LESSON

1.2

This lesson gives more experience with geometric sequences, both decreasing and increasing.

COMMON CORE STATE STANDARDS

APPLIED	DEVELOPED	INTRODUCED
	F.IF.3 F.BF.2 F.LE.1c F.LE.2	

Objectives

- Discover applications involving geometric sequences
- Use geometric sequences to model growth and decay situations
- Understand the physical limitations of models

Vocabulary

decay
growth
principal
simple interest
compound interest

Materials

- balls (racquetballs, basketballs, or handballs work well)
- video camera
- paper and colored markers
- motion sensors or meter sticks, *optional*
- Calculator Notes: Looking for the Rebound Using the EasyData App; Entering Data; Plotting Data; Tracing Data Plots; Sharing Data

Launch

Which of the following can be modeled as growth and which can be modeled as decay? Explain.

i. The temperature of hot chocolate
ii. Food left in a locker
iii. Amount of money in a savings account
iv. Value of a car
v. Height of a bouncing ball
vi. Population of a city

LESSON 1.2

Modeling Growth and Decay

Each sequence you generated in the previous lesson was either an arithmetic sequence with a recursive rule in the form $u_n = u_{n-1} + d$ or a geometric sequence with a recursive rule in the form $u_n = r \cdot u_{n-1}$. You compared consecutive terms to decide whether the sequence had a common difference or a common ratio.

In most cases you have used u_1 as the starting term of each sequence. In some situations, it is more meaningful to treat the starting term as a zero term, or u_0. The zero term represents the starting value before any change occurs. You can decide whether it would be better to begin at u_0 or u_1.

EXAMPLE A

An automobile depreciates, or loses value, as it gets older. Suppose that a particular automobile loses one-fifth of its value each year. Write a recursive formula to find the value of this car when it is 6 years old, if it cost $23,999 when it was new.

Consumer **CONNECTION**

The *Kelley Blue Book*, first compiled in 1926 by Les Kelley, annually publishes standard values of every vehicle on the market. Many people who want to know the value of an automobile will ask what its "Blue Book" value is. The *Kelley Blue Book* calculates the value of a car by accounting for its make, model, year, mileage, location, and condition.

Solution

Each year, the car will be worth $\frac{4}{5}$ of what it was worth the previous year. Therefore the sequence has a common ratio, which makes it a geometric sequence. It is convenient to start with $u_0 = 23{,}999$ to represent the value of the car when it was new so that u_1 will represent the value after 1 year, and so on. The recursive formula that generates the sequence of annual values is

$u_0 = 23,999$ Starting value.

$u_n = 0.8 \cdot u_{n-1}$ where $n \geq 1$ $\quad \frac{4}{5}$ is 0.8.

Use this rule to find the 6th term.

After 6 years, the car is worth $6,291.19.

fx = 0.8*B7

	A	B
1	n	u_n
2	0	23999
3	1	19199.2
4	2	15359.36
5	3	12287.488
6	4	9829.9904
7	5	7863.99232
8	6	6291.193856

In situations like the problem in Example A, it's easier to write a recursive formula than an equation using x and y.

ELL

You might start by asking whether the price of a car should go up, down, or stay the same as it gets older and why. What if the car is a classic sports car in mint condition? Link this to growth and decay. Check reading and verbal comprehension of terms like growth, decay, rebound, etc.

Support

Spend time on the Launch, discussing when each situation might be an example of growth and when it might be an example of decay. Link the specific notation of recursive rules (u_n notation) with the verbal description of a sequence by using starting value and rule. Use the **Whole Class** version of the Investigation.

Advanced

Focus on clear understanding of the specific notation of recursive rules. As an extension or challenge, you might explore finding a rule based on u_0 and u_2 or u_1 and u_4. Use the **One Step** version of the Investigation.

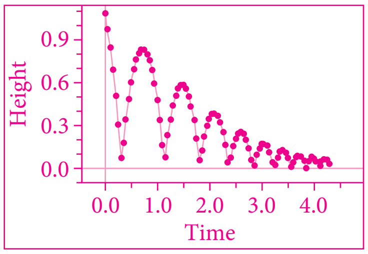
INVESTIGATION

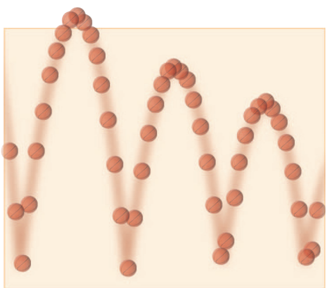
Looking for the Rebound

YOU WILL NEED
- a ball
- a video camera
- paper to make ruler
- color markers

When you drop a ball, the rebound height becomes smaller after each bounce. In this investigation you will write a recursive formula for the height of a real ball as it bounces.

Step 1





Procedure Note

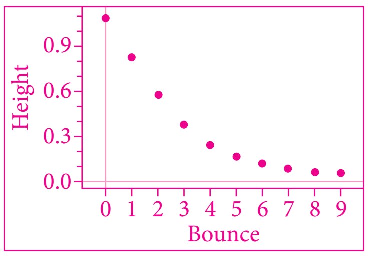
Collecting Data

1. Create a color ruler by marking each centimeter with rotating colors (Use at least three colors), clearly mark each 10 cm. Attach the ruler to a wall.
2. Sit or kneel with video camera at least 3 meters (10 feet) from the wall.
3. Drop the ball as close to the wall as you can. Record the initial drop and the first few bounces (approximately 5 seconds).

Step 2

Bounce Number	Rebound Height (m)
0	1.081
1	0.830
2	0.578
3	0.377
4	0.245
5	0.166
6	0.119
7	0.084
8	0.064
9	0.059

Step 3



Step 5

$u_0 = 1.00$ and $u_n = 0.64 \cdot u_{n-1}$ where $n \geq 1$; 1.00, 0.64, 0.41, 0.26, 0.17, 0.11

Step 1 Scroll through the video to record the initial height and subsequent heights when the ball reaches the top of each bounce. You will not be able to read the numbers on the ruler but should be able to use the colors to calculate the height.

Step 2 Transfer the data to your calculator in the form (x, y), where x is the time since the ball was dropped, and y is the height of the ball. Trace the data graphed by your calculator to find the starting height and the rebound height after each bounce. Record your data in a table.

Step 3 Graph a scatter plot of points in the form (*bounce number, rebound height*). Record the graphing window you use.

Step 4 Compute the rebound ratio for consecutive bounces.

$$\textit{rebound ratio} = \frac{\textit{rebound height}}{\textit{previous rebound height}}$$

sample answer: 0.77, 0.70, 0.65, …

Step 5 Decide on a single value that best represents the rebound ratio for your ball. Use this ratio to write a recursive formula that models your sequence of *rebound height* data, and use it to generate the first six terms.

Step 6 Compare your experimental data to the terms generated by your recursive formula. Answers will vary.

a. How close are they?

b. Describe some of the factors that might affect this experiment. (For examples how might the formula change if you used a different kind of ball.)

c. According to the recursive formula does the ball ever stop bouncing?

d. Realistically, how many bounces do you think there were before the ball stopped?

Answers could vary. The description will dictate the answer. For example, the population of a city could be either, depending on whether people are moving into or out of the city.

Investigate

Example A

Consider projecting the example from your ebook and having students work in pairs. Have students share their strategies along with their solutions. Emphasize the use of correct mathematical terminology and symbols. Whether student responses are correct or incorrect, ask other students if they agree and why. SMP 1, 3, 6

ASK "Why can we find a 20% depreciation by multiplying by $\frac{4}{5}$?" If necessary, suggest that they think about specific numbers, such as \$100. Point out that Example A seems to be claiming that $23{,}999 - (0.2)23{,}999 = 23{,}999(0.8)$. ASK "Is that reasoning justified? Explain." If needed, suggest they factor the equation. You might generalize to $u_{n-1} - (0.2)\,u_{n-1} = u_{n-1}(1 - 0.2) = u_{n-1}(0.8)$ as you review factoring. SMP 7

ASK "According to your rule, how low could the value of the car go? Realistically, what is the lowest value of the car?"

Guiding the Investigation

You can use the Investigation Worksheet Looking for the Rebound with Sample Data if you do not wish to conduct the investigation as an activity. There is also a desmos simulation of this investigation in the ebook. If you have motion sensors, students can collect data with the motion sensor. An Investigation Worksheet is available for use with motion sensors.

Introduce the investigation by demonstrating the bounce measurement process. If you are using small balls, balls without seams, such as racquetballs, will work best.

ALERT If the camera is too close to the ball or the ball is too far from the wall you get a bad line of sight any time the ball is above or below the camera.

Modifying the Investigation

Whole Class Have two students collect data in front of the class. Complete Steps 3–5 with student input. Discuss Step 6.

Shortened Use the sample data. Have students complete Steps 3–5. Discuss Step 6.

One Step Give the instructions for the investigation and pose this problem: "What is a recursive formula for the height of the ball at the top of each bounce?"

After students gather data, some will look for an additive formula (an arithmetic sequence) and others for a multiplicative formula (a geometric sequence). Have students discuss which models are best, based on differences between predicted and actual data. ASK "Does your model have a limit on the number of bounces before the ball stops? How realistic is that?"

Step 1 You can't really read measurements marked on the wall but, if your background is distinct, you can measure the peak heights by slowly scrolling through the video. Try to have an initial height of about 2 meters.

To minimize the parallax error from the camera angle you want the camera at the height of the ball when it is at the top of the bounce. As adjusting the camera height is difficult, you want the camera to be far away from the ball and the ball close to the ruler. You will have to translate the colors into numbers as you will not be able to read the ruler in the video.

Step 4 Let students decide what amount of error in the data is acceptable.

Have groups present their work, choosing presentations to include a variety of decimal places in the results, and question which number of decimals is most appropriate. SMP 3, 5, 6

ASK "Could you use the first or second rebound height as u_0?" [Yes, either; starting the sequence from the second height would produce an identical set of subsequent bounces.]

Example B

Consider projecting the example from your ebook and having students work in pairs. Have them share their solutions. Emphasize the use of correct mathematical terminology and symbols. Whether student responses are correct or incorrect, ask other students if they agree and why. SMP 1, 3, 6

Summarize

Have students present their solutions and explain their work in the Examples and the Investigation. Encourage a variety of approaches. Students may want to draw graphs or work with formulas. During the discussion, ASK "What kind of sequences did you see in Examples A and B and in the investigation?" [geometric sequences] Emphasize the use of correct mathematical terminology. Whether student responses are correct or incorrect, ask other students if they agree and why. SMP 1, 3, 5, 6

You may find it easier to think of the common ratio as the whole, 1, plus or minus a percent change. In place of r you can write $(1 + p)$ or $(1 - p)$. The car example involved a 20% (one-fifth) loss, so the common ratio could be written as $(1 - 0.20)$. Your bouncing ball may have had a common ratio of 0.75, which you can write as $(1 - 0.25)$ or a 25% loss per bounce.

In Example A, the value of the car decreased each year. Similarly, the rebound height of the ball decreased with each bounce. These and other decreasing geometric sequences are examples of **decay.** These examples of real world decay can be modeled with a geometric recursive rule but every model has error or variation in the application of the model. The value of the car will never be zero and the ball does not keep bouncing forever, yet the mathematical model tells us that the value of the car will keep going down and that the ball will still be bouncing after dozens of bounces. So, we can use models to understand and predict behavior, but every value from the sequence is only an estimate of the true value.

The next example is one of **growth,** or an increasing geometric sequence. Interest is a charge that you pay to a lender for borrowing money or that a bank pays you for letting it invest the money you keep in your bank account. **Simple interest** is a percentage paid on the **principal**, or initial balance, over a period of time. If you leave the interest in the account, then in the next time period you will receive interest on both the principal and the interest that were in your account. This is called **compound interest** because you are receiving interest on the interest.

EXAMPLE B

Gloria deposits $2,000 into a bank account that pays 7% annual interest compounded annually. This means the bank pays her 7% of her account balance as interest at the end of each year, and she leaves the original amount and the interest in the account. When will the original deposit double in value?

Solution

The balance starts at $2,000 and increases by 7% each year.

$u_0 = 2000$

$u_n = u_{n-1} + 0.07 \cdot u_{n-1} \quad \text{where } n \geq 1$ — The recursive rule that represents 7% growth.

$u_n = (1 + 0.07)\, u_{n-1} \quad \text{where } n \geq 1$ — Factor.

Use technology, such as a spreadsheet or calculator, to compute year-end balances recursively.

Term u_{11} is 4209.70, so the investment balance will more than double in 11 years.

fx = 1.07*B2

	A	B
1	n	u_n
2	0	$2,000.00
3	1	$2,140.00
4	2	$2,289.80
5	3	$2,450.09

fx = 1.07*B12

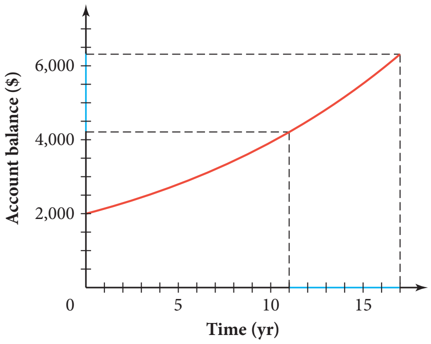
	A	B
1	n	u_n
11	9	$3,676.92
12	10	$3,934.30
13	11	$4,209.70
14	12	$4,504.38

ADVANCED You might discuss the difference between *nominal* and *effective interest rates*. If a 6.5% annual interest rate is compounded monthly, 6.5% is the nominal interest rate and $\left(1 + \frac{0.065}{12}\right)^{12} - 1$ is the effective annual interest rate.

Conceptual/Procedural

Conceptual The real world examples and Investigation situations help students conceptualize increasing and decreasing geometric sequences. Additionally, adding the component of looking at the limits of a mathematical model compared to the physical world makes analyzing and evaluating the model more conceptual.

Procedural Students practice the procedure of writing and evaluating geometric sequences.



Compound interest has many applications in everyday life. The interest on both savings and loans is almost always compounded, often leading to surprising results. This graph and spreadsheet show the account balance in Example B.

fx	=B18*(1+0.07)	
	A	B
1	n	u_n
16	14	5157.0683
17	15	5518.063081
18	16	5904.327497
19	17	6317.630422

Leaving just $2,000 in the bank at a good interest rate for 11 years can double your money. In another 6 years, the $2,000 will have tripled.

Some banks will compound the interest monthly. You can write the common ratio as $\left(1+\frac{0.07}{12}\right)$ to represent one-twelfth of the annual interest, compounding monthly. When you do this, n represents months instead of years. How would you change the rule to show that the interest is compounded 52 times per year? What would n represent in this situation?

$\left(1+\frac{0.07}{52}\right)$; n would represent the number of weeks

1.2 Exercises

You will need a graphing calculator for Exercise **18**.

Practice Your Skills

1. Find the common ratio for each sequence, and identify the sequence as growth or decay. Give the percent change for each.

a. 100, 150, 225, 337.5, 506.25, . . . @
1.5; growth; 50% increase

b. 73.4375, 29.375, 11.75, 4.7, 1.88, . . .
0.4; decay; 60% decrease

c. 80.00, 82.40, 84.87, 87.42, 90.04, . . .
1.03; growth; 3% increase

d. 208.00, 191.36, 176.05, 161.97, . . .
0.92; decay; 8% decrease

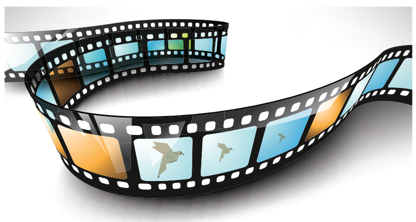
2. Write a recursive formula for each sequence in Exercise 1. Use u_0 for the first term and find u_{10}. @

3. Write each sequence or formula as described.

a. Write the first four terms of the sequence that begins with 2000 and has the common ratio 1.05. @
2000, 2100, 2205, 2315.25

b. Write the first four terms of the sequence that begins with 5000 and decays 15% with each term. What is the common ratio?
5000, 4250, 3612.5, 3070.625; common ratio = 0.85

c. Write a recursive formula for the sequence that begins 1250, 1350, 1458, 1574.64, ...
$a_0 = 1250$, $a_n = a_{n-1}(1 + 0.08)$ where $n \geq 1$



Films quickly display a sequence of photographs, creating an illusion of motion

CRITICAL QUESTION How do the geometric sequences in the Investigation, Example A, Example B compare with one another?

BIG IDEA The automobile depreciation in Example A and the bouncing ball in the investigation give a decreasing geometric sequence, described as *decay*. The bank account in Example B shows an increasing geometric sequence, described as *growth*.

CRITICAL QUESTION How would you describe the differences between the recursive formulas for geometric growth and decay sequences?

BIG IDEA In growth, the multiplier is greater than 1; in decay, it's less than 1.

Formative Assessment

As students work and present, assess their understanding of geometric sequences and common ratios, as well as their ability to calculate ratios. Observe how students relate situations to notation. Do they have a good understanding of the meaning of u_{n-1} context? Can they represent a loss of 25%?

Apply

Extra Example

1. You buy a pair of limited edition shoes, then immediately sell them on an online auction site. The bidding starts at $100 and each bid pushes the price up by 15%. Make a table. If the 10th bidder purchases the shoes, how much does that person pay? $351.79

2. Rewrite the expression $u_{n-1} + 2u_{n-1}$ so that the variable appears only once. $3u_{n-1}$

Closing Question

Write a recursive formula for the height of a ball that is dropped from 150 cm and has a 60% rebound ratio.
$u_0 = 150$; $u_n = 0.60 \cdot u_{n-1}$ where $n \geq 1$

Assigning Exercises

Suggested: 1 – 11, 14 – 16
Additional Practice: 12, 13, 17 – 21

Exercise 1 **ALERT** Students might offer the common ratio as the percent change.

2a. $u_0 = 100$ and $u_n = 1.5u_{n-1}$ where $n \geq 1$; $u_{10} \approx 5766.5$

2b. $u_0 = 73.4375$ and $u_n = 0.4u_{n-1}$ where $n \geq 1$; $u_{10} \approx 0.0077$

2c. $u_0 = 80.00$ and $u_n = 1.03u_{n-1}$ where $n \geq 1$; $u_{10} \approx 107.513$

2d. $u_0 = 208.00$ and $u_n = .92u_{n-1}$ where $n \geq 1$; $u_{10} \approx 90.35$

Exercises 6, 7 These exercises are related to the investigation. Students who missed the investigation may need assistance visualizing the graph of the data.

6d. Yes, when watching a ball bounce from 100 inches the ball stops moving before it has bounced between 20 to 30 times.

Exercises 7, 8 If students have difficulty understanding the recursive formulas because they are printed on one line, suggest that they write them out as they've seen them before. **ELL** Having students describe the real-world meanings for these exercises will give them the chance to practice their vocabulary and will also serve as a checkpoint for comprehension.

7. 100 is the initial height, but the units are unknown. 0.20 is the percent loss, so the ball loses 20% of its height each rebound.

Exercise 8 Because n is related to the year 20, students will probably conclude that the interest rate is annual. Be open to multiple interpretations: 0.025 could be $\frac{1}{12}$ of 30% compounded monthly or $\frac{1}{4}$ of 10% compounded quarterly. With these two options n would need to change to represent months or quarters.

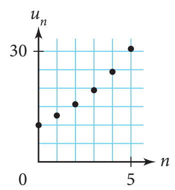
Exercise 9 **ALERT** Discourage students from answering with fractions of people.

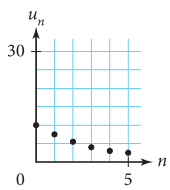
4. Match each recursive rule to a graph. Explain your reasoning.

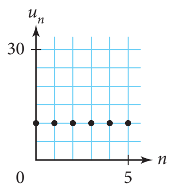
a. $u_0 = 10$ ii. decay
$u_n = (1 - 0.25) \cdot u_{n-1}$ where $n \geq 1$ @

b. $u_0 = 10$ i. growth
$u_n = (1 + 0.25) \cdot u_{n-1}$ where $n \geq 1$

c. $u_0 = 10$ iii. constant
$u_n = 1 \cdot u_{n-1}$ where $n \geq 1$

i.


ii.


iii.


5. Factor these expressions so that the variable appears only once. For example, $x + 0.05x$ factors into $x(1 + 0.05)$.

a. $x + Ax$ @ $x(1 + A)$

b. $A - 0.18A$ @ $(1 - 0.18)A$, or $0.82A$

c. $x + 0.08125x$ $(1 + 0.08125)x$, or $1.08125x$

d. $2u_{n-1} - 0.85u_{n-1}$ $(2 - 0.85)u_{n-1}$, or $1.15u_{n-1}$

Reason and Apply

6. Suppose the initial height from which a rubber ball drops is 100 in. The rebound heights to the nearest inch are 80, 64, 51, 41, ...

a. What is the rebound ratio for this ball? ⓗ 0.8

b. What is the height of the tenth rebound? 11 in.

c. After how many bounces will the ball rebound less than 1 in.? Less than 0.1 in.? 21 bounces; 31 bounces

d. Is there reason to suspect that these last two estimates are not correct? Explain.

7. Suppose the recursive formula $u_0 = 100$ and $u_n = (1 - 0.20)u_{n-1}$ where $n \geq 1$ models a bouncing ball. Give real-world meanings for the numbers 100 and 0.20.

8. Suppose the recursive formula $u_{2015} = 250{,}000$ and $u_n = (1 + 0.025)u_{n-1}$ where $n \geq 2016$ describes an investment made in the year 2015. Give real-world meanings for the numbers 250,000 and 0.025, and find u_{2019}. @ \$250,000 was invested at 2.5% annual interest in 2015. u_{2019} = \$27,595.32

9. **APPLICATION** A company with 12 employees is growing at a rate of 20% per year. It will need to hire more employees to keep up with the growth, assuming its business keeps growing at the same rate.

a. How many people should the company plan to hire in each of the next 5 years?
number of new hires for next 5 years: 2, 3, 3 (or 4), 4, and 5

b. How many employees will it have 5 years from now?
about 30 employees

10. **APPLICATION** The table below shows investment balances over time.

Elapsed time (yr)	0	1	2	3	. . .
Balance ($)	2,000	2,170	2,354.45	2,554.58	. . .

a. Write a recursive formula that generates the balances in the table. @ $u_0 = 2000$;
$u_n = (1 + 0.085)u_{n-1}$ where $n \geq 1$

b. What is the annual interest rate? 8.5%

c. How many years will it take before the original deposit triples in value? 14 years (\$6,266.81)

42 CHAPTER 1 Linear Modeling

11. **APPLICATION** Suppose you deposit $500 into an account that earns 6% annual interest. You don't withdraw or deposit any additional money for 3 years.

a. If the interest is paid once per year, what will the balance be after 3 years? ⓐ $595.51

b. If the interest is paid every six months, what will the balance be after 3 years? This is also referred to as 6% compounded semiannually. Divide the annual interest rate by 2 to find the semiannual interest rate. $597.03

c. What will the balance be if you receive 6% compounded *quarterly* for 3 years? ⓗ $597.81

d. What will the balance be if you receive 6% compounded *monthly* for 3 years? $598.34

12. **APPLICATION** Suppose $500 is deposited into an account that earns 6.5% annual interest and no more deposits or withdrawals are made.

a. If the interest is compounded monthly, what is the monthly rate? $\frac{6.5}{12} \approx 0.542\%$

b. What is the balance after 1 month? $502.71

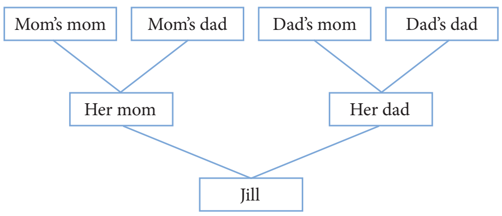
c. What is the balance after 1 year? $533.49

d. What is the balance after 29 months? $584.80

13. Suppose Jill's biological family tree looks like the diagram at right. You can model recursively the number of people in each generation.

a. Make a table showing the number of Jill's ancestors in each of the past five generations. Use u_0 to represent Jill's generation.

b. Look in your table at the sequence of the number of ancestors. Describe how to find u_n if you know u_{n-1}. Write a recursive formula.



c. Find the number of the term of this sequence that is closest to 1 billion. What is the real-world meaning of this answer?

d. If a new generation is born every 25 years, approximately when did Jill have 1 billion living ancestors in the same generation? 750 years ago

e. Your answer to 13c assumes there are no duplicates, that is, no common ancestors on Jill's mom's and Jill's dad's sides of the family. Look up Earth's population for the year you found in 13d. Describe any problems you notice with the assumption of no common ancestors.

The population of the planet at the time was less than 1 billion. Jill must have some common ancestors.

Cultural CONNECTION

Family trees are lists of family descendants and are used in the practice of genealogy. People who research genealogy may want to trace their family's medical history or national origin, discover important dates, or simply enjoy it as a hobby. Author Alex Haley (1921–1992), honored here in this statue in Annapolis, Maryland, told the powerful history of his family's prolonged slavery and decades of discrimination in his 1976 genealogical book, *Roots: The Saga of an American Family*. The book, along with the 1977 television miniseries, inspired many people to trace their family lineage.

13b. Start with 1 and recursively multiply by 2; $u_0 = 1$ and $u_n = 2u_{n-1}$ where $n \geq 1$.

13c. u_{30}; 30 generations ago, Jill had 1 billion living ancestors

Exercise 11 As needed, remind students that as the interval between payments (and therefore the interest rate) decreases, the number of payments increases.

Exercise 13 Encourage students to critique their answer in 13c and to compare it with their answer in 13d. **SUPPORT** Students may need to be reminded why u_0 should be used to represent Jill's generation, especially because her parents are considered to be one generation removed from Jill, u_1. Students are sometimes uncertain when to use u_0 versus u_1 as the starting value in a problem. Explain that the specific notation depends on the context of the problem and the question that is being considered.

ALERT This exercise can generate interesting class discussions. However, students may be sensitive about issues surrounding ancestry, and traditional and nontraditional families.

13a.

Generations back n	0	1	2	3	4	5
Ancestors in the generation u_n	1	2	4	8	16	32

LESSON 1.2 Modeling Growth and Decay 43

14. $u_0 = 1$ and $u_n = 0.8855u_{n-1}$ where $n \geq 1$; $u_{25} = 0.048$, or 4.8%. It would take about 25,000 years to reduce to 5%.

Context **Guitar Feedback** Some students will recognize feedback as the loud whine a microphone sometimes makes. Explain that this occurs when the output of the speakers becomes the input to the microphone. The feedback is the result of the system being unable to handle this recursive process.

14. **APPLICATION** Carbon dating is used to find the age of ancient remains of once-living things. Carbon-14 is found naturally in all living things, and it decays slowly after death. About 11.45% of it decays in each 1000-year period of time.

Let 100%, or 1, be the beginning amount of carbon-14. At what point will less than 5% remain? Write the recursive formula you used.

Anthropologist carefully revealing human remains at an ancient burial site.

15. **APPLICATION** Between 1980 and 2010, the population of Grand Traverse County in Michigan grew from 54,899 to 86,999.

a. Find the actual increase and the percent increase over the 30-year period. @ 32,100 and 58.47%

b. If you average that total and rate over 30 years, what is the change per year? Find the average rate of change and the average percent rate of change. 1,070 people and 1.95%

c. Since the growth is not linear, the actual change each year is different. However, overall it averages out to the value you found in 15b. What happens if you use the average percent rate you found in 15b starting in 1980 for 30 years? $u_{1980} = 54{,}899$ and $u_n = (1 + 0.0195)u_{n-1}$ where n ≥ 1981 yields 97,990.

d. Try this again with a rate of 1.55% for 30 years. 87,089 people – much closer

e. Using 1.55%, find the population estimates for 1990 and 2000. $u_{1990} = 64{,}027$, $u_{2000} = 74{,}673$

f. Using 1980 and the population estimates from 15e, what is the average rate of change in people per year between 1980 and 1990? 1980 and 2000? 1990 and 2000? 912; 989; 1064 people per year

g. Without computing, which do you think is larger:

i. The average rate of change from 2000 to 2010, or

ii. the average rate of change between 1990 and 2010?

Explain your thinking.

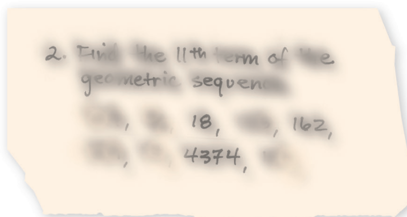
ii. While the % rate is fixed the number of people added increases each year so the average from the second ten years will be higher than the average over the whole 20 years.

16. Taoufik looks at the second problem of his wet homework that fell in a puddle.

a. What is the common ratio? How did you find it?

b. What are the missing terms?

c. What is the answer he needs to find?



16a. 3; $\frac{162}{18} = 9$, $3^2 = 9$
16b. 2, 6, …, 54, …, 486, 1458, …, 13122
16c. 118,098

Guitar feedback is a real-world example of recursion. When the amplifier is turned up loud enough, the sound is picked up by the guitar and amplified again and again, creating a feedback loop. Jimi Hendrix (1942–1970), a pioneer in the use of feedback and distortion in rock music, remains one of the most legendary guitar players of the 1960s.

44 CHAPTER 1 Linear Modeling

Review

17. The population of the United States grew 9.70% from 2000 to 2010. The population reported in the 2010 census was 308.7 million. What population was reported in 2000? Explain how you found this number. 281.4 million; $(1 + 0.097)x = 308.7$

18. An elevator travels at a nearly constant speed from the ground level to an observation deck at 160 m. This trip takes 40 s. The elevator's trip back down is also at this same constant speed.

a. What is the elevator's speed in meters per second? 4 m/s

b. How long does it take the elevator to reach the restaurants, located 40 m above ground level? @ 10 s

c. Graph the height of the elevator as it moves from ground level to the observation deck.

d. Graph the height of the elevator as it moves from the restaurant level, at 40 m, to the observation deck.

e. Graph the height of the elevator as it moves from the deck to ground level.

The CN Tower in Toronto is one of Canada's landmark structures and one of the world's tallest buildings. Built in 1976, it has six glass-fronted elevators that allow you to view the landscape as you rise above it at 15 mi/h. At 1136 ft, you can either brace against the wind on the outdoor observation deck or test your nerves by walking across a 256 ft^2 glass floor with a view straight down.

19. Consider the sequence 180, 173, 166, 159, ...

a. Write a recursive formula. Use $u_1 = 180$. $u_1 = 180$ and $u_n = u_{n-1} - 7$ where $n \geq 2$

b. What is u_{10}? $u_{10} = 117$

c. What is the first term with a negative value? $u_{27} = -2$

20. Solve each equation.

a. $-151.7 + 3.5x = 0$ @ $x \approx 43.34$

b. $0.88x + 599.72 = 0$ $x = -681.5$

c. $18.75x - 16 = 0$ $x \approx 0.853$

d. $0.5 \cdot 16 + x = 16$ $x = 8$

21. For the equation $y = 47 + 8x$, find the value of y when

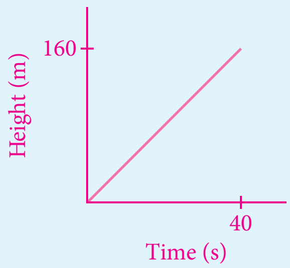
a. $x = 0$ $y = 47$

b. $x = 1$ @ $y = 55$

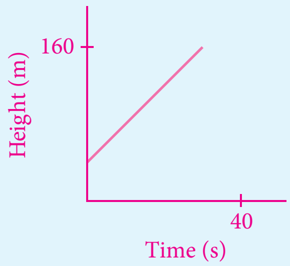
c. $x = 5$ $y = 87$

d. $x = -8$ $y = -17$

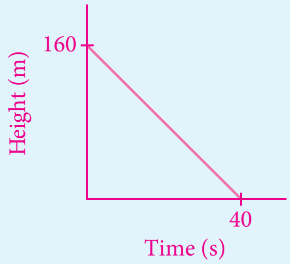
18c.



18d.



18e.



LESSON 1.2 Modeling Growth and Decay 45