with symbol: _____
- equation with solution: _____

SAMPLE COPY

Addressing CA CCSSM 3.OA.3-4, 3.OA.7, 3.OA.9; practicing MP8

More Factors, More Problems

Let's solve more multiplication problems.

Warm-up

Number Talk: Tens

Find the value of each expression mentally.

- 1×10
- 2×10
- 3×10
- 4×10

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Activity 1

Represent Situations with Equations

For each problem:

- Write a multiplication equation with a symbol for the unknown to represent the situation.
- Find the number that makes the equation true. Show your reasoning.

1. There are 15 bottles of paint. Han placed 5 bottles of paint on each table. How many tables have paint on them?
   a. equation:
   b. solution:

2. Lin's class has 6 tables. Each table has 2 bags of clay. How many bags of clay does the class have?
   a. equation:
   b. solution:

3. Han's class has 60 markers. There are 10 markers in a pack. How many packs of markers does the class have?
   a. equation:
   b. solution:

SAMPLE COPY

Activity 2

Multiplication Mashup

Solve each problem. Explain or show your reasoning.

1. Clare has 16 socks. She puts them in piles of 2. How many piles can she make?

2. Diego has 8 piles of socks. Each pile has 2 socks. How many socks does Diego have?

SAMPLE COPY

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3. Andre has 16 socks. He puts them in 8 groups that are the same size. How many socks are in each group?

4. A store has 9 boxes. Each box has 5 shirts. How many shirts are there?

5. There are 80 sweaters in piles on a shelf. Each pile has 8 sweaters. How many piles of sweaters are on the shelf?

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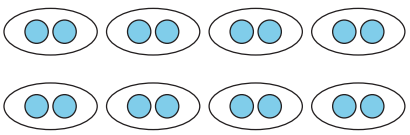
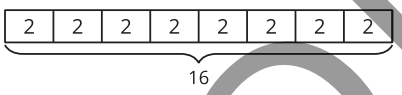
Section B Summary

We learned about equal groups. We created drawings and diagrams to represent situations that involve equal groups.

Situation	Drawing	Diagram
Diego has 8 piles of socks. Each pile has 2 socks.		

We wrote multiplication **expressions** and **equations** to represent equal groups.

Expression	Equation
8×2	$8 \times 2 = 16$

We learned that the numbers that are multiplied are called **factors** and the number that is the result of multiplying is called a **product**. In the equation $8 \times 2 = 16$, the numbers 8 and 2 are the factors and 16 is the product.

SAMPLE COPY

Practice Problems

10 Problems

1 from Unit 1, Lesson 9

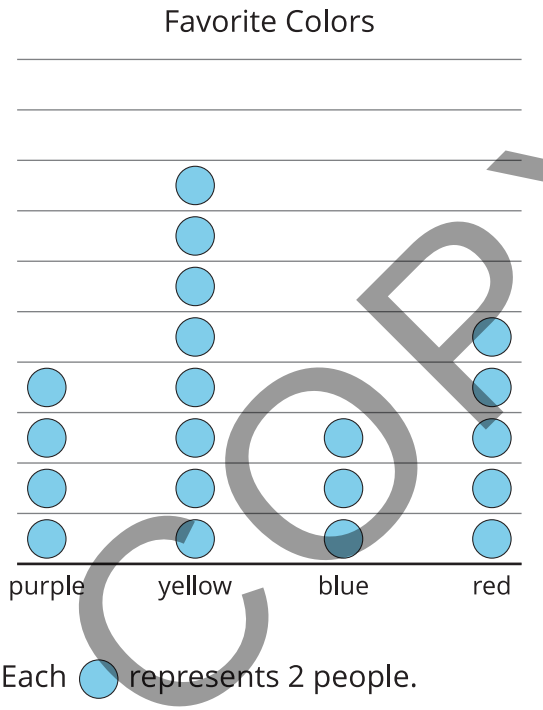
There are 6 tennis courts. There are 2 players on each tennis court.

Create a drawing or diagram to represent the tennis players. Then find how many players are on the tennis courts. Explain or show your reasoning.

SAMPLE COPY

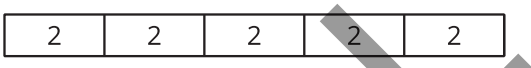
2 from Unit 1, Lesson 10

The picture graph shows the favorite colors of some people.



Match each diagram or drawing to the number of people who chose each color.

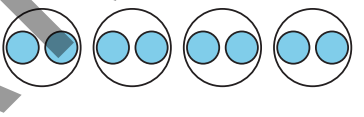
A.



B.

C.

D.



1. red
2. blue
3. purple
4. yellow

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3 from Unit 1, Lesson 11

Create a drawing or diagram to represent the expression 4×3.

4 from Unit 1, Lesson 12

There are 4 stacks of books on the table. Each stack has 5 books. How many books are on the table? Explain or show your reasoning.

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5 from Unit 1, Lesson 13

There are 6 basketball teams in the gym. There are 5 people on each team. How many people are on the basketball teams in the gym?

a. Write a multiplication equation with a symbol for the unknown to represent the situation.

b. Find the number that makes the equation true. Show your reasoning.

6 from Unit 1, Lesson 14

Write a multiplication equation for the situation. Use ? for the unknown. Find the number that makes the equation true.

There are 4 soccer teams. Each soccer team has 10 players. How many players are there altogether?

SAMPLE COPY

7 from Unit 1, Lesson 15

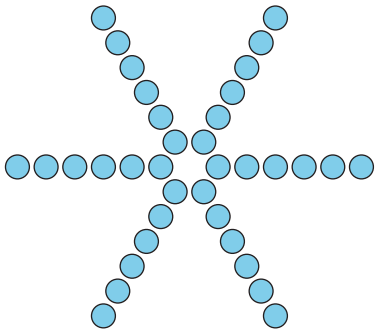
Solve each problem. Explain or show your reasoning.

a. There are 7 flowers. Each flower has 5 petals. How many petals are there?

b. There are 50 petals on some flowers. Each flower has 5 petals. How many flowers are there?

8 Exploration

Write an expression for the number of circles in the image. Then find the number of circles.



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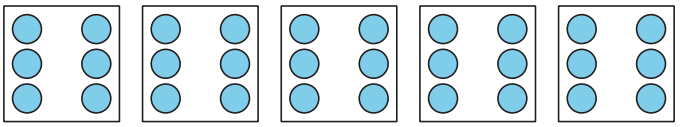
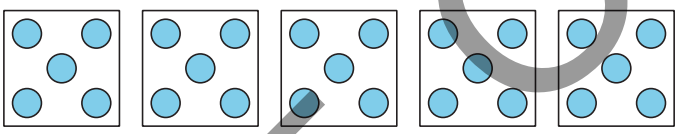
9 Exploration

For each image, determine if there is an even or odd number of circles. Explain or show your reasoning.

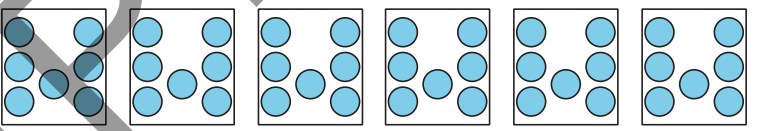
a.



b.



c.



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10 Exploration

Look outdoors or in your school or home to find some equal groups of objects.

a. Describe the objects.

b. Create a drawing to represent the objects.

c. Write an equation showing how many objects there are.

SAMPLE COPY

Unit 1, Lesson 16

Addressing CA CCSSM 3.OA.1; building on 2.OA.4; building towards 3.OA.1; practicing MP3 and MP7

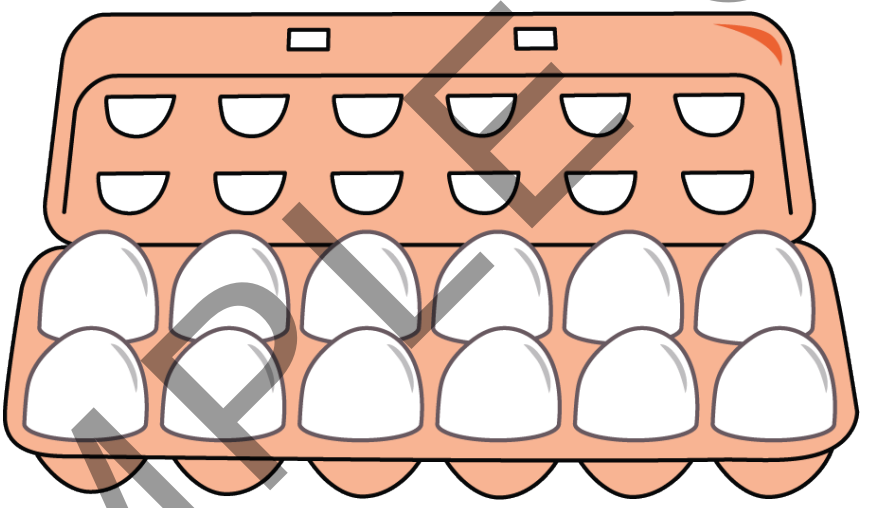
Arrange Objects into Arrays

Let's make some arrays.

Warm-up

Notice and Wonder: Eggs

What do you notice? What do you wonder?



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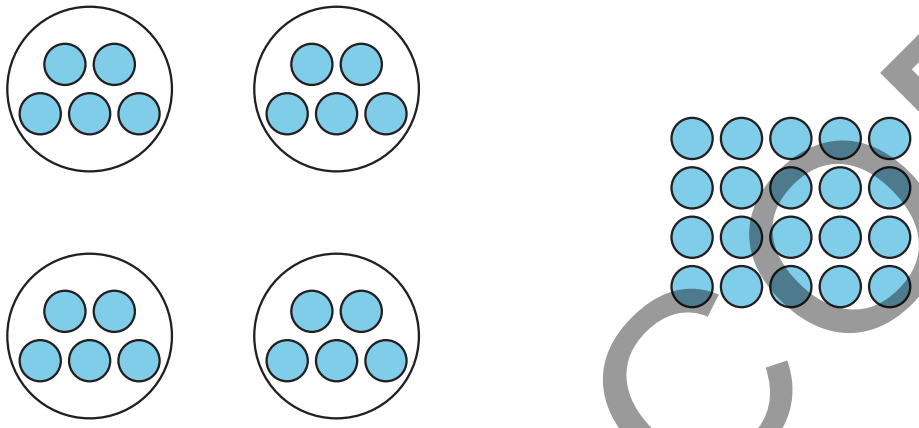
Activity 1

Compare Equal Groups and Arrays



1. How does arranging the dots into an array affect how you see the number?

2. Noah says he sees equal groups in the drawing with 4 circles and 5 dots in each circle, but he says there are no equal groups in the array. Do you agree with Noah? Explain your reasoning.

SAMPLE COPY

Activity 2

Arrange into Arrays

1. Use cubes to make 6 groups of 5.
   - Arrange them into an array.
   - Explain or show how the array is related to equal groups.

2. Count out 20 cubes.
   - Arrange them into as many arrays as you can.
   - Explain or show how each array is related to equal groups.

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3. Count out 24 cubes.
    - Arrange them into as many arrays as you can.
    - Explain or show how each array is related to equal groups.

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Addressing CA CCSSM 3.OA.1; building towards 3.OA.1; practicing MP6

Match and Draw Arrays

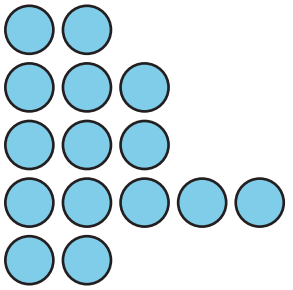
Let's match arrays to equal groups and draw arrays.

Warm-up

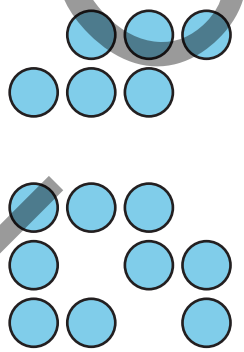
Which Three Go Together: Arrangements

Which 3 go together?

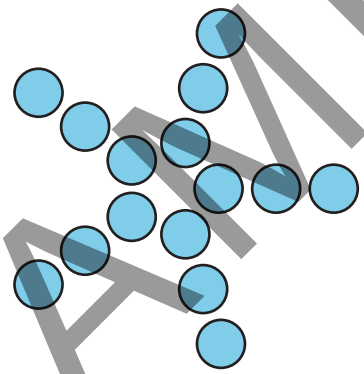
A



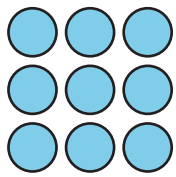
B



C



D



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Activity 1

Card Sort: Arrays

Your teacher will give you a set of cards.

1. Match each drawing of equal groups to an array. Be ready to explain your reasoning.
2. Choose a match you and your partner made. Write down how you know the drawing matches the array.

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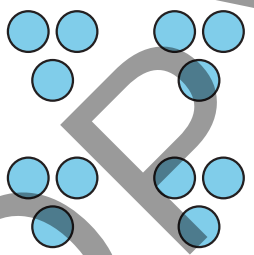
Activity 2

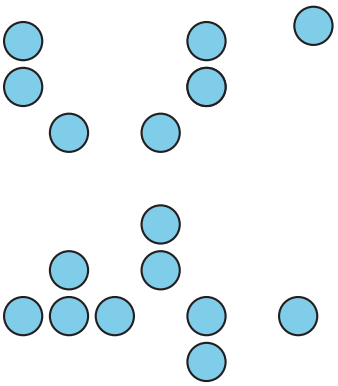
Draw Arrays

1. a. Draw 1 way the dots could be rearranged into an array.



b. Explain or show how the array is related to multiplication.

2. a. Draw ways that the dots could be arranged into arrays. Draw as many ways as you can.



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b. Explain or show how each array is related to multiplication.

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Unit 1, Lesson 18

Addressing CA CCSSM 3.OA.1; practicing MP2 and MP7

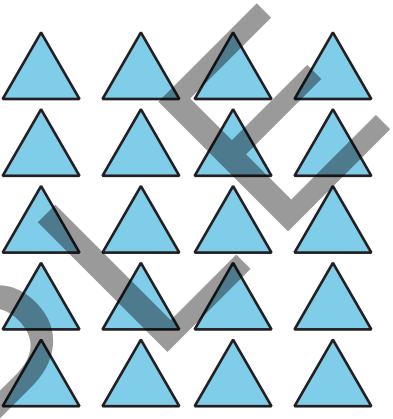
Represent Arrays with Expressions

Let's represent situations with arrays and expressions.

Warm-up

How Many Do You See: An Array of Shapes

How many do you see? How do you see them?



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Activity 1

Represent Array Situations

1. Use objects or drawings to represent each situation with an array.
   a. There are 3 rows of chairs. Each row has 5 chairs.

   b. There are 4 rows of cars. Each row has 5 cars in it.

   c. There are 2 rows of eggs. Each row has 6 eggs.

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d. There are 2 teams of students lined up. Each team has 10 students.

2. Write a multiplication expression to represent each situation.

   a. _____

   b. _____

   c. _____

   d. _____

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Activity 2

Connect Arrays to Expressions

Draw an array for each multiplication expression. Be prepared to share your reasoning.

1. 2×3

2. 5×2

3. 4×4

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Addressing CA CCSSM 3.OA.1, 3.OA.3, 3.OA.7, 3.OA.9; practicing MP8

Solve Problems Involving Arrays

Let's solve problems involving arrays.

Warm-up

Number Talk: One Less Group

Find the value of each expression mentally.

- 10×2
- 9×2
- 8×2
- 7×2

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Activity 1

Array of Colors

There are 7 rows. Each row has 5 crayons. How many crayons are there?

1. Solve this problem. Explain or show your reasoning.

2. Represent the situation with an array and a multiplication equation with a symbol for the unknown.

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Activity 2

Tyler's Trees

For each problem:

- Write a multiplication equation with a symbol for the unknown to represent the situation.
- Solve the problem. Show your reasoning.

1. A field of coconut trees in Mexico has 5 rows of trees. Each row has 9 trees. How many trees are there?

2. Tyler wants to plant coconut trees in a community garden in Florida. He will plant 2 rows of 4 trees. How many trees will Tyler plant?

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Addressing CA CCSSM 3.OA.5; building on 2.NBT.5; building towards 3.NBT.2; practicing MP2, MP6, MP7

The Commutative Property

Let's learn about the commutative property.

Warm-up

Number Talk: Subtraction

Find the value of each expression mentally.

- $70 - 10$
- $68 - 10$
- $70 - 12$
- $68 - 12$

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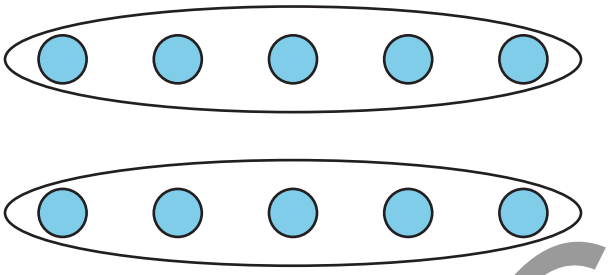
Activity 1

Learn More about Multiplication

What do you notice? What do you wonder?



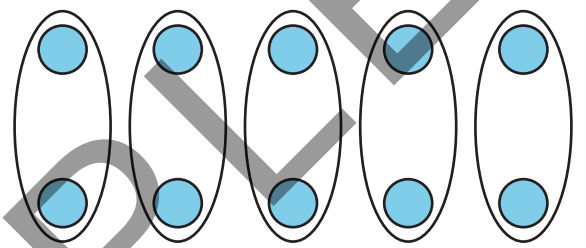


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Image A

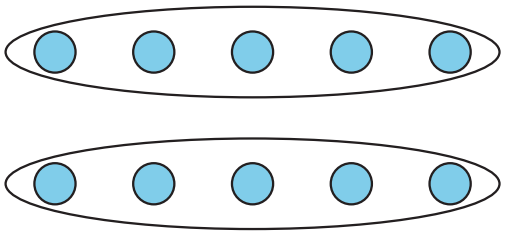
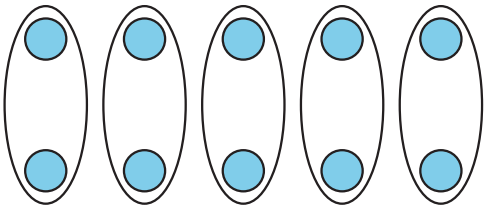


Image B



1. a. Write a description of a situation for each array.

   Image A

   Image B

   b. How are the situations alike? How are they different?

SAMPLE COPY

2. a. Write a multiplication equation for each situation.

Image A

Image B

b. How does your equation connect to the situation and array?

Image A

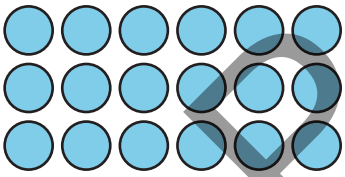
Image B

SAMPLE COPY

Activity 2

Revisit Arrays

1. Write 2 multiplication equations that represent the array.



2. Explain why both equations can represent the array.

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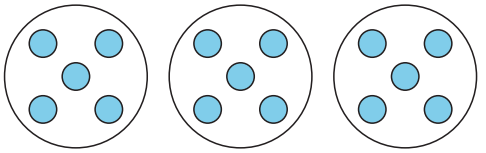
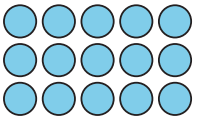
Section C Summary

We learned how equal groups are related to **arrays** and how to represent arrays with multiplication expressions and equations.

Equal groups:



Array:



Expression:

3×5

Equation:

$3 \times 5 = 15$

We also learned that we can multiply numbers in any order and get the same product.

$3 \times 5 = 15$ $\quad$ $5 \times 3 = 15$ $\quad$ $3 \times 5 = 5 \times 3$

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Addressing CA CCSSM 3.MD.3, 3.OA.1-4; practicing MP4

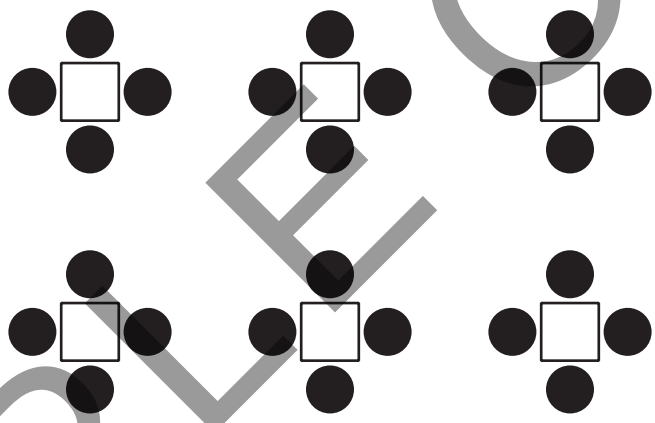
Game Night Seating Plan

Let's plan a game night.

Warm-up

Notice and Wonder: Squares and Circles

What do you notice? What do you wonder?



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Activity 1

Game Night

Your club is planning a game night.

Guests can play 1 of 4 games that require a different number of players:

- Game A - 2 players
- Game B - 4 players
- Game C - 5 players
- Game D - 10 players

The game room has 16 identical square tables. One person can sit on each side of the table.

1. Make a seating plan that shows a table arrangement so that each guest can play 1 of the games and all the tables are used.

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2. Make a poster that includes:

   a. a seating chart
   b. an explanation about how you decided on your seating plan
   c. how many people can play games in the room with your seating plan

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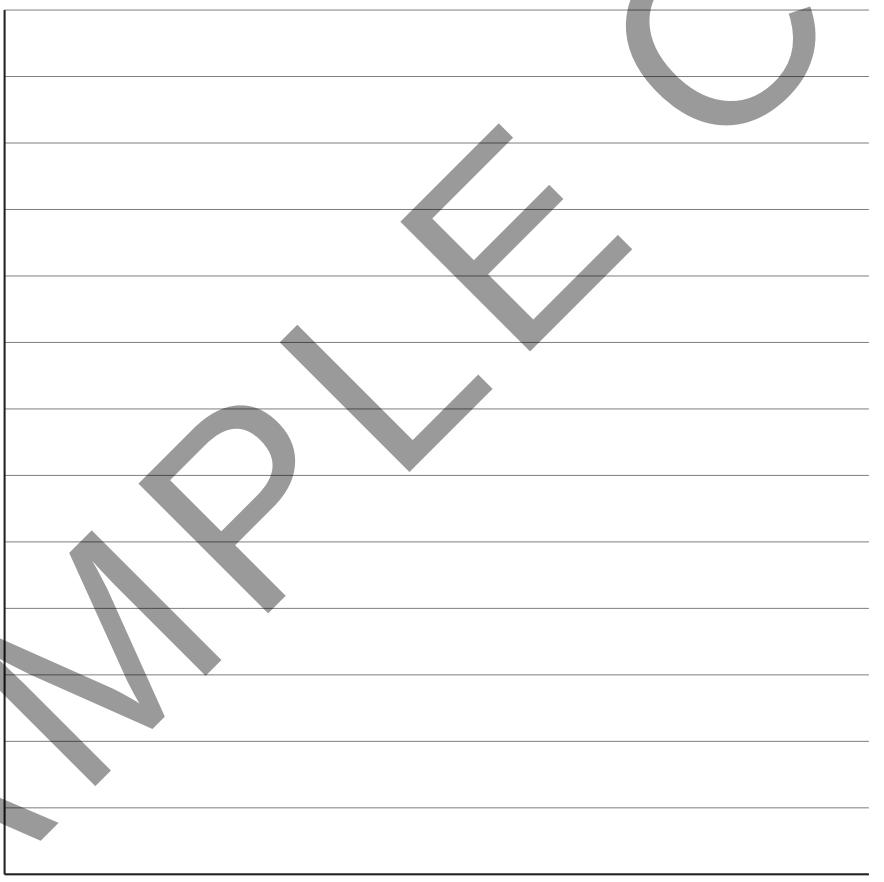
Activity 2

Game Night on a Graph

Make a scaled bar graph that shows the number of guests that can play each of the games A, B, C, and D. Be sure to include:

- a title and other labels
- a scale that counts by a number other than 1



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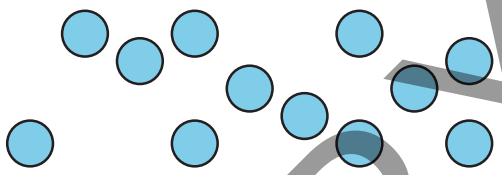
Practice Problems

7 Problems

1 from Unit 1, Lesson 17

Rearrange the circles to make an array in 2 different ways.



2 from Unit 1, Lesson 18

There are 4 rows of water bottles in the box. There are 5 bottles in each row.

a. Draw an array to represent the situation.

b. Write a multiplication expression to represent the number of bottles.

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3 from Unit 1, Lesson 19

There are 5 rows of chairs in the room. There are 4 chairs in each row. How many chairs are in the room?

a. Write a multiplication equation to represent the situation. Use a symbol for the unknown.

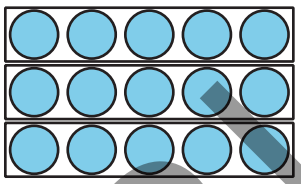
b. Find the value that makes your equation true.

Sec C

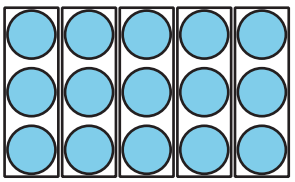
4 from Unit 1, Lesson 20

a. Write a multiplication equation that represents each array.

A



B



b. How are the arrays alike? How are they different?

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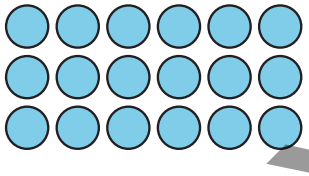
5 from Unit 1, Lesson 16

a. Explain or show 2 different ways that you see equal groups in the array.

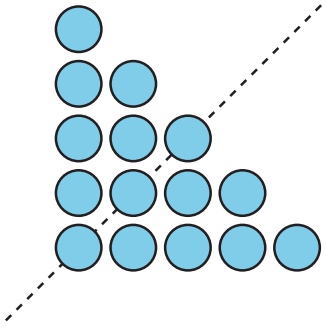


b. Rearrange the dots to make a different array.

6 Exploration

Andre says that there are an odd number of circles in this picture.

Do you agree with Andre? Explain or show your reasoning.



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7 Exploration

Find a collection of objects in the classroom or at home that is arranged in an array.

a. Describe the objects.

b. Create a drawing of the objects.

c. Write an equation showing how many objects there are.

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GRADE 3

UNIT

Area and Multiplication

Content Connections

In this unit you will explore the concept of area, relate the area of a rectangle to multiplication and solve problems involving area. You will make connections by:

- **Exploring Changing Quantities** while multiplying within 100 and representing your work with different visual representations.
- **Taking Wholes Apart, Putting Parts Together** while using square tiles to measure the area of shapes.

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Addressing the Standards

As you work your way through **Unit 2 Area and Multiplication,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 10
MP2 Reason abstractly and quantitatively.	Lessons 5 and 13
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 1, 10, 12, and 14
MP4 Model with mathematics.	Lessons 7 and 15
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 2, 4, 6, and 7
MP7 Look for and make use of structure.	Lessons 3, 4, 8, 9, 11, and 12
MP8 Look for and express regularity in repeated reasoning.	Lesson 11

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations	**3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	Lesson 1
• Number Flexibility to 100 for All Four Operations	**3.OA.5** Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Lessons 5, 8, 9, 11

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Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Lesson 10
• Patterns in Four Operations	**3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	Lessons 10, 11, 12
• Square Tiles	**3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	Lessons 1, 2, 15
• Square Tiles	**3.MD.5b** Recognize area as an attribute of plane figures and understand concepts of area measurement. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	Lesson 3
• Square Tiles	**3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	Lessons 3, 4, 6, 7
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.MD.7a** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	Lesson 8

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Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.MD.7b** Relate area to the operations of multiplication and addition. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	Lessons 5, 8, 9, 10, 11, 15
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.MD.7d** Relate area to the operations of multiplication and addition. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	Lessons 12, 13, 14, 15
• Patterns in Four Operations	**3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Lesson 13

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

SAMPLE COPY

Addressing CA CCSSM 3.MD.5, 3.OA.1; building towards 3.MD.5; practicing MP3

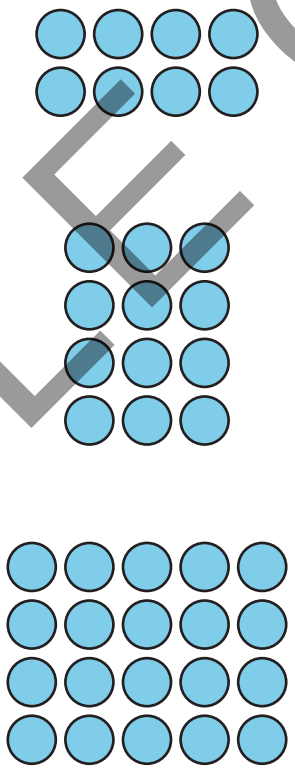
What Is Area?

Let's compare the sizes of shapes.

Warm-up

How Many Do You See: Arrays

How many do you see? How do you see them?

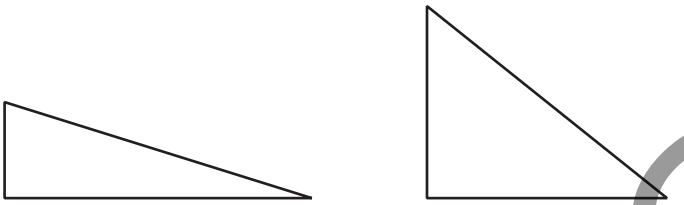


SAMPLE COPY

Activity 1

Compare Shapes

1. Here are 2 triangles. Which triangle is larger?

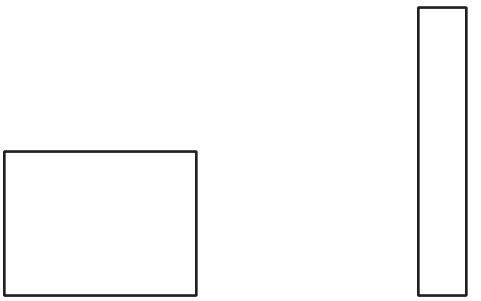


2. In each pair of shapes, which shape is larger? Be prepared to explain your reasoning.

   a.

   b.

   c.



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Activity 2

Pattern Blocks to Compare Shapes

Your teacher will give you handouts with some shapes on them.

Use pattern blocks to decide which shape covers the most space. Be ready to explain your reasoning.

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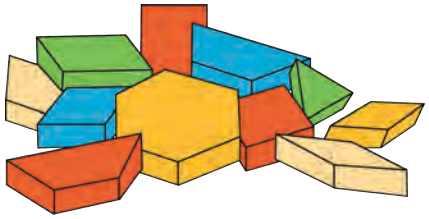


Addressing CA CCSSM 3.MD.5; building towards 3.MD.5; practicing MP6

How Do We Measure Area?

Let's use square tiles to measure area.

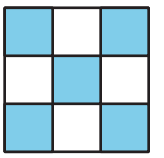
Warm-up

Which Three Go Together: Shapes with Squares

Which 3 go together?

A

B



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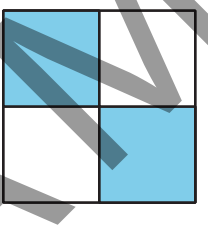

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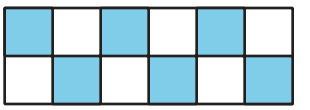

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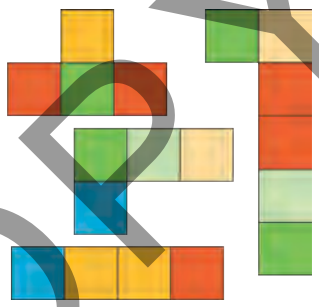
Activity 1

Create and Compare

1. Take a handful of square tiles.
2. Create a flat shape by connecting the tiles.
3. As a group, order the shapes from smallest to largest.



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Activity 2

Use Square Tiles to Measure Area

Your teacher will give you handouts with some figures on them.

Use square tiles to find the area of each figure. Each tile has an area of 1 square unit. Record your answers here. Be prepared to explain your reasoning.

1. Area: _____ square units

2. Area: _____ square units

3. Area: _____ square units

4. Area: _____ square units

5. Area: _____ square units

6. Area: _____ square units

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Addressing CA CCSSM 3.MD.5b, 3.MD.6; building towards 3.MD.6; practicing MP7

Tile Rectangles

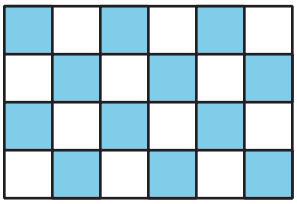
Let's count square tiles.

Warm-up

Which Three Go Together: Tiles

Which 3 go together?

A



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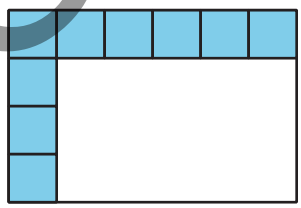

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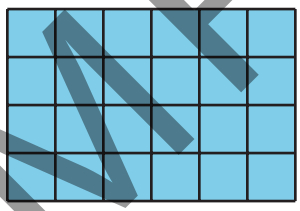

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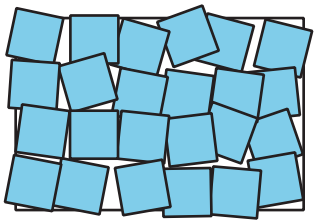


SAMPLE COPY

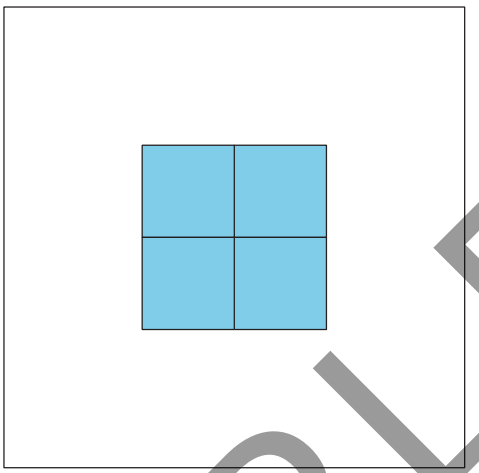
Activity 1

Time to Tile

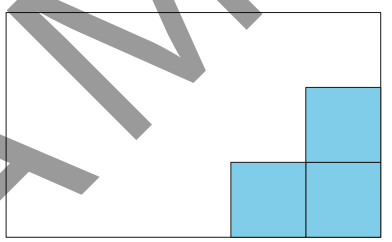
Your teacher will give you square tiles and a handout showing rectangles and squares.

1. Describe or show how to use the square tiles to measure the area of each rectangle. You can place square tiles on the handout where squares are already shown. You can also move the tiles, if needed.

   a.



   b.



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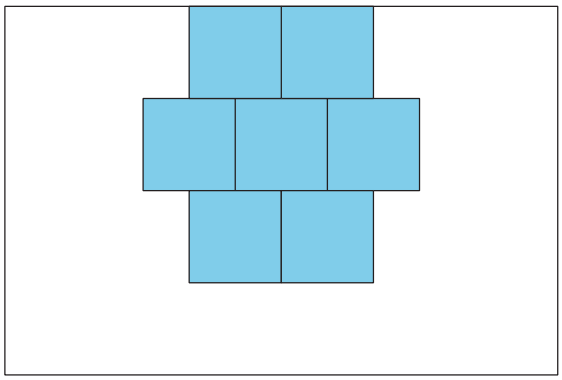
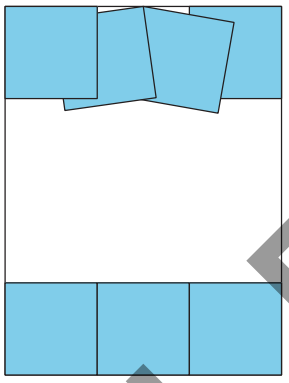
c.



d.



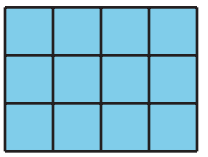
2. Describe how to use square tiles to find the area of any rectangle.

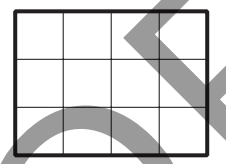
SAMPLE COPY

Activity 2

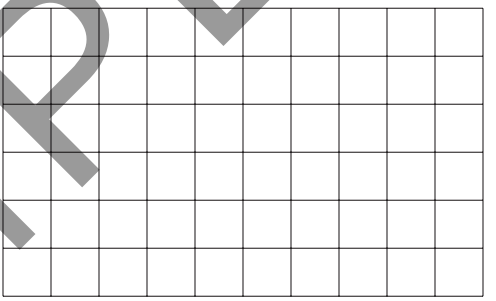
Card Sort: Rectangles

What do you notice? What do you wonder?





Draw a rectangle with an area of 8 square units on the grid.



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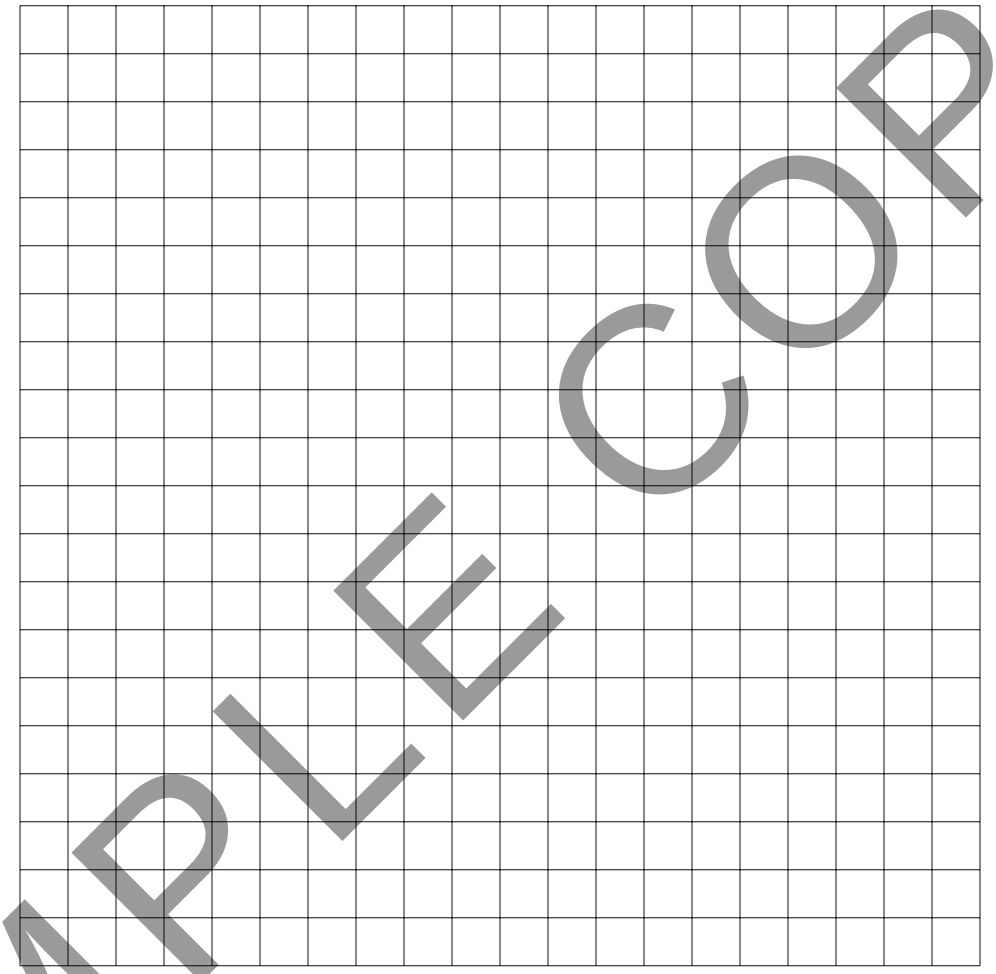
Your teacher will give you a set of cards that show rectangles.

1. Sort the cards into categories in a way that makes sense to you. Be ready to explain the meaning of your categories.
2. Draw rectangles on the grid to fit each of your categories.



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Addressing CA CCSSM 3.MD.6; building towards 3.MD.6; practicing MP6 and MP7

Area of a Rectangle

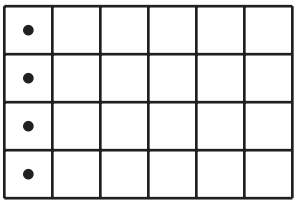
Let's find the areas of more rectangles.

Warm-up

Which Three Go Together: Area and Arrays

Which 3 go together?

A



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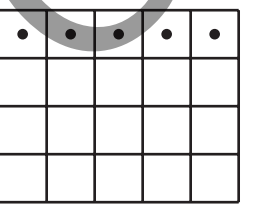
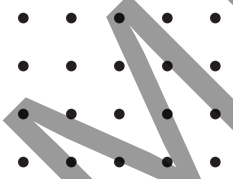


C



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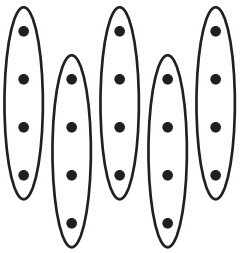

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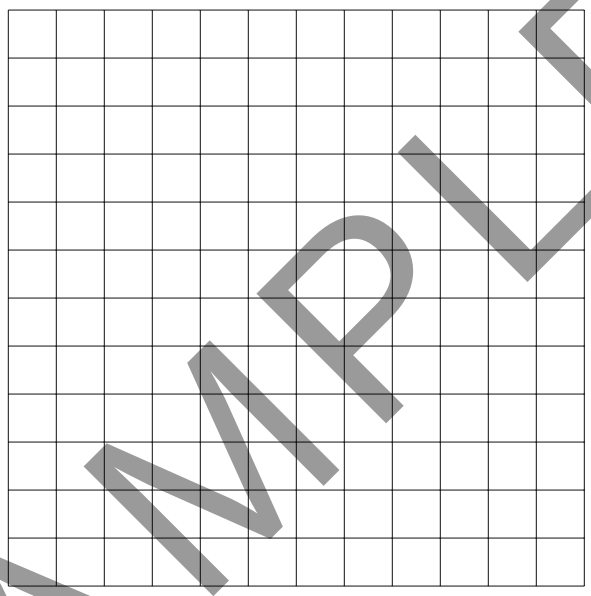
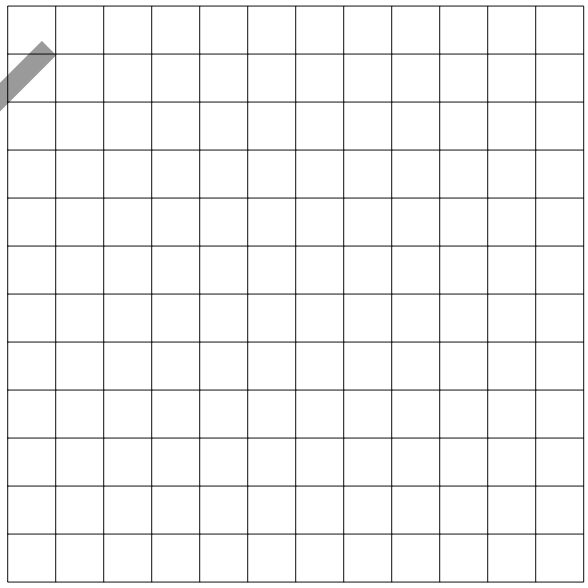
Activity 1

What Did I Create?

1. Can you and your partner draw the same rectangle without looking at each other's drawing?
    - Partner A: Draw a rectangle on 1 of the grids provided. Describe the rectangle to your partner without telling the total number of grid squares it covers.
    - Partner B: Draw the rectangle your partner describes to you.
2. Place your 2 rectangles next to each other. Discuss: What is the same? What is different?
3. Switch roles and repeat.





SAMPLE COPY

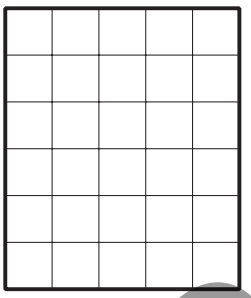
Activity 2

Find the Area

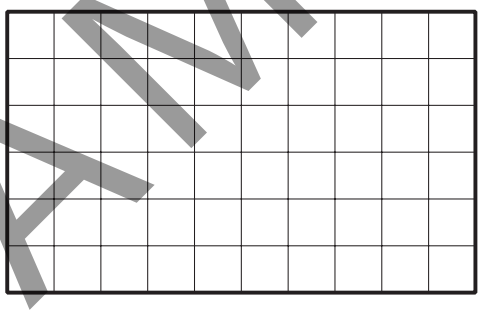
Find the area of each rectangle and include the units. Explain or show your reasoning.

1.

2.



3.



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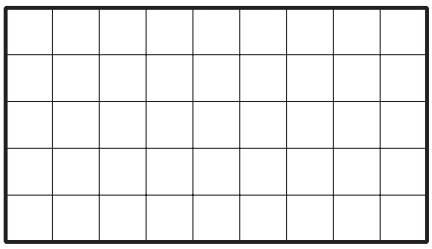
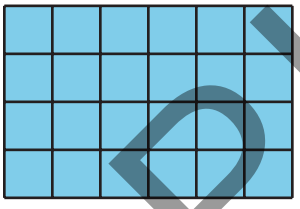
4.

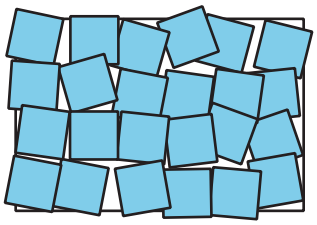


Section A Summary

We learned that **area** is the amount of space covered by a shape.

We saw that we can count tiled squares to measure area. When we tile a shape, we need to make sure that the squares are covering the whole shape without gaps or overlaps.





Area is measured in square units. Each square tile in this rectangle is 1 square unit. The area of the tiled rectangle is 24 square units.

SAMPLE COPY

Practice Problems

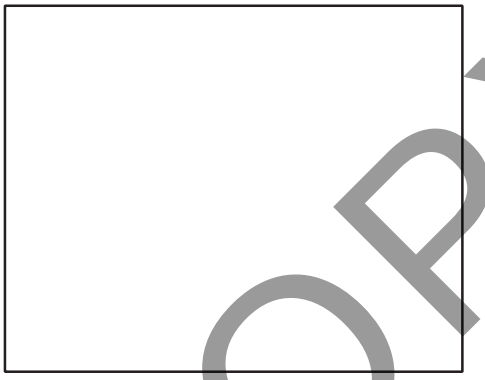
11 Problems

1

Pre-unit

a. Partition the rectangle into 4 equal rows and 5 equal columns.

b. How many small squares are there in the rectangle?



2

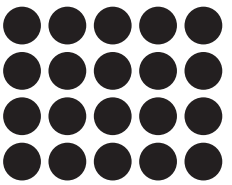
Pre-unit

Is the number of dots in each image even or odd? Explain how you know.

a.

b.

c.



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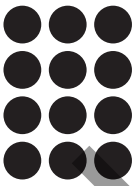
3 Pre-unit

How many dots are in each array? Explain or show your reasoning.

a.

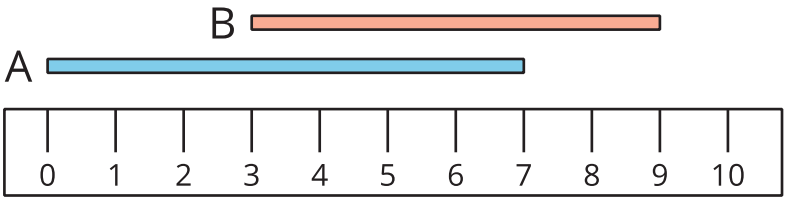
b.

c.



SAMPLE COPY

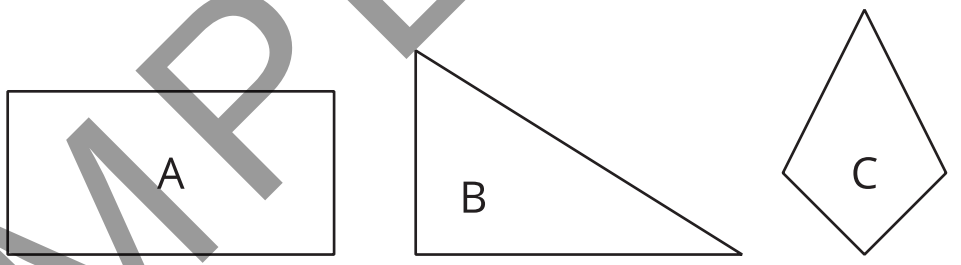
4 Pre-unit



Use the centimeter ruler to find the lengths of Rectangles A and B. Explain your reasoning.

5 from Unit 2, Lesson 1

Which shape is the largest? Which shape is the smallest? Explain your reasoning. You may trace and cut out the shapes if it is helpful.



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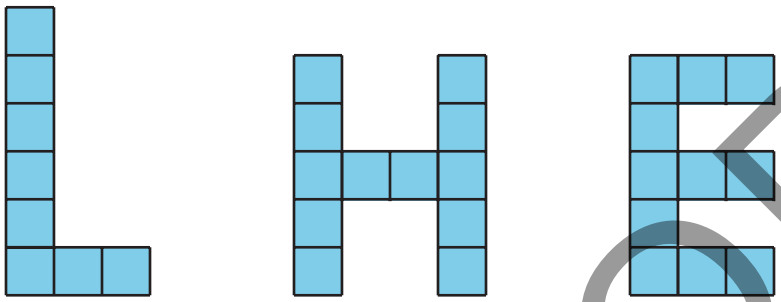
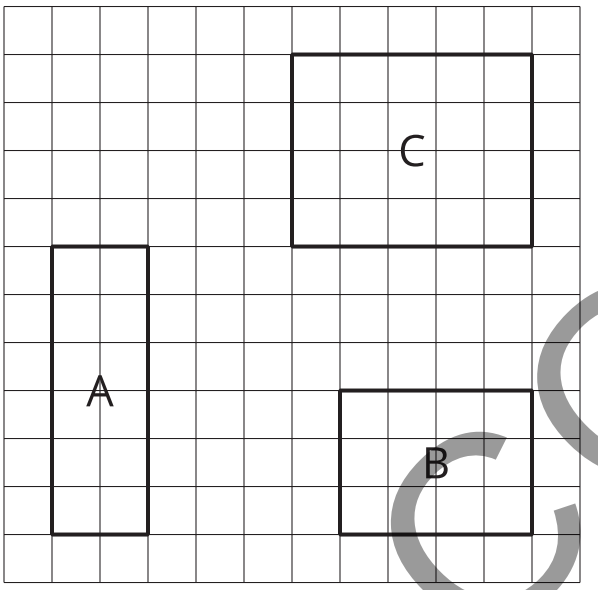
6 from Unit 2, Lesson 2

Lin, Han, and Elena made letters from squares. Put the letters in order from least area to greatest area. Explain your reasoning.



SAMPLE COPY

a. Find the area of each rectangle drawn on the grid.



b. Can rectangles with different shapes have the same area? Explain your reasoning.

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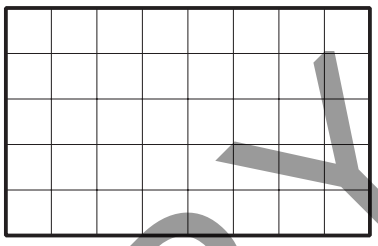
8

from Unit 2, Lesson 4

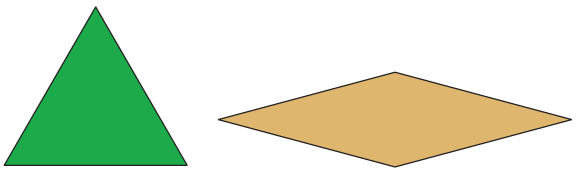
Find the area of the rectangle. Explain or show your reasoning.



9

Exploration

Which shape has the greater area, the green triangle pattern block or the tan rhombus pattern block? Explain your reasoning



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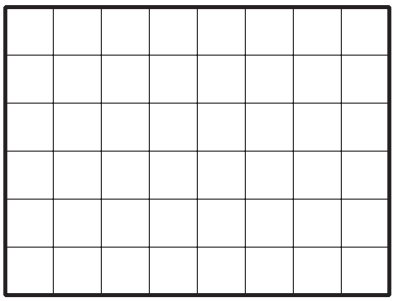
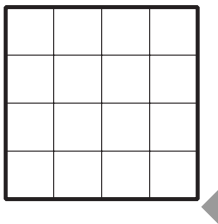
10 Exploration

Here are 2 rectangles.





a. What is the area of the larger rectangle?

b. What is the area of 3 of the smaller rectangles combined?

c. Can you completely cover the first rectangle with 3 of the smaller rectangles without cutting them up? Explain or show your reasoning.

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11 Exploration

a. How many different rectangles can you make with 36 square tiles? Describe or draw the rectangles.

b. How are the rectangles alike? How are they different?

SAMPLE COPY

Unit 2, Lesson 5

Addressing CA CCSSM 3.MD.7b, 3.OA.5; practicing MP2

Represent Products as Areas

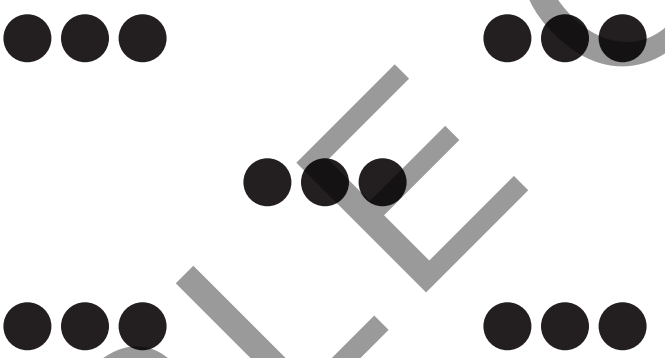
Let's connect multiplication expressions to areas.

Warm-up

Sec B

How Many Do You See: One More

How many do you see? How do you see them?



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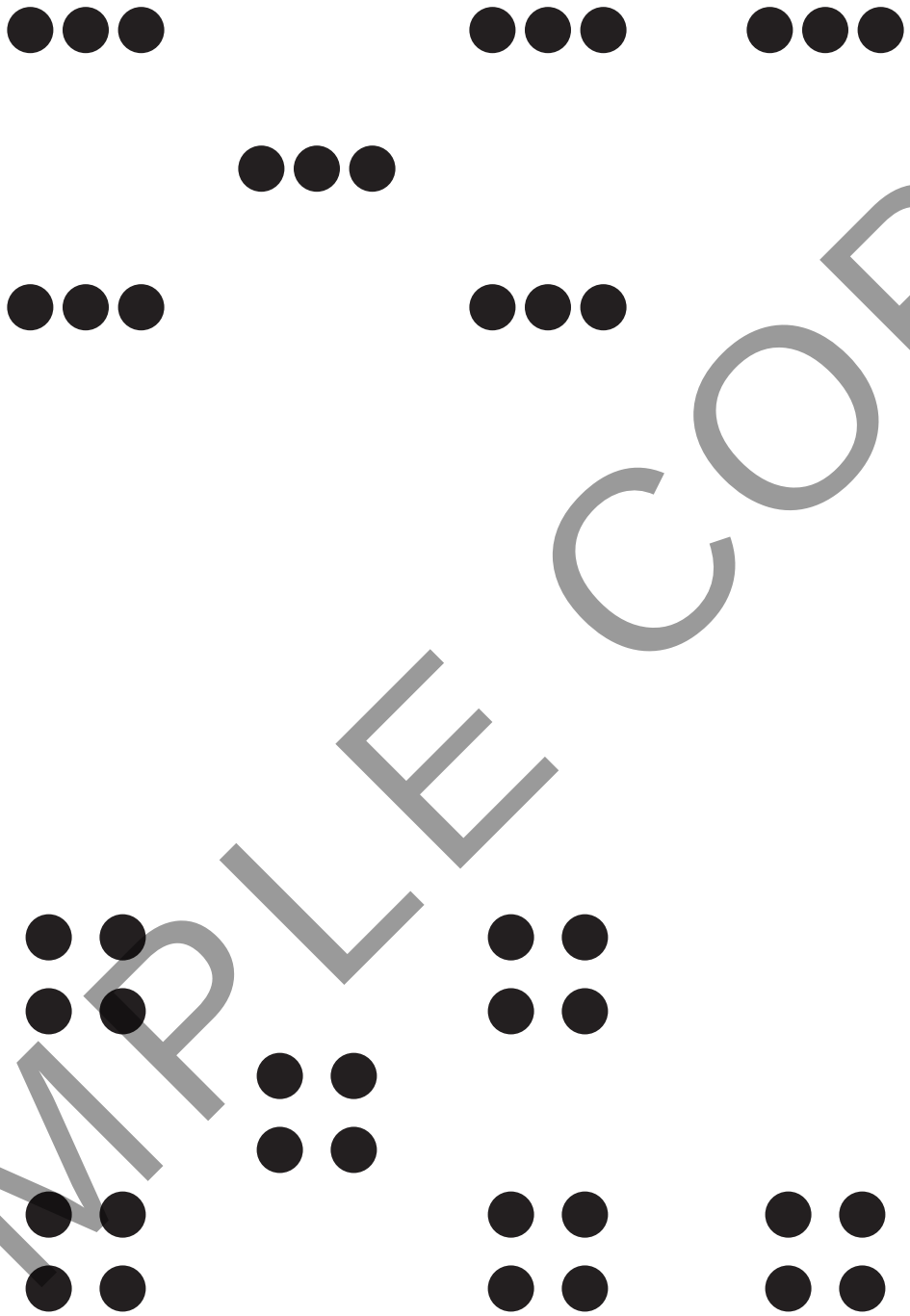

SAMPLE COPY

Activity 1

Match Expressions and Areas

Your teacher has posted images of rectangles around the room. Match each expression with a rectangle that can represent it. Be prepared to explain your reasoning.

1. 9×5

2. 8×2

3. 7×10

4. 3×3

5. 2×6

6. 8×4

7. 5×7

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Activity 2

Create from Expressions

1. The numbers in each expression represent the number of rows (or columns) in a rectangle and how many squares are in each row (or column).

   On the grid, draw each rectangle, label it with the numbers, and find its area.

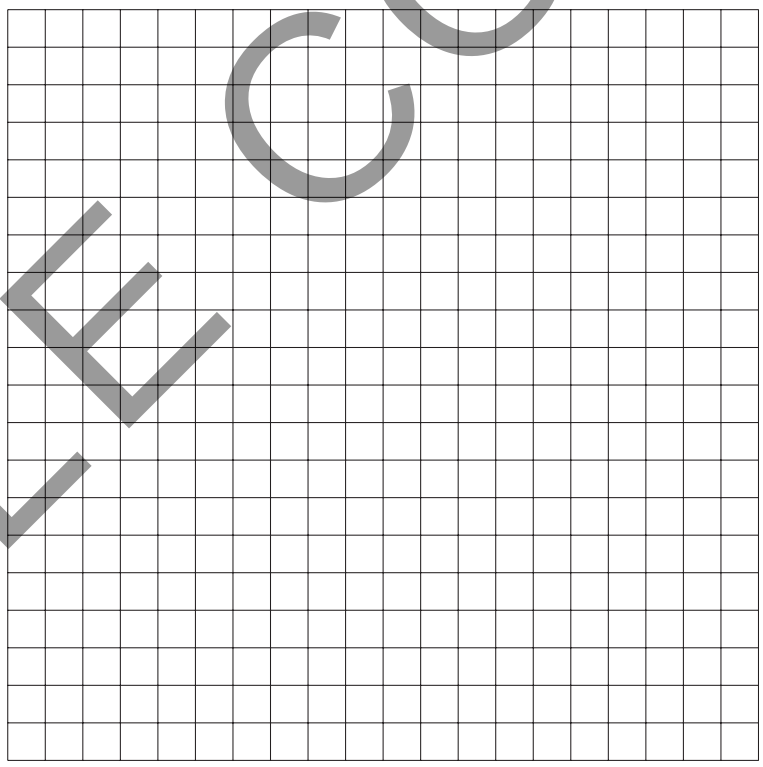
   a. 3×4

   b. 4×6

   c. 6×3

   d. 7×4

   e. 3×2



2. Explain why multiplying the numbers in each expression gives us the area of the rectangle.

SAMPLE COPY

Addressing CA CCSSM 3.MD.6; practicing MP6

Different Square Units (Part 1)

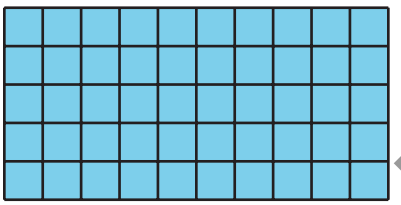
Let's learn about different units we can use to measure area.

Warm-up

Notice and Wonder: Squares, Squares

What do you notice? What do you wonder?

A



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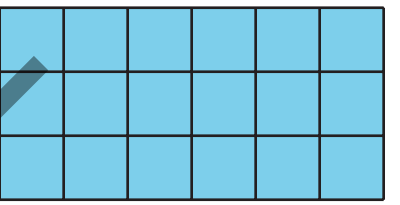

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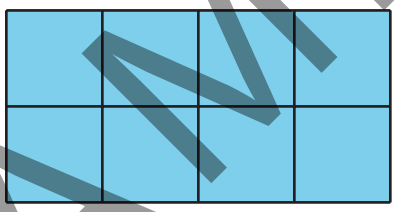

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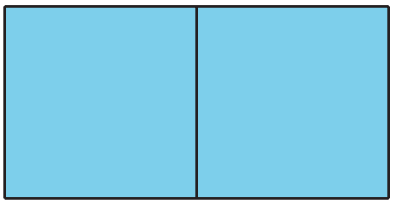


SAMPLE COPY

Activity 1

Same Rectangle, Different Units

Your teacher will give you 2 kinds of grid paper. Use them to create a rectangle for each expression.

Partner 1: Use grid 1.

Partner 2: Use grid 2.

1. 2×5

2. 7×3

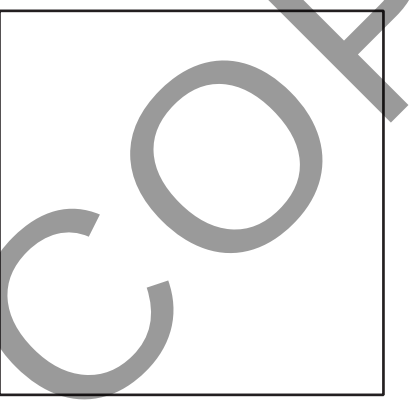
3. 6×8

SAMPLE COPY

Activity 2

What's the Area?

Estimate how many square centimeters and square inches it will take to tile this square.



- about _____ square inches
- about _____ square centimeters

1. Use the inch grid and the centimeter grid to find the area of the square.
   - _____ square inches
   - _____ square centimeters
2. Write a multiplication expression that describes the rows and the columns in the square, and represents the area in each unit.
   - _____ square inches
   - _____ square centimeters

SAMPLE COPY

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Addressing CA CCSSM 3.MD.6; building towards 3.MD.6; practicing MP4 and MP6

Different Square Units (Part 2)

Let's learn about larger square units.

Warm-up

Notice and Wonder: A Larger Square

What do you notice? What do you wonder?

SAMPLE COPY

Activity 1

Square Feet and Square Meters

1. This is a square meter.

What kinds of areas would make sense to measure with square meters? Be ready to explain your reasoning.

SAMPLE COPY

2. This is a square foot.

What kinds of areas would make sense to measure with square feet? Be ready to explain your reasoning.

SAMPLE COPY

Activity 2

Which Square Unit?

1. For each area, tell if you would use square centimeters, square inches, square feet, or square meters to measure it. Be ready to explain your choices.

   a. a baseball field

   _____

   b. a book cover

   _____

   c. our classroom floor

   _____

   d. a piece of paper

   _____

   e. the top of a table

   _____

   f. the screen on a phone

   _____

SAMPLE COPY

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2. Choose the area that best matches each item. Be ready to explain your reasoning.

- about 9 square inches
- about 3 square feet
- about 55 square centimeters
- about 55 square meters

a. a playing card

b. the floor of a classroom

c. a sticky note

d. the top of a student desk

SAMPLE COPY

Activity 3

Area Scavenger Hunt

Find an object or a space that you would measure with square inches, square centimeters, square feet, and square meters. Explain your unit choices.

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area	square unit and reasoning
Example: a piece of paper	*Example:* "I think it can fit about 8 inches across and 10 inches down, so square inches work well. It can be measured in square centimeters, too, but it would just take a lot more squares. Square feet and square meters would be too large."

SAMPLE COPY

Unit 2, Lesson 8

Addressing CA CCSSM 3.MD.7a-b, 3.OA.5; practicing MP7

Area of a Rectangle without a Grid

Let's solve area problems, without a grid.

Sec B

Warm-up

How Many Do You See: One More, One Less

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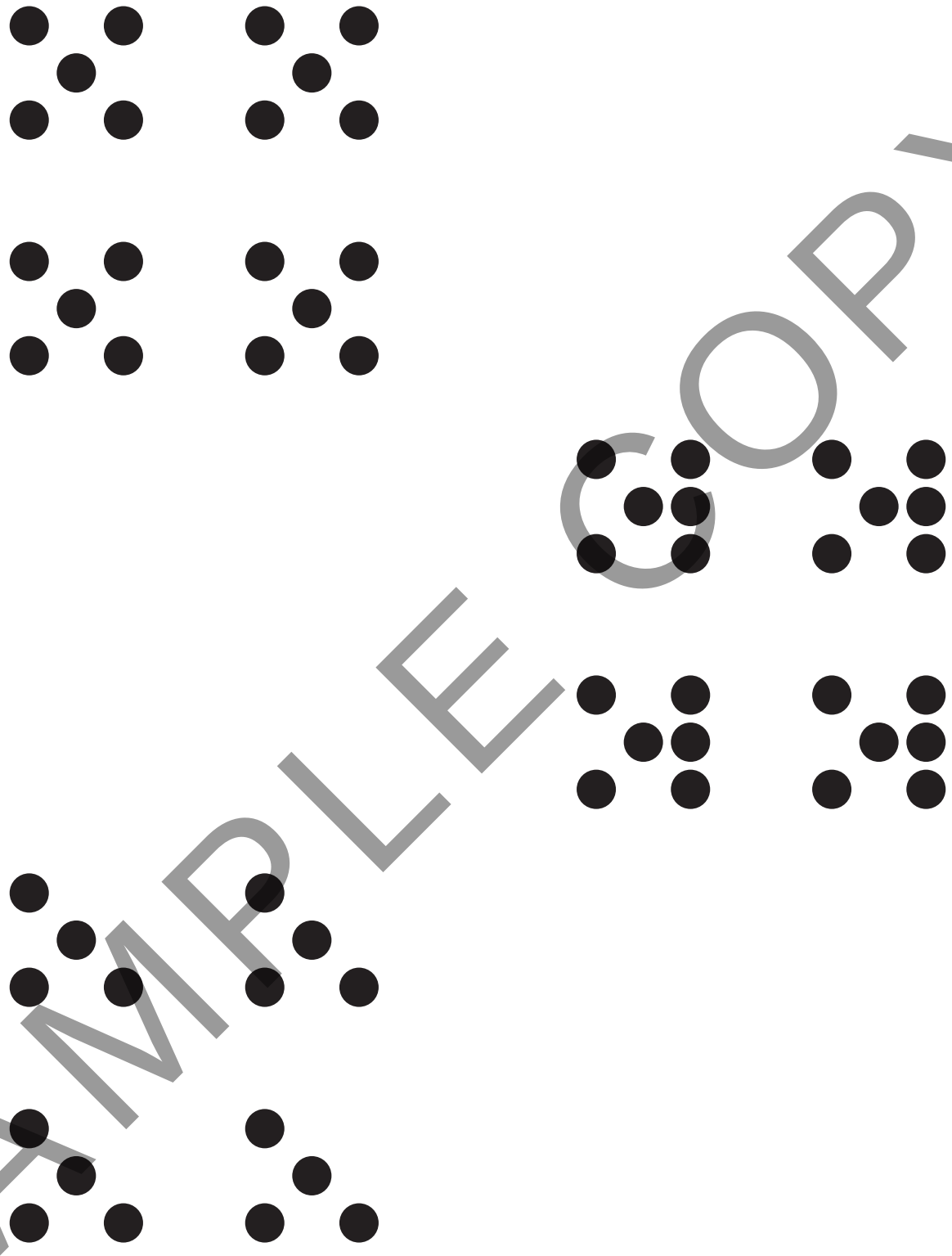
How many do you see? How do you see them?



SAMPLE COPY

Activity 1

Partially Tiled

What do you notice? What do you wonder?

SAMPLE COPY

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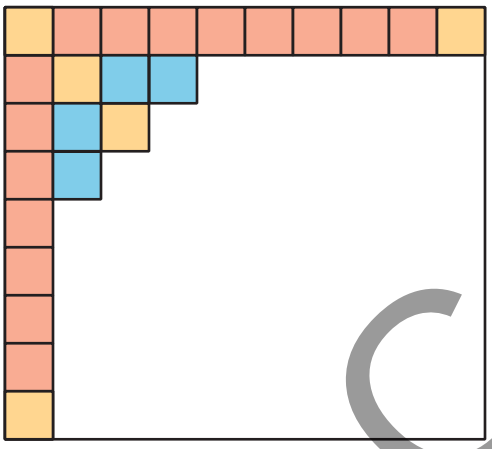
After learning about *azulejos* (ah-soo-LAY-hohs) in Portugal, Elena is making her own tile artwork. This rectangle shows the project Elena is tiling. Each square tile has a side length of 1 inch.

How many tiles are needed to tile the whole rectangle? Explain or show your reasoning.



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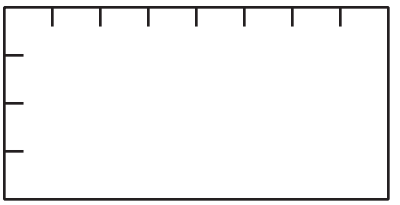
Activity 2

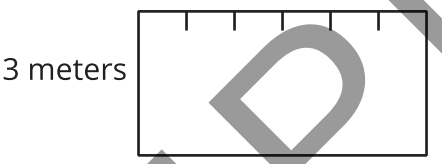
No More Squares

1. The tick marks on the sides of this rectangle are 1 meter apart.



What is the area of the rectangle in square meters?

2. The top side of this rectangle is marked off in meter lengths. The left side is labeled with the length in meters.



What is the area of the rectangle in square meters?

SAMPLE COPY

Unit 2, Lesson 9

Addressing CA CCSSM 3.MD.7b, 3.OA.5; practicing MP7

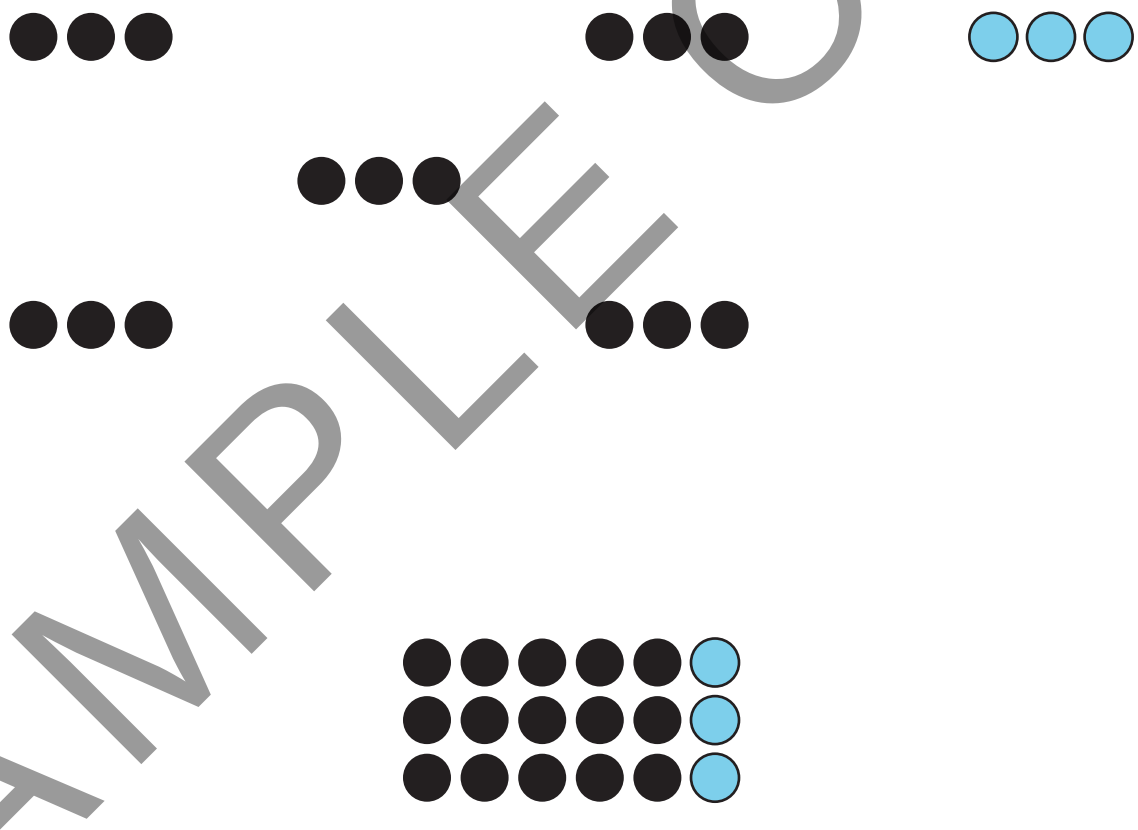
Measure to Find the Area

Let's measure the sides of rectangles and find the areas.

Warm-up

Notice and Wonder: Groups and Arrays

What do you notice? What do you wonder?



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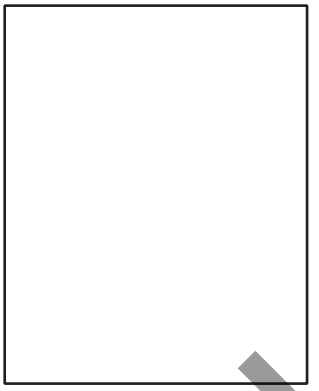
Activity 1

Measure to Find Area

Use a ruler to measure the sides of each rectangle in centimeters. Then calculate the area of each rectangle in square centimeters.

Sec B

A



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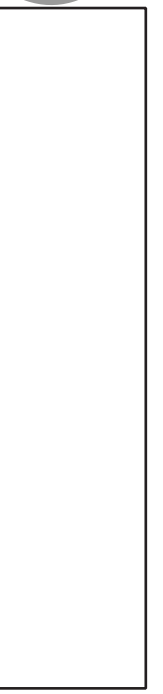

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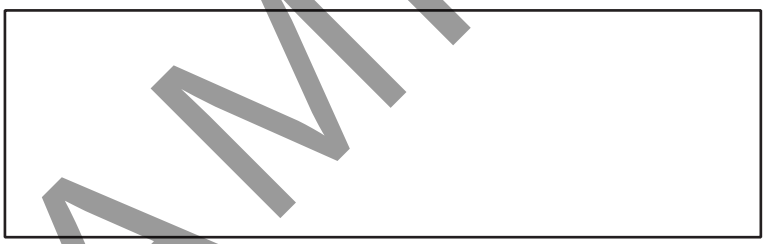

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Activity 2

Create a Rectangle

Your teacher will give you some tape. Use it to create a rectangle with your assigned area.

- Area A: 4 square feet
- Area B: 6 square feet
- Area C: 9 square feet
- Area D: 10 square feet
- Area E: 12 square feet
- Area F: 16 square feet

SAMPLE COPY

Addressing CA CCSSM 3.MD.7b, 3.OA.7, 3.OA.9; building on 3.OA.5; practicing MP1 and MP3

Solve Area Problems

Let's solve area problems.

Warm-up

Number Talk: One More Group

Find the value of each expression mentally.

- 5×2
- 6×2
- 5×6
- 6×6

SAMPLE COPY

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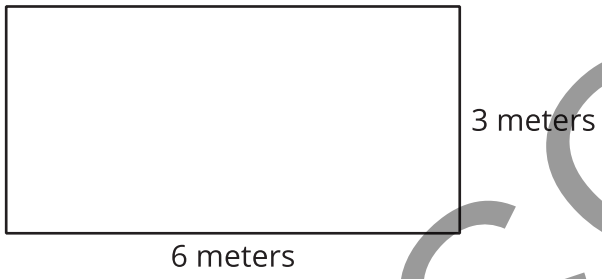
Activity 1

Paint a Wall

Noah is painting a wall in a community garden. The wall is shaped like a rectangle. A diagram of the wall is shown here.



Paint is sold in 3 different sizes:

- A small container will cover 3 square meters.
- A medium container will cover 10 square meters.
- A large container will cover 40 square meters.

What should Noah buy? Explain your reasoning.

SAMPLE COPY

Activity 2

Create a Garden

Sec B

1. Elena is planning to build a raised garden bed for the community garden. The garden bed will be rectangular. One side must be 3 feet long. To fit all her plants, Elena needs the bed to cover at least 20 square feet. She has enough soil for a garden bed that covers 30 square feet.

   What are some possible sizes for the garden bed?

Create a poster to show your thinking. Organize your poster so it can be followed by others.

SAMPLE COPY

2. As you look at the posters with your group:

   a. Describe how another group solved the problem, using a different way than your group did.

   b. Write 3 things you saw that helped you understand how other groups solved the problem.

SAMPLE COPY

Addressing CA CCSSM 3.MD.7b, 3.OA.5, 3.OA.9; building on 3.OA.5; building towards 3.MD.7d; practicing MP7 and MP8

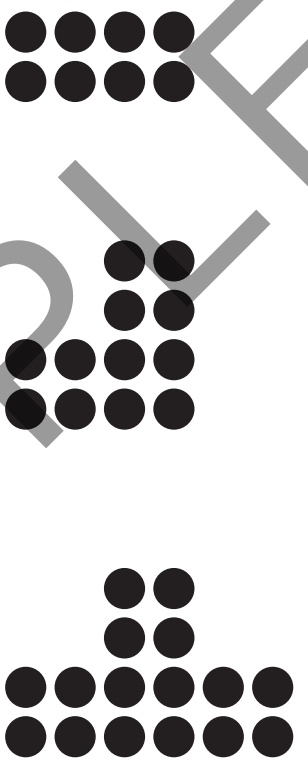
Area and the Multiplication Table

Let's explore area and the multiplication table.

Warm-up

How Many Do You See: Arrays That Grow

How many do you see? How do you see them?



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Activity 1

Area and the Multiplication Table

What do you notice? What do you wonder?

×	1	2	3	4	5
1					
2					
3	3				
4					
5					

×	1	2	3	4	5
1					
2					
3		6			
4					
5					

×	1	2	3	4	5
1					
2					
3			9		
4					
5					

×	1	2	3	4	5
1					
2					
3				12	
4					
5					

1. Use the blank table to create your own rectangle.

   Start from the top left corner. Record the product that the rectangle represents. Be prepared to explain your reasoning.

×	1	2	3	4	5
1					
2					
3					
4					
5					

2. Use the blank table to create a rectangle with an area of 24 square units.

   Start from the top left corner. Record the product that the rectangle represents. Be prepared to explain your reasoning.

×	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

SAMPLE COPY

Activity 2

Products in the Multiplication Table

What do you notice? What do you wonder?

×	1	2	3	4	5	6	7	8	9	10
1		2								
2	2	4	6	8	10					
3		6								
4		8								
5		10								
6										
7										
8										
9										
10										

1. Write as many other products in the table as you can. You may want to start with rows and columns that show products of 2, 5, and 10.

2. What patterns do you see in the row and the column that show products of 5?

SAMPLE COPY

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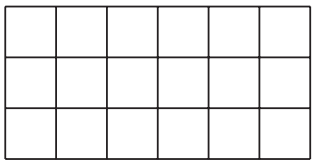
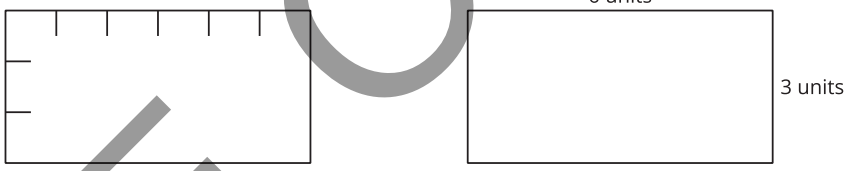
3. Write some equations that show 1 of the patterns that you see in the multiplication table. Explain or show why your pattern happens.

Section B Summary

We learned how area is related to multiplication. We multiplied the side lengths of a rectangle to find its area.

$$6 \times 3 = 18 \text{ square units}$$





We also learned how different square units are useful for measuring area in different situations and solved problems involving area.

square inch

square meter

SAMPLE COPY

Practice Problems

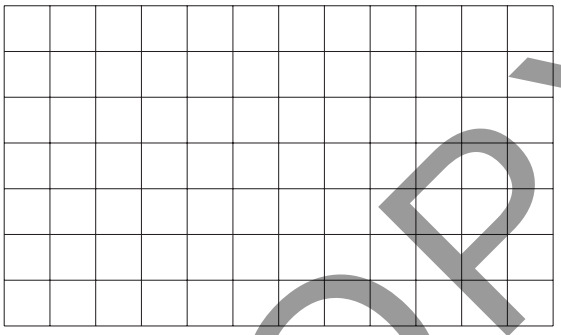
10 Problems

1

from Unit 2, Lesson 5

a. Draw a rectangle on the grid whose area can be represented by 5×7.

b. How does your rectangle represent the expression 5×7?

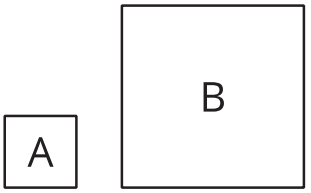


2

from Unit 2, Lesson 6

Here are 2 squares. One is a square centimeter, and the other is a square inch.

Which square is a square centimeter? Which square is a square inch? Explain how you know.



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3 from Unit 2, Lesson 7

For each object, decide if you would use square centimeters, square inches, square feet, or square meters to measure area. Explain your reasoning.

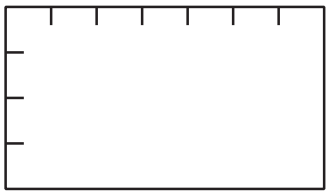
a. a baseball field

b. a table top

c. a cell-phone screen

4 from Unit 2, Lesson 8

The sides of the rectangle are marked in centimeters.



What is the area of the rectangle? Explain your reasoning.

SAMPLE COPY

5 from Unit 2, Lesson 9

a. Use a centimeter ruler.

i. Find the area of the rectangle in square centimeters.

ii. Draw a rectangle the area of which is 18 square centimeters.

6 from Unit 2, Lesson 10

Tyler has 40 carpet squares with sides of 1 foot. He wants to use all the squares to make a rectangle-shaped carpet.

The longest side of the carpet cannot be more than 12 feet. What could be the side lengths of Tyler's carpet?

SAMPLE COPY

7 from Unit 2, Lesson 11

a. Describe some patterns you see for the numbers in the table.

×	1	2	3	4	5	6	7	8	9
1			3			6			9
2			6			12			18
3			9			18			27
4			12			24			36
5			15			30			45
6			18			36			54
7			21			42			63
8			24			48			72
9			27			54			81

b. Write some equations that show 1 of the patterns you found. Explain or show why that pattern happens.

SAMPLE COPY

8 Exploration

a. Find in your classroom or at home an object or a space that is shaped like a rectangle. Describe the rectangle.

b. Would you use square centimeters, square inches, square feet, or square meters to measure the area of the rectangle? Explain your reasoning.

9 Exploration

What patterns do you notice in the 3 filled-in columns of the multiplication table?

×	1	2	3	4	5	6	7	8	9	10
1		2		4	5					
2		4		8	10					
3		6		12	15					
4		8		16	20					
5		10		20	25					
6		12		24	30					
7		14		28	35					
8		16		32	40					
9		18		36	45					
10		20		40	50					

SAMPLE COPY

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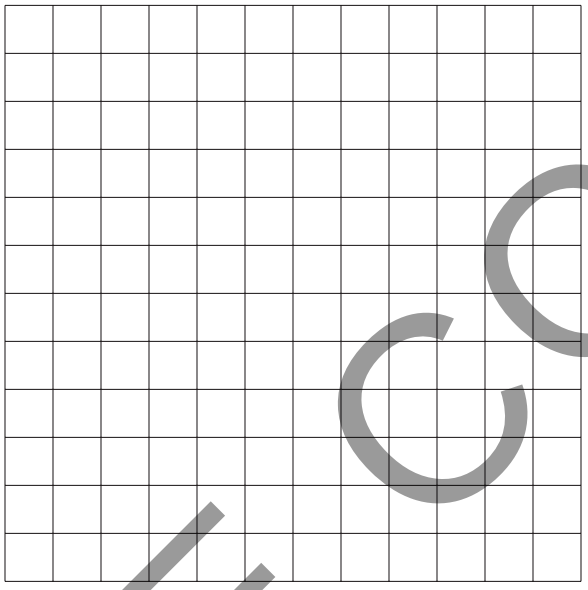
10 Exploration

Mai picks a mystery number that is less than 30. She says that she can draw, on this grid, 3 rectangles with different side lengths, whose areas in square units are the same as her mystery number.



What could be Mai's mystery number? Explain or show your reasoning.

SAMPLE COPY

Addressing CA CCSSM 3.MD.7d, 3.OA.9; building on 2.NBT.5; building towards 3.NBT.2; practicing MP3 and MP7

Area and Addition

Let's find the area of a figure made up of rectangles.

Warm-up

Number Talk: So Close

Find the value of each expression mentally.

- $9 + 6$
- $29 + 6$
- $59 + 6$
- $49 + 8$

SAMPLE COPY

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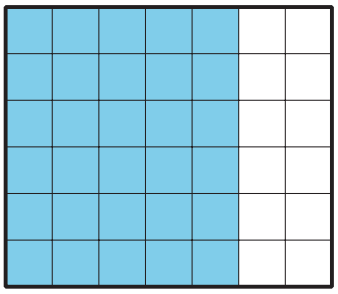
Activity 1

Rectangles in Rectangles

1. This rectangle represents space in a community garden. The shaded part has vegetables, and the unshaded part has flowers. Each grid square represents 1 square foot.



What is the area of the whole space?

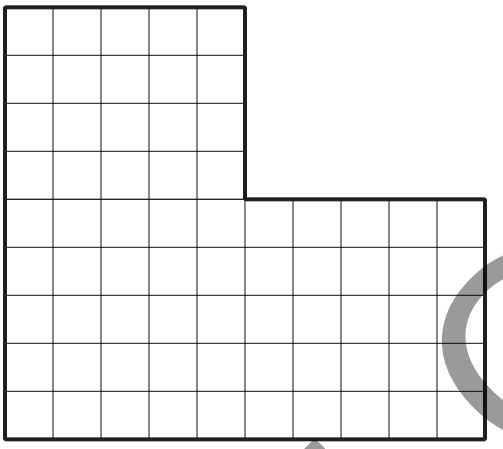
2. Design your own rectangular garden that has at least 2 parts. Find the area of each part of the garden and the area of the whole garden.

SAMPLE COPY

Activity 2

Find the Rectangles

What do you notice? What do you wonder?



Find the area of this figure. Explain or show your reasoning. Organize your explanation so it can be followed by others.

SAMPLE COPY

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Unit 2, Lesson 13

Addressing CA CCSSM 3.MD.7d, 3.NBT.2; practicing MP2

Find the Area of a Figure

Let's find the area of a figure.

Warm-up

Number Talk: Extend Make a Ten

Find the value of each expression mentally.

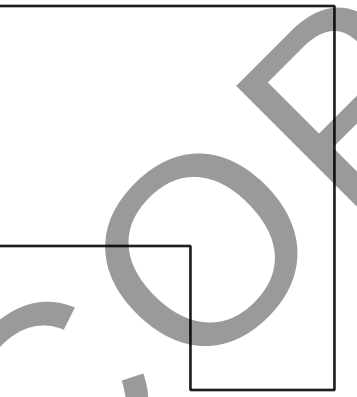
- $109 + 4$
- $109 + 14$
- $209 + 34$
- $219 + 34$

SAMPLE COPY

Activity 1

Bye-Bye Squares

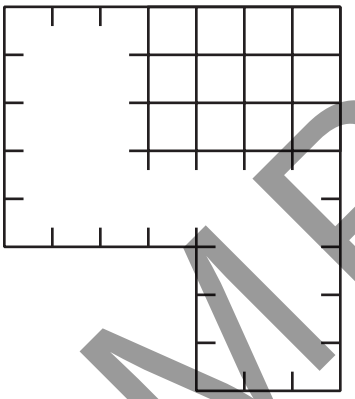
What do you notice? What do you wonder?



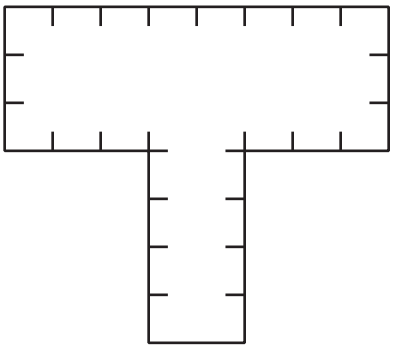
Sec C

Find the area of each figure. Explain or show your reasoning.

1.



2.



SAMPLE COPY

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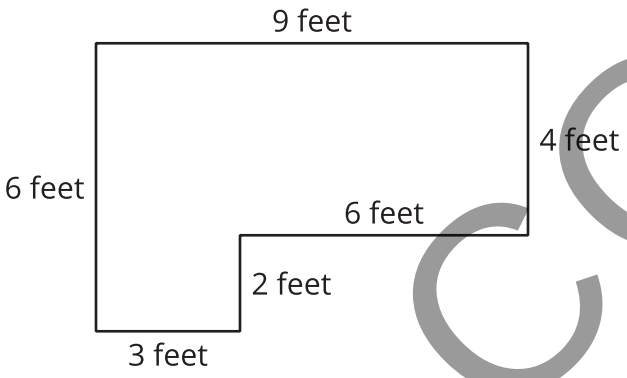
Activity 2

How Many Pavers Do We Need?

Noah wants to use square pavers to create a small patio in the community garden. A diagram of the patio is shown.



1. Each paver covers 1 square foot. How many pavers will Noah need to cover the whole patio?

2. What is the area of the patio? Explain or show your reasoning.

SAMPLE COPY

Unit 2, Lesson 14

Addressing CA CCSSM 3.MD.7d; building towards 3.MD.7d; practicing MP3

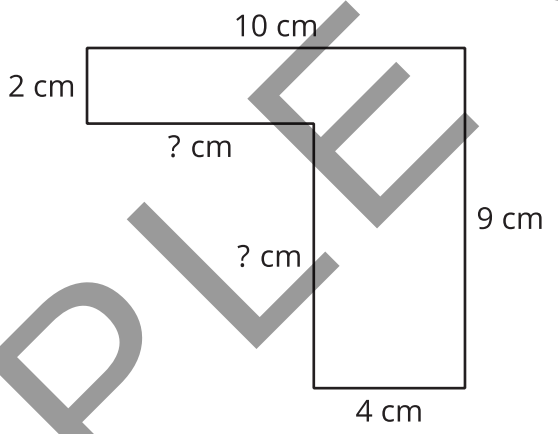
Find the Area of a Figure with Unknown Side Lengths

Let's find the area of a figure with unknown side lengths.

Warm-up

Notice and Wonder: Mystery Sides

What do you notice? What do you wonder?



SAMPLE COPY

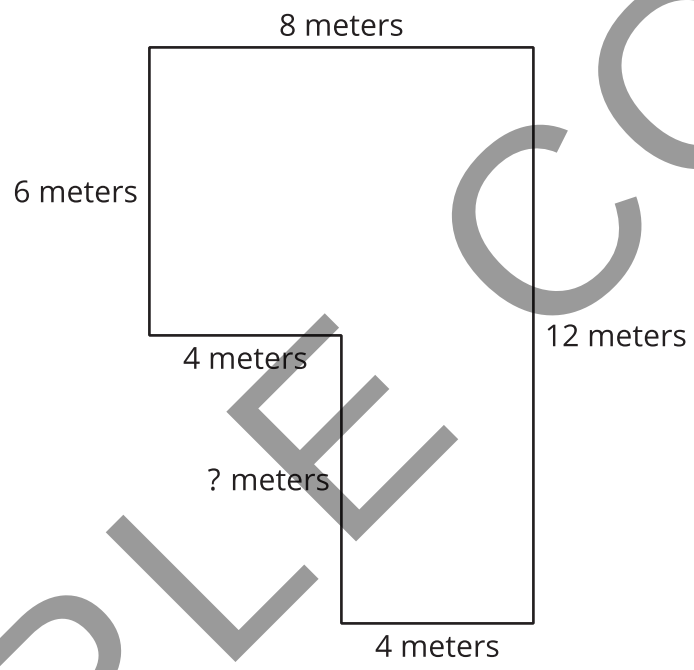
Illustrative Mathematics LEARN MATH FOR LIFE

Activity 1

The Mystery Side

Tyler says that this figure's unknown side length is 5 meters because it looks longer than the sides that are 4 meters long.

Do you agree or disagree? Be prepared to explain your reasoning.



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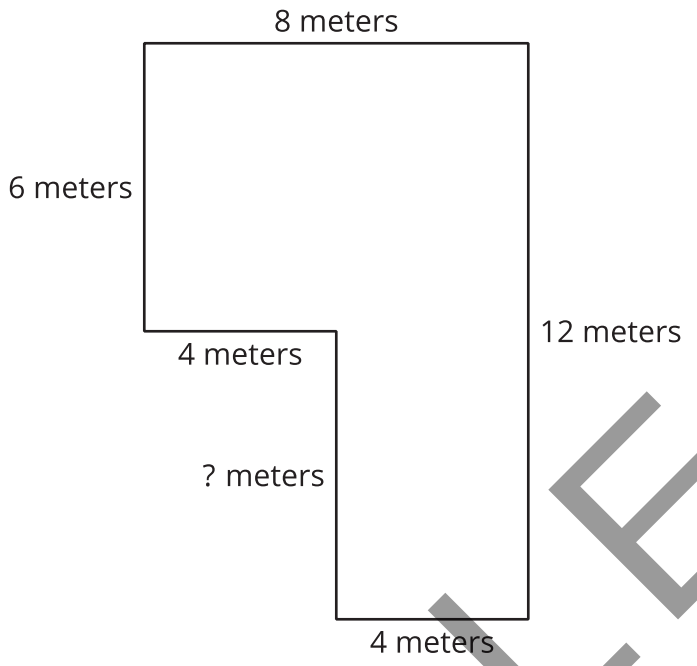
Activity 2

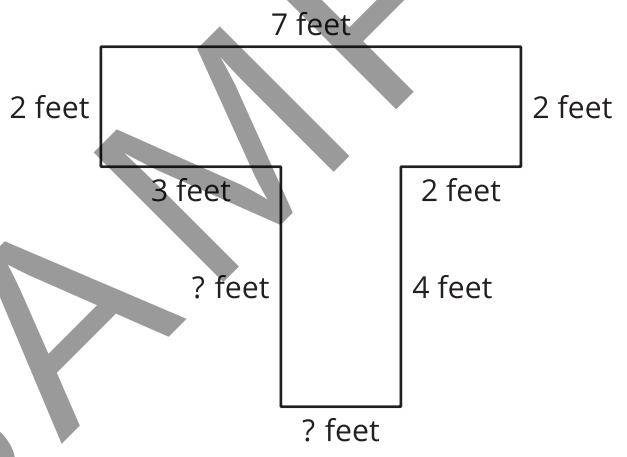
Practice with Mystery Sides

Find the area of each figure. Explain or show your reasoning.

A



B



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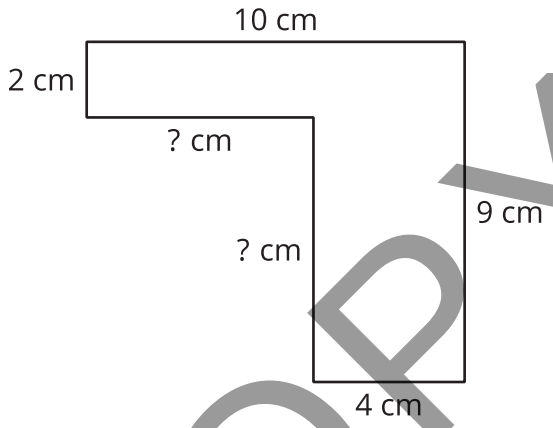
Section C Summary

We found the area of a figure that can be decomposed, or broken apart, into rectangles. We added the areas of the rectangles to find the area of the entire figure.

We also found unknown side lengths of figures by using what we know about opposite sides of rectangles.



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Addressing CA CCSSM 3.MD.5, 3.MD.7b, 3.MD.7d; building towards 3.MD.5, 3.MD.7b, 3.MD.7d; practicing MP4

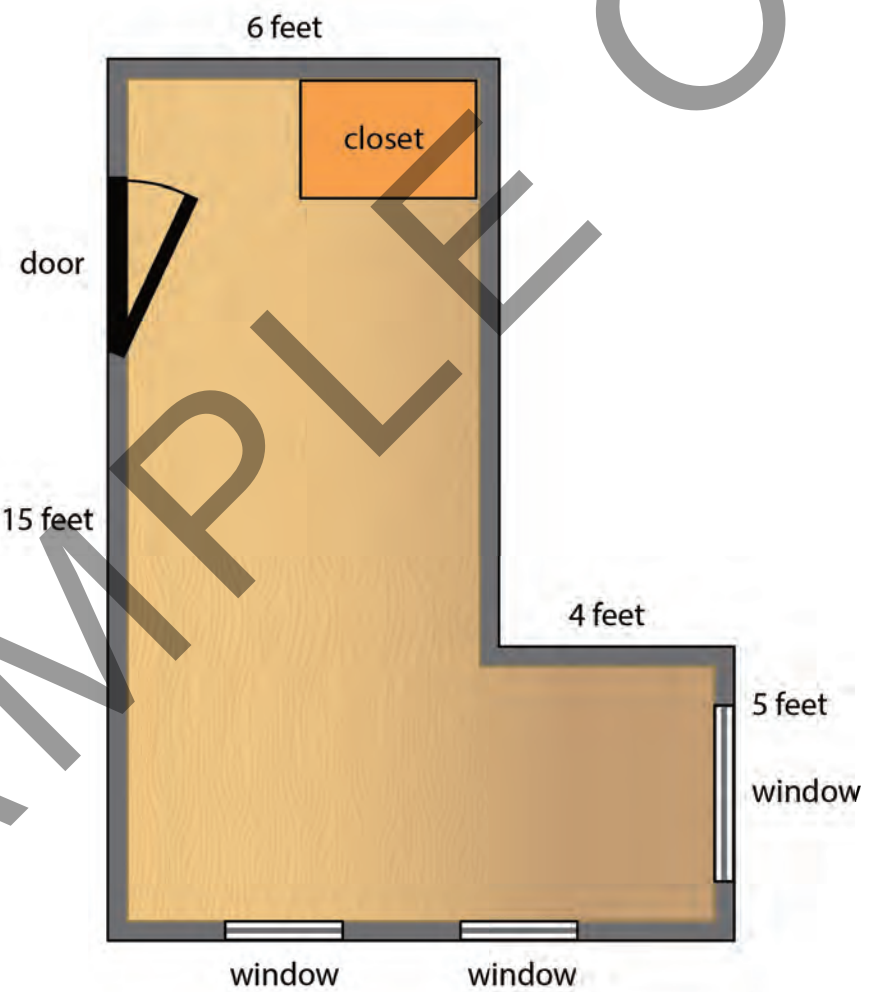
New Room

Let's fit furniture into a room.

Warm-up

Notice and Wonder: Floor Plan

What do you notice? What do you wonder?



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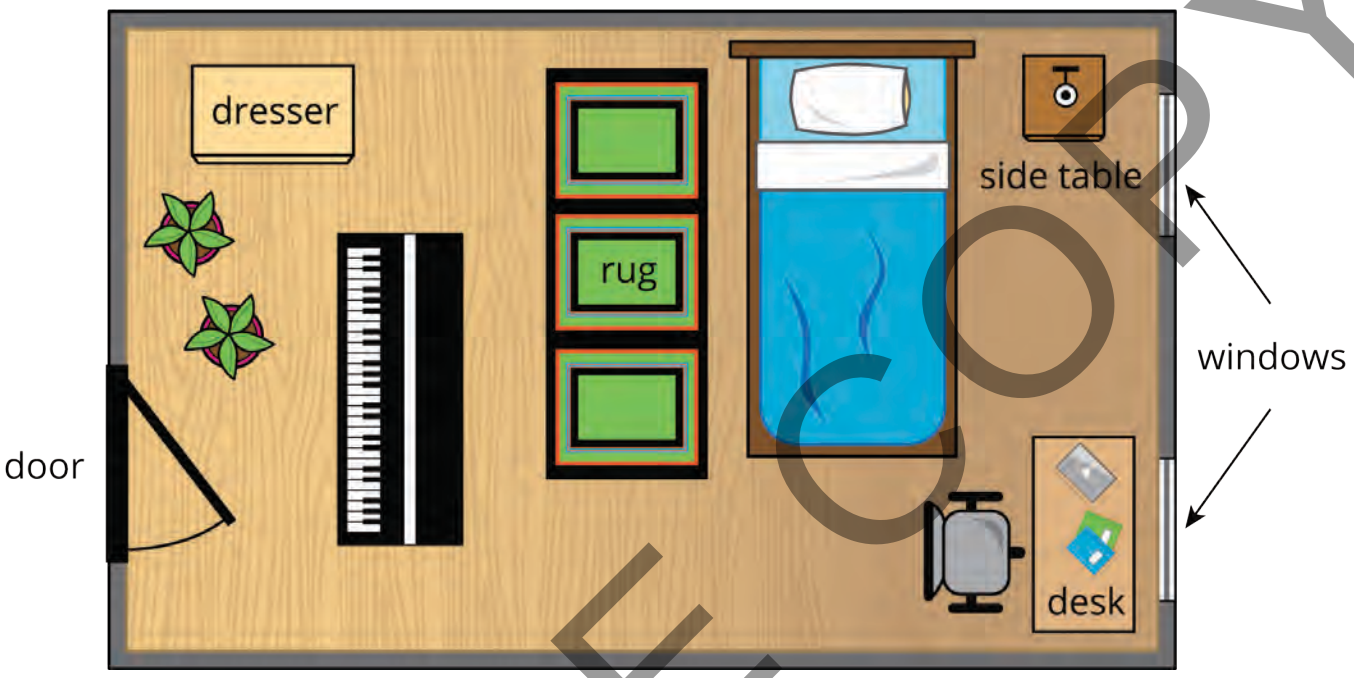
Activity 1

Floor Plans



The image shows the design of a floor plan for a bedroom. Discuss with your partner:

1. What is 1 thing you like about the design?

2. What is 1 thing you would change and why?

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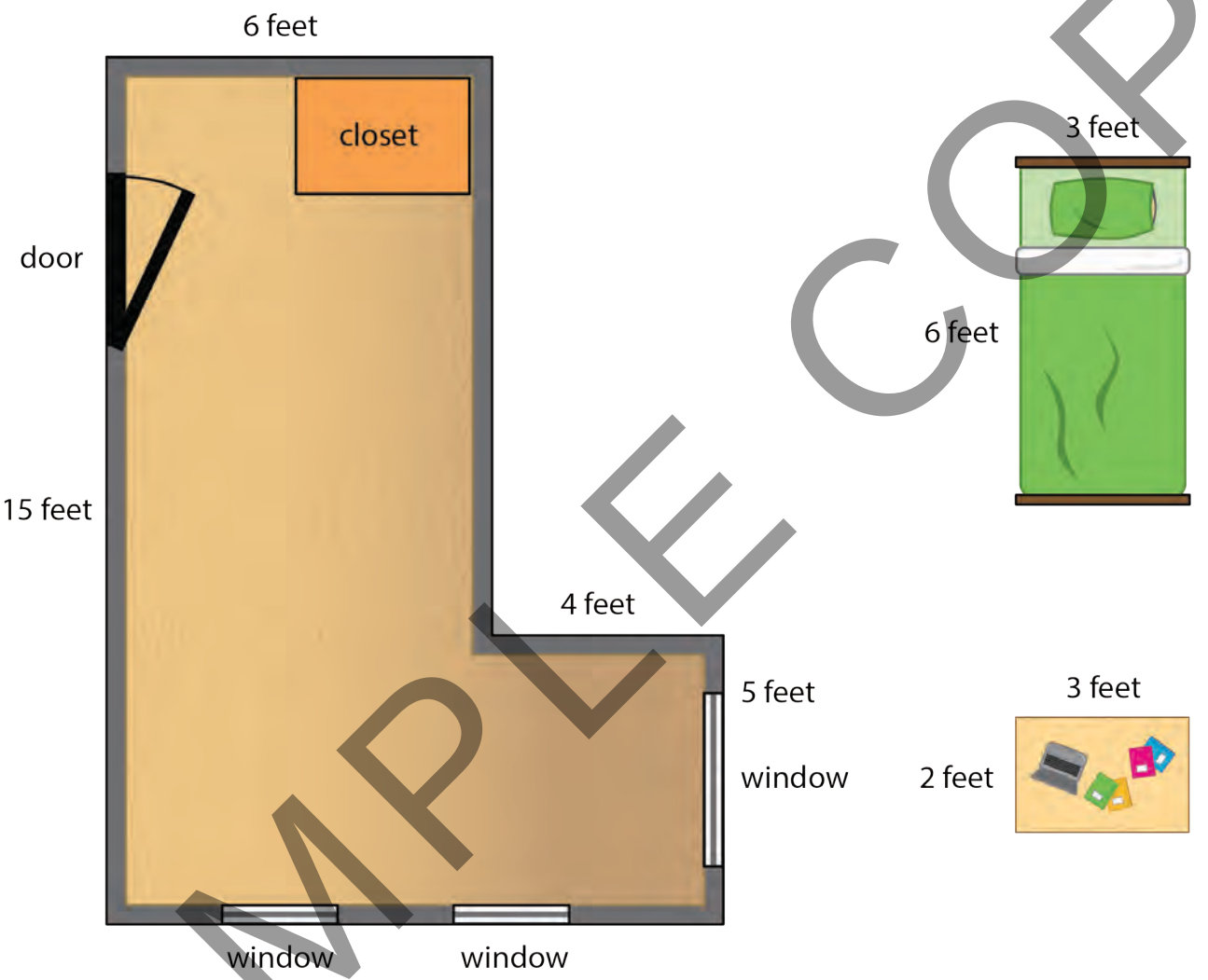
Activity 2

New Bed and Desk

This is a diagram of a bedroom, a desk, and a bed.



1. How would you arrange the bed and desk in the room?

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2. What is the area of the floorspace in the room that is not covered by furniture?

Create a poster to show your thinking. Organize it so it can be followed by others.

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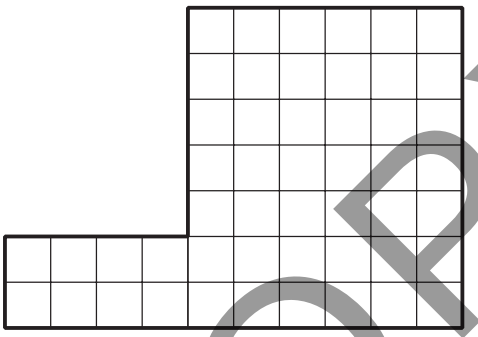
Practice Problems

5 Problems

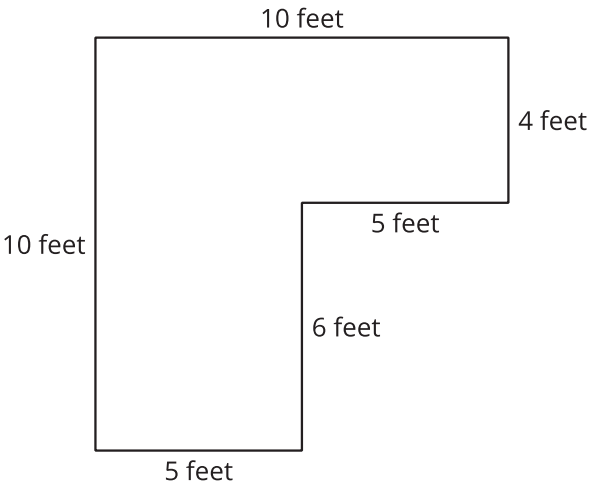
1 from Unit 2, Lesson 12

What is the area of this figure in square units? Explain or show your reasoning.



2 from Unit 2, Lesson 13

Find the area of this figure. Explain or show your reasoning.



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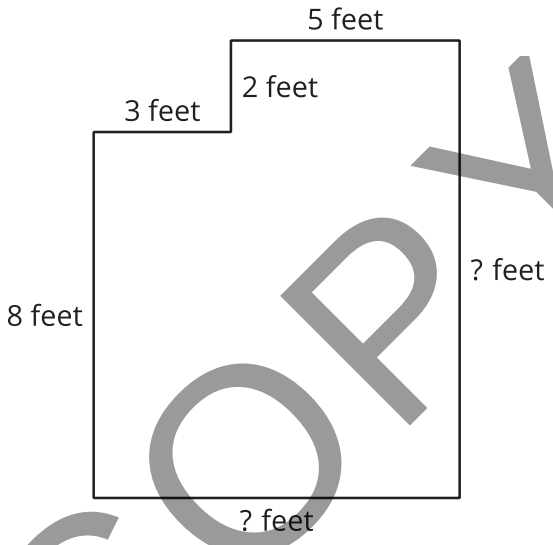
3 from Unit 2, Lesson 14

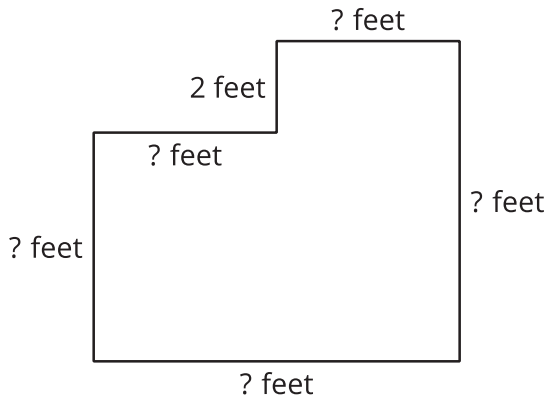
Find the area of this figure.
Explain or show your reasoning.



4 Exploration

Lin says that she knows how to find the area of this figure. Diego says there is not enough information to find the area.

Do you agree with Lin or with Diego? Explain your reasoning.



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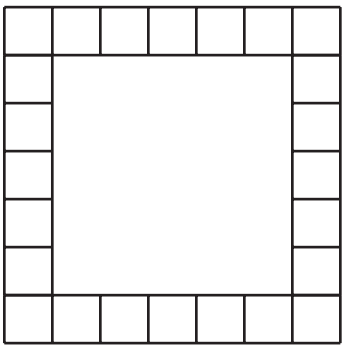
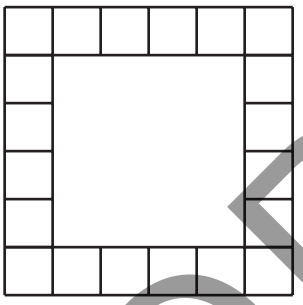
a. Each image shows part of a shape filled with squares.





For each image, which do you think is greater, the number of squares in the image or the number of squares still needed to fill the middle?

b. Check whether or not your answers are correct.

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Glossary

- area
  The number of square units that cover a flat figure without gaps or overlaps.

- array
  An arrangement of objects in rows and columns. All the rows have the same number of objects. All the columns have the same number of objects.

- bar graph
  A way to show data using the height or length of rectangles to represent how many in each group or category.

- equation
  A statement that includes an equal sign (=). It tells us that what is on 1 side of the equal sign is equal to what is on the other side.

- expression
  A statement that has at least 2 numbers and at least 1 math operation (such as addition, subtraction, multiplication and division).

- factor
  A whole number that is multiplied by at least 1 other whole number to get a product.

- key
  The part of a picture graph that tells what each picture or symbol represents.

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- multiplication
  An operation for finding the total number of objects when we have a certain number of equal groups.

- parentheses
  Grouping symbols that can be used in expressions or equations, such as: $(3 \times 5) + (2 \times 5)$, $(24 \div 2) + 5 = 17$.

- picture graph
  A way to show data using pictures or symbols to represent how many in each group or category.

- product
  The result of multiplying some numbers.

- scale
  A number line on one side of a bar graph that tells what data value each rectangle represents.

- scaled bar graph
  A bar graph marked in multiples of some number other than 1.

- scaled picture graph
  A picture graph where each picture represents an amount other than 1.

- square centimeter
  A square with side lengths of 1 centimeter.

- square foot

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A square with side lengths of 1 foot.

- square inch
  A square with side lengths of 1 inch.

- square meter
  A square with side lengths of 1 meter.

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California Common Core State Standards for Mathematics (CA CCSSM) Reference

3.G: Grade 3 – Geomery

Reason with shapes and their attributes.

3.G.1
Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

3.MD: Grade 3 – Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1
Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2
Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).Excludes compound units such as cm^3 and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.Excludes multiplicative comparison problems (problems involving notions of "times as much"); see Glossary, Table 2.

Represent and interpret data.

3.MD.3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

3.MD.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units--whole numbers, halves, or quarters.

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Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5
Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.5a
A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

3.MD.5b
A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7
Relate area to the operations of multiplication and addition.

3.MD.7a
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

3.MD.7b
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

3.MD.7c
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

3.MD.7d
Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8
Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

3.NBT: Grade 3 – Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.1
Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

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3.NBT.3
Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

3.NF: Grade 3 – Numbers and Operations—Fractions

Develop understanding of fractions as numbers.

3.NF.1
Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.2a
Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

3.NF.2b
Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

3.NF.3a
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

3.NF.3b
Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

3.NF.3c
Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

3.NF.3d
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

3.OA: Grade 3 – Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1
Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.

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3.OA.2
Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

3.OA.3
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.See Glossary, Table 2.

3.OA.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5
Apply properties of operations as strategies to multiply and divide.Students need not use formal terms for these properties. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

3.OA.6
Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

3.OA.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3.OA.9
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

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California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

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MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

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MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

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