IMKH California

GRADE 8

Student Edition

UNITS 1-3

Illustrative® Mathematics LEARN MATH FOR LIFE | Kendall Hunt

Book 1

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GRADE 8

UNIT

1

Rigid Transformations and Congruence

Content Connections

In this unit you will explore translations, rotations, and reflections of plane figures in order to understand the structure of rigid transformations. You will make connections by:

- **Discovering Shape and Space** while performing translations, rotations, and reflections and noticing when shapes maintain congruence.

Addressing the Standards

As you work your way through **Unit 1 Rigid Transformations and Congruence,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 1, 6, and 13
MP2 Reason abstractly and quantitatively.	
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 2, 11, 12, 13, and 14
MP4 Model with mathematics.	
MP5 Use appropriate tools strategically.	Lessons 7 and 11
MP6 Attend to precision.	Lessons 1, 2, 3, 4, 6, and 10
MP7 Look for and make use of structure.	Lessons 1, 3, 5, 8, 9, 10, 15, 16, and 17
MP8 Look for and express regularity in repeated reasoning.	Lessons 8 and 15

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Transformational Geometry	**8.G.1** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	Lessons 2, 3, 4, 6, 11, 14, and 17
• Transformational Geometry	**8.G.1a** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.	Lessons 7, 8, 9, 10, and 13

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Transformational Geometry	**8.G.1b** Verify experimentally the properties of rotations, reflections, and translations: b. Angles are taken to angles of the same measure.	Lessons 7, 8, 9, and 10
• Transformational Geometry	**8.G.1c** Verify experimentally the properties of rotations, reflections, and translations: c. Parallel lines are taken to parallel lines.	Lesson 9
• Transformational Geometry	**8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Lessons 11, 12, 13, 15, and 17
• Transformational Geometry	**8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Lessons 5, 6, and 17
• Transformational Geometry	**8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	Lessons 14, 15, and 16

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 1, Lesson 1
Building on CA CCSSM 4.MD.5; Building towards 8.G.1; practicing MP1, MP6, MP7

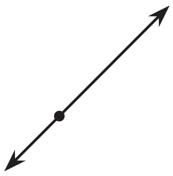
Moving in the Plane

Let's describe ways figures can move in the plane.

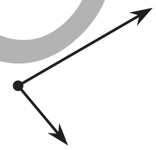
1.1 Which Three Go Together: Diagrams

Which three go together? Why do they go together?

A



B



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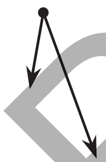
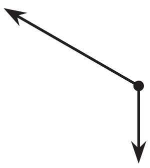


D



1.2 Triangle Square Dance

Sec A

Your teacher will give you three pictures. Each shows a different set of dance moves.

1. Arrange the three pictures so you and your partner can both see them right way up. Choose who will start the game.
   - The starting player mentally chooses A, B, or C and describes the dance to the other player.
   - The other player identifies which dance is being talked about: A, B, or C.
2. After one round, trade roles. When you have described all three dances, come to an agreement on the words you use to describe the moves in each dance.
3. With your partner, write a description of the moves in each dance.

Are you ready for more?

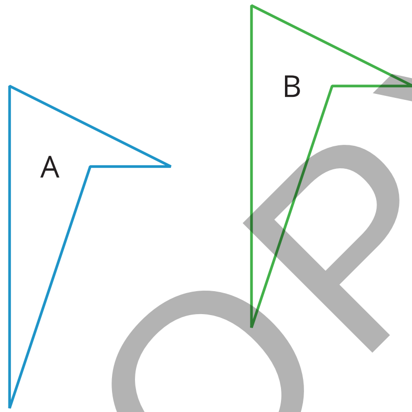
We could think of each dance as a new dance by running it in reverse, starting in the 6th frame and working backwards to the first.

1. Pick a dance and describe one of these reversed dances.
2. How do the directions for running your dance in the forward direction and the reverse direction compare?

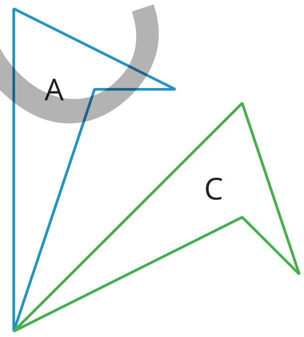
Lesson 1 Summary

Here are two ways for changing the position of a figure in a plane without changing its shape or size:

- Sliding or shifting the figure without turning it. Shifting Figure A to the right and up puts it in the position of Figure B.



- Turning or rotating the figure around a point. Figure A is rotated around the bottom vertex to create Figure C.

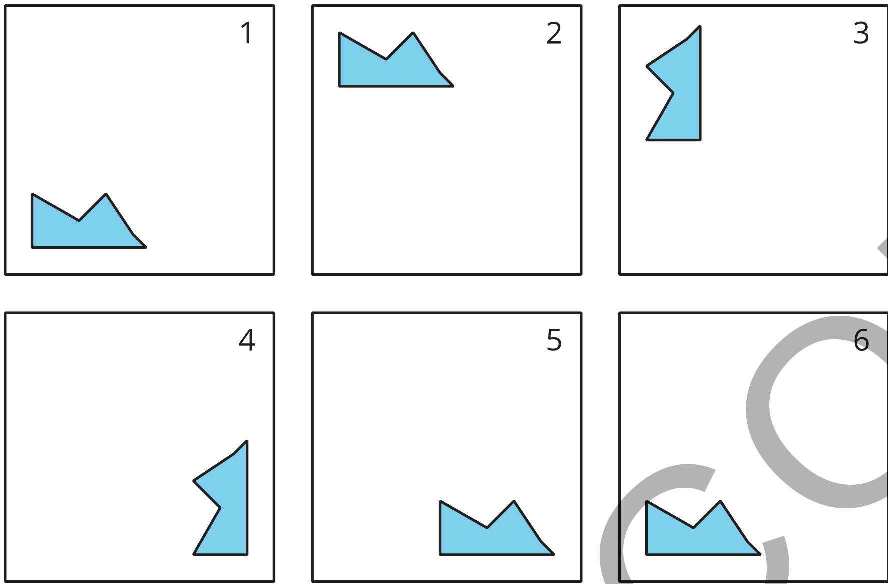


Glossary

- vertex

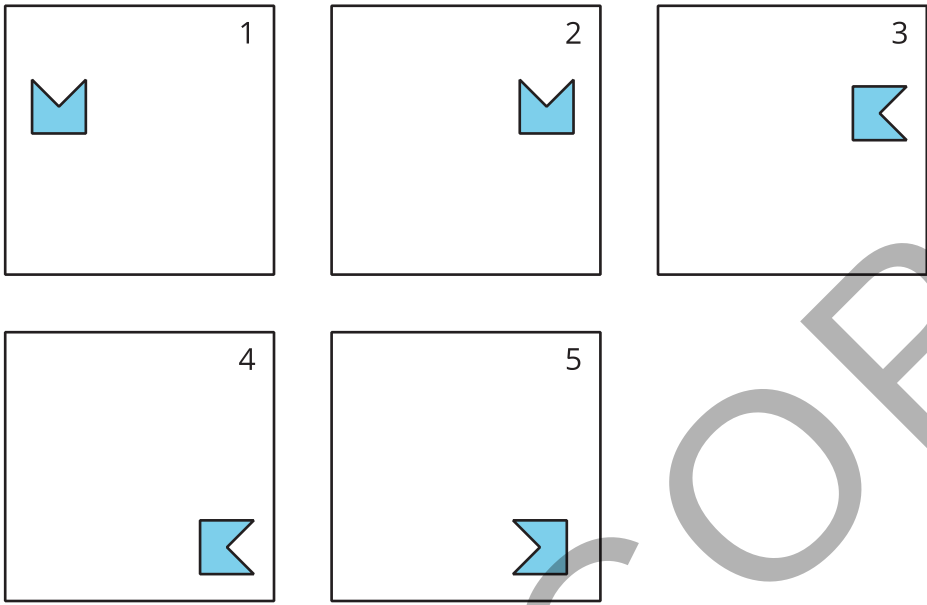
Practice Problems

1 The six frames show a shape's different positions:



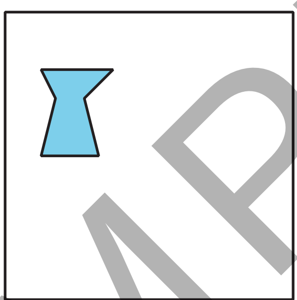
Describe how the shape moves to get from its position in each frame to the next.

2 These five frames show a shape's different positions:



Describe how the shape moves to get from its position in each frame to the next.

3 Diego started with this shape.



Diego moves the shape down, turns it 90 degrees clockwise, then moves the shape to the right. Draw the location of the shape after each move.

Unit 1, Lesson 2
Addressing CA CCSSM 8.G.1; building on 4.MD.5; practicing MP3 and MP6

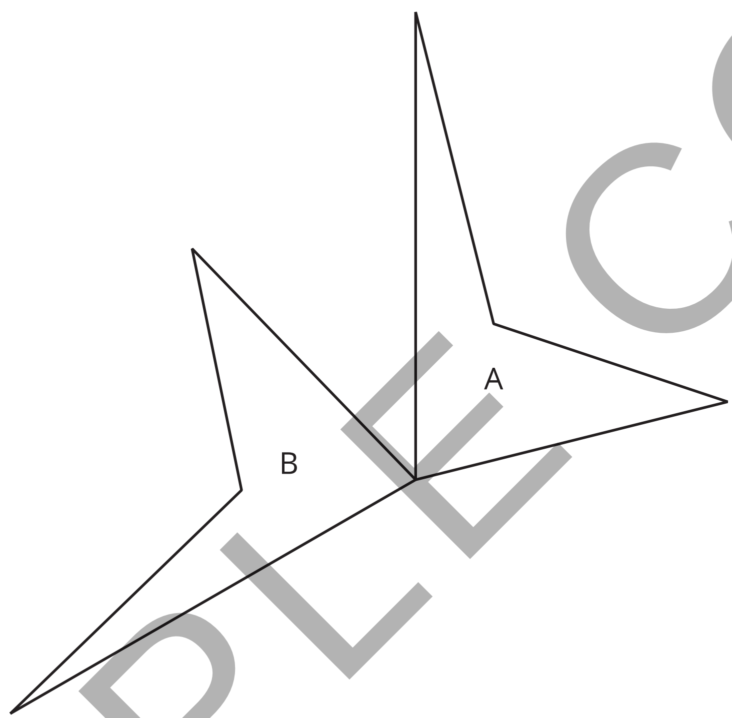
Naming the Moves

Let's be more precise about describing moves of figures in the plane.

Sec A

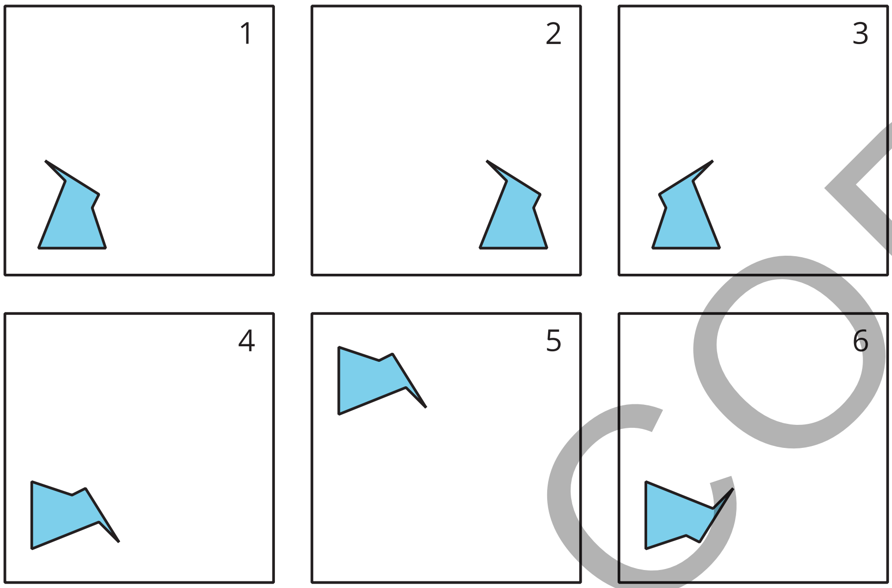
2.1 Notice and Wonder: A Pair of Quadrilaterals

What do you notice? What do you wonder?



2.2 How Did You Make That Move?

Here is a set of dance moves.



1. Describe each move and say if it is a new type of move.

   a. Frame 1 to Frame 2

   b. Frame 2 to Frame 3

   c. Frame 3 to Frame 4

   d. Frame 4 to Frame 5

   e. Frame 5 to Frame 6

2. How would you describe the new move?

Are you ready for more?

Create a new dance by putting the frames in a different order, then describe the moves. Are there any frames that are tricky to put next to each other and describe in a single move?

2.3 Card Sort: Move

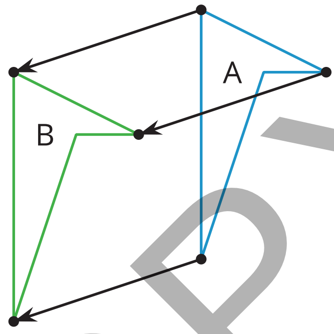
Your teacher will give you a set of cards. Take turns with your partner to sort the cards into categories according to the type of move they show. Be prepared to describe each category and why it is different from the others.

1. For each card, explain to your partner how you know which move it shows.
2. For each card that your partner describes, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

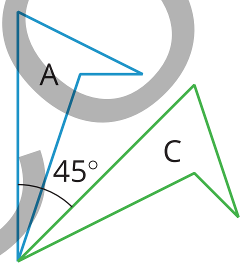
Lesson 2 Summary

Here are the moves we have learned about so far:

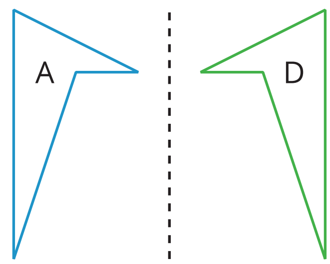
- A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.



- A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.



- A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.

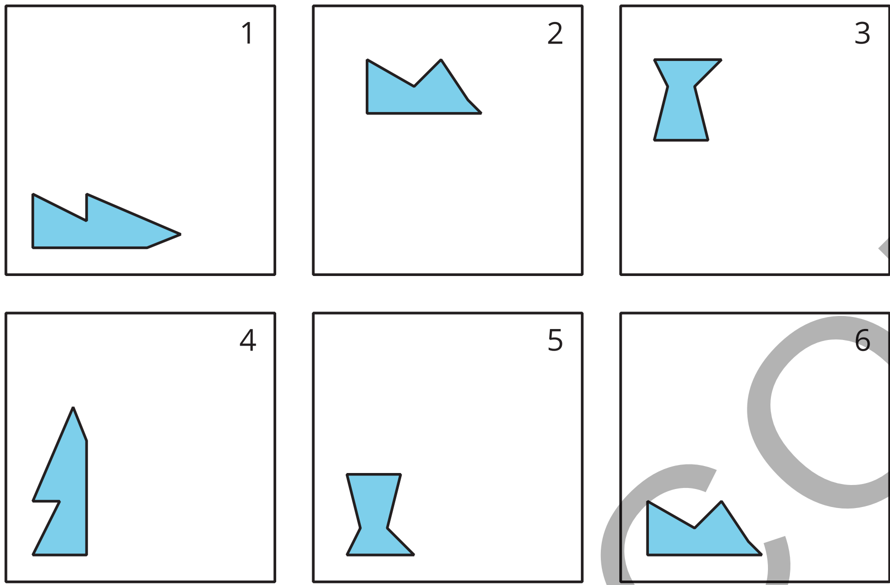


Glossary

- clockwise
- counterclockwise
- reflection
- rotation
- translation

Practice Problems

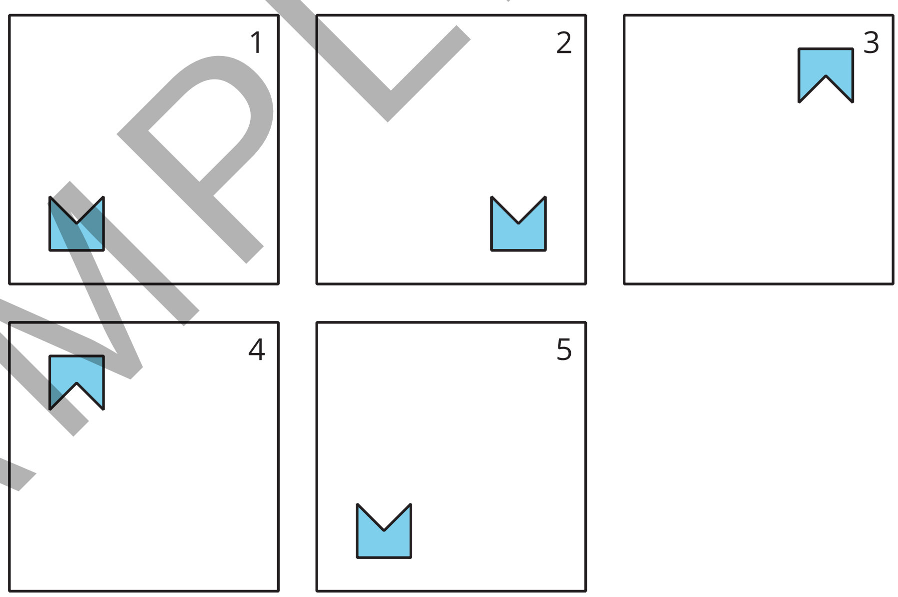
1 Each of the six cards shows a shape.



a. Which pair of cards shows a shape and its image after a rotation?

b. Which pair of cards shows a shape and its image after a reflection?

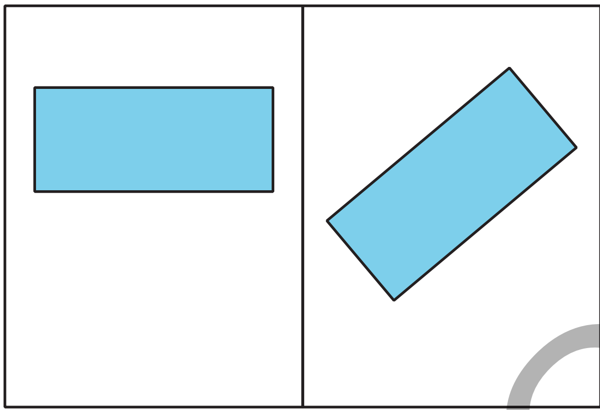
2 The five frames show a shape's different positions.



Describe how the shape could move to get from its position in each frame to the next.

3 from Unit 1, Lesson 1

The rectangle seen in Frame 1 is rotated to a new position, seen in Frame 2.



Frame 1 Frame 2

Select **all** the ways the rectangle could have been rotated to get from Frame 1 to Frame 2.

A. 40° clockwise

B. 40° counterclockwise

C. 90° clockwise

D. 90° counterclockwise

E. 140° clockwise

F. 140° counterclockwise

Unit 1, Lesson 3
Addressing CA CCSSM 8.G.1; building on 7.G.1; building towards 8.G.1; practicing MP6 and MP7

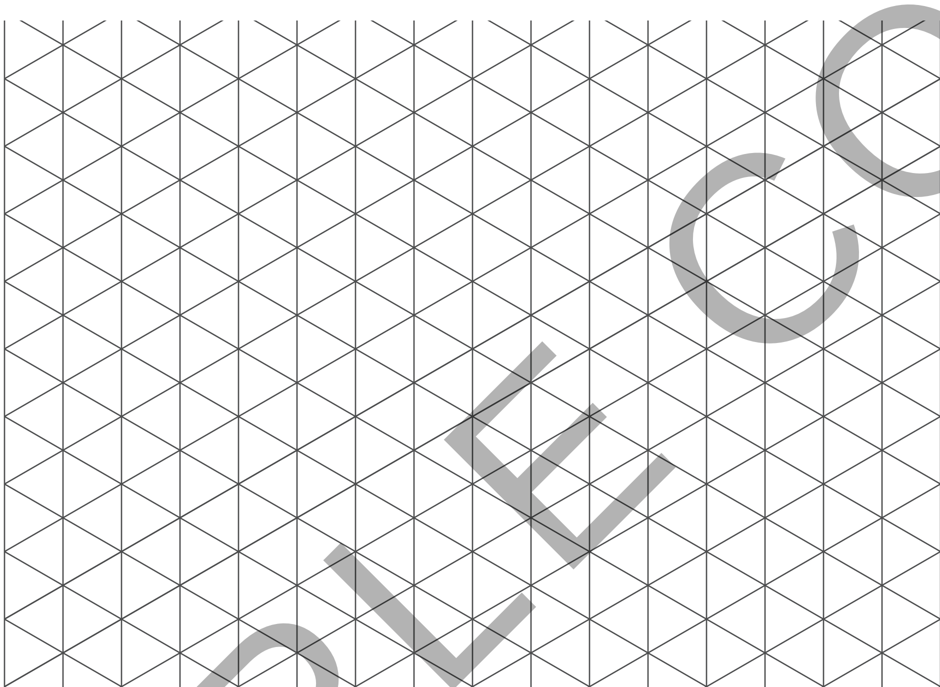
Grid Moves

Let's transform some figures on grids.

Sec A

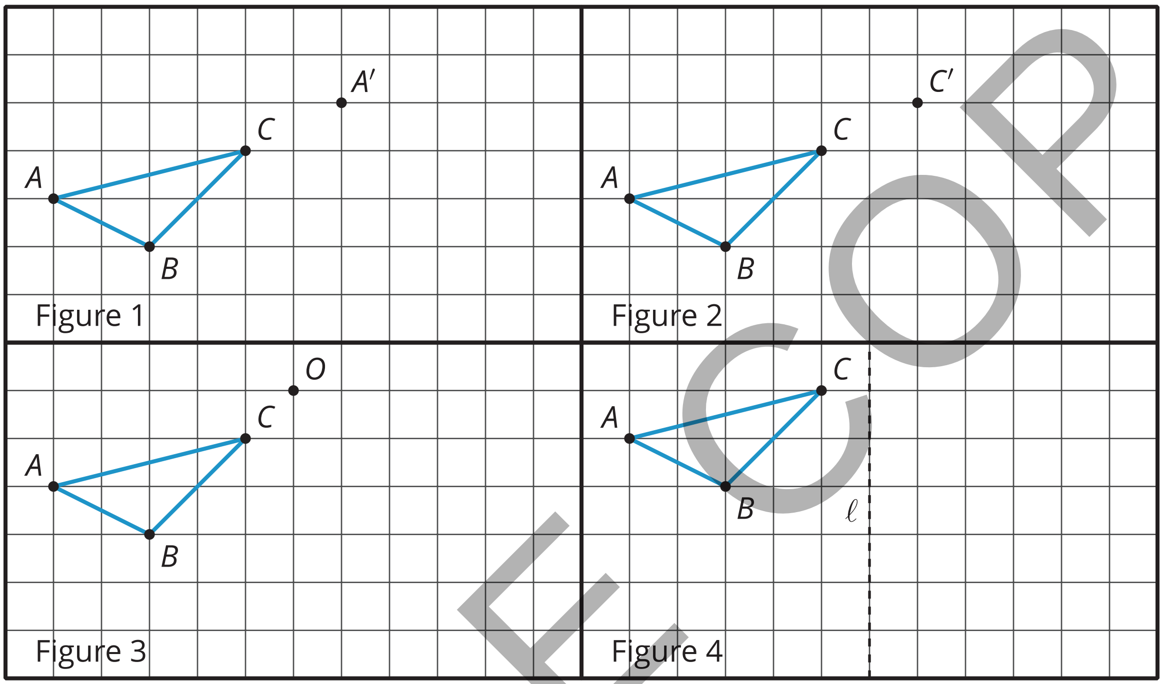
3.1 Notice and Wonder: The Isometric Grid

What do you notice? What do you wonder?

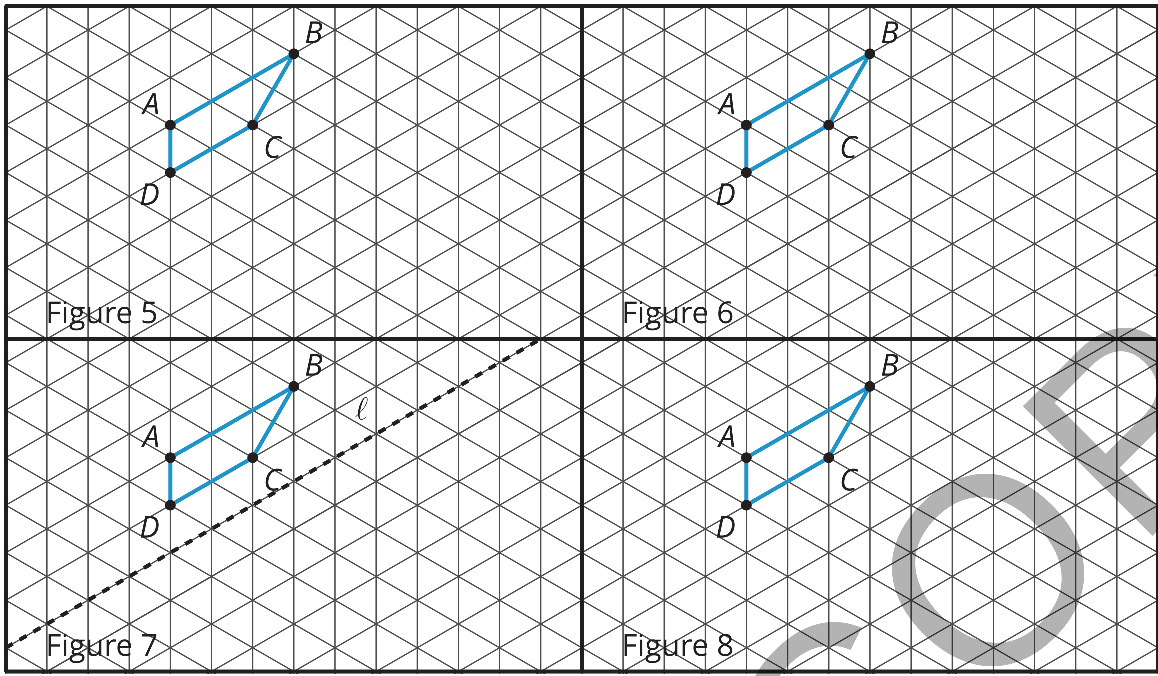


3.2 Image Information

Your teacher will give you tracing paper to carry out the moves specified. Use A', B', C', and D' to indicate vertices in the new figure that correspond to the points A, B, C, and D in the original figure.



1. In Figure 1, translate triangle ABC so that A goes to A'.
2. In Figure 2, translate triangle ABC so that C goes to C'.
3. In Figure 3, rotate triangle ABC 90° counterclockwise using center O.
4. In Figure 4, reflect triangle ABC using line ℓ.



5. In Figure 5, rotate quadrilateral $ABCD$ 60° counterclockwise using center B.
6. In Figure 6, rotate quadrilateral $ABCD$ 60° clockwise using center C.
7. In Figure 7, reflect quadrilateral $ABCD$ using line ℓ.
8. In Figure 8, translate quadrilateral $ABCD$ so that A goes to C.

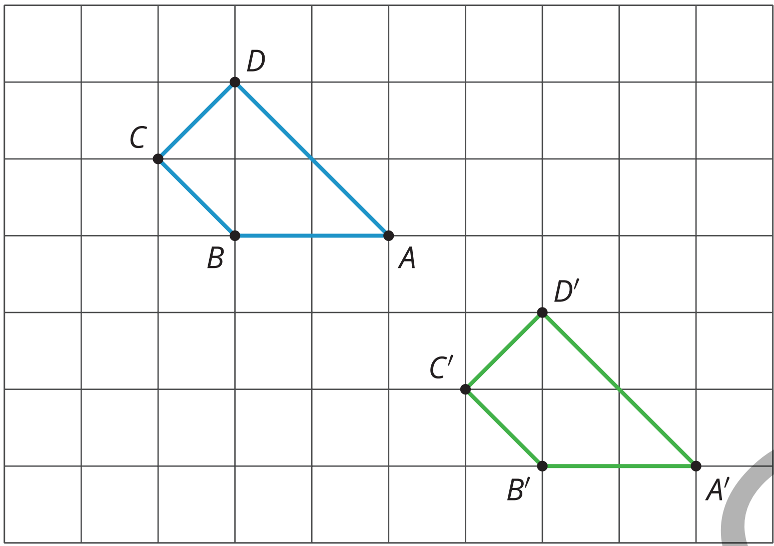
Are you ready for more?

The effects of each move can be "undone" by using another move. For example, to undo the effect of translating 3 units to the right, we could translate 3 units to the left. What move undoes each of the following moves?

1. Translate 3 units up
2. Translate 1 unit up and 1 unit to the left
3. Rotate 30° clockwise around a point P
4. Reflect across a line ℓ

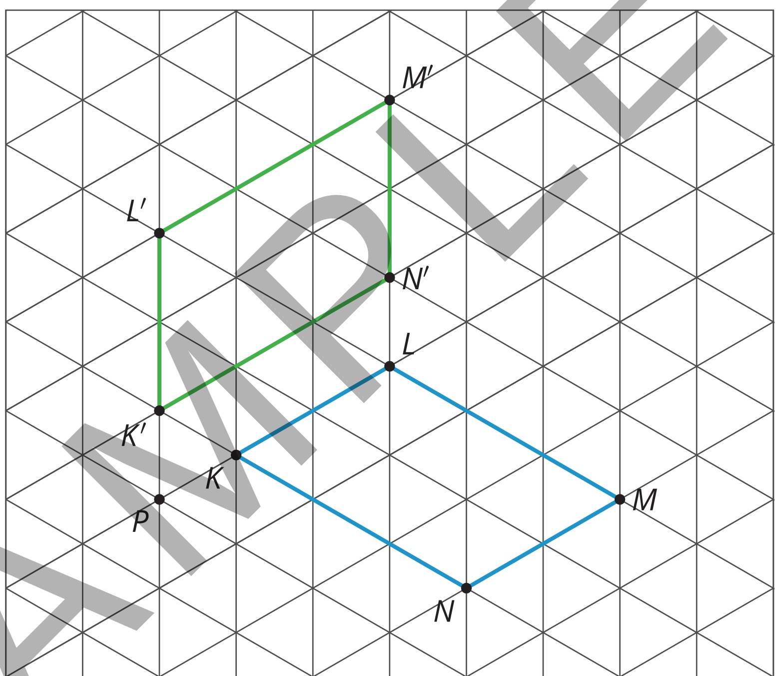
Lesson 3 Summary

When a figure is on a grid, we can use the grid to describe a move. For example, here is a figure and an **image** of the figure after a move.



Quadrilateral $ABCD$ is translated 4 units to the right and 3 units down to the position of quadrilateral $A'B'C'D'$.

This type of grid is called an *isometric grid*. The isometric grid is made up of equilateral triangles. The angles in the triangles all measure 60°, making the isometric grid convenient for showing rotations of 60°.



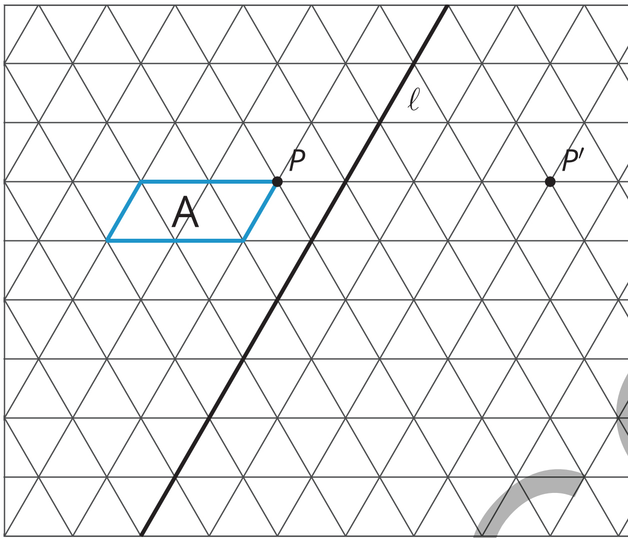
Here is quadrilateral $KLMN$ and its image $K'L'M'N'$ after a 60-degree counterclockwise rotation around a point P.

Glossary

- image

Practice Problems

1 Apply each move described to Figure A. If you get stuck, try using tracing paper.



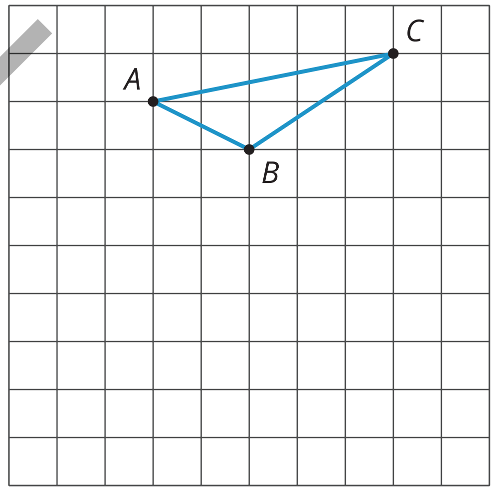
a. A translation which takes P to P'

b. A counterclockwise rotation of Figure A, using center P, of 60°

c. A reflection of Figure A across line ℓ

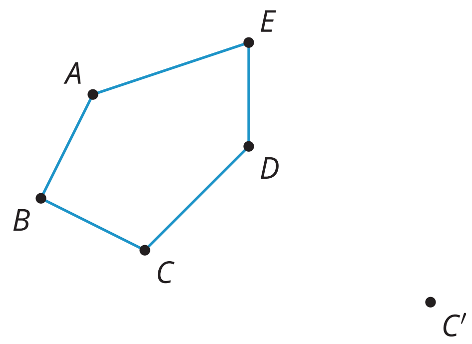
2 Here is triangle ABC drawn on a grid.

On the grid, draw a rotation of triangle ABC, a translation of triangle ABC, and a reflection of triangle ABC. Describe clearly how each was done.

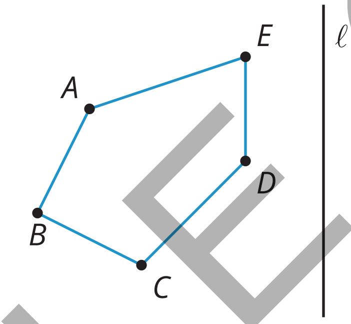


3 from Unit 1, Lesson 2

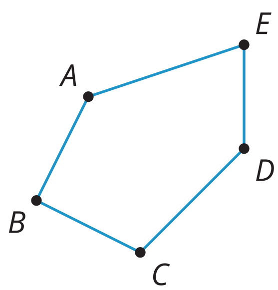
a. Draw the translated image of $ABCDE$ so that vertex C moves to C'. Tracing paper may be useful.



b. Draw the reflected image of pentagon $ABCDE$ with line of reflection ℓ. Tracing paper may be useful.



c. Draw the rotation of pentagon $ABCDE$ around C clockwise by an angle of 150°. Tracing paper and a protractor may be useful.

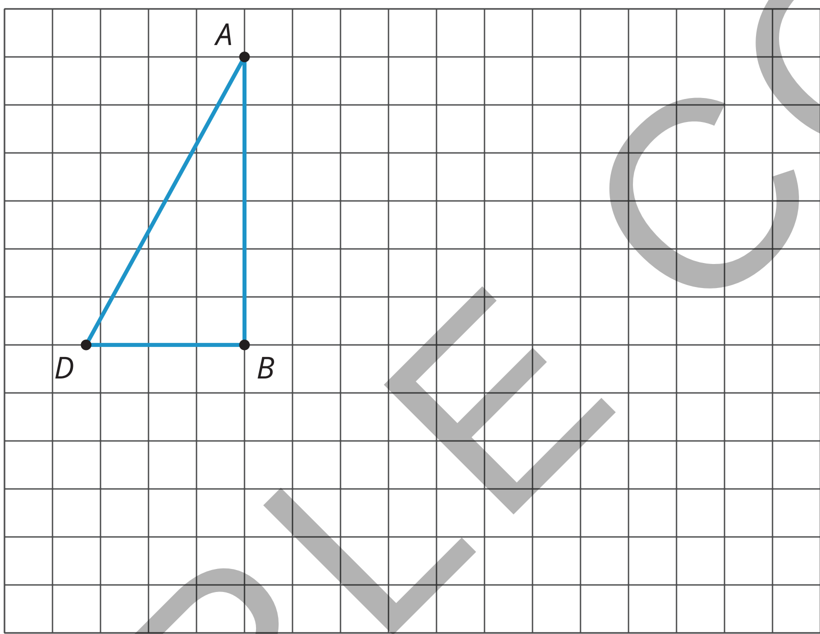


Making the Moves

Let's draw and describe translations, rotations, and reflections.

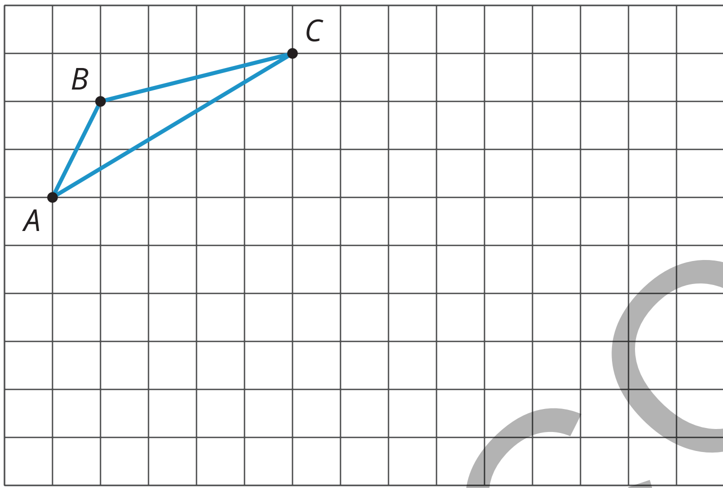
4.1 Reflection Quick Image

Here is an incomplete image. Your teacher will display the completed image twice, for a few seconds each time. Your job is to complete the image on your copy.



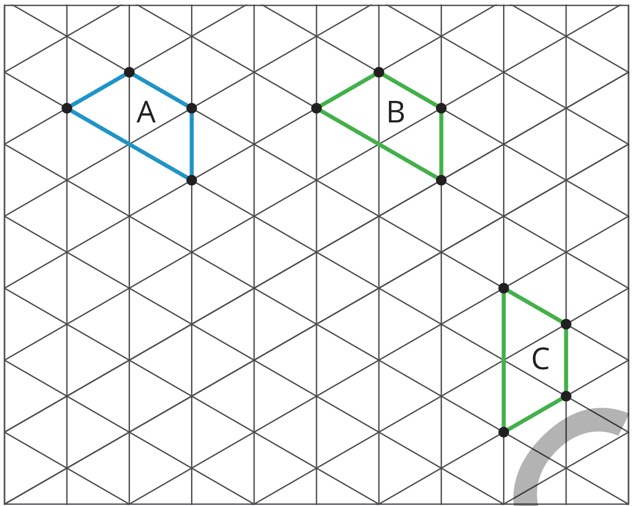
4.2 Make That Move

Your partner will describe the image of this triangle after a certain **transformation**. Sketch it here.



4.3 A to B to C

Here are some figures on an isometric grid.



1. Name a transformation that takes Figure A to Figure B.

2. Name a transformation that takes Figure B to Figure C.

3. What is one **sequence of transformations** that takes Figure A to Figure C? Explain how you know.

Are you ready for more?

Experiment with some other ways to take Figure A to Figure C. For example, can you do it with . . .

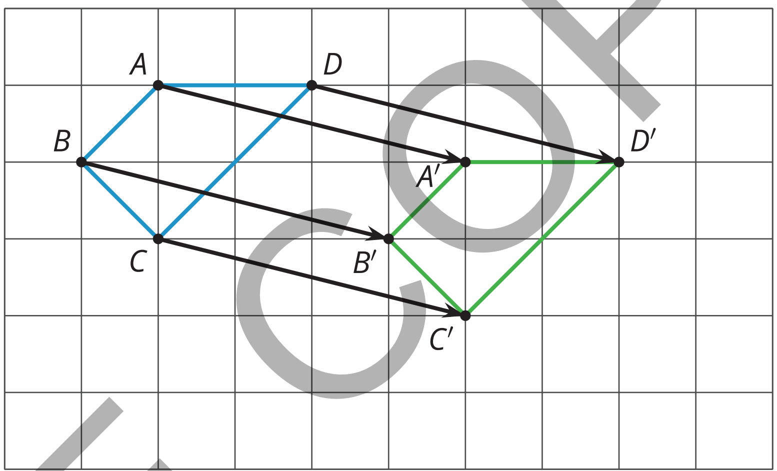
- No rotations?
- No reflections?
- No translations?

Lesson 4 Summary

A **transformation** is a translation, rotation, reflection, or dilation, or a combination of these. To distinguish an original figure from its image, points in the image are sometimes labeled with the same letters as the original figure, but with the symbol $'$ attached, as in A' (pronounced "A prime").

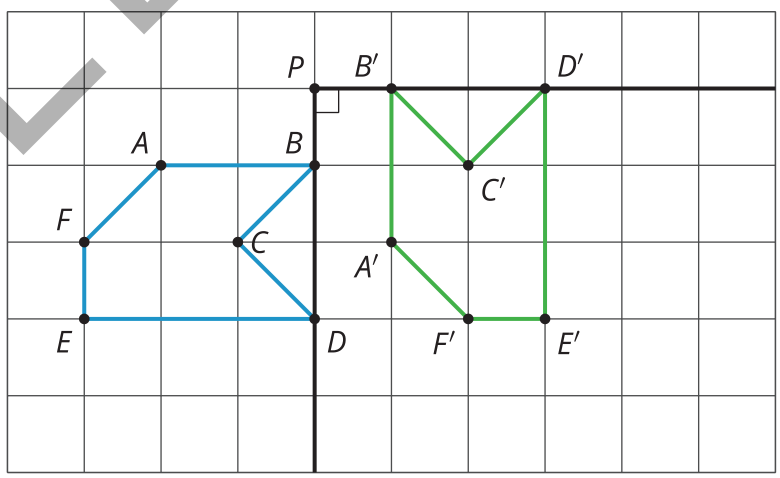
- A translation can be described by two points. If a translation moves point A to point A', it moves the entire figure the same distance and direction as the distance and direction from A to A'. The distance and direction of a translation can be shown by an arrow.

  For example, here is a translation of quadrilateral $ABCD$ that moves A to A'.



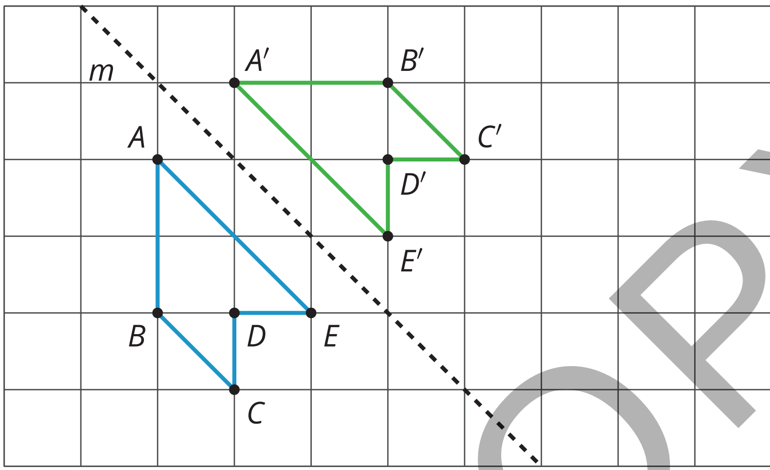
- A rotation can be described by an angle and a center. The direction of the angle can be clockwise or counterclockwise.

  For example, hexagon $ABCDEF$ is rotated 90° counterclockwise using center P.



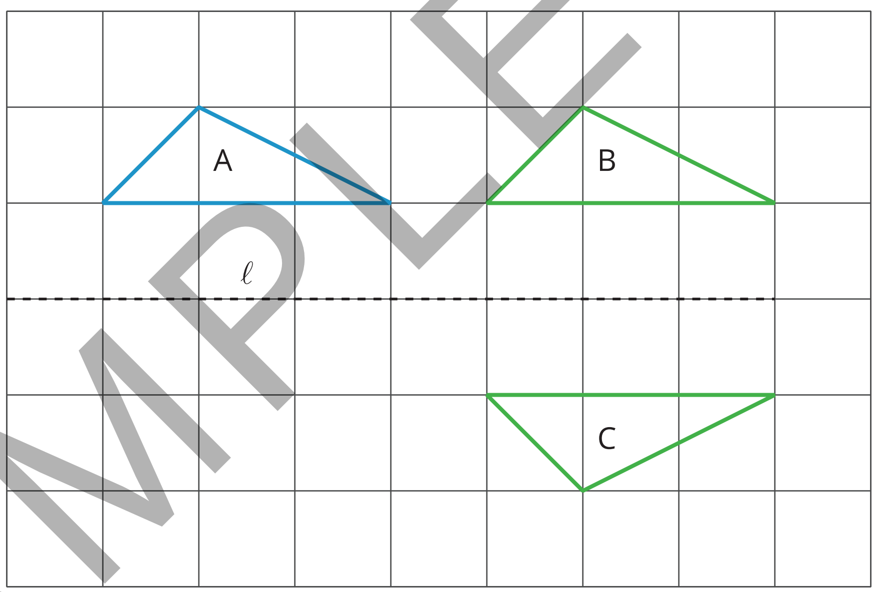
- A reflection can be described by a line of reflection (the “mirror”). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.

  For example, pentagon $ABCDE$ is reflected across line m.



When we do one or more moves in a row, we often call that a **sequence of transformations**. For example, a sequence of transformations taking Triangle A to Triangle C is to translate Triangle A 4 units to the right, then reflect over line ℓ.

There may be more than one way to describe or perform a transformation that results in the same image. For example, another sequence of transformations that would take Triangle A to Triangle C would be to reflect over line ℓ, then translate Triangle A 4 units to the right.



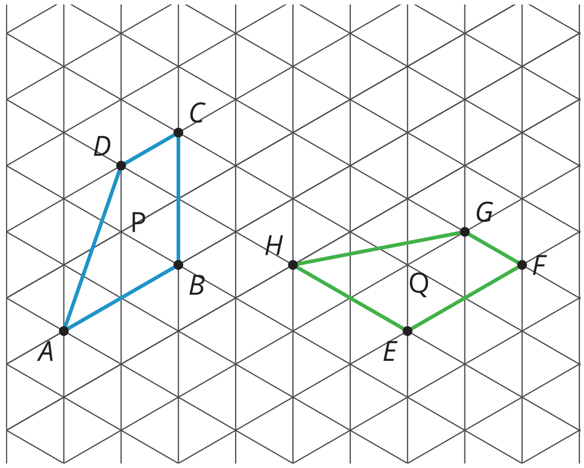
Glossary

- sequence of transformations
- transformation

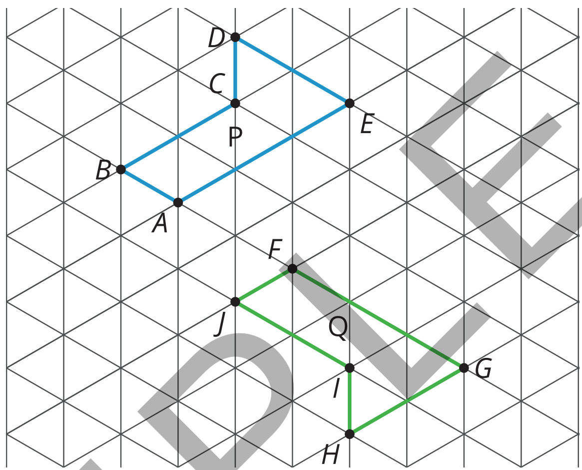
Practice Problems

1 For each pair of polygons, describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.

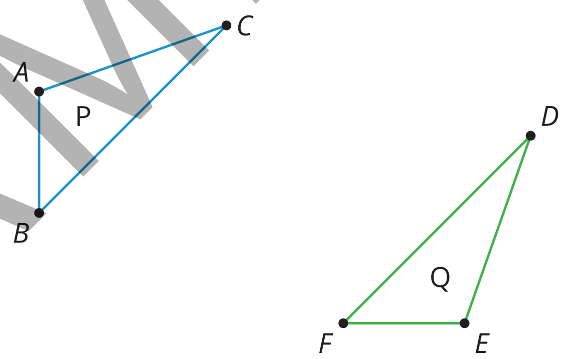
a.



b.

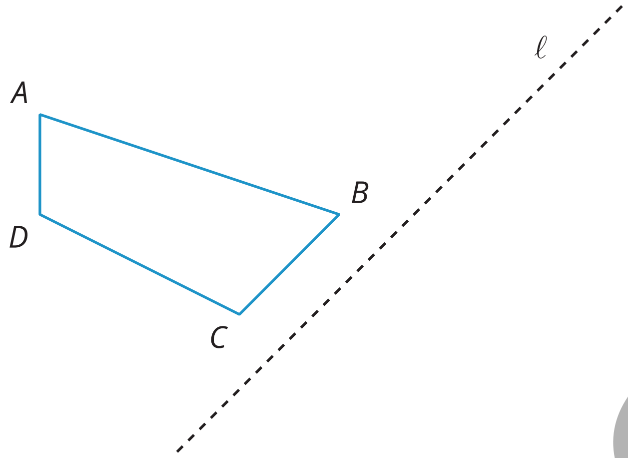


c.



2 from Unit 1, Lesson 2

Here is quadrilateral $ABCD$ and line ℓ.



Draw the image of quadrilateral $ABCD$ after reflecting it across line ℓ.

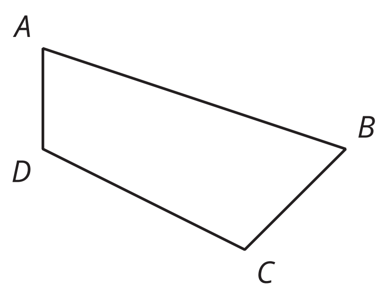
3 from Unit 1, Lesson 2

Here is quadrilateral $ABCD$. Draw the image of quadrilateral $ABCD$ after each rotation using B as center.

a. 90° clockwise

b. 120° clockwise

c. 30° counterclockwise



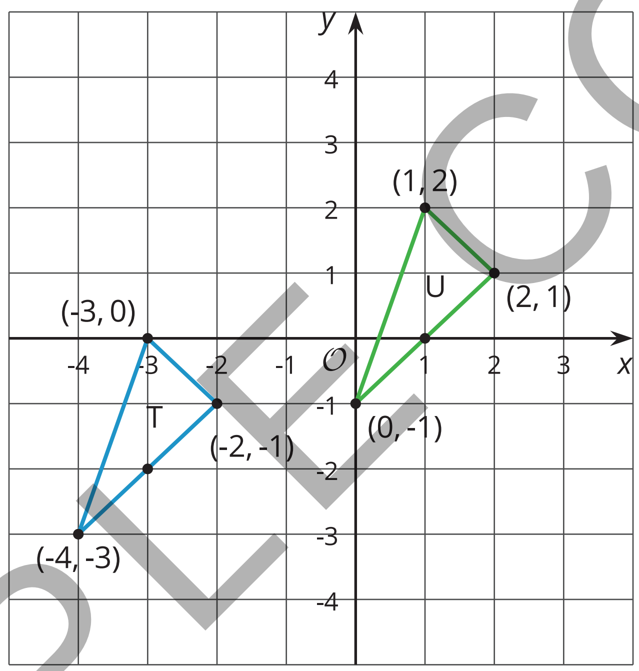
Addressing CA CCSSM 8.G.3; building on 8.G.1; building towards 8.G.3; practicing MP7

Coordinate Moves

Let's transform some figures and see what happens to the coordinates of points.

5.1 Translating Coordinates

Select all of the translations that take Triangle T to Triangle U. There may be more than one correct answer.



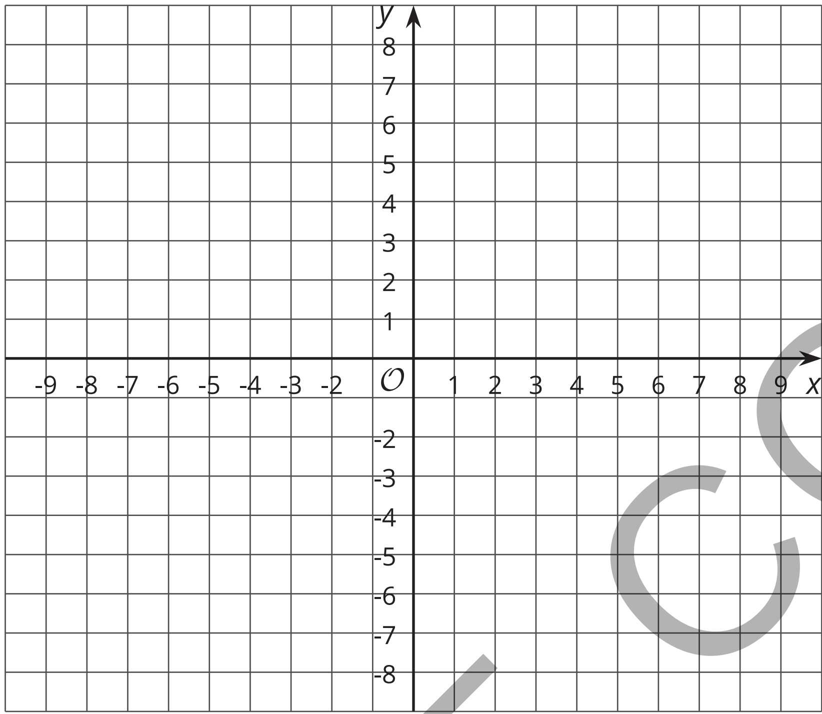
A. Translate $(-3, 0)$ to $(1, 2)$.

B. Translate $(2, 1)$ to $(-2, -1)$.

C. Translate $(-4, -3)$ to $(0, -1)$.

D. Translate $(1, 2)$ to $(2, 1)$.

5.2 Reflecting Points on the Coordinate Plane



1. Here is a list of points:
   A : $(0.5, 4)$, B : $(-4, 5)$, C : $(7, -2)$, D : $(6, 0)$

   On the coordinate plane:

   a. Plot each point and label each with its coordinates.

   b. Using the x-axis as the line of reflection, plot the image of each point.

   c. Label the image of each point with its coordinates.

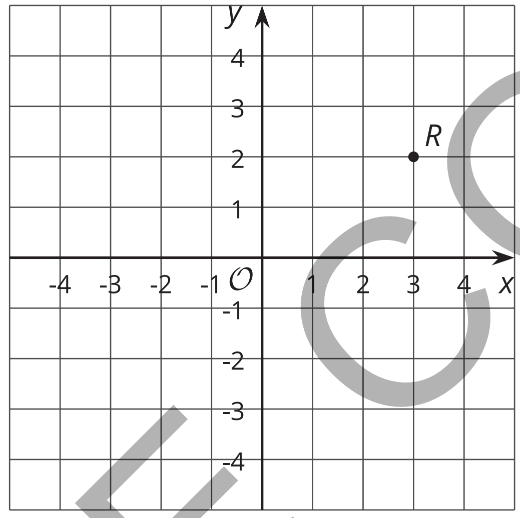
   d. Include a label using a letter. For example, the image of point A should be labeled A'.

2. If the point $(13, 10)$ were reflected using the x-axis as the line of reflection, what would be the coordinates of the image? What about $(13, -20)$? $(13, 570)$? Explain how you know.

3. The point R has coordinates $(3, 2)$.

   a. Without graphing, predict the coordinates of the image of point R if point R were reflected using the y-axis as the line of reflection.

   b. Check your answer by finding the image of R on the graph.

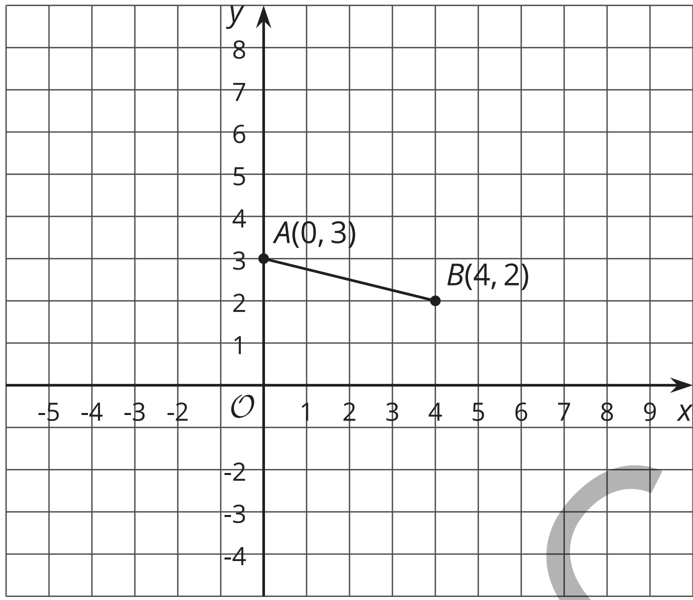


   c. Label the image of point R as R'.

   d. What are the coordinates of R'?

4. Suppose you reflect a point using the y-axis as the line of reflection. How would you describe its image?

5.3 Transformations of a Segment



Apply each of the following transformations to segment AB.

1. Rotate segment AB 90° counterclockwise around center B. Label the image of A as C. What are the coordinates of C?

2. Rotate segment AB 90° counterclockwise around center A. Label the image of B as D. What are the coordinates of D?

3. Rotate segment AB 90° clockwise around $(0, 0)$. Label the image of A as E and the image of B as F. What are the coordinates of E and F?

4. Compare the two 90° counterclockwise rotations of segment AB. What is the same about the images of these rotations? What is different?

Are you ready for more?

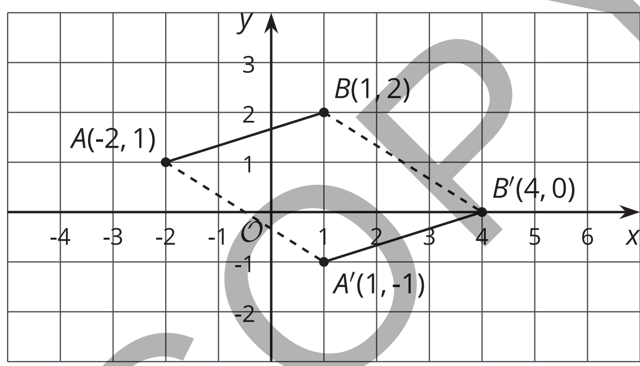
Suppose EF and GH are line segments of the same length. Describe a sequence of transformations that moves EF to GH.

Lesson 5 Summary

We can use coordinates to describe points and find patterns in the coordinates of transformed points.

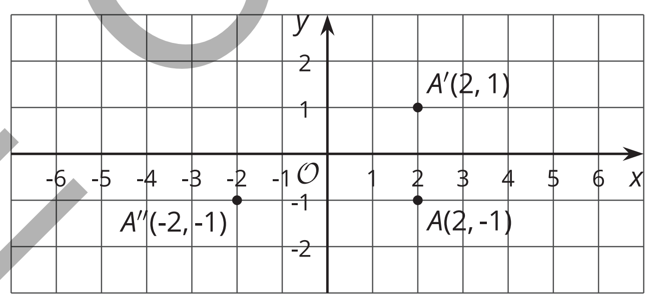
We can describe a translation by expressing it as a sequence of horizontal and vertical translations.

For example, segment AB is translated right 3 and down 2.



Reflecting a point across an axis changes the sign of one coordinate.

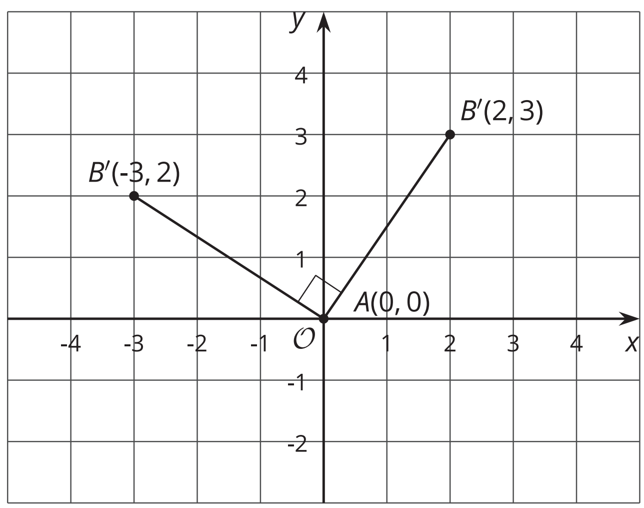
For example, reflecting the point A whose coordinates are $(2, \text{-}1)$ across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are $(2, 1)$. Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are $(\text{-}2, \text{-}1)$.



Reflections across other lines are more complex to describe.

We don't have the tools yet to describe rotations in terms of coordinates in general. Here is an example of a 90° rotation with center $(0, 0)$ in a counterclockwise direction.

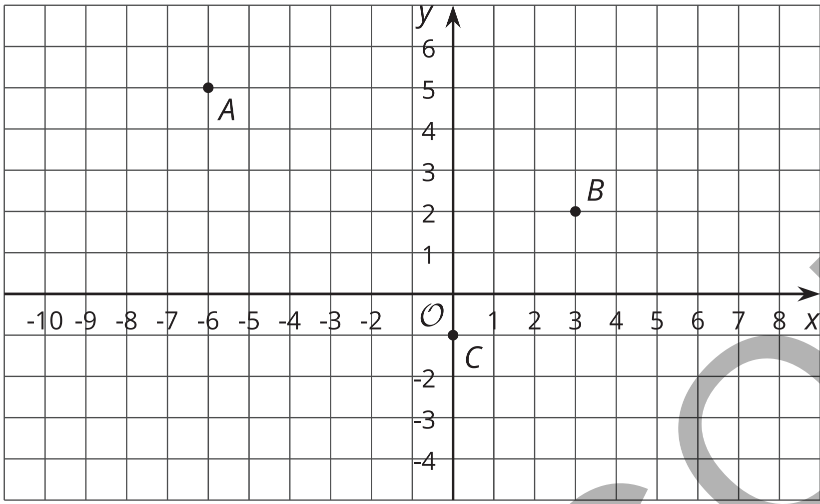
Point A has coordinates $(0, 0)$. Segment AB is rotated 90° counterclockwise around A. Point B with coordinates $(2, 3)$ rotates to point B' whose coordinates are $(\text{-}3, 2)$.



Practice Problems

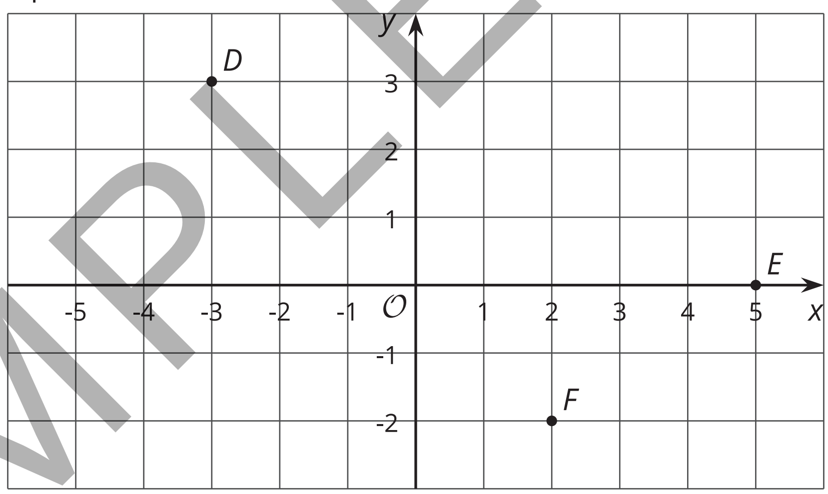
1

a. Here are some points:



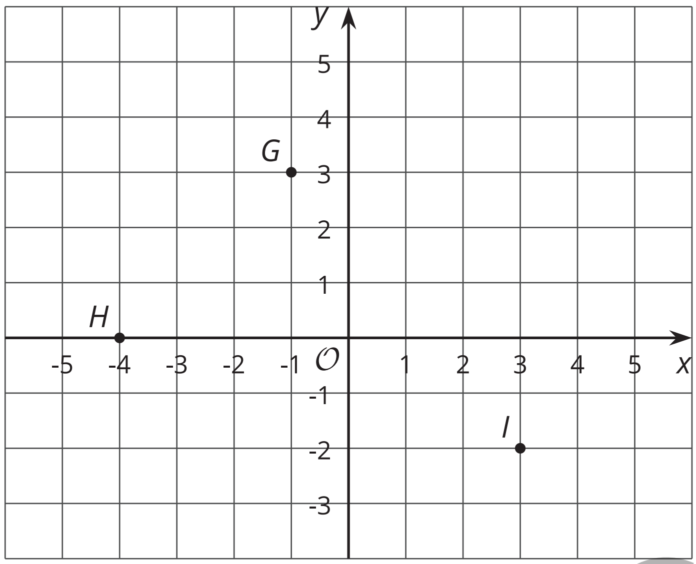
What are the coordinates of A, B, and C after a translation to the right by 4 units and up 1 unit? Plot these points on the grid, and label them A', B' and C'.

b. Here are some points:



What are the coordinates of D, E, and F after a reflection over the y axis? Plot these points on the grid, and label them D', E' and F'.

c. Here are some points:

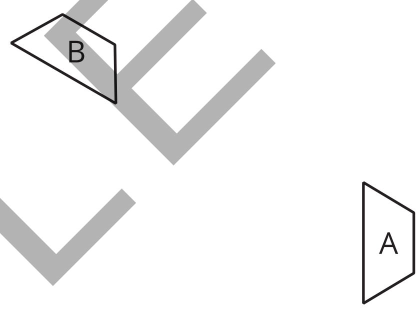


What are the coordinates of G, H, and I after a rotation about $(0, 0)$ by 90° clockwise? Plot these points on the grid, and label them G', H' and I'.

Sec A

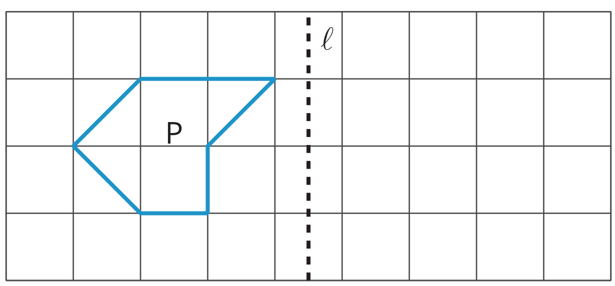
2 from Unit 1, Lesson 4

Describe a sequence of transformations that takes Trapezoid A to Trapezoid B.



3 from Unit 1, Lesson 3

Reflect Polygon P using line ℓ.

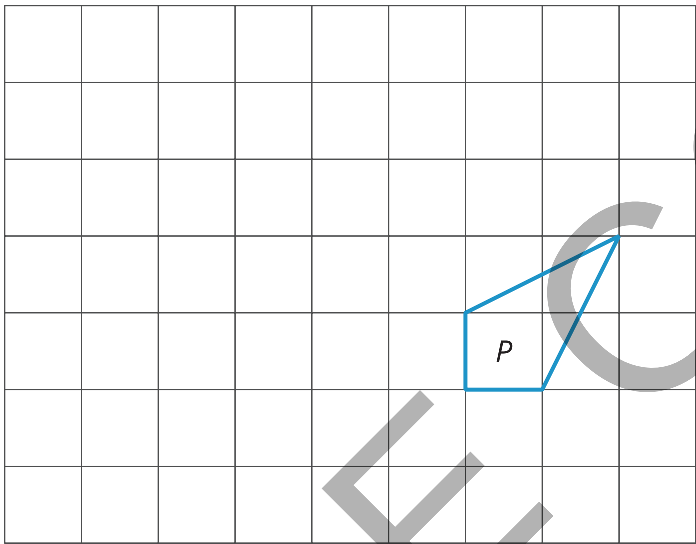


Addressing CA CCSSM 8.G.1, 8.G.3; practicing MP1 and MP6

Describing Transformations

Let's transform some polygons in the coordinate plane.

6.1 What Do You Want to Know?



P' is the image of P after some transformations.

What specific information do you need to be able to solve the problem?

6.2 Info Gap: Transformation Information

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ________?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ________ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ________?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Are you ready for more?

Sometimes two transformations, one performed after the other, have a simpler description as a single transformation. For example, instead of translating 2 units up followed by translating 3 units up, we could simply translate 5 units up. Instead of rotating 20° counterclockwise around the origin followed by rotating 80° clockwise around the origin, we could simply rotate 60° clockwise around the origin.

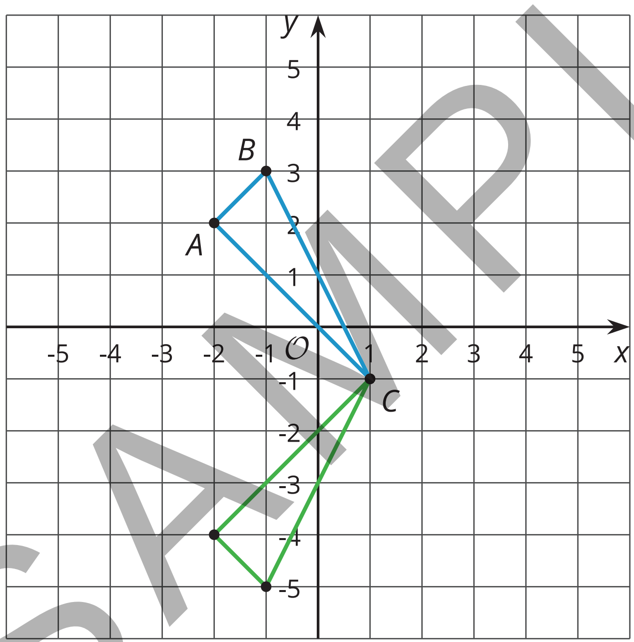
Can you find a simple description of reflecting across the x-axis followed by reflecting across the y-axis?

Lesson 6 Summary

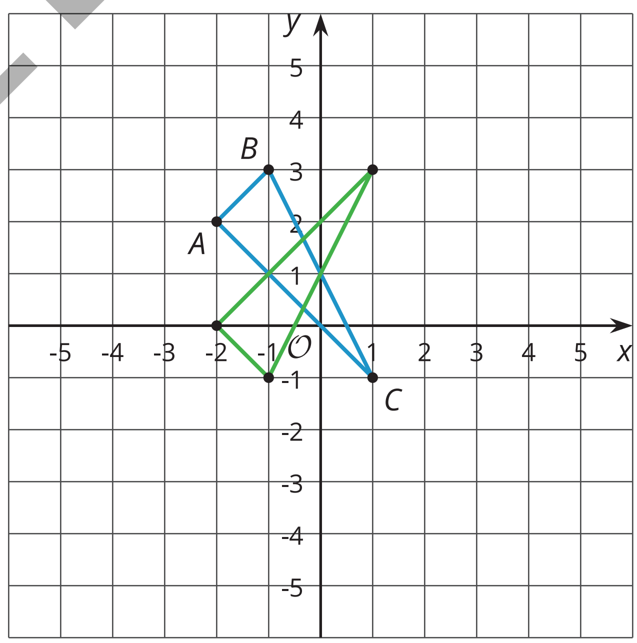
When describing a sequence of transformations, there are several pieces of information that are important to know. For a translation, we need to know distance and direction. For a rotation, we need the center of rotation, direction, and amount of rotation. For a reflection, we need a line of reflection. There is one more piece of information that is helpful though.

When we perform a sequence of transformations, the order of the transformations can be important.

Here is triangle ABC translated up two units and then reflected over the x-axis.



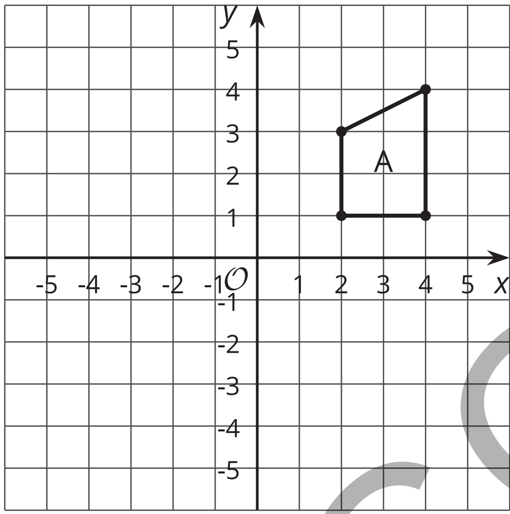
Here is triangle ABC reflected over the x-axis and then translated up 2 units.



Triangle ABC ends up in different places when the transformations are applied in the opposite order!

Practice Problems

1 Here is Polygon A in the coordinate plane:



a. Draw Polygon B, the image of A, using the y-axis as the line of reflection.

b. Draw Polygon C, the image of B, using the x-axis as the line of reflection.

c. Draw Polygon D, the image of C, using the x-axis as the line of reflection.

2 The point $(-4, 1)$ is rotated 180° counterclockwise using center $(-3, 0)$. What are the coordinates of the image?

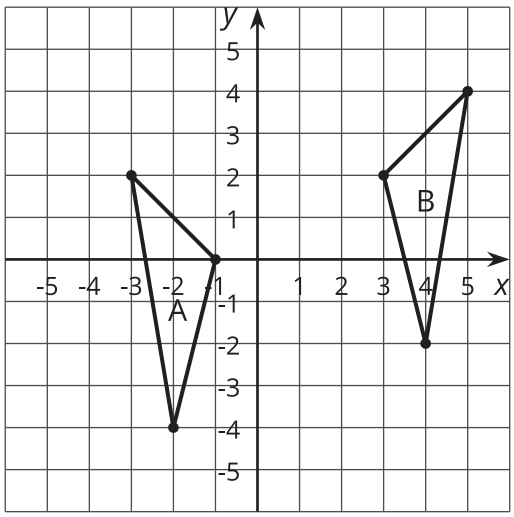
A. $(-5, 2)$

B. $(-4, -1)$

C. $(-2, -1)$

D. $(4, -1)$

3 Describe a sequence of transformations for which Triangle B is the image of Triangle A.



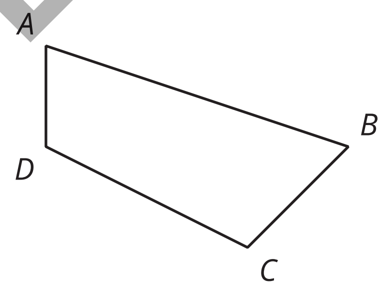
4 from Unit 1, Lesson 2

Here is quadrilateral $ABCD$. Draw the image of quadrilateral $ABCD$ after each transformation.

a. The translation that takes B to D.

b. The reflection over segment BC.

c. The rotation about point A by angle DAB, counterclockwise.



Addressing CA CCSSM 8.G.1a, 8.G.1b; building on 4.MD.1; practicing MP5

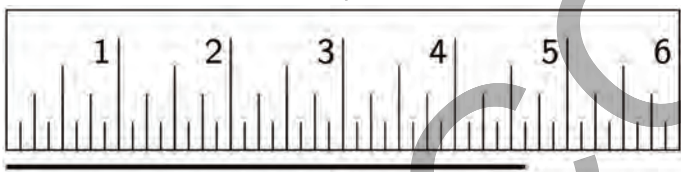
No Bending or Stretching

Let's compare measurements before and after translations, rotations, and reflections.

7.1 Measuring Segments

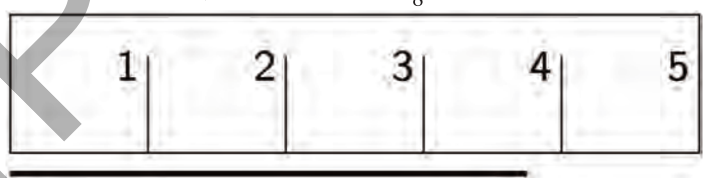
For each question, the unit is represented by the large tick marks with whole numbers.

1. Find the length of this segment to the nearest $\frac{1}{8}$ of a unit.



2. Find the length of this segment to the nearest 0.1 of a unit.

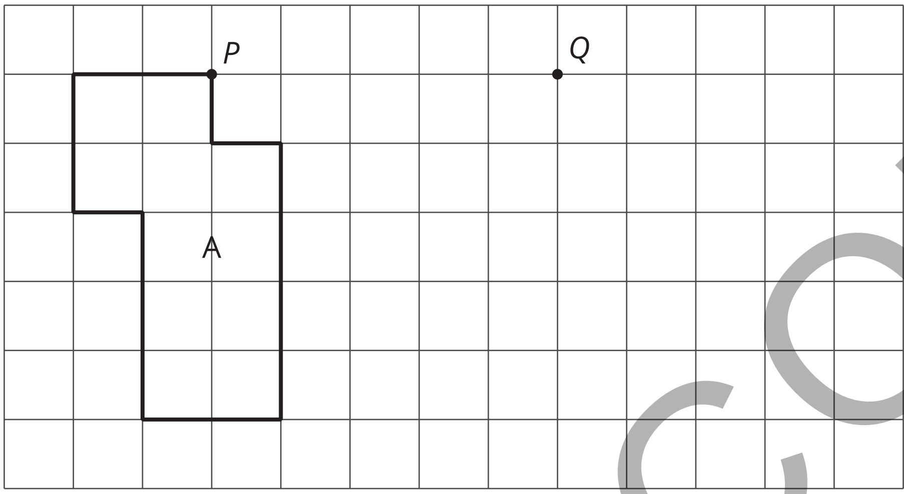
3. Estimate the length of this segment to the nearest $\frac{1}{8}$ of a unit.



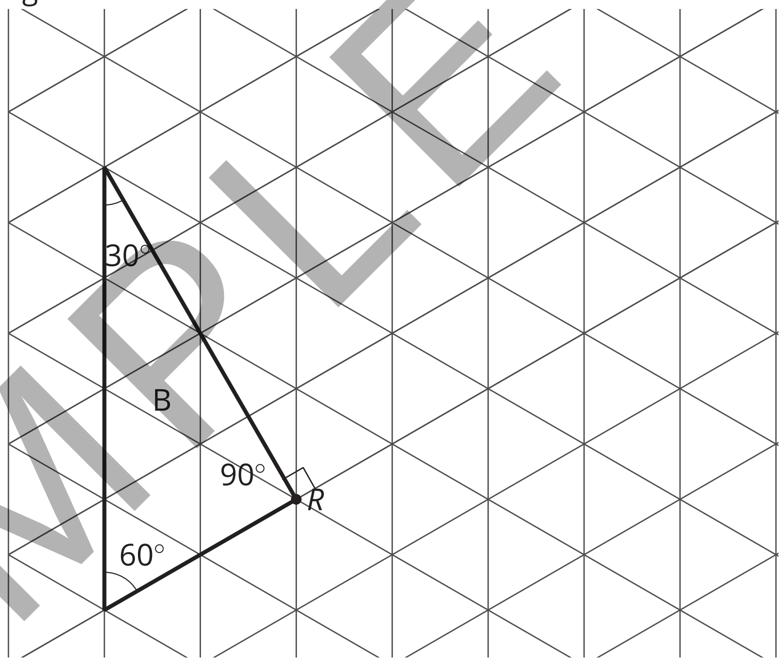
4. Estimate the length of the segment in the prior question to the nearest 0.1 of a unit.

7.2 Sides and Angles

1. Translate Polygon A so point P goes to point Q. In the image, write the length of each side, in grid units, next to the side.



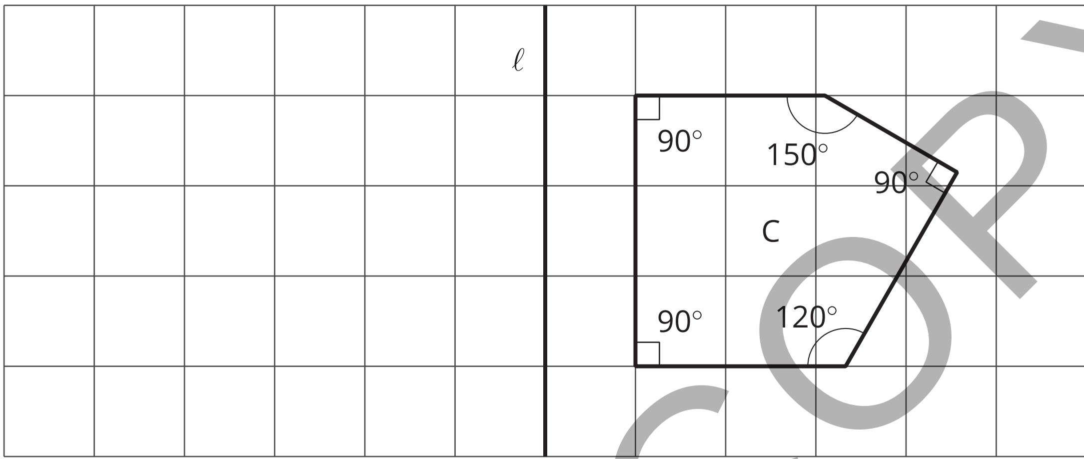
2. Rotate Triangle B 90° clockwise using R as the center of rotation. In the image, write the measure of each angle in its interior.



3. Reflect Pentagon C across line ℓ.

   a. In the image, write the length of each side, in grid units, next to the side. You may need to make your own ruler with tracing paper or a blank index card.

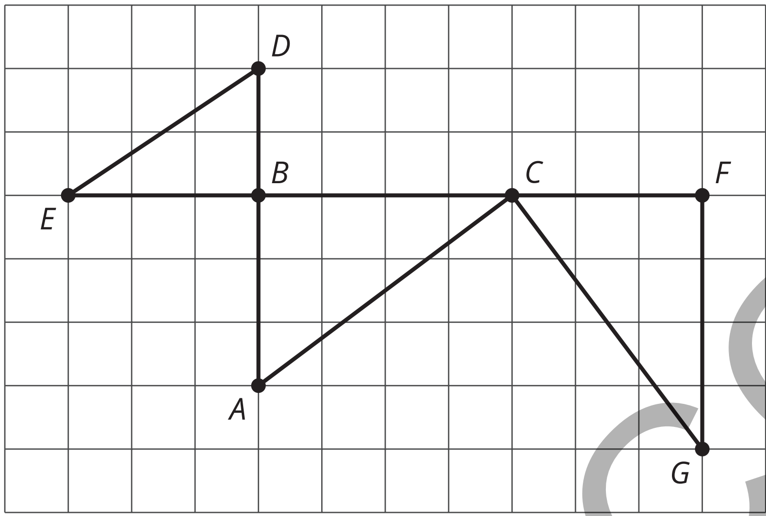
   b. In the image, write the measure of each angle in the interior.



Sec B

7.3 Which One?

Here is a grid showing triangle ABC and two other triangles.



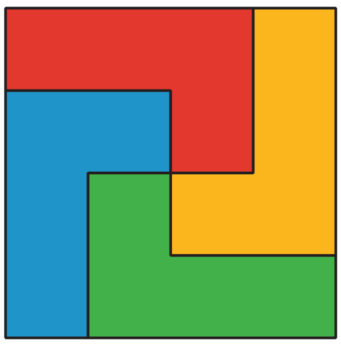
You can use a **rigid transformation** to take triangle ABC to one of the other triangles.

1. Which other triangle? Explain how you know.

2. Describe a rigid transformation that takes ABC to the triangle you selected.

Are you ready for more?

A square is made up of an L-shaped region and three transformations of the region. If the perimeter of the square is 40 units, what is the perimeter of each L-shaped region?

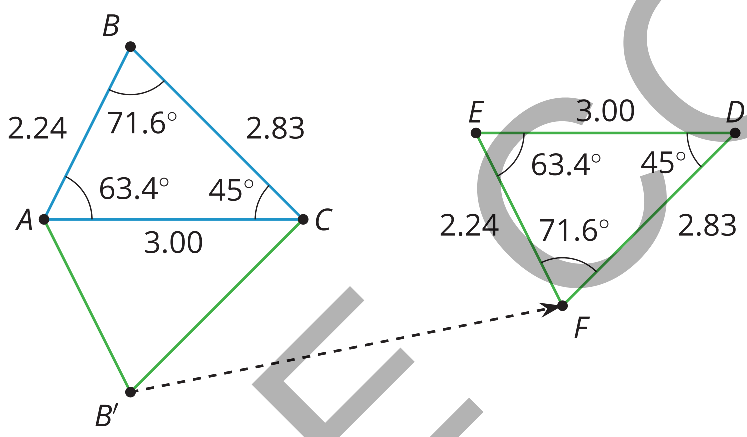


Lesson 7 Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of **rigid transformations**. A rigid transformation is a move that doesn't change measurements on any figure.

Earlier, we learned that a figure and its image have corresponding points. With a rigid transformation, figures like polygons also have **corresponding** sides and corresponding angles. These corresponding parts have the same measurements.

For example, triangle EFD was made by reflecting triangle ABC across a horizontal line, then translating. Corresponding sides have the same lengths, and corresponding angles have the same measures.



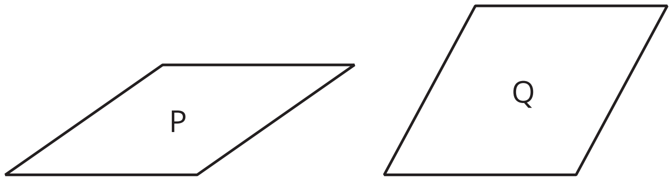
Measurements in triangle ABC	Corresponding measurements in image EFD
$AB = 2.24$	$EF = 2.24$
$BC = 2.83$	$FD = 2.83$
$CA = 3.00$	$DE = 3.00$
angle $ABC = 71.6^\circ$	angle $EFD = 71.6^\circ$
angle $BCA = 45.0^\circ$	angle $FDE = 45.0^\circ$
angle $CAB = 63.4^\circ$	angle $DEF = 63.4^\circ$

Glossary

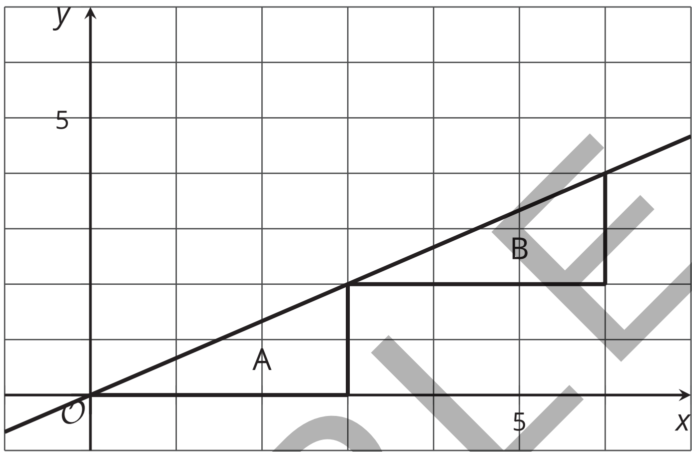
- corresponding
- rigid transformation

Practice Problems

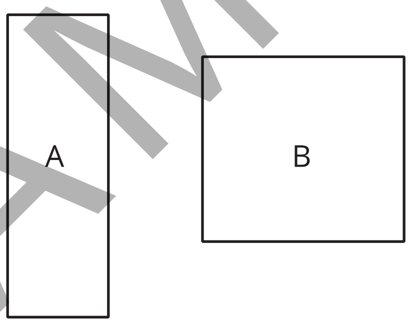
1 Is there a rigid transformation taking Rhombus P to Rhombus Q? Explain how you know.



2 Describe a rigid transformation that takes Triangle A to Triangle B.



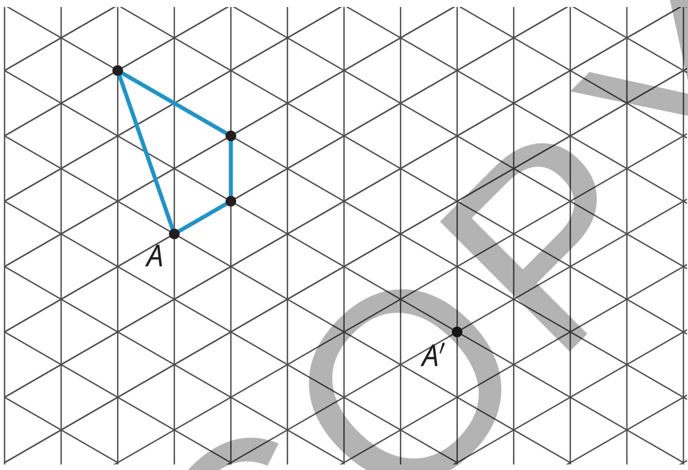
3 Is there a rigid transformation taking Rectangle A to Rectangle B? Explain how you know.



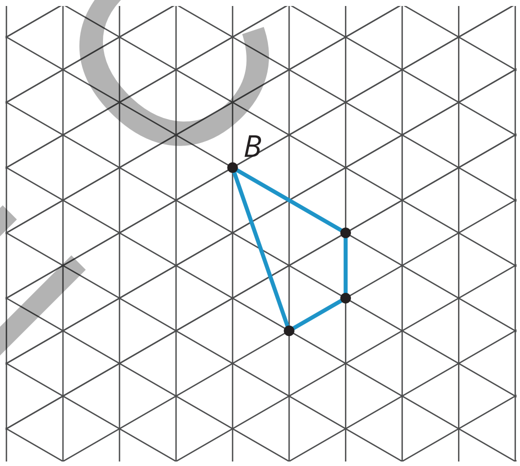
4 from Unit 1, Lesson 4

For each shape, draw its image after performing the transformation. If you get stuck, consider using tracing paper.

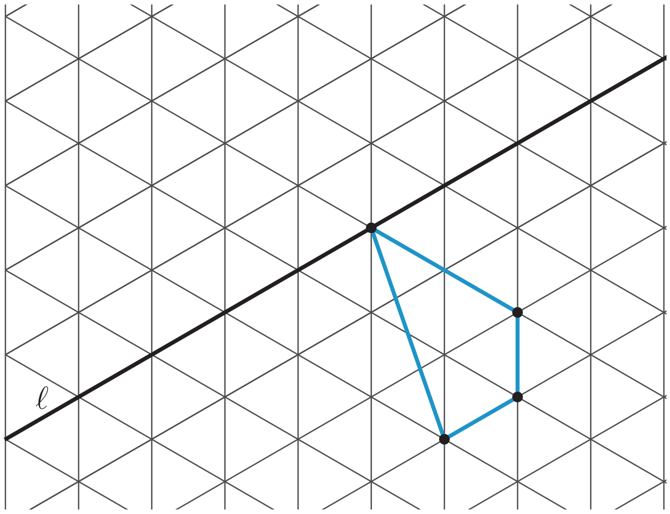
a. Translate the shape so that A goes to A'.



b. Rotate the shape 180° counterclockwise around B.



c. Reflect the shape over the line shown.



Sec B

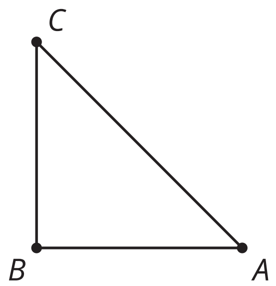
Addressing CA CCSSM 8.G.1a, 8.G.1b; building towards 8.G.1c; practicing MP7 and MP8

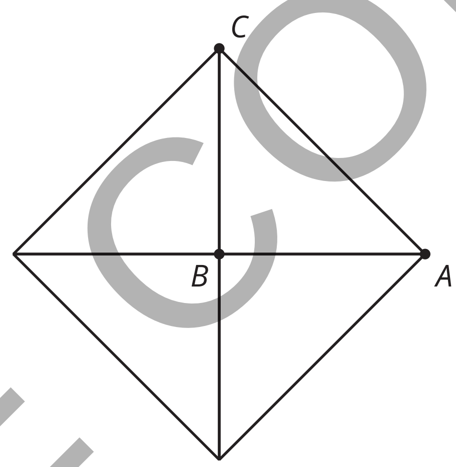
Rotation Patterns

Let’s rotate figures in a plane.

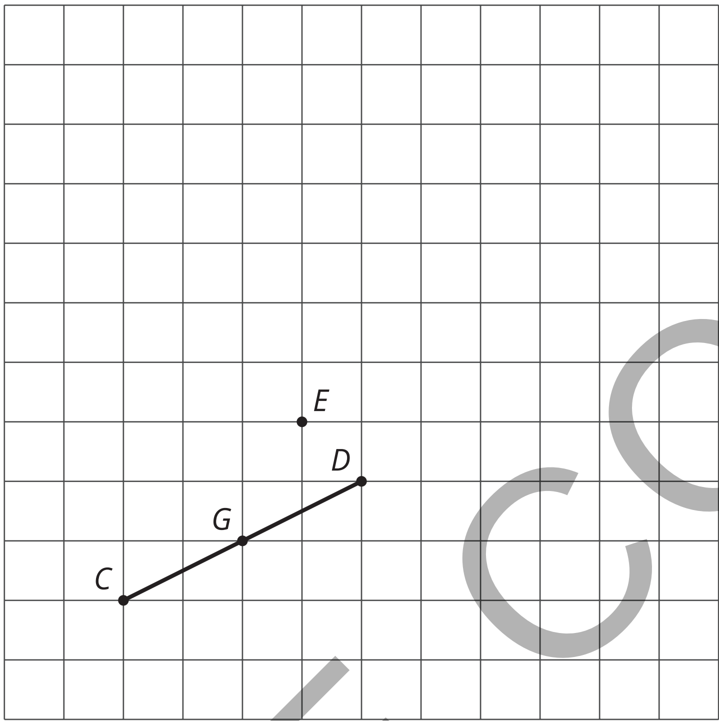
8.1 Notice and Wonder: Building a Quadrilateral

What do you notice? What do you wonder?



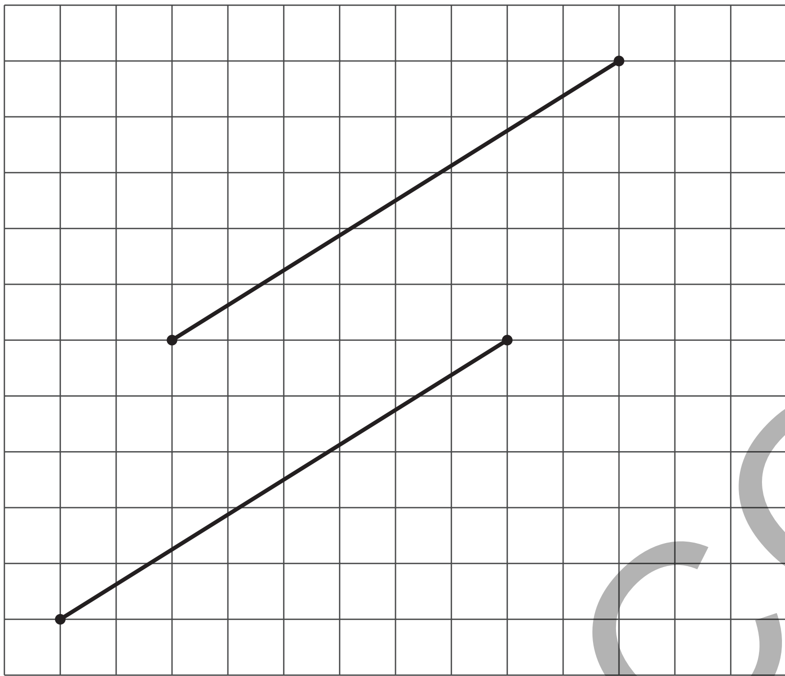


8.2 Rotating a Segment



1. Rotate segment CD 180° around point D. Draw its image and label the image of C as A.
2. Rotate segment CD 180° around point E. Draw its image and label the image of C as B and the image of D as F.
3. Rotate segment CD 180° around its midpoint, G. What is the image of C?>
4. What happens when you rotate a segment 180° around a point?

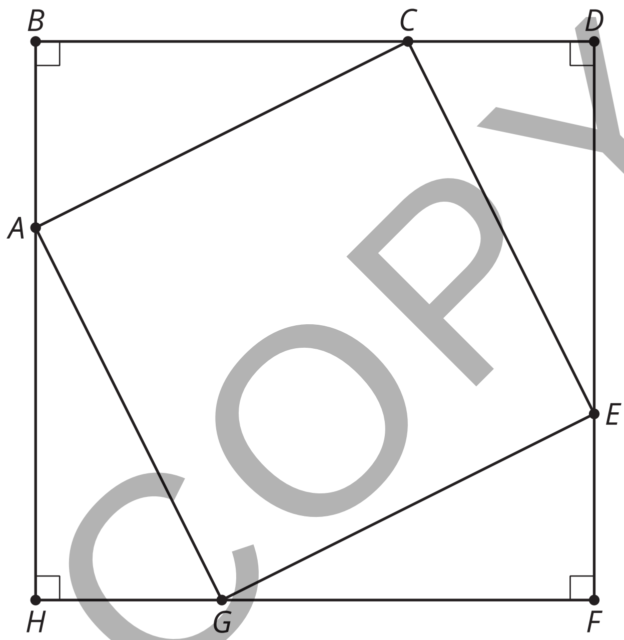
Are you ready for more?



Here are two line segments. Is it possible to rotate one line segment to the other? If so, find the center of such a rotation. If not, explain why not.

8.3 A Pattern of Four Triangles

You can use rigid transformations of a figure to make patterns. Here is a diagram built with three different transformations of triangle ABC.



1. Describe a rigid transformation that takes triangle ABC to triangle CDE.

2. Describe a rigid transformation that takes triangle ABC to triangle EFG.

3. Describe a rigid transformation that takes triangle ABC to triangle GHA.

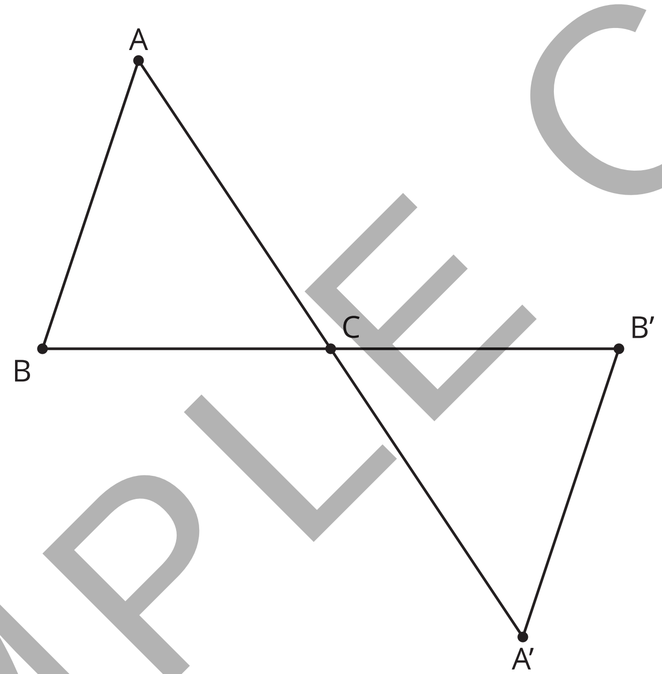
4. Do segments AC, CE, EG, and GA all have the same length? Explain your reasoning.

Lesson 8 Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

- The image of the segment maps is the same as the original (if the center of rotation is the midpoint of the segment).
- The image of the segment overlaps with the segment and lies on the same line (if the center of rotation is a point on the segment).
- The image of the segment does not overlap with the segment and is parallel to the original segment (if the center of rotation is *not* on the segment).

This can also tell us important information about a figure that has been rotated. In this example, triangle ABC has been rotated 180 degrees with point C as the center of rotation. If we think of side AB as a line segment, then we know that its image $A'B'$ must be parallel to it. If we think of side BC as a line segment, then we know that its image $B'C$ must be along the same line.



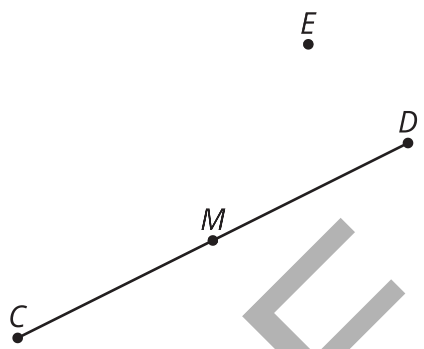
Practice Problems

1 For the figure shown here,

a. Rotate segment CD 180° around point D.

b. Rotate segment CD 180° around point E.

c. Rotate segment CD 180° around point M.

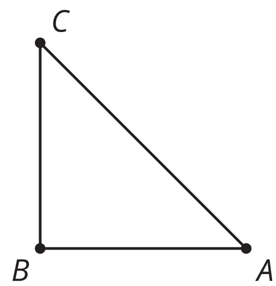


2 Here is an isosceles right triangle. Draw these three rotations of triangle ABC together.

a. Rotate triangle ABC 90° clockwise around A.

b. Rotate triangle ABC 180° around A.

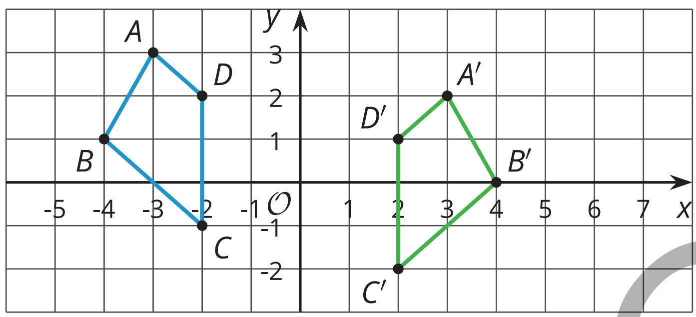
c. Rotate triangle ABC 270° clockwise around A.



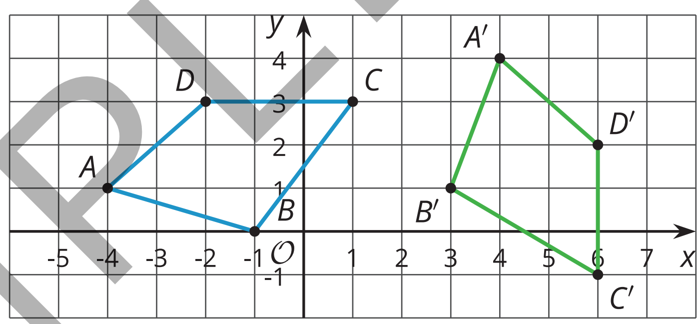
3 from Unit 1, Lesson 5

Each graph shows two polygons $ABCD$ and $A'B'C'D'$. In each case, describe a sequence of transformations that takes $ABCD$ to $A'B'C'D'$.

a.

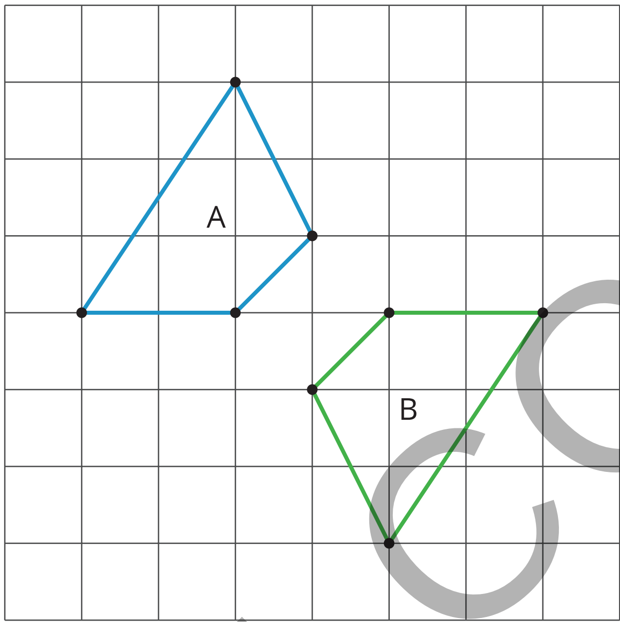


b.



4 from Unit 1, Lesson 4

Lin says that she can map Polygon A to Polygon B using *only* reflections. Do you agree with Lin? Explain your reasoning.



Sec B

Unit 1, Lesson 9

Addressing CA CCSSM 8.G.1a, 8.G.1b, 8.G.1c; building on 7.G.5; practicing MP7

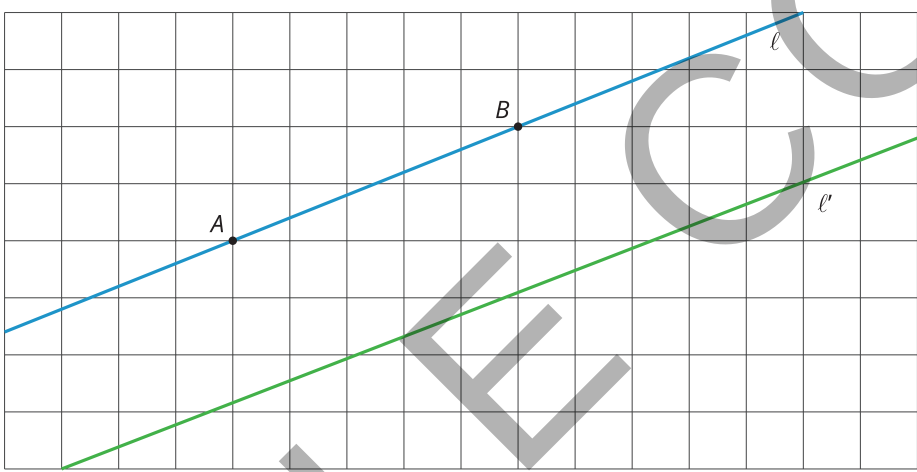
Moves in Parallel

Let's transform some lines.

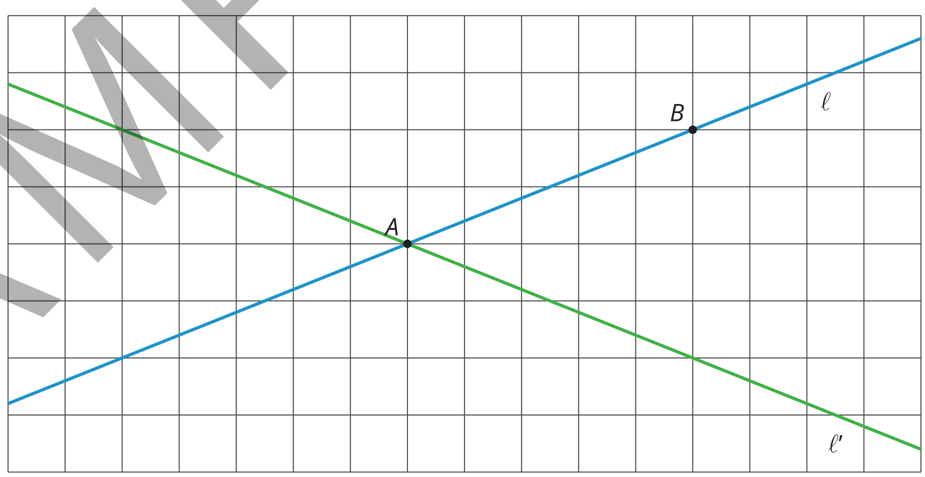
9.1 Line Moves

For each diagram, describe a translation, rotation, or reflection that takes line ℓ to line ℓ'. Then plot and label A' and B', the images of A and B.

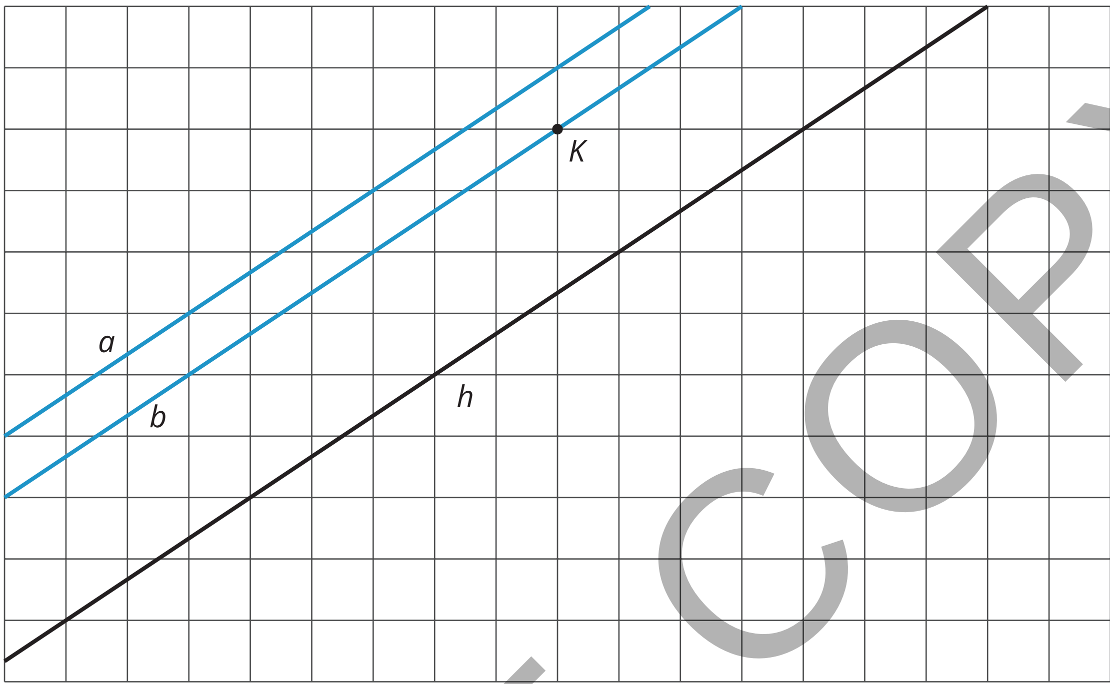
1.



2.



9.2 Parallel Lines



Use a piece of tracing paper to trace lines a and b and point K. Then use that tracing paper to draw the images of the lines under the three different transformations listed.

As you perform each transformation, think about the question:

What is the image of two parallel lines under a rigid transformation?

1. Translate lines a and b 3 units up and 2 units to the right.

   a. What do you notice about the changes that occur to lines a and b after the translation?

   b. What is the same in the original and the image?

2. Rotate lines a and b counterclockwise 180° using K as the center of rotation.

   a. What do you notice about the changes that occur to lines a and b after the rotation?

   b. What is the same in the original and the image?

Sec B

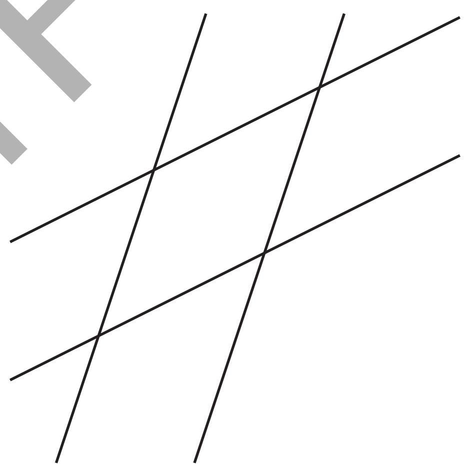
3. Reflect lines a and b across line h.

   a. What do you notice about the changes that occur to lines a and b after the reflection?

   b. What is the same in the original and the image?

Are you ready for more?

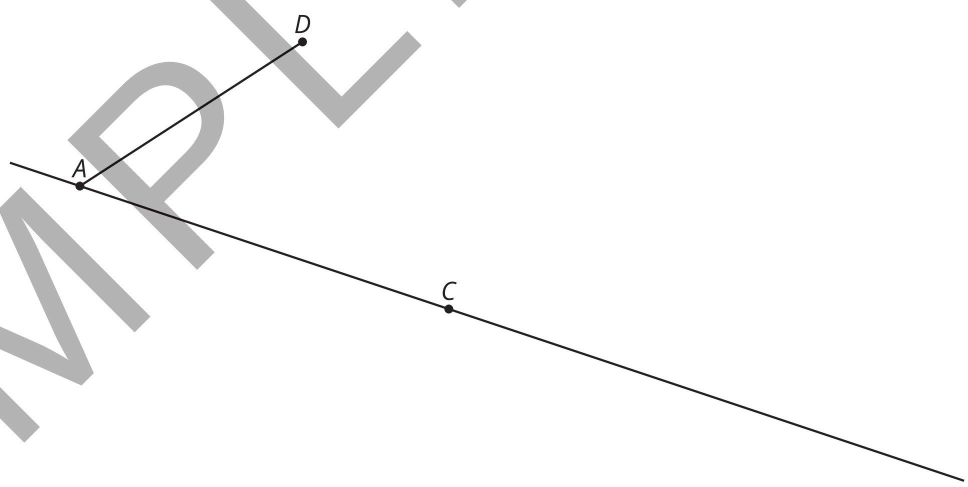
When you rotate two parallel lines, sometimes the two original lines intersect their images and form a quadrilateral. What is the most specific thing you can say about this quadrilateral? Can it be a square? A rhombus? A rectangle that isn't a square? Explain your reasoning.



9.3 Let's Do Some 180s

1. The diagram shows a line with points labeled A, C, D, and B.
   a. On the diagram, draw the image of the line and points A, C, and B after the line has been rotated 180° around point D.
   b. Label the images of the points A', B', and C'.
   c. What is the order of all seven points? Explain or show your reasoning.

2. The diagram shows a line with points A and C on the line and a segment AD where D is not on the line.
   a. Rotate the figure 180° about point C. Label the image of A as A' and the image of D as D'.
   b. What do you know about the relationship between angle CAD and angle $CA'D'$? Explain or show your reasoning.

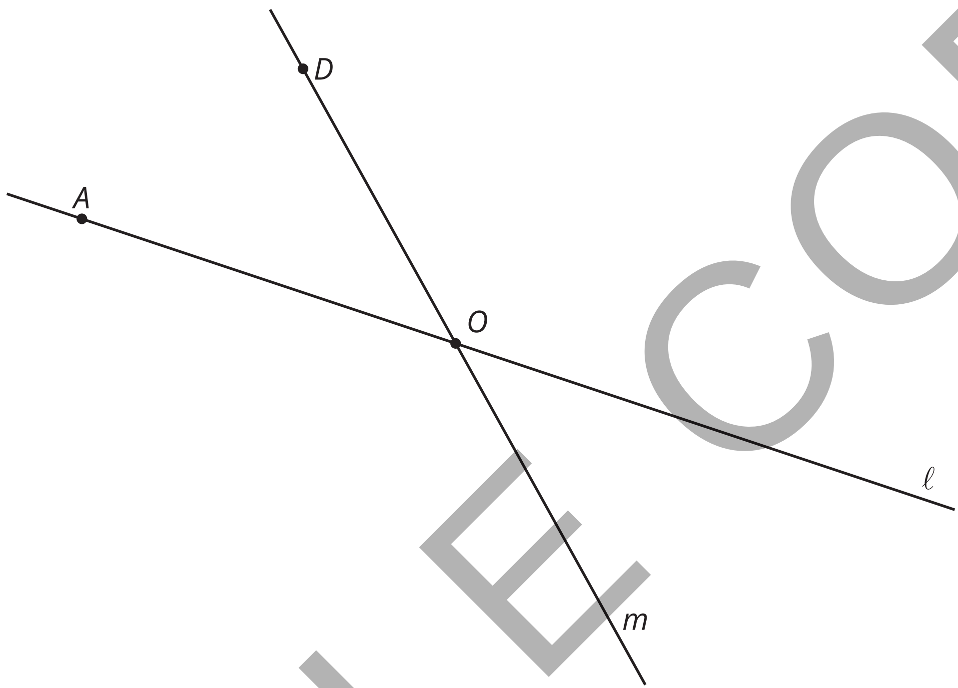


Sec B

3. The diagram shows two lines ℓ and m that intersect at a point O with point A on ℓ and point D on m.

   a. Rotate the figure 180° around O. Label the image of A as A' and the image of D as D'.

   b. What do you know about the relationship between the angles in the figure? Explain or show your reasoning.

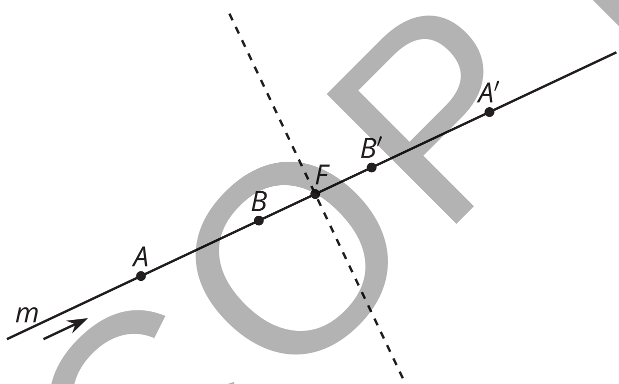


Sec B

Lesson 9 Summary

Rigid transformations have the following properties:

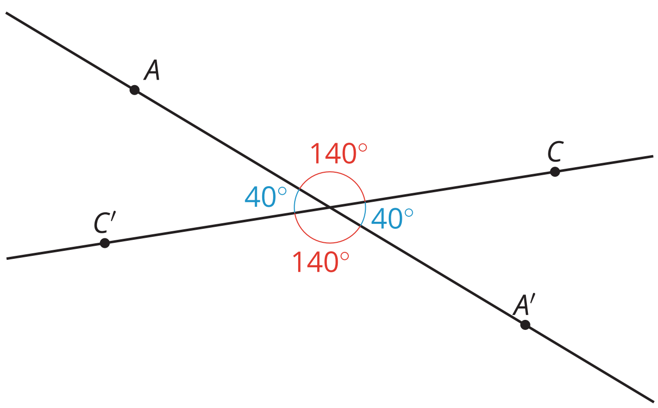
- A rigid transformation of a line is a line.
- A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.
- Sometimes, a rigid transformation takes a line to itself. For example:



  - A translation parallel to the line. The arrow shows a translation of line m that will take m to itself.
  - A rotation by 180° around any point on the line. A 180° rotation of line m around point F will take m to itself.
  - A reflection across any line perpendicular to the line. A reflection of line m across the dashed line will take m to itself.

These facts let us make an important conclusion.

If two lines intersect at a point, which we'll call O, then a 180° rotation of the lines with center O shows that **vertical angles** are congruent. Here is an example:



Rotating both lines by 180° around O sends angle AOC to angle $A'OC'$, therefore proving that they have the same measure. The rotation also sends angle AOC' to angle $A'OC$.

Glossary

- vertical angles

Practice Problems

1

a. Draw parallel lines AB and CD.

b. Pick any point E. Rotate AB 90° clockwise around E.

c. Rotate line CD 90° clockwise around E.

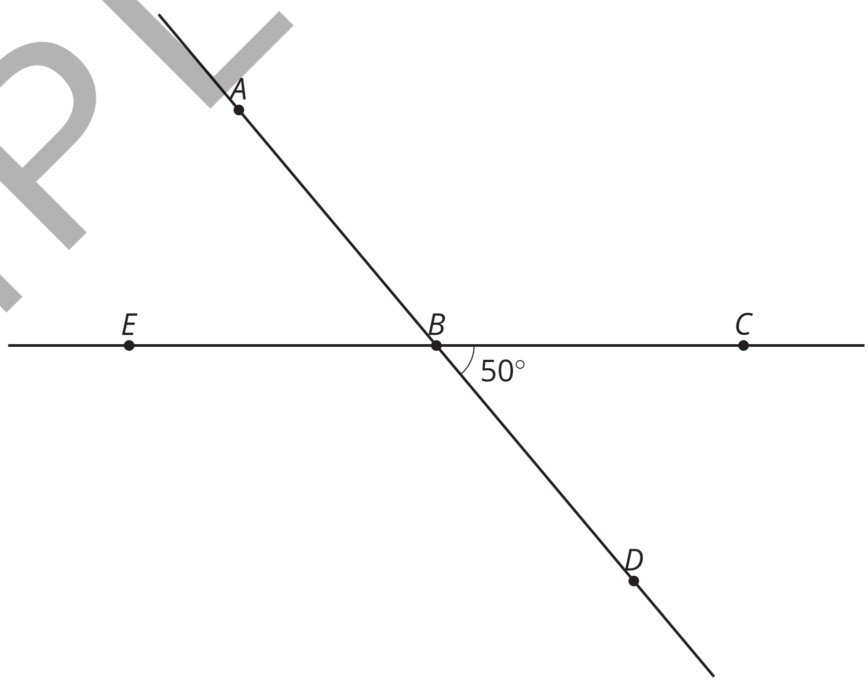
d. What do you notice?

2 Use the diagram to find the measures of each angle. Explain your reasoning.

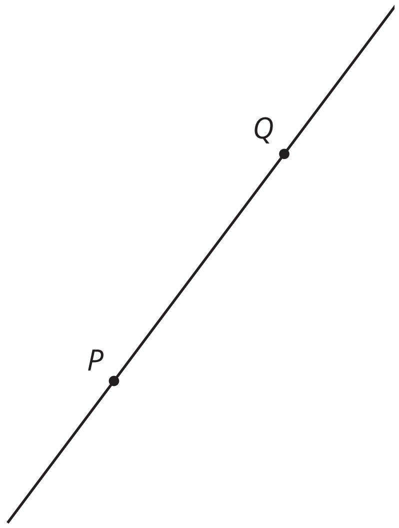
a. angle ABC

b. angle EBD

c. angle ABE



3 Points P and Q are plotted on a line.

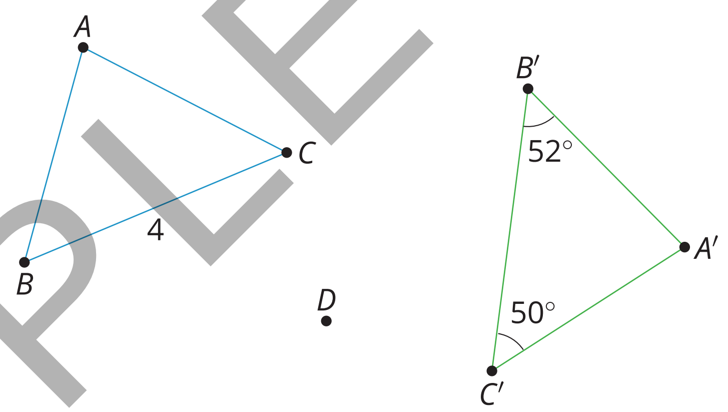


a. Find a point R so that a 180° rotation with center R sends P to Q and Q to P.

b. Is there more than one point R that works for part a?

4 from Unit 1, Lesson 7

In the picture triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is D.



a. What is the length of side $B'C'$? Explain how you know.

b. What is the measure of angle B? Explain how you know.

c. What is the measure of angle C? Explain how you know.

Sec B

5 from Unit 1, Lesson 6

The point $(-4, 1)$ is rotated 180° counterclockwise using center $(0, 0)$. What are the coordinates of the image?

A. $(-1, -4)$

B. $(-1, 4)$

C. $(4, 1)$

D. $(4, -1)$

Sec B

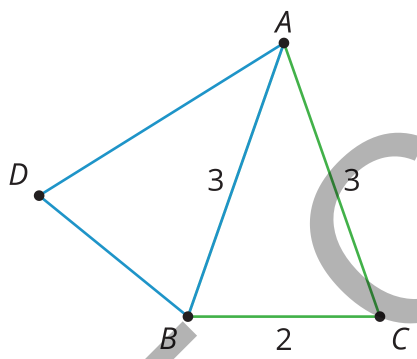
Addressing CA CCSSM 8.G.1a, 8.G.1b; practicing MP6 and MP7

Composing Figures

Let's use reasoning about rigid transformations to find measurements without measuring.

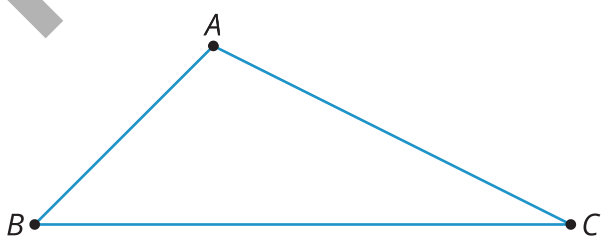
10.1 Notice and Wonder: Angles of an Isosceles Triangle

What do you notice? What do you wonder?



10.2 Triangle Plus One

Here is triangle ABC.



1. Draw midpoint M of side AC.
2. Rotate triangle ABC 180° using center M to form a new triangle. Draw this triangle, and label the new point D.
3. What kind of quadrilateral is $ABCD$? Explain how you know.

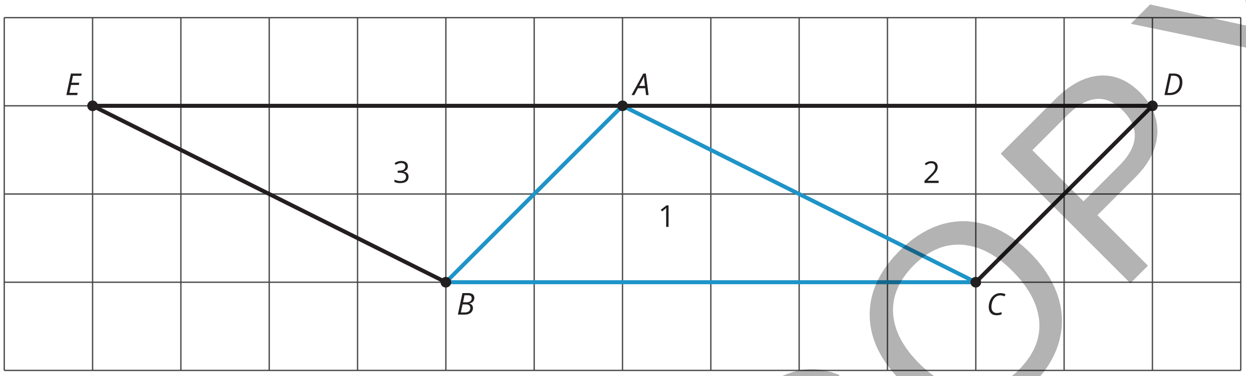
Are you ready for more?

In the activity, we made a parallelogram by taking a triangle and its image under a 180-degree rotation around the midpoint of a side. This picture helps you justify a well-known formula for the area of a triangle. What is the formula and how does the figure help justify it?

Sec B

10.3 Triangle Plus Two

The picture shows 3 triangles. Triangle 2 and Triangle 3 are images of Triangle 1 under rigid transformations.



1. Describe a rigid transformation that takes Triangle 1 to Triangle 2. What points in Triangle 2 correspond to points A, B, and C in the original triangle?

2. Describe a rigid transformation that takes Triangle 1 to Triangle 3. What points in Triangle 3 correspond to points A, B, and C in the original triangle?

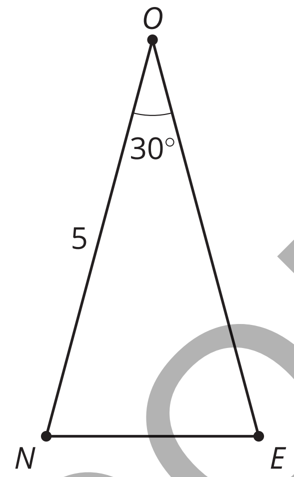
3. Find two pairs of line segments in the diagram that are the same length, and explain how you know they are the same length.

4. Find two pairs of angles in the diagram that have the same measure, and explain how you know they have the same measure.

Sec B

10.4 Triangle ONE Plus

Here is isosceles triangle ONE. Its sides ON and OE have equal lengths. Angle O is 30°. The length of ON is 5 units.

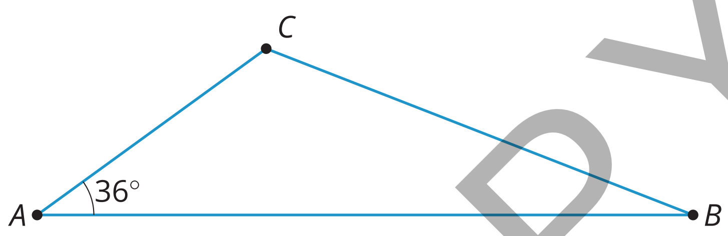


1. Reflect triangle ONE across segment ON. Label the new vertex M.
2. What is the measure of angle MON?
3. What is the measure of angle MOE?
4. Reflect triangle MON across segment OM. Label the point that corresponds to N as T.
5. How long is segment OT? How do you know?
6. What is the measure of angle TOE?
7. If you continue to reflect each new triangle this way to make a pattern, what will the pattern look like?

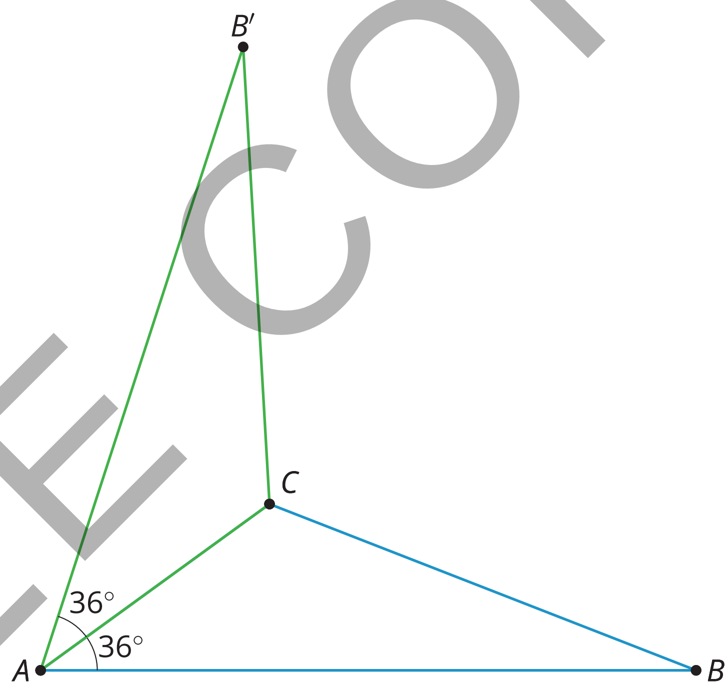
Lesson 10 Summary

Earlier, we learned that if we apply a sequence of rigid transformations to a figure, then corresponding sides have equal length and corresponding angles have equal measure. These facts let us figure out things without having to measure them!

For example, here is triangle ABC.



We can reflect triangle ABC across side AC to form a new triangle:



Because points A and C are on the line of reflection, they do not move. So the image of triangle ABC is $AB'C$. We also know that:

- Angle $B'AC$ measures 36° because it is the image of angle BAC.
- Segment AB' has the same length as segment AB.

When we construct figures using copies of a figure made with rigid transformations, we know that the measures of the images of segments and angles will be equal to the measures of the original segments and angles.

Sec B

Practice Problems

1 Here is the design for the flag of Trinidad and Tobago.

Describe a sequence of translations, rotations, and reflections that take the lower left triangle to the upper right triangle.

2 Here is a picture of an older version of the flag of Great Britain. There is a rigid transformation that takes Triangle 1 to Triangle 2, another that takes Triangle 1 to Triangle 3, and another that takes Triangle 1 to Triangle 4.

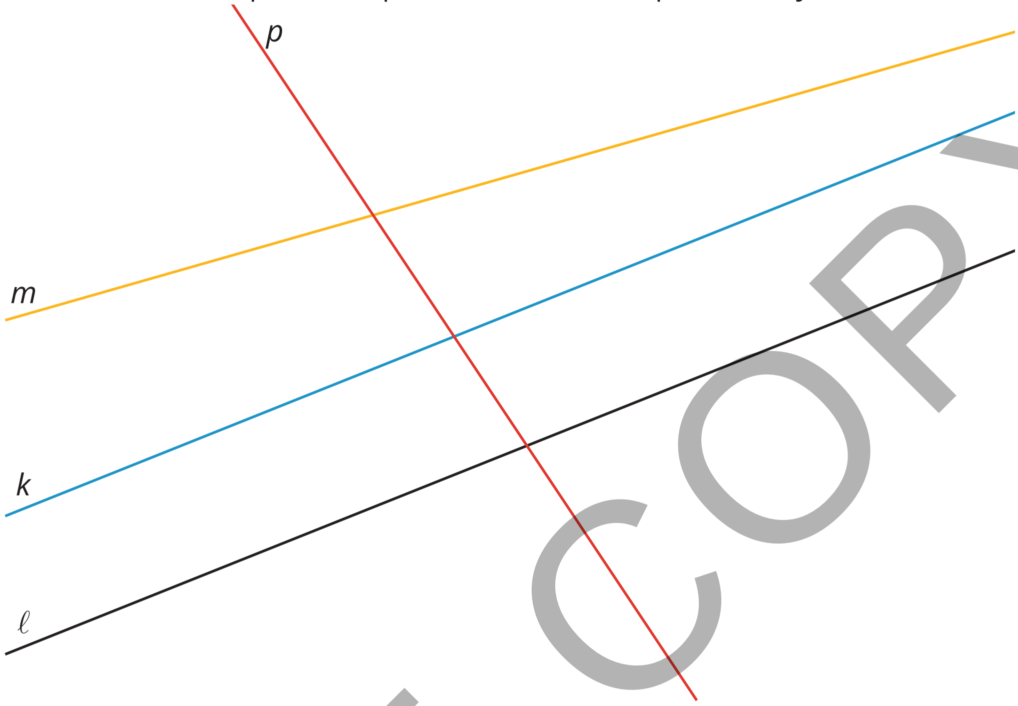
a. Measure the lengths of the sides in Triangles 1 and 2. What do you notice?

b. What are the side lengths of Triangle 3? Explain how you know.

c. Do all 8 triangles in the flag have the same area? Explain how you know.

3 from Unit 1, Lesson 9

a. Which of the lines in the picture is parallel to line ℓ? Explain how you know.



b. Explain how to translate, rotate or reflect line ℓ to obtain line k.

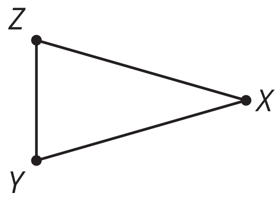
c. Explain how to translate, rotate or reflect line ℓ to obtain line p.

4 from Unit 1, Lesson 6

Point A has coordinates $(3, 4)$. After a translation 4 units left, a reflection across the x-axis, and a translation 2 units down, what are the coordinates of the image?

5 from Unit 1, Lesson 8

Here is triangle XYZ:



Draw these three rotations of triangle XYZ together.

a. Rotate triangle XYZ 90° clockwise around Z.

b. Rotate triangle XYZ 180° around Z.

c. Rotate triangle XYZ 270° clockwise around Z.

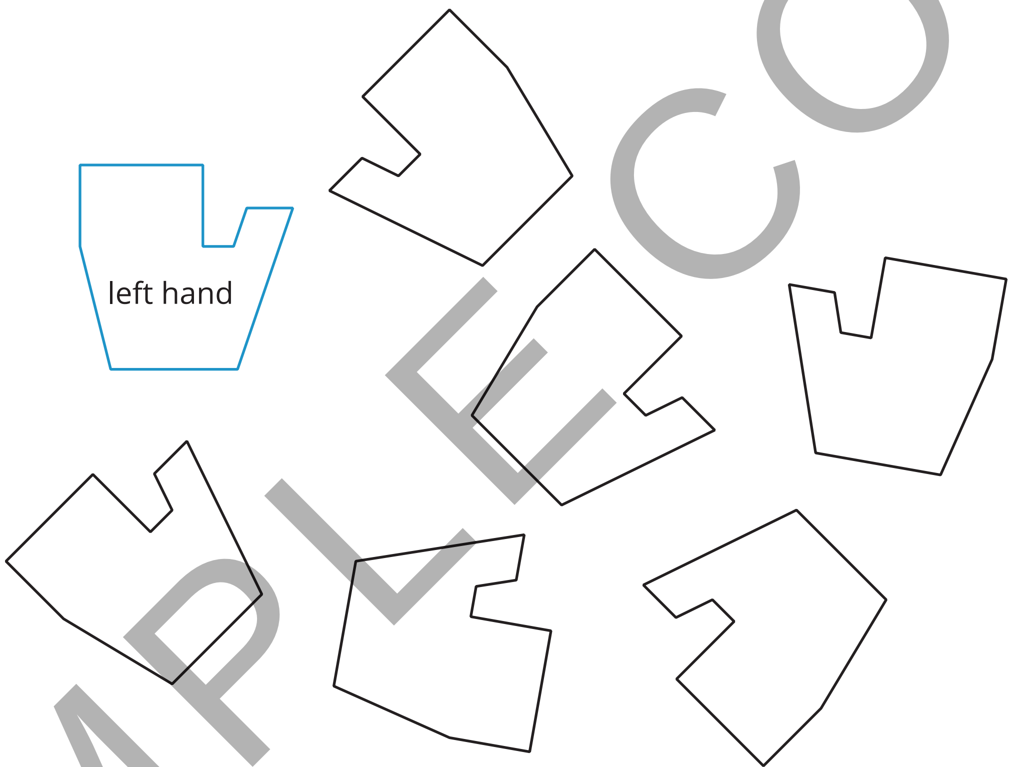
Unit 1, Lesson 11
Addressing CA CCSSM 8.G.1, 8.G.2; building towards 8.G.2; practicing MP3 and MP5

What Is the Same?

Let's decide whether shapes are the same.

11.1 Find the Right Hands

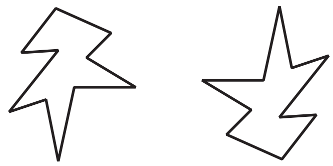
A person's hands are mirror images of each other. In the diagram, a left hand is labeled. Shade all of the right hands.



11.2 Are They the Same?

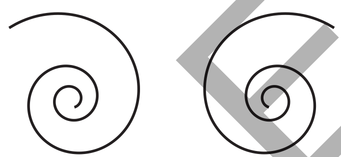
For each pair of shapes, decide whether or not they are the same.

A

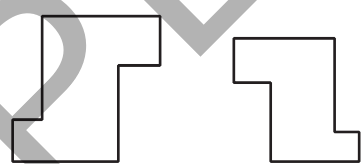


B

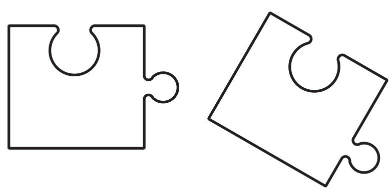
C



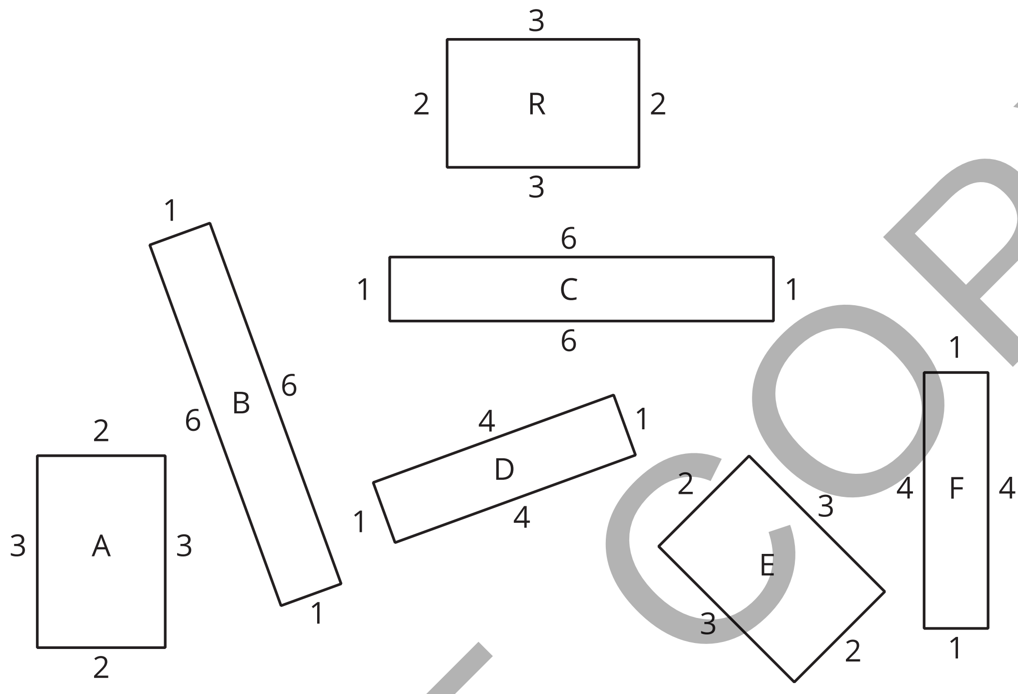
D



E



11.3 Area, Perimeter, and Congruence



1. Which of these rectangles have the same area as Rectangle R but a different perimeter?

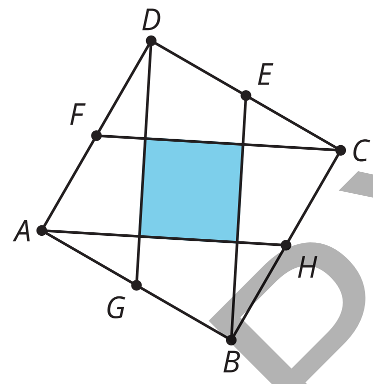
2. Which rectangles have the same perimeter as Rectangle R but a different area?

3. Which rectangles have the same area *and* the same perimeter as Rectangle R?

4. Decide which rectangles are **congruent**. Shade congruent rectangles with the same color.

Are you ready for more?

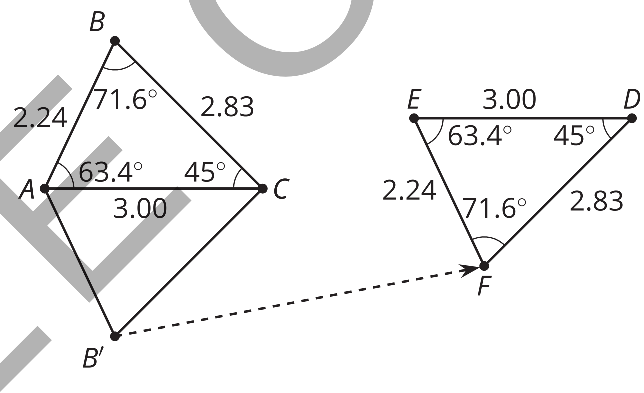
In square $ABCD$, points E, F, G, and H are midpoints of their respective sides. What fraction of square $ABCD$ is shaded? Explain your reasoning.



Lesson 11 Summary

Congruent is a new term for an idea we have already been using. We say that two figures are congruent if one can be lined up exactly with the other by a rigid transformation.

For example, triangle EFD is congruent to triangle ABC because they can be matched up by reflecting triangle ABC across AC followed by the translation shown by the arrow. Notice that all corresponding angles and side lengths are equal.



Here are some other facts about congruent figures:

- We don't need to check all the measurements to prove two figures are congruent. We just have to find a rigid transformation that matches up the figures.
- A figure that looks like a mirror image of another figure can be congruent to it. This means there must be a reflection in the sequence of transformations that matches up the figures.
- Since two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have a different area or perimeter.

Glossary

- congruent

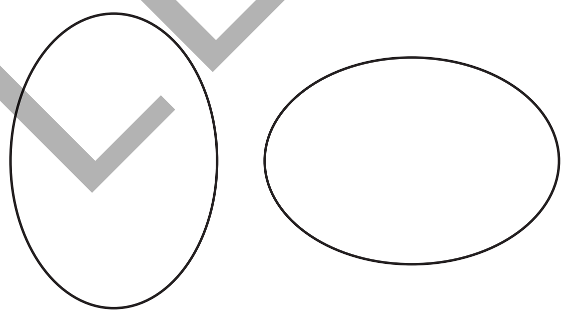
Practice Problems

1 If two rectangles have the same perimeter, do they have to be congruent? Explain how you know.

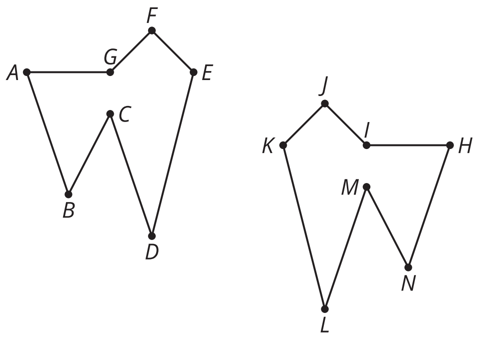
2 Draw two rectangles that have the same area, but are *not* congruent.

3 For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

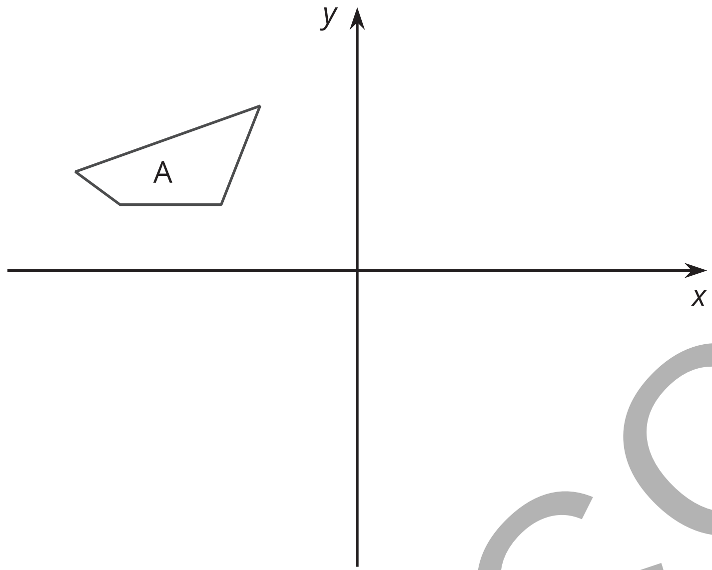
a.



b.



4 a. Reflect Quadrilateral A over the x-axis. Label the image Quadrilateral B. Reflect Quadrilateral B over the y-axis. Label the image C.



Sec C

b. Are Quadrilaterals A and C congruent? Explain how you know.

5 from Unit 1, Lesson 6

The point $(-2, -3)$ is rotated 90° counterclockwise using center $(0, 0)$. What are the coordinates of the image?

A. $(-3, -2)$

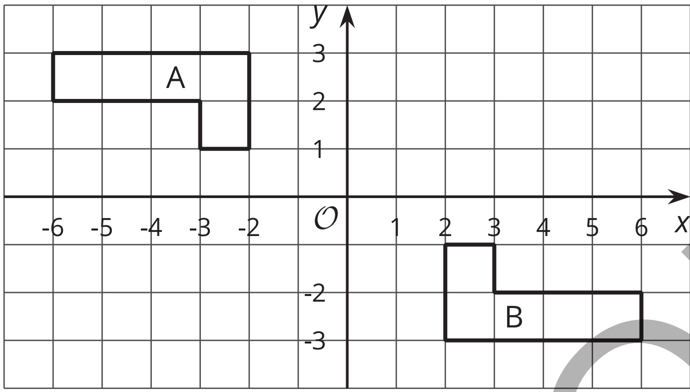
B. $(-3, 2)$

C. $(3, -2)$

D. $(3, 2)$

6 from Unit 1, Lesson 7

Describe a rigid transformation that takes Polygon A to Polygon B.



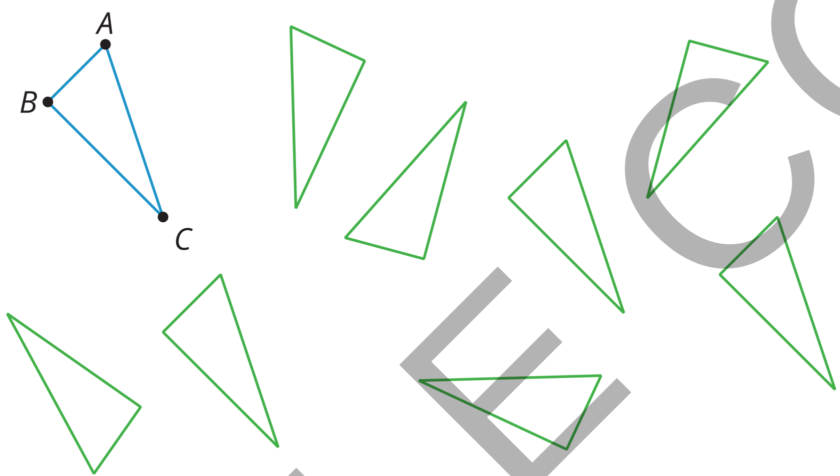
Unit 1, Lesson 12
Addressing CA CCSSM 8.G.2; practicing MP3

Congruent Polygons

Let's decide if two figures are congruent.

12.1 Translated Images

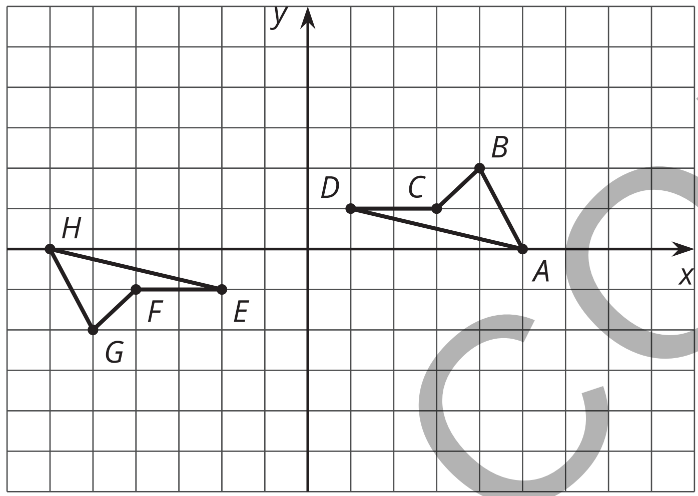
All of these triangles are congruent. Sometimes we can take one figure to another with a translation. Shade the triangles that are images of triangle ABC under a translation.



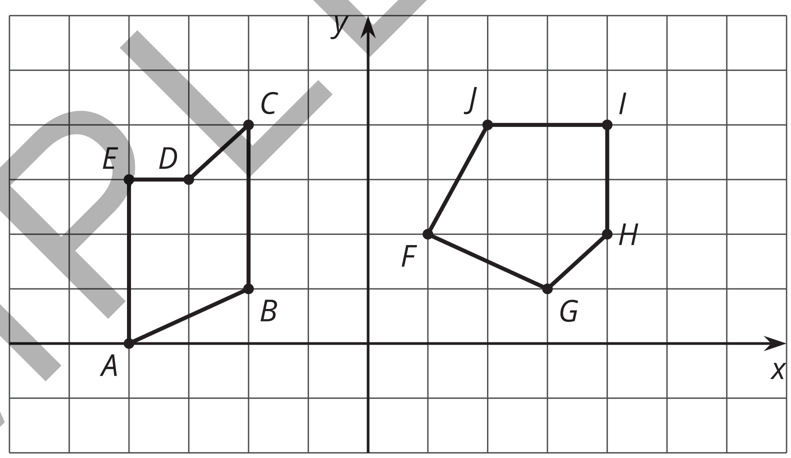
12.2 Congruent Pairs (Part 1)

For each of the following pairs of shapes, decide whether or not they are congruent. Explain your reasoning.

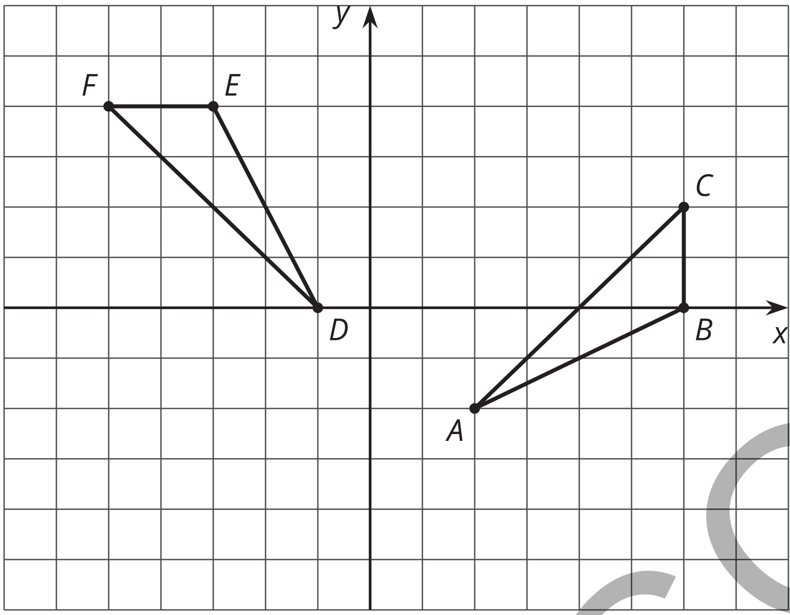
1.



2.

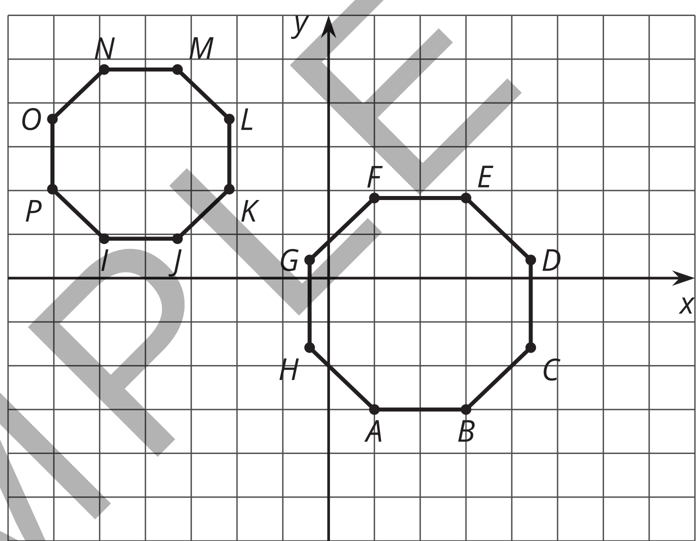


3.



Sec C

4.

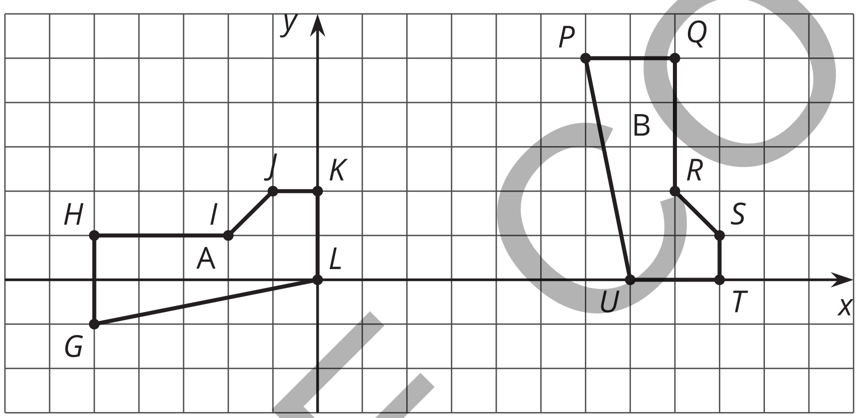


12.3 Congruent Pairs (Part 2)

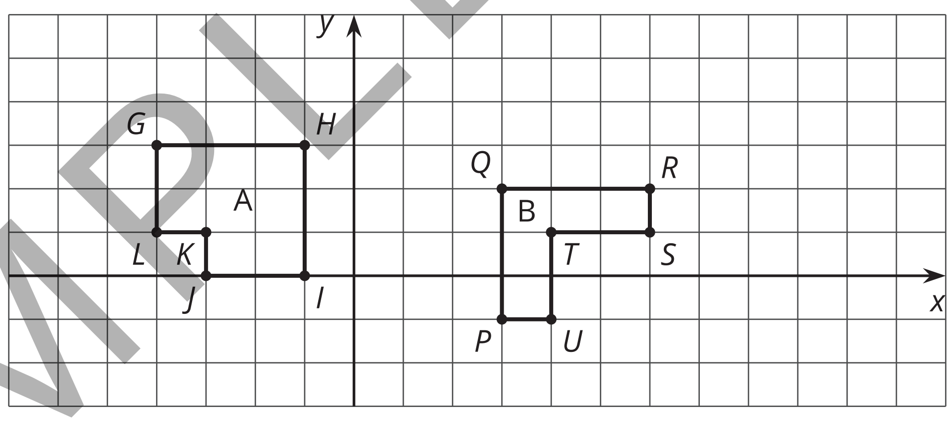
Take turns with your partner to decide whether Shape A is congruent to Shape B.

- For each pair of shapes that you decide is congruent or not congruent, explain to your partner how you know.
- For each pair of shapes that your partner decides is congruent or not congruent, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

1.

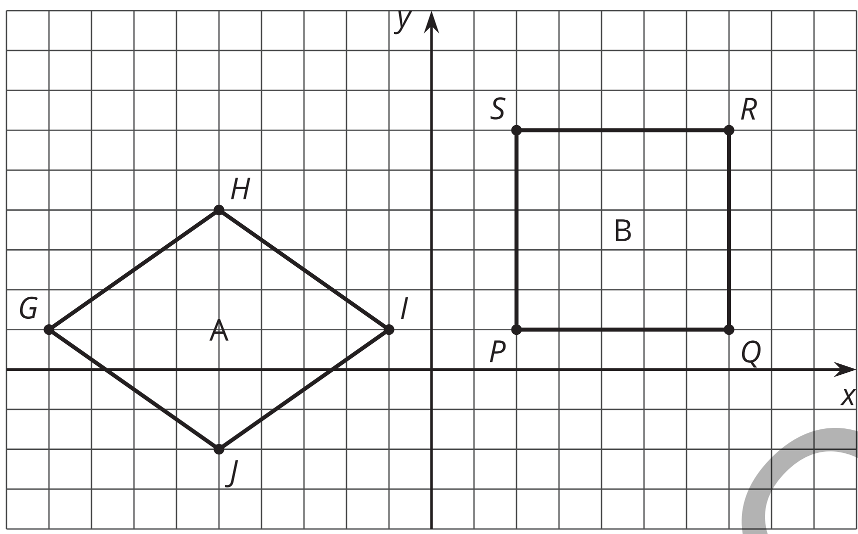


2.

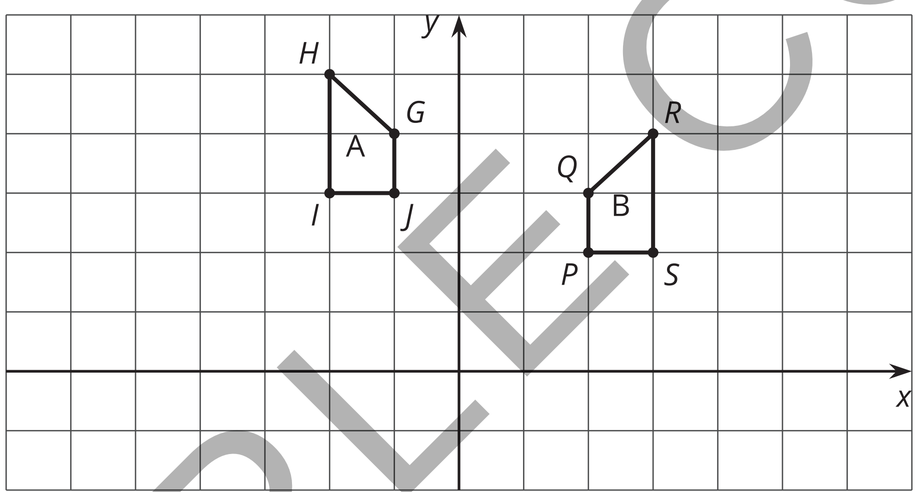


Sec C

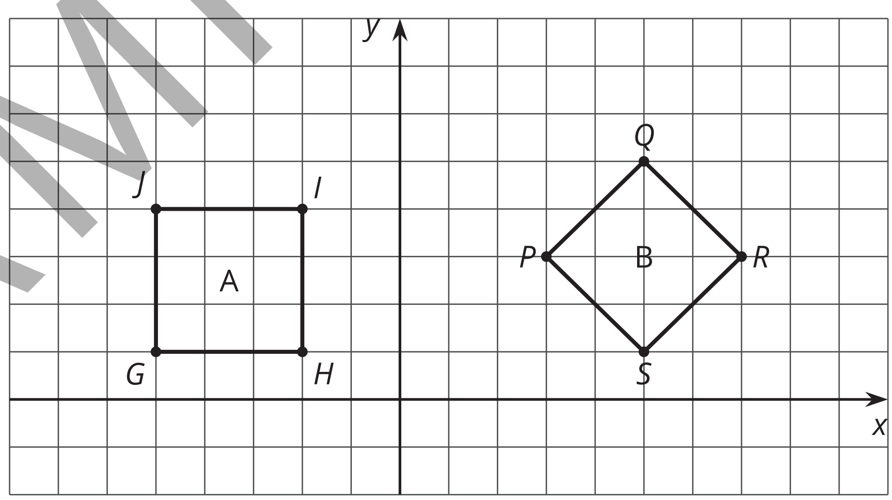
3.



4.



5.



Sec C

Are you ready for more?

A polygon has 8 sides: five of length 1, two of length 2, and one of length 3. All sides lie on grid lines. (It may be helpful to use graph paper when working on this problem.)

1. Find a polygon with these properties.

2. Is there a second polygon, not congruent to the first, with these properties?

Sec C

12.4 Building Quadrilaterals

Your teacher will give you a set of four objects.

1. Make a quadrilateral with your four objects and record what you have made.

2. Compare your quadrilateral with your partner's. Are they congruent? Explain how you know.

3. Repeat Steps 1 and 2, forming different quadrilaterals. If your first quadrilaterals were not congruent, can you build a pair that is? If your first quadrilaterals were congruent, can you build a pair that is not? Explain.

Lesson 12 Summary

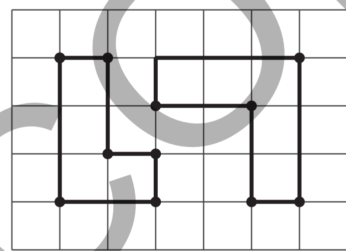
How do we know if two figures are congruent?

- If we copy one figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.
- If we can describe a sequence of translations, rotations, and reflections that move one figure onto the other so they match up exactly, they are congruent.

How do we know that two figures are *not* congruent?

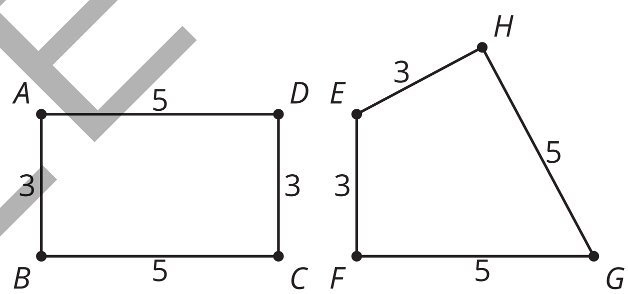
- If there is no correspondence between the figures where the parts have equal measure, that shows that the two figures are *not* congruent.
    - If two polygons have different sets of side lengths, they can't be congruent.

      For example, the figure on the left has side lengths 3, 2, 1, 1, 2, 1. The figure on the right has side lengths 3, 3, 1, 2, 2, 1. There is no way to make a correspondence between them where all corresponding sides have the same length.

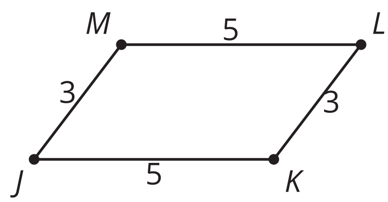
    - If two polygons have the same side lengths, but not in the same order, the polygons can't be congruent.

      For example, rectangle $ABCD$ can't be congruent to quadrilateral $EFGH$. Even though they both have two sides of length 3 and two sides of length 5, they don't correspond in the same order.

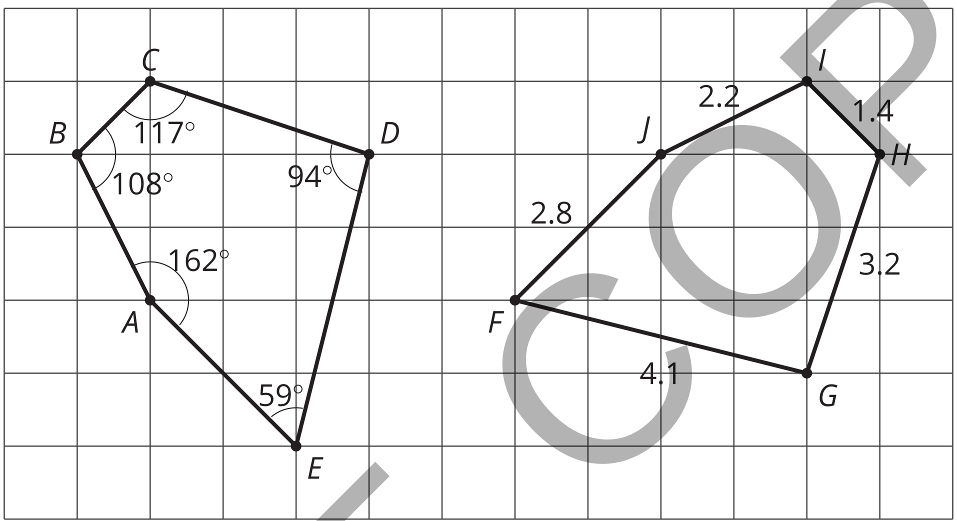
    - If two polygons have the same side lengths, in the same order, but different corresponding angles, the polygons can't be congruent.

      For example, parallelogram $JKLM$ can't be congruent to rectangle $ABCD$. Even though they have the same side lengths in the same order, the angles are different. All angles in $ABCD$ are right angles. In $JKLM$, angles J and L are less than 90 degrees and angles K and M are more than 90 degrees.

Practice Problems

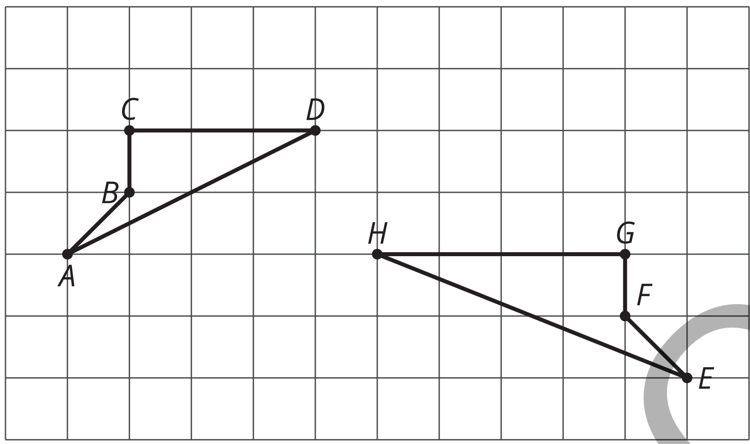
1 a. Show that the two pentagons in the following image are congruent.



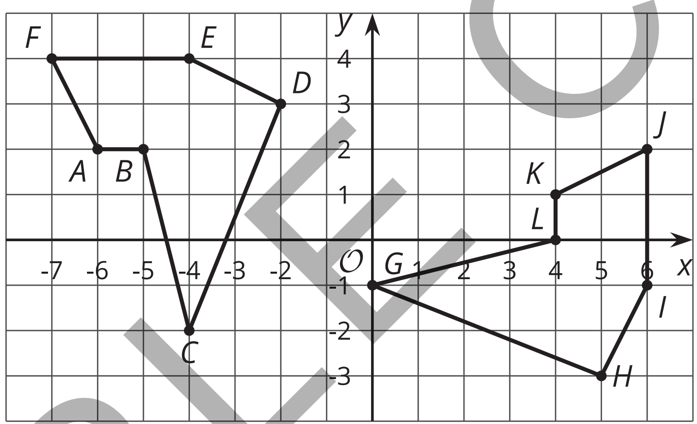
b. Find the side lengths of pentagon $ABCDE$ and the angle measures of pentagon $FGHIJ$.

2 For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

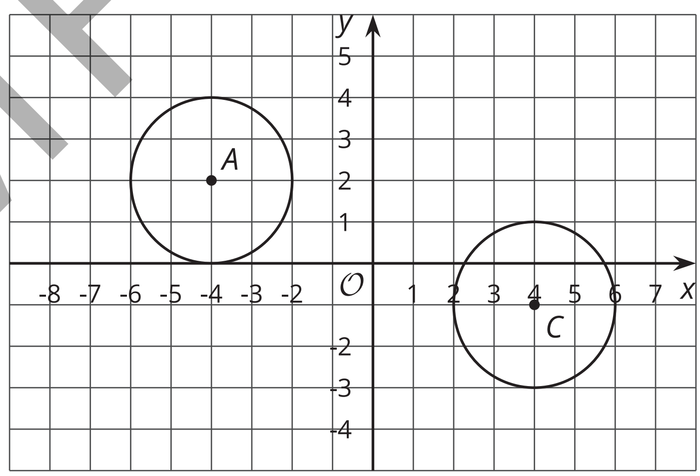
a.



b.



c.



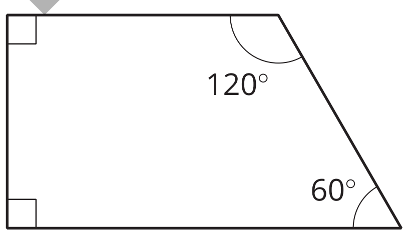
3 from Unit 1, Lesson 8

a. Draw segment PQ.

b. When PQ is rotated 180° around point R, the resulting segment is the same as PQ. Where could point R be located?

4 from Unit 1, Lesson 10

Here is a trapezoid.



Using rigid transformations on the trapezoid, build a pattern. Describe some of the rigid transformations you used.

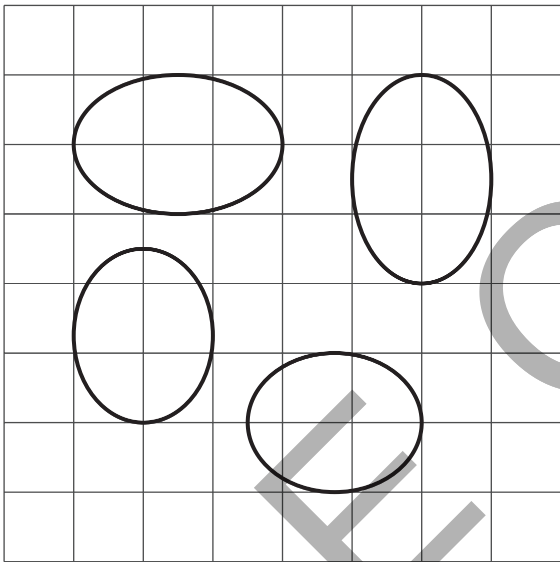
Unit 1, Lesson 13
Addressing CA CCSSM 8.G.1a, 8.G.2; practicing MP1 and MP3

Congruence

Let's find ways to test congruence of interesting figures.

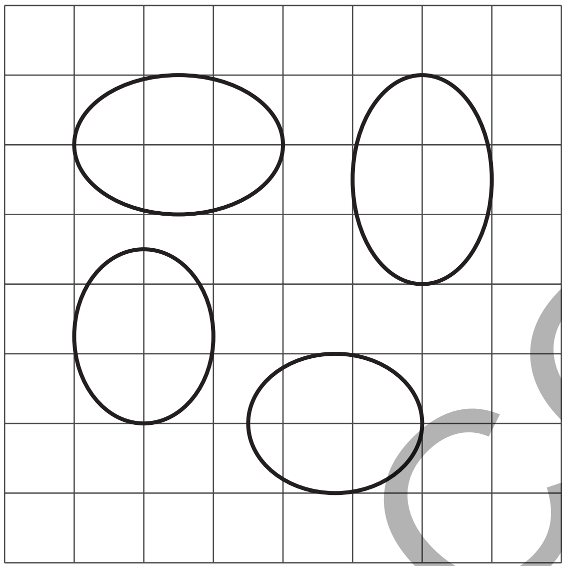
Sec C

13.1 Oval Questions



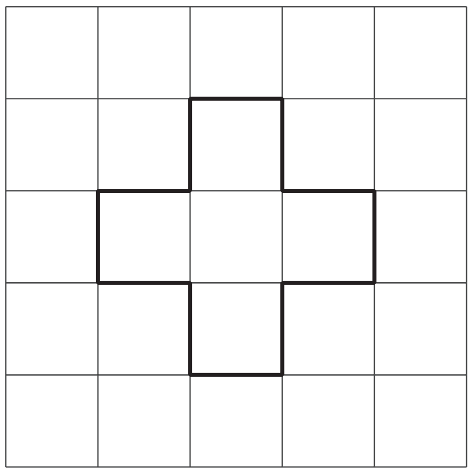
13.2 Congruent Ovals

Are any of the ovals congruent to one another? Explain how you know.



Are you ready for more?

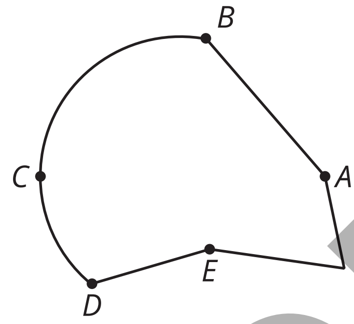
You can use 12 toothpicks to create a polygon with an area of five square toothpicks, like this:

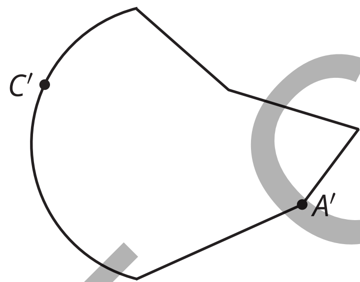


Can you use exactly 12 toothpicks to create a polygon with an area of four square toothpicks?

13.3 Corresponding Points in Congruent Figures

Here are two congruent shapes with some corresponding points labeled:

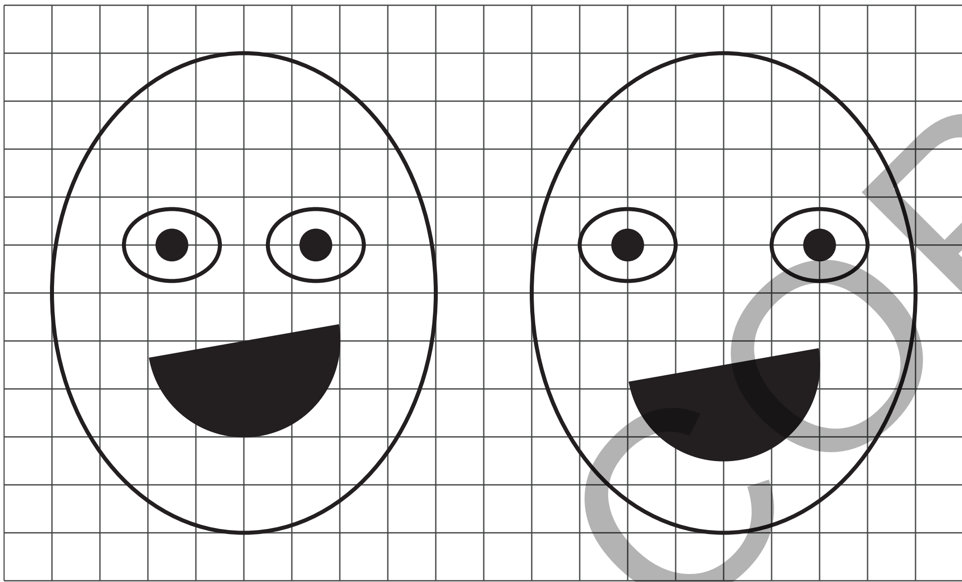




1. On the bottom figure, draw the points corresponding to B, D, and E, and label them B', D', and E'.
2. Draw line segments AD and $A'D'$ and measure them. Do the same for segments BC and $B'C'$ and for segments AE and $A'E'$. What do you notice?
3. Do you think there could be a pair of corresponding segments with different lengths? Explain.

13.4 Astonished Faces

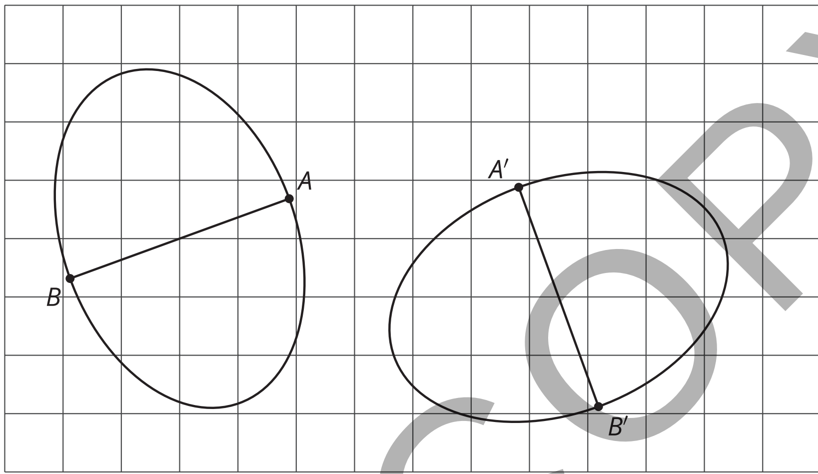
Are these faces congruent? Explain your reasoning.



Lesson 13 Summary

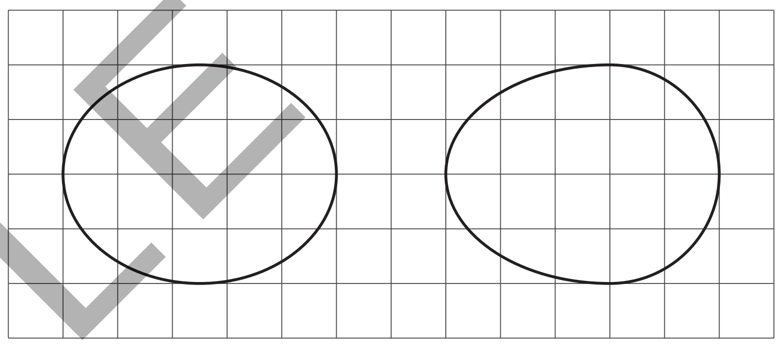
To show two figures are congruent, one is aligned with the other by a sequence of rigid transformations. This is true even for figures with curved sides. Distances between corresponding points on congruent figures are always equal, even for curved shapes.

For example, corresponding segments AB and $A'B'$ on these congruent ovals have the same length:

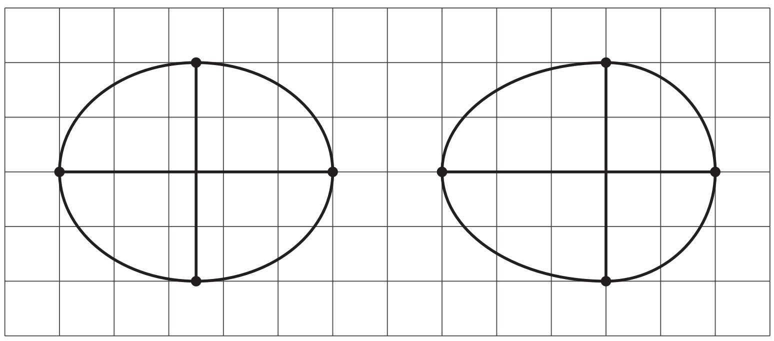


To show two figures are not congruent, you can find parts of the figures that should correspond but that have different measurements.

For example, these two ovals don't look congruent.



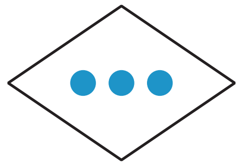
On both, the longest distance is 5 units across, and the longest distance from top to bottom is 4 units. The line segment from the highest to lowest point is in the middle of the left oval, but in the right oval, it's 2 units from the right end and 3 units from the left end. This shows they are not congruent.

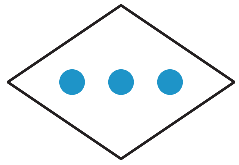


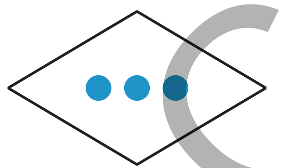
Sec C

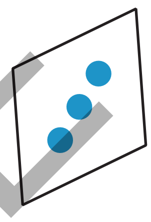
Practice Problems

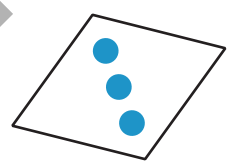
1 Which of these four figures is congruent to this figure?



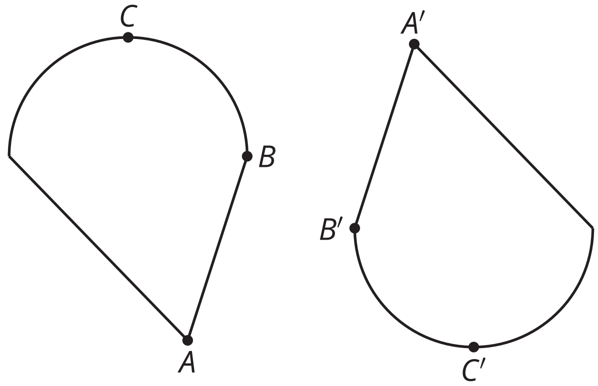
A. 

B. 

C. 

D. 

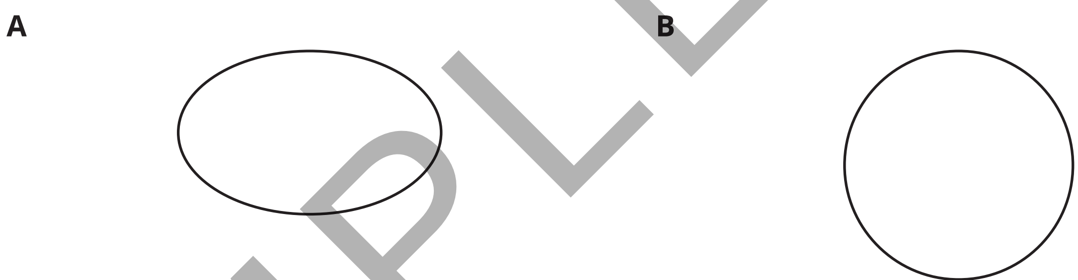
2 These two figures are congruent and have corresponding points marked.



a. Are angles ABC and $A'B'C'$ congruent? Explain your reasoning.

b. Measure angles ABC and $A'B'C'$ to check your answer.

3 Here are two figures.

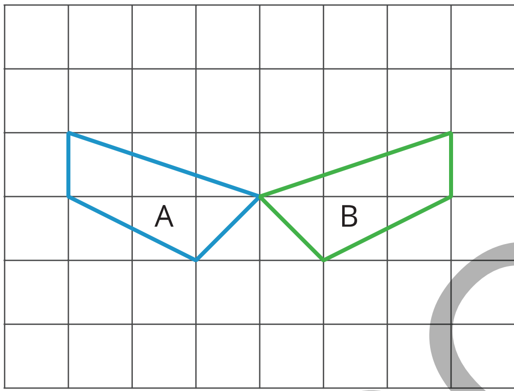


Explain why these two figures are *not* congruent.

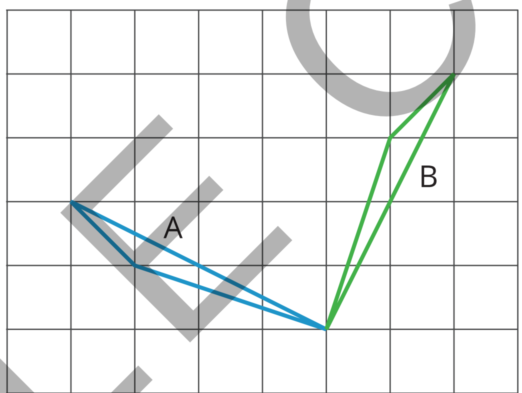
4 from Unit 1, Lesson 3

Each picture shows two polygons, one labeled Polygon A and one labeled Polygon B. Describe how to move Polygon A into the position of Polygon B using a transformation.

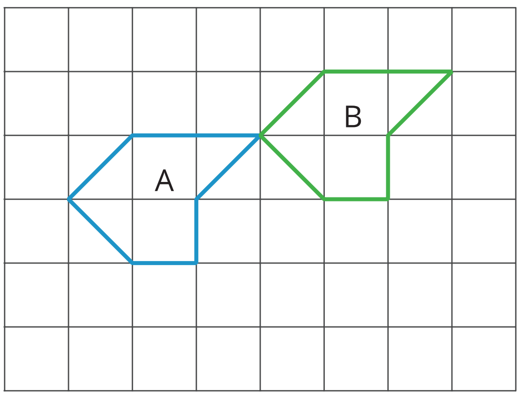
a.



b.



c.



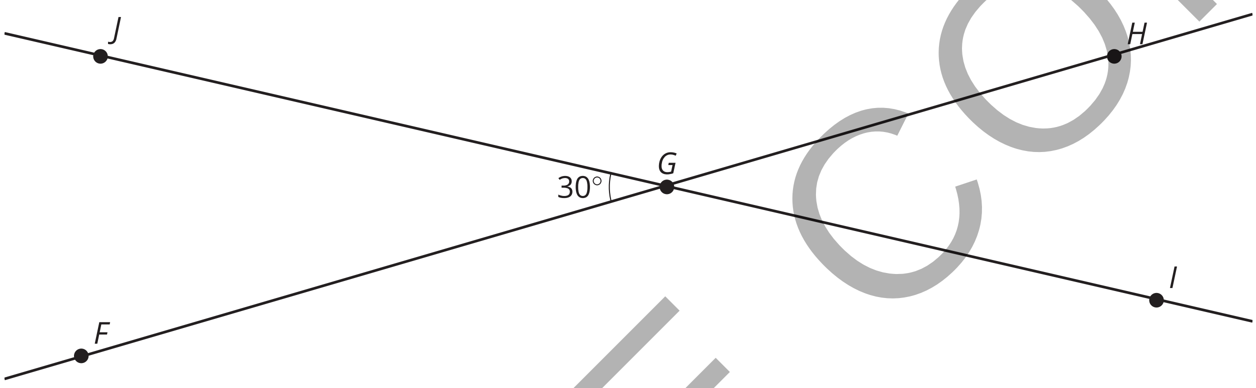
Unit 1, Lesson 14
Addressing CA CCSSM 8.G.1, 8.G.5; building on 7.G.5; practicing MP3

Alternate Interior Angles

Let's explore why some angles are always equal.

14.1 Angle Pairs

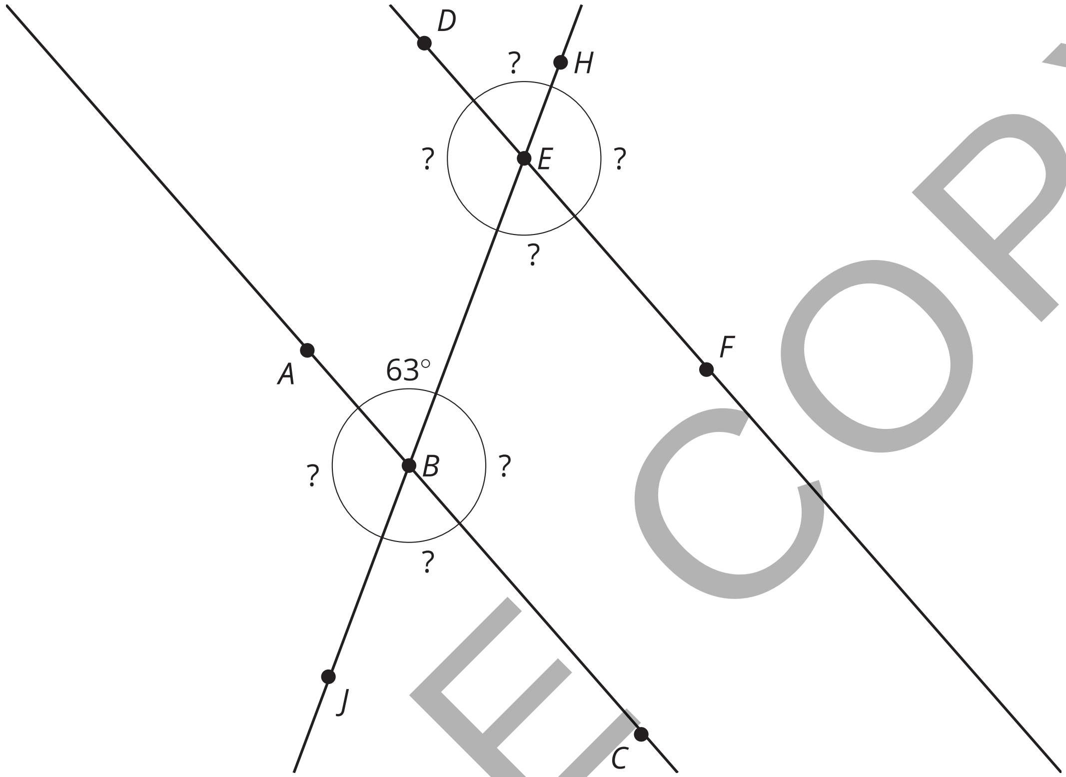
1. Find the measure of angle JGH. Explain or show your reasoning.



2. Find and label a second 30° angle in the diagram. Find and label an angle congruent to angle JGH.

14.2 Cutting Parallel Lines with a Transversal

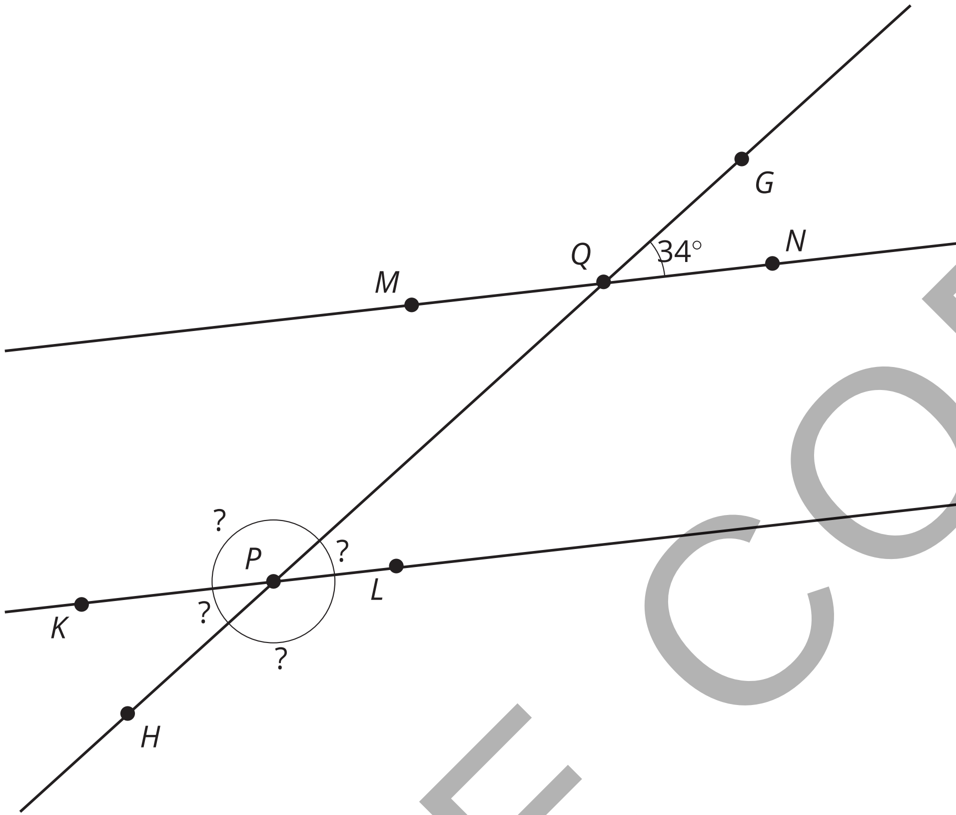
Lines AC and DF are parallel. They are cut by **transversal** HJ.



1. With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning.

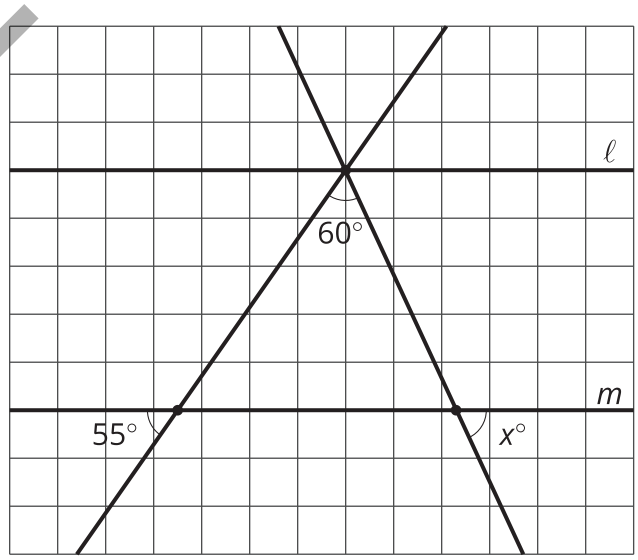
2. What do you notice about the angles with vertex B and the angles with vertex E?

3. Using what you noticed, find the measures of the four angles at point P in the diagram. Lines KL and MN are parallel.

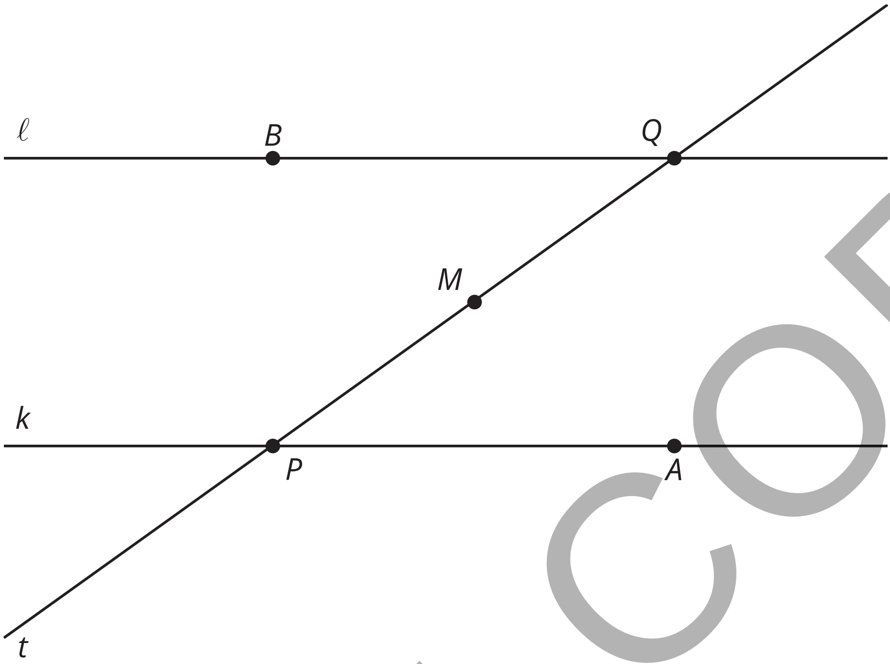


Are you ready for more?

Parallel lines ℓ and m are cut by two transversals that intersect ℓ at the same point. Two angles are marked in the figure. Find the measure x of the third angle.



14.3 Alternate Interior Angles Are Congruent

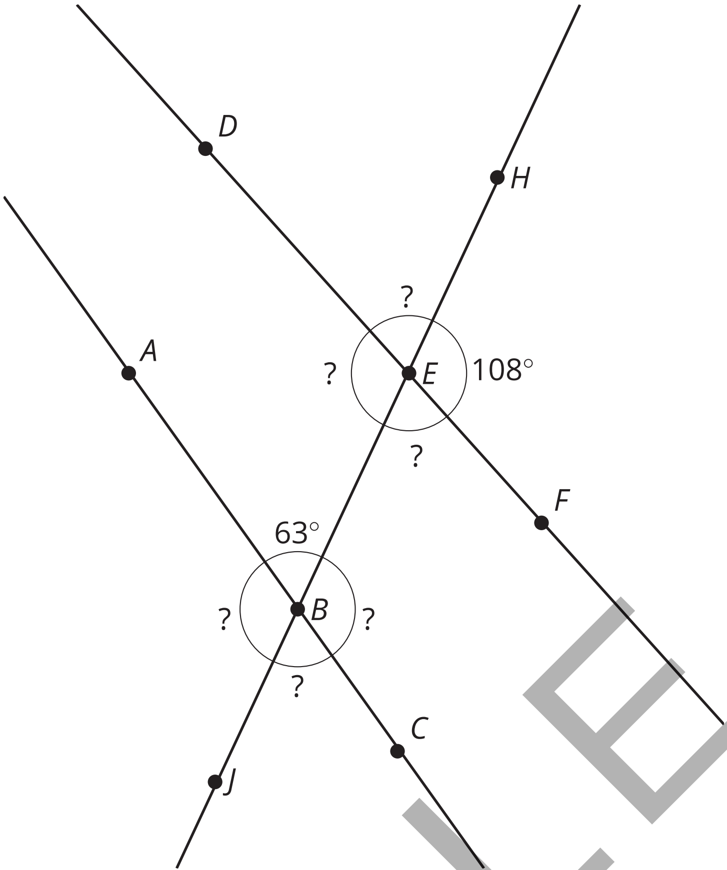


Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.

Find a rigid transformation showing that angles MPA and MQB are congruent.

14.4 Not Parallel

1. Lines DF and AC are not parallel in this image.

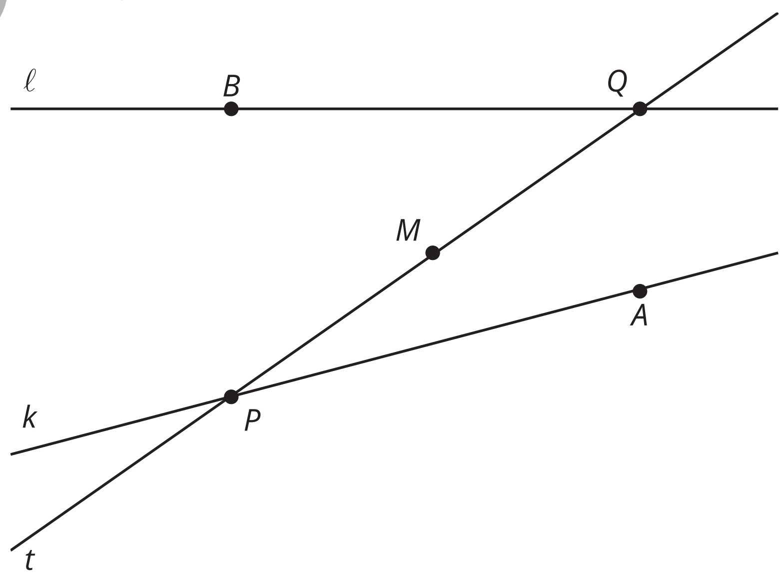


Find the missing angle measures around point E and point B.

What do you notice about the angles in this diagram?

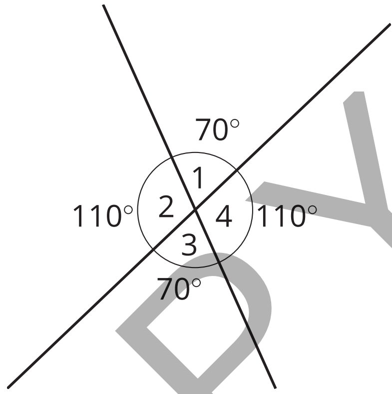
2. Point M is the midpoint of line segment QP.

Can you find a rigid transformation that shows angle BQM is congruent to angle MPA? Explain your reasoning.

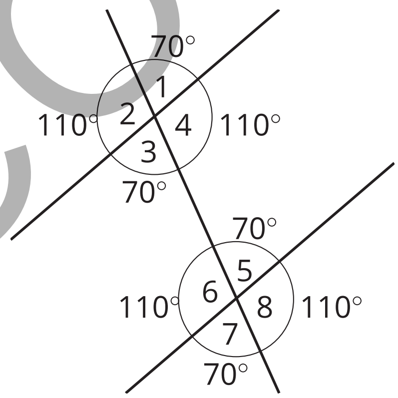


Lesson 14 Summary

When two lines intersect, vertical angles are congruent, and adjacent angles are supplementary, so their measures sum to 180. For example, in this figure angles 1 and 3 are congruent, angles 2 and 4 are congruent, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.



When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.



Alternate interior angles are equal because a 180° rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections. Can you see how rotating 180° about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is 70° we use vertical angles to see that angle 3 is 70°, then we use alternate interior angles to see that angle 5 is 70°, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is 110° since $180 - 70 = 110$. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure 70°, and angles 2, 4, 6, and 8 measure 110°.

Glossary

- alternate interior angles
- transversal

Practice Problems

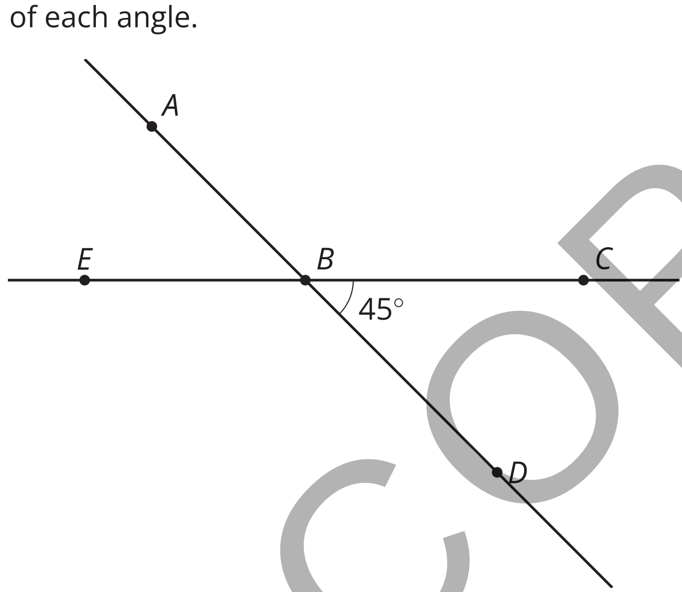
1 from Unit 1, Lesson 9

Use the diagram to find the measure of each angle.

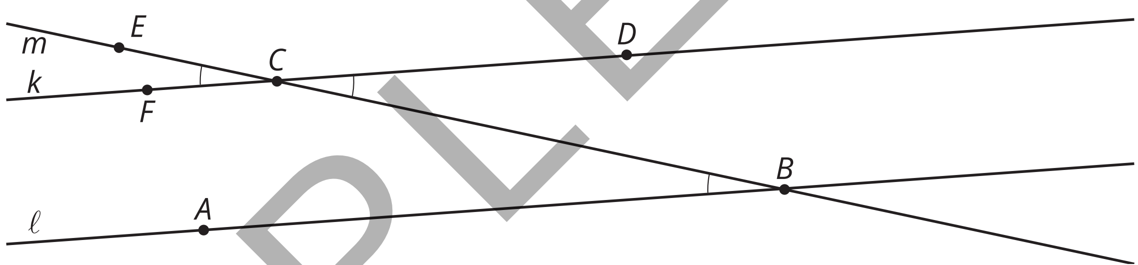
a. angle ABC

b. angle EBD

c. angle ABE



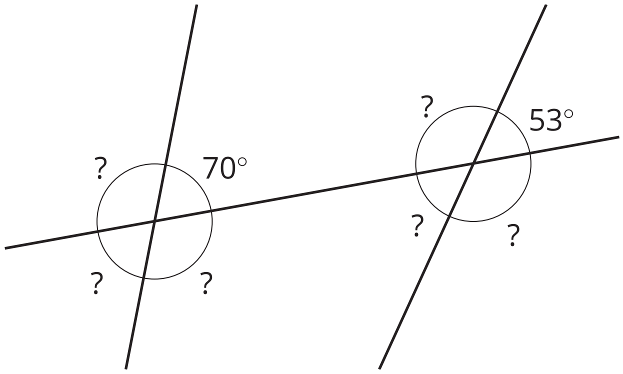
2 Lines k and ℓ are parallel, and the measure of angle ABC is 19°.



a. Explain why the measure of angle ECF is 19°. If you get stuck, consider translating line ℓ by moving B to C.

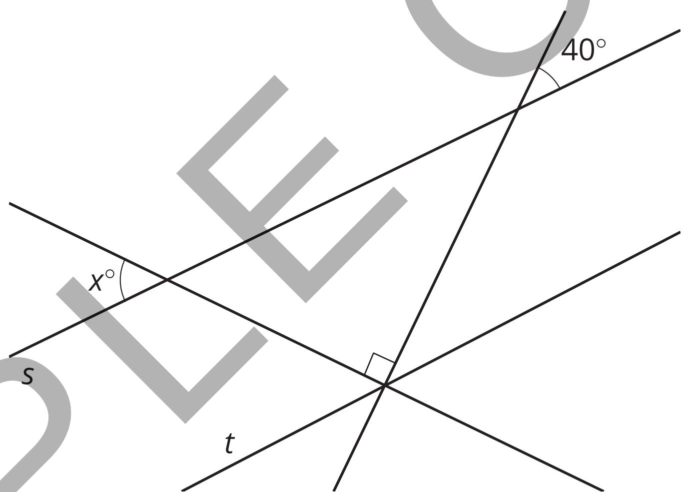
b. What is the measure of angle BCD? Explain.

3 The diagram shows three lines with some marked angle measures:



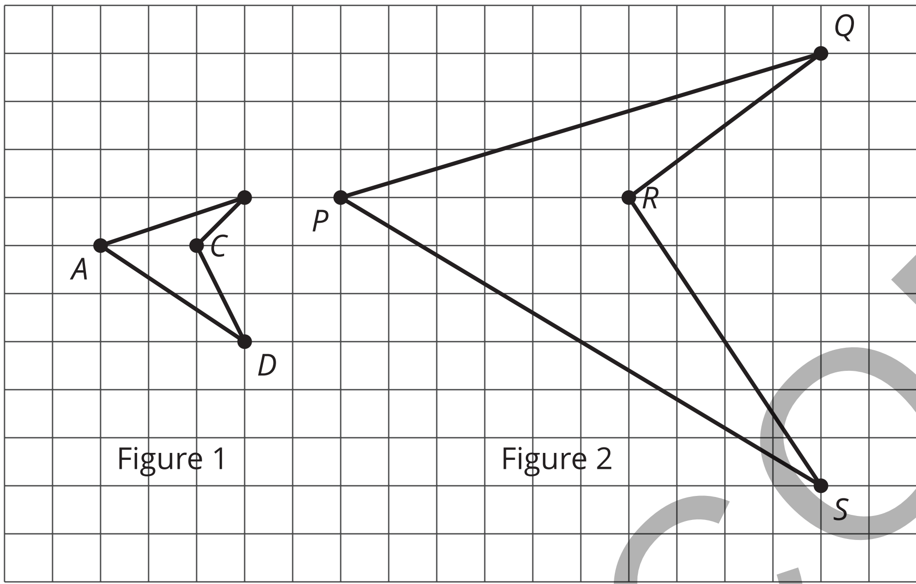
Find the missing angle measures marked with question marks.

4 Lines s and t are parallel. Find the value of x. Explain your reasoning.



Sec D

5 The two figures are scaled copies of each other.



a. What is the scale factor that takes Figure 1 to Figure 2?

b. What is the scale factor that takes Figure 2 to Figure 1?

Sec D

Unit 1, Lesson 15

Addressing CA CCSSM 8.G.2, 8.G.5; building on 7.G.2; practicing MP7 and MP8

Adding the Angles in a Triangle

Let’s explore angles in triangles.

15.1 Can You Draw It?

Draw 3 different types of triangles.

15.2 Find All Three

Your teacher will give you a card with a picture of a triangle.

1. The measurement of one of the angles is labeled. Mentally estimate the measures of the other two angles.
2. Find two other students with triangles congruent to yours but with a different angle labeled. Confirm that the triangles are congruent, that each card has a different angle labeled, and that the angle measures make sense.
3. Enter the three angle measures for your triangle on the table your teacher has posted.

15.3 Tear It Up

Your teacher will give you a page with three sets of angles and a blank space. Cut out each set of three angles. Can you make a triangle from each set that has these same three angles?

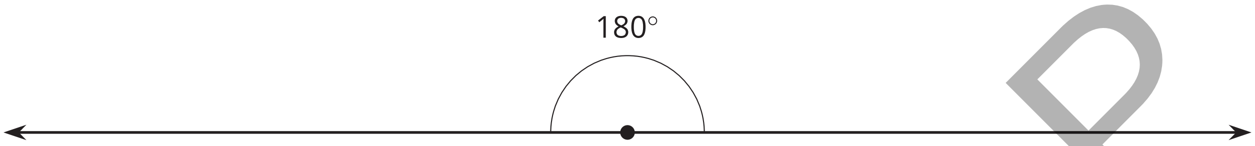
Are you ready for more?

1. Draw a quadrilateral. Cut it out, tear off its angles, and line them up. What do you notice?

2. Repeat this for several more quadrilaterals. Do you have a conjecture about the angles?

Lesson 15 Summary

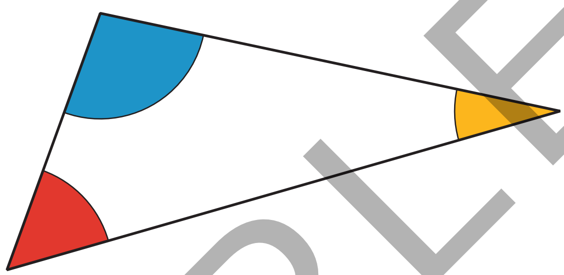
A 180° angle is called a straight angle because when it is made with two rays, they point in opposite directions and form a straight line.

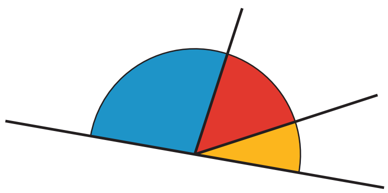


If we experiment with angles in a triangle, we find that the sum of the measures of the three angles in each triangle is 180° — the same as a straight angle!

Through experimentation we find:

- If we add the three angles of a triangle physically by cutting them off and lining up the vertices and sides, then the three angles form a straight angle.
- If we have a line and two rays that form three angles added to make a straight angle, then there is a triangle with these three angles.





Sec D

Practice Problems

1 In triangle ABC, the measure of angle A is 40°.

a. Give possible measures for angles B and C if triangle ABC is isosceles.

b. Give possible measures for angles B and C if triangle ABC is right.

2 For each set of angles, decide if there is a triangle whose angles have these measures in degrees:

a. 60, 60, 60

b. 90, 90, 45

c. 30, 40, 50

d. 90, 45, 45

e. 120, 30, 30

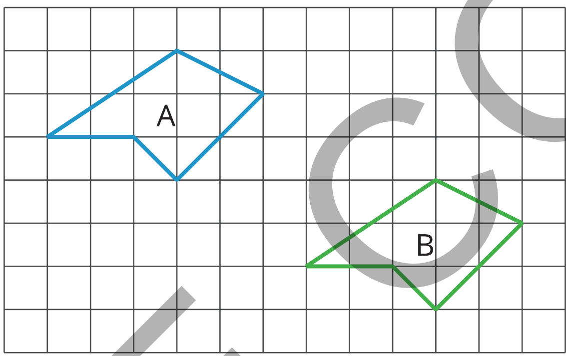
If you get stuck, consider making a line segment. Then use a protractor to measure angles with the first two angle measures.

3 Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your reasoning.

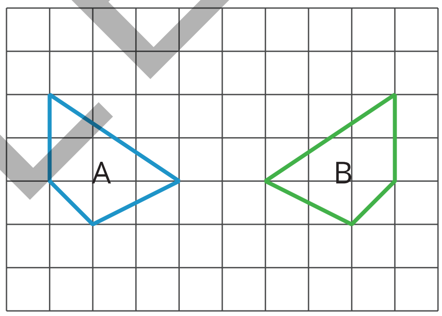
4 from Unit 1, Lesson 3

For each pair of polygons, describe the transformation that could be applied to Polygon A to get Polygon B.

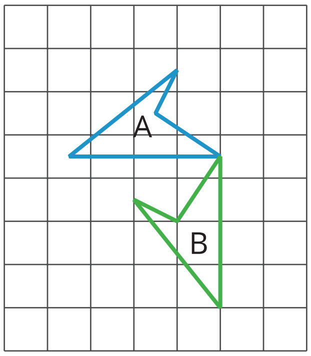
a.



b.

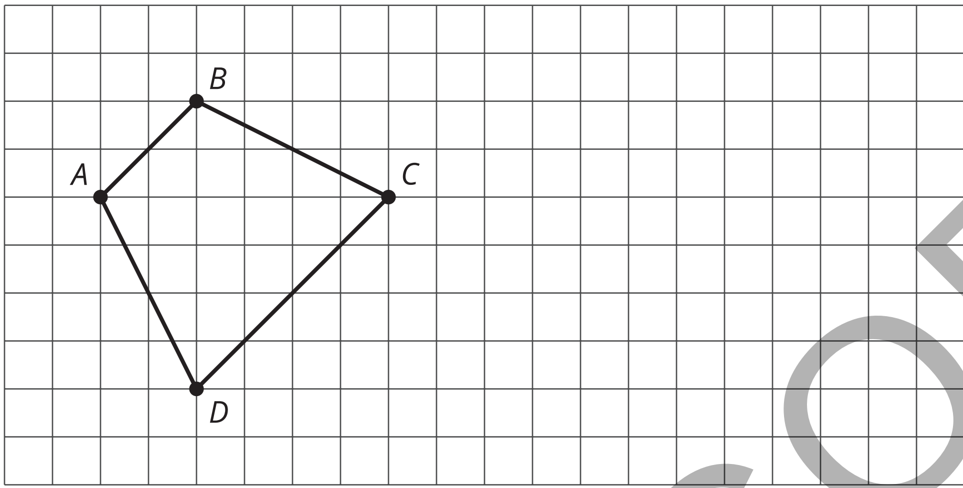


c.



5 from an earlier course

On the grid, draw a scaled copy of quadrilateral $ABCD$ using a scale factor of $\frac{1}{2}$.



Sec D

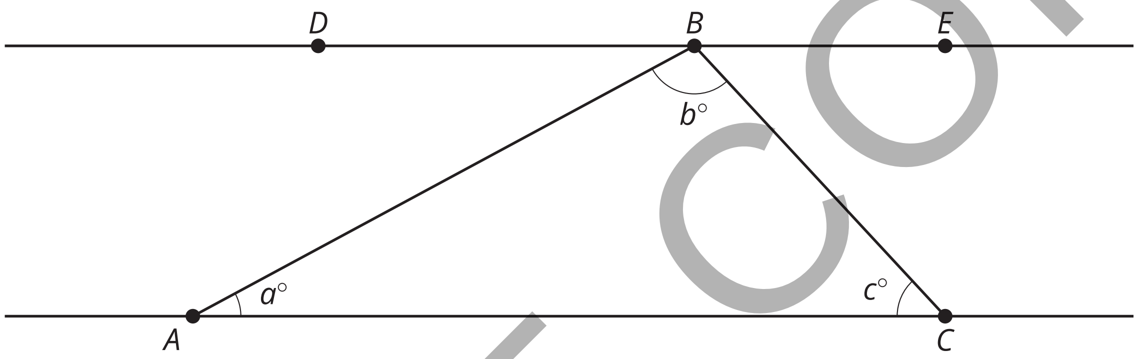
Unit 1, Lesson 16
Addressing CA CCSSM 8.G.5; building on 7.G.5, 8.G.1b; building towards 8.G.6; practicing MP7

Parallel Lines and the Angles in a Triangle

Let’s see why the angles in a triangle add to 180 degrees.

16.1 All the Angles

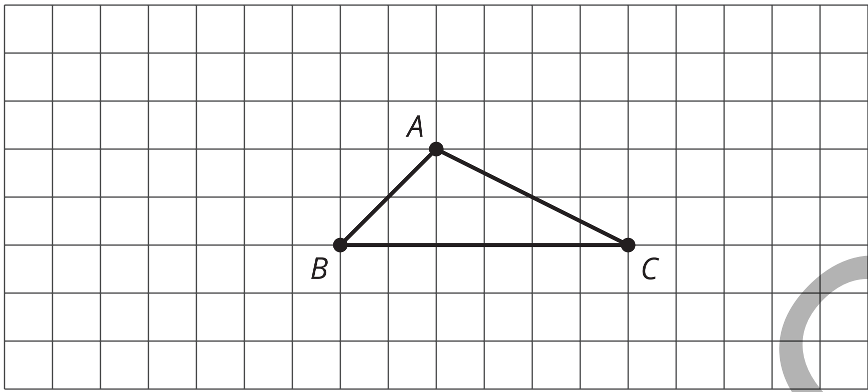
Here is triangle ABC. Line DE is parallel to line AC.



Sec D

16.2 Angle Plus Two

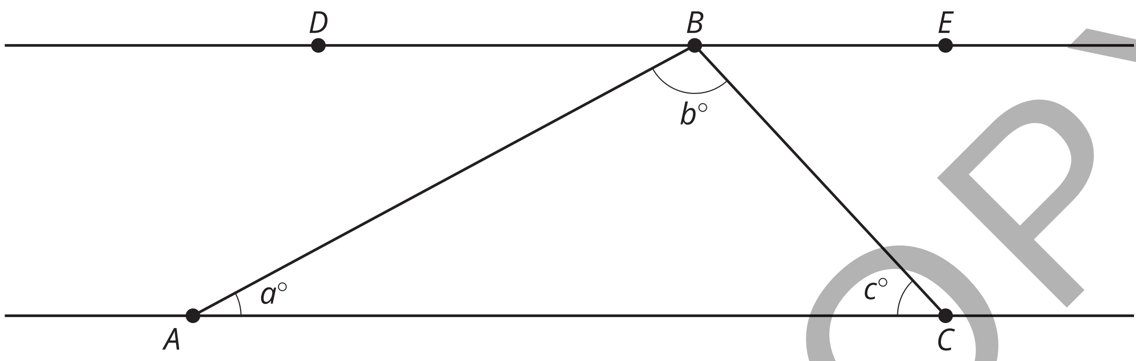
Here is triangle ABC.



1. Rotate triangle ABC 180° around the midpoint of side AC. Label the new vertex D.
2. Rotate triangle ABC 180° around the midpoint of side AB. Label the new vertex E.
3. Look at angles EAB, BAC, and CAD. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.
4. Is the measure of angle EAB equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.
5. Is the measure of angle CAD equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.
6. What is the sum of the measures of angles ABC, BAC, and ACB? Explain your reasoning.

16.3 Every Triangle in the World

Here is triangle ABC. Line DE is parallel to line AC.



1. What is the sum of the measures of angle DBA, angle ABC, and angle CBE?

2. Use your answer to explain why $a + b + c = 180$.

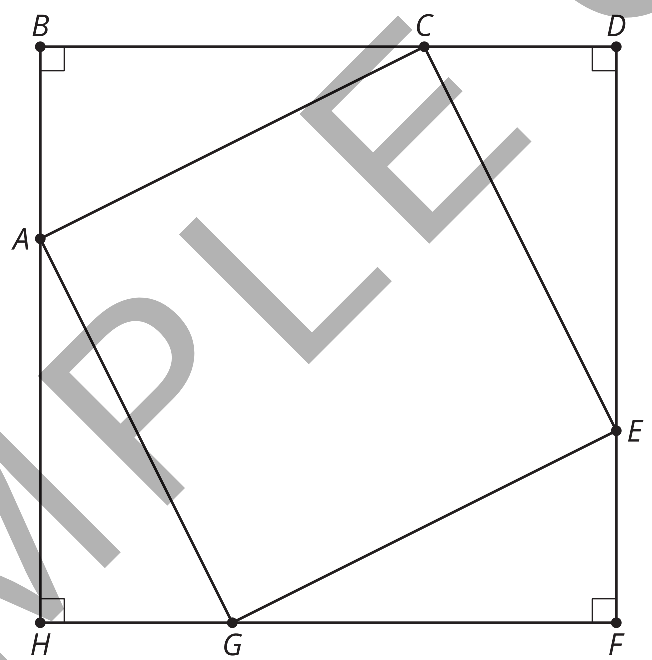
3. Explain why your argument will work for any triangle: that is, explain why the sum of the angle measures in any triangle is 180°.

Are you ready for more?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?
2. Come up with an explanation for why anything you notice must be true. (Hint: draw one diagonal in each quadrilateral.)

16.4 Four Triangles Revisited

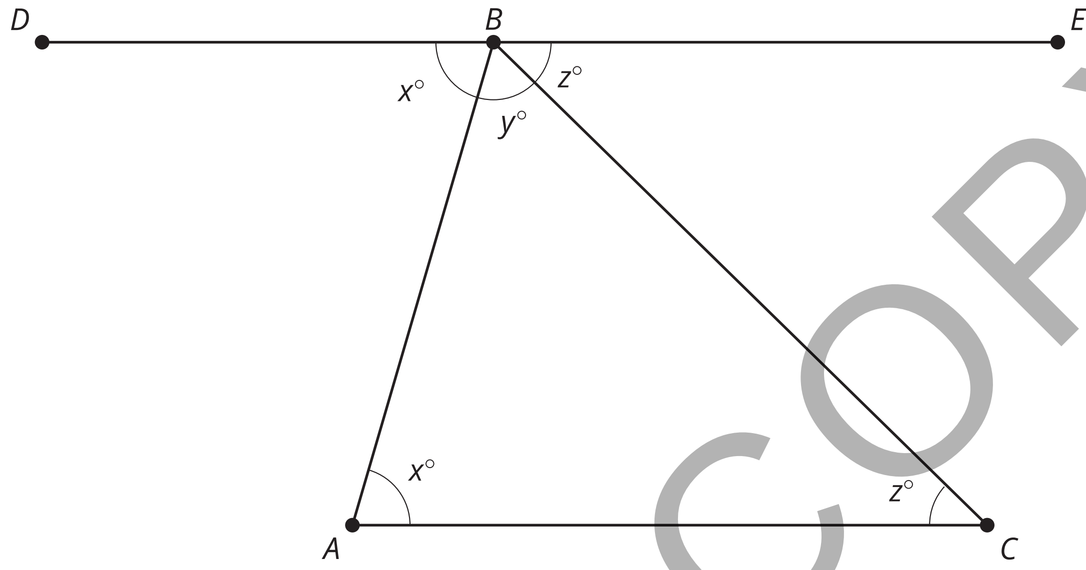
This diagram shows a square $BDFH$ that has been made by images of triangle ABC under rigid transformations.



Given that angle BAC measures 53°, find as many other angle measures as you can.

Lesson 16 Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to 180°. Here is triangle ABC. Line DE is parallel to AC and contains B.



A 180° rotation of triangle ABC around the midpoint of AB interchanges angles A and DBA so they have the same measure (in the picture these angles are marked as x°).

A 180° rotation of triangle ABC around the midpoint of BC interchanges angles C and CBE so they have the same measure (in the picture, these angles are marked as z°).

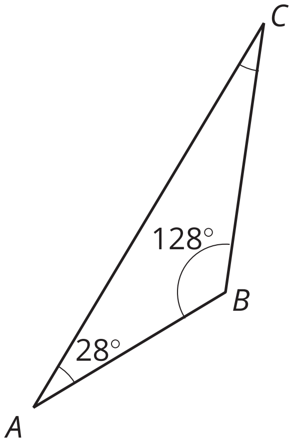
Also, DBE is a straight line because 180° rotations take lines to parallel lines.

So the three angles with vertex B make a line and they add up to 180° ($x + y + z = 180$). But x, y, z are the measures of the three angles in triangle ABC so the sum of the angles in a triangle is always 180°!

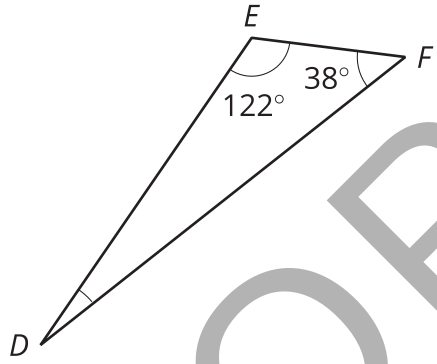
Practice Problems

1 For each triangle, find the measure of the missing angle.

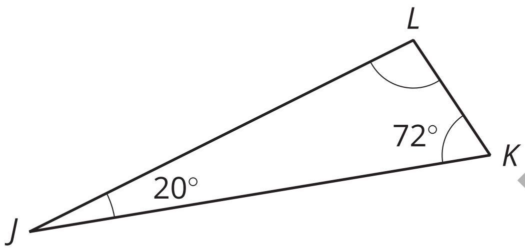
a.



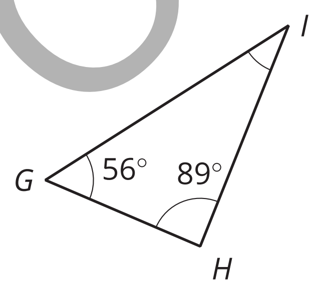
b.



c.



d.



2 Is there a triangle with *two* right angles? Explain your reasoning.

Sec D

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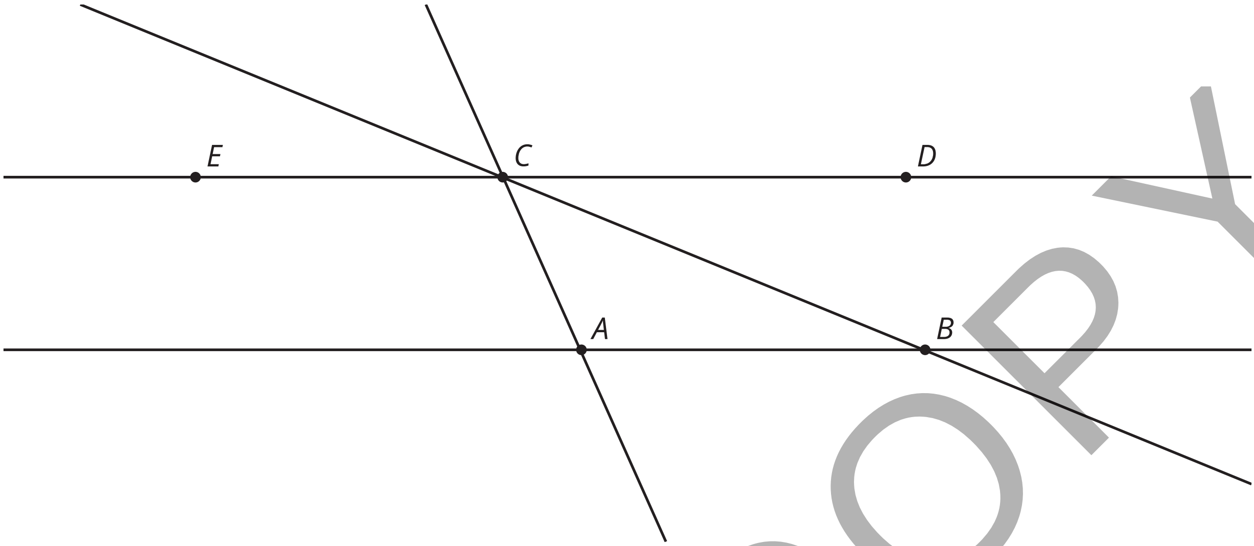
3 In this diagram, lines AB and CD are parallel.



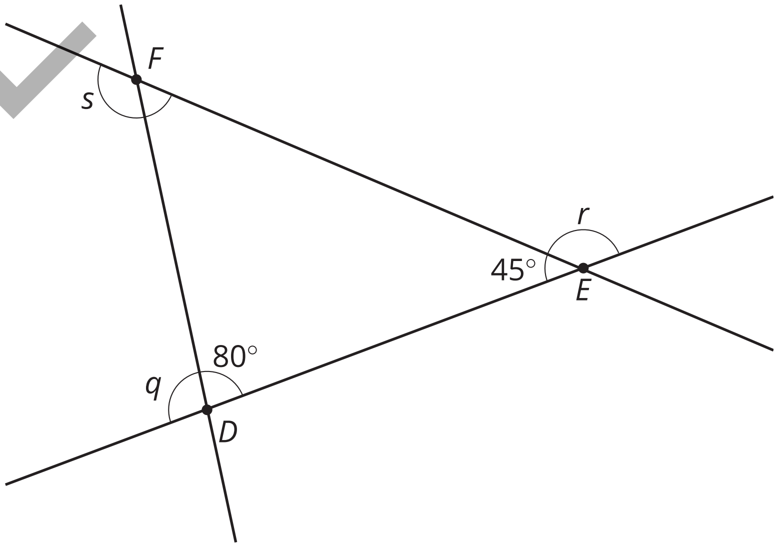
Angle ABC measures 35° and angle BAC measures 115°.

a. What is the measure of angle ACE?

b. What is the measure of angle DCB?

c. What is the measure of angle ACB?

4 Here is a diagram of triangle DEF.



a. Find the measures of angles q, r, and s.

b. Find the sum of the measures of angles q, r, and s.

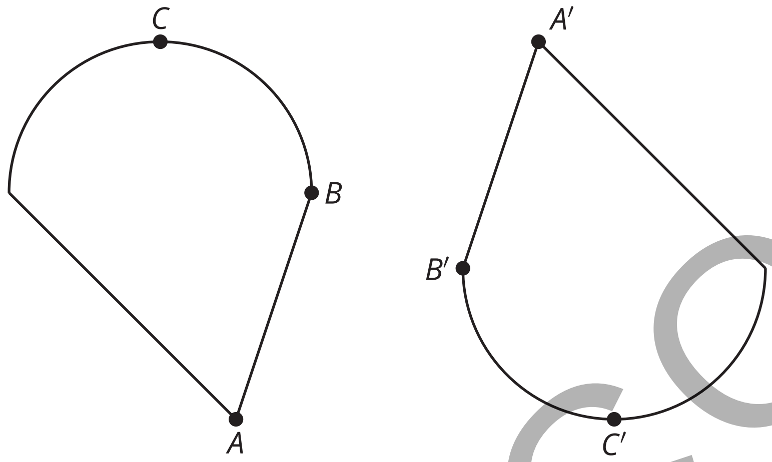
c. What do you notice about these three angles?

Sec D

5 from Unit 1, Lesson 13

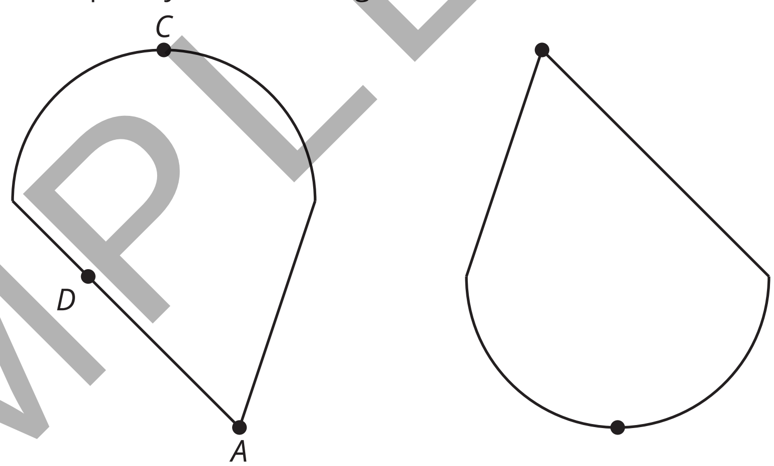
The two figures are congruent.

a. Label the points A', B' and C' that correspond to A, B, and C in the figure on the right.



b. If segment AB measures 2 cm, how long is segment $A'B'$? Explain.

c. The point D is shown in addition to A and C. How can you find the point D' that corresponds to D? Explain your reasoning.



Addressing CA CCSSM 8.G.1-3; building on 7.G.5; practicing MP7

Rotate and Tessellate

Let's make complex patterns using transformations.

17.1 Deducing Angle Measures

Your teacher will give you some shapes.

1. How many copies of the equilateral triangle can you fit together around a single vertex, so that the triangles' edges have no gaps or overlaps? What is the measure of each angle in these triangles?

2. What are the measures of the angles in the
   a. Square?
   b. Hexagon?
   c. Parallelogram?
   d. Right triangle?
   e. Octagon?
   f. Pentagon?

17.2 Tessellate This

1. Design your own **tessellation**. You will need to decide which shapes you want to use and make copies. Remember that a tessellation is a repeating pattern that goes on forever to fill up the entire plane.
2. Find a partner and trade pictures. Describe a transformation of your partner's picture that takes the pattern to itself. How many different transformations can you find that take the pattern to itself? Consider translations, reflections, and rotations.
3. If there's time, color and decorate your tessellation.

17.3 Rotate That

1. Make a design with rotational symmetry.

2. Find a partner who has also made a design. Exchange designs and find a transformation of your partner's design that takes it to itself. Consider rotations, reflections, and translations.

3. If there's time, color and decorate your design.

Glossary

- tessellation

Learning Targets

Lesson 1 Moving in the Plane

- I can describe how a figure moves and turns to get from one position to another.

Lesson 2 Naming the Moves

- I can identify corresponding points before and after a transformation.
- I know the difference between translations, rotations, and reflections.

Lesson 3 Grid Moves

- I can decide which type of transformations will work to move one figure to another.
- I can use grids to carry out transformations of figures.

Lesson 4 Making the Moves

- I can use the terms "translation," "rotation," and "reflection" to precisely describe transformations.

Lesson 5 Coordinate Moves

- I can apply transformations to points on a grid if I know their coordinates.

Lesson 6 Describing Transformations

- I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.

Lesson 7 No Bending or Stretching

- I can describe the effects of a rigid transformation on the lengths and angles in a polygon.

Lesson 8 Rotation Patterns

- I can describe how to move one part of a figure to another using a rigid transformation.

Lesson 9 Moves in Parallel

- I can describe the effects of a rigid transformation on a pair of parallel lines.
- If I have a pair of vertical angles and know the angle measure of one of them, I can find the angle measure of the other.

Lesson 10 Composing Figures

- I can find missing side lengths or angle measures using properties of rigid transformations.

Lesson 11 What Is the Same?

- I can decide whether or not two figures are congruent using rigid transformations.

Lesson 12 Congruent Polygons

- I can decide using rigid transformations whether or not two figures are congruent.

Lesson 13 Congruence

- I can use distances between points to decide if two figures are congruent.

Lesson 14 Alternate Interior Angles

- If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements.

Lesson 15 Adding the Angles in a Triangle

- I can determine whether three angles could make a triangle using their sum.

Lesson 16 Parallel Lines and the Angles in a Triangle

- I can explain using pictures why the sum of the angles in any triangle is 180 degrees.

Lesson 17 Rotate and Tessellate

- I can repeatedly use rigid transformations to make interesting repeating patterns of figures.
- I can use properties of angle sums to reason about how figures will fit together.

GRADE 8

UNIT

2

Dilations, Similarity, and Introducing Slope

Content Connections

In this unit you will learn what makes figures similar and justify claims of similarity. You will use properties of similar triangles to write equations that can describe all points on a given line. You will make connections by:

- **Discovering Shape and Space** while describing a sequence of translations, reflections, rotations, and dilations that take one figure to the other.
- **Exploring Changing Quantities** while using similar triangles to visualize slope and rate of change with equations containing rational number coefficients.

Addressing the Standards

As you work your way through **Unit 2 Dilations, Similarity, and Introducing Slope,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 3, 5, and 13
MP2 Reason abstractly and quantitatively.	Lesson 13
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 2, 6, 7, 8, and 13
MP4 Model with mathematics.	Lesson 13
MP5 Use appropriate tools strategically.	Lessons 3 and 4
MP6 Attend to precision.	Lessons 1, 4, 5, 6, 7, and 8
MP7 Look for and make use of structure.	Lessons 1, 2, 4, 10, 11, and 12
MP8 Look for and express regularity in repeated reasoning.	Lessons 6, 9, 10, 11, and 12

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Transformational Relationships	**8.G.1** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	Lessons 3 and 4
• Transformational Relationships	**8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Lessons 6 and 7

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Transformational Relationships	**8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Lessons 2, 3, 4, 5, and 12
• Transformational Relationships	**8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	Lessons 2, 5, 6, 7, 8, and 9
• Transformational Relationships	**8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	Lessons 8 and 13
• Multiple Representation of Functions	**8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	Lessons 10, 11, and 12

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 2, Lesson 1
Building on CA CCSSM 6.NS.1; building towards 8.G.1-2, 8.G.4; practicing MP6 and MP7

Projecting and Scaling

Let's explore scaling.

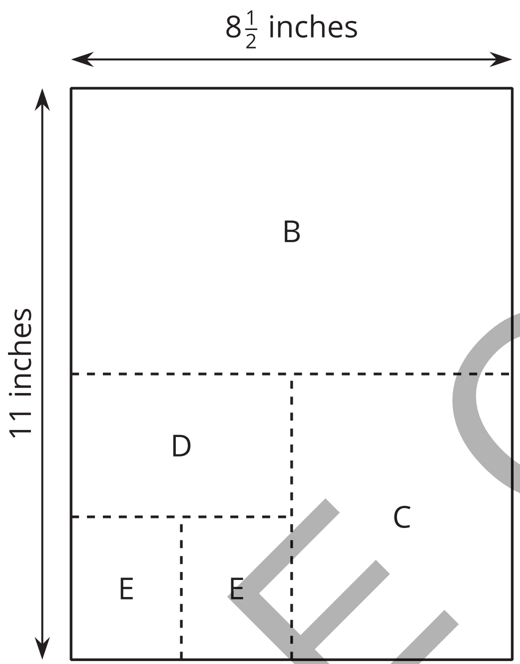
1.1 Math Talk: Remembering Fraction Division

Find the value of each expression mentally.

- $6\frac{1}{4} \div 2$
- $10\frac{1}{7} \div 5$
- $4\frac{1}{3} \div 8$
- $8\frac{1}{2} \div 11$

1.2 Sorting Rectangles

Rectangles are made by cutting an $8\frac{1}{2}$-inch by 11-inch piece of paper in half, in half again, and so on, as illustrated in the diagram. Find the lengths of each rectangle and enter them in the appropriate table.



1. Some of the rectangles are scaled copies of Rectangle A (the full sheet of paper). Record the measurements of those rectangles in this table.

rectangle	length of short side (inches)	length of long side (inches)
A	$8\frac{1}{2}$	11

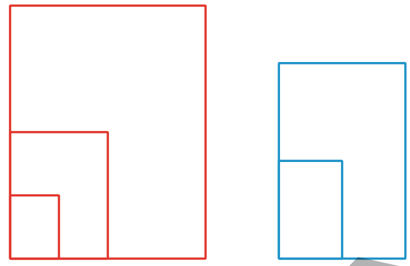
2. Some of the rectangles are *not* scaled copies of Rectangle A (the full sheet of paper). Record the measurements of those rectangles in this table.

rectangle	length of short side (inches)	length of long side (inches)

3. Look at the measurements for the rectangles that are scaled copies of the full sheet of paper. What do you notice?

4. Look at the measurements for the rectangles that are *not* scaled copies of the full sheet. What do you notice?

5. Stack the rectangles that are scaled copies of the full sheet so that they all line up at a corner, as shown in the diagram. Do the same with the other set of rectangles. On each stack, draw a line from the bottom left corner to the top right corner of the biggest rectangle.
What do you notice?



6. Stack *all* of the rectangles from largest to smallest so that they all line up at a corner. Compare the lines that you drew. Can you tell, from the drawn lines, which set each rectangle came from? What do you notice?

Are you ready for more?

In many countries, the standard paper size is not 8.5 inches by 11 inches (called “letter” size), but instead 210 millimeters by 297 millimeters (called “A4” size). Are these two rectangle sizes scaled copies of one another?

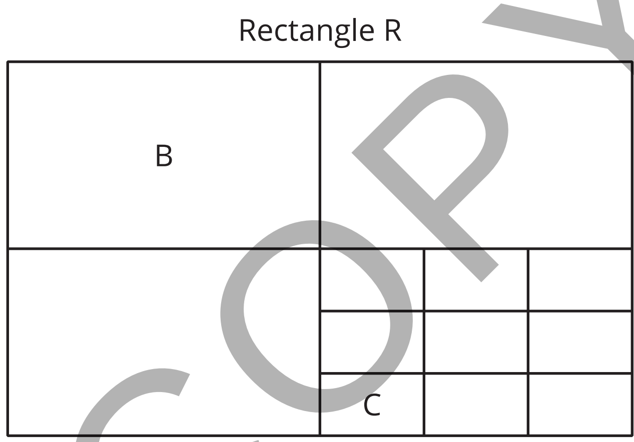
1.3 Scaled Rectangles

Here is a picture of Rectangle R, which has been evenly divided into smaller rectangles. Two of the smaller rectangles are labeled B and C.

1. Is B a scaled copy of R? If so, what is the scale factor?

2. Is C a scaled copy of B? If so, what is the scale factor?

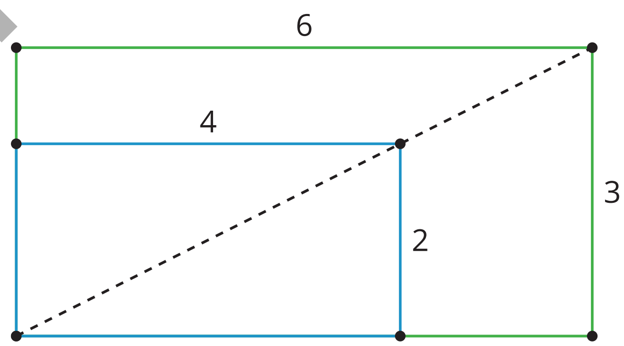
3. Is C a scaled copy of R? If so, what is the scale factor?



Sec A

Lesson 1 Summary

In this diagram, the larger rectangle is a scaled copy of the smaller one, and the scale factor is $\frac{3}{2}$ because $4 \cdot \frac{3}{2} = 6$ and $2 \cdot \frac{3}{2} = 3$. Scaled copies of rectangles have another interesting property: the diagonal of the large rectangle contains the diagonal of the smaller rectangle. This is the case for any two scaled copies of a rectangle if we line them up as shown. If two rectangles are not scaled copies of one another, then their diagonals would not match up.



Practice Problems

1 Rectangle A measures 12 cm by 3 cm. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

A. 6 cm by 1.5 cm

B. 10 cm by 2 cm

C. 13 cm by 4 cm

D. 18 cm by 4.5 cm

E. 80 cm by 20 cm

2 Rectangle A has length 12 units and width 8 units. Rectangle B has length 15 units and width 10 units. Rectangle C has length 30 units and width 15 units.

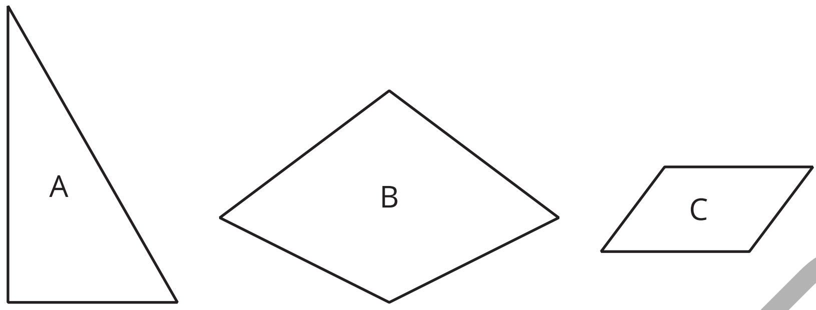
a. Is Rectangle A a scaled copy of Rectangle B? If so, what is the scale factor?

b. Is Rectangle B a scaled copy of Rectangle A? If so, what is the scale factor?

c. Explain how you know that Rectangle C is *not* a scaled copy of Rectangle B.

d. Is Rectangle A a scaled copy of Rectangle C? If so, what is the scale factor?

3 Here are 3 polygons.



a. Draw a scaled copy of Polygon A with scale factor $\frac{1}{2}$.

b. Draw a scaled copy of Polygon B with scale factor 2.

c. Draw a scaled copy of Polygon C with scale factor $\frac{1}{4}$.

4 from Unit 1, Lesson 15

Which of these sets of angle measures could be the 3 angles in a triangle?

A. $40^\circ, 50^\circ, 60^\circ$

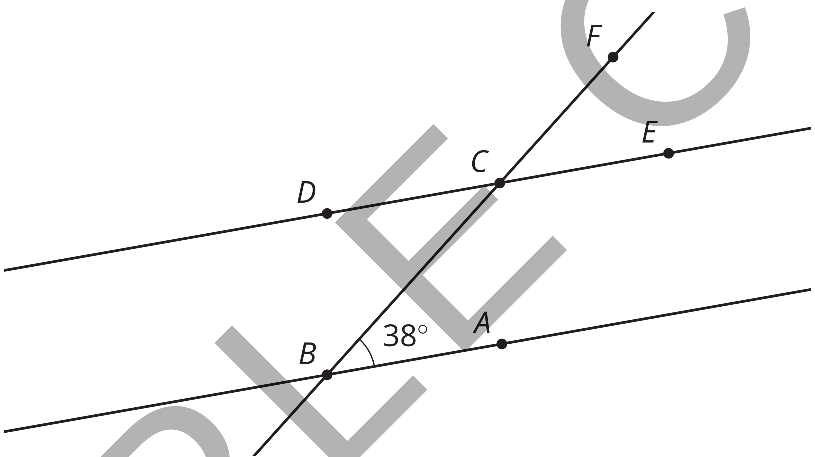
B. $50^\circ, 60^\circ, 70^\circ$

C. $60^\circ, 70^\circ, 80^\circ$

D. $70^\circ, 80^\circ, 90^\circ$

5 from Unit 1, Lesson 14

Lines AB and CD are parallel. Find the measures of the following angles. Explain your reasoning.



a. Angle BCD

b. Angle ECF

c. Angle DCF

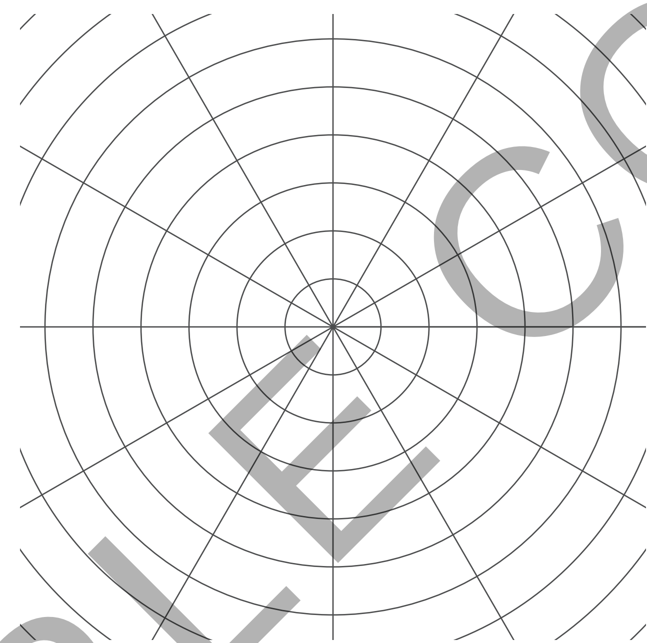
Addressing CA CCSSM 8.G.3-4; building on 4.MD.5; practicing MP3 and MP7

Circular Grid

Let's dilate figures on circular grids.

2.1 Notice and Wonder: Concentric Circles

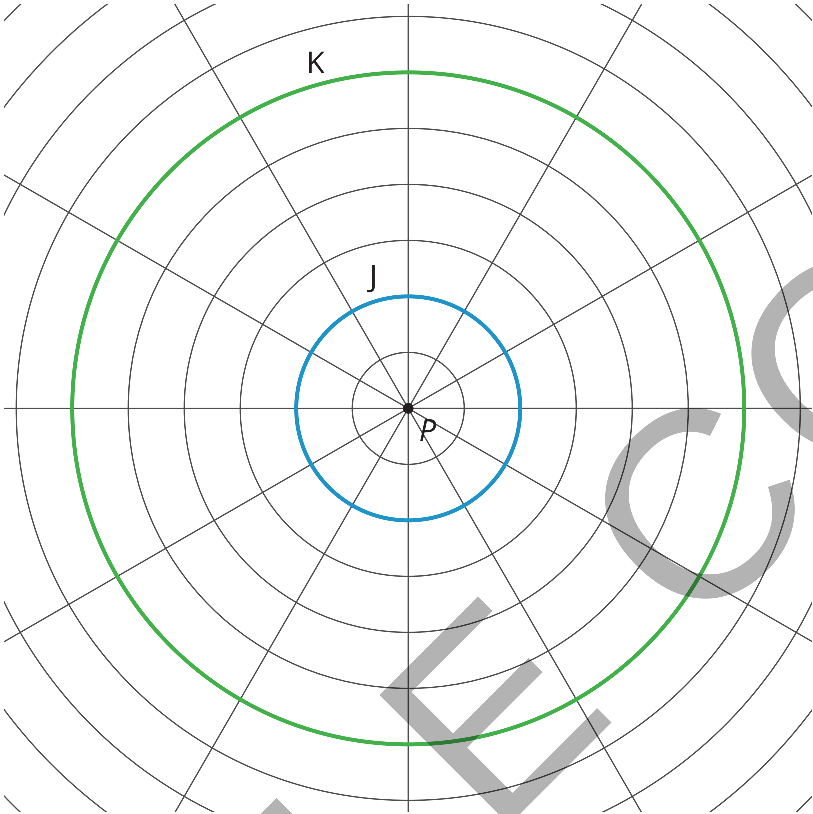
What do you notice? What do you wonder?



2.2 A Droplet on the Surface

Sec A

Here are two circles drawn on a circular grid with point P at the center.



1. Draw four points on Circle J (not inside the circle), and label them A, B, C, and D.
2. Draw a ray from P through each of your four points.
3. Mark the points where the rays intersect Circle K, and label them as E, F, G, and H.
4. In the first table, write the distance between point P and each point on the smaller circle. In the second table, write the distance between point P and each point on the larger circle.

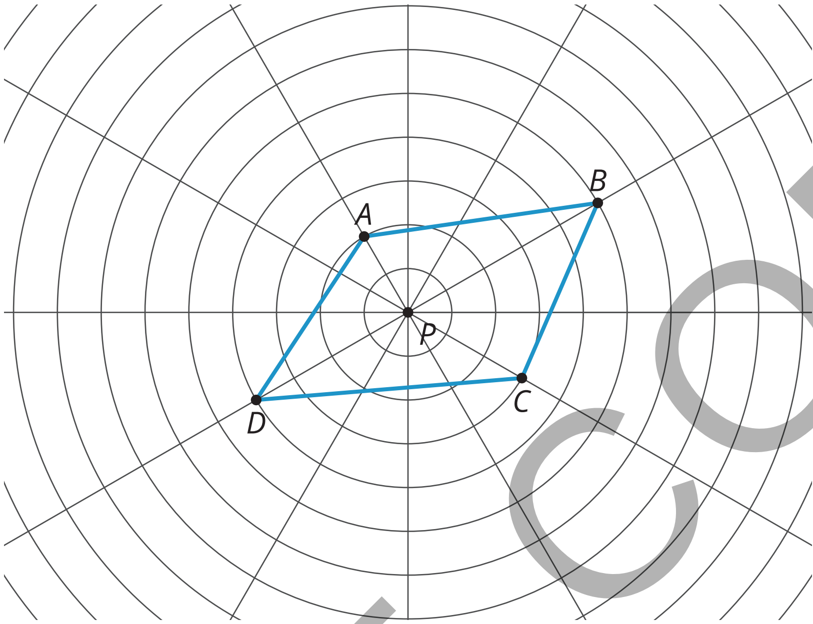
	A	B	C	D
P				

	E	F	G	H
P				

5. What is the scale factor that takes smaller Circle J to larger Circle K? Explain your reasoning.

2.3 Quadrilateral on a Circular Grid

Here is a polygon $ABCD$.



1. **Dilate** each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of 2. Label the image of A as E, the image of B as F, the image of C as G, and the image of D as H. Draw segments between the dilated points to create polygon $EFGH$.
2. What are some things you notice about the new polygon?

3. Choose a few more points on the sides of the original polygon and transform them using the same dilation. What do you notice?

4. Dilate each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of $\frac{1}{2}$. Label the image of A as I, the image of B as J, the image of C as K, and the image of D as L. Draw segments between the dilated points to create polygon $IJKL$.
5. What do you notice about polygon $IJKL$?

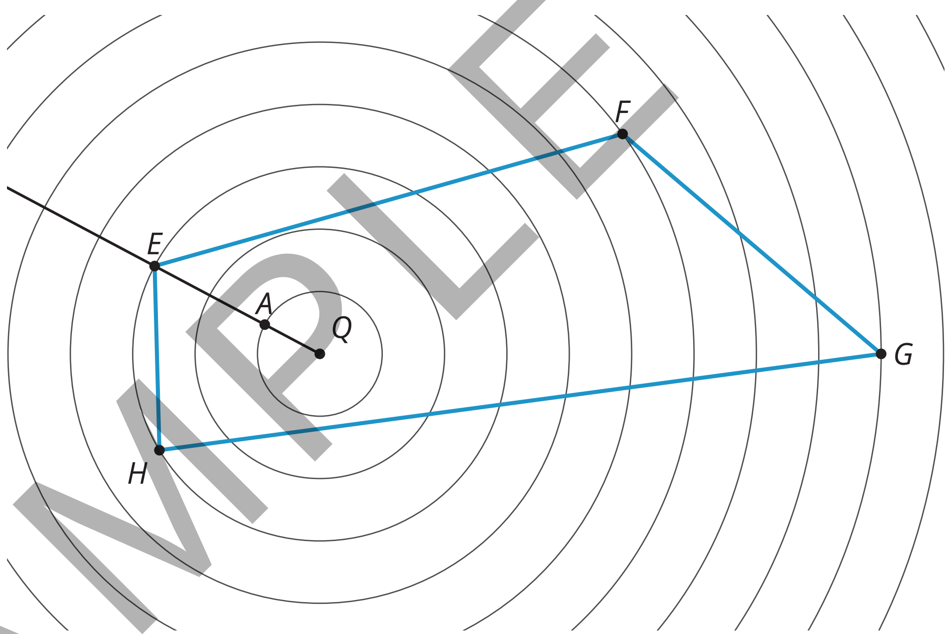
Are you ready for more?

Suppose P is a point that is not on line segment WX. Let line segment YZ be the dilation of line segment WX using P as the center with a scale factor of 2. Experiment using a circular grid to make predictions about whether each of the following statements is always true, sometimes true, or never true.

1. Line segment YZ is twice as long as line segment WX.
2. Line segment YZ is 5 units longer than line segment WX.
3. The point P is on line segment YZ.
4. Line segments YZ and WX intersect.

Sec A

2.4 A Quadrilateral and Concentric Circles



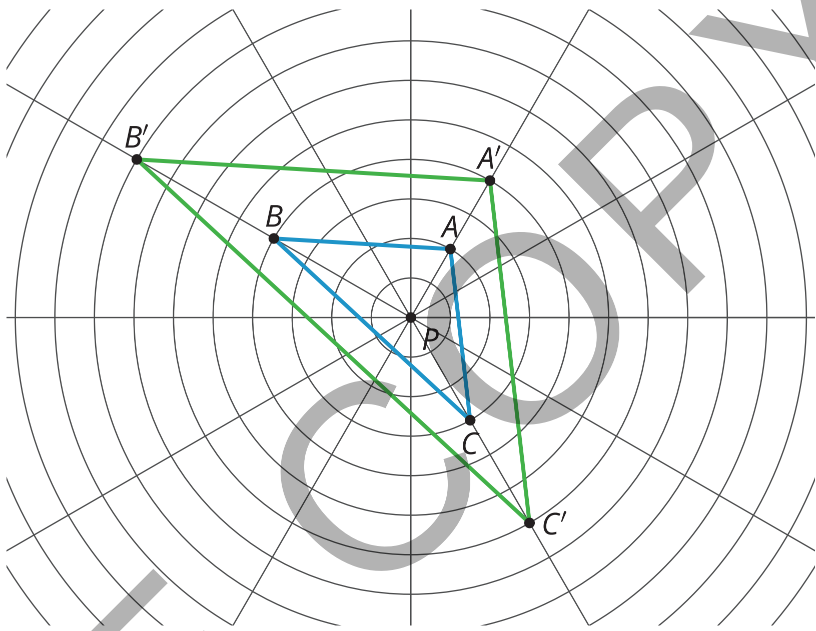
Dilate polygon $EFGH$ using Q as the center of dilation and a scale factor of $\frac{1}{3}$. A, the image of E, is already shown on the diagram. (You may need to use a straightedge to draw more rays from Q in order to find the images of other points.)

Lesson 2 Summary

A **dilation** is a transformation in which each point on a figure moves along a line and changes its distance from a fixed point, called the center of dilation.

All of the original distances are multiplied by the same scale factor.

In this diagram, P is the center of dilation and the scale factor is 2.



Each point of triangle ABC stays on the same ray from P, but its distance from P doubles.

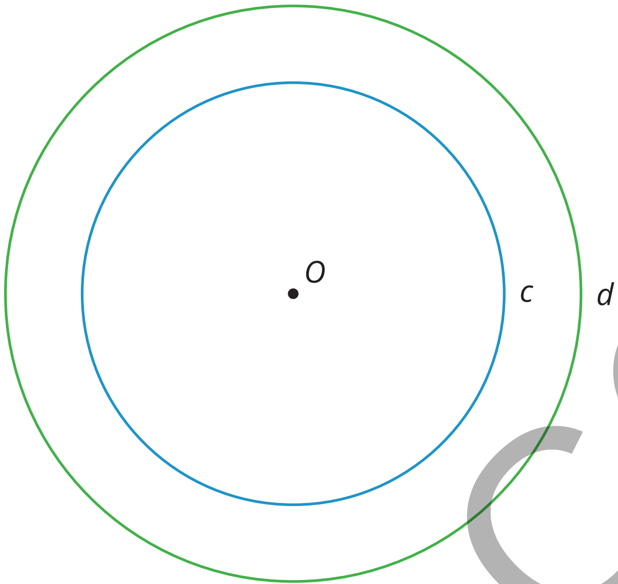
Since the circles on a circular grid are the same distance apart, we can simply count units from the center to a given point and use the scale factor to determine where the new point should be located, making the circular grid useful for performing dilations.

Glossary

- dilation

Practice Problems

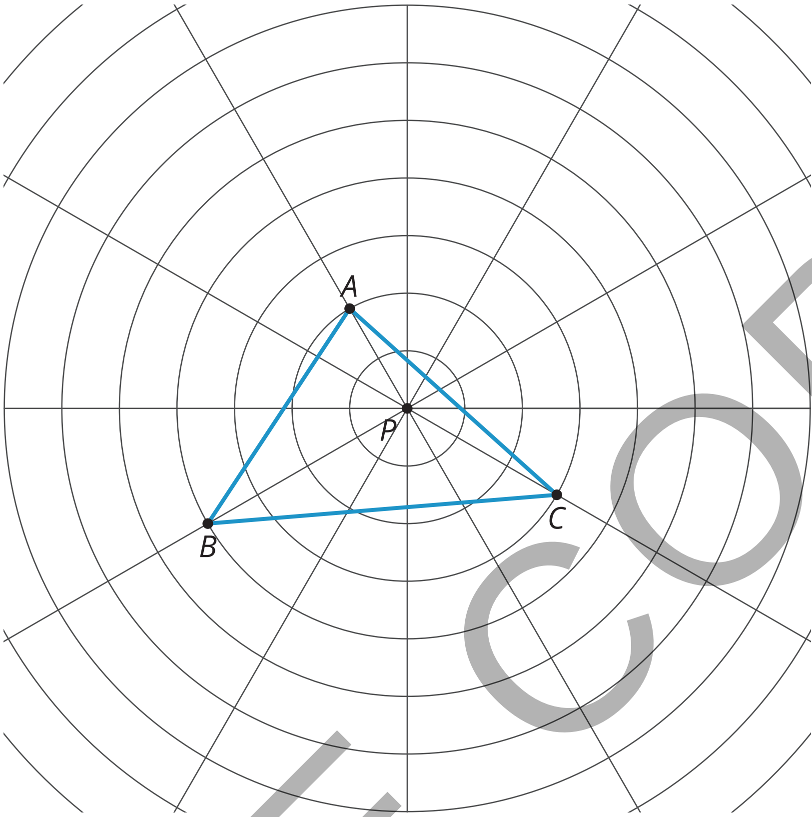
1 Here are Circles c and d. Point O is the center of dilation, and the dilation takes Circle c to Circle d.



a. Plot a point on Circle c. Label the point P. Plot where P goes when the dilation is applied and label the point P'.

b. Plot a point on Circle d. Label the point Q'. Plot the point that the dilation takes to Q' and label it Q.

2 Here is triangle ABC.



a. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of 2. Draw the triangle connecting the 3 new points.

b. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the triangle connecting the 3 new points.

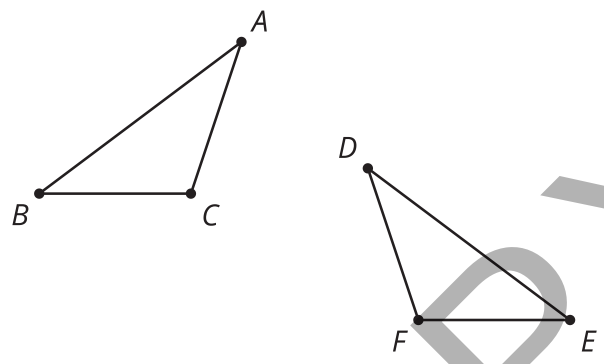
c. Measure the longest side of each of the 3 triangles. What do you notice?

d. Measure the angles of each triangle. What do you notice?

Sec A

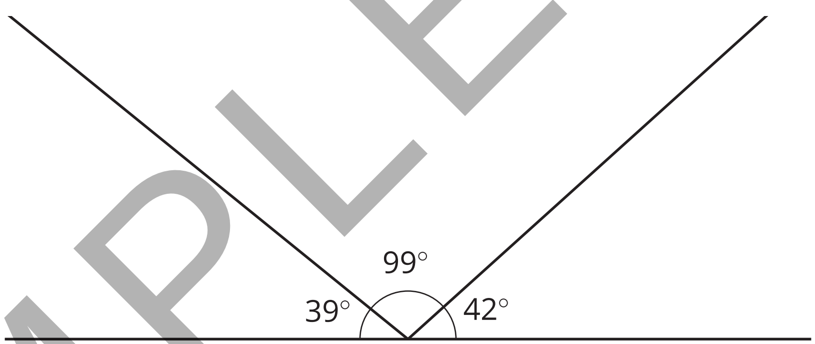
3 from Unit 1, Lesson 12

Describe a sequence of translations, rotations, or reflections that show the triangles are congruent.



4 from Unit 1, Lesson 15

The line has been partitioned into 3 angles.



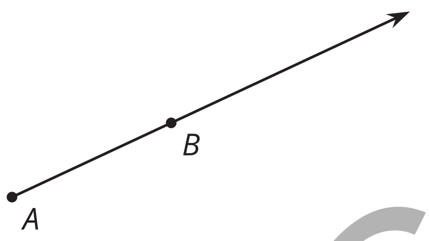
Is there a triangle with these 3 angle measures? Explain.

Addressing CA CCSSM 8.G.1, 8.G.3; practicing MP1 and MP5

Dilations with No Grid

Let's dilate figures not on grids.

3.1 Points on a Ray



1. Find a point on the ray whose distance from A is twice the distance from B to A and label it C.
2. Find a point on the ray whose distance from A is half the distance from B to A and label it D.

Sec A

3.2 Dilation Obstacle Course

Here is a diagram that shows 9 points.

1. Dilate B using a scale factor of 5 and A as the center of dilation. Which point is its image?
2. Using H as the center of dilation, dilate G so that its image is E. What scale factor did you use?
3. Using H as the center of dilation, dilate E so that its image is G. What scale factor did you use?
4. To dilate F so that its image is B, what point on the diagram can you use as the center of dilation?
5. Dilate H using A as the center of dilation and a scale factor of $\frac{1}{3}$. Which point is its image?
6. Describe a dilation that uses a labeled point as its center of dilation and that would take F to H.
7. Using B as the center of dilation, dilate H so that its image is itself. What scale factor did you use?

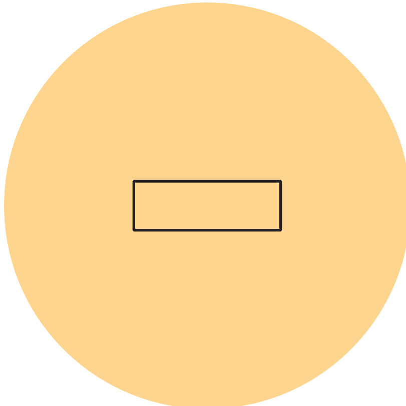
3.3 Getting Perspective

1. Draw the images of points P and Q using C as the center of dilation and a scale factor of 4. Label the new points P' and Q'.

2. Draw the images of points P and Q using C as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new points P'' and Q''.

   Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

3. Let's make a perspective drawing. Here is a rectangle.



a. Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the center of dilation. Label it C.

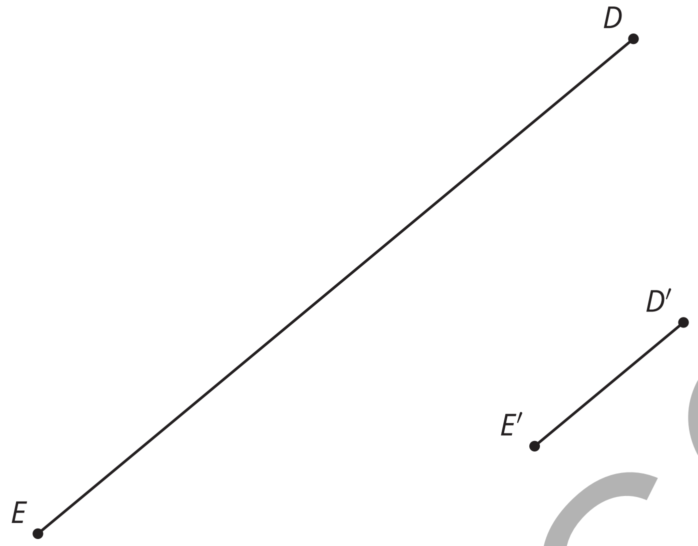
b. Use your center C and the scale factor you were given to draw the image under the dilation of each vertex of the rectangle, one at a time. Connect the dilated vertices to create the dilated rectangle.

c. Draw segments that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If time allows, you can shade the sides of the box to make it look more realistic.

d. Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your dilated rectangle closer to C than to the original rectangle, or farther away? How is that decided?

Are you ready for more?

Here is line segment DE and its image $D'E'$ under a dilation.



1. Use a ruler or straightedge to find and draw the center of dilation. Label it F.
2. What is the scale factor of the dilation?

Lesson 3 Summary

In the figure, point B is dilated with the center of dilation at A.

Since point C is farther away from A than B, the scale factor is larger than 1. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B, meaning the scale factor of the dilation is 2.

Since point D is closer to A than B, the scale factor is smaller than 1. If we measure the distance between A and D, we would find that it is one third the distance between A and B, meaning the scale factor of the dilation is $\frac{1}{3}$.

Practice Problems

1 Segment AB measures 3 cm. Point O is the center of dilation. How long is the image of AB after a dilation with:

a. Scale factor 5?

b. Scale factor 3.7?

c. Scale factor $\frac{1}{5}$?

d. Scale factor s?

2 Here are points A and B. Plot the points for each dilation described.

a. C is the image of B using A as the center of dilation and a scale factor of 2.

b. D is the image of A using B as the center of dilation and a scale factor of 2.

c. E is the image of B using A as the center of dilation and a scale factor of $\frac{1}{2}$.

d. F is the image of A using B as the center of dilation and a scale factor of $\frac{1}{2}$.

3 Make a perspective drawing. Include in your work the center of dilation, the shape you dilate, and the scale factor you use.

4 from Unit 2, Lesson 1

Triangle ABC and triangle DEF are scaled copies. Side AB measures 12 cm and is the longest side of ABC. Side DE measures 8 cm and is the longest side of DEF.

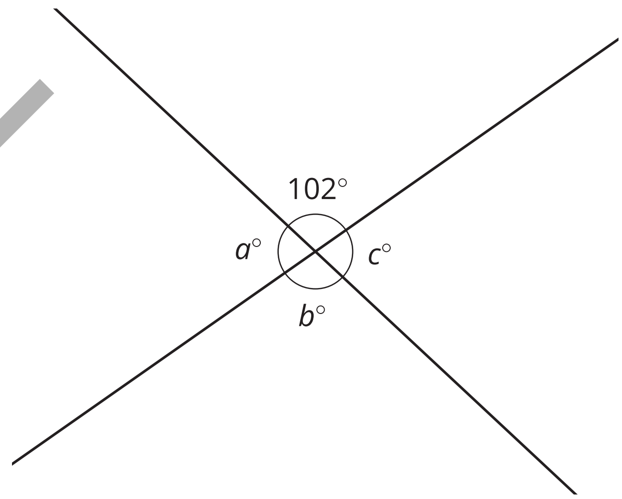
a. Triangle ABC is a scaled copy of triangle DEF with what scale factor?

b. Triangle DEF is a scaled copy of triangle ABC with what scale factor?

5 from Unit 1, Lesson 14

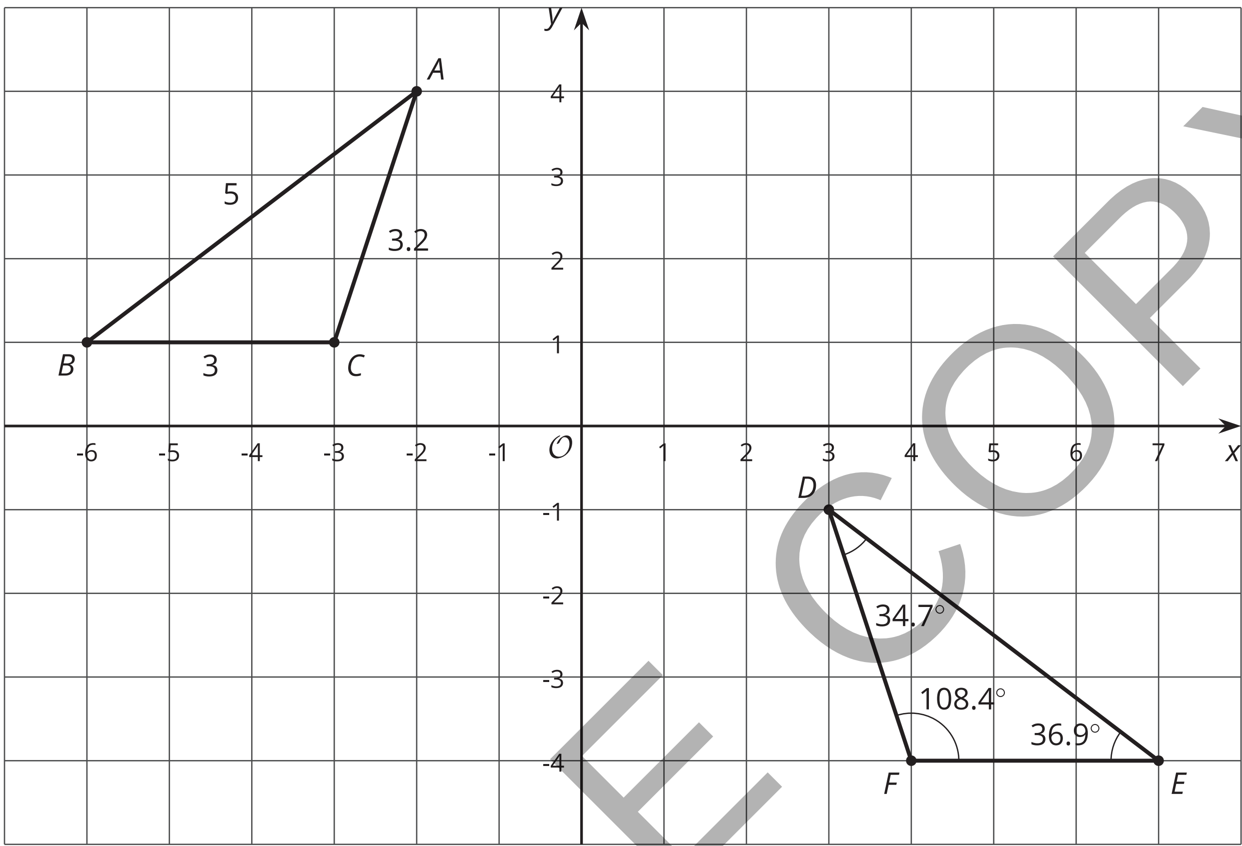
The diagram shows two intersecting lines.

Find the missing angle measures.



6 from Unit 1, Lesson 12

Here are 2 triangles.



a. Show that the 2 triangles are congruent.

b. Find the side lengths of triangle DEF and the angle measures of triangle ABC.

Addressing CA CCSSM 8.G.1, 8.G.3; practicing MP5, MP6, and MP7

Dilations on a Square Grid

Let's dilate figures on a square grid.

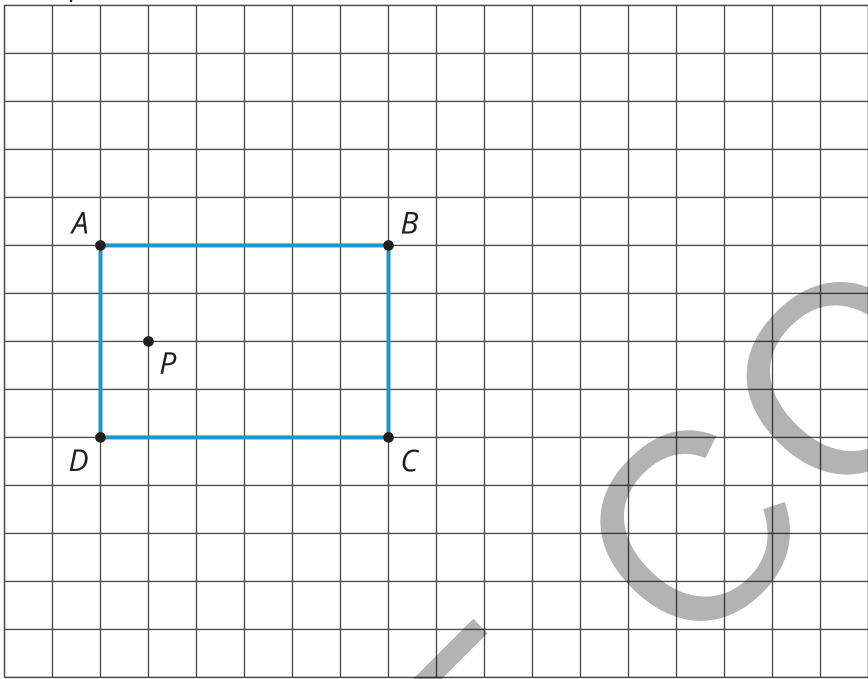
4.1 Estimating a Scale Factor

Point C is the dilation of point B with center of dilation A and scale factor s.

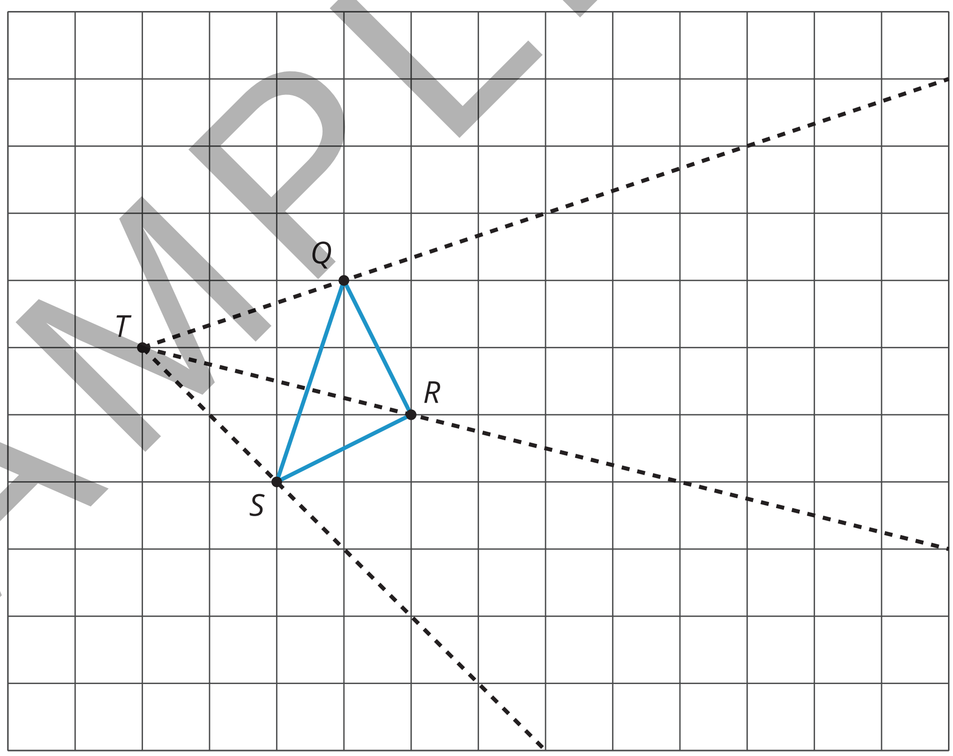
Estimate s. Be prepared to explain your reasoning.

4.2 Dilations on a Grid

1. Draw the image of quadrilateral $ABCD$ after a dilation with center P and scale factor 2.



2. Draw the image of triangle QRS after a dilation with center T and scale factor 2.
3. Draw the image of triangle QRS after a dilation with center T and scale factor $\frac{1}{2}$.



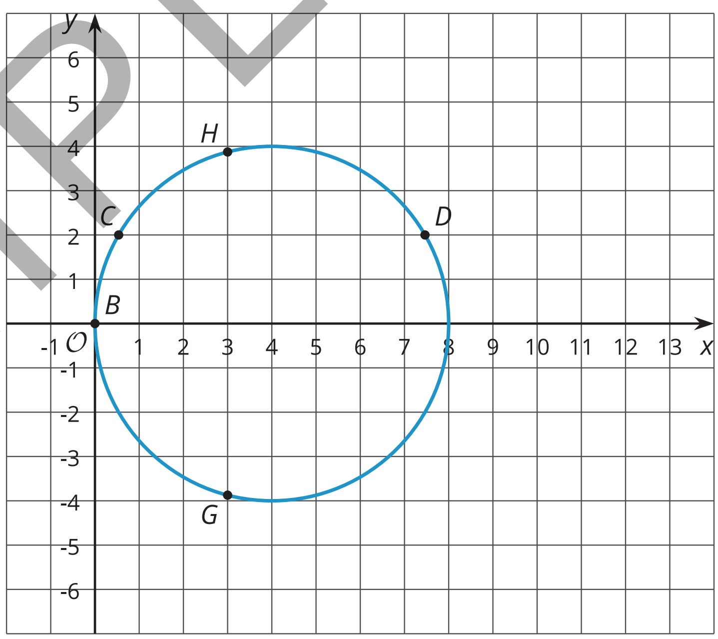
4.3 Card Sort: Matching Dilations on a Coordinate Grid

Your teacher will give you a set of cards containing descriptions of dilations and graphs. Match each number card showing a figure in the coordinate plane with a letter card describing the image after the given dilation. Record your matches and be prepared to explain your reasoning.

One of the number cards will not have a match. For this card, you will need to draw an image.

Are you ready for more?

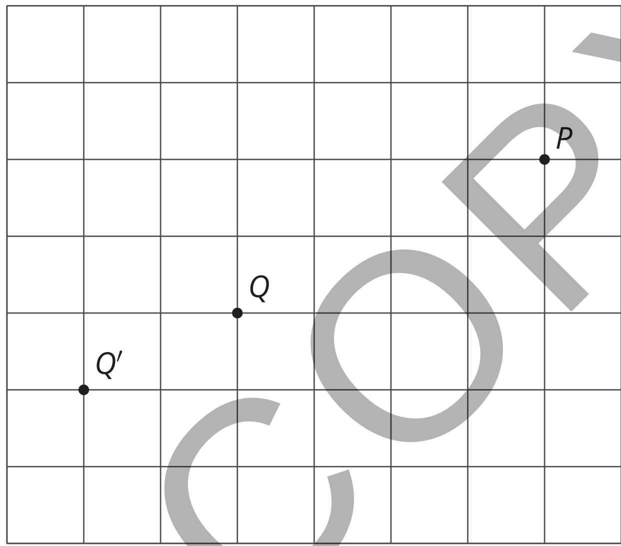
The image of a circle under dilation is a circle when the center of the dilation is the center of the circle. What happens if the center of dilation is a point on the circle? Using center of dilation $(0, 0)$ and scale factor 1.5, dilate the circle shown on the diagram. This diagram shows some points to try dilating.



Lesson 4 Summary

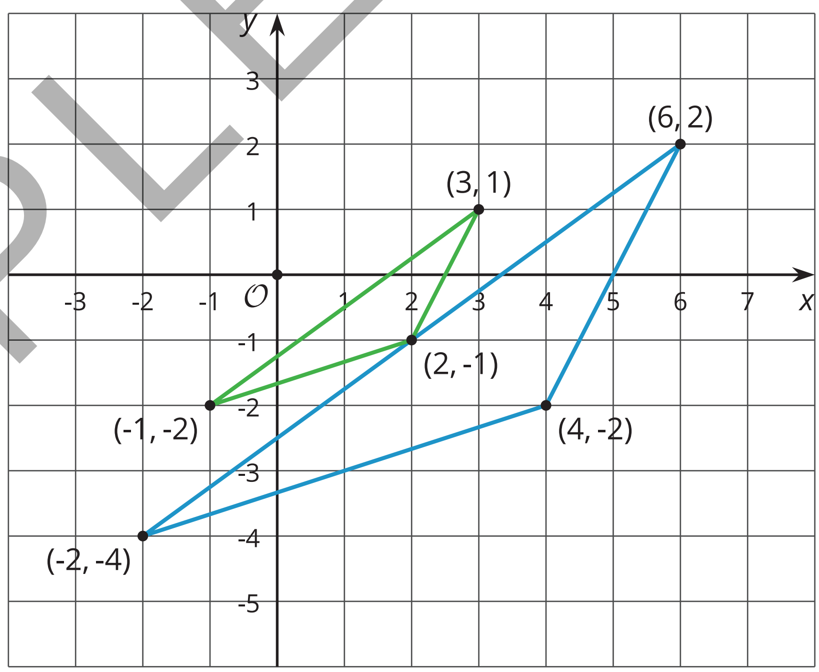
Square grids can be useful for showing dilations, especially when the center of dilation and the point(s) being dilated lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, the dilation of point Q with center of dilation P and scale factor $\frac{3}{2}$ will be 6 grid squares to the left and 3 grid squares down from P, since Q is 4 grid squares to the left and 2 grid squares down from P. The dilated image is marked as Q'.



Sometimes the square grid comes with coordinates, giving us a convenient way to name points. Sometimes the coordinates of the image can be found just using arithmetic, without having to measure.

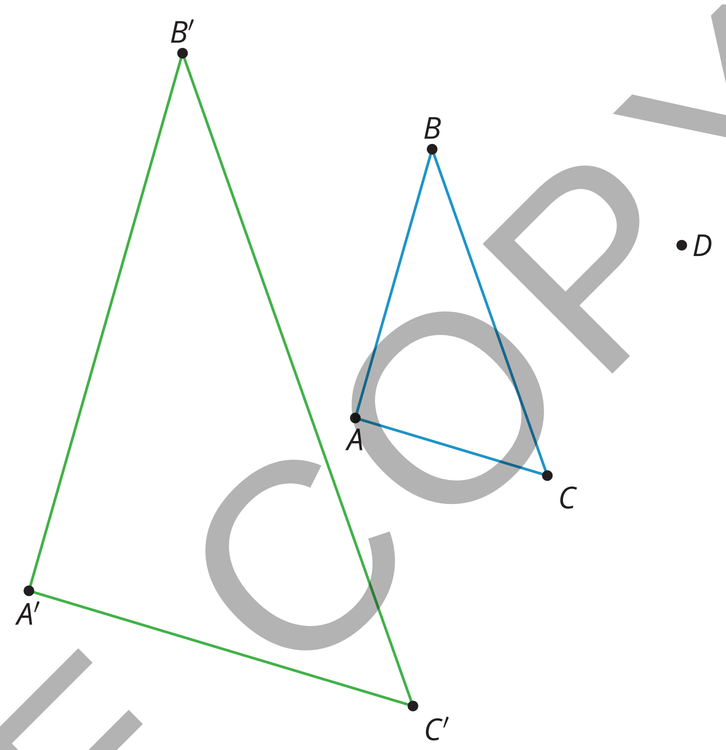
For example, to perform a dilation with center of dilation at $(0, 0)$ and scale factor 2 on the triangle with coordinates $(-1, -2)$, $(3, 1)$, and $(2, -1)$, we can just double the coordinates to get $(-2, -4)$, $(6, 2)$, and $(4, -2)$.



Practice Problems

1 Triangle ABC is dilated using D as the center of dilation with scale factor 2.

The image is triangle $A'B'C'$. Clare says the 2 triangles are congruent, because their angle measures are the same. Do you agree? Explain your reasoning.

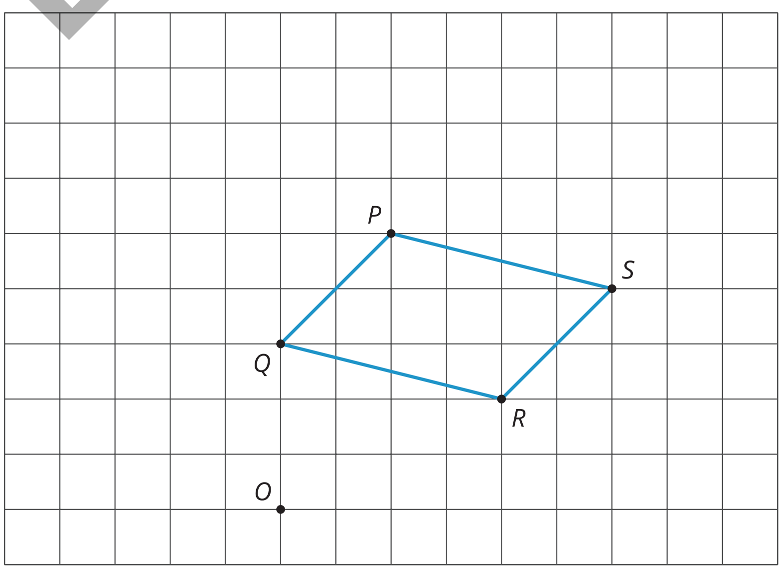


2 Sketch the image of quadrilateral $PQRS$ under the following dilations:

a. The dilation centered at R with scale factor 2.

b. The dilation centered at O with scale factor $\frac{1}{2}$.

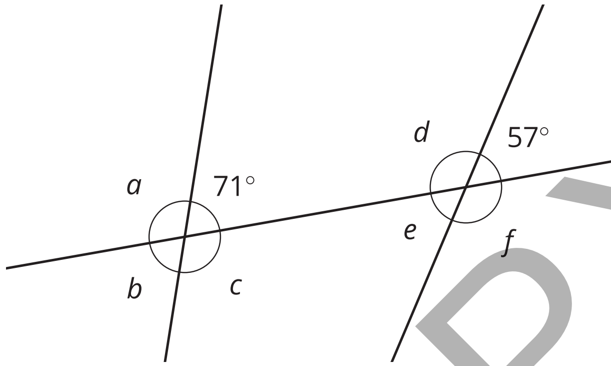
c. The dilation centered at S with scale factor $\frac{1}{2}$.



3 from Unit 1, Lesson 14

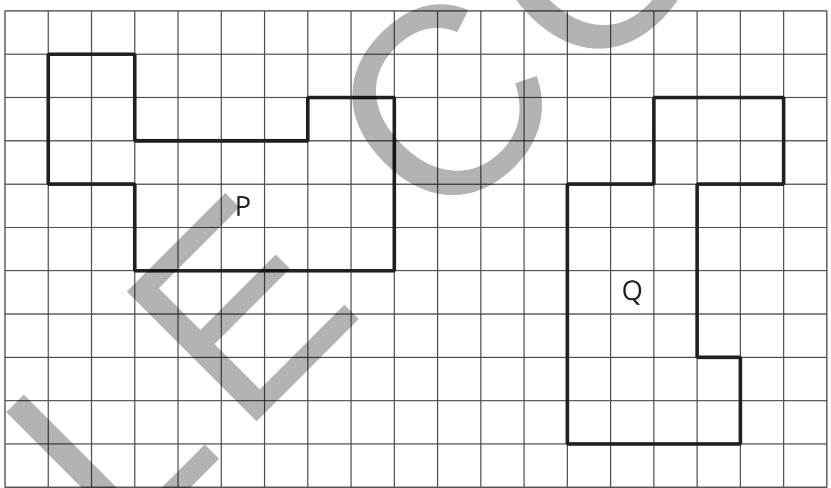
The diagram shows 3 lines with some marked angle measures.

Find the missing angle measures.



4 from Unit 1, Lesson 4

Describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.



5 from Unit 1, Lesson 6

Point B has coordinates $(-2, -5)$. After a translation 4 units down, a reflection across the y-axis, and a translation 6 units up, what are the coordinates of the image?

Unit 2, Lesson 5

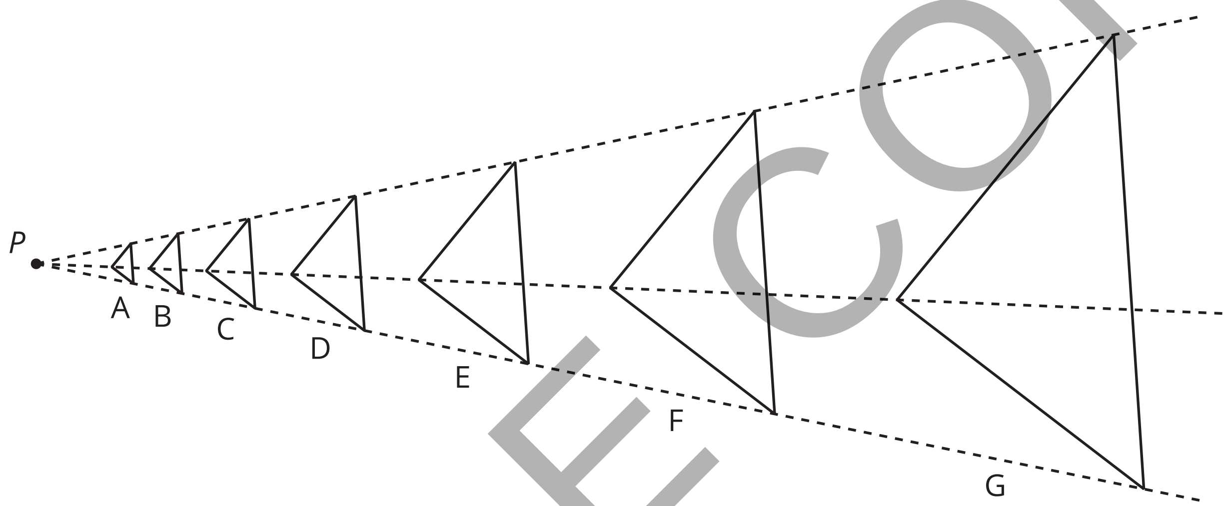
Addressing CA CCSSM 8.G.3-4; practicing MP1 and MP6

More Dilations

Let’s dilate figures in the coordinate plane.

5.1 Notice and Wonder: Many Dilations of a Triangle

All of the triangles are dilations of Triangle D. What do you notice? What do you wonder?



5.2 Info Gap: Dilations

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ________?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ________ because . . . .”

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

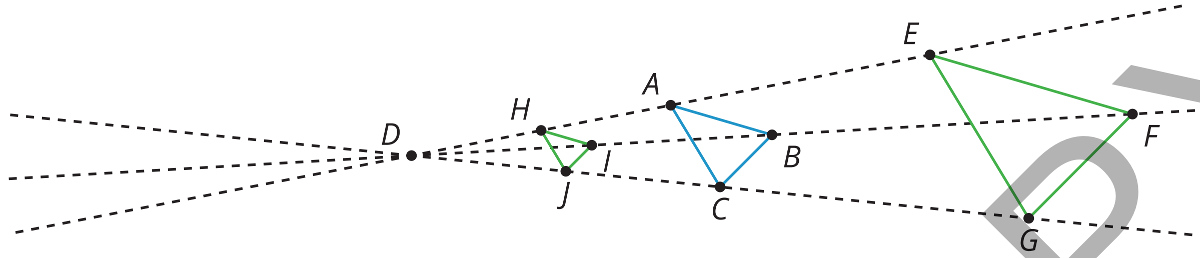
If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know ________?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Are you ready for more?

Triangle EFG was created by dilating triangle ABC using a scale factor of 2 and center D. Triangle HIJ was created by dilating triangle ABC using a scale factor of $\frac{1}{2}$ and center D.



1. What would the image of triangle ABC look like under a dilation with scale factor 0?

2. What would the image of triangle ABC look like under dilation with a scale factor of -1? If possible, draw it and label the vertices A', B', and C'. If it's not possible, explain why not.

3. If possible, describe what happens to a point if it is dilated with a negative scale factor. If dilating with a negative scale factor is not possible, explain why not.

Lesson 5 Summary

One important use of coordinates is to communicate geometric information precisely. Like an address in a city, they tell you exactly where to go. Because the plane is laid out in a grid, these "addresses" are simple, consisting of 2 signed numbers.

Consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires 3 vital pieces of information:

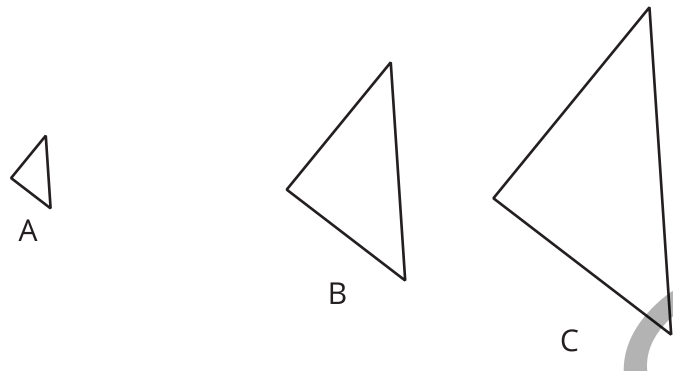
1. The coordinates of A, B, C, and D
2. The coordinates of the center of dilation
3. The scale factor

With this information, we can dilate each of the vertices A, B, C, and D and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.

Sec A

Practice Problems

1 Triangles B and C have been built by dilating Triangle A.



a. Find the center of dilation.

b. Triangle B is a dilation of A with approximately what scale factor?

c. Triangle A is a dilation of B with approximately what scale factor?

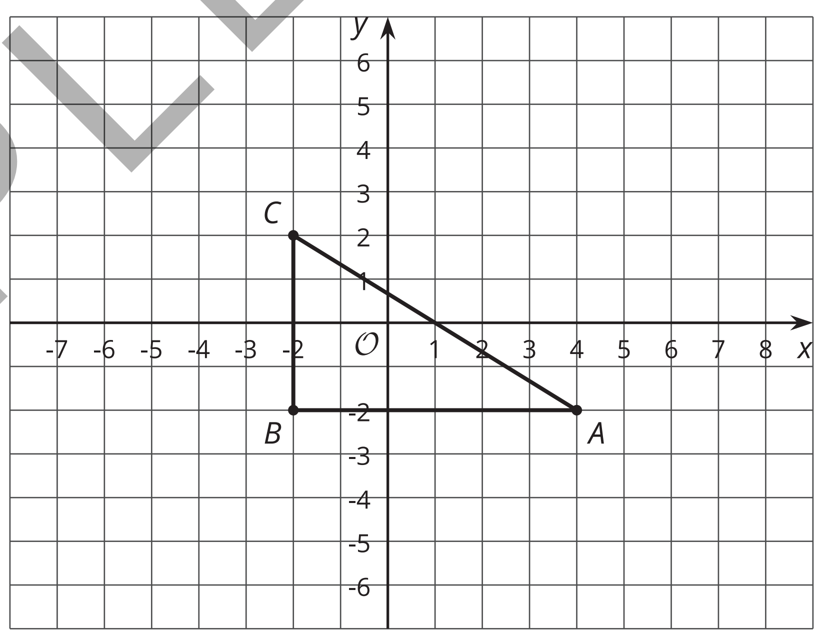
d. Triangle B is a dilation of C with approximately what scale factor?

2 Here is a triangle.

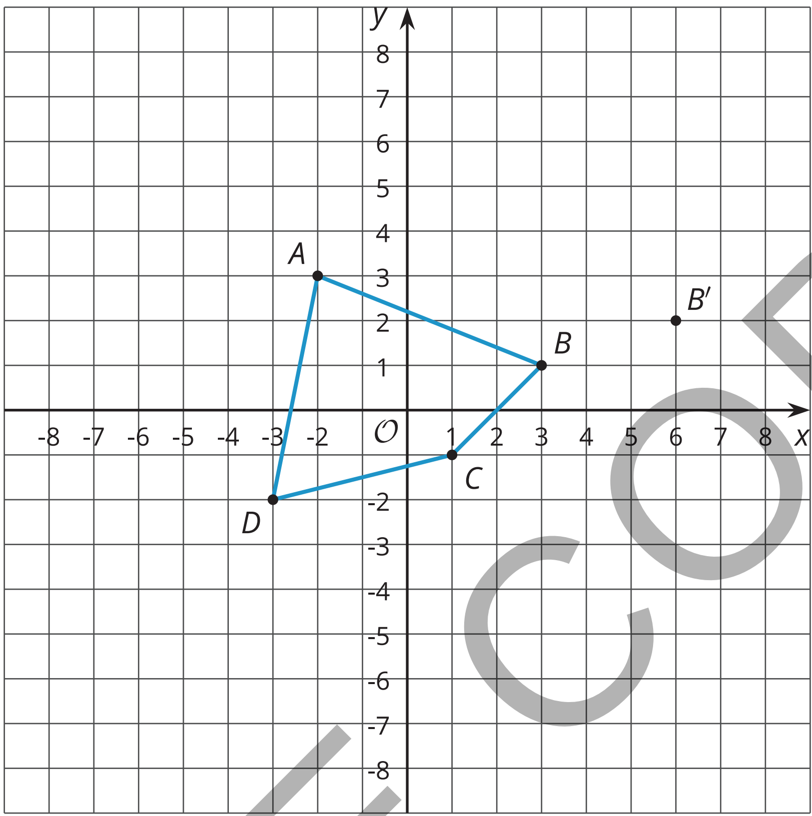
a. Draw the dilation of triangle ABC, with center $(0, 0)$, and scale factor 2. Label this triangle $A'B'C'$.

b. Draw the dilation of triangle ABC, with center $(0, 0)$, and scale factor $\frac{1}{2}$. Label this triangle $A''B''C''$.

c. Is $A''B''C''$ a dilation of triangle $A'B'C'$? If yes, what are the center of dilation and the scale factor?



3 Quadrilateral $ABCD$ is dilated with center $(0, 0)$, taking B to B'. Draw $A'B'C'D'$.



4 from Unit 1, Lesson 15

Triangle DEF is a right triangle, and the measure of angle D is 28°. What are the measures of the other two angles?

Unit 2, Lesson 6
Addressing CA CCSSM 8.G.2, 8.G.4; building on 8.G.2; building towards 8.G.4; practicing MP3, MP6, MP8

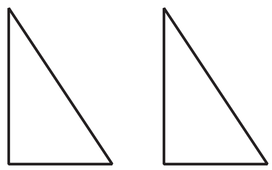
Similarity

Let’s explore similar figures.

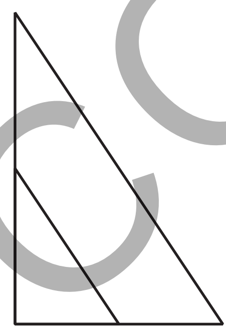
6.1 Which Three Go Together: Triangles

Which 3 go together? Why do they go together?

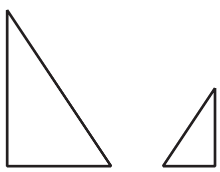
A



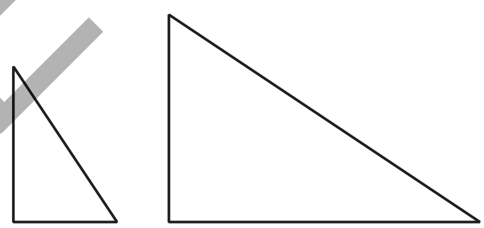
B



C

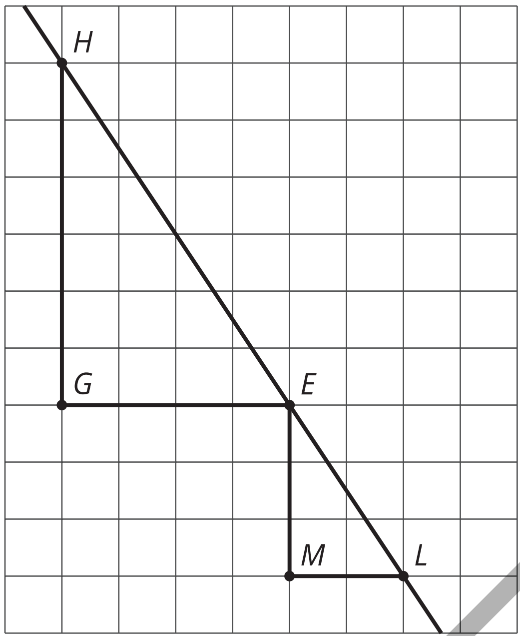


D

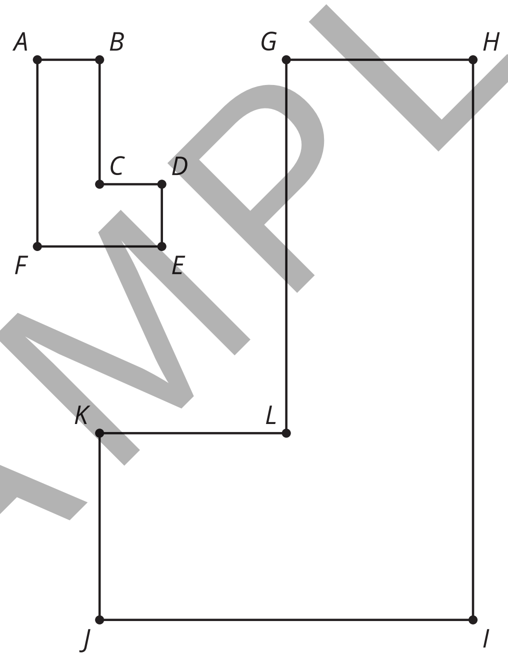


6.2 Similarity Transformations (Part 1)

1. Triangle EGH and triangle LME are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.



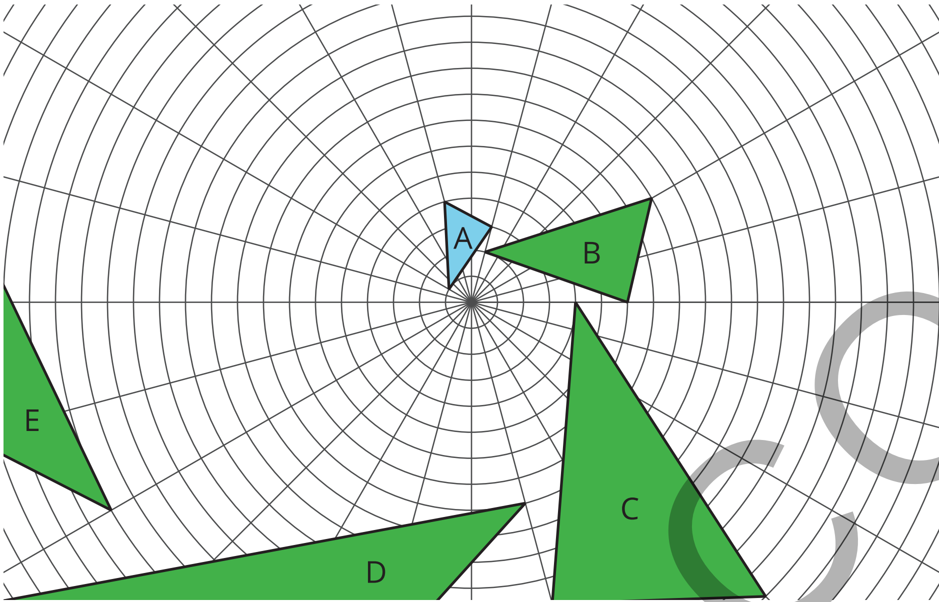
2. Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.



Sec B

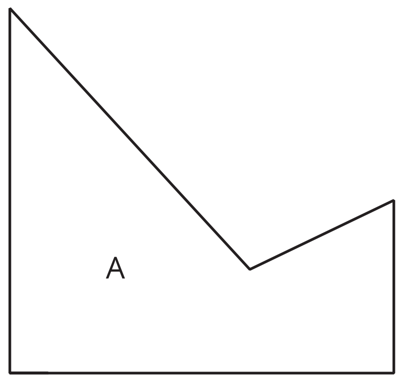
Are you ready for more?

The same sequence of transformations that takes Triangle A to Triangle B, also takes Triangle B to Triangle C, and so on. Describe a possible sequence of transformations.



6.3 Similarity Transformations (Part 2)

Sketch figures similar to Figure A that use only the transformations listed to show similarity.



1. A translation and a reflection. Label your sketch Figure B.
   Pause here so your teacher can review your work.
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.
3. A rotation and a reflection. Label your sketch Figure D.
4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

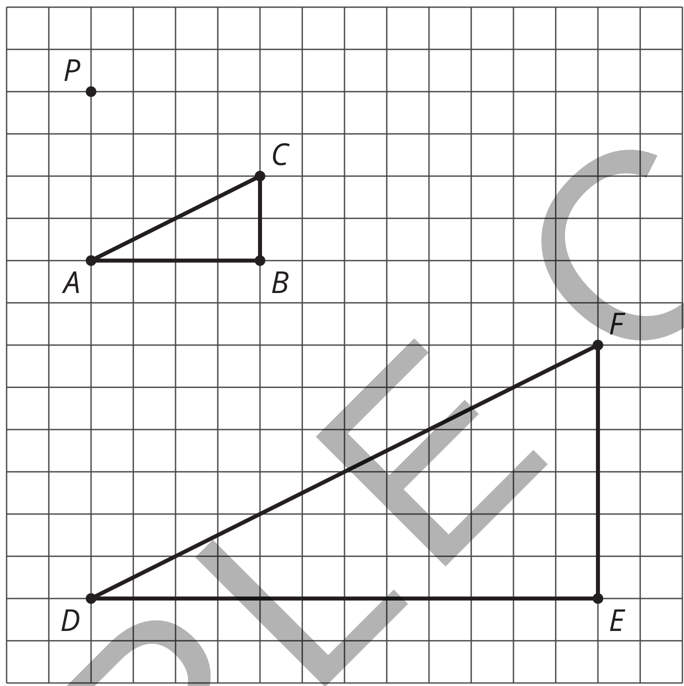
6.4 Methods for Translations and Dilations

Your teacher will give you and your partner a set of cards. Each set contains five cards for Partner A and a different set of five cards for Partner B.

Using only the cards in your set, find one or more ways to show that triangle ABC and triangle DEF are similar.

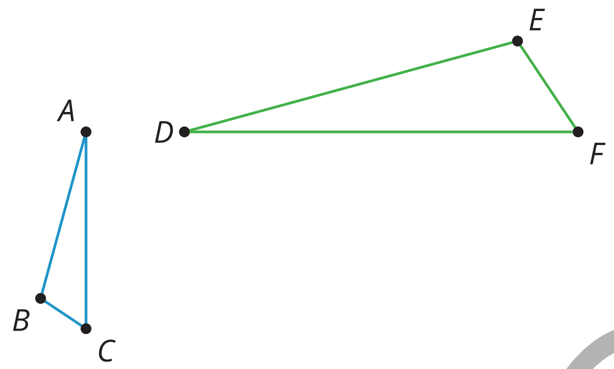
Compare your method with your partner's method. How are your methods similar? How are they different?

Sec B



Lesson 6 Summary

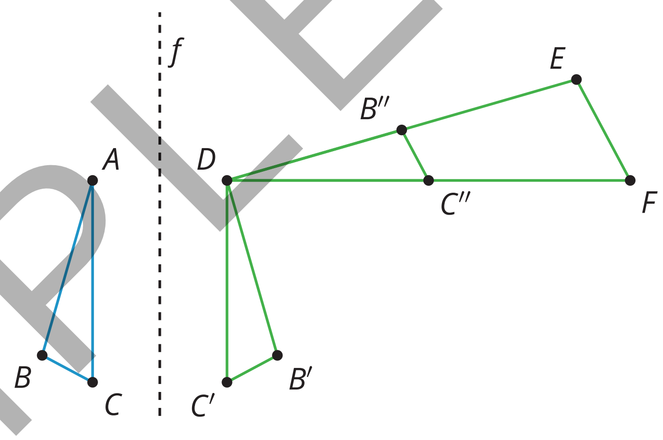
Let's show that triangle ABC is similar to triangle DEF:



Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from triangle ABC to triangle DEF follows these steps:

- Reflect triangle ABC across line f
- Rotate 90° counterclockwise around D
- Dilate with center D and scale factor 2



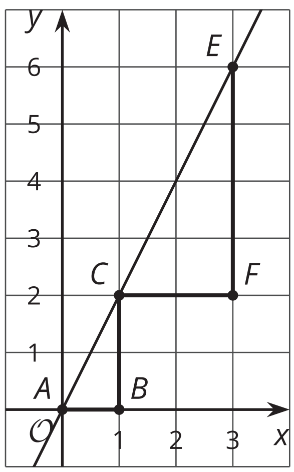
Another way to show that triangle ABC is similar to triangle DEF would be to dilate triangle DEF by a scale factor of $\frac{1}{2}$ with center of dilation at D, then translate D to A, then rotate it 90° clockwise around D, and finally reflect it across the vertical line containing DF so it matches up with triangle ABC.

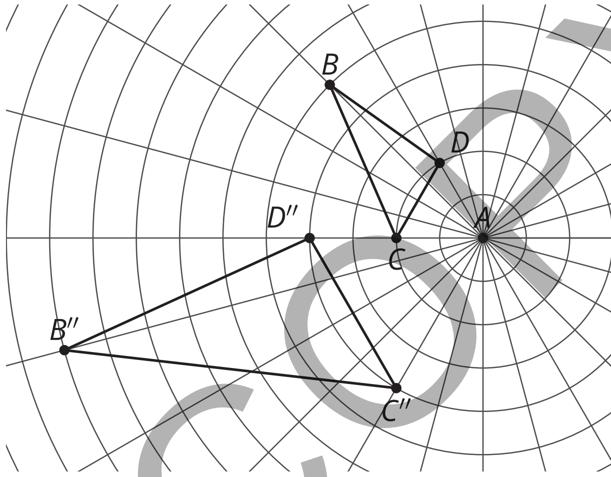
Glossary

- similar

Practice Problems

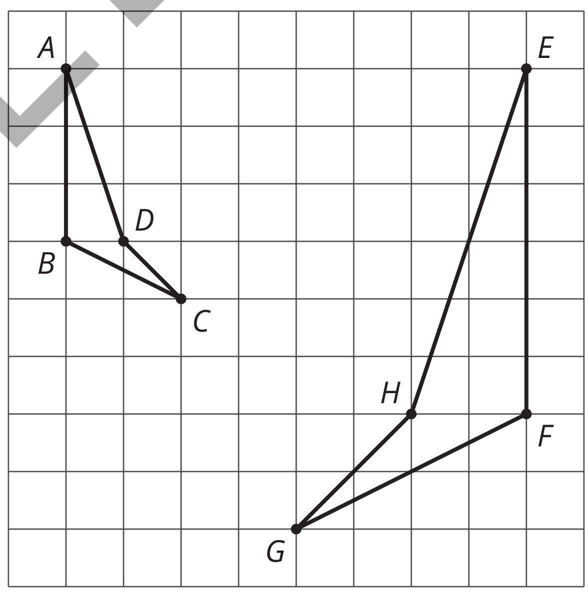
1 Each diagram has a pair of figures, one larger than the other. For each pair, show that the 2 figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the smaller figure to the larger one.



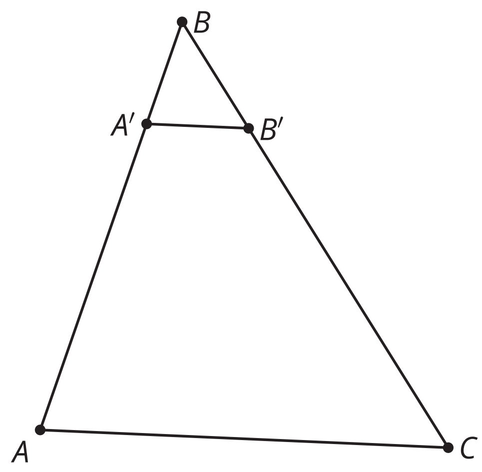


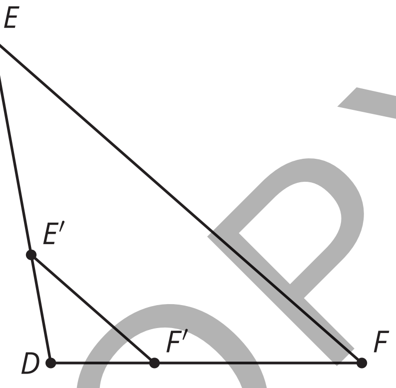
2 Here are two similar polygons.

Measure the side lengths and angles of each polygon. What do you notice?



3 Each figure shows a pair of similar triangles, one contained in the other. For each pair, describe a point and a scale factor to use for a dilation moving the larger triangle to the smaller one. Use a measurement tool to find the scale factor.





Unit 2, Lesson 7
Addressing CA CCSSM 8.G.2, 8.G.4; practicing MP3 and MP6

Similar Polygons

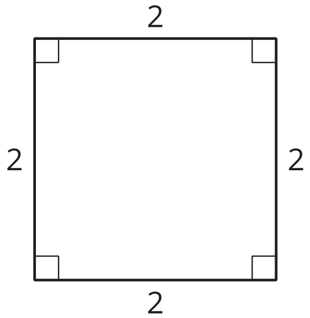
Let's look at sides and angles of similar polygons.

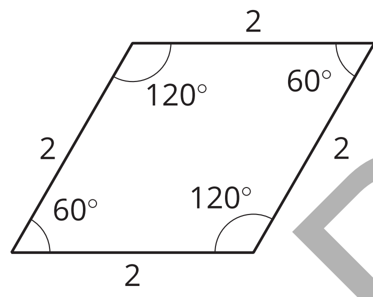
7.1 Math Talk: Congruence and Similarity

Decide mentally whether each statement is always true, sometimes true, or never true.

- If two figures are congruent, then they are similar.
- If two figures are similar, then they are congruent.
- If a triangle is dilated with the center of dilation at one of its vertices, the side lengths of the new triangle will change.
- If a triangle is dilated with the center of dilation at one of its vertices, the angle measures of the triangle will change.

7.2 Are They Similar?

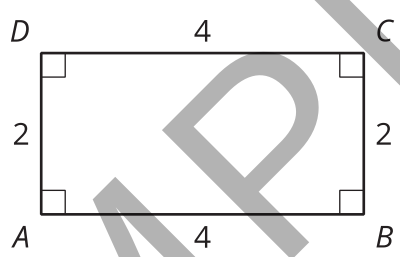


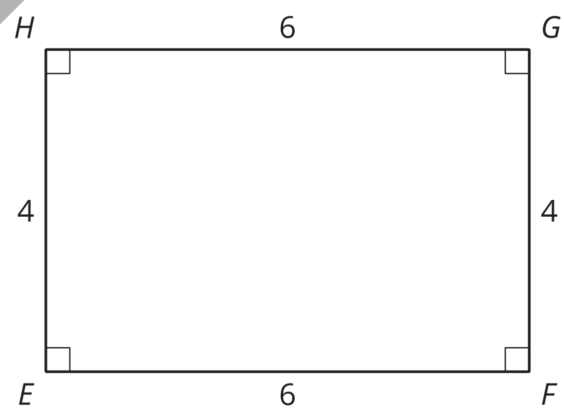


1. Let’s look at a square and a rhombus.

   Priya says, “These polygons are similar because their side lengths are all the same.” Clare says, “These polygons are not similar because the angles are different.” Do you agree with either Priya or Clare? Explain your reasoning.

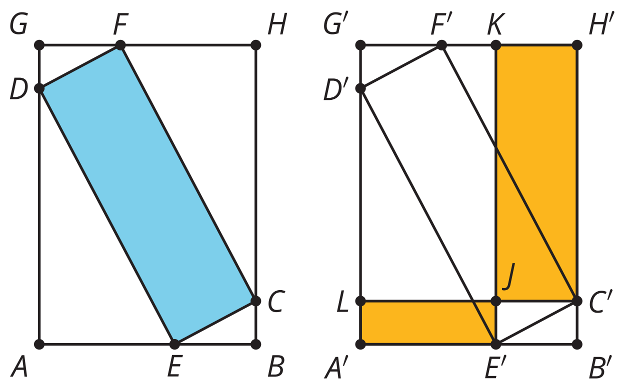
2. Now, let’s look at rectangles $ABCD$ and $EFGH$.





Jada says, “These rectangles are similar because all of the side lengths differ by 2.” Lin says, “These rectangles are similar. I can dilate AD and BC using a scale factor of 2 and AB and CD using a scale factor of 1.5 to make the rectangles congruent. Then I can use a translation to line up the rectangles.” Do you agree with either Jada or Lin? Explain your reasoning.

Are you ready for more?



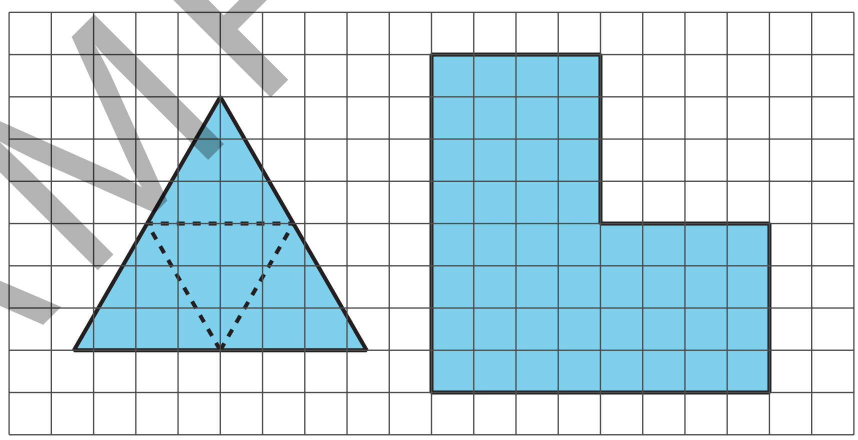
Points A through H are translated to the right to create points A' through H'. All of the following are rectangles: $GHBA$, $FCED$, $KH'C'J$, and $LJE'A'$. Which is greater, the area of blue rectangle $DFCE$ or the total area of yellow rectangles $KH'C'J$ and $LJE'A'$?

7.3 Find Someone Similar

Your teacher will give you a card. Find someone else in the room who has a card with a polygon that is similar but not congruent to yours. When you have found your partner, work with them to explain how you know that the two polygons are similar.

Are you ready for more?

On the left is an equilateral triangle where dashed lines have been added, showing how an equilateral triangle can be partitioned into smaller similar triangles.



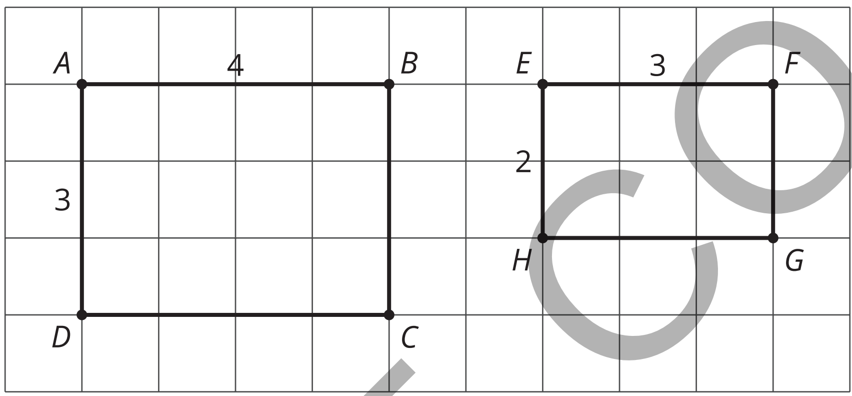
Find a way to do this for the figure on the right, partitioning it into smaller figures which are each similar to that original shape. What's the fewest number of pieces you can use? The most?

Lesson 7 Summary

When two polygons are similar:

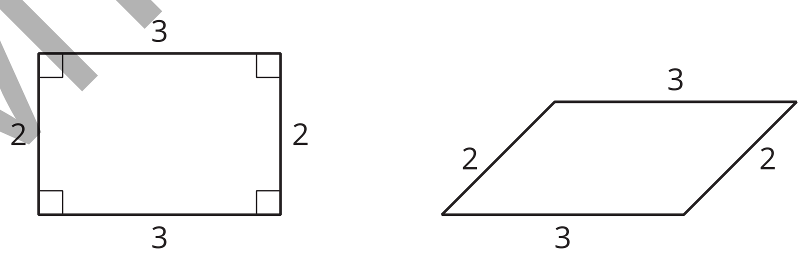
- Every angle and side in one polygon has a corresponding angle and side in the other polygon.
- All pairs of corresponding angles have the same measure.
- Each side length in one figure is multiplied by the same scale factor to get the corresponding side length in the other figure.

Consider the two rectangles shown here. Are they similar?



It looks like rectangles $ABCD$ and $EFGH$ could be similar, if you match the long edges and match the short edges. All the corresponding angles are congruent because they are all right angles. Calculating the scale factor between the sides is where we see that “looks like” isn't enough to make them similar. To scale the long side AB to the long side EF, the scale factor must be $\frac{3}{4}$, because $4 \cdot \frac{3}{4} = 3$. But the scale factor to match AD to EH has to be $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. So, the rectangles are not similar because the scale factors for all the parts must be the same.

Here is an example that shows how sides can correspond with a scale factor of 1, but the quadrilaterals are not similar because the corresponding angles don't have the same measure:



Practice Problems

1 Triangle DEF is a dilation of triangle ABC with scale factor 2. In triangle ABC, the largest angle measures 82°. What does the largest angle measure in triangle DEF?

A. 41°

B. 82°

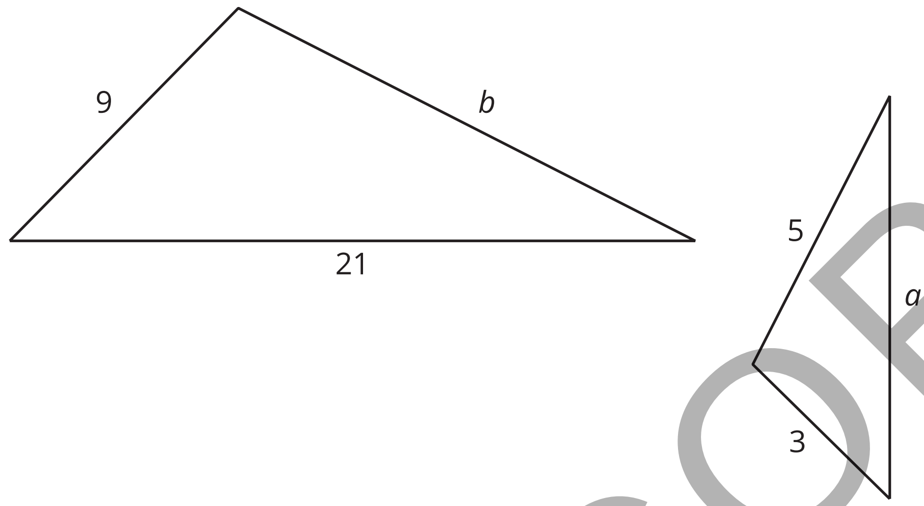
C. 98°

D. 164°

2 Draw 2 polygons that are similar but could be mistaken for not being similar. Explain why they are similar.

3 Draw 2 polygons that are *not* similar but could be mistaken for being similar. Explain why they are not similar.

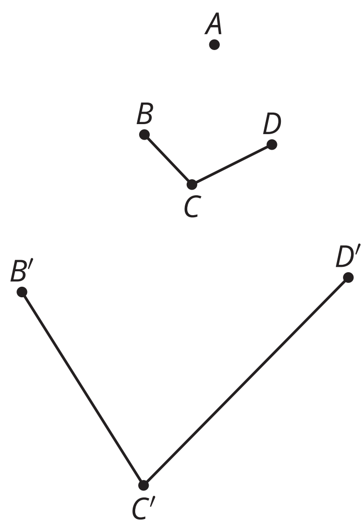
4 These 2 triangles are similar. Find side lengths a and b. Note: the 2 figures are not drawn to scale.



5 from Unit 2, Lesson 3

Jada claims that $B'C'D'$ is a dilation of BCD using A as the center of dilation.

What are some ways you can convince Jada that her claim is not true?



6 from Unit 1, Lesson 8

a. Draw a horizontal line segment AB.

b. Rotate segment AB 90° counterclockwise around point A. Label any new points.

c. Rotate segment AB 90° clockwise around point B. Label any new points.

d. Describe a transformation on segment AB you could use to finish building a square.

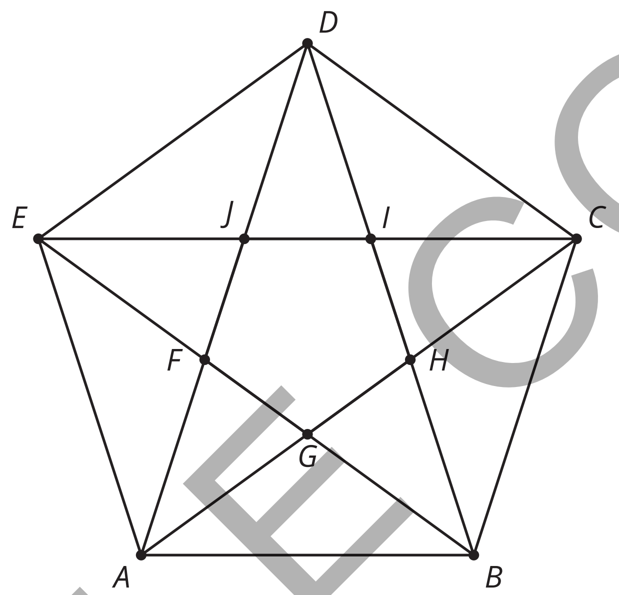
Unit 2, Lesson 8

Addressing CA CCSSM 8.G.4, 8.G.5; building on 7.RP.2a, 8.G.4, 8.G.5; practicing MP3 and MP7

Similar Triangles

Let’s look at similar triangles.

8.1 A Star and a Pentagon



Sec B

8.2 Making Pasta Angles and Triangles

Your teacher will give you dried pasta, a set of 3 angles labeled A, B, and C, blank paper, and tape.

1. Create a triangle using 3 pieces of pasta and angle A. Your triangle *must* include the angle you were given, but you are otherwise free to make any triangle you like. Tape your pasta triangle to a sheet of paper so it won't move.
   a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5° using a protractor and record these measurements on your paper.
   b. Find 2 others in the room who have the same angle A and compare your triangles. What is the same? What is different?
   c. Are the triangles congruent? Are the triangles similar? Explain your reasoning.

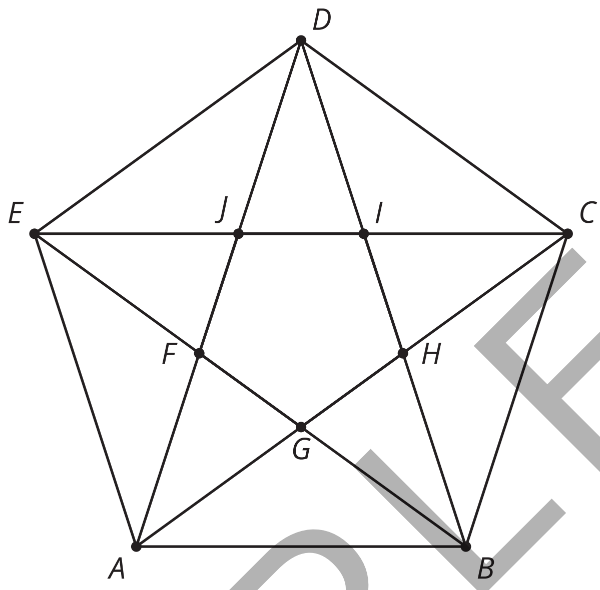
2. Now use more pasta and all 3 angles A, B, and C to create 1 new triangle. Tape this pasta triangle on a separate blank sheet of paper.
   a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5° using a protractor and record these measurements on your paper.
   b. Find 2 others in the room who used your same 3 angles and compare your triangles. What is the same? What is different?
   c. Are the triangles congruent? Are the triangles similar? Explain your reasoning.

Are you ready for more?

Quadrilaterals $ABCD$ and $EFGH$ have 4 angles with measures of 240°, 40°, 40°, and 40°. Do $ABCD$ and $EFGH$ have to be similar?

8.3 Similar Figures in a Regular Pentagon

1. This diagram has several triangles that are similar to triangle DJI.



   a. Three different scale factors were used to make triangles similar to DJI. In the diagram, find at least 1 triangle of each size that is similar to DJI.

   b. Explain how you know each of these 3 triangles is similar to DJI.

2. Find a triangle in the diagram that is not similar to DJI.

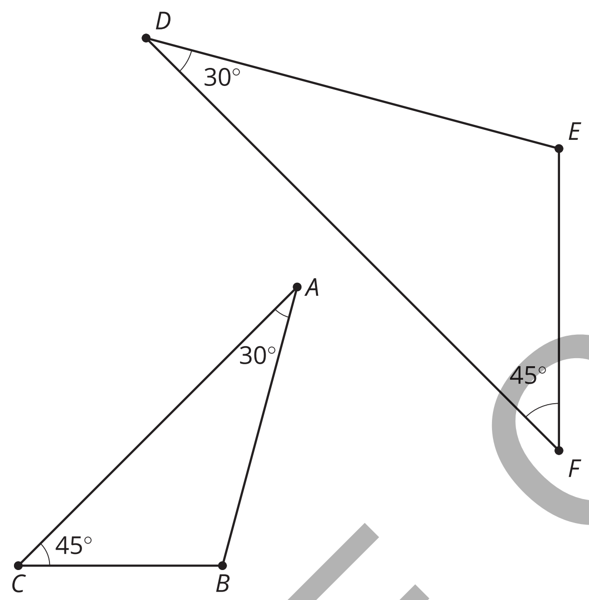
Are you ready for more?

How can you draw more lines to create additional triangles similar to triangle DJI?

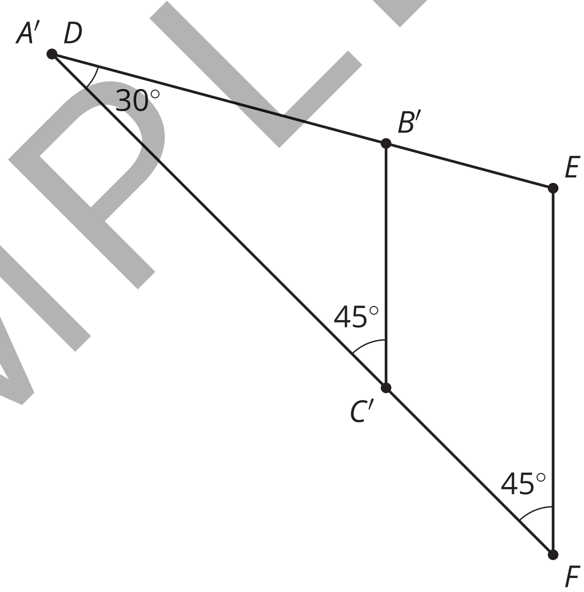
Lesson 8 Summary

Two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. When the polygons are triangles, we only need to check that both triangles have two corresponding angles to show they are similar.

For example, triangle ABC and triangle DEF both have a 30-degree angle and a 45-degree angle.



We can translate A to D and then rotate around point D so that the two 30-degree angles are aligned, giving this picture:



Then a dilation with center D and appropriate scale factor will move C' to F. This dilation also moves B' to E, showing that triangles ABC and DEF are similar.

Practice Problems

1 For each pair of triangles, some of the angle measures are given in degrees. Use the information to decide if the triangles are similar or not. Explain how you know.

- Triangle A: 53, 71, ________; Triangle B: 53, 71, ________
- Triangle C: 90, 37, ________; Triangle D: 90, 53, ________
- Triangle E: 63, 45, ________; Triangle F: 14, 71, ________
- Triangle G: 121, ________, ________; Triangle H: 70, ________, ________

2 a. Draw 2 equilateral triangles that are not congruent.

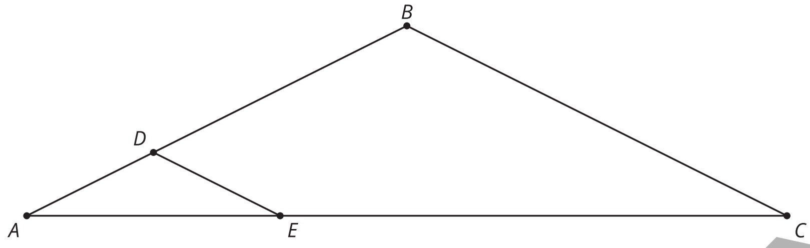
b. Measure the side lengths and angles of your triangles. Are the 2 triangles similar?

c. Do you think 2 equilateral triangles will be similar *always*, *sometimes*, or *never*? Explain your reasoning.

Sec B

3 In the figure, line segment BC is parallel to line segment DE.

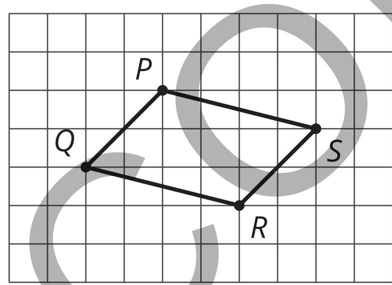
Explain why triangle ABC is similar to triangle ADE.



4 from Unit 2, Lesson 4

The quadrilateral $PQRS$ in the diagram is a parallelogram.

Let $P'Q'R'S'$ be the image of $PQRS$ after applying a dilation centered at a point O (not shown) with scale factor 3.



Which of the following is true?

A. $P'Q' = PQ$

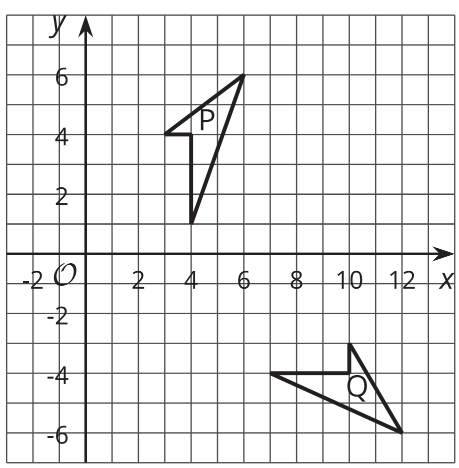
B. $P'Q' = 3PQ$

C. $PQ = 3P'Q'$

D. $P'Q' = \frac{1}{3}PQ$

5 from Unit 1, Lesson 6

Describe a sequence of transformations for which Quadrilateral P is the image of Quadrilateral Q.



Addressing CA CCSSM 8.G.4; building on 7.RP.2; practicing MP8

Side Length Quotients in Similar Triangles

Let's find missing side lengths in triangles.

9.1 Two-three-four and Four-five-six

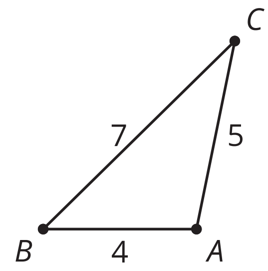
Triangle A has side lengths 2, 3, and 4. Triangle B has side lengths 4, 5, and 6.

Is Triangle A similar to Triangle B? Be prepared to explain your reasoning.

9.2 Quotients of Sides Within Similar Triangles

Triangle ABC is similar to triangles DEF, GHI, and JKL.

The scale factors for the dilations that show triangle ABC is similar to each triangle are in the table.



1. Find the side lengths of triangles DEF, GHI, and JKL. Record them in the table.

triangle	scale factor	length of short side	length of medium side	length of long side
ABC	1	4	5	7
DEF	2			
GHI	3			
JKL	$\frac{1}{2}$			

2. Your teacher will assign you 1 of the 3 columns. For all 4 triangles, find the quotient of the triangle side lengths assigned to you and record it in the table.

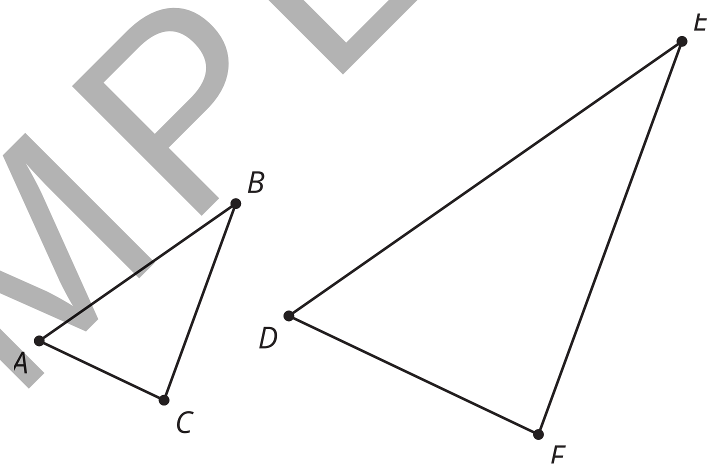
triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75	$\frac{7}{5}$ or 1.4	$\frac{5}{4}$ or 1.25
DEF			
GHI			
JKL			

What do you notice about the quotients?

3. Compare your results with your partners' and complete your table.

Are you ready for more?

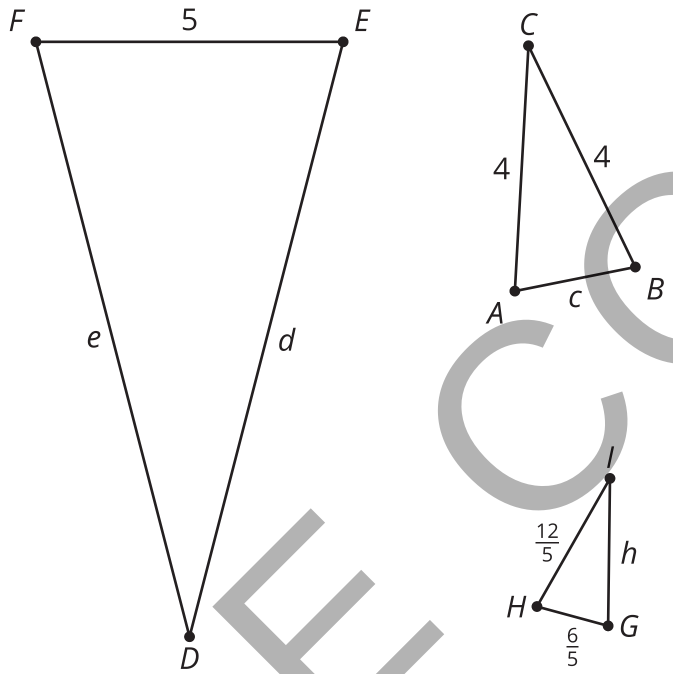
Triangles ABC and DEF are similar. Explain why $\frac{AB}{BC} = \frac{DE}{EF}$.



9.3 Using Side Quotients to Find Side Lengths of Similar Triangles

Triangles ABC, EFD, and GHI are all similar.

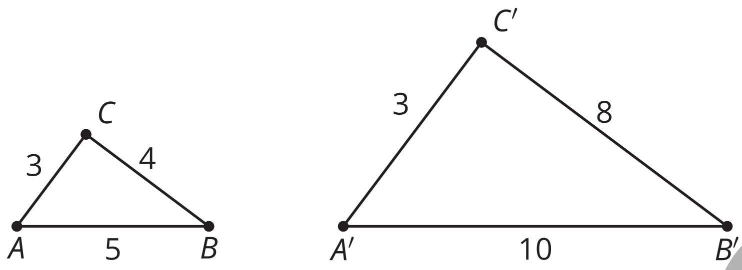
The side lengths of the triangles all have the same units. Find the unknown side lengths.



Lesson 9 Summary

If 2 polygons are similar, then the side lengths in one polygon are multiplied by the same scale factor to give the corresponding side lengths in the other polygon.

For these triangles the scale factor is 2:



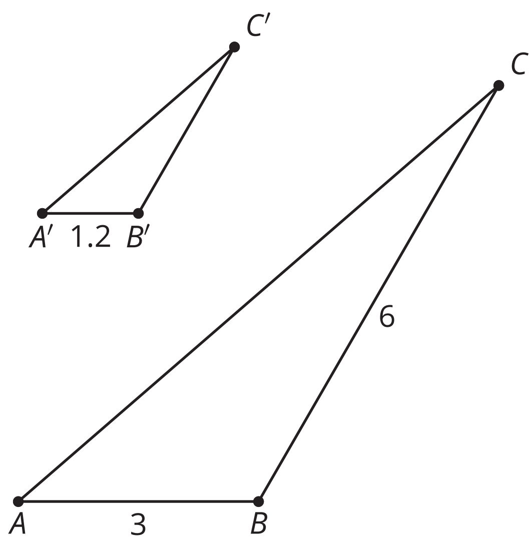
Here is a table that shows relationships between the lengths of the short and medium sides of the 2 triangles.

	small triangle	large triangle
medium side	4	8
short side	3	6
(medium side) ÷ (short side)	$\frac{4}{3}$	$\frac{8}{6} = \frac{4}{3}$

The lengths of the medium side and the short side are in a ratio of $4 : 3$. This means that the medium side in each triangle is $\frac{4}{3}$ as long as the short side. This is true for all similar polygons: the ratio between 2 sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

We can use these facts to calculate missing lengths in similar polygons. For example, triangles ABC and $A'B'C'$ are similar.

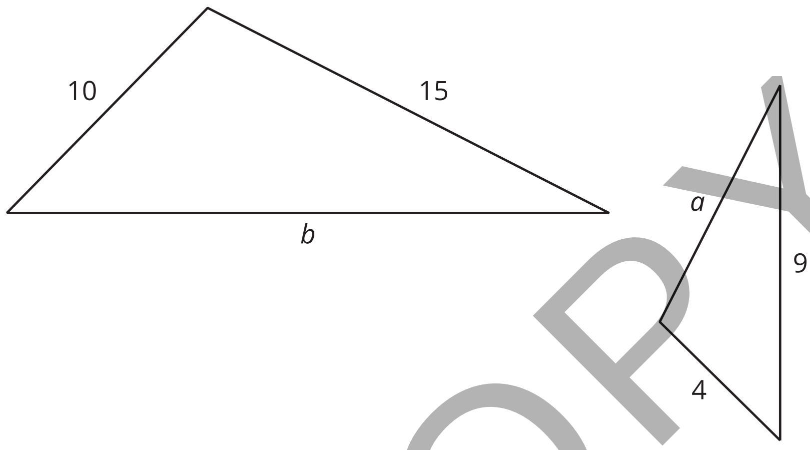
Since side BC is twice as long as side AB, side $B'C'$ must be twice as long as side $A'B'$. Since $A'B'$ is 1.2 units long and $2 \cdot 1.2 = 2.4$, the length of side $B'C'$ is 2.4 units.



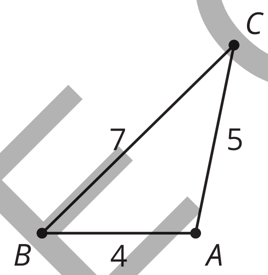
Practice Problems

1 These 2 triangles are similar. What are a and b?

Note: the 2 figures are not drawn to scale.



2 Here is triangle ABC. Triangle XYZ is similar to ABC with scale factor $\frac{1}{4}$.



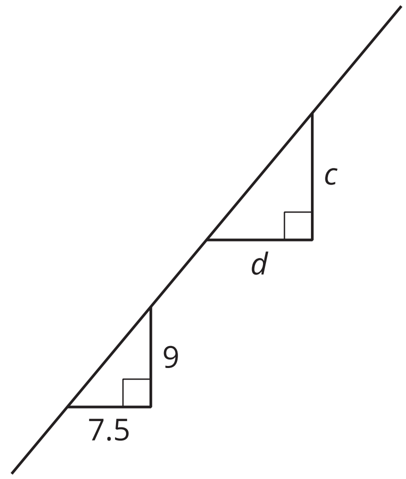
a. Draw what triangle XYZ might look like.

b. How do the angle measures of triangle XYZ compare to triangle ABC? Explain how you know.

c. What are the side lengths of triangle XYZ?

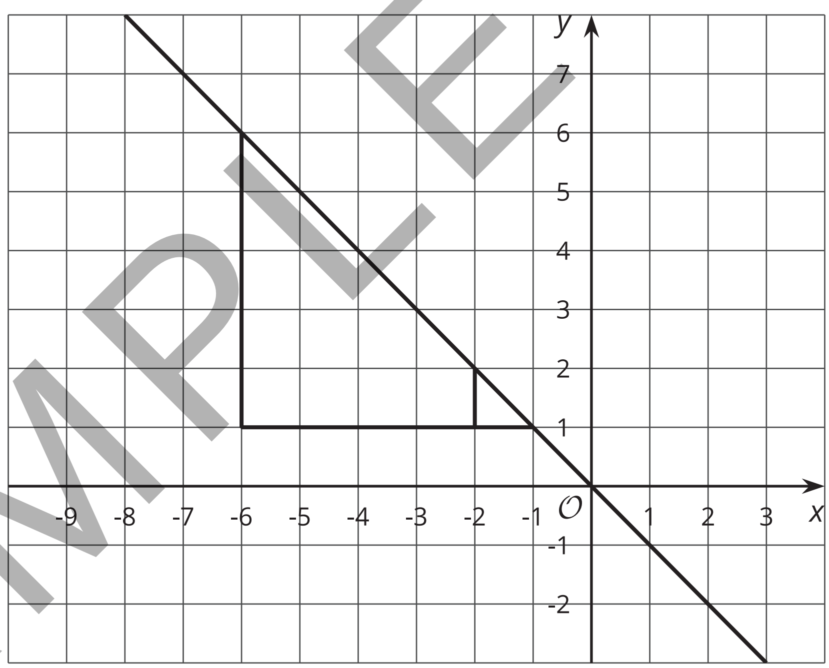
d. For triangle XYZ, calculate (long side) ÷ (medium side), and compare to triangle ABC.

3 The 2 triangles shown are similar. Find the value of $\frac{d}{c}$.



4 from Unit 2, Lesson 5

The diagram shows 2 nested triangles that share a vertex. Find a center and a scale factor for a dilation that would move the larger triangle to the smaller triangle.



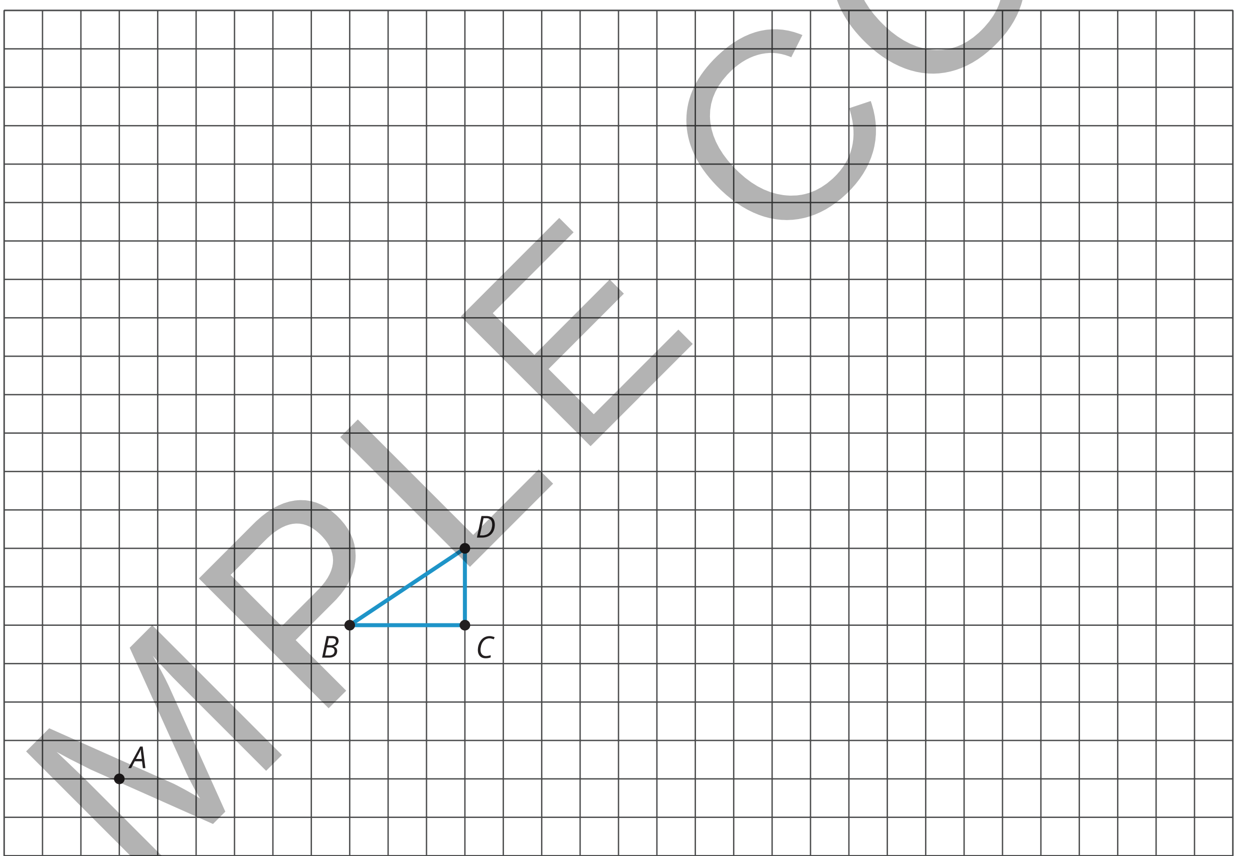
Addressing CA CCSSM 8.EE.6; building on 8.G.4; building towards 8.EE.6; practicing MP7 and MP8

Meet Slope

Let's learn about the slope of a line.

10.1 One Triangle, Many Scale Factors

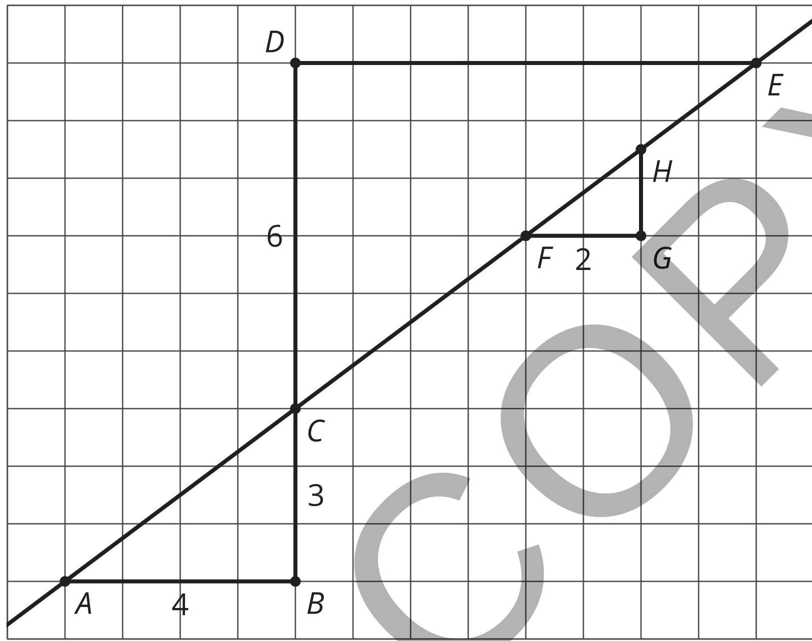
1. Choose a scale factor and draw a dilation of triangle BCD using point A as the center of dilation. What scale factor did you use?



2. Use a piece of tracing paper to trace point A and your dilated figure. Compare your dilation with your group. What do you notice?

10.2 Similar Triangles on the Same Line

1. The grid shows three right triangles, each with its longest side on the same line. Your teacher will assign you two of the triangles. Explain why the two triangles are similar.



2. Complete the table.

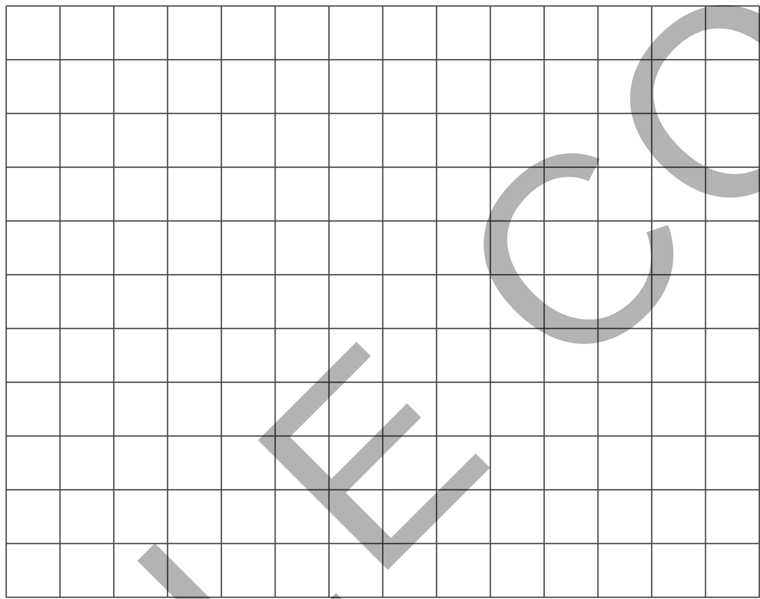
triangle	length of vertical side	length of horizontal side	(vertical side) ÷ (horizontal side)
ABC	3	4	$\frac{3}{4}$ or 0.75
CDE			
FGH			

3. What do you notice about the last column in the table? Why do you think this is true?

10.3 Multiple Lines with the Same Slope

1. Draw two lines with a **slope** of 3. What do you notice about the two lines?

2. Draw two lines with a slope of $\frac{1}{2}$. What do you notice about the two lines?

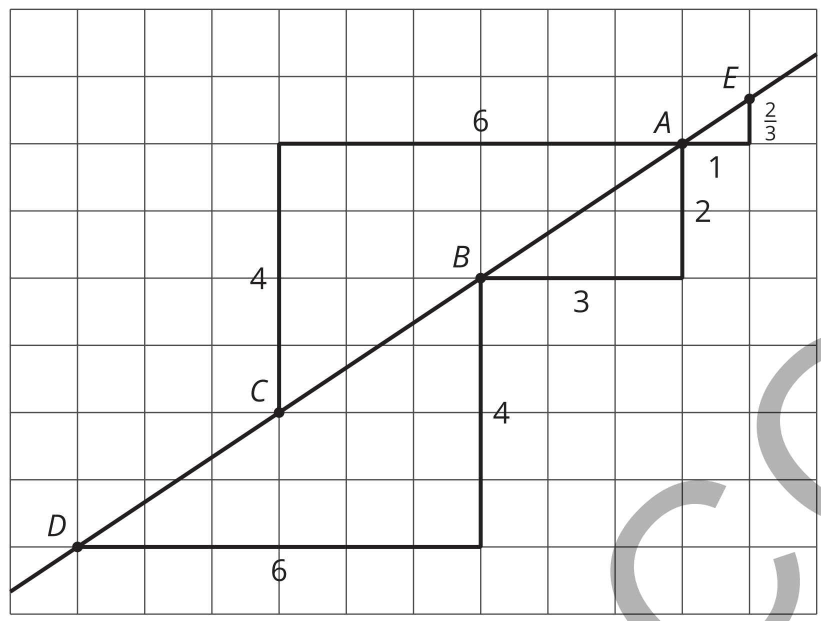


Are you ready for more?

As you learn more about lines, you will occasionally have to consider perfectly vertical lines as a special case and treat them differently. Think about applying what you have learned in the last couple of activities to the case of vertical lines. What is the same? What is different?

Lesson 10 Summary

Here is a line drawn on a grid. There are also four right triangles drawn.



These four triangles are all examples of *slope triangles*. The longest side of a slope triangle is on the line, one side is vertical, and another side is horizontal. The **slope** of the line is the quotient of the vertical length and the horizontal length of the slope triangle. This number is the same for all slope triangles for the same line because all slope triangles for the same line are similar.

In this example, the slope of the line is $\frac{2}{3}$. Here is how the slope is calculated using the slope triangles:

- Points A and B give $2 \div 3 = \frac{2}{3}$.
- Points D and B give $4 \div 6 = \frac{2}{3}$.
- Points A and C give $4 \div 6 = \frac{2}{3}$.
- Points A and E give $\frac{2}{3} \div 1 = \frac{2}{3}$.

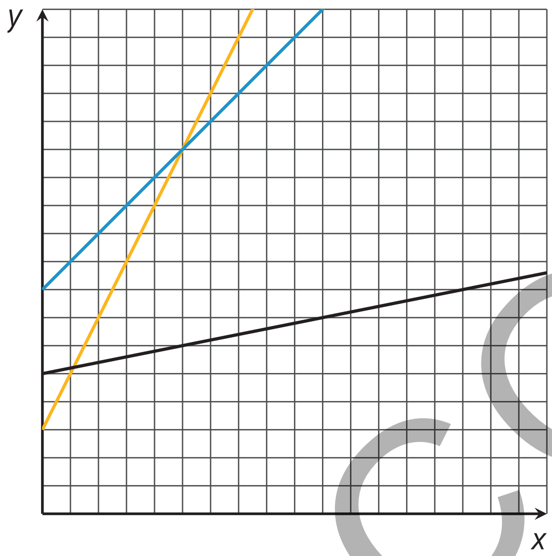
Glossary

- slope

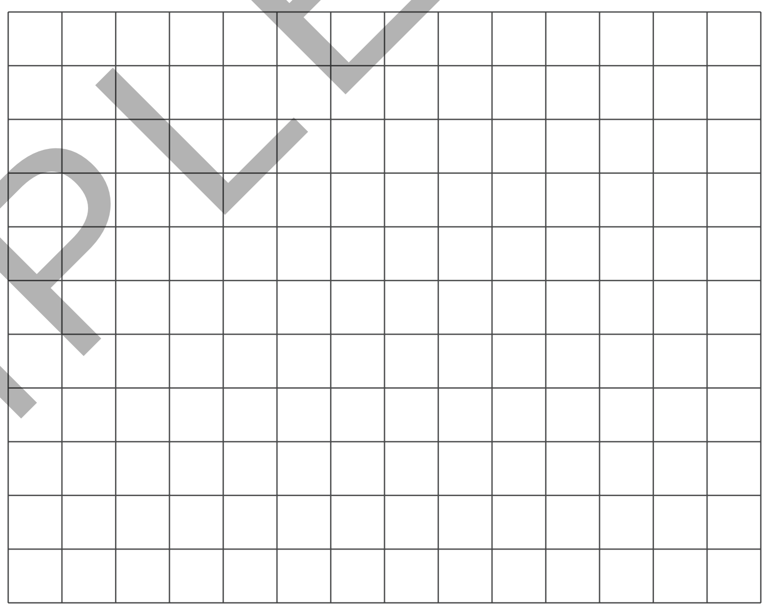
Sec C

Practice Problems

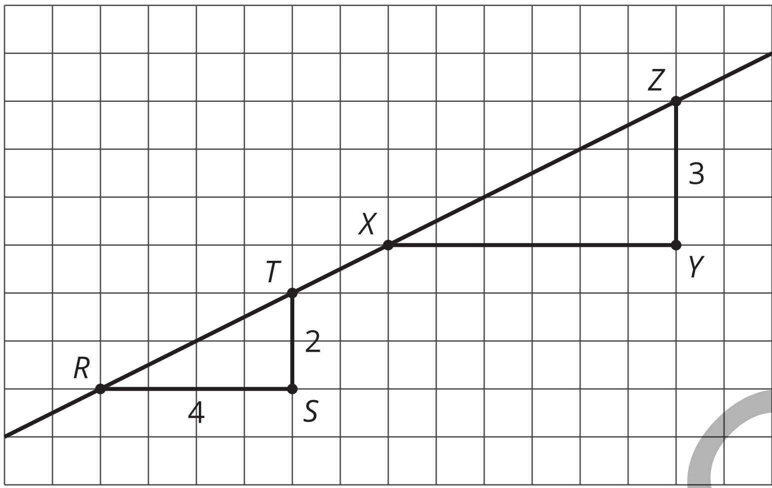
1 Here are three lines. One has slope 1, one has slope 2, and one has slope $\frac{1}{5}$. Label each line with its slope.



2 Draw three lines with slope 4, and three lines with slope $\frac{2}{5}$. What do you notice?



3 The grid shows two right triangles, each with its longest side on the same line.



a. Explain how you know the two triangles are similar.

Sec C

b. How long is XY?

c. For each triangle, calculate (vertical side) ÷ (horizontal side).

d. What is the slope of the line? Explain how you know.

4 from Unit 2, Lesson 9

Triangle A has side lengths 3 units, 4 units, and 5 units. Triangle B has side lengths 6 units, 7 units, and 8 units.

a. Explain how you know that Triangle B is *not* similar to Triangle A.

b. Give possible side lengths for Triangle B so that it is similar to Triangle A.

Unit 2, Lesson 11

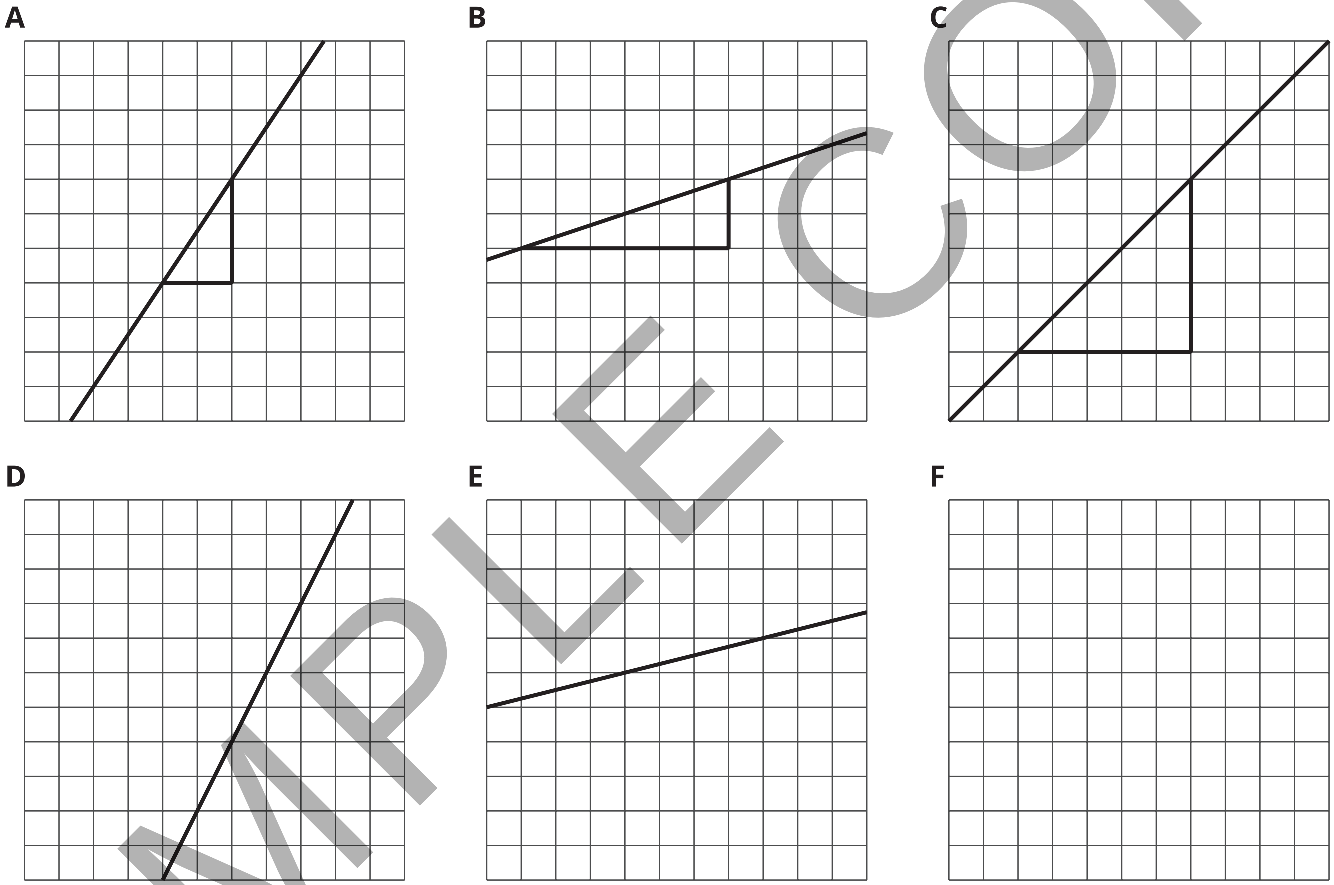
Addressing CA CCSSM 8.EE.6; building on 7.RP.2, 8.G.3; practicing MP7 and MP8

Writing Equations for Lines

Let's explore the relationship between points on a line and the slope of the line.

11.1 Different Slopes of Different Lines

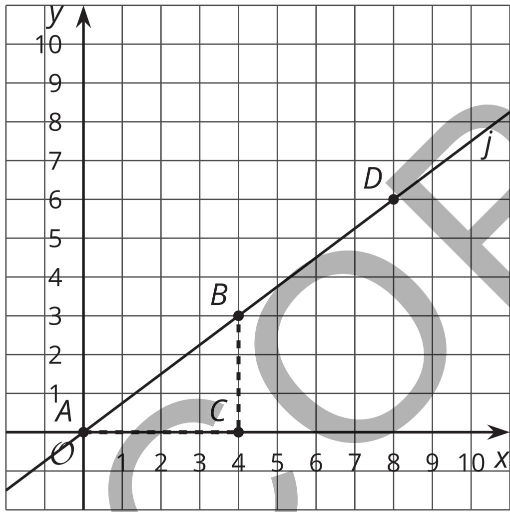
Here are several lines.



1. Match each line shown with a slope from this list: $\frac{1}{3}$, 2, $\frac{3}{5}$, 1, 0.25, $\frac{3}{2}$.
2. One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).

11.2 What We Mean by an Equation of a Line

Line j is shown in the coordinate plane.



1. What are the coordinates of B and D?

2. Is point $(20, 15)$ on line j? Explain how you know.

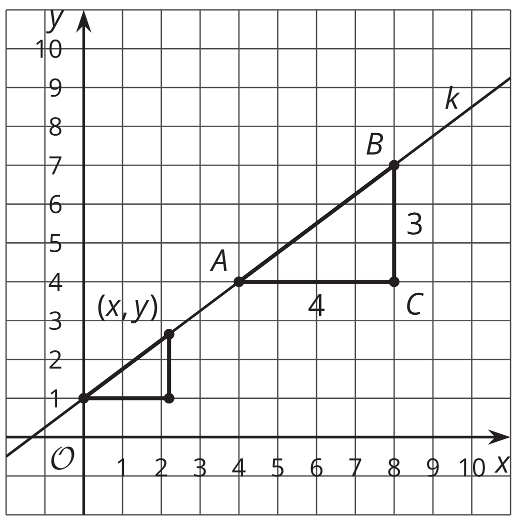
3. Is point $(100, 75)$ on line j? Explain how you know.

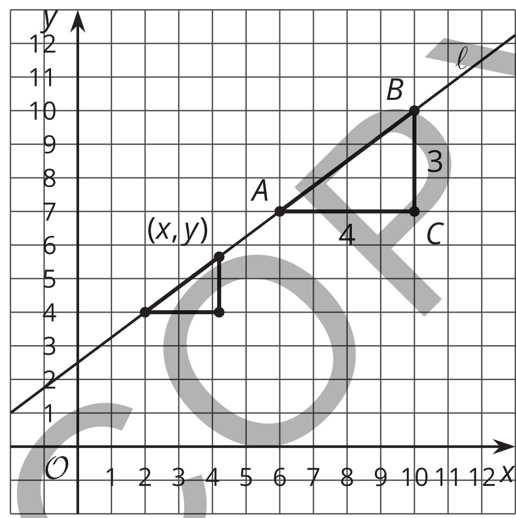
4. Is point $(90, 68)$ on line j? Explain how you know.

5. Suppose you know the x- and y-coordinates of a point. Write a rule that would allow you to test whether the point is on line j.

11.3 Writing Relationships from Slope Triangles

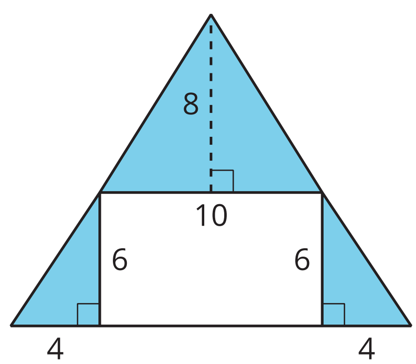
Here are two diagrams:





1. Complete each diagram so that all vertical and horizontal sides of the slope triangles have expressions for their lengths.
2. Use what you know about similar triangles to find an equation for the quotient of the vertical and horizontal side lengths of the smaller triangle in each diagram.

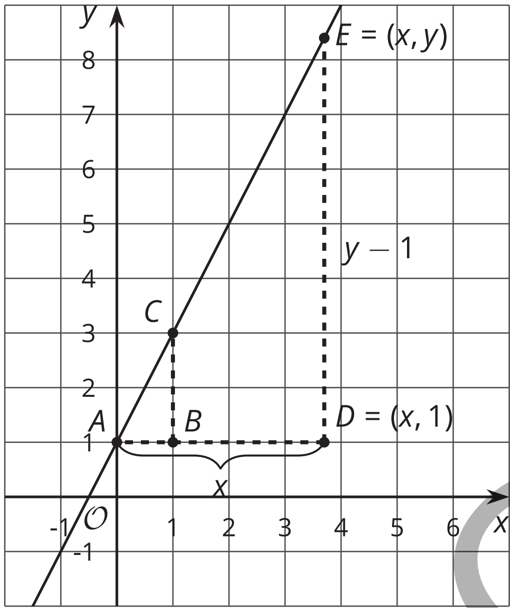
Are you ready for more?



1. Find the area of the shaded region by adding the areas of the shaded triangles.
2. Find the area of the shaded region by subtracting the area of the unshaded region from the large triangle.
3. What is going on here?

Lesson 11 Summary

Here is a line on the coordinate plane.



The points A, C, and E are on the same line. Triangles ABC and ADE are slope triangles for the line, so they are similar triangles. We can use their similarity to better understand the relationship between x and y, which are the coordinates of point E.

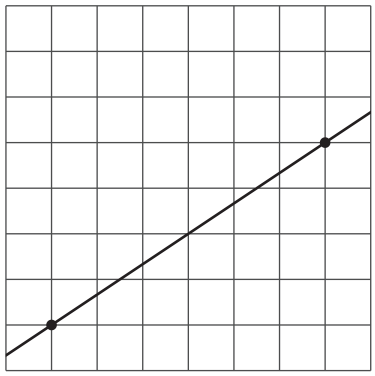
- The slope for triangle ABC is $\frac{2}{1}$ because the vertical side has length 2 and the horizontal side has length 1.
- For triangle ADE, the vertical side has length $y-1$ because y is the distance from point E to the x-axis, and side DE is 1 unit shorter than the distance. The horizontal side has length x. So, the slope for triangle ADE is $\frac{y-1}{x}$.
- The slopes for the two slope triangles are equal, meaning $\frac{2}{1}=\frac{y-1}{x}$.

The equation $\frac{2}{1}=\frac{y-1}{x}$ is true for all points on the line.

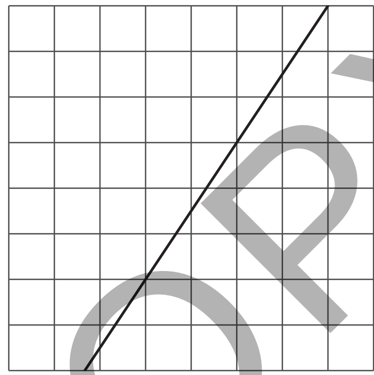
Practice Problems

1 Find the slope of each line.

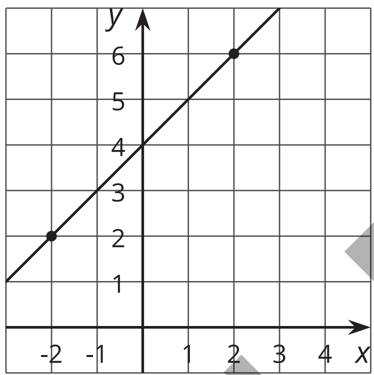
A



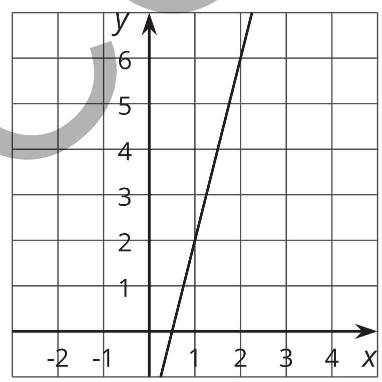
B



C

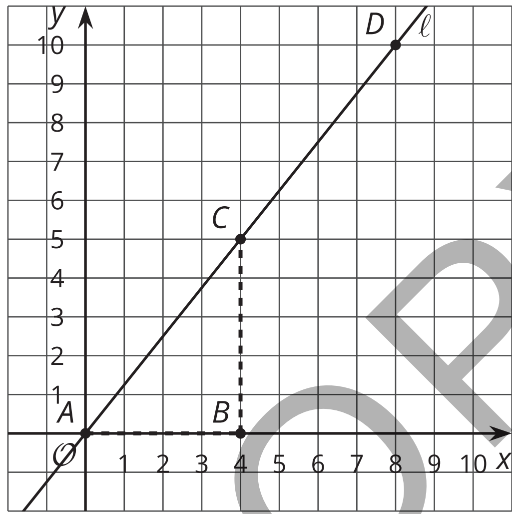


D



2 Line ℓ is shown in the coordinate plane.

a. What are the coordinates of points C and D?



b. Is the point $(16, 20)$ on line ℓ? Explain how you know.

c. Is the point $(20, 24)$ on line ℓ? Explain how you know.

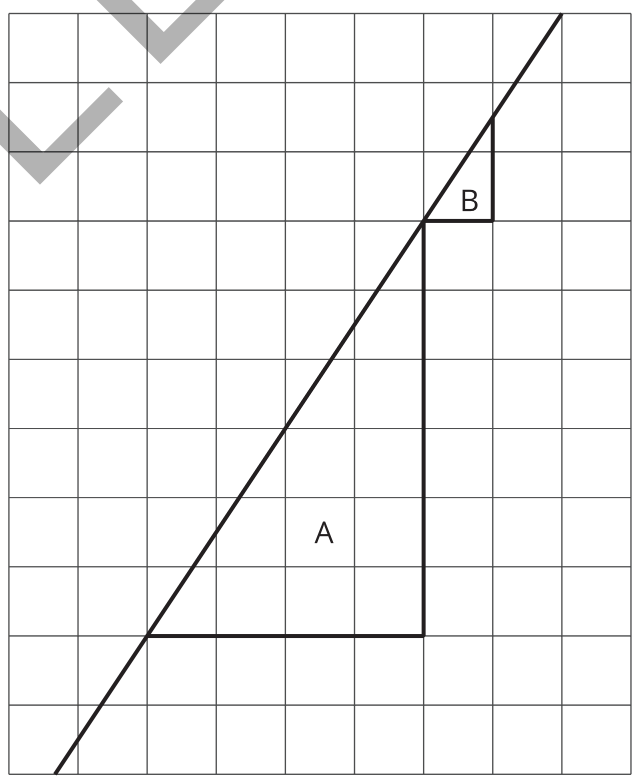
d. Is the point $(80, 100)$ on line ℓ? Explain how you know.

e. Write a rule that would allow you to test whether (x, y) is on line ℓ.

Sec C

3 Consider the graphed line.

Mai uses Triangle A and says the slope of this line is $\frac{6}{4}$. Elena uses Triangle B and says the slope of this line is 1.5. Do you agree with either of them? Explain or show your reasoning.



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4 from Unit 2, Lesson 7

A rectangle has dimensions 6 units by 4 units and is similar to quadrilateral $ABCD$. Select **all** statements that are true.

A. The length of side AB is the same as the length of side BC.

B. If the length of side AB is 9 units, then the length of side BC is 7 units.

C. The length of the shortest side of quadrilateral $ABCD$ is $\frac{2}{3}$ the length of the longest side.

D. Quadrilateral $ABCD$ is a rectangle.

E. The measure of angle ABC is 90°.

F. The measure of angle BCD is 105°.

Unit 2, Lesson 12
Addressing CA CCSSM 8.EE.6, 8.G.3; building towards 8.EE.6; practicing MP7 and MP8

Using Equations for Lines

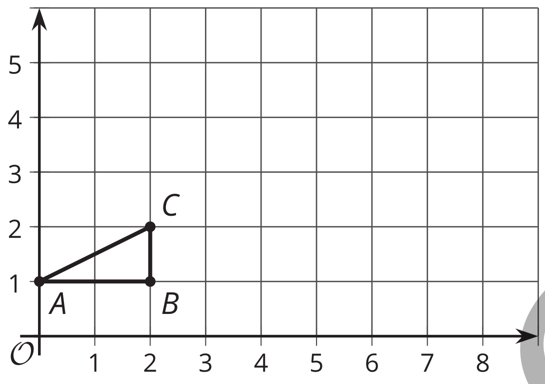
Let's write equations for lines.

12.1 Missing Center

A dilation with scale factor 2 sends A to B. Where is the center of the dilation?

12.2 Dilations and Slope Triangles

Here is triangle ABC.

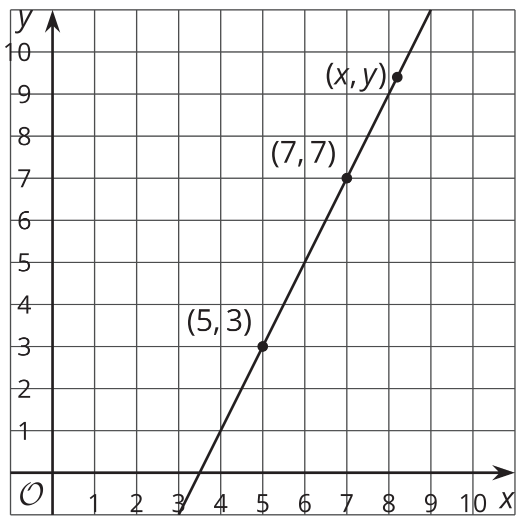


1. Draw the dilation of triangle ABC with center $(0, 1)$ and scale factor 2.
2. Draw the dilation of triangle ABC with center $(0, 1)$ and scale factor 2.5.
3. For which scale factor does the dilation with center $(0, 1)$ send point C to $(9, 5.5)$? Explain your reasoning.
4. What are the coordinates of point C after a dilation with center $(0, 1)$ and scale factor s?

Sec C

12.3 Writing Relationships from Two Points

Here is a line.



1. Using what you know about similar triangles, find an equation for the line in the diagram.

2. What is the slope of this line? Does it appear in your equation?

3. Is $(9, 11)$ also on the line? Explain your reasoning.

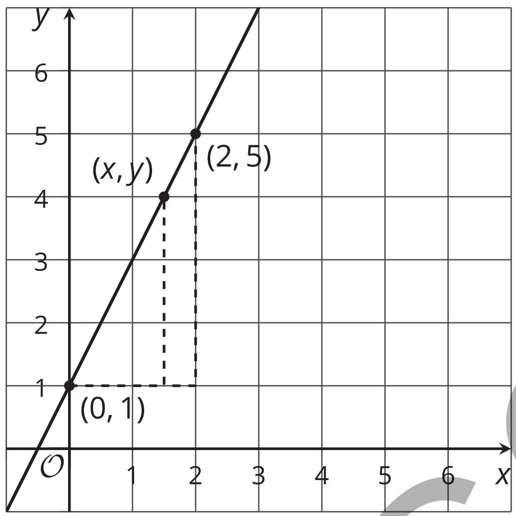
4. Is $(100, 193)$ also on the line? Explain your reasoning.

Are you ready for more?

There are many different ways to write an equation for a line like the one in the *Student Task* you just completed. Does $\frac{y-3}{x-6} = 2$ represent that line? What about $\frac{y-6}{x-4} = 5$? What about $\frac{y+5}{x-1} = 2$? Explain your reasoning.

Lesson 12 Summary

Here is a line with a few of the points labeled.



We can use what we know about slope to decide if a point lies on a line.

First, use points and slope triangles to write an equation for the line.

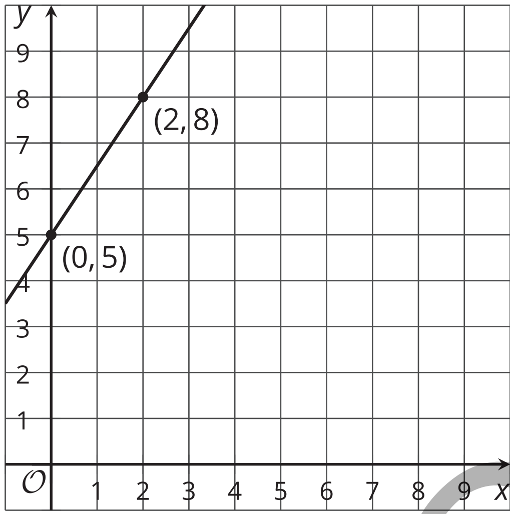
- The slope triangle with vertices $(0, 1)$ and $(2, 5)$ gives a slope of $\frac{5-1}{2-0} = 2$.
- The slope triangle with vertices $(0, 1)$ and (x, y) gives a slope of $\frac{y-1}{x}$.
- Since these slopes are the same, $\frac{y-1}{x} = 2$ is an equation for the line.

To check whether or not the point $(11, 23)$ lies on this line, we can check that $\frac{23-1}{11} = 2$. Since $(11, 23)$ is a solution to the equation, it's on the line!

Sec C

Practice Problems

1 Here is a line.



Select **all** the points that are on the line.

A. $(4, 11)$

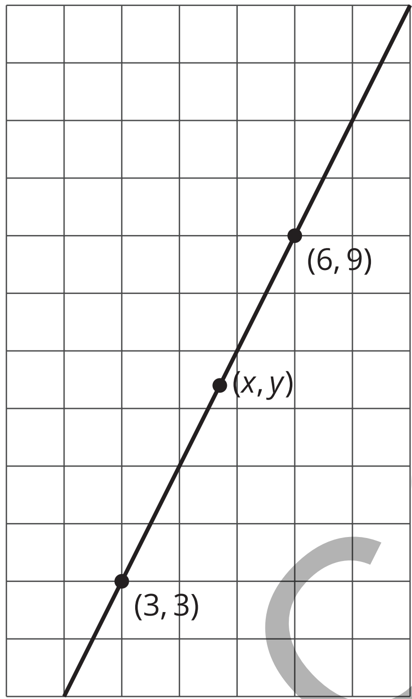
B. $(5, 10)$

C. $(6, 14)$

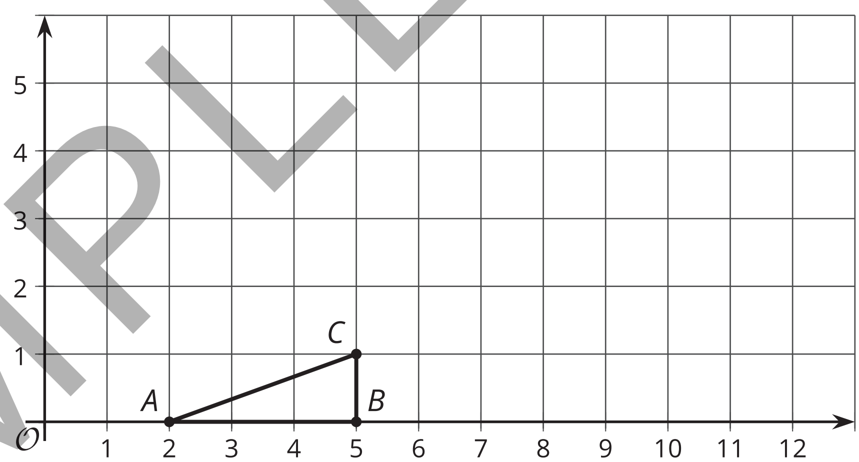
D. $(30, 50)$

E. $(40, 60)$

2 All three points displayed are on the line. Find an equation relating x and y.



3 Here is triangle ABC.



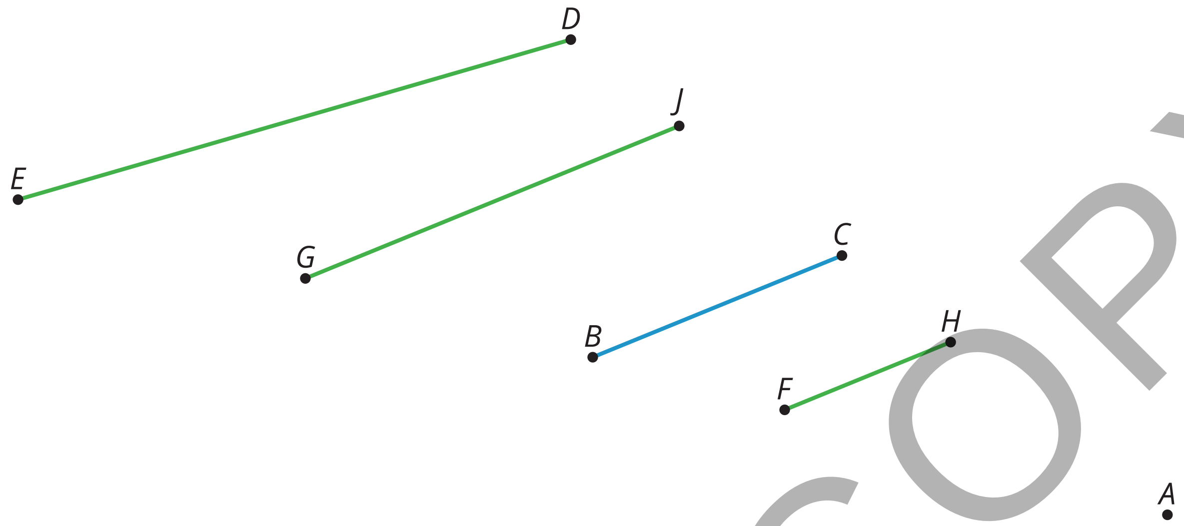
a. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor 3.

b. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor $\frac{1}{2}$.

c. What are the coordinates of the image of point C when triangle ABC is dilated with center $(2, 0)$ and scale factor s?

4 from Unit 2, Lesson 4

Here are some line segments.



a. Which segment is a dilation of segment BC using A as the center of dilation and a scale factor of $\frac{2}{3}$?

b. Which segment is a dilation of segment BC using A as the center of dilation and a scale factor of $\frac{3}{2}$?

c. Which segment is *not* a dilation of segment BC? Explain your reasoning.

Sec C

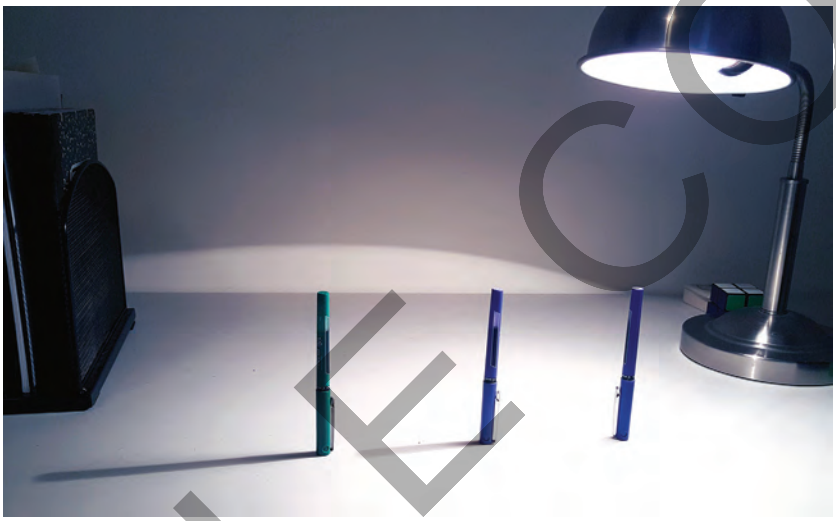
Unit 2, Lesson 13
Addressing CA CCSSM 8.G.5; building on 7.RP.2; building towards 8.G.4, 8.G.5; practicing MP1, MP2, MP3, MP4

The Shadow Knows

Let’s use shadows to find the height of an object.

13.1 Notice and Wonder: Long Shadows and Short Shadows

What do you notice? What do you wonder?



Sec D

13.2 Objects and Shadows

Here are some measurements that were taken when the photo was taken. It was impossible to directly measure the height of the lamppost, so that cell is blank.

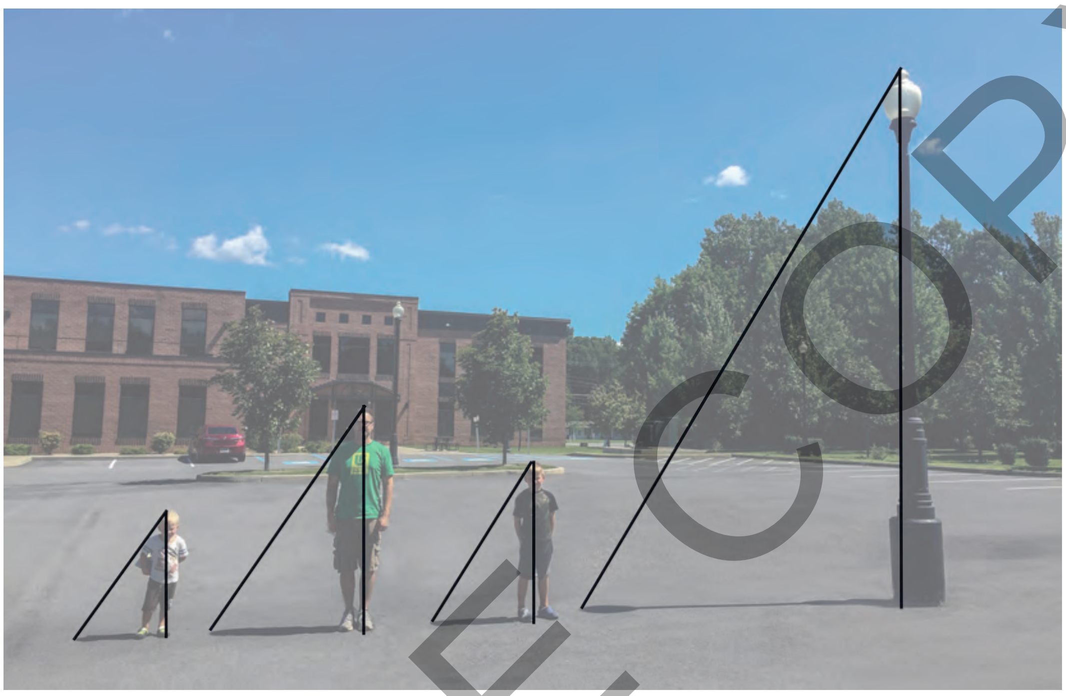
	height (inches)	shadow length (inches)
younger boy	43	29
man	72	48
older boy	51	34
lamppost		114

1. What relationships do you notice between an object's height and the length of its shadow?

2. Make a conjecture about the height of the lamppost and explain your thinking.

13.3 Justifying the Relationship

Explain why the relationship between the height of these objects and the length of their shadows is approximately proportional.



13.4 The Height of a Tall Object

1. Head outside. Make sure that it is a sunny day and you take a measuring device (like a tape measure or meter stick) as well as a pencil and some paper.
2. Choose an object whose height is too large to measure directly. Your teacher may assign you an object.
3. Use what you have learned to figure out the height of the object. Explain or show your reasoning.

Learning Targets

Lesson 1 Projecting and Scaling

- I can decide if one rectangle is a scaled copy of another rectangle.

Lesson 2 Circular Grid

- I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid.

Lesson 3 Dilations with No Grid

- I can apply a dilation to a polygon using a ruler.

Lesson 4 Dilations on a Square Grid

- I can apply dilations to figures on a square grid.

Lesson 5 More Dilations

- I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.

Lesson 6 Similarity

- I can apply a sequence of transformations to one figure to get a similar figure.
- I can use a sequence of transformations to explain why two figures are similar.

Lesson 7 Similar Polygons

- I can use angle measures and side lengths to conclude that two polygons are not similar.
- I know the relationship between angle measures and side lengths in similar polygons.

Lesson 8 Similar Triangles

- I know how to decide if two triangles are similar just by looking at their angle measures.

Lesson 9 Side Length Quotients in Similar Triangles

- I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides.
- I can find missing side lengths in a pair of similar triangles using quotients of side lengths.

Lesson 10 Meet Slope

- I can draw a line on a grid with a given slope.
- I can find the slope of a line on a grid.

Lesson 11 Writing Equations for Lines

- I can decide whether a point is on a line by finding quotients of horizontal and vertical distances.

Lesson 12 Using Equations for Lines

- I can find an equation for a line and use it to decide which points are on that line.

Lesson 13 The Shadow Knows

- I can model a real-world context with similar triangles to find the height of an unknown object.

GRADE 8

UNIT

3

Linear Relationships

Content Connections

In this unit you will use the information you know about rates, proportional relationships, and similarity and slope to interpret nonproportional linear relationships. You will make connections by:

- **Reasoning with Data** while creating graphs, tables, and equations in order to interpret the constant of proportionality in a context.
- **Exploring Changing Quantities** while considering what it means for a pair of values to be a solution to an equation and the correspondence between coordinates of points on a graph and solutions of an equation.

Addressing the Standards

As you work your way through **Unit 3 Linear Relationships,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 3 and 5
MP2 Reason abstractly and quantitatively.	Lessons 1, 4, 5, 6, 7, 8, 9, 12, and 15
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 10, 12, and 14
MP4 Model with mathematics.	Lessons 2, 3, and 8
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 1, 3, 9, 11, and 12
MP7 Look for and make use of structure.	Lessons 2, 6, 8, and 14
MP8 Look for and express regularity in repeated reasoning.	Lessons 7, 10, and 13

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Interpret Scatter Plots • Data, Graphs, and Tables • Data Explorations • Linear Equations • Multiple Representations of Functions • Slopes and Intercepts	**8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	Lessons 2, 3, 4, 5, 6, 7, 8, 9, and 13
• Multiple Representations of Functions	**8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	Lessons 7, 8, 9, 10, 11, 12, and 15

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Linear Equations	**8.EE.8** Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	Lessons 13 and 14
• Linear Equations	**8.EE.8a** Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	Lessons 14 and 15

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

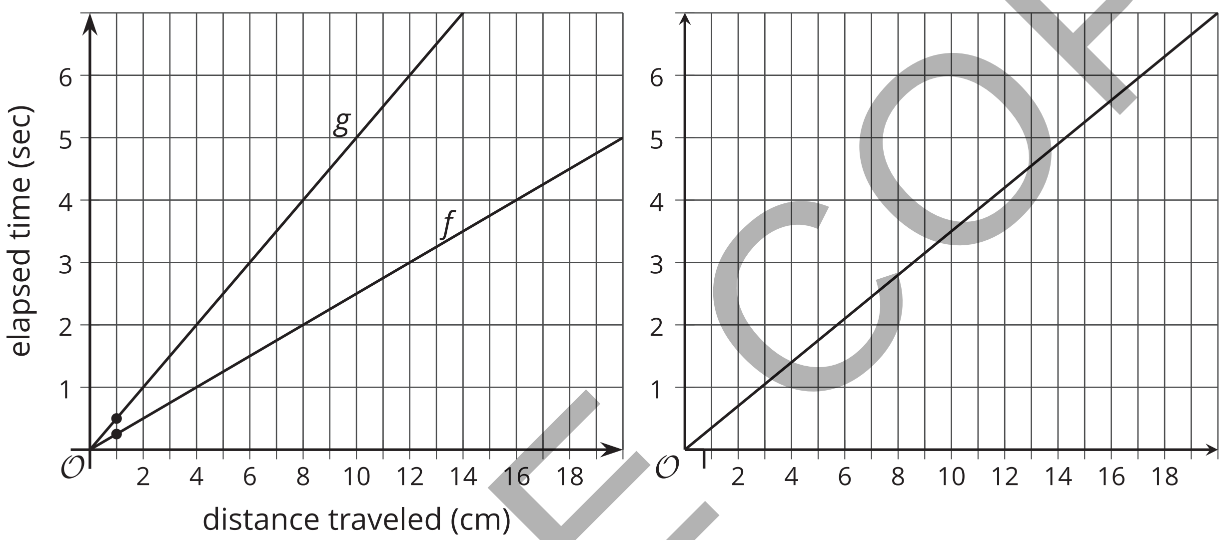
Unit 3, Lesson 1

Building on CA CCSSM 7.RP.2; building towards 8.EE.5; practicing MP2 and MP6

Understanding Proportional Relationships

Let's study some graphs.

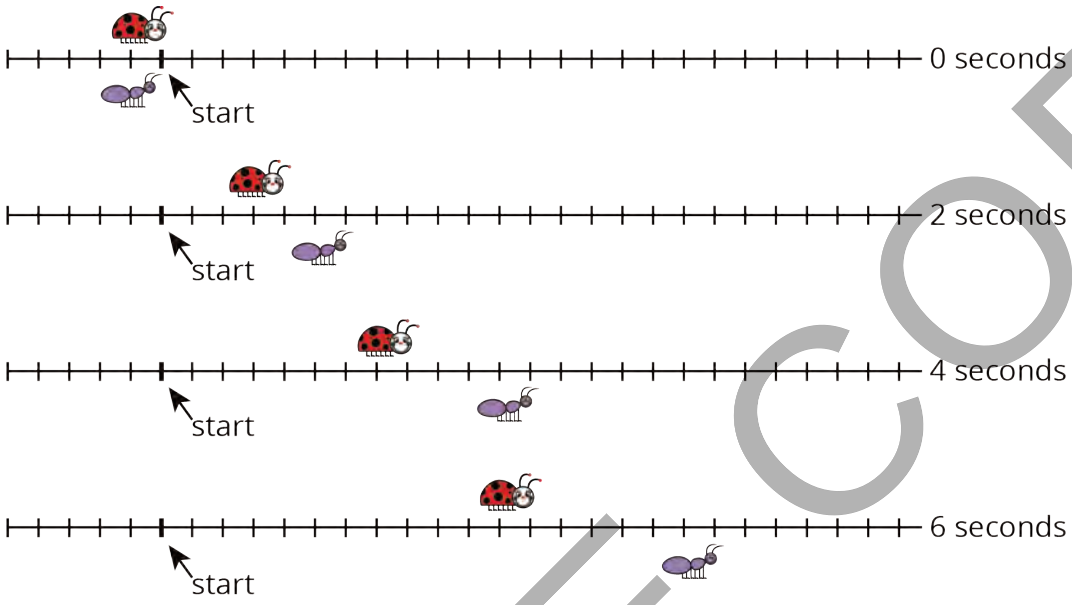
1.1 Notice and Wonder: Two Graphs

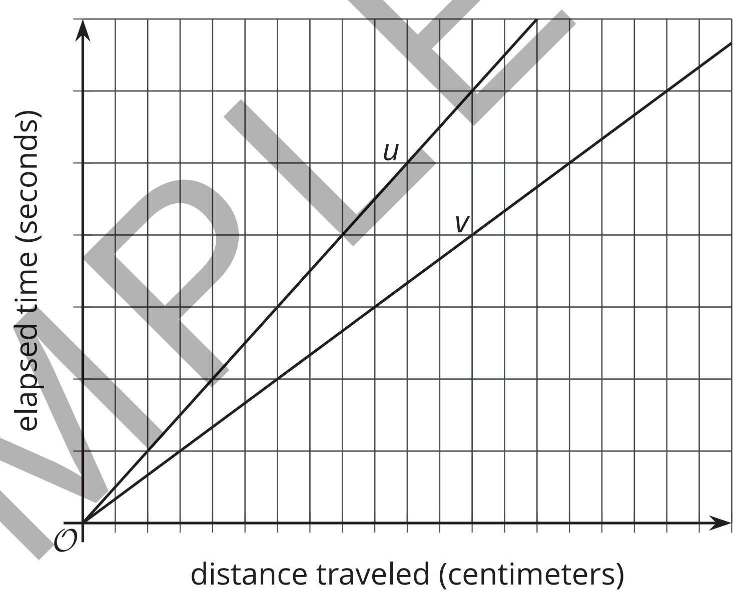


What do you notice? What do you wonder?

1.2 Moving Through Representations

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times, as measured by the front of each bug's head. Each tick mark represents 1 centimeter.





1. Lines u and v also show the positions of the two bugs. Which line shows the ladybug's movement? Which line shows the ant's movement? Explain your reasoning.

2. How long does it take the ladybug to travel 12 centimeters? The ant?

3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time and distance information shown in the tick-mark diagrams.
4. Mark and label the point on line u and the point on line v that represent the time and position of each bug after traveling 1 centimeter.

Are you ready for more?

1. How fast is each bug traveling?

2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning.

1.3 Moving Twice as Fast

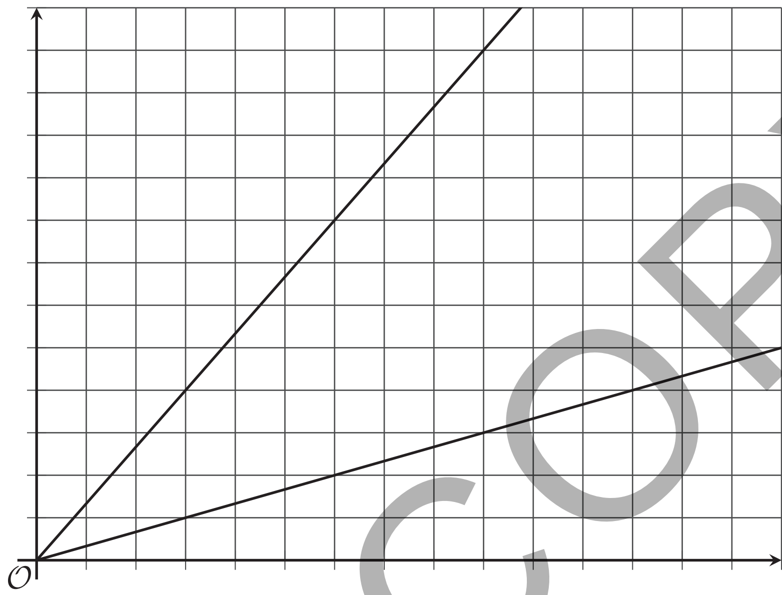
Refer to the tick-mark diagrams and graph in the earlier activity.

1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.
2. Plot this bug's positions on the coordinate axes with lines u and v, and connect them with a line.
3. Write an equation for each of the three lines where x represents the distance traveled by each bug and y represents the elapsed time.

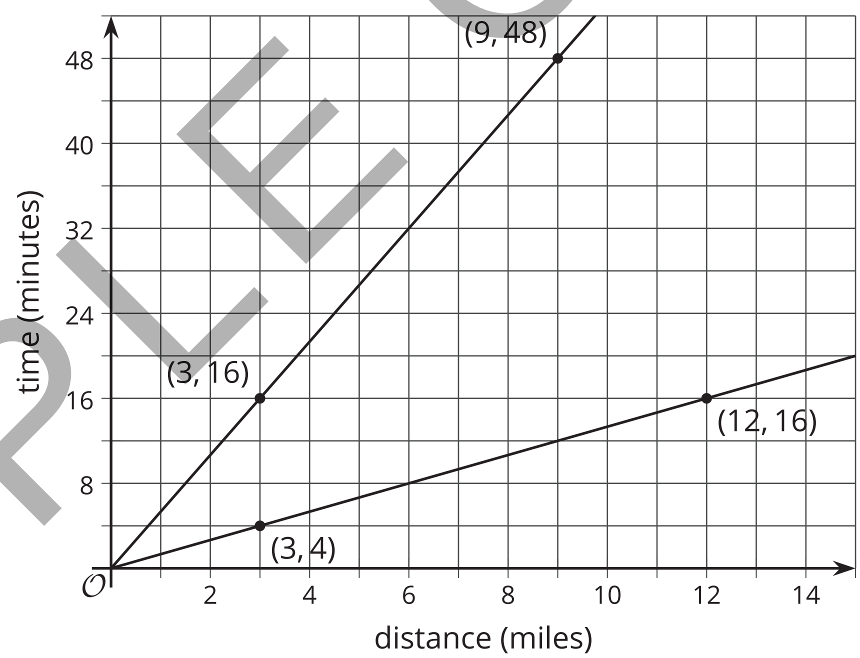
Lesson 1 Summary

Graphing is a way to help make sense of relationships.

But the graph of a line on a coordinate plane without labels or a scale isn't very helpful. Without labels, we can't tell what the graph is about or what units are being used. Without an appropriate scale, we can't tell any specific values.



Here are the same graphs, but now with labels and a scale:

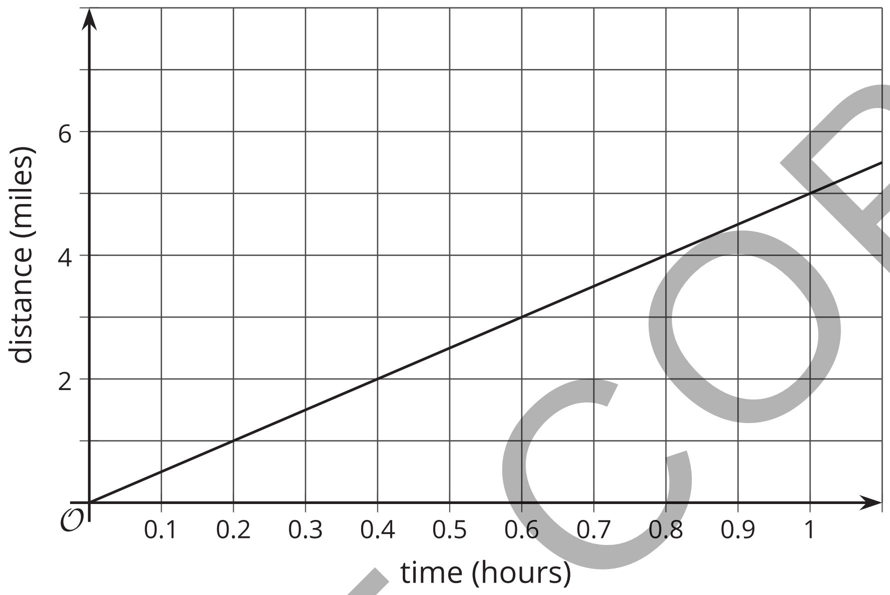


Notice how adding labels lets us know that the relationship compares time and distance and helps to understand both the speed and pace of two different items. When adding labels to axes, be sure to include units, such as minutes and miles.

Notice how adding a scale makes it possible to identify specific points and values. When adding a scale to an axis, be sure that the space between each grid line represents the same amount.

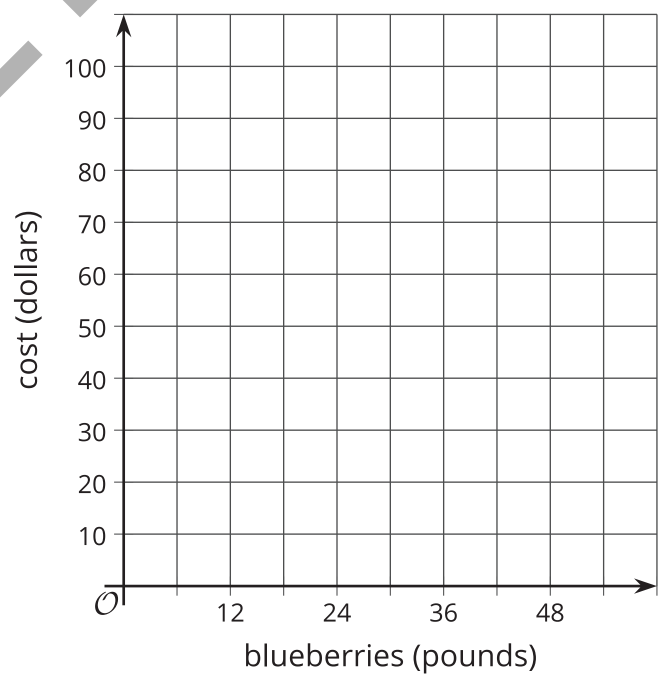
Practice Problems

1 Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between Diego's distance and time.



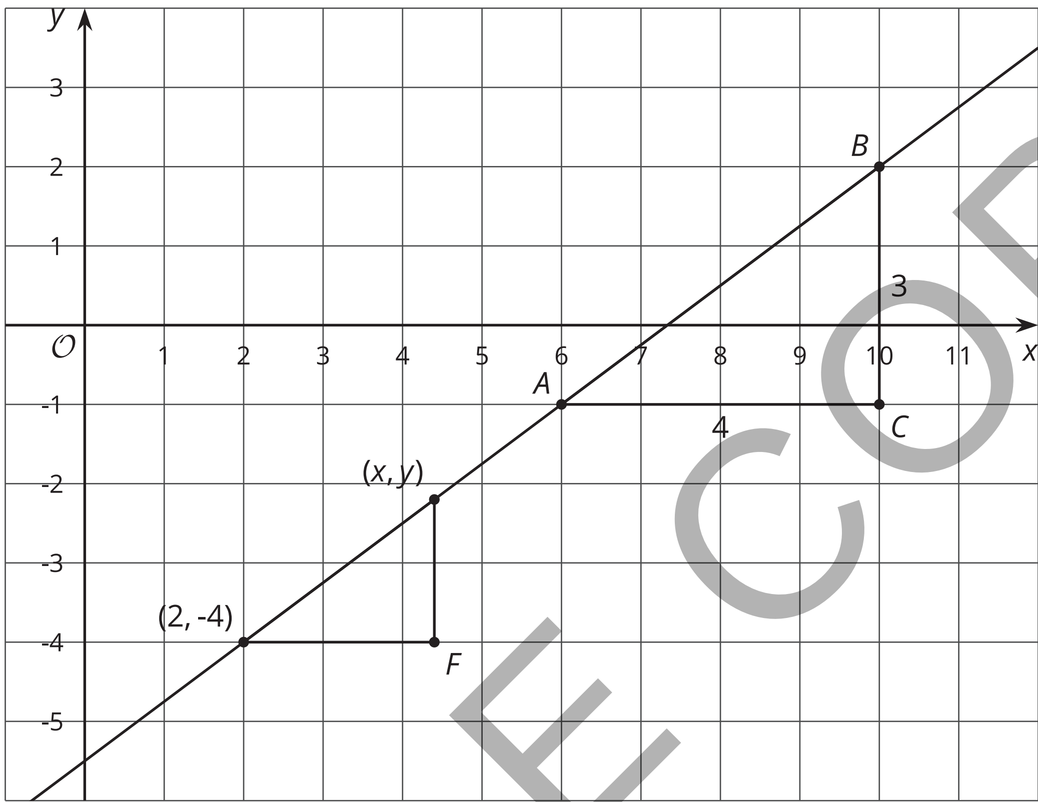
2 A blueberry farm offers 6 pounds of blueberries for $15.00.

Sketch a graph of the relationship between cost in dollars and pounds of blueberries.



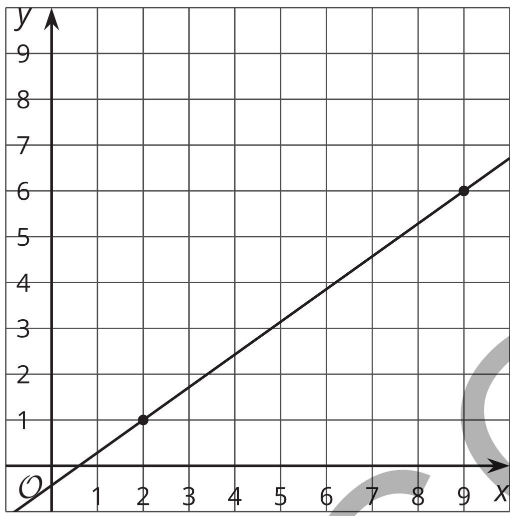
3 from Unit 2, Lesson 11

The points $(2, -4)$, (x, y), A, and B all lie on the line. Write an equation that describes the line.



4 from Unit 2, Lesson 12

The graph shows a line.



Select **all** points that are on this line.

A. $(0, 3)$

B. $(4, 2)$

C. $(30, 21)$

D. $(16, 11)$

E. $(-4, -5)$

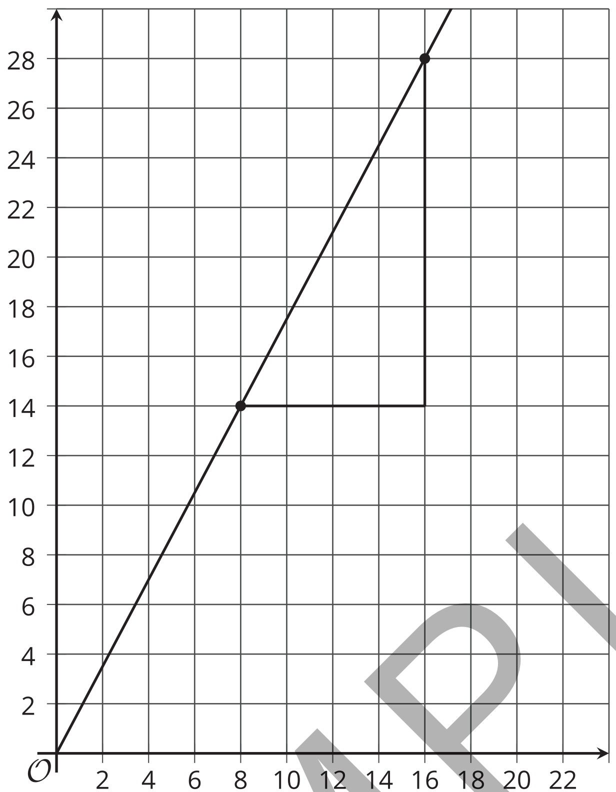
Addressing CA CCSSM 8.EE.5; building on 7.RP.2; building towards 8.EE.5; practicing MP4 and MP7

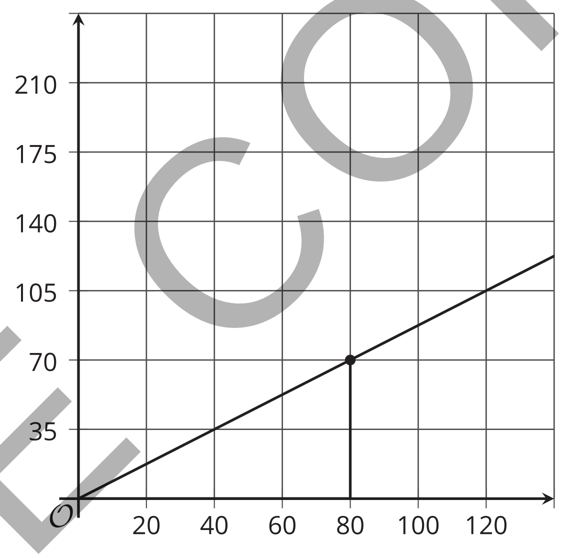
Graphs of Proportional Relationships

Let's think about scale.

2.1 Two Perspectives

Here are two graphs that could represent a variety of different situations.





Andre claims that the line in the graph on the left has a greater slope because it is steeper. Do you agree with Andre? Explain your reasoning.

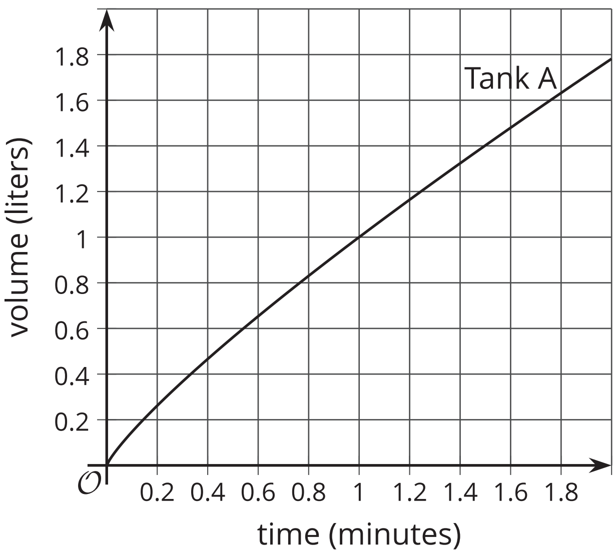
2.2 Card Sort: Proportional Relationships

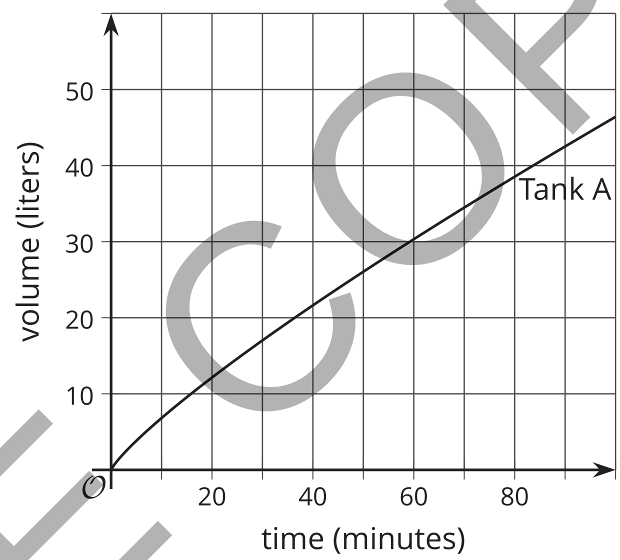
Your teacher will give you a set of cards. Each card contains a graph of a proportional relationship.

1. Sort the graphs into groups based on what proportional relationship they represent.
2. Write an equation for each *different* proportional relationship you find.

2.3 Different Scales

Two large water tanks are filling with water. Tank A is *not* filled at a constant rate, and the relationship between its volume of water and time is graphed on each set of axes. Tank B is filled at a constant rate of $\frac{1}{2}$ liters per minute. The relationship between its volume of water and time can be described by the equation $v = \frac{1}{2}t$, where t is the time in minutes, and v is the total volume in liters of water in the tank.





1. Sketch and label a graph of the relationship between the volume of water v and time t for Tank B on each of the coordinate planes.

2. Answer the following questions and say which graph you used to find your answer.

   a. After 30 seconds, which tank has the most water?

   b. At approximately what times do both tanks have the same amount of water?

   c. At approximately what times do both tanks contain 1 liter of water? 20 liters?

Are you ready for more?

A giant tortoise travels at 0.17 miles per hour and an arctic hare travels at 37 miles per hour.

1. Draw separate graphs that show the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.

2. Would it be helpful to try to put both graphs on the same pair of axes? Why or why not?

3. The tortoise and the hare start out together and after half an hour the hare stops to take a rest. How long does it take the tortoise to catch up?

Sec A

Lesson 2 Summary

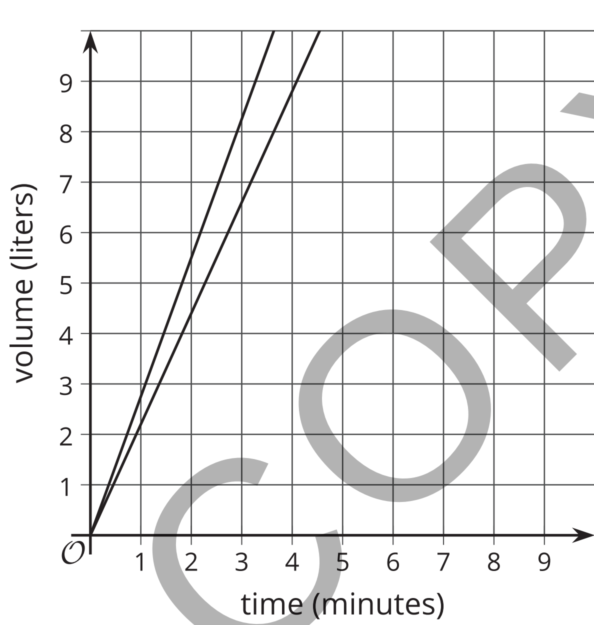
The scales we choose when graphing a relationship often depend on what information we want to know. For example, consider two water tanks filled at different constant rates.

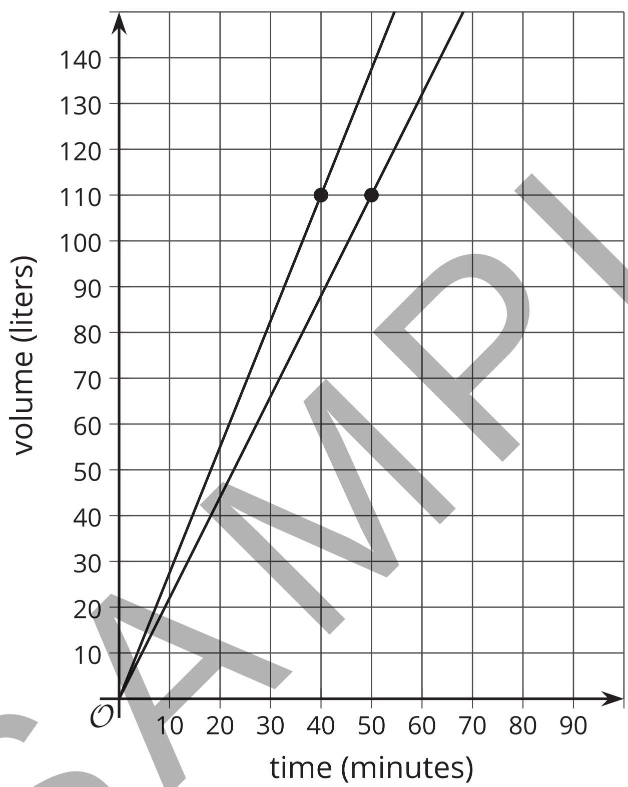
The relationship between time in minutes t and volume in liters v of Tank A can be described by the equation $v = 2.2t$.

For Tank B the relationship can be described by the equation $v = 2.75t$

These equations tell us that Tank A is being filled at a constant rate of 2.2 liters per minute and Tank B is being filled at a constant rate of 2.75 liters per minute.

If we want to use graphs to see at what times the two tanks will have 110 liters of water, then using an axis scale from 0 to 10, as shown here, isn't very helpful.





If we use a vertical scale that goes to 150 liters, a bit beyond the 110 we are looking for, and a horizontal scale that goes to 100 minutes, we get a much more useful set of axes for answering our question.

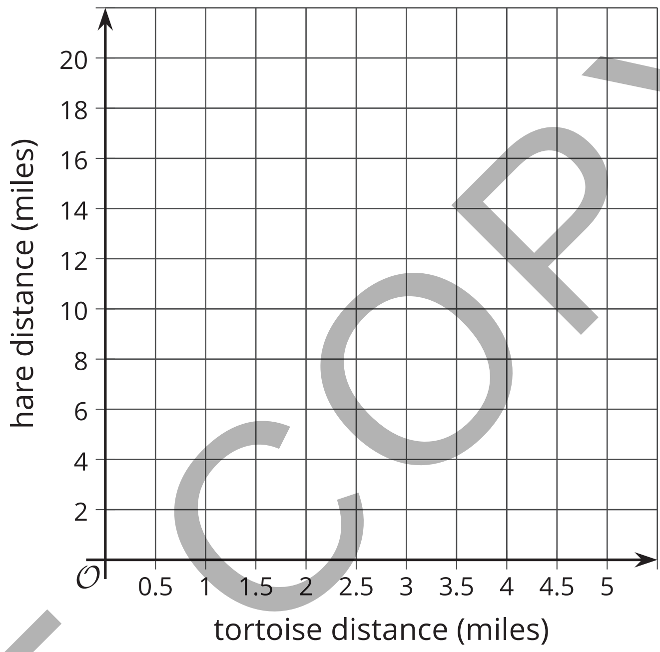
Now we can see that the two tanks will reach 110 liters 10 minutes apart—Tank B after 40 minutes of filling and Tank A after 50 minutes of filling.

It is important to note that both of these graphs are correct, but one uses a range of values that helps answer the question. In order to always pick a helpful scale, we should consider the situation and the questions asked about it.

Practice Problems

1 The tortoise and the hare are having a race. After the hare runs 16 miles the tortoise has only run 4 miles.

This relationship can be described by the equation $y = 4x$, where x is the distance tortoise "runs" in miles, and y is the distance the hare runs in miles. Create a graph of this relationship.



2 The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched.

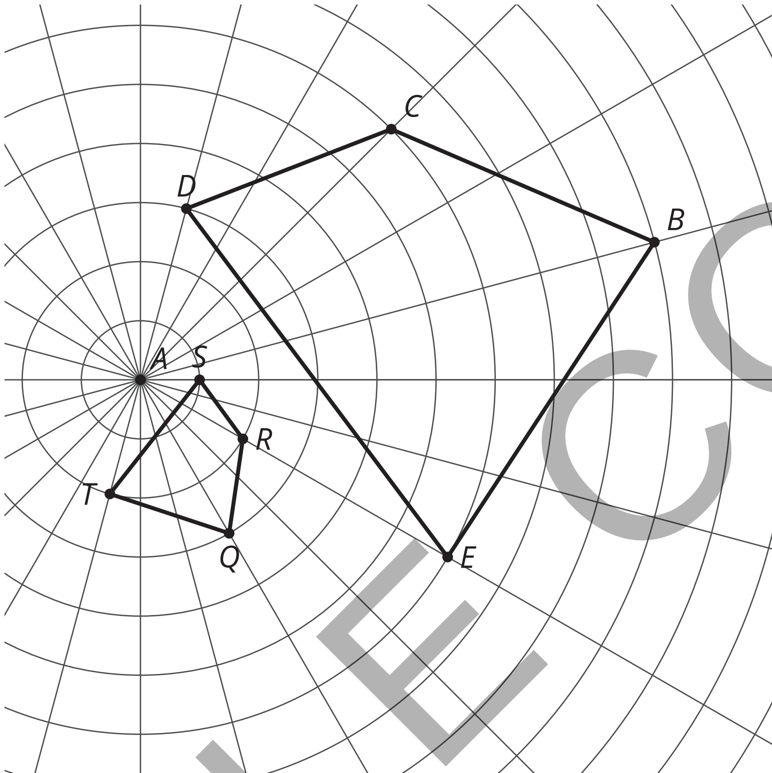
a. Complete the table.

b. Describe the scales you could use on the x- and y-axes of a coordinate plane that would show all the distances and weights in the table.

distance (cm)	weight (newtons)
20	28
55	
	140
1	

3 from Unit 2, Lesson 6

Describe a sequence of rotations, reflections, translations, and dilations that show one figure is similar to the other. Be sure to include the distance and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.



4 from Unit 2, Lesson 6

Andre said, “I found two figures that are congruent, so they can’t be similar.”

Diego said, “No, they are similar! The scale factor is 1.”

Do you agree with either of them? Use the definition of similarity to explain your answer.

Addressing CA CCSSM 8.EE.5; building towards 8.EE.5; practicing MP1, MP4, MP6

Representing Proportional Relationships

Let's graph proportional relationships.

3.1 A Car Wash

Here are two ways to represent a situation.

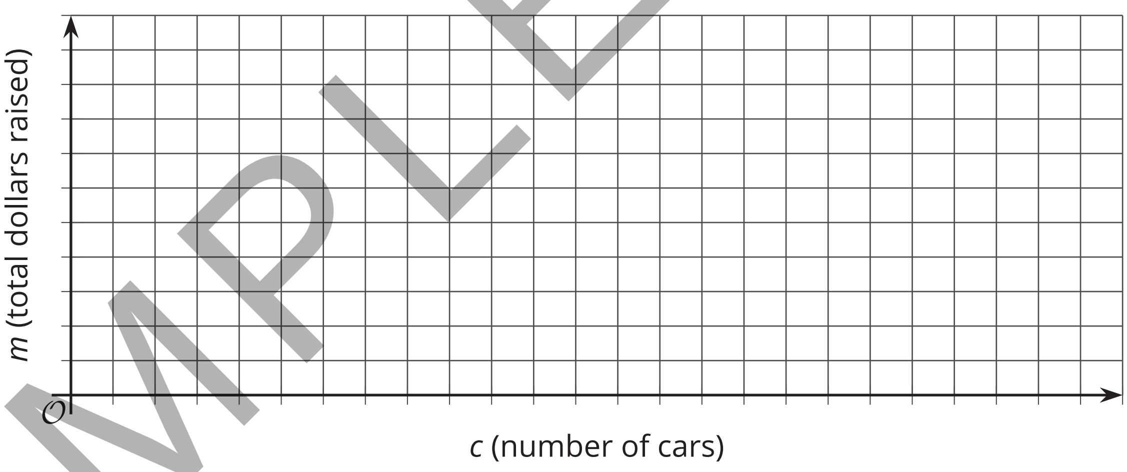
Description:

The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of $93.50. After 23 cars, they raised a total of $195.50.

Table:

number of cars	amount raised in dollars
11	93.50
23	195.50

Create a graph that represents this situation.



3.2 Info Gap: Graphing Proportional Relationships

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Sec A

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ________?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ________ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ________?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Are you ready for more?

Ten people can dig 5 holes in 3 hours. If n people digging at the same rate dig m holes in d hours:

1. Is n proportional to m when $d = 3$?
2. Is n proportional to d when $m = 5$?
3. Is m proportional to d when $n = 10$?

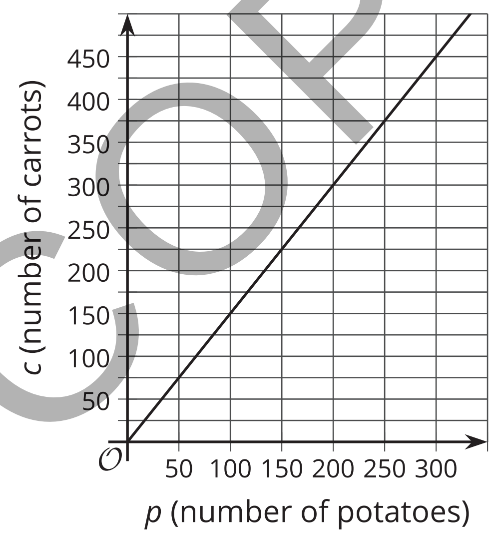
Lesson 3 Summary

Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are p potatoes and c carrots, then $c = \frac{3}{2}p$.

Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots, we could just use the equation: $\frac{3}{2} \cdot 150 = 225$ carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy of a day it is, using up to 300 potatoes at a time.

Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because $450 = \frac{3}{2} \cdot 300$. Then we can read how many carrots are needed for any number of potatoes up to 300.



Or if the recipe is used in a food factory that produces very large quantities and where the potatoes come in bags of 150, we might just make a table of values showing the number of carrots needed for different multiples of 150.

number of potatoes	number of carrots
150	225
300	450
450	675
600	900

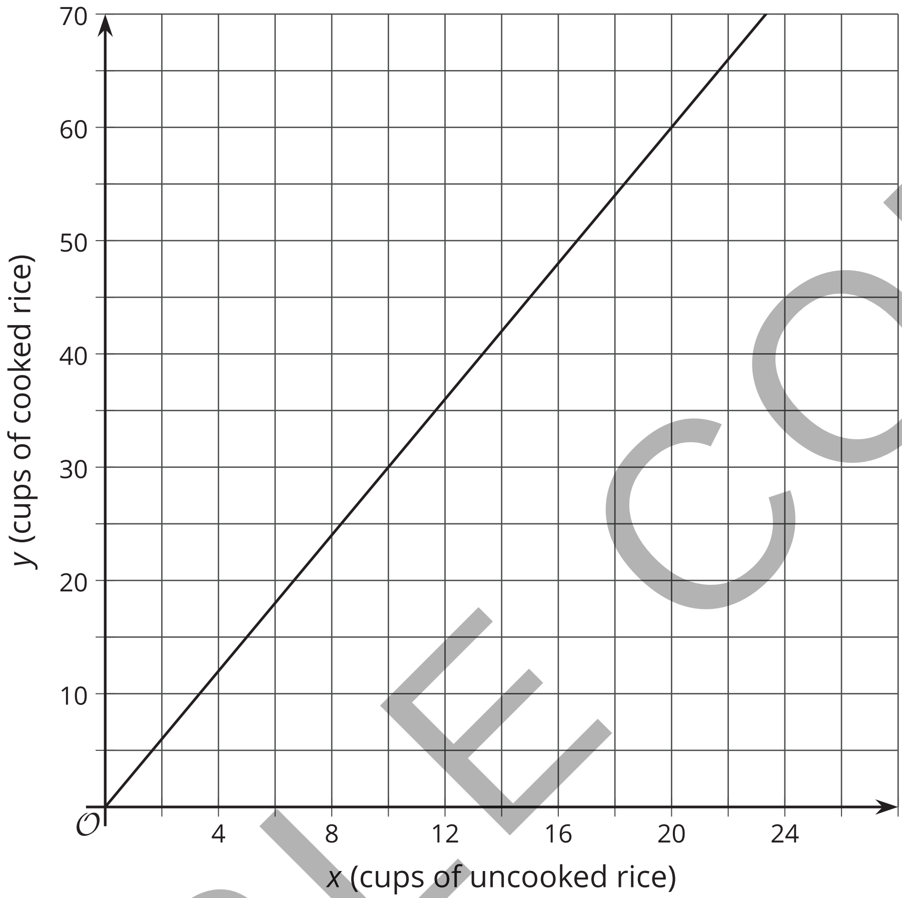
No matter the representation or the scale used, the constant of proportionality, $\frac{3}{2}$, is evident in each. In the equation it is the number we multiply p by. In the graph, it is the slope. In the table, it is the number we multiply values in the left column to get numbers in the right column. We can think of the constant of proportionality as a **rate of change**: the amount one variable changes by when the other variable increases by 1. In this case, the rate of change of c with respect to p is $\frac{3}{2}$ carrots per potato.

Glossary

- rate of change (in a linear relationship)

Practice Problems

1 This graph describes the relationship between the volume of uncooked rice and the volume of the rice after it is cooked.



a. Write an equation that reflects this relationship where x represents the volume in cups of uncooked rice and y represents the volume in cups of the rice after it is cooked.

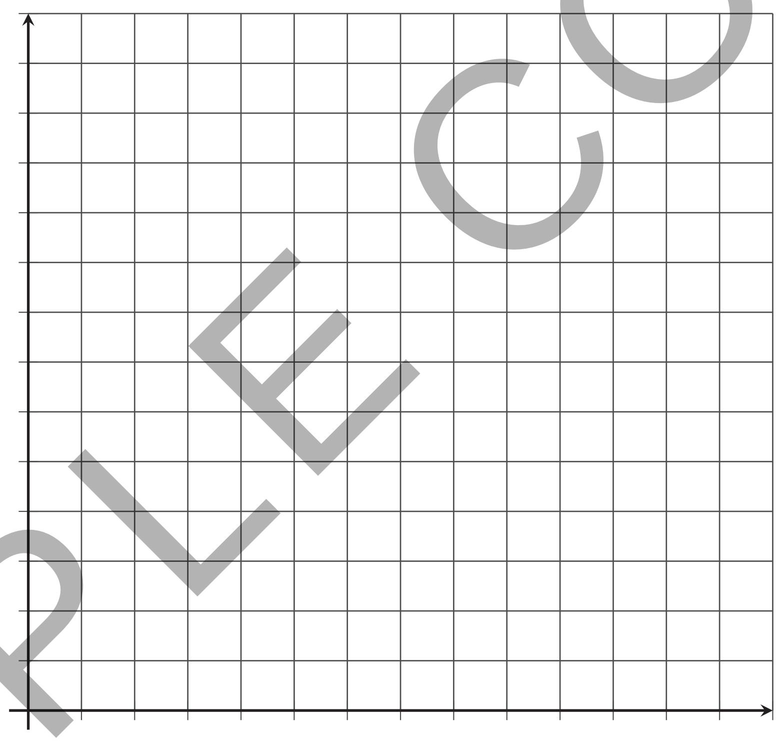
b. Complete the table:

cups of uncooked rice	cups of cooked rice
20	
	120
1	

2 Students are selling raffle tickets for a school fundraiser. They collect \$24 for every 10 raffle tickets they sell.

a. Suppose M is the amount of money the students collect for selling R raffle tickets. Write an equation that reflects the relationship between M and R.

b. Label and scale the axes and graph this situation with M on the vertical axis and R on the horizontal axis. Make sure the scale is large enough to see how much they would raise if they sell 1,000 tickets.



3 from Unit 2, Lesson 12

A line is represented by the equation $\frac{y}{x-2} = \frac{3}{11}$. For each point, determine if it is on this line.

a. $(3, 13)$

b. $(13, 3)$

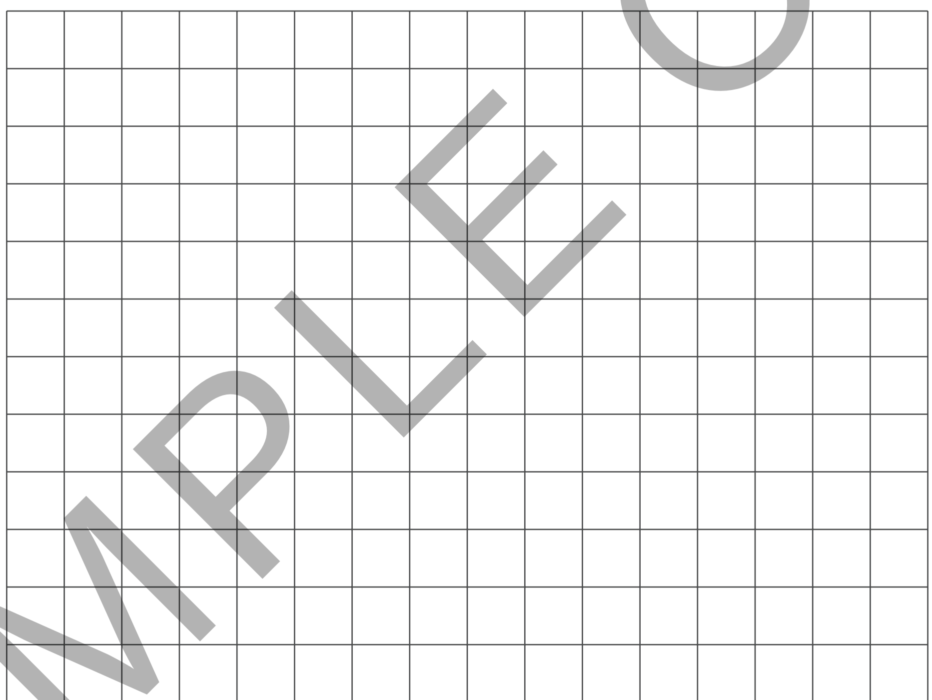
c. $(35, 9)$

d. $(40, 10)$

e. $(46, 12)$

4 from Unit 2, Lesson 10

Use a straightedge to draw two lines: one with slope $\frac{3}{7}$ and one with slope $\frac{7}{3}$.



Addressing CA CCSSM 8.EE.5; practicing MP2

Comparing Proportional Relationships

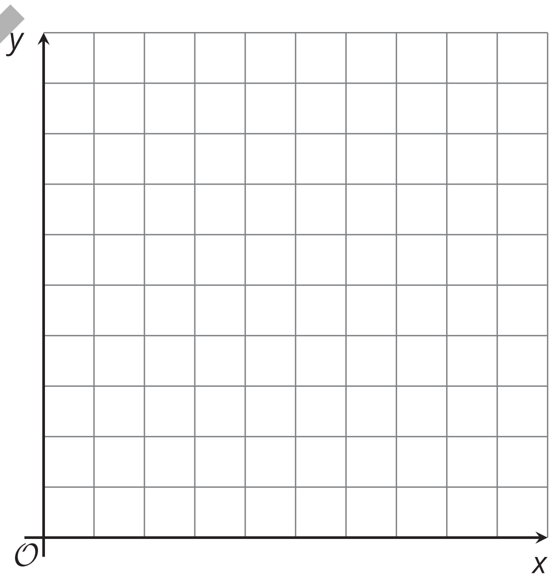
Let's compare proportional relationships.

4.1 What's the Relationship?

The equation $y = 4.2x$ could represent a variety of different situations.

1. Describe a situation that can be represented by this equation. What do the quantities x and y represent in your situation?

2. Create a table and a graph that represent the situation.



4.2 Comparing Two Different Representations

1. Elena babysits her neighbor's children. Her earnings are given by the equation $y = 8.40x$, where x represents the number of hours she worked, and y represents the amount of money she earned in dollars.

   Jada earns $7 per hour mowing her neighbors' lawns.

   a. Who makes more money after working 12 hours? How much more do they make? Explain your reasoning by creating a graph or a table.

   b. What is the value of the **rate of change** for each situation, and what does each value mean?

   c. Using your graph or table, determine how long it would take each person to earn $150.

2. Clare and Han have summer jobs stuffing envelopes for two different companies.

Han earns $15 for every 300 envelopes he finishes.

Clare's earnings can be seen in the table.

number of envelopes	**money earned in dollars**
400	40
900	90

a. By creating a graph, show how much money each person makes after stuffing 1,500 envelopes.

b. What is the value of the rate of change for each situation, and what does each value mean?

c. Using your graph, determine how much more money one person makes relative to the other after stuffing 1,500 envelopes. Explain or show your reasoning.

Sec A

3. Tyler plans to start a lemonade stand and is trying to perfect his recipe for lemonade. He wants to make sure the recipe doesn't use too much lemonade mix (lemon juice and sugar) but still tastes good.

Recipe 1 is given by the equation $y = 4x$, where x represents the amount of lemonade mix in cups, and y represents the amount of water in cups.

Recipe 2 is given in the table.

lemonade mix (cups)	water (cups)
10	50
13	65
21	105

a. If Tyler had 16 cups of lemonade mix, how many cups of water would he need for each recipe? Explain your reasoning by creating a graph or a table.

b. What is the value of the rate of change for each recipe, and what does each value mean?

c. Tyler has 16 cups of lemonade mix to use for his lemonade stand. Which lemonade recipe should he use? Explain or show your reasoning.

Are you ready for more?

Han and Clare are still stuffing envelopes. Han can stuff 20 envelopes in a minute, and Clare can stuff 10 envelopes in a minute. They start working together on a pile of 1,000 envelopes.

1. How long does it take them to finish the pile?

2. Who earns more money?

Lesson 4 Summary

When two proportional relationships are represented in different ways, we can compare them by finding a common piece of information.

For example, Clare's earnings are represented by the equation $y = 14.50x$, where y is her earnings in dollars for working x hours.

The table shows some information about Jada's earnings.

time worked (hours)	earnings (dollars)
7	92.75
4.5	59.63
37	490.25

If we want to know who makes more per hour, we can look at the rate of change for each situation.

In Clare's equation, we see that the rate of change is 14.50. This tells us that she earns \$14.50 per hour. For Jada, we can calculate the rate of change by dividing her earnings in one row by the hours worked in the same row. For example, using the last row, the rate of change is 13.25 since $490.25 \div 37 = 13.25$. This tells us that Clare earns 1.25 more dollars per hour than Jada.

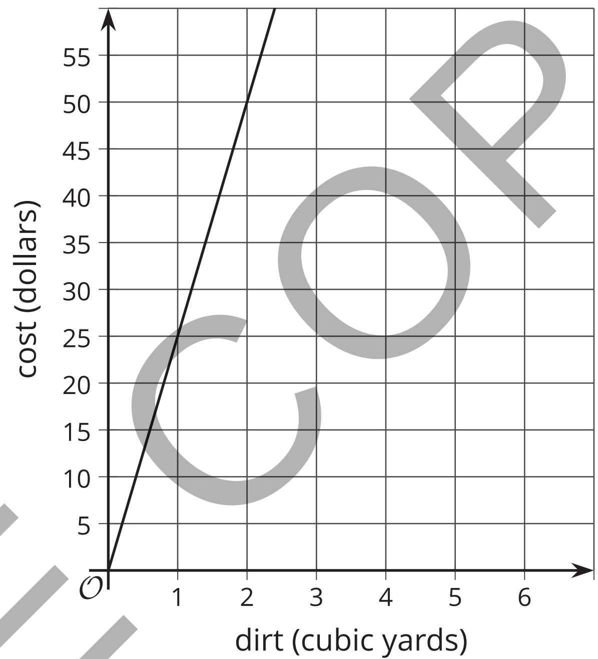
Practice Problems

1 A contractor must haul a large amount of dirt to a worksite. She collected information from two hauling companies.

EZ Excavation gives its prices in a table.

dirt (cubic yards)	cost (dollars)
8	196
20	490
26	637

Happy Hauling Service gives its prices in a graph.



a. How much would each hauling company charge to haul 40 cubic yards of dirt? Explain or show your reasoning.

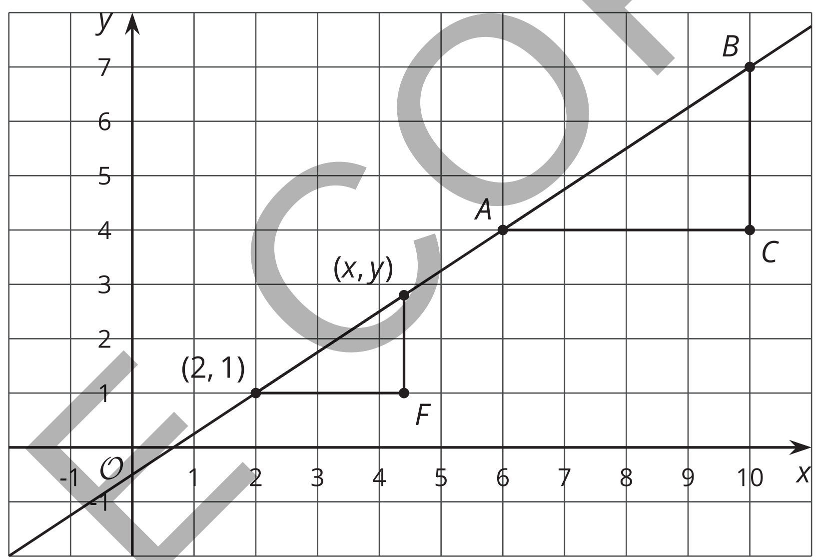
b. Calculate the rate of change for each relationship. What do they mean for each company?

c. If the contractor has 40 cubic yards of dirt to haul and a budget of $1,000, which hauling company should she hire? Explain or show your reasoning.

2 Andre and Priya are tracking the number of steps they walk. Andre records that he can walk 6,000 steps in 50 minutes. Priya writes the equation $y = 118x$, where y is the number of steps, and x is the number of minutes she walks, to describe her step rate. This week, Andre and Priya each walk for a total of 5 hours. Who walks more steps? How many more?

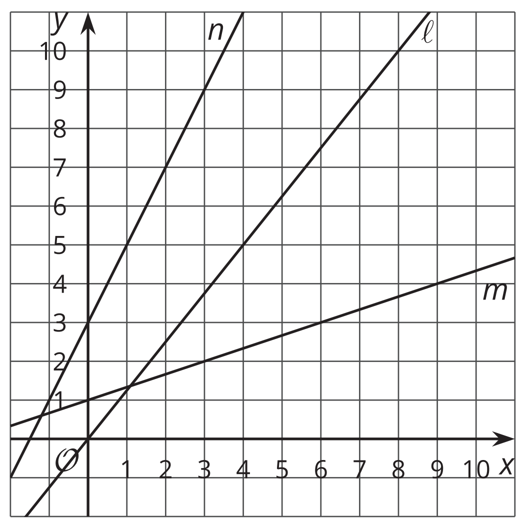
3 from Unit 2, Lesson 11

The points $(2, 1)$, (x, y), A, and B all lie on the displayed line. Find an equation for the line.



4 from Unit 2, Lesson 11

Calculate the slope of each line.



Unit 3, Lesson 5

Addressing CA CCSSM 8.EE.5; building on 7.RP.2a; building towards 8.EE.5; practicing MP1 and MP2

Introduction to Linear Relationships

Let’s explore some relationships between two variables.

5.1 Stacks of Cups

5.2 Stacking Cups

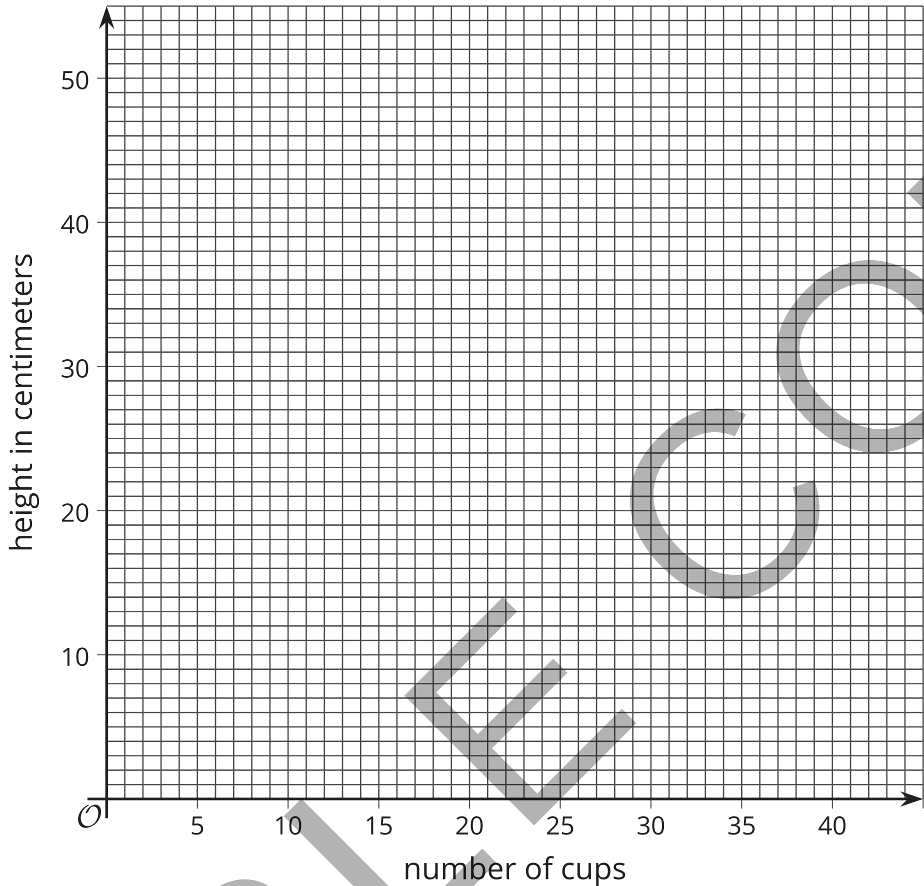
Here is information about the two stacks of styrofoam cups in the photo.

- One stack has 6 cups, and its height is 15 cm.
- The other stack has 12 cups, and its height is 23 cm.

How many cups are needed for a stack with a height of 50 cm?

5.3 Connecting Slope to Rate of Change

1. If you didn't create your own graph of the situation before, do so now.



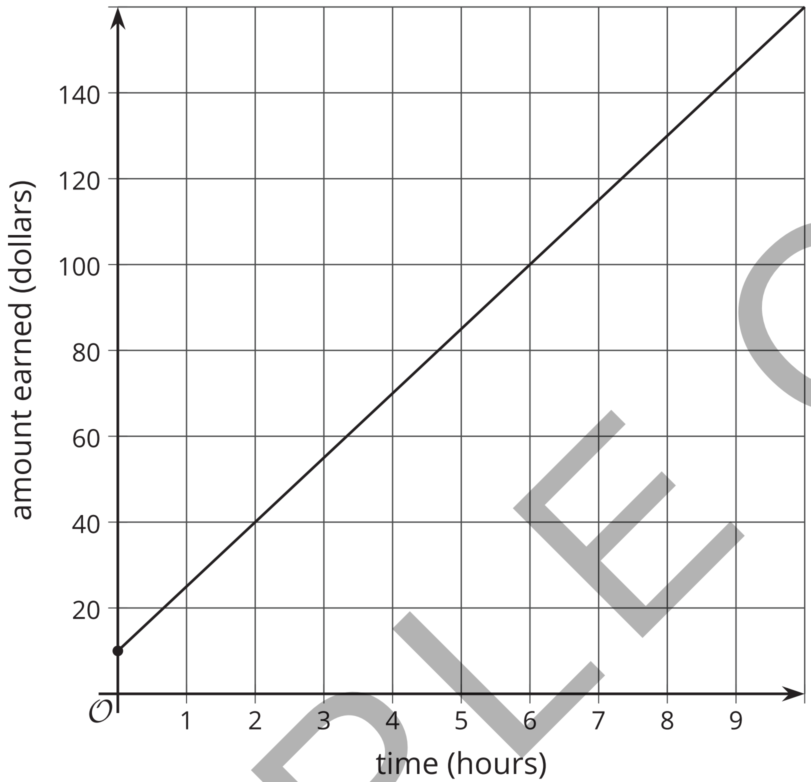
2. What are some ways to tell that the number of cups is not proportional to the height of the stack?

3. What is the **slope** of the line in your graph? What does the slope mean in this situation?

4. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the cups?

Lesson 5 Summary

A **linear relationship** is any relationship between two quantities where one quantity has a constant rate of change with respect to the other. For example, Andre babysits and charges a fee for traveling to and from the job, and then a set amount for every additional hour he works. Since the total amount he charges with respect to the number of hours he works changes at a constant rate, this is a linear relationship. But since Andre charges a fee for traveling, and the graph does not go through the point $(0, 0)$, this is not a proportional relationship. Here is a graph of how much Andre charges based on how many hours he works.



The rate of change can be calculated using the graph. Since the rate of change is constant, we can take any two points on the graph and divide the amount of vertical change by the amount of horizontal change. For example, the points $(2, 40)$ and $(6, 100)$ mean that Andre earns 40 dollars for working 2 hours and 100 dollars for working 6 hours. The rate of change is $\frac{100-40}{6-2} = 15$ dollars per hour. Andre's earnings go up 15 dollars for each hour of babysitting.

Notice that this is the same way we calculate the slope of the line. That's why the graph is a line and why we call this a "linear relationship." The **rate of change** of a linear relationship is the same as the slope of its graph.

Glossary

- linear relationship
- rate of change (in a linear relationship)

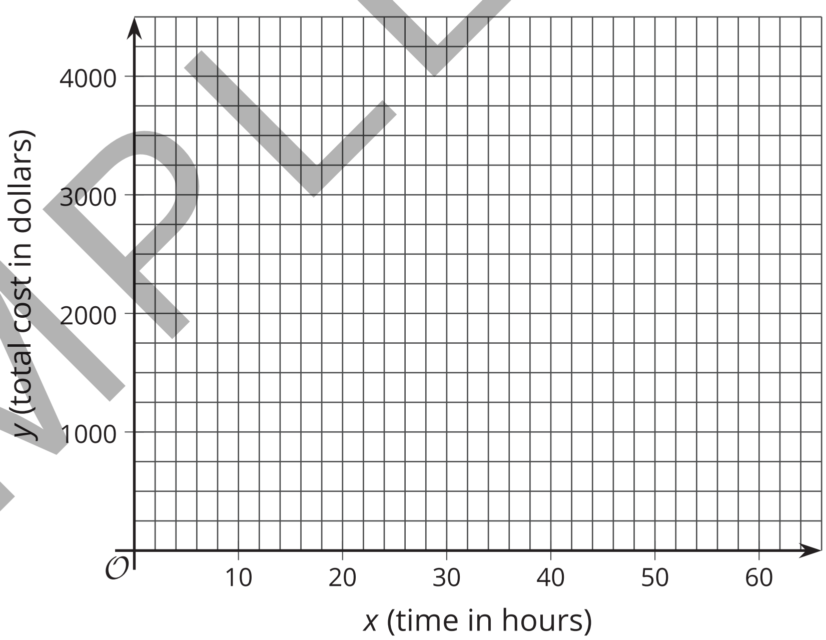
Practice Problems

1 A restaurant offers delivery for their pizzas for a fee added to the price of the pizzas. One customer pays \$25 to have 2 pizzas delivered. Another customer pays \$58 to have 5 pizzas delivered. How many pizzas are delivered to a customer who paid \$80?

2 To paint a house, a painting company charges a flat rate of \$500 for supplies, plus \$50 for each hour of labor.

a. How much would the painting company charge to paint a house that needs 20 hours of labor? A house that needs 50 hours?

b. Draw a line representing the relationship between x, the number of hours it takes the painting company to finish the house, and y, the total cost of painting the house. Label the two points from the earlier question on your graph.



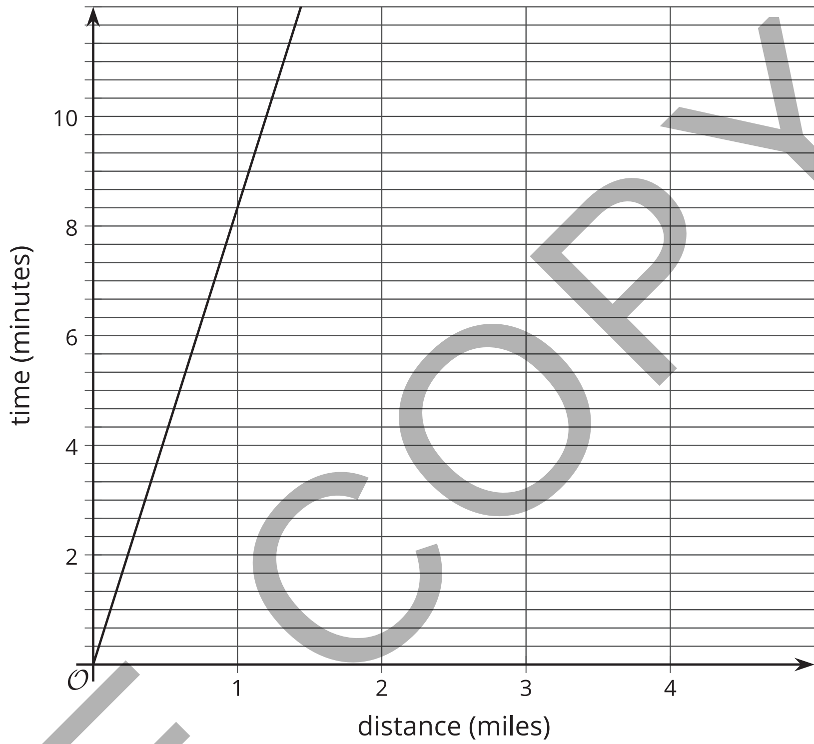
c. Find the slope of the line. What is the meaning of the slope in this context?

3

from Unit 3, Lesson 4

Tyler and Elena are on the cross country team.

Tyler's distances and times for a training run are shown on the graph.



Elena’s distances and times for a training run are given by the equation $y = 8.5x$, where x represents distance in miles, and y represents time in minutes.

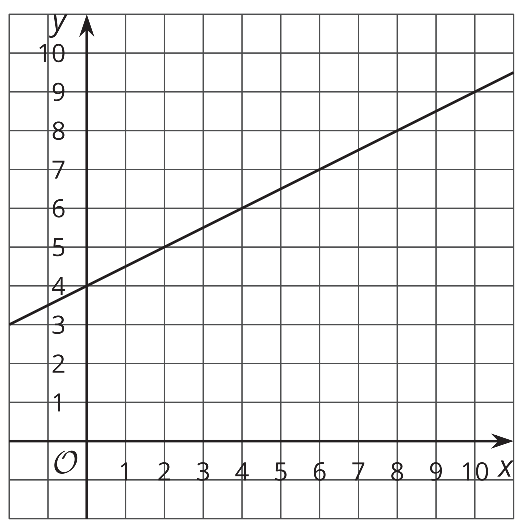
a. Calculate each runner's pace in minutes per mile.

b. Who ran faster during the training run? Explain or show your reasoning.

4

from Unit 2, Lesson 12

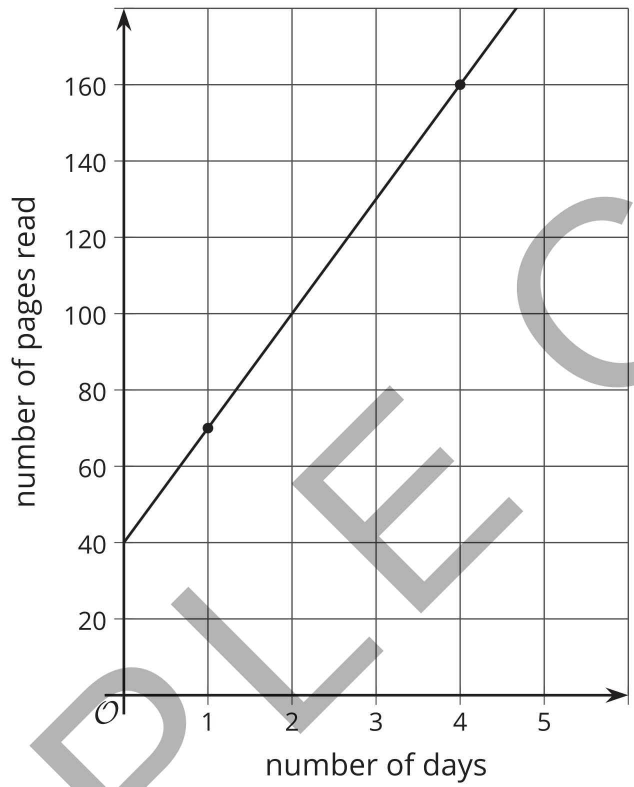
Write an equation for the line.



More Linear Relationships

Let's explore some more relationships between two variables.

6.1 Notice and Wonder: Daily Reading

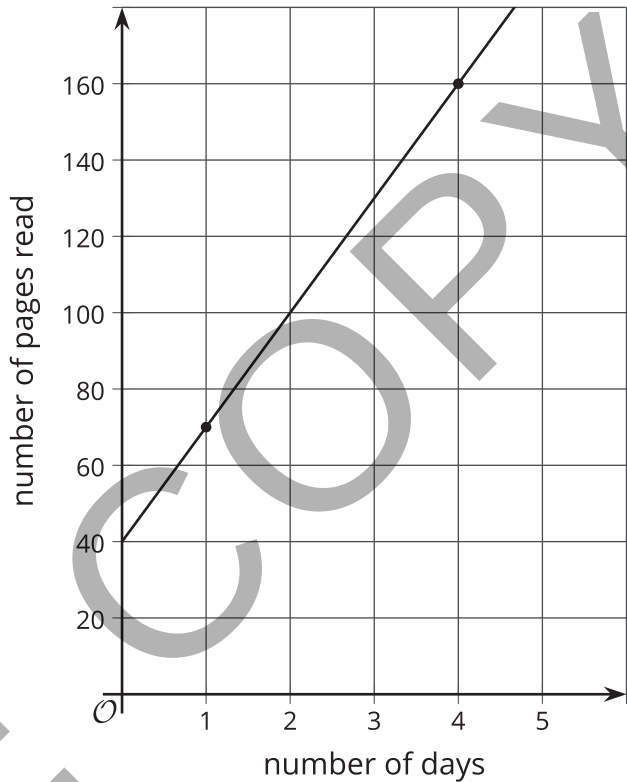


What do you notice? What do you wonder?

6.2 Summer Reading

Lin has a summer reading assignment. After reading the first 30 pages of the book, she plans to read 40 pages each day until she finishes. Lin makes the graph shown here to track how many total pages she'll read over the next few days.

After day 1, Lin reaches page 70, which matches the point $(1, 70)$ she made on her graph. After day 4, Lin reaches page 190, which does not match the point $(4, 160)$ she made on her graph. Lin is not sure what went wrong since she knows she followed her reading plan. Why doesn't Lin's reading progress match her graph?



6.3 Card Sort: Slopes, Vertical Intercepts, and Graphs

Your teacher will give you a set of cards containing descriptions of situations and graphs. Match each situation with a graph that represents it. Record your matches and be prepared to explain your reasoning.

Are you ready for more?

A savings account was opened in 2010. The table shows the amount in the account each year.

If this relationship is graphed with the year on the horizontal axis and the amount in dollars on the vertical axis, what is the **vertical intercept**? What does it mean in this context?

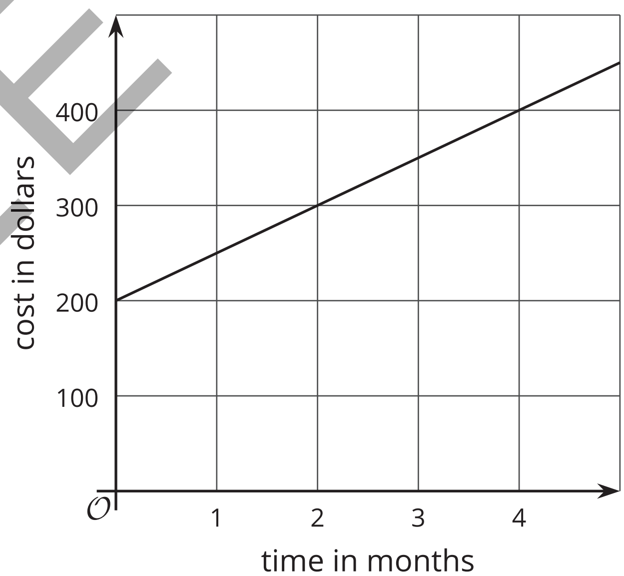
year	amount in dollars
2010	600
2012	750
2014	900
2016	1050

Sec B

Lesson 6 Summary

Lines drawn on a coordinate plane have a slope and a vertical intercept. The **vertical intercept** indicates where the graph of the line meets the vertical axis. Since the vertical axis is often referred to as the y-axis, the vertical intercept is often called the "y-intercept." A line represents a proportional relationship when the vertical intercept is 0.

Here is a graph of a line showing the amount of money paid for a new cell phone and monthly plan.



The vertical intercept for the graph is at the point $(0, 200)$ and means the initial cost for the phone was \$200.

A slope triangle connecting the two points $(0, 200)$ and $(2, 300)$ can be used to calculate the slope of this line. The slope of 50 means that the phone service costs \$50 per month in addition to the initial \$200 for the phone.

Glossary

- vertical intercept

Practice Problems

1 Explain what the slope and y-intercept mean in each situation.

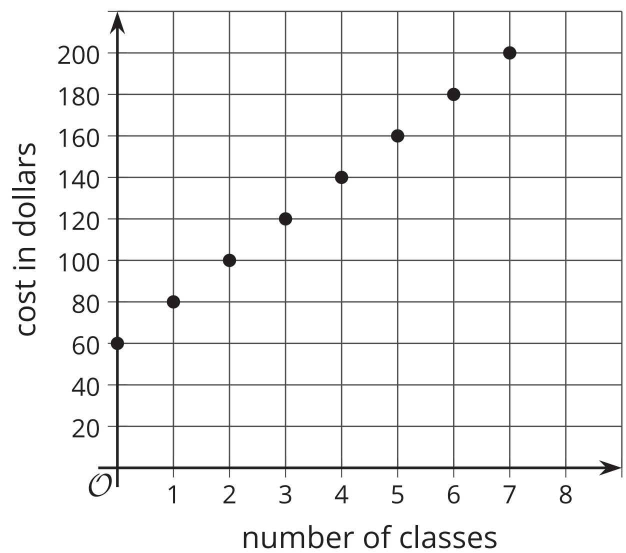
a. A graph represents the perimeter, y, in units, for an equilateral triangle with side length x units. The slope of the line is 3 and the y-intercept is 0.

b. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$ and the y-intercept is 8.

c. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$ and the y-intercept is 2.

d. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2 and the y-intercept is 3.

2 Customers at the gym pay a membership fee to join and then a fee for each class they attend. Here is a graph that represents the situation.



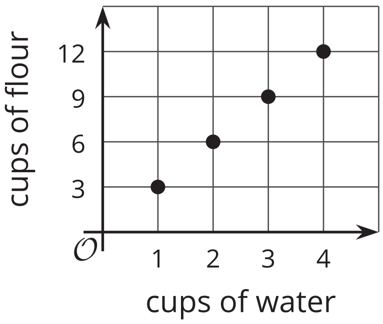
a. What does the slope of the line shown by the points mean in this situation?

b. What does the vertical intercept mean in this situation?

3 from Unit 3, Lesson 4

The graph shows the relationship between the number of cups of flour and the number of cups of water in a recipe for making tortillas.

Recipe 1:



The table shows the amounts of flour and water needed for a different tortilla recipe.

Recipe 2:

flour (cups)	water (cups)
$2\frac{1}{2}$	1
5	2
$7\frac{1}{2}$	3

a. How much flour is needed for each recipe if 6 cups of water are used?

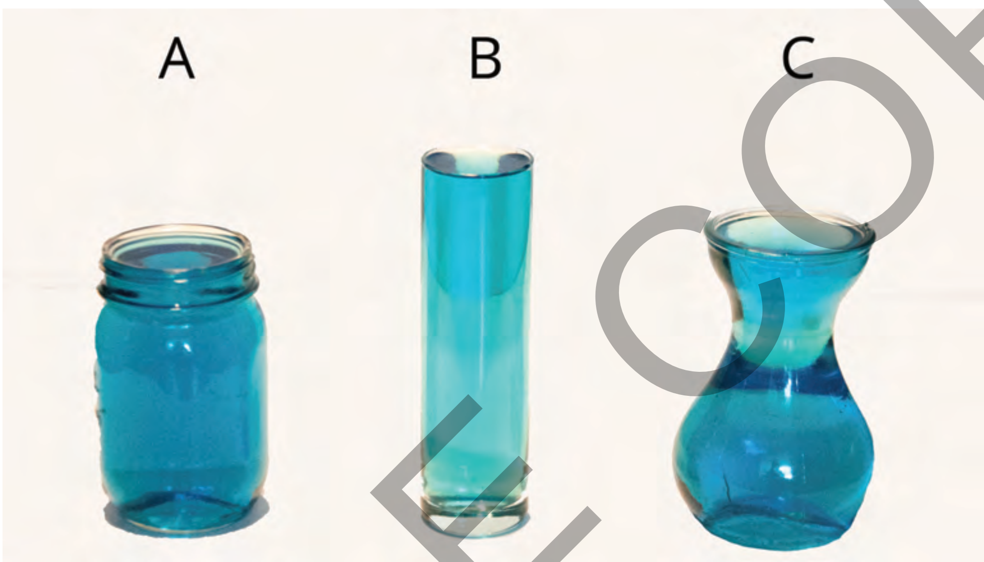
b. How much water is needed for each recipe if 5 cups of flour are used?

Sec B

Unit 3, Lesson 7
Addressing CA CCSSM 8.EE.5, 8.EE.6; building on 5.MD.3, 7.RP.2b; practicing MP2 and MP8
Representations of Linear Relationships

Let's write equations from real situations.

7.1 Which Holds More?



Which container holds the most liquid? The least?

7.2 Rising Water Levels

1. Record the data in the table. (You may not need all the rows.)

number of objects	volume in ml

2. What is the volume, V, in the cylinder after you add x objects? Explain your reasoning.

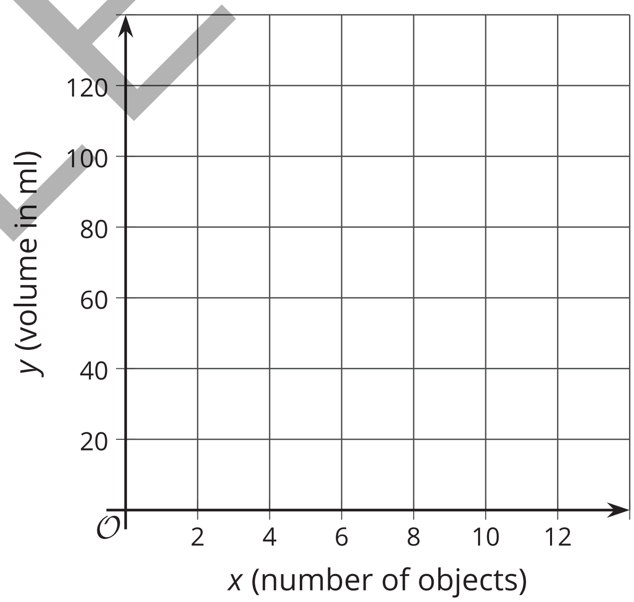
3. If you wanted to make the water reach the highest mark on the cylinder, how many objects would you need?

4. Plot and label points that show your measurements from the experiment.

5. Plot and label a point that shows the volume of the water before you added any objects.

6. The points should fall on a line. Use a straightedge to graph this line.

7. Calculate the slope of the line. What does the slope mean in this situation?



8. What is the vertical intercept? What does the vertical intercept mean in this situation?

7.3 Telling Stories

1. Consider a new cylinder that is filled with 25 ml of water. Identical beads that have a volume of 0.75 ml are dropped into the cylinder one at a time.

   a. Write an equation that describes the volume in the cylinder as beads are added. Use V for the total volume in the cylinder and b for the number of beads.

   b. How would your original equation change if the cylinder was only filled with 12 ml of water?

   c. How would your original equation change if larger beads that had a volume of 1.25 ml were used?

2. A situation is represented by the equation $y = 5 + \frac{1}{2}x$.

   a. Create a story for this situation.

   b. What does the 5 represent in your situation?

   c. What does the $\frac{1}{2}$ represent in your situation?

Lesson 7 Summary

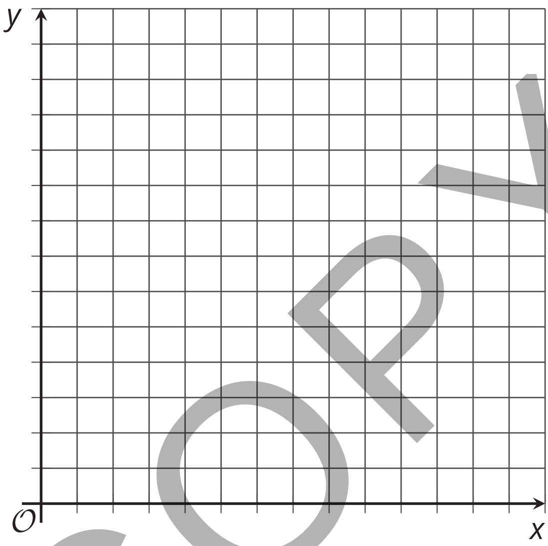
A glass cylinder is filled with 50 ml of water. Marbles, each with a volume of 3 ml, are dropped into the cylinder one at a time. With each marble, the water level increases in height by an amount equivalent to a volume of 3 ml. This constant rate of change means there is a linear relationship between the number of marbles and the total volume in the cylinder. If 1 marble is added, the total volume increases by 3 ml. If 2 marbles are added, the total volume increases by 6 ml. If x marbles are added, the total volume goes up $3x$ ml.

This means that the total volume, y, for x marbles is $y = 3x + 50$. The 3 represents the rate of change, or slope of the graph, and the 50 represents the initial amount, or vertical intercept of the graph.

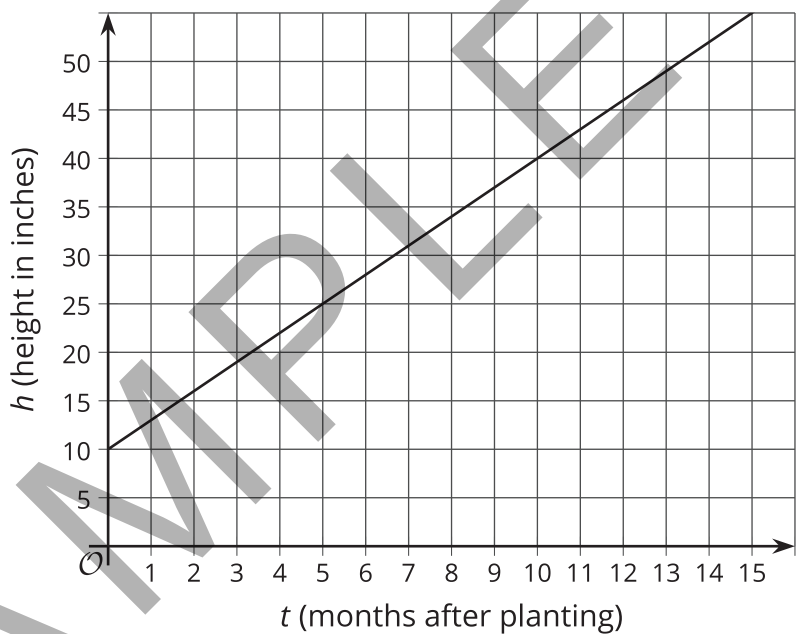
Any linear relationship can be expressed in the form $y = mx + b$ using just the rate of change, m, and the initial amount, b. For example, the equation $y = 5x + 20$ could be used to describe a different scenario where marbles, each with a volume of 5 ml, are added to a cylinder that initially had 20 ml of water.

Practice Problems

1 Create a graph that shows three lines with slopes $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{6}{5}$.



2 The graph shows the height in inches, h, of a bamboo plant t months after it has been planted.



a. Write an equation that describes the relationship between h and t.

b. How tall will the bamboo plant be after 18 months? Explain or show your reasoning.

3 from Unit 3, Lesson 4

Here are recipes for two different banana cakes. Information for the first recipe is shown in the table.

sugar (cups)	flour (cups)
$\frac{1}{2}$	$\frac{3}{4}$
$2\frac{1}{2}$	$3\frac{3}{4}$
3	$4\frac{1}{2}$

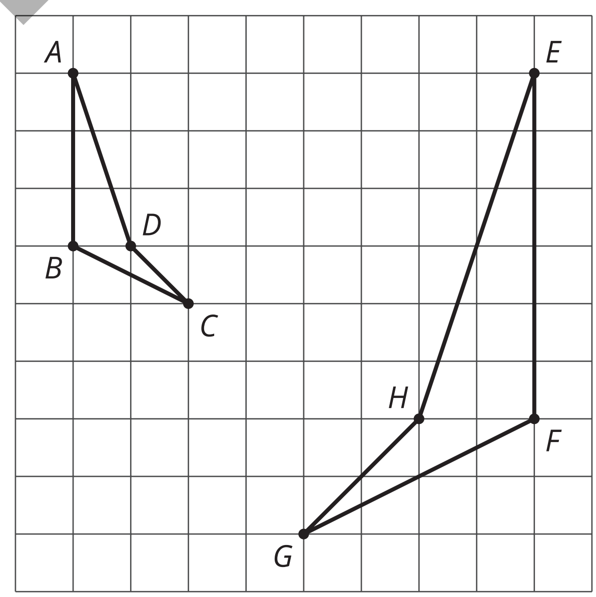
The relationship between cups of flour y and cups of sugar x in the second recipe is $y = \frac{7}{4}x$.

a. If 4 cups of sugar are used, how much flour does each recipe need?

b. What is the constant of proportionality for each situation and what does it mean?

4 from Unit 2, Lesson 6

Show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes one figure to the other.



Sec B

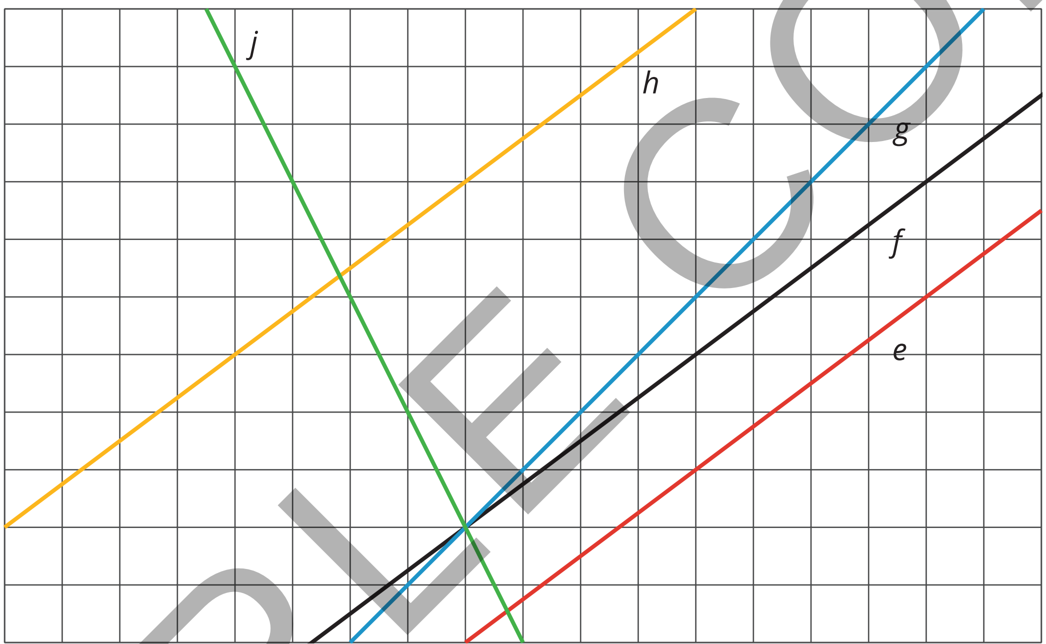
Addressing CA CCSSM 8.EE.5-6; building on 8.G.1, 8.G.1c; practicing MP2, MP4, MP7

Translating to $y = mx + b$

Let's see what happens to the equations of translated lines.

8.1 Lines that Are Translations

The diagram shows several lines. You can only see part of the lines, but they actually continue forever in both directions.

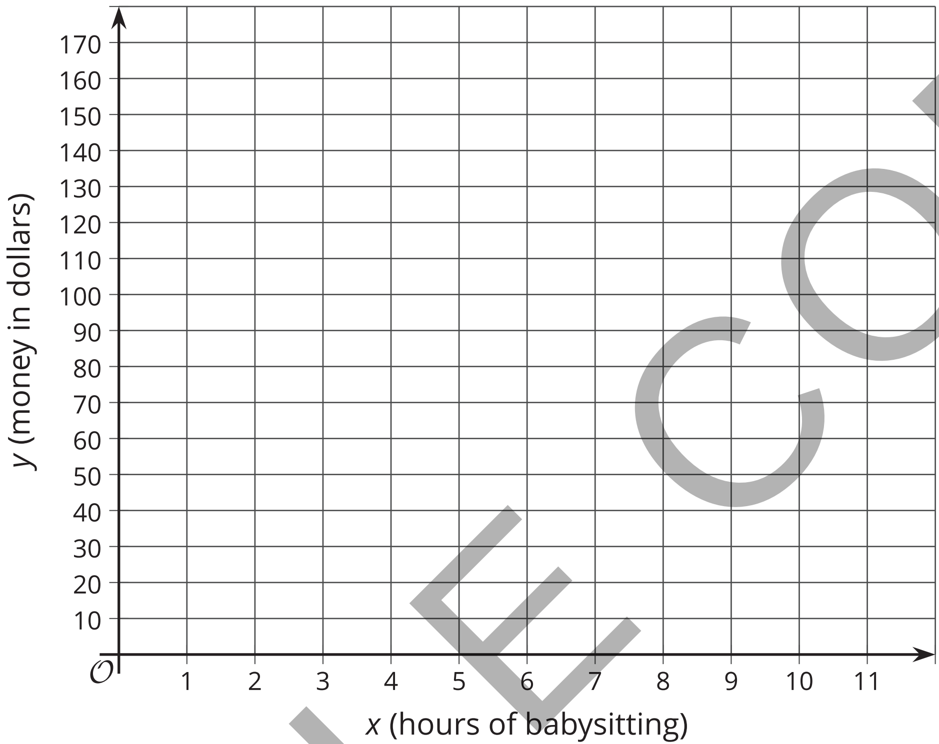


1. Which lines are images of line f after a translation?

2. For each line that is a translation of f, draw an arrow on the grid that shows the vertical translation distance.

8.2 Increased Savings

1. Diego earns $10 per hour babysitting. He has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting.



2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting.

3. Compare the second line with the first line. How much *more* money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours?

8.3 Translating a Line

Your teacher will give you a set of cards containing 4 graphs showing line a and its image, line h, after a translation. Match each graph with an equation describing the translation and either a table or description. Record your matches and be prepared to explain your reasoning. For the line with no matching equation, write one on the blank card.

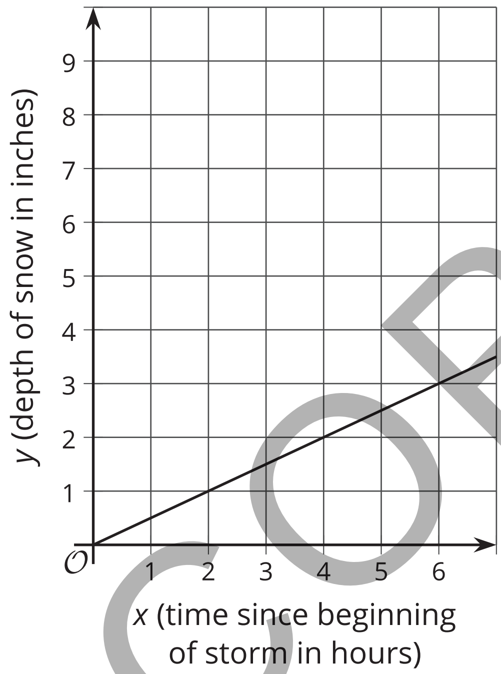
Are you ready for more?

A student says that the graph of the equation $y = 3(x + 8)$ is the same as the graph of $y = 3x$, only translated upwards by 8 units. Do you agree? Explain your reasoning.

Lesson 8 Summary

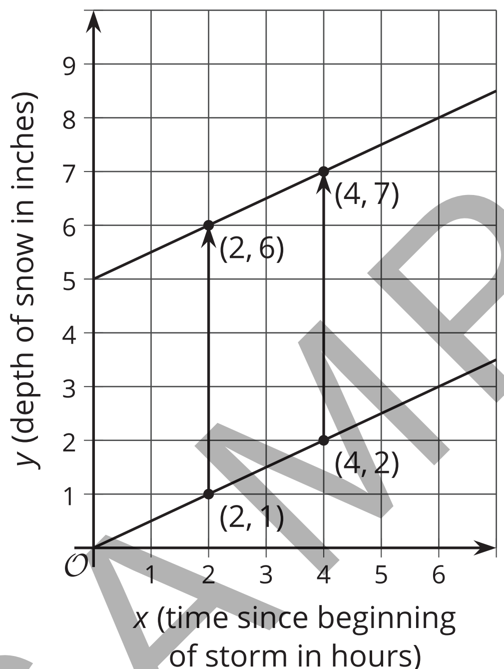
During an early winter storm, snow falls at a rate of $\frac{1}{2}$ inch per hour. The rate of change, $\frac{1}{2}$, can be seen in both the equation $y = \frac{1}{2}x$ and in the slope of the line representing this storm.

The time since the beginning of the storm and the depth of the snow is a linear relationship. This is also a proportional relationship since the depth of snow is 0 inches at the beginning of the storm.



Sec B

During a mid-winter storm, snow again falls at a rate of $\frac{1}{2}$ inch per hour, but this time there were already 5 inches of snow on the ground.



The rate of change, $\frac{1}{2}$, can still be seen in both the equation and in the slope of the line representing this second storm.

The 5 inches of snow that were already on the ground can be graphed by translating the graph of the first storm up 5 inches, resulting in a vertical intercept at $(0, 5)$. It can also be seen in the equation $y = \frac{1}{2}x + 5$.

This second storm is also a linear relationship, but unlike the first storm, is not a proportional relationship since its graph has a vertical intercept of 5.

Practice Problems

1 Select **all** the equations that have graphs with the same y-intercept as the graph of $y = 3x - 8$.

A. $y = 4x - 8$

B. $y = 3x - 9$

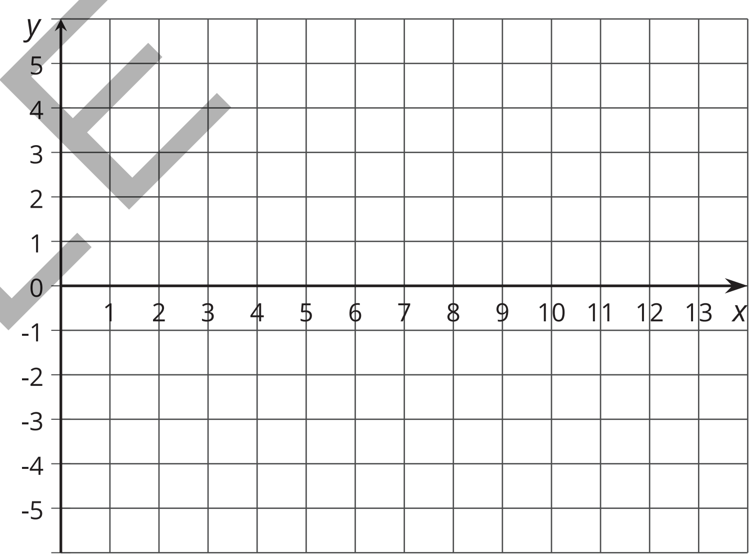
C. $y = 3x + 8$

D. $y = -8 + 5x$

E. $2x - 8 = y$

F. $y = 8 - 3x$

2 Create a graph showing the equations $y = \frac{1}{3}x$ and $y = \frac{1}{3}x - 4$. Explain how the graphs are the same and how they are different.



3 An internet company charges $70 per month for internet service to existing customers.

a. Write an equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by an existing customer.

b. For new customers, there is a one-time $100 installation fee. Write an equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by a new customer.

c. Explain how graphs of the lines representing each situation would be the same and how they would be different.

4 from Unit 3, Lesson 6

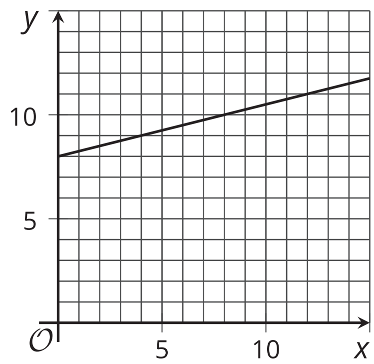
A mountain road is 5 miles long and gains elevation at a constant rate. At the start of the road, the elevation is 4,800 feet above sea level. After 2 miles, the elevation is 5,500 feet above sea level.

a. Find the elevation of the road after 5 miles.

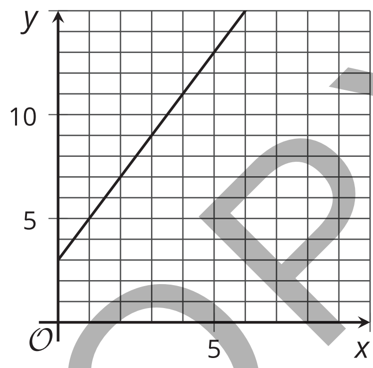
b. What is the vertical intercept and what does it represent in this situation?

Match each graph to a situation.

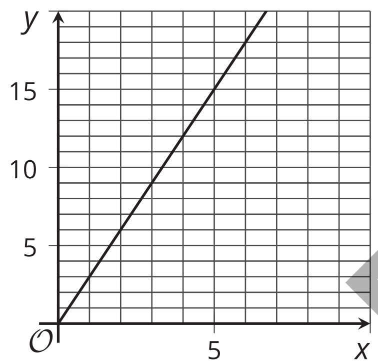
A



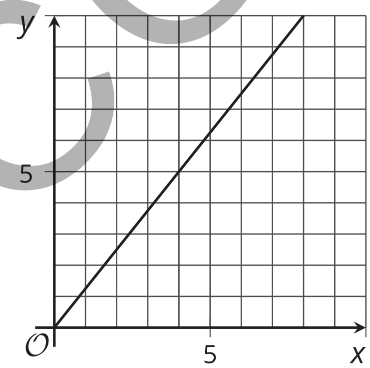
B



C



D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. The graph represents the perimeter, y, in units, for an equilateral triangle with side length of x units. The slope of the line is 3.

2. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$.

3. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$.

4. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2.

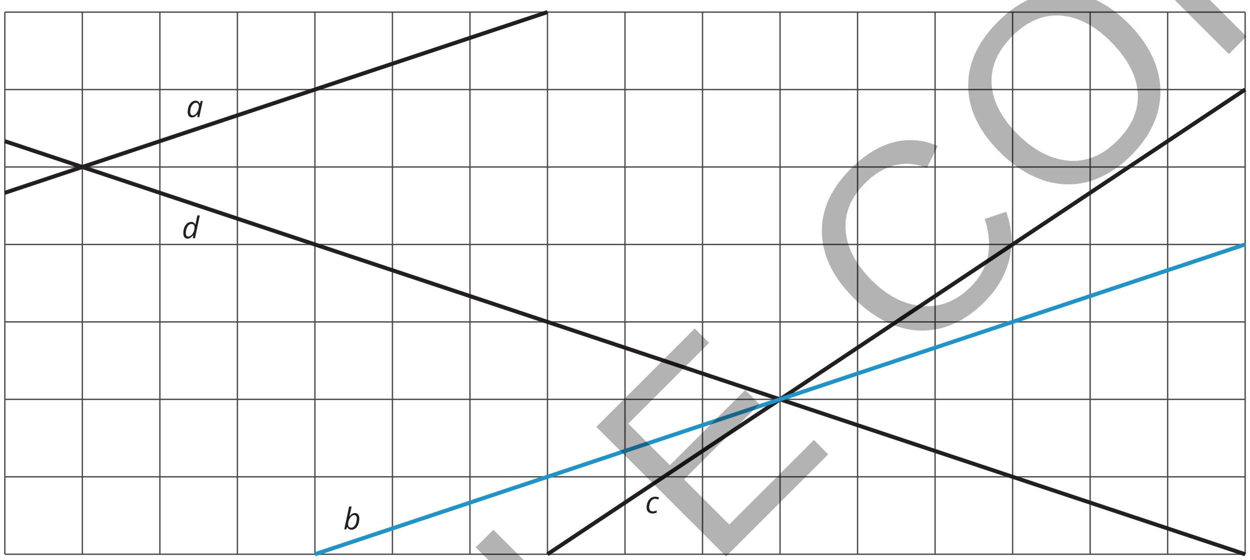
Unit 3, Lesson 9
Addressing CA CCSSM 8.EE.5-6; practicing MP2 and MP6

Slopes Don't Have to Be Positive

Let's find out what a negative slope means.

9.1 Which Three Go Together: Intersecting Lines

Which three go together? Why do they go together?



9.2 Stand Clear of the Closing Doors, Please

Noah has $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after he takes

   a. 0 rides?

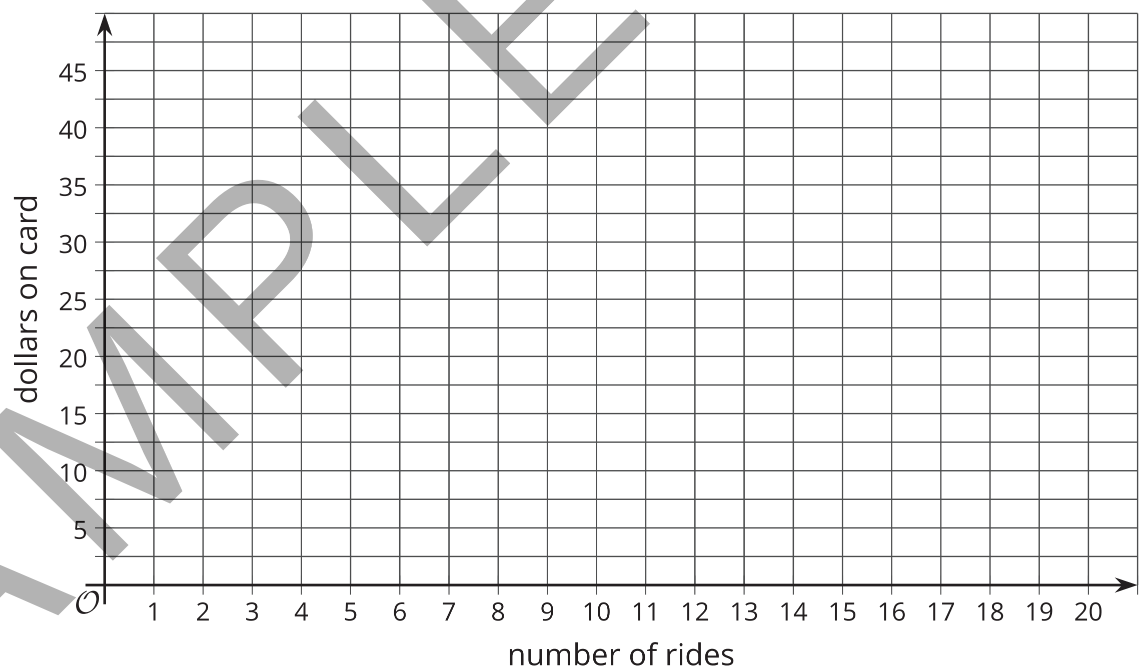
   b. 1 ride?

   c. 2 rides?

   d. x rides?

2. How many rides can Noah take before the card runs out of money? Where would you see this number of rides on a graph?

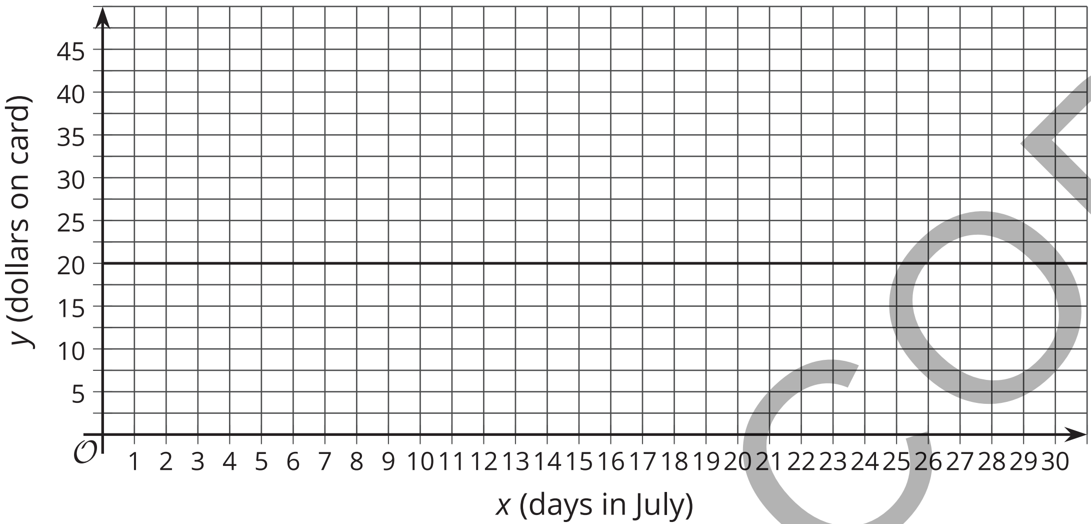
3. Graph the relationship between amount of money on the card and number of rides.



Sec C

9.3 Travel Habits in July

This graph shows the amount of money in dollars that is on Han's fare card for every day of last July.



1. Describe what happened with the money on Han's fare card in July.

2. Plot and label 3 different points on the line.
3. Write an equation where x represents the day in July and y represents the dollars on the card.

4. What value makes sense for the slope of the line that represents the money on Han's fare card in July?

Are you ready for more?

A loan was taken out and is being paid back in multiple payments. Which of the following situations would have a graph with a positive slope and which would have a negative slope? Explain your reasoning.

1. Amount paid on the vertical axis and time since payments started on the horizontal axis.

2. Amount owed on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

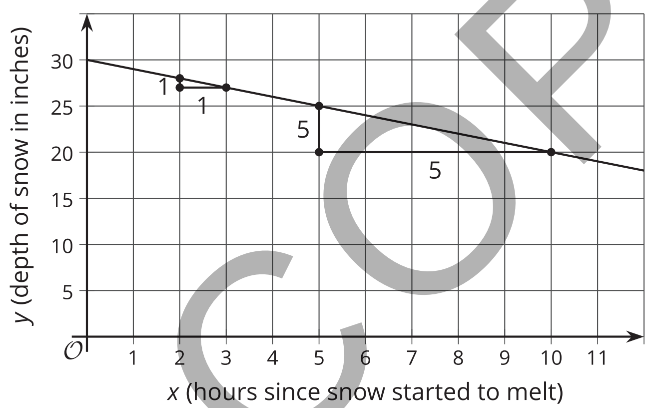
3. Amount paid on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

Lesson 9 Summary

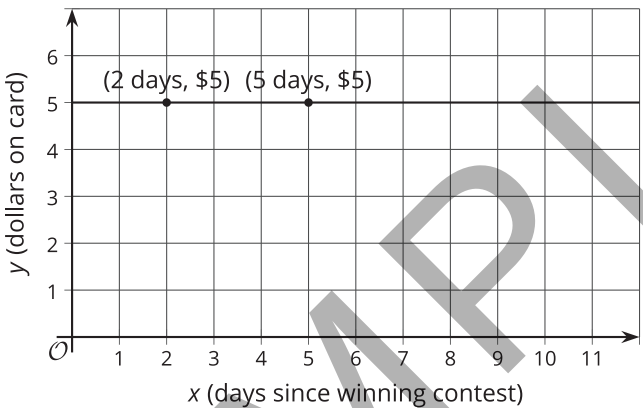
At the end of winter in Maine, the snow on the ground was 30 inches deep. Then there was a particularly warm day and the snow melted at the rate of 1 inch per hour. The graph shows the relationship between the time since the snow started to melt and the depth of the remaining snow.

Graphs with a negative slope often describe situations where some quantity is decreasing over time.

Since the depth of the snow decreases by 1 inch per hour, the rate of change is -1 inch per hour and the slope of this graph is -1. The vertical intercept is 30 since the snow was 30 inches high before it started to melt.



Graphs with a slope of 0 describe situations where there is no change in the y-value even though the x-value is changing.



For example, Elena wins a prize that gives her free bus rides for a year. Her fare card already had $5 on it when she won the prize. Here is a graph of the amount of money on her fare card after winning the prize. Since she doesn't need to add or use money from her fare card for the next year, the amount on her fare card will not change. The rate of change is 0 dollars per day and the slope of this graph is 0. All graphs of linear relationships with slopes of 0 are horizontal.

Sec C

Practice Problems

1 Suppose that during its flight, the elevation e (in feet) of a certain airplane and its time since takeoff t are related by a linear equation. Consider the graph of this equation, with time represented on the horizontal axis and elevation on the vertical axis. For each situation, decide if the slope is positive, zero, or negative.

a. The plane is cruising at an altitude of 37,000 feet above sea level.

b. The plane is descending at a rate of 1,000 feet per minute.

c. The plane is ascending at a rate of 2,000 feet per minute.

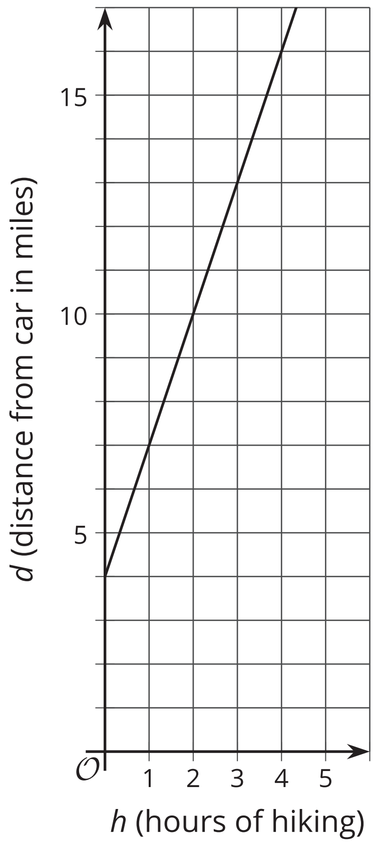
2 from Unit 3, Lesson 7

A group of hikers park their car at a trail head and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite walking at a steady rate. The graph shows their distance in miles, d, from the car after h hours of hiking.

a. How far is the campsite from their car? Explain how you know.

b. Write an equation that describes the relationship between d and h.

c. After how many hours of hiking will they be 16 miles from their car? Explain or show your reasoning.



Sec C

3 from Unit 3, Lesson 4

Elena's aunt pays her $1 for each call she makes to let people know about her aunt's new business

The table shows how much money Diego receives for washing windows for his neighbors.

number of windows	number of dollars
27	30
45	50
81	90

Select **all** the statements about the situation that are true.

A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.

B. Diego makes more money for washing each window than Elena makes for making each call.

C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.

D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.

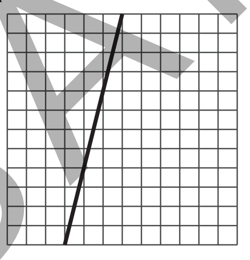
E. The equation $y = \frac{9}{10}x$, where y is the number of dollars, and x is the number of windows, represents Diego's situation.

F. The equation $y = x$, where y is the number of dollars and x is the number of calls, represents Elena's situation.

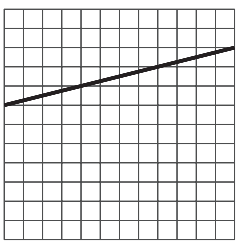
4

Each square on a grid represents 1 unit on each side. Match the graphs with the slopes of the lines.

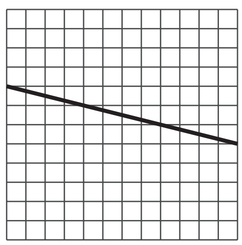
A



B



C



- $-\frac{1}{4}$
- $\frac{1}{4}$
- 4

Unit 3, Lesson 10

Addressing CA CCSSM 8.EE.6; building on 7.NS.2; building towards 8.EE.6; practicing MP3 and MP8

Calculating Slope

Let’s calculate slope from two points.

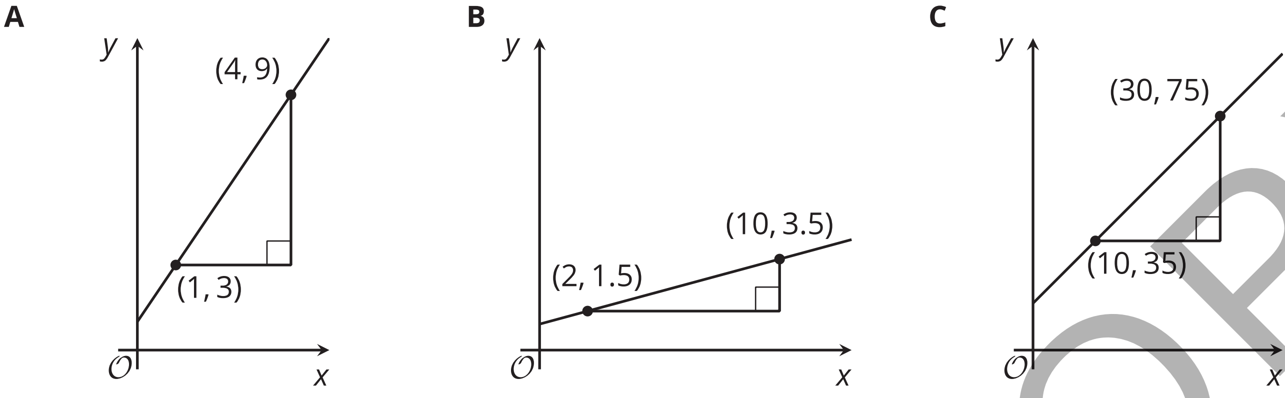
10.1 Math Talk: Integer Operations

Mentally find values for a and b that make each equation true.

- $a + b = -2$
- $a - b = -2$
- $\frac{a}{b} = 2$
- $\frac{a}{b} = -2$

Sec C

10.2 Toward a More General Slope Formula



1. For each graph, record:

	vertical change	horizontal change	slope
A			
B			
C			

2. Describe a procedure for finding the slope between any two points on a line.

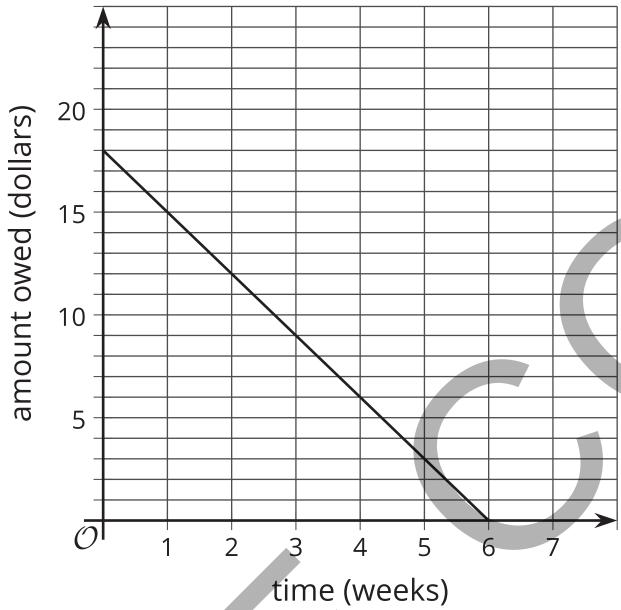
Are you ready for more?

Find the value of k so that the line passing through each pair of points has the given slope.

1. $(k, 2)$ and $(11, 14)$, slope = 2
2. $(1, k)$ and $(4, 1)$, slope = -2
3. $(3, 5)$ and $(k, 9)$, slope = $\frac{1}{2}$
4. $(\text{-}1, 4)$ and $(\text{-}3, k)$, slope = $\frac{\text{-}1}{2}$
5. $\left(\frac{\text{-}15}{2}, \frac{3}{16}\right)$ and $\left(\frac{\text{-}13}{22}, k\right)$, slope = 0

10.3 Payback Plan

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes after each week.

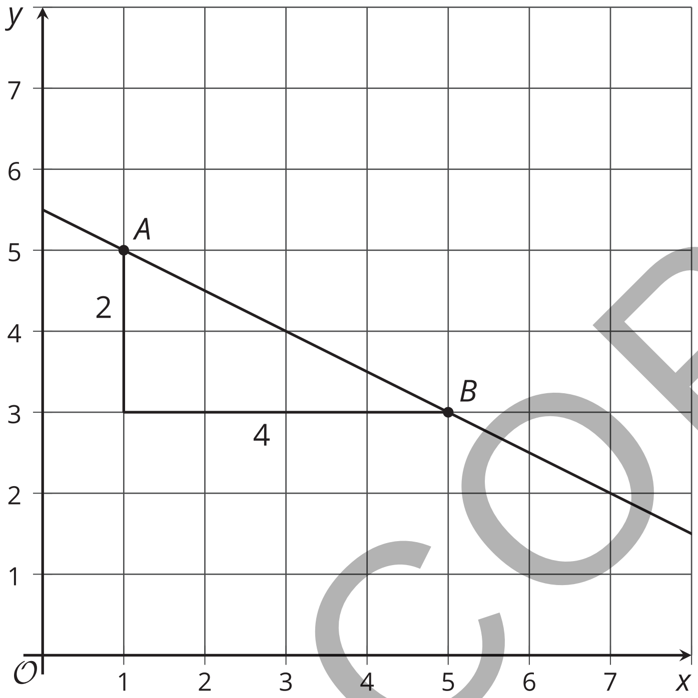


Answer and explain your reasoning for each question.

1. What is the slope of the line?
2. Explain how you know whether the slope is positive or negative.
3. What does the slope represent in this situation?
4. How much did Elena borrow?
5. How much time will it take for Elena to pay back all the money she borrowed?

Lesson 10 Summary

One way to calculate the slope of a line is by drawing a slope triangle. For example, using this slope triangle, the slope of the line is $-\frac{2}{4}$, or $-\frac{1}{2}$. The slope is negative because the line is decreasing from left to right.



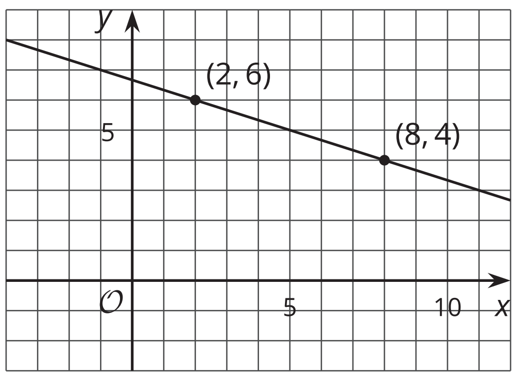
Another way to calculate the slope of this line uses just the points $A:(1,5)$ and $B:(5,3)$. The slope is the vertical change divided by the horizontal change, or the change in the y-values divided by the change in the x-values. Between points A and B, the y-value change is $3-5=-2$ and the x-value change is $5-1=4$. This means the slope is $-\frac{2}{4}$, or $-\frac{1}{2}$, which is the same value as the slope calculated using a slope triangle.

Notice that in each of the calculations, the value from point A was subtracted from the value from point B. If it had been done the other way around, then the y-value change would have been $5-3=2$ and the x-value change would have been $1-5=-4$, which still gives a slope of $-\frac{1}{2}$.

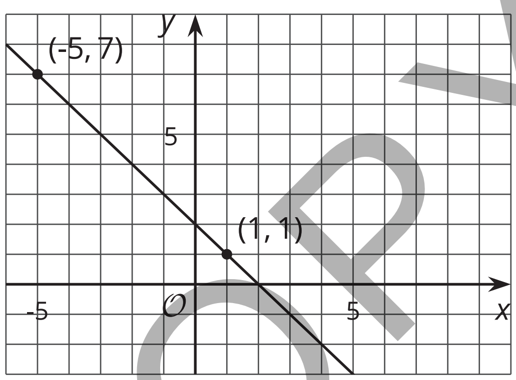
Practice Problems

1 For each graph, calculate the slope of the line.

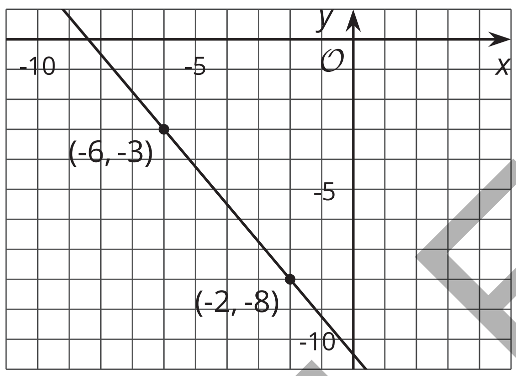
A



B



C



2 Match each pair of points with the slope of the line that passes through each pair.

A. $(9, 10)$ and $(7, 2)$

B. $(-8, -11)$ and $(-1, -5)$

C. $(5, -6)$ and $(2, 3)$

D. $(6, 3)$ and $(5, -1)$

E. $(4, 7)$ and $(6, 2)$

1. 4
2. -3
3. $-\frac{5}{2}$
4. $\frac{6}{7}$

Sec C

3 Select **all** of the points that could be on a line with the point $(10, 1)$ if the line has a negative slope.

A. $(2, 6)$

B. $(-1, 0)$

C. $(-4, 7)$

D. $(11, 0)$

E. $(2, -2)$

Sec C

Unit 3, Lesson 11

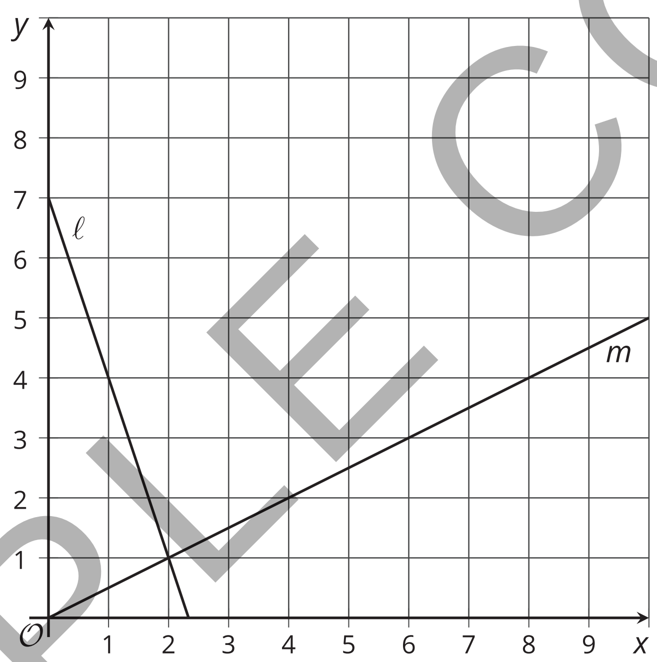
Addressing CA CCSSM 8.EE.6; practicing MP6

Line Designs

Let's describe lines.

11.1 Comparing Lines

What is the same about the two lines? What is different about the two lines?



11.2 Making Designs

Your teacher will give you either a design or a blank graph. Do not show your card to your partner.

If your teacher gives you the design:

1. Look at the design silently and think about how you could communicate what your partner should draw. Think about ways that you can describe what a line looks like, such as its slope or points that it goes through.
2. Describe each line, one at a time, and give your partner time to draw them.
3. Once your partner thinks they have drawn all the lines you described, only then should you show them the design.

If your teacher gives you the blank graph:

1. Listen carefully as your partner describes each line, and draw each line based on their description.
2. You are not allowed to ask for more information about a line than what your partner tells you.
3. Do not show your drawing to your partner until you have finished drawing all the lines they describe.

When finished, place the drawing next to the card with the design so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

Pause here so your teacher can review your work. When your teacher gives you a new set of cards, switch roles for the second problem.

Sec C

11.3 Calculate the Slope

Calculate the slope of the line that passes through each pair of points.

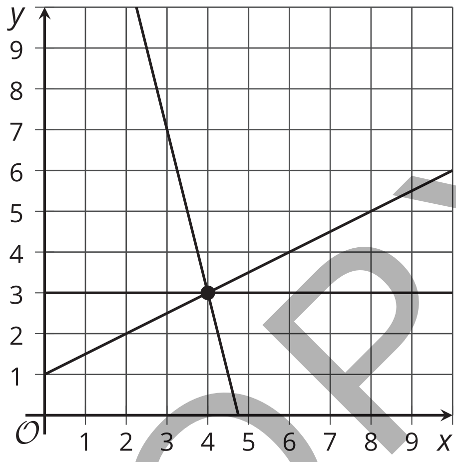
1. $(-5, -7)$ and $(1, -3)$

2. $(-10, 15)$ and $(-8, 1)$

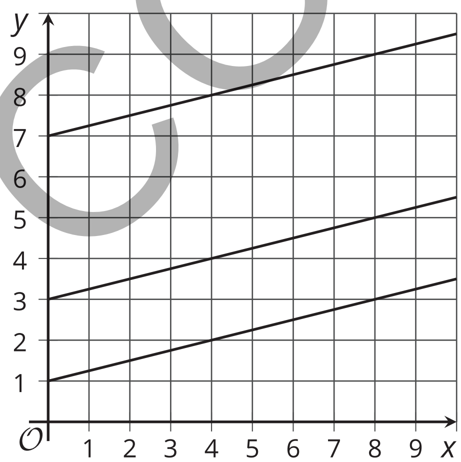
Lesson 11 Summary

In order to draw a specific line, knowing one point that the line passes through is not enough information. We would know where to draw the line, but it could have any slope.

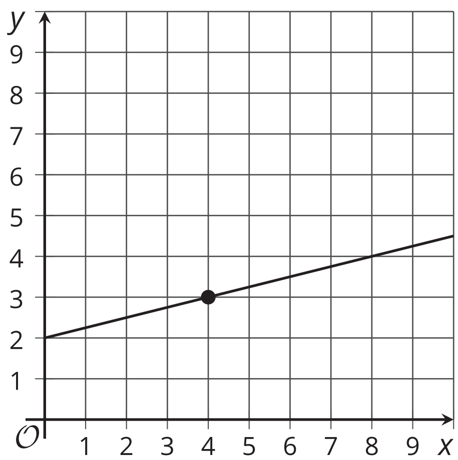
For example, these lines all go through the point $(4, 3)$.



If we knew only the slope of a line, we would know how to draw it but the line could be located anywhere. For example, these lines all have a slope of $\frac{1}{4}$.



To know the exact location of a line, we need either both the slope and the coordinates of one point on the line, or the locations of at least 2 points that are on the line. For example, this line goes through the point $(4, 3)$ and has a slope of $\frac{1}{4}$. This line could also be described as the line that passes through $(4, 3)$ and $(8, 4)$.

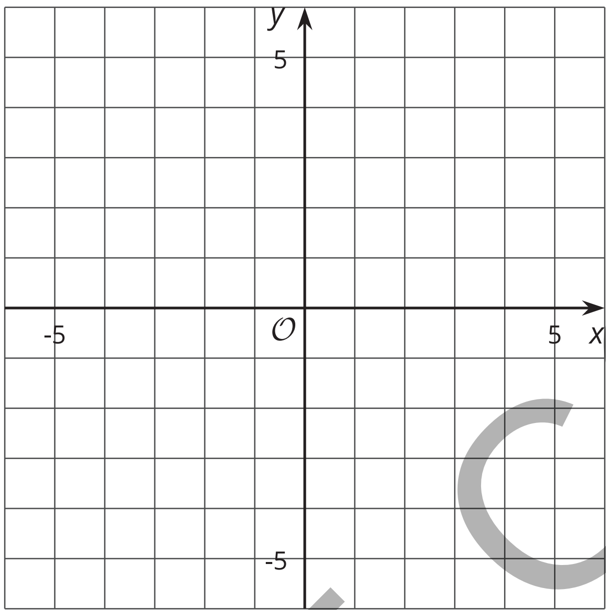


Sec C

Practice Problems

1 from Unit 3, Lesson 10

Graph the line that has a slope of -2 and a vertical intercept at $(0, -5)$.



2 from Unit 3, Lesson 10

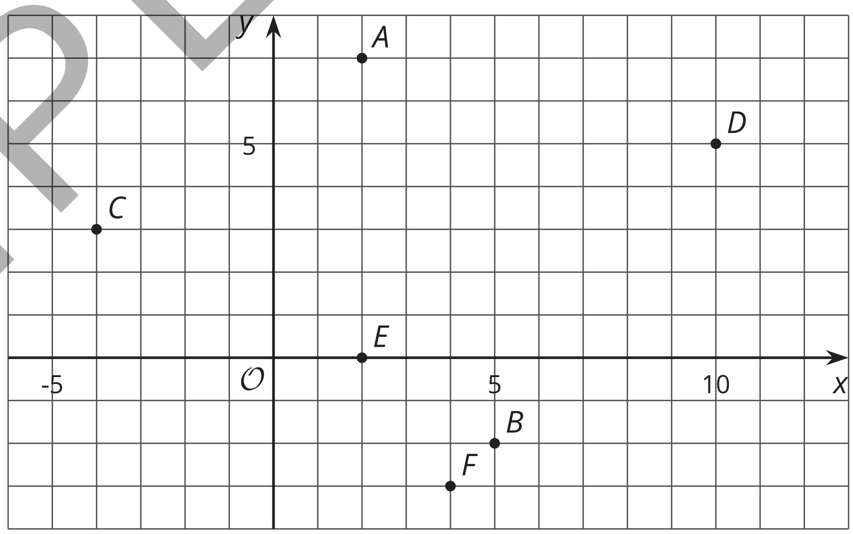
Draw a line with the given slope through the given point. What other point lies on that line?

a. Point A, slope = -3

b. Point A, slope = $\frac{-1}{4}$

c. Point C, slope = $\frac{-1}{2}$

d. Point E, slope = $\frac{-2}{3}$



3 from Unit 3, Lesson 8

Make a sketch of a linear relationship with a slope of 4 and a negative y-intercept. Show how you know the slope is 4 and write an equation for the line.

Sec C

Unit 3, Lesson 12

Addressing CA CCSSM 8.EE.6; building on 8.G.1; practicing MP2, MP3, MP6

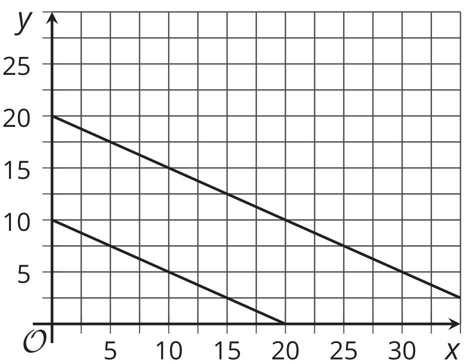
Equations of All Kinds of Lines

Let's write equations for vertical and horizontal lines.

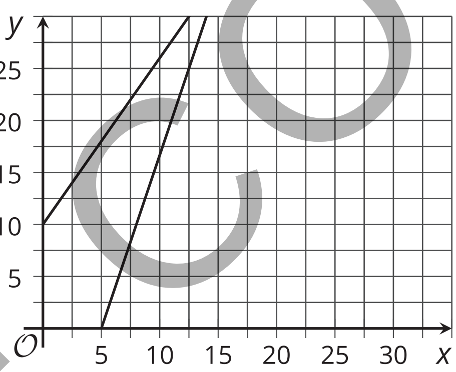
12.1 Which Three Go Together: Pairs of Lines

Which three go together? Why do they go together?

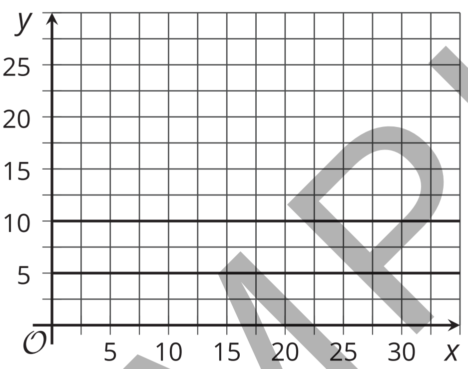
A



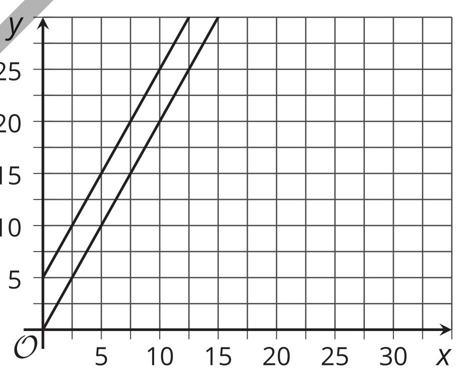
B



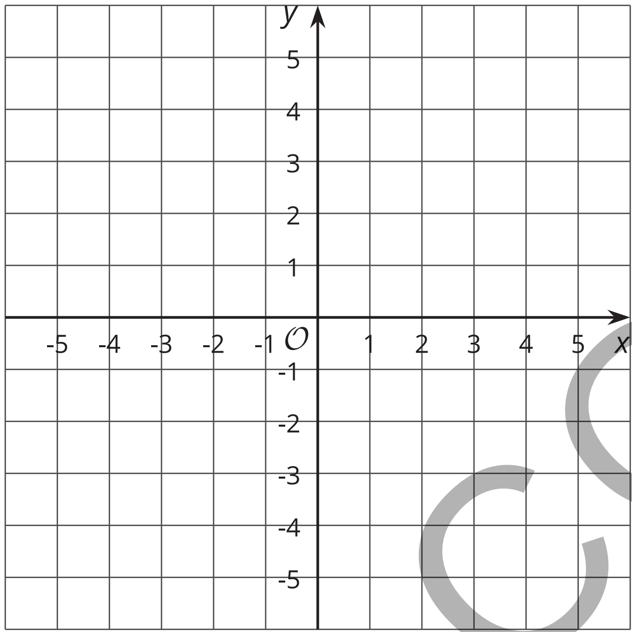
C



D



12.2 All the Same



1. Plot at least 10 points whose y-coordinate is -4. What do you notice about them?

2. Which equation makes the most sense to represent all of the points with y-coordinate -4? Explain how you know.
   $x = -4$ $\quad$ $y = -4x$ $\quad$ $y = -4$ $\quad$ $x + y = -4$

3. Plot at least 10 points whose x-coordinate is 3. What do you notice about them?

4. Which equation makes the most sense to represent all of the points with x-coordinate 3? Explain how you know.
   $x = 3$ $\quad$ $y = 3x$ $\quad$ $y = 3$ $\quad$ $x + y = 3$

5. Graph the equation $x = -2$.

6. Graph the equation $y = 5$.

Are you ready for more?

1. Draw the rectangle with vertices $(2, 1)$, $(5, 1)$, $(5, 3)$, and $(2, 3)$.

2. For each of the four sides of the rectangle, write an equation for a line containing the side.

Sec C

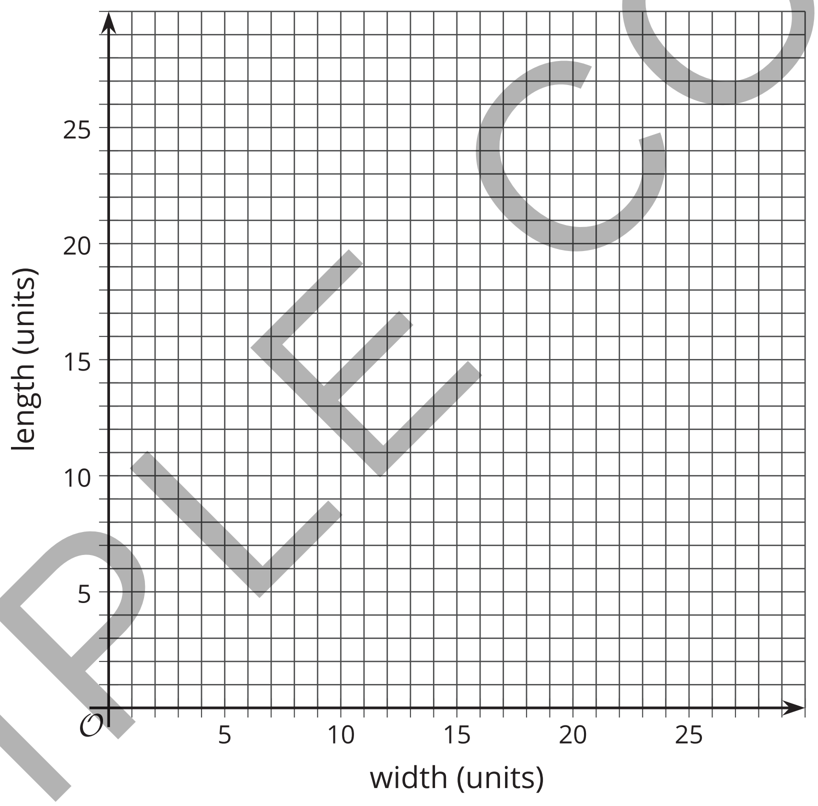
3. A rectangle has sides on the graphs of $x = -1$, $x = 3$, $y = -1$, $y = 1$. Find the coordinates of each vertex.

12.3 Same Perimeter

1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths, ℓ, and widths, w, of at least 10 such rectangles.

ℓ										
w										

2. On the graph, plot the length and width of rectangles whose perimeter is 50 units using the values from your table. Using a straightedge, draw the line that passes through these points.



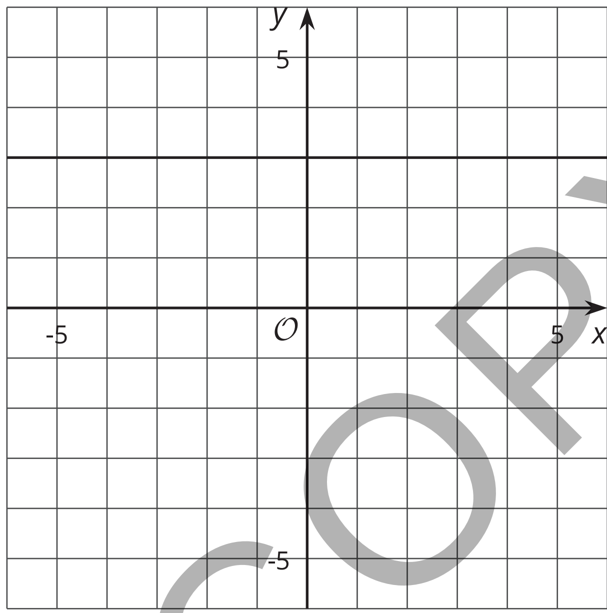
3. What is the slope of this line? What does the slope mean in this situation?

4. Write an equation for this line.

Lesson 12 Summary

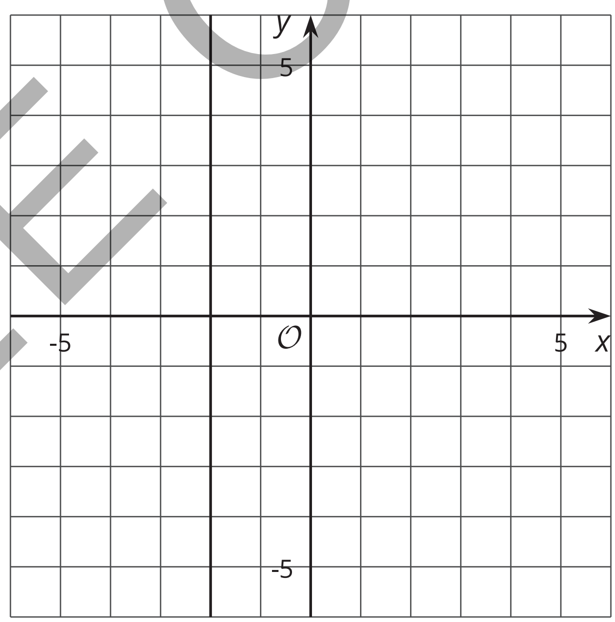
Horizontal lines in the coordinate plane represent situations where the y-value doesn't change at all while the x-value changes.

The horizontal line that goes through the point $(0, 3)$ can be described by saying that "for all points on the line, the y-value is always 3." Since horizontal lines are neither increasing or decreasing, they have a slope of 0, and so an equation for this horizontal line is $y = 0x + 3$, or just $y = 3$.



Vertical lines in the coordinate plane represent situations where the x-value doesn't change at all while the y-value changes.

The vertical line that goes through the point $(\text{-}2, 0)$ can be described by saying that "for all points on the line, the x-value is always -2." An equation that says the same thing is $x = \text{-}2$.



Practice Problems

1 Suppose you wanted to graph the equation $x = -3$. Describe the steps you would take to draw the graph.

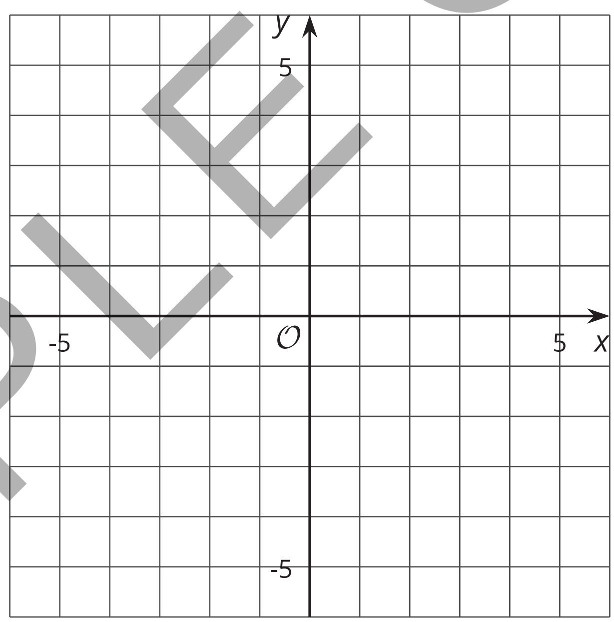
2 Draw and label a line for each given slope and y-intercept.

a. Line a: slope is 0, y-intercept is 5

b. Line b: slope is 2, y-intercept is -1

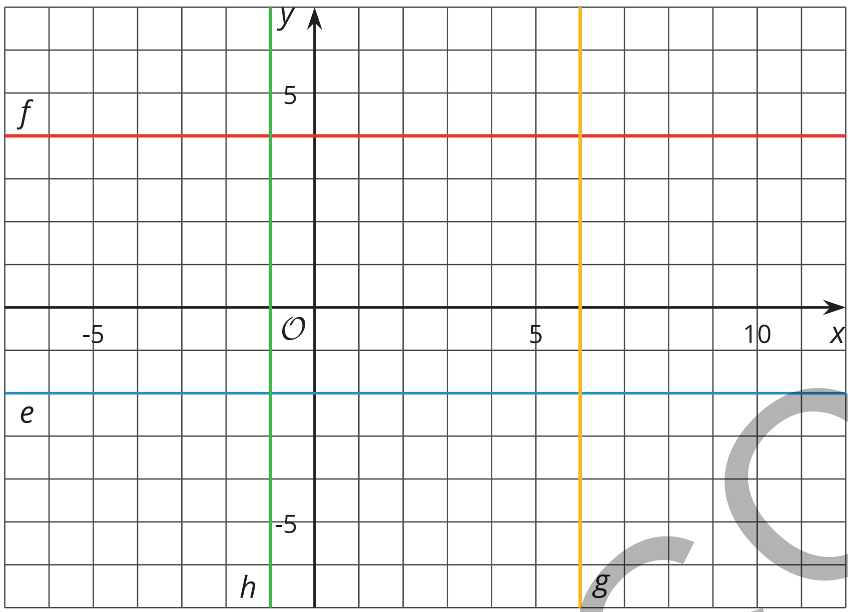
c. Line c: slope is -2, y-intercept is 1

d. Line d: slope is $-\frac{1}{2}$, y-intercept is -1



Sec C

3 Write an equation for each line.



Sec C

4 from Unit 3, Lesson 7

A publisher wants to figure out how thick their new book will be. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ of an inch. They have a choice of which type of paper to print the book on.

a. Bond paper has a thickness of $\frac{1}{4}$ inch per one hundred pages. Write an equation for the total thickness of the book in inches, y, if it has x hundred pages, printed on bond paper.

b. Ledger paper has a thickness of $\frac{2}{5}$ inch per one hundred pages. Write an equation for the total thickness of the book in inches, y, if it has x hundred pages, printed on ledger paper.

c. If they instead chose front and back covers of thickness $\frac{1}{3}$ of an inch, how would this change the equations in the previous two parts?

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Unit 3, Lesson 13

Addressing CA CCSSM 8.EE.5, 8.EE.8; building on 6.G.1; building towards 8.EE.8; practicing MP8

Solutions to Linear Equations

Let's think about what it means to be a solution to a linear equation with two variables in it.

13.1 Avocados and Pineapples

At the market, avocados cost \$1 each and pineapples cost \$2 each. Find the cost of:

1. 6 avocados and 3 pineapples

2. 4 avocados and 4 pineapples

3. 5 avocados and 4 pineapples

4. 8 avocados and 2 pineapples

13.2 More Avocados and Pineapples

At the market, avocados cost $1 each and pineapples cost $2 each.

1. Noah has $10 to spend at the produce market. Can he buy 7 avocados and 2 pineapples? Explain or show your reasoning.

2. What combinations of avocados and pineapples can Noah buy if he spends all of his $10?

3. Write an equation that represents $10 combinations of avocados and pineapples, using a for the number of avocados and p for the number of pineapples.

4. What are 3 combinations of avocados and pineapples that make your equation true? What are three combinations of avocados and pineapples that make it false?

Are you ready for more?

1. Create a graph relating the number of avocados and the number of pineapples that can be purchased for exactly $10.
2. What is the slope of the graph? What is the meaning of the slope in terms of the context?
3. Suppose Noah has $20 to spend. Graph the equation describing this situation. What do you notice about the relationship between this graph and the earlier one?

13.3 Solutions and Everything Else

There are two numbers. When the first number is doubled and added to the second number, the sum is 10.

1. Let x represent the first number and let y represent the second number. Write an equation showing the relationship between x, y, and 10.

2. Draw and label a coordinate plane.

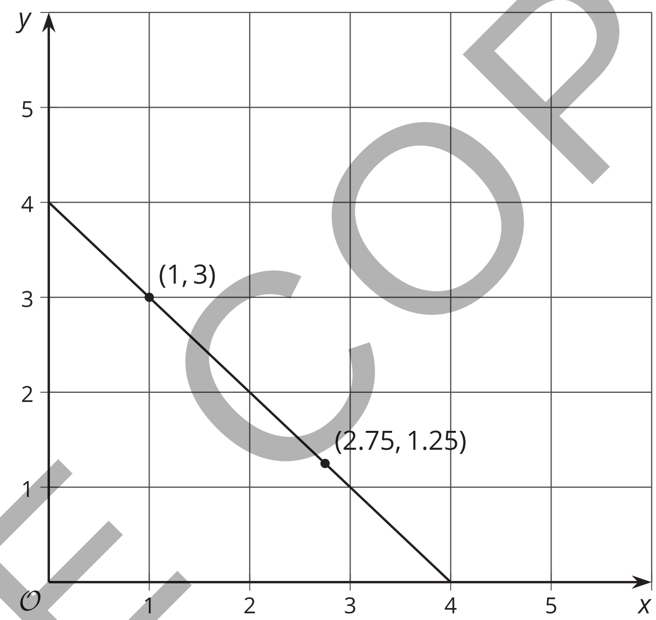
3. Find 5 pairs of x- and y-values that make the statement and your equation true. Plot each pair of values as a point (x, y) on the coordinate plane. What do you notice?

4. List 10 pairs of x- and y-values that do not make the statement and equation true. Using a different color, plot each pair of values as a point (x, y) on the coordinate plane. What do you notice about these points compared to your first set of points?

Lesson 13 Summary

A **solution to an equation with two variables** is any pair of values for the variables that make the equation true. For example, the equation $2x + 2y = 8$ represents the relationship between the width x and length y for rectangles with a perimeter of 8 units. One solution to the equation $2x + 2y = 8$ is that the width and length could be 1 and 3, since $2 \cdot 1 + 2 \cdot 3 = 8$. Another solution is that the width and length could be 2.75 and 1.25, since $2 \cdot (2.75) + 2 \cdot (1.25) = 8$. There are many other possible pairs of width and length that make the equation true.

The pairs of numbers that are solutions to an equation can be seen as points on the coordinate plane where every point represents a different rectangle whose perimeter is 8 units. Here is part of the line created by all the points (x, y) that are solutions to $2x + 2y = 8$. In this situation, it makes sense for the graph to only include positive values for x and y since there is no such thing as a rectangle with a negative side length.



Sec D

Glossary

- solution to an equation with two variables

Practice Problems

1 Select **all** of the ordered pairs (x, y) that are solutions to the linear equation $2x + 3y = 6$.

A. $(0, 2)$

B. $(0, 6)$

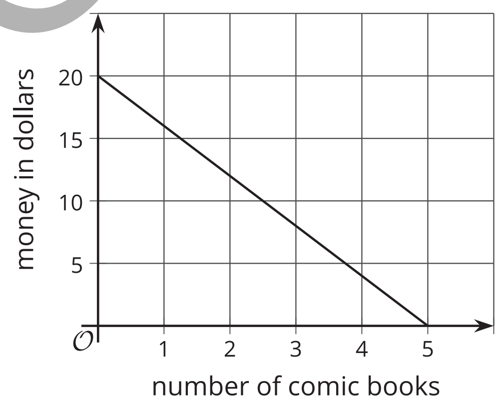
C. $(2, 3)$

D. $(3, -2)$

E. $(3, 0)$

F. $(6, -2)$

2 The graph shows the relationship between the number of comic books Priya buys at the store and the amount of money in dollars that Priya has left after buying the comic books.



a. What is the vertical intercept and what does it mean in this situation?

b. What is the horizontal intercept and what does it mean in this situation?

c. What is the slope of this line and what does it mean in this situation?

d. Write an equation that represents this line where x represents the number of comic books Priya buys and y represents how many dollars Priya has left.

e. If Priya buys 3 comic books, how much money will she have remaining?

Sec D

3 Match each equation with the set of points that are all solutions to the equation.

A. $y = 1.5x$

B. $2x + 3y = 7$

C. $x - y = 4$

D. $3x = \frac{y}{2}$

E. $y = -x + 1$

1. $(14, 21), (2, 3), (8, 12)$
2. $(-3, -7), (0, -4), (-1, -5)$
3. $\left(\frac{1}{8}, \frac{7}{8}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{4}\right)$
4. $\left(1, 1\frac{2}{3}\right), (-1, 3), \left(0, 2\frac{1}{3}\right)$
5. $(0.5, 3), (1, 6), (1.2, 7.2)$

4 from Unit 3, Lesson 10

A container of fuel empties at a rate of 5 gallons per second. A graph representing this situation is drawn in the coordinate plane, where x is the number of seconds that have passed since the container started emptying, and y is the amount of fuel remaining in the container.

Will the slope of the line representing this situation have a positive, negative, or zero slope? Explain your reasoning.

5 from Unit 3, Lesson 5

A sandwich shop charges a delivery fee to bring lunch to an office building. One office pays 33 dollars for 4 turkey sandwiches including the delivery fee. Another office pays 61 dollars for 8 turkey sandwiches including the delivery fee.

a. What is the cost of one turkey sandwich? Explain your reasoning.

b. What is the delivery fee? Explain your reasoning.

Unit 3, Lesson 14

Addressing CA CCSSM 8.EE.8, 8.EE.8a; practicing MP3 and MP7

More Solutions to Linear Equations

Let's find solutions to more linear equations.

14.1 Coordinate Pairs

For each equation choose a value for x and then find the corresponding y-value that makes that equation true.

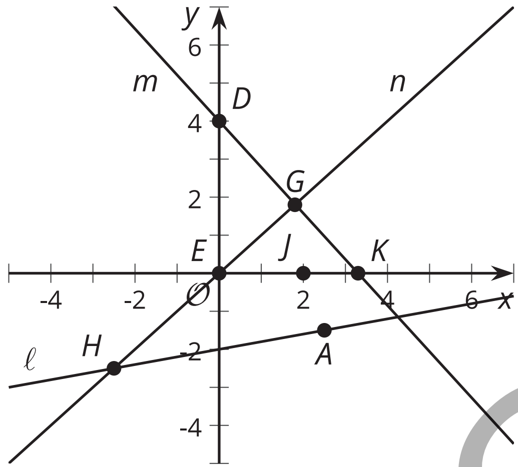
1. $6x = 7y$

2. $5x + 3y = 9$

3. $y + 5 - \frac{1}{3}x = 7$

14.2 Solutions in the Coordinate Plane

Here are graphs representing three linear relationships. These relationships could also be represented by equations.



Decide if each statement is true or false. Explain your reasoning.

1. The point $(4, 0)$ represents a solution to the equation for line m.

2. The coordinates of the point G make both the equation for line m and the equation for line n true.

3. $(2, 0)$ makes the equation for line m and the equation for line n true.

4. There is not a solution to the equation for line ℓ that has $y = 0$.

5. The coordinates of point H are a solution to the equation for line ℓ.

6. There are exactly two solutions to the equation for line ℓ.

7. There is a point whose coordinates make the equations of all three lines true.

8. $x = 0$ is a solution to the equation for line n.

Discuss your thinking with your partner. If you disagree, work to reach an agreement.

14.3 I'll Take an X, Please

Your teacher will give you a set of cards. One partner has 6 cards labeled A through F and one partner has 6 cards labeled a through f. Cards with the same letter, for example Cards A and a, have an equation on one card and a coordinate pair (x, y) that makes the equation true on the other card. Take turns asking your partner for either the x- or y-coordinate value and using it to solve your equation for the other value.

1. The partner with the equation asks the partner with a solution for either the x-value or the y-value.
2. The partner with the equation uses this value to find the other value, explaining each step as they go.
3. The partner with the coordinate pair checks their partner's work. If the coordinate pair does not match, both partners should look through the steps to find and correct any errors. Otherwise, both partners move onto the next set of cards.

Keep playing until you have finished all the cards.

Are you ready for more?

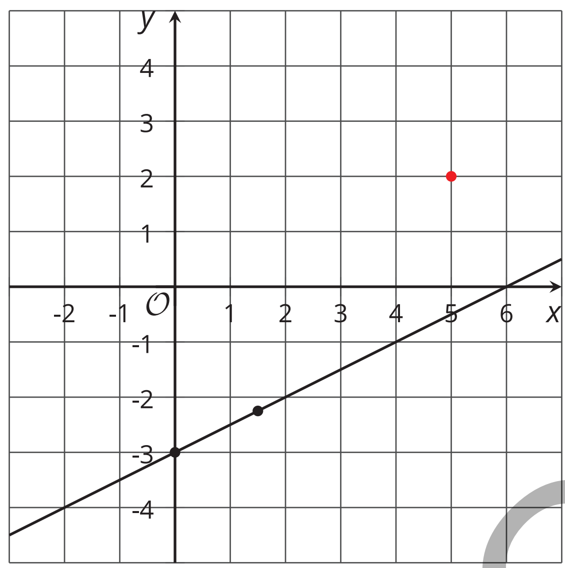
Consider the equation $ax + by = c$, where a, b, and c are positive numbers.

1. Find the coordinates of the x- and y-intercepts of the graph of the equation.

2. Find the slope of the graph.

Lesson 14 Summary

Consider the graph of the linear equation $2x - 4y = 12$.



Since $(0, -3)$ is a point on the graph of the equation, $(0, -3)$ is a solution to the equation. Any point not on the line is not a solution to the equation.

Sometimes the coordinates of a solution cannot be determined exactly by looking at the graph. For example, when $x = 1.5$, the y-value is somewhere between -2 and -3. If we have a value for one of the variables, we can use the equation to figure out the value of the other variable.

$$\begin{aligned} 2x - 4y &= 12 \\ 2(1.5) - 4y &= 12 \\ 3 - 4y &= 12 \\ -4y &= 9 \\ y &= -\frac{9}{4} \text{ or } -2\frac{1}{4} \end{aligned}$$

The equation can also be used to check whether a pair of values is a solution to the equation by seeing if the values make the equation true. For example, since the values $x = 5$ and $y = 2$ do not make the equation true, then the point $(5, 2)$ is not a solution and does not lie on the line.

Practice Problems

1 For each equation, find y when $x = -3$. Then find x when $y = 2$

a. $y = 6x + 8$

b. $y = \frac{2}{3}x$

c. $y = -x + 5$

d. $y = \frac{3}{4}x - 2\frac{1}{2}$

e. $y = 1.5x + 11$

2 Do the points $(6, 13)$, $(21, 33)$, and $(99, 137)$ lie on the graph of the line defined by the equation $y = \frac{4}{3}x + 5$? Explain or show your reasoning.

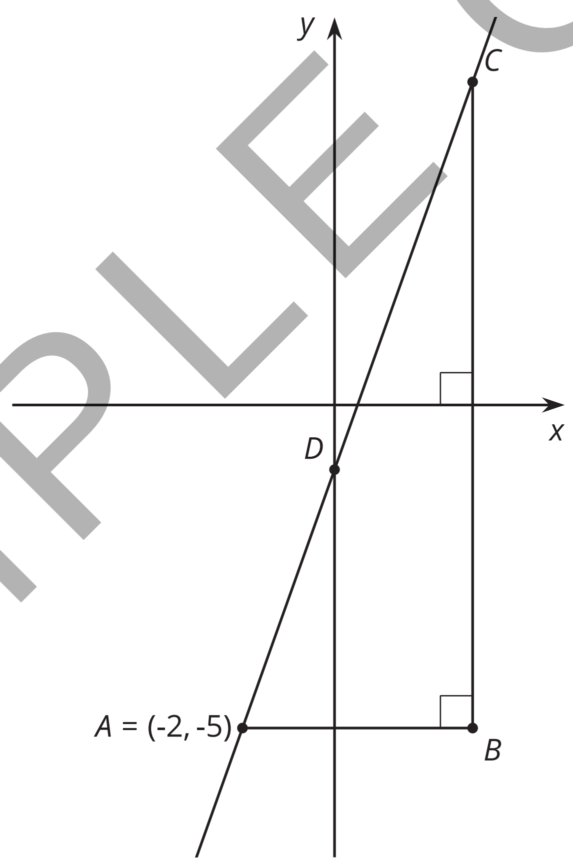
3 A line can be described by the equation $y = \frac{1}{4}x + \frac{5}{4}$.

a. Is $(1, 1.5)$ a solution to the equation? Explain your reasoning.

b. Does the point $(12, 4)$ lie on the graph of the line? Explain your reasoning.

4 from Unit 2, Lesson 11

Find the coordinates of B, C, and D if the length of segment AB is 5 units and the length of segment BC is 10 units.

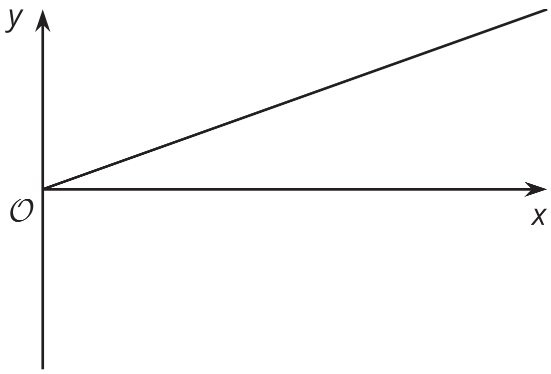


Sec D

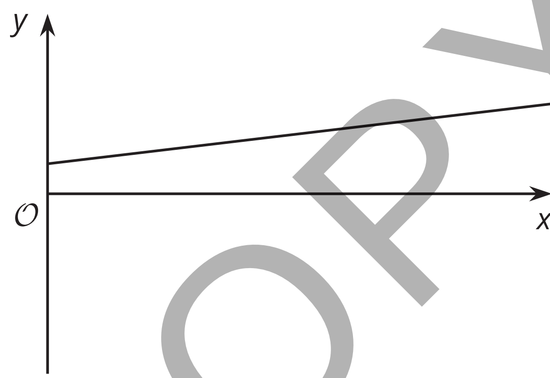
5 from Unit 3, Lesson 9

Match each graph of a linear relationship to a situation that most reasonably reflects its context.

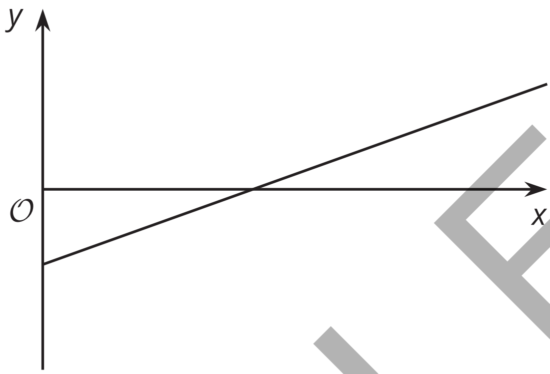
A



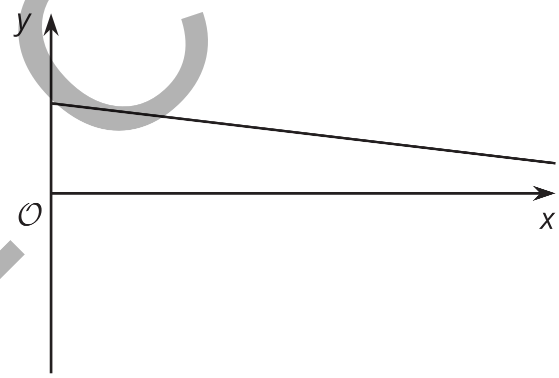
B



C



D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. y is the weight of a kitten x days after birth.

2. y is the distance left to go in a car ride after x hours of driving at a constant rate toward its destination.

3. y is the temperature, in degrees C, of a gas being warmed in a laboratory experiment.

4. y is the height in feet climbed after walking up x stairs.

Unit 3, Lesson 15

Addressing CA CCSSM 8.EE.6, 8.EE.8a; practicing MP2

Using Linear Relations to Solve Problems

Let's write equations for real-world situations and think about their solutions.

15.1 Writing Equations

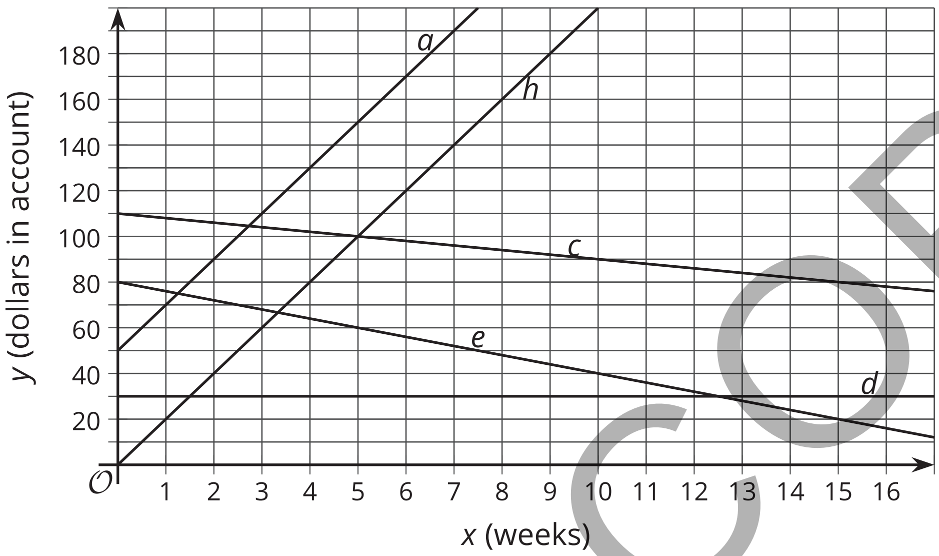
Write an equation to represent each relationship.

1. Grapes cost \$2.39 per pound. Papayas cost \$1.34 per pound. There are only \$15 to spend on g pounds of grapes and p pounds of papayas.

2. A savings account has \$50 in it at the start of the year and \$20 is deposited each week. After x weeks, there are y dollars in the account.

15.2 Five Savings Accounts

Each line represents one person’s weekly savings account balance from the start of the year.



1. Choose one line and write a description of what happens to that person's account over the first 17 weeks of the year. Do not tell your group which line you chose.

2. Share your story with your group and see if anyone can guess your line.
3. Write an equation for each line on the graph.

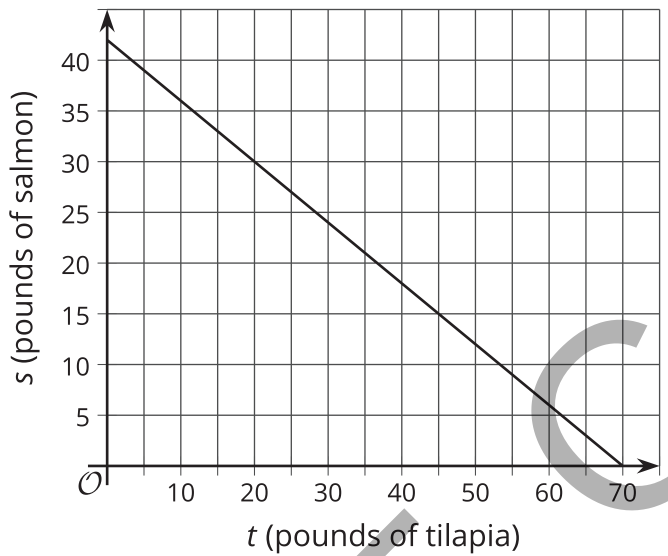
4. Predict the balance in each account after 20 weeks.

5. For which equation is $(5, 100)$ a solution? What does this solution represent in this situation?

Sec E

15.3 Fabulous Fish

The Fabulous Fish Market orders tilapia, which costs \$3 per pound, and salmon, which costs \$5 per pound. The graph shows how much of each type of fish can be purchased if the market budgets to spend \$210 on this order each day.



1. Write an equation that represents the relationship between pounds of tilapia, t, and the pounds of salmon, s, that can be purchased for \$210.

2. On the graph, plot and label the combinations A-F.

	A	B	C	D	E	F
pounds of tilapia	5	19	27	25	65	55
pounds of salmon	36	30.6	25	27	6	4

3. Which of these combinations is a possible order if the market plans to spend its entire budget of \$210? Explain your reasoning.

Sec E

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Learning Targets

Lesson 1 Understanding Proportional Relationships

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different animals.

Lesson 2 Graphs of Proportional Relationships

- I can graph a proportional relationship from an equation.
- I can tell when two graphs are of the same proportional relationship even if the scales are different.

Lesson 3 Representing Proportional Relationships

- I can scale and label coordinate axes in order to graph a proportional relationship.

Lesson 4 Comparing Proportional Relationships

- I can compare proportional relationships represented in different ways.

Lesson 5 Introduction to Linear Relationships

- I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.

Lesson 6 More Linear Relationships

- I can interpret the vertical intercept of a graph of a real-world situation.
- I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.

Lesson 7 Representations of Linear Relationships

- I can use patterns to write a linear equation to represent a situation.
- I can write an equation for the relationship between the total volume in a graduated cylinder and the number of objects added to the graduated cylinder.

Lesson 8 Translating to $y = mx + b$

- I can explain where to find the slope and vertical intercept in both an equation and its graph.
- I can write equations of lines using $y = mx + b$.

Lesson 9 Slopes Don't Have to Be Positive

- I can create a graph of a situation that has a negative slope.
- I can determine if a situation or a graph has a slope that is positive, negative, or zero and

explain how I know.

Lesson 10 Calculating Slope

- I can calculate positive and negative slopes given two points on the line.

Lesson 11 Line Designs

- Puedo describir una recta con suficiente precisión para que otro estudiante pueda dibujarla.

Lesson 12 Equations of All Kinds of Lines

- I can write equations of lines that have a positive or a negative slope.
- I can write equations of vertical and horizontal lines.

Lesson 13 Solutions to Linear Equations

- I know that the graph of an equation is a visual representation of all the solutions to the equation.
- I understand what the solution to an equation in two variables is.

Lesson 14 More Solutions to Linear Equations

- I can find solutions (x, y) to linear equations given either the x- or y-value to start from.

Lesson 15 Using Linear Relations to Solve Problems

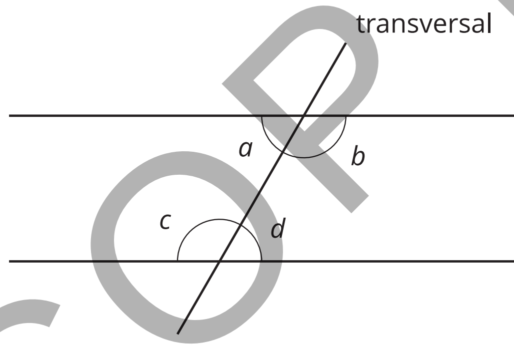
- I can write linear equations to reason about real-world situations.

Glossary

- alternate interior angles
  Alternate interior angles are created when 2 parallel lines are crossed by another line. This line is called a *transversal*. Alternate interior angles are inside the parallel lines and on opposite sides of the transversal.

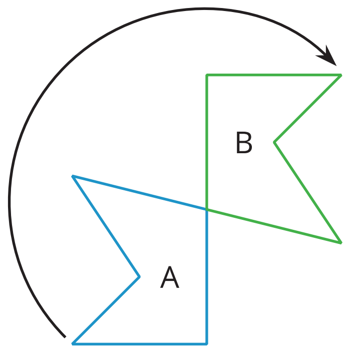
  This diagram shows 2 pairs of alternate interior angles:

  - Angles a and d
  - Angles b and c



- clockwise
  The word *clockwise* means to turn in the same direction as the hands of a clock. The top turns to the right.

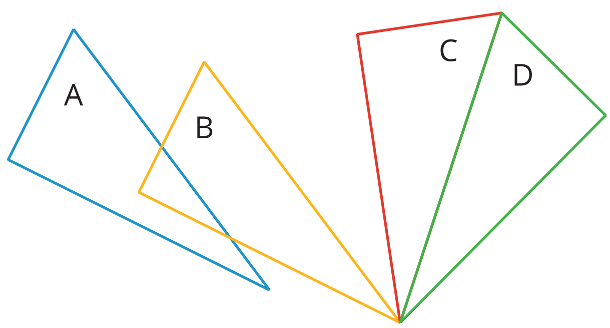
  This diagram shows that Figure A turns clockwise to make Figure B.



- congruent
  One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

  In this figure, Triangle A is congruent to Triangles B, C, and D.

  - A translation takes Triangle A to Triangle B.
  - A rotation takes Triangle B to Triangle C.
  - A reflection takes Triangle C to Triangle D.



- constant of proportionality

In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity. This number is called the *constant of proportionality*.

In this example, the constant of proportionality is 3.

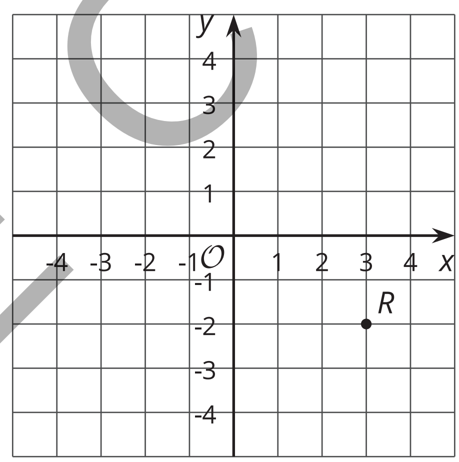
number of oranges	number of apples
2	6
3	9
5	15

$\cdot 3$ $\cdot 3$ $\cdot 3$

- coordinate plane
  The coordinate plane is one way to represent pairs of numbers. The plane is made of a horizontal number line and a vertical number line that cross at 0.

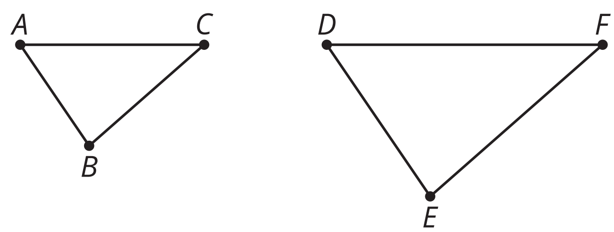
  Pairs of numbers can be used to describe the location of a point in the coordinate plane.

  Point R is located at $(3, -2)$. This means R is 3 units to the right and 2 units down from $(0, 0)$.



- corresponding
  Corresponding parts are the parts that match up between a figure and its scaled copy. They have the same relative position. Points, segments, angles, or distances can be corresponding.

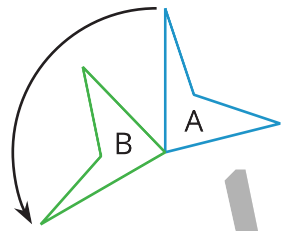
  Point B in the first triangle corresponds to point E in the second triangle. Segment AC corresponds to segment DF.



- counterclockwise

The term *counterclockwise* means to turn opposite of the way the hands of a clock turn. The top turns to the left.

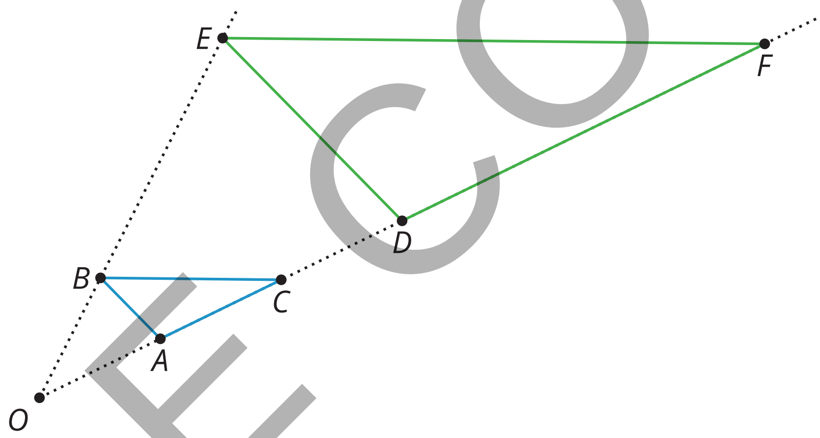
This diagram shows that Figure A turns counterclockwise to make Figure B.



- dilation
  A dilation is a transformation that can reduce or enlarge a figure. Each point on the figure moves along a line closer to or farther from a fixed point. That fixed point is the center of the dilation. All of the original distances are multiplied by the same scale factor.

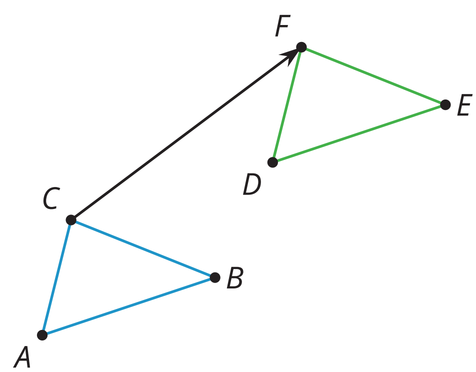
  Triangle DEF is a dilation of triangle ABC. The center of dilation is O. The scale factor is 3.

  Every point of triangle DEF is 3 times as far from O as every corresponding point of triangle ABC.



- image
  An image is the result of translations, rotations, and reflections on an object. Every part of the original object moves in the same way to match up with a part of the image.

  Triangle ABC has been translated up and to the right to make triangle DEF. Triangle DEF is the image of the original triangle ABC.

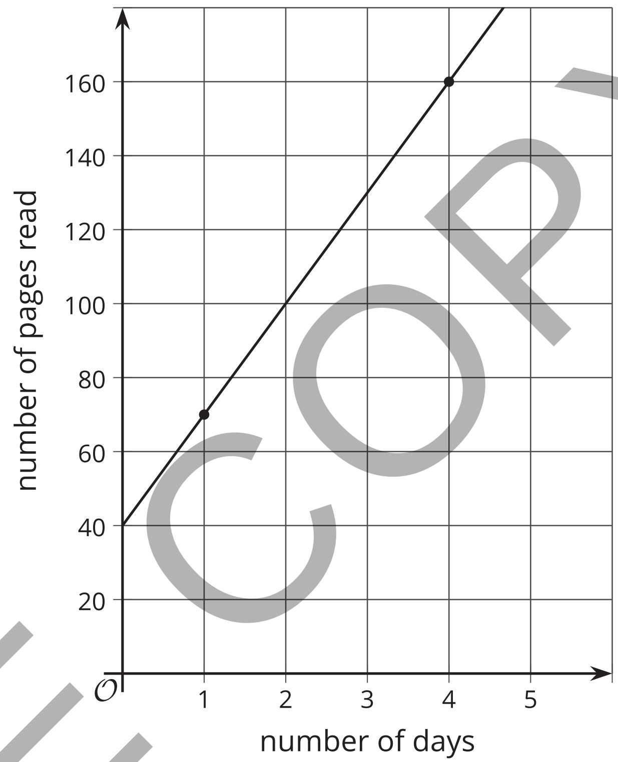


- linear relationship
  Two quantities have a linear relationship when:

- ◦ One quantity changes by a set amount, whenever the other quantity changes by a set amount.
  - ◦ The rate of change is constant.
  - ◦ The graph of the relationship is a line.

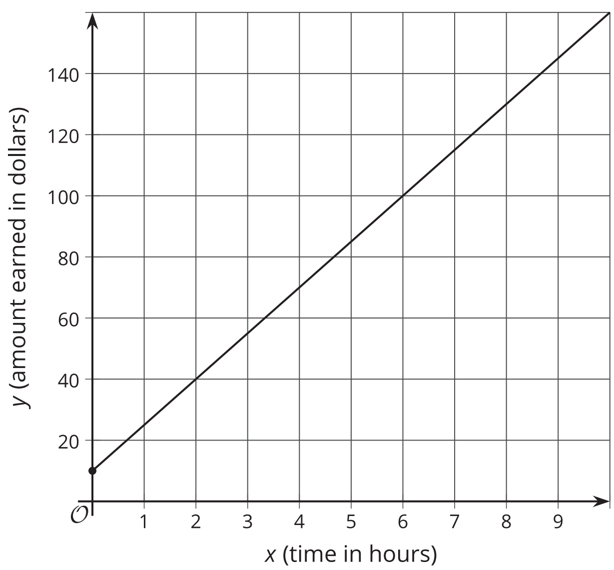
  This graph shows a linear relationship between number of days and number of pages read.

  When the number of days increases by 2, the number of pages read always increases by 60. The rate of change is constant, 30 pages per day.



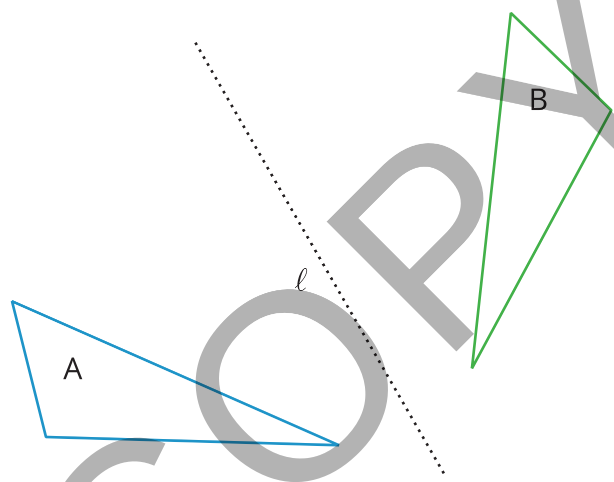
- rate of change (in a linear relationship)
  The rate of change is the amount y changes when x increases by 1. On a graph, the rate of change is the slope of the line.

  In this graph, y increases by 15 dollars when x increases by 1 hour. The rate of change is 15 dollars per hour.

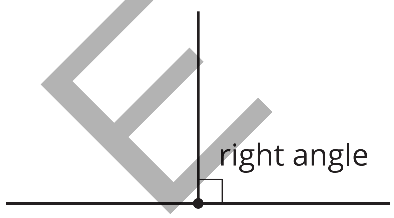


- reflection
  A reflection is a transformation that “flips” a figure over a line. Every point on the figure moves to a point directly on the opposite side of the line. The new points are the same distance from the line as they are in the original figure.

  This diagram shows a reflection of A over line ℓ that makes the mirror image B.



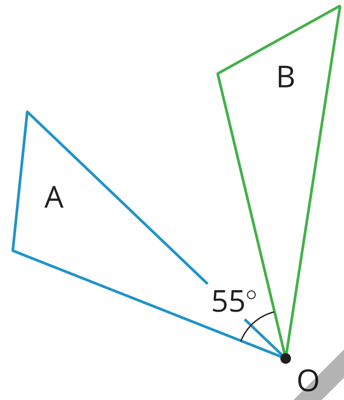
- right angle
  A right angle is half of a straight angle. It measures 90 degrees.



- rigid transformation
  A rigid transformation is a move that does not change any measurements of a figure. Translations, rotations, and reflections are rigid transformations. So is any sequence of these.

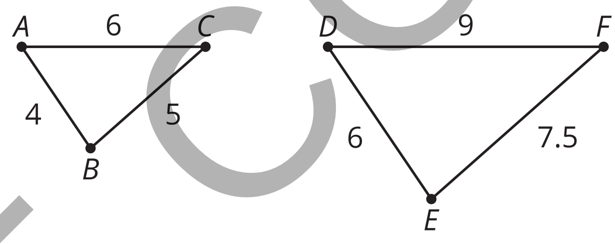
- rotation
  A rotation is a transformation that “turns” a figure. Every point on the figure moves around a center by a given angle in a specific direction.

This diagram shows Triangle A rotated around center O by 55 degrees clockwise to get Triangle B.



- scale factor
  To create a scaled copy of a figure, all the side lengths in the original figure are multiplied by the same number. This number is called the *scale factor*.

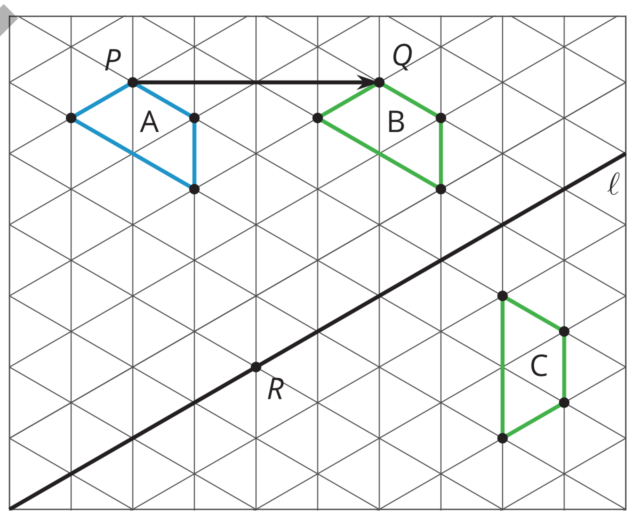
  In this example, the scale factor is 1.5, because $4 \cdot (1.5) = 6$, $5 \cdot (1.5) = 7.5$, and $6 \cdot (1.5) = 9$.



- sequence of transformations
  A sequence of transformations is a set of translations, rotations, reflections, and dilations on a figure. The transformations are performed in a given order.

  This diagram shows a sequence of transformations to move Figure A to Figure C.

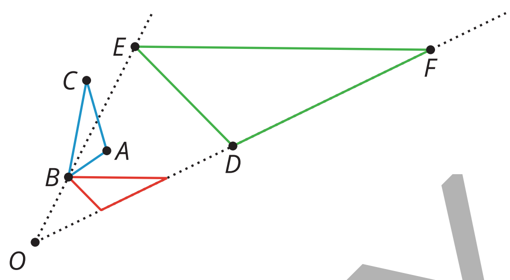
  First, A is translated to the right to make B. Next, B is reflected across line ℓ to make C.



- similar
  Two figures are similar if one can fit exactly over the other after transformations.

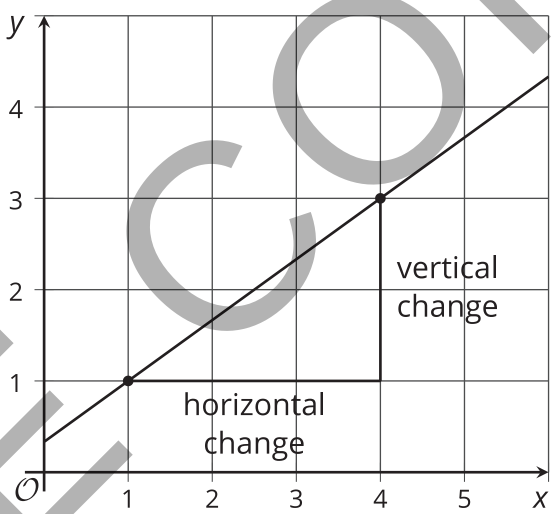
This figure shows triangle ABC is similar to triangle DEF.

- Rotate triangle ABC around point B.
- Then dilate it with center point O.
- The image will fit exactly over triangle DEF.



- slope
  Slope is a number that describes how steep a line is. To find the slope, divide the vertical change by the horizontal change for any 2 points on the line.

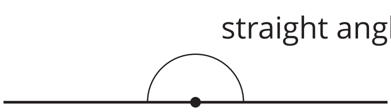
  The slope of this line is 2 divided by 3, or $\frac{2}{3}$.



- solution to an equation with two variables
  A solution to an equation with 2 variables is a pair of values for the variables that make the equation true.

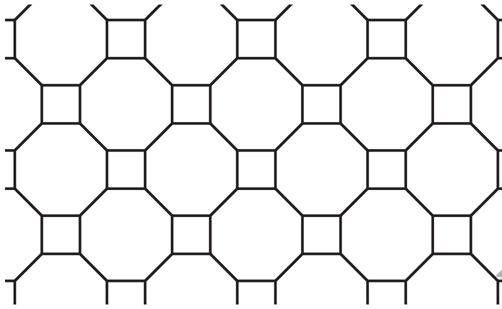
  For example, one solution to the equation $4x + 3y = 24$ is $(6, 0)$. Substituting 6 for x and 0 for y makes this equation true because $4(6) + 3(0) = 24$.

- straight angle
  A straight angle is an angle that forms a straight line. It measures 180 degrees.



- tessellation
  A tessellation is a repeating pattern of 1 or more shapes. The sides of the shapes fit together with no gaps or overlaps. The pattern goes on forever in all directions.

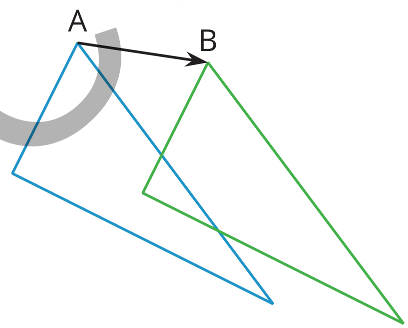
This diagram shows part of a tessellation.



- transformation
  A transformation is a translation, rotation, reflection, or dilation, or a combination of these.

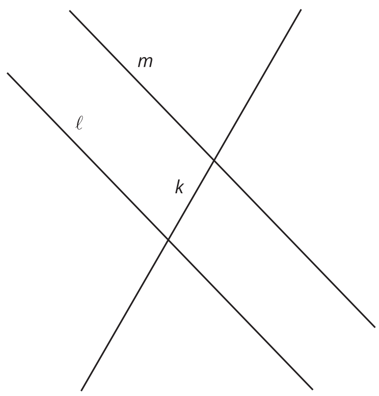
- translation
  A translation is a transformation that “slides” a figure along a straight line. Every point on the figure moves a given distance in a given direction.

  This diagram shows a translation of Figure A to Figure B using the direction and distance given by the arrow.



- transversal
  A transversal is a line that crosses parallel lines.

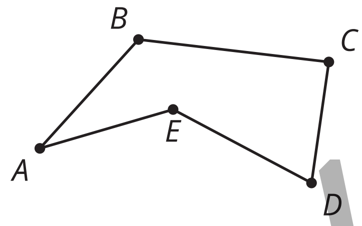
  This diagram shows a transversal line k intersecting parallel lines m and ℓ.



- vertex

A vertex is a point where 2 or more edges meet. When there is more than 1 vertex, they are called *vertices*.

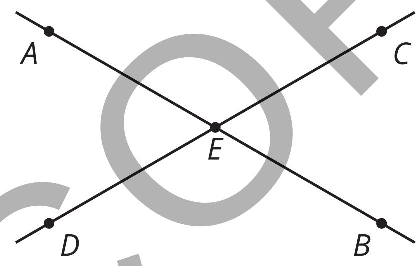
The vertices of this polygon are labeled A, B, C, D, and E.



- vertical angles
  Vertical angles are opposite angles that share the same vertex. They are formed when two lines cross each other. Their angle measures are equal.

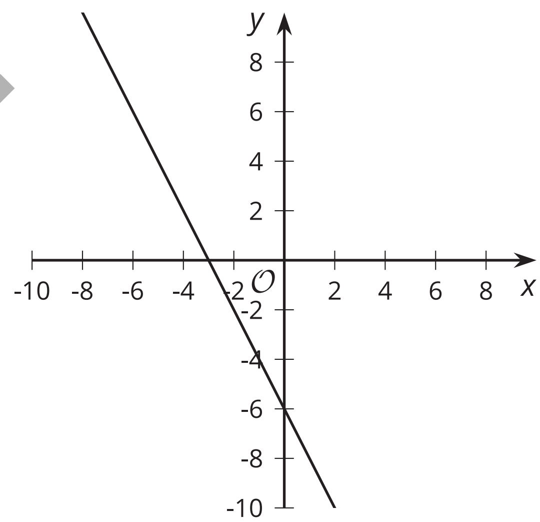
  Angles AEC and DEB are vertical angles. If angle AEC measures 120°, then angle DEB must also measure 120°.

  Angles AED and BEC are another pair of vertical angles.



- vertical intercept
  The vertical intercept is the point where the graph of a line crosses the vertical axis.

  The vertical intercept of this line is $(0, -6)$ or just -6.



Attributions

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California Common Core State Standards for Mathematics (CA CCSSM) References

8.EE: Grade 3 – Expressions and Equations

Work with radicals and integer exponents.

8.EE.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

8.EE.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

8.EE.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.6
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7
Solve linear equations in one variable.

8.EE.7a
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

8.EE.7b
Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8
Analyze and solve pairs of simultaneous linear equations.

8.EE.8a
Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.8b
Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

8.EE.8c
Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

8.F: Grade 8 – Functions

Define, evaluate, and compare functions.

8.F.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.

8.F.2
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.3
Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1, 1)$, $(2, 4)$ and $(3, 9)$, which are not on a straight line.

Use functions to model relationships between quantities.

8.F.4
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

8.G: Grade 8 – Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1
Verify experimentally the properties of rotations, reflections, and translations:

8.G.1a
Lines are taken to lines, and line segments to line segments of the same length.

8.G.1b
Angles are taken to angles of the same measure.

8.G.1c
Parallel lines are taken to parallel lines.

8.G.2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

8.G.5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

8.G.6
Explain a proof of the Pythagorean Theorem and its converse.

8.G.7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

8.NS: Grade 8 – The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

8.SP: Grade 8 – Statistics and Probability

Investigate patterns of association in bivariate data.

8.SP.1
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP.4
Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.