LESSON

5.5

Properties of Special Parallelograms

"You must know a great deal about a subject to know how little is known about it."

LEO ROSTEN

The legs of the lifting platform shown at right form rhombuses. Can you visualize how this lift would work differently if the legs formed parallelograms that weren't rhombuses?

In this lesson you will discover some properties of rhombuses, rectangles, and squares. What you discover about the diagonals of these special parallelograms will help you understand why these lifts work the way they do.

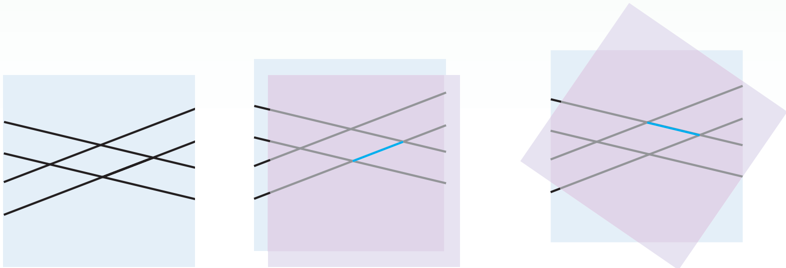
INVESTIGATION 1

What Can You Draw with the Double-Edged Straightedge?

YOU WILL NEED

- patty paper
- a double-edged straightedge

In this investigation you will discover the special parallelogram that you can draw using just the parallel edges of a straightedge.



Step 1 Step 2 Step 3

Step 1 On a piece of patty paper, use a double-edged straightedge to draw two pairs of parallel lines that intersect each other.

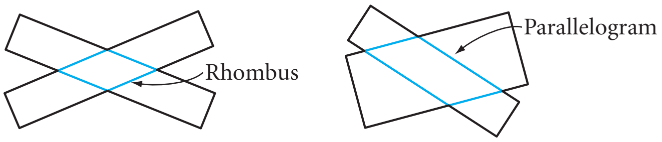
Step 2 Assuming that the two edges of your straightedge are parallel, you have drawn a parallelogram. Place a second patty paper over the first and copy one of the sides of the parallelogram.

Step 3 Compare the length of the side on the second patty paper with the lengths of the other three sides of the parallelogram. How do they compare? Share your results with your group. Copy and complete the conjecture.

Double-Edged Straightedge Conjecture **C-45**

If two parallel lines are intersected by a second pair of parallel lines that are the same distance apart as the first pair, then the parallelogram formed is a ?.

Recall that a **rhombus** is a parallelogram with four congruent sides, or an equilateral parallelogram. In Chapter 3, you learned how to construct a rhombus using a compass and straightedge, or using patty paper. Now you know a quicker and easier way, using a double-edged straightedge. To construct a parallelogram that is not a rhombus, you need two double-edged staightedges of different widths.



Now let's investigate some properties of rhombuses.

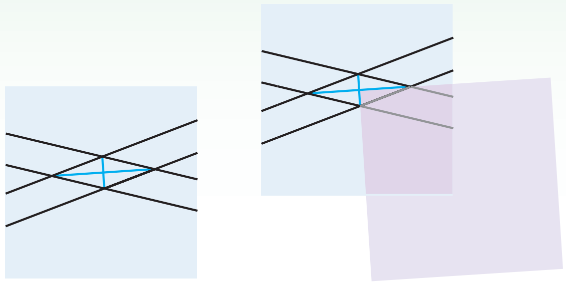
INVESTIGATION 2

Do Rhombus Diagonals Have Special Properties?

YOU WILL NEED

- patty paper
- a straightedge
- a protractor (optional)

View an interactive version of this lesson, see the **Dynamic Geometry Exploration** in your ebook.



Step 1 Step 2

Step 1 Draw in both diagonals of the rhombus you created in Investigation 1.

Step 2 Use the corner of a patty paper or a protractor to measure the angles formed by the intersection of the two diagonals. Are the diagonals perpendicular?

Compare your results with your group. Also, recall that a rhombus is a parallelogram and that the diagonals of a parallelogram bisect each other. Combine these two ideas into your next conjecture.

> **Rhombus Diagonals Conjecture** **C-46**
>
> The diagonals of a rhombus are _?_, and they _?_.

Step 3 The diagonals and the sides of the rhombus form two angles at each vertex. Fold your patty paper to compare each pair of angles. What do you observe? Compare your results with your group. Copy and complete the conjecture.

> **Rhombus Angles Conjecture** **C-47**
>
> The _?_ of a rhombus _?_ the angles of the rhombus.

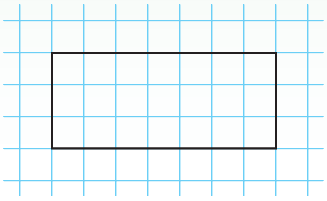
So far you've made conjectures about a rhombus, a quadrilateral with four congruent sides. Now let's look at quadrilaterals with four congruent angles. A rectangle is a parallelogram with four congruent angles, or an equiangular parallelogram. What special properties do they have?

INVESTIGATION 3

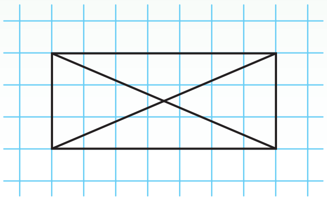
Do Rectangle Diagonals Have Special Properties?

YOU WILL NEED
- graph paper
- a compass

Now let's look at the diagonals of rectangles.



Step 1



Step 2

Step 1 Draw a large rectangle using the lines on a piece of graph paper as a guide.

Step 2 Draw in both diagonals. With your compass, compare the lengths of the two diagonals.

Compare results with your group. In addition, recall that a rectangle is also a parallelogram. So its diagonals also have the properties of a parallelogram's diagonals. Combine these ideas to complete the conjecture.

Rectangle Diagonals Conjecture **C-48**

The diagonals of a rectangle are _?_ and _?_.

INVESTIGATION 4

What Are the Properties of the Diagonals of a Square?

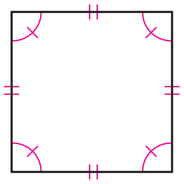
This final investigation is really a "thought experiment." What happens if you combine the properties of a rectangle and a rhombus? We call the shape a square. You can think of a square as a special rhombus and also a special rectangle. So you can define it in at least two different ways.

A **square** is an equiangular rhombus.

Or

A **square** is an equilateral rectangle.

A square is a parallelogram, as well as both a rectangle and a rhombus. Thus the square has the diagonal properties of all three. Discuss with your group members what you know about the diagonals of these three special parallelograms, then copy and complete this conjecture.

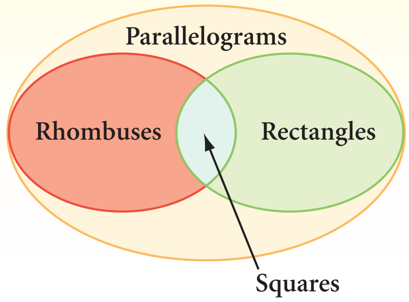


Square Diagonals Conjecture **C-49**

The diagonals of a square are __?__, __?__, and __?__.

DEVELOPING PROOF

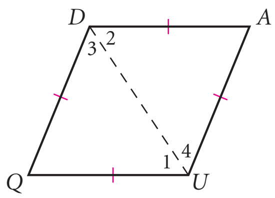
In Lesson 1.6 you arrived at the definitions for rhombuses and rectangles. It seemed like a neat organized way to categorize them as special parallelograms. You defined a rectangle as a parallelogram with all four angles congruent (an equiangular parallelogram). You defined a rhombus as a parallelogram with all four sides congruent (an equilateral parallelogram).



However, when we defined rhombus, we did not need the added condition of it being a parallelogram. We only needed to say that it is a quadrilateral with all four sides congruent (an equilateral quadrilateral). With your group members, follow along with the flowchart proof started for you below. Complete the flowchart proof by providing the missing reasons or write your own paragraph proof. The proof demonstrates logically that if a quadrilateral has four congruent sides then it is a parallelogram. If it is a parallelogram with four congruent sides then it is a rhombus.

Given: Quadrilateral *QUAD* has $\overline{QU} \cong \overline{UA} \cong \overline{AD} \cong \overline{DQ}$ with diagonal $\overline{DU}$

Show: *QUAD* is a rhombus



Flowchart Proof

1. $\overline{QU} \cong \overline{AD}$ — Given
2. $\overline{QD} \cong \overline{AU}$ — __?__
3. $\overline{DU} \cong \overline{DU}$ — Same segment
4. $\triangle QUD \cong \triangle ADU$ — __?__
5. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ — __?__
6. $\overline{QU} \parallel \overline{AD}$, $\overline{QD} \parallel \overline{AU}$ — Converse of the Parallel Lines Conjecture
7. *QUAD* is a parallelogram — Definition of parallelogram
8. $\overline{QU} \cong \overline{UA} \cong \overline{AD} \cong \overline{DQ}$ — Given
9. *QUAD* is a rhombus — __?__

Likewise, when we defined rectangle, we did not need the added condition of it being a parallelogram. We only needed to say that it is a quadrilateral with all four angles congruent (an equiangular quadrilateral). With your group members, follow along with the flowchart proof started for you below. Complete the flowchart proof by providing the missing statements and reasons or write your own paragraph proof. The proof demonstrates logically that if a quadrilateral has four congruent angles then it is a parallelogram. If it is a parallelogram with four congruent angles then it is a rectangle.

Given: Quadrilateral *ABCD* with $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$ with side AD extended to form exterior $\angle 5$.

Show: *ABCD* is a rectangle.

Flowchart Proof

$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4$
Given

$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90°$
Def. of Congruent Angles

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$
_?_

$\angle 5 \cong \angle 3$, $\angle 5 \cong \angle 1$
Algebra Operations

$\overline{AD} \parallel$ _?_, $\overline{AB} \parallel$ _?_
_?_

ABCD is a _?_ .
_?_

ABCD is a _?_ .
_?_

$m\angle 4 + m\angle 5 = 180°$
_?_

$m\angle 5 = 90°$
Substitution

5.5 Exercises

YOU WILL NEED

Construction tools for Exercises 17–19

DEVELOPING PROOF For Exercises 1–10, state whether each statement is always true, sometimes true, or never true. Use sketches or explanations to support your answers.

1. The diagonals of a parallelogram are congruent. *(h)*

2. The consecutive angles of a rectangle are congruent and supplementary.

3. The diagonals of a rectangle bisect each other.

4. The diagonals of a rectangle bisect the angles.

5. The diagonals of a square are perpendicular bisectors of each other.

6. A rhombus is a square.

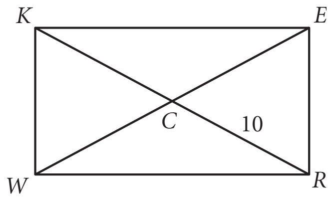
7. A square is a rectangle.

8. A diagonal divides a square into two isosceles right triangles.

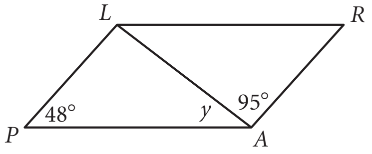
9. Opposite angles in a parallelogram are congruent.

10. Consecutive angles in a parallelogram are congruent.

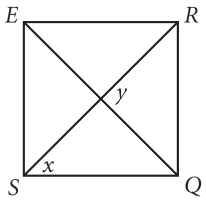
11. *WREK* is a rectangle.
$CR = 10$
$WE = \underline{?}$



12. *PARL* is a parallelogram.
$y = \underline{?}$ ⓗ

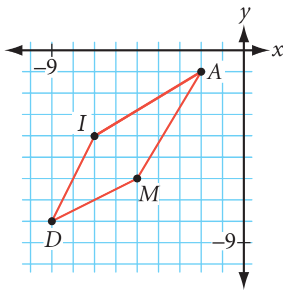


13. *SQRE* is a square.
$x = \underline{?}$
$y = \underline{?}$

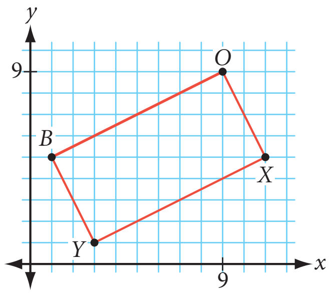


DEVELOPING PROOF For Exercises 14–16, use deductive reasoning to explain your answers.

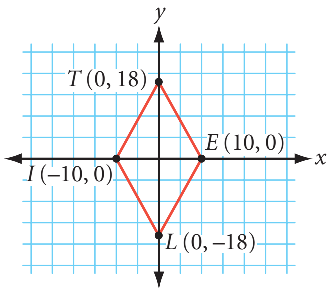
14. Is *DIAM* a rhombus? Why?



15. Is *BOXY* a rectangle? Why?

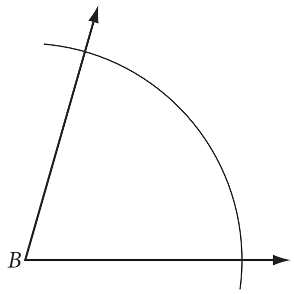


16. Is *TILE* a parallelogram? Why?

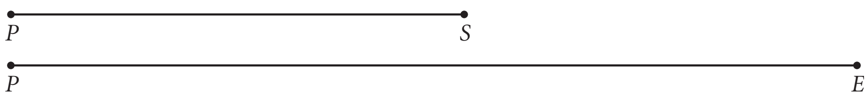


17. Given the diagonal $\overline{LV}$, construct square *LOVE*. ⓗ

18. Given diagonal $\overline{BK}$ and $\angle B$, construct rhombus *BAKE*. ⓗ

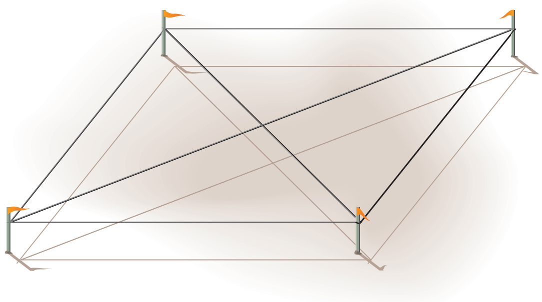


19. Given side $\overline{PS}$ and diagonal $\overline{PE}$, construct rectangle *PIES*.



20. **DEVELOPING PROOF** Write the converse of the Rectangle Diagonals Conjecture. Is it true? Prove it or show a counterexample. ⓗ

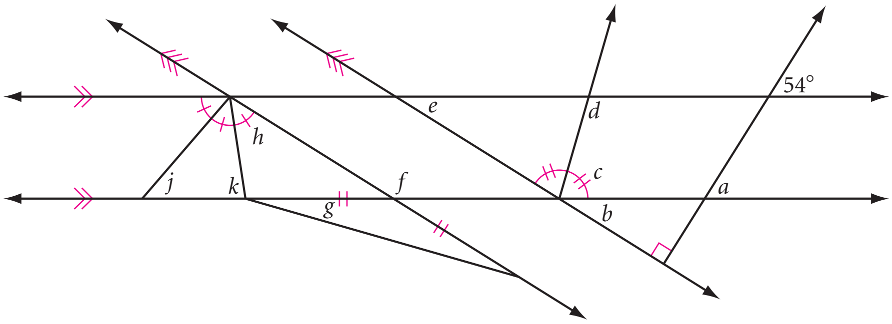
21. To make sure that a room is rectangular, builders check the two diagonals of the room as shown at right. Explain what they check about the diagonals, and why this works.



22. The platforms shown at the beginning of this lesson and here lift objects straight up. The platform also stays parallel to the floor. You can clearly see rhombuses in the picture, but you can also visualize the frame as the diagonals of rectangles. Explain why the diagonals of a rectangle guarantee this vertical movement.

Review

23. Trace the figure below. Calculate the measure of each lettered angle.



24. Find the coordinates of three more points that lie on the line passing through the points $(2, -1)$ and $(-3, 4)$.

25. Write the equation of the perpendicular bisector of the segment with endpoints $(-12, 15)$ and $(4, -3)$.

26. $\triangle ABC$ has vertices $A(0, 0)$, $B(-4, -2)$, and $C(8, -8)$. What is the equation of the median to side $\overline{AB}$?

DEVELOPING MATHEMATICAL REASONING

How Did the Farmer Get to the Other Side?

A farmer was taking her pet rabbit, a basket of prize-winning baby carrots, and her small—but hungry—rabbit-chasing dog to town. She came to a river and realized she had a problem. The little boat she found tied to the pier was big enough to carry only herself and one of the three possessions. She couldn't leave her dog on the bank with the little rabbit (the dog would frighten the poor rabbit), and she couldn't leave the rabbit alone with the carrots (the rabbit would eat all the carrots). But she still had to figure out how to cross the river safely with one possession at a time. How could she move back and forth across the river to get the three possessions safely to the other side?