Number Tricks

Big Mathematical Ideas

In this lesson the concepts of variable and equation introduced in Lesson 1 are revisited. Students practice using variables and writing equations. First, they analyze a mathematical trick, and then they represent the solution to the trick algebraically.

Objectives

- Students will write equations using variables.
- Students will learn about the sums of even and odd numbers.
- Students will solve equations.

Materials

Students

- "Mathematical Trickery!" (Student Mathematician's Journal p. 9)
- "Mathematician's Journal Think Deeply About…" (Student Mathematician's Journal pp. 11–13)

Teacher

- Magic Number Trick Cards (five cards prepared in advance; see directions in "Initiate" section)
- Blackline master "Hint Cards" (p. 89)
- Blackline master "Think Beyond Cards" (p. 90)

NOTES

Supplies

- Calculator
- Blank 3-by-5 index cards (at least 10 per student)
- Colored markers

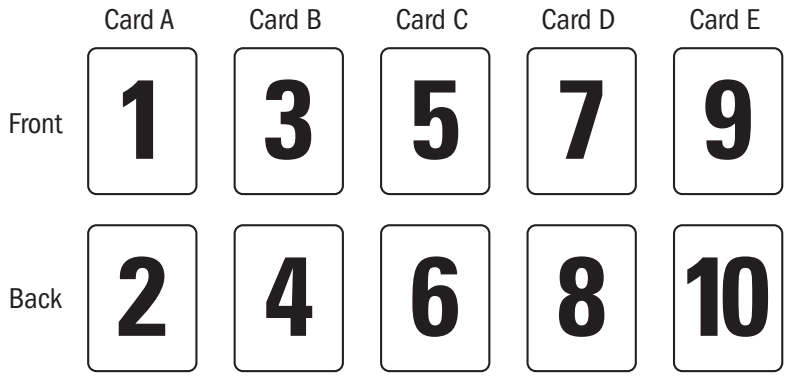
Mathematical Language

- **Variable** – a quantity whose value changes or varies. A variable could also be defined as a specific unknown in an equation. In algebra, letters often represent variables.

Initiate ($\frac{1}{2}$ day)

An Amazing Math Trick

Ask students if they have ever seen a magic number trick. Encourage anyone who knows a trick to share it with the class. Tell them that today they are going to learn a trick that they can use to impress their friends and family! Not only will they learn how to do the trick, but they also will learn why the trick works. Prior to having students analyze the trick, you'll show it to them. In advance, prepare five cards as shown below. (Blank index cards can be used as cards.) Note that the front and back of each card is different from its back.



Ask a student to select 5 of the 10 numbers on the cards and place the cards so the chosen numbers are face up. The cards should be in a location so that other students can see them but you cannot, perhaps at the back of the room. Tell the class that without having seen the cards, you will be able to predict the sum of the numbers after you ask just one question! Have students determine the sum but not tell you what it is. Then ask the question, "How many of the numbers showing are even?" When you are told how

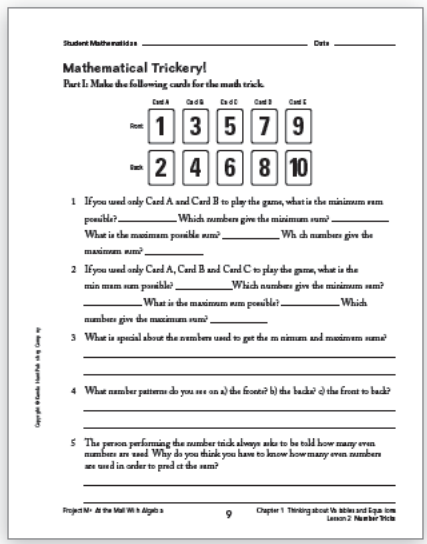
many of the numbers are even, add this amount to 25, and that will be the sum. Your students will be amazed and will want to know how you did it! Perform the trick two more times.

You might be interested in knowing why this trick works. *(Do not reveal this information to students yet!)* Notice that each card consists of a consecutive odd and even pair of numbers, one on each side. The smallest possible sum occurs when all the numbers are odd: $1 + 3 + 5 + 7 + 9 = 25$. If students say there are no even numbers showing on the cards, then you know that all numbers are odd and that their sum is 25. Because the cards consist of consecutive pairs of odd/even numbers, the number of even numbers chosen increases the sum by that amount (e.g., two evens raise the sum by 2 since now there are two numbers that are each 1 more than their odd partner). This card trick also works if you ask the question, "How many of the numbers showing are odd?" and use this information to determine the number of cards that are even. Students are challenged to uncover the mathematics behind this trick in the first Think Beyond Card.

Investigate ($1\frac{1}{2}$ days)

Understanding the Math Trick!

Distribute five blank 3-by-5 cards to each of the students and tell them they are going to make a set of number trick cards with a partner. Refer students to "Mathematical Trickery!" in the Student Mathematician's Journal and ask them to work individually on the questions in Part I. Then split the class into pairs or groups of three and ask them to discuss their answers. The focus of the first part of this lesson is on figuring out why the number trick works. Encourage students to look for patterns that will help them analyze the trick. Resist the temptation to tell them how the trick works, but do reinforce the idea that it isn't really a trick, just a nice use of mathematics. Instead, ask leading questions, such as "How are odd and even numbers the same, and how are they different?" or "Why do you think it is important that consecutive odd/even pairs are on each card?" Hold a class discussion in order for students to share their ideas with each other and further make sense of the mathematics behind the trick (see the sample dialogue for the second Think Deeply question below).


Student Mathematician's Journal p. 9

After discussing the questions on Part I of "Mathematical Trickery!" see if students are able to generalize a rule for finding the sum in this trick (Sum = 25 + number of even numbers). Reintroduce the concept of variable to the class as a quantity that varies. Ask students what varies in the number trick each time (the

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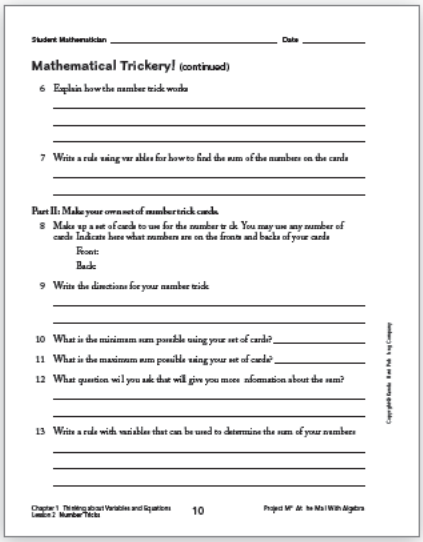

Student Mathematician _____ Date _____

Mathematical Trickery! (continued)

6 Explain how the number trick works

_____

_____

_____

7 Write a rule using variables for how to find the sum of the numbers on the cards

_____

_____

Part II: Make your own set of number trick cards.

8 Make up a set of cards to use for the number trick. You may use any number of cards. Indicate here what numbers are on the fronts and backs of your cards.

Front:

Back:

9 Write the directions for your number trick

_____

_____

10 What is the minimum sum possible using your set of cards? _____

11 What is the maximum sum possible using your set of cards? _____

12 What question will you ask that will give you more information about the sum?

_____

_____

13 Write a rule with variables that can be used to determine the sum of your numbers

_____

_____

_____

Chapter 1: Thinking about Variables and Equations
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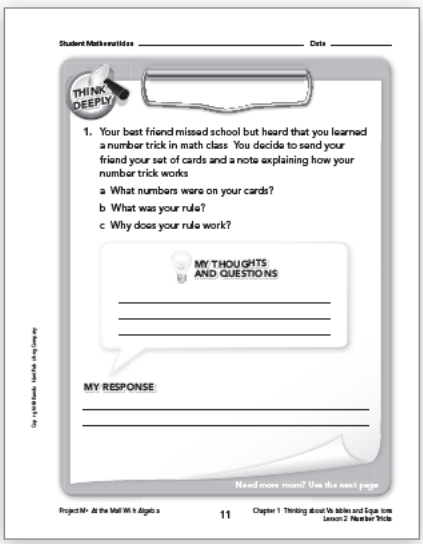
Student Mathematician's Journal p. 10

number of even numbers) and tell them we can represent this quantity using a letter (e.g., Sum = 25 + x) or another symbol (e.g., Sum = 25 + □). You might want to record the possibilities on the board so that students can see what parts of the equations are changing. Demonstrate how different values can be substituted for x or □, depending on the number of even cards. Some students might also mention that the sum varies each time and would like to use a letter to represent that quantity. They could use S to represent the sum, and thus the algebraic rule for the first equation would be $S = 25 + x$, where S represents the sum and x represents the number of even numbers.

Creating Your Own Math Trick

Part II of "Mathematical Trickery!" asks students to create a similar number trick using different cards. It is recommended that students use consecutive odd/even numbers on the fronts and backs of their cards. However, the number of cards used and the number pairs chosen for the cards can vary dramatically. Finally, ask students to write a rule for finding the sum for their trick. For example, a student might make up six different cards that have the following numbers on the front/back: 11/12, 13/14, 15/16, 17/18, 19/20 and 21/22. The minimum sum of these numbers is 96 (all odds). The rule would be Sum = 96 + n, where n is the number of even numbers. Afterwards, each student can practice his/her number trick with a classmate and then present it to the whole class.

Mathematical Communication (1 day)



Student Mathematician _____ Date _____

THINK DEEPLY

1. Your best friend missed school but heard that you learned a number trick in math class. You decide to send your friend your set of cards and a note explaining how your number trick works.
   a. What numbers were on your cards?
   b. What was your rule?
   c. Why does your rule work?

MY THOUGHTS AND QUESTIONS

MY RESPONSE

Need more room? Use the next page.

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Student Mathematician's Journal p. 11

After students have worked on understanding the number trick and have developed their own trick, you can discuss these Think Deeply questions as a whole class. Record students' ideas on the Math Messaging Board during the discussion; have students refer to the Board as they compose their responses to the Think Deeply questions.

1. Your best friend missed school but heard that you learned a number trick in math class. You decide to send your friend your set of cards and a note explaining how your number trick works.

   a. What numbers were on your cards?

   b. What was your rule?

   c. Why does your rule work?

NOTES

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- How do the numbers on the cards affect the minimum and maximum sums?
- What will my sum be if I just pick odd numbers?

What to Look for in Responses

- Students should clearly explain their number trick, including the numbers they used on the cards and the minimum and maximum sums. They might draw pictures of their cards or describe the number combinations in words.
- Students then need to explain what question they will ask and how they will interpret the answers. An advanced response includes an equation using a variable that represents the sum of the numbers on the card.

Possible Difficulties

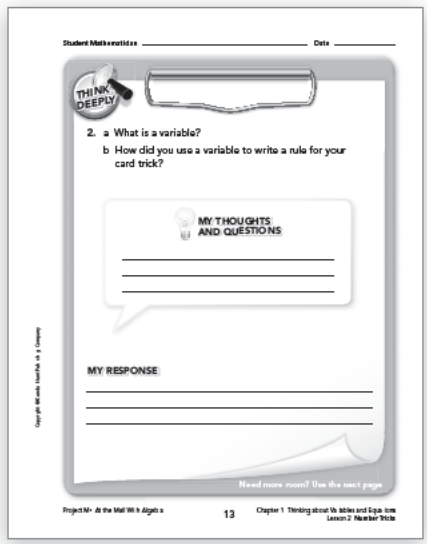
- Students might be confused by how to create their own number tricks and how to analyze them so as to use them convincingly. Suggest they make a game that is very similar to the one discussed in class. You may need to help them determine the minimum sum and the question they will ask their friends.

2. a. What is a variable?

b. How did you use a variable to write a rule for your card trick?

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- A variable has more than one meaning. It can be a value that varies or it can be a specific value.
- In the number trick, the value of the variable changes each time.


Student Mathematician's Journal p. 13

What to Look for in Responses

- Students should define a variable as a quantity that varies. They should give an example of an expression or equation that includes a variable.

NOTES

- Students should mention that a variable is represented in algebra using a variety of symbols, such as letters and shapes.
- Other characteristics of a variable that students might mention are that different letters can be used to represent the variable, and that they used the variable to represent the number of even numbers in the game.
- Students also need to give their number trick rule and explain how, in their case, the value of the variable changes each time the trick is performed. Be sure the number trick equation or expression accurately represents their trick.

Possible Difficulties

- Students may not understand the idea that any letter can be used to represent quantities. Using the problem in the dialogue below, the rules of $S = 25 + a$, $S = 25 + m$ and $S = 25 + x$ are the same, and any letter can be used to stand for the number of even numbers.
- Students often think that a variable is a label (e.g., b stands for "boys" rather than "the number of boys").
- In this situation, the variable represents a quantity that varies. (Students sometimes think that if during the first occurrence of the trick there are three even numbers, from then on, the variable will represent 3.)

A discussion centered on the second Think Deeply question about using a variable to make a rule might go something like this.

Teacher: Who can tell me what the term *variable* means? Scott?

Scott: It's the letter in math — the *x* or the *y*. That's the variable.

Teacher: Yes, a letter often stands for a variable, but what does it mean? Kinne, what can you add to Scott's response?

Kinne: Scott said that a variable is a letter, and I agree but I also think we can use a little square or something like that to be the variable, too.

Teacher: Ben?

Ben: I also agree with Kinne and Scott, but isn't a variable like what you don't know? Like in our trick, we don't know the number of even numbers at first.

Kinne: Yes, we don't know the number of even numbers, and the number of even numbers changes, and that's why it is a variable. A variable can change.

Teacher: Puja, would you please summarize the points that Scott, Kinne and Ben have made?

Puja: Well, a variable is what you don't know in a problem. You always use x to stand for the even numbers.

Teacher: Can we use a different letter, maybe a or m to stand for the number of even numbers in our number trick rule? Are Sum $= 25 + a$ and Sum $= 25 + m$ and Sum $= 25 + x$ [writes these number sentences on the board] the same rules?

Puja: I don't think so. I think x is what stands for the even numbers.

Teacher: What do other people think? Kyuri? Then Rudy.

Kyuri: I'm not sure, but I think you have to use the same letter all the time for the even numbers.

Teacher: Actually, both sentences are the same rule. We can use any letter we want to represent the even numbers as long as we indicate what the letter stands for. So, if we say that a represents the number of even numbers, we can use the rule Sum $= 25 + a$, and if we say that x represents the number of even numbers, we can use the rule Sum $= 25 + x$.

Rudy: Does that mean we can decide what letter to use?

Teacher: Yes, as long as you tell everyone else what quantity the letter is going to represent.

Teacher: So, a variable represents a quantity that changes, such as the number of even numbers in our trick. Sometimes there are zero even numbers, and other times there are five even numbers. The amount varies. We can use any letter or symbol we want to stand for the number of even numbers as long as we tell others, which is called "defining the variable." Let's write some of these key ideas on the board and then have everyone remember the ideas by writing in their Mathematician's Journal.

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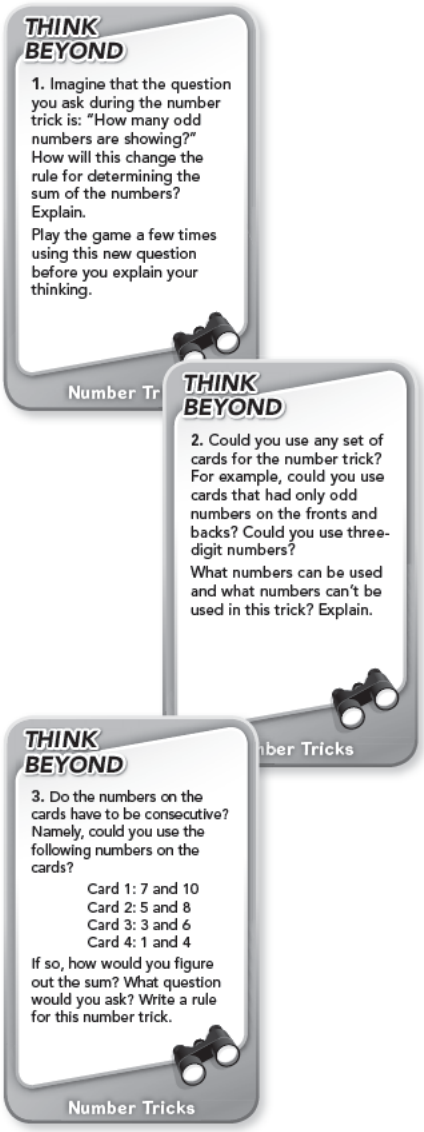

Developing number tricks and writing rules for the tricks is an enjoyable and challenging activity for talented students. In the first Think Beyond Card, students are asked to figure out how the number trick works when the question posed is "How many odd numbers are showing?" In this case, students need to know that the maximum sum is 30 when all even numbers are showing. They can then subtract 1 for every odd number showing. For example, if two odd numbers are showing, the student would subtract 2 from 30, and the sum would be 28.

The second Think Beyond Card addresses what is fundamental to this trick. That fact is that each card has one odd and one even number on it, and the even number is a predictable amount greater than the odd number. If all the numbers on both sides of the cards were odd (or even), it would be impossible to predict the sum. But if the odd/even pairs are consecutive, then the even numbers must be exactly one more than each of the odd numbers. If the even number in each odd/even pair is a set amount greater than the odd number, the trick will also work. For example, the third Think Beyond Card suggests using even numbers that are all three more than the odd number on each card (10, 8, 6, 4). The sum of the odd numbers (7, 5, 3, 1) is 16. So to answer the question "How many even numbers are there?" the number of even numbers must be multiplied by 3 and added to the sum of the odd numbers ($16 + 3x$).

In summary, there must be a predictable pattern between the numbers on the sides of the cards. However, it doesn't matter how many cards are used for the trick or what numbers are on the cards as long as the minimum sum is calculated prior to trying it out.

Name: _____ Date: _____

Mathematical Trickery! *ANSWER SHEET*

Part I: Make the following cards for the math trick.

	Card A	Card B	Card C	Card D	Card E
Front	1	3	5	7	9
Back	2	4	6	8	10

1. If you used only Card A and Card B to play the game, what is the minimum sum possible? 4 Which numbers give the minimum sum? 1 and 3 What is the maximum possible sum? 6 Which numbers give the maximum sum? 2 and 4

2. If you used only Card A, Card B and Card C to play the game, what is the minimum sum possible? 9 Which numbers give the minimum sum? 1, 3, 5 What is the maximum sum possible? 12 Which numbers give the maximum sum? 2, 4, 6

3. What is special about the numbers used to get the minimum and maximum sums?
   The minimum sum is the sum when all odd numbers are chosen, and the maximum sum is the sum when all even numbers are chosen.

4. What number patterns do you see on a) the fronts? b) the backs? c) the front to back?
   a) all odd numbers b) all even numbers
   c) the back is the next consecutive number

5. The person performing the number trick always asks to be told how many even numbers are used. Why do you think you have to know how many even numbers are used in order to predict the sum?
   Since the even numbers are 1 more than the odd numbers, you need to know how much extra you will add to the minimum sum.

Name: _____ Date: _____

Mathematical Trickery! (continued)

6. Explain how the number trick works.

   First you figure out the sum of the odd numbers, which is the minimum sum. When you know how many even numbers there are, you add that number to the minimum, and you get the sum.

7. Write a rule using variables for how to find the sum of the numbers on the cards.

   The rule is: Sum = 25 + x, where x stands for the number of even numbered cards showing.

Part II: Make your own set of number trick cards.

8. Make up a set of cards to use for the number trick. You may use any number of cards. Indicate here what numbers are on the fronts and backs of your cards.

   Front: Answers will vary.

   Back: Answers will vary.

9. Write the directions for your number trick.

   Answers will vary. Look for clarity.

10. What is the minimum sum possible using your set of cards? Answers will vary.

11. What is the maximum sum possible using your set of cards? Answers will vary.

12. What question will you ask that will give you more information about the sum?

    Students should ask a question that enables them to add or subtract an amount to either the minimum or maximum sum.

13. Write a rule with variables that can be used to determine the sum of your numbers.

    Answers will vary depending on the question and the numbers on the cards.

HINT CARDS

What is the sum of $1 + 3 + 5$? How is this sum different from $2 + 4 + 6$?

Number Tricks

HINT CARDS

The numbers 1, 3 and 5 are all odd numbers.

Number Tricks

HINT CARDS

Compare the two numbers that are on each individual card. For example, how are 3 and 4 related? How are 5 and 6 related? What about 7 and 8?

Number Tricks

HINT CARDS

The even numbers are one more than the odd numbers. How can you use this fact to help you determine the sum of the numbers on the cards?

Number Tricks

THINK BEYOND

1. Imagine that the question you ask during the number trick is: "How many odd numbers are showing?" How will this change the rule for determining the sum of the numbers? Explain.

Play the game a few times using this new question before you explain your thinking.

Number Tricks

THINK BEYOND

2. Could you use any set of cards for the number trick? For example, could you use cards that had only odd numbers on the fronts and backs? Could you use three-digit numbers?

What numbers can be used and what numbers can't be used in this trick? Explain.

Number Tricks

THINK BEYOND

3. Do the numbers on the cards have to be consecutive? Namely, could you use the following numbers on the cards?

Card 1: 7 and 10
Card 2: 5 and 8
Card 3: 3 and 6
Card 4: 1 and 4

If so, how would you figure out the sum? What question would you ask? Write a rule for this number trick.

Number Tricks

A QUICK LOOK

Chapter 1: Lesson 2
Number Tricks

NOTES

Objectives:

- Students will write equations using variables.
- Students will learn about the sums of even and odd numbers.
- Students will solve equations.

Initiate: ($\frac{1}{2}$ day)

1. Using blank index cards, prepare in advance the following five cards:

	Card A	Card B	Card C	Card D	Card E
Front	1	3	5	7	9
Back	2	4	6	8	10

2. Ask students to select 5 of the 10 numbers on the cards and place the cards so the chosen numbers are face up where you cannot see them.
5. Have students determine the sum of the numbers but not tell you what it is.
4. Ask, "How many of the numbers showing are even?" When you are told this amount, add this amount to 25, and the result is the sum of the numbers. Tell students the sum. Perform the trick two more times.

Investigate: ($1\frac{1}{2}$ days)

1. Have students work on Part I of "Mathematical Trickery!" in the Student Mathematician's Journal.

NOTES

2. To help students figure out the trick, conduct a discussion about patterns and sums of the numbers. Ask leading questions, such as "How are odd and even numbers the same and how are they different?" or "Why do you think it is important that consecutive odd/even pairs are on each card?"
3. Have students generalize a rule for finding the sum in this trick: Sum = 25 + number of even numbers. Reintroduce the idea of a variable and ask students what varies in this trick each time (the number of even numbers). Show students how to represent the variable with a letter or symbol:

   $\text{Sum} = 25 + x$ or $\text{Sum} = 25 + \square$

   Some students might also suggest that the sum varies each time; they'll want to use a letter to represent that quantity.

   $S = 25 + x$ or $S = 25 + \square$
4. Have students work individually to create a similar number trick using their own cards.
5. When finished, have students complete Part II of "Mathematical Trickery!" and write a rule for finding the sum for their trick.
6. Have each student practice his/her number trick with a classmate and then present the trick to the whole class.

Mathematical Communication: (1 day)

Assign and discuss the Think Deeply questions.

- Have students work individually on the first Think Deeply question. Students are asked to explain how their number trick works to a friend who was absent from math class. This question is a good one to use to formally assess the lesson.
- The second Think Deeply question asks students to define a variable and describe how a variable is used to write a rule for a magic number trick. If class time is limited, this question may be assigned for homework.