LESSON 1.2

Representing Balance with Scales, Bars and Equations

Suggested Pacing: 2 Days

In this lesson students will learn that a balance scale or a bar model can represent an algebraic equation. They will solve equations using symbolic expressions with variables as well as these visual models.

LESSON OBJECTIVES

- Students will understand equations and equality.
- Students will use balance scales and bar models to represent and solve equations with one variable.
- Students will explore the use of variables in expressions and equations.
- Students will identify the use of the commutative property of addition and multiplication.

DAY 1	MATERIALS*	ESSENTIAL *ON YOUR OWN* QUESTIONS
A Fish Story: Using Variables, Expressions, and Equations Solving Equations Using a Balance	**In Class** - Blank 100 grid with four 5 • 5 areas marked with a bold line (optional) - Computer/Internet access (optional) - 4 colors of beads (about 20 of each) per student (optional)	Questions 1–4, 10–11
DAY 2	**MATERIALS***	**ESSENTIAL *ON YOUR OWN* QUESTIONS**
Solving Equations Using Bar Diagrams Just Symbols	**In Class** - 4 colors of beads (about 20 of each) per student (optional)	Questions 5–8, 12–14

* The Think Like a Mathematician Daily Record Sheet should be used daily

MATHEMATICALLY SPEAKING

- bar diagram
- commutative property of multiplication
- equation
- expression
- solution
- solve (an equation)
- variable

Start It Off

1. Dan has six facts to learn: $6 \cdot 6 = 36$, $6 \cdot 7 = 42$, $6 \cdot 8 = 48$, $7 \cdot 7 = 49$, $7 \cdot 8 = 56$, and $8 \cdot 8 = 64$.

2. **Tip 1:** Break one of the factors into $5 +$ something and use what you already know. For example, $6 \cdot 6$ is the same as $(5 + 1) \cdot 6$. Dan already knows $5 \cdot 6 = 30$ and $1 \cdot 6 = 6$, so he can add those to find the product of $6 \cdot 6 = 30 + 6 = 36$.
   Tip 2: Compare the fact to a nine fact. He knows $9 \cdot 8 = 72$, so $8 \cdot 8$ must be 8 less, $72 - 8 = 64$.
   Tip 3: Break the 8 into 2×4. All the 8 facts could be learned by doubling the known facts with 4 as a factor. For example, $8 \times 7 = 2 \times (4 \times 7) = 2 \times 28 = 56$.
   Tip 4: Break the fact into any smaller pieces that he knows. For example, he knows $5 \cdot 8 = 40$ and $2 \cdot 8 = 16$, so $7 \cdot 8$ equals $40 + 16$.

3. The commutative property works for addition, too. $3 + 5 = 5 + 3 = 8$. The order of the addends does not affect the outcome. The commutative property does not work for division or subtraction. $3 \div 6 \neq 6 \div 3$ and $6 - 3 \neq 3 - 6$.

DAY 1 TEACHING THE LESSON

In this lesson, students will learn that the balance scale or a bar model can represent an algebraic equation. They will think about maintaining balance through compensation. For example, if one addend on the right is 1 more than one addend on the left, then the other addend on the right would have to be 1 less than the other addend on the left.

As you circulate during student work time, remember to record notes for a few students that you can transfer later to the Student Snapshot chart.

Differentiation

Think Differently: If students are having difficulty with performing mental operations involving two-digit numbers, have students simplify using multiples of ten. For instance, if they do not know how to simplify $32 \cdot 3$ have them write $30 \cdot 3$ and $2 \cdot 3$. Then write $30 \cdot 3 + 2 \cdot 3 = 32 \cdot 3 = 96$.

LESSON 1.2

Representing Balance with Scales, Bars and Equations

Start It Off

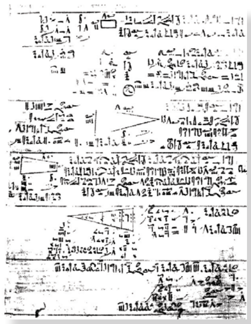
There are 100 single-digit multiplication facts. The fact with the smallest product is $0 \cdot 0 = 0$ and the fact with the largest product is $9 \cdot 9 = 81$. Dan knows his basic facts through the fives. He also knows all the nines facts and understands the commutative property of multiplication for whole numbers. It states that you can switch the order of the factors and not change their product. So if you know $4 \cdot 7 = 28$, you also know $7 \cdot 4 = 28$.

1. How many facts must Dan still learn? List them.
2. List three tips you might give Dan to learn the facts you listed.
3. Does the commutative property also work for addition, subtraction or division of whole numbers? Explain.

MATHEMATICALLY SPEAKING

- commutative property of multiplication
- variable

Thousands of years ago, people around the world began to realize they could use numbers and other symbols to record many of the things they did. Some of the symbols stood for operations like addition or multiplication. Other symbols were variables, which are letters or other symbols that stand for a number or set of numbers. One of the most famous early mathematical records is the Rhind papyrus from Egypt, which is over 3,700 years old. This is a mathematical record of many things from everyday Egyptian life.



A Fish Story: Using Variables, Expressions and Equations

In this part of the lesson students will model algebraic equations with balance scales. They will learn that the equal sign is represented by the base in the middle of the balance. Guide students to think about how the balance is affected when addends from each side have a small difference. Some students may think about moving pieces on one side to match the other side. For example, if you have $15 + 99 = 14 + n$, think about moving a piece off the 15 (to match the 14) and onto the 99 to make 100, so the right side would end up being $14 + 100$. Other students may think that if you have two more on one side, then you would have to compensate by having two more on the other side. Encourage students to articulate their thinking and to listen and paraphrase the thoughts of other students.

A Fish Story:
Using Variables, Expressions and Equations

Fish from the Nile were a big part of the Egyptian diet. Imagine Amal working in a fish market, keeping records. Amal could use variables, expressions and equations for his record keeping. An expression is a mathematical phrase made up of numbers, variables and/or operations. For example, Amal might use the variable n to stand for the weight of a swordfish in pounds. If a tuna weighs 13 pounds more than a swordfish, he might use the expression, $n + 13$ to represent the weight of the tuna.

1. The variable n stands for the weight of a swordfish in pounds. Match the expression in Column A with the words in Column B. Discuss your results with a partner.

Column A	Column B
a) $150n$	**i)** the weight of a tuna if the tuna and the swordfish together weigh 150 pounds
b) $n + 150$	**ii)** the number of swordfish if each one weighs n pounds and together they weigh 150 pounds
c) $150 - n$	**iii)** the weight of 150 swordfish that each weighs n pounds
d) $150 \div n$	**iv)** the weight of a tuna that weighs 150 pounds more than the swordfish

MATHEMATICALLY SPEAKING
- expression
- equation

When two expressions have the same value, you can write an equation. An equation is a mathematical sentence with an equal sign. An equation shows that the expressions on the two sides of the equal sign have the same value.

Solving Equations Using a Balance

Amal has three fish that weigh 12 pounds, 13 pounds and 23 pounds. Amal will use a balance and these three fish to find the weight of a fourth fish. Look at the fish on the balance below. Using the variable n to stand for the unknown weight in pounds, Amal can write the equation $12 + 23 = 13 + n$.

Differentiation

Think Differently: For students who need additional informal experience with equality, hide 2 different numbers behind a screen and another 2 different numbers behind another screen. One of the numbers under the second screen should be one less than one of the numbers under the first screen and the other should be one more. Tell students that you have the same total amount behind each screen. Reveal the value of 2 numbers behind the first and 1 of the numbers behind the second. Ask students to write an equation showing the relationship of the numbers behind both screensand then ask them to figure out the missing amount. Have them think about compensation for the difference of one between two of the numbers.

1. a) $150n$ — iii) the weight of 150 swordfish that each weighs n pounds

b) $n + 150$ — iv) the weight of a tuna that weighs 150 pounds more than the swordfish

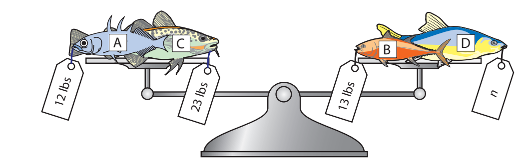
c) $150 - n$ — i) the weight of a tuna if the tuna and the swordfish together weigh 150 pounds

d) $150 \div n$ — ii) the number of swordfish if each one weighs n pounds and together they weigh 150 pounds

MATHEMATICALLY SPEAKING
- solve (an equation)
- solution

When you solve an equation, you find a value that makes the equation true. This value is called the solution of the equation.

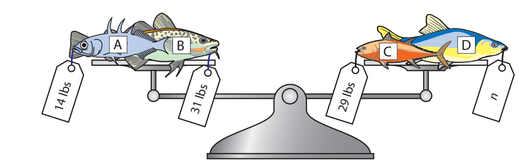
2. a) Write an equation for the weights of the fish on the scale below. Use n in your equation for the unknown weight in pounds.



   b) Solve your equation for the missing weight. How might you do this without finding the total weight of the fish on the left side?

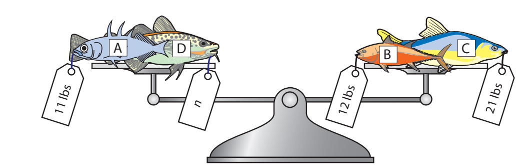
3. What value for n makes $12 + 23 = 13 + n$ a true statement? How might you determine this answer by thinking about balance and equality without using a pencil and paper or a calculator? How does your method relate to the balance scale?

4. a) Write an equation for the weights of the fish on the scale below. Use n in your equation for the unknown weight in pounds.



   b) Solve your equation for the missing weight. Explain how you could find this weight without using a pencil and paper or a calculator.

5. a) Write an equation for the weights of the fish on the scale below. Use n in your equation for the unknown weight in pounds.



   b) Solve your equation for the missing weight. Explain your reasoning to a partner.

Solving Equations Using Balance

2. a) $12 + 23 = 13 + n$

   b) $n = 22$

   Since fish B is one more pound that fish A, then fish D must be one less pound than fish C.

3. Possible answer: Since 13 is 1 more than 12, the value of the variable must be 1 less than 23, making it 22 pounds. This relates to a balance scale, because the left side of the equation is like the left side of the scale, the equal sign is like the fulcrum, and the right side of the equation is like the right side of the scale.

4. a) $14 + 31 = 29 + n$

   b) Possible answer: Since 29 is 2 less than 31, n must be 2 more than 14 or 16.

5. a) $11 + n = 12 + 21$

   b) Fish *D* would be 22 pounds. Possible explanation: Since fish *B* is one pound more than fish *A*, then the other two fish would also have a difference of one pound, with fish *D* being heavier than fish *C*. Since fish *C* is 21 pounds, then fish *D* must be $21 + 1 = 22$ pounds.

Summarize Day 1

Wrap up this lesson by asking students to make up their own balance problems and share the problems and solutions with the class.

DAY 2 TEACHING THE LESSON

Solving Equations Using Bar Diagrams

If you have Internet access and do not plan to do the optional lesson at the end of this section, you might start the lesson with one of the activities from the virtual Pan Balance from the Illuminations website. If not, ask students to discuss their answers to the Write About It, Question 1 from On Your Own.

Solving Equations Using Bar Diagrams

MATHEMATICALLY SPEAKING

- bar diagram

Mei Ling, who had attended school in Singapore, showed the class a different way to solve these problems. She used bar diagrams. To solve Question 2, Mei Ling drew the following bar diagram. Because the total weight on the left side of the scale is the same as the total weight on the right, she drew the bars for the two sides with the same length to show that the weights were equal. She also knew that 13 pounds is a little more than 12 pounds, so she made the 13-pound section of the bottom bar longer than the 12-pound section of the top bar.

12 pounds	23 pounds
13 pounds	*n* pounds

6. Explain how Mei Ling might find the weight of the fourth fish using her diagram.

7. Draw a bar diagram to solve the fish problem in Question 4 or Question 5. Show how you use the diagram to find the missing weight.

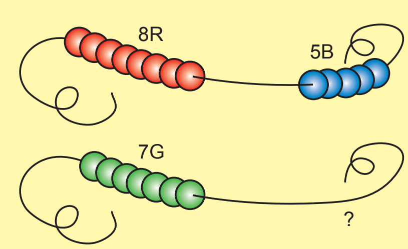
8. Bar diagrams can also be used for other problems. For the problems below, draw a bar diagram. Explain what the variable stands for. Then write an equation and solve for the variable.

a) Jerra's age plus her mother's age is the same as Jason's age plus his father's age. Jerra is 12, Jason is 11 and Jason's father is 42. How old is Jerra's mother?

b) Notebooks cost $2.95 each and two pens cost $1.95. A set of colored pencils and four pens have the same cost as three notebooks. How much does the set of colored pencils cost?

Differentiation

Think Differently: Use two beaded strings as an informal way to illustrate this situation: There are different numbers of beads of two colors on the top string that have an equivalent total number of beads to those of a third and fourth color on the bottom string. Leave one color hidden and challenge students to find the missing amount. Possible questions: 8 red and 5 blue is the same length as 7 green and ?; 9 red and 7 blue on the top and 8 green and ? on the bottom; 12 red and 3 blue; 13 green and ? on the bottom. As you use larger numbers students may decide that the bar diagram is more efficient: 29 red and 47 blue; 30 red and _?_.



In this section, assure students that the bar diagrams do not have to have exact proportional measurements, but the sections can be roughly estimated.

Differentiation

Think Differently: As Mei Ling introduces the bar model, this is a good time for students from other cultures to share any methods they might have learned to solve equations.

6. Mei Ling might notice that 13 is 1 more than 12, so *D* would be 1 less than 23, which is 22 pounds.

Just Symbols

In each of the following equations, the variable n is used to stand for a missing number. This is similar to representing the unknown weights of the fish above. For each equation, try to find the value of n just by reasoning about balance and equality. Draw a balance or bar diagram if it helps you. Be prepared to explain your reasoning.

9. $4832 + 197 = n + 200$

10. $49 + n = 73 + 50$

11. $23 + n = 14 + 24$

12. $51 - n = 50 - 25$

13. $78 + 32 = 80 + n$

Wrap It Up

How does the equation $23 + n = 18 + 22$ relate to balance scales and bar diagrams? Sketch a balance scale and a bar diagram to show this. Explain how you would solve this problem without using pencil and paper or a calculator. What is the meaning of the n in this equation?

MATHEMATICALLY SPEAKING

- bar diagram
- commutative property of multiplication
- equation
- expression
- solution
- solve (an equation)
- variable

7. Students will choose one of these to complete.

Bar diagram for Question 4:

$B = 31$	$A = 14$
$C = 29$	$D = n$

Since 31 is 2 more than 29, then D would be 2 more than 14. $D = 16$ pounds.

Bar diagram for Question 5:

$A = 11$	$D = n$
$B = 12$	$C = 21$

Since $11\frac{1}{2}$ is 1 less $12\frac{1}{2}$ then D would have to be 1 more than 21. $D = 22$ pounds.

NOTE Many students may need help interpreting and organizing the information in Questions 7 and 8. Note that the length of the bar on the top must be equal to the length of the bar on the bottom to represent the equality of the two sides of the equation. Guide students to first determine which parts go together in one row and then fill in the known values. Ask students to estimate about how long the bar should be to represent each part. Encourage students to write the equations by filling in the amounts they know and leaving the unknown listed as n.

8. **a)**

Jerra, $A = 12$	Mother, $M = n$
Jason, $B = 11$	Father, $F = 42$

The variable n represents Jerra's mother's age.

$12 + n = 11 + 42$

Since 12 is 1 more than 11, then n is 1 less than 42. The mother's age is 41.

b)

Notebook, $K = 2.95$	$K = 2.95$	$K = 2.95$
Pen, $P = 1.95$	$P = 1.95$	Colored Pencils, $c = n$

$3 \cdot 2.95 = 2 \cdot 1.95 + n$

Since two of the notebooks are each \$1.00 more than the pens, then the colored pencils must be \$2.00 more than the third notebook. The colored pencils cost \$4.95.

Just Symbols

Students may share strategies for such as inverse operations (for example, using subtraction to "undo" the addition), compensation (for example, adding one to the minuend since the subtrahend has one more or adjusting the first addend by the difference of the second addends), and some may use guess, test, and refine. As students offer solution methods, ask them if there is another way to think about these problems. Invite students who "guessed" to share their process of thinking as they "refined" their guess, and how they knew whether to make the next guess smaller or larger and by how much. Ask guiding questions about the relationship of the known addends from each side, and how that can help to solve for the missing addend. Make sure students demonstrate that their answer is correct by replacing the variable with their solution and simplifying the expressions into a true statement.

Differentiation

Think Differently: Some students may prefer to use a calculator to guess, test, and refine to get a closer guess for the solution for *n*.

Differentiation

Think Differently: Before introducing "Just Symbols" choose a few struggling students and pull them aside to go over the strategies of looking for compatible numbers to simplify the computation. Once these students are comfortable with the operations, give the equations to the whole class, and watch your struggling students shine as other students attempt complex computations.

Differentiation

Think Differently: Ask students to write their own equations with two values on one side and a value and a variable on the other. Require that one of the values on the left be 1 or 2 units away from one of the values on the right, meaning that the solution will be 1 or 2 units away from the other value. They may then trade with a partner.

9. $n = 4{,}829$

10. $n = 74$

11. $n = 15$

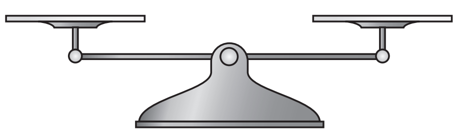
12. $n = 26$

13. $n = 30$

Wrap It Up

Talk Moves As students discuss their thinking about how the equation is like a balance scale or a bar diagram, remind them to use talk moves of listening and agree/disagree. Encourage students to share mental math strategies for finding the value of n, such as compensating for the difference between two of the addends. Make sure that students understand the meaning of n and can explain a strategy for solving the equation.

Possible Answer: $23 + n = 18 + 22$ is like having four fish on a balance scale. I know the weights of three of the fish, but not the fourth. The weight of the fish that is unknown is n; it is called the variable. To solve the equation, I know that 23 is 1 more than 22, so n must be 1 less than 18. $n = 17$.



Using a bar diagram I would put 23 and n on the top row and 22 and 18 on the bottom row. This diagram would show that each row has to have the same total length or value. To find the unknown value, I would compensate for the difference between 23 and 22 by subtracting 1 from 18. $n = 17$.

23	n
22	18

Reflect

Use these questions to help you reflect on the lesson and plan for future instruction.

- Did students demonstrate an understanding of how an equation is similar to a balance scale?
- Did students demonstrate an understanding of how an equation is similar to a bar model?
- Were students successful in using compensation when thinking about balance and equality?
- Did students correctly use the variable n?
- Were students able to solve for the variable and check their answer?
- Are all students, especially English language learners, effectively communicating their thinking?

LESSON 1.2 SECTION 1

On Your Own

Write About It

1. a) Solve for n by reasoning about balance and equality without using a diagram. $n + 19 = 20 + 82$. Explain your method.

   b) Explain how to solve the same equation using a bar diagram and then using a balance scale.

2. For each of the following, the variable a stands for the number of apples in a crate. Column A describes the number of oranges in a crate. Match each expression in Column A with the words in Column B.

Column A	Column B
a) $3a$	i) The number of oranges in a crate is three more than the number of apples in a crate.
b) $a + 3$	ii) There are three times as many oranges in a crate as apples.
c) $a - 3$	iii) There are three more apples in the crate than oranges.
d) $a \div 3$	iv) There are one-third as many oranges in a crate as apples.

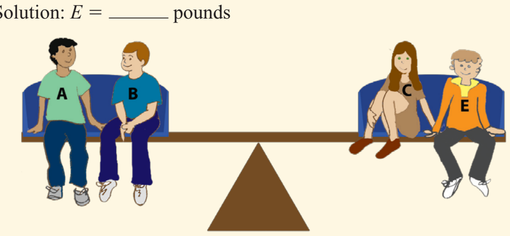
3. For each balance, write an equation. The first equation is written for you. Solve each equation for the unknown weight.

A = Antonio	85 pounds
B = Bob	97 pounds
C = Caitlin	80 pounds
D = Diane	102 pounds

E = Enrico	? pounds
F = Fran	? pounds
G = Gary	? pounds

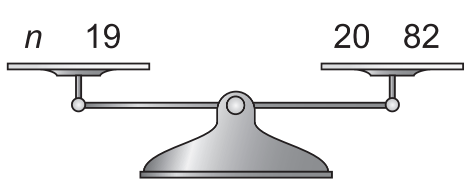
   a) Equation: $85 + 97 = 80 + E$

   Solution: E = ______ pounds



On Your Own

1. **Write About It**



a) Possible answer: Since 20 is 1 more than 19, the left side needs 1 more than 82 to balance. Therefore, n is 83.

Possible alternate answer: First I would find the total weight on the right side of the scale, $20 + 82 = 102$. Then I find the value that, when added to 19, equals 102. I use subtraction to find this: $102 - 19 = 83$. Or, I think 19 is close to 20; it is one less. In order to balance, n must be 1 more than 82, so $n = 83$.

Possible alternate answer: The unknown value in this equation is n. To solve the equation, there is only one value for n that makes the statement true. That is $n = 83$. It takes the place of the number that, when added to 19, is equal to 20 and 82.

b)

n	19
82	20

To use a bar diagram, I draw the diagram showing that the length of both bars is the same since the expressions are equal. Since 19 is one less than 20, n must be one more than 82, so $n = 83$.

To use a balance scale, I would draw a scale with $n$and 19 on one side and 82 and 20 on the other. If I take away 19 from both sides, the right side would read 82 and 1. This would show that n is equal to 83.

2. a) $3a$ ii) There are three times as many oranges in a crate as apples.

b) $a + 3$ i) The number of oranges in a crate is three more than the number of apples in a crate.

c) $a - 3$ iii) There are three more apples in the crate than oranges.

d) $a \div 3$ iv) There are one-third as many oranges in a crate as apples.

3. a) $E = 102$ pounds

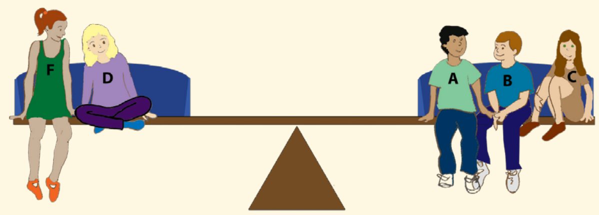
b) Equation: $F + 102 = 85 + 97 + 80$, Solution: $F = 160$ pounds

c) Equation: $80 + 102 = G + 85$, Solution: $G = 97$ pounds

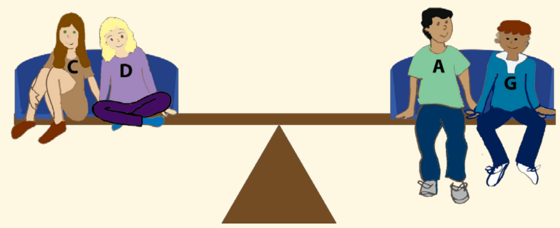
4. a) $p + \frac{1}{4} = 1$

b) $p + \frac{1}{4} - \frac{1}{4} = 1 - \frac{1}{4}$, $p = \frac{3}{4}$ pounds

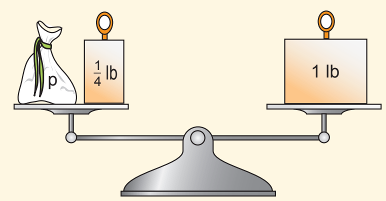
b)



c)



4. Mackenzie filled a bag with peanuts. The clerk put the peanuts on a scale and used weights to balance it as shown below.



a) Write an equation to show the balance shown on the scale. Use p as the variable for the unknown amount of peanuts.

b) Solve your equation for p.

5. Draw a bar diagram for each of your equations in Question 3. Solve each equation for the missing weight. Compare your answers to those in Question 3.

5. a) $E = 102$ pounds. This is the same as in Question 3.

$A = 85$ pounds	$B = 97$ pounds
$C = 80$ pounds	E

b) F = 160 pounds. This is the same as in Question 3.

F		$D = 102$ pounds
$A = 85$ pounds	$B = 97$ pounds	$C = 80$ pounds

6. For each of the following, draw a balance scale or a bar diagram. Write an equation to match your picture. Find the solution to each equation.

   a) Ceila had four ribbons of different lengths:

      Green: 18 inches, Blue: 23 inches, Red: 17 inches, Orange: ? inches

      The total length of the green and blue ribbons is the same as the total length of the red and orange ribbons. How long is the orange ribbon?

   b) Matt drove from his house to his cousin's house. He drove 82 miles and then stopped for gas. He then drove another 20 miles to his cousin's house. On the way back, he drove the same route. After driving for 22 miles, how much farther did he have to drive before getting home?

   c) Ali and Ray each bought the same amount of nuts. Ali bought $2\frac{1}{2}$ pounds of peanuts and 6 pounds of cashews. Ray bought 7 pounds of cashews. The rest of his nuts are peanuts. What is the weight of Ray's peanuts?

7. a) Write a word problem that might be solved using the equation $\$0.45 + n = \$0.82$.

   b) Draw either a bar diagram or a balance scale to illustrate your equation.

   c) Solve the equation for n.

8. Solve each of the following for n. Show your work or explain your thinking.

   a) $825 + n = 258 + 824$

   b) $n + 2\frac{1}{2} = 15 + 3\frac{1}{2}$

   c) $924 + 30 = 900 + n + 4$

   d) $2345 - 398 = n - 400$

   e) $4998 + 3786 = n + 5000$

   f) $8567 + 400 = 60 + 7 + 8000 + n$

c) $G = 97$ pounds. This is the same as in Question 3.

$C = 80$ pounds	$D = 102$ pounds
G	$A = 85$ pounds

6. a)

Green = 18"	Blue = 23"
Red = 17"	Orange = ?"

Equation:
$18 + 23 = 17 + o$.
Solution: $o = 24$ inches.

The green and blue ribbons together are the same length as the red and orange ribbons together. Since the green is one inch longer than the red, in order to be equal, the orange must be one inch longer than the blue, so orange is 24" long. 18" + 23" = 17" + 24".

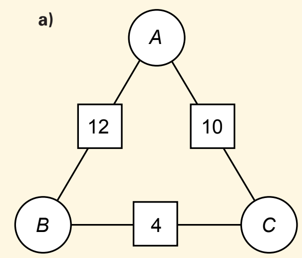
LESSON 1.2 SECTION 1

On Your Own

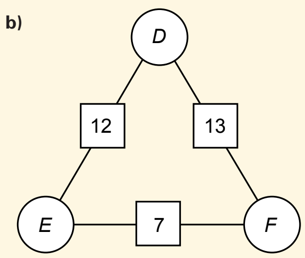
Think Beyond

9. In the following diagrams, the numbers in the corners sum to the number in the square between them. For example, in part a, $A + B = 12$; $A + C = 10$; and $B + C = 4$.
Find the value of each of the variables.

a)



b)



Think Back

10. Compute each of the following. Do not use a calculator and show your work.

a) $37 \cdot 38 =$

b) $21 \cdot 92 =$

c) $245 \div 34 =$

d) $496 \div 45 =$

e) $389 + 182 + 937 =$

f) $4028 - 329 =$

11. Write in simplest form.

a) $\frac{12}{16}$ **b)** $\frac{88}{12}$ **c)** $\frac{18}{10}$ **d)** $\frac{21}{6}$

12. Shamika bought 3 pounds of fish at \$3.98 per pound.

a) How much will her bill be?

b) How much change should she get back from a \$20 bill?

13. If Corinne paid \$15.96 for 4 pounds of fish, how much did the fish cost per pound?

14. True or False? All squares are rectangles. Explain.

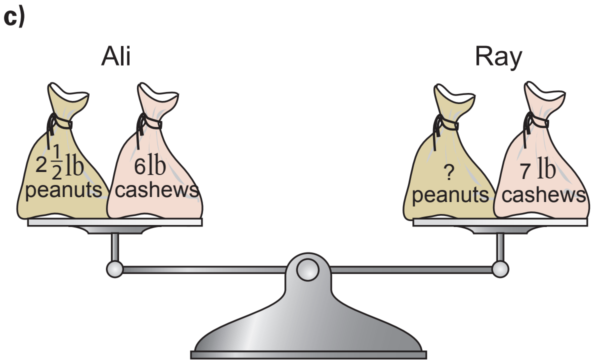
b)

82 miles	20 miles
? miles	22 miles

Equation:
$82 + 20 = 22 + m$.
Solution: $m = 80$ miles.

Following the same route, Matt will drive the same distance each way. On the way to his cousin's house, he drives two legs, 82 miles then 20 miles. On the first leg of his return trip he drives 22 miles; this is 2 miles longer than the last leg of his trip to Louisville. Therefore, he travels 2 miles less on the second leg of the return trip than on the first leg of the trip home, or 80 miles to go.

c)



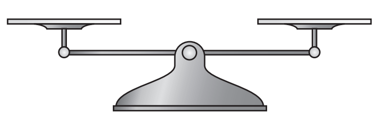
$2\frac{1}{2}$ lbs. peanuts	6 lbs. cashews
? lbs. peanuts	7 lbs. cashews

Equation: $2\frac{1}{2} + 6 = p + 7$.
Solution: $p = 1\frac{1}{2}$ pounds.

They bought the same total amount of nuts. Since Ray bought one more pound of cashews than Ali, Ali purchased one more pound of peanuts than Ray. Ali purchased $2\frac{1}{2}$ pounds of peanuts, so Ray purchased $1\frac{1}{2}$ pounds of peanuts.

7. **a)** Answers will vary. For example: Brian went to the bookstore and selected a notepad that cost \$0.45. He also saw a pen he wanted, but couldn't find the price of it. When he checked out, the total of these two items was \$0.82. What was the cost of the pen?

b) \$0.45, pen   \$0.82



\$0.45	pen $= n$
\$0.82	

c) $n = \$.37$

8. **a)** $825 + n$ must equal the sum of 258 and 824. Since 825 is one more than 824, n must be one less than 258. $n = 257$.

b) Since $3\frac{1}{2}$ is one more than $2\frac{1}{2}$, 15 must be one less than n. $n = 16$.

c) $924 + 30 = 954$ and
$954 - 900 - 4 = 50$.
$n = 50$. Once you get 954, think expanded notation. $954 = 900 + 50 + 4$.

d) 400 is 2 more than 398, so n must be 2 more than 2345 in order for the sides to balance and have equal differences. $n = 2347$.

e) 5000 is 2 more than 4998, so n must be 2 less than 3786. $n = 3784$.

f) Expand the first term and rewrite as $8567 + 400 = 8000 + 500 + 60 + 7 + 400$. Use remove and replace techniques and fill in $8000 + 500 + 60 + 7 + 400 = 60 + 7 + 8000 + n$. Remove 8000 from both sides, remove 60 from both sides, and remove 7 from both sides. What's left is $500 + 400 = n$, so $n = 900$.

9. **Think Beyond**

a) $A = 9$, $B = 3$, $C = 1$

b) $D = 9$, $E = 3$, $F = 4$

10. **Think Back**

a) 1,406

b) 1,932

c) 7.21 or $7\frac{7}{34}$

d) 11.02 or $11\frac{1}{45}$

e) 1,508

f) 3,699

11. **a)** $\frac{3}{4}$

b) $7\frac{1}{3}$

c) $1\frac{4}{5}$

d) $\frac{7}{2}$ or $3\frac{1}{2}$

12. **a)** \$11.94

b) \$8.06

13. \$3.99

14. True. Squares are a special category of rectangles; rectangles with all sides congruent.