IMKH California

GRADE 3

Student Edition

UNITS 5-6

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 3

Certified by Illustrative Mathematics®

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GRADE 3

UNIT

Fractions as Numbers

Content Connections

In this unit you will use different diagrams to represent, reason and compare fractions and relate them to whole numbers. You will make connections by:

- **Reasoning with Data** while using diagrams to represent and compare fractions.
- **Taking Wholes Apart, Putting Parts Together** while using fraction strips and tape diagrams to identify 1 whole and the fractions or parts that make up that 1 whole.
- **Discovering Shape and Space** while comparing unit fractions using different visual models and area models.
- **Exploring Changing Quantities** while comparing fractions that have the same denominator and those with the same numerator.

Addressing the Standards

As you work your way through **Unit 5 Fractions as Numbers,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	
MP2 Reason abstractly and quantitatively.	Lessons 4, 12, 15, and 17
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 6 and 11
MP4 Model with mathematics.	Lesson 18
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 1, 3, 5, 7, 9, 10, and 14
MP7 Look for and make use of structure.	Lessons 1, 3, 5, 8, 11, 13, and 16
MP8 Look for and express regularity in repeated reasoning.	Lessons 2, 8, and 13

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fractions of Shape and Time • Analyze Quadrilaterals	**3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	Lessons 1 and 2
• Fractions of Shape and Time • Square Tiles • Fractions as Relationships	**3.NF.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.	Lessons 1, 2, 3, and 4

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fractions of Shape and Time • Unit Fraction Models	**3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	Lessons 5, 6, 8, 9, 17, and 18
• Fractions of Shape and Time • Unit Fraction Models	**3.NF.2b** Understand a fraction as a number on the number line; represent fractions on a number line diagram. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	Lesson 7
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lessons 14 and 17

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3a** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	Lessons 10, 11, and 12
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3b** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.	Lessons 10, 11, 12
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3c** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	Lessons 8 and 13
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3d** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lessons 15 and 16

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations	**3.OA.5** Apply properties of operations as strategies to multiply and divide. 2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Lessons 4 and 11
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Lessons 4, 8, and 11

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Addressing CA CCSSM 3.G.2, 3.NF.1; building on 2.G.3; building towards 3.G.2; practicing MP6 and MP7

M M

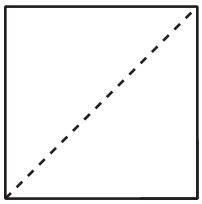
u D D D DD R

• Q

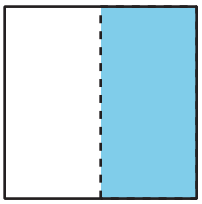
M M M dM M M

• DVD D h

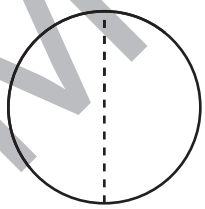
v



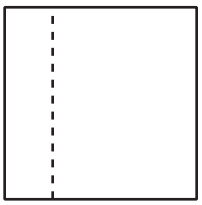
w



x



y



j DY

ff M M

o D D D D D D D D D DWPZPXPD DaD D PDn
D D D D D D D RDx D D D D
D D D D R

“ DYPDu DV

Addressing CA CCSSM 3.G.2, 3.NF.1; building on 2.G.3; building towards 3.NF.1; practicing MP8

M M Mf

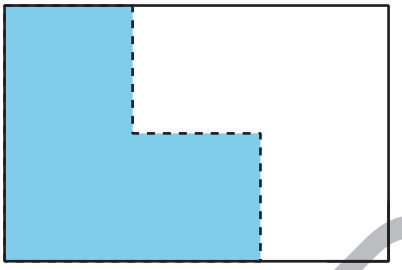
u D D D D D R

• Q

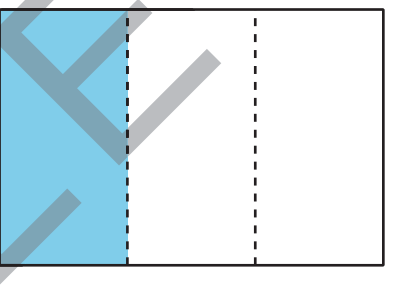
M M M dM M

• DVD D h

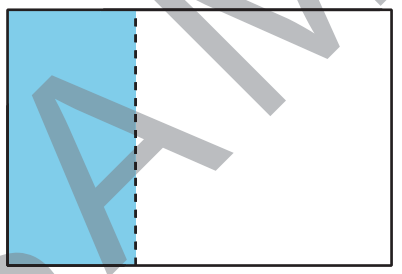
v



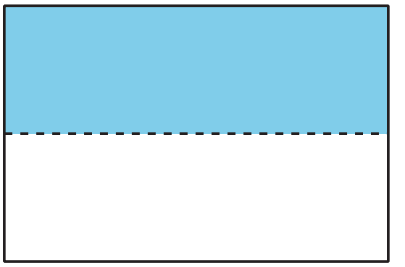
w



x



y



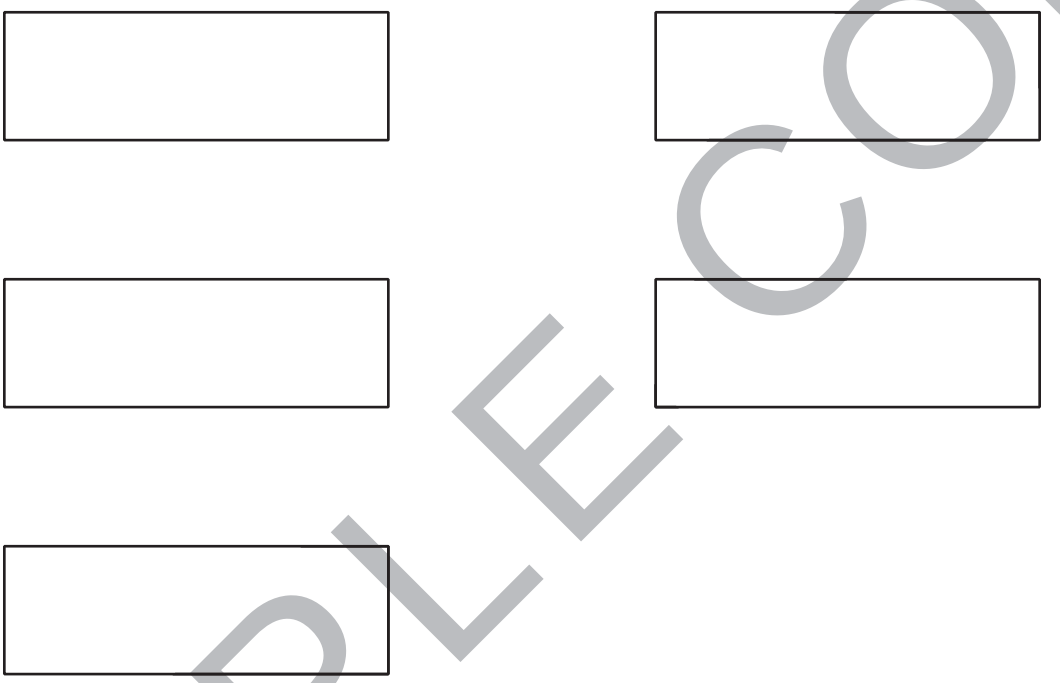
j DJ

M M

ff D D D D D D D D RDm D D DJR

o D D D D D D D D DJD D D D RD DJ
D D D R

When you finish folding, trace your folding lines with a pencil, and then label
D D D D D R



VR Ry D D D DVPDWPXPZPD DaD Q D Re DJ
D D R

R' D D DD RD D D D D UP
D D D D D hDm D D R

SAMPLE COPY

“ DYPDu DW

Addressing CA CCSSM 3.NF.1; practicing MP6 and MP7

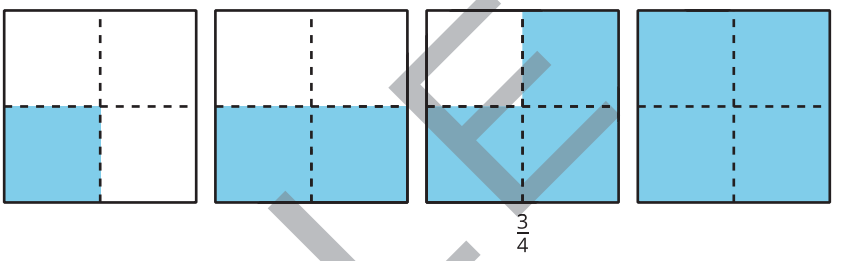
a Mf

u D D D Q D R

• Q

M M dM M M M

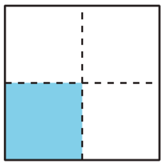
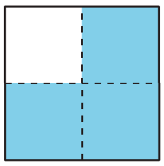
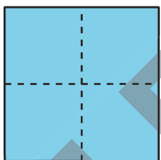
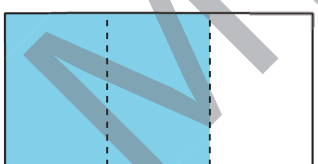
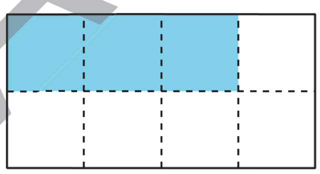
• D D D hD D D D h

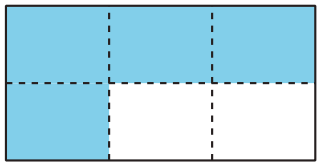
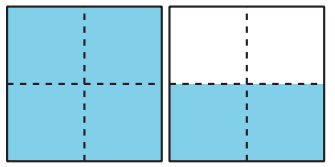
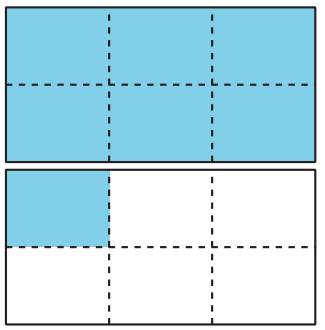


j DU

M M Mf

n D D D D D D D D DURO D D D D
D D D D D D RO D D D D
R

	number of shaded parts	size of each part	word name for the shaded parts	number name for the shaded parts
	1	$\frac{1}{4}$	one-fourth	$\frac{1}{4}$
				$\frac{3}{4}$
				
				
				

	number of shaded parts	size of each part	word name for the shaded parts	number name for the shaded parts
				
				
				

j DV

x M dff M

ff D D D D DD D D R

URv D D D DD RDk D D D D

D D D DD D R

VRl D D D D D R

WR“ D DCPDPPD DMD D DD D D D D

D D D D DD RDk D D D D D

D D D R

“ DYPDu DX

Addressing CA CCSSM 3.NF.1, 3.OA.5, 3.OA.7; building towards 3.NF.2; practicing MP2

w Mf M M

ff

u D D D D D D R

• Q

M dgM M Mf

o D D D D D D R

3×3

7×3

10×3

3×17

j DU

M Mf YW M a ff

The goal of the game is to be the first to build 3 secret fractions with unit R

URv DVD D D dDUD D D D DUD D D R
y D D D D R

VRn D c

- y DVD Q D D D D D PD D D D D D D D D D D D R
- y DYD Q D D D D D D D D D D R

WRy DPD D D c

- j D D DD D D DD Q D D D R
- ’ DUD D D Q D D D Q D D D D D D PDy D D D D D D D D R

XRy Dc

- rD D D D D Q D D D D D PD DD D PD D D D D DUPD D D D DUR
- rD D D D D D Q D D D D PD D D D D D DD D D D D D D Q D D DD D R

YR y]j dr D D D D D D DD D D
PD D D Q D D D D D D D
D D D Q D PD D D D D Q
D D R

ZR é D D D R

7. The first player to make 3 secret fractions wins.

r D D D D D Q D PD D D D D D D D D
D D D R

j DY

MVf M

q D DXD D D DY
lyrqQ Q MD DXD RDn D
D D D DD D D
D D R

© D D D DD RDx
D D D D R

UR j D D $\frac{4}{8}$ D D D D D D D DD R

VR j D D $\frac{2}{3}$ D D D D D D D DD R

WR j D D $\frac{3}{4}$ D D D D D D D DD R

XR j D D $\frac{5}{6}$ D D D D D D D DD R

YR′ D D D D D DD D R

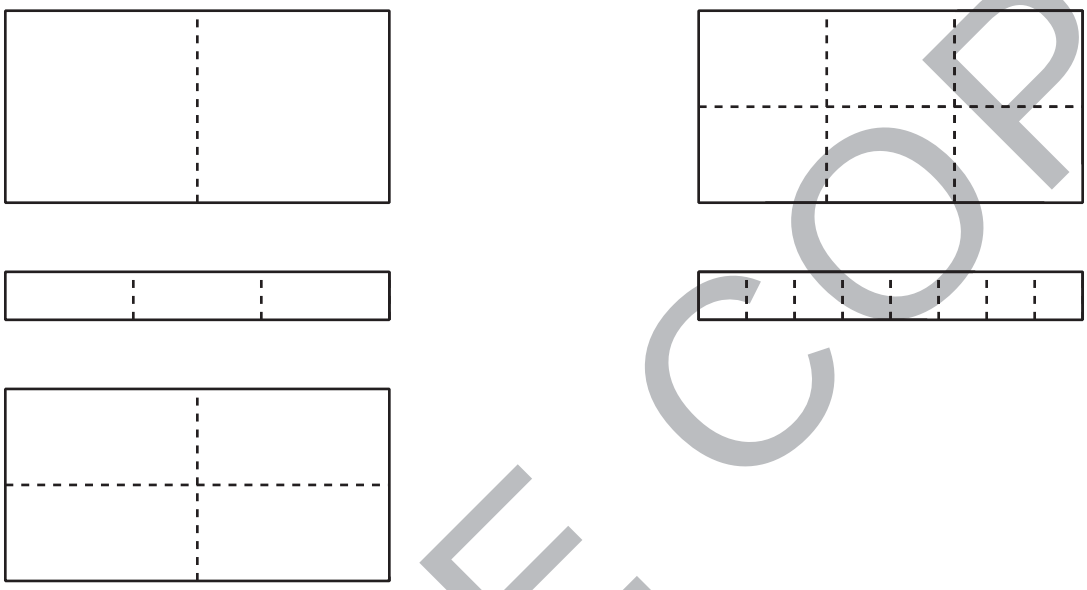
j D D D D D D D D D D D D D D

D h D D D D D R

e p

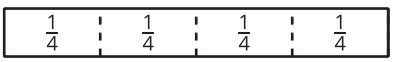
MM

• D D D D D D D PD PD PD PD
PD D D D D D D D D D D D D R



’ D D D D D D Q D D R

’ D $\frac{1}{4}$ D D Q D D



DUD D DXD D D DD
D D R

’ D $\frac{3}{4}$ D D Q D D

DVD D D D D Q D $\frac{1}{4}$
D R

o D D D D DUD D D D D D DD D D
RDn D D D c $\frac{1}{2}$P$\frac{1}{3}$P$\frac{1}{4}$P$\frac{1}{6}$P$\frac{1}{8}$R

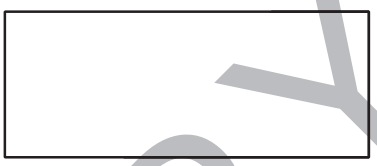
- D D D D D D D D D D D D D D
D D D D RD D D D D D D D D
D D D D D D D R

M

UTDy

1 y Q

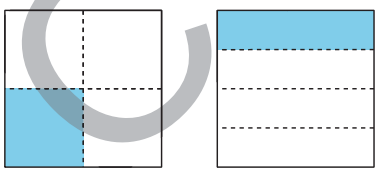
y D D D DUTD D R



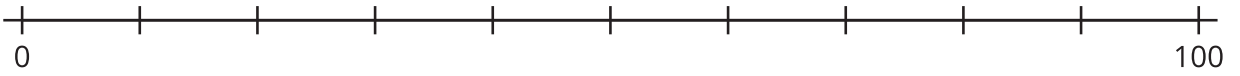
2 y Q

q D DVD Q D RD
D D D D D R

r D D D D D
D hDm D D
D R



3 y Q



Ru D D D D D D D R

Ru D D DXYD DZVD D D D R

4 y Q

• < > D D D D D D R

R 718 _____ 817

R 106 _____ 89

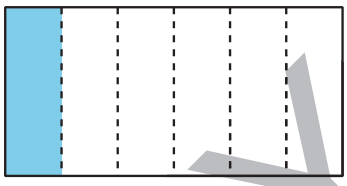
R806 _____ 809

5 D DYPDI DQ

y D D D DZD D R

6

R• D D D D D D D
h

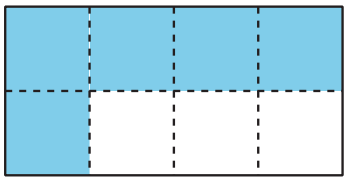


Ry D D D D D D R

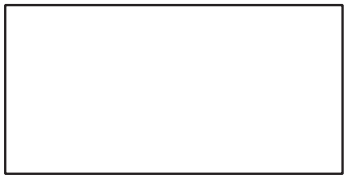
• D D D D D D
D D h

7

R• D D D D D D h
n D D D R



Ré $\frac{4}{6}$ D D R



8

s D D D D D DD D D $\frac{5}{6}$ D D D D

D D D D D D D D D D R

n D D R

9

n

• DD D D D D RDn D D D

D D R

10 n

u D D D D D D RD D D D D D
D hD D D D R

R

R

R

“ DYPDu DY

Addressing CA CCSSM 3.NF.2; practicing MP6 and MP7

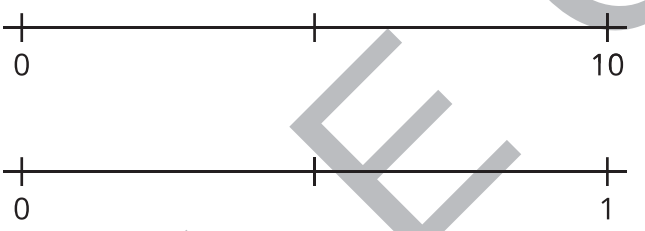
M M M

u D D D D D D D R

• Q

M M dM M M

• D D D hD D D D h



j DJ

x M dM M

ff D D D D D DD D D D D D D R D
D D D D DD D D D D D R D D D
D D D D D R

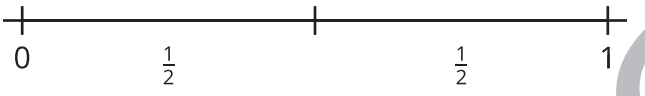
j DV

ff M M M M M

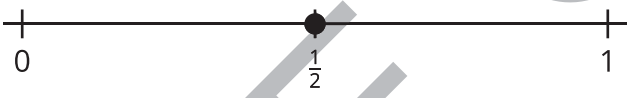
URj D D D D D D D D D D D D

R

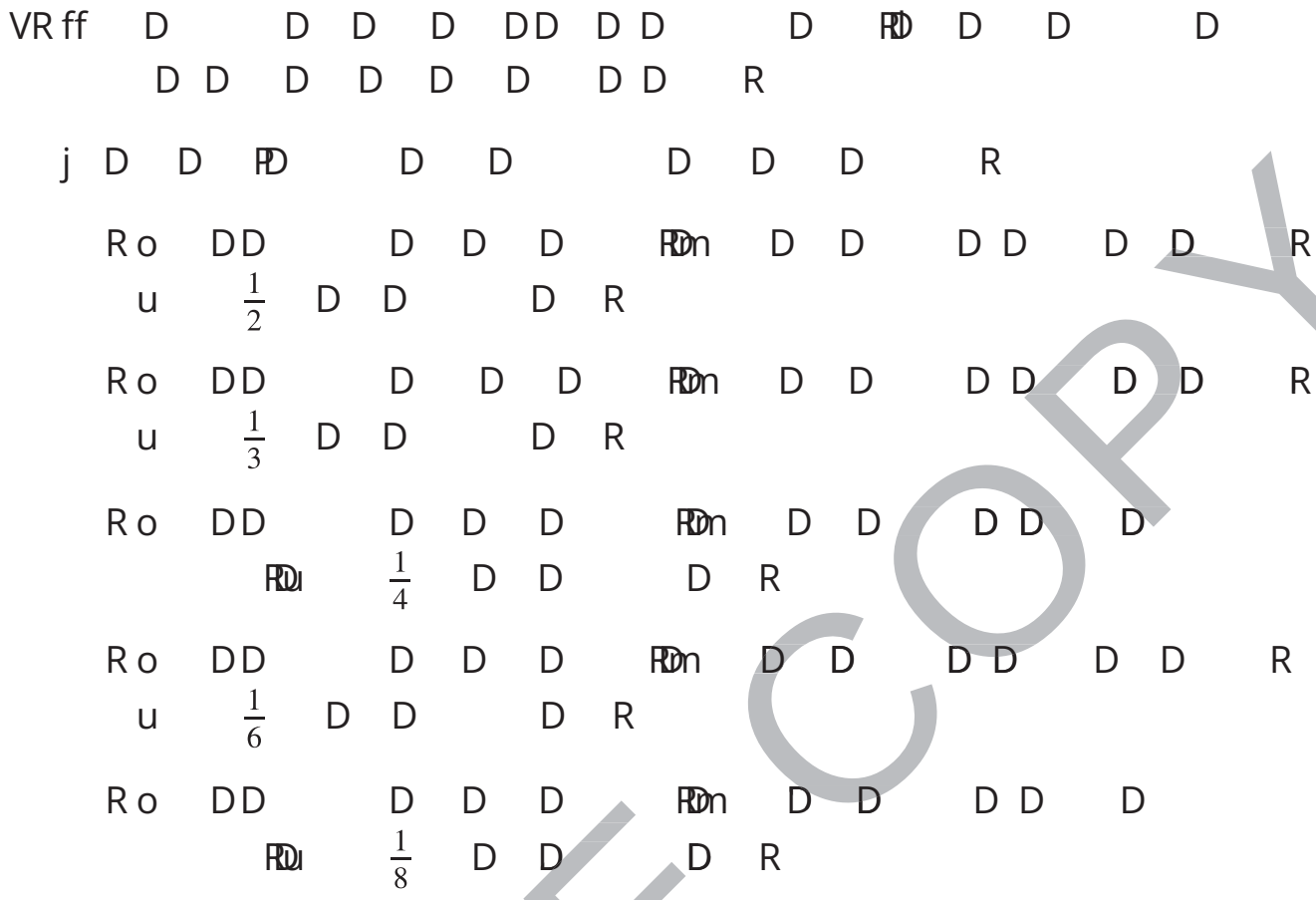
j D $\frac{1}{2}$ D D D D c



l D $\frac{1}{2}$ D D D D c



q D D D D D D h



“ DYPDu DZ

Addressing CA CCSSM 3.NF.2; practicing MP3

M Mff M M

M

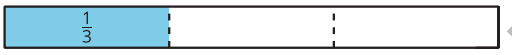
u D D D D D D D D R

• Q

M M M dff My

• DVD D h

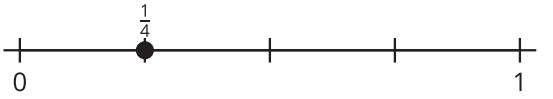
v



w

x

y

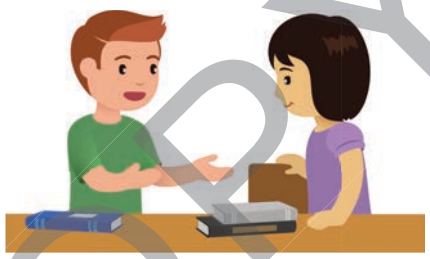


gh • p DV

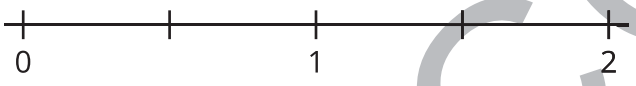
j DU

MMf

’ D D D DD D
RD D D D R



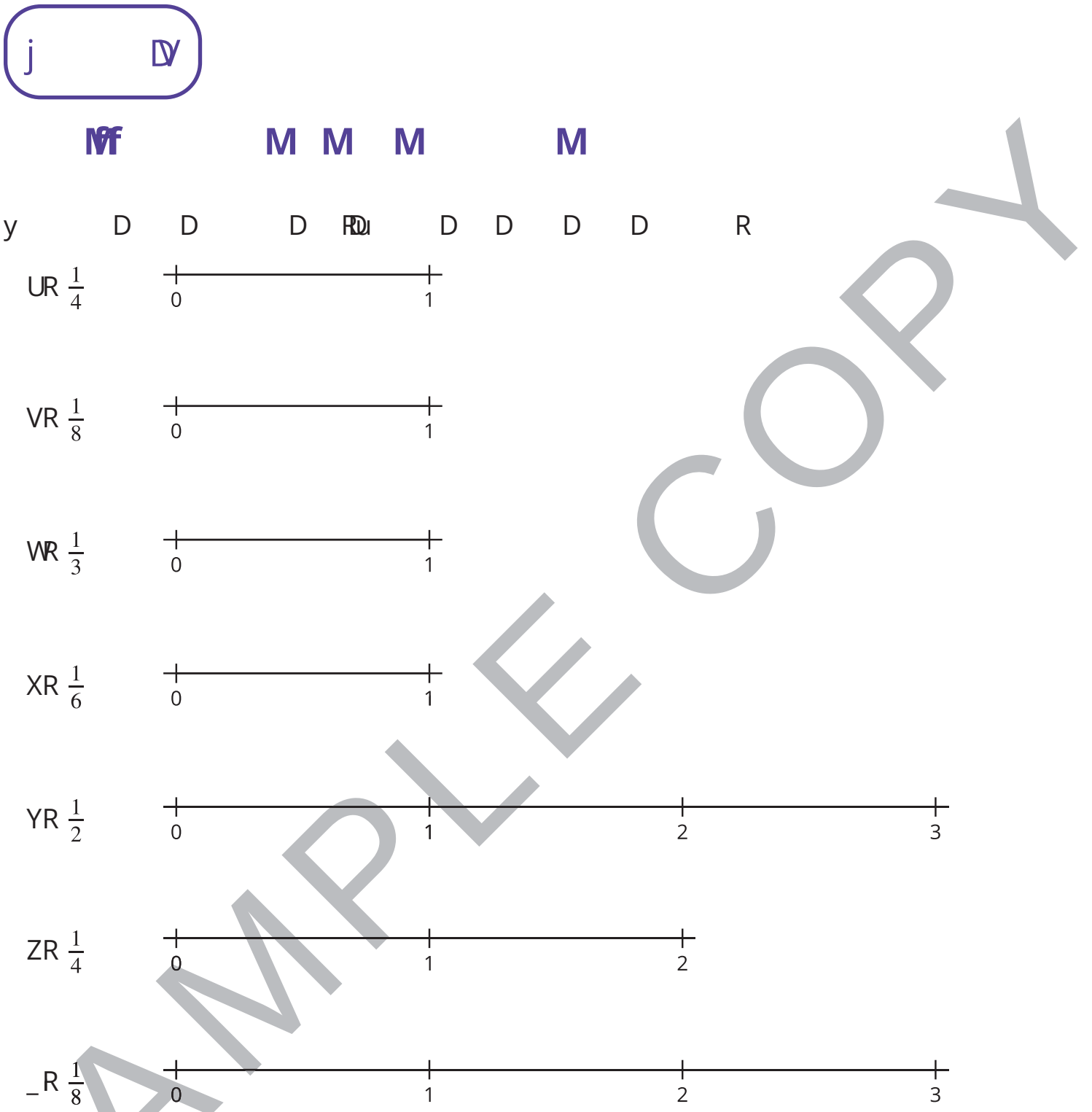
l D D c



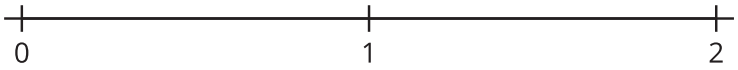
j D D c

m D D c

• D D D D D D D hDm D D R


UR $\frac{1}{4}$
0
1
VR $\frac{1}{8}$
0
1
WR $\frac{1}{3}$
0
1
XR $\frac{1}{6}$
0
1
YR $\frac{1}{2}$
0
1
2
3
ZR $\frac{1}{4}$
0
1
2
_R $\frac{1}{8}$
0
1
2
3

aR $\frac{1}{3}$



“ DYPDu D

Addressing CA CCSSM 3.NF.2b; building towards 3.NF.2; practicing MP6

a Mff M M M

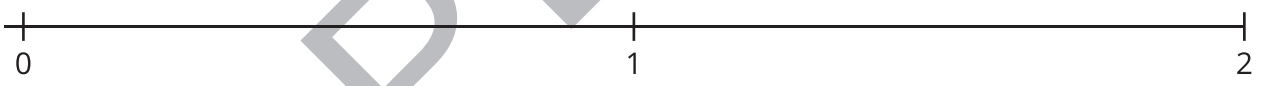
u D D Q D D D D D R

j DY

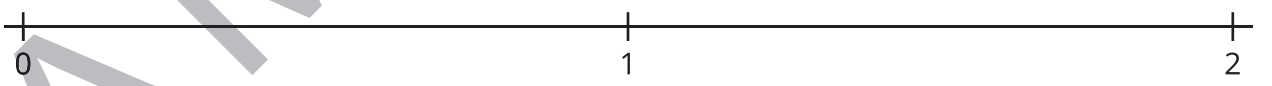
ff M M M M

UR u D D $\frac{3}{4}$ $\frac{6}{4}$ R

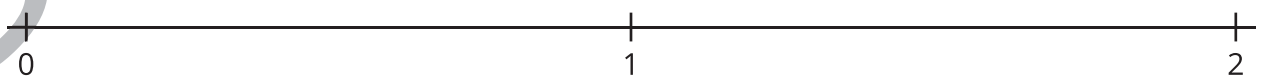
VR u D D $\frac{7}{8}$ $\frac{12}{8}$ R



WR u D D $\frac{2}{3}$ $\frac{4}{3}$ R



XR u D D $\frac{2}{6}$ $\frac{7}{6}$



gm p DW

YRq D D D D D D D D D D D

$\frac{7}{8}$ $\frac{12}{8}$ hDm D D R

ZR• D D D D D D D D D h

Addressing CA CCSSM 3.NF.2, 3.NF.3c, 3.OA.7; practicing MP7 and MP8

ff M M

u D D D D D D D D D D k

• Q

M dyM M MM

o D D D D D D R

12 ÷ 4

24 ÷ 4

60 ÷ 4

72 ÷ 4

j DJ

ff M M M M

URu D D D D D D D D D RDx

D D D D R

R $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \frac{7}{2}, \frac{8}{2}, \frac{9}{2}, \frac{10}{2}$

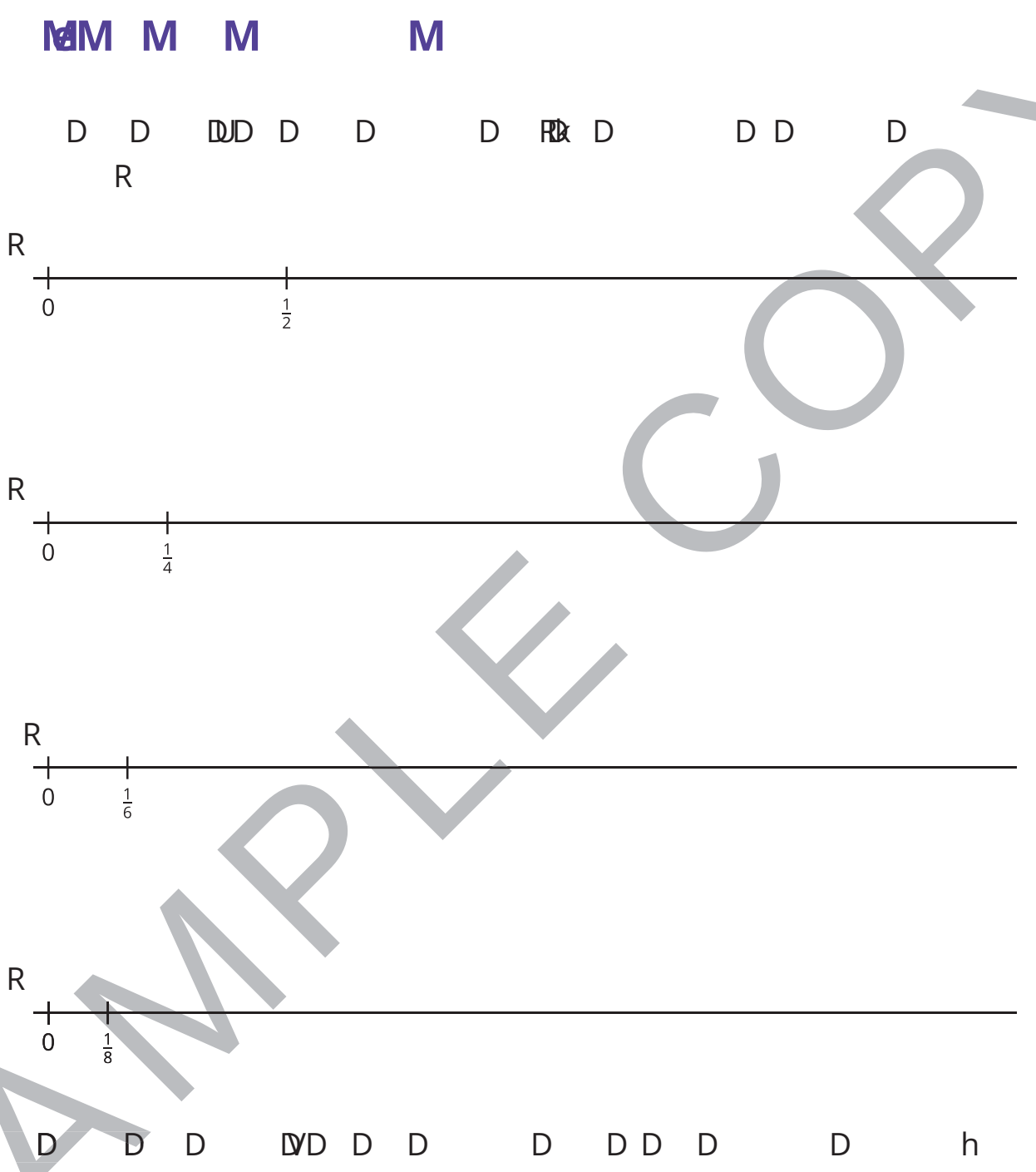
R $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}, \frac{10}{3}, \frac{11}{3}, \frac{12}{3}$

R $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, \frac{10}{4}, \frac{11}{4}, \frac{12}{4}$

VRu D D D D D D D D D D D DW

D D D D D R

WR • D D D D D D DWD D D h



Addressing CA CCSSM 3.NF.2; practicing MP6

v M M M M

M

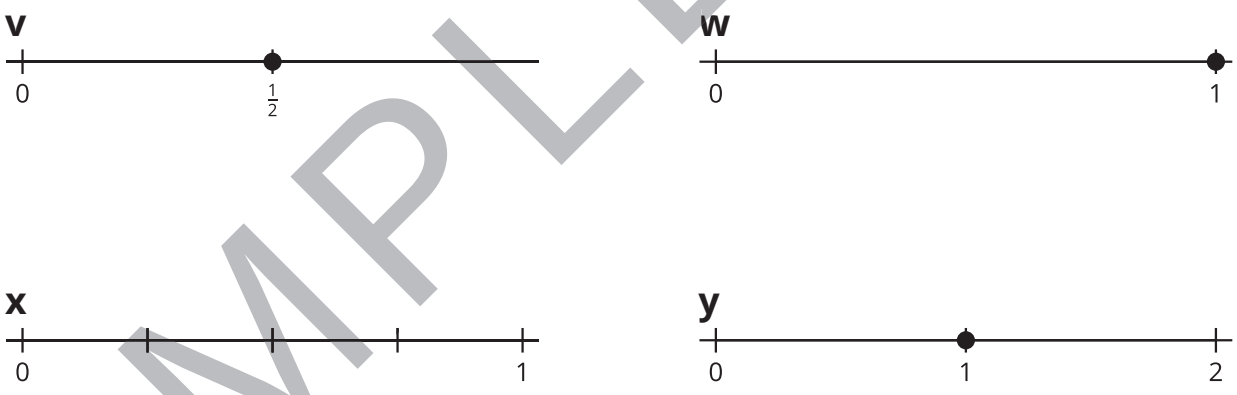
u D D D D D D D D D D D D D D DDJ

R

• Q

M M M dM M M

• DWD D h



j DJ

MdM

URu D D DJD D D D R

R

R

R

VR“ D D D D D D D D D D DJR

hk • p DV

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j DV

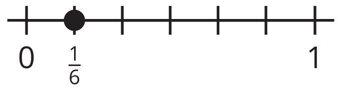
$\frac{3}{4}$

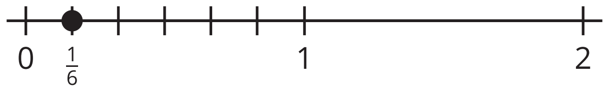
u D D $\frac{3}{4}$ D D D RDx D D D D

R

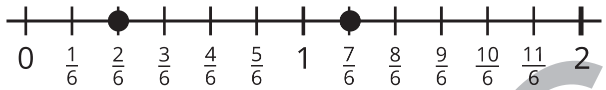
MM

- D D D D D D D D RD D D D
D D D D DTD DUD D D D R





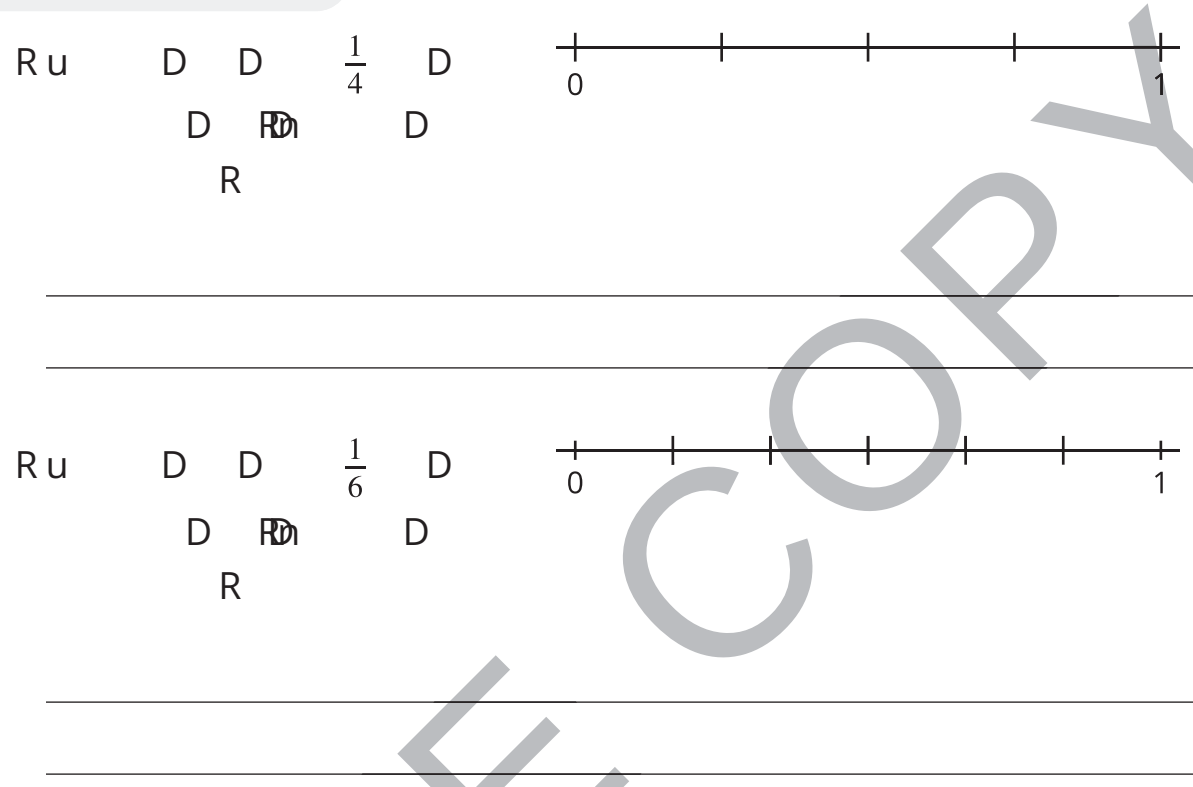
' D D D D D D D D D D D R



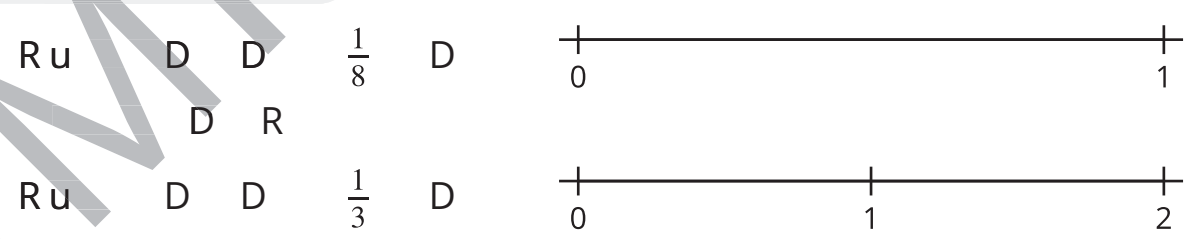
- D D D D D D D D D D D D
D D D D RDx D D D PD D D D $\frac{6}{6}$
D D D DUPD $\frac{12}{6}$ D D D D DVR

j D D D D D PD D D D D D D D
DUD D D D D D D D D D D D DD R

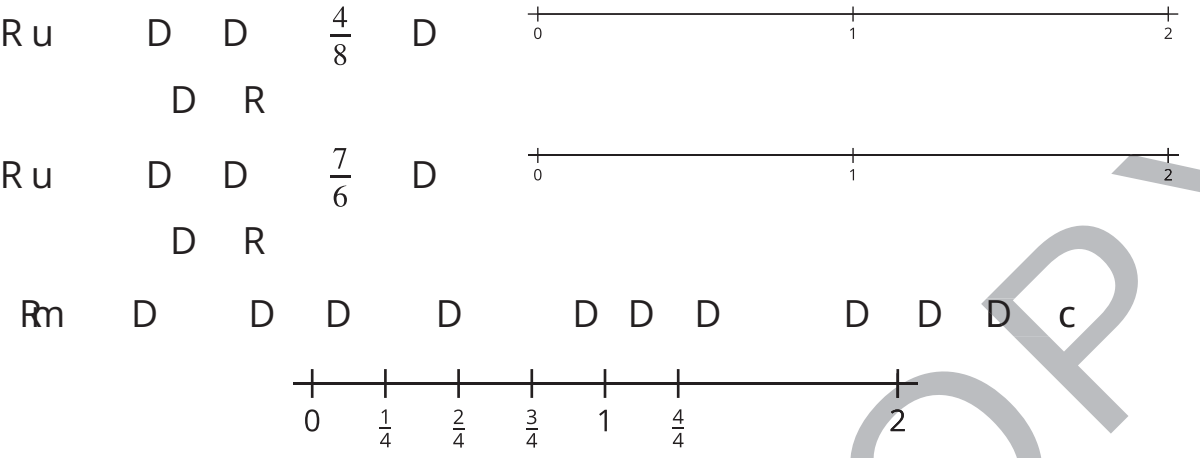
1



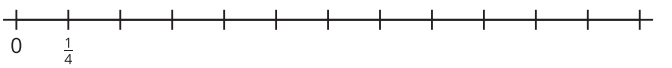
2



3



4

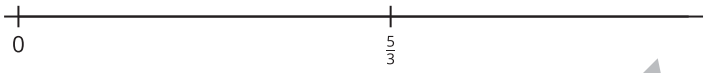


5

u D D DJD D

D R D

R



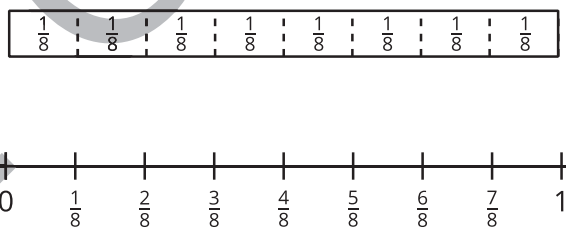
6

n

q D D D D D D

D D hDq D D

different?



7 n

Han says that he can find 1 on the number line, without finding $\frac{1}{8}$R

• D D Dq D h

Addressing CA CCSSM 3.NF.3a-b; building towards 3.NF.2; practicing MP6

’ Mf

u D D D R

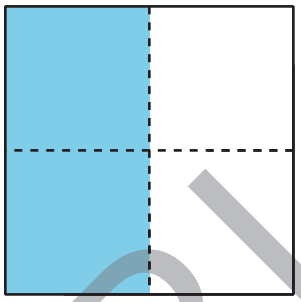
j DJ

’ M $\frac{1}{2}$

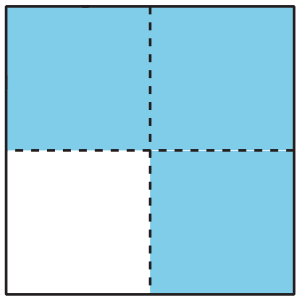
UR• D D D D $\frac{1}{2}$ D D hDk D D

D D R

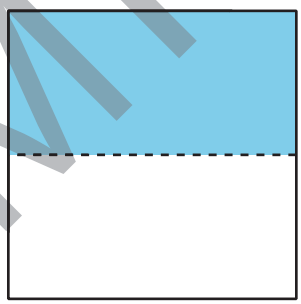
v



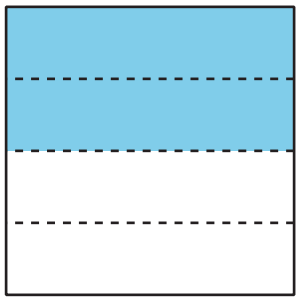
w



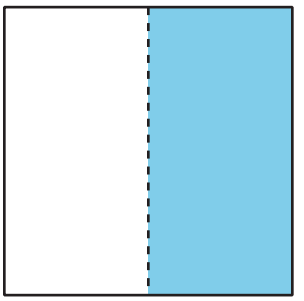
x



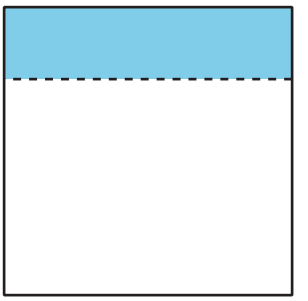
y



’



ff



VRq D D D D D DU D D DD D D $\frac{1}{2}$h

j DV

ff M Mf

Use your fraction strips from an earlier lesson to find as many fractions as
D D D D D c

UR $\frac{1}{2}$

VR $\frac{2}{3}$

WR $\frac{6}{6}$

XR $\frac{3}{4}$

k D D D D D D D D D R

“ DYPDU DDU

Addressing CA CCSSM 3.NF.3a-b, 3.OA.5, 3.OA.7; practicing MP3 and MP7

M

ff

u D D D R

• Q

M dM M Mn

o D D D D D D R

2×8

6×8

10×8

12×8

j DU

M

UR’ D D DUR

R• D D D D D D D D h

Rs D DD $\frac{4}{8}$RD DD D D P

m D D D Ds hD D PD D D D D D

D D Ds D D RD D PD D D D

R

VRn D D DDR

Ré D D D D D D D D D $\frac{1}{3}$

$\frac{2}{6}$R

Ré D D D D D D D D D $\frac{6}{8}$

$\frac{3}{4}$R

Ré D D D D D D D D $\frac{6}{6}$

$\frac{2}{2}$R

é D

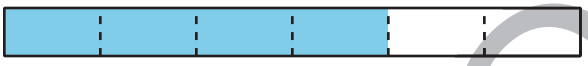
j DY

M M M

URn D D DURD DYD D D D D

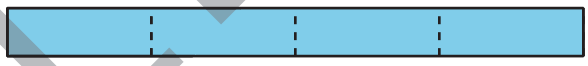
D D D R

R

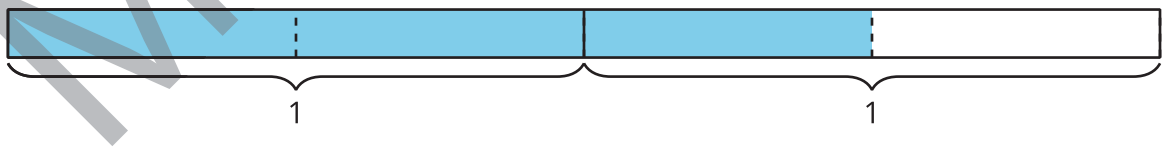


R

R



VRq D D R



R• D D D D D D D D D h

R• D D D DD R

kd • p DW

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Addressing CA CCSSM 3.NF.3a-b; building towards 3.NF.3; practicing MP2

Equivalent Fractions on a Number Line

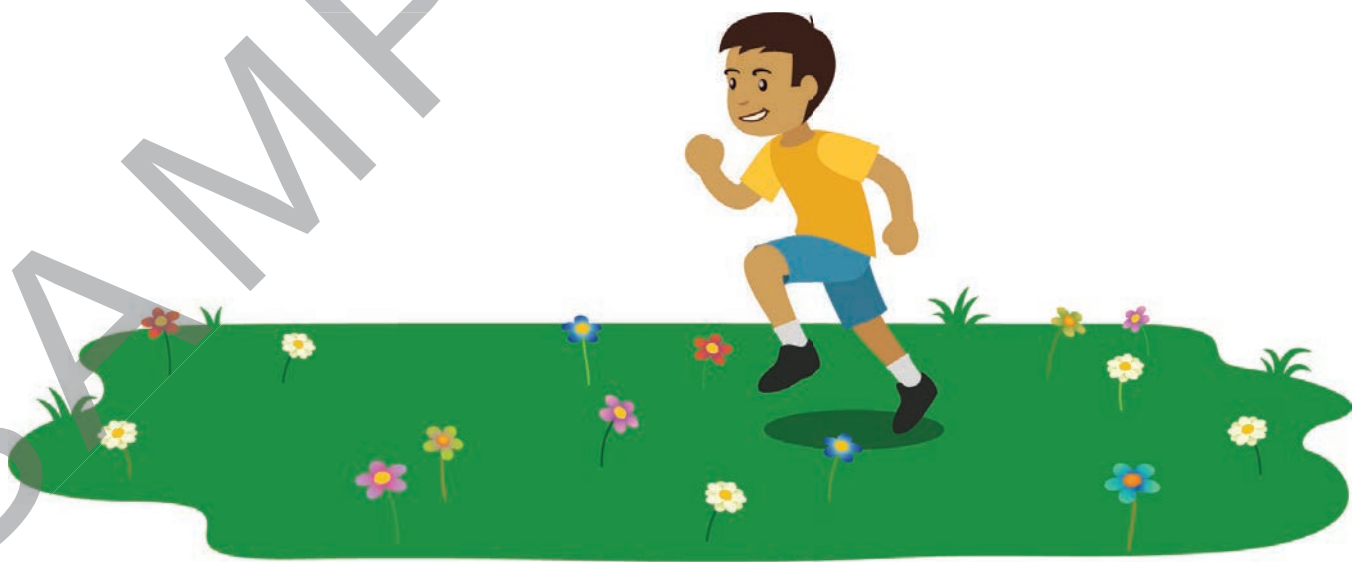
Let's find fractions at the same location.

Warm-up

Notice and Wonder: Running on a Trail

What do you notice? What do you wonder?

Tyler ran part of the length of a trail.
Han ran part of the length of the same trail.



j DJ

M MMM

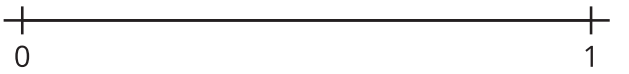
é D D D D D D DD RDn DD D D D
D D D R

ff D D D D DD D D D D R

URn D $\frac{3}{6}$ D D R

q D $\frac{1}{2}$ D D R

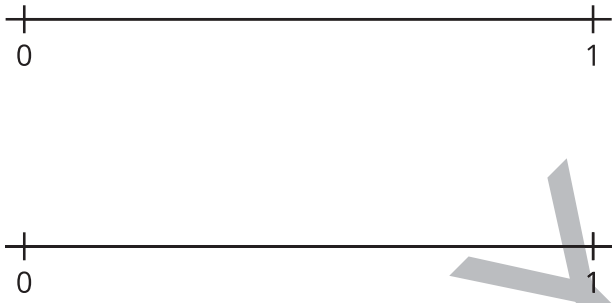
VRs D $\frac{1}{4}$ D D R



t D $\frac{2}{8}$ D D I

WRu D $\frac{2}{3}$ D D R

v D $\frac{5}{6}$ D D R

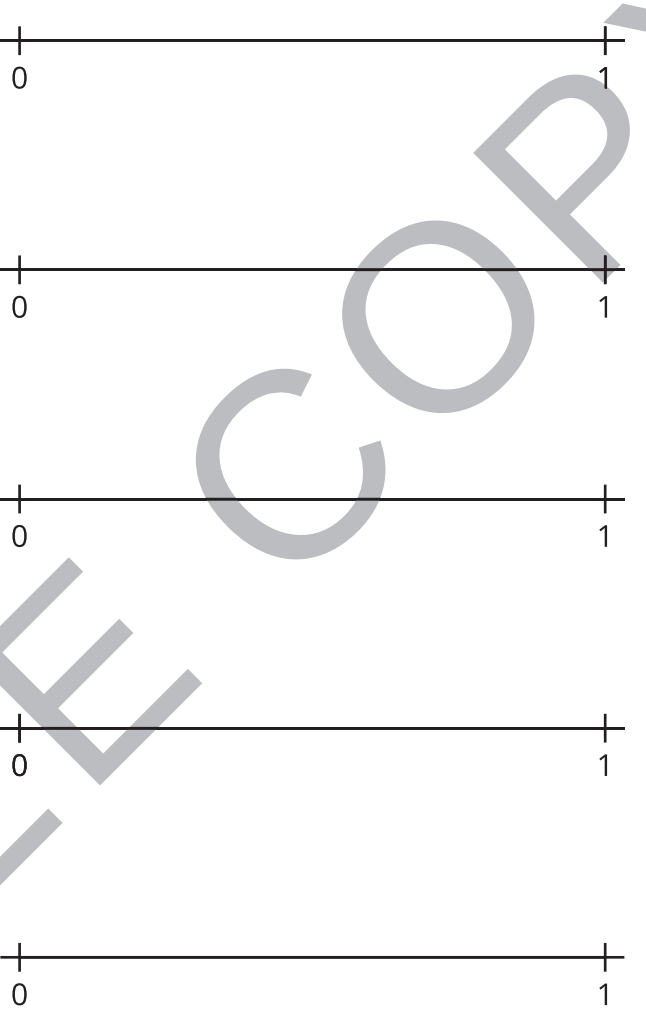


j DV

M M

UR u D D D
D D D
D R f D D
D DJD D D
D R

$\frac{1}{2}$ P $\frac{1}{3}$ P $\frac{1}{4}$ P $\frac{2}{3}$ P $\frac{2}{6}$ P $\frac{3}{8}$ P $\frac{3}{4}$ P $\frac{4}{6}$ P $\frac{4}{8}$ P $\frac{6}{8}$ P $\frac{7}{8}$



2. Use your labels to find 4 pairs of fractions that are equivalent. Write
D D D R

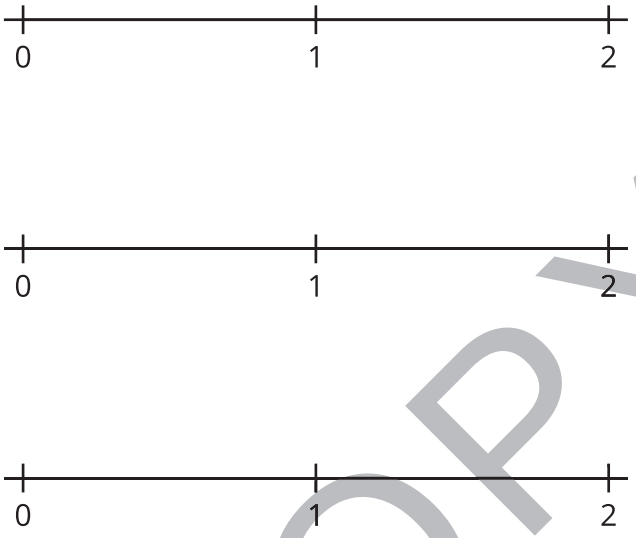
_____ = _____ _____ = _____ _____ = _____ _____ = _____

r D D D dD D

D D D D

D D D

D R



j DV

M M Mf

UR © DZD D RD D D D DY PD D D DD D D D
D D D R

VR l D D D D D D D D D D D D DD
that shows equivalent fractions? Work with your partner to find out.

WR r D D PD Q D D D D D D D RDf D D Q
D D D R

XR r D D D D D PD D D D D
D D D D D D D D RDf D DJD
D D D D D D D R

© DJc

$$\frac{\square}{\square} = \frac{\square}{\square}$$

é D D D D D
R

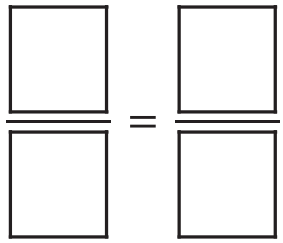
© DVc

$$\frac{\square}{\square} = \frac{\square}{\square}$$

é D D D D D
R

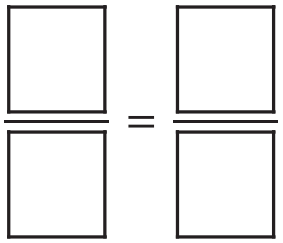
© DVc

© DXc



é D D D D D

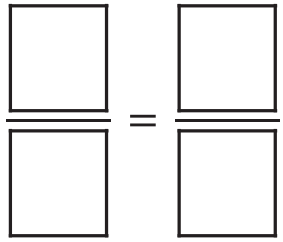
R



é D D D D D

R

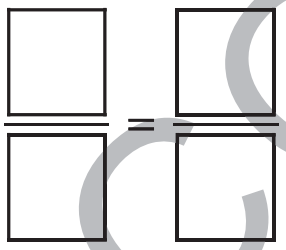
© DYc



é D D D D D

R

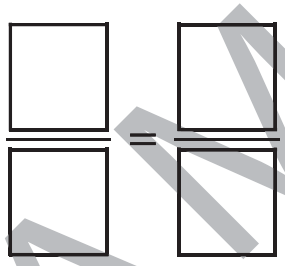
© DZc



é D D D D D

R

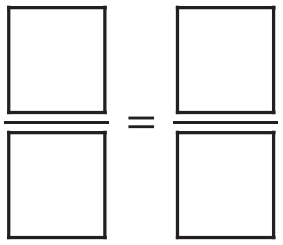
© D_c



é D D D D D

R

© Dac



é D D D D D

R

Addressing CA CCSSM 3.NF.3c; practicing MP7 and MP8

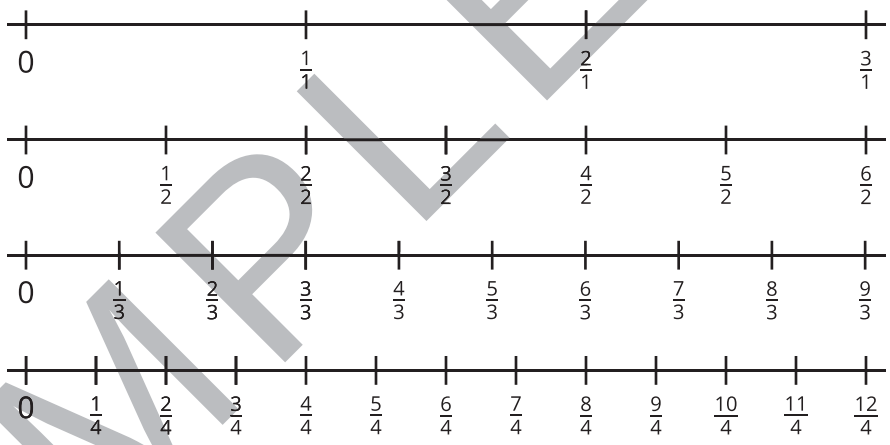
Whole Numbers and Fractions

Let's find fractions and whole numbers that are equivalent.

Warm-up

Notice and Wonder: Four Number Lines

What do you notice? What do you wonder?

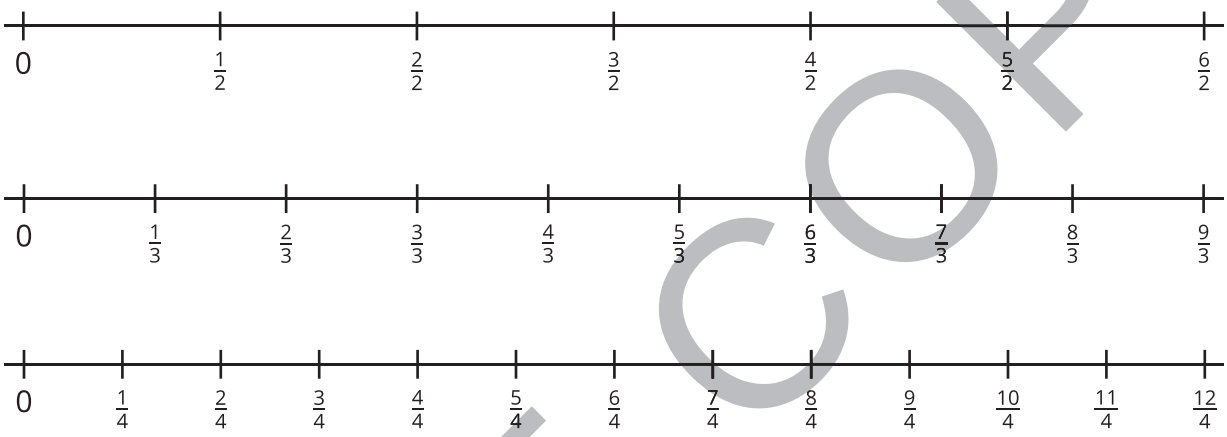


j DU

M M

URx D D D PD D D D D D D D D

RDn D D R



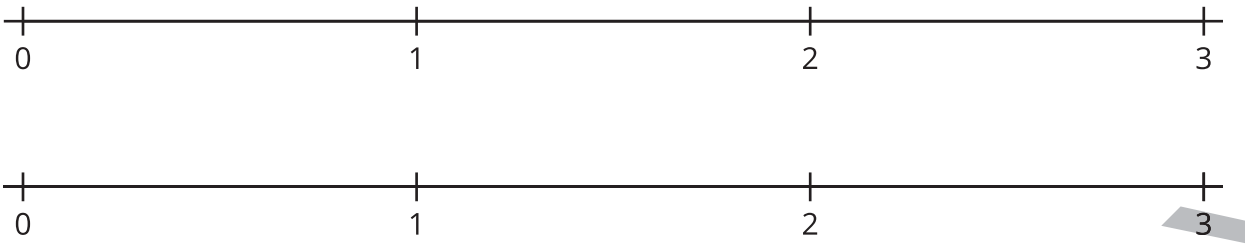
VR′ D $\frac{4}{2}$ D D D DV D D D D D

D PD D D D RD D D D D $\frac{4}{2} = 2$

D D DVD D D R

- DYD D D D D D D D D D

RD D D D DD D D R



m DD D DD D DD D R D

DD D D R

R $\frac{11}{2}$

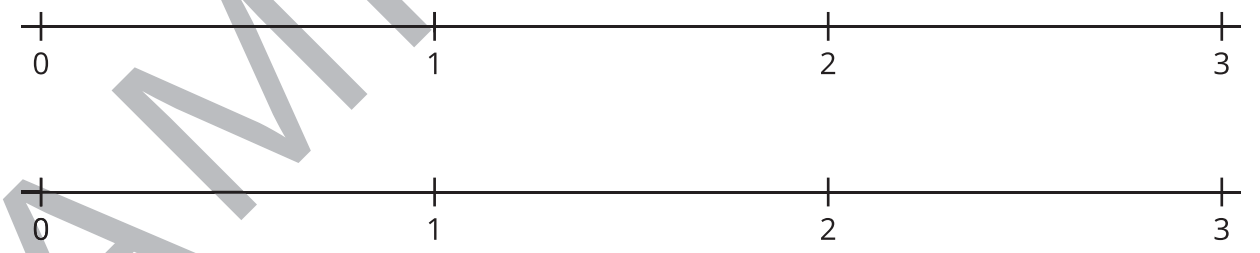
R $\frac{5}{1}$

R $\frac{12}{6}$

R $\frac{10}{3}$

R $\frac{12}{8}$

R $\frac{16}{4}$



j DY

M M Mf

- D D D D D D D RD D D PD D
  D D D D D D D D D D R

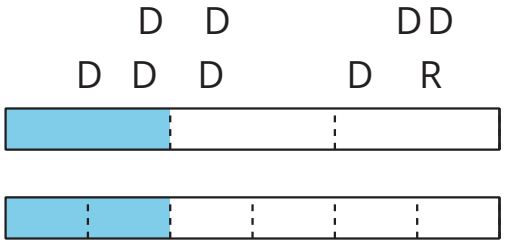
  é DdD DVD D D D D D D D DZ
  D D MR

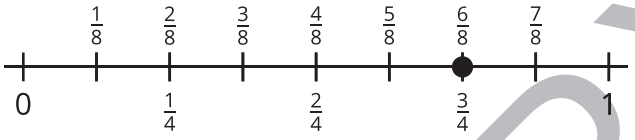
  é DVdy D D D D D RD D D D D
  PD DD D D D D D D DD D R

  © Dé DVD D D D D R

$\frac{4}{1}$		
	$\frac{\quad}{2}$	
		$\frac{\quad}{3}$
$\frac{\quad}{4}$		
	$\frac{30}{6}$	
		$\frac{48}{8}$

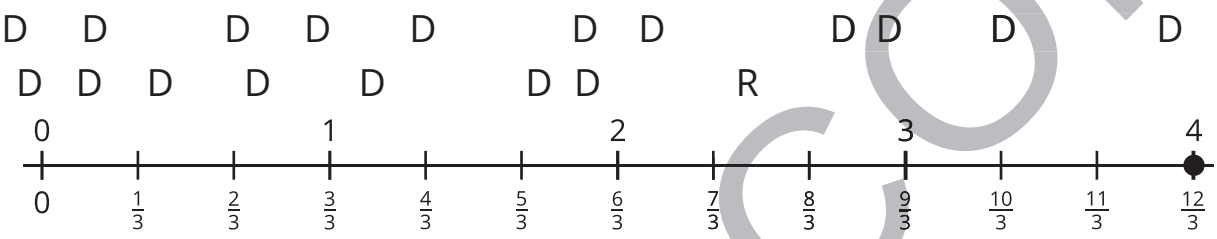
We learned that different fractions can be equivalent. We learned that





$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{6}{8} = \frac{3}{4}$$

- 

$$4 = \frac{12}{3}$$

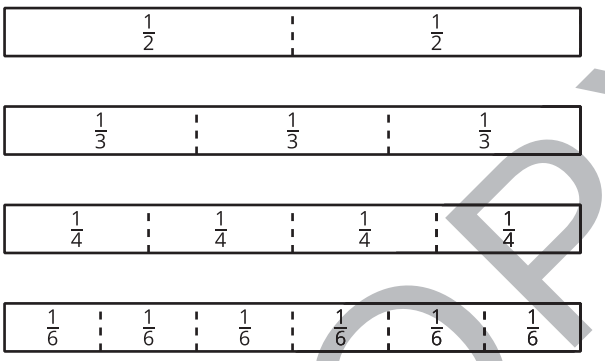
M

ZDy

1 D DYPDu DUT

é

R



j R $\frac{1}{2}$ D D $\frac{3}{6}$ R

k R $\frac{1}{2}$ D D $\frac{1}{3}$ R

l R $\frac{2}{2}$ D D $\frac{4}{4}$ R

mR $\frac{2}{2}$ D D $\frac{6}{6}$ R

nR $\frac{2}{3}$ D D $\frac{4}{6}$ R

oR $\frac{2}{3}$ D D $\frac{3}{4}$ R

2

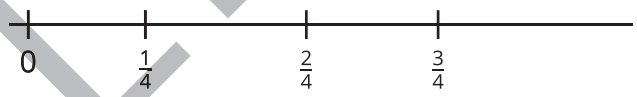
n D D DUR

• D D D D D D D D D D D D D D D D R

3

R′ D D D D D D D $\frac{3}{4}$ D D

$\frac{2}{3}$ RDn D D D D D R



Ro DD D D $\frac{2}{3}$R

Ro DD D D $\frac{3}{4}$R

4 D DYPDu DDW

a. Write 10 as a fraction in 2 different ways.

Rr $\frac{88}{8}$ D DD D hDn D D D

R

é D

5 n

m DD D D D D D Rn D

D D R

R $\frac{100}{2}$

R $\frac{100}{3}$

P$\frac{100}{4}$

$R\ \frac{100}{6}$

$R\ \frac{100}{8}$

é D

6 n

r D D D D DD D PD D D D D D D
hD D D D D DUTTD D hDn D
R

Unit 5, Lesson 14

Addressing CA CCSSM 3.NF.3; practicing MP6

How Do You Compare Fractions?

Let's represent and compare fractions.

Warm-up

Number Talk: Which Whole Numbers?

Find the whole number equivalent to each fraction.

- $\frac{16}{1}$
- $\frac{16}{2}$
- $\frac{16}{4}$
- $\frac{20}{4}$

j DU

’ M M t

j D D D hDé D D PD D PD Γ
D D R

UR $\frac{1}{2}$ $\frac{1}{3}$

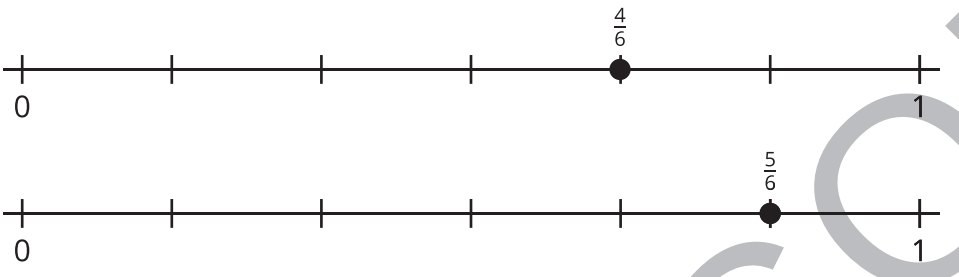
VR $\frac{4}{6}$ $\frac{5}{6}$

WR $\frac{3}{4}$ $\frac{6}{8}$

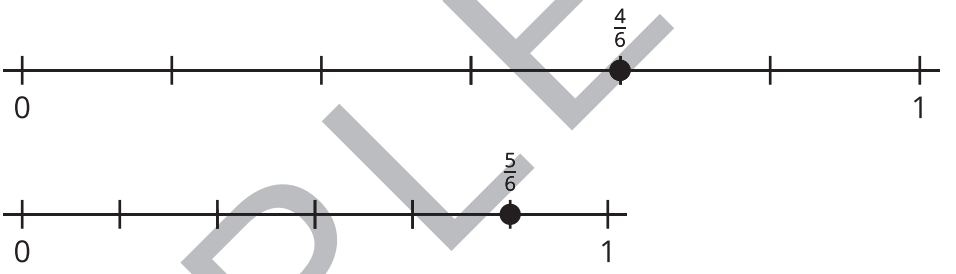
j DV

Same Fractions, Different Results?

q D $\frac{4}{6}$ D D $\frac{5}{6}$ RQq D D D R



u D $\frac{4}{6}$ D D $\frac{5}{6}$ RQq D D D R



Why might Han and Lin make different comparison statements for the same h

Addressing CA CCSSM 3.NF.3d; building towards 3.NF.3d; practicing MP2

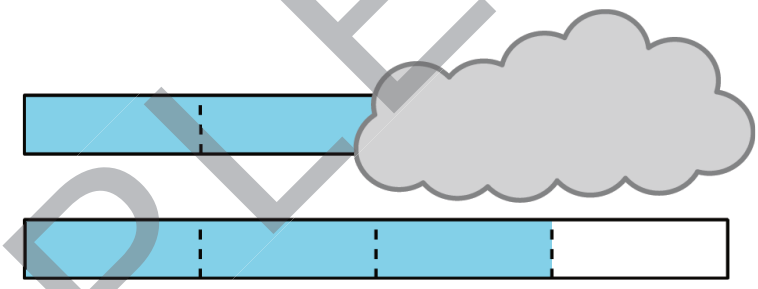
Compare Fractions with the Same Denominator

Let's compare 2 fractions with the same denominator.

Warm-up

Notice and Wonder: Two More Strips

What do you notice? What do you wonder?



j DJ

x Mf M M M My

UR o D D D D PD D D D D D RDn D
D D R

R $\frac{1}{2}$ $\frac{3}{2}$

R $\frac{3}{8}$ $\frac{2}{8}$

é Dn

VR “ D D gD DeD D D D D RDn D D
D R

R $\frac{1}{6}$ $\frac{4}{6}$

R $\frac{4}{4}$ $\frac{5}{4}$

R $\frac{2}{3}$ _____ $\frac{1}{3}$

R $\frac{4}{8}$ _____ $\frac{6}{8}$

r D D D c• DD D D D D D D D D D D D D D D R

UR $\frac{1}{2}$ < ___
2

VR $\frac{6}{4}$ > ___
4

WR $\frac{4}{3}$ < ___
3

XR $\frac{5}{8}$ > ___
8

j DV

M M dM MyM

r D D PD D D D D D D D RD DD
utensil in a color different than your partner's, so you can tell which fraction
D D D D D R

UR n D D D D RD D D D D
D Dy DUR

2. Player 1 chooses a denominator for the first round: 2, 3, 4, 6, or 8.

WR n D D D D D D D RrD D D
the same numerator, both players spin again until they are different.

XR n D D D D D D D D D D
D D R

YR k D D D D D Dg D De D D D D D
D D D R

ZR' D D D D D D D D D D
D D R

_R © D DUTD RD D D D D D D D
R

Unit 5, Lesson 16

Addressing CA CCSSM 3.NF.3d; practicing MP7

Compare Fractions with the Same Numerator

Let's compare 2 fractions with the same numerator.

Warm-up

True or False: Unit Fractions

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{2} > \frac{1}{4}$

- $\frac{1}{4} > \frac{1}{3}$

- $\frac{1}{6} > \frac{1}{8}$

j DJ

ff M M M

URy D D $\frac{5}{6}$ D D $\frac{5}{8}$R

’ D D $\frac{5}{8}$ D D $\frac{5}{6}$R

• D D D D hDe D D PD D D D
R

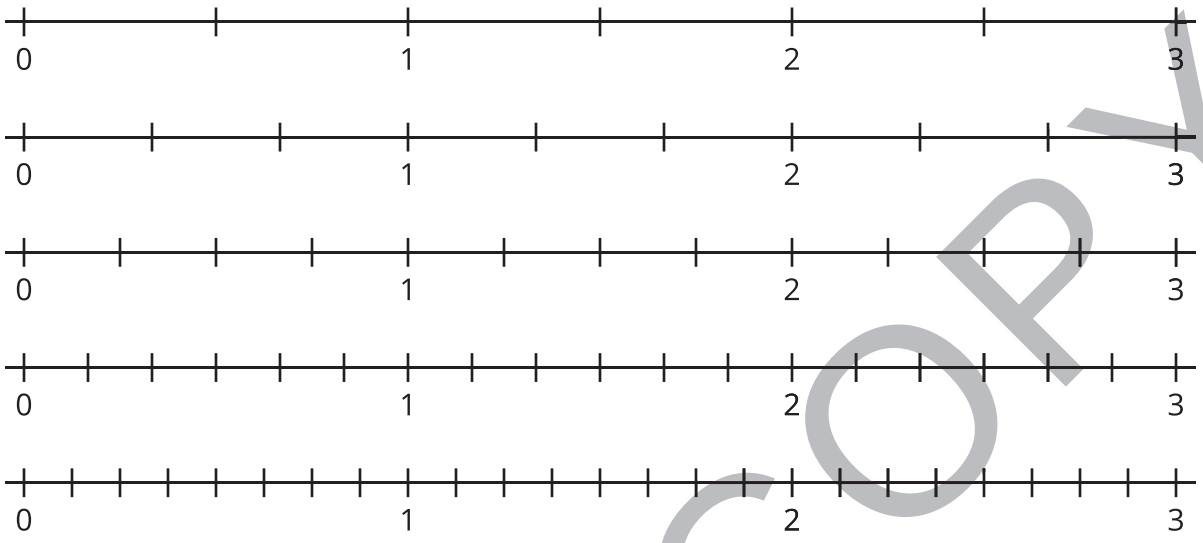
VRo D D D D PD D D D D D D h

R $\frac{5}{3}$ $\frac{5}{4}$

R $\frac{5}{8}$ $\frac{5}{2}$

R$\frac{5}{6}$ $\frac{5}{4}$

WRu D D D D D DD D c$\frac{5}{2}$P$\frac{5}{3}$P$\frac{5}{4}$P$\frac{5}{6}$P$\frac{5}{8}$R



• D D D D D D hDV DUD DV R

j DV

ff M M M M

UR o D D D D PD D D D D D D RDn D
D D R

R $\frac{1}{4}$ $\frac{1}{3}$

R $\frac{3}{4}$ $\frac{3}{8}$

R $\frac{5}{3}$ $\frac{5}{6}$

R $\frac{9}{8}$ $\frac{9}{6}$

VR “ D D gD DeD D D D D RDx D D
D D R

R $\frac{2}{2}$ ________ $\frac{2}{6}$

R $\frac{4}{3}$ ________ $\frac{4}{8}$

R $\frac{8}{8}$ ________ $\frac{8}{4}$

R $\frac{5}{4}$ ________ $\frac{5}{3}$

WR • DD D D D D D D D D D
D D RDx D D D D D R

R $\frac{1}{3} < \frac{1}{\_\_\_}$

R $\frac{6}{4} > \frac{6}{\_\_\_}$

R $\frac{4}{4} < \frac{4}{\_\_\_}$

R $\frac{2}{6} < \frac{2}{\_\_\_}$

Addressing CA CCSSM 3.NF.2-3; building on 3.NF.2; building towards 3.MD.4; practicing MP2

Compare Fractions

Let's compare more fractions in different situations.

Warm-up

Estimation Exploration: Ladybug Length

What is the length of this ladybug?

Record an estimate that is:

too low	about right	too high

j DJ

x M

o D D c

j D D D D D D D D k

© D D D DD D D D D gPePD DR

URj D D $\frac{2}{8}$ D D D DD RjD D $\frac{2}{3}$ D

D D D D RD D D D h

VRj D D DXD D RjD

D $\frac{12}{3}$ D RD

D h

4 centimeters

WRj D D $\frac{3}{8}$ D D D DD RQ D D $\frac{5}{8}$ D
D D D D RO D D D h

XRj D D $\frac{5}{8}$ D D D D D RQ D D
$\frac{5}{6}$ D D D D D D RO D D DD
h

j DV

Mf M M t

1. Oh, no! Some juice spilled on Noah's fractions. Help him figure out what
D D D D D D R

o D D D D D D D D D D D D Rm
D D D R

R

$\frac{2}{8} < \frac{\square}{8}$

R

$\frac{3}{6} = \frac{\square}{6}$

R

$\frac{4}{3} > \frac{4}{\square}$

VRo DD D D D D PDD D D D D PD D
D D D D D PD D D DD D
D D D gPePD D D D D D R

Rp D $\frac{4}{6}$cD é c

u D $\frac{4}{6}$cD é c

n D $\frac{4}{6}$cD é c

Rp D $\frac{3}{4}$cD é c

u D $\frac{3}{4}$cD é c

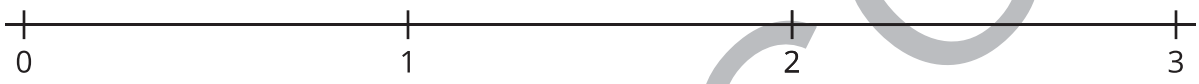
n D $\frac{3}{4}$cD é c

j DW

M M M

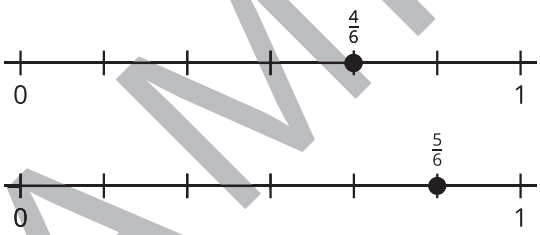
u D D D D D D D D R D D D D
R

$\frac{1}{2}, \frac{3}{8}, \frac{13}{8}, \frac{2}{4}, \frac{3}{4}, \frac{9}{8}, \frac{5}{4}, \frac{12}{6}, \frac{5}{2}, \frac{9}{3}, \frac{20}{8}$

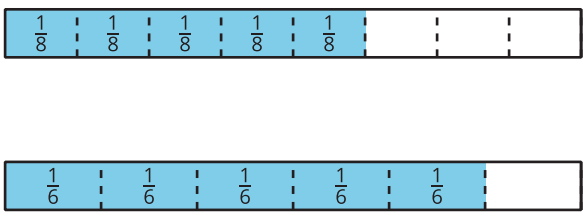


MyMM

• D D D D D D D D D D P
D D D >P<PD = D D D R• D D
D D D D D R



$\frac{4}{6} < \frac{5}{6}$



$\frac{5}{6} > \frac{5}{8}$

Addressing CA CCSSM 3.NF.2; building on 3.NF.2; practicing MP4

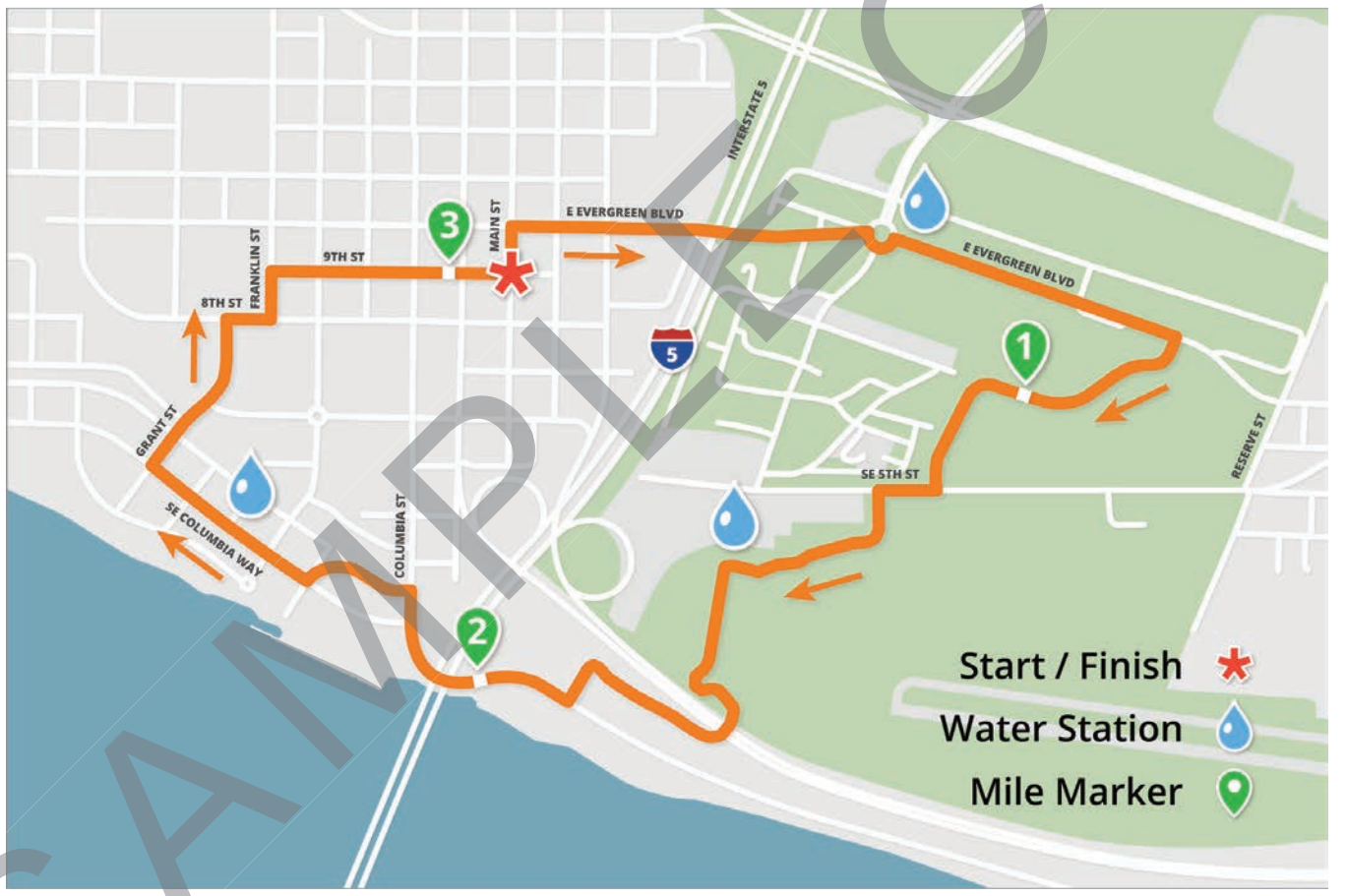
Plan a Fun Run

Let's use what we know about fractions to plan a 3-mile fun run.

Warm-up

Notice and Wonder: 3-Mile Run

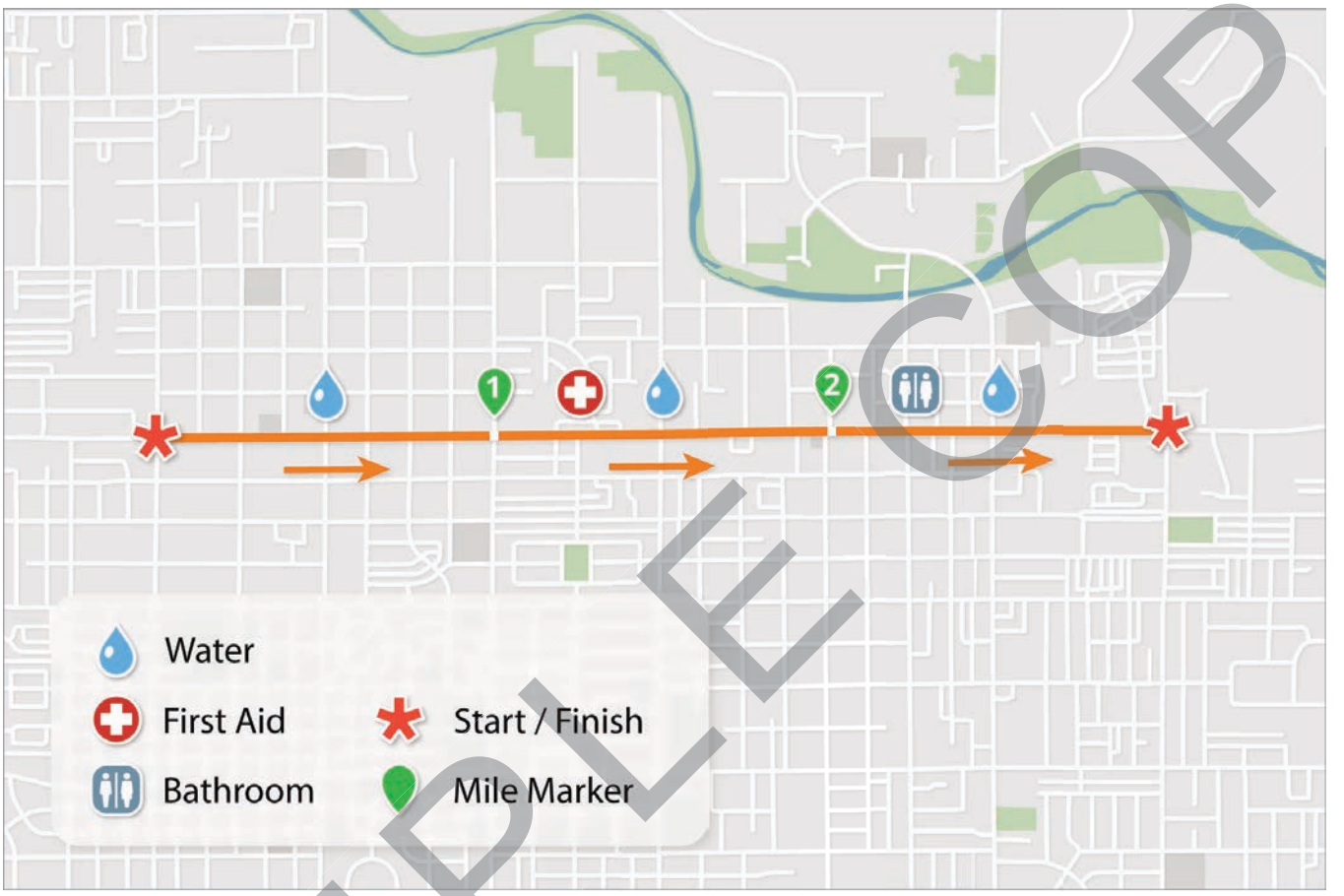
What do you notice? What do you wonder?



j DJ

TM Mv t

• D D D hD D D D h



URq D D D D D D D D h

R • D DUcD

R • D DVcD

R • D DWcD

R• D D D h

2. How far is the first-aid station from the starting line?

WR q D D D D D D D D h

XR ’ D D D D D D D D D D D D D
D Ré D D D D D D D D Rn D D
D D D D D D D D D D D D D R

j DV

M M Mf M

y D D D D D D D R

ff D D D D DD D D D D D D D

D D D R

ff D D D c

j D DD D D D D DDTD RX DVD D D

D D D D D D R

u D D D D D DD D D D D D D

RD D D D PD D D D DD I

u D D D D D D D DD D D D RD

D D R

After you finish your plan, be prepared to share it with others and explain:

D D D D

D D D D D D D D D

é Dx

• Dé

k

o Dp

x D

l Dfi

o D€

v

l Dn

é Dn

M

ZDy

1 D DYPDu DUX

Rj $\frac{2}{3}$ $\frac{4}{6}$ hde D D PD D P

PD D D R

Rj $\frac{6}{8}$ $\frac{7}{8}$ hde D D PD D P

PD D D R

2

D̓ DYPDu DUY

Han says there is no fraction with denominator 8 that's greater than $\frac{8}{8}$

$\frac{8}{8}$ D D DURDn D D D Dq hDn D

R

____

____

3

D̓ DYPDu DUZ

“ D D DgD DeD D D D D RDn D

R

R $\frac{5}{3}$ __________ $\frac{5}{2}$

____

____

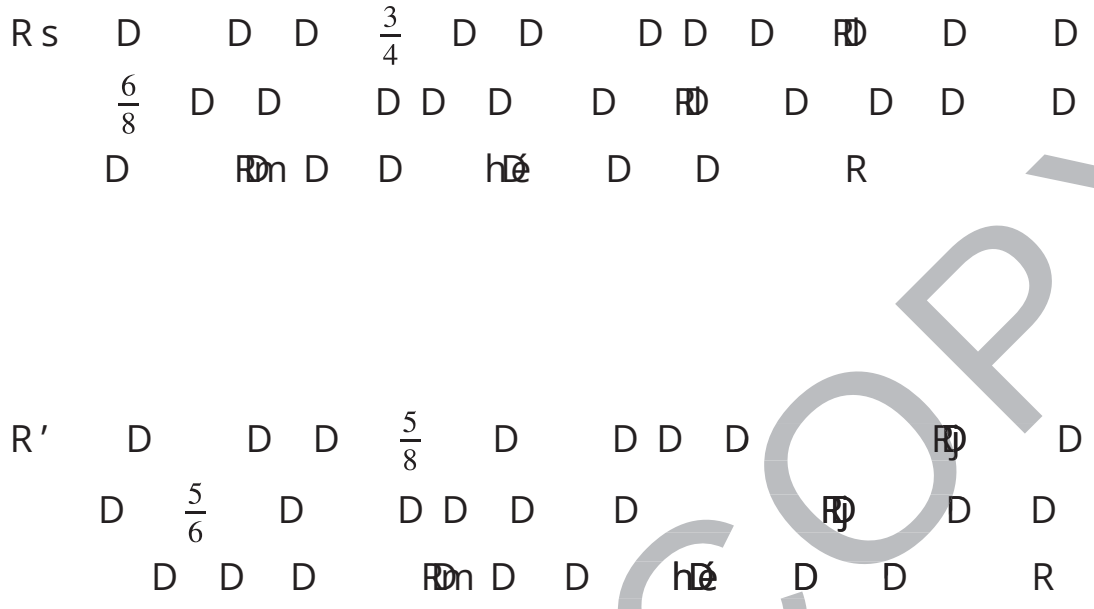
R $\frac{3}{4}$ __________ $\frac{5}{4}$

____

____

é Dn

4



5

n

l D $\frac{3}{4}$ D D D DD D $\frac{3}{6}$ D D

around a different park. Who walked farther? Explain your reasoning.

6 n

l DUD D D D D D D $\frac{3}{8}$ $\frac{5}{6}$ D

D D D D D D R

GRADE 3

UNIT

Measuring Length, Time, Liquid Volume, and Weight

Content Connections

In this unit you will measure length, weight, liquid volume, and time. You will make connections by:

- **Reasoning with Data** while collecting measurements and using line plots to compare and display data.
- **Taking Wholes Apart, Putting Parts Together** while using information learned about fractions to measure and compare measurements accurately.
- **Discovering Shape and Space** while using rounding strategies to approximate lengths when appropriate.
- **Exploring Changing Quantities** while solving problems that are related to length, weight, and time.

Addressing the Standards

As you work your way through **Unit 6 Measuring Length, Time, Liquid Volume, and Weight,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 7 and 13
MP2 Reason abstractly and quantitatively.	Lessons 5, 10, 11, 12, and 15
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 7, 8, 14, and 15
MP4 Model with mathematics.	Lessons 6, 7, 8, 11, 13, 14, and 16
MP5 Use appropriate tools strategically.	Lessons 4 and 6
MP6 Attend to precision.	Lessons 1, 2, 3, 6, 9, and 13
MP7 Look for and make use of structure.	Lessons 8, 12, 14, and 15
MP8 Look for and express regularity in repeated reasoning.	Lesson 10

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Represent Multivariable Data • Fractions of Shape and Time • Patterns in Four Operations • Unit Fraction Models	**3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	Lessons 9, 10, 11, 14, and 15
• Represent Multivariable Data • Measuring	**3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	Lessons 6, 7, 8, 12, 13, 14, and 15

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Represent Multivariable Data • Measuring	**3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	Lessons 1, 2, 3, 4, and 5
• Square Tiles • Fractions as Relationships	**3.NF.1** Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	Lesson 8
• Fractions of Shape and Time • Unit Fraction Models	**3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	Lesson 8
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lesson 8

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3a** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	Lesson 3
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3c** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*	Lesson 3
• Patterns in Four Operations	**3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Lesson 14
• Number Flexibility to 100 for All Four Operations	**3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Lesson 15
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Lessons 5, 8, and 15

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 6, Lesson 1

Addressing CA CCSSM 3.MD.4; building on 2.MD.3; building towards 3.MD.4; practicing MP6

Measure in Halves of an Inch

Let's measure the length of objects around the room.

Warm-up

What Do You Know about Inches?

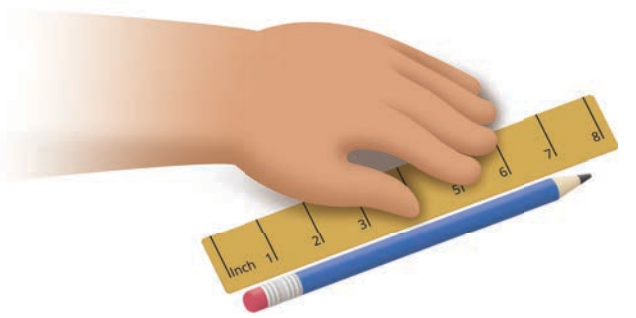
What do you know about inches?

j DJ

M M M

“ D D D D D D D D D D D D D D
RDx D D D D D R

	s



j DV

M M M

ff D D DU D D D D R

UR• D D D D D D D D D D D D D
D R

VR′ D D D D D D D D D D
Rv D D D D D D D R

	s

Addressing CA CCSSM 3.MD.4; practicing MP6

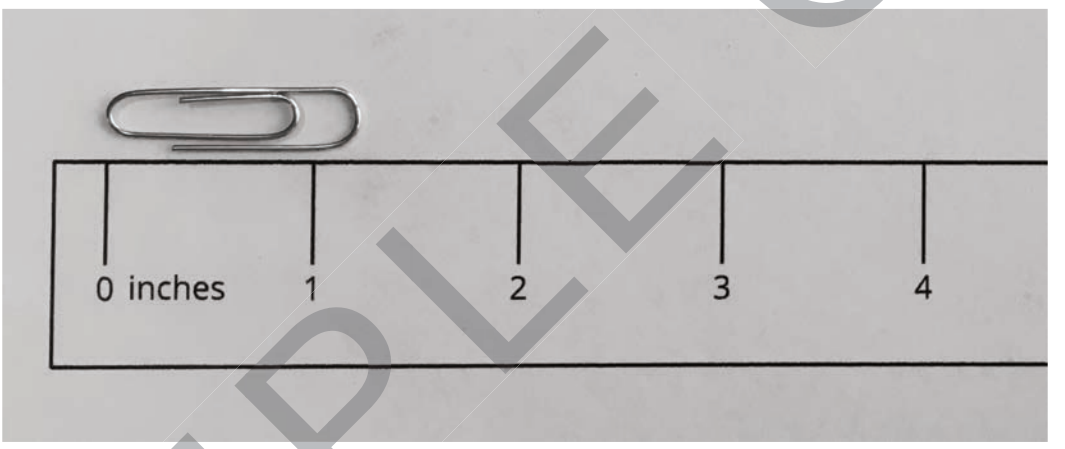
Measure in Fourths of an Inch

Let's measure lengths in quarters of an inch.

Warm-up

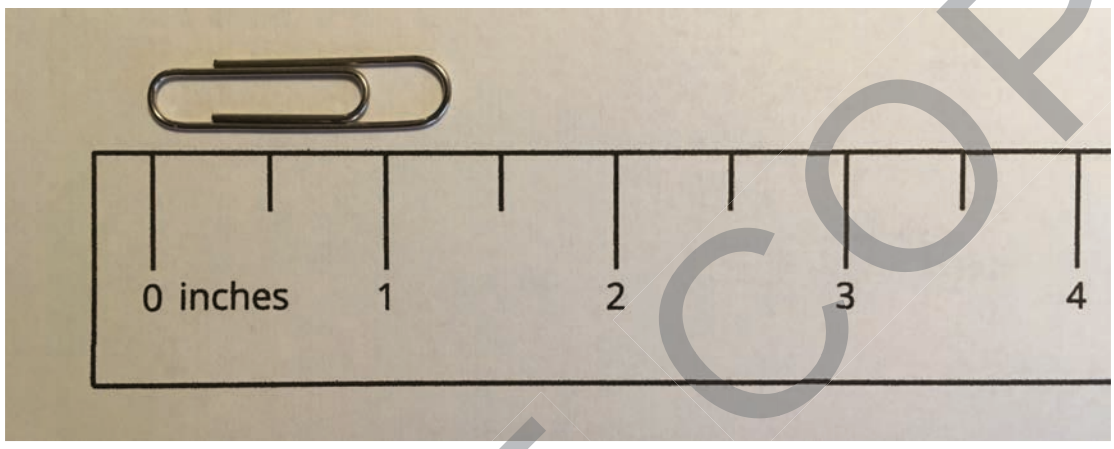
Estimation Exploration: Measure in Inches

What is the length of the paper clip?



Record an estimate that is:

too low	just right	too high



j DV

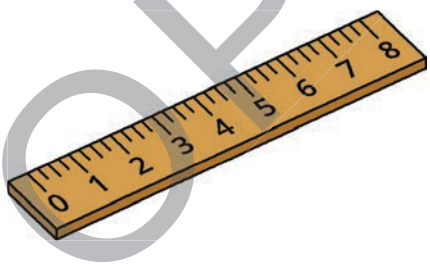
ff M M

ff D D D D D D D D D D D D D D
D R

- D D c

  o D D DXD D D D D
  D D D D D D R

  y D D D R



- Record the object in the table. If you find an
  D D DD D D DD
  D D PD D D D R

	$2\frac{1}{2}$
	$1\frac{1}{4}$
	$3\frac{3}{4}$
	$8\frac{1}{2}$
	D D D D
	D D $\frac{1}{4}$
	D D $\frac{2}{4}$
	D D $\frac{3}{4}$

Addressing CA CCSSM 3.MD.4, 3.NF.3a, 3.NF.3c; building towards 3.MD.4; practicing MP6

Measure in Halves and Fourths of an Inch

Let's measure lengths in halves of an inch and quarters of an inch.

Warm-up

Notice and Wonder: Rulers

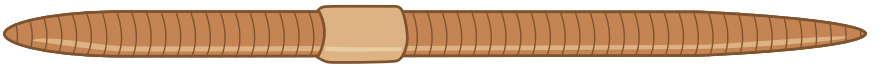
Look at the rulers you have been using to measure and the ruler your teacher gave you.

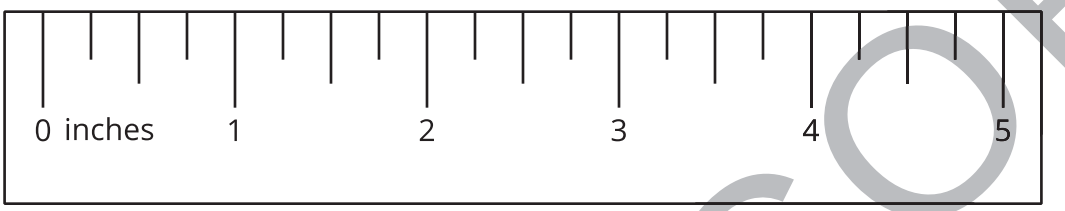
What do you notice? What do you wonder?

j DJ

M M

URt D D D D D DD R



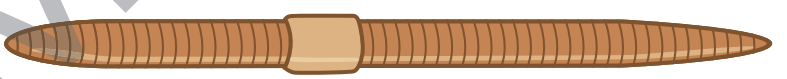


- t D D D D D $4\frac{2}{4}$ D R
- s D D D D D $4\frac{1}{2}$ D R

“ D D D D D D D D D D D R

VRv D D D D D R

v



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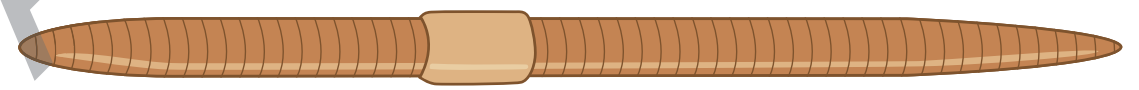
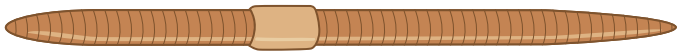


x

y



j DV

M My

URD D D D D D D D D D D D
R

o D D DJD D D D D D D D D D D D
DVD D D D D D D D D R

VR’ D D D D RD DD D D D D D
different way.

Addressing CA CCSSM 3.MD.4; building on 2.MD.9; building towards 3.MD.4; practicing MP5

Interpret Measurement Data on Line Plots

Let's make sense of line plots with lengths in half inches and quarter inches.

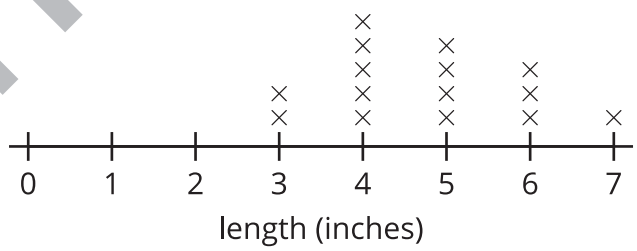
Warm-up

Notice and Wonder: A List and a Line Plot

What do you notice? What do you wonder?

Lengths in Inches

3 5 4 4 5 6 7 5 3 4 4 5 6 6 4



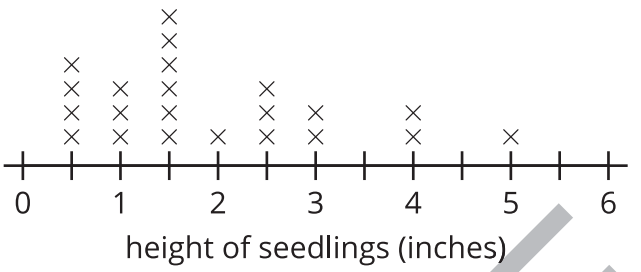
j DJ

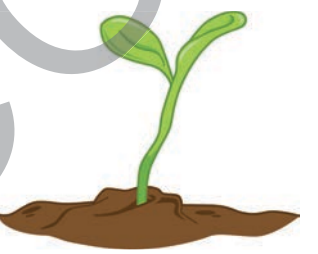
vM M M

q D Dé D D M

$\frac{1}{2}$ U U $\frac{1}{2}$ $1\frac{1}{2}$ $2\frac{1}{2}$ X $\frac{1}{2}$ W $1\frac{1}{2}$ Y $1\frac{1}{2}$ $1\frac{1}{2}$ $2\frac{1}{2}$ W

$\frac{1}{2}$ $2\frac{1}{2}$ $1\frac{1}{2}$ U $1\frac{1}{2}$ X V





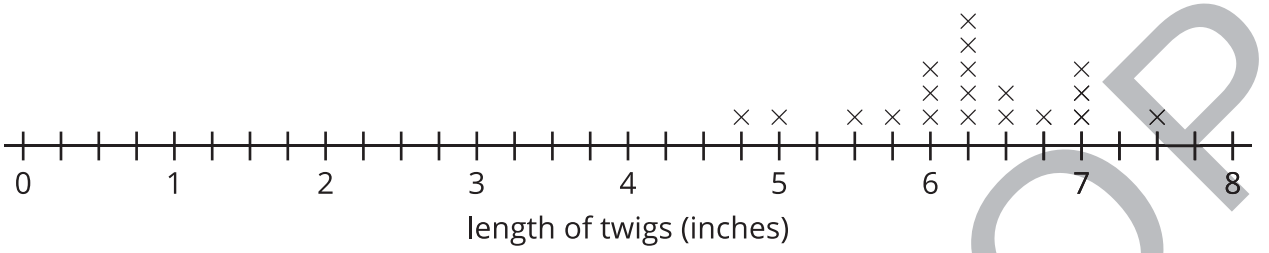
UR • DWD D D D D D D D

R

VR• D D D D D D D D D D D
D hD• D D DVD R

j DV

v M M



UR q D D D D D D D h

VR q D D D D D $6\frac{1}{2}$ D h

WR q D D D D D D D D D h

XR q D D D D D D D D D h

YR • D D D D D D D h

ZR • D D D D D D D h

_R• DD D D D D h

aRj D D D D D D D D
DD D DD D DW
DXD R

• DD D D D D D D
D D D h

Unit 6, Lesson 5

Addressing CA CCSSM 3.MD.4, 3.OA.7; practicing MP2

Represent Measurement Data on Line Plots

Let's collect measurement data and show them on a line plot.

Warm-up

Number Talk: Multiply Teen Numbers

Find the value of each expression mentally.

- 3×10
- 3×13
- 6×13
- 3×26

j DU

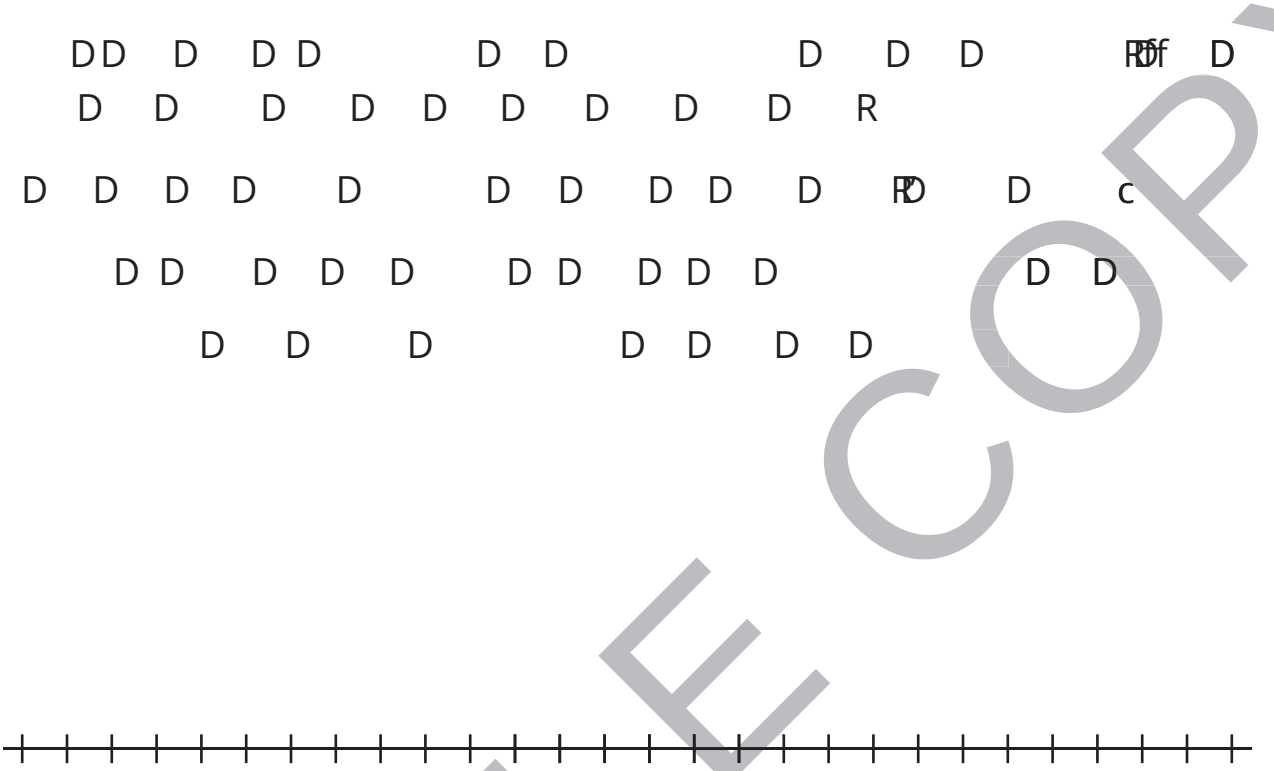
M M M M

UR• D D D D hDk D D D D D D D D D R

VR© D D D D D D D D D D D D D D NR

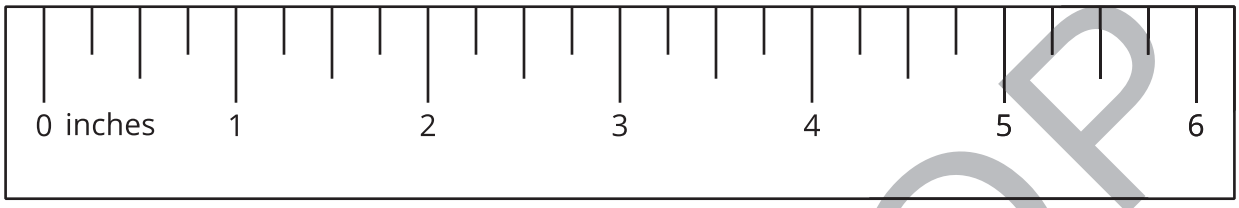
	s

	s

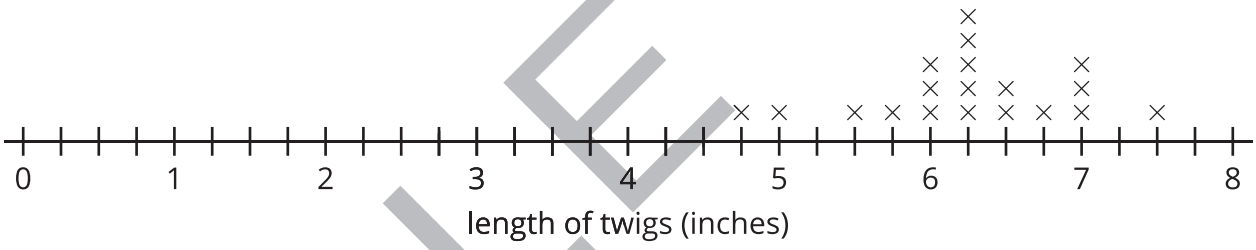


MM

- D D D D D D D D D D D
  D D D R



- D D D D D D D D D D D D D
  D R



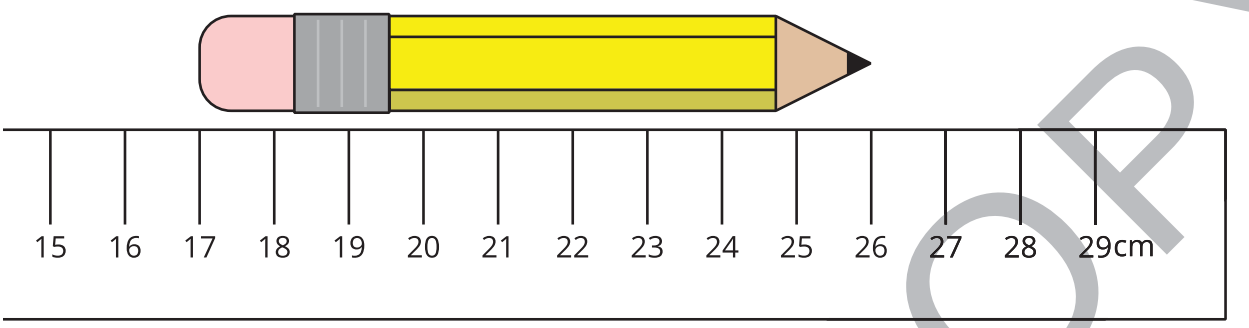
M

UVDy

1

y Q

• D D D D D D D D hDe D D R



2

y Q

q D D D D D D D D D D D D D D D D D R

a UT UW UW UW UY UY Ua

VU VV

3 y Q

Find the value of each sum or difference. Explain or show your

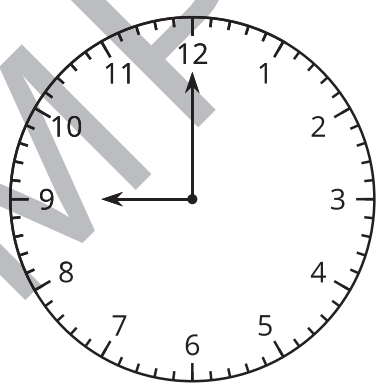
R

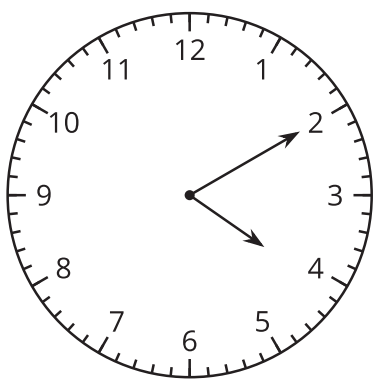
R $374 + 455$

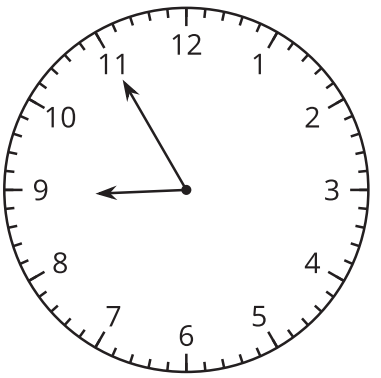
R $259 - 186$

4 y Q

• D D D D D D R







R

R

R

5 y Q

o D D D D D R

R 8×9

R 16×6

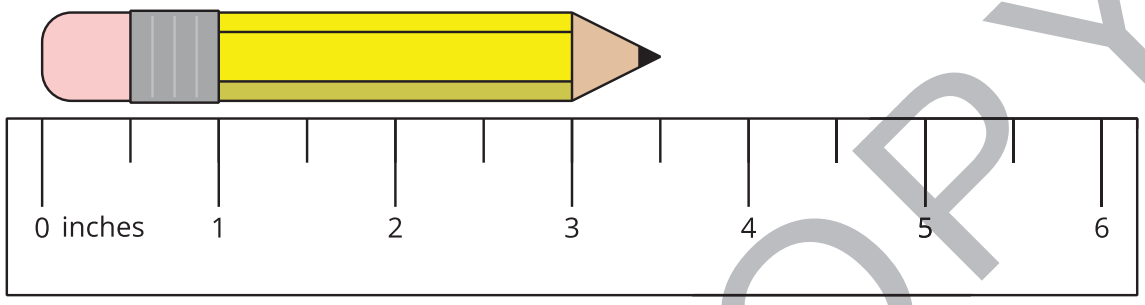
R $72 \div 8$

R $92 \div 4$

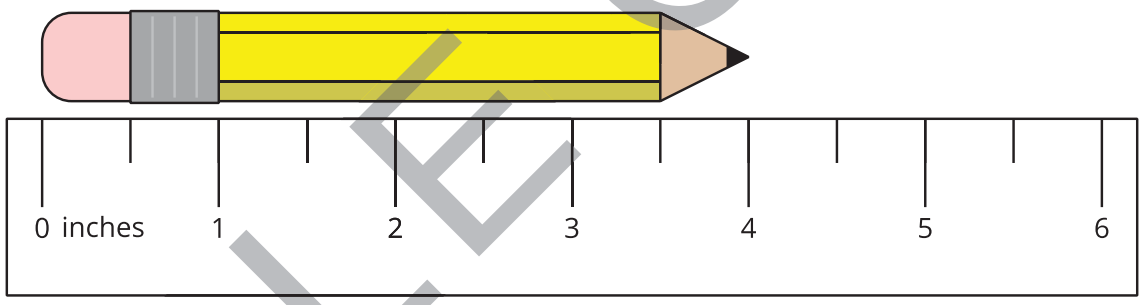
6 D DZPDu DJ

o D D D D D R

R

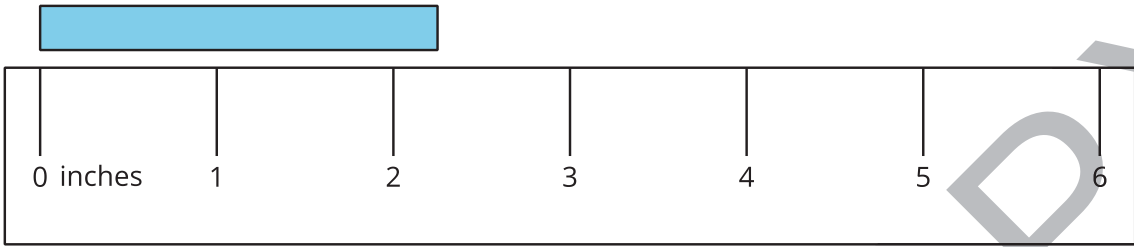


R



7 D DZPDı DY

Ry D D D D D D D D D D D D
R



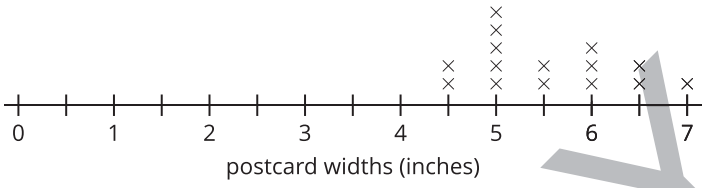
R• D D D D D D hDn D D D
R

8 D DZPDı DW

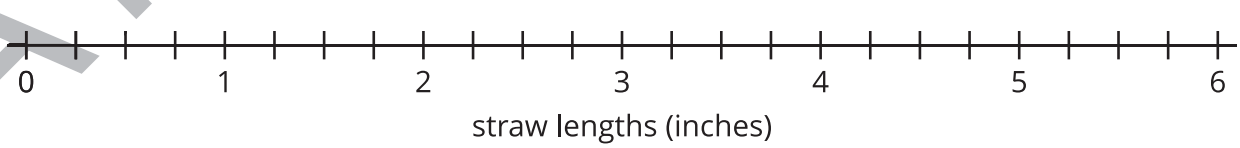
q D D D D D D D D D D R

• D D D D h $3\frac{2}{4}$ V $3\frac{1}{2}$ $\frac{4}{2}$
n D D D D R

9



10



11 n

ff D D DD D D $\frac{1}{4}$ D D D R

Ro D D D D D D D D D D D D D D D R

- $1\frac{1}{2}$
- _D
- WWD

Rv D D D DD D D $\frac{1}{4}$ RD D D D RD D RD D D h

12 n

l DD D D D D D D D D RÉ D D D DD D R

Addressing CA CCSSM 3.MD.2; building towards 3.MD.2; practicing MP4, MP5, MP6

Estimate and Measure Weight

Let's measure and estimate weight.

Warm-up

Notice and Wonder: Produce Stand

What do you notice? What do you wonder?

j DJ

’ M

’ D D D U ’ D D D D U
R R

1. For each weight measurement, find an example of something you think
D D D D R

R UD

R UTD

R UTTD

R UD

R VD

VRo D D D D D D D D D D D D D D D R

s s	s s	s s s s	s s

j DV

M M M

v D D D D D D D D D D D RDn D
R

UR D

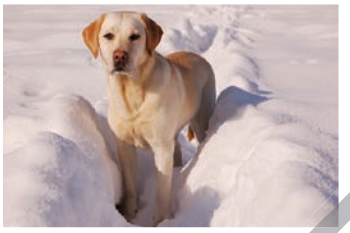
j R VTD

k R UD

l R WD

mR WD

VR



3. small fish

XR

Unit 6, Lesson 7

Addressing CA CCSSM 3.MD.2; building towards 3.MD.2; practicing MP1, MP3, MP4

Introduction to Liquid Volume

Let's learn about the volume of liquids and containers.

Warm-up

Notice and Wonder: The Bowl and the Jar

What do you notice? What do you wonder?

j DU

M M M M M

ff D D D D D D D j D Dk P D D
D D R

1. How many units do you think it takes to fill Container A?

   © D D D D c

s	s	s

2. How many units do you think it takes to fill Container B?

   © D D D D c

s	s	s

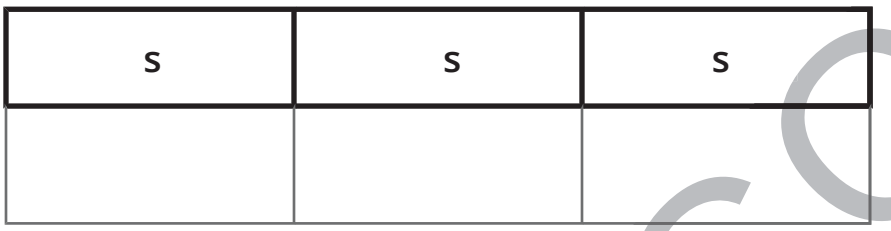
3. Use the unit container to compare the volume of water it takes to fill Containers A and B. Which container is filled with the greater volume of h D q D D D h

j DV

M M M M M M

How many liters of water fit in the large container?

© D D D D c



Addressing CA CCSSM 3.MD.2, 3.NF.1-3, 3.OA.7; practicing MP3, MP4, MP7

Estimate and Measure Liquid Volume

Let's measure and estimate the volume of liquids and containers.

Warm-up

Number Talk: Divide by 3

Find the value of each expression mentally.

- $30 \div 3$
- $60 \div 3$
- $63 \div 3$
- $54 \div 3$

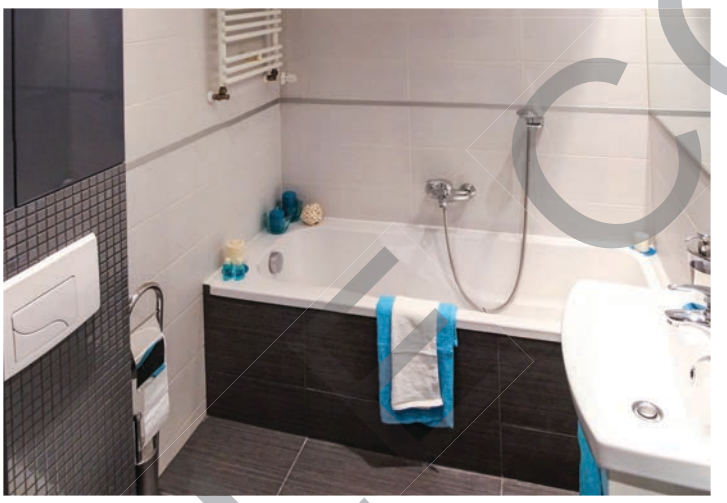
j DJ

’ M M M M

1. Clare says it takes about 2 liters of water to fill the bathtub.

   Jada says it takes about 20 liters of water to fill the bathtub.

   Kiran says it takes about 200 liters to fill the bathtub.



• D D D D D D D hDn D D D
R

2. About how many liters does it take to fill each container? Circle your

RDn D D D R

R VD VTD VTTD

R VD VTD VTTD

WRv D D D D D RDx D D D D D R

D D

R

- XD
- ZD
- YTTD
- UTD

R D

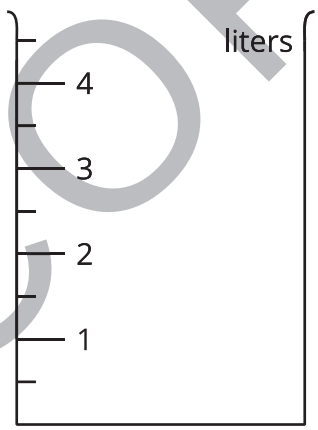
R

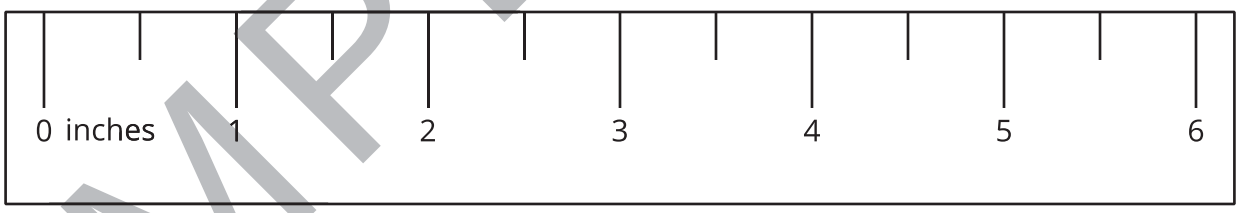
R D

j DV

M M M M

• D D D hD D D D h



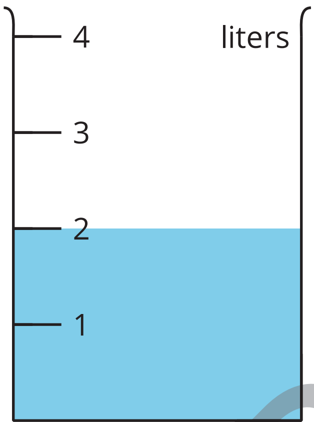


UR’ D D D D D D D D D Ro D D D D
R

v

w

x



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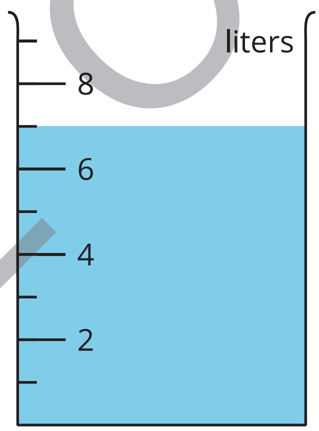
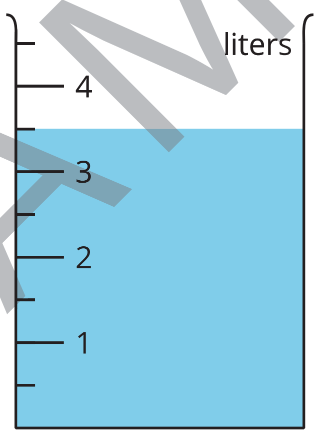
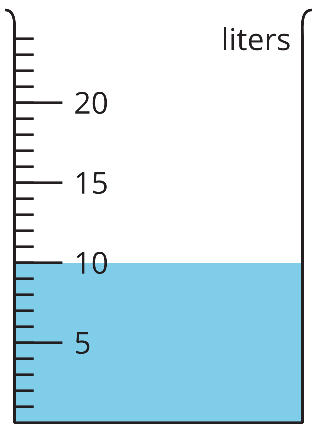
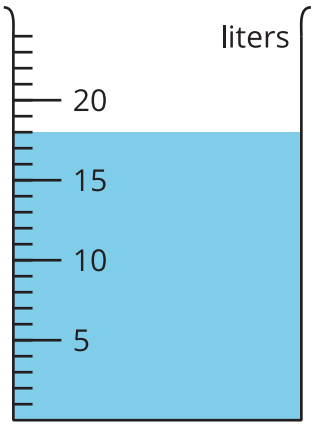


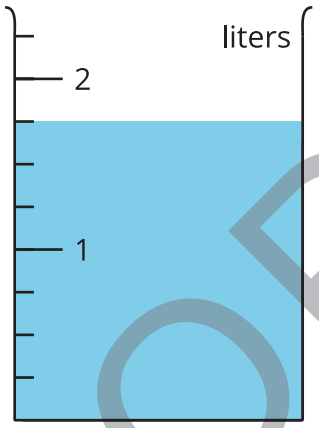
’



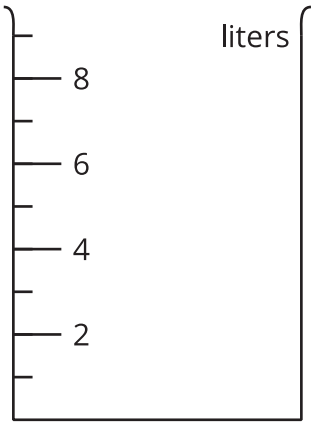
ff

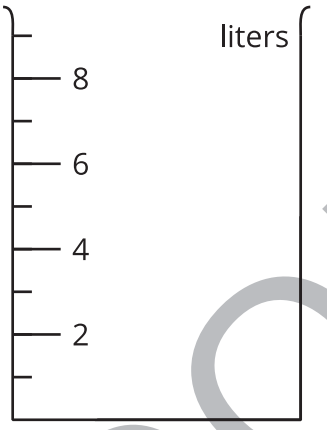


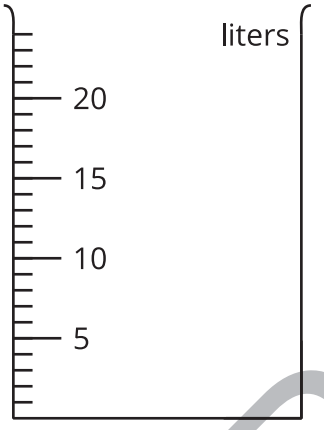


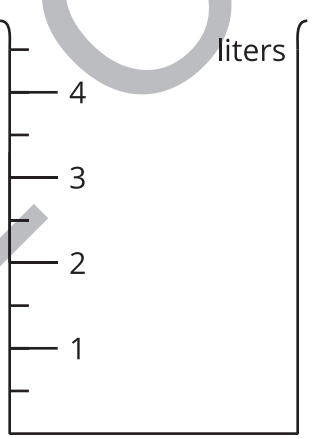


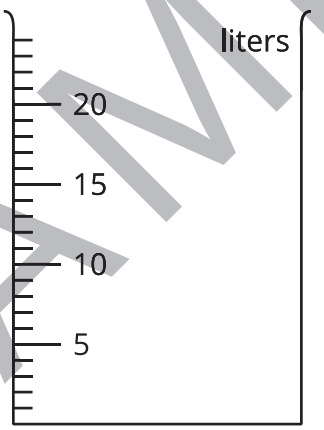
VR é D D D D D D D D D D D
R











r D D D c

x D D D D D D P D D Y D D D D D
h D q D D D D D D D D D D D D h
n D D D D R

MM

- D D D D D D D **b**

  ’ D D D ’ D D D D
  U R U R

- D D D D D D D D
  M M D **M M**
  R

  ’ D D D D D D D D Y D R

  ’ D D D D D D D T D R

M

YD y

1

D D ZPDu D Z

R l D D D D D D DUD R

D D D D D D D

R l D D D D D D DUD R

D D D D D

2

D D ZPDu D _

m D D D D D D D D D DU PD

DU PD D DU R

R D

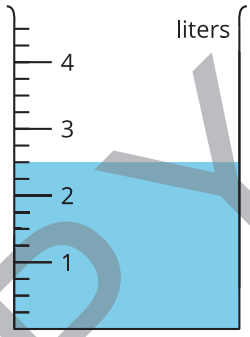
R D

R D D

R D D D

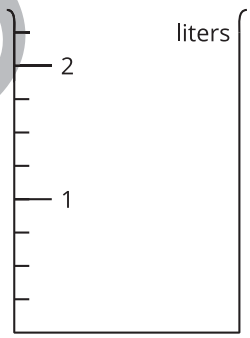
3

R•



Ré

$1\frac{3}{4}$



4

n

t

5

n

ff D DDVQ D D D D DDVQ D D D RQ D D

D DXD D D D D D D DVD h

Unit 6, Lesson 9

Addressing CA CCSSM 3.MD.1; building on 2.MD.7; practicing MP6

Time to the Nearest Minute

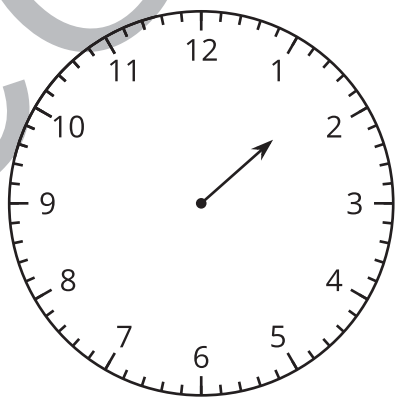
Let's tell and write time to the nearest minute.

Warm-up

Estimation Exploration: On the One Hand...

This clock only has an hour hand.

What time could it be?



Record an estimate for the time that is:

too early	about right	too late

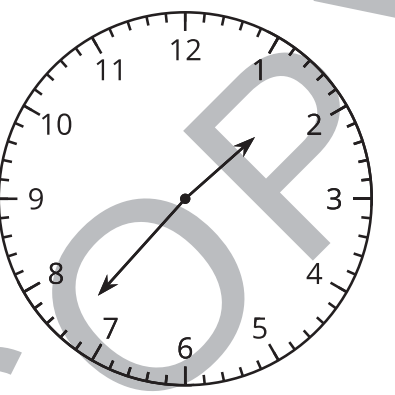
j DJ

M M M M M

URu D D D D D D D D DWV_
R R

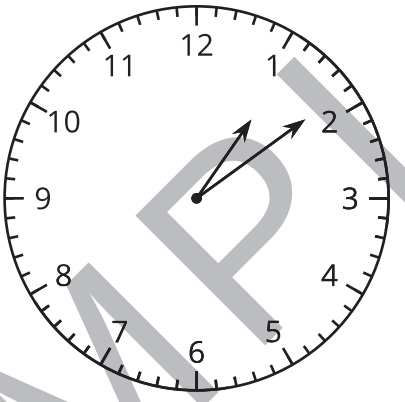
m D D D D DWV D R R

• D D D h Dm D D D
R

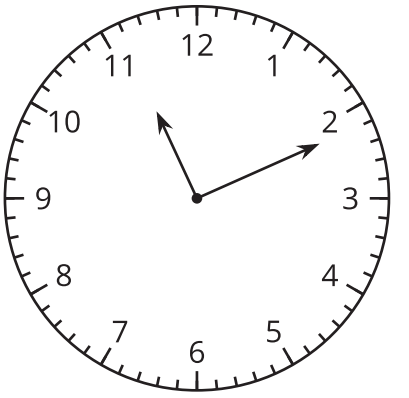


VR • D D D D D D h

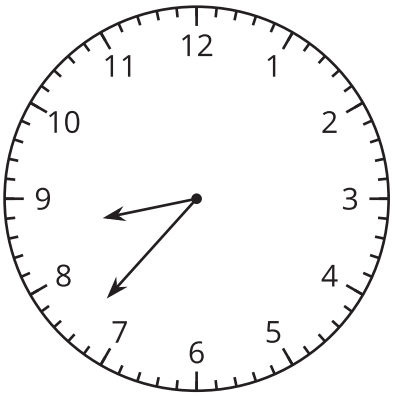
v



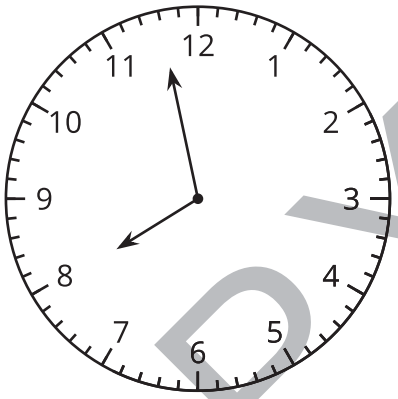
w



x



y

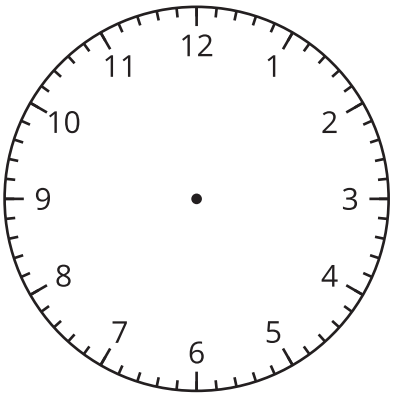


j DV

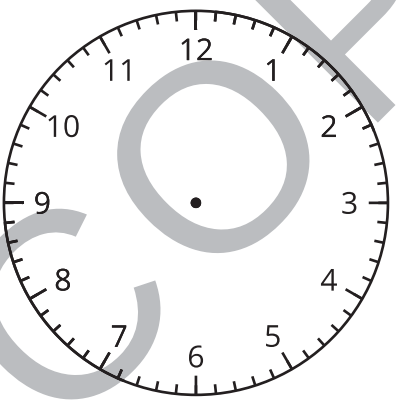
M

URm D D D D D D D D R

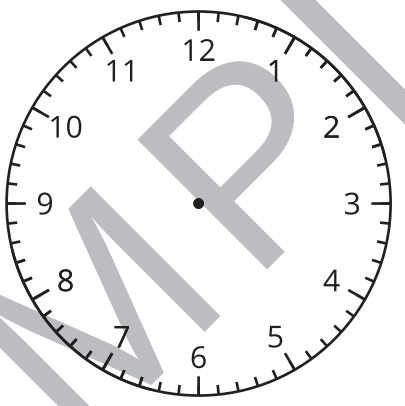
v fgkMb b



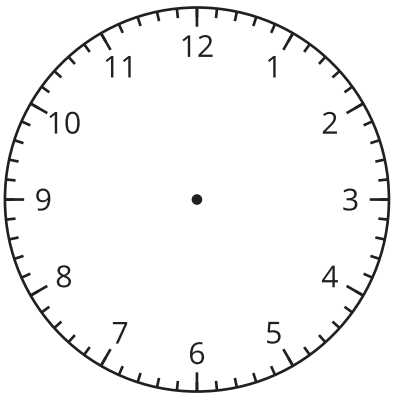
w gœmMb b



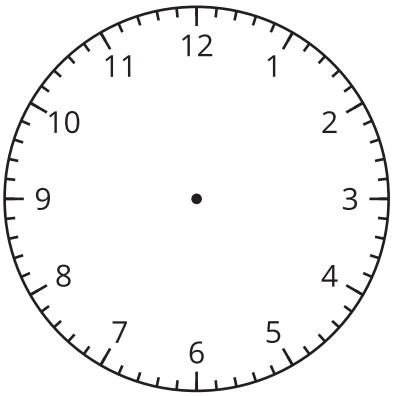
x efœl Mb b



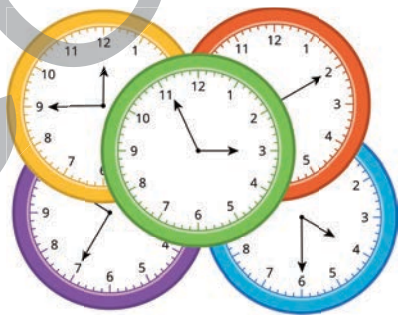
y nœdf Mb b



VRm D D D D D D DD RD D DD D D
D D D D D R



e



Unit 6, Lesson 10

Addressing CA CCSSM 3.MD.1; practicing MP2 and MP8

Solve Problems Involving Time (Part 1)

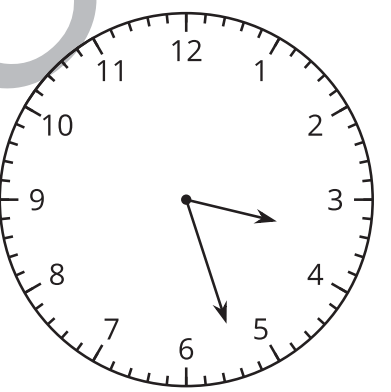
Let's solve problems involving time.

Activity 1

Time at the Bus Stop

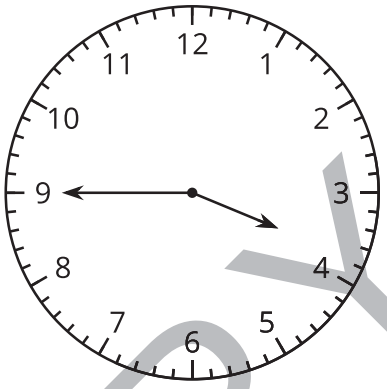
1. Kiran arrived at the bus stop at 3:27 p.m., as shown on this clock. He waited 24 minutes for his bus to arrive.

   What time did his bus arrive? Show your thinking. Organize your work so it can be followed by others.



VRn D D D D D D DWXYDR RD
D D D Re D D DX
D D D D D R

• D D D D D hDe D
RDx D D D DD D
D D R



j DV

M M MV

q D D D D c

At 6:32 p.m., Elena got on a bus to go home. She got off the bus
DdUTD R RDq D D D D D h

é D

• D D D D D D DD D
hDn D D R

Addressing CA CCSSM 3.MD.1; building towards 3.MD.1; practicing MP2 and MP4

Solve Problems Involving Time (Part 2)

Let's solve more problems involving time.

Warm-up

Notice and Wonder: Band Practice

What do you notice? What do you wonder?

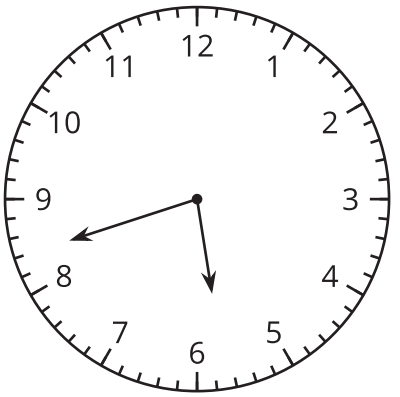
Han ate his dinner before he caught a bus.
When he got off the bus, he had to hurry to get to band practice on time.

j DJ

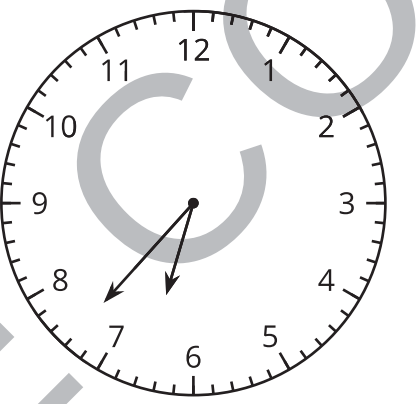
M Mv M

UR o D D D D Dq D D D hDn D D D
R

’ D Dq D D D
c



The time Han got off the
c

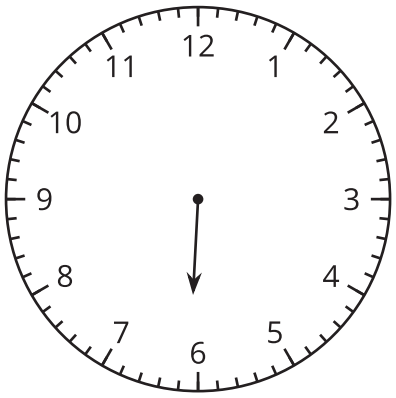


2. Draw the minute hand on the first clock to show that Elena waited for

D D DWD R

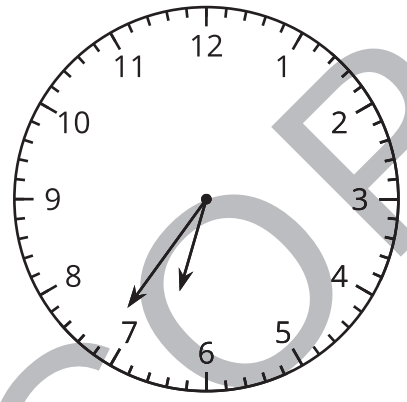
’ D Dn D

c



’ D Dn D D D

c



j DV

M M

o D D PD DD D D D D D D D RD D D
RÉe D D RDx D D D DD D D D
R

UR _____

_____ DadVYDR RD
finished at 8:50 a.m. How much time was spent doing that activity?

VR _____ finished

_____ DYdVaDR RD
DVTD RD D D D D h

WR _____

_____ DJTcXYDR RD
finished at 11:18 a.m. How much time was spent on that activity?

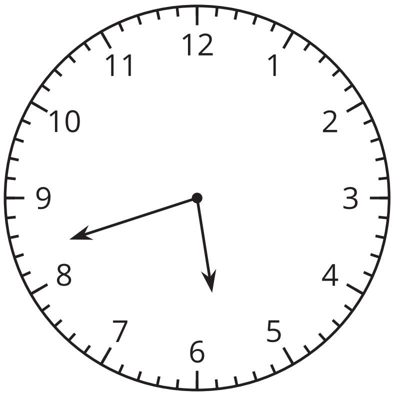
XR _____

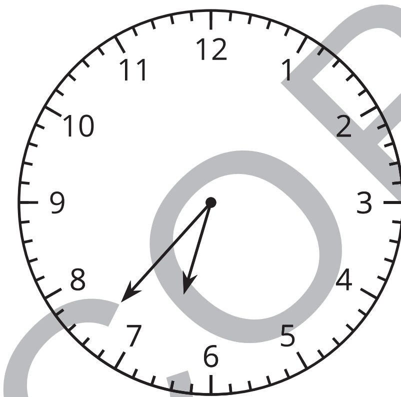
_____ DWcWTDR RD
took 45 minutes to complete it. What was the finish time?

MM

- D D D D D D D D D D PD D D
D D D D R

D Dq D D D D c the time Han got off the bus:





Here is 1 way to find out how much time Han spent on the bus:

- Identify the start and finish times shown on the clock: 5:42 and 6:37.

I D DWD D DYcXVD DYcXYPD D DUYD D DYcXYD ZcTTR

I D DWTD D DZcTTD DZcWTPD D D_D D D ZcW_R

j D D Q D P$3 + 15 + 30 + 7$PD D DYR

q D DYYD D D D R

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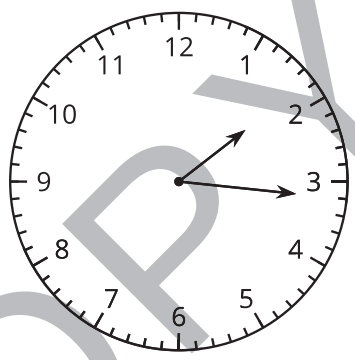
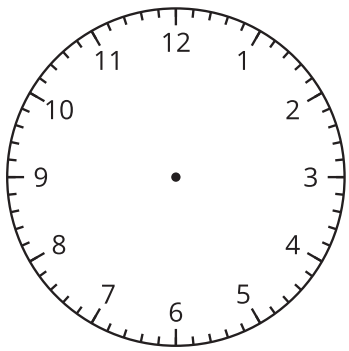
1 D DZPDu Db

R q D D D D DVdUZDR RDm D
D Dq hDm D D R



R m D D D D D D D DYdYY
R R



2 D DZPDu DUT

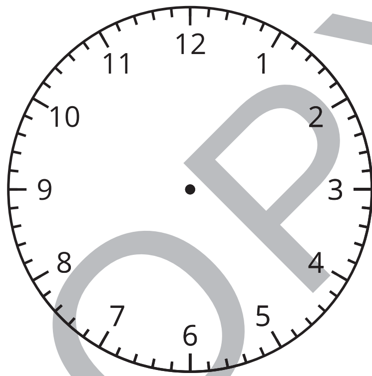
v D D D D D D DZdYTDR RD D D D DcXXDR R
q D D D Dv D D D D h

3 D' DZPu DUU

j D D D D DWXYD R Rq D D D_YD
D D R

R m D D D D D D D
D D D R

R j D D D D D DUT
RD D D D D
hDn D D D D R



4 n

R • D D D D D D D D D h

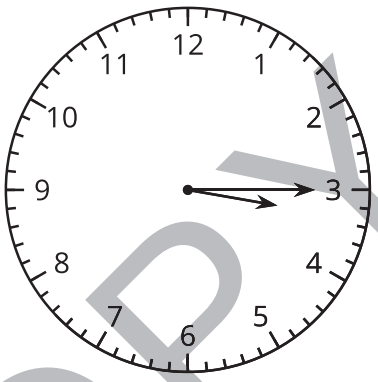
R • D D D D D D D D D h

R q D D D D D D D hDn D D D
h

5 n

y D D D D D D D D
WdUYR

m D D Dy D D D h
n D D D D R



Unit 6, Lesson 12

Addressing CA CCSSM 3.MD.2; building towards 3.MD.2; practicing MP2 and MP7

Ways to Represent Measurement Situations

Let's make sense of and represent measurement situations at the fair.

• Q

M M dM Mf

• D D D hD D D D h

j DJ

M M

UR• DD D D D D D D D D D
R

VR• D D D D D D D D D D D D D
RX DD PD D D D D D D D D
D D D D DR

el m • p DV

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j DV

x M dM M

ff D D D D DD D D D D D D R

v D D D DD RDx D D D D D R

Unit 6, Lesson 13

Addressing CA CCSSM 3.MD.2; practicing MP1, MP4, MP6

Problems with Missing Information

Let's find out what information is needed to solve problems about measurements at the fair.

• Q

’ M dM M

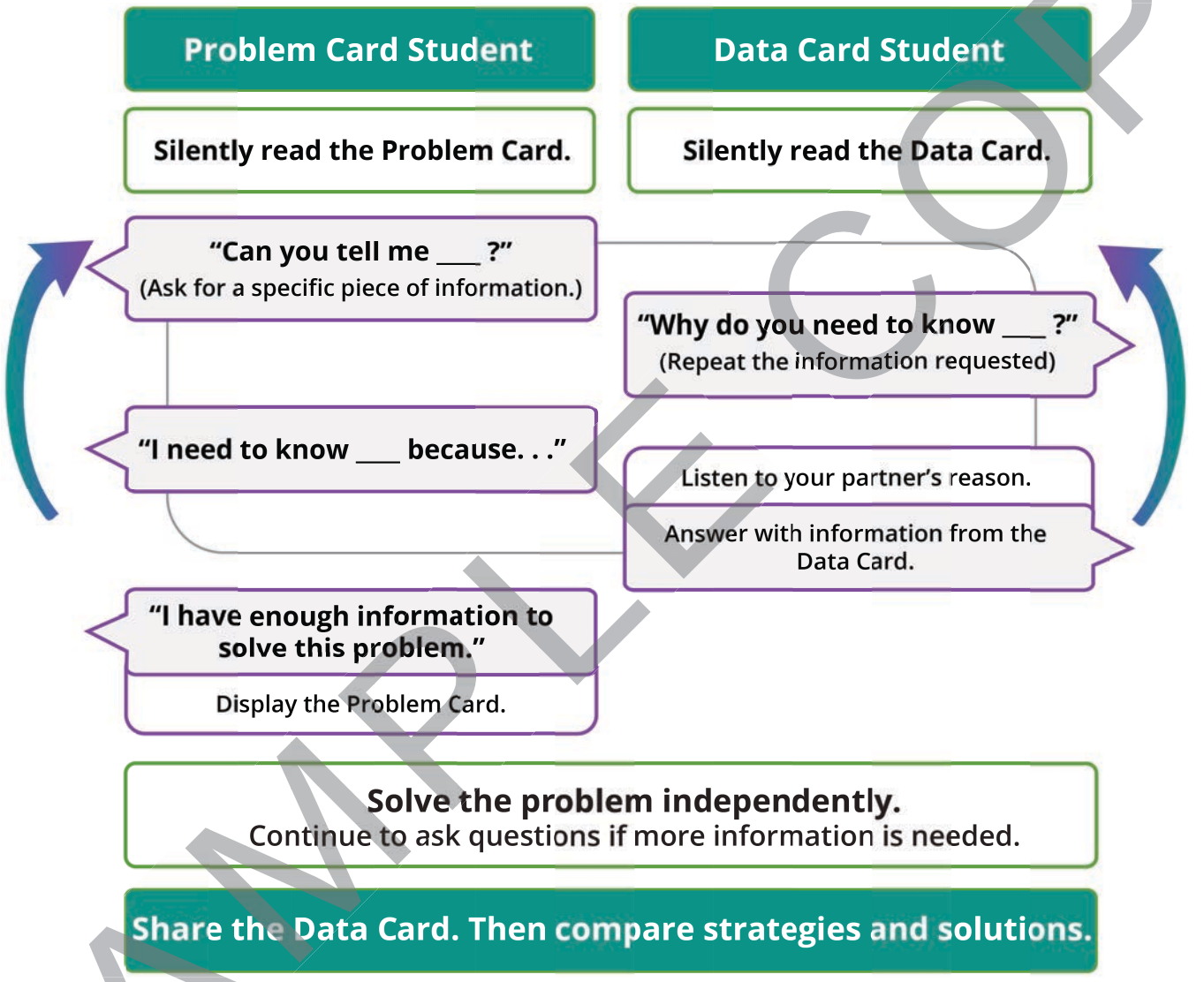
Each regular pumpkin in this field weighs between 4 and 10 kilograms.

q D D D D D D D D D D h

© D D D D c

s	s	s

Info Gap: Pumpkin Weigh-Off



j DY

Info Gap: Pig Weigh-Off

ff D D D D DD Dy D D Dn D RDn D D D
D D D D D R

“ D D r Dp D D D D RD D D
D D D D R

j D D D DD D D D D D D D PD D
D D R

Unit 6, Lesson 14

Addressing CA CCSSM 3.MD.1-2, 3.NBT.2; practicing MP3, MP4, MP7

What Makes Sense in the Problem?

Let's think about what numbers and questions make sense in problems.

Warm-up

Number Talk: Give and Take

Find the value of each expression mentally.

- $306 + 199$
- $318 + 297$
- $275 + 325$
- $275 + 329$

j DU

x M M Mx

q D DVD D D D DD RDn D D D D

R

URr D D PD D D D D D D D D D D D

D D D D R

Rl D D D' D D D D Do D RD D D

D D D RDq D

D D D D h

R• D D D D PD D D D D D D D D D D

RD D RDj PD

D D D RD• D D D D D D h

Rt D D D D D D D RDj

PD D DD D D D RD• D D

D D D D h

VRé D D D D D D D D D D D D D D D D D R

WR• D D D D D D D RDx D D D D R

j DV

MV

j D D D D D D DWD D
DD RD D D D D D
D PD D DTD D R

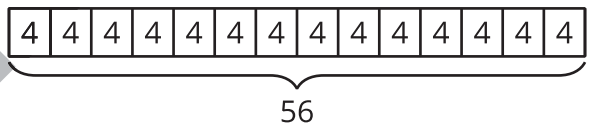
j D D D D D PD D DYZD D
D PD D D D D D D
XQ D R

“ D D D D D D D D D DD D D
D D D D D D R

UR $132 - 90 = 42$

z c

VR



z c

Unit 6, Lesson 15

Addressing CA CCSSM 3.MD.1-2, 3.OA.3, 3.OA.7; practicing MP2, MP3, MP7

Ways to Solve Problems and Show Solutions

Let's solve problems about spending a day at the fair and think about how to best show our solutions.

Warm-up

Number Talk: Divide by 8

Find the value of each expression mentally.

- $80 \div 8$
- $72 \div 8$
- $96 \div 8$
- $96 \div 4$

j DJ

vMyM M M Mff

ff D DD D D D RÉ DX D D D D D D
D D D D D D R

UR ff D D D D E

n D D D D DHbDD RÐf D D D DZ
D RQq D D DD D D D
D D h

VR q D D D D D hDl DJR

You arrived at the giant pumpkin weigh-off at 11:12
R RD D D DJVcVYD R RQq D D D D h

ff D DXaD D D D D D D DJVcJT
R RP D D D D D D D h

WR• D D hDl DJRM

ff D DD D D_D RD PD D
D DbUD D D DD RÐn D D D
D D RQq D D D D D
D D h

ff D DD Q D D DD D D

PD D D D DV_WD PD D

V__D D D D RQq D

D D D D h

XRk D D D PD D D D R

j D D D D PD D DVD

D DbD D RQq D D D D

D h

1. Look for a problem that was solved using a strategy that is different from yours. What made it different? Describe the strategy.

and division, as well as different reasoning strategies.

−8 −30 −10
11:22 ← 11:30 ← 12:00 ← 12:10

o l D D D D D DUUcVV

R R

oj D D D D D DV_W

RD D DV__D D

D D RDq D D D

D D h

o′ D D DYYTD R

oj D D DaXD D D

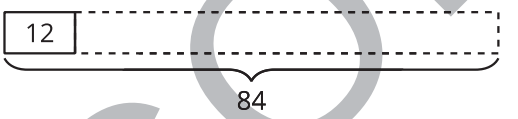
D D RDn D

UVD RDq D D D

h

o′ D D_DD R

1 1
2 7 3
+ 2 7 7
5 5 0



Unit 6, Lesson 16

Building on 3.MD.1-2, 3.MD.4, 3.NBT.2, 3.OA.1-4; building towards 3.MD.4; practicing MP4

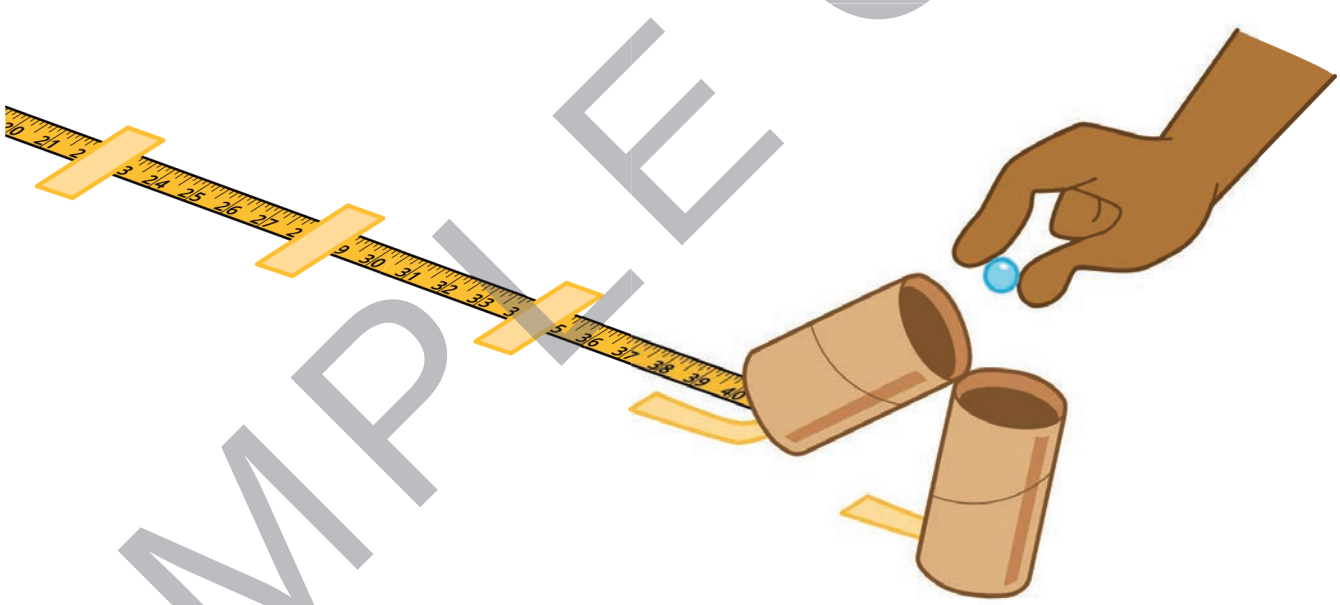
Design a Game

Let's design a game.

Warm-up

Notice and Wonder: Games

What do you notice? What do you wonder?



j DU

x M M M

UR“ D D D D D D D R

R• D D D D D D h

Rq D D D D D h

VR’ D D D D D DU R

WR© D D D D D D DVD D D c

- D D D D D
- D D D
- D D D DJTT
- D D D DJPTTT

rD D D PD D D D D D R

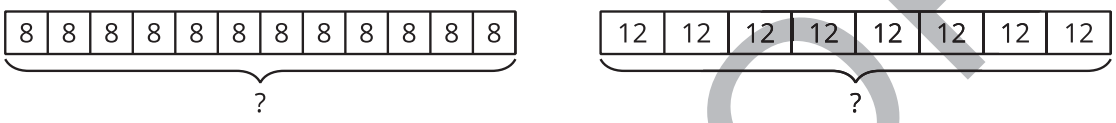
1

u D D Da D D DD D D Rm

D DVD Rq D D D D

D D h

R• D D D D D h



Ré D D Rm D D D D R

2

A family brought 2 pigs to the fair. The first pig weighed 153 kg. The

D D D Rq D D D D D D h

• D D D D D D D D hm D D

D R

3

Dʻ DZPDu DUX

v D D D D DD D D D D D̲cXWD R RjD D D D
D DUYD RDv D D D D D D D D D D D
D D D DacTTD R R

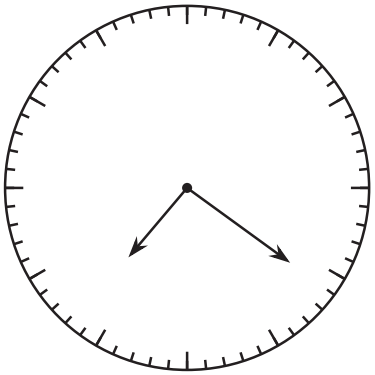
m D D hDn D D D D R

4

Dʻ DZPDu DUY

m D D D D D D D DaD D D D RDq D
D DXTD D D D RDq D D D D D
m D D D hDn D D D D R

5 n



s D D D D DD D D

R• D D DhDm D

R

_____

_____

6 n

’ D D D D D DD D DDPTTTD R

R• D DDD D D D D D D h

R’ D D D DWD D DDYYYD R• D DD D

D D D D D D DYD h

R D D D DWD D DDXVaD RQ D D D

D D h

j D D D D D D D D D D D D D D D D
R

The number of square units that cover a flat figure without gaps or
R

j D D D D D D D RD D D D D
D D D RD D D D D D D D
R

D
j D D D D D D D D D D D D D D
D D D D D D R

’ D D D DD D D D D D D D D
D D R

An operation for finding the number of groups or finding the size of
D D D D PD D DD D D D D D R

’ D D D D D PD D D D D D D
D D D D D R n D D_D D D DYPD

DYD D R

x D D DD D D D DaD D R

j D D D D D D DfMD D D D DD D
D D D D D D D D D D D D R

D
o D D D D D D D D D D D
D RD c$\frac{1}{2}$ $\frac{2}{4}$ D D R

D
j D D D DD D DD D D D D D D R

n dXaVD D D D $400 + 80 + 2$R

j D D D D D DVD D D D DUD D
L D D PD PD D D MR

j D D D D D D D D DUD D D D
DD R

j D D D D D D D DD D D D
D D D R

j D D D D D D D D D D R

’ D DJPTTTD D DJD R

’ D D DD D D D D D D D D
R

j D D D D D D D D D D R

’ D DJPTTTD D DJD R

j D D D D D D D D D D D R

D
j D D D DD D D DD D D DJR

An operation for finding the total number of objects when we have a
D D D D R

’ D D D DD D D D D D D D D D
D R

p D D D D D D D D D PD
$c(3 \times 5) + (2 \times 5), (24 \div 2) + 5 = 17$R

D

j D D D D D D D R

D

j D D D D D D D R

D

j D D D D D D D R

D

j D D D D D D D R

D

j D D DD D D R

D DD

The amount of liquid it takes to completely fill a container.

D DD

’ D D D DD D D R

q D D D R

j

' YDy D Dé PDv RéRDj Dé PDv RDRDVTUUVRYD D D D
D RDv D D D' D Dv R

w D D• rDv SrD• D D D Dv D D D D
Mathematics, reflecting approaches developed by the Math Forum (cSS R R S
SVMD D D D D R

r D D D D D D D D Dv D D D D D D D
D DD D Dj DlI QkffVD PD D D D
RD D D D D D D Dv Dv D D D D D
R

© D D D D DD D D D D D D D
D D D D D D PD D D D D D D D
r Dv D D D PD PD D D D D PD PD
RD Dv DD D D D D D D R

M

y D PD Dk RDlTRD• D PD cSS R R S S
o cy TWR R

k Dv Dé RDy RDy RDj D D cSS R S S
Q Q QUaUVW_SR

j Dk Dp DL MD D Do RDlDkffDWRD• RDj D
cSS R R S Sp GS So cj k p R R

k D PD 'MjRDlDkffCéj DXRD• D PD cSS R R S S
o dk R R

k D TRDy RDy PD cSS R S S Q Q Q Q
QUaU_b_SR

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California Common Core State Standards for Mathematics (CA CCSSM) Reference

3.G: Grade 3 – Geomery

Reason with shapes and their attributes.

3.G.1
Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

3.MD: Grade 3 – Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1
Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2
Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).Excludes compound units such as cm^3 and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.Excludes multiplicative comparison problems (problems involving notions of "times as much"); see Glossary, Table 2.

Represent and interpret data.

3.MD.3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

3.MD.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units--whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5
Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.5a
A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

3.MD.5b
A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7
Relate area to the operations of multiplication and addition.

3.MD.7a
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

3.MD.7b
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

3.MD.7c
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

3.MD.7d
Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8
Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

3.NBT: Grade 3 – Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.1
Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3
Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

3.NF: Grade 3 – Numbers and Operations—Fractions

Develop understanding of fractions as numbers.

3.NF.1
Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.2a
Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

3.NF.2b
Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

3.NF.3a
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

3.NF.3b
Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

3.NF.3c
Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

3.NF.3d
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

3.OA: Grade 3 – Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1
Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.

3.OA.2
Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

3.OA.3
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.See Glossary, Table 2.

3.OA.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5
Apply properties of operations as strategies to multiply and divide.Students need not use formal terms for these properties. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

3.OA.6
Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

3.OA.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3.OA.9
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.