IMKH California

GRADE 3

Student Edition

UNITS 7-8

Illustrative Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 4

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GRADE 3

UNIT

Two-Dimensional Shapes and Perimeter

Content Connections

In this unit you will describe and compare two-dimensional shapes and solve problems involving the perimeter and area of shapes. You will make connections by:

- **Discovering Shape and Space** while describing, analyzing and comparing quadrilaterals and solve problems about the perimeter and area of shapes.
- **Exploring Changing Quantities** while solving problems about perimeter and area and using estimation and rounding when appropriate.
- **Taking Wholes Apart, Putting Parts Together** while using square tiles to measure the area of shapes.

Addressing the Standards

As you work your way through **Unit 7 Two-Dimensional Shapes and Perimeter,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 10 and 14
MP2 Reason abstractly and quantitatively.	Lessons 9 and 13
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 8 and 10
MP4 Model with mathematics.	Lessons 13 and 15
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 1, 3, 5, 10, and 14
MP7 Look for and make use of structure.	Lessons 1, 2, 4, 7, 8, 10, 11, and 12
MP8 Look for and express regularity in repeated reasoning.	Lesson 6

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Analyze Quadrilaterals	**3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	Lessons 1, 2, 3, 4, 5, and 14
• Number Flexibility to 100 for All Four Operations	**3.OA.5** Apply properties of operations as strategies to multiply and divide. 2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Lesson 10
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Lessons 5, 8, 10, 11, and 12
• Patterns in Four Operations • Number Flexibility to 100 for All Four Operations • Analyze Quadrilaterals	**3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Lessons 10 and 15
• Patterns in Four Operations	**3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	Lesson 5

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Square Tiles	**3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	Lesson 15
• Analyze Quadrilaterals	**3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Lessons 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15
• Patterns in Four Operations	**3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Lesson 7

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Addressing CA CCSSM 3.G.1; building on 2.G.1; building towards 3.G.1; practicing MP6 and MP7

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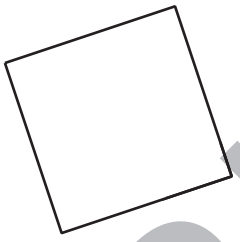

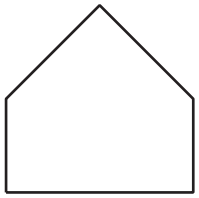

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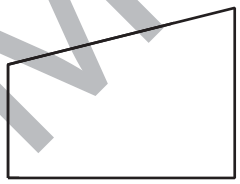

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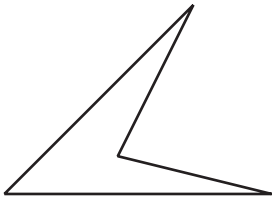

Addressing CA CCSSM 3.G.1, 3.NBT.3; practicing MP7

x P P P

Let's sort shapes into more specific categories.

fi R

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p E E E E E E E SEn E E E E

S

$3 \times 60 = 9 \times 10$

$180 = 3 \times 60$

$6 \times 40 = 24 \times 10$

$24 \times 10 = 240$

m EW

— P qP

ffi E E E E EE E E E E E S

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Addressing CA CCSSM 3.G.1, 3.NBT.3; practicing MP6

Attributes that Define Shapes

Let's play Mystery Quadrilateral.

Warm-up

Number Talk: Multiply Multiples of Ten

Find the value of each expression mentally.

- 4×40
- 8×40
- 7×40
- 9×40

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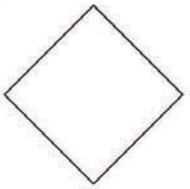
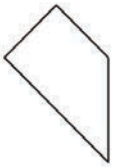
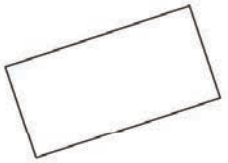
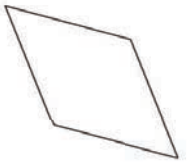
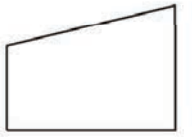
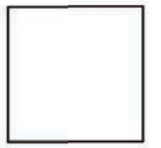
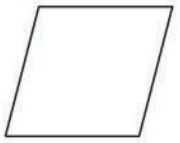
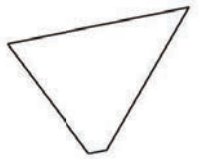
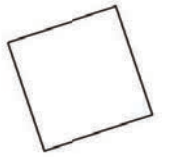
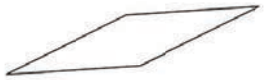
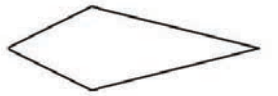
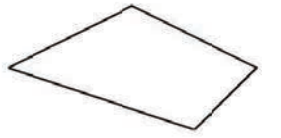
4. Play again with a different quadrilateral.

M 	N 	O 	P 
Q 	R 	S 	T 
U 	V 	W 	X 
Y 	Z 	AA 	BB 
CC 	DD 	EE 	FF 

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Unit 7, Lesson 4

Addressing CA CCSSM 3.G.1; practicing MP7

Attributes of Rectangles, Rhombuses, and Squares

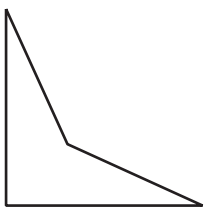
Let's find out what makes rectangles, rhombuses, and squares what they are.

Warm-up

Which Three Go Together: More Attributes

Which 3 go together?

A



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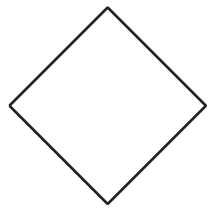

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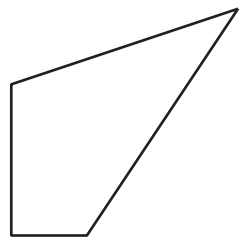

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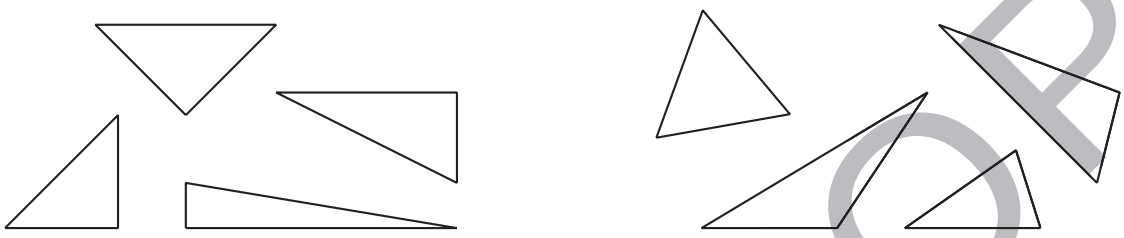

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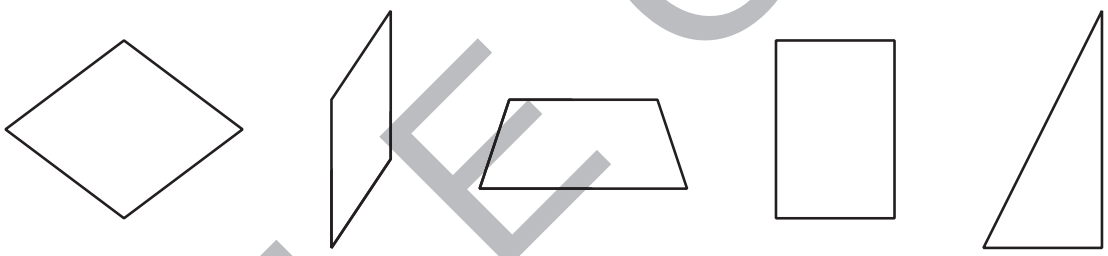

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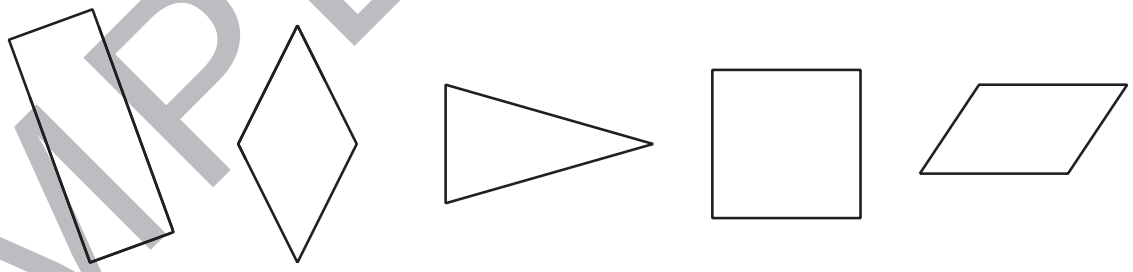

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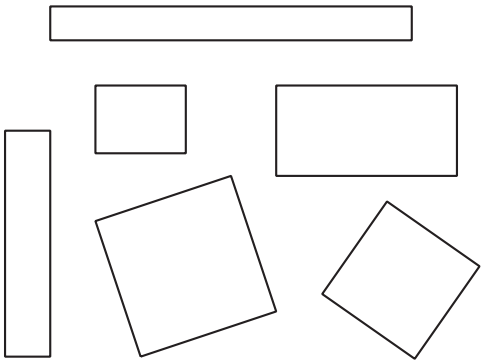

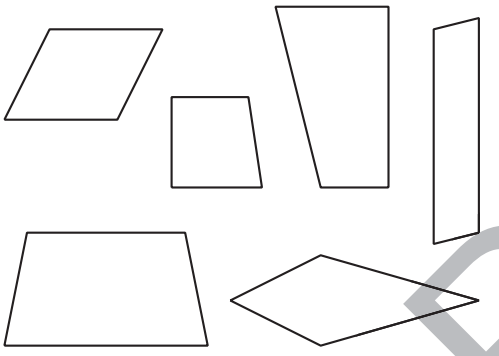

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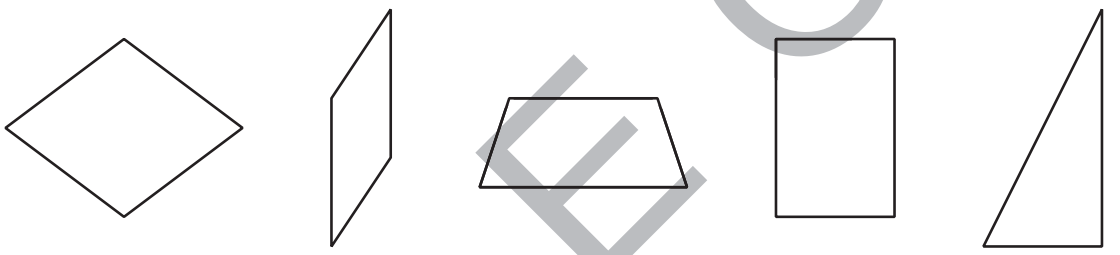

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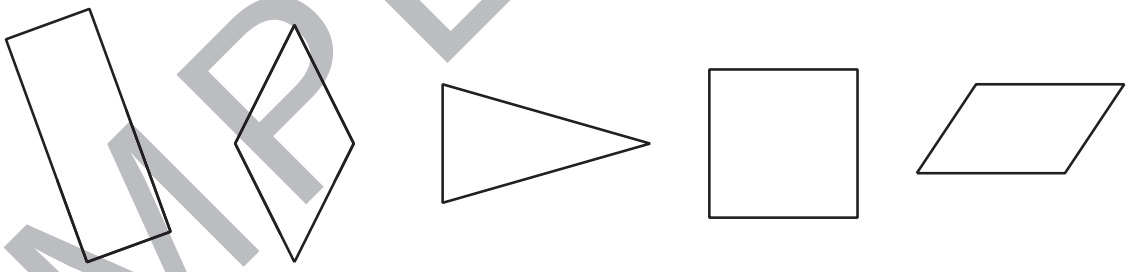

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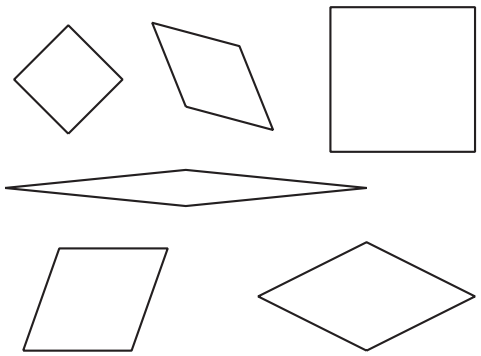

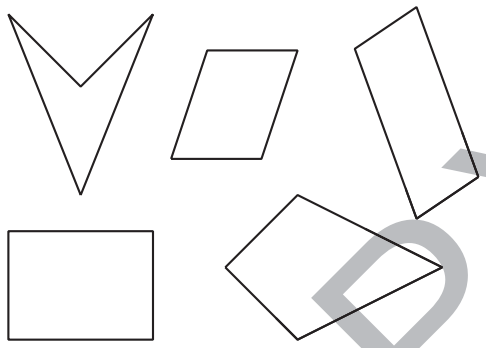

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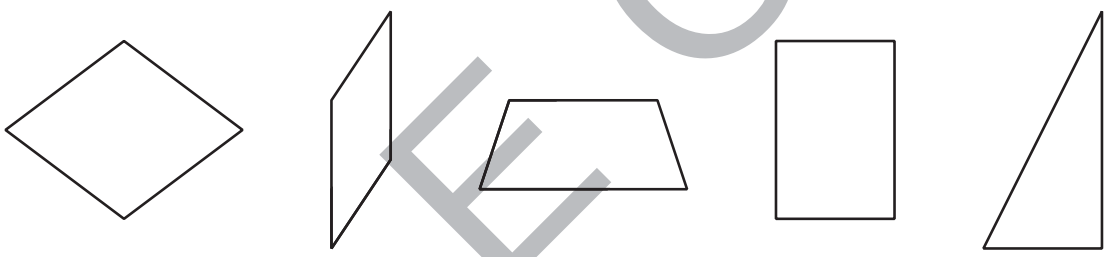

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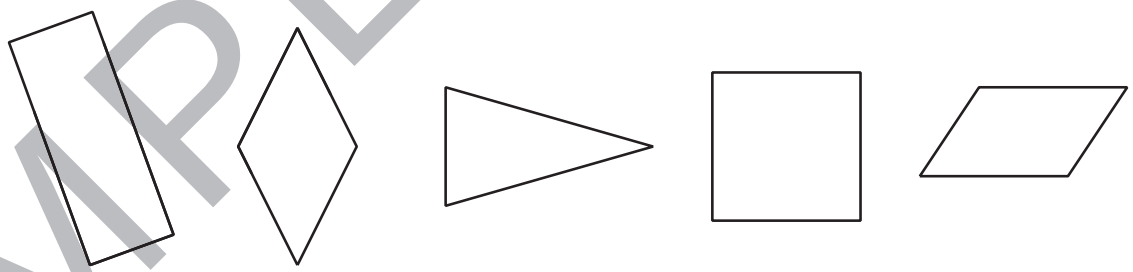

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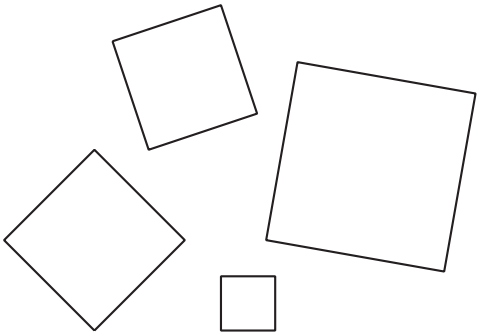

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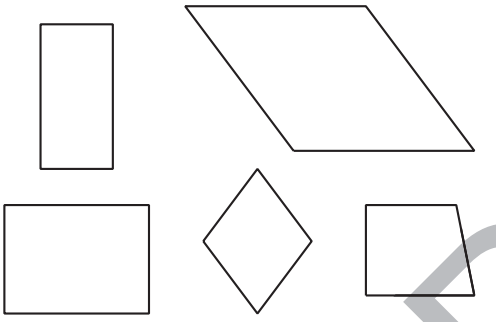

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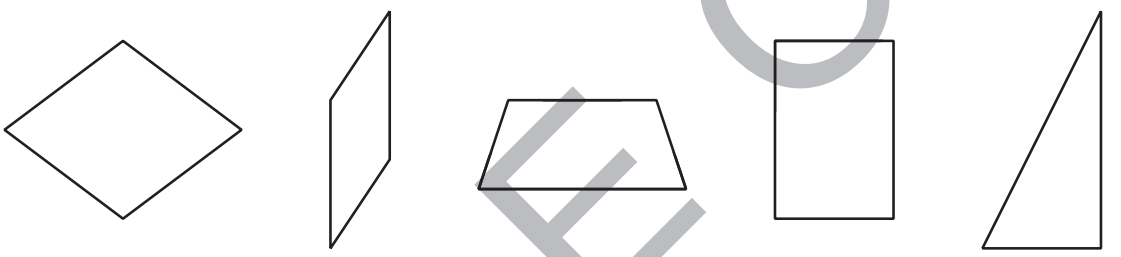

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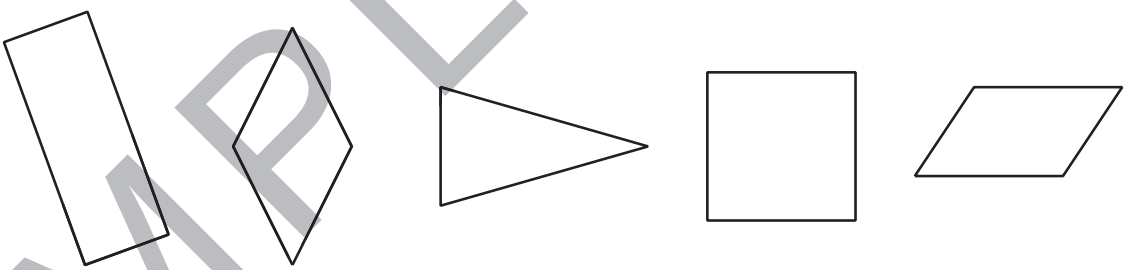

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Addressing CA CCSSM 3.G.1, 3.OA.7, 3.OA.9; practicing MP6

Attributes of Other Quadrilaterals

Let's describe and draw shapes in specific groups.

Warm-up

Number Talk: Divide by 7

Find the value of each expression mentally.

- $70 \div 7$
- $77 \div 7$
- $63 \div 7$
- $56 \div 7$

x P P

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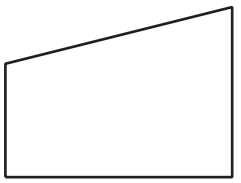

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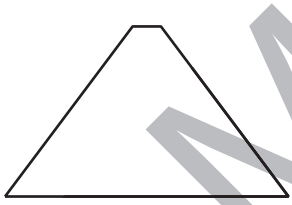

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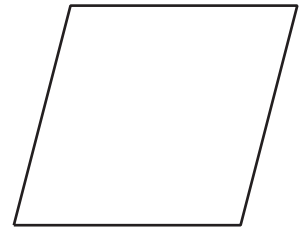

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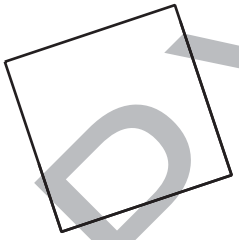

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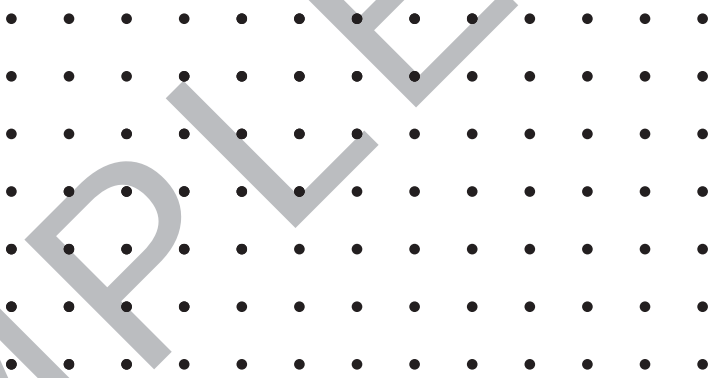

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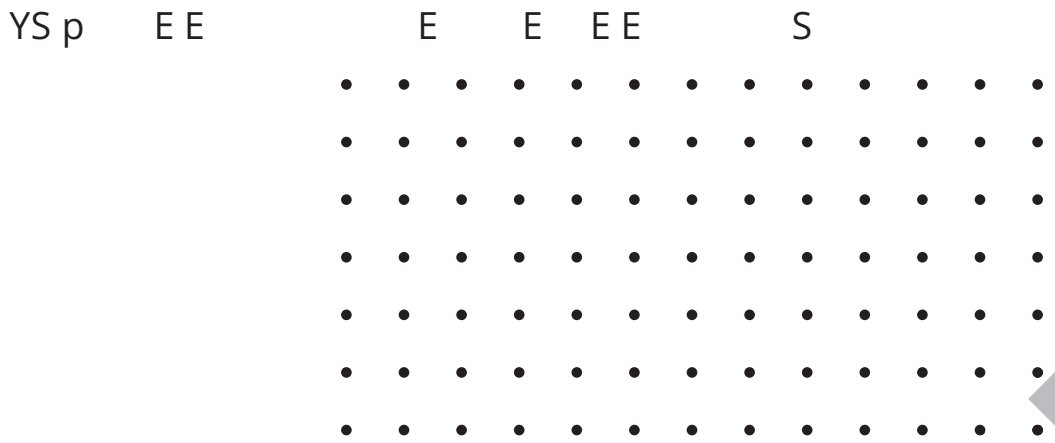

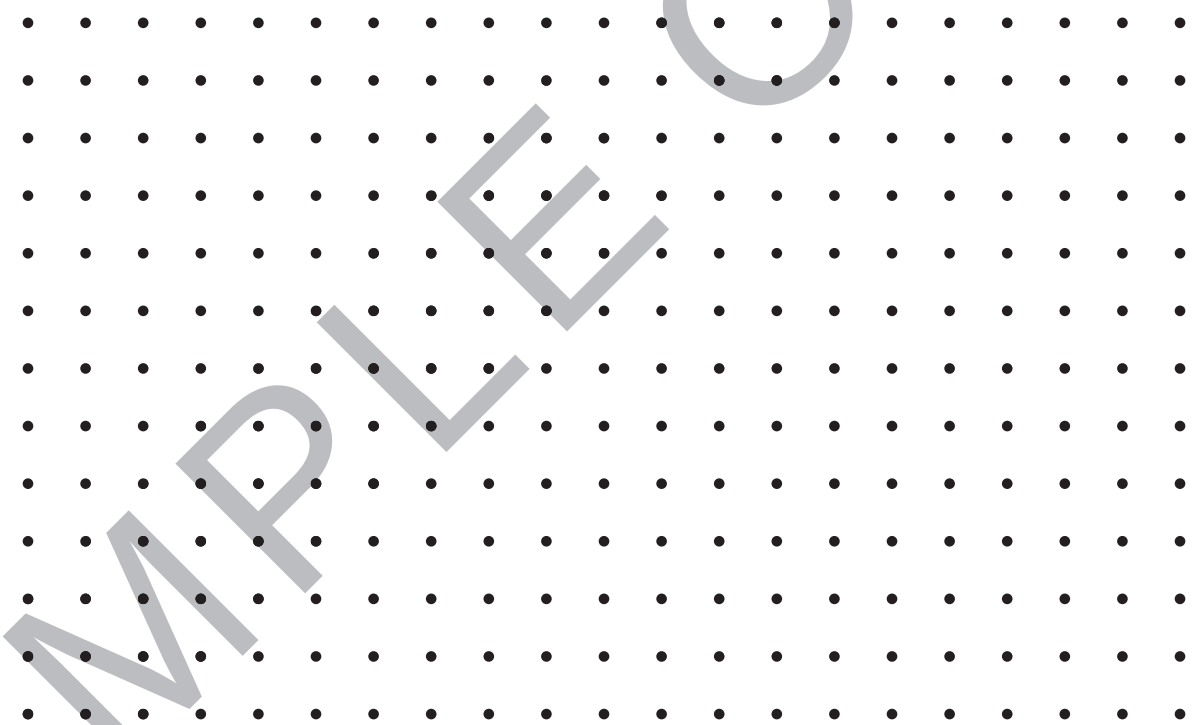

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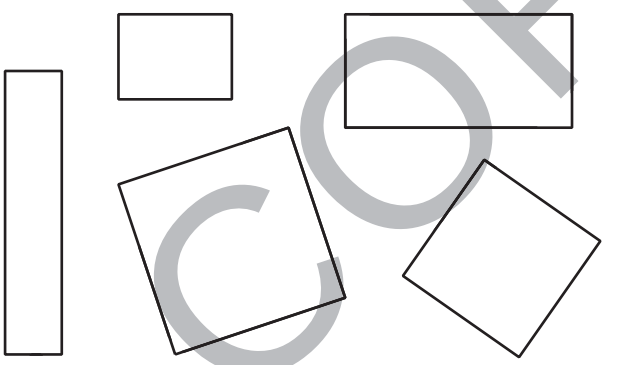

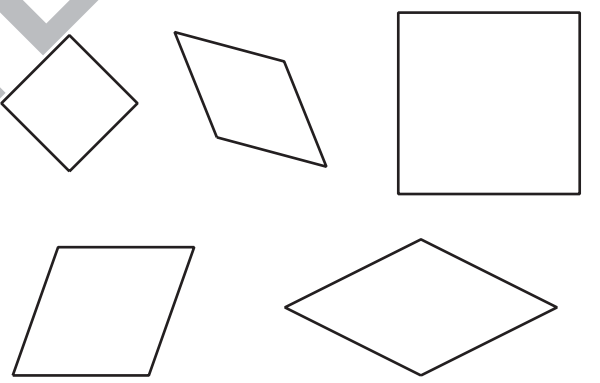

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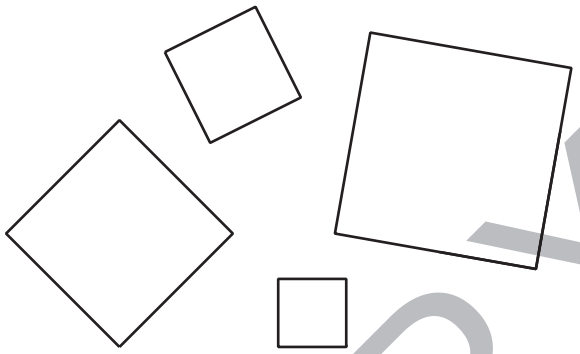


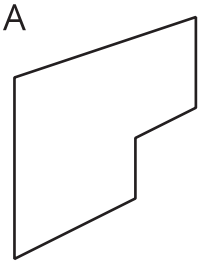
P

VVE

1 – R

S fi E E E kE

A



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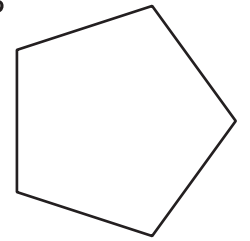

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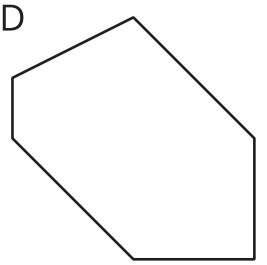

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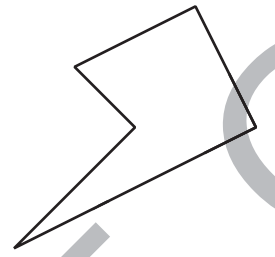

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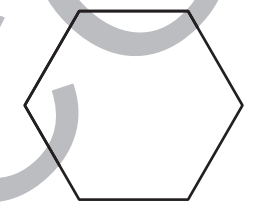
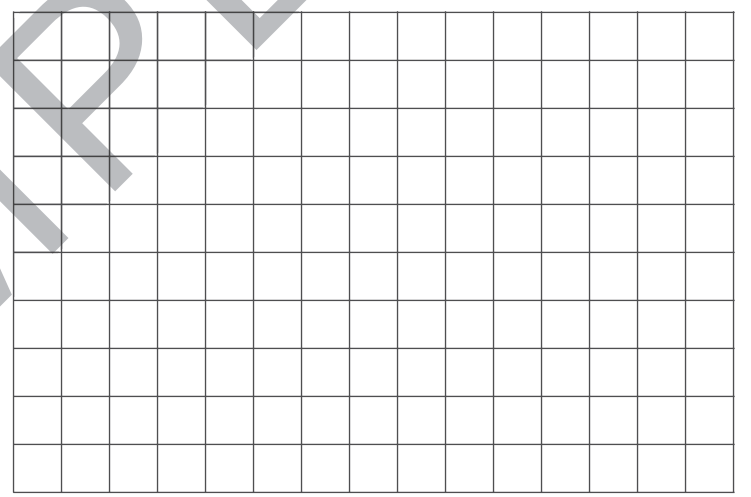
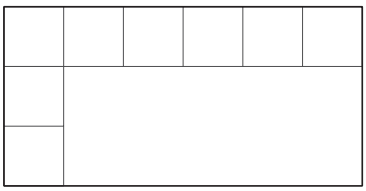


S p E E E E E–Sp E E E E E

–S



2 – R



Sr E E E E R
E S
Sfi E E E E E
E E E E E E
S

Sfi E E E E
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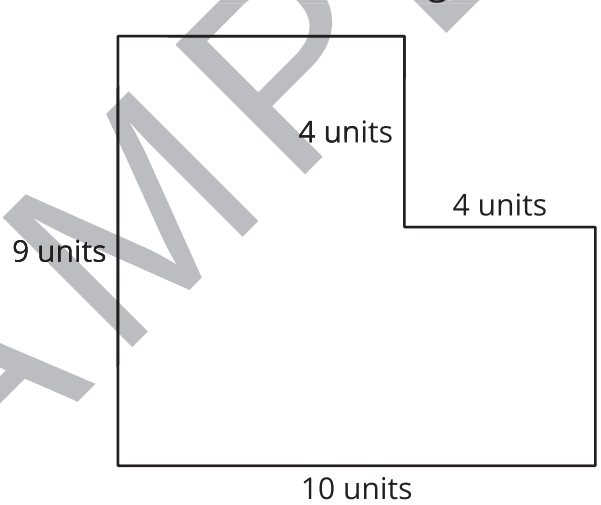
S

3 – R

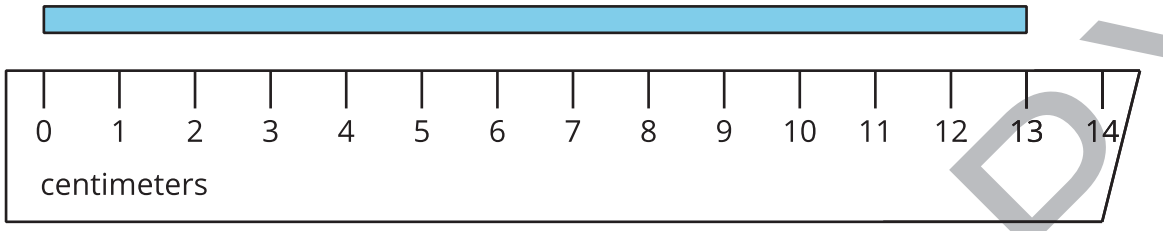
What is the area of the figure?



4 – R

r E E E E E S

S



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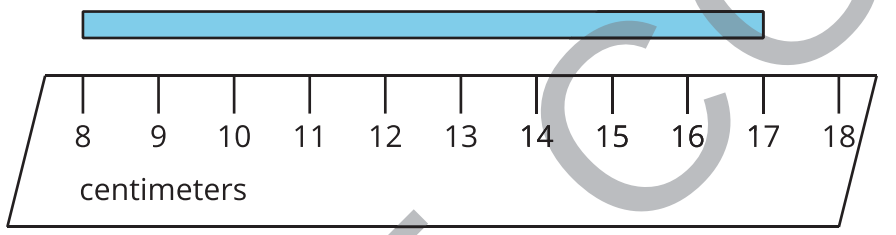

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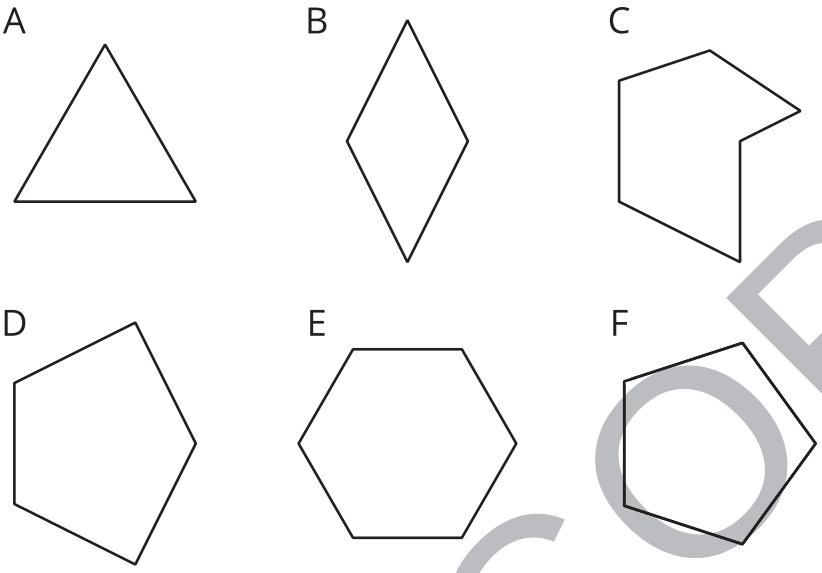
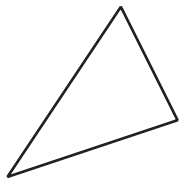


S fi E E Ea E k

S fi E E E E E E k

6

A



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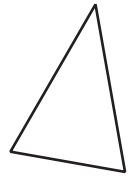

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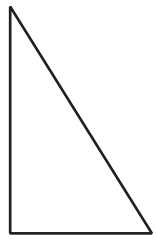

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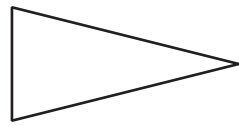


F

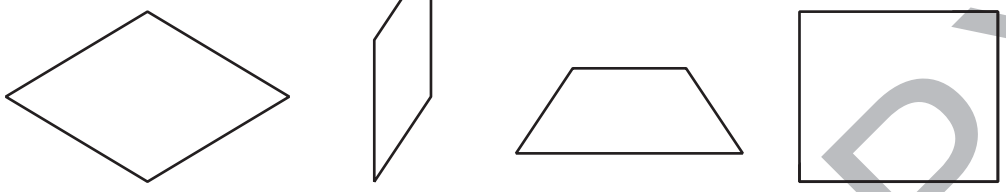
S fi E E EVE E kE

S fi E E EWE E E kE

7

© E E E E Ep E E S

A B C D



x E E E E E E E Ep E

quadrilateral. How would the answers help you figure out which

E E k

“ E E E E E S

mS

nS

oS

pS

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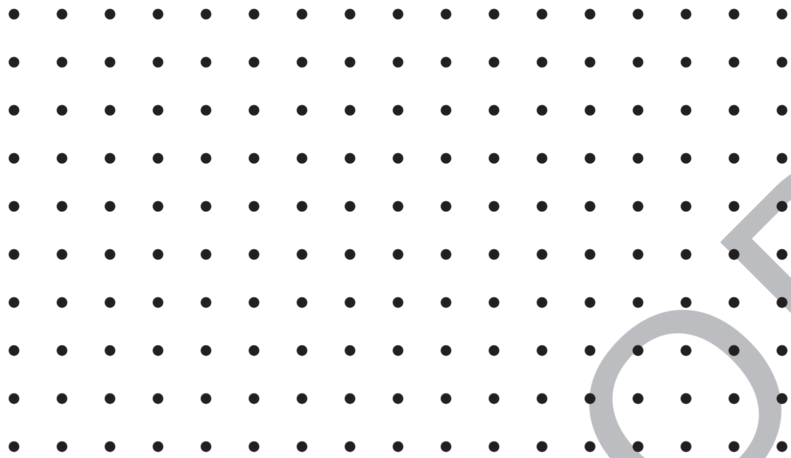

Sp EE E E E E E E EmSq
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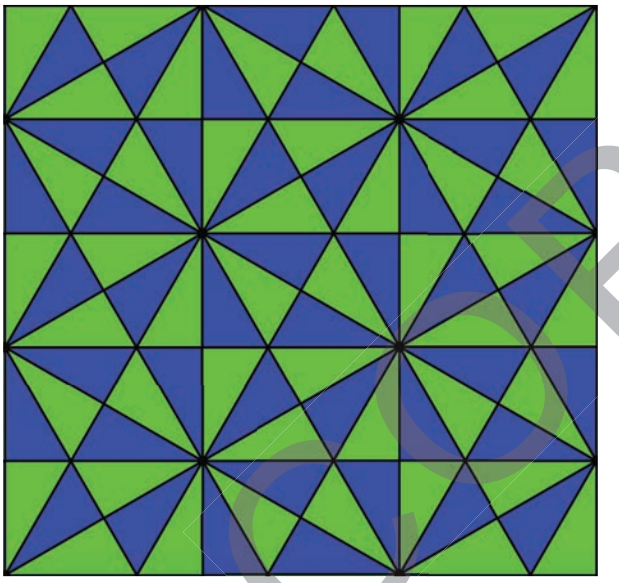

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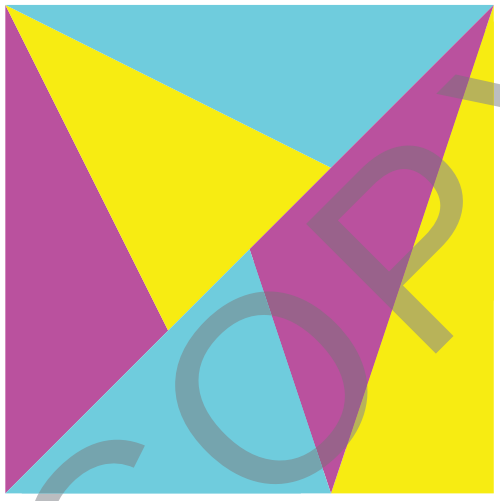


S WE E k

S YE E k

Addressing CA CCSSM 3.MD.8; building towards 3.MD.8; practicing MP8

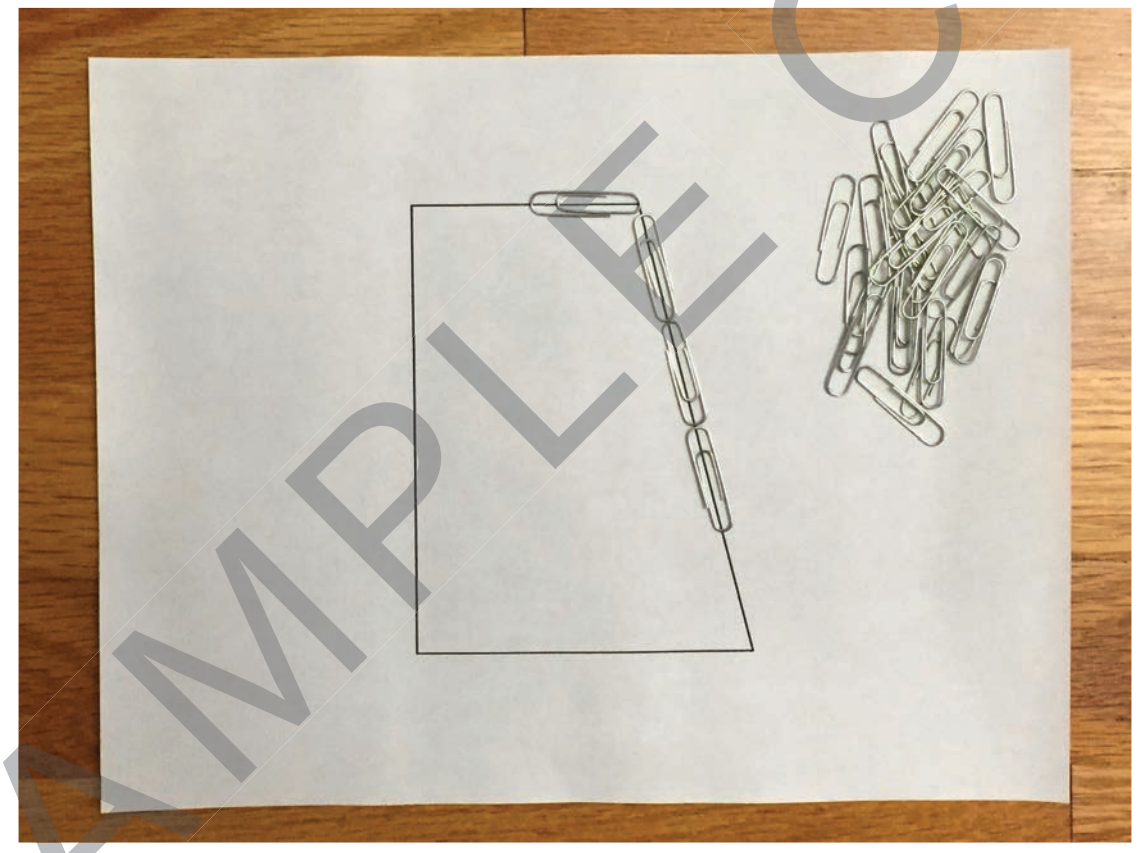
Distance around Shapes

Let's find the distance around shapes.

Warm-up

Notice and Wonder: Paper Clips and Shapes

What do you notice? What do you wonder?



m E

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ffi E E E E E E E E E E E S

Work with your group to find out which shape takes the most paper clips to build. Explain or show how you know. Record your findings here. Draw

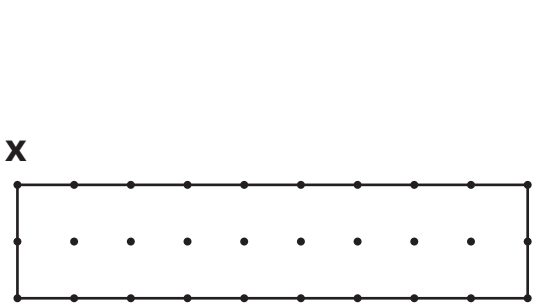
E E E E S

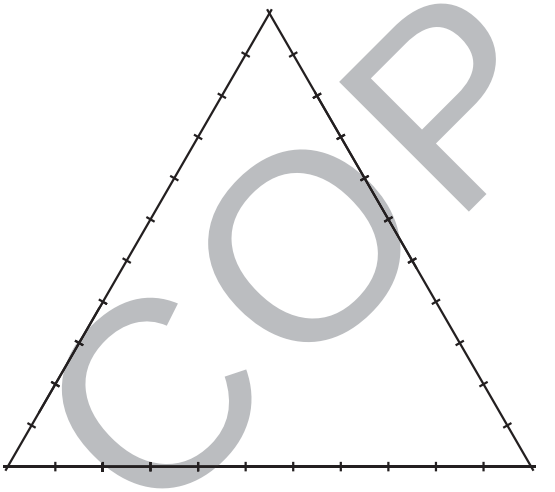
m

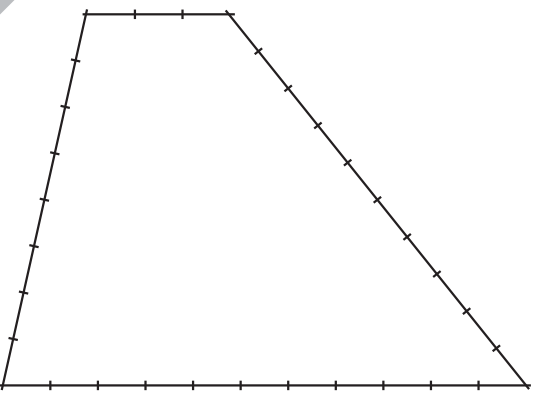
n

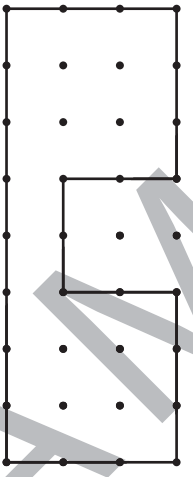
o

p









Addressing CA CCSSM 3.MD.8, 3.NBT.2; building towards 3.MD.8; practicing MP7

Same Perimeter, Different Shapes

Let's learn about shapes with the same perimeter.

Warm-up

True or False: Sums of Four Numbers

Decide whether each statement is true or false. Be prepared to explain your reasoning.

- $123 + 75 + 123 + 75 = 100 + 100 + 70 + 70 + 5 + 5 + 3 + 3$
- $123 + 75 + 123 + 75 = (2 \times 123) + (2 \times 75)$
- $123 + 75 + 123 + 75 = 208 + 208$
- $123 + 75 + 123 + 75 = 246 + 150$

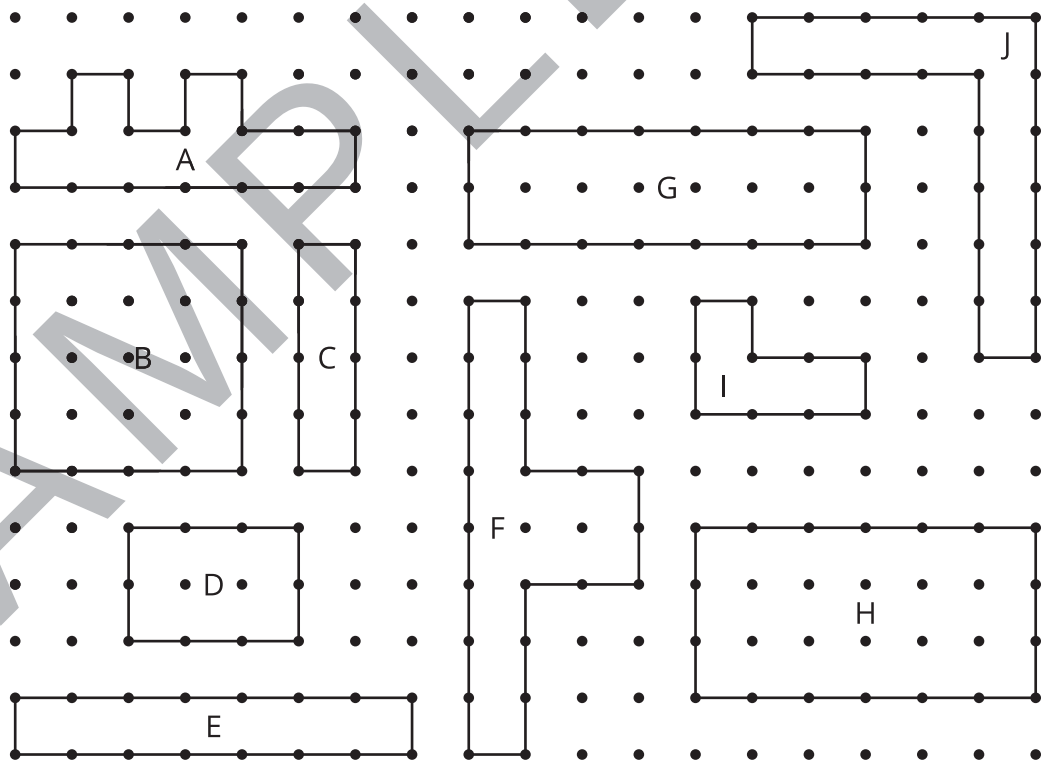
m E

x P P Ffi

1. Choose any 3 of the figures labeled A-J. Find the perimeter of each figure. Explain or show your reasoning.

2. Find 1 labeled figure that has the same perimeter as 1 of the 3 figures

E S



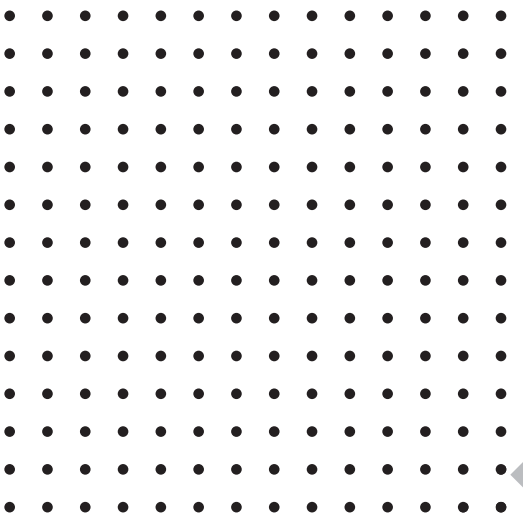
m EW

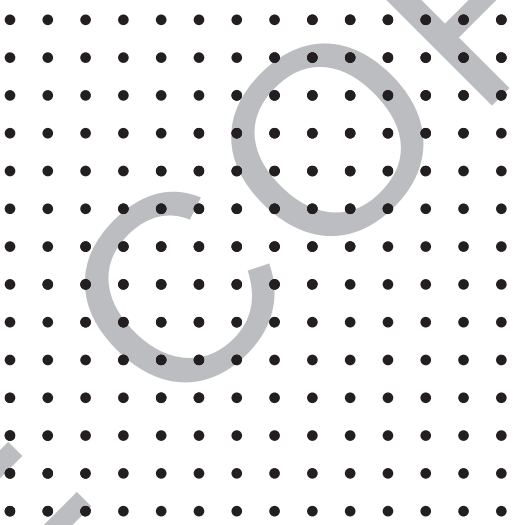
’ P P

VS p EWE E E E S

VWE

WbE





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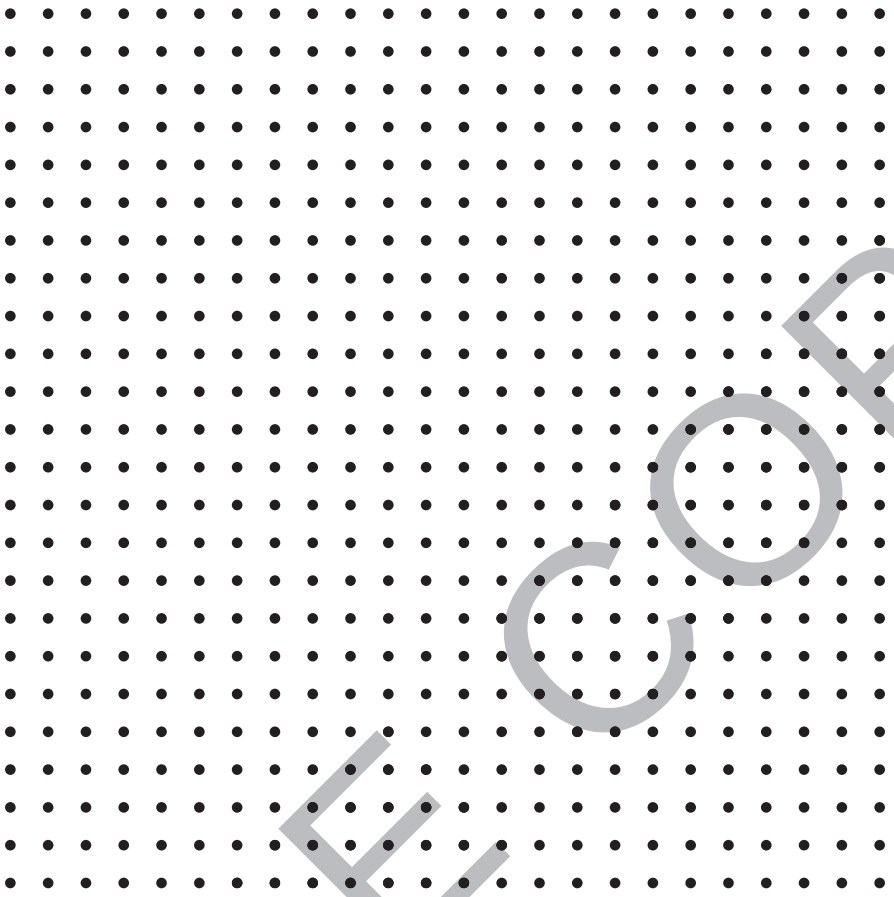
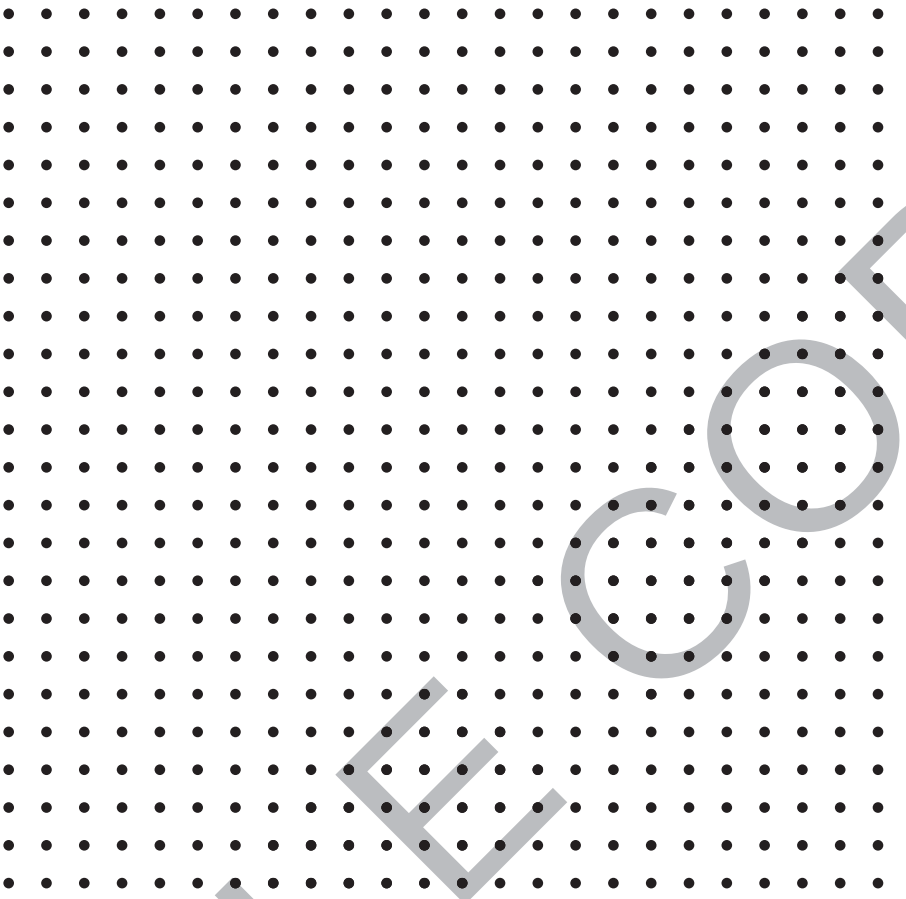
KH | iM Illustrative® Mathematics LEARN MATH FOR LIFE





they are different.

Addressing CA CCSSM 3.MD.8, 3.OA.7; practicing MP3 and MP7

Find the Perimeter

Let's find the perimeter of more shapes.

Warm-up

Number Talk: Decreasing Dividend

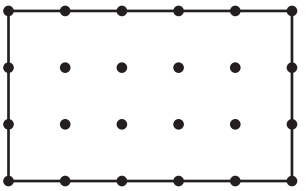
Find the value of each expression mentally.

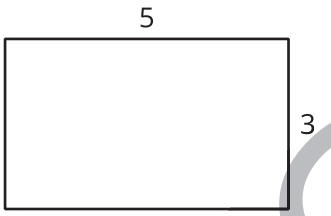
- $90 \div 9$
- $81 \div 9$
- $45 \div 9$
- $54 \div 9$

m E V

P Ffi P

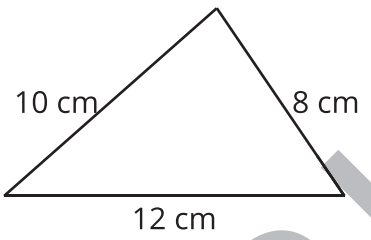
fi E E E kEfi E E E k



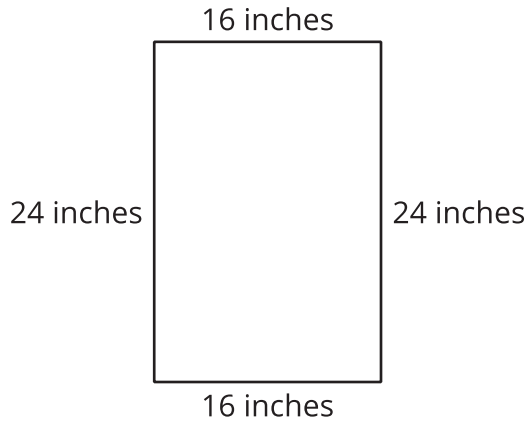


r E E E E E Sq E E E E S

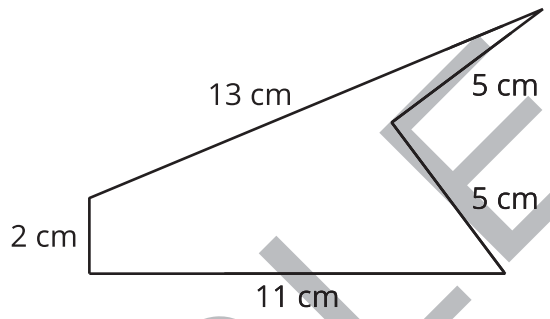
VS



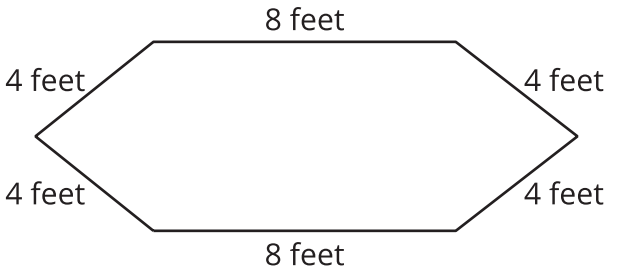
W8



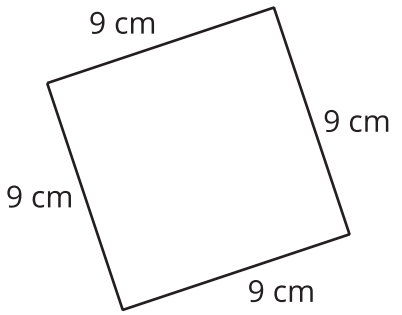
Y8



_S



aS



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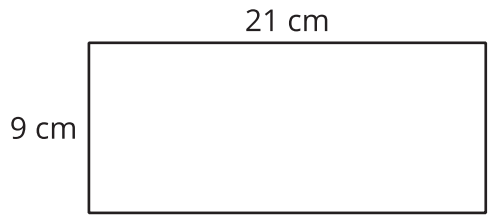
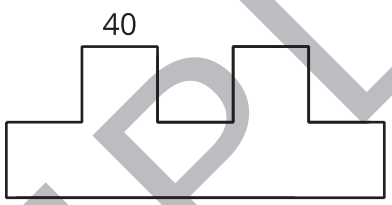
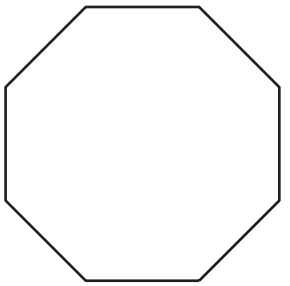
VS r E E E E E S q E E E E S



2. All the short sides of this figure are the same length, and all the angles are right angles. Find the perimeter of the figure. Explain or show your

S





Addressing CA CCSSM 3.MD.8; practicing MP2

Perimeter Problems

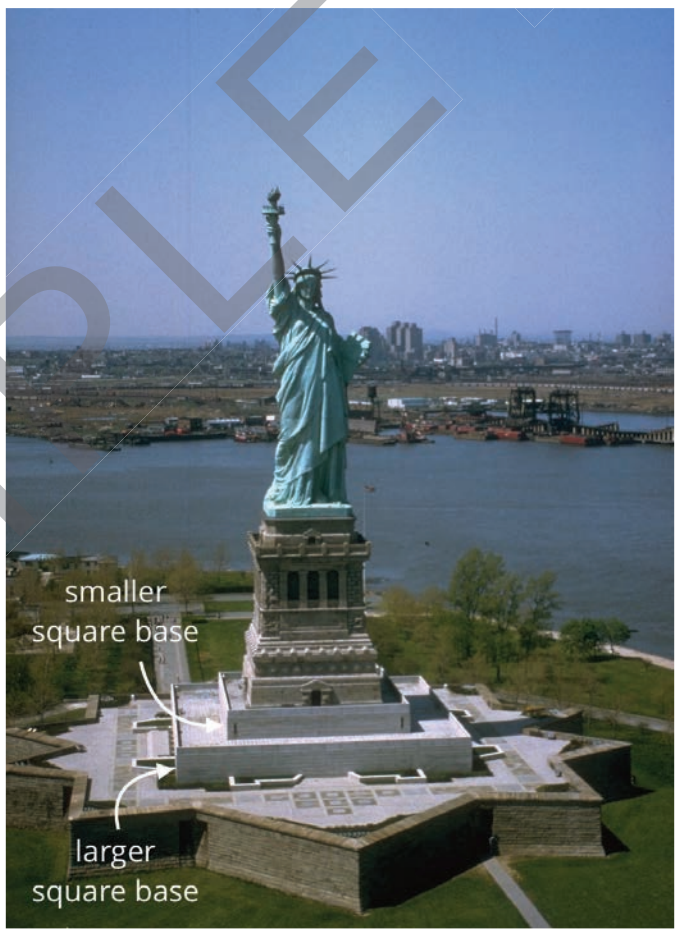
Let's solve problems about perimeter.

Warm-up

Estimation Exploration: Statue of Liberty

The Statue of Liberty has 2 square bases—1 larger than the other. The larger base has side lengths of 132 feet each.

Estimate the perimeter of the smaller square base.



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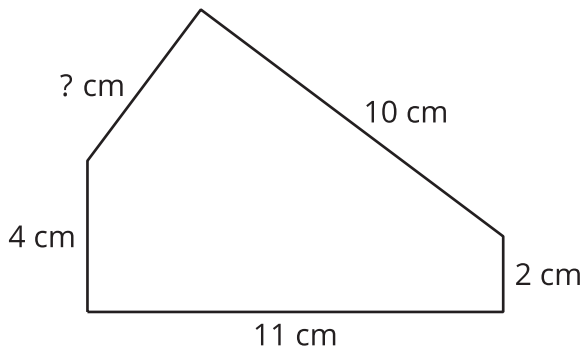

WS” E E E E E EabE SEfi E E E E E

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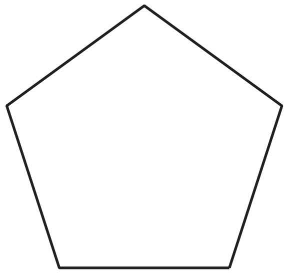

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— PP P v

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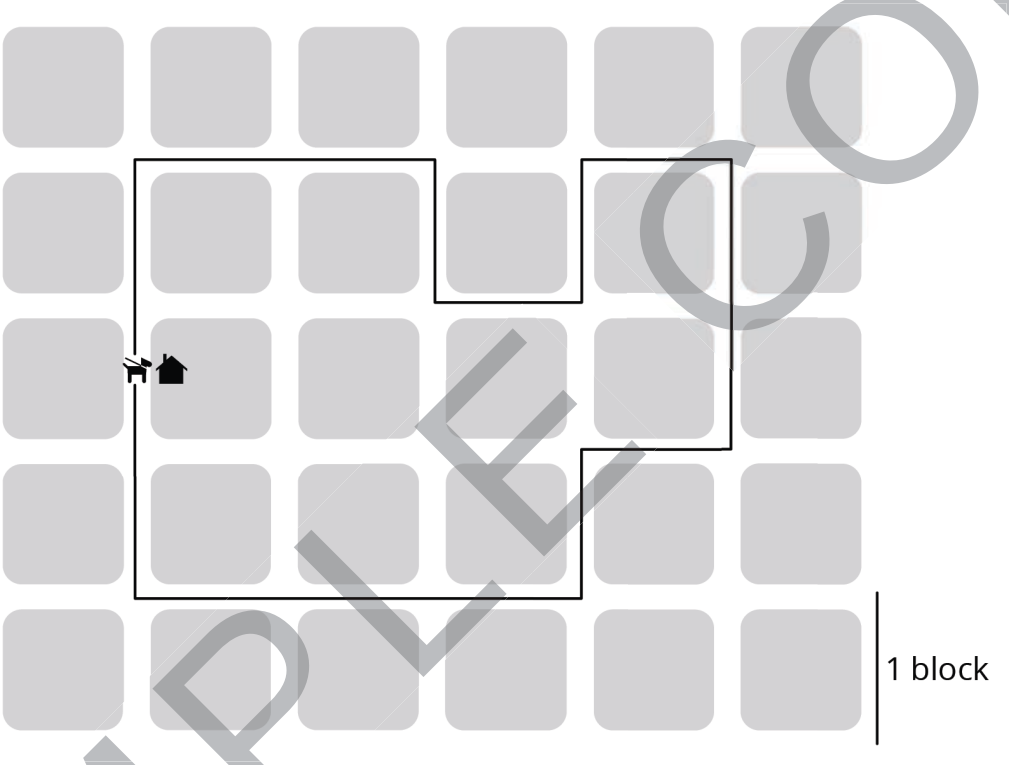
VS mE E E EcUE E E E E E E EVWUE E E
E St E E E E E E E E E E
E E E k

WS – E E E E SkC© E E EeE E St E E
E E E– E E E E E E E E k

3. A rectangular flower bed has a fence that measures 32 feet around. One side of the flower bed measures 12 feet. What are the lengths of the
E k

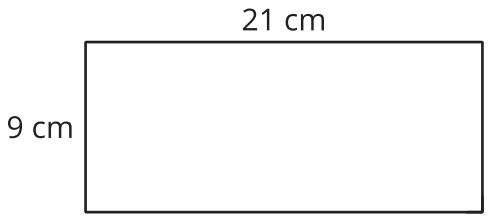
_S w E E E E E E SE E E E E Sft E E E
E k



aS mE E E EVUE E EdE Sq E E EVE E Sft
many tiles are needed to cover the floor of the room?

fi E E is the boundary of a flat shape.

We can find the length of a perimeter by adding the lengths of all the sides of E SEfi E E E E E E E E E E E E S

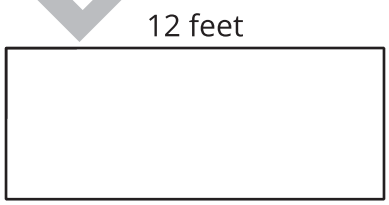


$$9 + 9 + 21 + 21$$

$$(2 \times 9) + (2 \times 21)$$

We used our knowledge of shapes to find the perimeter even when some side lengths were missing, and to use the perimeter to find missing side S

r E QE E E E E E E E EYWE QE E find the lengths of the three unlabeled sides.



mE E EWE E E E SE QE E EVE E E E EVW S

$$32 - 12 - 12 = 8$$

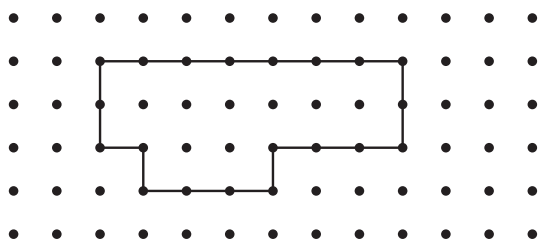
z E E E E EWE E E E E E EdE S

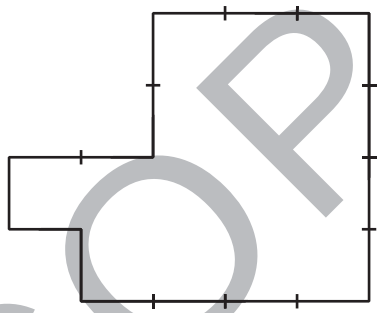
$$8 \div 2 = 4$$

P

1

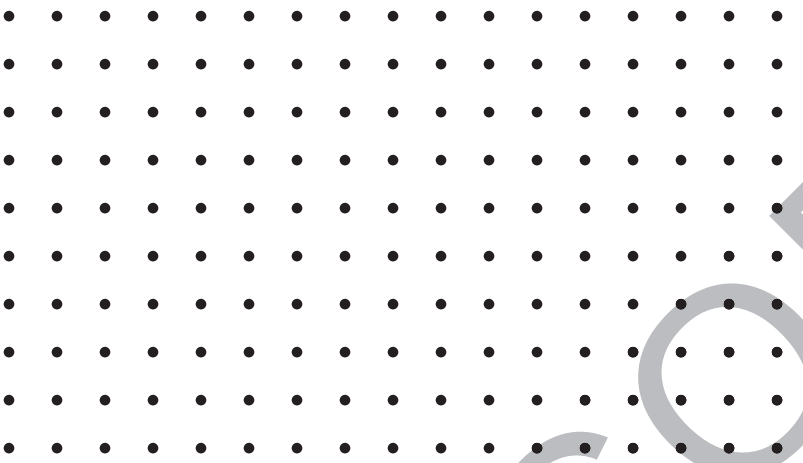
Find the perimeter of each figure.





2

a. Draw 2 different shapes with a perimeter of 20 units.



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3 E Ec Q x Ed

r E E E E E S q E E E E S

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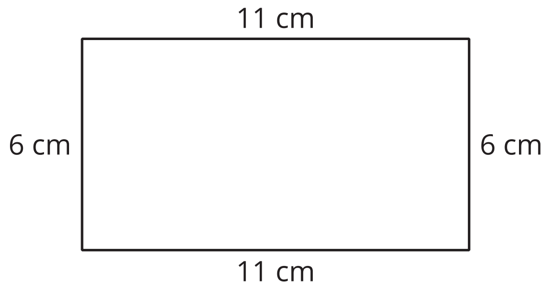

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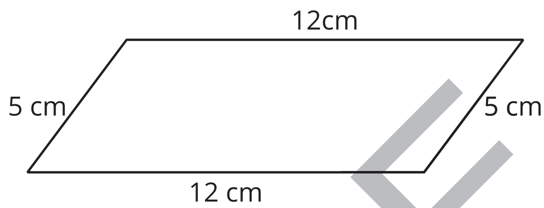

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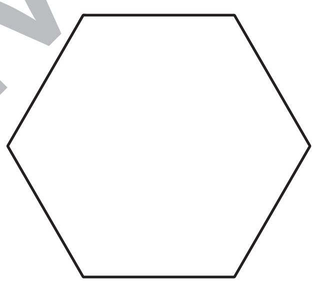

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a. What is the perimeter of the entire figure formed by all the
kEq E E S

b. What is the perimeter of the figure if you remove the skinny
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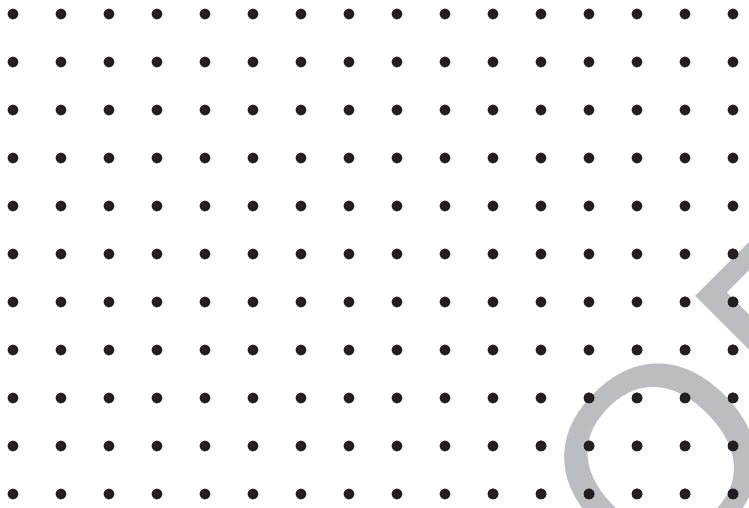

a. Draw some different shapes that you can find the perimeter of. Then find their perimeters.

So E E EE E E EE E E E

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c. Can you draw a pentagon or hexagon (or a figure with even more

NE E EE E E E E k

Unit 7, Lesson 10

Addressing CA CCSSM 3.MD.8, 3.OA.5, 3.OA.7, 3.OA.8; practicing MP1, MP3, MP6, MP7

Problem Solving with Perimeter and Area

Let's solve problems involving perimeter and area.

Warm-up

True or False: Divide in Parts

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $60 \div 6 = 10$

- $72 \div 6 = (60 \div 6) + (12 \div 6)$

- $78 \div 6 = (60 \div 10) + (18 \div 6)$

- $96 \div 8 = (80 \div 8) - (16 \div 8)$

m EV

Rope Off the Garden

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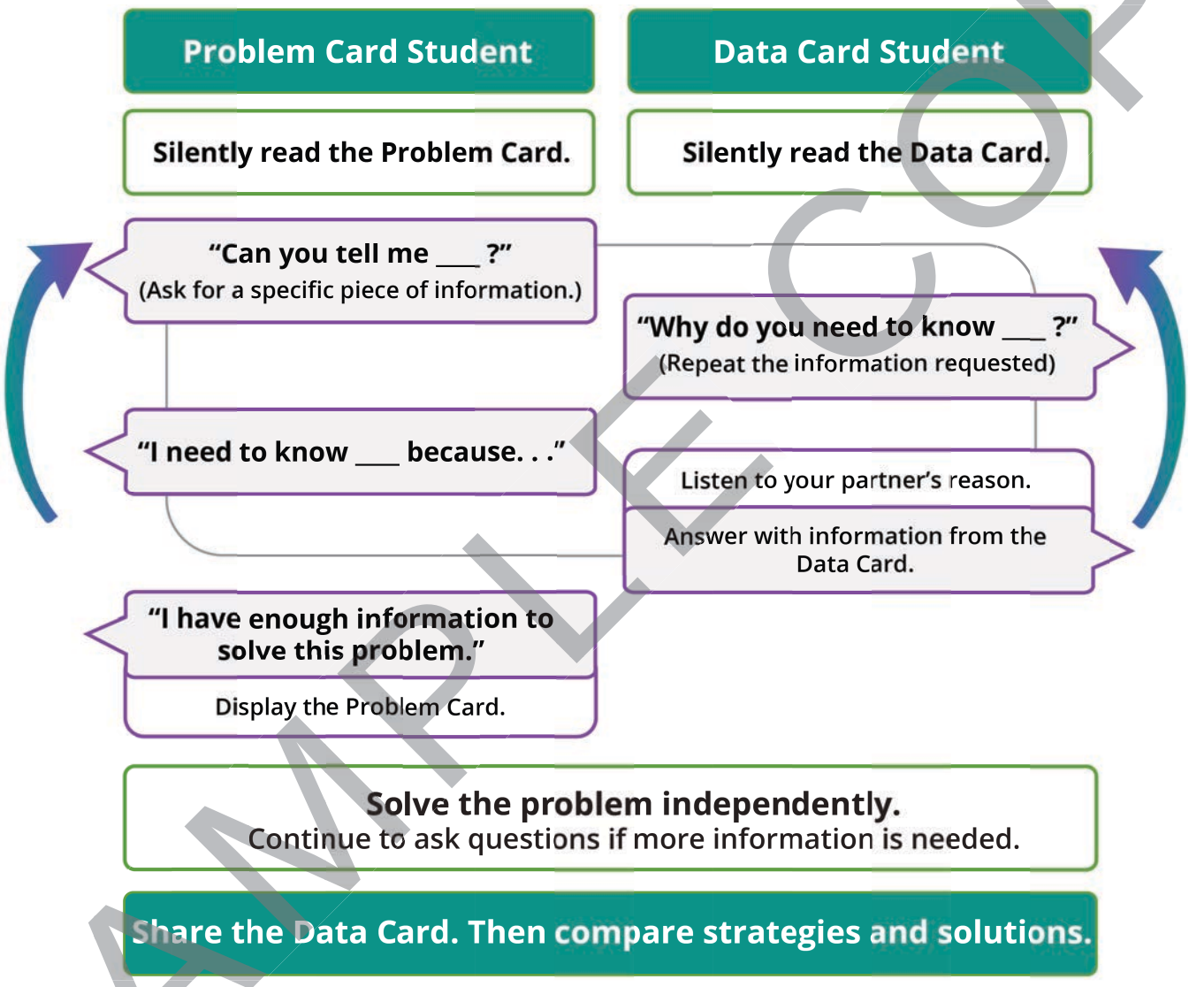
- Clare says she can use multiplication to find the length of rope Andre
  S
- Diego says he can use addition to find the length of rope Andre needs.

fi E E E E kEq E E E E S


Problem Card Student
Silently read the Problem Card.
"Can you tell me ___ ?"
(Ask for a specific piece of information.)
"I need to know ___ because. . ."
"I have enough information to solve this problem."
Display the Problem Card.
Data Card Student
Silently read the Data Card.
"Why do you need to know ___ ?"
(Repeat the information requested)
Listen to your partner's reason.
Answer with information from the Data Card.
Solve the problem independently.
Continue to ask questions if more information is needed.
Share the Data Card. Then compare strategies and solutions.

Addressing CA CCSSM 3.MD.8, 3.OA.7; practicing MP7

Rectangles with the Same Perimeter

Let's explore rectangles with the same perimeter.

Warm-up

Number Talk: Multiply to Divide

Find the value of each expression mentally.

- 5×5
- 10×5
- 2×5
- $85 \div 5$

m EV

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1. Draw as many different rectangles with a perimeter of 16 units as you
S

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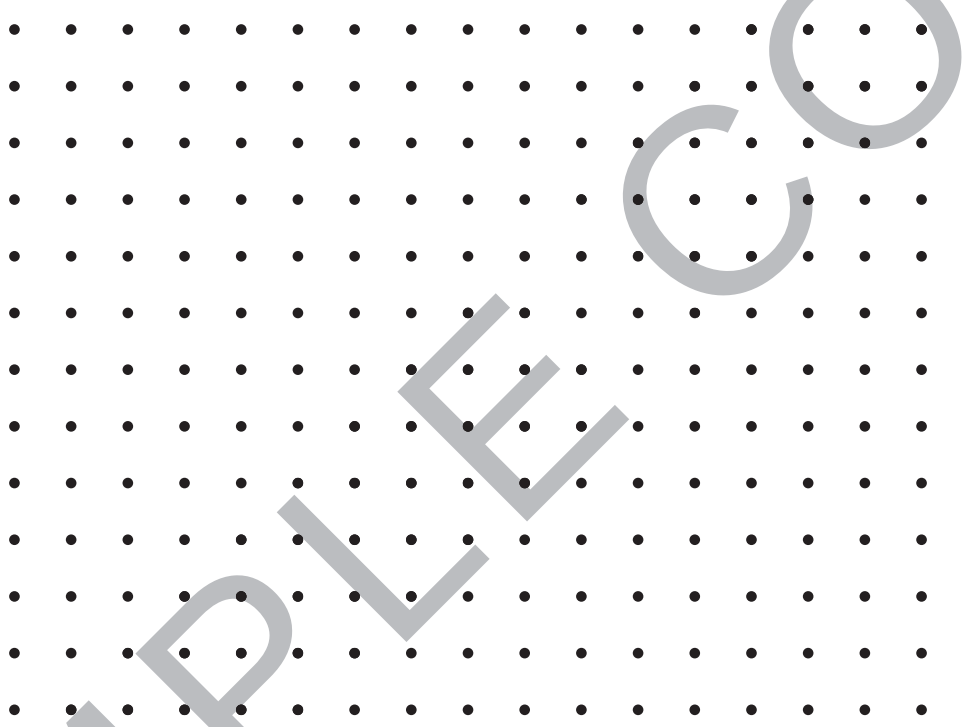
S



m EW

Same Perimeter, Different Area

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1. Draw 2 rectangles that each have the given perimeter but different
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   S VWE

   S WUE

   SWbE

   S Y_E

   So E E E S

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appropriate poster. Try to look for rectangles that are different from
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Unit 7, Lesson 12

Addressing CA CCSSM 3.MD.8, 3.OA.7; practicing MP7

Rectangles with the Same Area

Let's explore rectangles with the same area.

Warm-up

Number Talk: Divide in Parts

Find the value of each expression mentally.

- $40 \div 4$
- $60 \div 4$
- $80 \div 4$
- $96 \div 4$

m EV

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1. Draw as many different rectangles as you can with an area of 24 square
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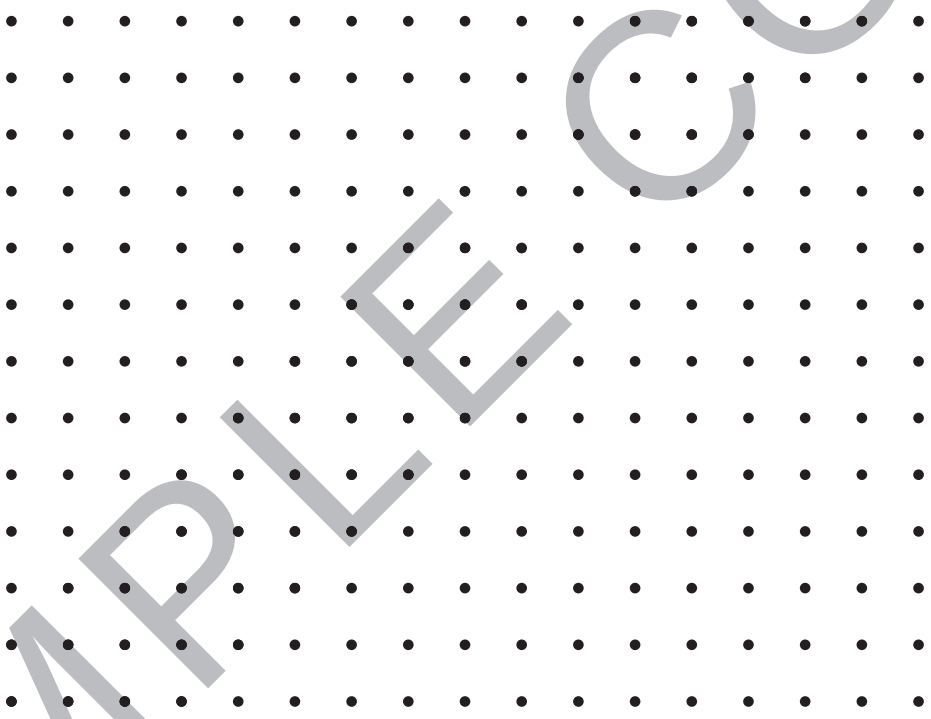

m EW

Same Area, Different Perimeter

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1. Draw 2 rectangles that each have the given area but different
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   S WUE E
   S_WE E
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WS o E E E E E E E E E E E E E
appropriate poster. Try to look for rectangles that are different from
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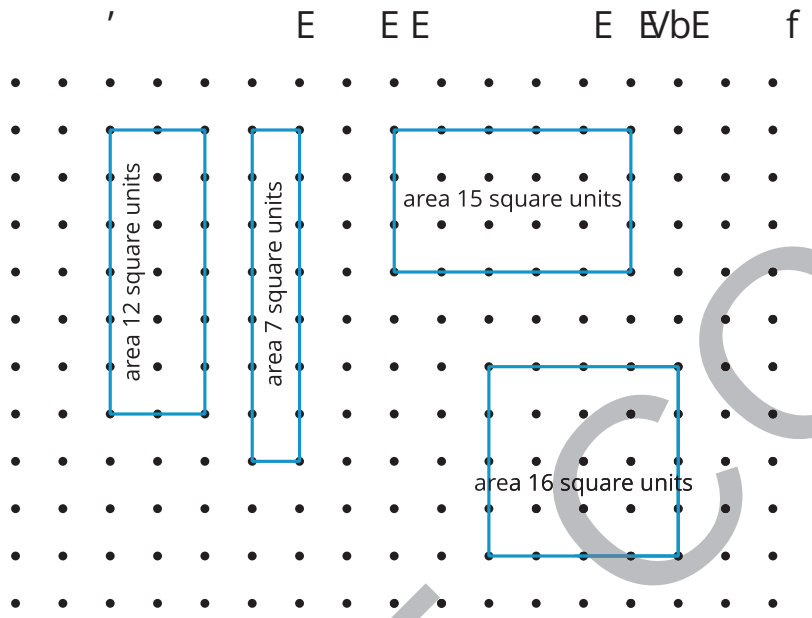
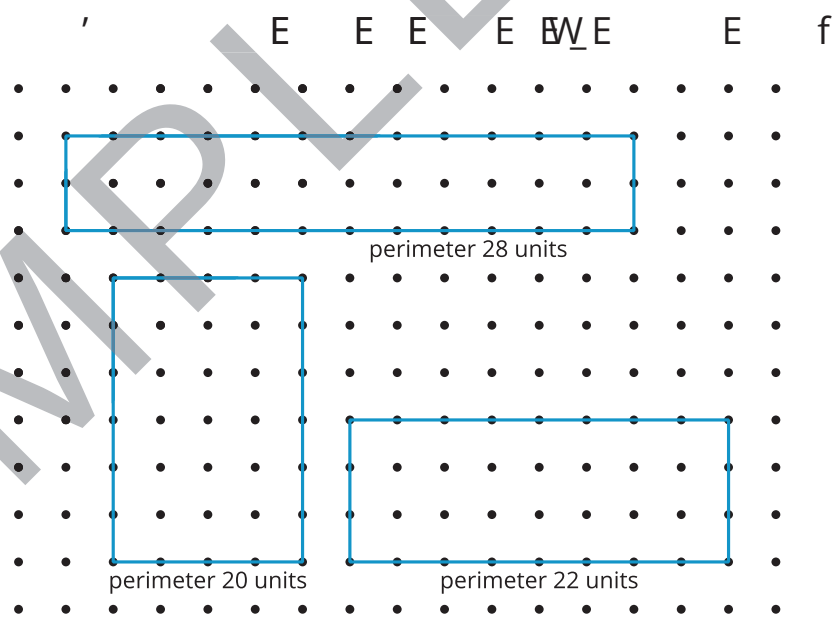
We drew rectangles with the same perimeter and different areas. We also drew rectangles with the same area and different perimeters.





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2 E EcQx EVV

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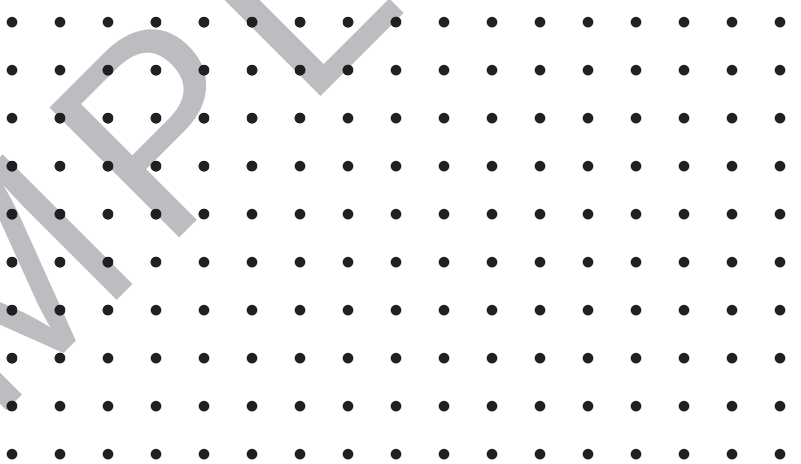
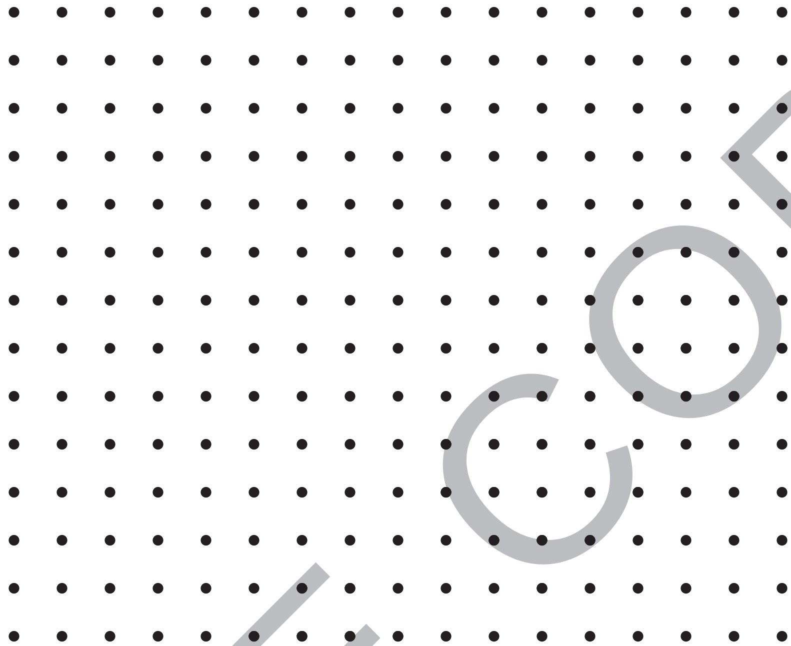
are different. What is the area of each rectangle?



3 E EcQEx EVW

p EWE E E E E E E E EYUE E E perimeters are different. What is the perimeter of each rectangle?



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Unit 7, Lesson 13

Addressing CA CCSSM 3.MD.8; building towards 3.MD.8; practicing MP2 and MP4

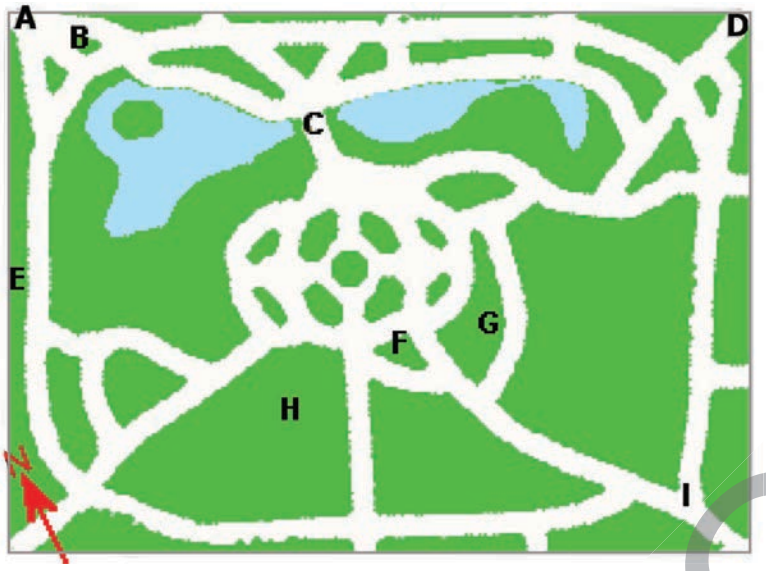
Shapes and Play

Let's design a park.

Warm-up

Notice and Wonder: A Park

What do you notice? What do you wonder?



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have the same perimeter, but a different area.

2. A planned rectangular field in a park will have an area of 48 square yards. Give 2 possible perimeters for the field.

Addressing CA CCSSM 3.G.1, 3.MD.8; building towards 3.G.1; practicing MP1 and MP6

Wax Prints

Let's analyze and make wax prints.

Warm-up

Notice and Wonder: Textiles

What do you notice? What do you wonder?

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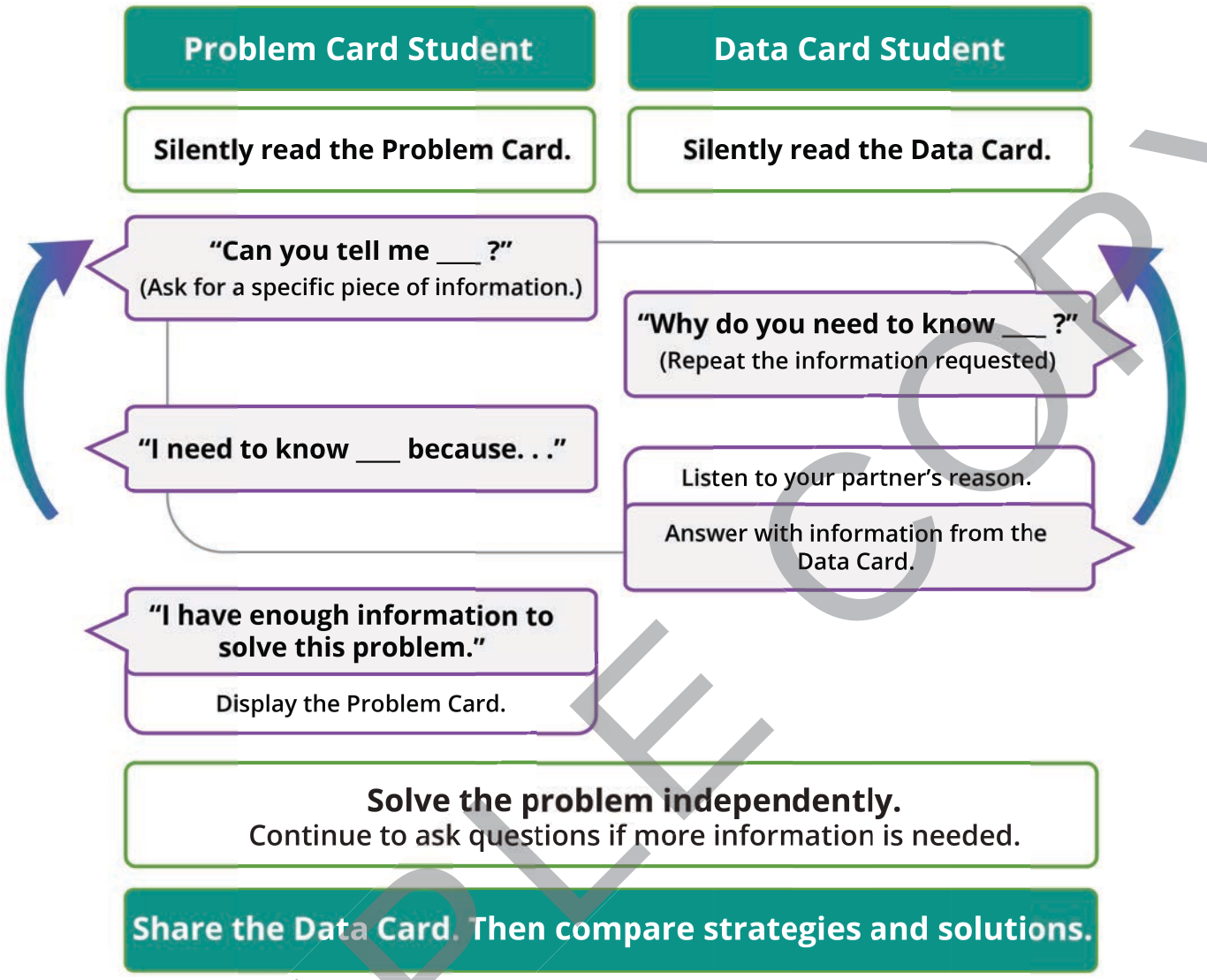
Problem Card Student
Data Card Student
Silently read the Problem Card.
Silently read the Data Card.
"Can you tell me ___ ?"
(Ask for a specific piece of information.)
"Why do you need to know ___ ?"
(Repeat the information requested)
"I need to know ___ because. . ."
Listen to your partner's reason.
Answer with information from the Data Card.
"I have enough information to solve this problem."
Display the Problem Card.
Solve the problem independently.
Continue to ask questions if more information is needed.
Share the Data Card. Then compare strategies and solutions.

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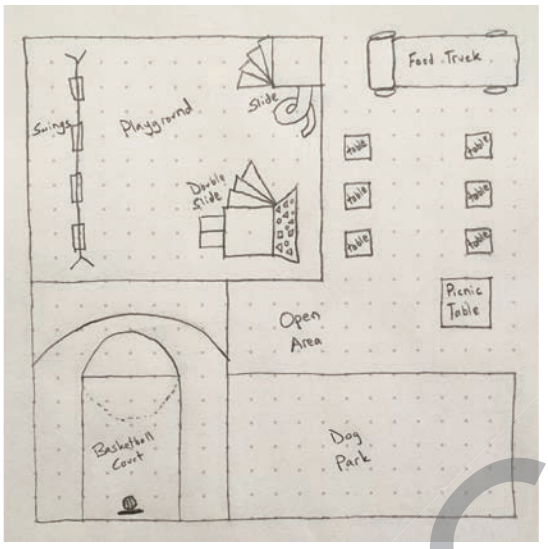

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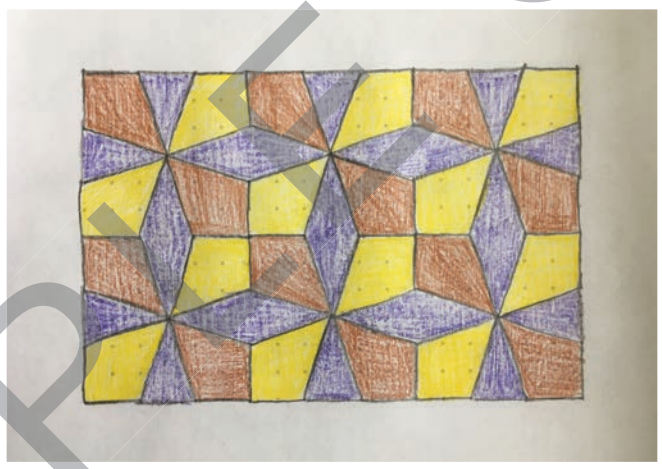

Addressing CA CCSSM 3.MD.5, 3.MD.8, 3.OA.8; building towards 3.MD.5, 3.MD.8; practicing MP4

A Space for Chickens

Let's plan a space to keep chickens.

Warm-up

Notice and Wonder: Chickens

What do you notice? What do you wonder?

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- Maya has a coop that will fit up to 10 chickens.

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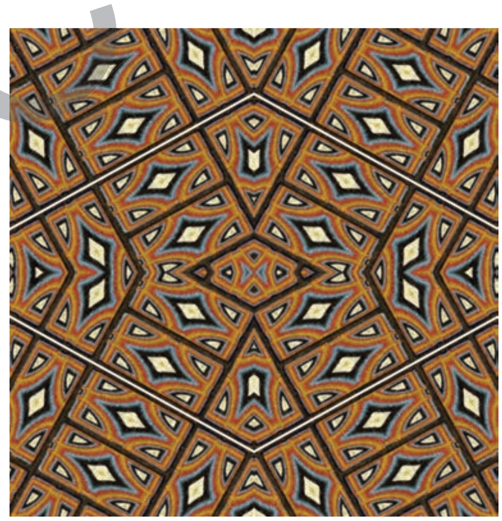

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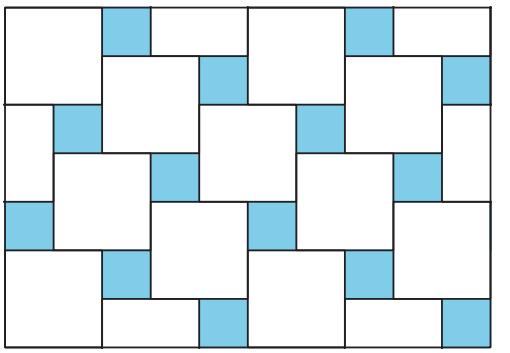


GRADE 3

UNIT

8

Putting It All Together

Content Connections

In this unit you will review what you have learned throughout the year including fractions, perimeter, area, measurement and data, and multiplication and division. You will make connections by:

- **Reasoning with Data** while using diagrams to represent and compare fractions and collecting measurements and using line plots to compare and display data.
- **Taking Wholes Apart, Putting Parts Together** while solving problems about perimeter and area and using information learned about fractions to measure and compare measurements accurately.
- **Discovering Shape and Space** while describing, analyzing and comparing quadrilaterals and comparing unit fractions using different visual models and area models.
- **Exploring Changing Quantities** while playing games to practice multiplication and division.

Addressing the Standards

As you work your way through **Unit 8 Putting It All Together,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 9, 12, and 14
MP2 Reason abstractly and quantitatively.	Lessons 4, 7, and 10
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3 and 4
MP4 Model with mathematics.	Lessons 4, 5, and 6
MP5 Use appropriate tools strategically.	Lesson 6
MP6 Attend to precision.	Lessons 1, 2, 3, 6, 7, 9, 12, and 15
MP7 Look for and make use of structure.	Lessons 1, 2, 8, and 11
MP8 Look for and express regularity in repeated reasoning.	Lessons 9 and 13

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fractions of Shape and Time • Square Tiles • Fractions as Relationships	**3.NF.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.	Lessons 1 and 3
• Fractions of Shape and Time • Unit Fraction Models	**3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	Lessons 1, 2, and 3

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Fractions of Shape and Time • Fractions as Relationships • Unit Fraction Models	**3.NF.3** Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., ½ = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* D. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Lessons 2 and 3
• Patterns in Four Operations	**3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Lessons 5 and 15
• Number Flexibility to 100 for All Four Operations	**3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	Lessons 12 and 13
• Number Flexibility to 100 for All Four Operations	**3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8.*	Lessons 12 and 13

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations	**3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Lessons 10, 12, and 13
• Number Flexibility to 100 for All Four Operations	**3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?.*	Lessons 12 and 13
• Number Flexibility to 100 for All Four Operations	**3.OA.5** Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Lesson 9
• Number Flexibility to 100 for All Four Operations	**3.OA.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	Lesson 10
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Lessons 8, 9, and 11
• Patterns in Four Operations • Number Flexibility to 100 for All Four Operations • Analyze Quadrilaterals	**3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Lesson 5

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Represent Multivariable Data	**3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	Lesson 7
• Represent Multivariable Data • Measuring	**3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	Lesson 14
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.MD.7** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	Lesson 9
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.MD.7b** Relate area to the operations of multiplication and addition. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning.	Lesson 4

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Number Flexibility to 100 for All Four Operations • Square Tiles	**3.MD.7d** Relate area to the operations of multiplication and addition. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	Lesson 4
• Analyze Quadrilaterals	**3.MD.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Lesson 4

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Addressing CA CCSSM 3.NF.1-2; practicing MP6 and MP7

Estimation Explorations with Fractions

Let's explore estimations with fractions.

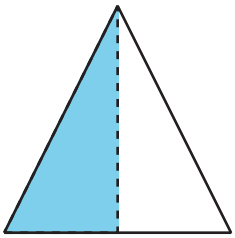
Warm-up

Which Three Go Together: Fractions

Which 3 go together?

A

B



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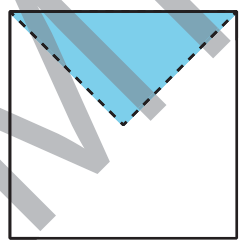

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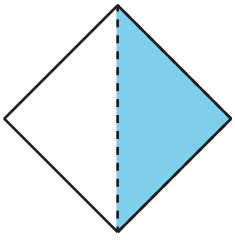

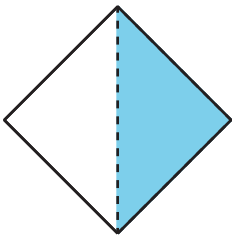

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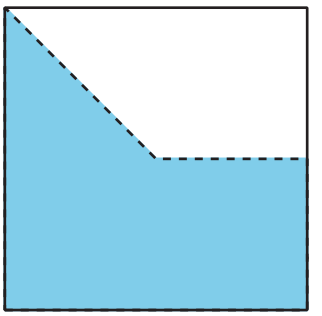

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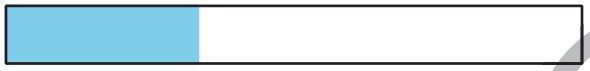


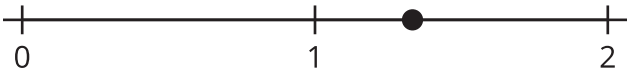
' E E E E f

y	y	y

m

ff Fff qP P

fi E E E E E E E E E k



’ E E E E f

y	y	y

Addressing CA CCSSM 3.NF.2-3; practicing MP6 and MP7

Create Your Own Number Line

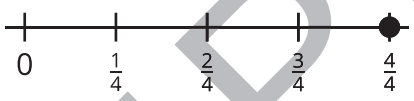
Let's create number lines and compare fractions.

Warm-up

Which Three Go Together: Fractions on Number Lines

Which 3 go together?

A



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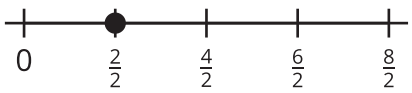

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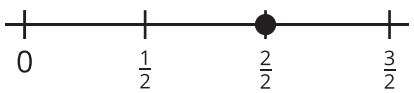

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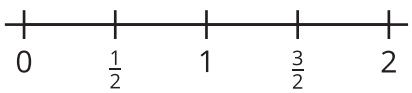


m EV

P P P P

Create a long number line on the floor.

x E E E E E E E E E E E Sn
E E E E S

U V W $\frac{1}{2}$ $\frac{1}{3}$ $\frac{6}{2}$ $\frac{12}{3}$ $\frac{1}{4}$ $\frac{5}{4}$ $\frac{6}{6}$ $\frac{5}{6}$ $\frac{9}{8}$ $\frac{15}{8}$ $\frac{5}{3}$ $\frac{18}{6}$ $\frac{2}{8}$

m EW

P P

fi EbE E E E E E E E E
SEu EWE E E E EV4Q<QE =NS

VS

WS

YS

_S

aS

bS

o EWE E E SE• E QE QE E E E
E E E S

Addressing CA CCSSM 3.NF.1-3; building on 3.NF.1-3; practicing MP3 and MP6

Fractions Round Table

Let's discuss fractions.

Warm-up

What Do You Know about $\frac{1}{8}$?

What do you know about $\frac{1}{8}$?

m EV

fi P P

p E E E EYE E E E S

’ EVfEs E E E E E E E E E Q Q
E E E E E E E E E E Sffi E
E E E E E S

’ EVfEs E E E E E E E E E Q Q
E E E E E E E E E E E E E

first round. You can change your response in the next round.

’ EYfE” E E E E E E E E E Q Q
E E E E E E E E E E E S

’ E E E E E E E E E E S

statement	round 1	round 2	round 3
A A fraction is a number less than 1.	agree disagree unsure	agree disagree unsure	agree disagree unsure
B A fraction can be located on a number line.	agree disagree unsure	agree disagree unsure	agree disagree unsure
C The numerator tells us the size of the part.	agree disagree unsure	agree disagree unsure	agree disagree unsure

statement	round 1	round 2	round 3
D The denominator tells us the number of parts.	agree disagree unsure	agree disagree unsure	agree disagree unsure
E Whole numbers are fractions.	agree disagree unsure	agree disagree unsure	agree disagree unsure
F Fractions are whole numbers.	agree disagree unsure	agree disagree unsure	agree disagree unsure
G One half is always greater than one third.	agree disagree unsure	agree disagree unsure	agree disagree unsure
H Fractions can be used to describe a length.	agree disagree unsure	agree disagree unsure	agree disagree unsure

Addressing CA CCSSM 3.MD.7b, 3.MD.7d, 3.MD.8; building on 3.MD.5-7, 3.MD.8; practicing MP2, MP3, MP4

Tiny House: Design and Solve

Let's design a tiny house.

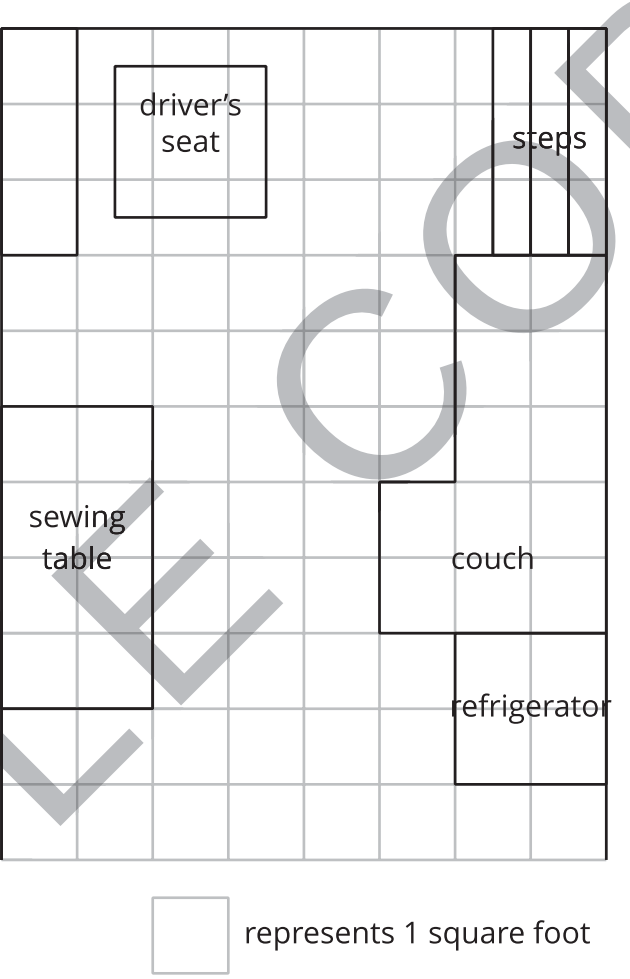
Warm-up

Notice and Wonder: Tiny Houses

What do you notice? What do you wonder?

’ P P P

t E E E E E E E E E E S



VSo E E E E E E E E E SEn E E
E E E E E E E E E E
E E S

E

dE × WUE

dE × VUE

cE × V_E

dE × dE

W S o E E E E E S n E E f

- Different rooms.
- fi E E E S
- r E E E E E E E E S

m EW

x bP bP Px

VS fi EW E E E E E S

S © E E E E S

S © E E E E S

WS fi E E E E E E E E E E E
E Sy E E E E E E E S

YS r E E E Sm E E E E E E
S

Unit 8, Lesson 5

Addressing CA CCSSM 3.NBT.2, 3.OA.8; practicing MP4

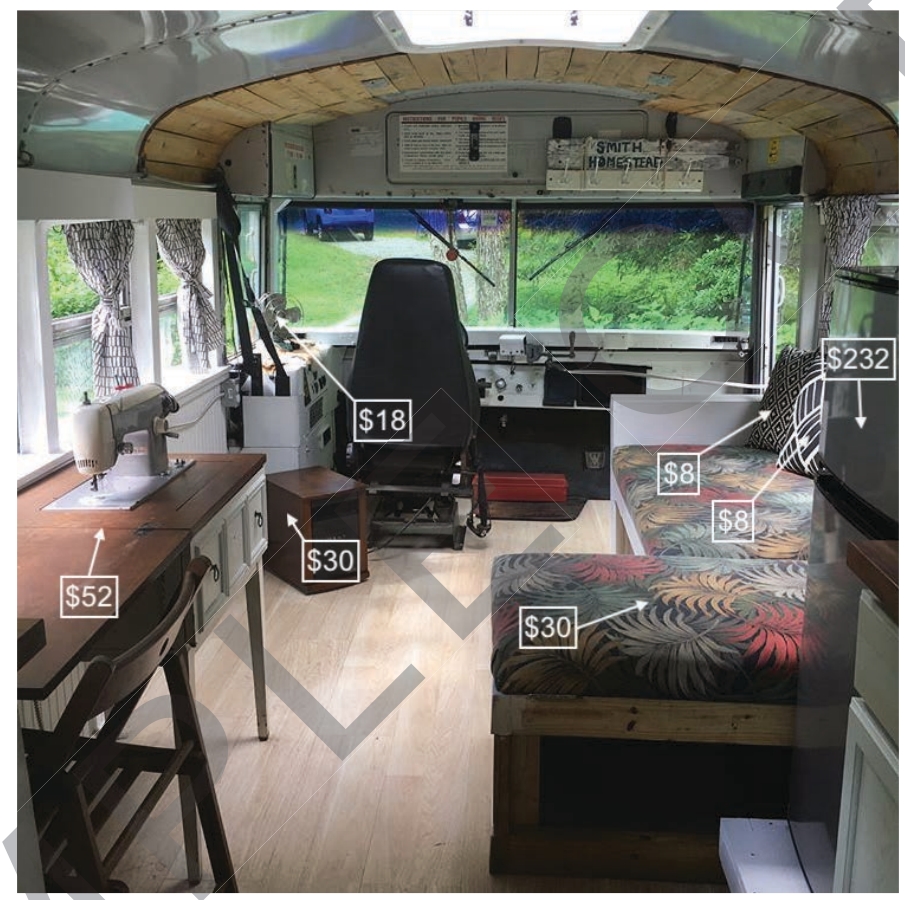
Tiny House: Cost

Let's calculate the cost of finishing a room in a tiny house.

fi R

ff Fff qP— P P P

fi E E E E E E E E E E E E k



’ E E E E f

y	y	y

m E

P P— v

Choose a room from your tiny house to finish. Use the cost sheet to calculate the cost of finishing the room in your tiny house. Your budget is $1,000.

flooring:	$5 for each square foot $4 for each square foot $2 for each square foot
	$25 for each gallon (up to 400 square feet)
E	$379
	$138
E E E	$112
E	$253
E E E	$227
	$90 for each foot
	$20 for each foot

Q Q Q Q S	– E S E E S
	$389
E	$174
	– E S E E S
E T	$225
	$189

Building towards CA CCSSM 3.MD.3; practicing MP4, MP5, MP6

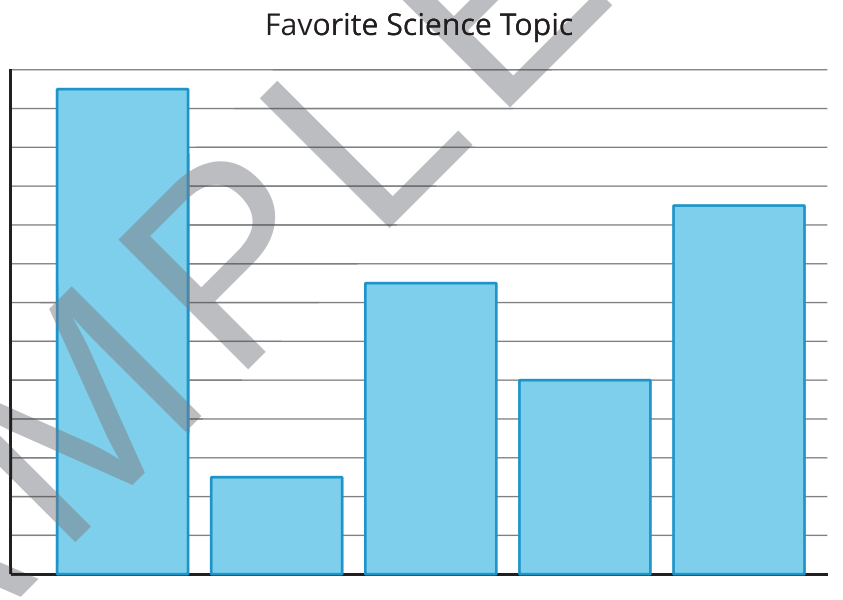
Survey the Class, Survey the School

Let's survey a large group.

Warm-up

Notice and Wonder: Survey

What do you notice? What do you wonder?



m EV

— P P

o E E E E E E E E E E E SEuE
f

E

E EbE E

Unit 8, Lesson 7

Addressing CA CCSSM 3.MD.3; building towards 3.MD.3; practicing MP2 and MP6

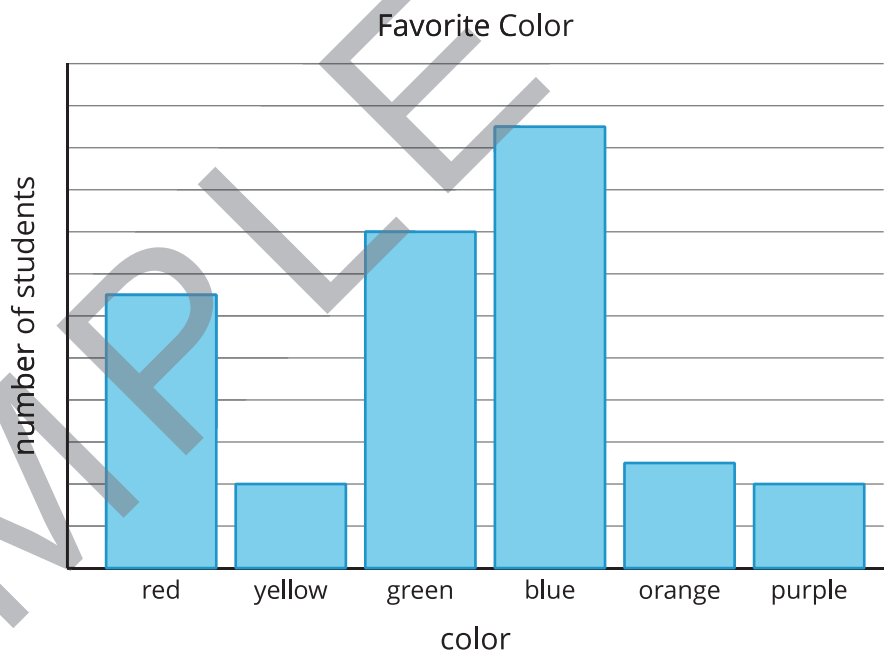
Graph and Answer

Let's represent our data on scaled bar graphs and answer questions about the data.

Warm-up

Notice and Wonder: Graph

What do you notice? What do you wonder?



m EV

’ P P Ry P

fi E E E E E E E E E E E E S
fi E E E E Q E f

fi E E E E E E S

t E E E E E E E E E E E E E E E
E S

” E E E E E S

m EW

x P Px P

1. Write questions that could be answered with your bar graph by filling in
E S

St E E E E E

E E k

St E E E E E

E k

St E E E E E

E E E k

Unit 8, Lesson 8

Addressing CA CCSSM 3.OA.7; practicing MP7

Multiplication Center Day

Let's sort multiplication facts and play a multiplication game.

Warm-up

Number Talk: Products

Find the value of each expression mentally.

- 2×4
- 5×4
- 10×4
- 17×4

m E V

P P Ffi

— E E E E E E E E E E E E E E E E f

VS E E E

2. can find it quickly

YS E E E

y E Eu E E E f

VS

WS

YS

_S

aS

m EW

P— P Pgff

– Eo E EWE S

VS“ E E E E E S

WSq E E E EV S

YSo E E SE E E E E E E E E S

_SuE E E E E QE E E E EVE E SE
E E E E E E E_E S

aS– E E E E E E SE E E E E E E E
E E E E S

Unit 8, Lesson 9

Addressing CA CCSSM 3.MD.7, 3.OA.5, 3.OA.7; practicing MP1, MP6, MP8

Multiplication Game Day

Let's play multiplication games.

Warm-up

Number Talk: Multiplying Large Factors

Find the value of each expression mentally.

- 4×10
- 4×20
- 4×21
- 4×24

m EV

P P P P P— v

p E E E f

- Both partners choose a different color to write with.

© E E f

- “ E E E E E E E S E EWE E E E EWE S
- “ E E E E E E E E E E EW S
- If your product won't fit on the gameboard, spin again.
- Take turns until the grid can’t fit any more rectangles.

r E E E E E S E E E E E E S

p E E E kf

’ E E E E EVUE E E E S

© E E f

- ” E E S
- o EW E E S
- fi E E E E E E E E E E S
- Your score for each round is the difference between your product EVUUS

m EW

— P qP P

o E E E E E E VUUS

o y E EVUUfEp E E E E E
S

t Eo k y E EVUUfEo EWE EYE E E E E E
E E EVUUS

’ E r EV VUfEy E E E
rectangular areas to fill a grid with more squares.

Addressing CA CCSSM 3.OA.3, 3.OA.6; practicing MP2

Multiplication and Division

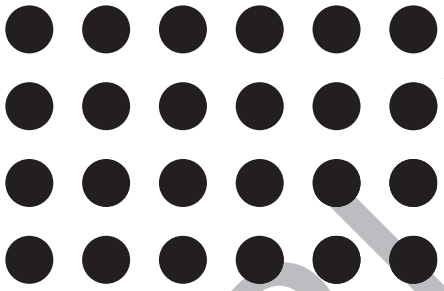
Let's represent equal groups and write situations with equal groups.

Warm-up

Which Three Go Together: Multiplication and Division

Which 3 go together?

A



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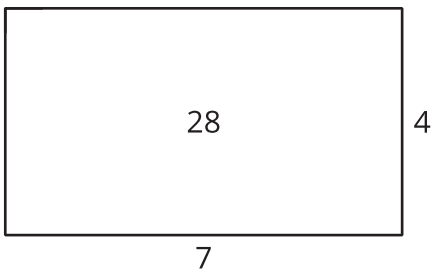

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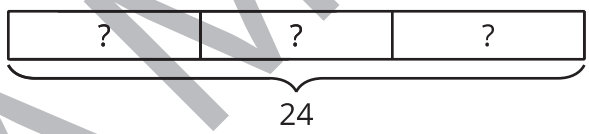

D

$24 \div 6$

b. a different diagram that your division equation could represent

m E W

P qfi P P

m E E E E E E E Q E E E S n
E E E E S

1. How is a representation you see different from those showed in your
k

W S fi E E E E E E E E E E
E E k

Unit 8, Lesson 11

Addressing CA CCSSM 3.OA.7; practicing MP7

Division Game Day

Let's play division games.

Warm-up

Number Talk: Divide 48

Find the value of each expression mentally.

- $48 \div 2$
- $48 \div 3$
- $48 \div 4$
- $48 \div 6$

m EV

P P Pg

p E E E EV

On your first turn:

VS ′ E E E S E E E E E E E SEVE E E EaQ
E SN

WSo EVE E E E E E E E E SEp E E E
S

YS fi E E E E E E E E E E SEffi
E E S

© E E E E f

VS ′ E E E S E E E E E E E SEVE E E EaQ
E SN

WSo E EE E E E E E E E E E E
E E E E E E E SEp E E E E S

YS fi E E E E E E E E E E SEffi
E E S

4. The first player to get a quotient of 1 wins.

Jada rolled 3 on her first turn, and then rolled 2 on her next three turns.

y				
E	$12 \div 3$	$16 \div 2$		$24 \div 2$
	_	d		VW
E				$12 \div 2$
				b
E				

s E

y				
E				
E				
E				
E				
E				
E				
E				
E				

s EW

y				
E				
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E				
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E				
E				
E				

m EW

P— ’

– Eo E EWE S

VS“ E E E E E S

WSq E E E EV S

YSo E E S“ E E E E E E E E S

_S– E E E E E E S“ E E E E E E E E E S

Unit 8, Lesson 12

Addressing CA CCSSM 3.OA.1-4; building towards 3.OA.1-4; practicing MP1 and MP6

Notice and Wonder

Let's create a "Notice and Wonder" routine.

Warm-up

Notice and Wonder: Equal Groups

What do you notice? What do you wonder?

m E

’ P R P P R

VS r E E E E E E E E E E E E S

WS fi E E E E E E E E E E S

“ E E f

- ○
- ○
- ○

“ E E f

- ○
- ○
- ○

m EW

fi P RR P P R

VSp E E E E E S

WSm E Q fi E E E kfi E E E k

YSs E E EVE E E E S

_Sm E E E E E E E S

aS' E E S

Unit 8, Lesson 13

Addressing CA CCSSM 3.OA.1-4; practicing MP8

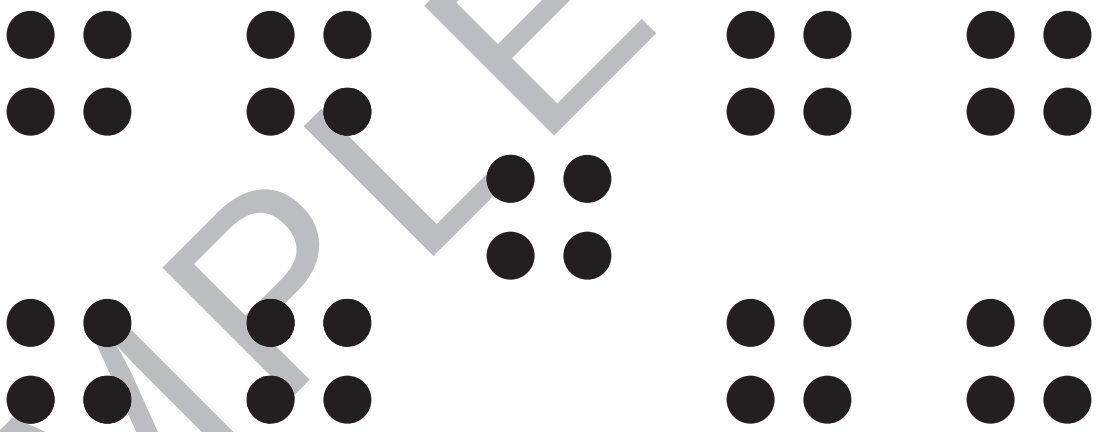
How Many Do You See?

Let's create a "How Many Do You See?" routine.

Warm-up

How Many Do You See: Equal Groups

How many do you see? How do you see them?



m EV

’ P RR P P P P vR

VSp E E E E E E E E E E E E S

WS fi E E E E E E E E E E E S

fi E E E E E f

- o
- o
- o

m E W

fi P R P P P P vR

VS m E E Q t E E E E k E E E E k

W S r E E S

Y S s E Y U E E E E E S

_ S p E E S

a S s E E E V E E E E S

b S m E E E E E E E E E E E E S

c S ’ E E S

Addressing CA CCSSM 3.MD.4; building on 3.MD.4; practicing MP1

Estimation Exploration

Let's create an "Estimation Exploration" routine.

Warm-up

Estimation Exploration: Fractional Measurement

What is the length of this earthworm?

Record an estimate that is:

too low	about right	too high

m EV

’ P PRff Fff R

VS r E E E E E E E E E E E
E SEfi E E E E E E E
E E E E E E E E E E E E E E S

WS ” E E E E E E E E E E E E
S

’ E E E E f

y	y	y

YS y E E E E E E E E E E E E
E SEuE QE E E S

m EW

fi P HRff Fff R

VS p E E E E E S

WS m E Q fi E E E E E E kE E kEm E k

YS s E E EVE E E E E S

_S s E EVE E E E S

aS m E E E E E S

bS ' E E S

gl f • s EY

Unit 8, Lesson 15

Addressing CA CCSSM 3.NBT.2; building on 3.NBT.2; practicing MP6

Number Talk

Let's create a "Number Talk" routine.

Warm-up

Number Talk: Addition

Find the value of each expression mentally.

- $300 + 400$
- $300 + 437$
- $321 + 437$
- $425 + 514$

m EV

P cP Pg

t E EV E S

600 – 400

600 – 399

500 – 399

1. Think of at least 2 different strategies you can use to find the value of each difference mentally.

WS fi EVE E E E E E E E E EV
E E E E E S

m E W

P q P h

t E E W E S

500 + 199

504 + 199

1. Think of at least 2 different strategies you can use to find the value of
E E S

W S fi E W E E E E E E E E E E E E E E E S

m EY

P cP R

fi EY E E E EGz E" SG

900 – 400

gl k • s EY

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m E

P qP Pk

fi E E E E E EGz E" SG

SAMPLE COPY

S

mE E E E E E E E E E E E E E E
S

E E E
mE E E E E E EWE E E E S

The number of square units that cover a flat figure without gaps or
S

m E E E E E E E Sm E E E E
E E E Sm E E E E E E E
S

E
mE E E E E E E E E E E E E
E E E E E E S

” E E E E E E E E E E E E E E
E E S

An operation for finding the number of groups or finding the size of
E E E E QE E E E E E E E E S

mE E E E E E E E E E E E
E E E S

mE E E E E E E E E E S

” E EVQUUE E EVE S

” E E E E E E E E E E E E E
S

mE E E E E E E E E E S

” E EVQUUE E EVE S

mE E E E E E E E E E E S

E

mE E E E E E E E E E E EVS

An operation for finding the total number of objects when we have a
E E E E S

” E E E E E E E E E E E E E E E
E S

s E E E E E E E E E Q
f $(3 \times 5) + (2 \times 5), (24 \div 2) + 5 = 17$ S

The boundary of a flat shape. We can find the length of the perimeter by finding the sum of the side lengths of the shape.

E
mE E E E E E E E E E E E E
E E E S

" E E E E S

A flat shape with 4 straight sides and 4 angles.

" E E E E E E S

mE E E E E E E E E S

mE E E E E S

E E E E
" E E E E E E E E E E E E S

mE E E E E EVE S

E
mE E E E E EVE S

A flat shape with 3 straight sides and 3 angles.

E
mE E E E E EVS

E E E
The amount of liquid it takes to completely fill a container.

E E E
” E E E E E E S

t E E E S

" o Eo E" E" E E Eo Eo E" E"
u SE Eo EWUVUSEz Es Em Eo E En E– E Eo
of Chief State School Officers. All rights reserved. fTT S S T

" aE– E E" Qy SE" SQK E" Qy SwSEWUVVNSaE E E E
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z E Efi uz TuEfi E E E Ez Eo E E" E
Mathematics, reflecting approaches developed by the Math Forum (fTT S S T
TNQ E E E E S

u E E E E E E E Eu Ey E E E E E E
E E Eo Eo Em EMoRnffNE Q E E E
SEu E E E E E E Eu Ey E E E E E
S

' E E E E E E E E E E E E E E
E E E E E E E SE" E E E E E E E E
u Ey E E E Q Q E E E E Q Q
SEu Ey E E E E E E E E S

Ex

n Er SEooRnffR'mEYSUSEfi SE fTT S S T Tr f" ff_b S S

n Ey SEooRnffR'mEYSUSEfi SE fTT S S T Tq HT T
r f" b S S

" E" E Ex E E Ex Eu Q Ez –"SE– Ep Sr E" E– S S
m E E fTT S S T TVVaccS

n Em SEooEnffR'mE_SUSEfi SE fTT S S T T– n HT T
r f" S S

n E SEooEnffEWSUSEfi Eo SE fTT S S T T
r f– p " " J Wc s NWWeaVUUbYaUNS S

n Em E" SEooEnffR'mE_SUSEfi Eo SE fTT S S T T

r fm ” S S

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_YYd_V_eWcS

m En Eu Ew QE E- E” SooUSE- p – S S
fTT S S T T R S k iY_VbcK i R R
R S

mE Ru E” Et En E E Efi QE En Eo SooEnffR mEWSUSEfi S
fTT S S T T” HT Tr fo R
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r VR” “ n J WdYWbV_dcbdbaJ We J WdYJ WeS S

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California Common Core State Standards for Mathematics (CA CCSSM) Reference

3.G: Grade 3 – Geomery

Reason with shapes and their attributes.

3.G.1
Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

3.MD: Grade 3 – Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1
Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2
Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).Excludes compound units such as cm^3 and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.Excludes multiplicative comparison problems (problems involving notions of "times as much"); see Glossary, Table 2.

Represent and interpret data.

3.MD.3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

3.MD.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units--whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5
Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.5a
A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

3.MD.5b
A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7
Relate area to the operations of multiplication and addition.

3.MD.7a
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

3.MD.7b
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

3.MD.7c
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

3.MD.7d
Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8
Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

3.NBT: Grade 3 – Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.1
Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3
Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

3.NF: Grade 3 – Numbers and Operations—Fractions

Develop understanding of fractions as numbers.

3.NF.1
Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.2a
Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

3.NF.2b
Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

3.NF.3a
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

3.NF.3b
Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

3.NF.3c
Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

3.NF.3d
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

3.OA: Grade 3 – Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1
Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.

3.OA.2
Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

3.OA.3
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.See Glossary, Table 2.

3.OA.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5
Apply properties of operations as strategies to multiply and divide.Students need not use formal terms for these properties. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

3.OA.6
Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

3.OA.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3.OA.9
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.