PROBLEM STRINGS LESSON

5.1 Graphing Quadratic Functions 1

At a Glance

$y = x^2$

$y = 3x$

$y = x^2 + 3x$

$y = x(x+3)$*

$y = x^2 - 3x$

*optional problem

Objectives

The goal of this Graphing Quadratic Functions series of problem strings is to help students develop a network of understandings about quadratic functions, connecting and using multiple representations. This string begins to develop the graphing strategies of adding ordinates and factoring to find zeros.

Placement

This is the first in a series of 2 strings to develop the graphing strategies of adding ordinates and factoring to find zeros. This string works with the form $y = x^2 + bx$. The next string will continue the work and allow students the opportunity to apply the strategies to the form $y = x^2 + bx + c$.

You could deliver this string after students have experience with the parent function $y = x^2$ and transformations and as students are graphing quadratic functions.

This string would be a nice pre-chapter problem string for textbook Chapter 5: Quadratic Functions and Relations.

Guiding the Problem String

This problem string is an opportunity to assess students' prior understanding. Listen, watch, and ask probing questions. As you deliver the third problem, wonder aloud about the combination of the first two problems: What does it even mean to add a parabola and a line? The fourth problem is provided if no students suggest using the factored form to graph the third problem. For the last problem, explore with students the adding ordinates strategy both by adding $-3x$ to x^2 and by subtracting $3x$ from x^2.

About the Mathematics

The strategy we are calling adding ordinates comes from the (abscissa, ordinate) language of ordered pairs. It simply means combining the y-values from the parts of a combination function. In this string, we look at combining a linear function with a quadratic function. For $y = x^2 + 3x$, you look to add all of the y-values of $y = x^2$ to the y-values of $y = 3x$ at each respective x-value. For example, (1, 1) for $y = x^2$ and (1, 3) for $y = 3x$ becomes (1, 4) for $y = x^2 + 3x$.

The ideas therein, that the quadratic function dominates the whole function, thus making the combination function also a quadratic function, portends that general meaning of polynomials, that is that the term of highest degree dominates the polynomial.

But this form, $y = x^2 + bx$, also begs to be factored into $y = x(x+b)$, where the roots are $x = 0, -b$. The vertex can then be found by considering symmetry between the roots or as always by $\frac{-b}{2a}$ for the x-coordinate, though we acknowledge the use of $\frac{-b}{2a}$ if students use it, we don't model it for the rest of the class yet as typically it is not widely developed at the curricular point where we would deliver this string. However, if it has been developed, then it might be appropriate to model for the whole class.

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Sample Interactions

Use the following as you plan how to elicit and model student strategies. This is not meant as a script, but as a view into the relationships involved and the intent of the problem string.

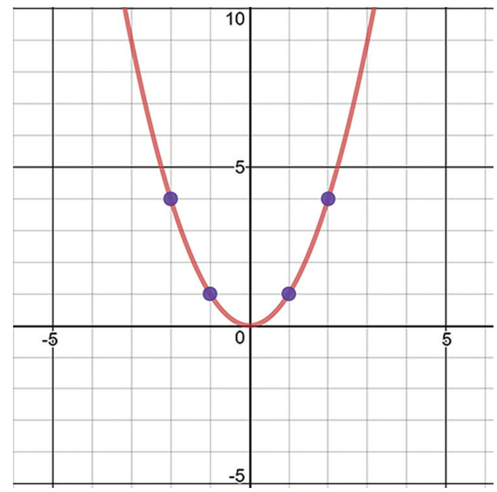
Teacher: *You have seen this function before,* $y = x^2$*. Sketch a quick graph on your paper and label some important points.*

Students work and the teacher circulates, looking for students who have graphed the points shown.

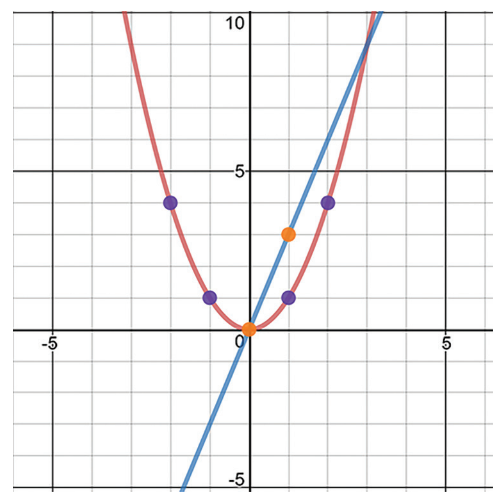
Teacher: *Please tell us which points you chose.*

Student: (0, 0), (1, 1), (2, 4)

Student: *And the points on the other side,* (–1, 1), (–2, 4).



Teacher: *Okay, great. What about this function,* $y = 3x$*? What does it look like? Label a couple of important points.*



Teacher: *I wonder what the function would look like that is a combination of the two functions we have already? That is our next problem:* $y = x^2 + 3x$ *. Predict first. What do you think the graph might look like? Turn and talk about your prediction.*

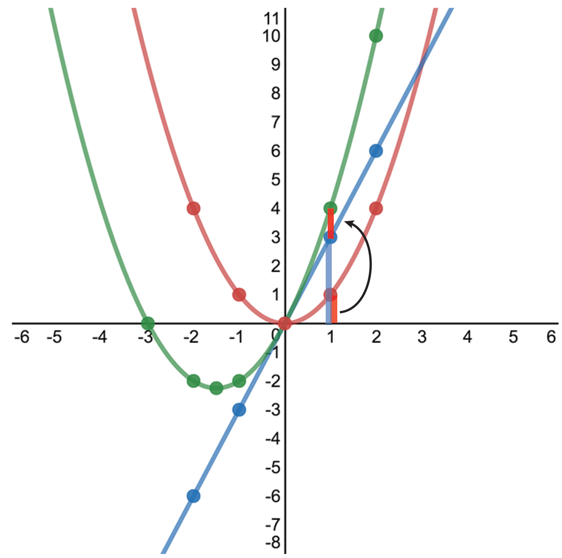
Students partner talk, while the teacher listens in.

Teacher: *What does* $y = x^2 + 3x$ *look like? Sketch a graph.*

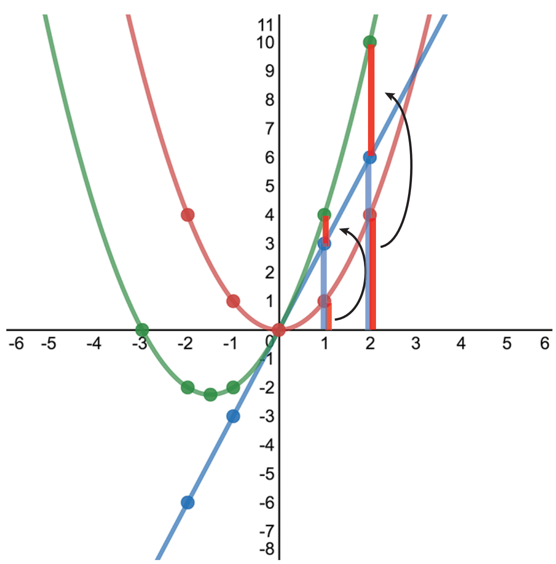
Students work and the teacher circulates, looking for students who are adding ordinates and asking them to share. The teacher uses braces to highlight the *y*-values of each component function, $y = x^2$ and $y = 3x$ and how they add together to be the *y*-value of the combination function, $y = x^2 + 3x$.

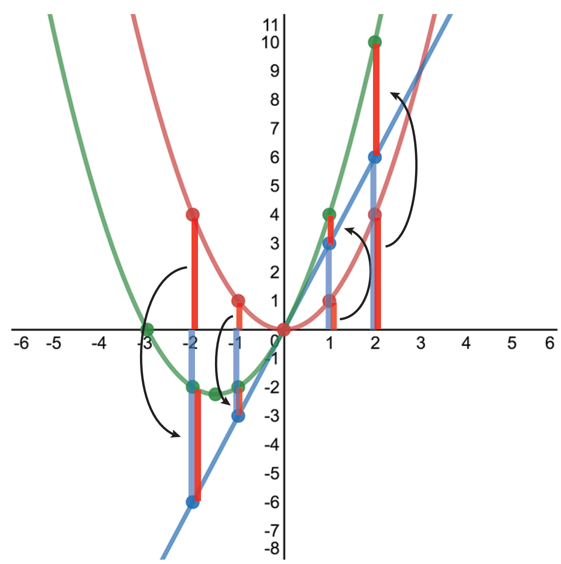
(If no students add ordinates, the teacher could ask:

How do the points at $x = 1$ on all three functions relate? What about at other *x*-values? Can we think about the function $y = x^2 + 3x$ as the addition of all of the y = values of $y = x^2$ and $y = 3x$?)



The teacher continues to have students share adding ordinates, highlighting the values for 3–4 key points.





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Teacher: *So, what's happening? Why does adding a parabola to a line result in a parabola? Think about that. When your partner is ready, turn and talk to your partner about your thinking.*

Students discuss in pairs while the teacher listens in.

Teacher: *Let's focus on the first quadrant? Why does the combination function increase? In other words, why is the green function positive and increasing in the first quadrant?*

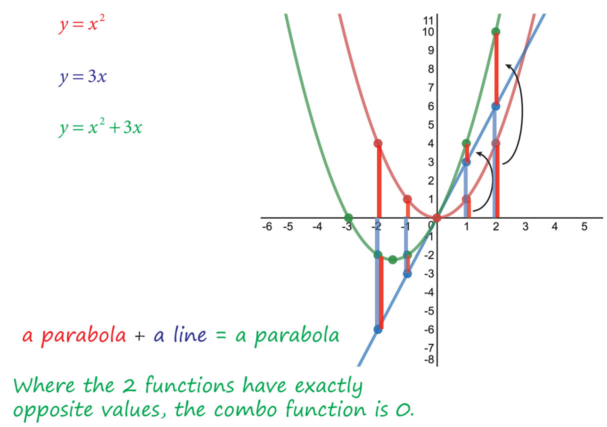
Why does the combination function increase again over in the second and third quadrants? Shouldn't it be decreasing because the line is decreasing?

Which function is growing or decreasing faster in the third and fourth quadrant?

Based on all of that, what could we say about a parabola plus a line?

What's happening at $x = 0$ and $x = -3$? Why?

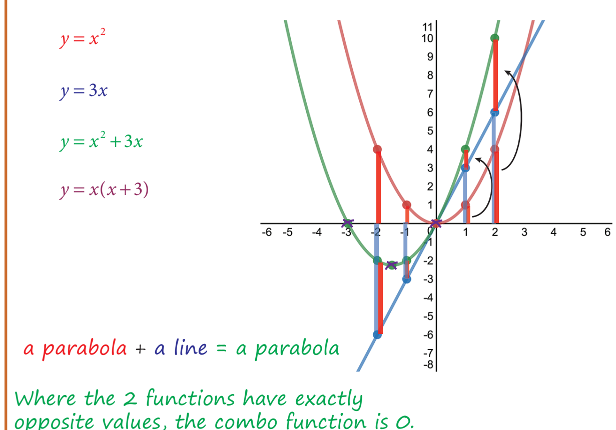
The teacher records student generalizations.



No students had used a "factoring to find zeros" strategy, so the teacher uses the optional next problem, $y = x(x+3)$.

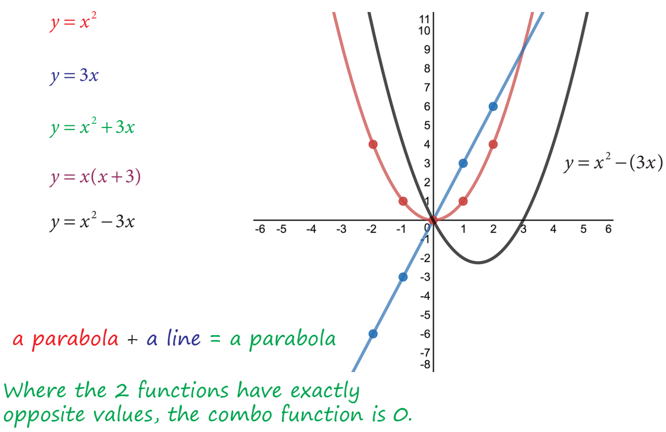
Teacher: *The next problem today is* $y = x(x+3)$. *Tell me about the graph. I wonder which strategy you might use.*

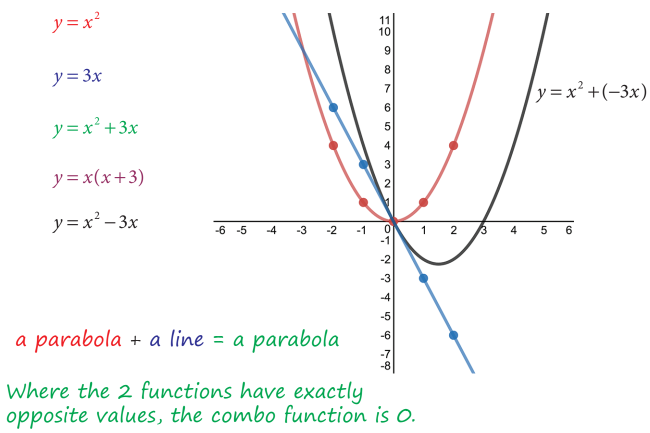
Students work, while the teacher circulates to finds students to share as the teacher models solving for the zeros $x = 0, -3$ and then using symmetry to find the vertex. Notice the purple x's to mark the x-intercepts and the vertex.



Teacher: *The last problem today is* $y = x^2 - 3x$ *. Tell me about the graph. I wonder which strategy you might use.*

Students work, then the teacher models both an adding ordinates strategy and a factoring strategy. When discussing the adding ordinates strategy, the teacher elicits from students both the ideas of subtracting $3x$ from x^2 and adding $-3x$ to x^2.

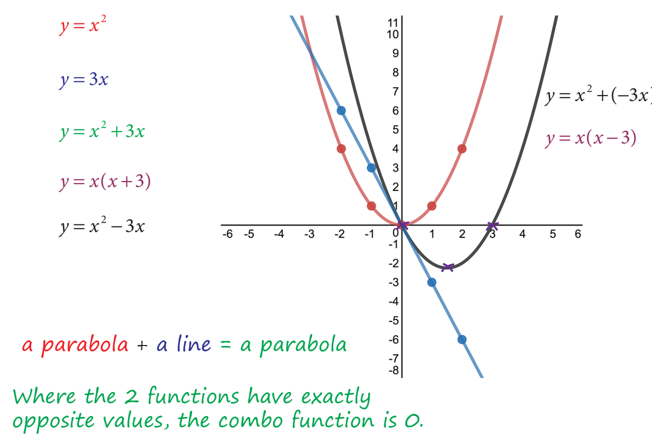




Teacher: *What about the factoring strategy? Let's put that up here too.*

What are the x-intercepts?

How did you find the vertex?



Teacher: *How would you summarize some of the things that came up in this string today?*

Elicit the following:

- *A quadratic function combined with a linear function behaves like a quadratic function and that is why the combination function is a quadratic function.*
- *The long run behavior of the linear part pulls on the long run behavior of the quadratic part but does not change the overall long run behavior of the combination function.*
- *You can find the graph of a combination function by adding the y-values of the parts.*
- *If the expression factors nicely, you can use the factors to find the x-intercepts and halfway between x-intercepts is the vertex.*

Sample Final Display

Your display could look like this at the end of the problem string:

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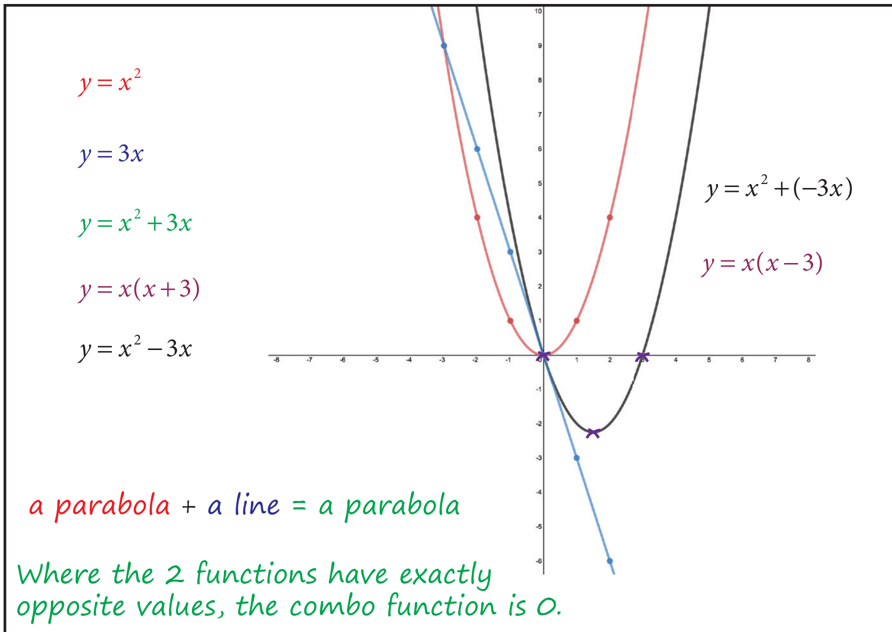

Facilitation Notes

This version of the problem string lists short notes for important teacher moves during the string. After you've done the string yourself and studied the relationships involved, you might make similar notes for the things you want a reminder of or deem important.

$y = x^2$	Seen before. Quickly graph, note important points. Sketch graph on board.
$y = 3x$	Seen before too! Quickly graph, important points. Add graph to previous on board.
$y = x^2 + 3x$	I wonder what it would look like to combine? Sketch. Could you use the important points to help? How? How does end behavior of parts affect end behavior of combo? If student uses factored form, share, add zeros to graph.
$(y = x(x+3))$	Optional Sketch a graph. Share. How is this connected to previous? Why? Which strategy do you prefer, factor-zeros or adding functions?
$y = x^2 - 3x$	I wonder how you'd sketch this? Share. Add to graph. Share factor-zeros strategy. Wondere about -3x versus +(-3x). How is this problem connected to previous? Why? How does end behavior of parts affect end behavior of combo?