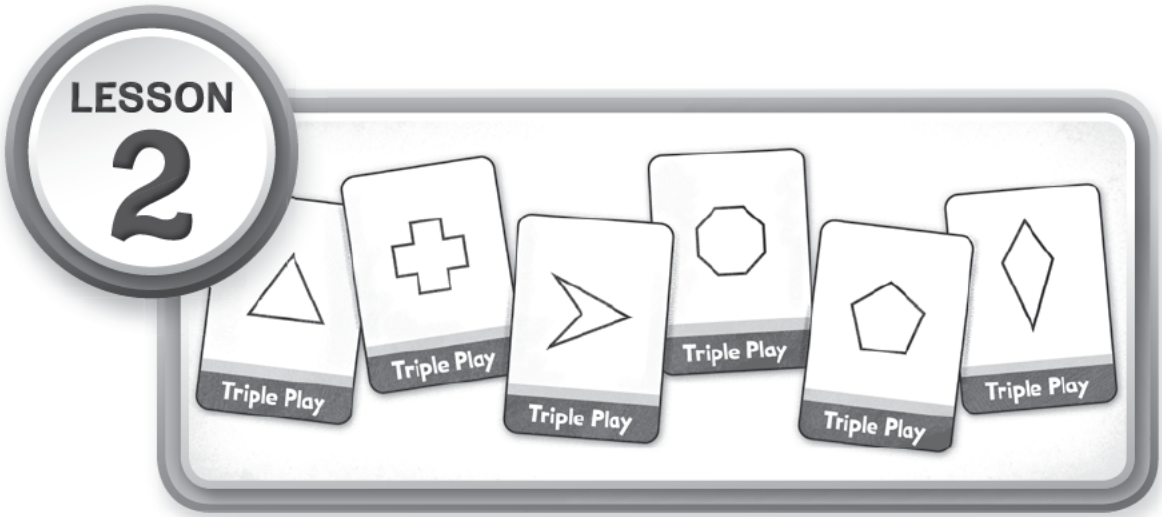


Triple Play with Shapes

Big Mathematical Ideas

This lesson reinforces the characteristics of different shapes and also makes students more aware of the fact that a square is a subset of both rectangles and rhombi. Students move from analyzing the characteristics of individual shapes to focusing on the relationship among the shapes. This requires a higher level of thinking. For example, they become more aware of the fact that a square is also a rectangle and a rhombus.

Objectives

- Students will compare and contrast different types of shapes.
- Students will classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
- Students will understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
- Students will classify two-dimensional shapes in a hierarchy based on properties.

NOTES

Materials

Students

- "Some, All or None" (Student Mathematician's Journal p. 23)
- "Three of These Things Belong Together" (Student Mathematician's Journal pp. 25–26)
- "Triple Play Game Mat" (Student Mathematician's Journal p. 27)
- "Mathematician's Journal Think Deeply About…" (Student Mathematician's Journal pp. 29–32)

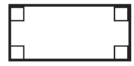
Teacher

- Blackline master "Three of These Things Belong Together" (pp. 104–105)
- Blackline master "Triple Play Game Cards" (copy, preferably on cardstock, and cut out one complete deck for every two students; pp. 107–112)
- Blackline master "Hint Cards" (p. 113)
- Blackline master "Think Beyond Cards" (p. 114)

Supplies

- Paper for students to keep score on
- Set® game (optional)

Mathematical Language

- **Counterexample** – a specific example that proves a mathematical statement is false
- **Equilateral** – a shape in which all sides are equal in length. For example, a square. 
- **Parallelogram** – a quadrilateral whose opposite sides are equal in length and parallel 
- **Polygon** – a simple, closed shape with three or more line segments as sides. For example, 
- **Rectangle** – a parallelogram with four right angles. Note that a square is also a rectangle. 
- **Regular polygons** – polygons that have all angles congruent and all sides congruent. For example, an equilateral triangle. 

NOTES

- **Rhombus (pl. rhombi or rhombuses)** – a parallelogram with four sides of equal length. Note that a square is also a rhombus.

- **Right angle** – an angle that measures exactly 90°

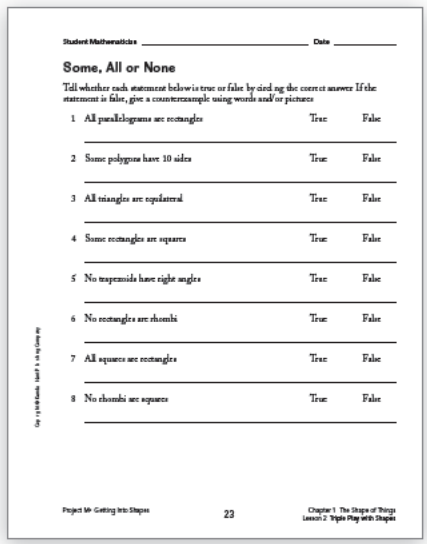
- **Square** – a parallelogram with four sides of equal length and four right angles

- **Trapezoid** – a quadrilateral with exactly one pair of opposite sides parallel. (Note that some texts define trapezoid as a quadrilateral with at least one pair of parallel sides. Thus, all parallelograms are trapezoids. We are not using this definition as it often causes confusion.)

Initiate (1 day)

Some, All or None

The initiation is a two-part activity. Both activities engage students in examining the relationships among various shapes. This is probably the first time that they have done this, and it involves high-level analysis. Thus we have provided two different opportunities for them to explore. The first activity uses the "Some, All or None" worksheet in the Student Mathematician's Journal. For this worksheet, students must decide if each statement is true or false. Discuss with students when to consider a statement false. If they can find just one example that does not fit the statement, a counterexample, then the statement is false. (If students are not yet familiar with the term "counterexample," introduce it to them at this time. Be sure to add it to your word wall.) For example, a statement might be, "All girls have blue eyes." This is false, since students can give an example of someone in the class or a girl that they know who might have brown or green eyes. On the worksheet, if a statement is false, students must justify why by providing a counterexample. Do the first example with students. They should find that the statement "All parallelograms are rectangles" is false and then provide a counterexample. They might state that a parallelogram does not have to have four right angles and draw one that does not; thus it is not a rectangle. This would be a good opportunity to discuss how a parallelogram and rectangle are alike and how they are different. Ask students to make a chart that lists similarities in one column and differences in the other, and have them discuss the chart. This gives students a deeper understanding of the relationship between parallelograms and rectangles. Then ask students to complete the rest of the "Some, All or None" worksheet.



Student Mathematician's Journal p. 23

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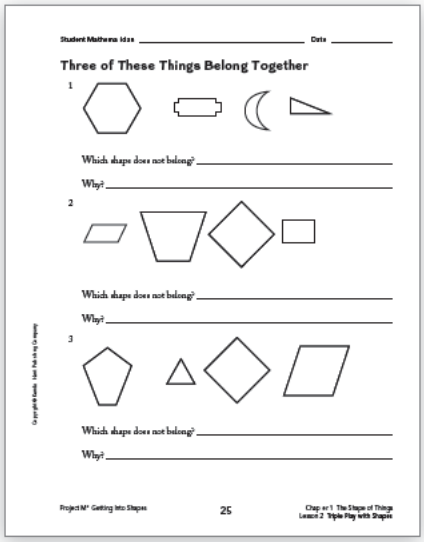
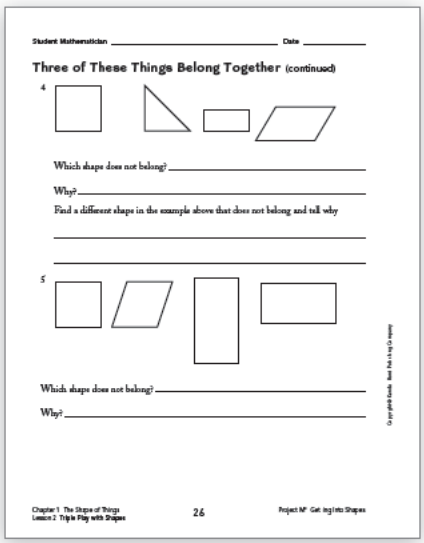

Student Mathematician's Journal p. 25


Student Mathematician's Journal p. 26

Three of These Things Belong Together

In the second activity, students are given four shapes; three belong together, and one does not fit. They need to find the one that does not fit and tell what characteristics the three shapes have in common, and also why the fourth shape does not fit. Use the worksheet "Three of These Things Belong Together." It is a good idea to display this worksheet on the IWB or on a document camera so you can view and discuss shapes together. Do the first question together and then let students finish working individually or in pairs. Some students may find different answers using size as a factor. Encourage them to find another shape to rule out, based on the characteristics of the shape itself that they learned in the previous lesson. Encourage them to use the terms rectangle, rhombus, square, trapezoid, parallelogram and polygon in their explanations.

Investigate ($1\frac{1}{2}$ days)

Triple Play

Next students use the properties of the different shapes that they explored in Lesson 1 to play a card game called Triple Play. In this game students compare and contrast different quadrilaterals as they try to make three-of-a-kind matches. The cards contain a variety of parallelograms, rectangles, squares, rhombi, trapezoids and quadrilaterals that are not parallelograms. As students try to put three cards together based on a similar property, they need to understand concepts such as, a square can also be categorized as, a rhombus, rectangle, regular polygon or parallelogram.

Tell students that the deck of cards is composed of six different configurations of each of the following shapes:

- squares,
- rhombi that are not squares,
- trapezoids,
- rectangles that are not squares,
- polygons that are not parallelograms (some polygons are regular polygons) and
- parallelograms that are not any of the other shapes.

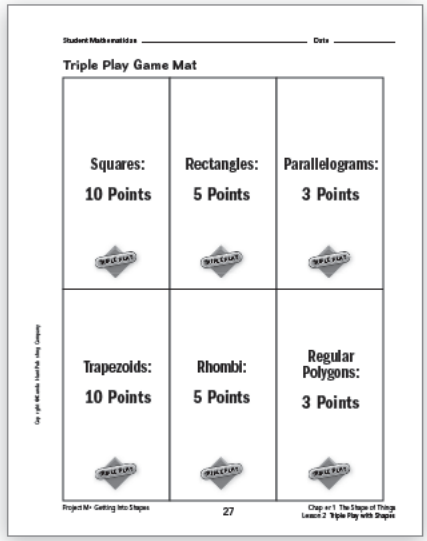
Note that in the following discussion of this game, for ease of discussion, a rectangle refers to a non-square rectangle, a rhombus refers to a non-square rhombus and a parallelogram refers to one that is neither a rectangle nor rhombus. Sets are made up of three of the following: regular polygons, parallelograms, rectangles, squares, rhombi or trapezoids.

PLEASE NOTE! If you have not discussed the meaning of regular polygons (polygons that have all angles and all sides congruent), this is an appropriate time to do so.

NOTES

The game may be played by two players or by two-person teams. To begin play, the deck is shuffled and dealt out one card at a time until each side has 18 cards. The object of the game is to maximize point totals by creating as many sets of three-of-a-kind shapes as possible using the 18 cards that have been dealt.

Scoring is based on the following rubric: 10 points is awarded for each set of three squares or three trapezoids; 5 points is awarded for each set of three rectangles or three rhombi; and 3 points is awarded for each set of three parallelograms or three regular polygons. After the cards have been dealt, players or teams are given a short time (1–2 minutes) to sort their cards into sets of three of the same shape, using the game mat in their Student Mathematician's Journal on p. 27 as a tool. At the end of the time period, each side must place any unmatched cards face up in the middle of the playing surface. The player or team who had the lowest number of unmatched cards is then given a short period (10–15 seconds) to select three cards from the unmatched cards in order to create another set of three of the same shape. The second player or team is then given the same amount of time to make a selection from any remaining unmatched cards. The opposition may challenge any three-of-a-kind set. Each side then tallies its point total and records its score. The cards are then reshuffled and dealt out again. The first player or team to reach 100 points is the winner.


Student Mathematician's Journal p. 27

Before beginning play, model an example for students. Deal out 18 cards, record the shapes on the cards for all to see, and have students sort together with you. Ask students to talk with a partner about the different sets that can be made. They should decide on which sets would give them the most points and which cards will remain unmatched. Depending on the cards dealt, discuss any interesting possibilities. For example, if in the group of cards there are two squares, two rectangles, one rhombus and one parallelogram, ask students how they would create sets to get the most points. Obviously, you cannot make three squares or trapezoids that are worth the most points, but you can make three rhombi using two squares for 5 points. You can also put the square, rectangle and parallelogram together to make a set of parallelograms for 3 points. It also is possible to match the square with the two rectangles for a 5-point set. At first, students may not see these combinations since it requires them to recognize that a shape can fit into more than one classification. You may need to create similar examples for more practice in order to increase student understanding before play begins.

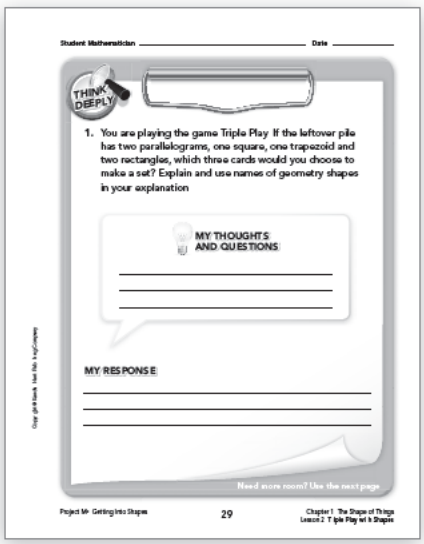
ONLINE RESOURCE

Triple Play

NOTES

Mathematical Communication (1 day)

Students should have an opportunity to play the game several times before answering the Think Deeply questions. The game gives them the opportunity to sort and to classify the shapes and recognize the relationships among them.


Student Mathematician _____ Date _____

THINK DEEPLY

1. You are playing the game Triple Play. If the leftover pile has two parallelograms, one square, one trapezoid and two rectangles, which three cards would you choose to make a set? Explain and use names of geometry shapes in your explanation.

MY THOUGHTS AND QUESTIONS

MY RESPONSE

Need more room? Use the next page.

Project M³ Getting Into Shapes 29 Chapter 1 The Shape of Things Lesson 2 Triple Play with Shapes

Student Mathematician's Journal p. 29

1. You are playing the game Triple Play. If the leftover pile has two parallelograms, one square, one trapezoid and two rectangles, which three cards would you choose to make a set? Explain and use names of geometry shapes in your explanation.

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- I can't get 10 points since I don't have three squares or three trapezoids.
- Can I get 5 points with the cards by putting some shapes into different categories?
- I need to find three rectangles or three rhombi.

Using the Math Messaging Board

The Math Messaging Board can help you organize the class discussion and helps students prepare to write their responses to the Think Deeply Question. Discuss with students what they need to find out and write this as the topic in the Talk About It section on the Math Messaging Board. Write down their ideas under Keep A Record on the Math Messaging Board. Record different ideas including misconceptions. Some students may get confused and think all parallelograms can be considered rectangles rather than all rectangles are parallelograms. For Wrap It Up, record all suggestions that make correct sets.

A class discussion might be similar to the following:

Teacher: Which do you think would be the best set to pick? Allison?

Allison: I think I would choose two rectangles and one parallelogram because then I would have a set.

NOTES

Teacher: And what kind of set would that be?

Allison: It would be a set of rectangles, so I would get 5 points.

Agree/ Disagree and Why?

Teacher: Does anyone agree or disagree with Allison's idea?

Pedro: I disagree. I think you would have three parallelograms because the two rectangles are parallelograms but the parallelogram is not a rectangle.

Repeat/ Rephrase

Teacher: Can someone repeat what Pedro just said?

Melissa: He said that Allison would not have three rectangles because the parallelogram is not a rectangle.

Teacher: How do you know that the parallelogram is not a rectangle?

Pedro: Because it does not have right angles.

Repeat/ Rephrase

Teacher: Okay, so if we chose two rectangles and a parallelogram, we would not have a set of three rectangles. Pedro, can you repeat again what you said this set would be?

Pedro: It would be a set of three parallelograms because the two rectangles are also parallelograms.

Agree/ Disagree and Why?

Teacher: Does anyone agree or disagree with this?

Discussion continues and the students come to the conclusion that this would be a set of parallelograms that is worth only 3 points. The teacher continues the discussion with the question, "Can we find a set that would be worth more points?" This discussion should lead to the set of two rectangles and the square. Students will need to verbalize that a square is also a rectangle. The discussion here should not be rushed and is very important in helping students clarify their thinking. These relationships are difficult for them to conceptualize and articulate. A lot of discussion is needed before students can internalize the relationships among the quadrilaterals.

NOTES

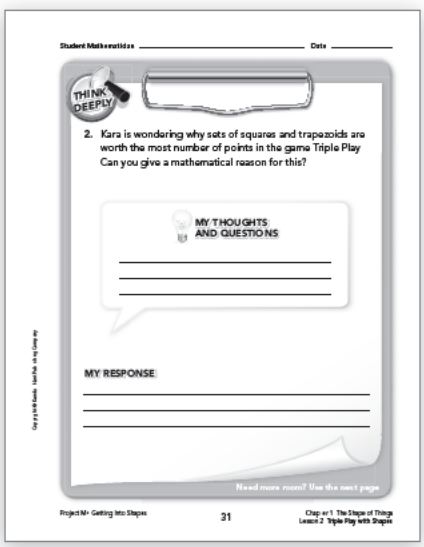
What to Look for in Responses

- Students should recognize that the square and two rectangles are the best choice to make a set since this would be worth the most number of points (5 points for a set of rectangles).
- Students should state that a square is also a rectangle, and that is why these three cards can be considered a set of rectangles.

Possible Difficulties

- Students may be confused and think that two rectangles and a parallelogram make a set of rectangles. They may not understand that every rectangle is a parallelogram but not every parallelogram is a rectangle.
- Students who are struggling with the idea of a square belonging to the set of rectangles may find it easier to try to make a threesome of parallelograms using the square or one of the rectangles for the third parallelogram. Although this shows that they recognize that all shapes are part of the set of parallelograms, they have not selected the best set worth the most number of points. A set of parallelograms is worth only 3 points, while a set of rectangles is worth 5 points.

2. Kara is wondering why sets of squares and trapezoids are worth the most number of points in the game Triple Play. Can you give a mathematical reason for this?


Student Mathematician's Journal p. 31

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- I think there are fewer cards that are squares and trapezoids.
- I wonder if any other cards can be used with the squares or trapezoids to make a set worth 10 points.
- Can it be that squares and trapezoids have more characteristics than the other shapes?

What to Look for in Responses

- This question really challenges students to think about the relationships among all the shapes. There are a lot of ways to make parallelograms, since most of the shapes (parallelograms, rectangles, rhombi and squares) are parallelograms.

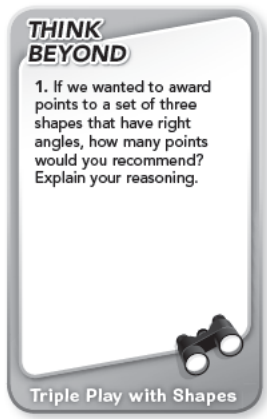
There are fewer options to make rectangles since you can only use combinations of six non-square rectangles and six squares. The same is true for rhombi that use combinations of the six non-square rhombi and six squares. The squares and trapezoids have the fewest cards available to make combinations, only six squares or six trapezoids respectively, so it is hardest to make sets for these combinations.

Possible Difficulties

- Some students may not understand the rationale for awarding the most number of points to these kinds of sets. They may think that each shape in the deck has a total of six cards and that all combinations should be awarded the same number of points. They have not considered the relationship among the shapes that are all parallelograms.
- Some students may be struggling with the concept that every rectangle, rhombus and square is a parallelogram, but the converse is not true. So, every parallelogram is not a rectangle, nor is every parallelogram a square, nor is every parallelogram a rhombus. This can be confusing for some students, even for some adults! Thus, a general parallelogram that is not one of these shapes cannot help in making a threesome to get a set of rectangles, squares or rhombi.

Students might make the claim that a parallelogram is neither a rectangle nor a rhombus nor a square. This is not correct. Some parallelograms are indeed rectangles, some are rhombi, and some are squares. But all parallelograms are not these shapes. It is important to make sure students state this properly. Do not let these statements go unnoticed with the idea that you really understand what the student means to say. It is important that students communicate their ideas precisely and that they are mathematically correct.

In the first Think Beyond Card, the rules of the game Triple Play are changed as points are awarded for a set of three shapes that have right angles, and students are challenged to see how this affects the outcome of the game. There are 15 shapes that have right angles. Students must justify their answer in relation to the number of shapes needed to create sets that are worth 3, 5 or 10 points. Clearly the number of points should be less than 5, since there were only 12 shapes available for each of these two sets.



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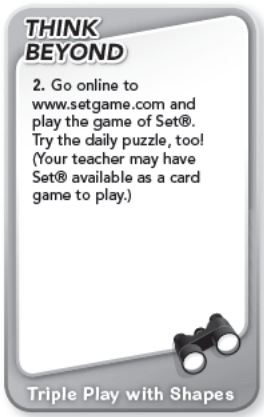


The second Think Beyond Card encourages students to play the game Set®. The object of this game is to identify a "Set" of 3 cards from 12 that are laid out on the table. Each card has a variation of the following four features:

1. Color: Each card is red, green or purple.
2. Symbol: Each card contains ovals, squiggles or diamonds.
3. Number: Each card has one, two or three symbols.
4. Shading: Each card is solid, open or striped.

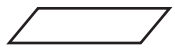
A "Set" consists of three cards in which each feature is EITHER the same on each card OR is different on each card. That is to say, any feature in the "Set" of three cards is either common to all three cards or is different on each card. This game can be purchased as a card game to be played in class; the website www.setgame.com also offers daily Set® puzzles, which are excellent brainteasers, as well as a link to a multi-player online version of the game.

Student Mathematician: _____ Date: _____

Some, All or None *ANSWER SHEET*

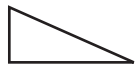
Tell whether each statement below is true or false by circling the correct answer. If the statement is false, give a counterexample using words and/or pictures.

1. All parallelograms are rectangles. True (False)

   This shape is a parallelogram but not a rectangle: 

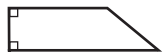
2. Some polygons have 10 sides. (True) False

3. All triangles are equilateral. True (False)

   There are triangles whose sides are not all congruent. 

4. Some rectangles are squares. (True) False

5. No trapezoids have right angles. True (False)

   This is a trapezoid and it has two right angles: 

6. No rectangles are rhombi. True (False)

   When a rectangle is a square, it is also a rhombus.

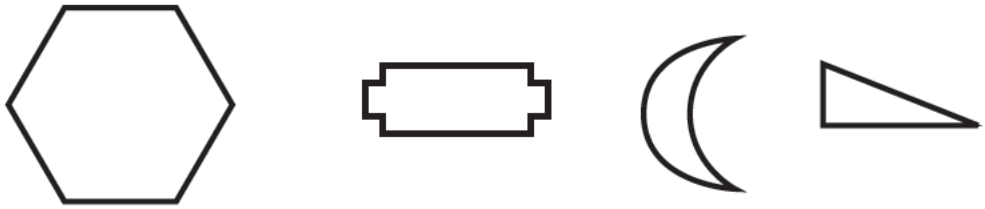
7. All squares are rectangles. (True) False

8. No rhombi are squares. True (False)

   Some rhombi are squares; a square is a rhombus.

Student Mathematician: _____ Date: _____

Three of These Things Belong Together

1. 

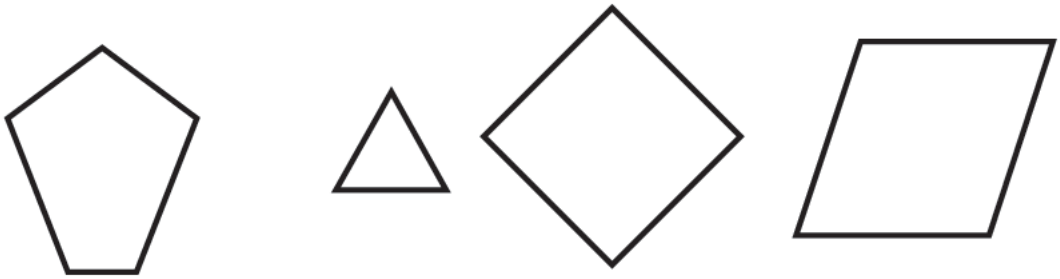
Which shape does not belong? _____

Why? _____

2. 

Which shape does not belong? _____

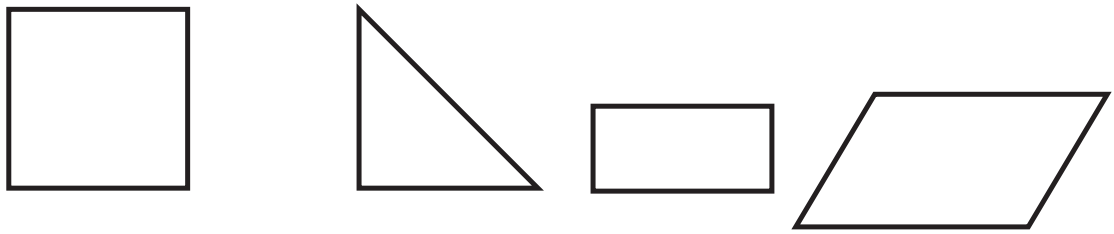
Why? _____

3. 

Which shape does not belong? _____

Why? _____

Three of These Things Belong Together (continued)

4. 

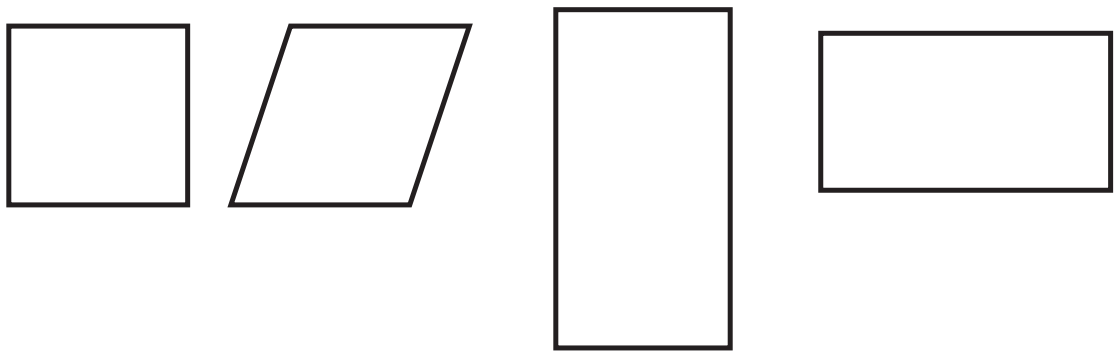
Which shape does not belong? _____

Why? _____

Find a different shape in the example above that does not belong and tell why.

_____

_____

5. 

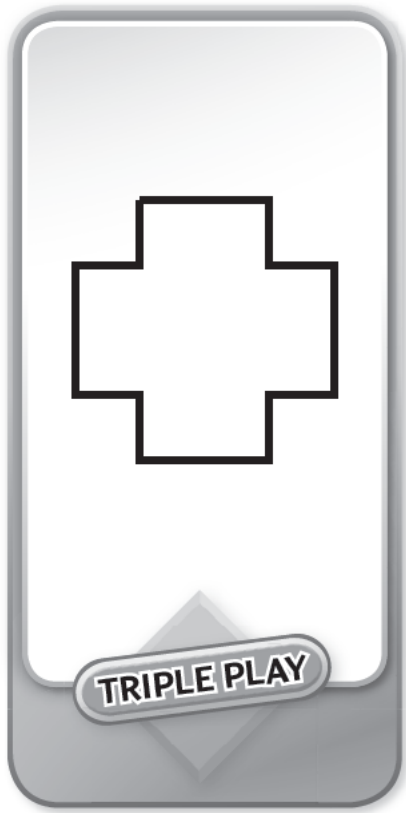
Which shape does not belong? _____

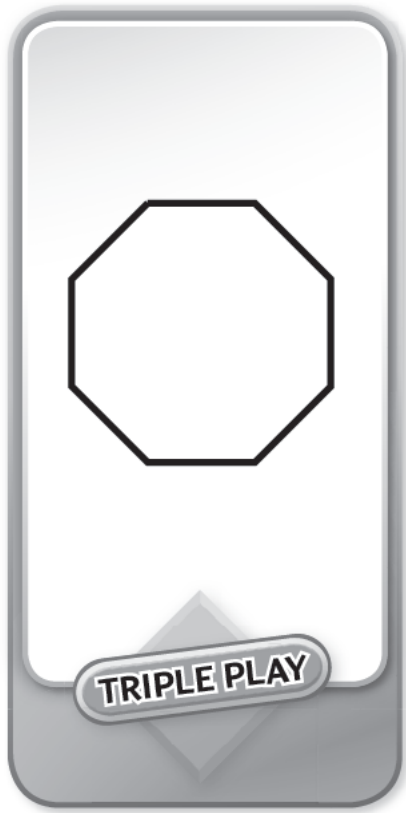
Why? _____

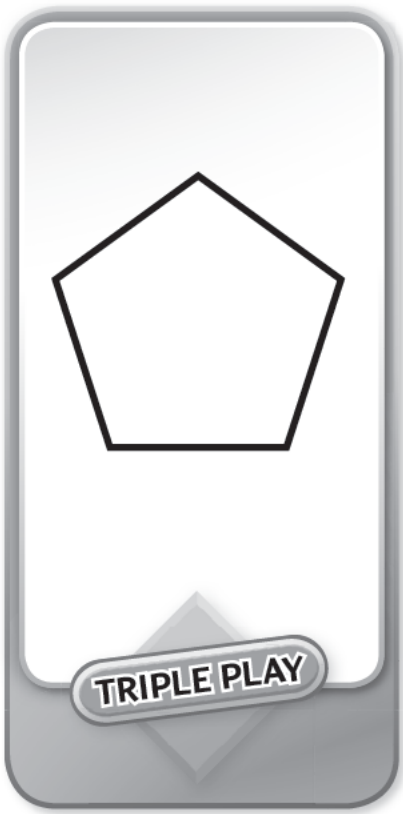
Student Mathematician: _____ Date: _____

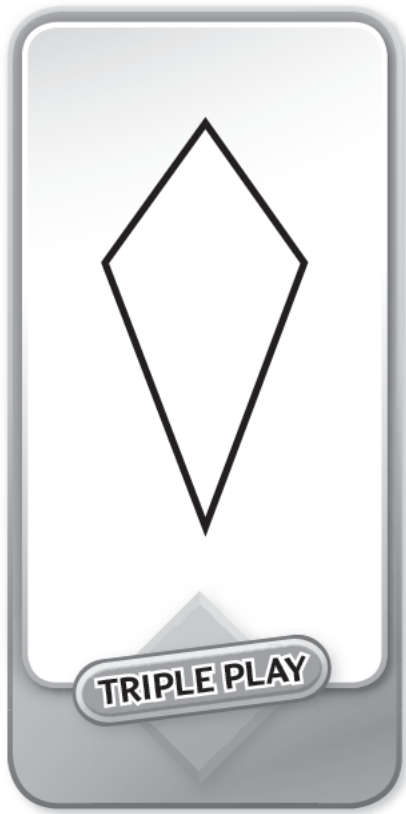
Three of These Things Belong Together (continued) *ANSWER SHEET*

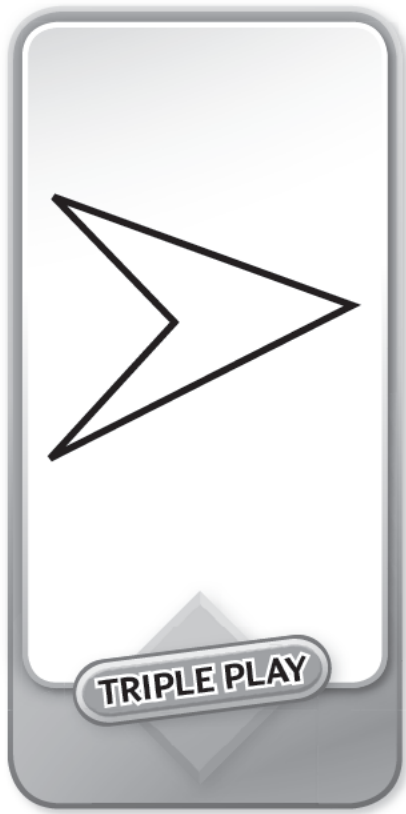
1. The moon shape does not belong since the rest are polygons.
2. The trapezoid does not belong since the rest are parallelograms.
3. The pentagon does not belong since the rest all have congruent sides or are equilateral. (This is a good place to introduce the geometric term "congruent" and add it to your word wall.) The triangle and the square are also regular polygons because they have both congruent sides and congruent angles. OR, the square does not belong because it is the only polygon that has a right angle.
4. The parallelogram does not belong since the rest all have right angles, OR the triangle does not belong since the rest are parallelograms.
5. The rhombus does not belong since the rest are rectangles.

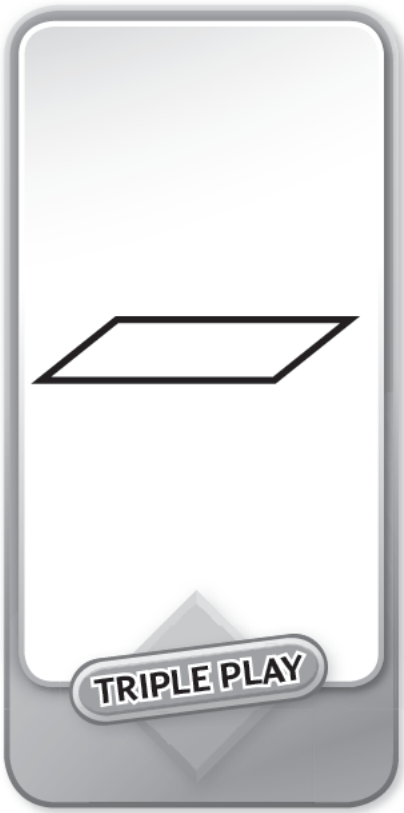


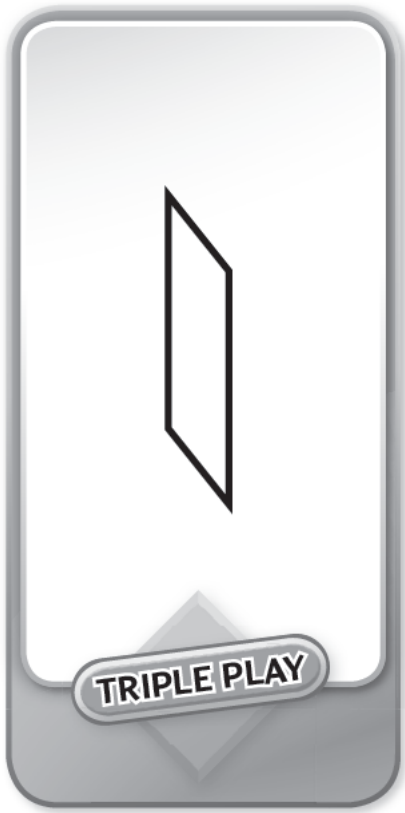


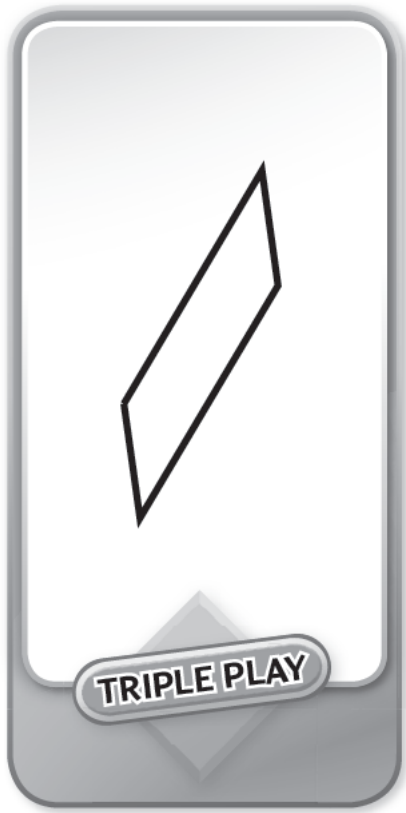


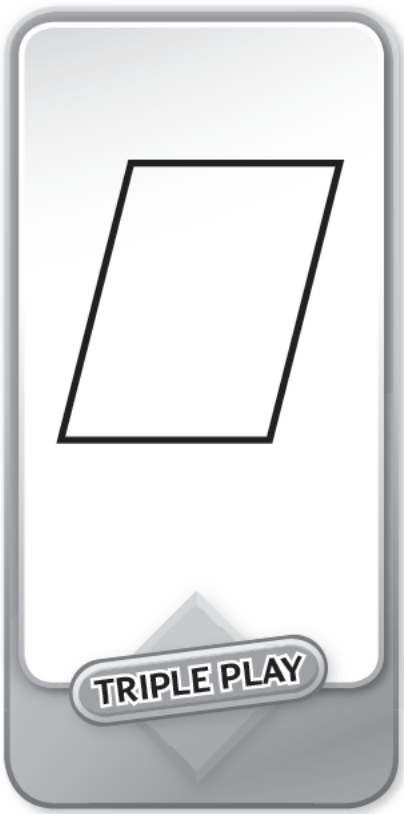


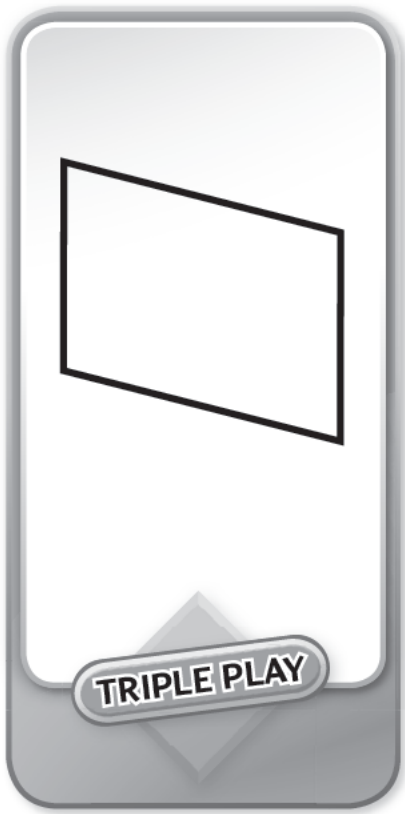


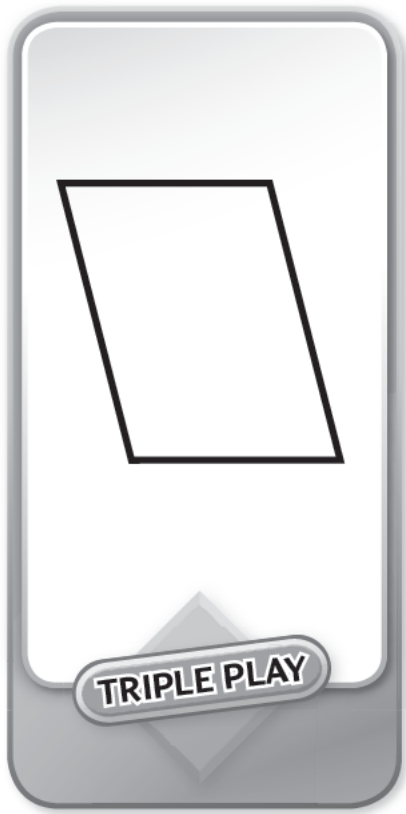


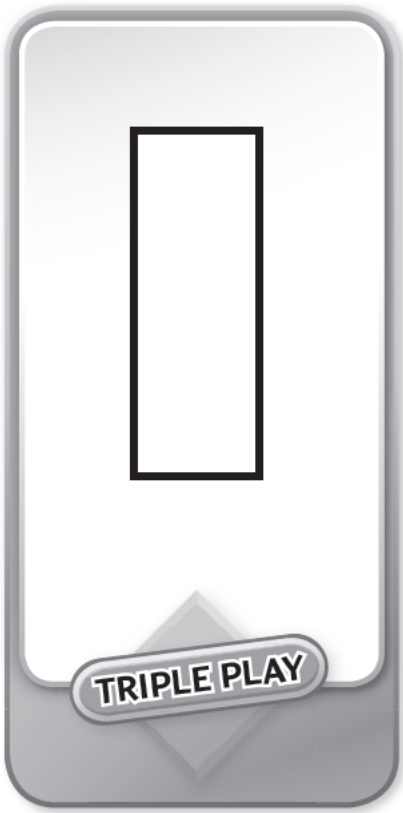


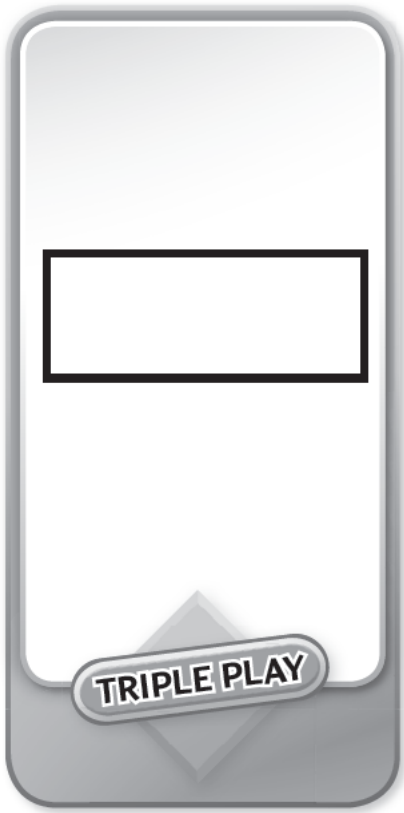


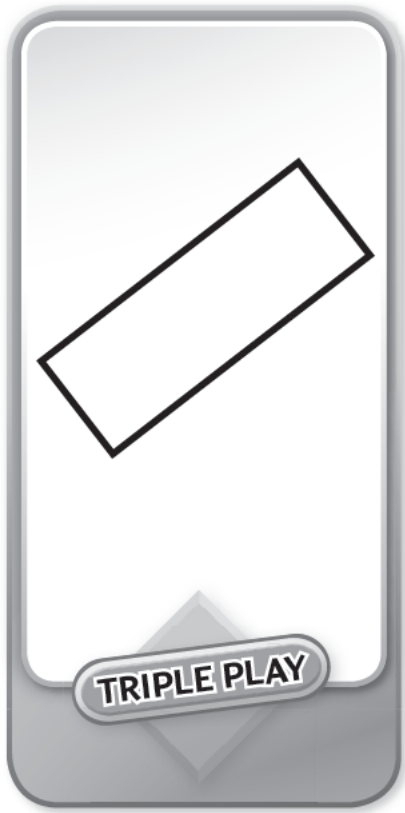


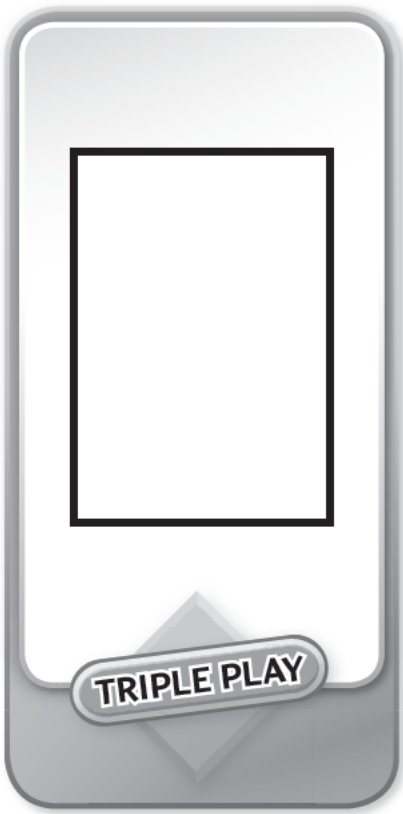


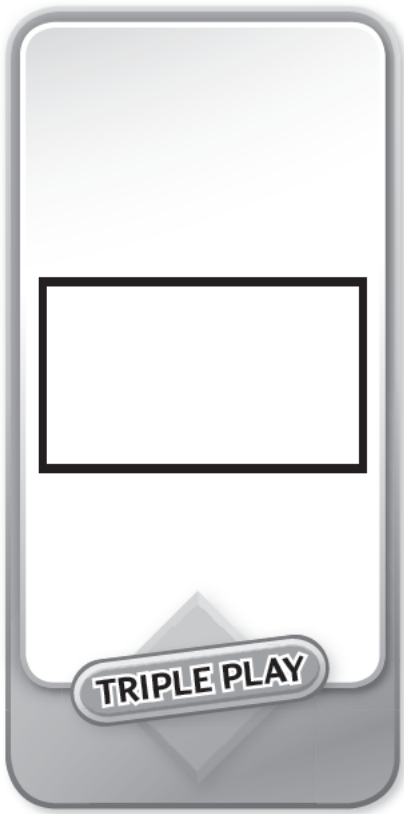


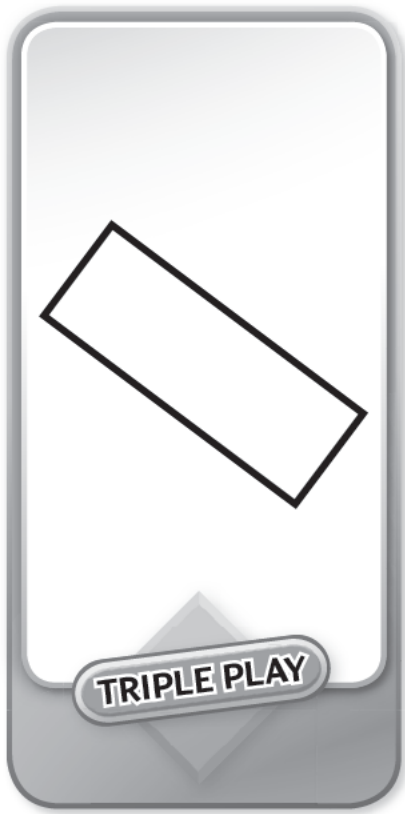


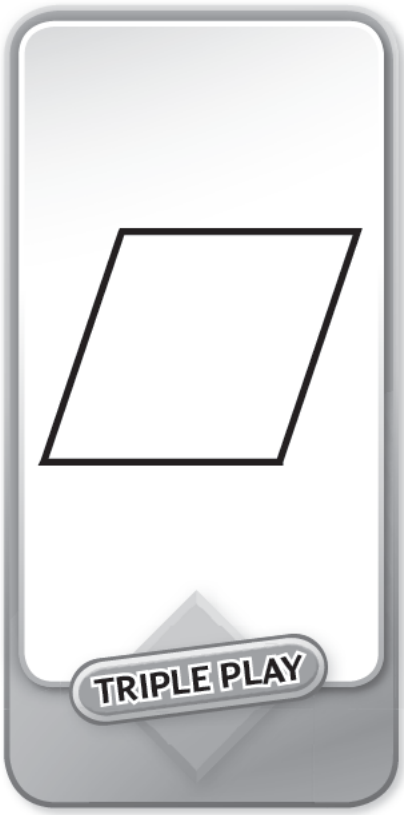


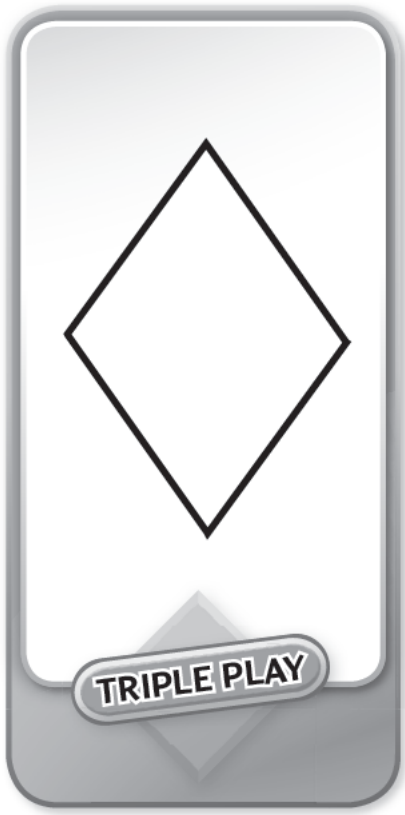


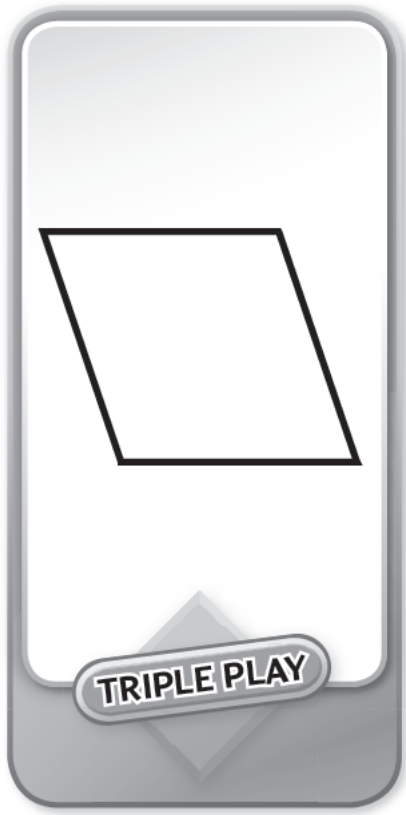


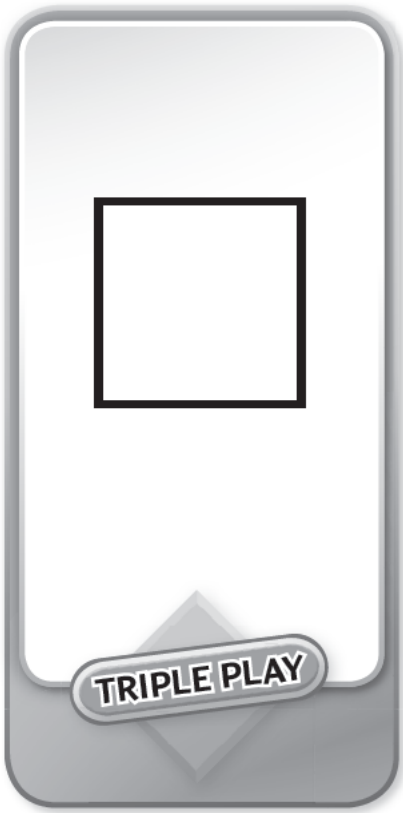


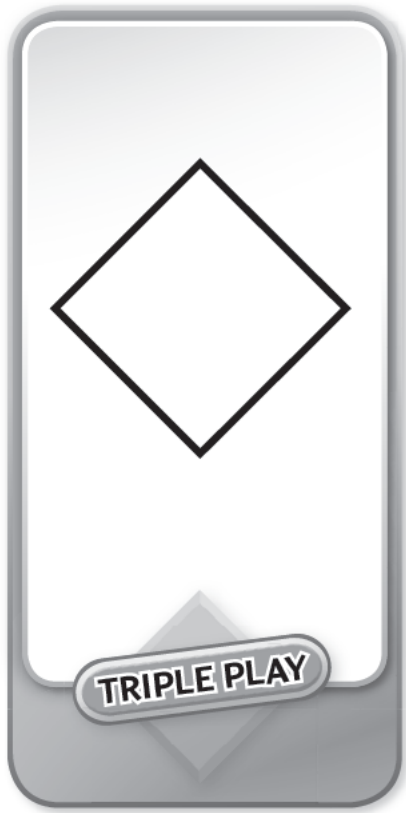


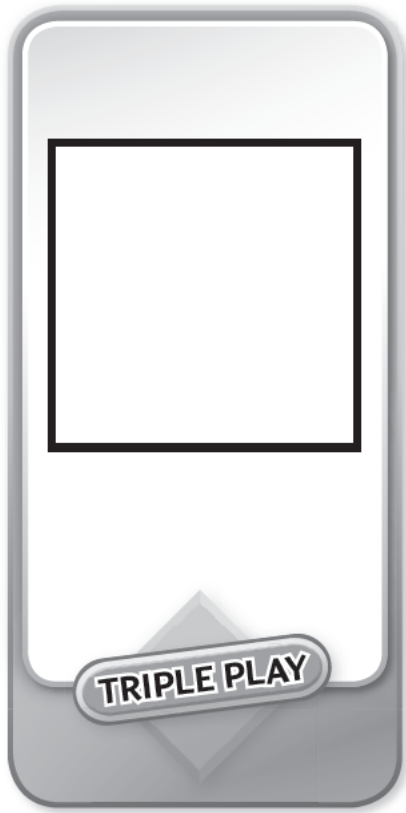


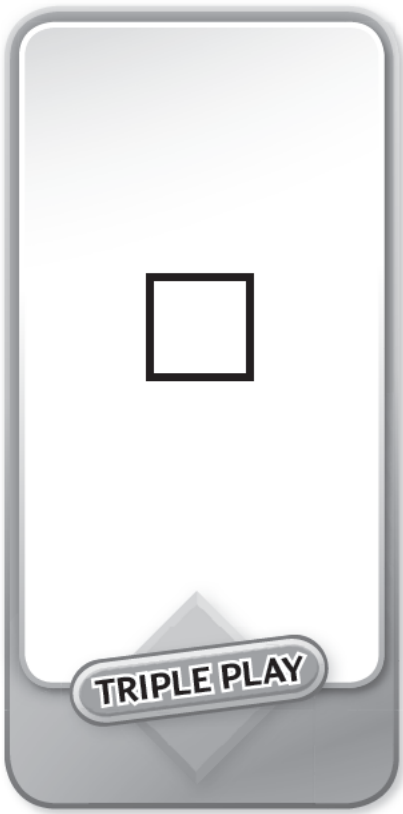


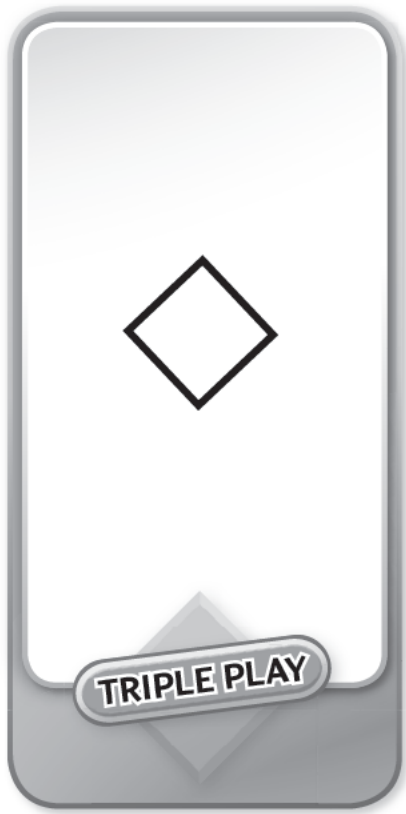


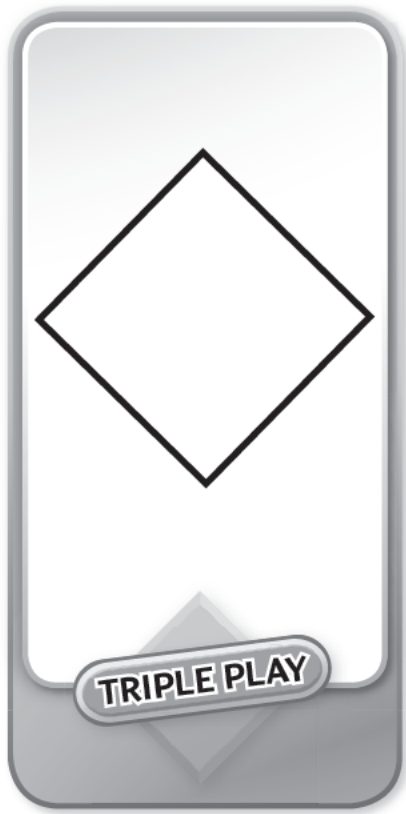


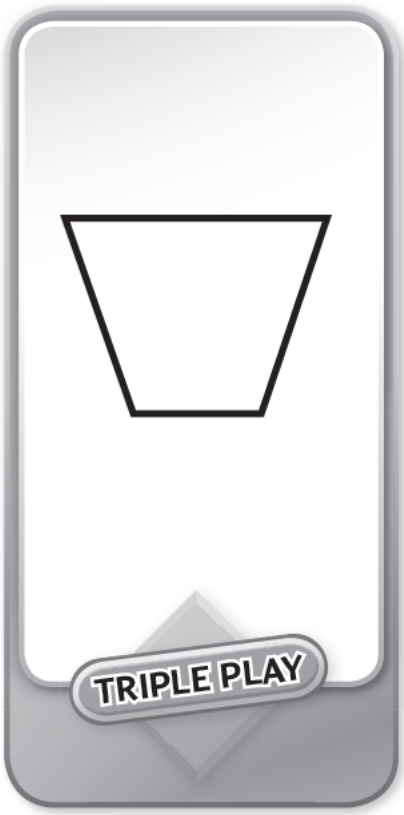


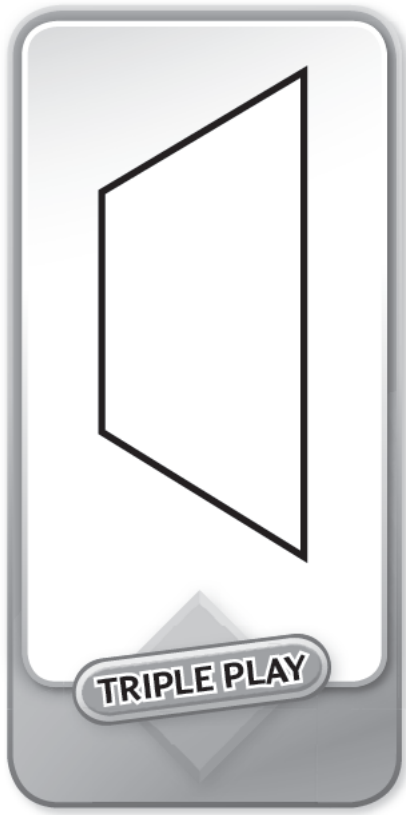


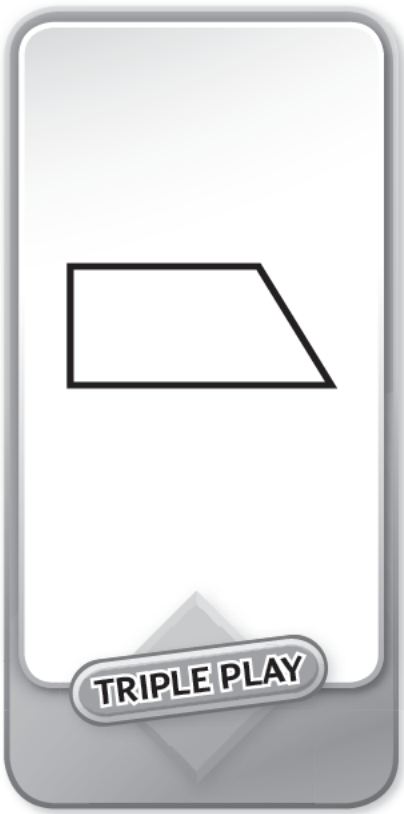


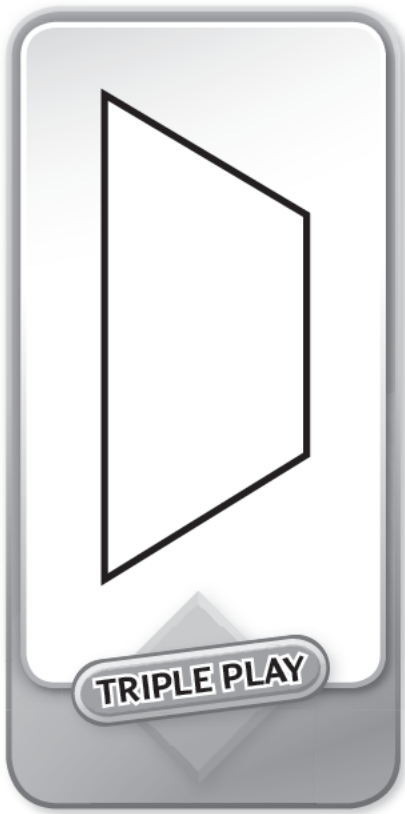


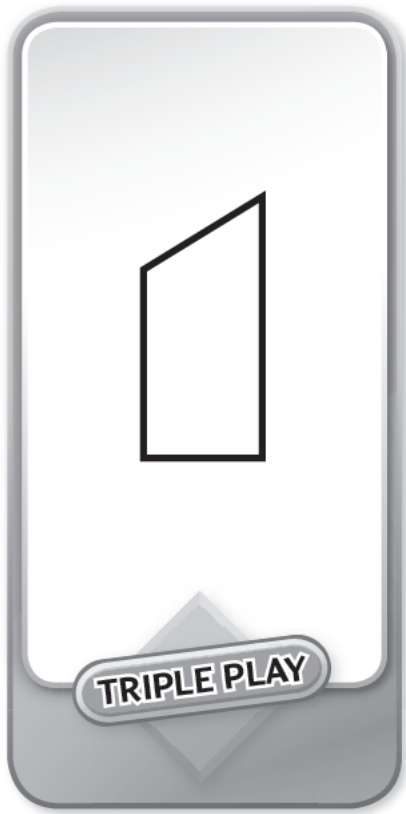












HINT CARDS

If you had two cards with rectangles on them, what kinds of shapes could you add to make a set of three?

Triple Play with Shapes

HINT CARDS

If you had two cards with rhombi on them, what kinds of shapes could you add to make a set of three?

Triple Play with Shapes

HINT CARDS

A square is also a rectangle because it is a parallelogram that has right angles.

Triple Play with Shapes

HINT CARDS

A trapezoid is different from a parallelogram because the trapezoid only has <u>one</u> pair of parallel sides whereas the parallelogram has <u>two</u> pairs of parallel sides.

Triple Play with Shapes

THINK BEYOND

1. If we wanted to award points to a set of three shapes that have right angles, how many points would you recommend? Explain your reasoning.

Triple Play with Shapes

THINK BEYOND

2. Go online to www.setgame.com and play the game of Set®. Try the daily puzzle, too! (Your teacher may have Set® available as a card game to play.)

Triple Play with Shapes

A QUICK LOOK

NOTES

Chapter 1: Lesson 2
Triple Play with Shapes

Objectives:

- Students will compare and contrast different types of shapes.
- Students will classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
- Students will understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
- Students will classify two-dimensional shapes in a hierarchy based on properties.

Initiate: (1 day)

1. Begin the lesson by having students turn to the "Some, All or None" worksheet in the Student Mathematician's Journal. Tell students that they must decide if each statement is true or false. Draw students' attention to the fact that a statement is considered false if they can find just one example that does not fit the statement. This is known as a counterexample.
2. Complete the first example on the worksheet together with students. Have students discuss how a parallelogram and rectangle are alike and different. Have students make a chart that lists similarities in one column and differences in another, then discuss.
3. Have students complete the "Some, All, or None" worksheet individually or in pairs. Then ask students to share their ideas.
4. Have students turn to "Three of These Things Belong Together" in the Student Mathematician's Journal. Do the first example together as a class. Have students complete the remaining examples individually or in pairs.

Investigate: ($1\frac{1}{2}$ days)

1. Tell students that they will play a game called "Triple Play." They will be comparing different quadrilaterals to try and make three-of-a-kind matches.

NOTES

Scoring	
Three-of-a-Kind Set	**Points**
Set of three squares	10
Set of three trapezoids	10
Set of three rectangles	5
Set of three rhombi	5
Set of three parallelograms	3
Set of three regular polygons	3

2. In playing the game, sets of three are made up of the following: regular polygons, parallelograms, rectangles, squares, rhombi or trapezoids. *(Note: If you have not discussed with students the meaning of regular polygons, do so at this time.)*
3. Go over the directions for playing the game with students. The object of the game is to earn the greatest amount of points by creating as many sets of three-of-a-kind shapes as possible using the 18 cards that have been dealt. Scoring is based on the rubric shown in the margin. The first player or team to reach 100 points is the winner.
4. Before beginning play, model a round of the game with students. Have students play a few rounds of the game and then share any strategies that helped students produce higher point values during the game.

Mathematical Communication: (1 day)

Assign and discuss at least one of the two Think Deeply questions. The other question can be used as a homework assignment. To make sense of the question, students first think about what is being asked and write down some initial reflections and/or questions. Use the Math Messaging Board as you facilitate the discussion of the first Think Deeply question.

- The first Think Deeply question asks students to make a three-of-a-kind set in the game Triple Play, given the following leftover cards: two parallelograms, one square, one trapezoid and two rectangles and explain how they created the set. This is a good question to use to formally assess the lesson.
- The second Think Deeply question challenges students to think about the relationships among all the quadrilaterals studied. In answering this question, students need to consider the number of cards that have squares and trapezoids in the deck.