IM California





Student Edition

UNITS

12



Kendall Hunt

Book 1
Certified by Illustrative Mathematics®



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Math in Our World

Content Connections

In this unit you will explore mathematical tools, notice numbers, and build a mathematical community. You will make connections by:

- Reasoning with Data while sorting, counting, classifying, comparing, and describing objects using numbers.
- **Exploring changing quantities** while learning number names, the sequence of numbers, and counting objects.

- Taking Wholes Apart, Putting Parts Together while modeling abstract problems and numbers using fingers or other manipulatives/models.
- Discovering Shape and Space while learning shape names and exploring similarities and differences between shapes.

Addressing the Standards

As you work your way through **Unit 1 Math in Our World,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

Mathematical Practices	Where You Use these MPs
MP1 Make sense of problems and persevere in solving them.	Lesson 7, 8, 9, 11
MP2 Reason abstractly and quantitatively.	Lesson 5, 7, 10, 14, 15
MP3 Construct viable arguments and critique the reasoning of others.	Lesson 3, 5, 12, 13
MP4 Model with mathematics.	Lesson 4, 6, 7, 17
MP5 Use appropriate tools strategically.	Lesson 1, 12, 13, 14, 15, 16
MP6 Attend to precision.	Lesson 2, 4, 5, 6, 8

MP7 Look for and make use of structure.		
MP8 Look for and express regularity in repeated reasoning.	Lesson 11	
repeated reasoning.		

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
• Shapes in the world	K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	Lesson 5
Shapes in the world	K.G.2 Correctly name shapes regardless of their orientations or overall size.	Lesson 5
• Shapes in the world	K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	Lesson 5

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
 Sort and Describe Data Bigger or Equal? Shapes in the world Making Shapes from parts 	K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	Lesson 5, 12, 14, and 17
Shapes in the worldMaking Shapes from parts	K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Lesson 4, 5, 12, 14, and 17
 Being Flexible within 10 Shapes in the World Making Shapes from parts 	K.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"	Lesson 5, 12, 14, and 17
How Many?	K.CC.1 Count to 100 by ones and by tens.	Lesson 12, 13, 14, 15, and 16

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
 Sort and Describe Data How Many? Bigger or Equal? 	K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	Lesson 9, 12, 13, 14, 15, and 16

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
Sort and	K.CC.5	Lesson 13,
Describe	Count to answer "how many?"	14, 15, 16,
Data	questions about as many as	17
• How Many?	20 things arranged in a line, a	
Bigger or	rectangular array, or a circle,	
Equal?	or as many as 10 things in a	
Place and	scattered configuration; given	
Position of	a number from 1–20, count	
numbers	out that many objects.	





Building Towards CA CCSSM K.CC.4-5, K.G.4-6, K.MD.1-2, and K.MD.3; practicing MP5

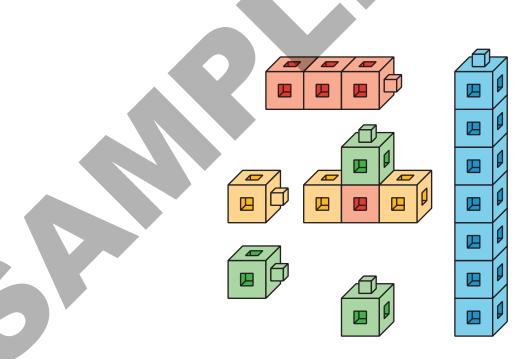
Explore Connecting Cubes

Let's explore connecting cubes.



Notice and Wonder: Connecting Cubes

What do you notice? What do you wonder?





Building towards CA CCSSM K.CC.4-5, K.G.4-6, K.MD.3; practicing MP6

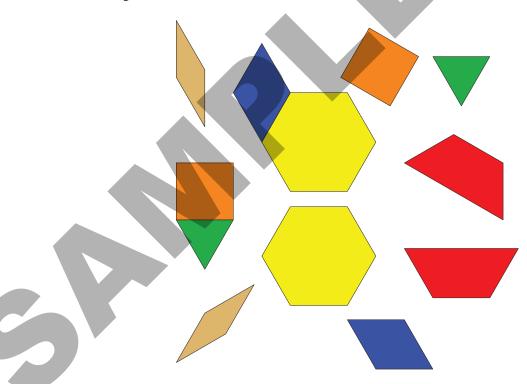
Explore Pattern Blocks

Let's explore pattern blocks.

(Warm-up

Notice and Wonder: Pattern Blocks

What do you notice? What do you wonder?







Building towards CA CCSSM K.CC.4-5 and K.MD.3; practicing MP3

Explore 2-Color Counters and 5-Frames

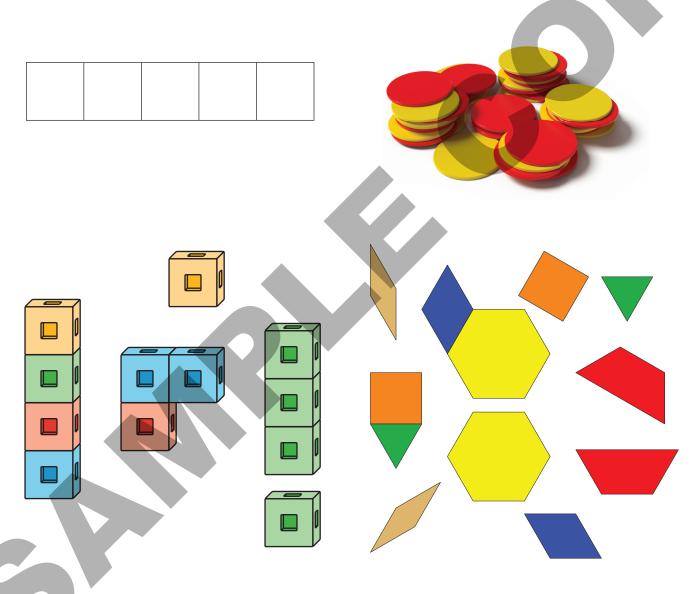
Let's explore 2-color counters and 5-frames.





Notice and Wonder: Counters and 5-Frames

What do you notice? What do you wonder?







Addressing CA CCSSM K.G.5; building towards K.CC.4-5, K.G.1-3, K.G.4-6, K.MD.3; practicing MP4 and MP6

Explore Geoblocks

Let's explore geoblocks.



Notice and Wonder: Geoblocks

What do you notice? What do you wonder?





Introduce Geoblocks—Build to Match

Use geoblocks to build a house.







Addressing CA CCSSM K.G.1-3 and K.G.4-6; building towards K.CC.4-5, K.G.1-3, K.G.4-6, and K.MD.3; practicing MP2, MP3, and MP6

Explore Math Tools

Let's explore our math tools.





Notice and Wonder: Using Different Tools

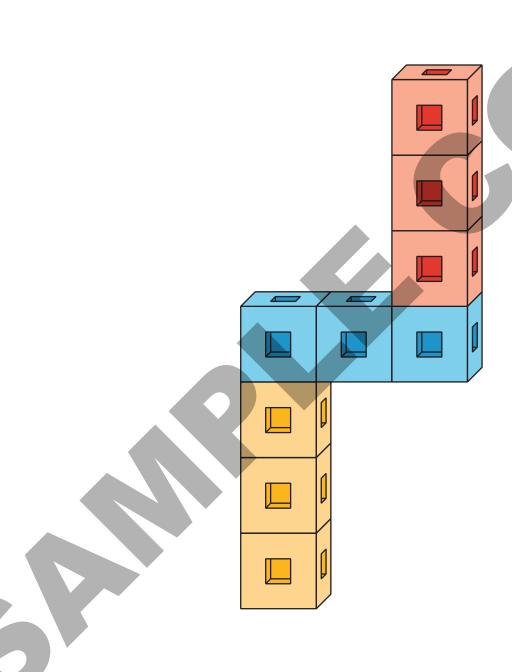
What do you notice? What do you wonder?







Introduce Connecting Cubes—Build to Match





Introduce Pattern Blocks—Puzzles



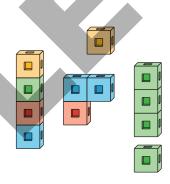
Centers: Choice Time

Choose a center.

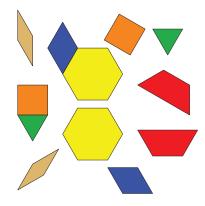
Geoblocks



Connecting Cubes

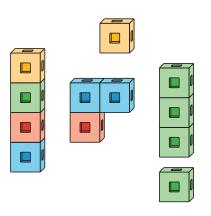


Pattern Blocks

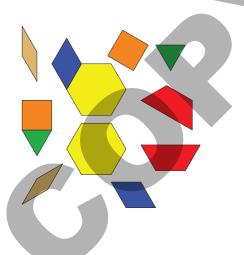


Section A Summary

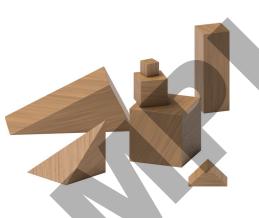
We can use many math tools. connecting cubes







geoblocks



2-color counters



5-frames





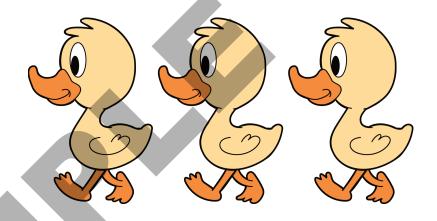
Building towards CA CCSSM K.CC.4; practicing MP4 and MP6

Look for Small Groups

Let's look for small groups of objects.



Act It Out: Introduction



3 little ducks went out one day, over the hill and far away. Mother duck said, "Quack, quack, quack." Then 3 little ducks came back.



How Many Do You See: Introduction

How many do you see? How do you see them?





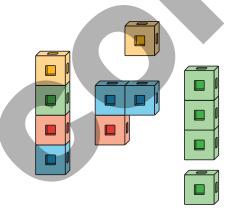
Centers: Choice Time

Choose a center.

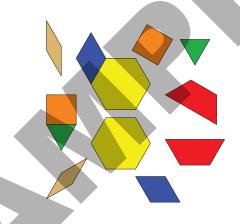
Geoblocks



Connecting Cubes



Pattern Blocks



Picture Books







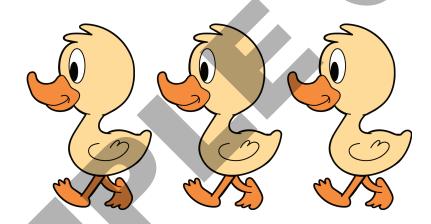
Building towards CA CCSSM K.CC.4; practicing MP1, MP2, and MP4

Classroom Scavenger Hunt

Let's look for groups of objects in the classroom.



Act It Out: How Can We Show It?



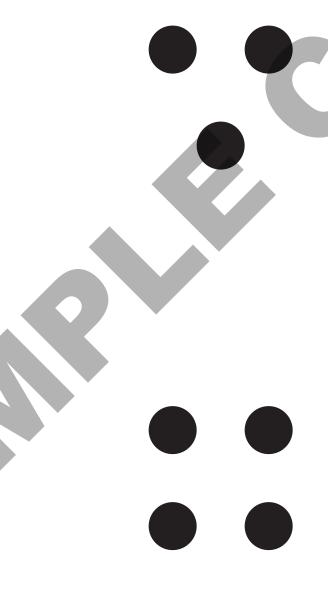
3 little ducks went out one day, over the hill and far away.
Mother duck said, "Quack, quack, quack."
Then 3 little ducks came back.





How Many Do You See: Two Images

How many do you see? How do you see them?



Activity 3

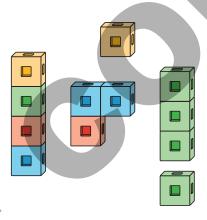
Centers: Choice Time

Choose a center.

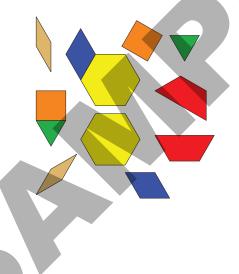
Geoblocks



Connecting Cubes



Pattern Blocks



Picture Books









Building towards CA CCSSM K.CC.4 and K.CC.6; practicing MP1 and MP6

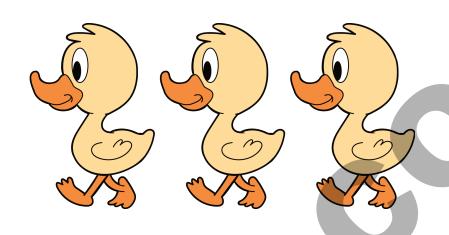
Different Groups, Same Quantity

Let's find groups that have the same number of things.





Act It Out: Another Way



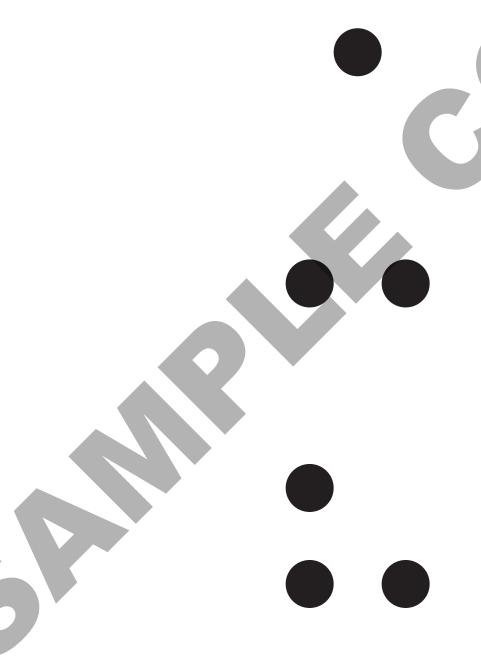
3 little ducks went out one day, over the hill and far away.
Mother duck said, "Quack, quack, quack."
Then 3 little ducks came back.





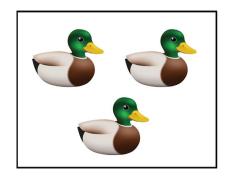
How Many Do You See: 1, 2, 3

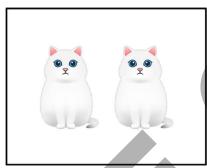
How many do you see? How do you see them?

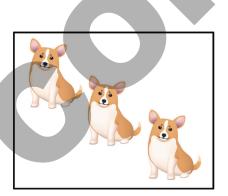




Card Sort: Different Groups, Same Quantity









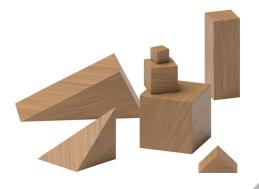




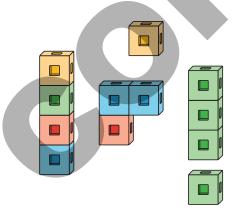
Centers: Choice Time

Choose a center.

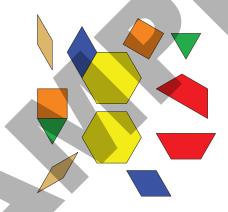
Geoblocks



Connecting Cubes



Pattern Blocks



Picture Books







Addressing CA CCSSM K.CC.4; building towards K.CC.4; practicing MP1

Create Picture Books

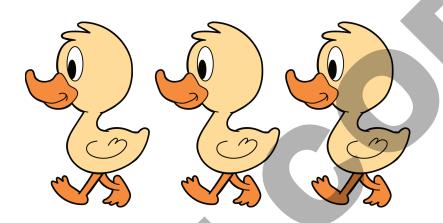
Let's make picture books about our classroom.







Act It Out: The Story Changes



3 little ducks went out one day, over the hill and far away.
Mother duck said, "Quack, quack, quack."
Then 3 little ducks came back.

3 little ducks went out one day, over the hill and far away. Mother duck said, "Quack, quack, quack." Then 2 little ducks came back.

How Many Do You See: What Do You Notice?

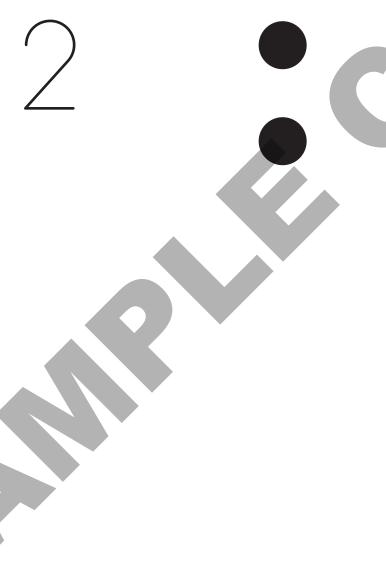
How many do you see? How do you see them?







Introduce Picture Books—Create



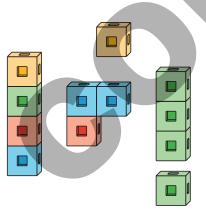
Centers: Choice Time

Choose a center.

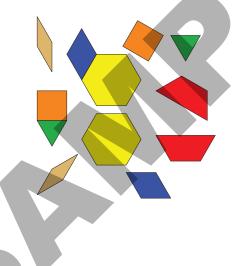
Geoblocks



Connecting Cubes



Pattern Blocks







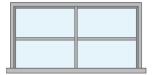


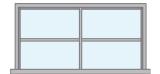


Section B Summary

We can find groups.

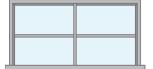
We can tell how many.

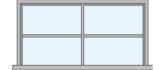




 \bigcirc

We can find different groups that have the same number of things.







2 windows

2 tables



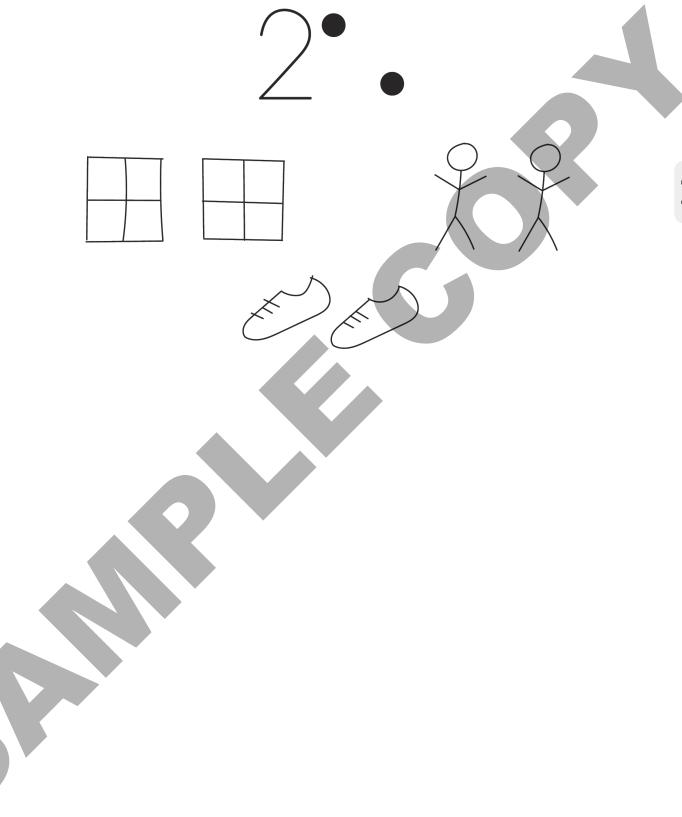




3 balls

Sec

groups of 2 in our classroom





Building towards CA CCSSM K.CC.4 and K.CC.6; practicing MP4

Are There Enough?

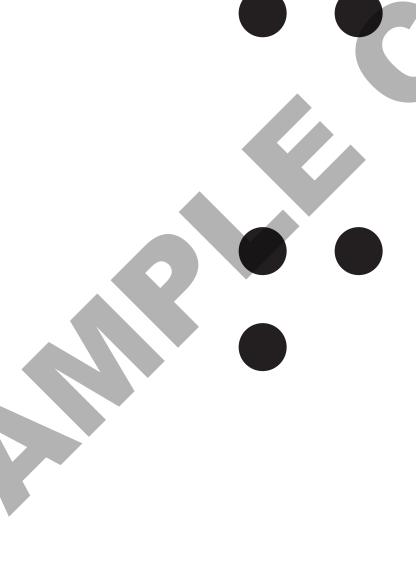
Let's figure out if there are enough supplies for everyone.





How Many Do You See: Building On

How many do you see? How do you see them?



Act It Out: 4 Frogs (Part 1)



4 little speckled frogs sat on a speckled log, eating the most delicious bugs. Yum! Yum!
1 jumped into the pool, where it was nice and cool.
Now there are 3 green speckled frogs. Glub! Glub!





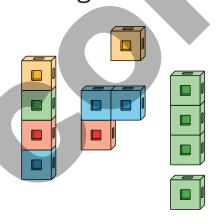
Centers: Choice Time

Choose a center.

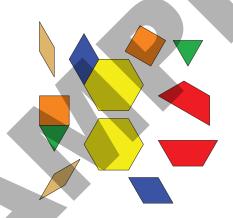
Geoblocks



Connecting Cubes



Pattern Blocks









Building towards CA CCSSM K.CC.4 and K.CC.6; practicing MP1 and MP8

Get Enough

Let's get enough pencils for everyone.





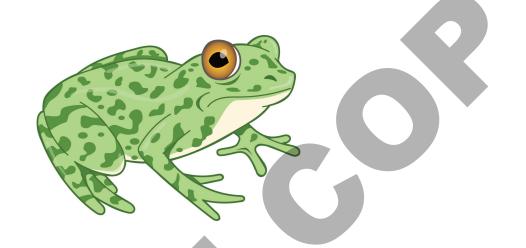


How Many Do You See: In a Flash

How many do you see? How do you see them?



Act It Out: 4 Frogs (Part 2)



4 little speckled frogs sat on a speckled log, eating the most delicious bugs. Yum! Yum!
1 jumped into the pool, where it was nice and cool.
Now there are 3 green speckled frogs. Glub! Glub!







Get Enough













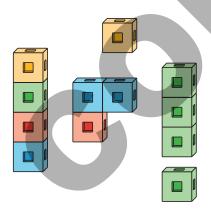
Centers: Choice Time

Choose a center.

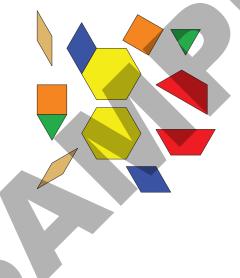
Geoblocks



Connecting Cubes



Pattern Blocks

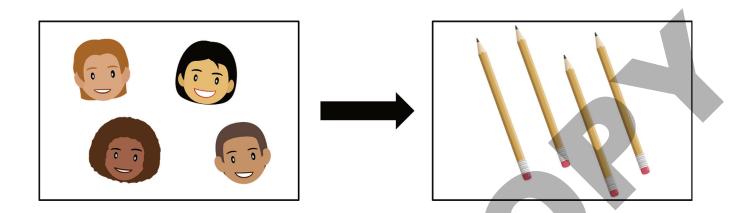








Section C Summary



We can tell if there are enough.

1 pencil for each person









Addressing CA CCSSM K.CC.1, K.CC.4, and K.G.4-6; building towards K.CC.4; practicing MP3 and MP5

How Many Are There? (Part 1)

Let's count collections of objects.

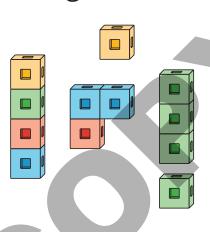


Choose a center.

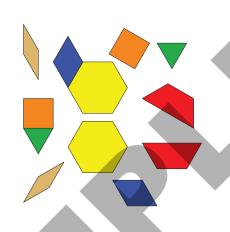
Geoblocks



Connecting Cubes



Pattern Blocks







Addressing CA CCSSM K.CC.1 and K.CC.4-5; building towards K.CC.4; practicing MP3 and MP5

How Many Are There? (Part 2)

Let's count collections of objects.

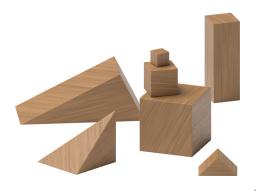




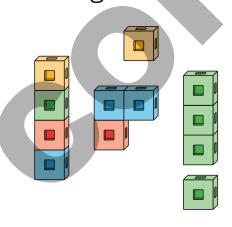
Centers: Choice Time

Choose a center.

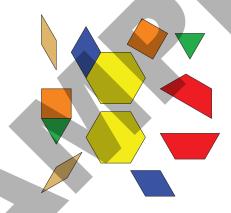
Geoblocks



Connecting Cubes



Pattern Blocks













Addressing CA CCSSM K.CC.1, K.CC.4-5, and K.G.4-6; building towards K.CC.4-5 and K.OA.1; practicing MP2 and MP5

Answer "How Many?" Questions

Let's count to figure out how many objects are in our collections.



























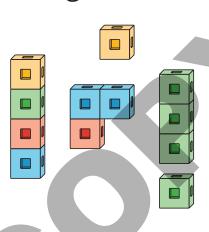


Choose a center.

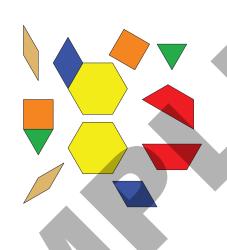
Geoblocks



Connecting Cubes



Pattern Blocks



Picture Books





Addressing CA CCSSM K.CC.1 and K.CC.4-5; building towards K.CC.4-5; practicing MP2 and MP5

Explain How You Counted

Let's count collections of objects and tell our partners how we counted.



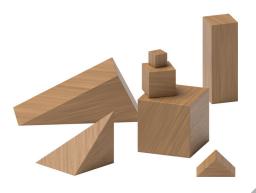




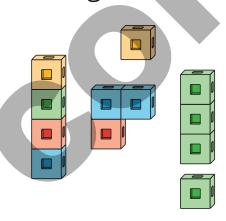
Centers: Choice Time

Choose a center.

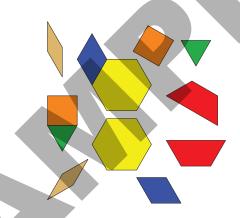
Geoblocks



Connecting Cubes



Pattern Blocks









Addressing CA CCSSM K.CC.1 and K.CC.4-5; building towards K.CC.4-5; practicing MP5

Represent Our Collections

Let's count collections of objects and show how we counted.

Activity 1

Counting Collections: Show How Many

Show how many objects are in your collection.





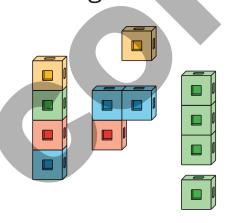
Centers: Choice Time

Choose a center.

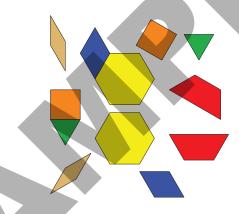
Geoblocks



Connecting Cubes



Pattern Blocks

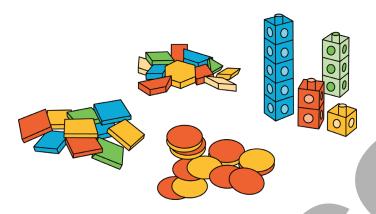




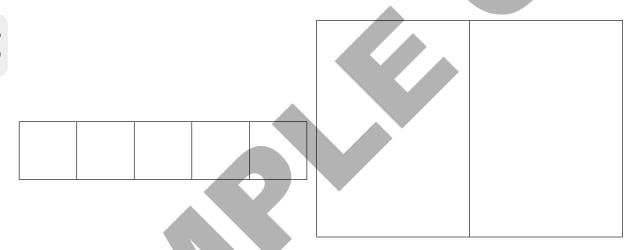


Section D Summary

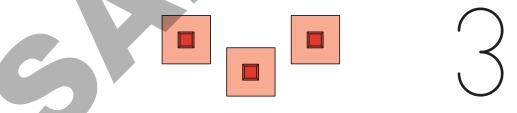
We can count groups of objects.



We can use 5-frames and counting mats to help us.



We can say a number to tell how many objects.







Addressing CA CCSSM K.CC.4-5 and K.G.4-6; building towards K.G.4-6; practicing MP4

Model with Math Tools

Let's use math tools to create things in our classroom.



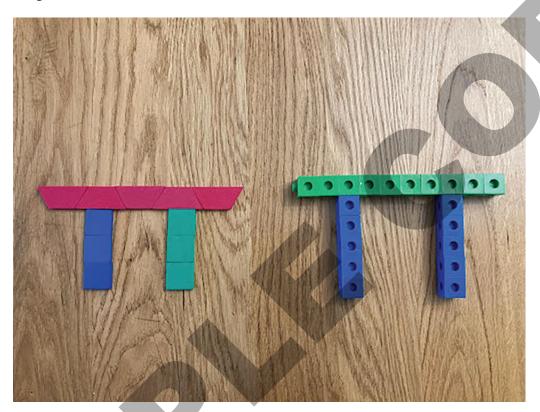
Notice and Wonder: Made of Bricks

What do you notice? What do you wonder?



Creating Classroom Objects

What object?



What tool?





Numbers 1-10

Content Connections

In this unit you will develop counting skills by comparing groups of objects and representing quantities with objects, pictures and numbers. You will make connections by:

- Reasoning with Data while sorting, counting, classifying, comparing, and describing objects using numbers.
- **Exploring Changing Quantities** while counting a group of objects arranged in a line, array, or circle.

• Taking Wholes Apart, Putting Parts Together while using fingers and 5-frames to represent numbers to see the structure of numbers 6-10.

Addressing the Standards

As you work your way through **Unit 2 Numbers 1-10,** you will use some mathematical practices that you may have started using in kindergarten and have continued strengthening over your school career. These practices describe types of thinking or behaviors that you might use to solve specific math problems.

	Where You Use
Mathematical Practices	these MPs
MP1 Make sense of problems and	Lesson 4, 19,
persevere in solving them.	20, and 21
MP2 Reason abstractly and quantitatively.	Lesson 1, 15,
	and 18
MP3 Construct viable arguments and	Lesson 3
critique the reasoning of others.	
MP4 Model with mathematics.	Lesson 24
MP5 Use appropriate tools strategically.	Lesson 5, 22,
	and 23
MP6 Attend to precision.	Lesson 4, 6, 7,
	8, 12, 14, 16, 17,
	and 22
MP7 Look for and make use of structure.	Lesson 10, 11,
	12, 20, and 21
MP8 Look for and express regularity in	Lesson 2, 9, 12,
repeated reasoning.	13, and 15

The California Common Core State Standards for Mathematics (CA CCSSM) describe the topics you will learn in this unit. Many of these topics build upon knowledge you already have and challenge you to expand upon that knowledge. The table below shows what standards are being addressed in this unit.

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
How Many?	K.CC.1 Count to 100 by ones and by tens.	Lesson 1, 2, 3
How Many?Place and Position of numbers	K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	Lesson 7. 8, 9, 14, 17, 18, 22, and 23

Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
 Sort and describe Data How Many? Bigger or Equal? 	K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. A. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. B. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	Lesson 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24
	C. Understand that each successive number name refers to a quantity that is one larger.	

 Describe Data questions about as many as questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. How Many? How Many? Equal? Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? K.CC.7 Beson 3 Jay 10, 12 15, 16, 11 18, and 2 18, and 2 19, 20, 2 22, and 2 22, and 2 23 24 25 26 27 28 29 20 21 22 22 23 24 25 26 27 28 29 29 20 21 22 22 23 24 25 26 27 28 29 29 20 21 22 22 23 24 25 26 27 28 29 29 20 21 22 22 23 24 25 26 27 28 29 29 20 21 22 22 23 24 25 26 27 28 29 29 20 21 22 23 24 25 26 27 27 28 29 29 20 21 22 23 24 25 26 27 28 29 29 29 20 21 21 22 22 23 24 25 26 27 28 29 29 29 20 20 20 21 21 22 23 24 24 24 25 	Big Ideas You Are Studying	California Content Standards	Lessons Where You Learn This
 Data How Many? Bigger or Equal? Place and Position of numbers Bigger or Equal? Place and Position of numbers Bigger or Equal? Being Flexible within 10 How Many? Being Greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? K.CC.7 Lesson 3 5, 6, 10, 12, 13, 14 15, 16, 17 18, and 2 19,			Lesson 1, 2,
 How Many? Bigger or Equal? Place and Position of numbers How Many? Bigger or Equal? Place and Position of numbers How Many? Bigger or Equal? Being Flexible within 10 How Many? Being Flexible within 10 How Many? Equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? Lesson 3 5, 6, 10, 12 12, 13, 13 19, 20, 2 22, and 2 Lesson 3 Lesson 3 Equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? Lesson 2 		,	3, 9, 10, 11,
 Bigger or Equal? Place and Position of numbers Bigger or Equal? Place and Position of numbers Bigger or Equal? Bigger or Equal? Being Flexible Within 10 How Many? Bigger or Equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? Equal? Being Flexible Equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? K.CC.7 Lesson 2 Lesson 2 			12, 13, 14,
 Equal? Place and Position of numbers How Many? Being Flexible within 10 How Many? K.CC.7 How Many? Lesson 3 5, 6, 10, 10 12, 13, 1 19, 20, 2 22, and 2 22, and 2 Lesson 2 		9	15, 16, 17,
 Place and Position of numbers How Many? Being Flexible within 10 How Many? How Many? Being Flexible within 10 How Many? K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? K.CC.7 Lesson 3 5, 6, 10, 12 12, 13, 13 19, 20, 2 22, and 2 Lesson 2 			18, and 22
Position of numbers • How Many? • Bigger or Equal? • Being greater than, less than, or Flexible within 10 in another group, e.g., by using matching and counting strategies. • How Many? • Lesson 3 5, 6, 10, 12, 13, 14 19, 20, 2 22, and 2 22, and 2 Lesson 3 5, 6, 10, 12 12, 13, 14 19, 20, 2 22, and 2 22, and 2 23 • Lesson 2			
 numbers How Many? Bigger or Equal? Being Flexible within 10 How Many? How Many? Many? Many? Many? Many? Many objects Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. Lesson 3 5, 6, 10, 10, 12 12, 13, 10 19, 20, 2 22, and 2 22, and 2 Lesson 2 			
 How Many? Bigger or Equal? Being Flexible within 10 How Many? K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? K.CC.7 Lesson 3 5, 6, 10, 12 12, 13, 13 19, 20, 2 22, and 2 Lesson 2 			
 Bigger or Equal? Being Flexible within 10 How Many? Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? Identify whether the number of, 12, 13, 1, 19, 20, 2 19, 20, 2 22, and 2 Lesson 2 			
 Equal? Being greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by Lesson 2 	- J -		Lesson 3, 4,
 Being greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. How Many? greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. Lesson 2 	-		5, 6, 10, 11,
Flexible equal to the number of objects in another group, e.g., by using matching and counting strategies. • How Many? K.CC.7	•		12, 13, 16,
within 10 in another group, e.g., by using matching and counting strategies. • How Many? K.CC.7 Lesson 2	•		19, 20, 21,
using matching and counting strategies. • How Many? K.CC.7 Lesson 2			22, and 23
strategies. • How Many? K.CC.7 Lesson 2			
How Many? K.CC.7 Lesson 2			
		O	
Compare two numbers 22 and 3			Lesson 21,
		Compare two numbers	22, and 23
between 1 and 10 presented		•	
as written numerals.		as written numerals.	





Addressing CA CCSSM K.CC.1 and K.CC.4-5; practicing MP2

Fingers as a Math Tool

Let's show numbers with our fingers.



Show Some Fingers

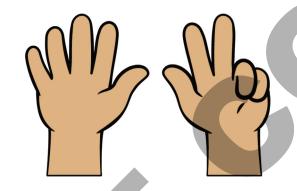




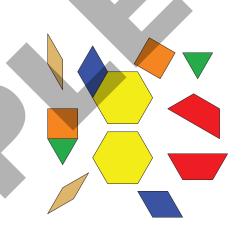
Introduce Math Fingers—Show and Say

Choose a center.

Math Fingers



Pattern Blocks



Picture Books





Addressing CA CCSSM K.CC.1 and K.CC.4-5; practicing MP8

Count and Arrange

Let's figure out how many objects we have.



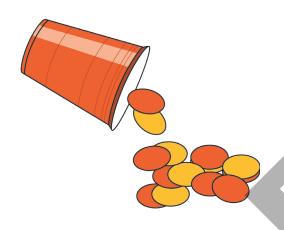


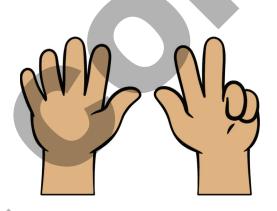


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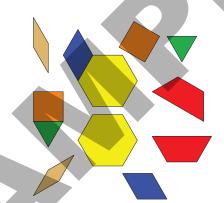
Shake and Spill

Math Fingers





Pattern Blocks



Picture Books







Addressing CA CCSSM K.CC.1, K.CC.4-5, K.CC.6; Practicing MP3

Groups That Look Very Different

Let's figure out if there are more green triangles or more orange squares.



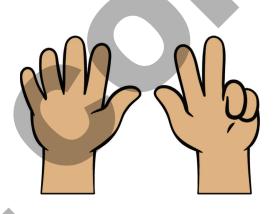


Choose a center.

Shake and Spill

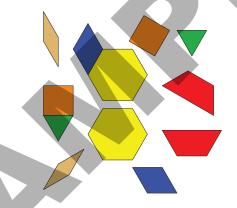






Pattern Blocks

Picture Books









Addressing CA CCSSM K.CC.4, K.CC.6; practicing MP1 and MP6

Groups That Look Alike

Let's find groups that have more, fewer, or the same number.

Activity 1

Which Has More?

There are 4 people.

There are 6 spoons.

Can each person get 1 spoon?





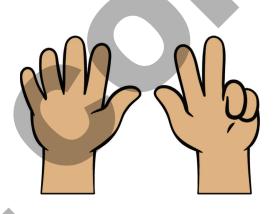


Choose a center.

Shake and Spill

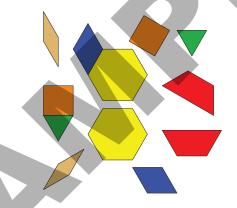






Pattern Blocks











Addressing CA CCSSM K.CC.4 and K.CC.6; practicing MP5

Make Groups of More, Fewer, or the Same

Let's make groups of objects that have more, fewer, or the same number of objects as another group.

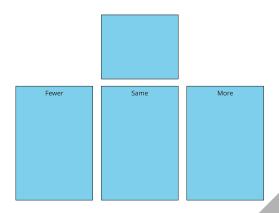






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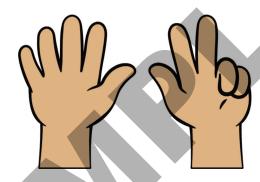
Fewer, Same, More



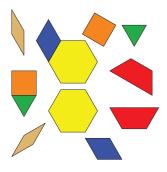
Shake and Spill



Math Fingers



Pattern Blocks



Picture Books





Sec A

Addressing CA CCSSM K.CC.4 and K.CC.6; practicing MP6

Use "More," "Fewer," or "the Same Number" to Describe Groups

Let's use "more," "fewer," and "the same number" to describe groups.

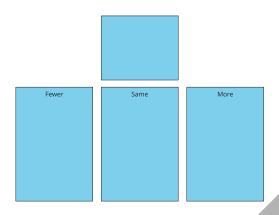






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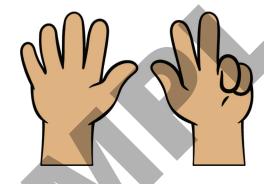
Fewer, Same, More



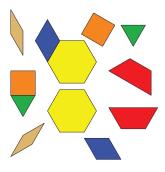
Shake and Spill



Math Fingers



Pattern Blocks



Picture Books

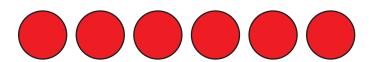




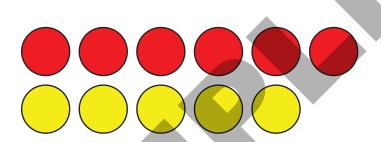
Section A Summary

We can count groups of objects.

We can tell which group has more or fewer.



more red counters



fewer yellow counters









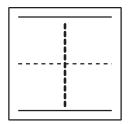
Addressing CA CCSSM K.CC.3 and K.CC.4; practicing MP6

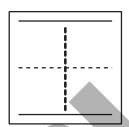
Write Numbers 1-5

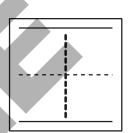
Let's write numbers.

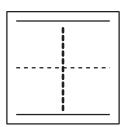


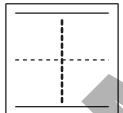
Write Numbers 1, 2, and 3

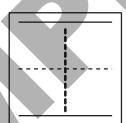


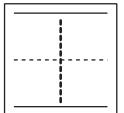


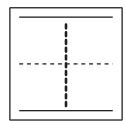




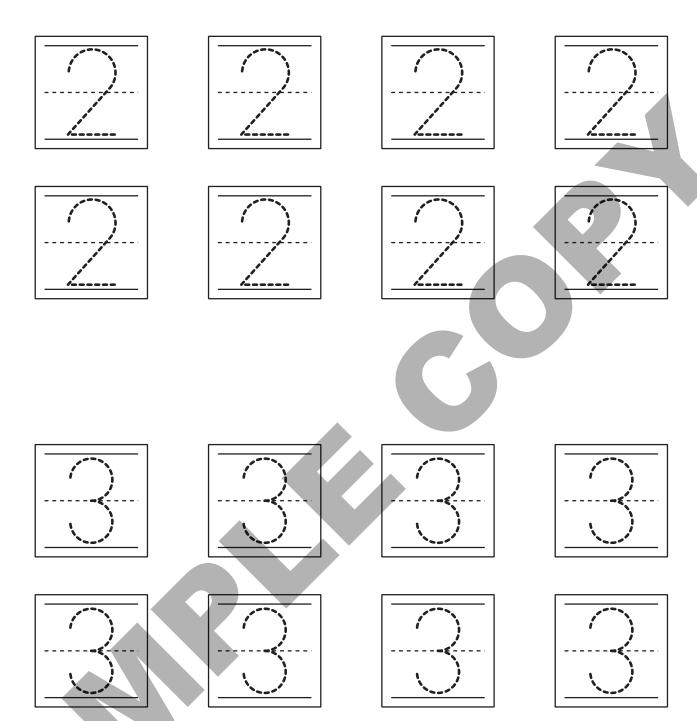






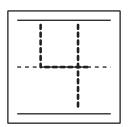


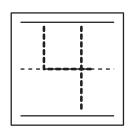


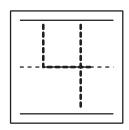


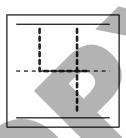


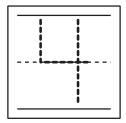
Write Numbers 4 and 5

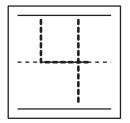


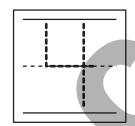


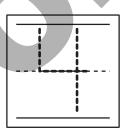


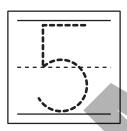




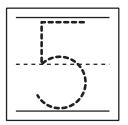


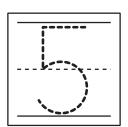




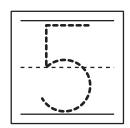


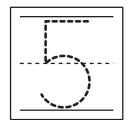


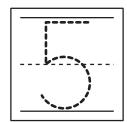










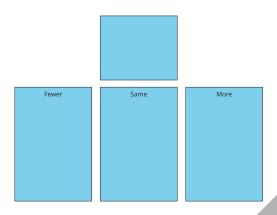






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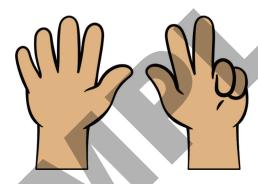
Fewer, Same, More



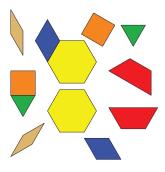
Shake and Spill



Math Fingers



Pattern Blocks



Picture Books





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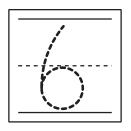
Addressing CA CCSSM K.CC.3 and K.CC.4; practicing MP6

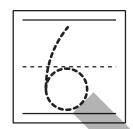
Write Numbers 6-10

Let's write more numbers.

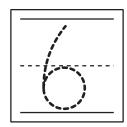


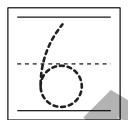
Write Numbers 6, 7, and 8



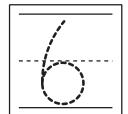


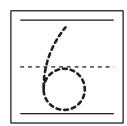






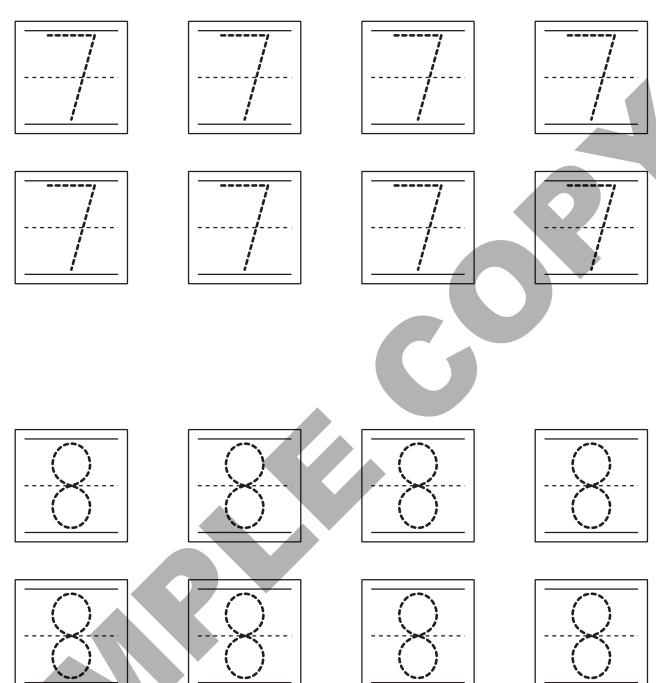






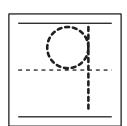


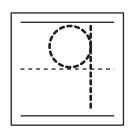


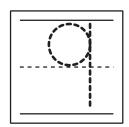




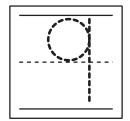
Write Numbers 9 and 10

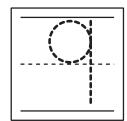


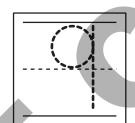




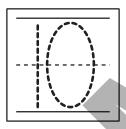




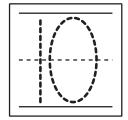


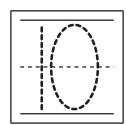




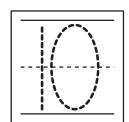


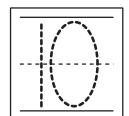


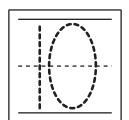










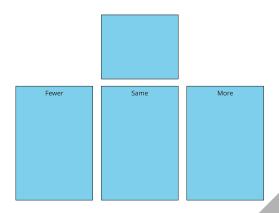






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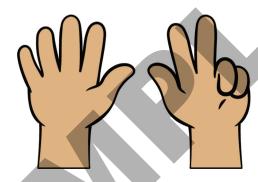
Fewer, Same, More



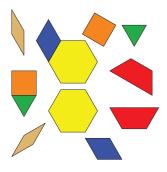
Shake and Spill



Math Fingers



Pattern Blocks



Picture Books







Addressing CA CCSSM K.CC.3 and K.CC.4-5; practicing MP8

Count Images in Different Arrangements

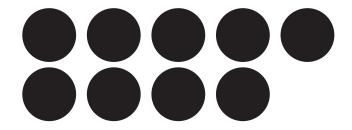
Let's figure out how many images there are.

Activity 2

Count Images

1.









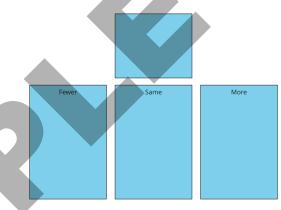
Introduce Number Race—Numbers 1-10

Choose a center.

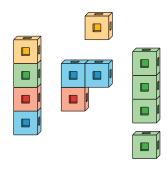
Number Race



Fewer, Same, More



Connecting Cubes





Addressing CA CCSSM K.CC.5 and K.CC.6; practicing MP7

Compare Matching Images

Let's figure out what we have more and fewer of.





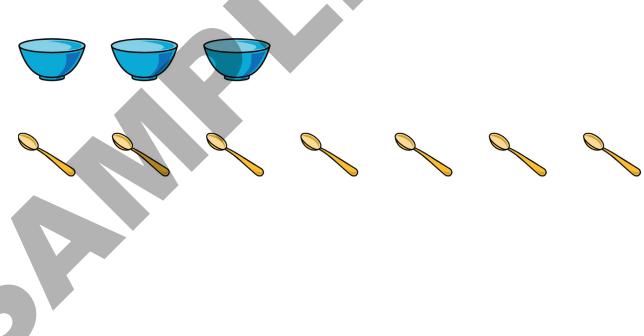


Are There Enough?



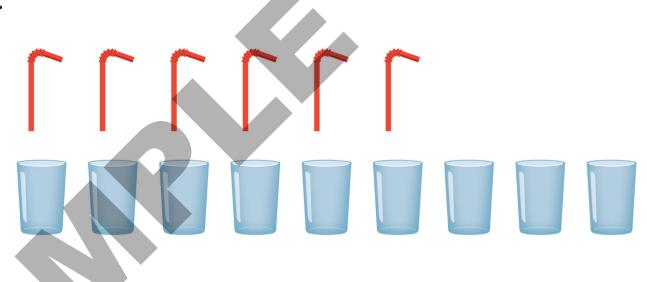








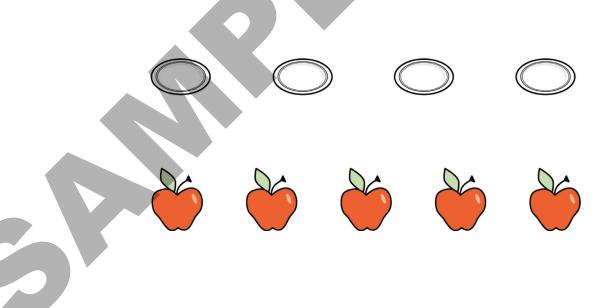




Compare Images That Aren't Matched

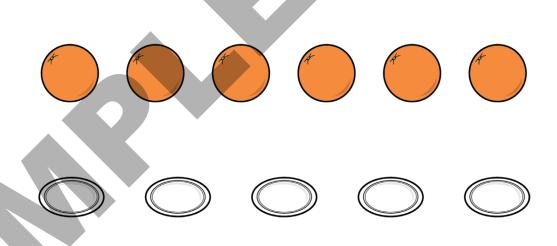
1.













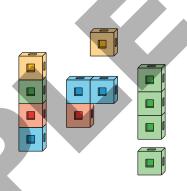
Introduce Math Stories—How Many?

Choose a center.

Math Stories



Connecting Cubes



Number Race







Addressing CA CCSSM K.CC.4-5 and K.CC.6; practicing MP7

More, Fewer, or the Same

Let's figure out if there are more, fewer, or the same number of images.

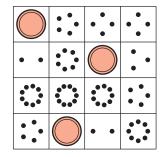


Activity 3

Introduce Bingo—Images

Choose a center.

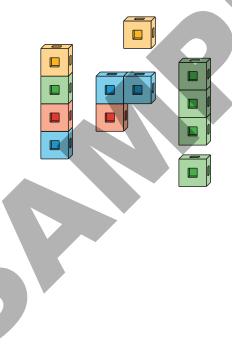
Bingo





Math Stories

Connecting Cubes



Number Race







Addressing CA CCSSM K.CC.4-5 and K.CC.6; practicing MP6, MP7, and M8

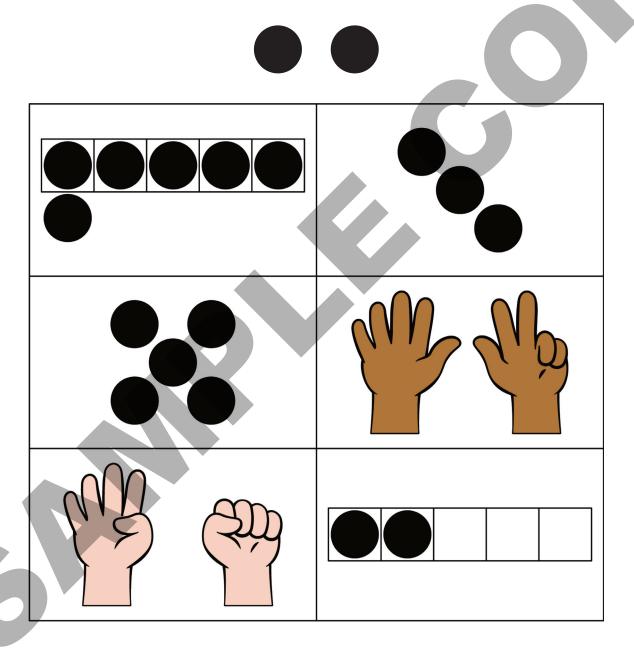
Find More, Fewer, or the Same

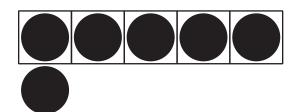
Let's compare groups of images using the words "more," "fewer," or "the same number."

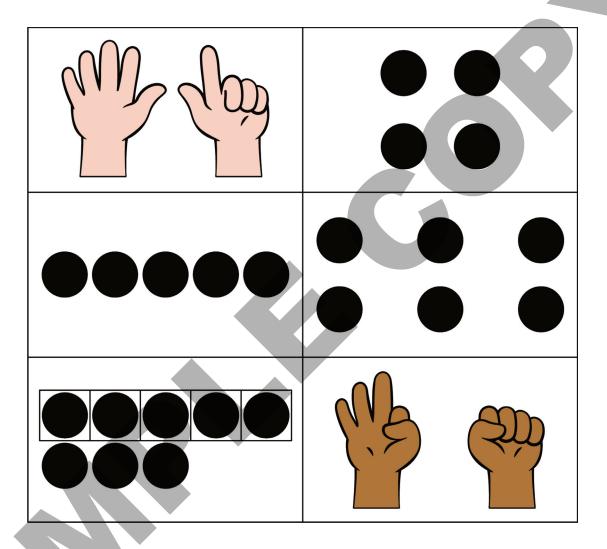


Circle More, Circle Fewer

1.





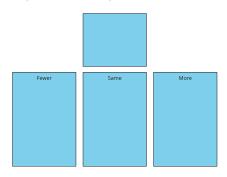




Centers: Choice Time

Choose a center.

Fewer, Same, More



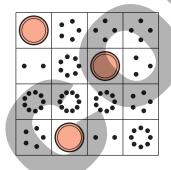
Math Stories



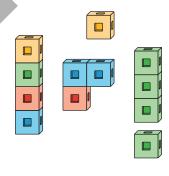
Number Race







Connecting Cubes









Addressing CA CCSSM K.CC.4-5 and K.CC.6; building towards K.CC.6; practicing MP8

Create Groups of Images

Let's make groups that have more, fewer, or the same number of images.





Draw Groups of Images

my group	my partner's group



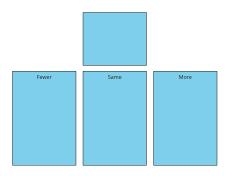




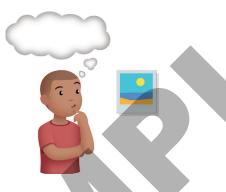
Centers: Choice Time

Choose a center.

Fewer, Same, More



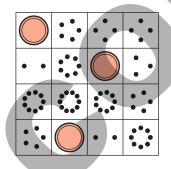
Math Stories



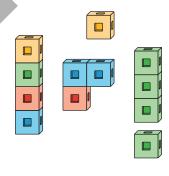
Number Race



Bingo



Connecting Cubes





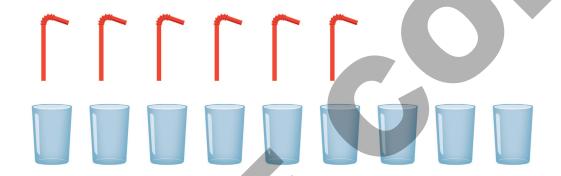


Section B Summary

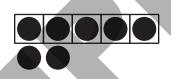
We can match to compare groups.

We can count to compare groups.

We say "more," "fewer," or "the same number" to compare.



There are more cups than straws.





6 dots is fewer than 7 dots.

How many dots are there?

a.

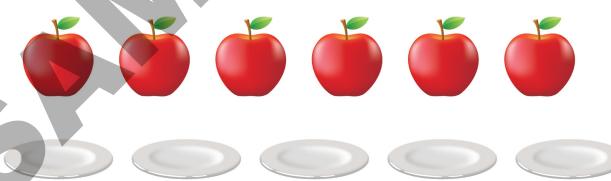


b.



2 from Unit 2, Lesson 10

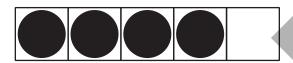
Are there more apples or plates?



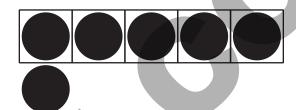


Circle the group that has more dots.

A.

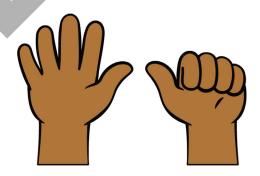


B.

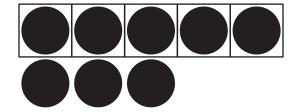


Circle the group that shows fewer things.

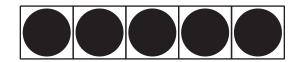
A.

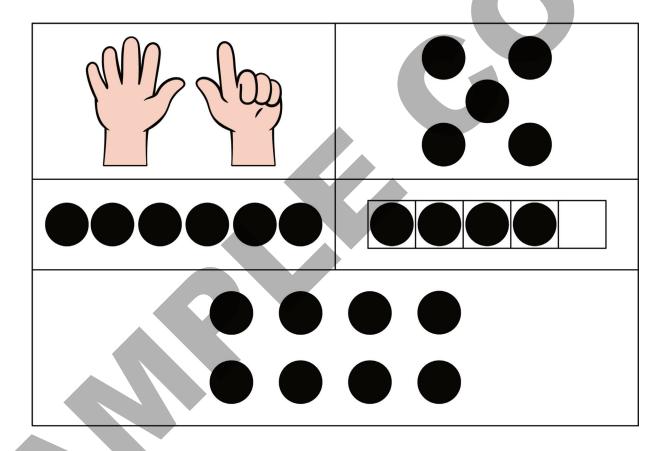


B.



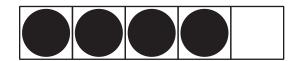
Circle the **3** groups that show more than this 5-frame.

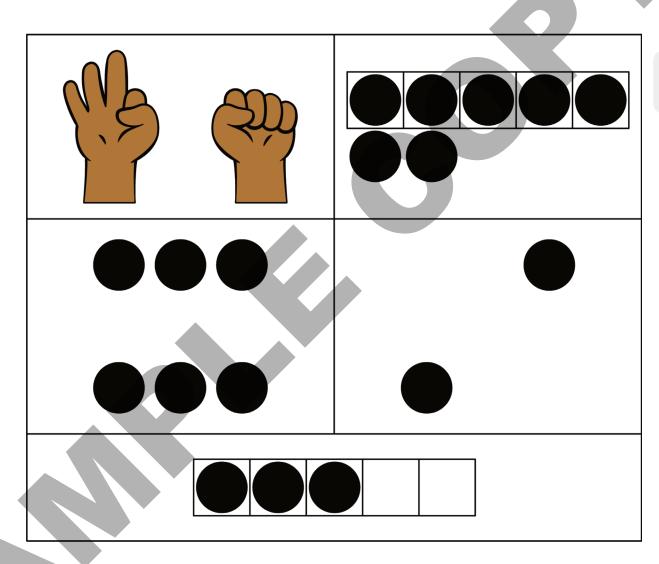






Circle the **3** groups that show fewer than this 5-frame.





Here is a drawing.













a. Draw a picture with more things.

b. Draw a picture with fewer things.



6 Exploration

Make a set of cards with images.

Some cards can have the same number of images and some cards can have different numbers of images.

Trade your cards with a partner.

Turn over 2 cards.

Tell if they show the same number or if one shows fewer or more.



7 Exploration

Are there fewer students than chairs? Explain how you know.







Addressing CA CCSSM K.CC.3 and K.CC.4-5; building towards K.CC.3; practicing MP6

Connect Quantities and Numbers

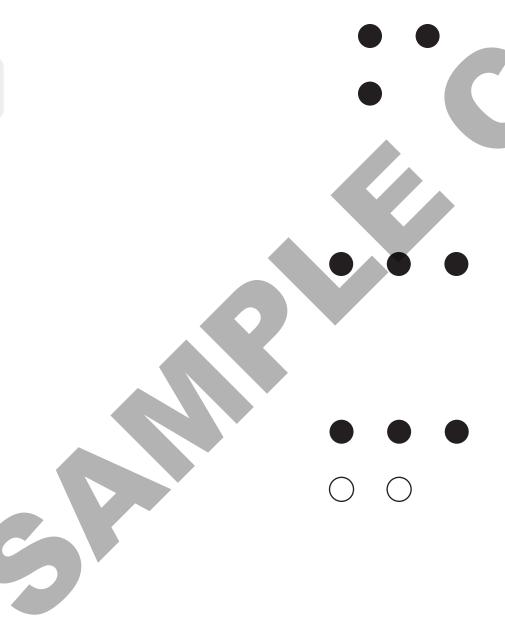
Let's figure out which groups of objects go with which numbers.





How Many Do You See: Image Flash

How many do you see? How do you see them?







Counting Stations

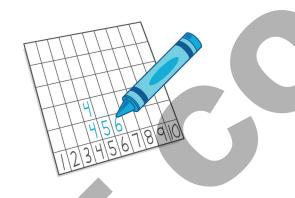
number of objects	bag name
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Activity 3

Revisit Number Race—Numbers 1-10

Choose a center.

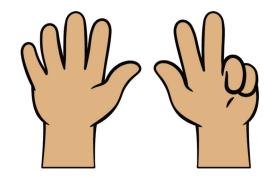
Number Race



Geoblocks



Math Fingers







Addressing CA CCSSM K.CC.4-5; building towards K.CC.3; practicing MP2 and MP8

Numbers in Many Ways

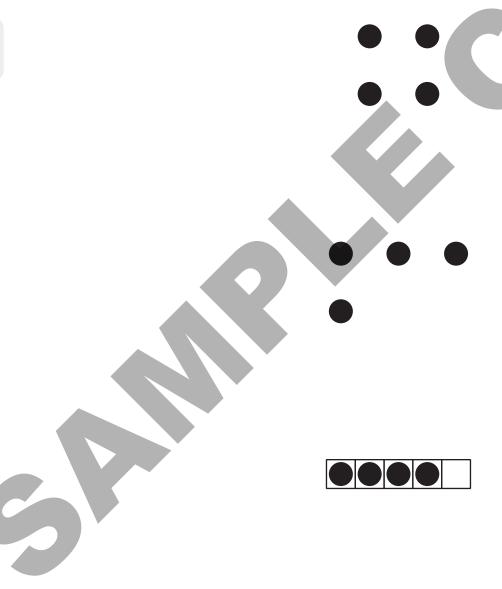
Let's figure out how many things there are.





How Many Do You See: Different Arrangements of 4

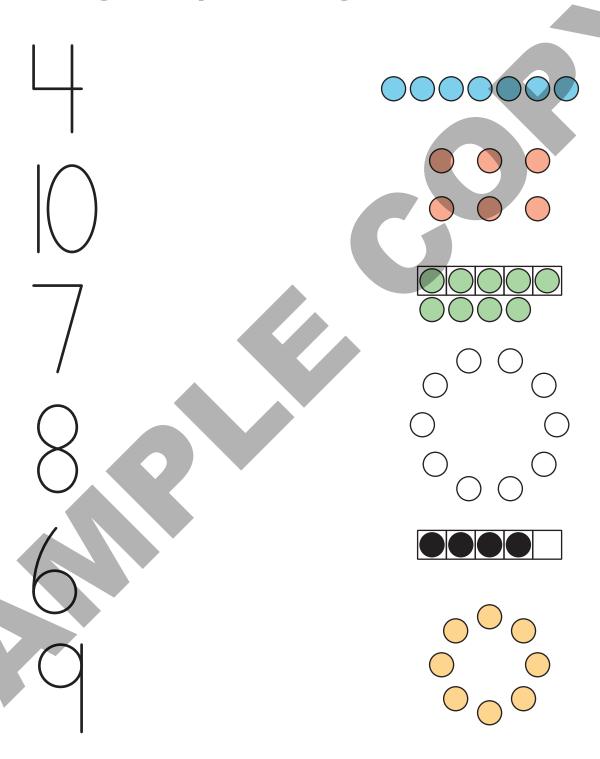
How many do you see? How do you see them?







Matching Groups of Images and Numbers

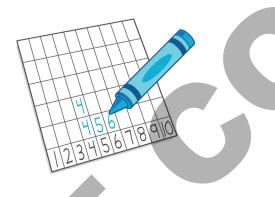


Activity 3

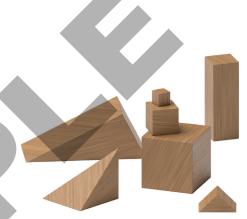
Centers: Choice Time

Choose a center.

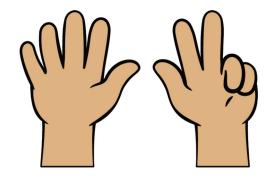
Number Race



Geoblocks



Math Fingers







Addressing CA CCSSM K.CC.4-5 and K.CC.6; practicing MP6

Count Out Objects

Let's figure out how many plants we need to put in our garden.

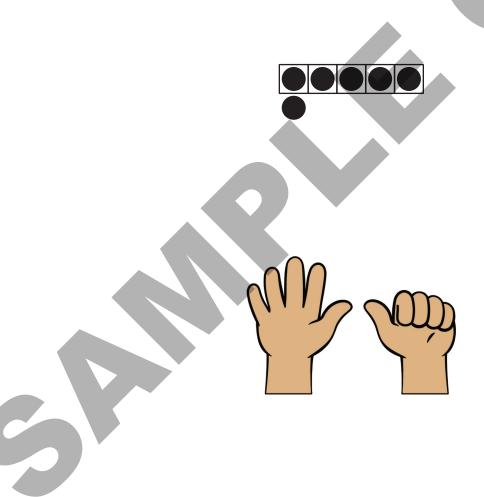




How Many Do You See: 5-Frames and Fingers

How many do you see? How do you see them?

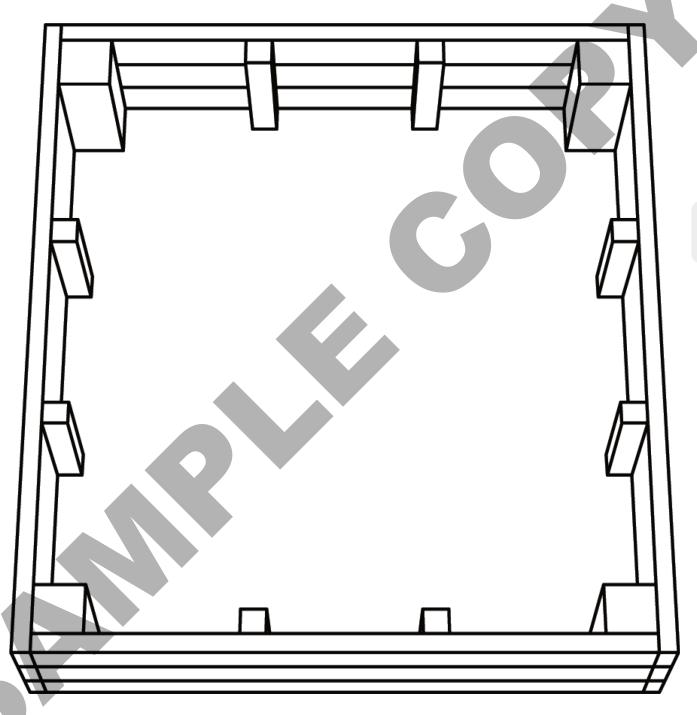








Plants in a Garden

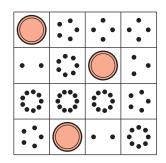


Activity 3

Introduce Bingo—Images and Numbers

Choose a center.

Bingo



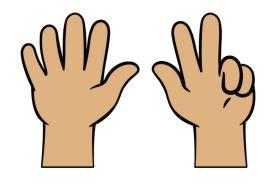
Number Race







Math Fingers







Addressing CA CCSSM K.CC.3 and K.CC.4-5; building towards K.CC.3; practicing MP6

Draw Groups of Things

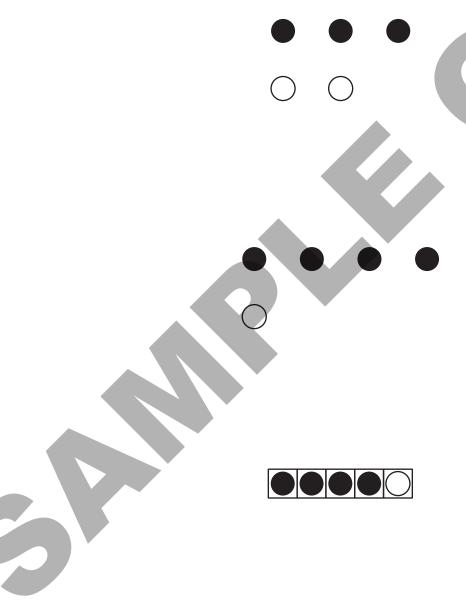
Let's figure out how many things to draw.





How Many Do You See: Many Ways to Show 5

How many do you see? How do you see them?



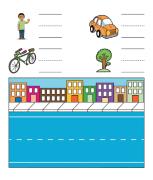




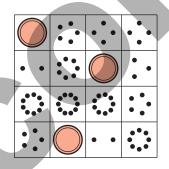
Centers: Choice Time

Choose a center.

Math Libs



Bingo



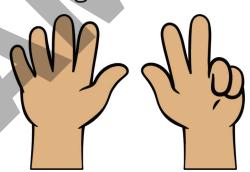
Number Race



Geoblocks



Math Fingers





Addressing CA CCSSM K.CC.3 and K.CC.4-5; building towards K.CC.4c; practicing MP2

Write Numbers to Represent Quantities

Let's write numbers to show how many there are.

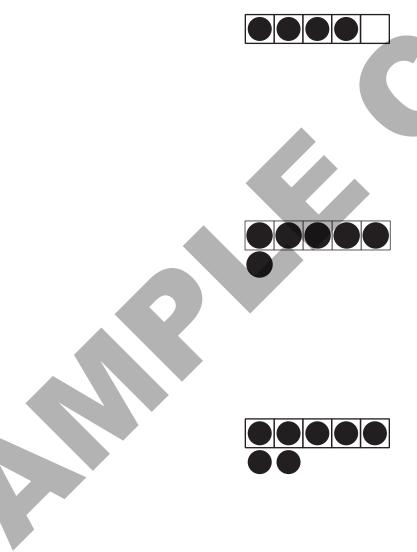






How Many Do You See: 1 More with a 5-Frame

How many do you see? How do you see them?



Activity 3

Centers: Choice Time

Choose a center.

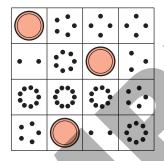
Math Stories



Math Libs



Bingo



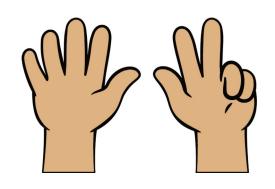
Number Race



Geoblocks



Math Fingers



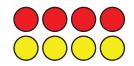


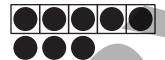
SecC

Section C Summary

We can match numbers with groups of objects.

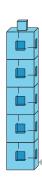






We can show numbers with objects and pictures.

5











We can write numbers to show how many.



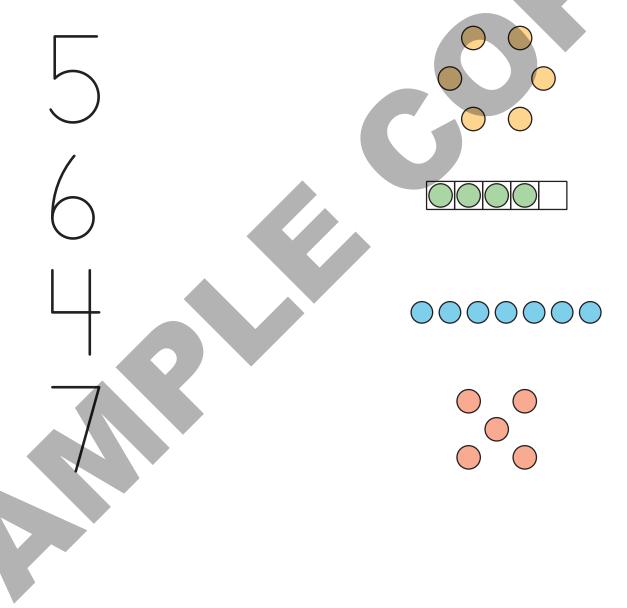






1 from Unit 2, Lesson 15

Draw a line from each number to the group of dots it matches.



Sec

2 from Unit 2, Lesson 16

Here is the number 6.



Count out 6 counters.

You can use a 5-frame if you want.



3 from Unit 2, Lesson 17

Here is the number 8.



Draw a picture to show 8 things.





Sec C

4 from Unit 2, Lesson 18

Ask a "how many?" question about this picture.

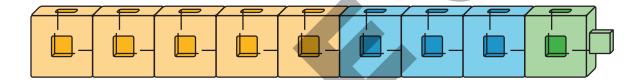


Write a number to answer your question.

5 Exploration

Make a shape with 2 handfuls of pattern blocks. Ask a "how many?" question about your shape. Trade questions with a partner. Answer your partner's question.

6 Exploration



Han sees 5.

Lin sees 4.

Tyler sees 3.

Explain or show how Han, Lin, and Tyler can all be correct.



Unit 2, Lesson 19



Addressing CA CCSSM K.CC.4 and K.CC.6; building towards K.CC.7; practicing MP1

Order Towers and Numbers

Let's put numbers and cube towers in order from 1 to 10.



Act It Out: Family Dinner

Han puts 8 plates on a table.



Activity 3

Centers: Choice Time

Choose a center.

Math Stories



Number Race







Unit 2, Lesson 20



Addressing CA CCSSM K.CC.4 and K.CC.6; building towards K.CC.7; practicing MP1 and MP7

1 More or 1 Less with Towers and Numbers

Let's find 1 more or 1 less than a number.

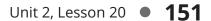


Act It Out: Forks for Dinner

There are 9 people.

Mai has 7 forks.

Can each person get 1 fork?



Activity 2

1 Less, 1 More

my tower	my partner's tower
my tower	my partner's tower
my tower	my partner's tower
my tower	my partner's tower
Tity tovici	





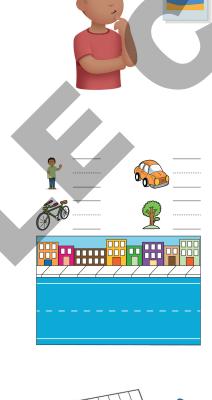
Centers: Choice Time

Choose a center.

Math Stories



Number Race





Unit 2, Lesson 21

Addressing CA CCSSM K.CC.4 and K.CC.6-7; practicing MP1 and MP7

Compare Numbers and Images

Let's figure out which number is more.

(Warm-up)

Act It Out: Pass Out Books

Diego has 5 books. He gets 1 more book. How many books does Diego have now?



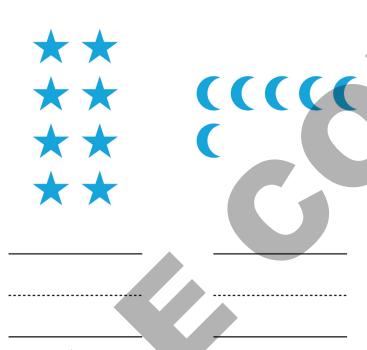




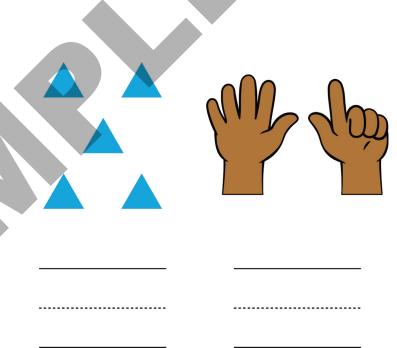
Activity 1

Which Has More?

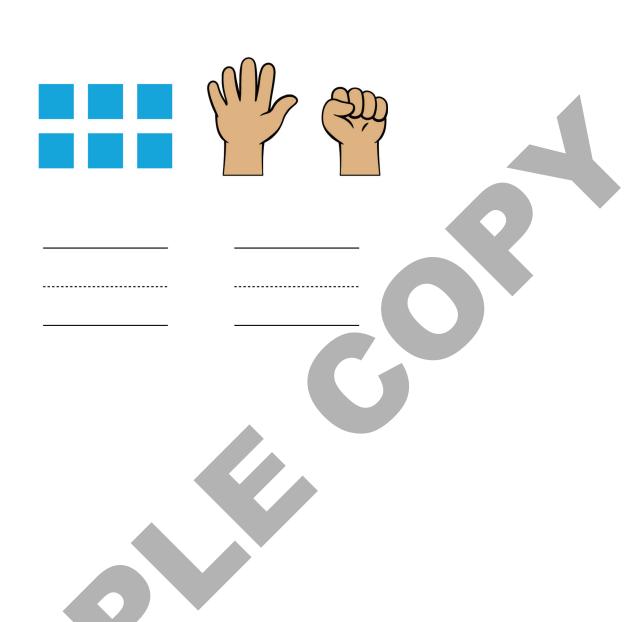
1.



2.







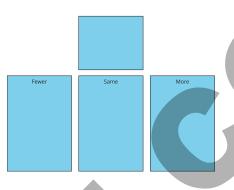




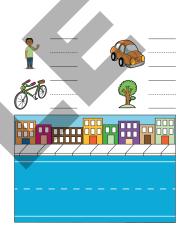
Centers: Choice Time

Choose a center.

Fewer, Same, More



Math Libs



Number Race



Unit 2, Lesson 22



Addressing CA CCSSM K.CC.3, K.CC.5, and K.CC.6-7; practicing MP5 and MP6

Represent and Compare Numbers

Let's show numbers in different ways and make comparison statements.



Act It Out: Plates and Cups for Dinner

Lin put 8 cups on the table.

Tyler put 6 plates on the table.

Who put more things on the table?





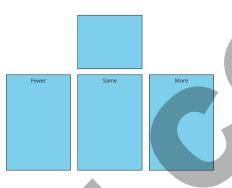




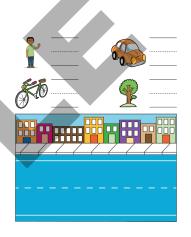
Centers: Choice Time

Choose a center.

Fewer, Same, More



Math Libs



Number Race







Unit 2, Lesson 23



Addressing CA CCSSM K.CC.3 and K.CC.6-7; practicing MP5

Compare Numbers

Let's figure out which number is more and which is less.



Act It Out: Hand Out Paper

9 kids get blue paper.

5 kids get red paper.

Did fewer kids get blue or red?





Which Number Is More?

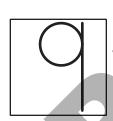
Circle the number that is more.

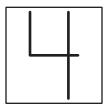
1.





2.









Which Number Is Less?

Write the numbers you roll. Circle the number that is less. Write the numbers you roll. Circle the number that is less.

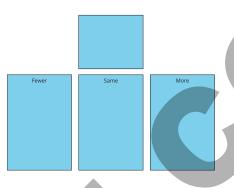




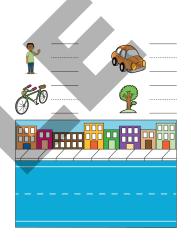
Centers: Choice Time

Choose a center.

Fewer, Same, More



Math Libs

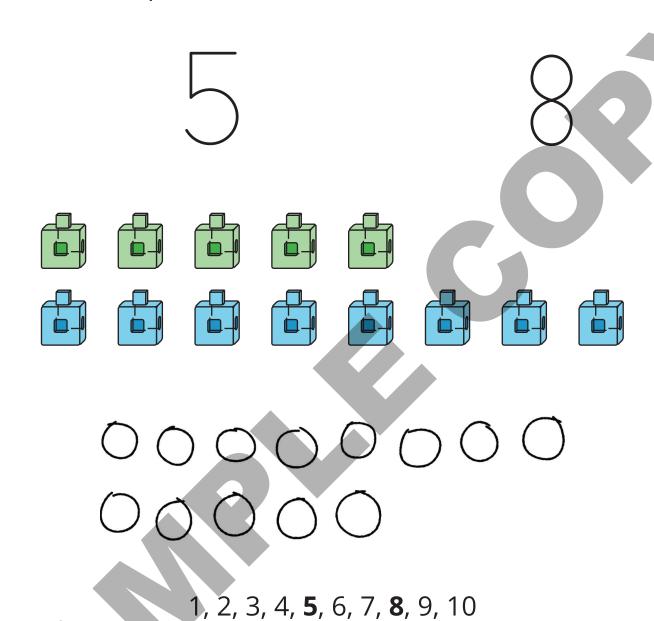


Number Race



Section D Summary

We can compare numbers.



5 is less than 8.

8 is more than 5.



Unit 2, Lesson 24



Addressing CA CCSSM K.CC.4; building towards K.CC.4 and K.CC.6; practicing MP4

Set the Table

Let's figure out and show what we need to set the table.



Act It Out: Spoons and Bowls

Jada put 6 bowls on the table.
How many spoons should Jada put on the table?





Your Family's Table

Show items you use to set the table.

Write numbers to tell how many of each item.



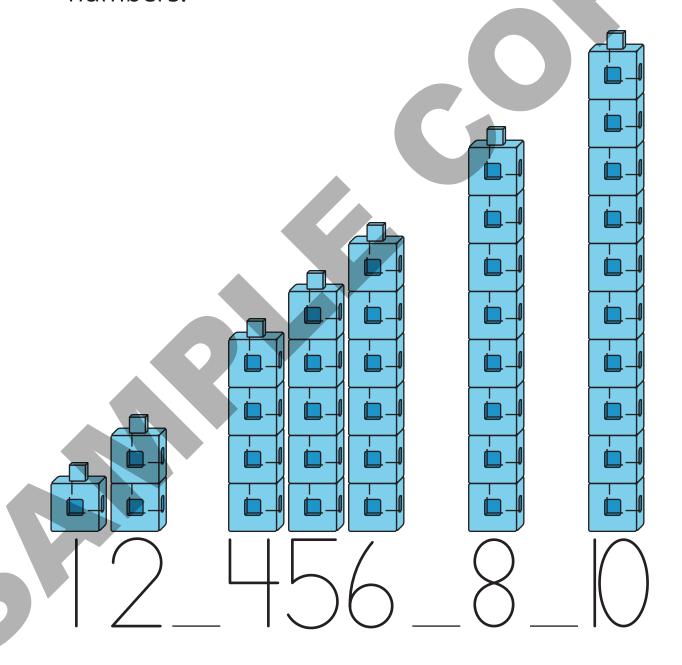


7 Problems

Practice Problems

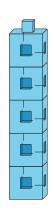
1 from Unit 2, Lesson 19

The towers and numbers are in order from 1 to 10. Draw the 3 missing towers. Write the 3 missing numbers.



2 from Unit 2, Lesson 20

Mai made this tower.



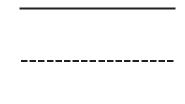
a. Write a number to show how many cubes are in Mai's tower.



b. Build a tower that shows 1 more cube than Mai's tower.Write a number to show how many cubes.



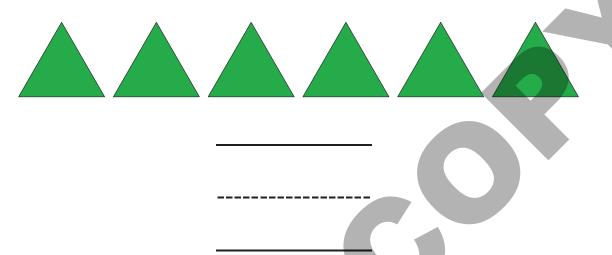
c. Build a tower that shows 1 less cube than Mai's tower. Write a number to show how many cubes.



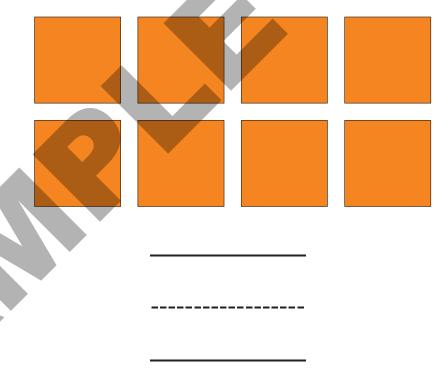


3 from Unit 2, Lesson 21

a. How many triangles?



b. How many squares?



c. Circle the group that has fewer shapes.

a. Show 5 in as many ways as you can.

 $\begin{bmatrix}
\end{bmatrix}$

b. Show 7 in as many ways as you can.

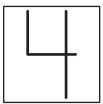
7

c. Circle the number that is more.



5 from Unit 2, Lesson 23

a. Circle the number that is more.





b. Circle the number that is less.





6 Exploration

How can you show 6 on your fingers? Find as many ways as you can.

How are they the same? How are they different?

Grab a handful of small objects.

Count the objects.

Write how many.

Are there more than 7 or less than 7?





Glossary

• fewer

fewer



less3 is less than 9

3



more9 is more than 3.

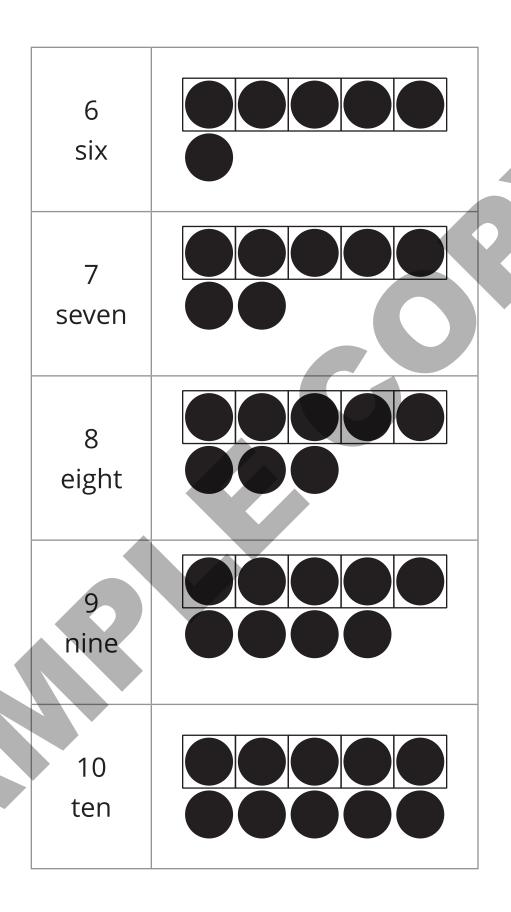




number writing reference

1 one	
2 two	
3 three	
4 four	
5 five	





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Notes



Notes



Notes



California Common Core State Standards for Mathematics (CA CCSSM) Reference

K.CC: Kindergarten-Counting and Cardinality Know number names and the count sequence.

K.CC.1

Count to 100 by ones and by tens.

K.CC.2

Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC.3

Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4

Understand the relationship between numbers and quantities; connect counting to cardinality.

K.CC.4a

When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

K.CC.4b

Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

K.CC.4c

Understand that each successive number name refers to a quantity that is one larger.

K.CC.5

Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

K.CC.6

Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects



in another group, e.g., by using matching and counting strategies. Include groups with up to ten objects.

K.CC.7

Compare two numbers between 1 and 10 presented as written numerals.

K.G Kindergarten-Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1

Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

K.G.2

Correctly name shapes regardless of their orientations or overall size.

K.G.3

Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

K.G.4

Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

K.G.5

Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

K.G.6

Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

K.MD Kindergarten—Measurement and Data Describe and compare measurable attributes.

K.MD.1

Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.



K.MD.2

Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Classify objects and count the number of objects in each category.

K.MD.3

Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. Limit category counts to be less than or equal to 10.

K.NBT Kindergarten—Number and Operations in Base Ten

Work with numbers 11-19 to gain foundations for place value.

K.NBT.1

Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 1810+8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

K.OA Kindergarten—Operations and Alegbraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1

Represent addition and subtraction with objects, fingers, mental images, drawings. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

K.OA.2

Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).



K.OA.4

For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects of drawings, and record the answer with a drawing or equation.

K.OA.5

Fluently add and subtract within 5.



California Common Core State Standards for Mathematics Standards for Mathematical Practice

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous



problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own,

without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments



using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

 Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify

important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and



compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same

amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2×7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle



school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Practices to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

