SECTION 1

THINKING LIKE A MATHEMATICIAN

Making Sense of Equality

When you think like a mathematician, you use mathematics to make sense of the world. Every human brain is designed to look for patterns. You will use patterns in your mathematical investigations to solve problems with balance, equality and equations.

LESSON 1.1

A Balancing Act: Understanding Equivalence and Patterns

Start It Off

The following table lists the results of multiplying the one-digit counting numbers by 9.

First Factor	Second Factor	Product
9	1	9
9	2	18
9	3	27
9	4	36
9	5	45
9	6	54
9	7	63
9	8	72
9	9	81

1. List at least three patterns you notice. Look for patterns within each column and across all three columns.
2. Test whether your patterns hold when you multiply 9 by a two-digit number. Use several examples.

On the Balance: Solving Balance Problems

The ancient Greeks, Pythagoras, Theano and the other Pythagoreans believed that mathematics is the key to understanding the universe. They believed that the study of patterns is essential to all mathematics. They saw patterns in everyday objects that could be touched and counted. This grew to abstract concepts of numbers and patterns that existed only in the mind. We will start as the Pythagoreans did, using concrete objects and studying patterns.

For centuries, scales and balances have been used to weigh things. Look at the balance below. Note that the two sides are in balance when the weight of the objects on one side is equal to the weight of the objects on the other side.

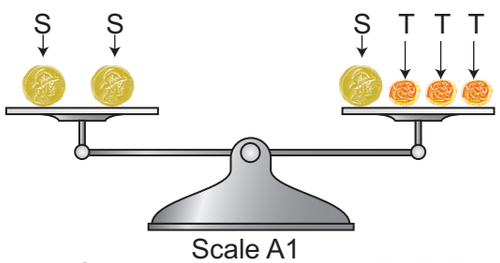
MATHEMATICALLY SPEAKING

- **equal**

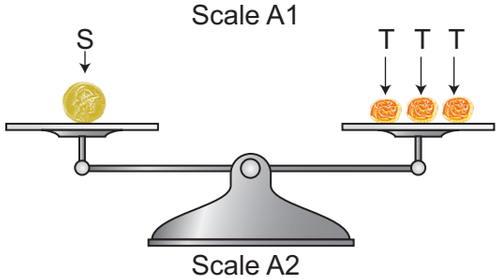
In the time of Pythagoras, scales and balances were often used to find the weight of coins to make sure trades were fair. The value of a coin was determined by its weight and the type of metal it was made from. For example, a silver stater weighed twice as much as a silver half-stater and was also worth twice as much. Other coins used in ancient Greece were trites, hektes, drachms and obols.

Example

Helena has Scale A below in front of her. She wants to know how many trites are needed to equal the value of one stater. What should she do? On one side she sees two staters (S). This balances with one stater and three trites (T). Helena removed a stater from each side to find the weight of a stater. Using this "remove" strategy, she found that one stater had the same weight as three trites.



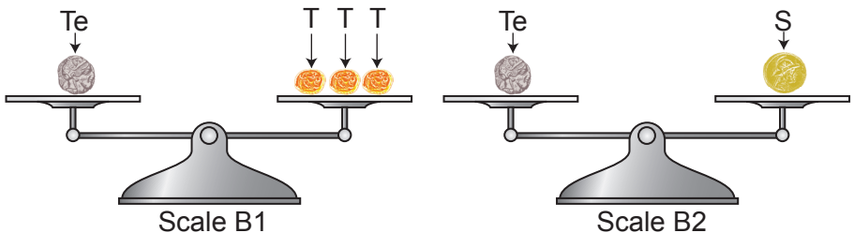
Scale A1



Scale A2

Example continued

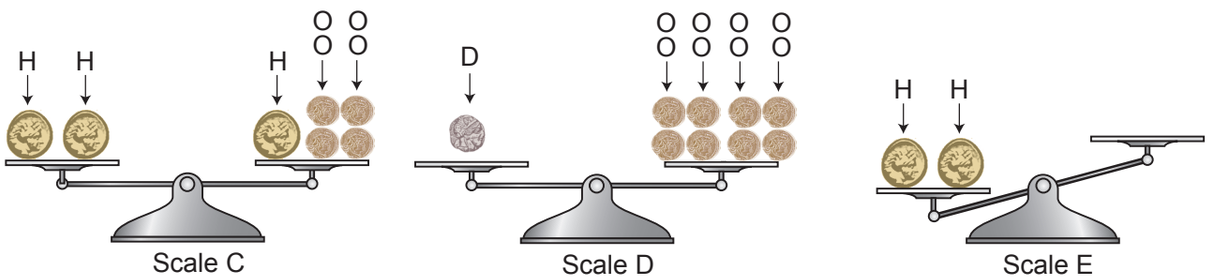
Helena saw on Scale B that three trites also weighed the same as another coin she had, a tetradrachm (Te). She wondered how the weight of a tetradachm compared to the weight of a stater. She knew from Scale A that three trites had the same weight as one stater. So, she replaced the three trites on the right with one stater. Using this "replace" strategy, Helena found that one stater had the same weight as one tetradrachm.



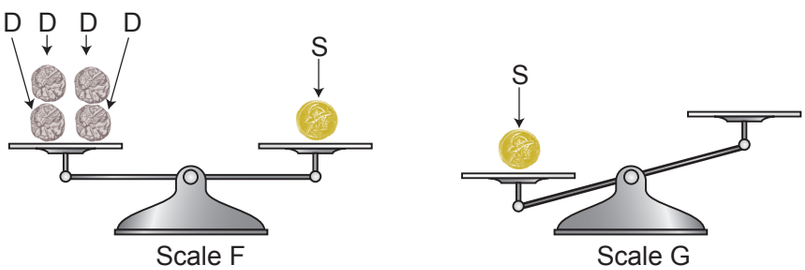
Use the strategies of remove and replace to solve the following coin problems about the coins from ancient Greece.

1. Alexander could find the value of staters (S), hektes (H), obols (O), and drachm (D) coins using the scales below.

a) What might Alexander use to balance the two hektes on Scale E? Can he balance the scale with one coin? How can you use the strategies of remove and replace to solve this problem?

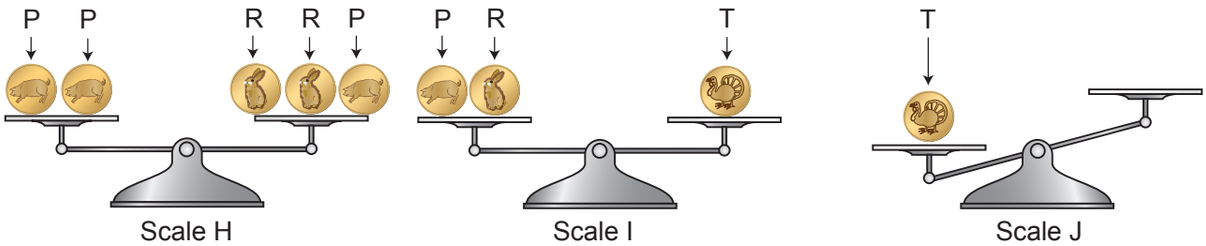


b) Use what you learned about the weights of the coins in Part a to determine how many hektes it would take to balance the stater on Scale G below. Explain how you found your answer.

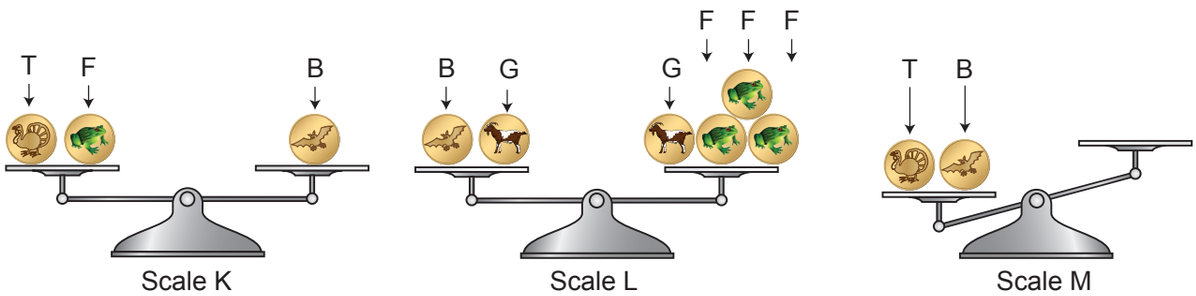


Jake decided to make up his own coin puzzles for Questions 2–4. His coins were named after the animals shown on them: pigs (P), rabbits (R), turkeys (T), goats (G), frogs (F), monkeys (M) and bats (B). All coins with the same animal weigh the same.

2. Use Scales H–I to find the number of rabbit coins it would take to balance a turkey coin. Explain how you solved this.

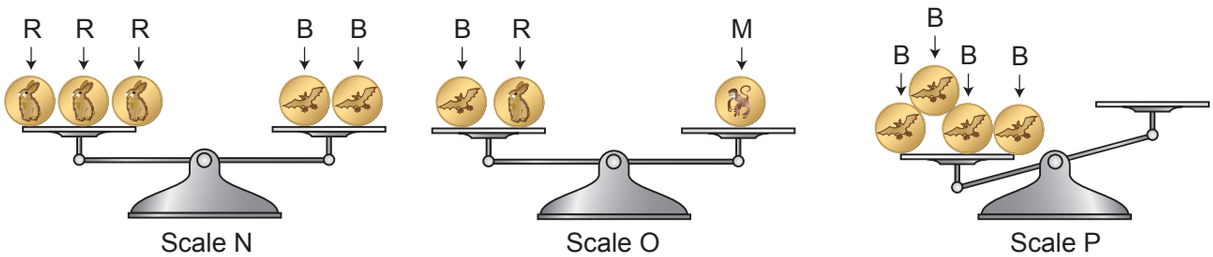


3. How many frog coins would you need to put on the right side of Scale M? Discuss the strategies you used with a partner.



4. **a)** What coins might you put on the right side of Scale P to balance it? Explain.

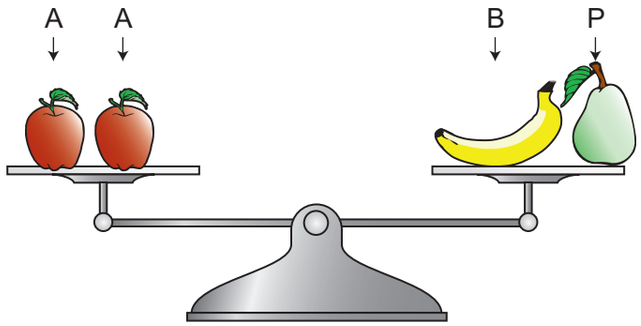
b) Is there another way you might do this? Compare your answer to a partner's.



Scale It Up: Patterns and Tables

Brody and Jake found an old scale at home and decided to weigh fruit. They found that all fruit of the same type weighed the same.

5. Look at Scale Q. The weight of the two apples is the same as the weight of one banana and one pear.



Scale Q

a) If each apple weighs 4 ounces, list at least five possible pairs of weights for the banana and the pear. Use a chart like the one below.

Banana	Pear

b) What patterns do you notice on your chart?

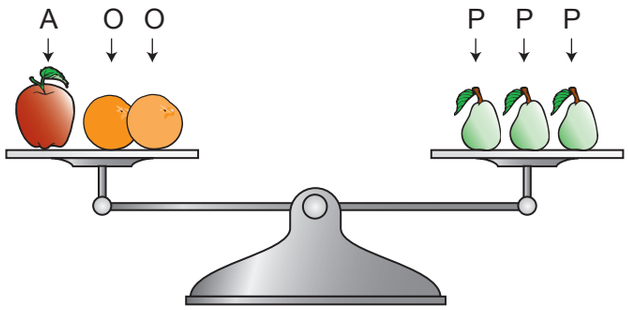
c) If each fruit weighs a whole number of ounces, how many pairs of weights are possible for the banana and the pear?

d) If the fruit could weigh a fractional number of ounces, how many pairs of weights are possible?

e) Suppose you know the weight of one apple and the banana. How would you find the weight of the pear? Is there more than one correct answer?

f) If you know the weight of the pear and the banana, how would you find the weight of three apples?

6. Use Scale R to answer the questions below. Note that all fruits of the same type will have the same weight. (Since this is a new problem, the weights might be different than in other problems.)



Scale R

a) If each pear weighs 5 ounces, give at least five possible pairs of weights for each apple and orange. Use a chart like the one below.

Apple	Orange

b) What patterns do you notice on this chart?

c) If one apple weighs 3 ounces and one pear weighs 4 ounces, what is the weight of one orange? Is there more than one correct answer?

d) If you know the weight of one apple and one orange, how can you find the weight of one pear? Is there more than one correct answer?

e) If you know the weight of one pear and one orange, how can you find the weight of three apples? Is there more than one correct answer?

f) If the weights of two types of fruit are given, is there a single answer for the weight of the third type of fruit?

g) If the weight of one type of fruit is given, is there a single answer for the weights of the other two types of fruit?

Think Beyond

7. **a)** Make up a balance puzzle that has a single correct answer. Trade puzzles with a partner and solve. Compare answers.

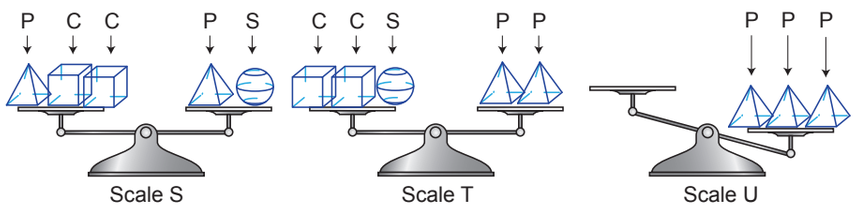
   b) Make up a balance puzzle that has many possible answers and trade with a partner. If you have a different answer than your partner, is each answer correct? How do you know? If you have the same answers, see whether you can find others.

Wrap It Up

MATHEMATICALLY SPEAKING

- equal

Explain how you solve balance puzzles using the strategies of remove and replace. Use the Scales S-U below in your explanation. Same shapes have the same weight. How many cubes will balance the three pyramids?



LESSON 1.1

SECTION 1

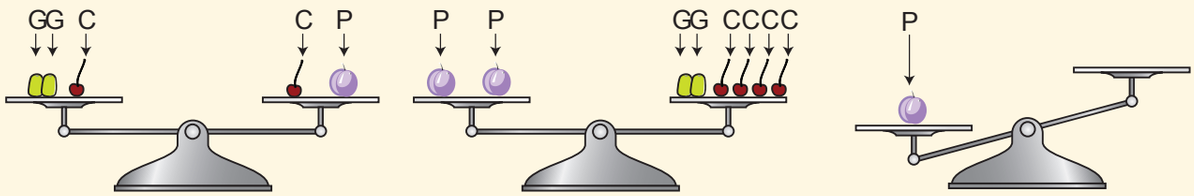
On Your Own

MATERIALS LIST

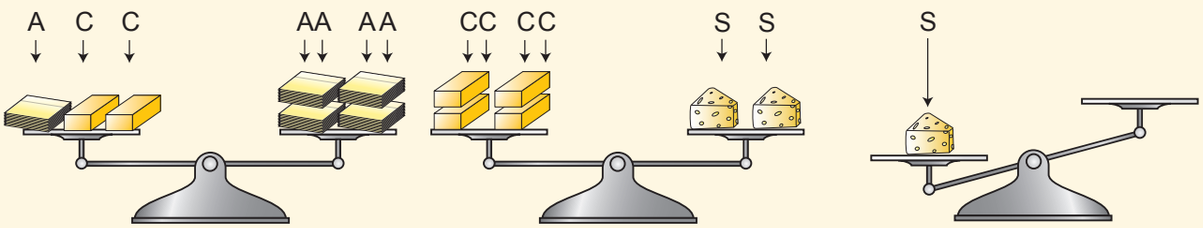
- Lesson Guide 1.1A (optional)

Write About It

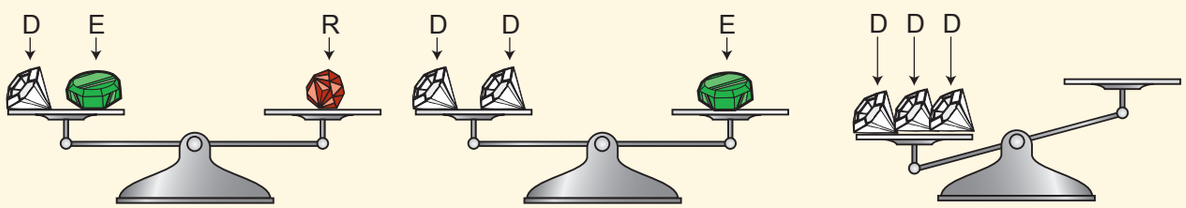
1. In the following balance puzzle, fruits of the same type have the same weight. How many cherries does it take to balance a plum? Use "remove" and "replace" in your explanation.



2. Ms. Martines bought several types of cheese for a party: American cheese (A), cheddar cheese (C) and Swiss cheese (S). All packages of the same type of cheese weigh the same. How many packages of American cheese would it take to balance one package of Swiss cheese?



3. In the book *1001 Arabian Nights*, one of the most popular stories is "Aladdin." Imagine that Aladdin uses scales to weigh the diamonds (D), rubies (R) and emeralds (E) he finds. All jewels of the same type have the same weight.



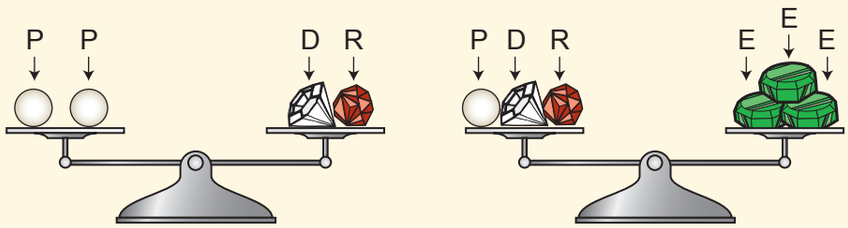
a) What single jewel balances three diamonds?

b) What combination of jewels could balance three diamonds?

c) If the diamond weighs 4 grams, can the emerald weigh 10 grams? Explain.

d) If the diamond weighs 4 grams, what is the weight of the ruby? Is there more than one correct answer? Explain.

4. Aladdin has found pearls (P) that each weigh 3 grams.



a) What do the diamond and the ruby weigh together?

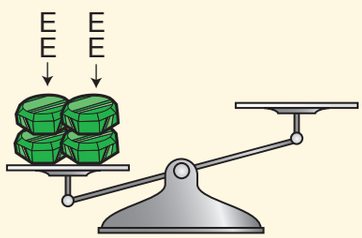
b) What is the weight of one emerald? Is there more than one correct answer?

c) List at least five different possible weights for one diamond and one ruby.

Diamond	Ruby

d) What patterns do you notice in the weights of one diamond and one ruby?

e) Make a chart listing three different ways that you can balance the four emeralds using diamonds, pearls and/or rubies.



5. Marisol loves to fish. She caught two trout, each of the same size, and one catfish. Each trout weighs $1\frac{1}{2}$ pounds. Together all the fish weigh 6 pounds. How much does the catfish weigh?

6. Mrs. Collins bought two bags of cashews that weigh 3 ounces each. Together the cashews balance the combined weight of a bag of peanuts and a bag of almonds.

a) If the bag of peanuts weighs $2\frac{3}{4}$ ounces, how much does the bag of almonds weigh?

b) Make a chart to show three different combinations of weights for the bag of peanuts and the bag of almonds.

Think Beyond

7. When Sue and her father get on the scale together, they find that they weigh the same amount as when her older brother John gets on the scale with their mother. John's father weighs 180 pounds and when he gets on the scale with his wife, the total is 300 pounds. John weighs four times as much as Sue. How much does John weigh? Show your reasoning.

Think Beyond

8. Balance puzzles are seen around the world. Work through these questions.

a) Jameel has a scale and nine coins. All nine of the coins look alike. Eight of the coins are real gold and have the same weight. The ninth coin is a fake and is heavier than the other coins. Jameel tells you that if you can identify the fake coin, then you get to keep all of the coins. However, you can only use the balance scale at most two times. How might you identify the fake coin?

b) Erin also has nine coins that look identical. Eight of the coins are real gold and have the same weight. The ninth coin is a fake, but you do not know whether it is lighter or heavier than the real coins. If you find the fake, you get to keep all the coins. However, you can only use the balance scale at most three times. How might you identify the fake coin?

Think Back

9. Compute each of the following without a calculator. Show your work.

a) $45 \cdot 18 =$

b) $125 \div 34 =$

c) $2.45 + 8.2 + 0.97 =$

d) $20.08 - 2.79 =$

10. Fill in three more terms in each number list below. Explain the pattern you used.

a) 1, 2, 4, 8, _____ , _____ , _____

b) 1, 2, 4, 7, _____ , _____ , _____

c) 64, 32, 16, 8, _____ , _____ , _____

d) $3, 3\frac{1}{2}, 4, 4\frac{1}{2},$ _____ , _____ , _____

11. If you know that $47 + 52$ is equal to some number plus 49, how would you find the missing number?

12. Complete the following:

a) 1 pint = _____ cups

b) 1 yard = _____ inches

c) 1 meter = _____ centimeters

d) 1 mile = _____ feet

13. What is the probability of rolling a prime number on a number cube with sides labeled 1, 2, 3, 4, 5 and 6?