LESSON 1.2

Representing Balance with Scales, Bars and Equations

Start It Off

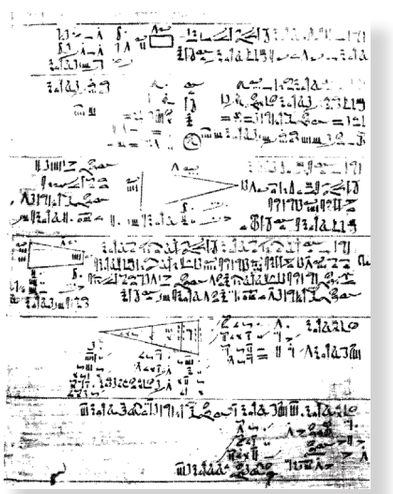
There are 100 single-digit multiplication facts. The fact with the smallest product is $0 \cdot 0 = 0$ and the fact with the largest product is $9 \cdot 9 = 81$. Dan knows his basic facts through the fives. He also knows all the nines facts and understands the commutative property of multiplication for whole numbers. It states that you can switch the order of the factors and not change their product. So if you know $4 \cdot 7 = 28$, you also know $7 \cdot 4 = 28$.

1. How many facts must Dan still learn? List them.
2. List three tips you might give Dan to learn the facts you listed.
3. Does the commutative property also work for addition, subtraction or division of whole numbers? Explain.

MATHEMATICALLY SPEAKING

- commutative property of multiplication
- variable

Thousands of years ago, people around the world began to realize they could use numbers and other symbols to record many of the things they did. Some of the symbols stood for operations like addition or multiplication. Other symbols were variables, which are letters or other symbols that stand for a number or set of numbers. One of the most famous early mathematical records is the Rhind papyrus from Egypt, which is over 3,700 years old. This is a mathematical record of many things from everyday Egyptian life.



A Fish Story: Using Variables, Expressions and Equations

Fish from the Nile were a big part of the Egyptian diet. Imagine Amal working in a fish market, keeping records. Amal could use variables, expressions and equations for his record keeping. An expression is a mathematical phrase made up of numbers, variables and/or operations. For example, Amal might use the variable n to stand for the weight of a swordfish in pounds. If a tuna weighs 13 pounds more than a swordfish, he might use the expression, $n + 13$ to represent the weight of the tuna.

1. The variable n stands for the weight of a swordfish in pounds. Match the expression in Column A with the words in Column B. Discuss your results with a partner.

Column A	Column B
a) $150n$	**i)** the weight of a tuna if the tuna and the swordfish together weigh 150 pounds
b) $n + 150$	**ii)** the number of swordfish if each one weighs n pounds and together they weigh 150 pounds
c) $150 - n$	**iii)** the weight of 150 swordfish that each weighs n pounds
d) $150 \div n$	**iv)** the weight of a tuna that weighs 150 pounds more than the swordfish

MATHEMATICALLY SPEAKING

- expression
- equation

When two expressions have the same value, you can write an equation. An equation is a mathematical sentence with an equal sign. An equation shows that the expressions on the two sides of the equal sign have the same value.

Solving Equations Using a Balance

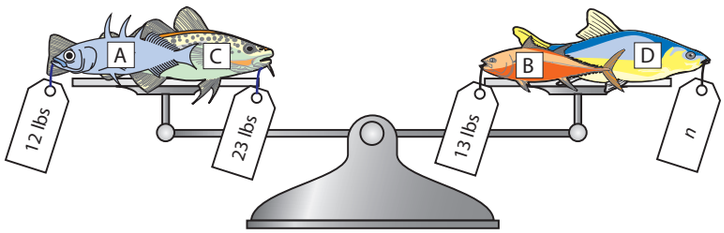
Amal has three fish that weigh 12 pounds, 13 pounds and 23 pounds. Amal will use a balance and these three fish to find the weight of a fourth fish. Look at the fish on the balance below. Using the variable n to stand for the unknown weight in pounds, Amal can write the equation $12 + 23 = 13 + n$.

MATHEMATICALLY SPEAKING

- solve (an equation)
- solution

When you solve an equation, you find a value that makes the equation true. This value is called the solution of the equation.

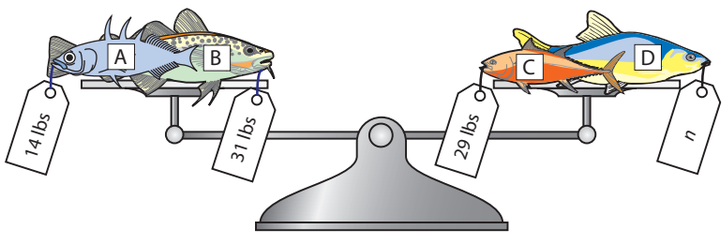
2. a) Write an equation for the weights of the fish on the scale below. Use n in your equation for the unknown weight in pounds.



b) Solve your equation for the missing weight. How might you do this without finding the total weight of the fish on the left side?

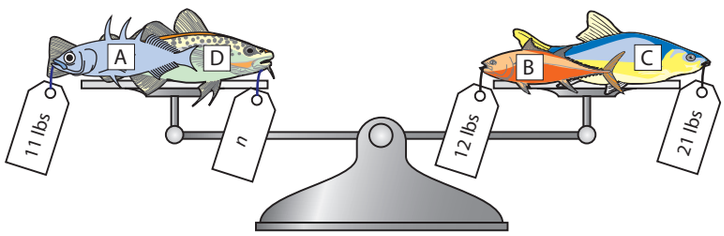
3. What value for n makes $12 + 23 = 13 + n$ a true statement? How might you determine this answer by thinking about balance and equality without using a pencil and paper or a calculator? How does your method relate to the balance scale?

4. a) Write an equation for the weights of the fish on the scale below. Use n in your equation for the unknown weight in pounds.



b) Solve your equation for the missing weight. Explain how you could find this weight without using a pencil and paper or a calculator.

5. a) Write an equation for the weights of the fish on the scale below. Use n in your equation for the unknown weight in pounds.



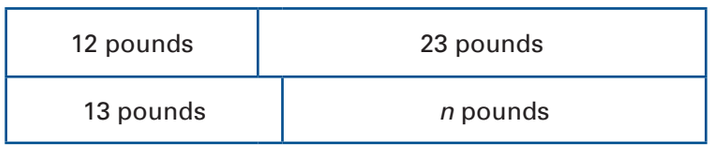
b) Solve your equation for the missing weight. Explain your reasoning to a partner.

Solving Equations Using Bar Diagrams

MATHEMATICALLY SPEAKING

▸ bar diagram

Mei Ling, who had attended school in Singapore, showed the class a different way to solve these problems. She used bar diagrams. To solve Question 2, Mei Ling drew the following bar diagram. Because the total weight on the left side of the scale is the same as the total weight on the right, she drew the bars for the two sides with the same length to show that the weights were equal. She also knew that 13 pounds is a little more than 12 pounds, so she made the 13-pound section of the bottom bar longer than the 12-pound section of the top bar.



6. Explain how Mei Ling might find the weight of the fourth fish using her diagram.

7. Draw a bar diagram to solve the fish problem in Question 4 or Question 5. Show how you use the diagram to find the missing weight.

8. Bar diagrams can also be used for other problems. For the problems below, draw a bar diagram. Explain what the variable stands for. Then write an equation and solve for the variable.

a) Jerra's age plus her mother's age is the same as Jason's age plus his father's age. Jerra is 12, Jason is 11 and Jason's father is 42. How old is Jerra's mother?

b) Notebooks cost $2.95 each and two pens cost $1.95. A set of colored pencils and four pens have the same cost as three notebooks. How much does the set of colored pencils cost?

Just Symbols

In each of the following equations, the variable n is used to stand for a missing number. This is similar to representing the unknown weights of the fish above. For each equation, try to find the value of n just by reasoning about balance and equality. Draw a balance or bar diagram if it helps you. Be prepared to explain your reasoning.

9. $4832 + 197 = n + 200$

10. $49 + n = 73 + 50$

11. $23 + n = 14 + 24$

12. $51 - n = 50 - 25$

13. $78 + 32 = 80 + n$

Wrap It Up

How does the equation $23 + n = 18 + 22$ relate to balance scales and bar diagrams? Sketch a balance scale and a bar diagram to show this. Explain how you would solve this problem without using pencil and paper or a calculator. What is the meaning of the n in this equation?

MATHEMATICALLY SPEAKING

- bar diagram
- commutative property of multiplication
- equation
- expression
- solution
- solve (an equation)
- variable

LESSON 1.2

SECTION 1

On Your Own

Write About It

1. **a)** Solve for n by reasoning about balance and equality without using a diagram. $n + 19 = 20 + 82$. Explain your method.

   b) Explain how to solve the same equation using a bar diagram and then using a balance scale.

2. For each of the following, the variable a stands for the number of apples in a crate. Column A describes the number of oranges in a crate. Match each expression in Column A with the words in Column B.

Column A	Column B
a) $3a$	**i)** The number of oranges in a crate is three more than the number of apples in a crate.
b) $a + 3$	**ii)** There are three times as many oranges in a crate as apples.
c) $a - 3$	**iii)** There are three more apples in the crate than oranges.
d) $a \div 3$	**iv)** There are one-third as many oranges in a crate as apples.

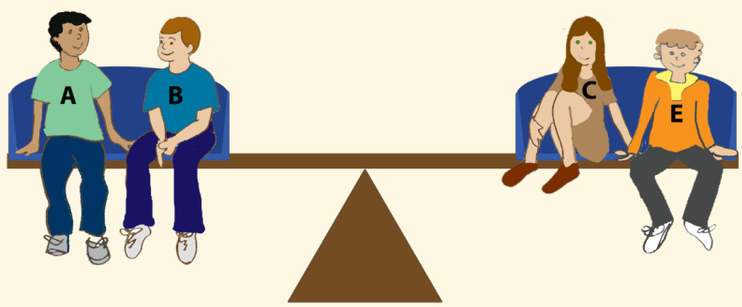
3. For each balance, write an equation. The first equation is written for you. Solve each equation for the unknown weight.

A = Antonio	85 pounds
B = Bob	97 pounds
C = Caitlin	80 pounds
D = Diane	102 pounds

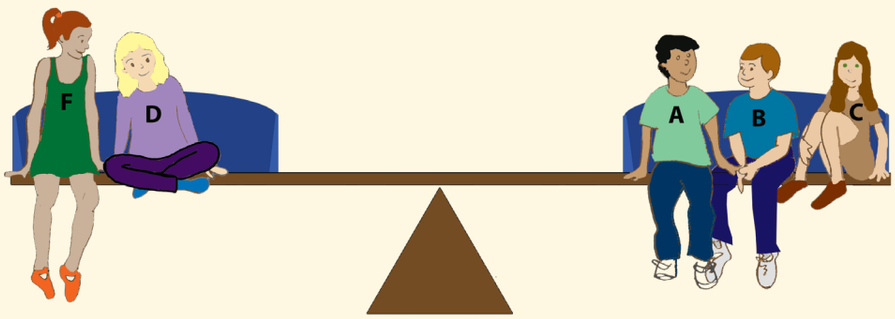
E = Enrico	? pounds
F = Fran	? pounds
G = Gary	? pounds

a) Equation: $85 + 97 = 80 + E$

Solution: $E =$ _____ pounds



b)



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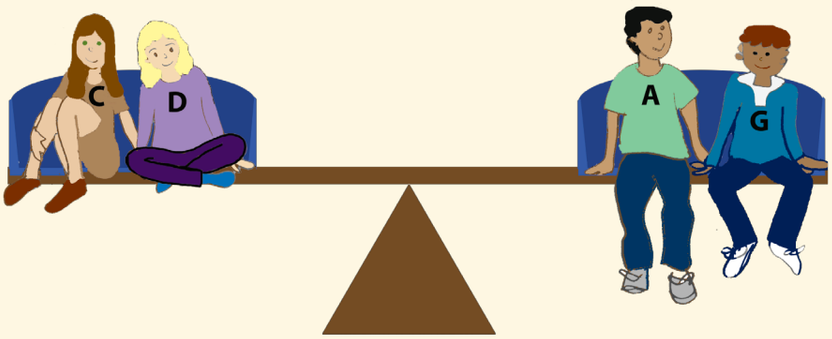
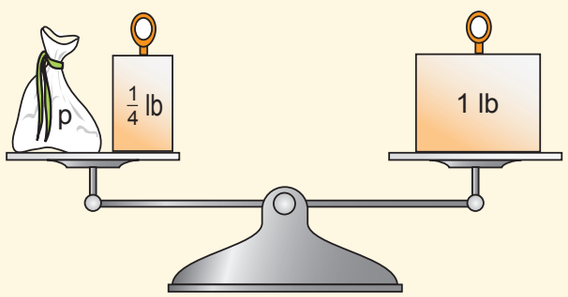


4. Mackenzie filled a bag with peanuts. The clerk put the peanuts on a scale and used weights to balance it as shown below.



a) Write an equation to show the balance shown on the scale. Use p as the variable for the unknown amount of peanuts.

b) Solve your equation for p.

5. Draw a bar diagram for each of your equations in Question 3. Solve each equation for the missing weight. Compare your answers to those in Question 3.

6. For each of the following, draw a balance scale or a bar diagram. Write an equation to match your picture. Find the solution to each equation.

a) Ceila had four ribbons of different lengths:

Green: 18 inches, Blue: 23 inches, Red: 17 inches, Orange: ? inches

The total length of the green and blue ribbons is the same as the total length of the red and orange ribbons. How long is the orange ribbon?

b) Matt drove from his house to his cousin's house. He drove 82 miles and then stopped for gas. He then drove another 20 miles to his cousin's house. On the way back, he drove the same route. After driving for 22 miles, how much farther did he have to drive before getting home?

c) Ali and Ray each bought the same amount of nuts. Ali bought $2\frac{1}{2}$ pounds of peanuts and 6 pounds of cashews. Ray bought 7 pounds of cashews. The rest of his nuts are peanuts. What is the weight of Ray's peanuts?

7. a) Write a word problem that might be solved using the equation $\$0.45 + n = \$0.82$.

b) Draw either a bar diagram or a balance scale to illustrate your equation.

c) Solve the equation for n.

8. Solve each of the following for n. Show your work or explain your thinking.

a) $825 + n = 258 + 824$

b) $n + 2\frac{1}{2} = 15 + 3\frac{1}{2}$

c) $924 + 30 = 900 + n + 4$

d) $2345 - 398 = n - 400$

e) $4998 + 3786 = n + 5000$

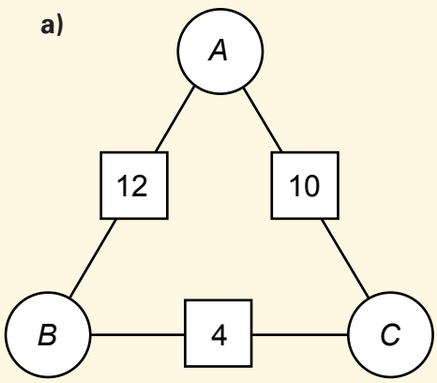
f) $8567 + 400 = 60 + 7 + 8000 + n$

On Your Own

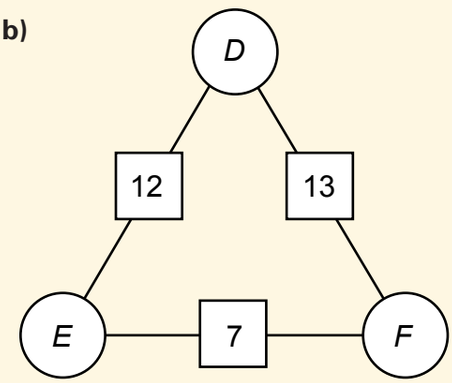
Think Beyond

9. In the following diagrams, the numbers in the corners sum to the number in the square between them. For example, in part a, $A + B = 12$; $A + C = 10$; and $B + C = 4$.
Find the value of each of the variables.

a)



b)



Think Back

10. Compute each of the following. Do not use a calculator and show your work.

a) $37 \cdot 38 =$

b) $21 \cdot 92 =$

c) $245 \div 34 =$

d) $496 \div 45 =$

e) $389 + 182 + 937 =$

f) $4028 - 329 =$

11. Write in simplest form.

a) $\frac{12}{16}$ **b)** $\frac{88}{12}$ **c)** $\frac{18}{10}$ **d)** $\frac{21}{6}$

12. Shamika bought 3 pounds of fish at $3.98 per pound.

a) How much will her bill be?

b) How much change should she get back from a $20 bill?

13. If Corinne paid $15.96 for 4 pounds of fish, how much did the fish cost per pound?

14. True or False? All squares are rectangles. Explain.