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An IM K–12 Math Curriculum

GRADE 8

Student Edition

UNITS 7-9

Illustrative® Mathematics
LEARN MATH FOR LIFE
Kendall Hunt

Book 3

Certified by Illustrative Mathematics®

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Table of Contents

UNIT 9 PUTTING IT ALL TOGETHER

GRADE 8

UNIT

7

Exponents and Scientific Notation

Content Connections

In this unit you will work with powers of 10, exponents, place value, and scientific notation. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while using scientific notation to investigate problems that include measurements of big and small numbers.
- **Discovering Shape and Space** while investigating and using the connections between integer exponents and area and volume.
- **Reasoning with Data** while conducting explorations involving numbers expressed in scientific notation using technology.

Addressing the Standards

As you work your way through **Unit 7 Exponents and Scientific Notation,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 1 and 14
MP2 Reason abstractly and quantitatively.	Lessons 2, 10, 11, 12, 14, 15, and 16
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 4, 13, and 15
MP4 Model with mathematics.	Lessons 12 and 16
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lessons 1, 7, 8, 9, 11, 14, 15, and 16
MP7 Look for and make use of structure.	Lessons 4, 5, 7, 8, 10, 11, and 13
MP8 Look for and express regularity in repeated reasoning.	Lessons 1, 2, 3, 4, 5, 6, and 8

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows the standards being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Pythagorean Explorations • Big and Small Numbers • Shape, Number, and Expressions	**8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, 32 × 3–5 = 3–3 = 1/33 = 1/27.*	Lessons 2, 3, 4, 5, 6, 7, 8, 9, 11, and 14
• Big and Small Numbers	**8.EE.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.*	Lessons 9, 10, 11, 12, 14, and 16

Big Ideas You Are Studying	**California Content Standards**	**Lessons Where You Learn This**
• Data Explorations • Big and Small Numbers	**8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	Lessons 10, 11, 12, 13, 14, 15, and 16

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 7, Lesson 1

Building on CA CCSSM 6.EE.1; building towards 8.EE.1; practicing MP1, MP6, MP8

Exponent Review

Let's review exponents.

1.1 Which Three Go Together: Fours

Which three go together? Why do they go together?

A

2^4

B

4^2

C

$4^4 - 240$

D

4^4

1.2 Building Blocks

Mai and Andre found some connecting cubes and took turns building towers made of single cubes stacked on top of each other.

- Mai went first and built a tower 2 cubes tall.
- Andre went second and built a tower 4 cubes tall.
- Mai went third and built a tower 8 cubes tall.
- They each tried to build a tower that was double the height of the previous tower.

1. How many cubes would be needed to build the 7th tower? Explain your reasoning.

2. The number of cubes needed to build the 25th tower is very, very large. Write an expression to represent this number without computing its value.

3. The 28th tower would require even more cubes than the 25th tower. How many times more cubes are needed to build the 28th tower compared to the 25th tower?

1.3 Shrinking Tower

Imagine a tall tower that is different from any other tower. One day this tower is only half as tall as it was the day before!

- On the second day, the tower is $\frac{1}{4}$ of its original height.
- On the third day, the tower is $\frac{1}{8}$ of its original height.

1. What fraction of the original height is the tower after 6 days?

2. What fraction of the original height is the tower after 28 days? Write an expression to describe this without computing its value.

3. Will the tower ever disappear completely? If so, after how many days?

Are you ready for more?

A rancher is tracking the ancestry of his prize cattle. Each cow has 2 parents and each parent also has two parents.

1. Draw a family tree showing a cow, its parents, its grandparents, and its great-grandparents.

2. We say that the cow's eight great-grandparents are “three generations back” from the cow. At which generation back would a cow have 262,144 ancestors?

Lesson 1 Summary

Consider the expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. Written this way using multiplication, we would need to count the number of factors. Written as 2^6, the **base** of 2 and the **exponent** of 6 make it easy to see that there are 6 factors of 2 being multiplied together. Exponents make it easy to show repeated multiplication — imagine writing out 2^{100} using multiplication!

Here is another example. Let's say that you start out with one grain of rice and that each day the number of grains of rice you have doubles. So on day one, you have 2 grains, on day two, you have 4 grains, and so on. When we write 2^{25}, we can see from the expression that the rice has doubled 25 times. So this notation is not only convenient, but it also helps us see structure: In this case, we can see right away that we have been doubling the amount of rice each day for 25 days! That's a lot of rice (more than a cubic meter)!

Glossary

- base (of an exponent)
- exponent

Practice Problems

1 Write each expression using an exponent:

a. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

b. $\left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

c. $(9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$

d. The number of coins Jada will have on the eighth day, if Jada starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.)

2 Evaluate each expression:

a. 2^5

b. 3^3

c. 4^3

d. 6^2

e. $\left(\frac{1}{2}\right)^4$

f. $\left(\frac{1}{3}\right)^2$

3 Clare made $160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare doesn't touch the money in her account, she can find the amount she'll have the next year by multiplying her current amount by 1.03.

a. How much money will Clare have in her account after 1 year? After 2 years?

b. How much money will Clare have in her account after 5 years? Explain your reasoning.

c. Write an expression for the amount of money Clare would have after 30 years if she never withdraws money from the account.

4 from Unit 3, Lesson 1

The equation $y = 5{,}280x$ gives the number of feet, y, in x miles. What does the number 5,280 represent in this relationship?

5 from Unit 3, Lesson 5

The points $(2, 4)$ and $(6, 7)$ lie on a line. What is the slope of the line?

A. 2

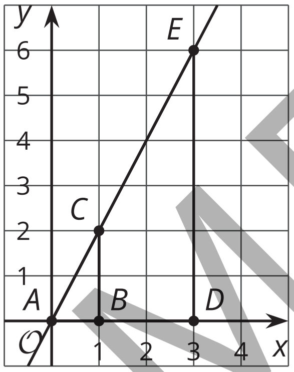
B. 1

C. $\frac{4}{3}$

D. $\frac{3}{4}$

6 from Unit 2, Lesson 6

The diagram shows a pair of similar figures, triangle and triangle . What is the center of dilation and the scale factor that moves the triangle to triangle ?



Unit 7, Lesson 2

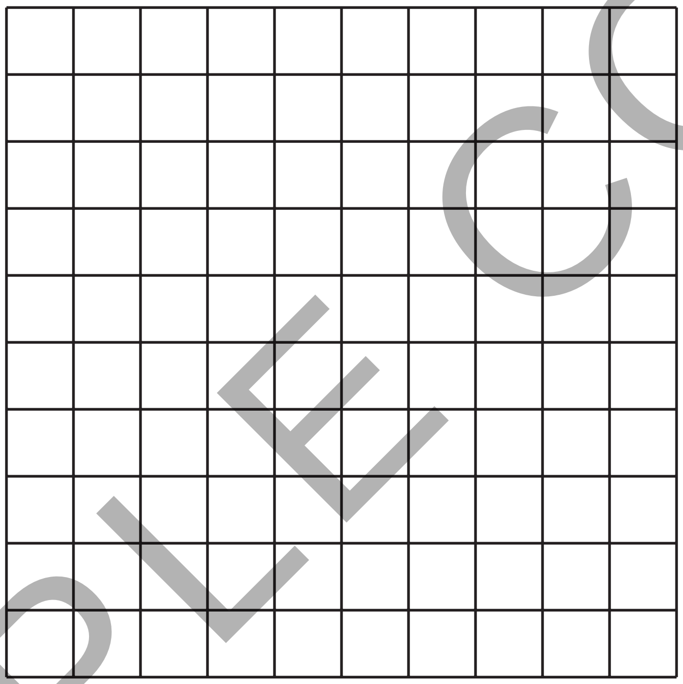
Addressing CA CCSSM 8.EE.1; building on 5.NBT.3a; building towards 8.EE.1, 8.EE.3, 8.EE.4; practicing MP2 and MP8

Multiplying Powers of 10

Let's explore patterns with exponents when we multiply powers of 10.

2.1 100, 1, or $\frac{1}{100}$?

Clare, Tyler, and Mai are looking at the diagram.

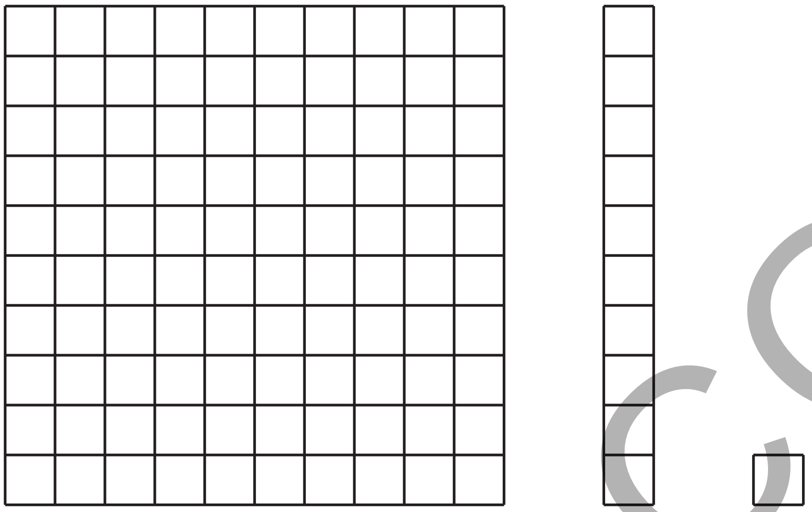


- Clare said she sees 100.
- Tyler says he sees 1.
- Mai says she sees $\frac{1}{100}$.

Whom do you agree with? Be prepared to explain your reasoning.

2.2 Picture a Power of 10

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could the large square be represented as a power of 10? Explain your reasoning.

2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?

3. If the medium rectangle represents 10^5, then what does the large square represent? The small square?

4. If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?

2.3 Multiplying Powers of Ten

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Are you ready for more?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?
2. How about 10^7? 10^8?

Lesson 2 Summary

In this lesson, we developed a rule for multiplying powers of 10: Multiplying powers of 10 corresponds to adding the exponents together.

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} \cdot \underbrace{(10 \cdot 10 \cdot 10)}_{\text{three factors that are ten}} = 10^5$ two factors that are ten · three factors that are ten = five factors that are ten

To see this, multiply 10^2 and 10^3. We know that 10^2 has two factors that are 10 and that 10^3 has three factors that are 10. That means that $10^2 \cdot 10^3$ has 5 factors that are 10.

This will work for other powers of 10, too. For example, $10^{14} \cdot 10^{47} = 10^{(14+47)} = 10^{61}$.

Practice Problems

1 Write each expression as a single power of 10:

a. $10^3 \cdot 10^9$

b. $10 \cdot 10^4$

c. $10^{10} \cdot 10^7$

d. $10^3 \cdot 10^3$

e. $10^5 \cdot 10^{12}$

f. $10^6 \cdot 10^6 \cdot 10^6$

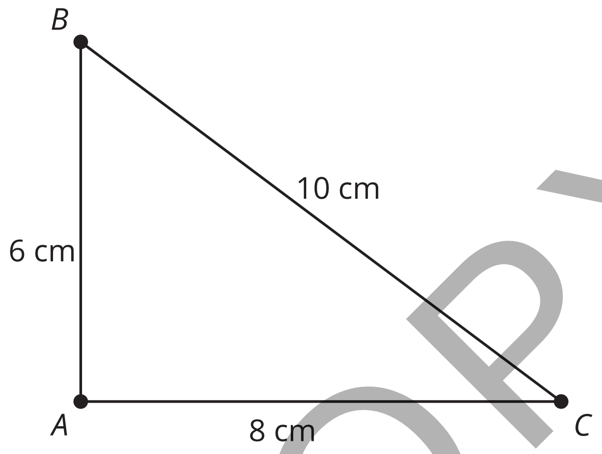
2 A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water. Express your answers both as a single power of 10 and as a number in standard form.

a. What is the area of the surface of the water in the pool?

b. How much water does the pool hold?

3 from Unit 2, Lesson 7

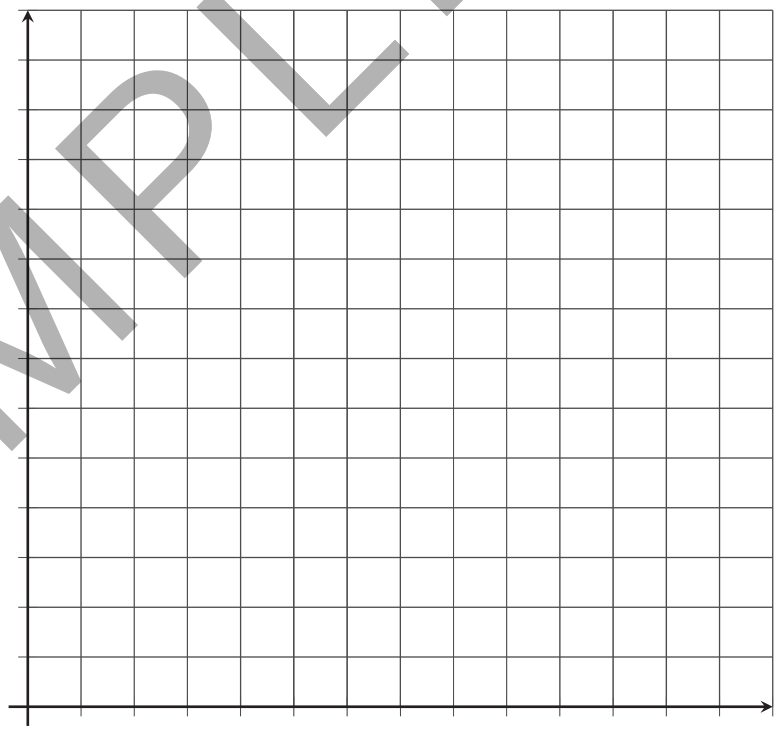
Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



4 from Unit 3, Lesson 3

Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



Unit 7, Lesson 3

Addressing CA CCSSM 8.EE.1; building towards 8.EE.4; practicing MP8

Powers of Powers of 10

Let's look at powers of powers of 10.

3.1 Big Cube

What is the volume of a giant cube that measures 10,000 km on each side? Be prepared to explain your reasoning.

3.2 Raising Powers of 10 to Another Power

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^n)^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

Are you ready for more?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Lesson 3 Summary

In this lesson, we developed a rule for raising a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents.

Rule	Example showing how it works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = \underline{(10 \cdot 10)} \cdot \underline{(10 \cdot 10)} \cdot \underline{(10 \cdot 10)} = 10^6$ three groups of two factors that are ten = six factors that are ten

To understand this, take 10^2 and raise it to the power of 3. We know that 10^2 has two factors that are 10. Raising 10^2 to the power of 3 means that there are three groups of two factors that are 10, for a total of 6 factors that are 10, or 10^6.

This works for any power of 10 raised to another power. For example, $(10^6)^{11} = 10^{(6 \cdot 11)} = 10^{66}$.

Practice Problems

1 Write each expression with a single exponent:

a. $(10^7)^2$

b. $(10^9)^3$

c. $(10^6)^3$

d. $(10^2)^3$

e. $(10^3)^2$

f. $(10^5)^7$

2 You have 1,000,000 number cubes, each measuring one inch on a side.

a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

3 Select **all** the expressions that are equivalent to 10^{10}

A. $(10^5)^2$

B. $10^5 \cdot 10^2$

C. $(10^2)^5$

D. $10^5 \cdot 10^5$

E. $(10^3)^7$

4 from Unit 7, Lesson 1

An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

a. How many amoebas are there after 1 hour?

b. How many amoebas are there after 2 hours?

c. Write an expression for the number of amoebas after 6 hours.

d. Write an expression for the number of amoebas after 24 hours.

e. Why might exponential notation be preferable to answer these questions?

5 from Unit 3, Lesson 13

You have two numbers, (x, y). If you triple the second number and subtract it from the first, the difference is 12.

a. Write an equation that describes the statement.

b. Is $(9, 1)$ a solution to your equation? Explain how you know.

c. Find the second number if the first is 18.

d. Find the first number if the second number is $\frac{1}{3}$.

Unit 7, Lesson 4

Addressing CA CCSSM 8.EE.1; building on 5.NF.5b; building towards 8.EE.1; practicing MP3, MP7, MP8

Dividing Powers of 10

Let's explore patterns with exponents when we divide powers of 10.

4.1 A Surprising One

What is the value of the expression? Be prepared to explain your reasoning.

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

4.2 Dividing Powers of Ten

1. a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the “expanded” column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Are you ready for more?

expression	expanded	single power
$10^4 \div 10^6$		

4.3 Zero Exponent

So far we have looked at powers of 10 with exponents greater than 0. Consider what would happen to our patterns if we included 0 as a possible exponent?

1. a. Write $10^{12} \cdot 10^0$ as a single power of 10. Explain or show your reasoning.

   b. What number could you multiply 10^{12} by to get this same answer?

2. a. Write $\frac{10^8}{10^0}$ as a single power of 10. Explain or show your reasoning.

   b. What number could you divide 10^8 by to get this same answer?

3. In order for the exponent rules we found to work even when the exponent is 0, what does the value of 10^0 have to be?

4.4 Making Millions

Write as many expressions as you can that have the same value as 10^8. Focus on using exponents, multiplication, and division.

Lesson 4 Summary

In this lesson, we developed a rule for dividing powers of 10: Dividing powers of 10 is the same as subtracting the exponent of the denominator from the exponent of the numerator. To see this, take 10^5 and divide it by 10^2.

Rule	Example showing how it works
$\frac{10^n}{10^m} = 10^{n-m}$	$\frac{10^5}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10^3 = 10^3$ five factors that are ten ÷ two factors that are ten = three factors that are ten

We know that 10^5 has 5 factors that are 10, and 2 of these factors can be divided by the 2 factors of 10 in 10^2 to make 1. That leaves $5 - 2 = 3$ factors of 10, or 10^3.

This will work for other powers of 10, too. For example $\frac{10^{56}}{10^{23}} = 10^{56-23} = 10^{33}$.

This rule also extends to 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0 doesn't change its value. That means if we want the rule to work when the exponent is 0, then 10^0 must equal 1.

Rule	Example showing how it works
$10^0 = 1$	$\frac{10^6}{10^0} = 10^{6-0} = 10^6$ this value must be equal to 1

Practice Problems

1 Write each expression as a single power of 10.

a. $\frac{10^6}{10^3}$

b. $\frac{10^{15}}{10^9}$

c. $\frac{10^{57}}{10^{19}}$

d. $\frac{10^{11} \cdot 10^{14}}{10^8}$

2 Write each expression as a single power of 10.

a. $\frac{10^3 \cdot 10^4}{10^5}$

b. $(10^4) \cdot \frac{10^{12}}{10^7}$

c. $\left(\frac{10^5}{10^3}\right)^4$

d. $\frac{10^4 \cdot 10^5 \cdot 10^6}{10^3 \cdot 10^7}$

e. $\frac{(10^5)^2}{(10^2)^3}$

3 The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain how you know.

4 from Unit 5, Lesson 3

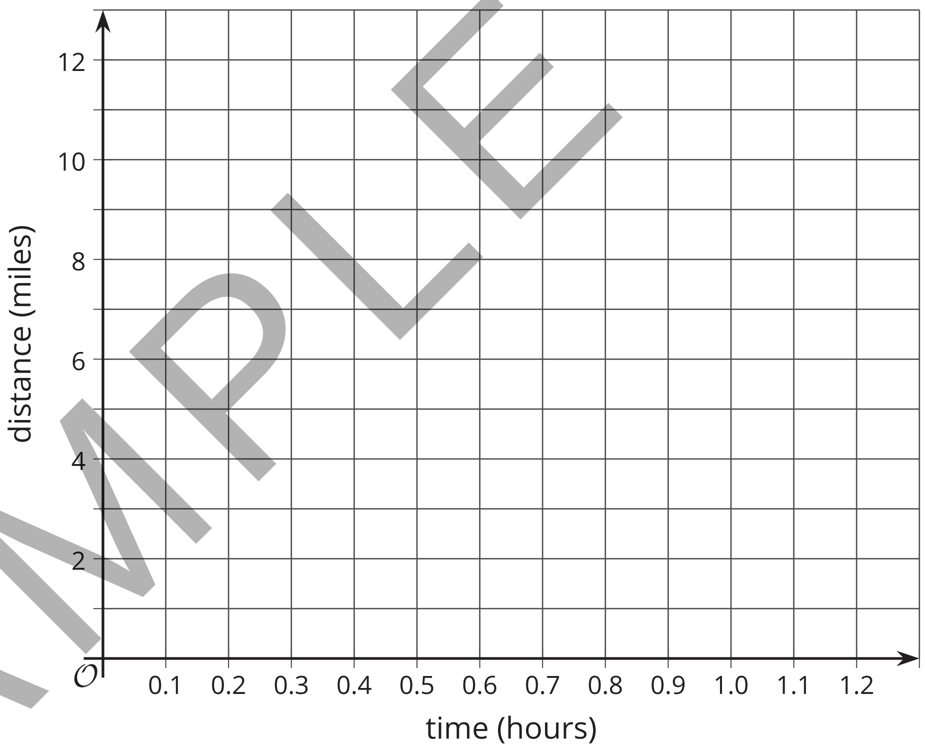
Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

a. Write an equation relating the two variables.

b. How many pounds of bananas can Kiran buy if he buys 2 pounds of guavas?

5 from Unit 3, Lesson 1

Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.



Unit 7, Lesson 5

Addressing CA CCSSM 8.EE.1; building on 5.NBT.2; practicing MP7 and MP8

Negative Exponents with Powers of 10

Let’s see what happens when exponents are negative.

5.1 Math Talk: What's That Exponent?

Find the value of x mentally.

- $\frac{100}{1} = 10^x$
- $\frac{100}{x} = 10^1$
- $\frac{x}{100} = 10^0$
- $\frac{100}{1000} = 10^x$

5.2 Negative Exponent Table

· 10 · 10 · 10 · 10 · 10 · 10

using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$

· ? · ? · ? · ? · ? · ?

1. Complete the table to explore what negative exponents mean.

2. As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?

3. How does a multiplier of 10 affect the exponent? How does it affect the value of the decimal and fraction?

4. How does the other multiplier affect the exponent? How does it affect the value of the decimal and fraction?

5. Use the patterns you found in the table to write 10^{-7} as a fraction.

6. Use the patterns you found in the table to write 10^{-5} as a decimal.

7. Write $\frac{1}{100,000,000}$ using a single exponent.

8. Use the patterns in the table to write 10^{-n} as a fraction.

5.3 Follow the Exponent Rules

1. a. Match each exponential expression with an equivalent multiplication expression:

$(10^2)^3$

$(10^2)^{-3}$

$(10^{-2})^3$

$(10^{-2})^{-3}$

$\frac{1}{(10\cdot 10)} \cdot \frac{1}{(10\cdot 10)} \cdot \frac{1}{(10\cdot 10)}$
$\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)$
$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$
$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$

b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$

$\frac{10^2}{10^{-5}}$

$\frac{10^{-2}}{10^5}$

$\frac{10^{-2}}{10^{-5}}$

$\frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$
$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{\frac{1}{10} \cdot \frac{1}{10}}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4 \cdot 10^3$

$10^4 \cdot 10^{-3}$

$10^{-4} \cdot 10^3$

$10^{-4} \cdot 10^{-3}$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (10 \cdot 10 \cdot 10)$
$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

b. Write $10^{-4} \cdot 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Are you ready for more?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, “I knew I’d be the only one left, since I went first.”

1. Record this game on paper a few times with different numbers of players. Does the person who starts always win?

2. Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Lesson 5 Summary

In this lesson, we observed that when we multiply a positive power of 10 by $\frac{1}{10}$, the exponent decreases by 1. For example, $10^8 \cdot \frac{1}{10} = 10^7$. This is true for any power of 10.

By using the rule $10^n \cdot 10^m = 10^{n+m}$ with this example, we see that: $10^8 \cdot 10^{-1} = 10^7$.

Notice that for the exponent rules we have developed to work, then $\frac{1}{10}$ must equal 10^{-1}.

Rule	Example showing how it works
$10^{-n} = \frac{1}{10^n}$	$10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^3}$
	three factors that are one tenth

Practice Problems

1 Write each expression using a single negative exponent.

a. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

b. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

c. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^2$

d. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^3$

e. $(10 \cdot 10 \cdot 10)^{-2}$

2 Write each expression as a single power of 10.

a. $10^{-3} \cdot 10^{-2}$

b. $10^{4} \cdot 10^{-1}$

c. $\frac{10^5}{10^7}$

d. $(10^{-4})^5$

e. $10^{-3} \cdot 10^{2}$

f. $\frac{10^{-9}}{10^5}$

3 Select **all** of the following that are equivalent to $\frac{1}{10{,}000}$:

A. $(10{,}000)^{-1}$

B. $(-10{,}000)$

C. $(100)^{-2}$

D. $(10)^{-4}$

E. $(-10)^{2}$

4 from Unit 3, Lesson 2

Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

Equations:

a. $y = 3x$

b. $\frac{1}{2}x = y$

c. $y = 3.5x$

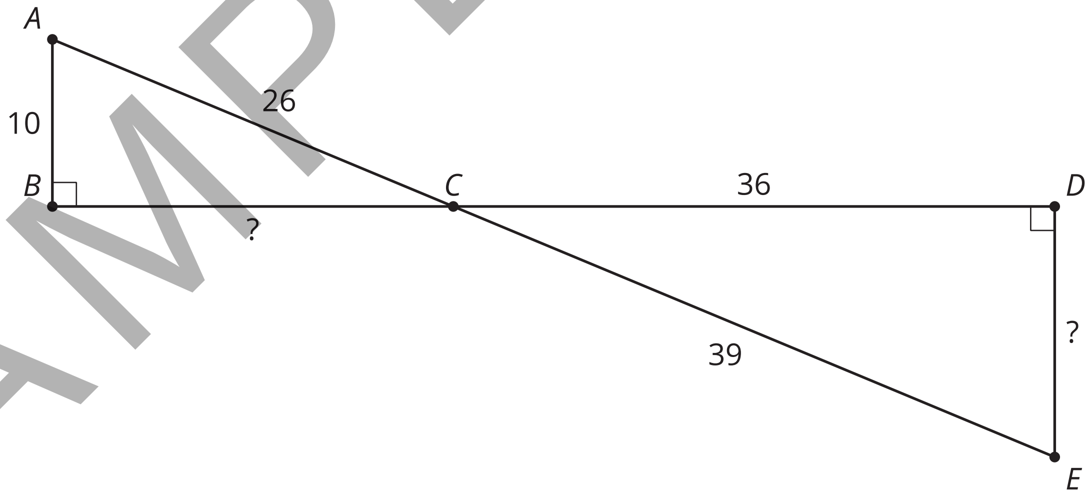
d. $y = \frac{5}{2}x$

Situations:

- ○ A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.
- ○ I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
- ○ A store has a “4 for $10” sale on hats.
- ○ For every 48 apples I pick, my students get 24.

5 from Unit 2, Lesson 8

a. Explain why triangle ABC is similar to EDC.



b. Find the missing side lengths.

Unit 7, Lesson 6

Addressing CA CCSSM 8.EE.1; practicing MP8

What About Other Bases?

Let’s explore exponent patterns with bases other than 10.

6.1 Math Talk: Comparing Expressions with Exponents

Decide mentally whether each statement is true.

- $3^5 < 4^6$
- $(-3)^2 < 3^2$
- $(-3)^3 = 3^3$
- $(-5)^2 > -5^2$

6.2 What Happens with Zero and Negative Exponents?

Complete the table.

$\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$

value	16					$\frac{1}{2}$			
exponent form	2^4								

$\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right?
2. Use the patterns you found in the table to write 2^{-6} as a fraction.
3. Write $\frac{1}{32}$ as a power of 2 with a single exponent.
4. What is the value of 2^0?
5. From the work you have done with negative exponents, how would you write 5^{-3} as a fraction?
6. How would you write 3^{-4} as a fraction?

Are you ready for more?

1. Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.

2. Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.

3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

6.3 Exponent Rules with Bases Other than 10

Lin, Noah, Diego, and Elena each chose an expression to start with and then came up with a new list of expressions — some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is 5^{-9} and her list is:

   $(5^3)^{-3}$ -5^9 $\frac{5^{-6}}{5^3}$ $(5^3)^{-2}$ $\frac{5^{-4}}{5^{-5}}$ $5^{-4} \cdot 5^{-5}$

2. Noah's original expression is 3^{10} and his list is:

   $3^5 \cdot 3^2$ $(3^5)^2$ $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$

   $\left(\frac{1}{3}\right)^{-10}$ $3^7 \cdot 3^3$ $\frac{3^{20}}{3^{10}}$ $\frac{3^{20}}{3^2}$

3. Diego's original expression is x^4 and his list is:

   $\frac{x^8}{x^4}$ $x \cdot x \cdot x \cdot x$ $\frac{x^{-4}}{x^{-8}}$ $\frac{x^{-4}}{x^8}$

   $(x^2)^2$ $4 \cdot x$ $x \cdot x^3$

4. Elena's original expression is 8^0 and her list is:

   1 0 $8^3 \cdot 8^{-3}$

   $\frac{8^2}{8^2}$ 10^0 11^0

Lesson 6 Summary

We can keep track of repeated factors using exponent rules. These rules also help us make sense of negative exponents and why a number to the power of 0 is defined as 1. These rules can be written symbolically where the base a can be any positive number:

Rule	Example showing how it works
$a^n \cdot a^m = a^{n+m}$	$a^2 \cdot a^3 = (a \cdot a) \cdot (a \cdot a \cdot a) = a^5$ two factors that are *a* · three factors that are *a* = five factors that are *a*
$\left(a^n\right)^m = a^{n \cdot m}$	$\left(a^2\right)^3 = \underline{(a \cdot a)} \cdot \underline{(a \cdot a)} \cdot \underline{(a \cdot a)} = a^6$ three groups of two factors that are *a* = six factors that are *a*
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a = 1 \cdot a^3 = a^3$ five factors that are *a* ÷ two factors that are *a* = three factors that are *a*
$a^0 = 1$	$\frac{a^6}{a^0} = a^{6-0} = a^6$ this value must be equal to 1
$a^{-n} = \frac{1}{a^n}$	$a^{-3} = \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} = \frac{1}{a^3}$ three factors that are one over *a*

Practice Problems

1 Priya says "I can figure out 5^0 by looking at other powers of 5. For example, $5^3 = 125$, $5^2 = 25$, and $5^1 = 5$."

a. What pattern do you notice?

b. If this pattern continues, what should be the value of 5^0? Explain your reasoning.

c. If this pattern continues, what should be the value of 5^{-1}? Explain your reasoning.

2 Select **all** the expressions that are equivalent to 4^{-3}.

A. -12

B. 2^{-6}

C. $\frac{1}{4^3}$

D. $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

E. 12

F. $(-4) \cdot (-4) \cdot (-4)$

G. $\frac{8^{-1}}{2^2}$

3 Write each expression using a single exponent.

a. $\frac{5^3}{5^6}$

b. $(14^3)^6$

c. $8^3 \cdot 8^6$

d. $\frac{16^6}{16^3}$

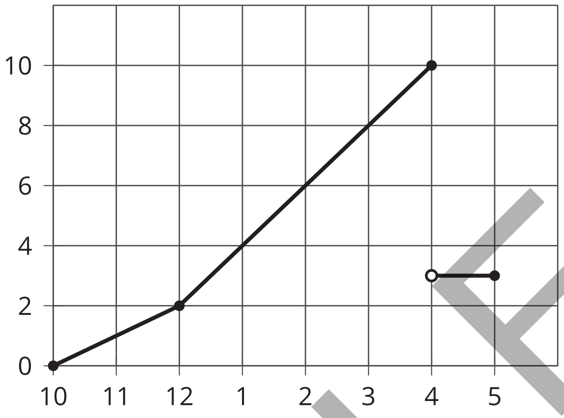
e. $(21^3)^{-6}$

4 from Unit 5, Lesson 6

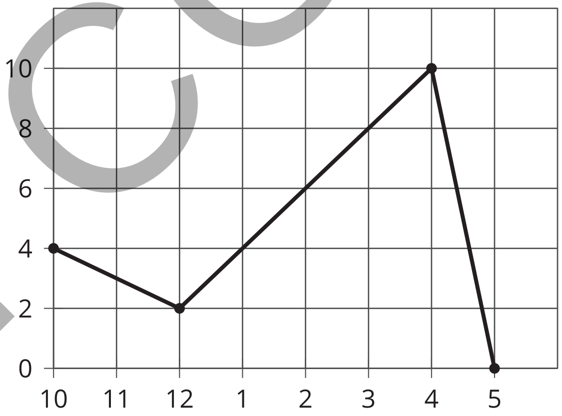
Andre sets up a rain gauge to measure rainfall in his backyard. On Tuesday, it rains off and on all day.

- He starts at 10 a.m. with an empty gauge when it starts to rain.
- Two hours later, he checks, and the gauge has 2 cm of water in it.
- It starts raining even harder, and at 4 p.m., the rain stops, so Andre checks the rain gauge and finds it has 10 cm of water in it.
- While checking it, he accidentally knocks the rain gauge over and spills most of the water, leaving only 3 cm of water in the rain gauge.
- When he checks for the last time at 5 p.m., there is no change.

Graph A



Graph B



a. Which of the two graphs could represent Andre's story? Explain your reasoning.

b. Label the axes of the correct graph with appropriate units.

c. Use the graph to determine how much total rain fell on Tuesday.

Unit 7, Lesson 7

Addressing CA CCSSM 8.EE.1; practicing MP6 and MP7

Practice with Rational Bases

Let's practice with exponents.

7.1 Which Three Go Together: Exponents

Which three go together? Why do they go together?

A

$$\frac{2^8}{2^5}$$

B

$$(4^{-5})^8$$

C

$$\left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8$$

D

$$\frac{10^{-8}}{5^5}$$

7.2 Exponent Rule Practice

For each expression, write at least 3 different equivalent expressions.

1. $(6^2)^4$

2. $\frac{4^5}{4^{-8}}$

3. 3^{-12}

7.3 Inconsistent Bases

Mark each equation as true or false. What could you change about the false equations to make them true?

1. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^6$

2. $5^4 + 5^5 = 5^9$

3. $\left(\frac{1}{2}\right)^4 \cdot 10^3 = 5^7$

4. $3^2 \cdot 5^2 = 15^2$

Are you ready for more?

Solve this equation: $3^{x-5} = 9^{x+4}$. Explain or show your reasoning.

Lesson 7 Summary

We can keep track of repeated factors using exponent rules. These exponent rules work with other bases in exactly the same way as they did with a base of 10. For example,

$$\begin{aligned} 7^5 \cdot 7^3 &= 7^{5+3} \\ (2^4)^3 &= 2^{4 \cdot 3} \\ \frac{\left(\frac{1}{3}\right)^4}{\left(\frac{1}{3}\right)^2} &= \left(\frac{1}{3}\right)^{4-2} \end{aligned}$$

The exponent rules also work with negative exponents. For example, to write 5^{-6} with a single positive exponent, we can write $\frac{1}{5^6}$.

These rules do not work when the bases are not the same. For example $\frac{6^5}{3^2} \neq 2^3$. We can check this by expanding the factors: $\frac{6^5}{3^2} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3}$, which is not equal to 3 factors that are 2.

Practice Problems

1 Write with a single exponent:

a. $\frac{7^6}{7^2}$

b. $(11^4)^5$

c. $4^2 \cdot 4^6$

d. $6 \cdot 6^8$

e. $(12^2)^7$

f. $\frac{3^{10}}{3}$

g. $(0.173)^9 \cdot (0.173)^2$

h. $\frac{0.87^5}{0.87^3}$

i. $\frac{(\frac{5}{2})^8}{(\frac{5}{2})^6}$

2 Determine whether each of the following equations is true or false? Explain or show your reasoning.

a. $(7^2)^3 = 7^5$

b. $2^4 \cdot 2 = 2^5$

c. $2^2 \cdot 3^4 = 6^6$

d. $((\frac{2}{3})^2)^4 = (\frac{2}{3})^8$

3 Noah says that $2^4 \cdot 3^2 = 6^6$. Tyler says that $2^4 \cdot 4^2 = 16^2$.

a. Do you agree with Noah? Explain or show your reasoning.

b. Do you agree with Tyler? Explain or show your reasoning.

4 from Unit 4, Lesson 13

Lin says that the system of equations $2x + 6y = 10$ and $3x + 9y = 15$ has an infinite number of solutions.

a. Do you agree with Lin? Explain your reasoning.

b. Write a new equation that makes a system with infinite solutions together with $2x + 6y = 10$.

c. Describe the graph of the new system.

Unit 7, Lesson 8

Addressing CA CCSSM 8.EE.1; practicing MP6, MP7, MP8

Combining Bases

Let’s multiply expressions with different bases.

8.1 Same Exponent, Different Base

1. Evaluate $5^3 \cdot 2^3$.

2. Evaluate 10^3.

8.2 Power of Products

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the “expanded” column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. Can you write $2^3 \cdot 3^4$ with a single exponent? Explain or show your reasoning.

3. What happens when multiplying bases where neither the exponents nor the bases are the same?

8.3 How Many Ways Can You Make 3,600?

Your teacher will give your group tools for creating a visual display to play a game. The goal is to write as many expressions as you can that equal a specific number, using any of the exponent rules that we have learned:

$a^n \cdot a^m = a^{n+m}$	$(a^n)^m = a^{n \cdot m}$	$\frac{a^n}{a^m} = a^{n-m}$
$a^n \cdot b^n = (a \cdot b)^n$	$a^0 = 1$	$a^{-n} = \frac{1}{a^n}$

When the time is up, compare your expressions with another group to see how many points you earned.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rules. If the other group has the same expression, neither group earns any points.
- If your *unique* expression uses negative exponents, your group gets 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the exponent rules.

Are you ready for more?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

Lesson 8 Summary

In this lesson, we developed a rule for combining expressions with the same exponent but different bases: The factors can be regrouped and raised to the same exponent.

Rule	Example showing how it works
$a^n \cdot b^n = (a \cdot b)^n$	$2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) = 10 \cdot 10 \cdot 10 = 10^3$
	three factors that are 2 · three factors that are 5 = regrouped factors = three factors that are ten

To see this, expand $2^3 \cdot 5^3$ into three factors that are 2 and three factors that are 5. Regroup the factors into three groups of $2 \cdot 5$, or three groups of 10.

Practice Problems

1 Select **all** the true statements:

A. $2^8 \cdot 2^9 = 2^{17}$

B. $8^2 \cdot 9^2 = 72^2$

C. $8^2 \cdot 9^2 = 72^4$

D. $2^8 \cdot 2^9 = 4^{17}$

E. $72^3 \cdot 72^2 = 72^5$

2 Select **all** the expressions that have the same value as 12^4.

A. $(2 \cdot 6)^4$

B. $6^4 \cdot 2^4$

C. $3^2 \cdot 4^2$

D. $12^3 \cdot 12$

E. $4^4 \cdot 3^4$

3 Elena and Diego are evaluating $(3 \cdot 2)^3$.

a. Diego begins by evaluating $3 \cdot 2$, which is 6. His next step is to find 6^3. Does Diego's method work? Explain your reasoning.

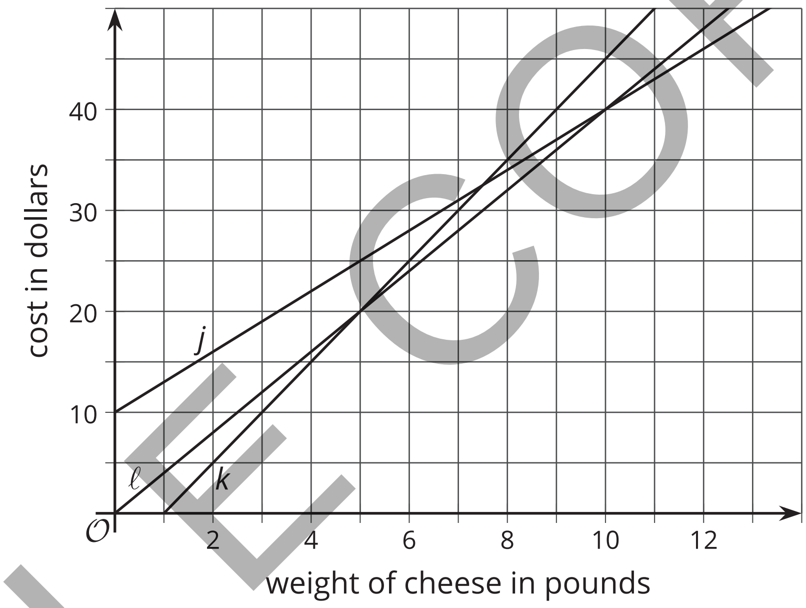
b. Elena starts by writing $(3 \cdot 2)^3 = 3^3 \cdot 2^3$. Her next step is to multiply 3^3 by 2^3. Does Elena's method work? Explain your reasoning.

4 from Unit 5, Lesson 8

The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.

- Store A sells the cheese for a dollars per pound.
- Store B sells the same cheese for b dollars per pound, and a customer has a coupon for $5 off the total purchase at that store.
- Store C is an online store, selling the same cheese at c dollars per pound, but with a $10 delivery fee.

This graph shows the total cost functions for stores A, B, and C after discounts are applied.



a. Match Stores A, B, and C with Graphs j, k, and ℓ.

b. What is the price per pound for cheese at each store?

c. How many pounds of cheese does the coupon for Store B pay for?

d. At which store will the customer pay the lowest amount for a half a pound of cheese?

e. A customer wants to buy 5 pounds of cheese for a party. Which store has the lowest total purchase price for 5 pounds of cheese?

f. How many pounds would a customer need to order to make Store C a good option?

Unit 7, Lesson 9

Addressing CA CCSSM 8.EE.3; building on 5.NBT.2, 5.NBT.3a; building towards 8.EE.3, 8.EE.4; practicing MP6

Describing Large and Small Numbers Using Powers of 10

Let’s find out how to use powers of 10 to write large or small numbers.

9.1 Thousand Million Billion Trillion

1. Match each expression with its corresponding value and word.

expression
10^{-3}
10^{6}
10^{9}
10^{-2}
10^{12}
10^{3}

value
1,000,000,000,000
$\frac{1}{100}$
1,000
1,000,000,000
1,000,000
$\frac{1}{1,000}$

word
billion
milli-
million
thousand
centi-
trillion

2. For each of the numbers, think of something in the world that is described by that number.

9.2 Base-Ten Representations Matching

1. Match each expression to one or more diagrams that could represent it. For each match, explain what the value of a single small square would have to be.

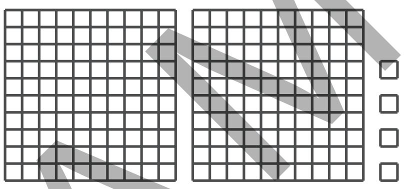
   a. $2 \cdot 10^{-1} + 4 \cdot 10^{-2}$

   b. $2 \cdot 10^{-1} + 4 \cdot 10^{-3}$

   c. $2 \cdot 10^{3} + 4 \cdot 10^{1}$

   d. $2 \cdot 10^{3} + 4 \cdot 10^{2}$

E



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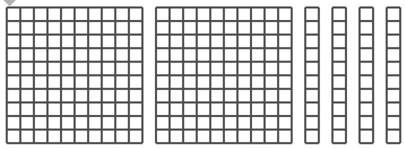

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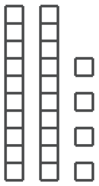
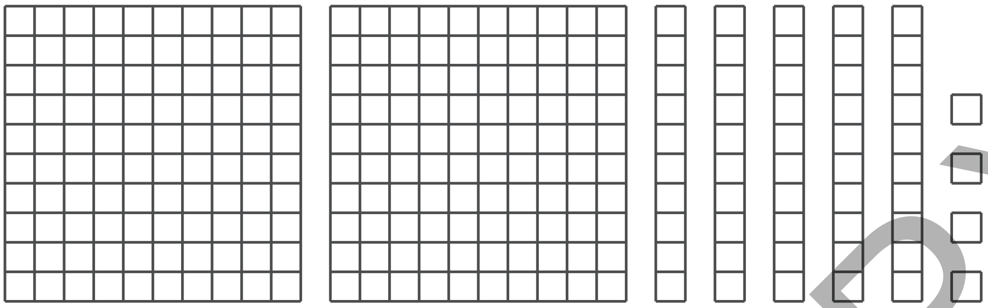


2. a. Write an expression to describe the base-ten diagram if each small square represents 10^{-4}. What is the value of this expression?



b. Write an expression to describe the base-ten diagram if each small square represents 10^{-3}. What is the value of this expression?

c. Write an expression to describe the base-ten diagram if each small square represents 10^{6}. What is the value of this expression?

d. How does changing the value of the small square change the value of the expression? Explain or show your reasoning.

9.3 Using Powers of 10 to Describe Large and Small Numbers

Your teacher will give you a card that tells you whether you are Partner A or B and gives you the information that is missing from your partner's statements. Do not show your card to your partner.

Read each statement assigned to you, ask your partner for the missing information, and write the number your partner tells you.

Partner A's statements:

1. Around the world, about ____________________ pencils are made each year.
2. The mass of a proton is ____________________ kilograms.
3. The population of Russia is about ____________________ people.
4. The diameter of a bacteria cell is about ____________________ meter.

Partner B's statements:

1. Light waves travel through space at a speed of ____________________ meters per second.
2. The population of India is about ____________________ people.
3. The wavelength of a gamma ray is ____________________ meters.
4. The tardigrade (water bear) is ____________________ meters long.

Are you ready for more?

A "googol" is a name for a really big number: a 1 followed by 100 zeros.

1. If you square a googol, how many zeros will the answer have? Show your reasoning.

2. If you raise a googol to the googol power, how many zeros will the answer have? Show your reasoning.

Lesson 9 Summary

Sometimes powers of 10 are helpful for expressing quantities, especially very large or very small quantities.

For example, the United States Mint has made over 500,000,000,000 pennies. To understand this number we can look at the number of zeros to know it is equivalent to 500 billion pennies. Since 1 billion can be written as 10^9, we can say that there are over $500 \cdot 10^9$ pennies.

Sometimes we may need to rewrite a number using a different power of 10. We can say that $500 \cdot 10^9 = 5 \cdot 10^{11}$. Since the factor 10^9 was multiplied by 100 to get 10^{11}, the factor of 500 was divided by 100 to keep the value of the entire expression the same.

The same is true for very small quantities. For example, a single atom of carbon weighs about 0.0000000000000000000000199 grams. If we write this as a fraction we get $\frac{199}{10,000,000,000,000,000,000,000,000}$. Using powers of 10, it becomes $199 \cdot 10^{-25}$, which is a lot easier to write!

Just as we did with large numbers, small numbers can be rewritten as an equivalent value with a different power of 10. In this example we can divide the factor 199 by 100 and multiply the factor 10^{-25} by 100 to get $1.99 \cdot 10^{-23}$.

Practice Problems

1 Match each number to its name.

A. 1,000,000
B. 0.01
C. 1,000,000,000
D. 0.000001
E. 0.001
F. 10,000

1. One hundredth
2. One thousandth
3. One millionth
4. Ten thousand
5. One million
6. One billion

2 Write each expression as a multiple of a power of 10:

a. 42,300
b. 2,000
c. 9,200,000
d. Four thousand
e. 80 million
f. 32 billion

3 Each statement contains a quantity. Rewrite each quantity using a power of 10.

a. There are about 37 trillion cells in an average human body.
b. The Milky Way contains about 300 billion stars.
c. A sharp knife is 23 millionths of a meter thick at its tip.
d. The wall of a certain cell in the human body is 4 nanometers thick. (A nanometer is one billionth of a meter.)

4 from Unit 5, Lesson 20

A fully inflated basketball has a radius of 12 centimeters. Your basketball is inflated only halfway. How many more cubic centimeters of air does your ball need to fully inflate? Express your answer in terms of π. Then estimate how many cubic centimeters this is by using 3.14 to approximate π.

5 from Unit 4, Lesson 5

Solve each of these equations. Explain or show your reasoning.

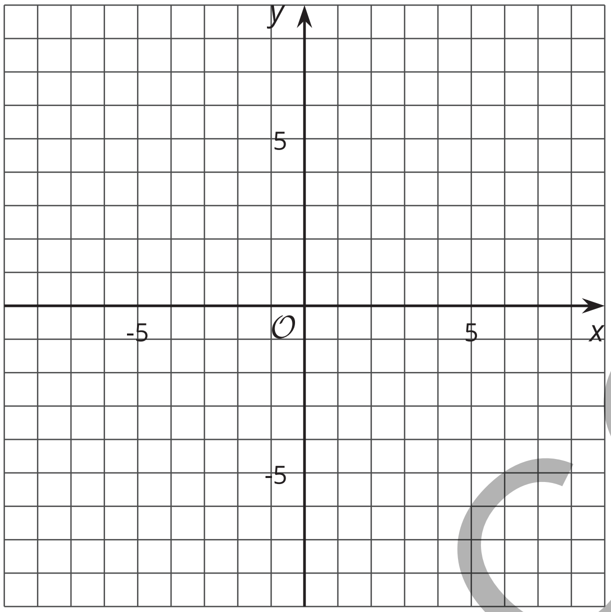
$2(3 - 2c) = 30$

$3x - 2 = 7 - 6x$

$31 = 5(b - 2)$

6 from Unit 3, Lesson 10

Graph the line going through $(-6, 1)$ with a slope of $\frac{-2}{3}$, and write its equation.



Unit 7, Lesson 10

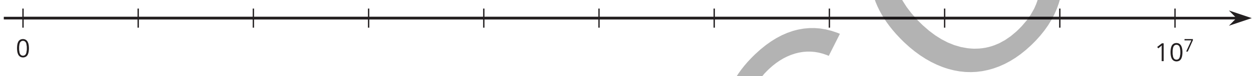
Addressing CA CCSSM 8.EE.3, 8.EE.4; building towards 8.EE.4; practicing MP2 and MP7

Representing Large Numbers on the Number Line

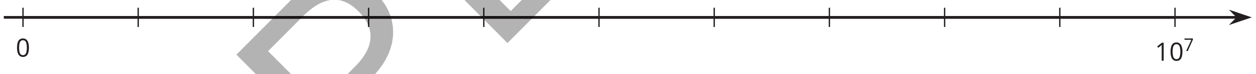
Let’s visualize large numbers on the number line using powers of 10.

10.1 Labeling Tick Marks on a Number Line

Label the tick marks on the number line. Be prepared to explain your reasoning.



10.2 Comparing Large Numbers with a Number Line



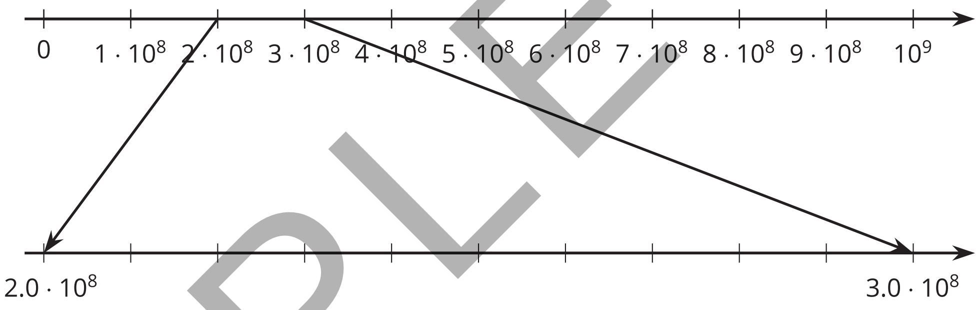
1. Place the numbers on the number line. Be prepared to explain your reasoning.
   a. 4,000,000
   b. $5 \cdot 10^6$
   c. $5 \cdot 10^5$
   d. $75 \cdot 10^5$
   e. $(0.6) \cdot 10^7$
2. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.
3. Compare number lines with a partner, and discuss how you each decided where each point should go. If you disagree about a placement, work to reach an agreement.

10.3 The Speeds of Light

The table shows how fast light waves can travel through different materials.

	material	speed (meters per second)
A	space	300,000,000
B	water	$(2.25) \cdot 10^8$
C	copper wire (electricity)	280,000,000
D	diamond	$124 \cdot 10^6$
E	ice	$(2.3) \cdot 10^8$
F	olive oil	200,000,000

Let's zoom in to highlight the values between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.



1. Label the tick marks between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$. Then plot a point for the speed of light through each material A–F on one of the number lines.
2. There is one material whose speed that you cannot plot on the bottom number line. Which is it? If you haven't already, plot the point for this material on the top number line.
3. Which travels faster — light through diamond or light through ice? How can you tell from the given expressions for the speed of light? How can you tell from the number line?

Are you ready for more?

Find a four-digit number using only the digits 0, 1, 2, or 3 and all of the following are true:

- The first digit tells you how many zeros are in the number.
- The second digit tells you how many ones are in the number.
- The third digit tells you how many twos are in the number.
- The fourth digit tells you how many threes are in the number.

The number 2,100 is close, but doesn't quite work. The first digit is 2, and there are 2 zeros. The second digit is 1, and there is 1 one. The fourth digit is 0, and there are no threes. But the third digit, which is supposed to count the number of 2's, is zero.

1. Can you find more than one number like this?
2. How many solutions are there to this problem? Explain or show your reasoning.

Lesson 10 Summary

Suppose we want to compare the number of pennies the U.S. Mint made in 2020, about 7,600,000,000, to the number of one dollar bills printed by the U.S. Bureau of Engraving and Printing in the same year (about 1.6 billion). There are many ways to do this.

We could write 1.6 billion as a decimal value, 1,600,000,000, and then we can tell that in 2020 there were more pennies made than one dollar bills printed.

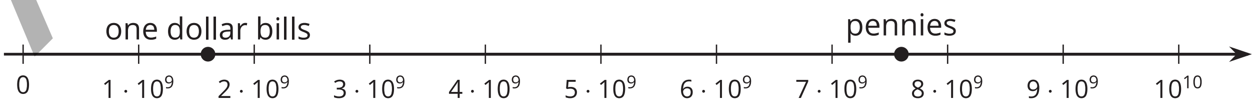
Or we could use powers of 10 to write these numbers:

$(7.6) \cdot 10^9$ for the number of pennies and

$(1.6) \cdot 10^9$ for the number of one dollar bills.

Since both numbers are written using the same power of 10, we can compare 7.6 to 1.6 and confirm that there were more pennies made than one dollar bills printed in 2020.

We could also plot these two numbers on a number line. We would need to carefully choose our end points to make sure that both numbers can be plotted. Here is a number line with the two values plotted:



Practice Problems

1 Write the number $437,000$ three different ways using powers of 10.

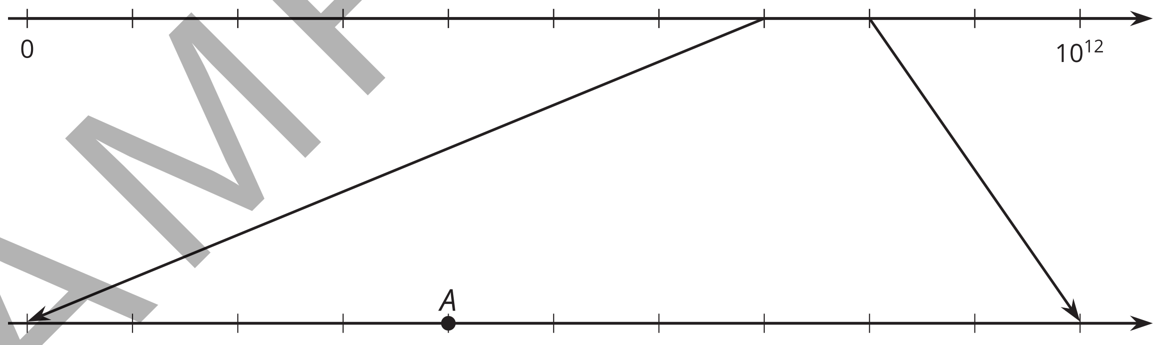
2 For each pair of numbers, circle the number with the larger value. Estimate about how many times larger.

a. $17 \cdot 10^8$ or $4 \cdot 10^8$

b. $2 \cdot 10^6$ or $7.839 \cdot 10^6$

c. $42 \cdot 10^7$ or $8.5 \cdot 10^8$

3 What number is represented by point A? Explain or show your reasoning.



4 from Unit 7, Lesson 8

Which expression has the same value as $5^3 \cdot 6^3$?

A. 11^6

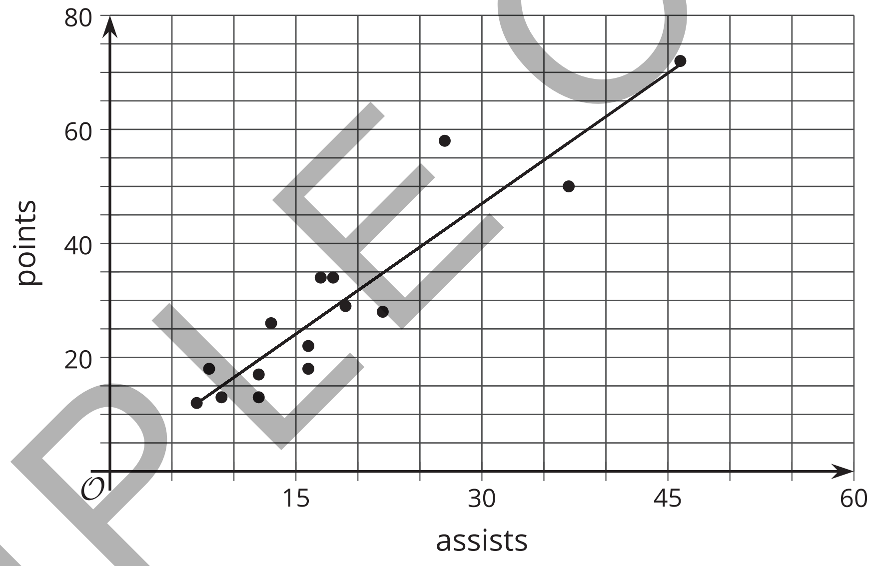
B. 30^6

C. 30^3

D. 30^9

5 from Unit 6, Lesson 7

This scatter plot shows the number of points and assists by a set of hockey players. Select **all** the phrases that describe the association in the scatter plot:



A. Linear association

B. Non-linear association

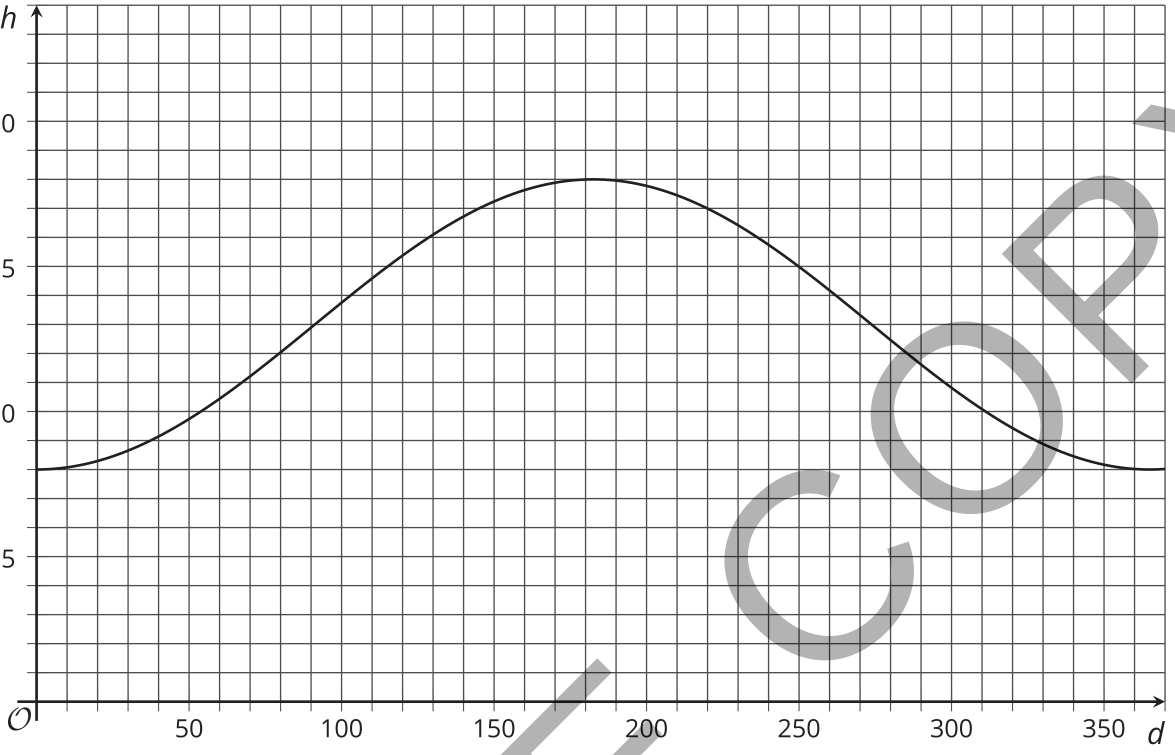
C. Positive association

D. Negative association

E. No association

6 from Unit 5, Lesson 5

Here is a graph of the day of the year, d, and the predicted hours of sunlight, h, on that day.



a. Is hours of sunlight a function of days of the year? Explain how you know.

b. For what days of the year is the number of hours of sunlight increasing? For what days of the year is the number of hours of sunlight decreasing?

c. Which day of the year has the greatest number of hours of sunlight?

Unit 7, Lesson 11

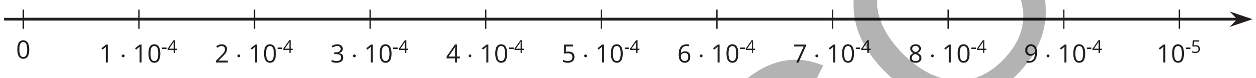
Addressing CA CCSSM 8.EE.1, 8.EE.3, 8.EE.4; practicing MP2, MP6, MP7

Representing Small Numbers on the Number Line

Let’s visualize small numbers on the number line using powers of 10.

11.1 Small Numbers on a Number Line

Kiran drew this number line.



Andre said, “That doesn’t look right to me.”

Explain why Kiran is correct or explain how he can fix the number line.

11.2 Comparing Small Numbers on a Number Line

1. Label the tick marks on the number line.
2. Plot the following numbers on the number line:

A. $6 \cdot 10^{-6}$ B. $6 \cdot 10^{-7}$ C. $29 \cdot 10^{-7}$ D. $(0.7) \cdot 10^{-5}$

3. Which is larger, $29 \cdot 10^{-7}$ or $6 \cdot 10^{-6}$? Estimate how many times larger.

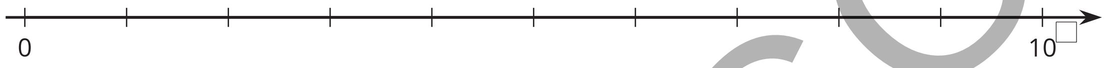
4. Which is larger, $7 \cdot 10^{-8}$ or $3 \cdot 10^{-9}$? Estimate how many times larger.

11.3 Atomic Scale

1. The radius of an electron is about 0.0000000000003 cm.
   a. Write this number as a multiple of a power of 10.

   b. Decide what power of 10 to put on the right side of this number line and label it.

   c. Label each tick mark as a multiple of a power of 10.

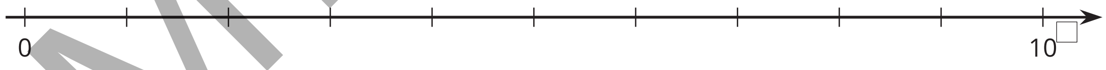


   d. Plot the radius of the electron in centimeters on the number line.

2. The mass of a proton is about 0.0000000000000000000000017 grams.
   a. Write this number as a multiple of a power of 10.

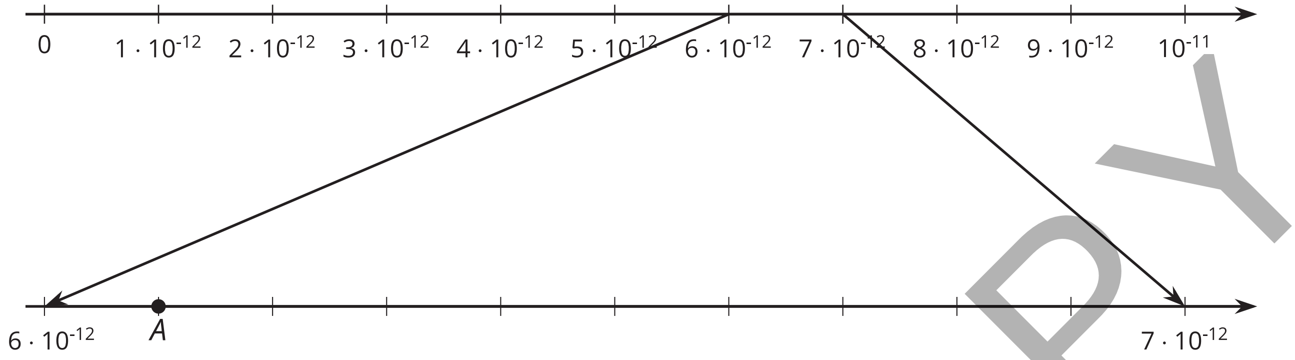
   b. Decide what power of 10 to put on the right side of this number line and label it.

   c. Label each tick mark as a multiple of a power of 10.



   d. Plot the mass of the proton in grams on the number line.

3. Point A on the zoomed-in number line describes the wavelength of a certain X-ray in meters.

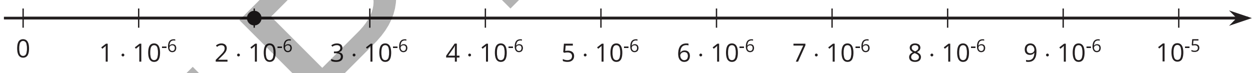


   a. Write the wavelength of the X-ray as a multiple of a power of 10.

   b. Write the wavelength of the X-ray as a decimal.

Lesson 11 Summary

The width of a bacterium cell is about $2 \cdot 10^{-6}$ meters. If we want to plot this on a number line, we need to find which two powers of 10 it lies between. We can see that $2 \cdot 10^{-6}$ is a multiple of 10^{-6}. So our number line will be labeled with multiples of 10^{-6}.



Note that the right side is labeled 10^{-5} because $10^{-6} \cdot 10 = 10^{-5}$.

The power of ten on the right side of the number line is always *greater* than the power on the left. This is true for powers with positive or negative exponents.

Practice Problems

1 Select **all** the expressions that are equal to $4 \cdot 10^{-3}$.

A. $4 \cdot (\frac{1}{10}) \cdot (\frac{1}{10}) \cdot (\frac{1}{10})$

B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$

C. $4 \cdot 0.001$

D. $4 \cdot 0.0001$

E. 0.004

F. 0.0004

2 Write each expression as a multiple of a power of 10:

a. 0.04

b. 0.072

c. 0.0000325

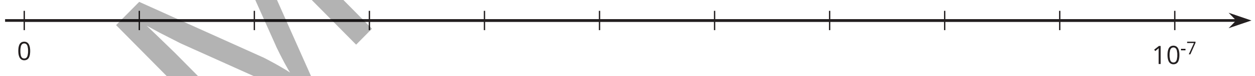
d. Three thousandths

e. 23 hundredths

f. 729 thousandths

g. 41 millionths

3



Plot the following points on the number line:

a. 10^{-8}

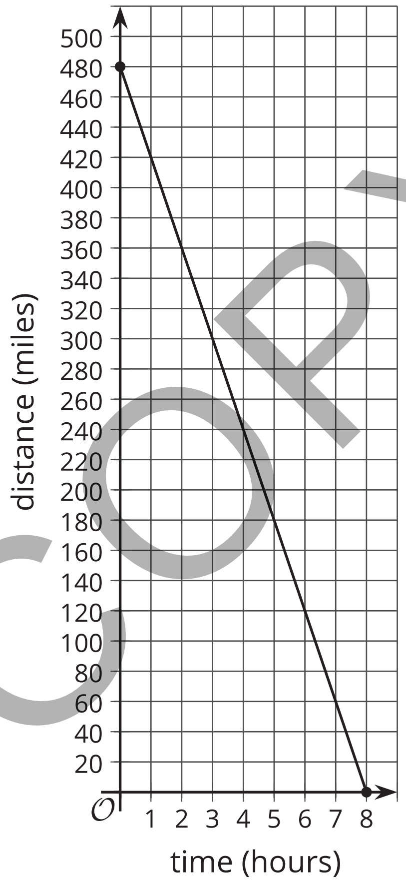
b. $3 \cdot 10^{-8}$

c. $5 \cdot 10^{-9}$

d. $25 \cdot 10^{-9}$

4 from Unit 3, Lesson 9

A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.



a. How fast are they traveling?

b. Is the slope positive or negative? Explain how you know and why that fits the situation.

c. How far is the trip and how long did it take? Explain how you know.

5 from Unit 7, Lesson 8

Select **all** of the expressions that have the same value as $(3 \cdot 5)^2$.

A. 15^2

B. $6 \cdot 10$

C. $3 \cdot 5 \cdot 3 \cdot 5$

D. $3 \cdot 3 \cdot 5 \cdot 5$

E. $9 \cdot 25$

F. 60

G. 225

Unit 7, Lesson 12

Addressing CA CCSSM 8.EE.3, 8.EE.4; building towards 8.EE.4; practicing MP2 and MP4

Applications of Arithmetic with Powers of 10

Let's use powers of 10 to help us make calculations with large and small numbers.

12.1 What Information Do You Need?

What information would you need to answer these questions?

1. Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?

2. Which has more mass, the Burj Khalifa or the mass of pennies it cost to build the Burj Khalifa?

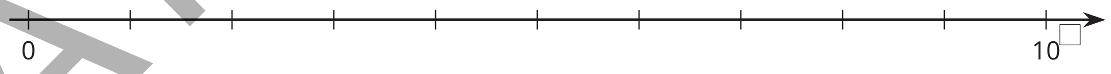
12.2 That’s a Tall Stack of Cash

In 2010, the Burj Khalifa became the tallest building in the world. It was very expensive to build.

1. Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa? Ask your teacher for the information you need to be able to answer this question, and record the information here.

2. Answer the question “Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?” and explain or show your reasoning.

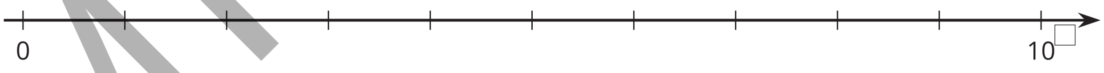
3. Decide what power of 10 to use to label the rightmost tick mark of the number line so that both the height of the stack of money and the height of the Burj Khalifa can be plotted on the same number line. Label the tick marks, and plot and label both values.



4. Which has more mass, the Burj Khalifa or the mass of the pennies it cost to build the Burj Khalifa? Ask your teacher for the information you need to be able to answer this question and record the information here.

5. Answer the question “Which has more mass, the Burj Khalifa or the mass of the pennies it cost to build the Burj Khalifa?” and explain or show your reasoning.

6. Decide what power of 10 to use to label the rightmost tick mark of the number line so that both the mass of the Burj Khalifa and the mass of the pennies it cost to build the Burj Khalifa can be plotted on the same number line. Label the tick marks and plot and label both values.



12.3 Meter Sticks to the Moon

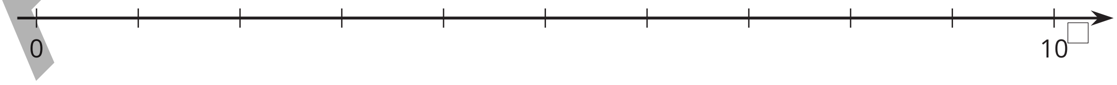
1. How many meter sticks does it take to equal the mass of the Moon? Ask your teacher for the information you need to be able to answer this question, and record the information here.

2. Answer the question “How many meter sticks does it take to equal the mass of the Moon?” and explain or show your reasoning.

3. Label the number line and plot your answer for the number of meter sticks.

4. If you took all the meter sticks from the last question and lined them up end to end, would they reach the Moon? Would they reach beyond the Moon? If yes, how many times farther will they reach? Explain your reasoning.

5. One light year is approximately 10^{16} meters. How many light years away would the meter sticks reach? Label the number line, and plot your answer.



Are you ready for more?

Here is a problem that will take multiple steps to solve. You may not know all the facts you need to solve the problem. That is okay. Take a guess at reasonable answers to anything you don't know. Your final answer will be an estimate.

If everyone alive on Earth right now stood very close together, how much area would they take up?

Lesson 12 Summary

Powers of 10 can be helpful for making calculations with large or small numbers. For example, in 2014, the United States had 318,586,495 people who used the equivalent of 2,203,799,778,107 kilograms of oil in energy.

The amount of energy used per person is the total energy divided by the total number of people. We can use powers of 10 to estimate the total energy as $2 \cdot 10^{12}$ and the population as $3 \cdot 10^{8}$. So the amount of energy per person in the U.S. is roughly $(2 \cdot 10^{12}) \div (3 \cdot 10^{8})$. That is the equivalent of $\frac{2}{3} \cdot 10^{4}$ kilograms of oil in energy. That's a lot of energy—the equivalent of almost 7,000 kilograms of oil per person!

In general, when we want to perform arithmetic with very large or very small quantities, estimating with powers of 10 and using exponent rules can help simplify the process. If we wanted to find the exact quotient of 2,203,799,778,107 by 318,586,495, then using powers of 10 would not simplify the calculation.

Practice Problems

1 Which is larger: the number of meters across the Milky Way, or the number of cells in all humans? Explain or show your reasoning.

Some useful information:

- The Milky Way is about 100,000 light years across.
- There are about 37 trillion cells in a human body.
- One light year is about 10^{16} meters.
- The world population is about 8 billion.

2 What information would you need to know in order to solve the following problem?

Suppose that someone leaves all the lights on at your school. Before leaving for the day, the principal goes around and turns off all the lights. How long does this take?

3 Lin’s mother has a car that she uses for work, shopping, and trips. About how many times does one tire on her car revolve in a year? Explain or show your reasoning.

Some useful information:

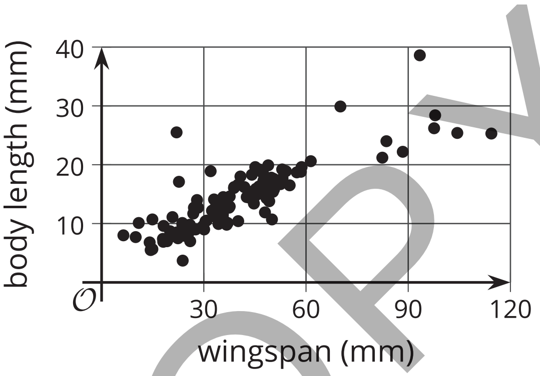
- The diameter of her tire is 27 inches.
- She drives about 10,000 miles in one year.
- There are 5,280 feet in one mile.
- There are 12 inches in one foot.

4 from Unit 6, Lesson 5

Ecologists measure the body length and wingspan of 127 butterfly specimens caught in a single field.

a. Draw a line that you think is a good fit for the data.

b. Write an equation for the line.



c. What does the slope of the line tell you about the wingspans and lengths of these butterflies?

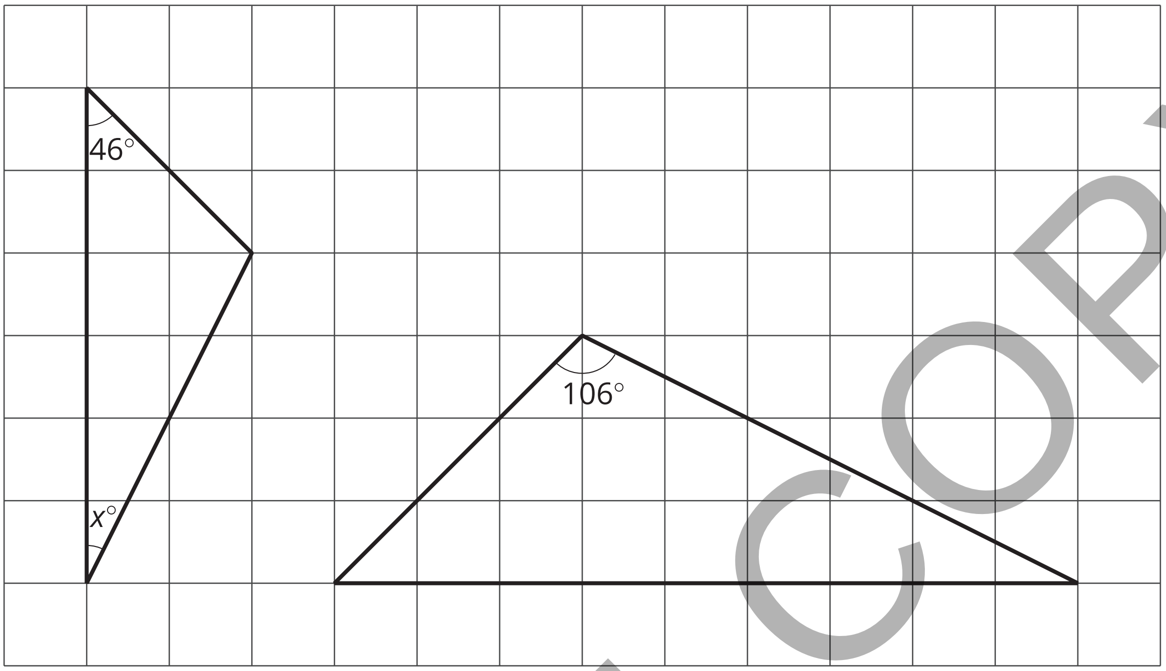
5 from Unit 4, Lesson 5

Diego was solving an equation, but when he checked his answer, he saw his solution was incorrect. He knows he made an error, but he can't find it. Where is Diego's error and what is the solution to the equation?

$$\begin{aligned} -4(7-2x) &= 3(x+4) \\ -28-8x &= 3x+12 \\ -28 &= 11x+12 \\ -40 &= 11x \\ -\frac{40}{11} &= x \end{aligned}$$

6 from Unit 2, Lesson 7

The two triangles are similar. Find x.



Unit 7, Lesson 13

Addressing CA CCSSM 8.EE.4; building on 5.NBT.2; building towards 8.EE.3; practicing MP3 and MP7

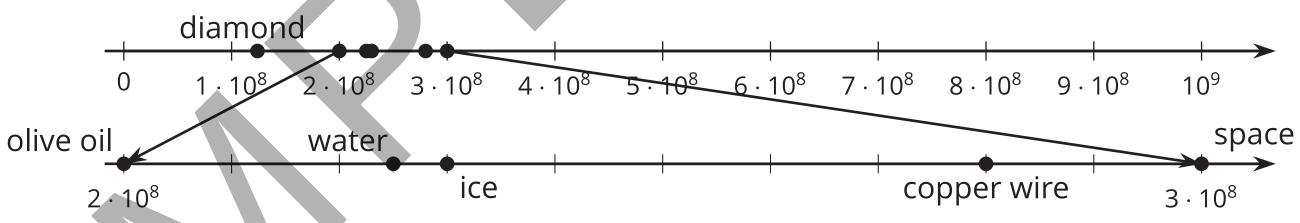
Definition of Scientific Notation

Let’s use scientific notation to describe large and small numbers.

13.1 Notice and Wonder: Scientific Notation

What do you notice? What do you wonder?

material	speed (meters per second)
space	300,000,000
water	2.25×10^8
copper (electricity)	280,000,000
diamond	124×10^6
ice	2.3×10^8
olive oil	0.2×10^9



13.2 The “Science” of Scientific Notation

The table shows the speed of light through different materials.

material	speed (meters per second)
space	300,000,000
water	2.25×10^8
copper (electricity)	280,000,000
diamond	124×10^6
ice	2.3×10^8
olive oil	0.2×10^9

1. Circle the speeds that are written in scientific notation.
2. Write the others using scientific notation.

13.3 Card Sort: Scientific Notation Matching

Your teacher will give you a set of cards. Take turns with your partner to match a number written as a decimal with a number written as a multiple of a power of 10.

1. For each match that you find, explain to your partner how you know it’s a match.
2. For each match that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking, and work to reach an agreement.

Are you ready for more?

1. What is $9 \times 10^{-1} + 9 \times 10^{-2}$? Express your answer as:
   a. A decimal

   b. A fraction

2. What is $9 \times 10^{-1} + 9 \times 10^{-2} + 9 \times 10^{-3} + 9 \times 10^{-4}$? Express your answer as:
   a. A decimal

   b. A fraction

3. The answers to the two previous questions should have been close to 1. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1,000,000}$ off?

4. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1,000,000,000}$ off? Can you keep adding numbers in this pattern to get as close to 1 as you want? Explain or show your reasoning.

5. Imagine a number line that goes from your current position (labeled 0) to the door of the room you are in (labeled 1). In order to get to the door, you will have to pass the points 0.9, 0.99, 0.999, etc. The Greek philosopher Zeno argued that you will never be able to go through the door, because you will first have to pass through an infinite number of points. What do you think? How would you reply to Zeno?

Lesson 13 Summary

The total value of all the quarters made in 2014 was 400 million dollars. There are many ways to express this using powers of 10. We could write this as $400 \cdot 10^6$ dollars, $40 \cdot 10^7$ dollars, $0.4 \cdot 10^9$ dollars, or many other ways. One special way to write this quantity is called **scientific notation**, where the first factor is a number greater than or equal to 1, but less than 10, and the second factor is an integer power of 10

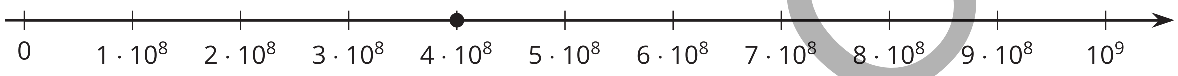
In scientific notation,

400 million dollars

would be written as

4×10^8 dollars.

Writing the number this way shows exactly where it lies between two consecutive powers of 10. The 10^8 shows us the number is between 10^8 and 10^9. The 4 shows us that the number is 4 tenths of the way to 10^9.



For scientific notation, the "×" symbol is the standard way to show multiplication instead of the dot symbol. Some other examples of scientific notation are 1.2×10^{-8}, 9.99×10^{16}, and 7×10^{12}.

Glossary

- scientific notation

Practice Problems

1 Write each number in scientific notation.

a. 14,700

b. 0.00083

c. 760,000,000

d. 0.038

e. 0.38

f. 3.8

g. 3,800,000,000,000

h. 0.0000000009

2 Elena is going to write the number 0.0000025 in scientific notation. She writes "$2.5 \times 10^{?}$" but isn't sure what to write for the exponent. Explain how Elena can decide which power of 10 to use.

3 The following numbers are all written in scientific notation. Write the value of each expression.

a. 8.5×10^6

b. 4.54×10^{10}

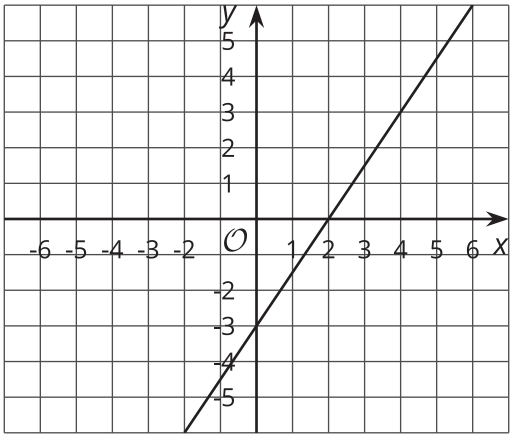
c. 9.03×10^2

d. 3.714×10^{-5}

e. 5.82×10^{-8}

4 from Unit 4, Lesson 12

Here is the graph for one equation in a system of equations.



a. Write a second equation for the system so it has infinitely many solutions.

b. Write a second equation whose graph goes through $(0, 2)$ so that the system has no solutions.

c. Write a second equation whose graph goes through $(2, 2)$ so that the system has one solution at $(4, 3)$.

5 from Unit 7, Lesson 6

Write each expression using a single exponent:

a. $\frac{5^3}{5^7}$

b. $5^3 \cdot 5^3 \cdot 5^2$

c. $(5^3)^7$

d. $5 \cdot 5^0 \cdot 5^1$

Unit 7, Lesson 14

Addressing CA CCSSM 8.EE.1, 8.EE.3, 8.EE.4; building towards 8.EE.4; practicing MP1, MP2, MP6

Estimating with Scientific Notation

Let's multiply and divide with scientific notation to answer questions about animals, careers, and planets.

14.1 Scientific Notation and Technology

Diego and Priya were calculating $(5 \times 10^{13}) \cdot (8 \times 10^{25})$.

- Diego used a calculator and his display read $4\text{E}39$.
- Priya used her knowledge of exponent rules and got 40×10^{38}.
- Clare used an online calculator and the screen showed $4e + 39$.

What do you think these different results mean?

14.2 Biomass

Use the table to answer questions about different creatures on the planet. Be prepared to explain your reasoning.

creature	number on planet	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. Which creature is least numerous? Estimate how many times more ants there are than this creature.

2. Which creature is the least massive? Estimate how many times more massive a human is than this creature.

3. Which is more massive, the total mass of all the humans or the total mass of all the ants? About how many times more massive is it?

4. Which is more massive, the total mass of all the krill or the total mass of all the blue whales? About how many times more massive is it?

14.3 Info Gap: Distances in the Solar System

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ________?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ________ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ________?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Are you ready for more?

Choose two celestial objects and create a scale drawing of them.

object	diameter (km)
Sun	1.392×10^6
Mercury	4.878×10^3
Venus	1.21×10^4
Earth	1.28×10^4
Mars	6.785×10^3
Jupiter	1.428×10^5
Saturn	1.199×10^5
Uranus	5.149×10^4
Neptune	4.949×10^4

Lesson 14 Summary

Multiplying numbers in scientific notation extends what we do when we multiply regular decimal numbers. For example, one way to find $(80)(60)$ is to view 80 as 8 tens and to view 60 as 6 tens. The product $(80)(60)$ is 48 hundreds or 4,800. Using scientific notation, we can write this calculation as

$$(8 \times 10^1)(6 \times 10^1) = 48 \times 10^2$$

To express the product in scientific notation, we would rewrite it as 4.8×10^3.

Calculating using scientific notation is especially useful when dealing with very large or very small numbers. For example, there are about 39 million, or 3.9×10^7 residents in California. The state has a water consumption goal of 42 gallons of water per person each day. To find how many gallons of water California would need each day if they met their goal, we can find the product $(42)(3.9 \times 10^7) = 163.8 \times 10^7$, which is equal to 1.638×10^9. That's more than 1 billion gallons of water each day.

Comparing very large or very small numbers by estimation also becomes easier with scientific notation. For example, how many ants are there for every human? There are 5×10^{16} ants and 8×10^9 humans. To find the number of ants per human, look at $\frac{5 \times 10^{16}}{8 \times 10^9}$. Rewriting the numerator to have the number 50 instead of 5, we get $\frac{50 \times 10^{15}}{8 \times 10^9}$. This gives us $\frac{50}{8} \times 10^6$. Since $\frac{50}{8}$ is roughly equal to 6, there are about 6×10^6 or 6 million ants per person!

Practice Problems

1 Evaluate each expression. Use scientific notation to express your answer.

a. $(1.5 \times 10^2)(5 \times 10^{10})$

b. $\dfrac{4.8 \times 10^{-8}}{3 \times 10^{-3}}$

c. $(5 \times 10^8)(4 \times 10^3)$

d. $(7.2 \times 10^3) \div (1.2 \times 10^5)$

2 How many bucketloads would it take to empty out the world's oceans? Write your answer in scientific notation.

Some useful information:

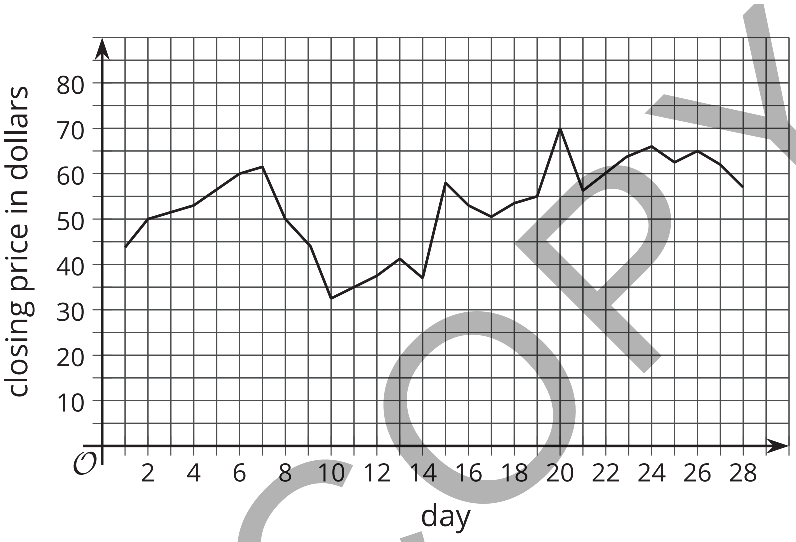
- The world's oceans hold roughly 1.4×10^9 cubic kilometers of water.
- A typical bucket holds roughly 20,000 cubic centimeters of water.
- There are 10^{15} cubic centimeters in a cubic kilometer.

3 from Unit 5, Lesson 5

The graph represents the closing price per share of stock for a company each day for 28 days.

a. In the first week, was the stock price generally increasing or decreasing?

b. During which period did the closing price of the stock decrease for at least 3 days in a row?



4 from Unit 3, Lesson 12

Write an equation for the line that passes through $(-8.5, 11)$ and $(5, -2.5)$.

5 from Unit 7, Lesson 8

Select **all** the expressions that have the same value as $4^{10} \cdot 5^{10}$

A. 9^{10}

B. 20^{20}

C. 20^{10}

D. $2^{10} \cdot 2^{10} \cdot 5^{10}$

E. 20^{100}

Unit 7, Lesson 15

Addressing CA CCSSM 8.EE.4; practicing MP2, MP3, MP6

Adding and Subtracting with Scientific Notation

Let's add and subtract using scientific notation to answer questions about animals and the solar system.

15.1 Math Talk: Non-zero Digits

Decide mentally how many non-zero digits each number will have.

- $(3 \times 10^9)(2 \times 10^7)$
- $(3 \times 10^9) \div (2 \times 10^7)$
- $3 \times 10^9 + 2 \times 10^7$
- $3 \times 10^9 - 2 \times 10^7$

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15.2 Measuring the Planets

Diego, Kiran, and Clare were wondering:

"If Neptune and Saturn were side by side, would they be wider than Jupiter?"

1. They start by trying to add 4.9×10^4 km and 1.2×10^5 km, the diameters of Neptune and Saturn. Here are the ways they approached the problem. Do you agree with any of them? Explain your reasoning.

   a. Diego says, "When we add the distances, we will get $4.9 + 1.2 = 6.1$. The exponent will be 9. So the two planets are 6.1×10^9 km side by side."

   b. Kiran wrote 4.9×10^4 as 49,000 and 1.2×10^5 as 120,000 and added them:

$$\begin{array}{r} 120{,}000 \\ +49{,}000 \\ \hline 169{,}000 \end{array}$$

   c. Clare says, "I think you can't add unless they are the same power of 10." She adds 4.9×10^4 km and 12×10^4 to get 16.9×10^4.

15.3 A Celestial Dance

object	diameter (km)	distance from the Sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

1. When you add the distances from the Sun of Mercury, Venus, Earth, and Mars, would you reach as far as Jupiter? Explain or show your reasoning.

2. Add the diameters of all the objects on the table except the Sun. Which is wider, all of these planets side-by-side, or the sun? Explain or show your reasoning.

Are you ready for more?

Standard aluminum foil has a thickness of about 6×10^{-3} inches. A sheet of tissue paper has a thickness of 3.9×10^{-4} inches. How thick would a stack of 1 sheet of aluminum foil and 2 sheets of tissue paper be, in inches?

15.4 A Massive Farm

The table shows the average mass of one individual creature and an estimated total number of those creatures on Earth. Use the table to answer each question, and explain or show your reasoning.

creature	total number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. On a farm there was a cow. And on the farm there were 2 sheep. There were also 3 chickens. What is the total mass of the 1 cow, the 2 sheep, the 3 chickens, and the 1 farmer on the farm?

2. What is the total mass of a human, a blue whale, and 6 ants all together?

3. Which is greater, the number of bacteria, or the number of all the other animals in the table put together?

Lesson 15 Summary

When adding decimal numbers, we need to pay close attention to place value. For example, when we calculate $13.25 + 6.7$, we need to make sure to add hundredths to hundredths (5 and 0), tenths to tenths (2 and 7), ones to ones (3 and 6), and tens to tens (1 and 0).

$$\begin{array}{r} 13.25 \\ +6.7 \\ \hline 19.95 \end{array}$$

We need to take the same care when we add or subtract numbers in scientific notation. For example, suppose we want to find how much further Earth is from the Sun than Mercury is from the Sun. Earth is about 1.5×10^8 km from the Sun, while Mercury is about 5.8×10^7 km. In order to find

$$1.5 \times 10^8 - 5.8 \times 10^7$$

we can rewrite this as

$$1.5 \times 10^8 - 0.58 \times 10^8$$

Now that both numbers are written in terms of 10^8, we can subtract 0.58 from 1.5 to get

$$0.92 \times 10^8$$

Rewriting this in scientific notation, Earth is

$$9.2 \times 10^7$$

km further from the Sun than Mercury is from the Sun.

Practice Problems

1 Evaluate each expression, giving the answer in scientific notation:

a. $5.3 \times 10^4 + 4.7 \times 10^4$

b. $3.7 \times 10^6 - 3.3 \times 10^6$

c. $4.8 \times 10^{-3} + 6.3 \times 10^{-3}$

d. $6.6 \times 10^{-5} - 6.1 \times 10^{-5}$

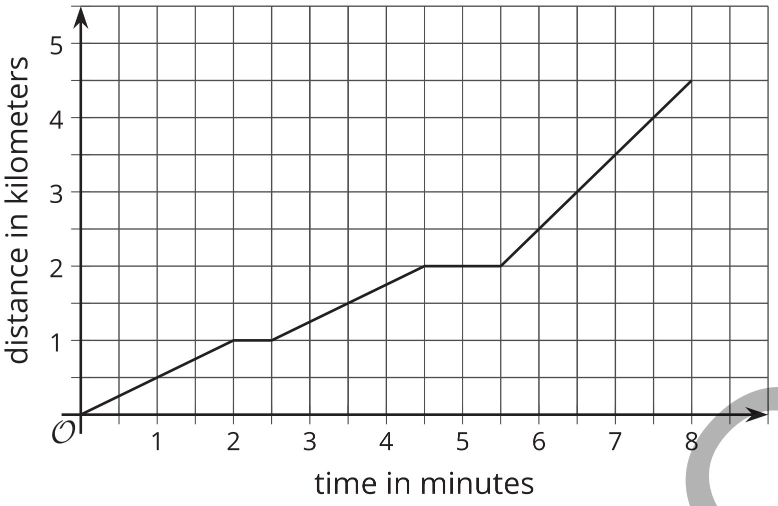
2 Here are the areas for each of the five Great Lakes:

- Superior: 8.2×10^4 square km
- Huron: 6.0×10^4 square km
- Michigan: 5.8×10^4 square km
- Erie: 2.6×10^4 square km
- Ontario: 1.9×10^4 square km

a. How much larger is Lake Huron than Lake Michigan? Give your answer with and without scientific notation.

b. Which is larger — Lake Michigan and Lake Ontario combined, or Lake Superior? Give the difference with and without scientific notation.

3 from Unit 5, Lesson 10



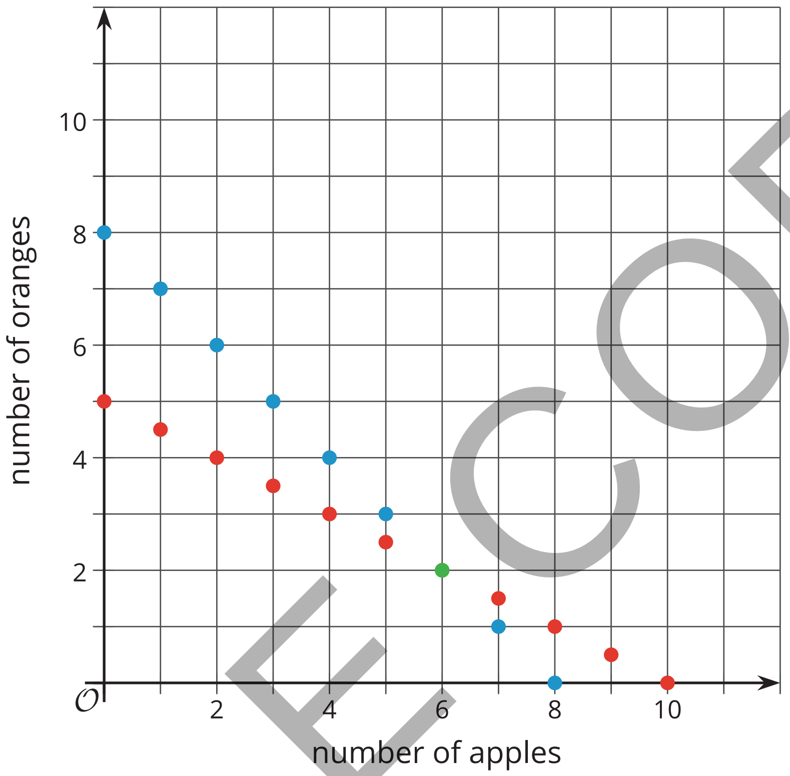
a. Write a scenario that describes what is happening in the graph.

b. What is happening at 5 minutes?

c. What does the slope of the line between 6 and 8 minutes mean?

4 from Unit 4, Lesson 10

Apples cost \$1 each. Oranges cost \$2 each. You have \$10 and want to buy 8 pieces of fruit. One graph shows combinations of apples and oranges that total to \$10. The other graph shows combinations of apples and oranges that total to 8 pieces of fruit.



a. Name one combination of 8 fruits shown on the graph whose cost does *not* total to \$10.

b. Name one combination of fruits shown on the graph whose cost totals to \$10 that are *not* 8 fruits all together.

c. How many apples and oranges would you need to have 8 fruits that cost \$10 at the same time?

5 from Unit 4, Lesson 5

Solve each equation and check your solution.

$-2(3x - 4) = 4(x + 3) + 6$

$\frac{1}{2}(z + 4) - 6 = -2z + 8$

$4w - 7 = 6w + 31$

Unit 7, Lesson 16

Addressing CA CCSSM 8.EE.3, 8.EE.4; practicing MP2, MP4, MP6

Is a Smartphone Smart Enough to Go to the Moon?

Let’s compare digital media and computer hardware using scientific notation.

16.1 Old Hardware, New Hardware

In 1966, the Apollo Guidance Computer was developed to make the calculations that would put humans on the Moon.

Your teacher will give you information for different devices from 1966 to 2023. Choose one device, and compare the specifications of that device with the 1966 Apollo Guidance Computer. If you get stuck, consider using scientific notation to help with the calculations.

For reference, storage is measured in bytes, processor speed is measured in hertz, and memory is measured in bytes. “Kilo” stands for 1,000, “mega” stands for 1,000,000, “giga” stands for 1,000,000,000, and “tera” stands for 1,000,000,000,000.

1. Which device did you choose?
2. How many times more information than the 1966 Apollo Guidance Computer can this device store?
3. How many times faster than the 1966 Apollo Guidance Computer is this device's processor speed?
4. How many times more memory than the 1966 Apollo Guidance Computer can this device store?

16.2 A Bit More on Bytes

For each question, think about what information you would need to figure out an answer. Your teacher may provide some of the information you ask for. Write your answers using scientific notation.

1. Mai found a 1980's computer magazine with an advertisement for a machine with hundreds of kilobytes of storage! Mai was curious and asked, "How many kilobytes would my dad's new computer hold?"

2. The old magazine showed another ad for a 750-kilobyte floppy disk, a device used in the past to store data. How many gigabytes is this?

3. Mai is writing a 1-page essay for school on her computer. Estimate how many 1-page essays it would take for Mai to fill up a floppy disk. Explain or show your reasoning.

4. Mai likes to go to the movies with her friends and knows that a high-definition film takes up a lot of storage space on a computer. Estimate how many floppy disks it would take to store a high-definition movie. Explain or show your reasoning.

5. How many seconds of a high-definition movie would one floppy disk be able to hold?

6. If you fall asleep watching a movie streaming service, and it streams movies all night while you sleep, how many floppy disks of information would that be?

Are you ready for more?

Humans tend to work with numbers using powers of 10, but computers work with numbers using powers of 2. A "binary kilobyte" is 1,024 bytes instead of 1,000, because $1{,}024 = 2^{10}$. Similarly, a "binary megabyte" is 1,024 binary kilobytes, and a "binary gigabyte" is 1,024 binary megabytes.

1. Which is bigger, a binary gigabyte or a regular gigabyte? How many more bytes is it?

2. Which is bigger, a binary terabyte or a regular terabyte? How many more bytes is it?

Learning Targets

Lesson 1 Exponent Review

- I can use exponents to describe repeated multiplication.
- I understand the meaning of a term with an exponent.

Lesson 2 Multiplying Powers of 10

- I can explain and use a rule for multiplying powers of 10.

Lesson 3 Powers of Powers of 10

- I can explain and use a rule for raising a power of 10 to a power.

Lesson 4 Dividing Powers of 10

- I can evaluate 10^0 and explain why it makes sense.
- I can explain and use a rule for dividing powers of 10.

Lesson 5 Negative Exponents with Powers of 10

- I can use the exponent rules with negative exponents.
- I know what it means if 10 is raised to a negative power.

Lesson 6 What About Other Bases?

- I can use the exponent rules for bases other than 10.

Lesson 7 Practice with Rational Bases

- I can change an expression with a negative exponent into an equivalent expression with a positive exponent.
- I can choose an appropriate exponent rule to rewrite an expression to have a single exponent.

Lesson 8 Combining Bases

- I can use and explain a rule for multiplying terms that have different bases but the same exponent.

Lesson 9 Describing Large and Small Numbers Using Powers of 10

- Given a very large or very small number, I can write an expression equal to it using a power of 10.

Lesson 10 Representing Large Numbers on the Number Line

- I can plot a multiple of a power of 10 on such a number line.

- I can subdivide and label a number line between 0 and a power of 10 with a positive exponent into 10 equal intervals.
- I can write a large number as a multiple of a power of 10.

Lesson 11 Representing Small Numbers on the Number Line

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals.
- I can write a small number as a multiple of a power of 10.

Lesson 12 Applications of Arithmetic with Powers of 10

- I can apply what I learned about powers of 10 to answer questions about real-world situations.

Lesson 13 Definition of Scientific Notation

- I can tell whether or not a number is written in scientific notation.

Lesson 14 Estimating with Scientific Notation

- I can multiply and divide numbers given in scientific notation.
- I can use scientific notation and estimation to compare very large or very small numbers.

Lesson 15 Adding and Subtracting with Scientific Notation

- I can add and subtract numbers given in scientific notation.

Lesson 16 Is a Smartphone Smart Enough to Go to the Moon?

- I can use scientific notation to compare different amounts and answer questions about real-world situations.

GRADE 8

UNIT

8

Pythagorean Theorem and Irrational Numbers

Content Connections

In this unit you will work with irrational numbers, focusing on connecting geometric and algebraic representations of square roots, cube roots, and the Pythagorean Theorem. You will make connections by:

- **Taking Wholes Apart, Putting Parts Together** while conducting investigations in the coordinate plane with right triangles to show that the areas of the squares of each leg combine to create the square of the hypotenuse.
- **Discovering Shape and Space** while composing and decomposing shapes to find the areas of tiles squares while using square root.
- **Reasoning with Data** while representing cube root as a decimal approximation and as a point on the number line.
- **Exploring Changing Quantities** while using the Pythagorean Theorem to solve real-world problems that include rational numbers.

Addressing the Standards

As you work your way through **Unit 8 Pythagorean Theorem and Irrational Numbers,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lessons 1, 8, 12, and 18
MP2 Reason abstractly and quantitatively.	Lesson 14
MP3 Construct viable arguments and critique the reasoning of others.	Lessons 3, 4, 5, 6, and 11
MP4 Model with mathematics.	Lesson 18
MP5 Use appropriate tools strategically.	Lesson 2
MP6 Attend to precision.	Lessons 3, 7, 9, 12, 13, 15, and 16
MP7 Look for and make use of structure.	Lessons 1, 2, 5, 6, 8, 10, 13, 14, and 17
MP8 Look for and express regularity in repeated reasoning.	Lessons 8, 16, and 17

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Cylindrical Investigations • Pythagorean Explorations • Big and Small Numbers • Shape, Number, and Expressions	**8.NS.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	Lessons 4, 16, and 17
• Cylindrical Investigations • Pythagorean Explorations • Big and Small Numbers • Shape, Number, and Expressions	**8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,π 2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	Lessons 2, 5, 6, 11, 14, and 15

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Interpret Scatter Plots • Data, Graphs, and Tables • Pythagorean Explorations • Big and Small Numbers • Shape, Number, and Expressions	**8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x2 = p$ and $x3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	Lessons 3, 4, 5, 6, 11, 14, 15, and 16
• Data Explorations • Big and Small Numbers	**8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	Lesson 12
• Data, Graphs, and Tables • Data Explorations • Linear Equations	**8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	Lesson 3
• Data, Graphs, and Tables • Data Explorations • Linear Equations	**8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	Lesson 3
• Cylindrical Investigations • Shape, Number, and Expressions • Transformational Geometry	**8.G.6** Explain a proof of the Pythagorean Theorem and its converse.	Lessons 7, 8, and 10
• Cylindrical Investigations • Pythagorean Explorations • Shape, Number, and Expressions • Transformational Geometry	**8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Lessons 7, 8, 9, 10, 11, 12, and 18

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Cylindrical Investigations • Pythagorean Explorations • Shape, Number, and Expressions • Transformational Geometry	**8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Lesson 13

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 8, Lesson 1

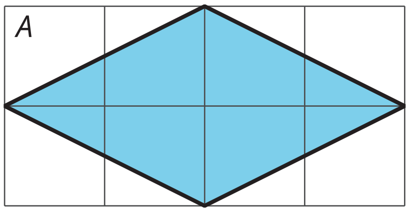
Building on 6.EE.1, 6.G.1; building towards 8.EE.2, 8.G.6, 8.NS.2; practicing MP1 and MP7

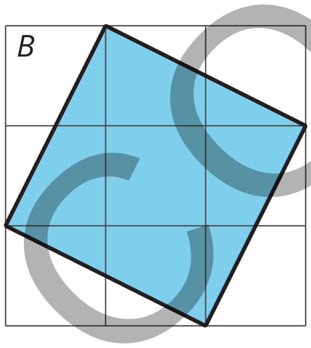
The Areas of Squares

Let’s investigate the areas of squares.

1.1 Two Regions

Which shaded region is larger? Explain your reasoning.





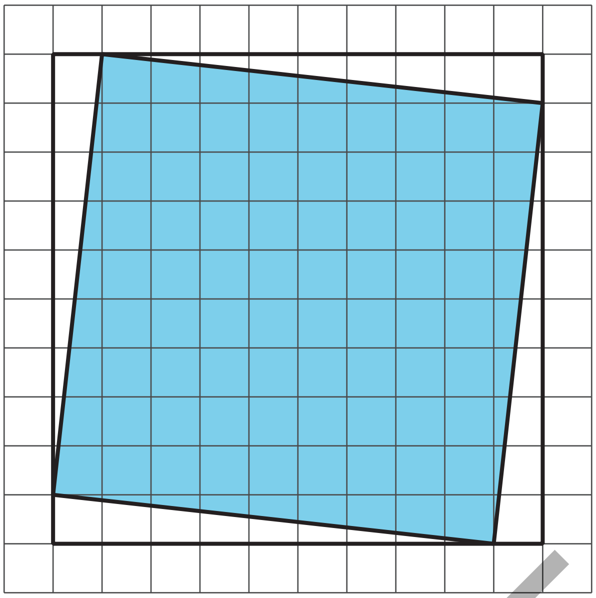
1.2 Making Squares

Your teacher will give your group a sheet with three squares and 5 cut-out shapes labeled D, E, F, G, and H. Use the squares to find the total area of the five shapes. Assume each small square is equal to 1 square unit.

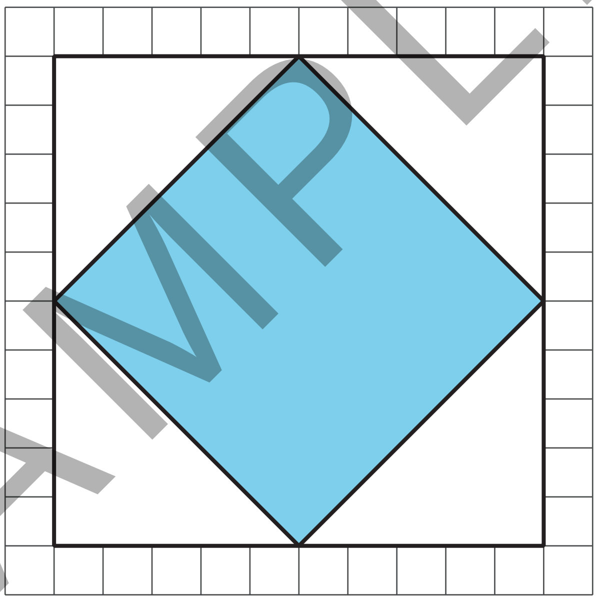
1.3 Decomposing to Find Area

Find the area of each shaded square (in square units).

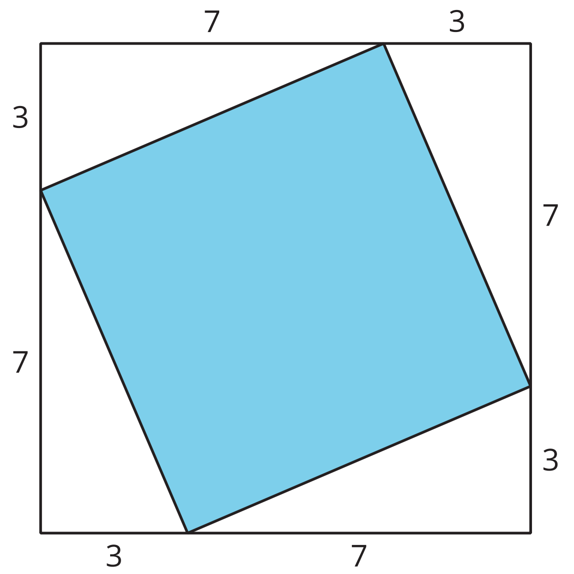
1.



2.

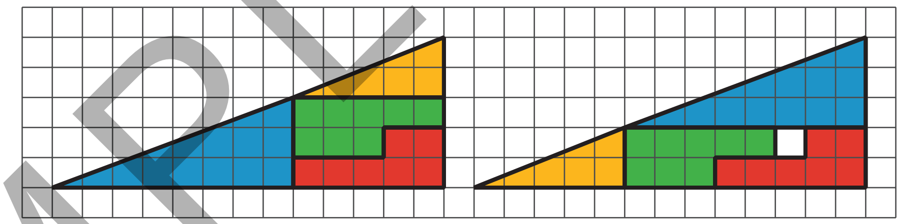


3.



Are you ready for more?

Any triangle with a base of 13 and a height of 5 has an area of $\frac{65}{2}$.

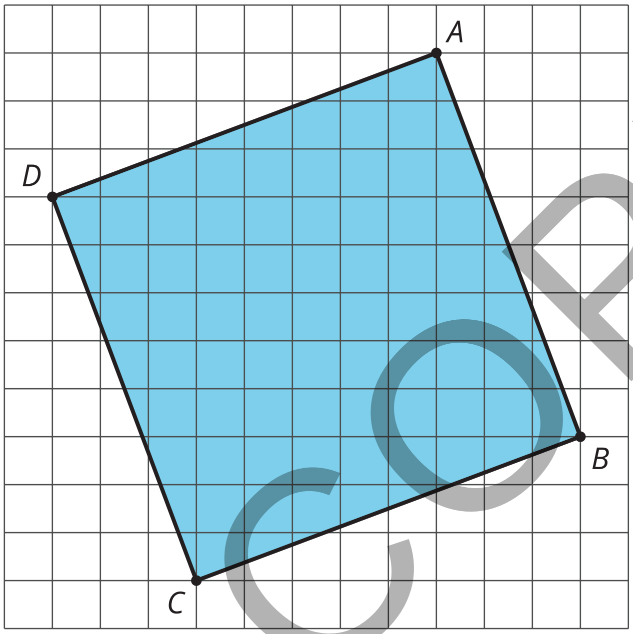


Both shapes in the figure have been partitioned into the same four pieces. Find the area of each of the pieces, and verify the corresponding parts are the same in each picture. There appears to be one extra square unit of area in the right figure. If all of the pieces have the same area, how is this possible?

Lesson 1 Summary

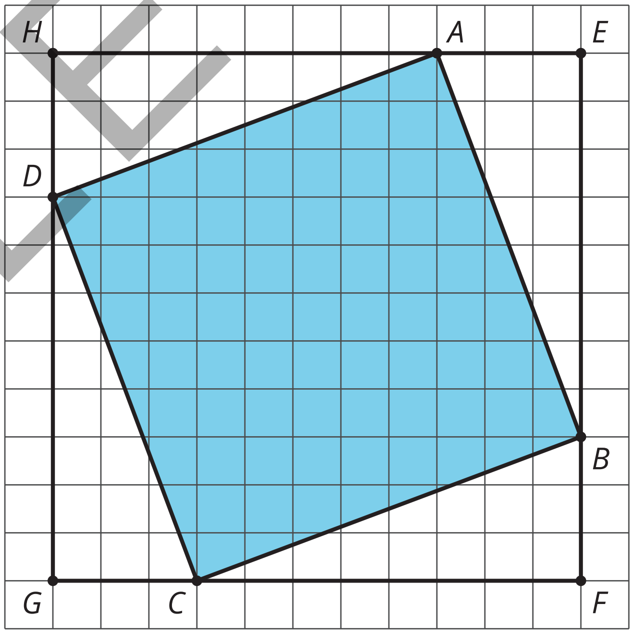
The area of a square with a side length of 12 units is 12^2 or 144 units^2.

Sometimes we want to find the area of a square but we don't know the side length. For example, how can we find the area of square $ABCD$?



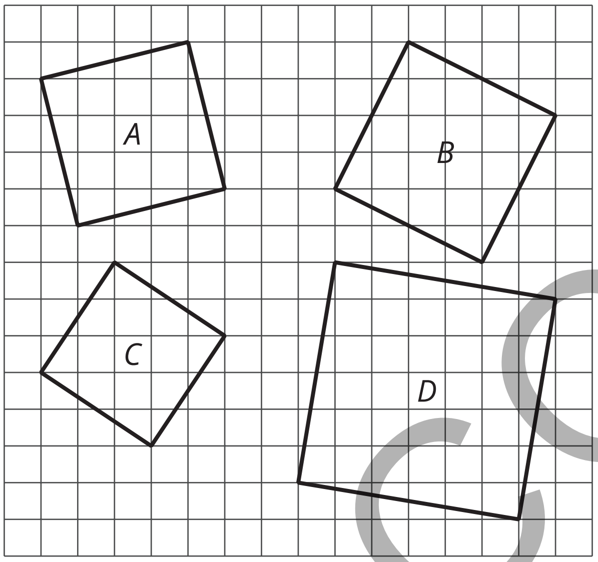
One way is to enclose it in a square whose side lengths we do know.

The outside square $EFGH$ has side lengths of 11 units, so its area is 121 units^2. The area of each of the four triangles is $\frac{1}{2} \cdot 8 \cdot 3 = 12$, so the area of all four together is $4 \cdot 12 = 48$ units^2. To get the area of the shaded square, we can take the area of the outside square and subtract the areas of the 4 triangles. So the area of the shaded square $ABCD$ is $121 - 48 = 73$ units^2.



Practice Problems

1 Find the area of each square. Each grid square represents 1 square unit.



2 Find the area of a square if its side length is:

a. 3 inches.

b. 7 units.

c. 100 centimeters.

d. 40 inches.

e. x units.

3 from Unit 7, Lesson 14

What is the value of $(3.1 \times 10^4) \cdot (2 \times 10^6)$?

A. 5.1×10^{10}

B. 5.1×10^{24}

C. 6.2×10^{10}

D. 6.2×10^{24}

4 from Unit 7, Lesson 15

Noah reads the problem, “Evaluate each expression and give the answer in scientific notation.” The first problem part is: $5.4 \times 10^5 + 2.3 \times 10^4$.

Noah says, “I can rewrite 5.4×10^5 as 54×10^4. Now I can add the numbers: $54 \times 10^4 + 2.3 \times 10^4 = 56.3 \times 10^4$.”

Do you agree with Noah’s solution to the problem? Explain your reasoning.

5 from Unit 7, Lesson 6

Select **all** the expressions that are equivalent to 3^8.

A. $(3^2)^4$

B. 8^3

C. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

D. $(3^4)^2$

E. $\frac{3^6}{3^{-2}}$

F. $3^6 \cdot 10^2$

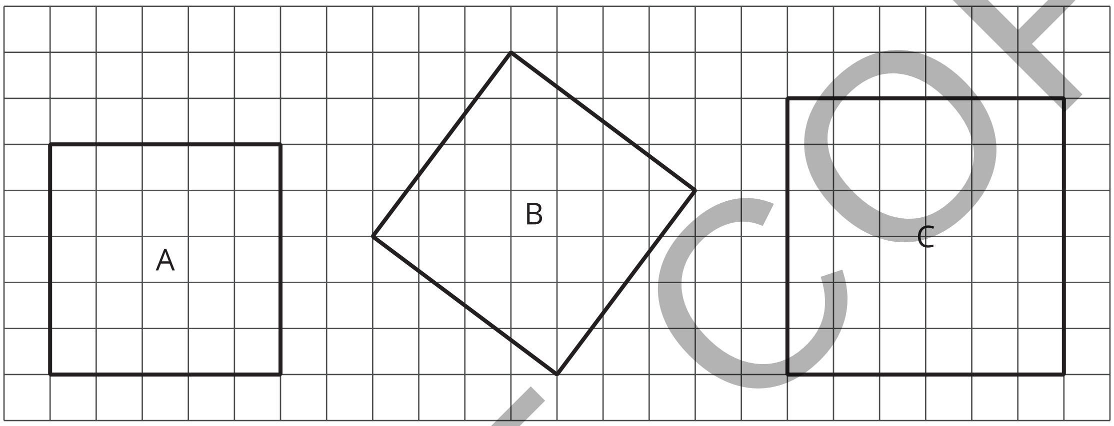
Unit 8, Lesson 2

Addressing CA CCSSM 8.NS.2; building on 6.EE.1, 6.G.1; building towards 8.EE.2; practicing MP5 and MP7

Side Lengths and Areas

Let's investigate some more squares.

2.1 Estimating Side Lengths from Areas (Part 1)



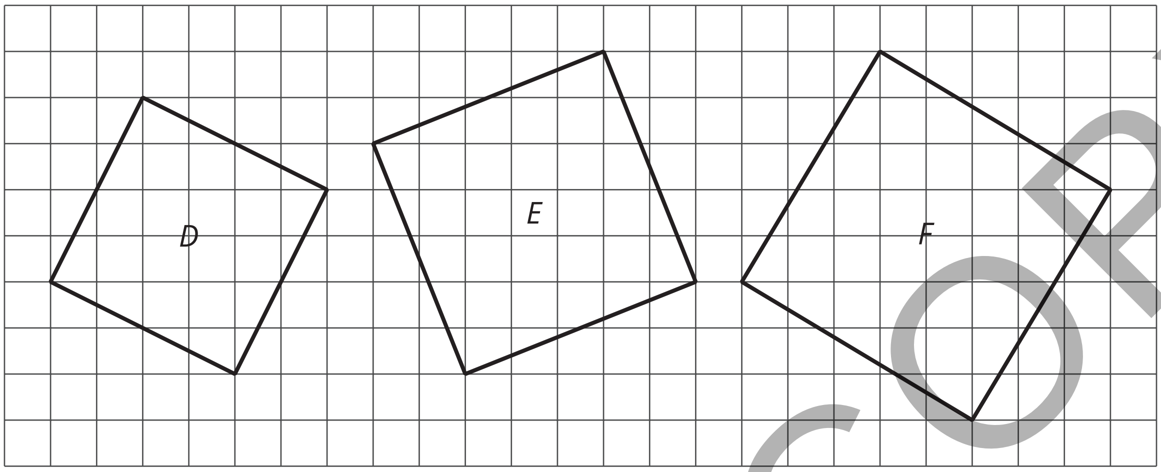
1. What is the side length of Square A? What is its area?

2. What is the side length of Square C? What is its area?

3. What is the area of Square B? What is its side length? (Use tracing paper to check your answer to this.)

2.2 Estimating Side Lengths from Areas (Part 2)

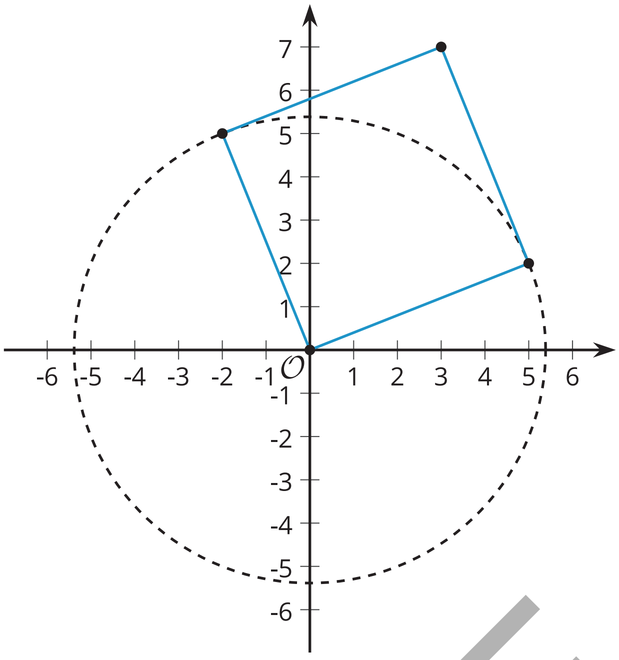
1. Find the areas of Squares D, E, and F.



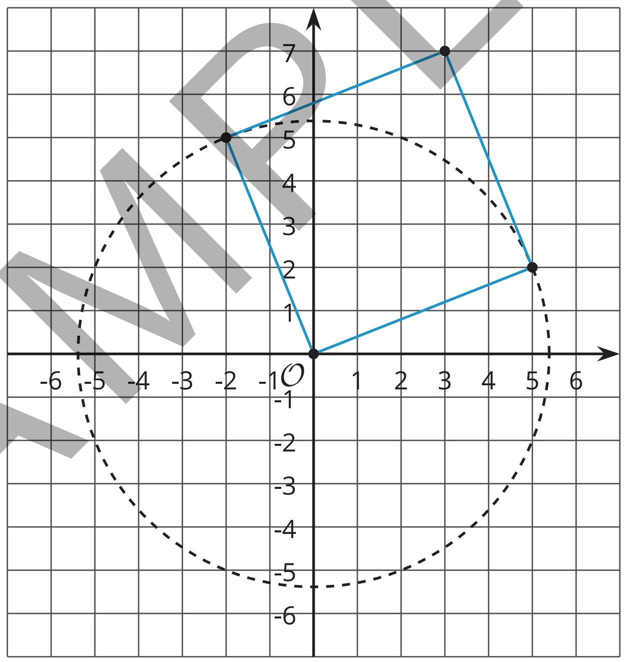
2. Which of these squares must have a side length that is greater than 5 but less than 6? Explain how you know.

2.3 One Square

1. Use the circle to estimate the area of the square shown here. Explain your reasoning.

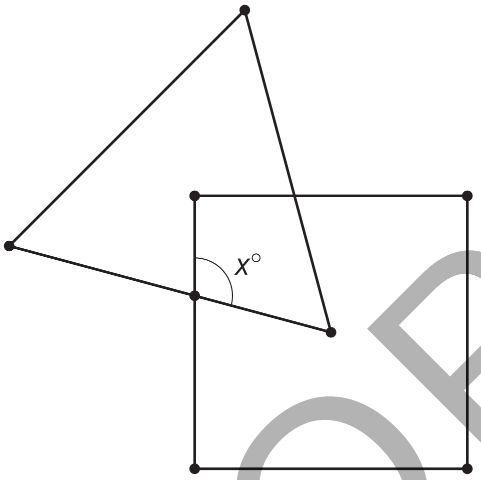


2. Use the grid to check your answer to the first problem.



Are you ready for more?

One vertex of the equilateral triangle is in the center of the square, and one vertex of the square is in the center of the equilateral triangle. What is x?

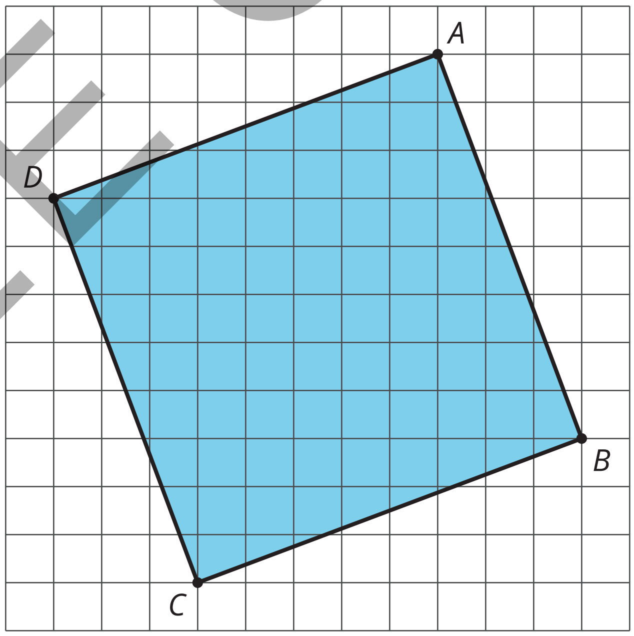


Lesson 2 Summary

The area of square $ABCD$ is 73 units^2.

Since the area is between $8^2 = 64$ and $9^2 = 81$, the side length must be between 8 units and 9 units. We can use tracing paper to trace a side length and compare it to the grid, which also shows the side length is between 8 units and 9 units.

When we want to talk about the exact side length, we can use the square root symbol. We say "the **square root** of 73," which is written as $\sqrt{73}$ and means "the side length of a square with area 73 square units." It is also true that $(\sqrt{73})^2 = 73$.



Glossary

- square root

Practice Problems

1 A square has an area of 81 square feet. Select **all** the expressions that equal the side length of this square, in feet.

A. $\frac{81}{2}$

B. $\sqrt{81}$

C. 9

D. $\sqrt{9}$

E. 3

2 Write the exact value of the side length, in units, of a square whose area in square units is:

a. 36

b. 37

c. $\frac{100}{9}$

d. $\frac{2}{5}$

e. 0.0001

f. 0.11

3 from Unit 8, Lesson 1

Find the area of a square if its side length is:

a. $\frac{1}{5}$ centimeter

b. $\frac{3}{7}$ unit

c. $\frac{11}{8}$ inches

d. 0.1 meter

e. 3.5 centimeters

4 from Unit 7, Lesson 15

Here is a table showing the areas of the seven largest countries.

country	area (km^2)
Russia	1.71×10^7
Canada	9.98×10^6
China	9.60×10^6
United States	9.53×10^6
Brazil	8.52×10^6
Australia	6.79×10^6
India	3.29×10^6

a. How much larger is Russia than Canada?

b. The Asian countries on this list are Russia, China, and India. The American countries are Canada, the United States, and Brazil. Which has the greater total area: the three Asian countries, or the three American countries?

5 from Unit 7, Lesson 5

Select **all** the expressions that are equivalent to 10^{-6}.

A. $\frac{1}{1,000,000}$

B. $\frac{-1}{1,000,000}$

C. $\frac{1}{10^6}$

D. $10^8 \cdot 10^{-2}$

E. $\left(\frac{1}{10}\right)^6$

F. $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

Unit 8, Lesson 3

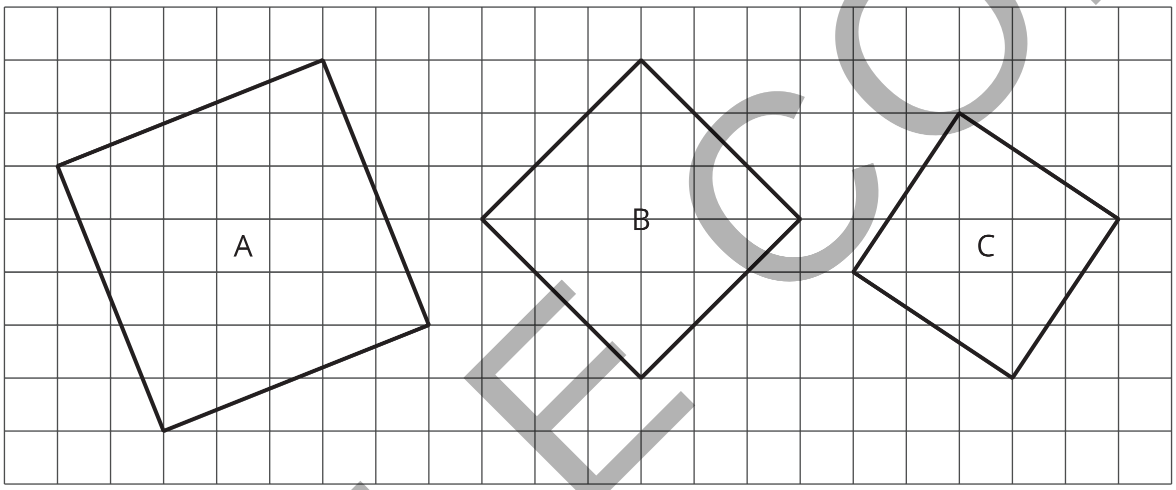
Addressing CA CCSSM 8.EE.2, 8.F.4-5; building on 8.G.2; building towards 8.G.7; practicing MP3 and MP6

Square Roots

Let's find some side lengths of squares.

3.1 The Sides and Areas of Tilted Squares

Find the area of each square and estimate the side lengths using your geometry toolkit. Then write the exact length for the sides of each square.



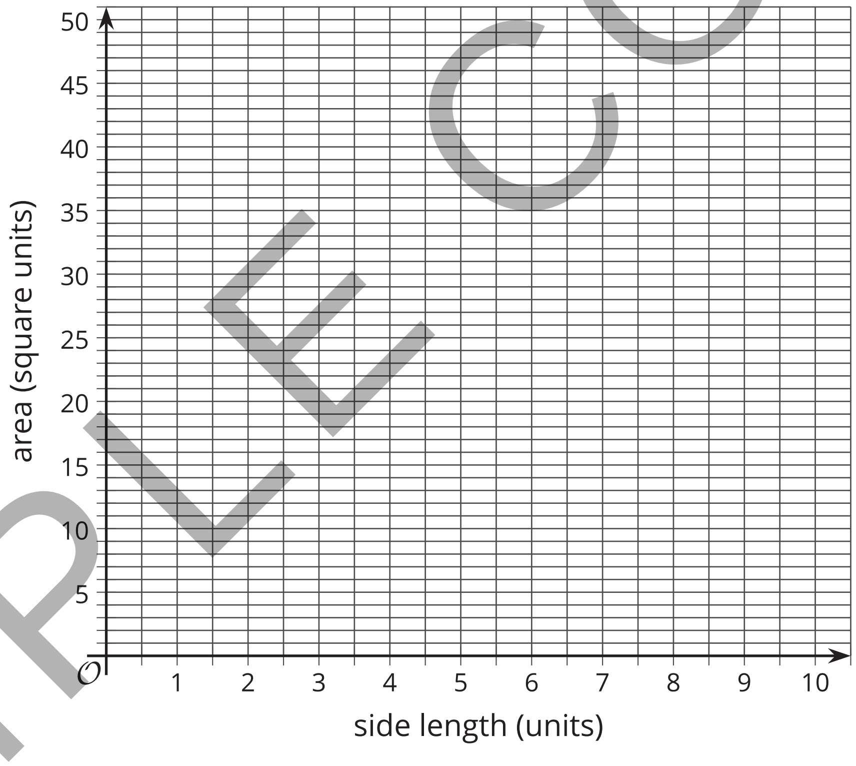
3.2 Side Lengths and Areas of Squares

1. Complete the tables with the missing side lengths and areas.

side length, s	0.5		1.5		2.5		3.5	
area, a		1		4		9		16

side length, s	4.5		5.5		6.5		7.5	
area, a		25		36		49		64

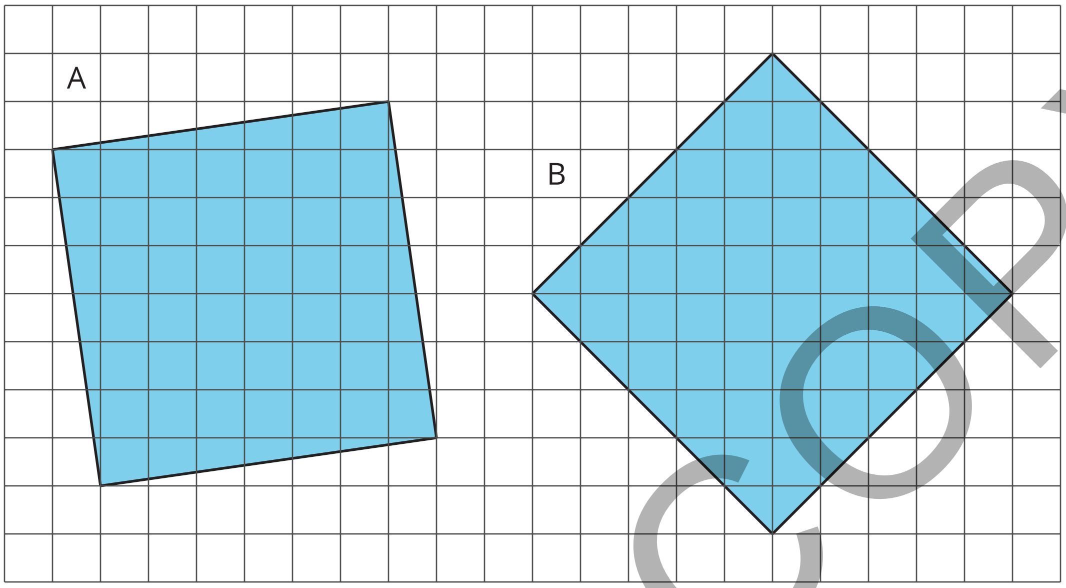
2. Plot the points, (s, a), on the coordinate plane shown here.



3. Use the graph to estimate the side lengths of Squares A, B, and C from the previous activity. How do your estimates from the graph compare to the estimates you made earlier using your geometry toolkit?

4. Use the graph to approximate $\sqrt{45}$.

3.3 Comparing Areas of Squares



1. Find the area of each square and estimate their side lengths.

2. Write the exact length for the side of each square.

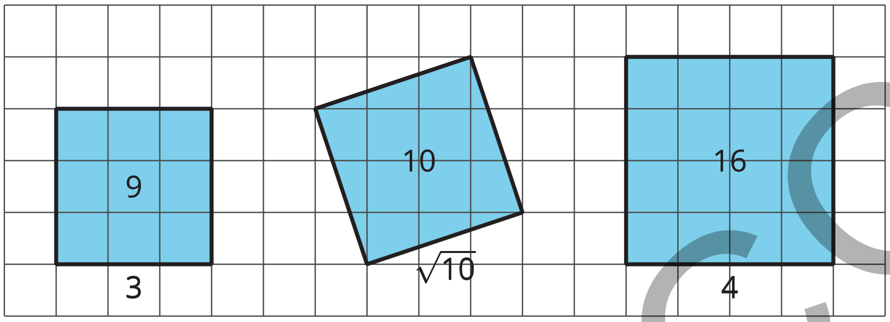
3. Which square has the larger area? Verify using your geometry toolkit.

Lesson 3 Summary

We know that:

- $\sqrt{9} = 3$ because $3^2 = 9$.
- $\sqrt{16} = 4$ because $4^2 = 16$.

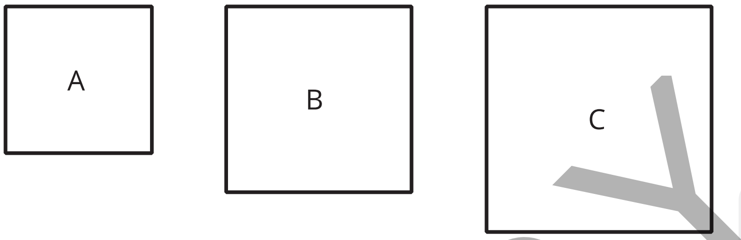
The value of $\sqrt{10}$ must be between 3 units and 4 units because it is between the values of $\sqrt{9}$ and $\sqrt{16}$.



Practice Problems

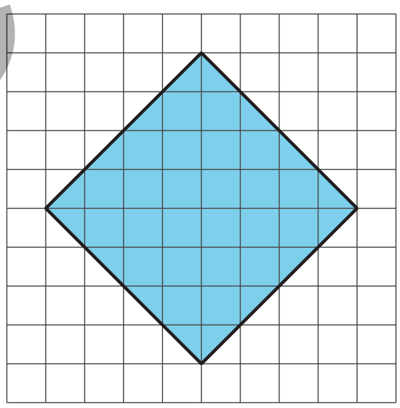
1 Square A is smaller than Square B. Square B is smaller than Square C.

The three squares' side lengths are $\sqrt{26}$, 4.2, and $\sqrt{11}$.



What is the side length of Square A? Square B ? Square C? Explain how you know.

2 Each grid square represents 1 square unit. What is the exact side length of the shaded square? Estimate the side length of the shaded square to the nearest tenth.



3 A square has a side length of $\sqrt{30}$ centimeters. Select **all** the expressions that equal the area of this square in square centimeters.

A. 30

B. $2 \cdot \sqrt{30}$

C. 90

D. $\sqrt{60}$

E. $\sqrt{30} \cdot \sqrt{30}$

F. $\sqrt{30}^2$

4 from Unit 8, Lesson 2

Find the length of a side of a square if its area is:

a. 81 square inches

b. $\frac{4}{25}$ square centimeters

c. 0.49 square units

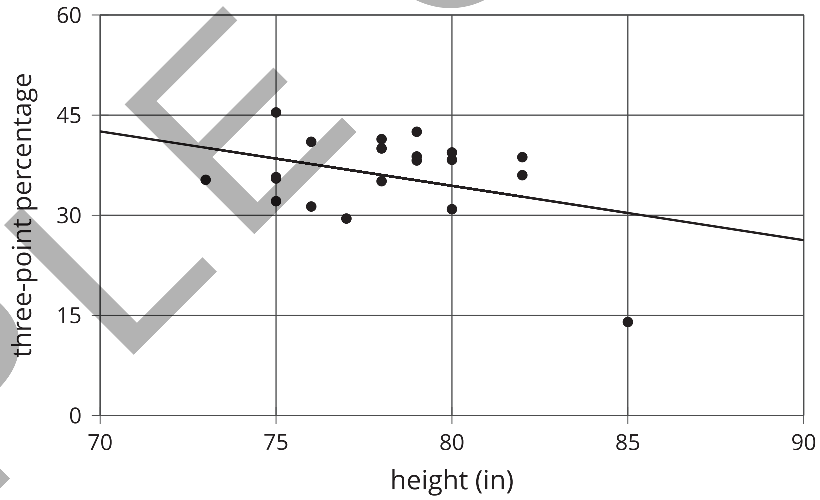
d. m^2 square units

5 from Unit 6, Lesson 4

The scatter plot shows the heights (in inches) and three-point percentages for different basketball players last season.

a. Circle any data points that appear to be outliers.

b. Compare any outliers to the values predicted by the model.



Unit 8, Lesson 4
Addressing CA CCSSM 8.EE.2, 8.NS.1; building on 5.NF.4, 6.EE.1; building towards 8.EE.2, 8.NS.1, 8.NS.2; practicing MP3

Rational and Irrational Numbers

Let’s learn about irrational numbers.

4.1 Math Talk: Positive Solutions

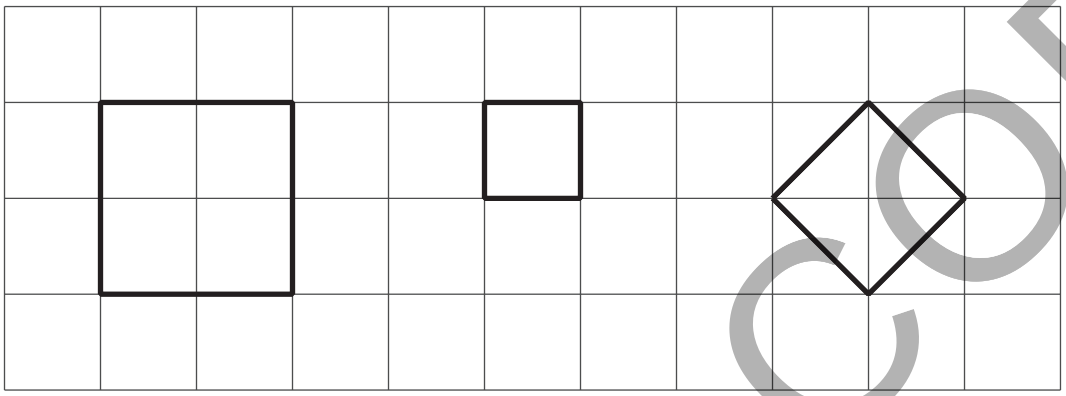
Solve each equation mentally.

- $x^2 = 36$
- $x^2 = \frac{9}{4}$
- $x^2 = \frac{1}{4}$
- $x^2 = \frac{49}{25}$

4.2 Three Squares

For each square:

1. Label the area.
2. Label the side length.
3. Write an equation that shows the relationship between the side length and the area.



4.3 Looking for a Solution

Are any of these numbers a solution to the equation $x^2 = 2$? Explain your reasoning.

- 1
- $\frac{1}{2}$
- $\frac{3}{2}$
- $\frac{7}{5}$

4.4 Looking for $\sqrt{2}$

A **rational number** is a number that can be expressed as a positive or negative fraction.

1. Find some more rational numbers that are close to $\sqrt{2}$.

2. Can you find a rational number that is exactly $\sqrt{2}$?

Are you ready for more?

If you have an older calculator and evaluate the expression $\left(\frac{577}{408}\right)^2$, it will tell you that the answer is 2, which might lead you to think that $\sqrt{2}=\frac{577}{408}$.

1. Explain why you might be suspicious of the calculator's result.

2. Find an explanation for why $408^2 \cdot 2$ could not possibly equal 577^2. How does this show that $\left(\frac{577}{408}\right)^2$ could not equal 2?

3. Repeat these questions for $\left(\frac{1414213562375}{1000000000000}\right)^2 \neq 2$, an equation that even many modern calculators and computers will get wrong.

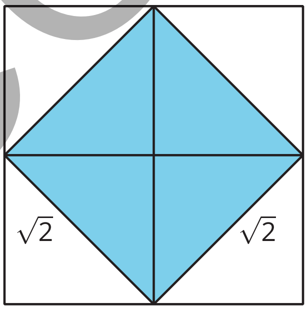
Lesson 4 Summary

A square whose area is 25 square units has a side length of $\sqrt{25}$ units, which means that $\sqrt{25} \cdot \sqrt{25} = 25$. Since $5 \cdot 5 = 25$, we know that $\sqrt{25} = 5$.

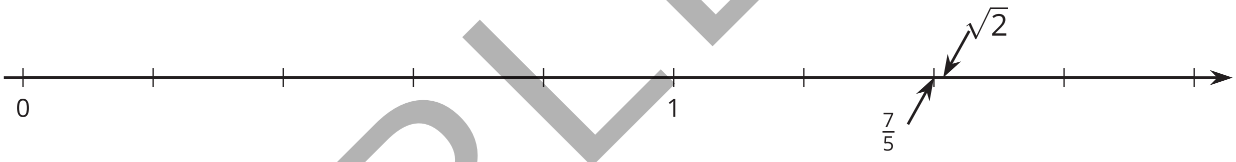
$\sqrt{25}$ is an example of a rational number. A **rational number** is a fraction or its opposite. In an earlier grade we learned that $\frac{a}{b}$ is a point on the number line found by dividing the interval from 0 to 1 into b equal parts and finding the point that is a of them to the right of 0. We can always write a fraction in the form $\frac{a}{b}$, where a and b are integers (and b is not 0), but there are other ways to write them. For example, we can write $\sqrt{25} = \frac{5}{1} = 5$ or $-\frac{1}{\sqrt{4}} = -\frac{1}{2}$. Because fractions and *ratios* are closely related ideas, fractions and their opposites are called *rational* numbers.

Here are some examples of rational numbers: $\frac{7}{4}, 0, \frac{6}{3}, 0.2, -\frac{1}{3}, -5, \sqrt{9}, -\frac{\sqrt{16}}{\sqrt{100}}$

Now consider a square whose area is 2 square units with a side length of $\sqrt{2}$ units. This means that $\sqrt{2} \cdot \sqrt{2} = 2$.



An **irrational number** is a number that is not rational, meaning it cannot be expressed as a positive or negative fraction. For example, $\sqrt{2}$ has a location on the number line (it's a tiny bit to the right of $\frac{7}{5}$), but its location can not be found by dividing the segment from 0 to 1 into b equal parts and going a of those parts away from 0.



$\frac{17}{12}$ is close to $\sqrt{2}$ because $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$, which is very close to 2 since $\frac{288}{144} = 2$. We could keep looking forever for rational numbers that are solutions to $x^2 = 2$, and we would not find any since $\sqrt{2}$ is an irrational number.

The square root of any whole number is either a whole number, like $\sqrt{36} = 6$ or $\sqrt{64} = 8$, or an irrational number. Here are some examples of irrational numbers: $\sqrt{10}, -\sqrt{3}, \frac{\sqrt{5}}{2}, \pi$.

Glossary

- irrational number
- rational number

Practice Problems

1 Decide whether each number in this list is *rational* or *irrational*.

$\frac{-13}{3}$ $\quad 0.1234 \quad \sqrt{37} \quad -77 \quad -\sqrt{100} \quad -\sqrt{12}$

2 Which value is an exact solution of the equation $m^2 = 14$?

A. 7

B. $\sqrt{14}$

C. 3.74

D. $\sqrt{3.74}$

3 from Unit 7, Lesson 8

Rewrite each expression in an equivalent form that uses a single exponent.

a. $(10^2)^{-3}$

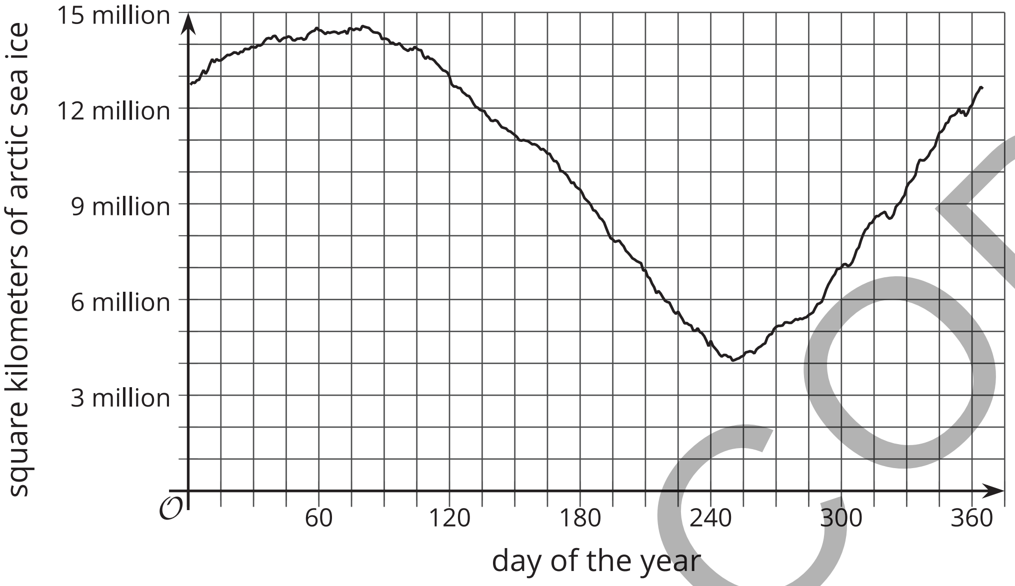
b. $(3^{-3})^2$

c. $3^{-5} \cdot 4^{-5}$

d. $2^5 \cdot 3^{-5}$

4 from Unit 5, Lesson 5

The graph represents the area of arctic sea ice in square kilometers as a function of the day of the year in 2016.



a. Give an approximate interval of days when the area of arctic sea ice was decreasing.

b. On which days was the area of arctic sea ice 12 million square kilometers?

5 from Unit 4, Lesson 14

A high school is hosting an event for seniors but will also allow some juniors to attend. The principal approved the event for 200 students and decided the number of juniors should be 25% of the number of seniors. How many juniors will be allowed to attend? If you get stuck, try writing two equations that each represent the number of juniors and seniors at the event.

Unit 8, Lesson 5

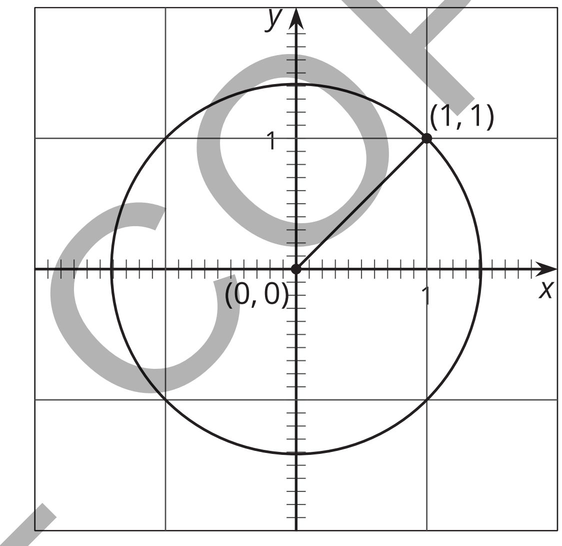
Addressing CA CCSSM 8.EE.2, 8.NS.2; practicing MP3 and MP7

Square Roots on the Number Line

Let’s explore square roots.

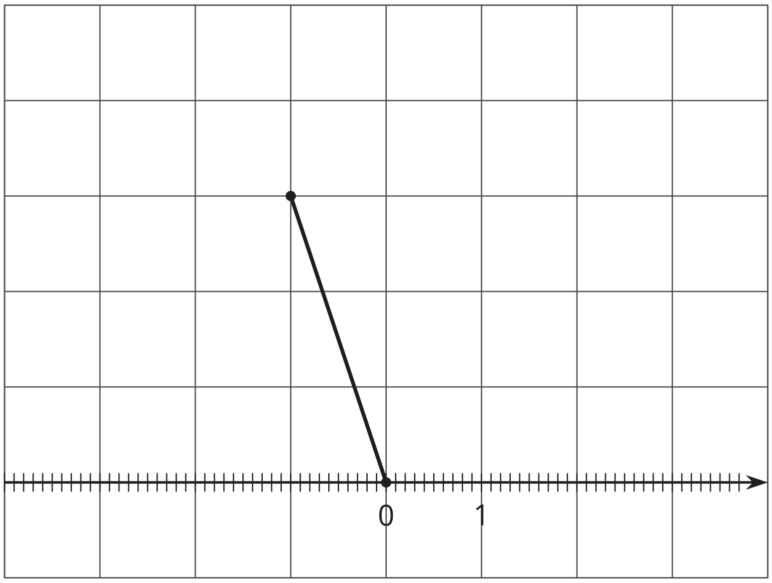
5.1 Notice and Wonder: Diagonals

What do you notice? What do you wonder?



5.2 Squaring Lines

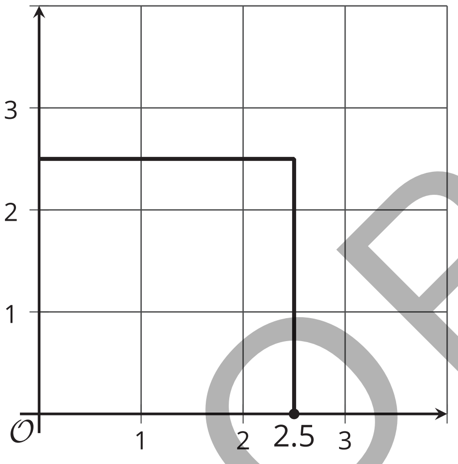
Find the length of the segment.



5.3 Square Root of 3

1. Diego says that $\sqrt{3} \approx 2.5$.

   Use the square to explain why 2.5 is not a very good approximation for $\sqrt{3}$.

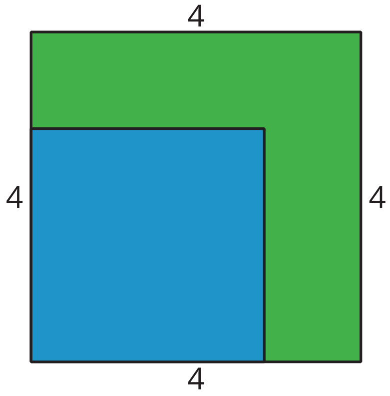


2. Find a point on the number line that is closer to $\sqrt{3}$. Draw a new square on the coordinate plane and use it to explain how you know the point you plotted is a good approximation for $\sqrt{3}$.

Are you ready for more?

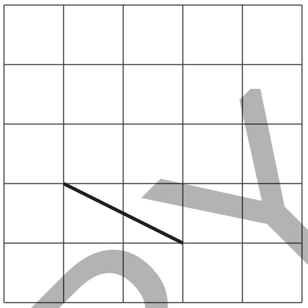
A farmer has a grassy patch of land enclosed by a fence in the shape of a square with a side length of 4 meters. To make it a suitable home for some animals, the farmer would like to carve out a smaller square to be filled with water, as shown in the figure.

What should the side length of the smaller square be so that half of the area is grass and half is water?

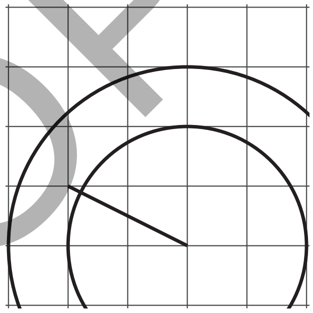


Lesson 5 Summary

Here is a line segment on a grid. How can we determine the length of this line segment?

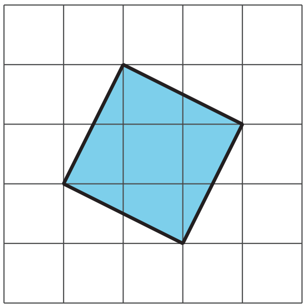


By drawing some circles, we can tell that it's longer than 2 units, but shorter than 3 units.



To find an exact value for the length of the segment, we can build a square on it, using the segment as one of the sides of the square.

The area of this square is 5 square units. That means the exact value of the length of its side is $\sqrt{5}$ units.



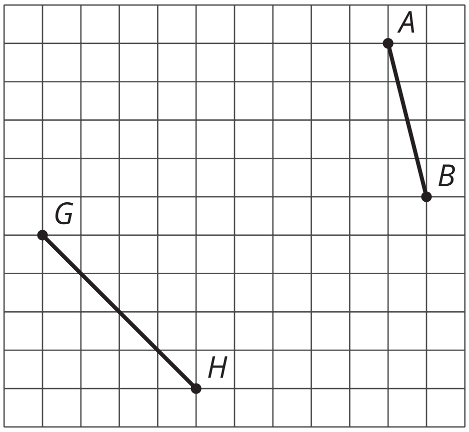
Notice that 5 is greater than 4, but less than 9. That means that $\sqrt{5}$ is greater than 2, but less than 3. This makes sense because we already saw that the length of the segment is in between 2 and 3.

With some arithmetic, we can get an even more precise idea of where $\sqrt{5}$ is on the number line. The image with the circles shows that $\sqrt{5}$ is closer to 2 than 3, so let's find the value of 2.1^2 and 2.2^2 and see how close they are to 5. It turns out that $2.1^2 = 4.41$ and $2.2^2 = 4.84$, so we need to try a larger number. If we increase our search by a tenth, we find that $2.3^2 = 5.29$. This means that $\sqrt{5}$ is greater than 2.2, but less than 2.3. If we wanted to keep going, we could try 2.25^2 and eventually narrow the value of $\sqrt{5}$ to the hundredths place. Calculators do this same process to many decimal places, giving an approximation like $\sqrt{5} \approx 2.2360679775$. Even though this is a lot of decimal places, it is still not exact because $\sqrt{5}$ is irrational.

Practice Problems

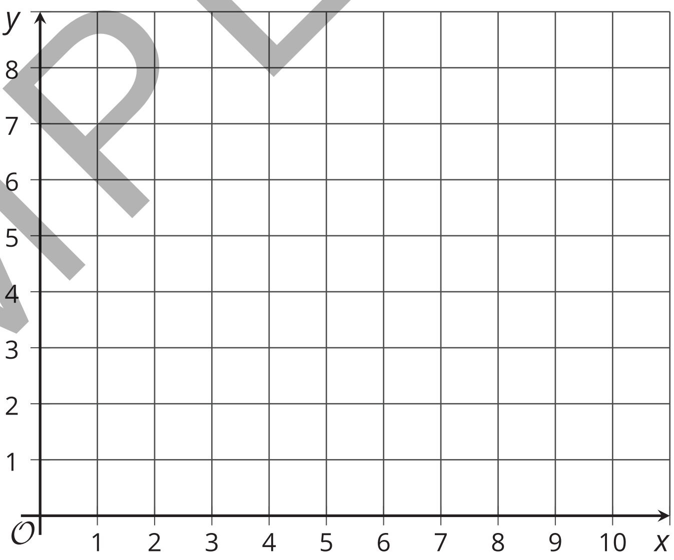
1

a. Find the exact length of each line segment.



b. Estimate the length of each line segment to the nearest tenth of a unit. Explain your reasoning.

2 Plot each number on the x-axis: $\sqrt{16}$, $\sqrt{35}$, $\sqrt{66}$. Consider using the grid to help.



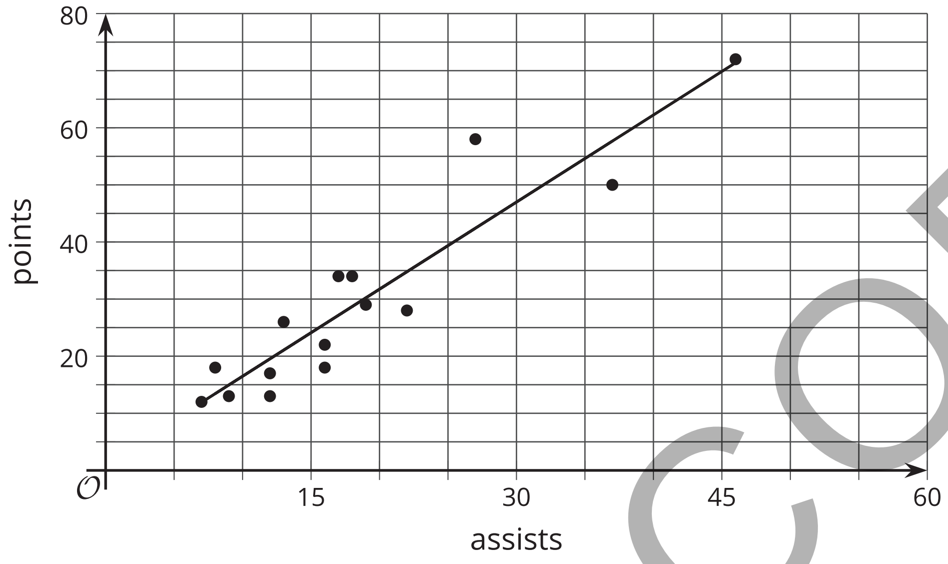
3 Use the fact that $\sqrt{7}$ is a solution to the equation $x^2 = 7$ to find a decimal approximation of $\sqrt{7}$, whose square is between 6.9 and 7.1.

4 from Unit 7, Lesson 14

Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometers, or 200×10^{-12} meters, thick. How many layers of graphene are there in a 1.6-mm-thick piece of graphite? Express your answer in scientific notation.

5 from Unit 6, Lesson 6

Here is a scatter plot that shows the number of assists and points for a group of hockey players. The model, represented by $y = 1.5x + 1.2$, is graphed with the scatter plot.



a. What does the slope mean in this situation?

b. Based on the model, how many points will a player have if he has 30 assists?

6 from Unit 3, Lesson 5

The points $(12, 23)$ and $(14, 45)$ lie on a line. What is the slope of the line?

Unit 8, Lesson 6

Addressing CA CCSSM 8.EE.2, 8.NS.2; practicing MP3 and MP7

Reasoning about Square Roots

Let's approximate square roots.

6.1 Math Talk: Squared

Decide mentally whether or not each statement is true.

- $\left(\sqrt{5}\right)^2 = 5$
- $\left(\sqrt{9}\right)^2 = 3$
- $\left(\sqrt{10}\right)^2 = 100$
- $\left(\sqrt{16}\right) = 2^2$

6.2 Square Root Values

The value of a square root of a number lies between two consecutive whole numbers. Which are those consecutive whole numbers for the following? Be prepared to explain your reasoning.

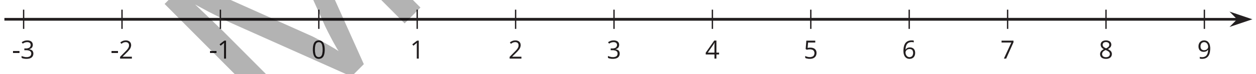
1. $\sqrt{7}$
2. $\sqrt{23}$
3. $\sqrt{50}$
4. $\sqrt{98}$

Are you ready for more?

Can we do any better than "between 3 and 4" for $\sqrt{12}$? Explain a way to figure out if the value is closer to 3.1 or closer to 3.9.

6.3 Solutions on a Number Line

The numbers x, y, and z are positive, and $x^2 = 3$, $y^2 = 16$, and $z^2 = 30$.



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.
2. Plot $-\sqrt{2}$ on the number line.

Lesson 6 Summary

In general, we can approximate the value of a square root by observing the whole numbers around it and remembering the relationship between square roots and squares. Here are some examples:

- $\sqrt{65}$ is a little more than 8 because $\sqrt{65}$ is a little more than $\sqrt{64}$, and $\sqrt{64} = 8$.
- $\sqrt{80}$ is a little less than 9 because $\sqrt{80}$ is a little less than $\sqrt{81}$, and $\sqrt{81} = 9$.
- $\sqrt{75}$ is between 8 and 9 (it's 8 point something) because 75 is between 64 and 81.
- $\sqrt{75}$ is approximately 8.67 because $8.67^2 = 75.1689$.

If we want to find the square root of a number between two whole numbers, we can work in the other direction. For example, since $22^2 = 484$ and $23^2 = 529$, then we know that $\sqrt{500}$ (to pick one possibility) is between 22 and 23. Many calculators have a square root command, which makes it simple to find an approximate value of a square root.

Practice Problems

1

a. Explain how you know that $\sqrt{37}$ is a little more than 6.

b. Explain how you know that $\sqrt{95}$ is a little less than 10.

c. Explain how you know that $\sqrt{30}$ is between 5 and 6.

2 Plot each number on the number line: $6, \sqrt{83}, \sqrt{40}, \sqrt{64}, 7.5$

3 The equation $x^2 = 25$ has *two* solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So 5 is a solution, and -5 is also a solution.

Select **all** the equations that have a solution of -4:

A. $10 + x = 6$

B. $10 - x = 6$

C. $-3x = -12$

D. $-3x = 12$

E. $8 = x^2$

F. $x^2 = 16$

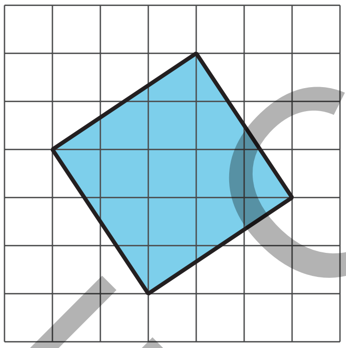
4 from Unit 8, Lesson 4

Select all the irrational numbers in the list.

$\frac{2}{3}$ $\frac{-123}{45}$ $\sqrt{14}$ $\sqrt{64}$ $\sqrt{\frac{9}{1}}$ $-\sqrt{99}$ $-\sqrt{100}$

5 from Unit 8, Lesson 2

Each grid square represents 1 square unit. What is the exact side length of the shaded square?



6 from Unit 7, Lesson 10

For each pair of numbers, which of the two numbers is larger? Estimate how many times larger.

a. $0.37 \cdot 10^6$ and $700 \cdot 10^4$

b. $4.87 \cdot 10^4$ and $15 \cdot 10^5$

Unit 8, Lesson 7

Addressing CA CCSSM 8.G.6, 8.G.7; building on 5.G.4, 7.G.1, 8.EE.2; building towards 8.G.6-7; practicing MP6

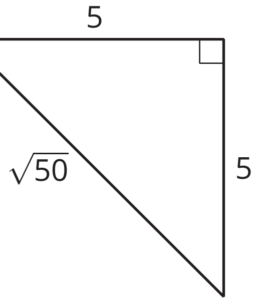
Finding Side Lengths of Triangles

Let’s find triangle side lengths.

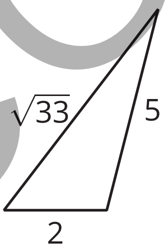
7.1 Which Three Go Together: Triangles

Which three go together? Why do they go together?

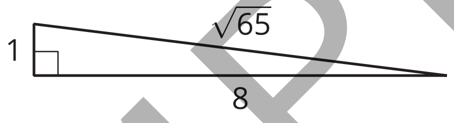
A



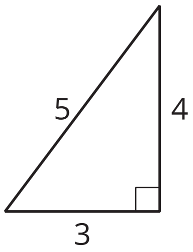
B



C



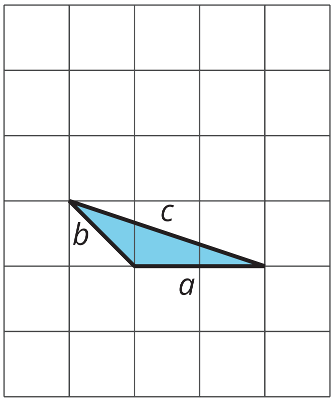
D



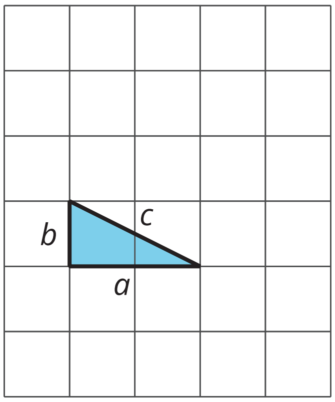
7.2 A Table of Triangles

1. Complete the tables for these three triangles:

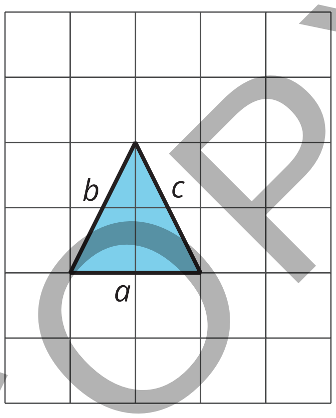
D



E



F



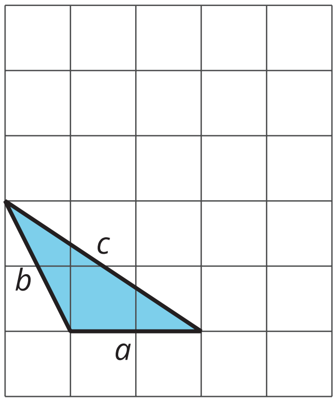
	a	b	c
Triangle D			
Triangle E			
Triangle F			

	a^2	b^2	c^2
Triangle D			
Triangle E			
Triangle F			

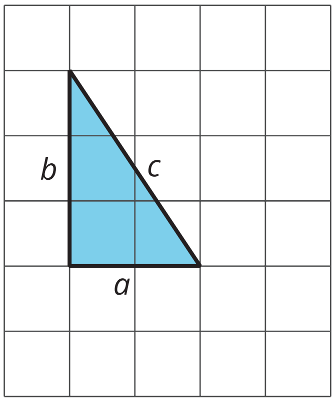
2. What do you notice about the values in the table for Triangle E but not for Triangles D and F?

3. Complete the tables for three more triangles:

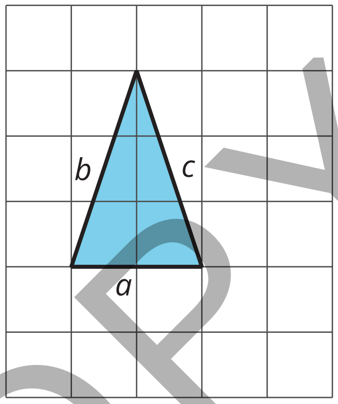
P



Q



R



	a	b	c
Triangle P			
Triangle Q			
Triangle R			

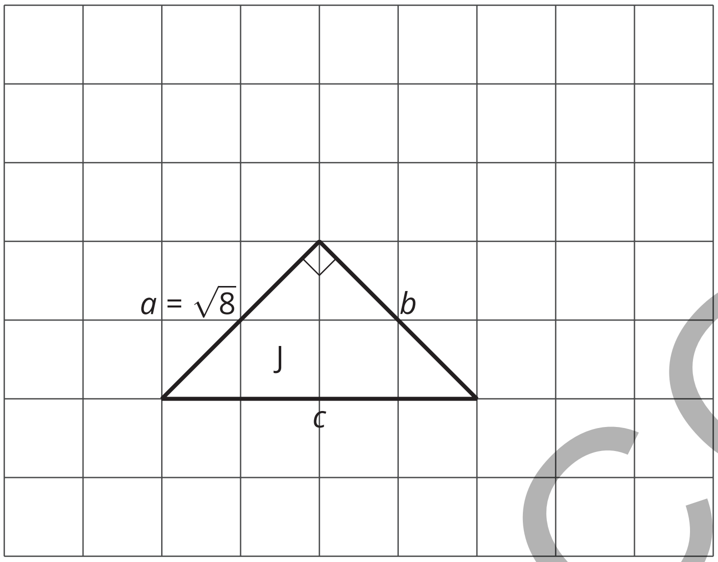
	a^2	b^2	c^2
Triangle P			
Triangle Q			
Triangle R			

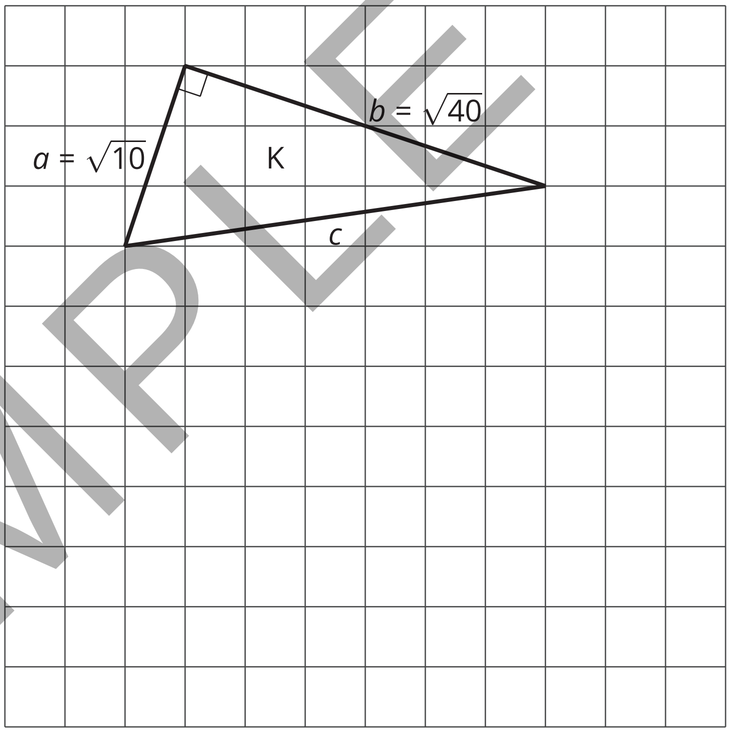
4. What do you notice about the values in the table for Triangle Q but not for Triangles P and R?

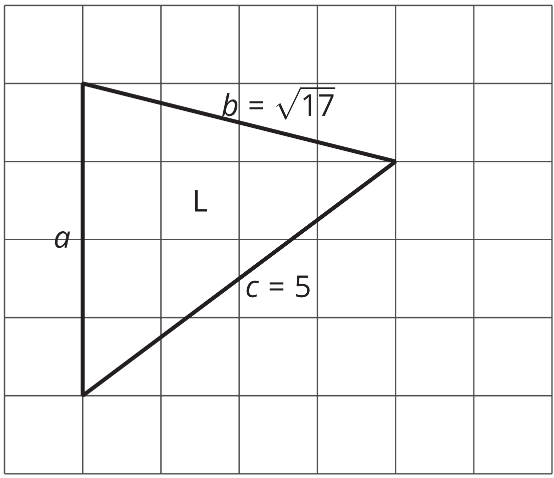
5. What do Triangle E and Triangle Q have in common?

7.3 Meet the Pythagorean Theorem

1. Find the missing side lengths. Be prepared to explain your reasoning.



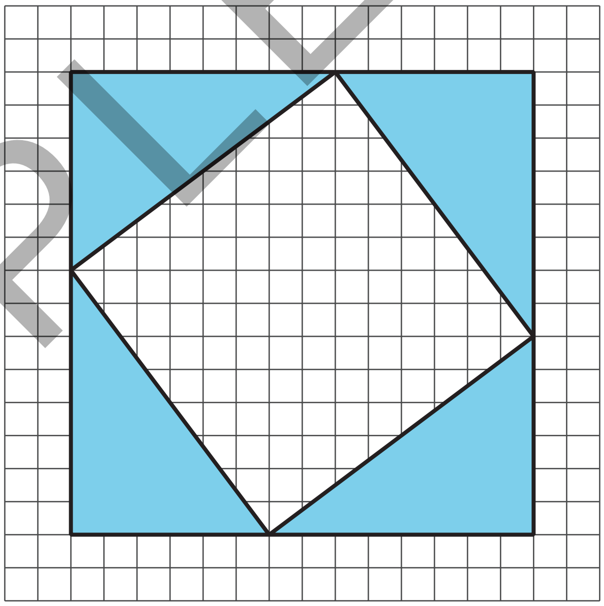




2. For which triangles does $a^2 + b^2 = c^2$?

Are you ready for more?

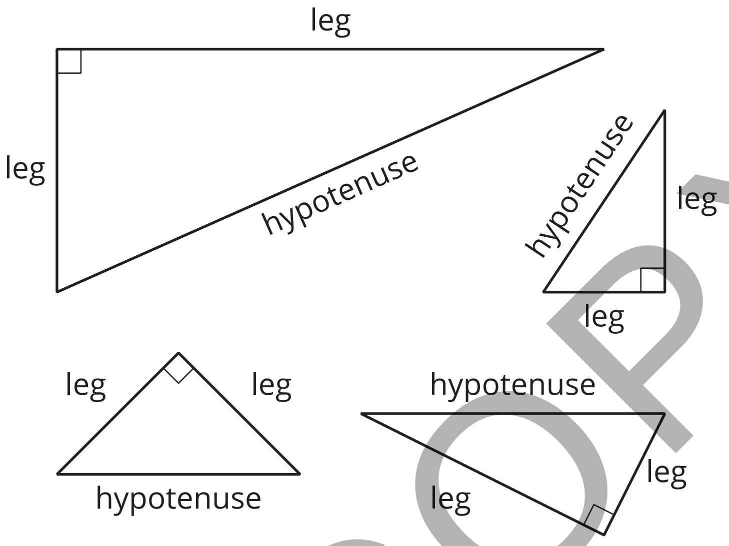
If the four shaded triangles in the figure are congruent right triangles, does the inner quadrilateral have to be a square? Explain how you know.

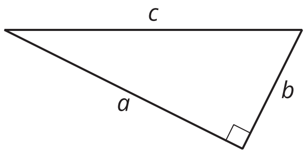


Lesson 7 Summary

A *right triangle* is a triangle with a right angle. In a right triangle, the side opposite the right angle is called the **hypotenuse**, and the two other sides that make the right angle are called its **legs**.

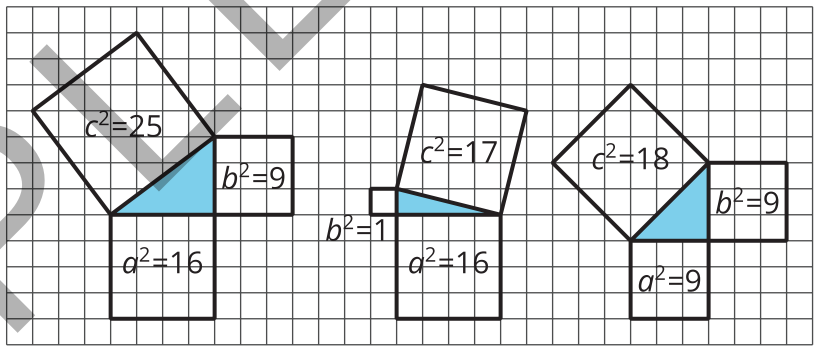
Here are some right triangles with the hypotenuse and legs labeled:





If the triangle is a right triangle, then a and b are used to represent the lengths of the legs, and c is used to represent the length of the hypotenuse. The hypotenuse is always the longest side of a right triangle.

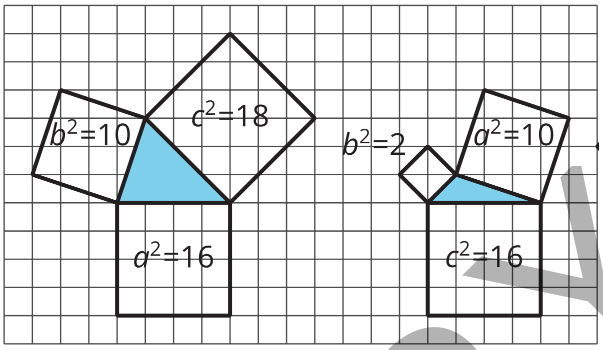
Here are some other right triangles:



Notice that for these examples of right triangles, the square of the hypotenuse is equal to the sum of the squares of the legs. In the first right triangle in the diagram, $16 + 9 = 25$, in the second, $16 + 1 = 17$, and in the third, $9 + 9 = 18$. Expressed another way, we have

This is a property of all right triangles, not just these examples, and is often known as the **Pythagorean Theorem**. The name comes from a mathematician named Pythagoras who lived in ancient Greece around 2,500 BCE, but this property of right triangles was also discovered independently by mathematicians in other ancient cultures including Babylon, India, and China. In China, a name for the same relationship is the Shang Gao Theorem.

It is important to note that this relationship does not hold for *all* triangles. Here are some triangles that are not right triangles. Notice that the lengths of their sides do not have the special relationship $a^2 + b^2 = c^2$. That is, $16 + 10$ does not equal 18, and $10 + 2$ does not equal 16.

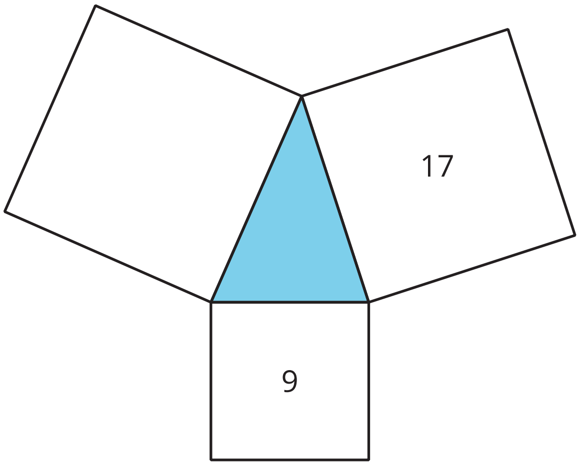


Glossary

- hypotenuse
- legs
- Pythagorean Theorem

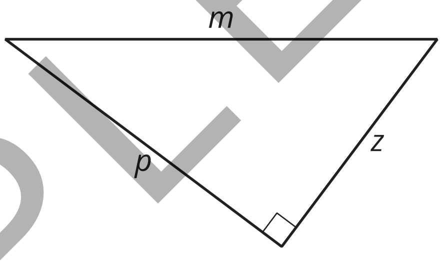
Practice Problems

1 Here is a diagram of an acute triangle and three squares.



Priya says the area of the large unmarked square is 26 square units because $9 + 17 = 26$. Do you agree? Explain your reasoning.

2 m, p, and z represent the lengths of the three sides of this right triangle.



Select **all** the equations that represent the relationship between m, p, and z.

A. $m^2 + p^2 = z^2$

B. $m^2 = p^2 + z^2$

C. $m^2 = z^2 + p^2$

D. $p^2 + m^2 = z^2$

E. $z^2 + p^2 = m^2$

F. $p^2 + z^2 = m^2$

3 The lengths of the three sides (in units) are given for several right triangles. For each, write an equation that expresses the relationship between the lengths of the three sides.

a. 10, 6, 8

b. $\sqrt{5}, \sqrt{3}, \sqrt{8}$

c. 5, $\sqrt{5}, \sqrt{30}$

d. 1, $\sqrt{37}$, 6

e. 3, $\sqrt{2}, \sqrt{7}$

4 from Unit 4, Lesson 1

Order the following expressions from least to greatest.

$25 \div 10$ $\quad 250{,}000 \div 1{,}000$ $\quad 2.5 \div 1{,}000$ $\quad 0.025 \div 1$

5 from Unit 8, Lesson 4

Which is the best explanation for why $-\sqrt{10}$ is irrational?

A. $-\sqrt{10}$ is irrational because it is a square root.

B. $-\sqrt{10}$ is irrational because it is less than zero.

C. $-\sqrt{10}$ is irrational because it is not a whole number.

D. $-\sqrt{10}$ is irrational because it cannot be written as a positive or negative fraction.

6 from Unit 7, Lesson 15

A teacher tells her students she is just over 1 and $\frac{1}{2}$ billion seconds old.

a. Write her age in seconds using scientific notation.

b. What is a more reasonable unit of measurement for this situation?

c. How old is she when you use a more reasonable unit of measurement?

Unit 8, Lesson 8

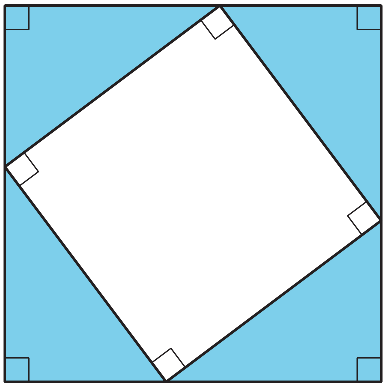
Addressing CA CCSSM 8.G.6, 8.G.7; building on 8.EE.7b; building towards 8.G.6; practicing MP1, MP7, MP8

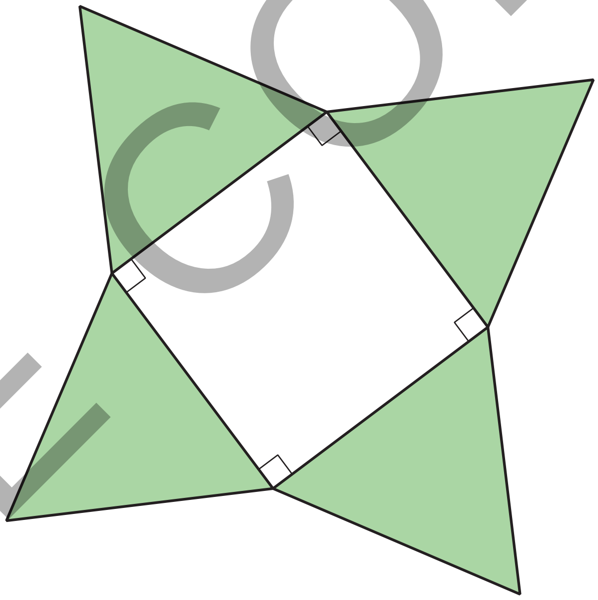
A Proof of the Pythagorean Theorem

Let’s prove the Pythagorean Theorem.

8.1 Notice and Wonder: A Square and Four Triangles

What do you notice? What do you wonder?

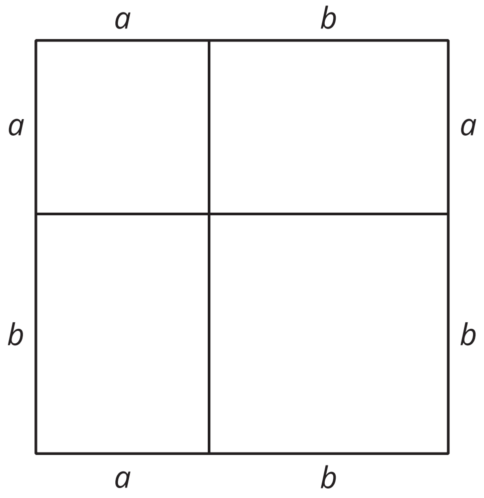




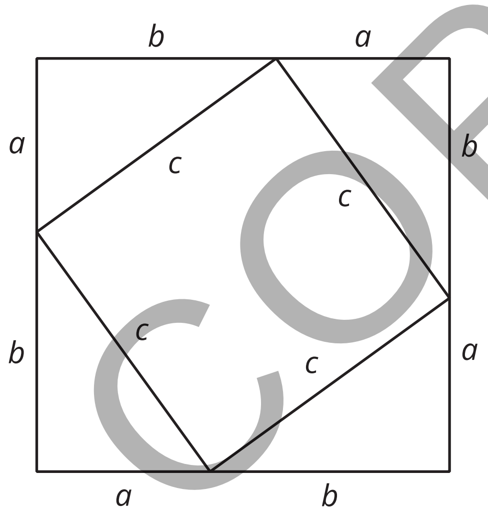
8.2 Adding Up Areas

Both figures shown here are squares with a side length of $a + b$. Notice that the first figure is divided into two squares and two rectangles. The second figure is divided into a square and four right triangles with **legs** of lengths a and b. Let's call the **hypotenuse** of these triangles c.

F



G



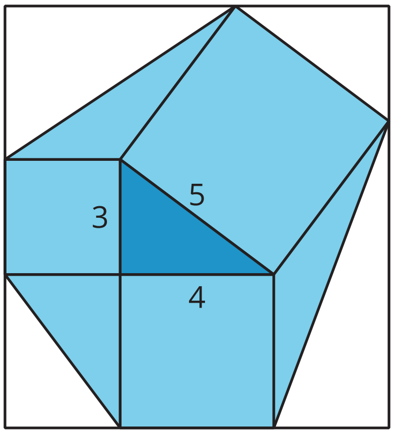
1. What is the total area of each figure?

2. Find the area of each of the 9 smaller regions shown in the figures and label them.

3. Add up the area of the 4 regions in Figure F and set this expression equal to the sum of the areas of the 5 regions in Figure G. If you rewrite this equation using as few terms as possible, what do you have?

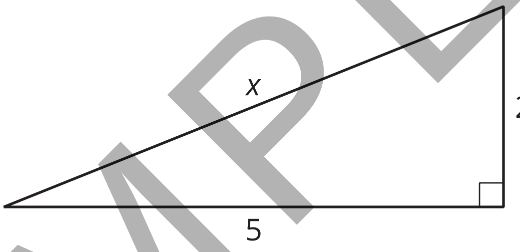
Are you ready for more?

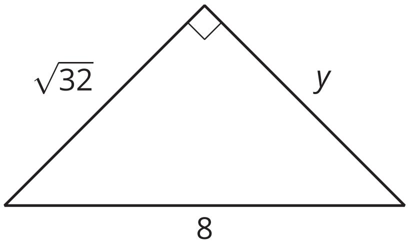
Take a 3-4-5 right triangle, add on the squares of the side lengths, and form a hexagon by connecting vertices of the squares as in the image. What is the area of this hexagon?



8.3 Let’s Take It for a Spin

Find the unknown side lengths in these right triangles.





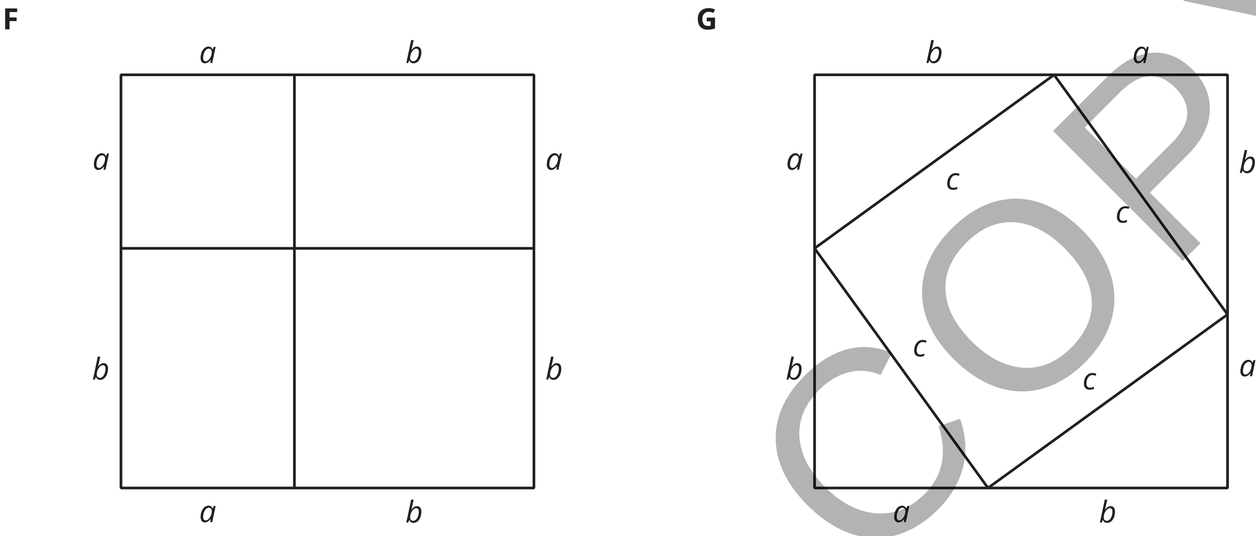
8.4 A Transformational Proof

Your teacher will give your group a sheet with 4 figures. Cut out the 5 shapes in Figure 1.

1. Arrange the 5 cut out shapes to fit inside Figure 2.
2. Now arrange the shapes to fit inside Figure 3.
3. Check to see that Figure 3 is congruent to the large square in Figure 4.
4. Check to see that the 5 cut out shapes fit inside the two smaller squares in Figure 4.
5. If the right triangle in Figure 4 has legs a and b and hypotenuse c, what have you just demonstrated to be true?

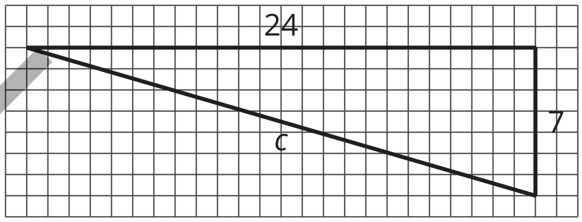
Lesson 8 Summary

The figures shown can be used to see why the Pythagorean Theorem is true. Both large squares have the same area, but they are broken up in different ways. When the sum of the four areas in Square F is set equal to the sum of the 5 areas in Square G, the result is $a^2 + b^2 = c^2$, where c is the hypotenuse of the triangles in Square G and also the side length of the square in the middle.



This is true for any right triangle. If the legs are a and b and the hypotenuse is c, then $a^2 + b^2 = c^2$.

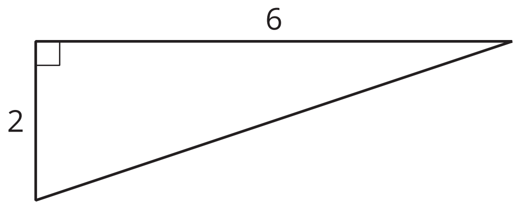
For example, to find the length of side c in this right triangle, we know that $24^2 + 7^2 = c^2$. The solution to this equation (and the length of the side) is $c = 25$.



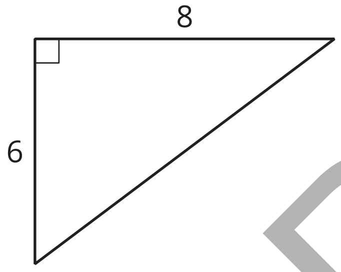
Practice Problems

1 a. Find the lengths (in units) of the unlabeled sides.

i.

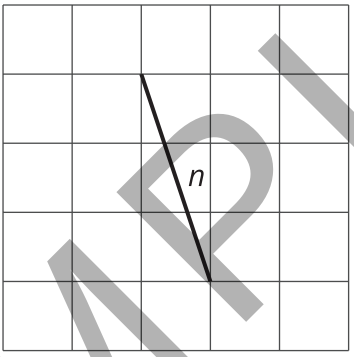


ii.

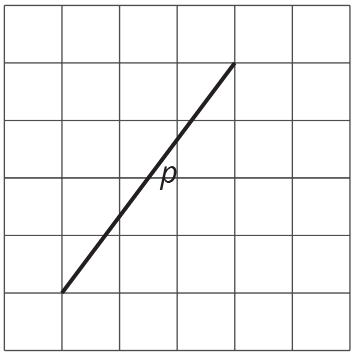


b. One segment is n units long and the other is p units long. Find the value of n and p. (Each small grid square is 1 square unit.)

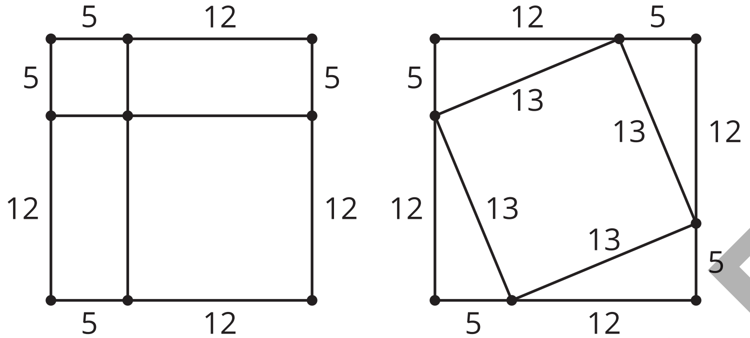
i.



ii.



2 Use the areas of the two identical squares to explain why $5^2 + 12^2 = 13^2$ without doing any calculations.



3 from Unit 8, Lesson 6

Each number is between which two consecutive integers?

a. $\sqrt{10}$

b. $\sqrt{54}$

c. $\sqrt{18}$

d. $\sqrt{99}$

e. $\sqrt{41}$

4 from Unit 8, Lesson 4

a. Give an example of a rational number, and explain how you know it is rational.

b. Give three examples of irrational numbers.

5 from Unit 7, Lesson 4

Write each expression as a single power of 10.

a. $10^5 \cdot 10^0$

b. $\frac{10^9}{10^0}$

6 from Unit 4, Lesson 15

Andre is ordering ribbon to make decorations for a school event. He needs a total of exactly 50.25 meters of blue and green ribbon. Andre needs 50% more blue ribbon than green ribbon for the basic design, plus an extra 6.5 meters of blue ribbon for accents. How much of each color of ribbon does Andre need to order?

Unit 8, Lesson 9

Addressing CA CCSSM 8.G.7; building towards 8.G.8; practicing MP6

Finding Unknown Side Lengths

Let’s find missing side lengths of right triangles.

9.1 Which Three Go Together: Equations

Which three go together? Why do they go together?

A

$$5^2 = 3^2 + b^2$$

B

$$5^2 - 3^2 = b^2$$

C

$$3^2 + 5^2 = b^2$$

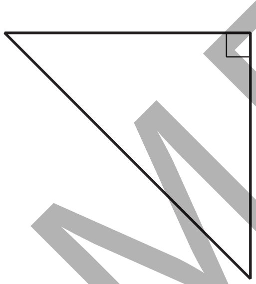
D

$$3^2 + 4^2 = 5^2$$

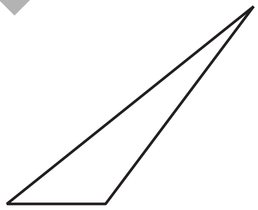
9.2 Which One Is the Hypotenuse?

Label all the hypotenuses with c.

A

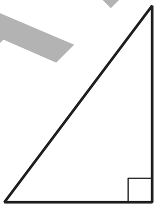


B

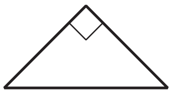


C

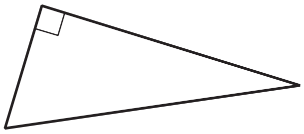
D



E

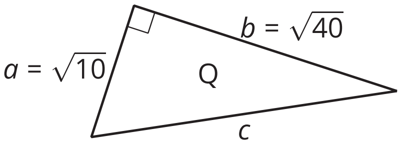


F

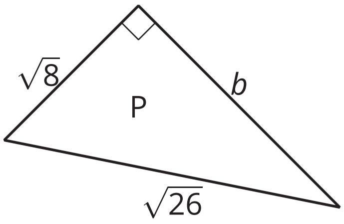


9.3 Find the Missing Side Lengths

1. Find c.



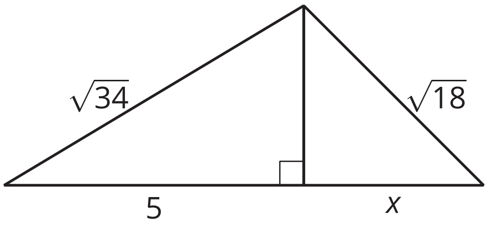
2. Find b.



3. A right triangle has sides of length 2.4 cm and 6.5 cm. What is the length of the hypotenuse?

4. A right triangle has a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$. What is the length of the other side?

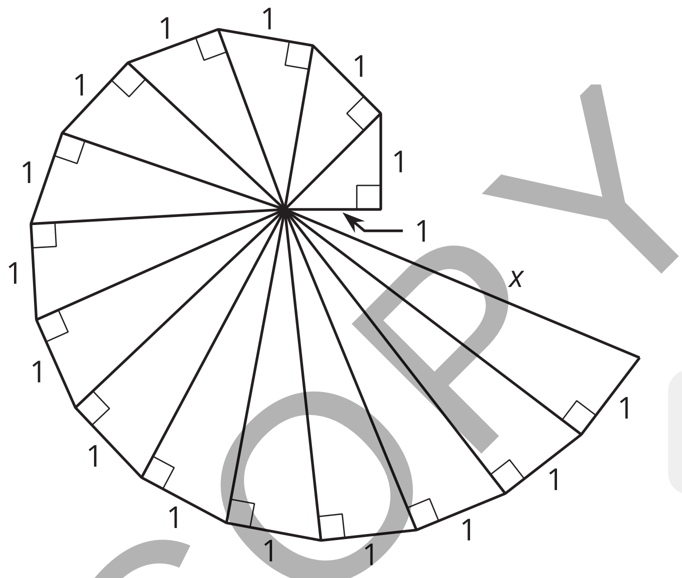
5. Find the value of x in the figure.



Are you ready for more?

The spiral in the figure is made by starting with a right triangle with both legs measuring one unit each. Then a second right triangle is built with one leg measuring 1 unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle's hypotenuse, again with the other leg measuring 1 unit, and so on.

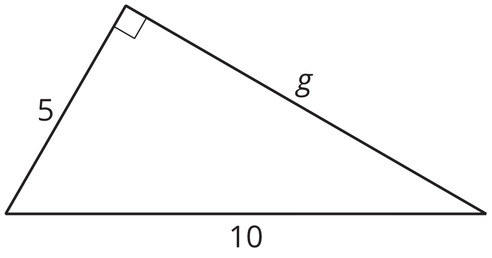
Find the length, x, of the hypotenuse of the last triangle constructed in the figure.



Lesson 9 Summary

The Pythagorean Theorem can be used to find an unknown side length in a right triangle as long as the length of the other two sides is known.

For example, here is a right triangle, where one leg has a length of 5 units, the hypotenuse has a length of 10 units, and the length of the other leg is represented by g.



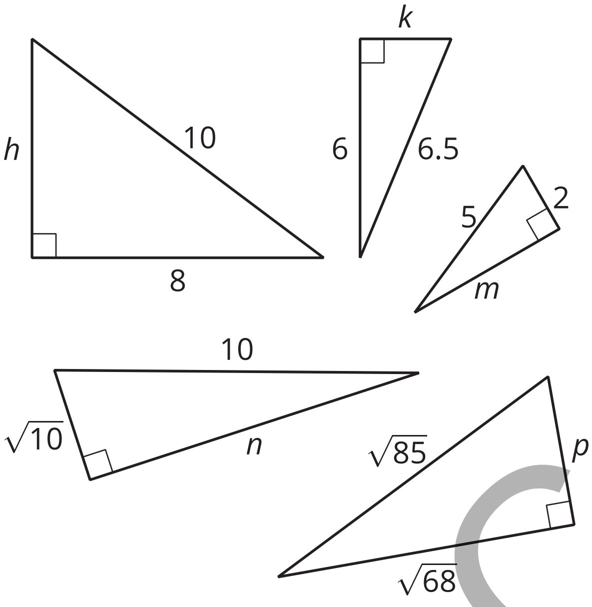
Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse, the side opposite the right angle. For this triangle, the hypotenuse is 10.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + g^2 &= 10^2 \\ g^2 &= 10^2 - 5^2 \\ g^2 &= 100 - 25 \\ g^2 &= 75 \\ g &= \sqrt{75} \end{aligned}$$

Use estimation strategies to know that the length of the other leg is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.

Practice Problems

1 Find the exact value of each variable that represents a side length in a right triangle.



2 In each part, a and b represent the length of a leg of a right triangle, and c represents the length of its hypotenuse. Find the missing length, given the other two lengths.

a. $a = 3, b = 1, c = ?$

b. $a = ?, b = 2, c = \sqrt{29}$

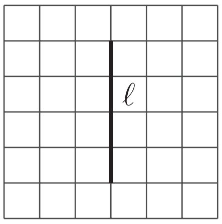
c. $a = \sqrt{5}, b = \sqrt{7}, c = ?$

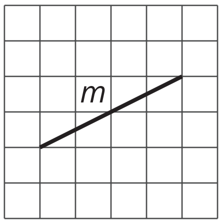
d. $a = \sqrt{8}, b = ?, c = \sqrt{13}$

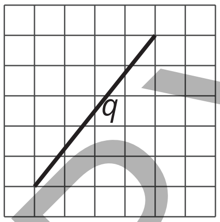
e. $a = ?, b = \sqrt{13}, c = 4$

3 from Unit 8, Lesson 8

What is the exact length of each line segment? Explain or show your reasoning. (Each grid square represents 1 square unit.)







4 from Unit 7, Lesson 15

In 2015, there were roughly 1×10^6 high school football players and 2×10^3 professional football players in the United States. About how many times more high school football players are there? Explain how you know.

5 from Unit 7, Lesson 6

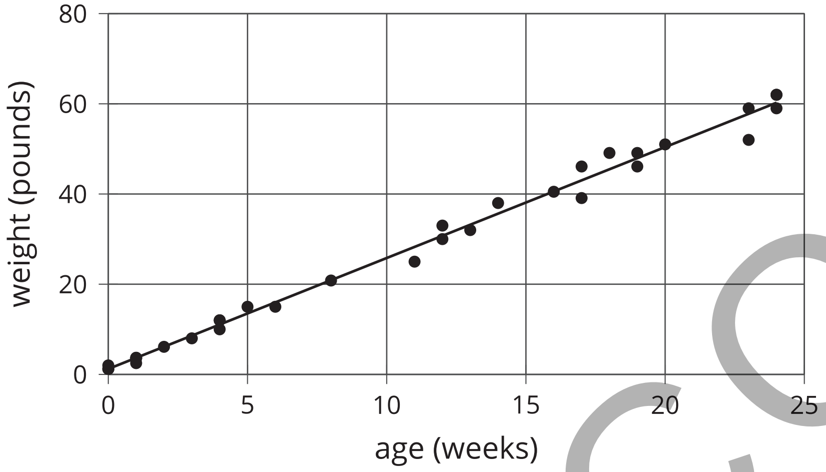
Evaluate:

a. $\left(\frac{1}{2}\right)^3$

b. $\left(\frac{1}{2}\right)^{-3}$

6 from Unit 6, Lesson 6

Here is a scatter plot of weight vs. age for different Dobermanns. The model represented by $y = 2.45x + 1.22$ is graphed with the scatter plot. Here, x represents age in weeks, and y represents weight in pounds.



a. What is the slope and what does it mean in this situation?

b. Based on this model, how heavy would you expect a newborn Dobermann to be?

Unit 8, Lesson 10

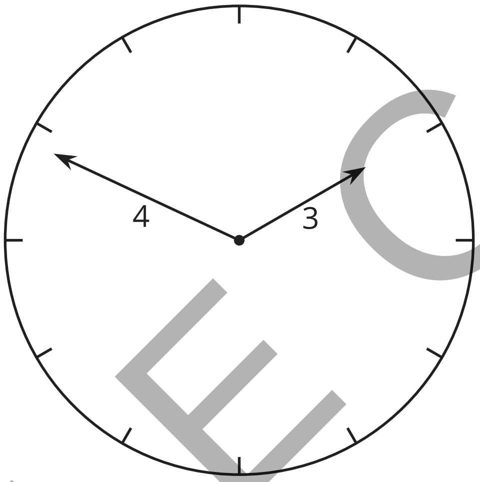
Addressing CA CCSSM 8.G.6-7; building towards 8.G.6; practicing MP7

The Converse

Let's figure out if a triangle is a right triangle.

10.1 The Hands of a Clock

Consider the tips of the hands of an analog clock that has an hour hand that is 3 centimeters long and a minute hand that is 4 centimeters long.



Over the course of a day:

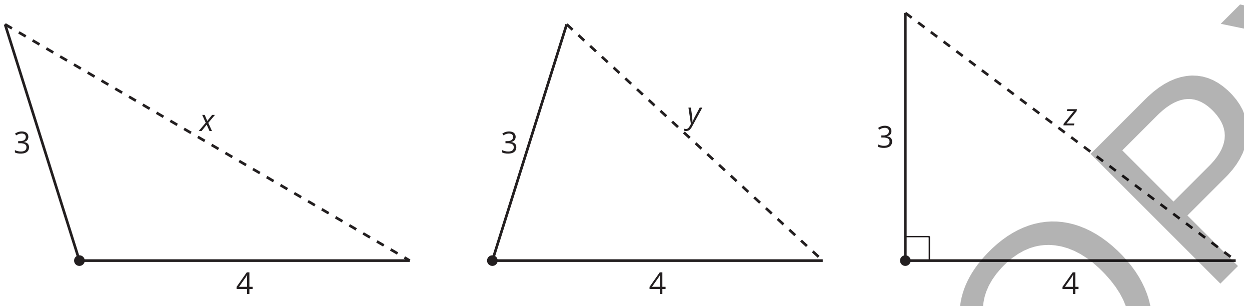
1. What is the farthest distance apart the two tips get?

2. What is the closest distance the two tips get?

3. Are the two tips ever exactly five centimeters apart? Explain your reasoning.

10.2 Proving the Converse

Here are three triangles with two side lengths measuring 3 and 4 units, and the third side of unknown length.



Order the following six values from smallest to largest. Put an equal sign between any you know to be equal. Be prepared to explain your reasoning.

$$1 \quad 5 \quad 7 \quad x \quad y \quad z$$

Are you ready for more?

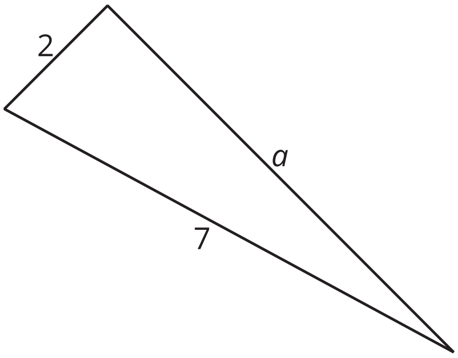
A related argument also lets us distinguish acute from obtuse triangles using only their side lengths.

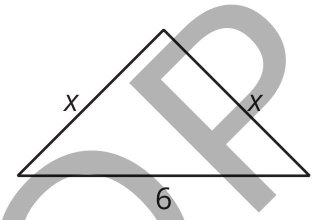
Decide if triangles with the following side lengths are acute, right, or obtuse. In right or obtuse triangles, identify which side length is opposite the right or obtuse angle.

1. $x = 15, y = 20, z = 8$
2. $x = 8, y = 15, z = 13$
3. $x = 17, y = 8, z = 15$

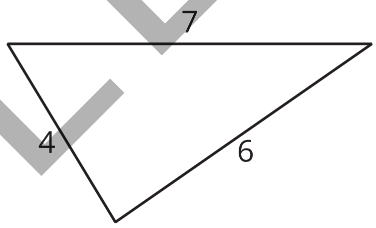
10.3 Calculating Legs of Right Triangles

1. Given the information provided for the right triangles shown here, find the unknown leg lengths to the nearest tenth.



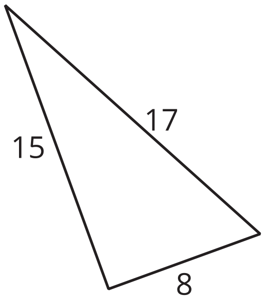


2. The triangle shown here is not a right triangle. What are two different ways you change *one* of the values so it would be a right triangle? Sketch these new right triangles, and clearly label the right angle.



Lesson 10 Summary

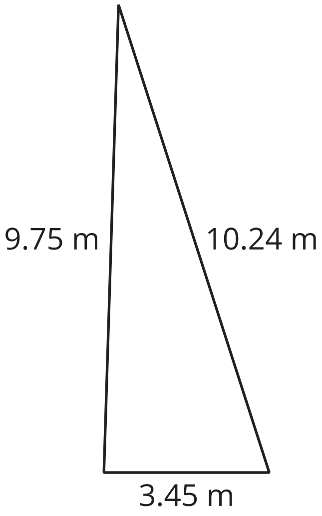
How can we tell whether a triangle is a right triangle or not? For example, in this triangle it isn't clear just by looking, and it may be that the sides aren't drawn to scale.



If we have a triangle with side lengths a, b, and c, with c being the longest of the three, then the converse of the Pythagorean Theorem tells us that any time we have $a^2 + b^2 = c^2$, we must have a right triangle. Since $8^2 + 15^2 = 64 + 225 = 289 = 17^2$, any triangle with side lengths 8, 15, and 17 must be a right triangle.

What about the jib sail on this boat? It is a triangle, but is it a right triangle?

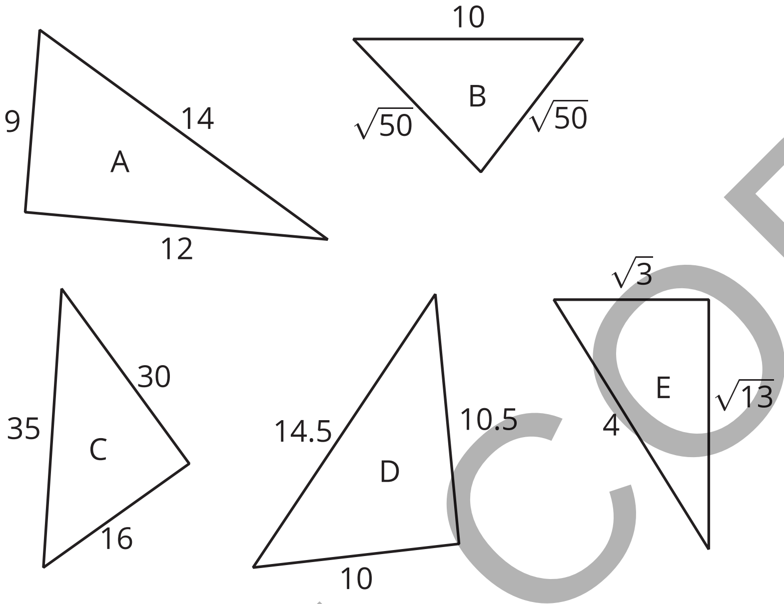
The measurements of the jib sail are shown here. The sum of the squares of the two shorter sides is 106.965 square meters, and the square of the longest side is 104.8576 square meters. So by the converse of the Pythagorean Theorem, it is not a right triangle, but it is close to one.



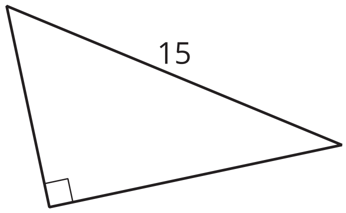
Together, the Pythagorean Theorem and its converse provide a way to check if a triangle is a right triangle just using its side lengths. If $a^2 + b^2 = c^2$, it is a right triangle. If $a^2 + b^2 \neq c^2$, it is not a right triangle.

Practice Problems

1 Which of these triangles are definitely right triangles? Explain how you know. (Note that not all triangles are drawn to scale.)



2 A right triangle has a hypotenuse of 15 cm. What are possible lengths for the two legs of the triangle? Explain your reasoning.



3 from Unit 8, Lesson 9

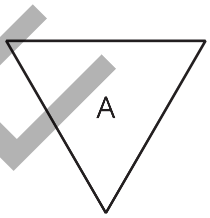
In each part, a and b represent the length (in units) of a leg of a right triangle, and c represents the length of its hypotenuse. Find the missing length, given the other two lengths.

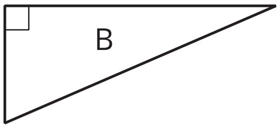
a. $a = 12, b = 5, c = ?$

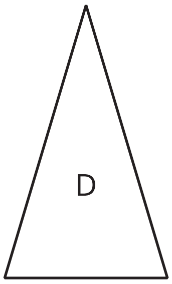
b. $a = ?, b = 21, c = 29$

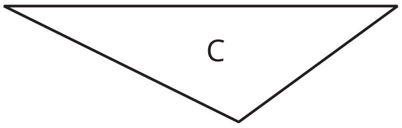
4 from Unit 8, Lesson 7

For which triangle does the Pythagorean Theorem express the relationship between the lengths of its three sides?









A. A

B. B

C. C

D. D

5 from Unit 4, Lesson 5

Andre makes a trip to Mexico. He exchanges some dollars for pesos at a rate of 20 pesos per dollar. While in Mexico, he spends 9,000 pesos. When he returns, he exchanges his pesos for dollars (still at 20 pesos per dollar). He gets back $\frac{1}{10}$ the amount he started with. Find how many dollars Andre exchanges for pesos and explain your reasoning. If you get stuck, try writing an equation representing Andre's trip using a variable for the number of dollars he exchanges.

Unit 8, Lesson 11

Addressing CA CCSSM 8.EE.2, 8.G.7, 8.NS.2; practicing MP3

Applications of the Pythagorean Theorem

Let’s explore some applications of the Pythagorean Theorem.

11.1 Math Talk: Square Roots

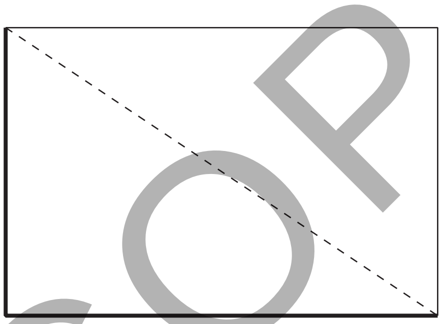
Mentally find the value of each expression to the nearest half.

- $\sqrt{24}$
- $\sqrt{7}$
- $\sqrt{42}$
- $\sqrt{10} + \sqrt{97}$

11.2 Cutting Corners

Mai and Tyler are standing at one corner of a large rectangular field and decide to race to the opposite corner. Since Mai has a bike and Tyler does not, they think it would be a fairer race if Mai rode along the sidewalk that surrounds the field (the bolded edges in the diagram) while Tyler ran the shorter distance directly across the field.

The field is 100 meters long and 80 meters wide. Tyler can run at around 5 meters per second, and Mai can ride her bike at around 7.5 meters per second.



1. Before making any calculations, who do you think will win? By how much? Explain your thinking.

2. Who wins? Show your reasoning.

Are you ready for more?

A calculator may be necessary to answer the following questions. Round answers to the nearest hundredth.

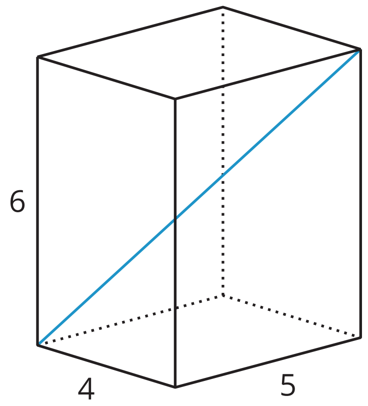
1. If you could give the loser of the race a head start, how much time would they need in order for both people to arrive at almost the exact same time?

2. If you could make the winner go slower, how slow would they need to go in order for both people to arrive at almost the exact same time?

11.3 Internal Dimensions

Here are two rectangular prisms:

K



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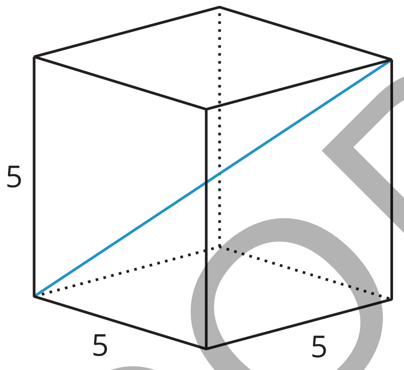


1. Which figure do you think has the longer diagonal? Why? Note that the figures are not drawn to scale.

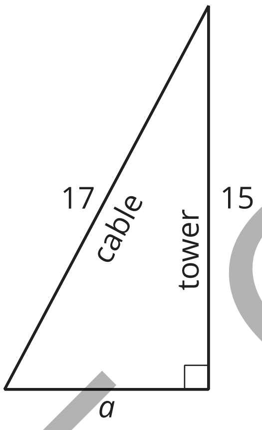
2. Calculate the lengths of both diagonals. Which one is actually longer?

Lesson 11 Summary

The Pythagorean Theorem can be used to solve any problem that can be modeled with a right triangle where the lengths of two sides are known and the length of the other side needs to be found.

For example, let's say a cable is being placed on level ground to support a tower. It's a 17-foot cable, and the cable should be connected 15 feet up the tower. How far away from the bottom of the tower should the other end of the cable connect to the ground?

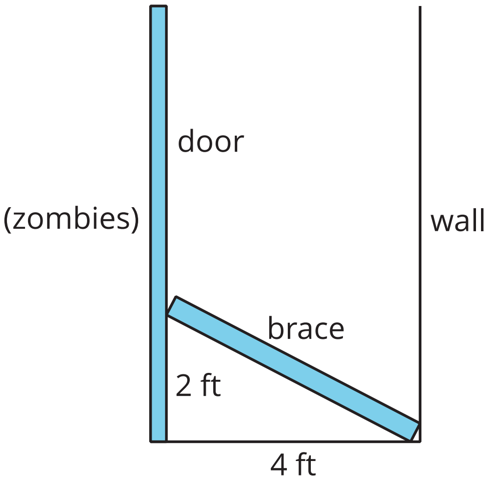
It is often very helpful to draw a diagram of a situation, such as the one shown here:



It's assumed that the tower makes a right angle with the ground. Since this is a right triangle, the relationship between its sides is $a^2 + b^2 = c^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides. The hypotenuse is the side opposite the right angle. Making substitutions gives $a^2 + 15^2 = 17^2$. Solving this for a gives $a = 8$. So the other end of the cable should connect to the ground 8 feet away from the bottom of the tower.

Practice Problems

1 A man is trying to zombie-proof his house. He wants to cut a length of wood that will brace a door against a wall. The wall is 4 feet away from the door, and he wants the brace to rest 2 feet up the door. About how long should he cut the brace?



2 At a restaurant, a trash can's opening is rectangular and measures 7 inches by 9 inches. The restaurant serves food on trays that measure 12 inches by 16 inches. Jada says it is impossible for the tray to accidentally fall through the trash can opening because the shortest side of the tray is longer than either edge of the opening.

Do you agree or disagree with Jada's explanation? Explain your reasoning.

3 from Unit 8, Lesson 10

Select **all** the sets that are the three side lengths of right triangles.

A. 8, 7, 15

B. 4, 10, $\sqrt{84}$

C. $\sqrt{8}$, 11, $\sqrt{129}$

D. $\sqrt{1}$, 2, $\sqrt{3}$

4 from Unit 7, Lesson 10

For each pair of numbers, which of the two numbers is larger? How many times larger?

a. $12 \cdot 10^9$ and $4 \cdot 10^9$

b. $1.5 \cdot 10^{12}$ and $3 \cdot 10^{12}$

c. $20 \cdot 10^4$ and $6 \cdot 10^5$

5 from Unit 3, Lesson 10

A line contains the point $(3, 5)$. If the line has a negative slope, which of these points could also be on the line?

A. $(2, 0)$

B. $(4, 7)$

C. $(5, 4)$

D. $(6, 5)$

6 from Unit 3, Lesson 4

Noah and Han are preparing for a jump rope contest. Noah can jump 40 times in 0.5 minute. Han can jump y times in x minutes, where $y = 78x$. If they both jump for 2 minutes, who jumps more times? How many more?

Unit 8, Lesson 12

Addressing CA CCSSM 8.EE.4, 8.G.7; practicing MP1 and MP6

More Applications of the Pythagorean Theorem

Let's solve problems using the Pythagorean Theorem.

12.1 Pythagorean Triples

A Pythagorean triple is a set of three integers a, b, and c where $a^2 + b^2 = c^2$. An example of a Pythagorean triple is 3, 4, and 5 because $3^2 + 4^2 = 5^2$. Find other Pythagorean triples.

12.2 Info Gap: Pythagorean Theorem

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ________?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ________ because . . . .”

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

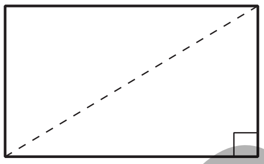
1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know ________?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

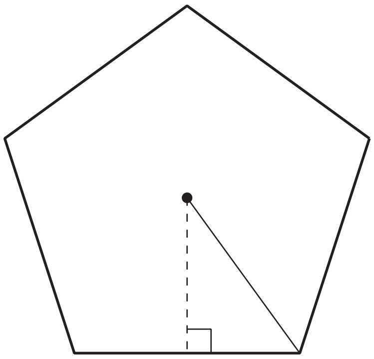
Lesson 12 Summary

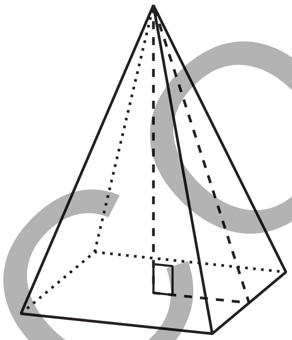
The Pythagorean Theorem can be used to find one side of a right triangle when you know the lengths of the other two sides. Sometimes the right triangle is not always obvious.

For example, if Kiran is trying to hang decorations from one corner of her rectangular classroom to the opposite corner, drawing in a diagonal helps to show how two right triangles are formed.



Consider these other figures and how a right triangle could be used to solve related problems.





Practice Problems

1 The dimensions of a rectangle are given. For each rectangle, find the exact length of its diagonal (a segment whose endpoints lie on opposite vertices of the rectangle).

a. 10 by 24

b. 16 by 30

c. 7 by 7

d. $\sqrt{13}$ by 6

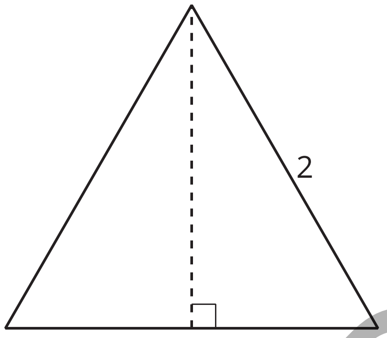
e. $\sqrt{3}$ by 1

f. $\frac{1}{2}$ by $\frac{\sqrt{3}}{2}$

2 A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference in length between cutting across the diagonal through the park compared to going around the park, along the streets.

How much shorter would her walk be going through the park? Round your answer to the nearest meter.

3 Here is an equilateral triangle. The length of each side is 2 units. A height is drawn. In an equilateral triangle, the height divides the opposite side into two pieces of equal length.



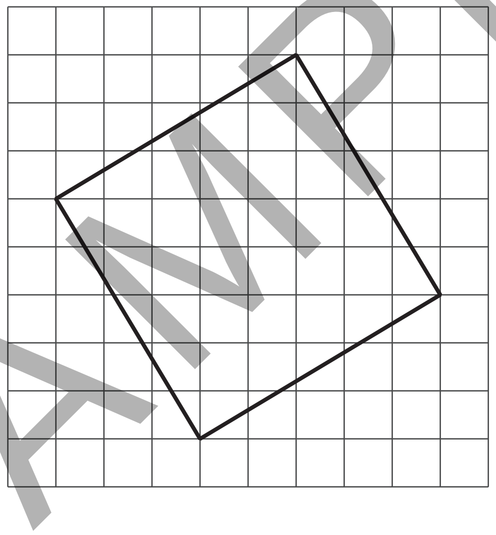
a. Find the exact height.

b. Find the area of the equilateral triangle.

c. (Challenge) Using x for the length of each side in an equilateral triangle, express its area in terms of x.

4 from Unit 8, Lesson 5

What is the side length of this square? Explain your reasoning.



5 from Unit 8, Lesson 6

Find all the solutions to each equation.

a. $x^2 = 81$

b. $x^2 = 100$

c. $\sqrt{x} = 12$

6 from Unit 7, Lesson 2

What is the value of the "?" in the following equation? Explain or show your reasoning.

$10^3 \cdot 10^? = 10^8$

Unit 8, Lesson 13

Addressing CA CCSSM 8.G.8; building towards 8.G.8; practicing MP6 and MP7

Finding Distances in the Coordinate Plane

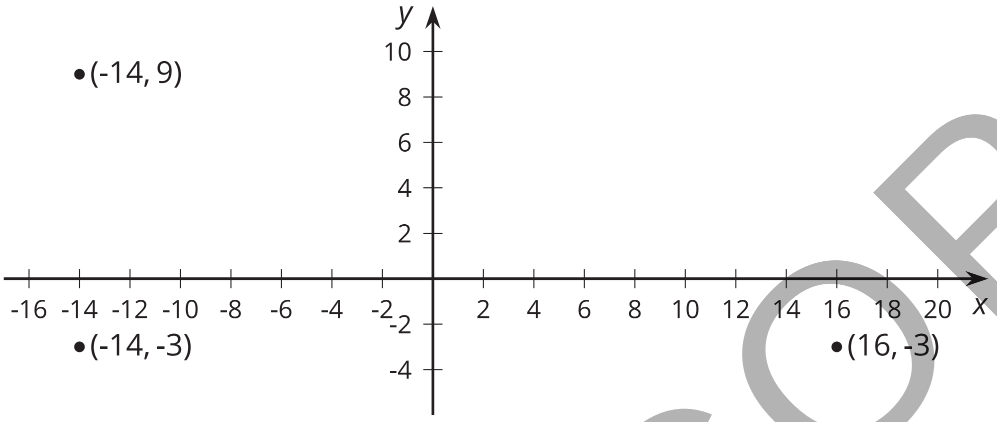
Let's find distances in the coordinate plane.

13.1 Closest Distance

1. Order the following pairs of coordinates from closest to farthest apart. Be prepared to explain your reasoning.
   a. $(2, 4)$ and $(2, 10)$
   b. $(-3, 6)$ and $(5, 6)$
   c. $(-12, -12)$ and $(-12, -1)$
   d. $(7, 0)$ and $(7, -9)$
   e. $(1, -10)$ and $(-4, -10)$
2. Name another pair of coordinates that would be closer together than the first pair on your list.
3. Name another pair of coordinates that would be farther apart than the last pair on your list.

13.2 How Far Apart?

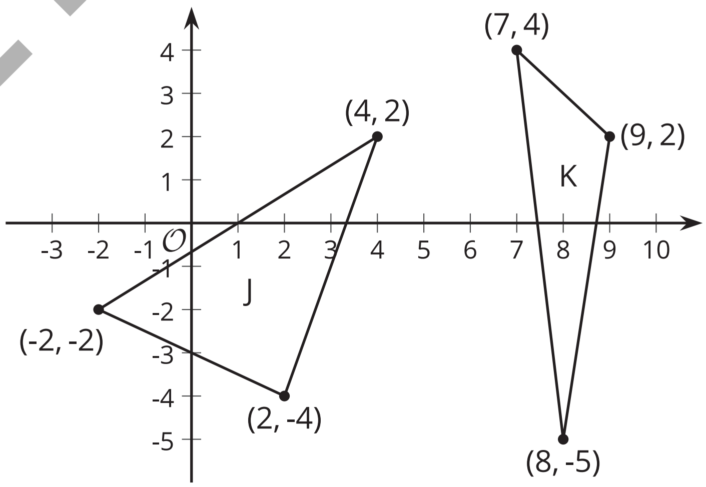
Find the distances between the three points shown.



13.3 Perimeters with Pythagoras

1. Which triangle do you think has the longer perimeter? Be prepared to explain your reasoning.

2. Select one triangle and calculate its perimeter. Your partner will calculate the perimeter of the other. Were you correct about which figure had the longer perimeter?



Are you ready for more?

Quadrilateral $ABCD$ has vertices at $A = (-5, 1)$, $B = (-4, 4)$, $C = (2, 2)$, and $D = (1, -1)$.

1. Use the Pythagorean Theorem to find the lengths of sides AB, BC, CD, and AD.
2. Use the Pythagorean Theorem to find the lengths of the two diagonals, AC and BD.
3. Explain why quadrilateral $ABCD$ is a rectangle.

13.4 Finding the Right Distance

Have each person in your group select one of the sets of coordinate pairs shown. Then calculate the distance between those two coordinates. Once the distances are calculated, have each person in the group briefly share how they did their calculations.

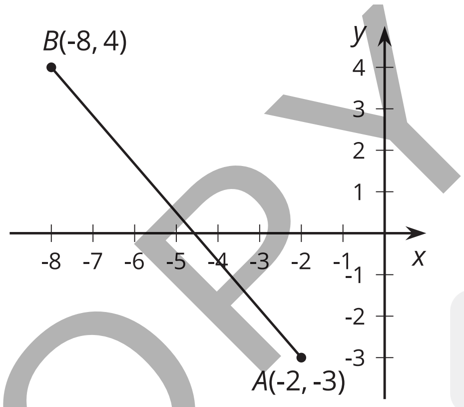
- $(-2, 1)$ and $(6, -5)$
- $(-1, -6)$ and $(5, 2)$
- $(-1, 2)$ and $(5, -6)$
- $(-2, -5)$ and $(6, 1)$

1. How does the distance between your coordinate pairs compare to the distances for the rest of your group?
2. In your own words, write an explanation to another student of how to find the distance between any two coordinate pairs.

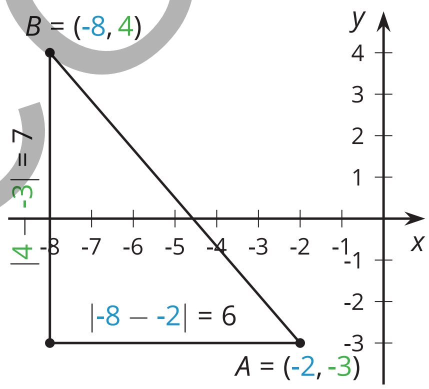
Lesson 13 Summary

We can use the Pythagorean Theorem to find the distance between any two points in the coordinate plane.

For example, if the coordinates of point A are $(-2, -3)$, and the coordinates of point B are $(-8, 4)$, the distance between them is also the length of line segment AB. It is a good idea to plot the points first.



Think of the segment AB as the hypotenuse of a right triangle. The legs can be drawn in as horizontal and vertical line segments.



The length of the horizontal leg is 6, which can be seen in the diagram. This is also the distance between the x-coordinates of A and B ($|-8 - -2| = 6$).

The length of the vertical leg is 7, which can be seen in the diagram. This is also the distance between the y-coordinates of A and B ($|4 - -3| = 7$).

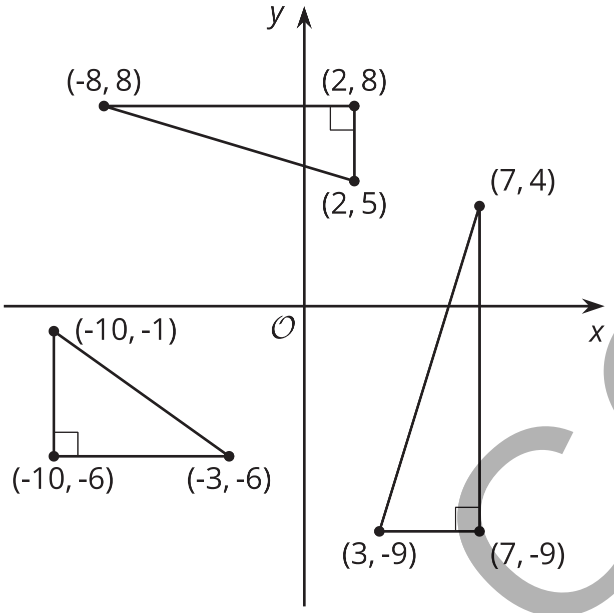
Once the lengths of the legs are known, we use the Pythagorean Theorem to find the length of the hypotenuse, AB, which we can represent with c:

$$\begin{aligned} 6^2 + 7^2 &= c^2 \\ 36 + 49 &= c^2 \\ 85 &= c^2 \\ \sqrt{85} &= c \end{aligned}$$

This length is a little longer than 9, since 85 is a little longer than 81. Using a calculator gives a more precise answer, $\sqrt{85} \approx 9.22$.

Practice Problems

1 The right triangles are drawn in the coordinate plane, and the coordinates of their vertices are labeled. For each right triangle, label each leg with its length.



2 Find the distance (in units) between each pair of points. If you get stuck, try plotting the points on graph paper.

a. $M = (0, -11)$ and $P = (0, 2)$

b. $A = (0, 0)$ and $B = (-3, -4)$

c. $C = (8, 0)$ and $D = (0, -6)$

3 from Unit 8, Lesson 10

a. Find an object that contains a right angle. This can be something in nature or something that was made by humans or machines.

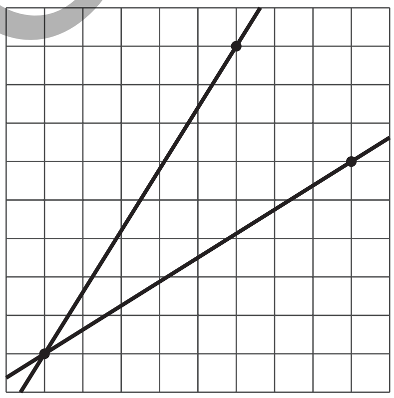
b. Measure the two sides that make the right angle. Then measure the distance from the end of one side to the end of the other.

c. Draw a diagram of the object, including the measurements.

d. Use the Pythagorean Theorem to show that your object really does have a right angle.

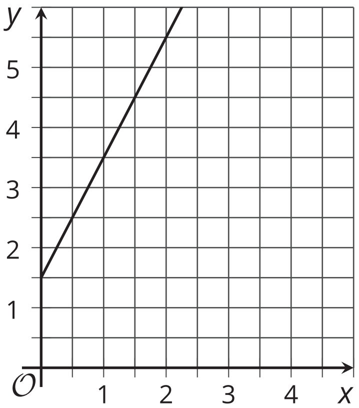
4 from Unit 2, Lesson 10

Which line has a slope of 0.625, and which line has a slope of 1.6? Explain why the slopes of these lines are 0.625 and 1.6.



5 from Unit 3, Lesson 7

Write an equation for the graph.



Unit 8, Lesson 14

Addressing CA CCSSM 8.EE.2, 8.NS.2; practicing MP2 and MP7

Edge Lengths and Volumes

Let's explore the relationship between volume and edge lengths of cubes.

14.1 Ordering Squares and Cubes

Let a, b, c, d, e, and f be positive numbers.

Given these equations, arrange a, b, c, d, e, and f from least to greatest. Explain your reasoning.

- $a^2 = 9$
- $b^3 = 8$
- $c^2 = 10$
- $d^3 = 9$
- $e^2 = 8$
- $f^3 = 7$

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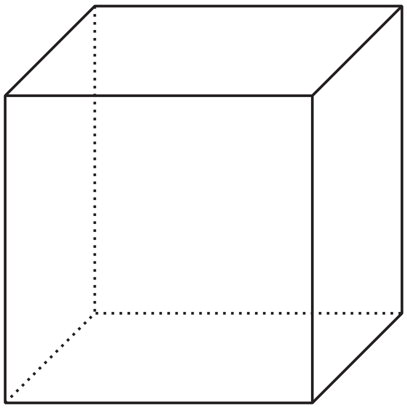
14.2 Name That Edge Length!

Fill in the missing values using the information provided:



sides	volume	volume equation
	27 in^3	
$\sqrt[3]{5}$ in		
		$(\sqrt[3]{16})^3 = 16$

Are you ready for more?

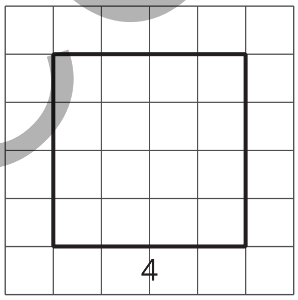
A cube has a volume of 8 cubic centimeters. A square has the same value for its area as the value for the surface area of the cube. How long is each side of the square?

14.3 Card Sort: Rooted in the Number Line

Your teacher will give you a set of cards. Each card has a number line with a plotted point, an equation, or a square or **cube root** value.

For each card with a letter and square or cube root value, match it with the location on a number line where the value exists, and the equation that the value makes true. Record your matches and be prepared to explain your reasoning.

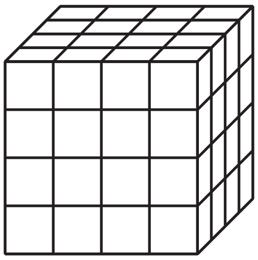
Lesson 14 Summary



For a square, its side length is the square root of its area. For example, this square has an area of 16 square units and a side length of 4 units.

Both of these equations are true:

$$4^2 = 16 \qquad \sqrt{16} = 4$$



For a cube, the edge length is the **cube root** of its volume. For example, this cube has a volume of 64 cubic units and an edge length of 4 units:

Both of these equations are true:

$$4^3 = 64 \qquad \sqrt[3]{64} = 4$$

$\sqrt[3]{64}$ is pronounced "the cube root of 64." Here are some other values of cube roots:

$$\sqrt[3]{8} = 2 \text{ because } 2^3 = 8$$

$$\sqrt[3]{27} = 3 \text{ because } 3^3 = 27$$

$$\sqrt[3]{125} = 5 \text{ because } 5^3 = 125$$

Glossary

- cube root

Practice Problems

1 a. What is the volume of a cube with an edge length of

   i. 4 centimeters?

   ii. $\sqrt[3]{11}$ feet?

   iii. s units?

b. What is the edge length of a cube with a volume of

   i. 1,000 cubic centimeters?

   ii. 23 cubic inches?

   iii. v cubic units?

2 Write an equivalent expression that doesn't use a cube root symbol.

a. $\sqrt[3]{1}$

b. $\sqrt[3]{216}$

c. $\sqrt[3]{8,000}$

d. $\sqrt[3]{\frac{1}{64}}$

e. $\sqrt[3]{\frac{27}{125}}$

f. $\sqrt[3]{0.027}$

g. $\sqrt[3]{0.000125}$

3 from Unit 8, Lesson 13

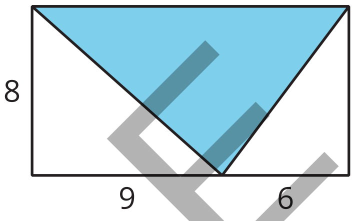
Find the distance (in units) between each pair of points. If you get stuck, try plotting the points on graph paper.

a. $X = (5, 0)$ and $Y = (-4, 0)$

b. $K = (-21, -29)$ and $L = (0, 0)$

4 from Unit 8, Lesson 10

Here is a 15-by-8 unit rectangle divided into triangles. Is the shaded triangle a right triangle? Explain or show your reasoning.



5 from Unit 7, Lesson 2

Express each of the following as a single power of 10:

a. 100

b. 100,000

c. 100,000,000

d. $100 \cdot 10$

e. $1,000 \cdot 1,000$

Unit 8, Lesson 15

Addressing CA CCSSM 8.EE.2, 8.NS.2; practicing MP6

Cube Roots

Let’s compare cube roots.

15.1 Math Talk: Cubed

Decide mentally whether each statement is true.

- $\left(\sqrt[3]{5}\right)^3 = 5$
- $\left(\sqrt[3]{27}\right)^3 = 3$
- $7 = \left(\sqrt[3]{7}\right)^3$
- $\left(\sqrt[3]{64}\right) = 2^3$

15.2 Cube Root Values

The value of a cube root of a number lies between two integers. Which are those consecutive whole numbers for the following? Be prepared to explain your reasoning.

1. $\sqrt[3]{5}$
2. $\sqrt[3]{23}$
3. $\sqrt[3]{81}$
4. $\sqrt[3]{999}$

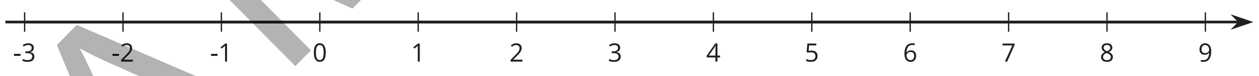
15.3 Solutions on a Number Line

The numbers x, y, and z are positive, and:

$x^3 = 5$

$y^3 = 27$

$z^3 = 700$



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.
2. Plot $-\sqrt[3]{2}$ on the number line.

Are you ready for more?

Diego knows that $8^2 = 64$ and that $4^3 = 64$. He says that this means the following are all true:

- $\sqrt{64} = 8$
- $\sqrt[3]{64} = 4$
- $\sqrt{-64} = -8$
- $\sqrt[3]{-64} = -4$

Is he correct? Explain how you know.

Lesson 15 Summary

Like square roots, most cube roots of whole numbers are irrational. The only time the cube root of a number is a rational number is when the number we are taking the cube root of is a perfect cube. For example, 8 is a perfect cube, and $\sqrt[3]{8} = 2$.

We can approximate the values of the cube root of a number by observing the integers around it and remembering the relationship between cubes and cube roots. For example, $\sqrt[3]{20}$ is between 2 and 3 since $2^3 = 8$ and $3^3 = 27$, and 20 is between 8 and 27. Similarly, since 100 is between 4^3 and 5^3, we know $\sqrt[3]{100}$ is between 4 and 5.

Many calculators have a cube root function which can be used to approximate the value of a cube root more precisely. Using our numbers from before, a calculator will show that $\sqrt[3]{20} \approx 2.7144$ and that $\sqrt[3]{100} \approx 4.6416$.

Practice Problems

1 Find the positive solution to each equation. If the solution is irrational, write the solution using square root or cube root notation.

a. $t^3 = 216$

b. $a^2 = 15$

c. $m^3 = 8$

d. $c^3 = 343$

e. $f^3 = 181$

2 For each cube root, find the two whole numbers that it lies between.

a. $\sqrt[3]{11}$

b. $\sqrt[3]{80}$

c. $\sqrt[3]{120}$

d. $\sqrt[3]{250}$

3 Order the following values from least to greatest:

$$\sqrt[3]{530},\ \sqrt{48},\ \pi,\ \sqrt{121},\ \sqrt[3]{27},\ \frac{19}{2}$$

4 Select **all** the equations that have a solution of $\frac{2}{7}$:

A. $x^2 = \frac{2}{7}$

B. $x^2 = \frac{4}{14}$

C. $x^2 = \frac{4}{49}$

D. $x^3 = \frac{6}{21}$

E. $x^3 = \frac{8}{343}$

F. $x^3 = \frac{6}{7}$

5 The equation $x^2 = 25$ has *two* solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So 5 is a solution, and -5 is also a solution. But! The equation $x^3 = 125$ only has one solution, which is 5. This is because $5 \cdot 5 \cdot 5 = 125$, and there are no other numbers you can cube to make 125. (Think about why -5 is not a solution!)

Find all the solutions to each equation.

a. $x^3 = 8$

b. $\sqrt[3]{x} = 3$

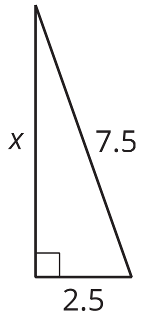
c. $x^2 = 49$

d. $x^3 = \frac{64}{125}$

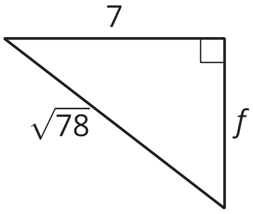
6 from Unit 8, Lesson 9

Find the value of each variable, in units, to the nearest tenth.

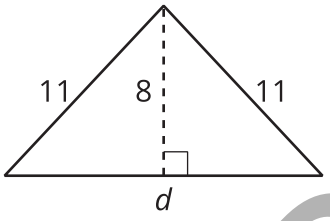
a.



b.



c.



Unit 8, Lesson 16

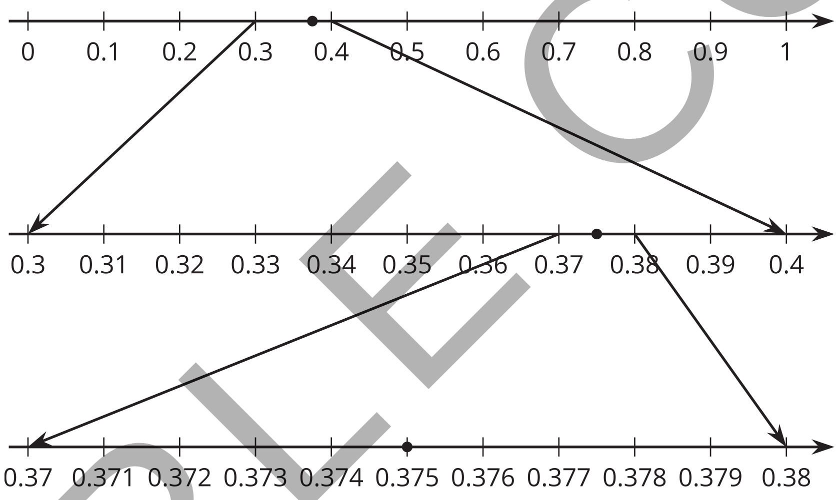
Addressing CA CCSSM 8.EE.2, 8.NS.1; building on 7.NS.2d; building towards 8.NS.2; practicing MP6 and MP8

Decimal Representations of Rational Numbers

Let's learn more about how rational numbers can be represented.

16.1 Notice and Wonder: Number Lines

What do you notice? What do you wonder?



16.2 Rational Numbers as Fractions

Rational numbers can be written as positive or negative fractions. All of these numbers are rational numbers. Show that they are rational by writing them in the form $\frac{a}{b}$ or $-\frac{a}{b}$.

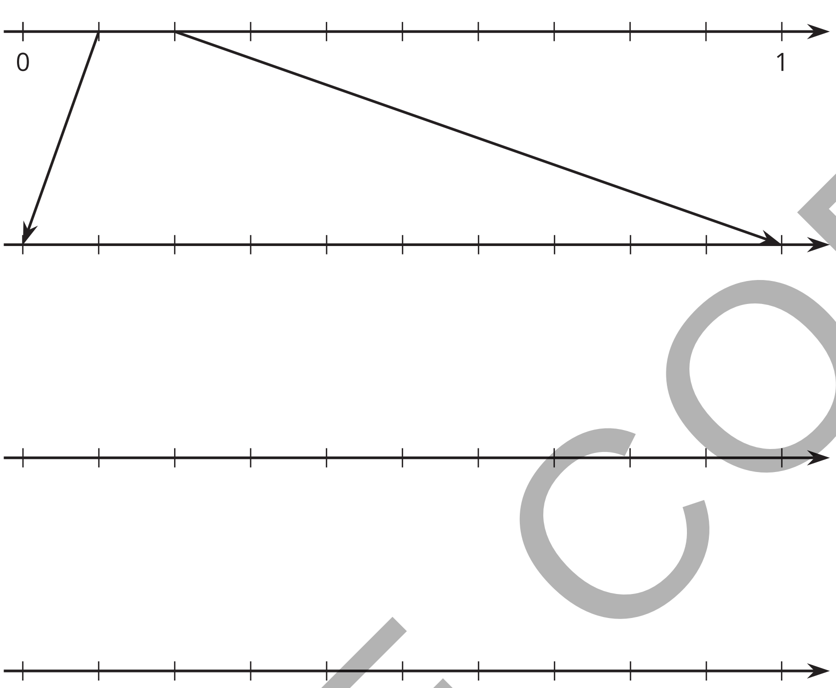
1. 0.2
2. $-\sqrt{4}$
3. 0.333
4. $\sqrt[3]{1,000}$
5. -1.000001
6. $\sqrt{\frac{1}{16}}$

16.3 Rational Numbers as Decimals

All rational numbers have decimal representations, too. Find the decimal representation of each of these rational numbers.

1. $\frac{7}{5}$
2. $\frac{999}{1,000}$
3. $\frac{111}{2}$
4. $\sqrt[3]{\frac{1}{8}}$
5. $\frac{3}{8}$

16.4 Zooming In on $\frac{2}{11}$



1. On the topmost number line, label the tick marks. Next, find the first decimal place of $\frac{2}{11}$ using long division and estimate where $\frac{2}{11}$ should be placed on the top number line.
2. Label the tick marks of the second number line. Find the next decimal place of $\frac{2}{11}$ by continuing the long division and estimate where $\frac{2}{11}$ should be placed on the second number line. Add arrows from the second to the third number line to zoom in on the location of $\frac{2}{11}$.
3. Label the tick marks of the remaining number lines. Continue using long division to calculate the next two decimal places, and plot them on the remaining number lines.
4. What do you think the decimal expansion of $\frac{2}{11}$ is?

Are you ready for more?

Let $x = \frac{25}{11} = 2.272727...$ and let $y = \frac{58}{33} = 1.75757575...$.

For each of the following questions, first decide whether the fraction or decimal representations of the numbers are more helpful to answer the question, and then find the answer.

- Which of x or y is closer to 2?

- Find x^2.

Lesson 16 Summary

We learned earlier that rational numbers can be written as a positive or negative fraction. For example, $\frac{3}{4}$ and $-\frac{5}{2}$ are both rational numbers. A complicated-looking numerical expression can also be a rational number as long as the value of the expression is a positive or negative fraction. For example, $\sqrt{64}$ and $-\sqrt[3]{\frac{1}{8}}$ are rational numbers because $\sqrt{64} = 8$ and $-\sqrt[3]{\frac{1}{8}} = -\frac{1}{2}$.

Rational numbers can also be written using decimal notation. Some have finite decimal expansions, like 0.75, -2.5, or -0.5. Other rational numbers have infinite decimal expansions, like 0.7434343 . . . , where the 43s repeat forever. This is called a **repeating decimal**. A repeating decimal has digits that keep going in the same pattern over and over, and these repeating digits are marked with a line above them. For example, we would write 0.7434343 . . . as $0.7\overline{43}$. The bar tells us which part repeats forever.

The decimal expansion of a number helps us plot it accurately on a number line divided into tenths. For example, $0.7\overline{43}$ should be between 0.7 and 0.8. Each additional decimal digit increases the accuracy of our plotting. So the number $0.7\overline{43}$ is between 0.743 and 0.744.

Glossary

- repeating decimal

Practice Problems

1 Andre and Jada are discussing how to write $\frac{17}{20}$ as a decimal.

- Andre says he can use long division to divide 17 by 20 to get the decimal.
- Jada says she can write an equivalent fraction with a denominator of 100 by multiplying by $\frac{5}{5}$, then writing the number of hundredths as a decimal.

a. Do both of these strategies work?

b. Which strategy do you prefer? Explain your reasoning.

c. Write $\frac{17}{20}$ as a decimal. Explain or show your reasoning.

2 Write each fraction as a decimal.

a. $\sqrt{\frac{9}{100}}$

b. $\frac{99}{100}$

c. $\sqrt{\frac{9}{16}}$

d. $\frac{23}{10}$

3 Write each decimal as a fraction.

a. $\sqrt{0.81}$

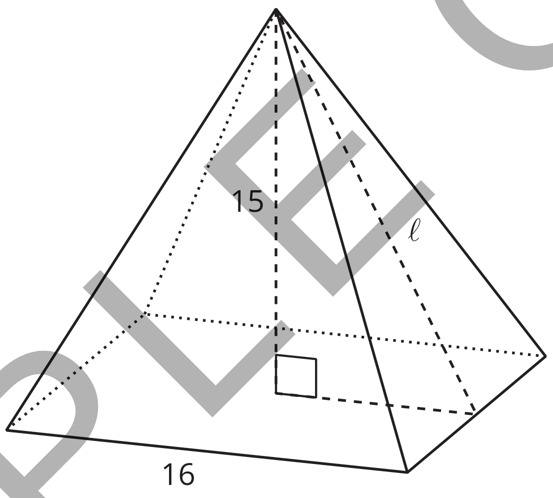
b. 0.0276

c. $\sqrt{0.04}$

d. 10.01

4 from Unit 8, Lesson 11

Here is a right square pyramid.



a. What is the measurement of the slant height ℓ of the triangular face of the pyramid? If you get stuck, use a cross section of the pyramid.

b. What is the surface area of the pyramid?

5 from Unit 8, Lesson 6

Order the values from least to greatest. Explain or show your reasoning.

$$\frac{31}{3},\ \sqrt{29},\ 10.4,\ \sqrt{100},\ 9,\ \frac{28}{3}$$

6 from Unit 8, Lesson 14

What is the exact side length of a cube with a volume of:

a. 8 cubic meters?

b. 3^3 cubic centimeters?

c. 31 cubic inches?

d. w cubic units?

Unit 8, Lesson 17

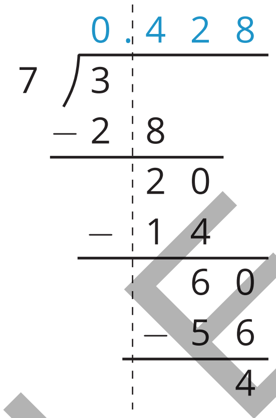
Addressing CA CCSSM 8.NS.1; building on 7.NS.2d, 8.EE.7b; practicing MP7 and MP8

Infinite Decimal Expansions

Let’s think about infinite decimals.

17.1 Searching for Digits

The first 3 digits after the decimal for the decimal expansion of $\frac{3}{7}$ have been calculated. Find the next 4 digits.



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17.2 Some Numbers Are Rational

Your teacher will give your group a set of cards. Each card will have a calculations side and an explanation side.

1. The cards show Noah's work calculating the fraction representation of $0.4\overline{85}$. Arrange these in order to see how he figured out that $0.4\overline{85} = \frac{481}{990}$ without needing a calculator.
2. Use Noah's method to calculate the fraction representation of:
   a. $0.1\overline{86}$

   b. $0.7\overline{88}$

Are you ready for more?

Use this technique to find fractional representations for $0.\overline{3}$ and $0.\overline{9}$.

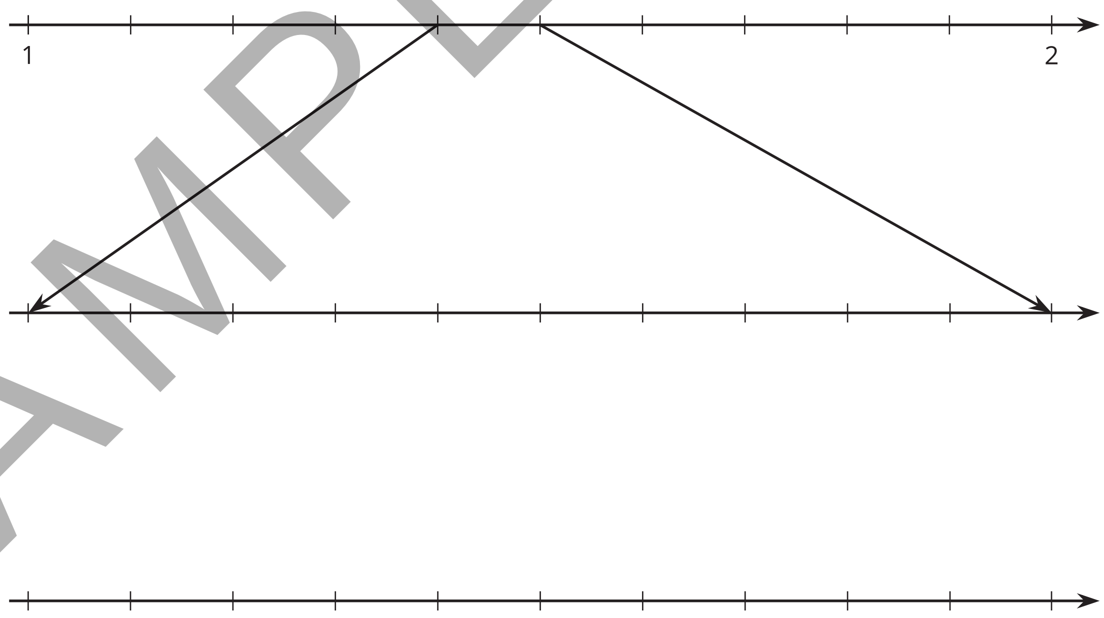
17.3 Some Numbers Are Not Rational

1. a. Why is $\sqrt{2}$ between 1 and 2 on the number line?

   b. Why is $\sqrt{2}$ between 1.4 and 1.5 on the number line?

   c. How can you figure out an approximation for $\sqrt{2}$ accurate to 3 decimal places?

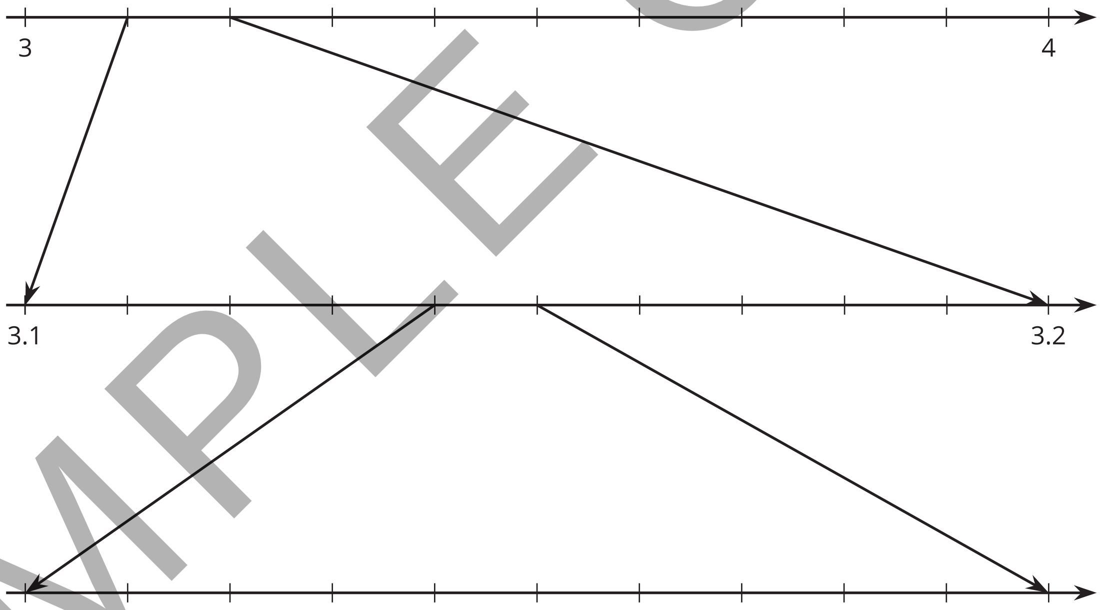
   d. Label all of the tick marks. Plot $\sqrt{2}$ on all three number lines. Make sure to add arrows from the second to the third number line.



2. a. Elena notices that a beaker in science class says it has a diameter of 9 cm and measures its circumference to be 28.3 cm. Using these values and the equation for circumference, $C = 2\pi r$, what value do you get for π?

   b. Diego learns that one of the space shuttle fuel tanks has a diameter of 840 cm and a circumference of 2,639 cm. Using these values and the equation for circumference, $C = 2\pi r$, what value do you get for π?

   c. Label all of the tick marks on the number lines. Use a calculator to get a very accurate approximation of π and plot that number on all three number lines.



   d. How can you explain the differences between these calculations of π?

Lesson 17 Summary

Not every number is rational. Earlier we tried to find a fraction whose square is equal to 2. That turns out to be impossible, although we can get pretty close (try squaring $\frac{7}{5}$). Since there is no fraction equal to $\sqrt{2}$, it is not a rational number, so we call it an "irrational number." Another well-known irrational number is π.

Every number, rational or irrational, has a decimal expansion. For example, the rational number $\frac{2}{11}$ has the decimal expansion 0.181818 . . . with the 18s repeating forever. Irrational numbers also have infinite decimal expansions, but they don't end up having a repeating pattern.

Practice Problems

1 How are the numbers 0.444 and $0.\overline{4}$ the same? How are they different?

2 Write each fraction as a decimal.

a. $\frac{5}{9}$

b. $\frac{5}{4}$

c. $\frac{48}{99}$

d. $\frac{5}{99}$

e. $\frac{7}{100}$

f. $\frac{53}{90}$

3 Write each decimal as a fraction.

a. $0.\overline{7}$

b. $0.\overline{2}$

c. $0.1\overline{3}$

d. $0.\overline{14}$

e. $0.\overline{03}$

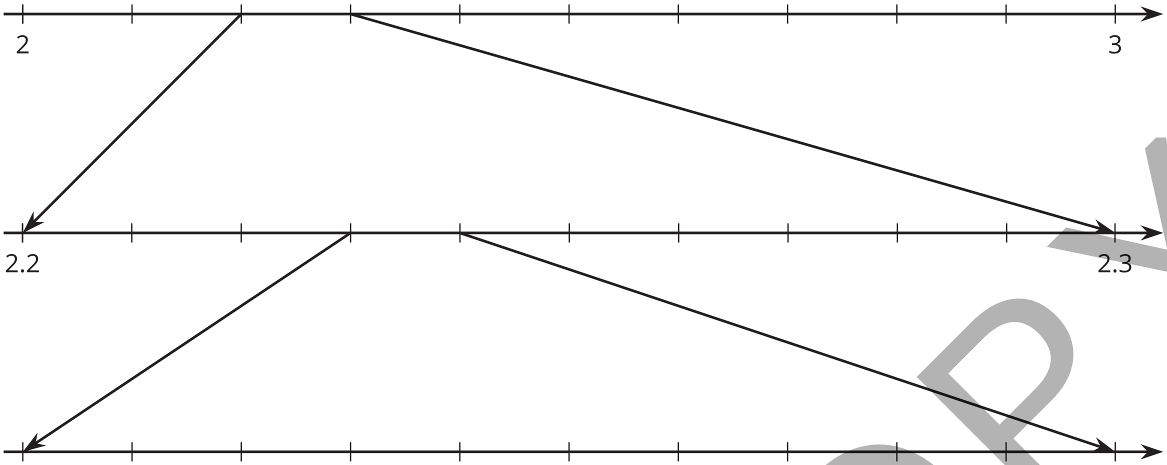
f. $0.6\overline{38}$

g. $0.52\overline{4}$

h. $0.1\overline{5}$

4 $2.2^2 = 4.84$ and $2.3^2 = 5.29$. This gives some information about $\sqrt{5}$.

Without directly calculating the square root, plot $\sqrt{5}$ on all three number lines using successive approximation.



5 from Unit 8, Lesson 4

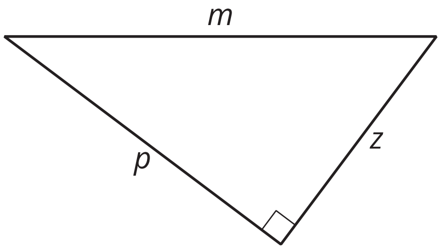
Which of the following numbers are rational?

$$\frac{9}{4}, -9, \sqrt{9}, \sqrt{11}, \sqrt{\frac{16}{25}}, \sqrt{\frac{100}{7}}$$

6 from Unit 8, Lesson 7

m, p, and z represent the lengths of the three sides of this right triangle.

Select **all** the equations that represent the relationship between m, p, and z.



A. $m^2 - z^2 = p^2$

B. $z^2 - p^2 = m^2$

C. $m^2 + p^2 - z^2 = 0$

D. $m^2 + z^2 - p^2 = 0$

E. $m^2 - p^2 = z^2$

F. $m^2 - z^2 - p^2 = 0$

Unit 8, Lesson 18

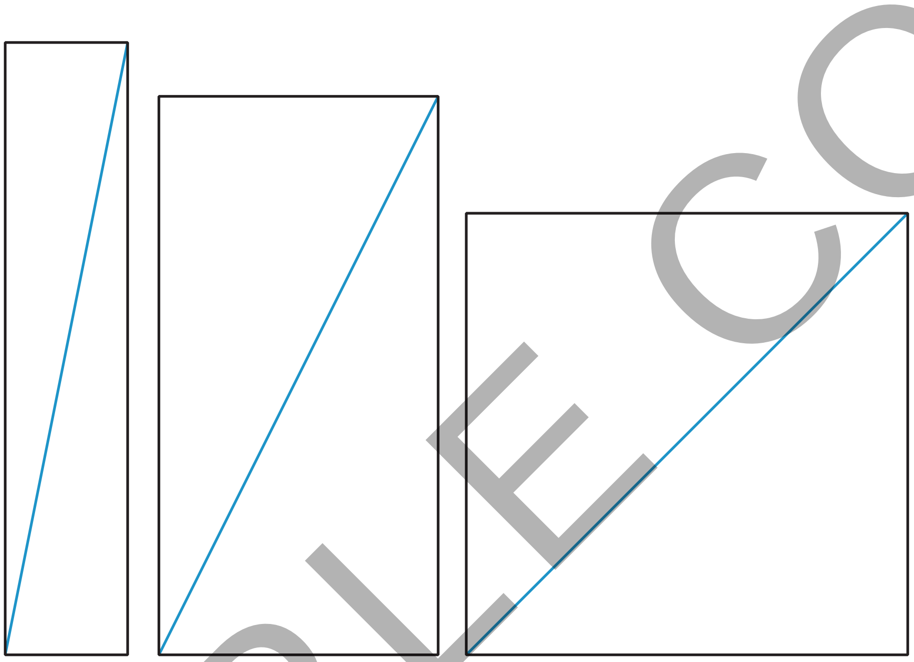
Addressing CA CCSSM 8.G.7; building on 6.G.1, 6.RP.1; practicing MP1 and MP4

When Is the Same Size Not the Same Size?

Let's figure out how aspect ratio affects screen area.

18.1 Three Figures

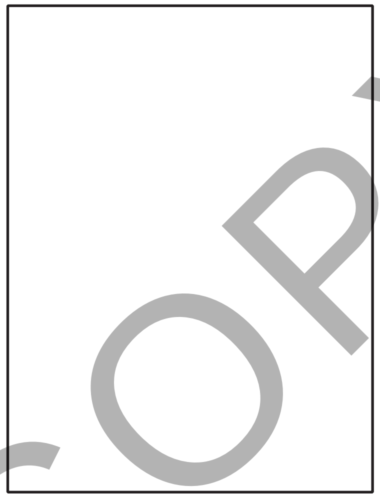
What do you notice? What do you wonder?



18.2 A $4:3$ Rectangle

A typical aspect ratio for photos is $4:3$.

Here's a rectangle with a $4:3$ aspect ratio.



1. What does it mean that the aspect ratio is $4:3$? Mark up the diagram to show what it means.
2. If the shorter side of the rectangle measures 15 inches:
    a. What is the length of the longer side?
    b. What is the length of the rectangle's diagonal?
3. If the diagonal of the $4:3$ rectangle measures 10 inches, how long are its sides?
4. If the diagonal of the $4:3$ rectangle measures 6 inches, how long are its sides?

18.3 The Screen Is the Same Size . . . Or Is It?

Before 2017, a smart phone manufacturer's phones had a diagonal length of 5.8 inches and an aspect ratio of $16:9$. In 2017, they released a new phone that also had a 5.8-inch diagonal length, but an aspect ratio of $18.5:9$.

Some customers complained that the new phones had a smaller screen. Were they correct? If so, how much smaller was the new screen compared to the old screen?

Learning Targets

Lesson 1 The Areas of Squares

- I can find the area of a tilted square on a grid by using methods like “decompose and rearrange” and “surround and subtract.”
- I can find the area of a triangle.

Lesson 2 Side Lengths and Areas

- I can explain what a square root is.
- If I know the area of a square, I can express its side length using square root notation.

Lesson 3 Square Roots

- I can find exact and approximate side lengths of squares.

Lesson 4 Rational and Irrational Numbers

- I know what an irrational number is and can give an example.
- I know what a rational number is and can give an example.

Lesson 5 Square Roots on the Number Line

- I can find a decimal approximation for square roots.
- I can plot square roots on the number line.

Lesson 6 Reasoning about Square Roots

- When I have a square root, I can reason about which two whole numbers it is between.

Lesson 7 Finding Side Lengths of Triangles

- I can explain what the Pythagorean Theorem says.

Lesson 8 A Proof of the Pythagorean Theorem

- I can explain why the Pythagorean Theorem is true.

Lesson 9 Finding Unknown Side Lengths

- If I know the lengths of two sides, I can find the length of the third side in a right triangle.
- When I have a right triangle, I can identify which side is the hypotenuse and which sides are the legs.

Lesson 10 The Converse

- I can explain why it is true that if the side lengths of a triangle satisfy the equation

$a^2 + b^2 = c^2$ then it must be a right triangle.

- If I know the side lengths of a triangle, I can determine if it is a right triangle or not.

Lesson 11 Applications of the Pythagorean Theorem

- I can use the Pythagorean Theorem to solve problems.

Lesson 12 More Applications of the Pythagorean Theorem

- I can recognize situations where the Pythagorean Theorem can be used to solve a problem.

Lesson 13 Finding Distances in the Coordinate Plane

- I can find the distance between two points in the coordinate plane.
- I can find the length of a diagonal line segment in the coordinate plane.

Lesson 14 Edge Lengths and Volumes

- I can approximate cube roots.
- I know what a cube root is.
- I understand the meaning of expressions like $\sqrt[3]{5}$.

Lesson 15 Cube Roots

- When I have a cube root, I can reason about which two whole numbers it is between.

Lesson 16 Decimal Representations of Rational Numbers

- I can write a fraction as a repeating decimal.
- I understand that every number has a decimal expansion.

Lesson 17 Infinite Decimal Expansions

- I can write a repeating decimal as a fraction.
- I understand that every number has a decimal expansion.

Lesson 18 When Is the Same Size Not the Same Size?

- I can apply what I have learned about the Pythagorean Theorem to solve a more complicated problem.
- I can decide what information I need to know to be able to solve a real-world problem using the Pythagorean Theorem.

GRADE 8

UNIT

9

Putting It All Together

Content Connections

In this unit you will solve complex problems involving tessellations of the plane and use properties of shapes and transformations to make inferences about regular tessellations. You will make connections by:

- **Discovering Shape and Space** while using scatter plots and lines of best fit to model the association between temperature and latitude.
- **Reasoning with Data** while constructing and interpreting data visualizations including scatter plots and constructing graphs of relationships between two variables.
- **Exploring Changing Quantities** while interpreting the trend(s) in change of data over time.

Addressing the Standards

As you work your way through **Unit 9 Putting It All together,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use These MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 4
MP2 Reason abstractly and quantitatively.	Lessons 2 and 5
MP3 Construct viable arguments and critique the reasoning of others.	Lesson 3
MP4 Model with mathematics.	Lessons 4, 5, and 6
MP5 Use appropriate tools strategically.	
MP6 Attend to precision.	Lesson 1
MP7 Look for and make use of structure.	Lessons 1, 2, and 3
MP8 Look for and express regularity in repeated reasoning.	

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Transformational Geometry	**8.G.1** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	Lessons 1, 2, and 3
• Transformational Geometry	**8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Lessons 1, 2, and 3

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Transformational Geometry	**8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Lessons 2 and 3
• Transformational Geometry	**8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	Lessons 1 and 2
• Transformational Geometry	**8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	Lessons 2 and 3
• Interpret Scatter Plots • Data Explorations	**8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	Lesson 4
• Data, Graphs, and Tables • Data Explorations • Linear Equations	**8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	Lesson 4
• Data, Graphs, and Tables • Data Explorations • Linear Equations	**8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	Lesson 6

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Interpret Scatter Plots • Data Explorations • Slopes and Intercepts	**8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Lessons 5 and 6
• Interpret Scatter Plots • Data Explorations • Slopes and Intercepts	**8.SP.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Lessons 4, 5, and 6
• Interpret Scatter Plots • Data, Graphs, and Tables • Data Explorations • Slopes and Intercepts	**8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	Lessons 4 and 5
• Data, Graphs, and Tables • Data Explorations	**8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	Lesson 4

Note: For a full explanation of the California Common Core State Standards for Mathematics (CA CCSSM) refer to the standards section at the end of this book.

Unit 9, Lesson 1

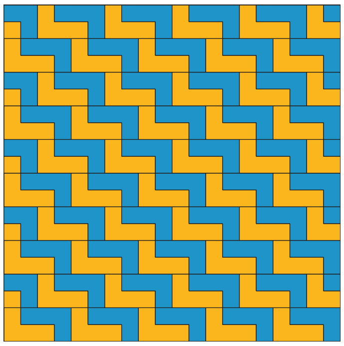
Addressing CA CCSSM 8.G.1, 8.G.2, 8.G.4; building on 7.G.1; practicing MP6 and MP7

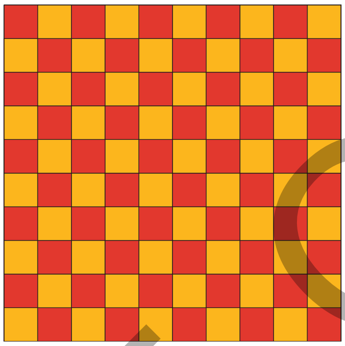
Tessellations of the Plane

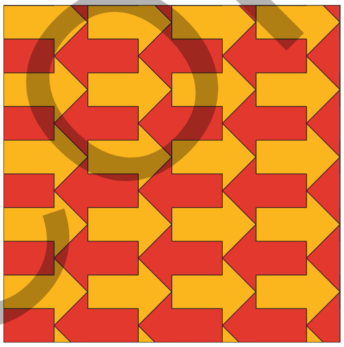
Let’s explore geometric patterns!

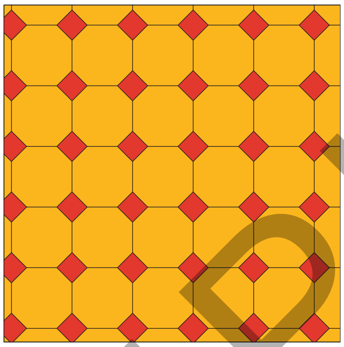
1.1 Notice and Wonder: Polygon Patterns

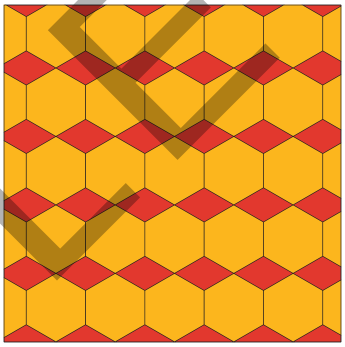
What do you notice? What do you wonder?

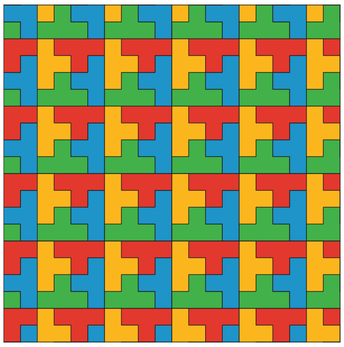




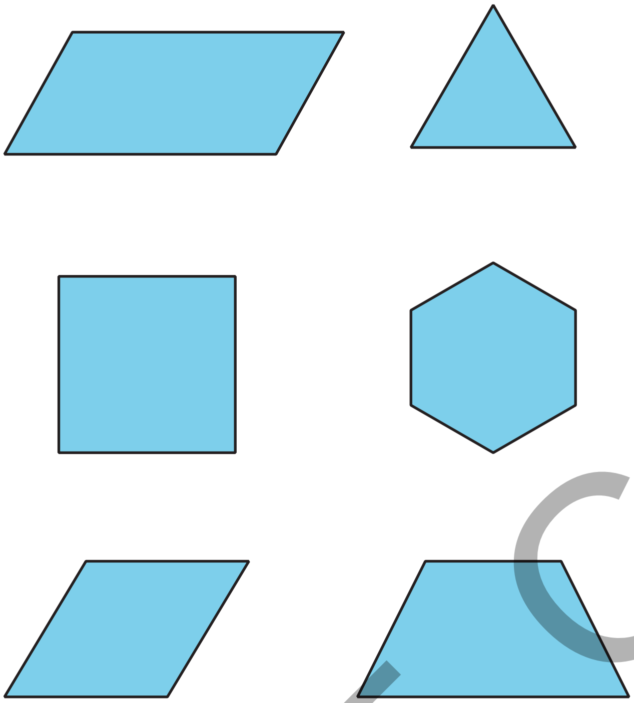








1.2 Tessellations



1. Pick one of the shapes. Create a **tessellation** by tracing copies of your shape. Make sure to use the same shape as your partner.

2. Compare your tessellation to your partner's. How are they similar? How are they different?

3. If possible, make a third tessellation (that is different from the ones you and your partner already created) of the plane with your shape. If not possible, explain why it is not possible.

1.3 Describing a Tessellation

1. You and your partner each have a card with a tessellation. Describe what is on your card so that your partner can produce the tessellation. (This should be done so that you cannot see your partner's work until it is complete.)

2. Check together to see if your partner's tessellation agrees with your card and discuss any differences.
3. Change roles so your partner describes a tessellation, which you try to produce.
4. Check the accuracy of your construction and discuss any discrepancies.

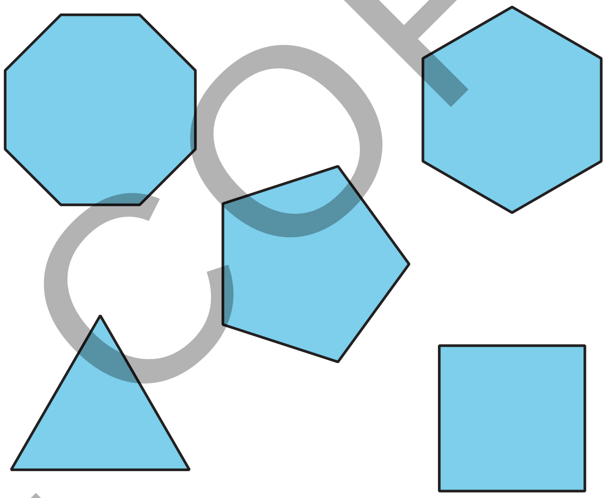
Addressing CA CCSSM 8.G.1-5; building on 7.G.5; practicing MP2 and MP7

Regular Tessellations

Let's make some regular tessellations.

2.1 Regular Tessellations

1. For each shape (triangle, square, pentagon, hexagon, and octagon), decide if you can use that shape to make a regular tessellation of the plane. Explain your reasoning.



2. For the polygons that do not work, what goes wrong? Explain your reasoning.

2.2 Equilateral Triangle Tessellation

1. What is the measure of each angle in an equilateral triangle? How do you know?

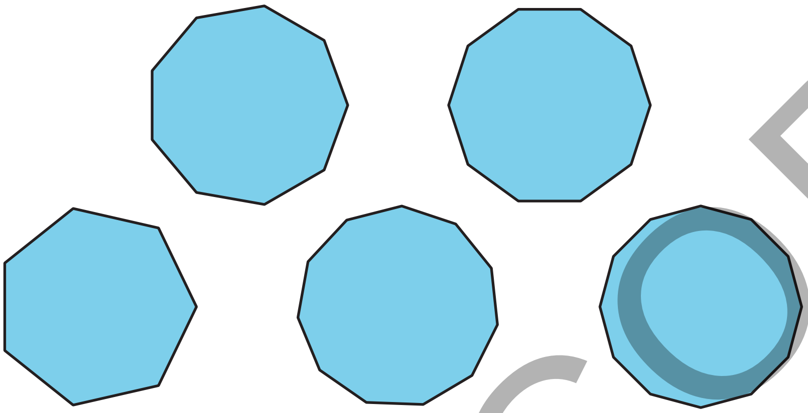
2. How many triangles can you fit together at 1 vertex? Explain why there is no space between the triangles.

3. Explain why you can continue the pattern of triangles to tessellate the plane.

4. How can you use your triangular tessellation of the plane to show that regular hexagons can be used to give a regular tessellation of the plane?

2.3 Regular Tessellation for Other Polygons

1. Can you make a regular tessellation of the plane using regular polygons with 7 sides? What about 9 sides? 10 sides? 11 sides? 12 sides? Explain.



2. How does the measure of each angle in a square compare to the measure of each angle in an equilateral triangle? How does the measure of each angle in a regular 8-sided polygon compare to the measure of each angle in a regular 7-sided polygon?

3. What happens to the angles in a regular polygon as you add more sides?

4. Which polygons can be used to make regular tessellations of the plane?

Unit 9, Lesson 3

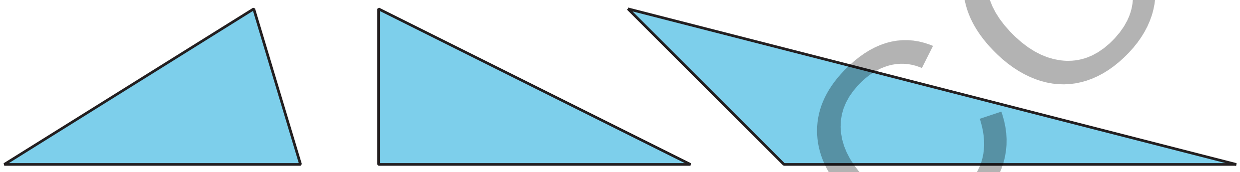
Addressing CA CCSSM 8.G.1-3, 8.G.5; practicing MP3 and MP7

Tessellating Polygons

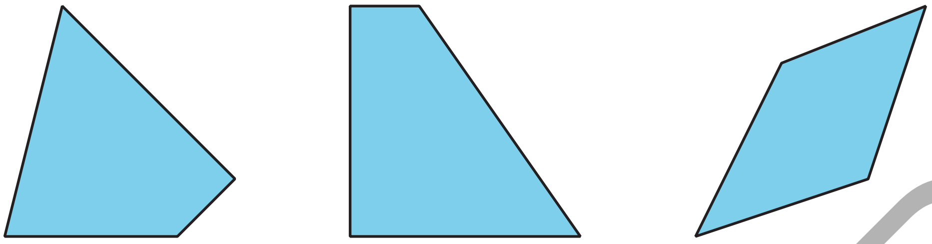
Let’s make tessellations with different polygons.

3.1 Triangle Tessellations

Your teacher will assign you one of the three triangles. You can use the picture to draw copies of the triangle on tracing paper. Your goal is to find a tessellation of the plane with copies of the triangle.



3.2 Quadrilateral Tessellations



1. Can you make a tessellation of the plane with copies of the trapezoid? Explain.

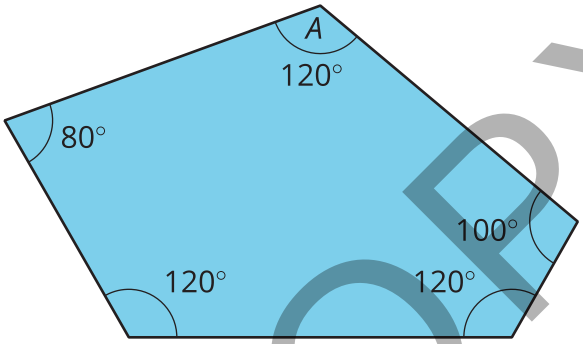
2. Choose and trace a copy of one of the other two quadrilaterals. Next, trace images of the quadrilateral rotated 180 degrees around the midpoint of each side. What do you notice?

3. Can you make a tessellation of the plane with copies of the quadrilateral from the earlier problem? Explain your reasoning.

3.3 Pentagonal Tessellations

1. Can you tessellate the plane with copies of this pentagon? Explain why or why not.

   Note that the two sides making angle A are congruent.



   Pause your work here.

2. Take the pentagon and rotate it 120 degrees clockwise about the vertex at angle A, and trace the new pentagon. Next, rotate the pentagon 240 degrees clockwise about the vertex at angle A, and trace the new pentagon.

3. Explain why the three pentagons make a full circle at the central vertex.

4. Explain why the shape that the three pentagons make is a hexagon (that is, the sides that look like they are straight really are straight).

Unit 9, Lesson 4

Addressing CA CCSSM 8.F.1, 8.F.4, 8.SP.2-4; practicing MP1 and MP4

What Influences Temperature?

Let’s see if we can predict the temperature.

4.1 Temperature Changes

What factors or variables can influence the outside temperature in North America?

- Make a list of different factors.
- Write a sentence for each factor describing how changing it could change the temperature.

Example: One factor is time of day. Often, after sunrise, the temperature increases, reaches a peak in the early afternoon, and then decreases.

4.2 Predicting the Temperature

Andre says, “I think temperature is a function of the latitude of the location, as long as we keep the time the same when we are measuring the temperature. I know that the temperature varies from summer to winter and throughout the day. By fixing the time, we remove that variability.”

Lin says, “Good idea to fix the time. Now what if we have 2 places with the same latitude? Look at this weather map for Washington State. Seattle and Spokane have the same latitude but different temperatures.”

What do Andre and Lin mean?

4.3 Is There an Association Between Latitude and Temperature?

1. Data collection:
    a. Latitude ______________________________
    b. Temperature ______________________________
2. Complete the table to collect your data. Your table should include the cities or locations chosen, the latitude (in degrees north), and the temperature (in degrees Fahrenheit).

Unit 9, Lesson 5

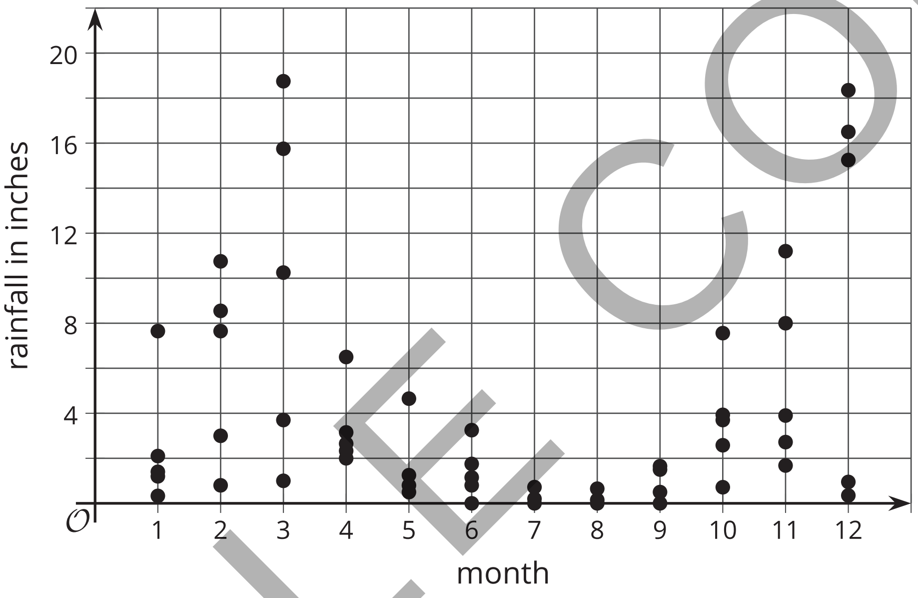
Addressing CA CCSSM 8.SP.1-3; practicing MP2 and MP4

Plotting the Temperature

Let's construct a model.

5.1 Notice and Wonder: California Rain

What do you notice? What do you wonder?



5.2 Data Snooping

The table shows the average high temperature in September for cities with different latitudes. Examine the data in the table.

city	latitude (degrees north)	temperature (degrees Fahrenheit)
Atlanta, GA	33.75	84
Belmopan, BZ	17.31	87
Boston, MA	42.36	73
Chinandega, NI	12.36	91
Dallas, TX	32.78	87
Denver, CO	39.74	81
Edmonton, AB	53.55	62
Fairbanks, AK	64.84	55
Juneau, AK	58.34	56
Kansas City, MO	39.10	80
Lincoln, NE	40.81	80
Miami, FL	25.76	89
Minneapolis, MN	44.98	73
New York City, NY	40.71	76
Orlando, FL	28.54	92
Philadelphia, PA	39.95	79
Portland, ME	43.66	71
Puerto Morelos, MX	20.89	89
San Antonio, TX	29.42	90
San Francisco, CA	37.78	70
Seattle, WA	47.61	72
Tampa, FL	27.95	89
Tucson, AZ	32.25	96
Yellowknife, NT	62.45	50

1. What information does each row hold?

2. What is the range for each variable?

3. Do you see an association between the two variables? If so, describe the association.

5.3 Temperature vs. Latitude

1. Make a scatter plot of the data.

2. Describe any patterns of association that you notice.

3. Draw a line that fits the data. Write an equation for this line.

Unit 9, Lesson 6

Addressing CA CCSSM 8.F.5, 8.SP.1-2; practicing MP4

Using and Interpreting a Mathematical Model

Let’s use a model to make some predictions.

6.1 Using a Mathematical Model

In an earlier activity, you found the equation of a line to represent the association between latitude and temperature. This is a *mathematical model*.

1. Use your model to predict the average high temperature in September for the following cities that were not included in the original data set:
   a. Detroit (Lat: 42.33 degrees north)
   b. Albuquerque (Lat: 35.09 degrees north)
   c. Nome (Lat: 64.50 degrees north)
   d. Your own location (if available)
2. Draw points that represent the predicted temperatures for each city on the scatter plot.
3. The actual average high temperature in September in these cities were:
   - Detroit: 74 degrees Fahrenheit
   - Albuquerque: 82 degrees Fahrenheit
   - Nome: 49 degrees Fahrenheit
   - Your own location (if available):

   How well does your model predict the temperature? Compare the predicted and actual temperatures.

4. If you added the actual temperatures for these 4 cities to the scatter plot, would you move your line?

5. Are there any outliers in the data? What might be the explanation?

6.2 Interpreting a Mathematical Model

Refer to your equation for the line that models the association between latitude and temperature of the cities.

1. What does the slope mean in the context of this situation?

2. Find the vertical and horizontal intercepts and interpret them in the context of the situation.

3. What is the model not good at predicting? Explain your reasoning.

Learning Targets

Lesson 1 Tessellations of the Plane

- Puedo describir patrones geométricos.

Lesson 2 Regular Tessellations

- Puedo crear teselaciones con polígonos regulares.

Lesson 3 Tessellating Polygons

- Puedo crear teselaciones con otros polígonos.

Lesson 4 What Influences Temperature?

- Puedo identificar distintas variables que influyen en la temperatura de un lugar.

Lesson 5 Plotting the Temperature

- Puedo modelar datos.

Lesson 6 Using and Interpreting a Mathematical Model

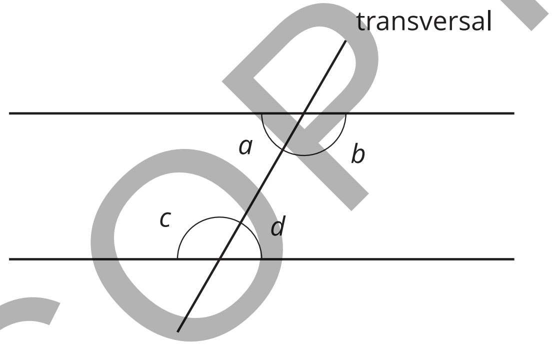
- Puedo hacer predicciones basándome en un modelo.

Glossary

- alternate interior angles
  Alternate interior angles are created when 2 parallel lines are crossed by another line. This line is called a *transversal*. Alternate interior angles are inside the parallel lines and on opposite sides of the transversal.

  This diagram shows 2 pairs of alternate interior angles:

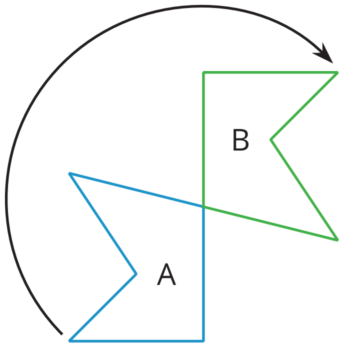
  - Angles a and d
  - Angles b and c



- base (of an exponent)
  In expressions like 5^3 and 8^2, the 5 and the 8 are bases. They tell what factor is multiplied repeatedly. For example, $5^3 = 5 \cdot 5 \cdot 5$, and $8^2 = 8 \cdot 8$.

- clockwise
  The word *clockwise* means to turn in the same direction as the hands of a clock. The top turns to the right.

  This diagram shows that Figure A turns clockwise to make Figure B.



- coefficient
  A coefficient is a number that is multiplied by a variable.

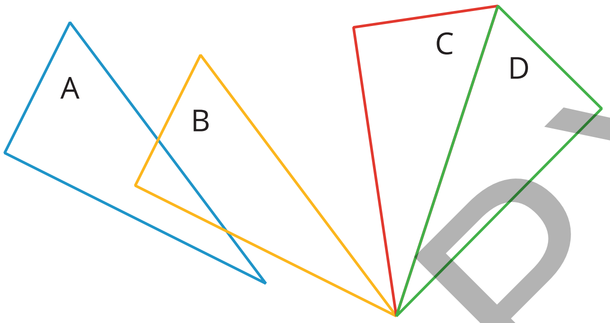
  - In the expression $3x + 5$, the coefficient of x is 3.
  - In the expression $y + 5$, the coefficient of y is 1, because $y = 1 \cdot y$.

- cone
  A cone is a three-dimensional figure like a pyramid, but the 1 base is a circle.

- congruent
  One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

  In this figure, Triangle A is congruent to Triangles B, C, and D.

  - A translation takes Triangle A to Triangle B.
  - A rotation takes Triangle B to Triangle C.
  - A reflection takes Triangle C to Triangle D.



- constant of proportionality
  In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity. This number is called the *constant of proportionality*.

  In this example, the constant of proportionality is 3.

number of oranges	**number of apples**
2 $\xrightarrow{\cdot 3}$	6
3 $\xrightarrow{\cdot 3}$	9
5 $\xrightarrow{\cdot 3}$	15

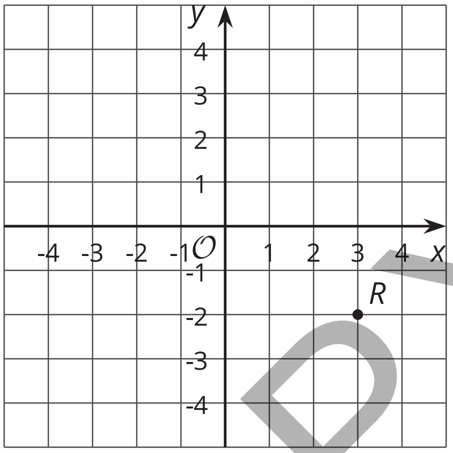
- constant term
  In an expression like $5x + 2$, the number 2 is called the *constant term*. It doesn't change when the variable x changes.

  - In the expression $7x + 9$, the constant term is 9.
  - In the expression $5x + (\text{-}8)$, the constant term is -8.
  - In the expression $12 - 4x$, the constant term is 12.

- coordinate plane
  The coordinate plane is one way to represent pairs of numbers. The plane is made of a horizontal number line and a vertical number line that cross at 0.

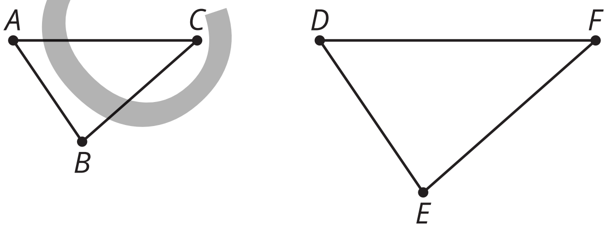
Pairs of numbers can be used to describe the location of a point in the coordinate plane.

Point R is located at $(3, -2)$. This means R is 3 units to the right and 2 units down from $(0, 0)$.



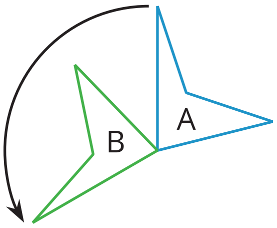
- corresponding
  Corresponding parts are the parts that match up between a figure and its scaled copy. They have the same relative position. Points, segments, angles, or distances can be corresponding.

  Point B in the first triangle corresponds to point E in the second triangle. Segment AC corresponds to segment DF.



- counterclockwise
  The term *counterclockwise* means to turn opposite of the way the hands of a clock turn. The top turns to the left.

  This diagram shows that Figure A turns counterclockwise to make Figure B.



- cube root
  The cube root of a number n is the number whose cube is n. It is also the edge length of a cube with a volume of n. The cube root of n is written as $\sqrt[3]{n}$.

  The cube root of 64 is written as $\sqrt[3]{64}$. Its value is 4 because 4^3 is 64.

  $\sqrt[3]{64}$ is also the edge length of a cube that has a volume of 64.

- cylinder
  A cylinder is a three-dimensional figure like a prism, but with 2 bases that are circles.

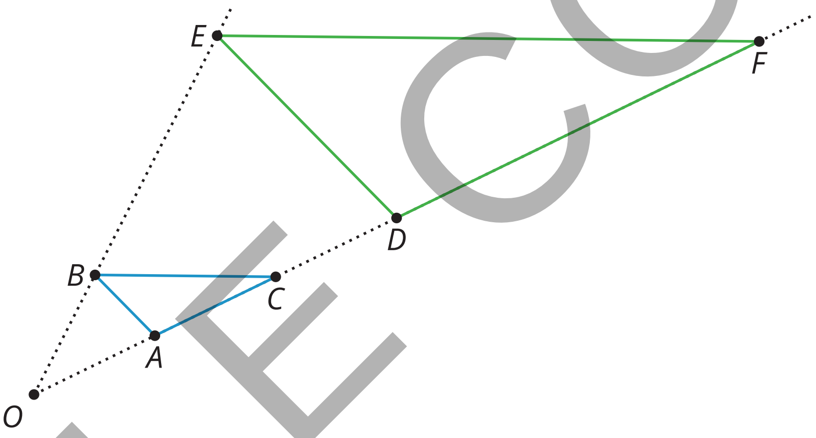
- dependent variable
  A dependent variable represents the output of a function.

  For example, someone needs to buy 20 pieces of fruit and decides to buy apples and bananas. If they select the number of apples first, the equation $b = 20 - a$ shows the number of bananas they can buy. The number of bananas is the dependent variable because it depends on the number of apples.

- dilation
  A dilation is a transformation that can reduce or enlarge a figure. Each point on the figure moves along a line closer to or farther from a fixed point. That fixed point is the center of the dilation. All of the original distances are multiplied by the same scale factor.

  Triangle DEF is a dilation of triangle ABC. The center of dilation is O. The scale factor is 3.

  Every point of triangle DEF is 3 times as far from O as every corresponding point of triangle ABC.



- exponent
  In expressions like 5^3 and 8^2, the numbers 3 and the 2 are called *exponents*. They tell how many times a number is used as a factor.

  For example, $5^3 = 5 \cdot 5 \cdot 5$, and $8^2 = 8 \cdot 8$.

- function
  A function is a rule that has exactly 1 output for each possible input.

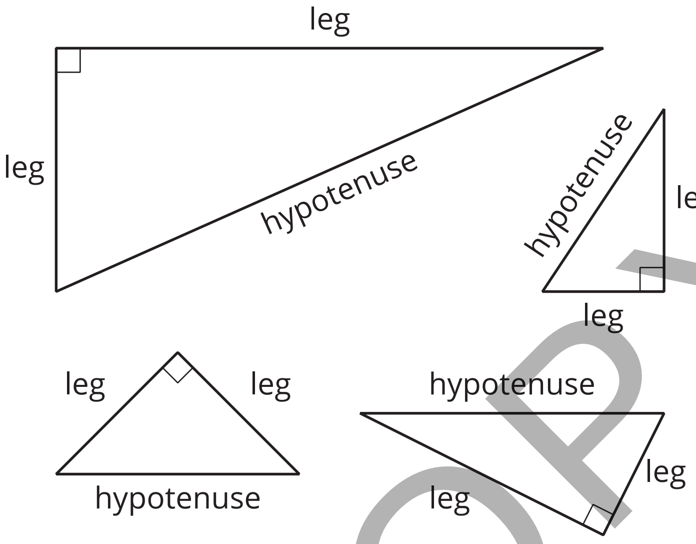
  In the function $y = 6x + 4$, x is the input and y is the output. When x is 5, y has one value, 34.

  $$y = 6x + 4$$
  $$y = 6(5) + 4$$
  $$y = 34$$

- hypotenuse

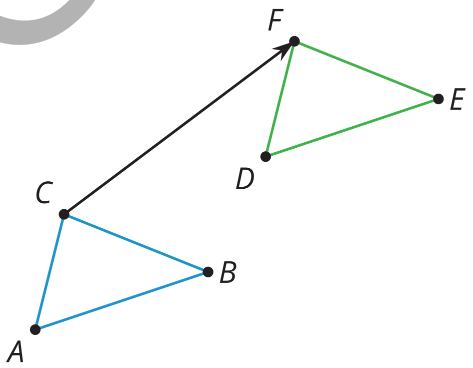
The hypotenuse is the side of a right triangle that is opposite the right angle. It is the longest side of a right triangle.

Here are some right triangles. Each hypotenuse is labeled.



- image
  An image is the result of translations, rotations, and reflections on an object. Every part of the original object moves in the same way to match up with a part of the image.

  Triangle ABC has been translated up and to the right to make triangle DEF. Triangle DEF is the image of the original triangle ABC.

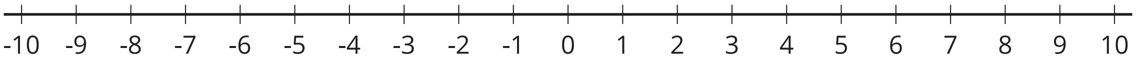


- independent variable
  An independent variable represents the input of a function.

  For example, suppose someone needs to buy 20 pieces of fruit and decides to buy some apples and bananas. If they select the number of apples first, the equation $b = 20 - a$ shows the number of bananas they can buy. The number of apples is the independent variable because any number can be chosen for it.

- integer
  An integer is a type of number. All whole numbers and their opposites are integers.

  The labels on this number line show all the integers from -10 to 10.

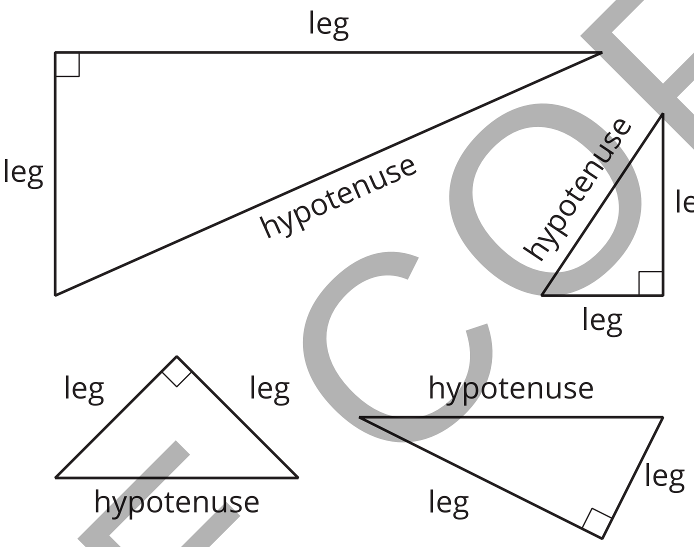


- irrational number
  An irrational number is a number that is not rational. It cannot be written as a positive fraction, a negative fraction, or zero.

  Pi (π) and $\sqrt{2}$ are examples of irrational numbers.

- legs
  The legs of a right triangle are the sides that make the right angle.

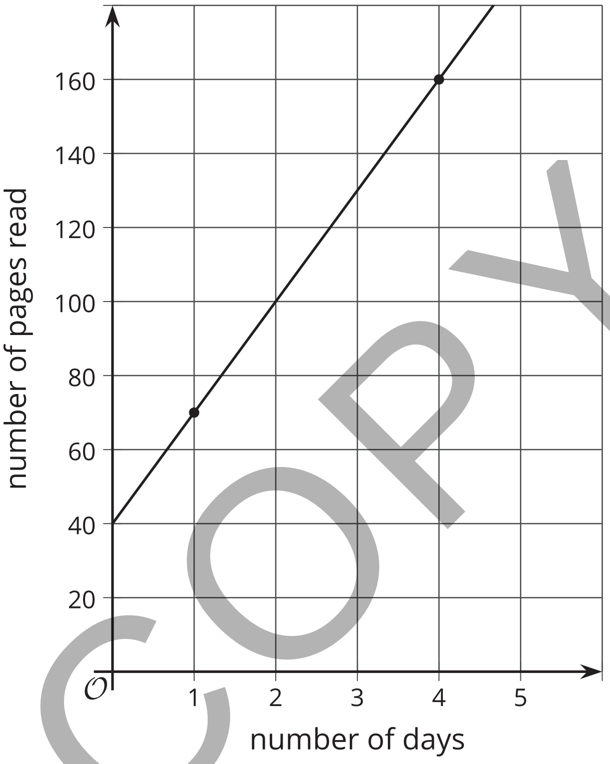
  Here are some right triangles. Each leg is labeled.



- linear relationship
  Two quantities have a linear relationship when:
  - One quantity changes by a set amount, whenever the other quantity changes by a set amount.
  - The rate of change is constant.
  - The graph of the relationship is a line.

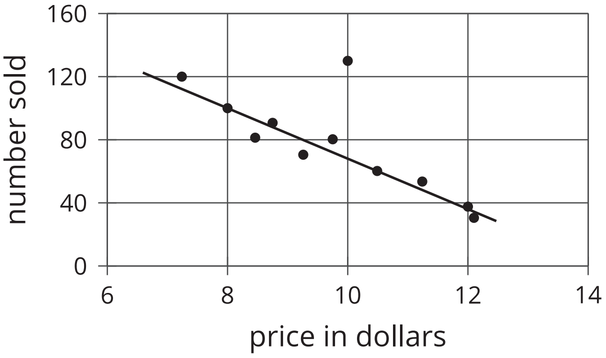
This graph shows a linear relationship between number of days and number of pages read.

When the number of days increases by 2, the number of pages read always increases by 60. The rate of change is constant, 30 pages per day.



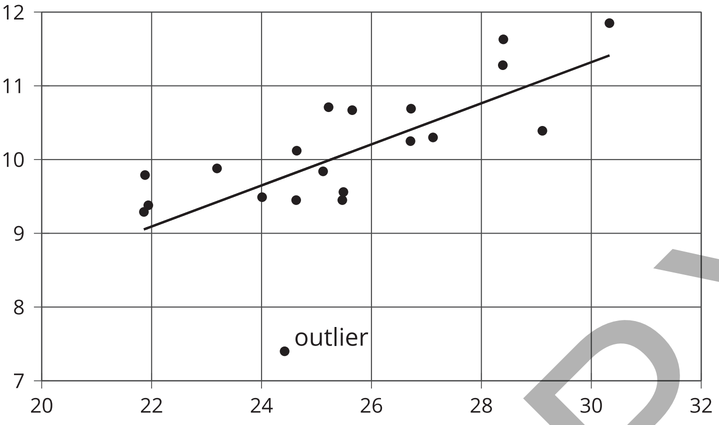
- negative association
  A negative association is a relationship between 2 quantities where one tends to decrease as the other increases. In a scatter plot, the data points tend to group around a line with negative slope.

  This scatter plot shows a negative association between the the price of a book and the number of books sold.



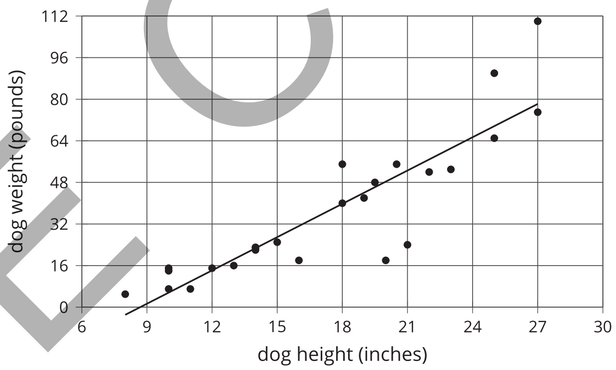
- outlier
  An outlier is a data value that is far from the other values in the data set.

This scatter plot shows 1 outlier.



- positive association
  A positive association is a relationship between 2 quantities where one tends to increase as the other increases. In a scatter plot, the data points tend to group around a line with positive slope.

  This scatterplot shows a positive association between dog height and dog weight.

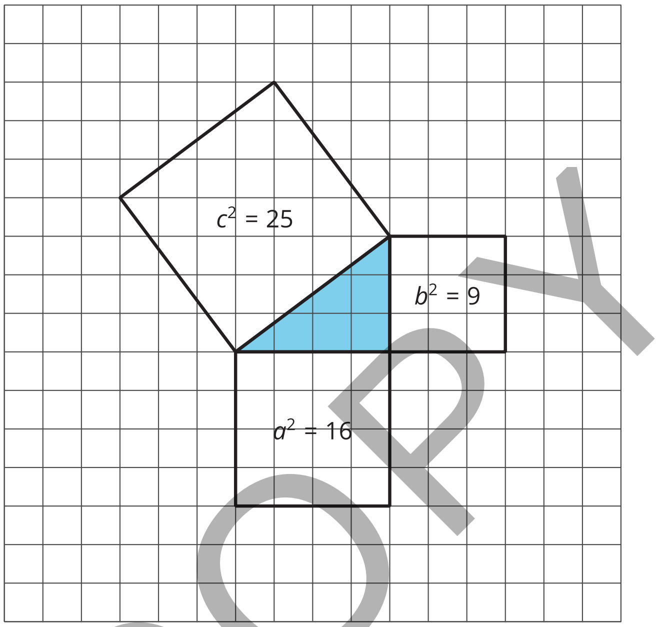


- Pythagorean Theorem

The Pythagorean Theorem describes the relationship between the side lengths of right triangles.

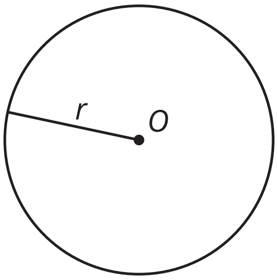
The square of the hypotenuse is equal to the sum of the squares of the legs. This is written as $a^2 + b^2 = c^2$.

This diagram shows the relationship.



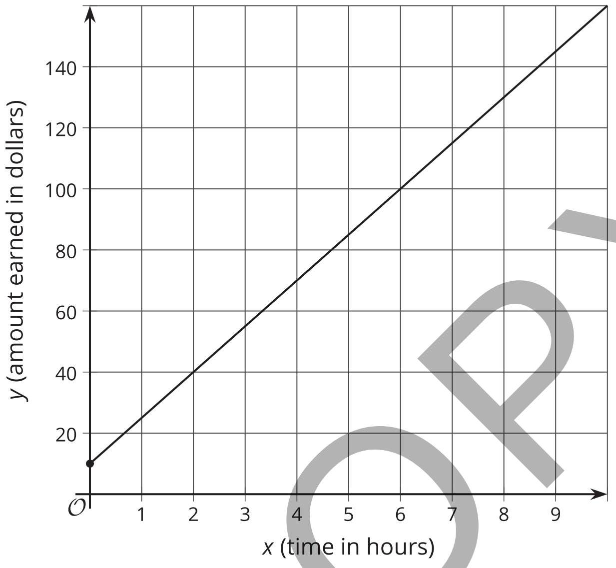
- radius
  A radius is a line segment that goes from the center of a circle to any point on the circle. The length of this segment is also called the *radius*. Every radius of a circle is the same length.

  For example, r is the radius of this circle with center O.



- rate of change (in a linear relationship)
  The rate of change is the amount y changes when x increases by 1. On a graph, the rate of change is the slope of the line.

In this graph, y increases by 15 dollars when x increases by 1 hour. The rate of change is 15 dollars per hour.



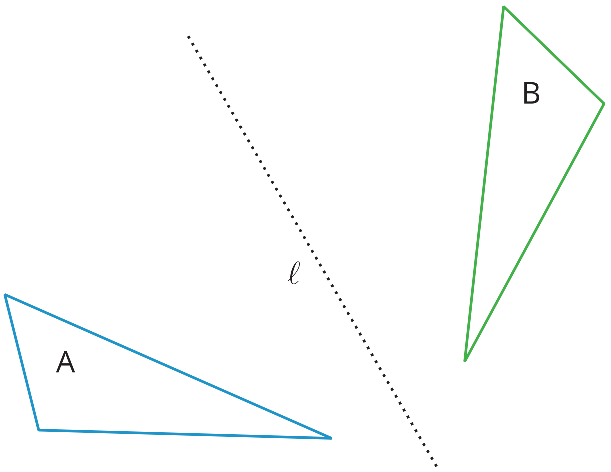
- rational number
  A rational number is a number that can be written as a positive fraction, a negative fraction, or zero. It can be written in the form $\frac{a}{b}$ where a and b are integers and b is not equal to 0.

  For example, 0.7 is a rational number because it can be written as $\frac{7}{10}$.

  Some examples of rational numbers: $\frac{7}{4}, 0, \frac{6}{3}, 0.2, -\frac{1}{3}, -5, \sqrt{9}$

- reflection
  A reflection is a transformation that “flips” a figure over a line. Every point on the figure moves to a point directly on the opposite side of the line. The new points are the same distance from the line as they are in the original figure.

  This diagram shows a reflection of A over line ℓ that makes the mirror image B.



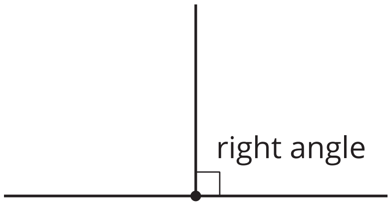
- relative frequency
  The relative frequency of a category tells the proportion at which the category occurs in the data set. It is written as a fraction, decimal, or percentage of the total number.

  For example, there were 21 dogs in a park. This table shows the frequency and the relative frequency of each color.

dog color	**frequency**	**relative frequency**
white	5	$\frac{5}{21}$
brown	7	$\frac{7}{21}$
black	3	$\frac{3}{21}$
multi-color	6	$\frac{6}{21}$

- repeating decimal
  A repeating decimal has digits that keep going in the same pattern over and over. The repeating digits are marked with a line above them.
  - The decimal representation for $\frac{1}{3}$ is $0.\overline{3}$, which means 0.3333333 . . . .
  - The decimal representation for $\frac{25}{22}$ is $1.1\overline{36}$, which means 1.136363636 . . . .

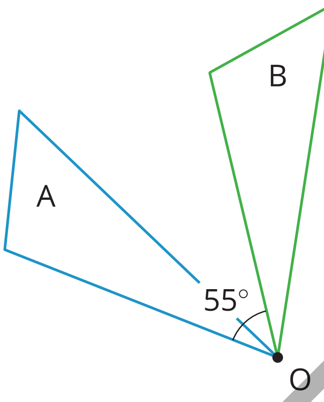
- right angle
  A right angle is half of a straight angle. It measures 90 degrees.



- rigid transformation
  A rigid transformation is a move that does not change any measurements of a figure. Translations, rotations, and reflections are rigid transformations. So is any sequence of these.

- rotation
  A rotation is a transformation that “turns” a figure. Every point on the figure moves around a center by a given angle in a specific direction.

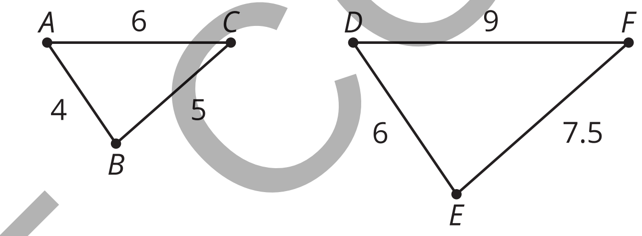
This diagram shows Triangle A rotated around center O by 55 degrees clockwise to get Triangle B.



- scale factor

  To create a scaled copy of a figure, all the side lengths in the original figure are multiplied by the same number. This number is called the *scale factor*.

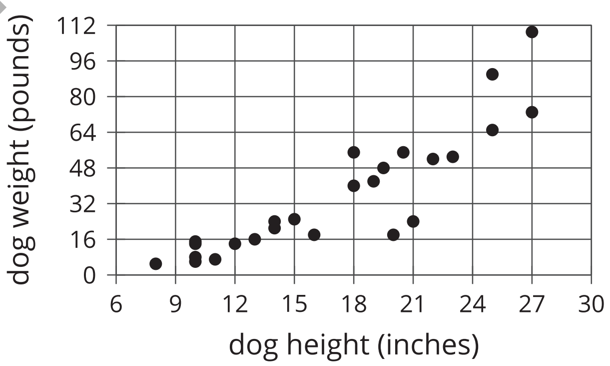
  In this example, the scale factor is 1.5, because $4 \cdot (1.5) = 6$, $5 \cdot (1.5) = 7.5$, and $6 \cdot (1.5) = 9$.



- scatter plot

  A scatter plot is a graph that shows values of 2 variables on a coordinate plane. It can be used to look for relationships between the 2 variables.

  Each point on this scatter plot represents the height and weight of 1 dog.



- scientific notation

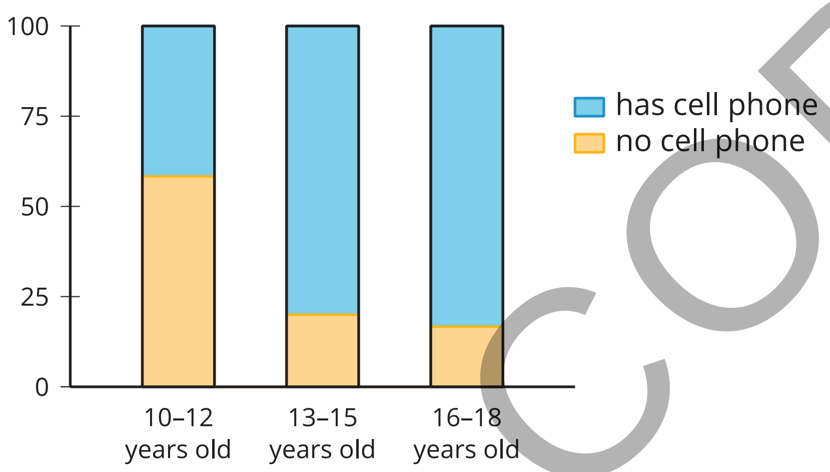
  Scientific notation is a way to write very large or very small numbers. They are written as a product. The first factor is a number greater than or equal to 1, but less than 10. The second factor is a power of 10.

  - The number 425,000,000 in scientific notation is 4.25×10^8.

  - The number 0.0000000000783 in scientific notation is 7.83×10^{-11}.

- segmented bar graph

  A segmented bar graph shows categories within a data set. Each whole bar represents all the data in one main category. Each bar is separated into parts (segments) that show subcategories.

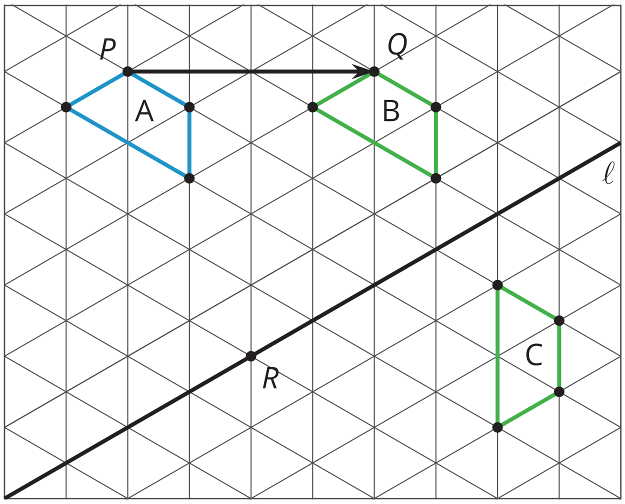
  This segmented bar graph shows the percentage of people in different age groups that do and do not have a cell phone. For example, among people ages 10 to 12, about 40% have a cell phone and 60% do not.

- sequence of transformations

  A sequence of transformations is a set of translations, rotations, reflections, and dilations on a figure. The transformations are performed in a given order.

  This diagram shows a sequence of transformations to move Figure A to Figure C.

  First, A is translated to the right to make B. Next, B is reflected across line ℓ to make C.

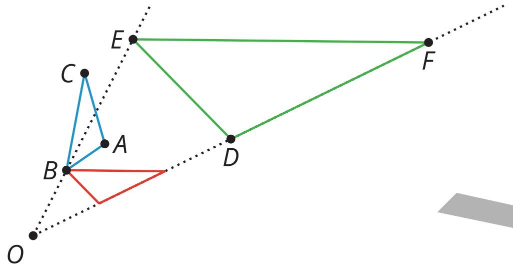
  

- similar

Two figures are similar if one can fit exactly over the other after transformations.

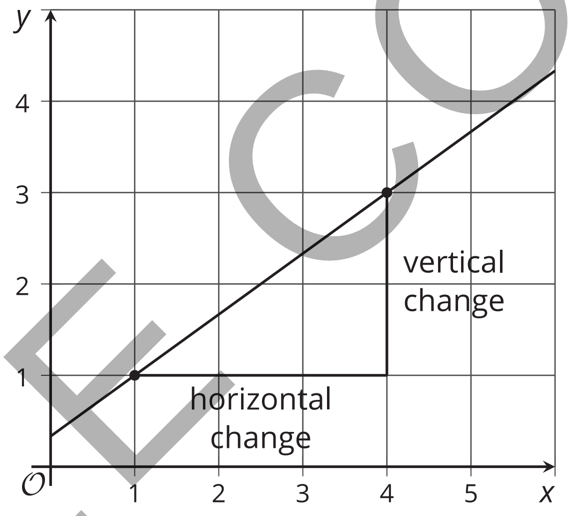
This figure shows triangle ABC is similar to triangle DEF.

  - Rotate triangle ABC around point B.
  - Then dilate it with center point O.
  - The image will fit exactly over triangle DEF.



- slope
  Slope is a number that describes how steep a line is. To find the slope, divide the vertical change by the horizontal change for any 2 points on the line.

  The slope of this line is 2 divided by 3, or $\frac{2}{3}$.



- solution to an equation with two variables
  A solution to an equation with 2 variables is a pair of values for the variables that make the equation true.

  For example, one solution to the equation $4x + 3y = 24$ is $(6, 0)$. Substituting 6 for x and 0 for y makes this equation true because $4(6) + 3(0) = 24$.

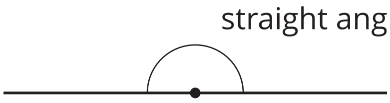
- sphere
  A sphere is a three-dimensional figure where any cross section is a circle.

- square root
  The square root of a positive number n is the positive number whose square is n. It is also the side length of a square whose area is n. The square root of n is written as $\sqrt{n}$.

  The square root of 16 is written as $\sqrt{16}$. Its value is 4 because 4^2 is 16.

$\sqrt{16}$ is also the side length of a square that has an area of 16.

- straight angle
  A straight angle is an angle that forms a straight line. It measures 180 degrees.



- system of equations
  A system of equations is a set of 2 or more equations. Each equation has 2 or more variables. A solution to the system is values for the variables that make all the equations true.

  These equations make up a system of equations:

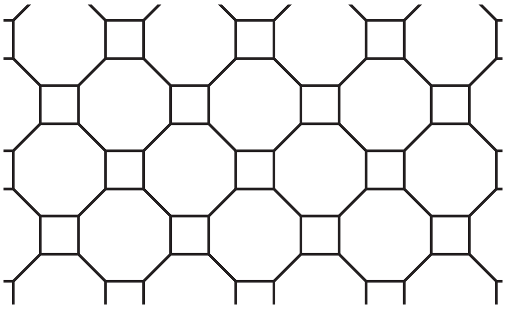
$$\begin{cases} x + y = \text{-}2 \\ x - y = 12 \end{cases}$$

  The solution to this system is $x = 5$ and $y = \text{-}7$. When these values are substituted for x and y, both equations are true: $5 + (\text{-}7) = \text{-}2$ and $5 - (\text{-}7) = 12$.

- term
  Terms are the parts of an expression that are added together. They can be a single number, a variable, or a number and a variable that are multiplied together.
  - 7, y, and $9a$ are examples of terms.
  - The expression $5x + 3 - 18$ has 3 terms: $5x$, 3, and -18.

- tessellation
  A tessellation is a repeating pattern of 1 or more shapes. The sides of the shapes fit together with no gaps or overlaps. The pattern goes on forever in all directions.

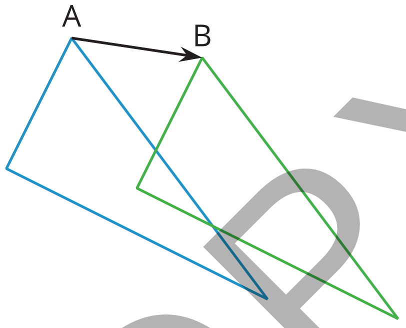
  This diagram shows part of a tessellation.



- transformation
  A transformation is a translation, rotation, reflection, or dilation, or a combination of these.

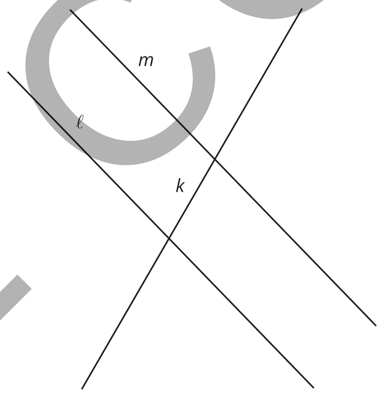
- translation
  A translation is a transformation that “slides” a figure along a straight line. Every point on the figure moves a given distance in a given direction.

  This diagram shows a translation of Figure A to Figure B using the direction and distance given by the arrow.



- transversal
  A transversal is a line that crosses parallel lines.

  This diagram shows a transversal line k intersecting parallel lines m and ℓ.



- two-way table
  A two-way table shows data for 2 categorical variables. One variable is shown in rows and the other in columns. Each entry is the frequency or relative frequency of the category shown by the column and row headings.

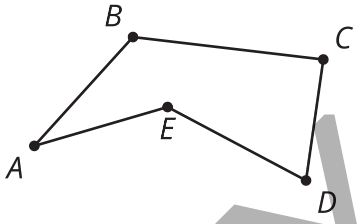
  This two-way table shows the results of a study. The study looked at how meditation affects the way athletes feel.

	meditated	**did not meditate**	**total**
calm	45	8	53
agitated	23	21	44
total	68	29	97

- vertex

  A vertex is a point where 2 or more edges meet. When there is more than 1 vertex, they are called *vertices*.

  The vertices of this polygon are labeled A, B, C, D, and E.

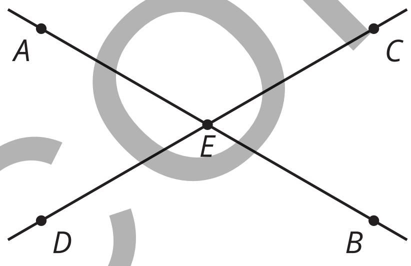


- vertical angles

  Vertical angles are opposite angles that share the same vertex. They are formed when two lines cross each other. Their angle measures are equal.

  Angles AEC and DEB are vertical angles. If angle AEC measures 120°, then angle DEB must also measure 120°.

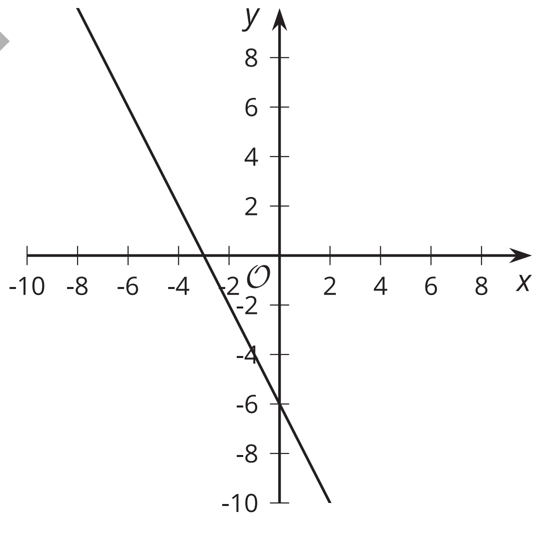
  Angles AED and BEC are another pair of vertical angles.



- vertical intercept

  The vertical intercept is the point where the graph of a line crosses the vertical axis.

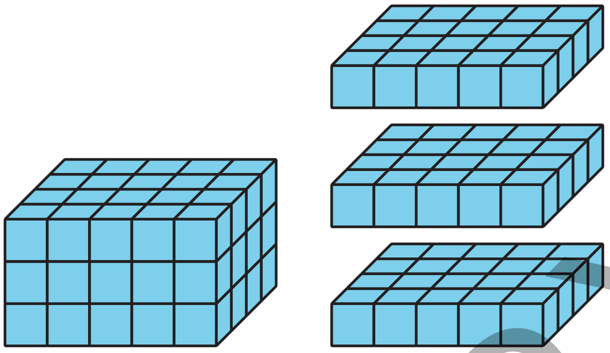
  The vertical intercept of this line is $(0, \text{-}6)$ or just -6.



- volume

  Volume is the number of cubic units that fill a three-dimensional region with no gaps or overlaps.

This rectangular prism has 3 layers that are each 20 units^3. So, the volume of the prism is 60 units^3.



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California Common Core State Standards for Mathematics (CA CCSSM) References

8.EE: Grade 3 – Expressions and Equations

Work with radicals and integer exponents.

8.EE.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

8.EE.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

8.EE.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.6
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7
Solve linear equations in one variable.

8.EE.7a
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

8.EE.7b
Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8
Analyze and solve pairs of simultaneous linear equations.

8.EE.8a
Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.8b
Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x+2y=5$ and $3x+2y=6$ have no solution because $3x+2y$ cannot simultaneously be 5 and 6.

8.EE.8c
Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

8.F: Grade 8 – Functions

Define, evaluate, and compare functions

8.F.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.

8.F.2
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.3
Interpret the equation $y=mx+b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A=s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.

Use functions to model relationships between quantities.

8.F.4
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

8.G: Grade 8 – Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1
Verify experimentally the properties of rotations, reflections, and translations:

8.G.1a
Lines are taken to lines, and line segments to line segments of the same length.

8.G.1b
Angles are taken to angles of the same measure.

8.G.1c
Parallel lines are taken to parallel lines.

8.G.2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

8.G.5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

8.G.6
Explain a proof of the Pythagorean Theorem and its converse.

8.G.7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

8.NS: Grade 8 – The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

8.SP: Grade 8 – Statistics and Probability

Investigate patterns of association in bivariate data.

8.SP.1
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP.4
Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.