Making Sense of Equality

LESSON 1.1 A Balancing Act: Understanding Equivalence and Patterns

Suggested Pacing: 2 Days

In this lesson students are introduced to the Pythagorean society who studied patterns in the world around them. This search for patterns is designed to help students begin to "think like mathematicians". They start by looking for patterns in the multiplication facts and then use patterns to solve balance scale problems as an introduction to solving equations symbolically.

LESSON OBJECTIVES

- Students will analyze and extend patterns and make generalizations.
- Students will understand that algebra can be thought of as generalized arithmetic and is often used to represent mathematical situations and structures for analysis and problem solving.
- Students will deepen understanding of balance and its relationship to equality and equations.
- Students will informally explore the use of variables in expressions and equations.
- Students will model equations using a balance scale and solve real-world and mathematical problems with simple equations.

DAY 1	MATERIALS*	ESSENTIAL *ON YOUR OWN* QUESTIONS
On The Balance: Solving Balance Problems	**In Class** ■ Lesson Guide 1.1A: *On the Balance* (optional) ■ Internet Access	Questions 1–2, 3a, b, 9–11
DAY 2	**MATERIALS***	**ESSENTIAL *ON YOUR OWN* QUESTIONS**
Scale It Up: Patterns and Tables	**In Class** ■ Lesson Guide 1.1A: *Scaling It Up* (optional) ■ Pan Balance (optional)	Questions 3c, 3d, 4–6, 12–13

* The Think Like a Mathematician Daily Record Sheet should be used daily

MATHEMATICALLY SPEAKING

- equal

Making Sense of Equality

When you think like a mathematician, you use mathematics to make sense of the world. Every human brain is designed to look for patterns. You will use patterns in your mathematical investigations to solve problems with balance, equality and equations.

LESSON 1.1 A Balancing Act: Understanding Equivalence and Patterns

Start It Off

The following table lists the results of multiplying the one-digit counting numbers by 9.

First Factor	Second Factor	Product
9	1	9
9	2	18
9	3	27
9	4	36
9	5	45
9	6	54
9	7	63
9	8	72
9	9	81

1. List at least three patterns you notice. Look for patterns within each column and across all three columns.
2. Test whether your patterns hold when you multiply 9 by a two-digit number. Use several examples.

Start It Off

1. Answers will vary. Examples: As the second factor increases by one, the tens digit of the product increases by one. As the second factor increases by one, the ones digit of the product decreases by one. The tens digit is one less than the second factor. When you add the digits of the product, the sum is nine. The sum of the second factor and the ones digit of the product is ten.

2.
$9 \cdot 10 = 90$ $9 \cdot 14 = 126$ $9 \cdot 18 = 162$
$9 \cdot 11 = 99$ $9 \cdot 15 = 135$ $9 \cdot 19 = 171$
$9 \cdot 12 = 108$ $9 \cdot 16 = 144$ $9 \cdot 20 = 180$
$9 \cdot 13 = 117$ $9 \cdot 17 = 153$ $9 \cdot 21 = 189$

This might also be written as:

First Factor	Second Factor	Product
9	10	90
9	11	99
9	12	108
9	13	117
9	14	126
9	15	135
9	16	144
9	17	153
9	18	162
9	19	171
9	20	180
9	21	189

As the second factor increases by one, the tens digit of the product increases by one. This holds true except when the second factor changes from a multiple of ten to the next consecutive number.

As the second factor increases by one, the ones digit of the product decreases by one. This also holds true. When the ones digit reaches 0, start over at 9 and count down again.

The tens digit of the product is one less than the second factor. This holds for the second factor of 10. Beginning with the second factor of 11, if you consider the hundreds and tens digit of the product together, they are two less than the second factor until the second factor is 21. At 21, the difference becomes 3. At 31, the difference becomes four, etc.

When you add the digits of the product, the sum is nine. This will always be true if you continue to add the digits of the result until you have a one-digit sum.

The sum of the second factor and the ones digit of the product is ten. After $9 \cdot 10$, this sum changes to 20. At $9 \cdot 21$, this sum changes to 30. The sum will increase by ten whenever the factor has a ones digit equal to 1.

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DAY 1 TEACHING THE LESSON

On the Balance: Solving Balance Problems

This lesson begins with looking at patterns in familiar multiplication facts. Students are introduced to Pythagoras, his wife Theano, and the Pythagorean society who studied patterns in the world around them. Students then use patterns to solve balance scale problems, including comparisons of the weights and values of coins that were used in ancient Greece. Students will consider the relationships of known weights and use that knowledge to create other equivalent relationships. They will solve puzzles by using the "remove strategy": taking off pieces that are identical on each side of the balance. They will also use the "replace strategy": replace unknown amounts with known equivalent amounts that will lead to a solution. The major concept is that objects that balance are equivalent and the sum of the weights of the objects on one side of the balance is equal to the sum the weights of the objects on the other side. Further, students will find that the exact solution can be determined when there is only one unknown weight, but that there is more than one possible solution when there are two or more different unknown weights. The "remove strategy" will establish a basis for students to subtract the same term from both sides of an equation to simplify or solve it. The "replace strategy" develops students' abilities to later use substitution when solving equations.

On the Balance: Solving Balance Problems

The ancient Greeks, Pythagoras, Theano and the other Pythagoreans believed that mathematics is the key to understanding the universe. They believed that the study of patterns is essential to all mathematics. They saw patterns in everyday objects that could be touched and counted. This grew to abstract concepts of numbers and patterns that existed only in the mind. We will start as the Pythagoreans did, using concrete objects and studying patterns.

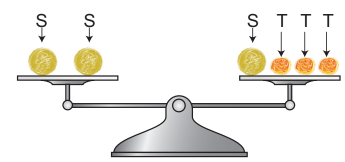
MATHEMATICALLY SPEAKING
- equal

For centuries, scales and balances have been used to weigh things. Look at the balance below. Note that the two sides are in balance when the weight of the objects on one side is **equal** to the weight of the objects on the other side.

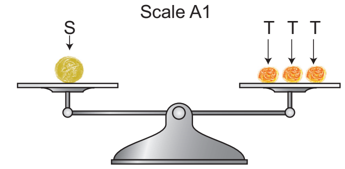
In the time of Pythagoras, scales and balances were often used to find the weight of coins to make sure trades were fair. The value of a coin was determined by its weight and the type of metal it was made from. For example, a silver stater weighed twice as much as a silver half-stater and was also worth twice as much. Other coins used in ancient Greece were trites, hektes, drachms and obols.

Example

Helena has Scale A below in front of her. She wants to know how many trites are needed to equal the value of one stater. What should she do? On one side she sees two staters (S). This balances with one stater and three trites (T). Helena removed a stater from each side to find the weight of a stater. Using this "remove" strategy, she found that one stater had the same weight as three trites.



Scale A1



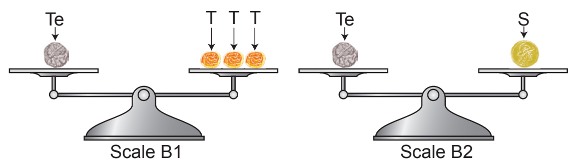
Scale A2

If students choose to write the variables as they work, make sure they use proper notation. For example, 3 times the weight of one stater should be written as $3S$ or $S + S + S$, not SSS (which means $S \cdot S \cdot S$). Similarly, 2 times the weight of one trite should be written as $2T$ or $T + T$, not $T\,T$ Note that the letters on the diagrams in this section are used simply for identification on the balance scales.

As you circulate during student work time, remember to record notes for a few students that you can transfer later to the Student Snapshot chart. You might suggest students work with a partner on the first question or two and then explore others on their own.

Example continued

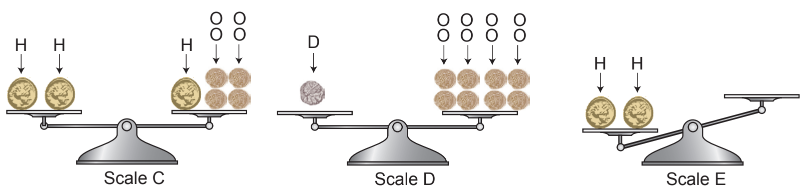
Helena saw on Scale B that three trites also weighed the same as another coin she had, a tetradrachm (Te). She wondered how the weight of a tetradachm compared to the weight of a stater. She knew from Scale A that three trites had the same weight as one stater. So, she replaced the three trites on the right with one stater. Using this “replace” strategy, Helena found that one stater had the same weight as one tetradrachm.



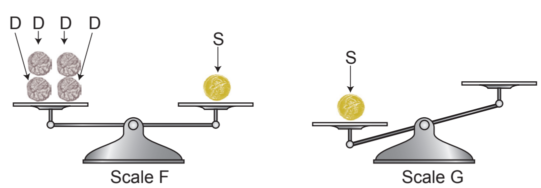
Use the strategies of remove and replace to solve the following coin problems about the coins from ancient Greece.

1. Alexander could find the value of staters (S), hektes (H), obols (O), and drachm (D) coins using the scales below.

a) What might Alexander use to balance the two hektes on Scale E? Can he balance the scale with one coin? How can you use the strategies of remove and replace to solve this problem?



b) Use what you learned about the weights of the coins in Part a to determine how many hektes it would take to balance the stater on Scale G below. Explain how you found your answer.



Working with a balance scale will allow students to establish a foundational concept of equality that will contribute to skills in writing students might and solving algebraic equations. As a class, students might explore the NCTM Illuminations Pan Balance Shapes (Fixed Values) http://illuminations.nctm.org/ActivityDetail.aspx?ID5131 as well as the other NCTM Illuminations Pan Balances to activate students’ prior knowledge while establishing an equal sign so they become familiar with the mathematical context.

Differentiation

Think Differently: Provide a physical pan balance for students who may need to explore relationships using a concrete model. To familiarize them with the setting, ask students to find sums of weights that are equivalent to sums of other weights. For example, hide one weight in a cup on the left and ask the student to find two different weights for the right side that will make it balance. Ask students to record the weights with an equal sign so they become familiar with the mathematical context.

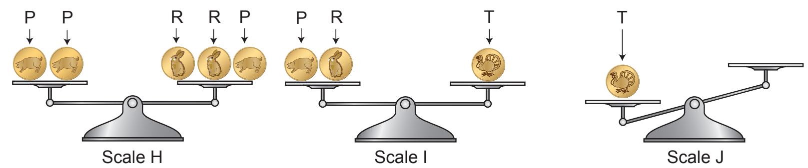
Differentiation

Think Differently: Provide Lesson Guide 1.1A so that students will have a place to keep track of the pieces they "remove" and "replace" as they think through the equivalent relationships they discover. *Accommodation Guides* (Lesson Guides which contain an A after the number) are designed to provide additional support for those students experiencing difficulty with a lesson activity. They can mark with an *x* any coins that they remove and write in the new coins above the replaced item. Encourage students to write the relationships using numbers, symbols and an equal sign.

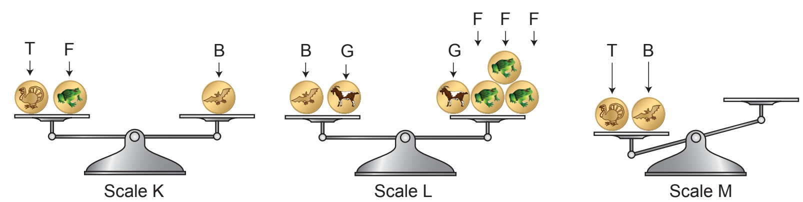
Talk Moves As you circulate during student explorations, listen to student explanations and the partner(s) paraphrasing of the explanation. (Learn more about the "talk moves" in the Teaching and Learning Strategies on page T5.) Pay special attention to English language learners as they discuss the meaning of the relationships and the symbols that represent them.

Jake decided to make up his own coin puzzles for Questions 2–4. His coins were named after the animals shown on them: pigs (P), rabbits (R), turkeys (T), goats (G), frogs (F), monkeys (M) and bats (B). All coins with the same animal weigh the same.

2. Use Scales H–I to find the number of rabbit coins it would take to balance a turkey coin. Explain how you solved this.

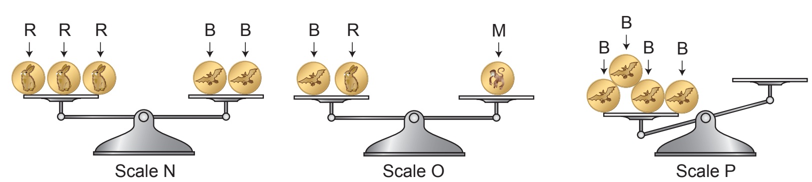


3. How many frog coins would you need to put on the right side of Scale M? Discuss the strategies you used with a partner.



4. **a)** What coins might you put on the right side of Scale P to balance it? Explain.

b) Is there another way you might do this? Compare your answer to a partner's.



1. **a)** If you remove a hekte (H) from each side of Scale C, you are left with $H = 4$ Obols (O). On Scale D, since $D = 8$ Obols (O), $D = 2H$ (because $H = 4$ Obols). Therefore, the single coin that would be on Scale E would be D, or one drachm.

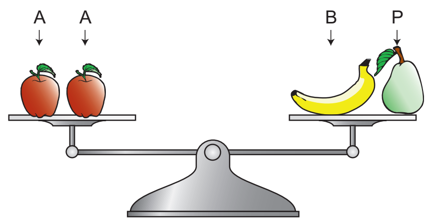
b) Since each drachm has the same weight as 2 hektes, 8 hektes would have the same weight as 4 drachms or one stater.

2. On the first balance, if you remove P from both sides, you are left with $P = 2R$ (one pig coin has the same weight as 2 rabbit coins). On the second balance, you can replace the P with $2R$ (as found from the first balance) leaving $3R = T$ (3 rabbit coins have the same weight as one turkey coin), which would be the third balance. You would need 3 rabbit coins to balance the turkey coin.

Scale It Up: Patterns and Tables

Brody and Jake found an old scale at home and decided to weigh fruit. They found that all fruit of the same type weighed the same.

5. Look at Scale Q. The weight of the two apples is the same as the weight of one banana and one pear.



Scale Q

a) If each apple weighs 4 ounces, list at least five possible pairs of weights for the banana and the pear. Use a chart like the one below.

Banana	Pear

b) What patterns do you notice on your chart?

c) If each fruit weighs a whole number of ounces, how many pairs of weights are possible for the banana and the pear?

d) If the fruit could weigh a fractional number of ounces, how many pairs of weights are possible?

e) Suppose you know the weight of one apple and the banana. How would you find the weight of the pear? Is there more than one correct answer?

f) If you know the weight of the pear and the banana, how would you find the weight of three apples?

3. If you remove the goat coin (G) from each side of the second balance, you get $B = 3F$ (the bat coin has the same weight as 3 frog coins). Given that $B = 3F$, you could replace the bat coin (B) from the first balance with 3 frog coins (F). Then on the first balance you could remove one F from each side, leaving $T = 2F$. On the third balance, you could balance the T with $2F$ and balance the B with $3F$, so you would need to add 5 frog coins on the third balance.

4. **a)** and **b)** Possible solution method 1: It is clear that $3R = 2B$ as shown on the first balance, and the third balance has twice as many bat coins as the first balance. You could double the number of rabbit coins on the first balance to get the number of rabbit coins on the third balance. You would need 6 rabbit coins on the right side of the third scale, so $4B = 6R$.

Possible solution method 2: Since $2B = 3R$, you could place $2B$ and $3R$ (based on the equality shown on the first balance) on the right side of the third balance, so $4B = 2B + 3R$.

Possible solution method 3: Based on the reasoning in the solution method 2 above, you could then use the relationship of the second balance to replace $2B$ and $2R$ with $2M$, leaving the third balance as $4B = 2M + R$.

Possible solution method 4: Based on the relationship seen in the first balance, you could have the $4B$ equal to all of the coins on the first balance, leaving $4B = 3R + 2B$. Since $B + R = M$ on the second scale, then $2B + 2R = 2M$. To make $4B$, you would need $2M + R$. Hence, a possible solution for the third balance would be $4B = 2M + R$. Although this is the same solution as method 3, it is a different reasoning process.

Possible solution method 5: For each $2\frac{1}{2}$ rabbit coins you remove from the right side of the third balance, you could substitute one monkey coin. Since $4B = 6R$, then $4B = M + 3\frac{1}{2}R$. If you remove another $1\frac{1}{2}$ R you can replace it with B. You could have $4B = M + 2R + B$.

Summarize Day 1

Talk Moves Wrap up this part of the lesson by encouraging students to share a variety of methods for solving the coin problems. This is a good chance to practice the talk moves of repeat, rephrase, add on, and agree or disagree. If time permits, encourage students to make up their own coin balance problems.

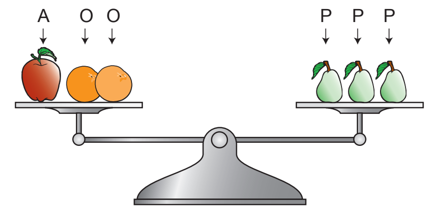
DAY 2 TEACHING THE LESSON

Scale It Up: Patterns and Tables

To start the day, you might choose one of the balance problems from On Your Own and ask students to talk to a partner about different solution methods.

Make sure students are aware that the two apples have identical weights. While it is possible that the banana and pear have weights equivalent to each other or to the apples, there are also several other possibilities. Make sure they understand that the other fruits do not need to have whole number weights, but could weigh fractional amounts. As students work on this part of the lesson, remind them that you will be listening to their discussions of their mathematical thinking, specifically for their explanations of how and why they solved the problems.

6. Use Scale R to answer the questions below. Note that all fruits of the same type will have the same weight. (Since this is a new problem, the weights might be different than in other problems.)



Scale R

a) If each pear weighs 5 ounces, give at least five possible pairs of weights for each apple and orange. Use a chart like the one below.

Apple	Orange

b) What patterns do you notice on this chart?

c) If one apple weighs 3 ounces and one pear weighs 4 ounces, what is the weight of one orange? Is there more than one correct answer?

d) If you know the weight of one apple and one orange, how can you find the weight of one pear? Is there more than one correct answer?

e) If you know the weight of one pear and one orange, how can you find the weight of three apples? Is there more than one correct answer?

f) If the weights of two types of fruit are given, is there a single answer for the weight of the third type of fruit?

g) If the weight of one type of fruit is given, is there a single answer for the weights of the other two types of fruit?

Differentiation

Think Differently: Distribute Lesson Guide 1.1A—*Scaling it Up*, which students can use to organize their thinking before completing the possible weights in the tables for Questions 5 and 6. Students can then work on the tables in Questions 5 and 6, once they have completed the visual models with possible fruit weights.

5 **a)** Answers will vary. Possible answers:

Banana (1)	Pear (1)
7 ounces	1 ounce
6 ounces	2 ounces
5 ounces	3 ounces
4 ounces	4 ounces
3 ounces	5 ounces
2 ounces	6 ounces
1 ounce	7 ounces
4.5 ounces	3.5 ounces

b) The sum of the weights of the pear and banana is always 8 ounces. Each time the banana weighs one ounce less, the pear must weigh one ounce more.

c) If each fruit has a whole number weight then there are 7 possible solutions ($1 + 7$, $2 + 6$, $3 + 5$, $4 + 4$, $5 + 3$, $6 + 2$, and $7 + 1$).

d) Using fractional weights, there are an infinite number of correct answers, because there are an infinite number of real numbers that add to 8.

e) You would double the weight of the apple and then subtract the weight of the banana to get the weight of the pear. There would only be one correct weight for the pear if you knew the weights of the apples and banana.

f) If the banana weighs 3 ounces and the pear weighs 6 ounces, then the two apples must weigh a total of 9 ounces. Assuming that both apples have the same weight, there would be one correct answer—each would weigh 4.5 ounces, which is half of 9 ounces. If the apples could be different weights, there would be an infinite number of weights that could add to 9 ounces.

6. **a)** Answers will vary. Possible answers:

Apple (1)	Orange (2)
2 ounces	6.5 ounces
3 ounces	6 ounces
4 ounces	5.5 ounces
5 ounces	5 ounces
6 ounces	4.5 ounces
7 ounces	4 ounces
8 ounces	3.5 ounces
9 ounces	3 ounces

b) Double the weight of the orange plus the weight of the apple equals 15 ounces, which is the weight of three pears. Each time the apple is one ounce more, each orange must be 0.5 ounces less.

c) Since 3 pears would equal 12 ounces, the two oranges would weigh this amount minus 3 ounces for the apple. Divide the result by two to get the weight of one orange. Since $12 - 3 = 9$, each orange would weigh 4.5 ounces, half of 9 ounces. There would be only one correct answer.

d) You would double the given weight of the orange and add that to the weight of the apple. Divide this sum by 3 to get the weight of one pear. There would be only one correct answer.

e) You would multiply the weight of one pear by 3 and the weight of an orange by 2. Then you would subtract the total weight of the oranges from the total weight of the pears to get the weight of one apple. For three apples, you would just multiply the weight of one apple by 3. There would only be one correct answer.

f) Yes, if the weights of two types of the fruits are given, you can find a single answer for the weight of the third type of fruit.

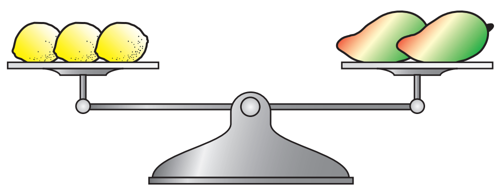
g) No, there will be many answers. In general, a balance puzzle will have many correct answers if you do not know the values of two or more of the variables (in this case fruits) in the puzzle.

7. **Think Beyond**

a) Answers will vary.

Possible answer: Each lemon weighed 4 ounces.

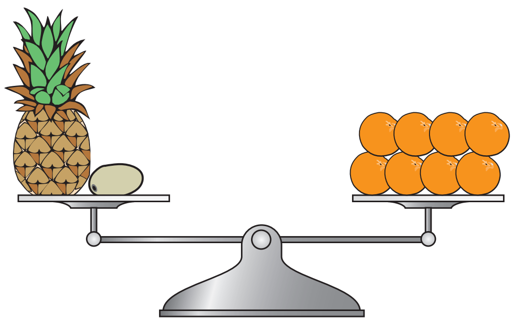
How much did each mango weigh?



Solution: Each mango weighed 6 ounces (half of 12 ounces).

b) Possible answer: Each orange weighed 5 ounces.

How much might the pineapple and kiwi weigh?



Solution: The pineapple and kiwi can have any weights that sum to 40 ounces (determined from 8 oranges at 5 ounces each).

Wrap It Up

Talk Moves Ask students to explain the "remove" and "replace" strategies and to justify their decisions of which pieces to remove and replace. Ask other students to listen carefully and to repeat/rephrase. Ask students to offer varied strategies for finding the values of the weights. Guide students to generalize how they know which pieces to remove first. Guide students to explicitly state that the goal of "removing" and "replacing" is to find the simplest relationships. This is similar to the goal of solving equations; to isolate one variable on one side.

Possible answer: I remove a pyramid from each side of the first scale. This shows that one sphere has the same weight as two cubes. On the second scale, I see that one sphere and two cubes are balanced by two pyramids. I can replace the sphere with two cubes, so four cubes balance two pyramids. If I halve the number of shapes on both sides, two cubes will balance one pyramid. Since it takes two cubes to balance one pyramid, six cubes are needed to balance three pyramids.

Think Beyond

7. a) Make up a balance puzzle that has a single correct answer. Trade puzzles with a partner and solve. Compare answers.

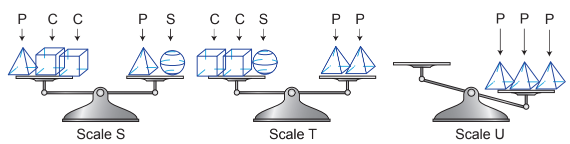
b) Make up a balance puzzle that has many possible answers and trade with a partner. If you have a different answer than your partner, is each answer correct? How do you know? If you have the same answers, see whether you can find others.

Wrap It Up

Explain how you solve balance puzzles using the strategies of remove and replace. Use the Scales S-U below in your explanation. Same shapes have the same weight. How many cubes will balance the three pyramids?

MATHEMATICALLY SPEAKING

- equal



Reflect

Use these questions to help you reflect on the lesson and plan for future instruction.

- Did students understand the concept of balance? Were they able to relate that to equations?
- Were students successful in being able to find the values of unknown weights?
- Could students use an organized list to find several possible values for variables?
- Were all students, especially English language learners, able to explain the “remove” and “replace” strategies?
- Did students discover that one solution is possible when there is one unknown value and that more than one solution is possible when there is more than one unknown value?
- What concepts need to be strengthened?
- What vocabulary words were used correctly and which need more focus?

On Your Own

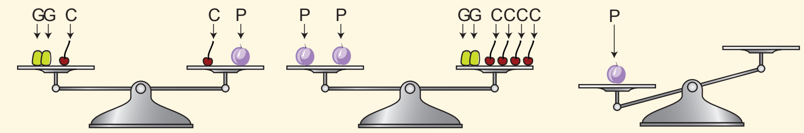
LESSON 1.1 SECTION 1

On Your Own

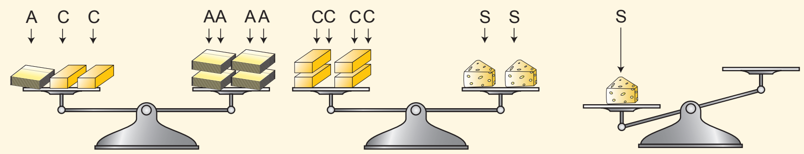
MATERIALS LIST
- Lesson Guide 1.1A (optional)

Write About It

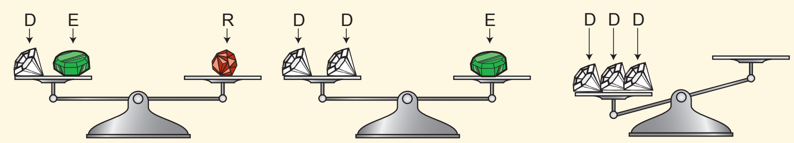
1. In the following balance puzzle, fruits of the same type have the same weight. How many cherries does it take to balance a plum? Use "remove" and "replace" in your explanation.



2. Ms. Martines bought several types of cheese for a party: American cheese (A), cheddar cheese (C) and Swiss cheese (S). All packages of the same type of cheese weigh the same. How many packages of American cheese would it take to balance one package of Swiss cheese?



3. In the book *1001 Arabian Nights*, one of the most popular stories is "Aladdin." Imagine that Aladdin uses scales to weigh the diamonds (D), rubies (R) and emeralds (E) he finds. All jewels of the same type have the same weight.



a) What single jewel balances three diamonds?

b) What combination of jewels could balance three diamonds?

c) If the diamond weighs 4 grams, can the emerald weigh 10 grams? Explain.

d) If the diamond weighs 4 grams, what is the weight of the ruby? Is there more than one correct answer? Explain.

1. **Write About It** If I remove a cherry from each side of the first scale, one plum has a weight equal to two grapes. Using the second scale, I remove the two grapes from the left side and replace them with one plum. Then I remove one plum from each side and see that one plum has the same weight as four cherries. So one plum will balance with four cherries on the last scale.

2. If I remove one package of American cheese from each side of the first scale, two packages of cheddar equal three packages of American. Then I could replace the four packages of cheddar with six packages of American, twice the weights on the first scale. Then I can halve the weights of both sides to get three packages of American cheese to balance one package of Swiss cheese.

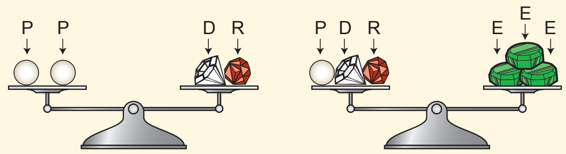
3. **a)** 1 ruby

   b) 1 diamond and 1 emerald, OR three diamonds

   c) No, if the diamond weighs 4 grams, the emerald cannot weigh 10 grams. Since the second scale shows that two diamonds weigh the same as one emerald, if a diamond weighs 4 grams, an emerald must weigh 8 grams.

   d) If a diamond weighs 4 grams then a ruby weighs 3(4) = 12 grams. There is only one correct answer since the ruby weighs as much as three diamonds.

4. Aladdin has found pearls (P) that each weigh 3 grams.



a) What do the diamond and the ruby weigh together?

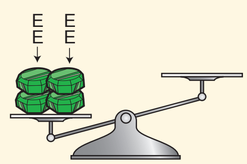
b) What is the weight of one emerald? Is there more than one correct answer?

c) List at least five different possible weights for one diamond and one ruby.

Diamond	Ruby

d) What patterns do you notice in the weights of one diamond and one ruby?

e) Make a chart listing three different ways that you can balance the four emeralds using diamonds, pearls and/or rubies.



5. Marisol loves to fish. She caught two trout, each of the same size, and one catfish. Each trout weighs $1\frac{1}{2}$ pounds. Together all the fish weigh 6 pounds. How much does the catfish weigh?

4. a) 6 g

b) 3 g. This is the only correct answer. Replace the diamond and ruby on the second scale with two pearls from the first scale. The shows that the 3 emeralds together must equal 9 g (3*P*) and they are equal to each other in weight, so they must each weigh 3 g.

c) Any of the combinations where one diamond and one ruby together equal 6 grams. Some possibilities are listed here.

Pearl	Diamond	Ruby
3 grams (6 total)	1 gram	5 grams
3 grams (6 total)	2 grams	4 grams
3 grams (6 total)	3 grams	3 grams
3 grams (6 total)	4 grams	2 grams
3 grams (6 total)	5 grams	1 gram
3 grams (6 total)	$\frac{1}{2}$ gram	$5\frac{1}{2}$ grams
3 grams (6 total)	$2\frac{1}{2}$ grams	$3\frac{1}{2}$ grams

d) As the weight of the diamond increases, the weight of the ruby decreases, and vice versa. If the diamond's weight decreases by 1, the ruby's weight increases by 1. Because it must balance the weight of two pearls (3 grams each, 6 grams total), the total weight of one diamond and one ruby will always be 6 grams.

e) Answers may vary.

Emeralds	Pearl	Diamond	Ruby
4	4	0	0
4	2	1	1
4	0	2	2

5. The catfish weighed 3 pounds.

6. a) $3\frac{1}{4}$ ounces.

b)

Cashews	Peanuts	Almonds
6 ounces	1 ounce	5 ounces
6 ounces	2 ounces	4 ounces
6 ounces	3 ounces	3 ounces
6 ounces	1.5 ounces	4.5 ounces

7. Think Beyond John weighs 80 pounds. The father weighs 180 pounds. The father and mother together weigh 300 pounds, so the mother weighs 120 pounds (300 pounds – 180 pounds = 120 pounds). John and the mother together weigh as much as the father and Sue together, so John + 120 pounds is the same weight as Sue + 180 pounds. John weighs four times as much as Sue, so if I could "remove and replace" John with 4 Sues, I would see that one Sue + 180 pounds balances with 4 Sues + 120 pounds. Now if I remove 120 pounds from each side, and remove one Sue from each side, I am left with three Sues on one side balancing with 60 pounds on the other, so one Sue weighs 20 pounds. Since John weighs four times as much, John weighs 80 pounds.

8. Think Beyond

a) Divide the coins into 3 groups of 3. Label the coins *A*, *B*, *C*; *D*, *E*, *F*; *G*, *H*, and *I*.

Possibility 1: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If they balance you know the fake coin is one of *GHI*. 2nd weighing: Place *G* into the left pan and *H* into the right pan. The heavier coin of *G* and *H* is the fake coin. If *G* and *H* balance, you know the fake coin is I.

Possibility 2: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If *ABC* is heavier, you know the heavier coin is one of *ABC*. 2nd weighing: Place A into the left pan and *B* into the right pan. The heavier coin of A and *B* is the fake coin. If A and *B* balance, you know the fake coin is C.

Possibility 3: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If *DEF* is heavier, you know the heavier coin is one of *DEF*. 2nd weighing: Place *D* into the left pan and *E* into the right pan. The heavier coin of *D* and *E* is the fake coin. If *D* and *E* balance, you know the fake coin is *F*.

b) Divide the 9 coins into 3 groups of 3. Label the coins *A*, *B*, *C*; *D*, *E*, *F*; *G*, *H*, and *I*.

Possibility 1: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If they balance, you know the fake coin is one of *GHI*. 2nd weighing: Place *G* into the left pan and *H* into the right pan. If *G* and *H* balance, you know the fake coin is *I* but need to determine if it is heavier or lighter. 3rd weighing: Place *G* (or *H*) into the left pan and *I* into the right pan. If *I* is heavier, the fake coin is heavier. If *I* is lighter, the fake coin is lighter.

Possibility 2: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If they balance, you know the fake coin is one of *GHI*. 2nd weighing: Place *G* into the left pan and *H* into the right pan. If they don't balance, note which is heavier. 3rd weighing: Remove *H* from the right pan and place *I* into the right pan. If *G* and *I* balance, *H* is the fake coin. If *H* was heavier than *G*, then the fake coin is heavier, if *H* was lighter than *G*, then the fake coin is lighter. If on the third weighing, *G* and *I* did not balance, then *G* is the fake coin (*H* and *I* will be real, so equal). If *G* was heavier than *H*, then the fake coin is heavier; if *G* was lighter than *H*, then the fake coin is lighter.

Possibility 3: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If they don't balance, note which group is heavier (for this example, we'll say *ABC*). 2nd weighing: Put *DEF* into the left pan and *GHI* into the right pan. If they balance, *ABC* contains the fake coin and it is heavier. 3rd weighing: Put *A* into the left pan and *B* into the right pan. If they balance, *C* is the fake coin and it is heavier. If they don't balance, the heavier coin is the fake one. (Change heavier for lighter in the above explanation to see if the fake coin is *A*, *B* or *C* and it is lighter.)

Possibility 4: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If they don't balance, note which group is heavier (for this example, we'll say *ABC*). 2nd weighing: Put *DEF* into the left pan and *GHI* into the right pan. If they don't balance, *GHI* must be heavier (all other possibilities are already discussed above and *DEF* cannot be lighter than *ABC* and heavier than *GHI* if 8 of the coins have equal weights). This means *DEF* is lighter than both *ABC* and *GHI*, so the fake coin is one of *DEF* and it is lighter. 3rd weighing: Place *D* in the left pan and *E* in the right pan. If they balance, *F* is the fake coin and it is lighter. If they don't balance, the lighter coin is the fake one.

Possibility 5: 1st weighing: Put *ABC* into the left pan and *DEF* into the right pan. If they don't balance, note which group is heavier (for this example, we'll say *DEF*). 2nd weighing: Put *DEF* into the left pan and *GHI* into the right pan. If they don't balance, *DEF* must be heavier than *GHI* (all other possibilities are already discussed above and *DEF* cannot be heavier than ABC and lighter than *GHI* since 8 of the coins have equal weights). This means *DEF* is heavier than *ABC* and *GHI*, so the fake coin is one of *DEF* and it is heavier. 3rd weighing: Place *D* into the left pan and *E* into the right pan. If they balance, *F* is the fake coin and it is heavier. If they don't balance, the heavier coin is the fake one.

LESSON 1.1 SECTION 1

On Your Own

6. Mrs. Collins bought two bags of cashews that weigh 3 ounces each. Together the cashews balance the combined weight of a bag of peanuts and a bag of almonds.

a) If the bag of peanuts weighs $2\frac{3}{4}$ ounces, how much does the bag of almonds weigh?

b) Make a chart to show three different combinations of weights for the bag of peanuts and the bag of almonds.

Think Beyond

7. When Sue and her father get on the scale together, they find that they weigh the same amount as when her older brother John gets on the scale with their mother. John's father weighs 180 pounds and when he gets on the scale with his wife, the total is 300 pounds. John weighs four times as much as Sue. How much does John weigh? Show your reasoning.

Think Beyond

8. Balance puzzles are seen around the world. Work through these questions.

a) Jameel has a scale and nine coins. All nine of the coins look alike. Eight of the coins are real gold and have the same weight. The ninth coin is a fake and is heavier than the other coins. Jameel tells you that if you can identify the fake coin, then you get to keep all of the coins. However, you can only use the balance scale at most two times. How might you identify the fake coin?

b) Erin also has nine coins that look identical. Eight of the coins are real gold and have the same weight. The ninth coin is a fake, but you do not know whether it is lighter or heavier than the real coins. If you find the fake, you get to keep all the coins. However, you can only use the balance scale at most three times. How might you identify the fake coin?

Think Back

9. **a)** 810

b) 3.68 or $3\frac{23}{34}$

c) 11.62

d) 17.29

10. **a)** 16, 32, 64

b) 11, 16, 22

c) 4, 2, 1

d) 5, $5\frac{1}{2}$, 6

Think Back

9. Compute each of the following without a calculator. Show your work.

a) $45 \cdot 18 =$

b) $125 \div 34 =$

c) $2.45 + 8.2 + 0.97 =$

d) $20.08 - 2.79 =$

10. Fill in three more terms in each number list below. Explain the pattern you used.

a) 1, 2, 4, 8, ______ , ______ , ______

b) 1, 2, 4, 7, ______ , ______ , ______

c) 64, 32, 16, 8, ______ , ______ , ______

d) 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, ______ , ______ , ______

11. If you know that $47 + 52$ is equal to some number plus 49, how would you find the missing number?

12. Complete the following:

a) 1 pint = ______ cups

b) 1 yard = ______ inches

c) 1 meter = ______ centimeters

d) 1 mile = ______ feet

13. What is the probability of rolling a prime number on a number cube with sides labeled 1, 2, 3, 4, 5 and 6?

11. $47 + 52 = n + 49$. 49 is two more than 47, so n must be 2 less than 52; the number is 50.

12. **a)** 2

b) 36

c) 100

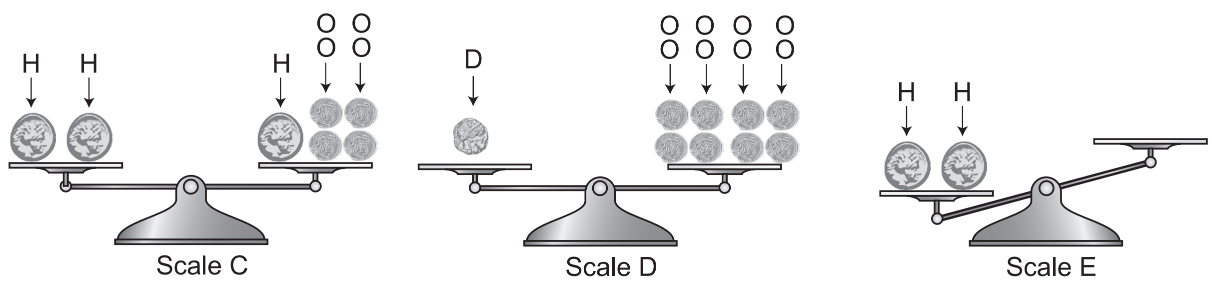
d) 5280

13. $\frac{1}{2}$ or 50%. Prime numbers are 2, 3 and 5. 4 and 6 are composite, and 1 is neither prime nor composite.

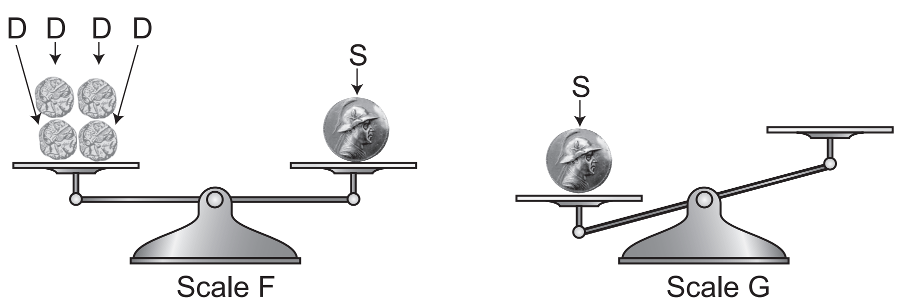
Lesson Guide 1.1A *On the Balance*

1. Alexander could find the value of staters (*S*), hektes (*H*), obols (*O*), and drachm (*D*) coins using the scales below.

a) What might Alexander use to balance the two hektes on Scale E? Can you do this with one coin? How can you use the strategies of remove and replace to solve this?



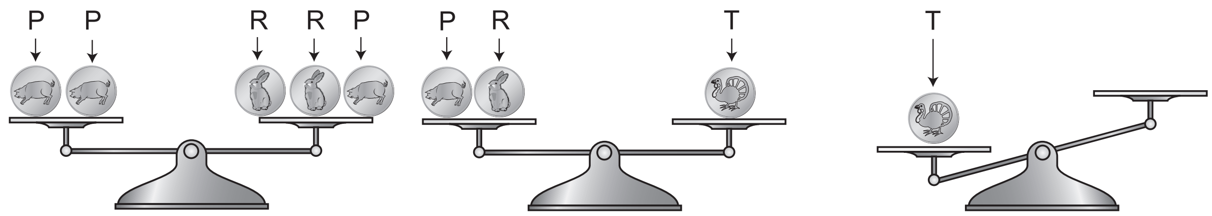
b) How many hektes would it take to balance the stater on scale G below? Explain what you know about the weight of the coins from Part a.



Jake decided to make up his own coin puzzles for Questions 2–4. His coins were named after the animals shown on them: pigs (*P*), rabbits (*R*), turkeys (*T*), goats (*G*), frogs (*F*), monkeys (*M*) and bats (*B*). All coins with the same animal weigh the same.

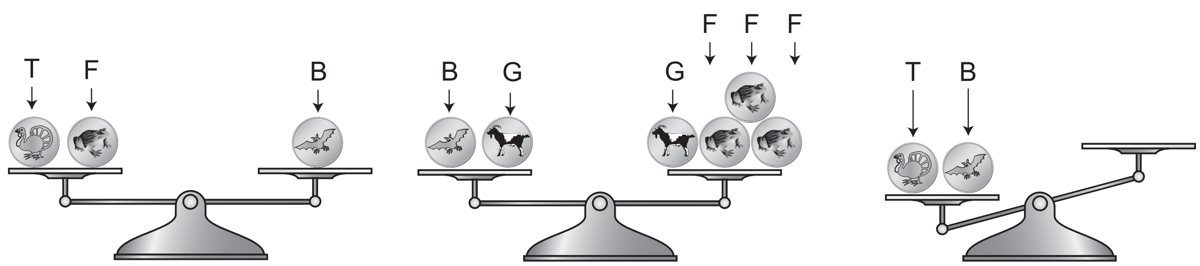
Lesson Guide *1.1A continued*

2. Use the scales to find the number of rabbit coins it would take to balance a turkey coin. Explain how you solved this.



Scale H Scale I Scale J

3. How many frog coins would it take to balance the third scale? Discuss the strategies you used with a partner.

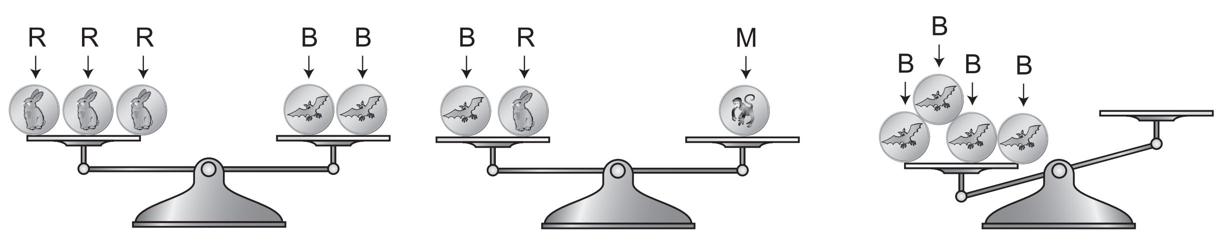


Scale K Scale L Scale M

4. Use the scales below for your answers.

a) What coins might you put on the right side of the third scale to balance it? Explain.

b) Is there another way you might do this? Compare your answer to a partner's.



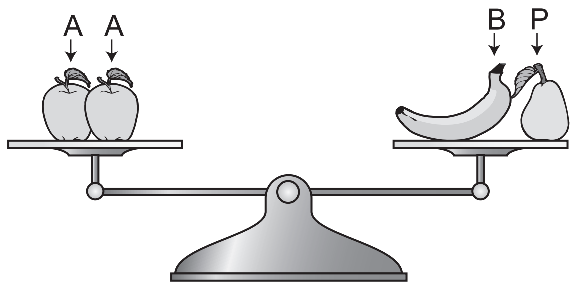
Scale N Scale O Scale P

Lesson Guide *1.1A continued*

Scaling It Up

Brody and Jake found an old scale at home and decided to weigh some fruit. They found that all fruit of the same type weighed the same. We say that if two items on a scale are in balance, then their weights are equal. This means both have the same weight.

5. Look at the following scale. Note that two apples weigh the same. The weight of the two apples is the same as the weight of one banana and one pear.



Scale Q

a) If each apple weighs 4 ounces, list at least five possible weights for the banana and the pear. Use a chart like the one below.

Banana (1)	Pear (1)

b) What patterns do you notice on the chart above?

c) If each fruit weighs a whole number of ounces, how many weights are possible for the banana and the pear?

d) If the fruit could weigh a fractional number of ounces, how many weights are possible?

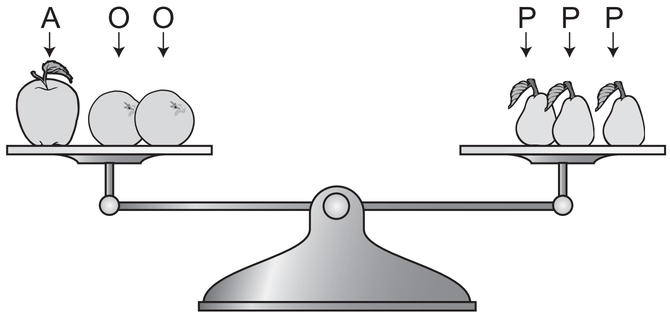
e) In one situation, you know that one banana weighs 3 ounces and one pear weighs 6 ounces. Together they balance two apples of equal weight. What is the weight of each apple?

f) You know the weight of one apple and the banana. How would you find the weight of the pear? Is there more than one correct answer?

g) If you know the weight of the pear and the banana, how would you find the weight of three apples?

Lesson Guide *1.1A continued*

6. Use the following scale to answer the questions below. Note that all fruits of the same type will have the same weight.



Scale R

a) If each pear weighs 5 ounces, give at least five possible weights for each apple and orange. Use a chart like the one below.

Apple (1)	Orange (1)

b) What patterns do you notice on this chart?

c) If one apple weighs 3 ounces and one pear weighs 4 ounces, what is the weight of one orange? Is there more than one correct answer?

d) If you know the weight of one apple and one orange, how can you find the weight of one pear? Is there more than one correct answer?

e) If you know the weight of one pear and one orange, how can you find the weight of three apples? Is there more than one correct answer?

f) If the weights of two types of fruit are given, is there a single answer for the weight of the third type of fruit?

g) If the weight of one type of fruit is given, is there a single answer for the weights of the other two types of fruit?