PROBLEM STRINGS LESSON

8.1 Squaring and Unsquaring

At a Glance

(6×6)*

(-6×-6)*

(12×12)*

(-12×-12)*

$x^2=64$

$x^2-17=64$

$x^2+15=64$

$(x+3)^2=49$

$(x-7)^2=25$

$(x+2)^2+1=82$

*optional problems

Objectives

The goal of this string is to bridge students' earlier work finding the square and square roots of values and expressions with current work solving quadratic functions by using inverse operations—a strategy also known as "solving by inspection."

Placement

You can use this problem string as students begin to solve quadratic equations.

Students have had previous experiences with the squaring function (lesson 7.3) but may not have considered how quadratic models (such as the throwing of a ball) can be seen as a transformation of the squaring function. Students will be asked to solve quadratic functions symbolically in Chapter 8 of the textbook, so use this string at the beginning of the chapter to help prepare them to use inverse operations with flexibility and ease.

Guiding the Problem String

As you are leading the string be mindful of going a bit deeper than simply soliciting solutions. This means probing students' understandings and giving them the chance to articulate to the class how they know something is true. This also means creating a bit of puzzlement so that students are really invited to express what they know and what makes sense. To be clear, this does not mean (a) pretending to not know the mathematics yourself or (b) having students trying to convince you.

For example, instead of the question "Does it work?", think about asking "Does it make sense?", followed by "Why?" or "Why not?" In this way we assure kids that just because the algebra of quadratic functions may feel different and potentially more challenging than linear functions, the role of sense-making in mathematics still matters.

Another way to keep students thinking critically about the meaning of squaring is to consistently ask, "Could it be anything else?" Early in the string students may wonder why you are asking this, but later it will become an important question, especially to those students who are satisfied by only one solution. More importantly, you are offering students an important question for them to consider when doing all kinds of mathematics.

About the Mathematics

When solving quadratic functions using inverse operations, students will encounter the need to undo the squaring operation. Without a strong sense of the meaning of the square root symbol and absolute value symbol, they are likely to make errors or not consider multiple values that could make the function true. A simple example of this is when students assume that when $x^2 = 16$ then x must be 4, ignoring the value of -4 as a valid solution.

The structure of the string is designed to move slowly, but deliberately, from friendly values towards more complex functions that resemble the kinds of quadratic functions they will soon encounter.

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Sample Interactions

Use the following as you plan how to elicit and model student strategies. This is not meant as a script, but as a view into the relationships involved and the intent of the problem string.

Teacher: *Okay, everyone. Let's get our brains warmed up with a problem string. You might not need paper and pencil, but be sure you are sitting near your math partner so you can talk together. I'll put a problem on the board and ask that you think about the answer or answers and be able to explain how you know. Here we go! The first problem is x squared is 64. What is x?*	$x^2 = 64$
Student: *So, this is 8 because you are asking what number squared is 64 and that is 8.* **Teacher:** *What do other people think?* **Student:** *Same idea but squaring is like what number multiplied by itself is 64. I agree, it's got to be 8.*	$x^2 = 64$ $8^2 = 64$ $x \cdot x = 64$ $8 \cdot 8 = 64$
Teacher: *Seems like we agree. Could it have another solution? Is there anything else x could be?* **Student:** *Ahhh, it could be negative. Negative 8 also works.* **Teacher:** *So this is also true?* **Students:** *Yes!*	$x^2 = 64$ $(-8)^2 = 64$ $-8 \cdot -8 = 64$ $(-8)^2$ or -8^2?
Teacher: *Turn and talk to your partner for 30 seconds about how I wrote that. Why did I use parentheses here? Is there a difference between $(-8)^2$ and -8^2?* **Student:** *So my partner and I discussed that the difference might be about order of operations, in the first expression it's clear from what you wrote that we are supposed to square negative 8, and in the second one it's a little murky. …maybe that means square the 8, then make the answer negative which would not give us 64, but −64.* **Teacher:** *What do other people think?* **Student:** *The first notation is just clearer so I get why you used it. And we don't have to use it with positive 8 because it's not necessary.*	
Teacher: *In fact, mathematicians agree with you that this notation could be murky and so they have decided to use parentheses to mean negative 8 times negative 8 and when there are no parentheses, it means 8 squared and that is negative, so −64. We'll use these order of operations from now on.*	$(-8)^2 = -8 \cdot -8 = 64$ $-8^2 = -(64) = -64$
Teacher: *And, so we are all clear going forward, I'm going to use this symbol to capture the idea of a positive and negative solution. Everyone okay with that and understand what it means? We mean positive and negative 8 when we write this, but sometimes mathematicians would say, "plus or minus eight" when they use this symbol.*	$x = \pm 8$

Teacher: *Okay, now I'm going to tinker with things a little bit. Be thinking about how this solution, or these solutions, will relate to the positive and negative eight.*	$x^2 - 17 = 64$
Teacher: *What is x?* **Student:** *I got* 9. **Student:** *I got both plus and minus* 9. **Teacher:** *How did you find x?* **Student:** *I wasn't sure about this one, so I added* 17 *to both sides and got x squared equals* 81 *and then it was way easier.* **Teacher:** *Can you say why that made sense to you?* **Student:** *Well, the way it is written I couldn't figure it out. I wanted just a variable on one side and a number on the other, like the last one.* **Student:** *I thought that if something minus* 17 *is* 64, *than that number must be* 17 *more than* 64. **Teacher:** *Nice. Can someone finish this thinking?* **Student:** *Sure, so based on the last one, x has to be* 9 *or* −9. **Teacher:** *Sometimes people call this solving by inspection or using inverse operations.*	$x = -9, 9$ $x^2 - 17 = 64$ $+17 \quad +17$ $x^2 = 81$ $x = \pm 9$
Teacher: *Now what? What if this time we add* 15 *to x squared to get* 64. *How will that relate? What is x now?* **Student:** *If something plus* 15 *is* 64, *the number has to be* 15 *less than* 64. **Student:** *I first subtracted* 15 *from both sides to isolate the* x^2 *and I was really happy that I got* 49. *Then I knew it would be* 7 *and* −7. **Teacher:** *Okay. Let me capture your strategy. Who understands why Jeffrey was excited about the* 49? **Student:** *It's a perfect square and we know how to find the square roots without a calculator. Makes life easier.*	$x^2 + 15 = 64$ $x^2 + 15 = 64$ $-15 \quad -15$ $x^2 = 49$ $x = \pm 7$
Teacher: *How would you solve this one? Just notice what's the same and what's different here. This time it's the sum of x and* 3 *squared to get* 49. *What is x?* Students work. The teacher circulates, looking for students who found only one solution, $x = 4$ and students who found both solutions. After most students have a solution, the teacher asks students to share their thinking with their partner.	$(x+3)^2 = 49$

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Teacher: *Who can tell us how their partner solved this one?* Asking students to explain their partner's thinking can help encourage students to pay attention to other strategies and make sense of other's thinking. **Student:** *My partner thought about this as "what number plus three, when you square it, would give you 49." Since that number is 7, we knew x was 4 so 4 plus 3 is 7. Square it and you get 49.* **Teacher:** *Thoughts on this strategy?*	$(x+3)^2=49$ $(\_+3)^2=49,\ \ (7)^2=49$ so $x=4$
Student: *I like it, but I feel like there's another answer. In all of the previous ones there were two answers not one. So I'm just wondering if we mean plus and minus 4?* **Teacher:** *Interesting. What do we think?* **Student:** *Uhhh, −4 won't work, but I like the idea of checking for a negative solution. What about −10?* **Teacher:** *Say more about that.* **Student:** *In the previous problems when a number was squared we had two solutions, one negative and one positive. That is true here, but we are kinda thinking about how to get x plus 3 to equal 7 and how to get x plus 3 to equal −7.* **Teacher:** *Raise your hand if you are convinced by this idea. Anybody not convinced?*	Since $(x+3)^2=49$ $x+3=\pm 7$ $x+3=7$ or $x+3=-7$ $x=4$ $x=-10$
Teacher: *Using the ideas we've talked about so far, how would you solve this one?* Students work. The teacher circulates. **Teacher:** *Turn and tell your partner how you are thinking about this one?* Teacher records conversation like the last one, being sure to wonder about a second solution if only one solution is mentioned.	$(x-7)^2=25$ $x-7=\pm 5$ $x-7=5$ *or* $x-7=-5$ $x=12$ $x=2$
Teacher: *Alright my friends, let's end with this one. Try to make it friendlier to solve, like we always do. This time it's the sum of x and 2, that quantity squared, then add 1 to get 82. What's x?* Students work. The teacher circulates. **Teacher:** *Go ahead and hear how your partner solved this one.* **Teacher:** *Luis, will you share your strategy with all of us.* **Student:** *Sure. We talked about how getting rid of the one made sense—so now we have x plus two squared equals 81. Then we saw the perfect square and knew we were looking for plus or minus nine.* **Teacher:** *Am I capturing your thinking here?* **Student:** *Yeah, exactly. So then it's just a linear equation. Where x will equal 7 or −11.*	$(x+2)^2+1=82$ $(x+2)^2=81$ $(x+2)=\pm 9$ $x+2=9$ or $x+2=-9$ $x=7$ $x=-11$ $x=-11, 7$

Teacher: *How would you summarize some of the things that came up in this string today?*

Elicit the following:

- *When you are looking for what squared is a number, you have to think of both the positive and negative possibilities.*
- *You can often find one value by guessing and checking but the other one might need some more work to find.*
- *Just like you can add/subtract/multiply/divide to both sides of an equation, you can "unsquare" or take the square root of both sides of an equation.*

Sample Final Display

Your display could look like this at the end of the problem string:

$6 \times 6 = 36$
$-6 \times -6 = 36$

$x^2 = 64$ $x = -8, 8$ $x = \pm 8$	$x^2 = 64$ $8^2 = 64$ $x \cdot x = 64$ $8 \cdot 8 = 64$	$x^2 = 64$ $(-8)^2 = 64$ $-8 \cdot -8 = 64$ $(-8)^2$ or -8^2?	$(-8)^2 = -8 \cdot -8 = 64$ $-8^2 = -(64) = -64$
$x^2 - 17 = 64$ $x = \pm 9$	$x^2 - 17 = 64$ $+17 \quad +17$ $x^2 = 81$ $x = \pm 9$		
$x^2 + 15 = 64$ $x = \pm 7$	$x^2 + 15 = 64$ $-15 \quad -15$ $x^2 = 49$ $x = \pm 7$		
$(x+3)^2 = 49$ $x = -10, 4$	$(\_+3)^2 = 49$, $(7)^2 = 49$ so $x = 4$		Since $(x+3)^2 = 49$ $x + 3 = \pm 7$ $x + 3 = 7$ or $x + 3 = -7$ $x = 4$ $\quad$ $x = -10$
$(x-7)^2 = 25$ $x = 2, 12$	$x - 7 = \pm 5$	$x - 7 = 5$ *or* $x - 7 = -5$ $x = 12$ $\quad$ $x = 2$	
$(x+2)^2 + 1 = 82$ $x = -11, 9$		$(x+2)^2 = 81$ $(x+2) = \pm 9$ $x + 2 = 9$ or $x + 2 = -9$ $x = 7$ $\quad$ $x = -11$ $x = -11, 7$	

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Facilitation Notes

This version of the problem string lists short notes for important teacher moves during the string. After you've done the string yourself and studied the relationships involved, you might make similar notes for the things you want a reminder of or deem important.

(6×6)* (-6×-6)* (12×12)* (-12×-12)*	*Use these only if we need them.* *Will students consider both possibilities when solving?*
$x^2=64$	*Friendly problem.* *Could there be another solution?* *Pause to clarify notation here: $(-8)^2$ or -8^2* *Name and use this symbol: ±8*
$x^2-17=64$	*Tinkering with the structure.* *Could there be another solution?*
$x^2+15=64$	*Keeping structure the same—but subtracting instead of adding.* *Could there be another solution?*
$(x+3)^2=49$	*New structure—now what? Sum of x and 3, that quantity squared is 49, what's x?*
$(x-7)^2=25$	*Related structure. Difference between x and 7, that quantity squared is 25, what's x?* *Be sure to have students articulate why two solutions and how to find them.*
$(x+2)^2+1=82$	*Challenge problem, but friendly enough to solve mentally.*