Student Edition & Teacher Guide

IMKH

An IM K–12 Math™ Curriculum

iM v.360 by Kendall Hunt

PRINT SAMPLER

GRADE 3

Illustrative Mathematics CERTIFIED

An IM K–12 Math™ Curriculum

All-embracing, all-encompassing, and all-inclusive

IM® v.360, the new version of the IM K-12 Math curriculum has undergone significant upgrades, enhancements, and revisions based upon feedback from school leaders, teachers and students nationwide. This updated version introduces fresh activities, lessons, problems, and titles.

What is different with IM® v.360?

Upgrades to the K–5 curriculum include:

- *NEW!* Language Learning Goals, End of Unit Guidance, Checklist Guidance
- Strengthened representations of diverse cultures
- Revisions to the Course Guide content, Instructional Routines, and blackline masters
- 2 lessons added in Kinder for number writing/sense (previously found in centers but do direct lesson)
- More blackline masters included in SE so teachers don't need to copy and distribute (alleviates lift)
- Reviewing activities that could create stress (especially food/recipes - when scarcity is a real issue in urban districts)

Upgrades to the 6–12 curriculum include:

- NEW! Narrative Structures, Section-level Assessments (Checkpoints), Instructional Goals, and Teacher Reflection Questions
- Embedded guidance for building a classroom community
- Embedded Math Language Routines and revised Instructional Routine language, including for 5 Practices activities
- Revised context and activity launches to invite more students into the mathematics, including more representations of diverse cultures
- Revised lesson contexts to align with the California framework, including environmental literacy enhancements
- Unit Narratives being revised for accuracy, clarity, and length
- More guidance around BLM's which to laminate and reuse
- More blackline masters included in SE so teachers don't need to copy and distribute (alleviates lift)

Illustrative Mathematics
LEARN MATH FOR LIFE
Kendall Hunt

GRADE 3

Student Edition

UNIT 1

Book 1
Certified by Illustrative Mathematics®

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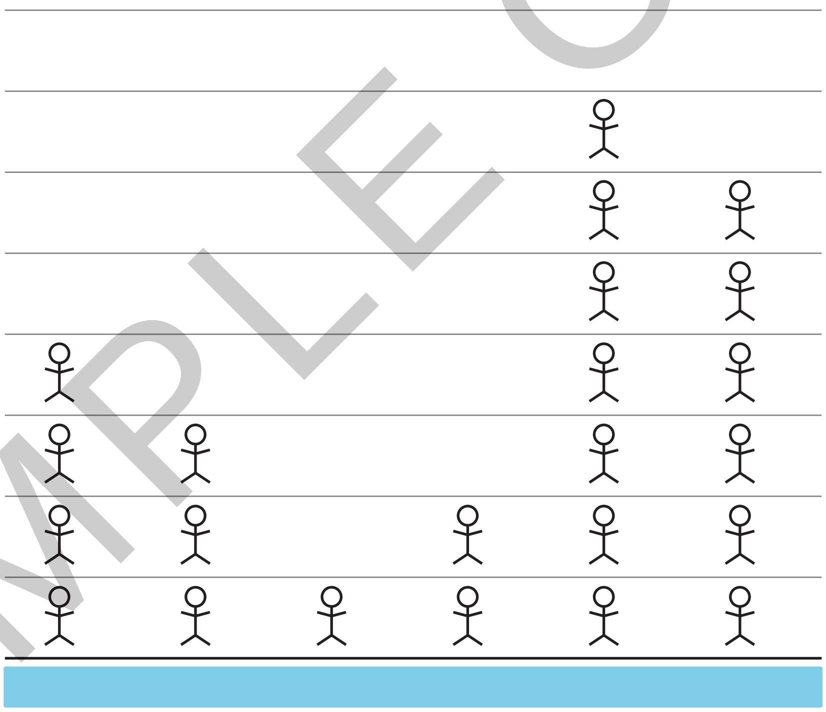
Make Sense of Data

Let’s read and ask questions about data.

Warm-up

Notice and Wonder: Graphs

What do you notice? What do you wonder?



Each represents 1 student.

Activity 1

Picture Time

Sec A

What could the categories be for this picture graph?

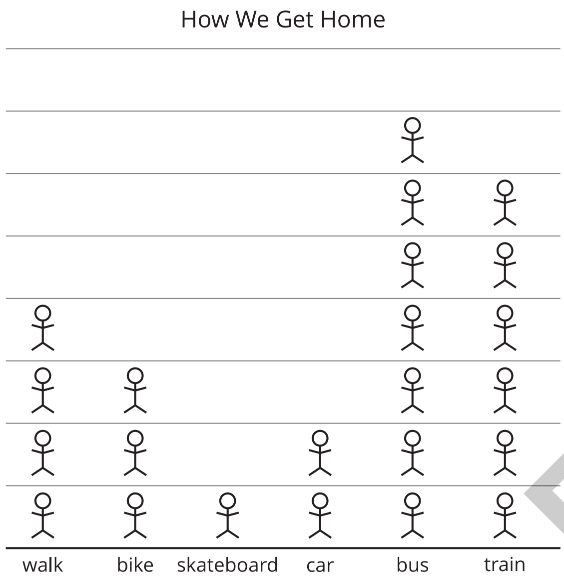
Be prepared to explain your reasoning.

Each [person symbol] represents 1 student.

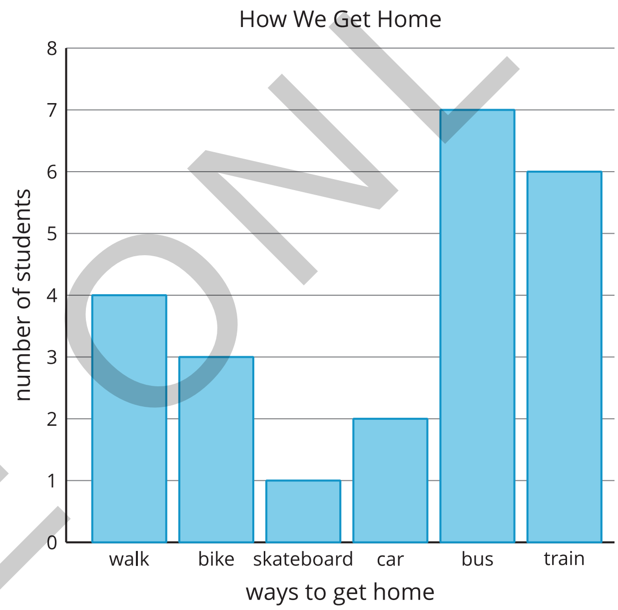
Activity 2

Picture Graphs and Bar Graphs

A group of students were asked, “How do you get home?” Their responses are shown in a picture graph and a bar graph.



Each ♀ represents 1 student.



1. How are the 2 graphs alike? How are they different?

2. What can you learn about how students get home based on the graphs?

3. Write 2 questions that can be answered by reading the graphs.

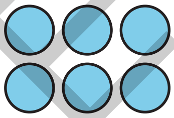
Represent Data and Solve Problems

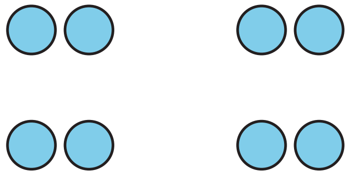
Let's create graphs and answer questions.

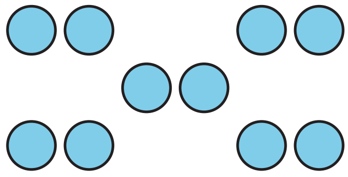
Warm-up

How Many Do You See: Dots in Groups

How many do you see? How do you see them?



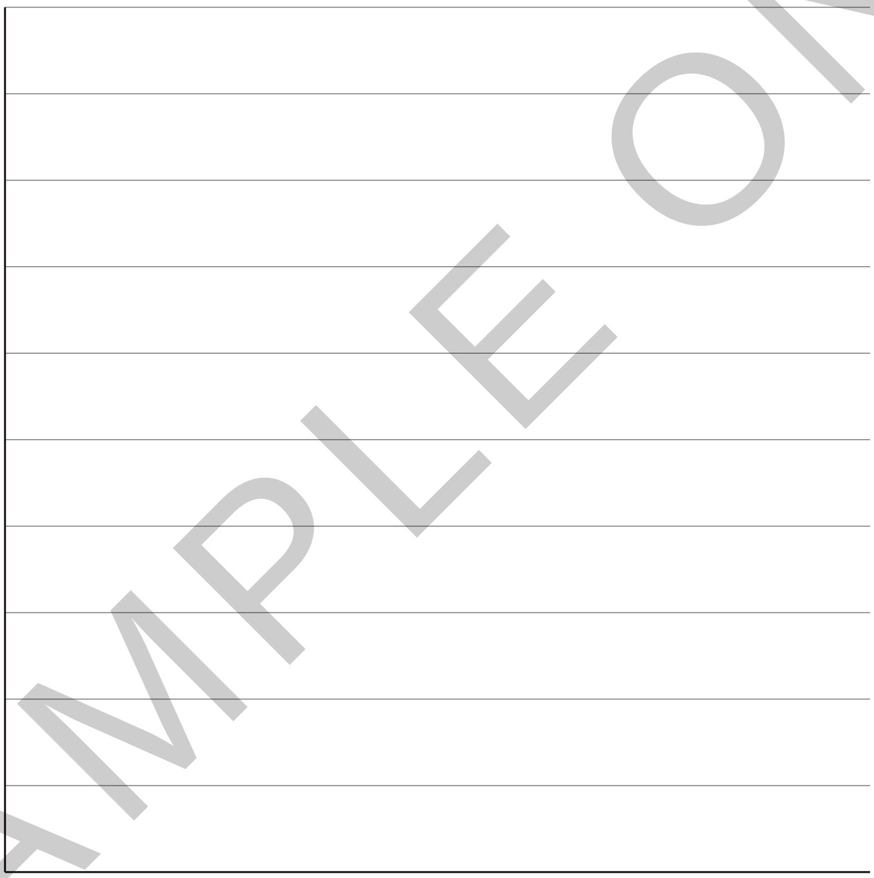




Activity 1

How We Get Home

1. Follow your teacher's instructions to represent the class data in a picture graph.
2. Represent the same data in a bar graph.



Activity 2

Questions about a Bar Graph

1. Decide if each statement about how our class gets home is true or false. Use the graph to explain your reasoning to your partner.

   a. More students walk than go home any other way.

   ____

   b. More students ride home on a bus than in a car.

   ____

   c. Fewer students walk home than ride their bikes.

   ____

   d. More students walk or ride their bikes than ride in a van.

   ____

2. Fill in the blanks as directed by your teacher. Then answer each question.

   a. "How many more students ______________ than ______________?"

   b. "How many more students ______________ or ______________ than ______________?"

Unit 1, Lesson 3

Scaled Picture Graphs

Let's explore scaled picture graphs.

Sec A

Warm-up

Number Talk: Addition

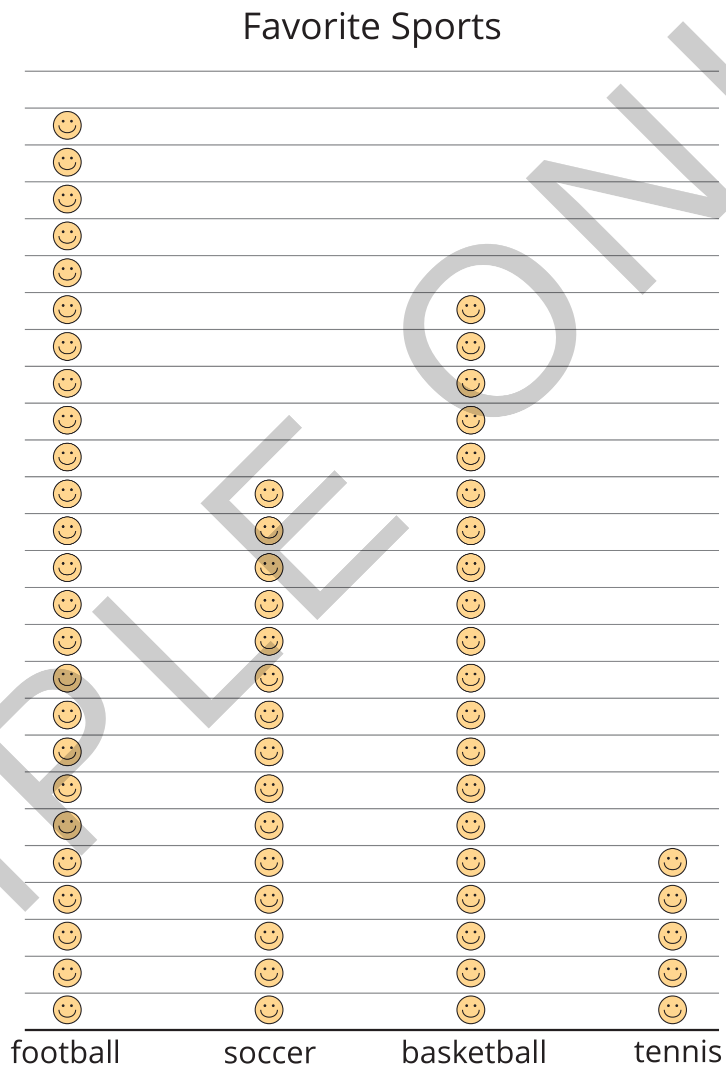
Find the value of each expression mentally.

- $50 + 10$
- $50 + 12$
- $60 + 13$
- $65 + 13$

Activity 1

So Many Responses

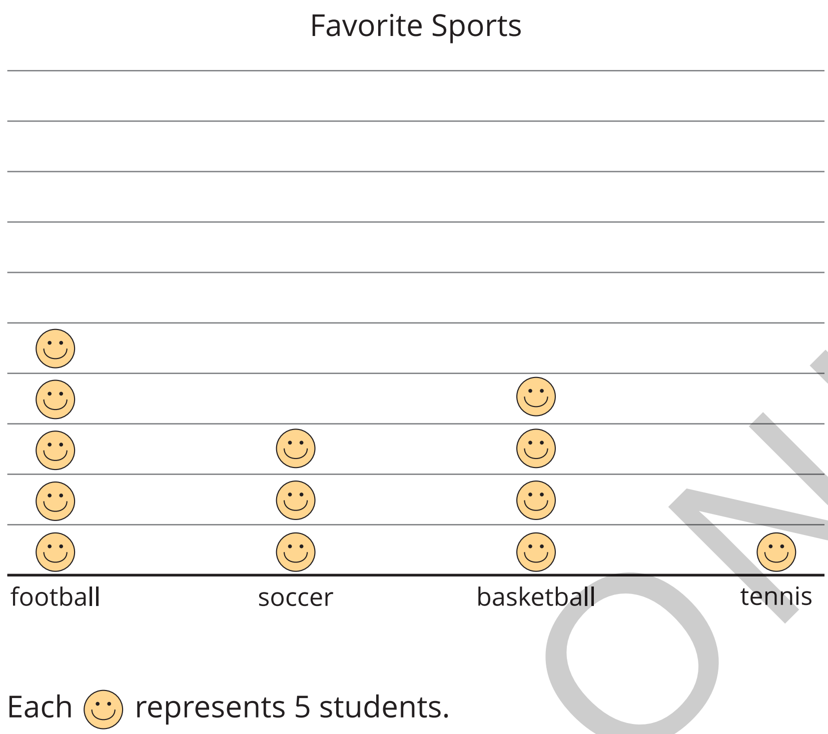
1. A group of students were asked, “Which of these 4 sports is your favorite?” Their responses are shown in this picture graph:



Each ☺ represents 1 student.

How many students are represented in the graph? ______________

2. The students' responses are also shown in this picture graph:



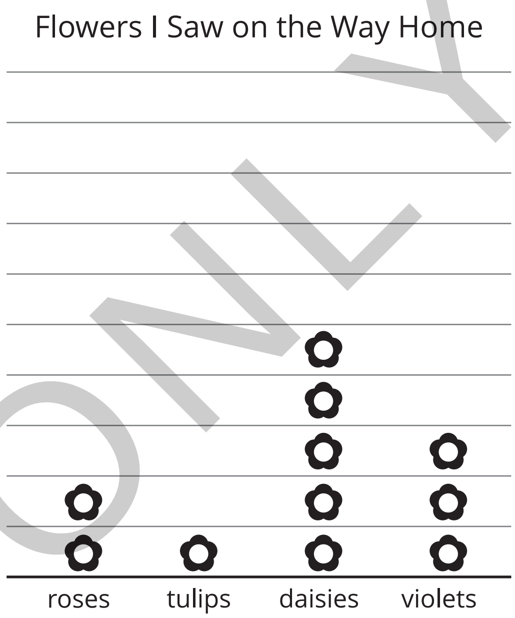
How is counting the total number of students in this graph different from counting the total number of students in the first graph?

Activity 2

Questions about Scaled Picture Graphs

Sec A

1. Andre collects data about 4 types of flowers he sees on the way home. The data is shown in this picture graph:



Each ✿ represents 5 flowers.

a. How many of each type of flower did Andre see on the way home?

roses _____ tulips _____ daisies _____ violets _____

b. Write 2 questions that can be answered by reading the graph.

____

____

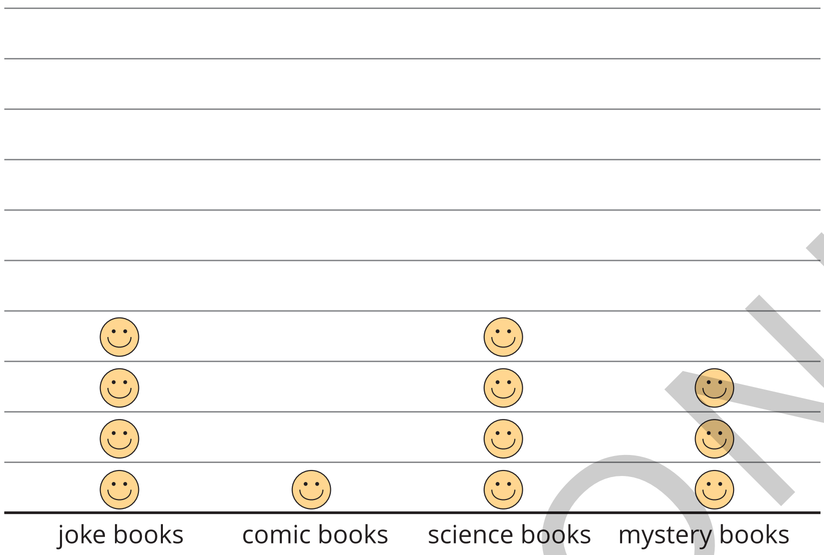
____

____

Sec A

2. A group of students were asked, “Which is your favorite type of book?” Their responses are shown in this picture graph:

Favorite Type of Books



Each represents 2 students.

a. How many students chose each type of book? How do you know?

b. Write 2 questions that can be answered by reading the graph.

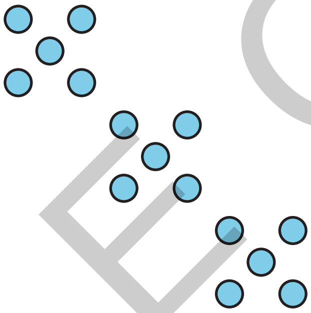
Create Scaled Picture Graphs

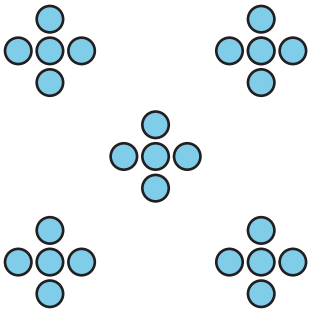
Let's make a scaled picture graph.

Warm-up

How Many Do You See: More Groups of Dots

How many do you see? How do you see them?





Activity 1

Ways to Travel

Sec A

How would you like to travel?

- car (C)
- train (T)
- boat (B)
- balloon (Bal)
- plane (P)
- helicopter (H)

student's name	way of traveling

Activity 2

Create a Scaled Picture Graph

Represent the class survey data in a scaled picture graph. Have each picture represent 2 students.

Represent Data in Scaled Bar Graphs

Sec A

Let's make a scaled bar graph.

Warm-up

Number Talk: Twos and Fives

Find the value of each expression mentally.

- $2 + 2 + 2 + 2$

- $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$

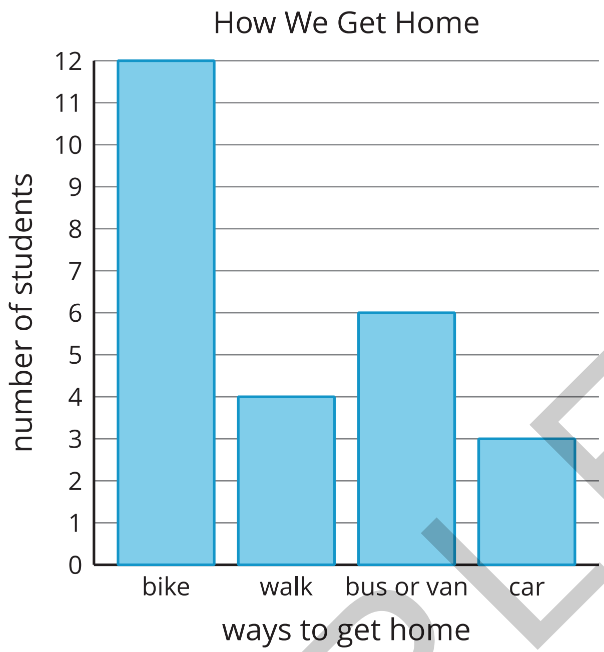
- $5 + 5 + 5 + 5$

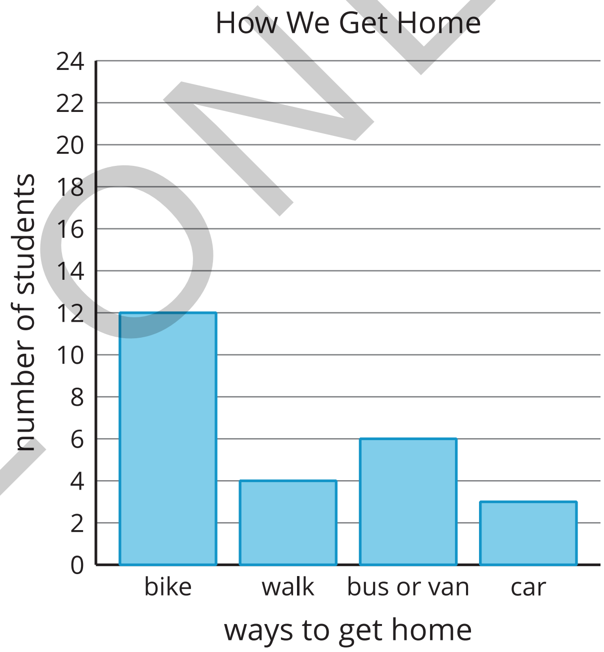
- $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$

Activity 1

Compare Bar Graphs

All the students in a class were asked, “How do you get home from school?” Their responses are shown in these 2 bar graphs:





Discuss with your partner:

- How are the 2 graphs alike?
- How are they different?

Activity 2

Create a Scaled Bar Graph

Represent the data we collected earlier about travel choices in a **scaled bar graph.** Use the graph with a scale of 2 or the graph with a scale of 5. If you have time, you can make 2 graphs. Be sure to label your title and categories.

Unit 1, Lesson 6

Choose a Scale

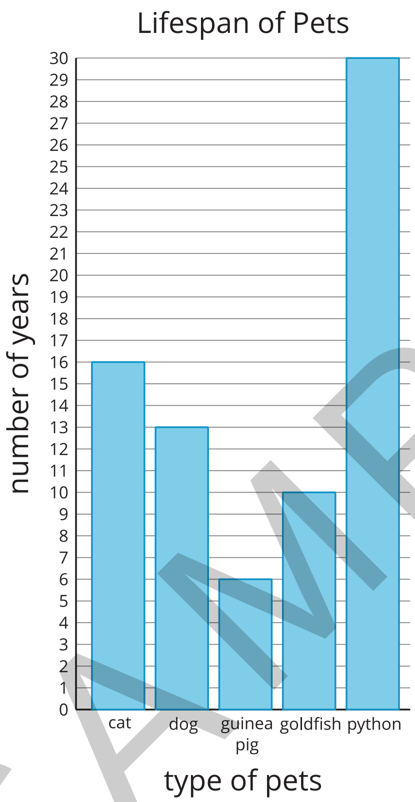
Let’s choose a scale for our bar graph.

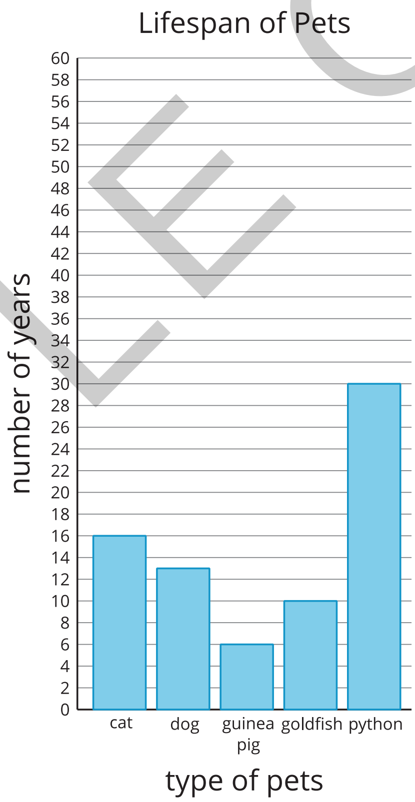
Sec A

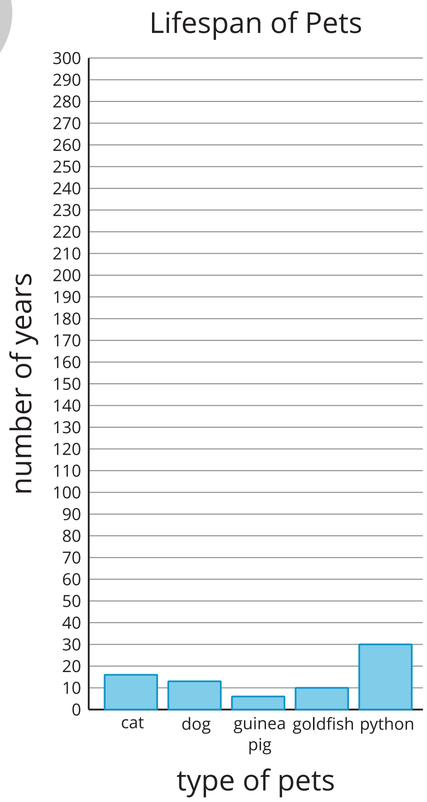
Warm-up

Notice and Wonder: Bar Graph Scales

What do you notice? What do you wonder?





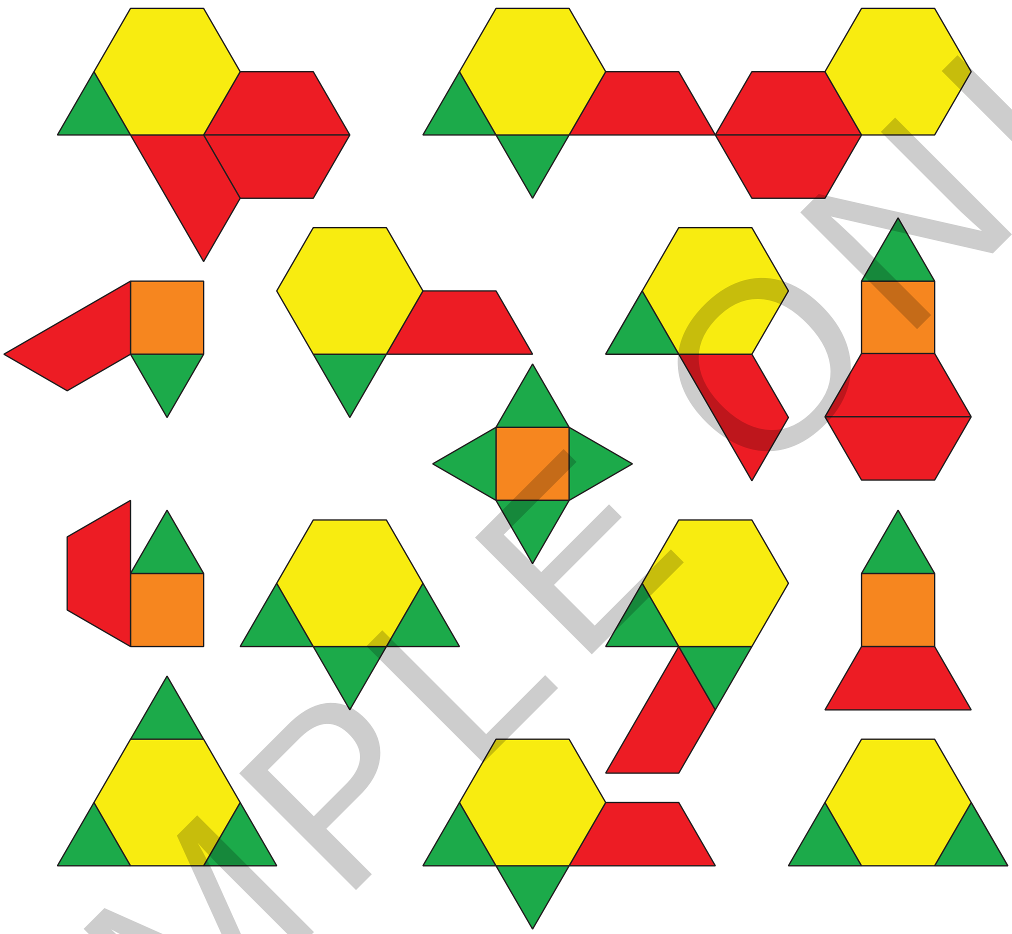


Activity 1

Represent Pattern Blocks

Sec A

Here is a collection of pattern blocks.



Mai, Noah, and Priya want to make a bar graph to represent the number of triangles, squares, trapezoids, and hexagons in the collection.

- Mai says the scale of the bar graph should be 2.
- Noah says the scale of the bar graph should be 5.
- Priya says the scale of the bar graph should be 10.

1. Who do you agree with? Explain your reasoning.

2. Use the scale that you chose to create a scaled bar graph to represent the collection.

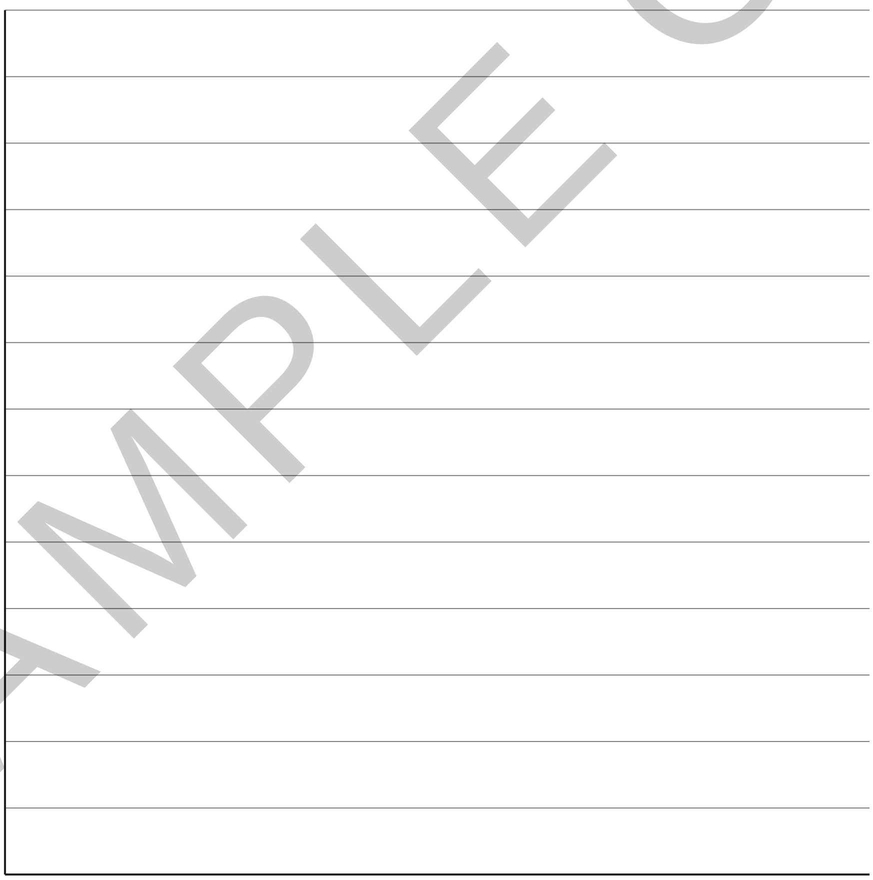
Activity 2

Represent More Data in a Scaled Bar Graph

All the third-grade students at a school were asked, "What is your favorite season?" Their responses are shown in this table.

favorite season of the year	winter	spring	summer	fall
number of students	24	13	40	22

Use the data from the table to create a scaled bar graph.



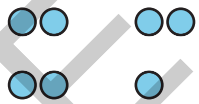
Answer Questions about Scaled Bar Graphs

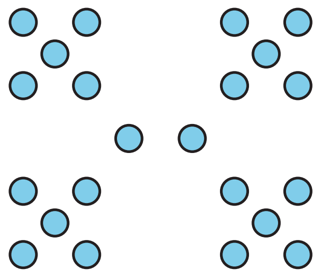
Let’s solve problems based on data represented in bar graphs.

Warm-up

How Many Do You See: Groups of Dots

How many do you see? How do you see them?





Activity 1

Questions about Favorite Season

Sec A

Use your Favorite Season bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many students are represented in the graph?

2. How many students chose spring or fall as their favorite season?

3. How many more students chose summer than winter?

4. How many fewer students chose spring than fall?

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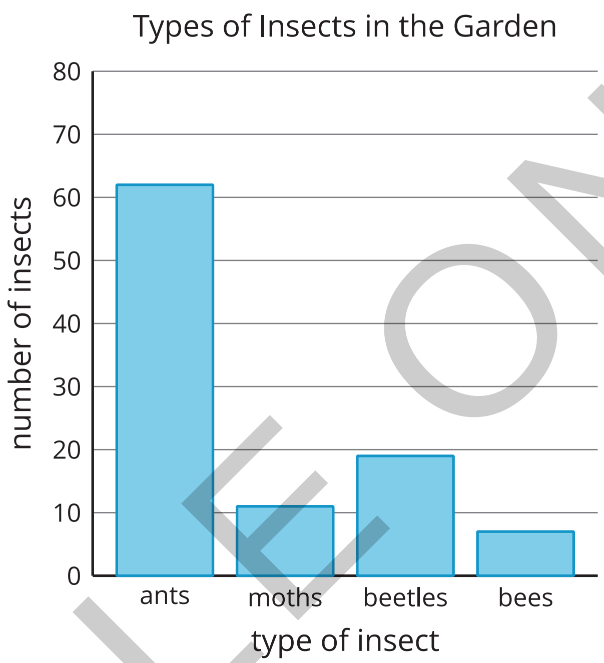
Activity 2

Questions about Insects in the Garden

Data was collected to see how many of the 4 types of insect were in a garden. The data is shown in this bar graph:



Use the bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many insects were in the garden?

2. How many more ants were in the garden than bees?

3. How many fewer moths were there than ants?

Sec A

4. Work with your partner to write 2 other questions that can be answered by reading the graph.

5. Trade with another group and answer each other's questions.

More Questions about Scaled Bar Graphs

Let's solve problems using data shown on bar graphs.

Warm-up

Number Talk: Repeated Addition

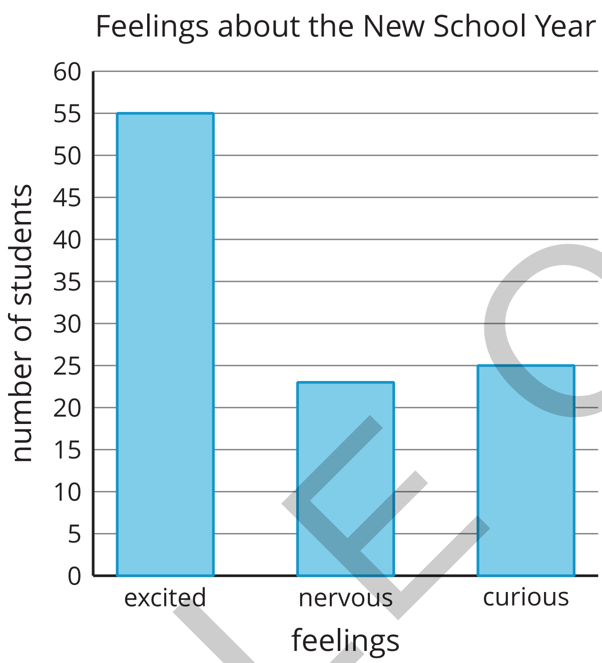
Find the value of each expression mentally.

- $2 + 2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5$

Activity 1

New School Year

A group of students were asked, “Which way do you feel about the new school year?” Their responses are shown in this bar graph:

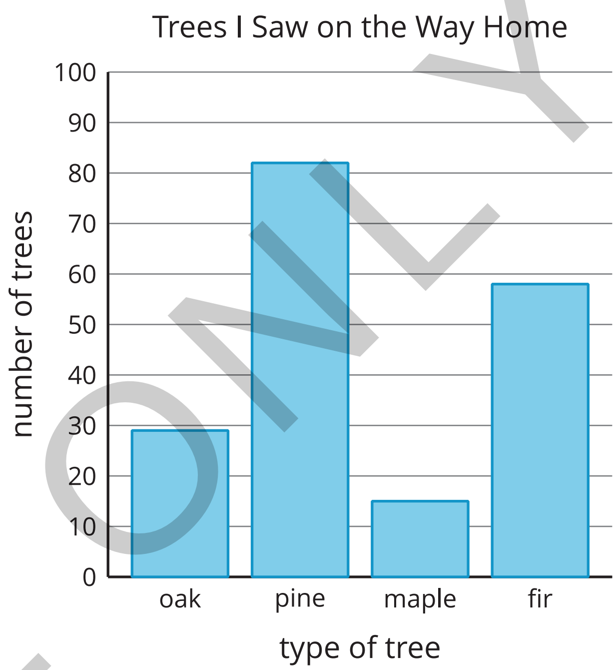


How many more students are excited about the new school year than are nervous or curious?

Activity 2

Use Bar Graphs to Solve Problems

The bar graph shows how many of the 4 types of trees Clare saw on the way home. Use the graph to answer the questions. Show your thinking using expressions or equations.



1. How many more pine trees did Clare see than fir trees?

2. How many more pine trees did Clare see than oak or maple trees?

3. How many fewer oak trees did Clare see than pine trees?

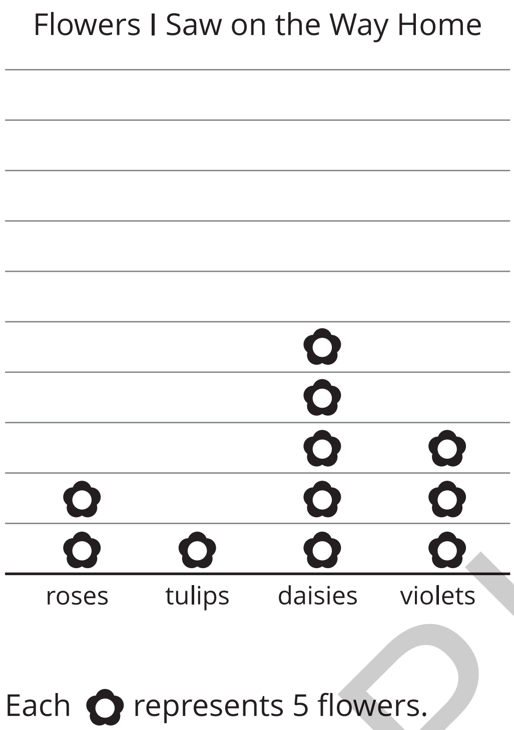
4. How many fewer maple or oak trees did Clare see than fir trees?

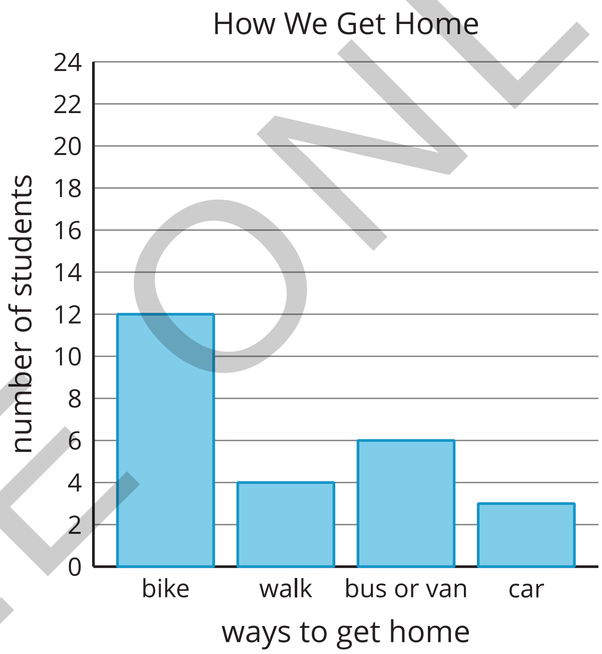
Section A Summary

We created **scaled picture graphs** and **scaled bar graphs**.

The **key** tells what each picture represents in a picture graph.

The **scale** tells what number each bar represents in a bar graph.





We asked and answered questions about data represented in the graphs.

- How many more daisies were seen than violets?
- How many fewer students walk home than bike home?
- How many more students bike home than walk or ride in a car?

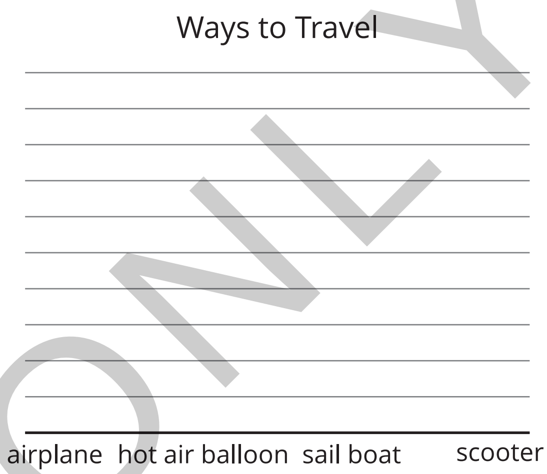
Practice Problems

15 Problems

1 Pre-unit

The table shows how a group of students chose between 4 ways they would most like to travel. Use the table to complete the picture graph.

way to travel	number of students
airplane	4
hot air balloon	7
sail boat	6
scooter	3

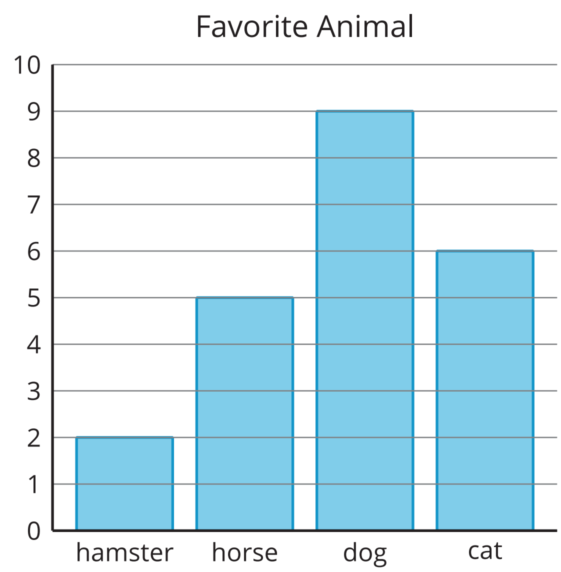


2 Pre-unit

The bar graph shows students' choices for favorite animals.

a. How many students recorded their favorite animal?

b. How many fewer students chose hamsters than dogs?



3 Pre-unit

The table shows the favorite summer vacation activity for a group of students.

vacation activities	number of students
family time	6
playing sports	8
sleeping in	5
reading	3

Use the table to complete the bar graph.

4

Pre-unit

Find each sum or difference. Show your reasoning.

a. $25 + 62$

b. $37 - 9$

c. $24 + 47$

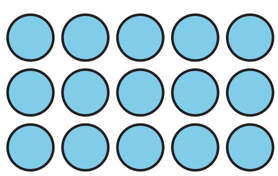
d. $84 - 59$

5

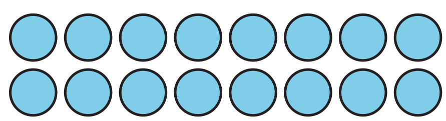
Pre-unit

How many objects are in each array? Explain or show your reasoning.

a.



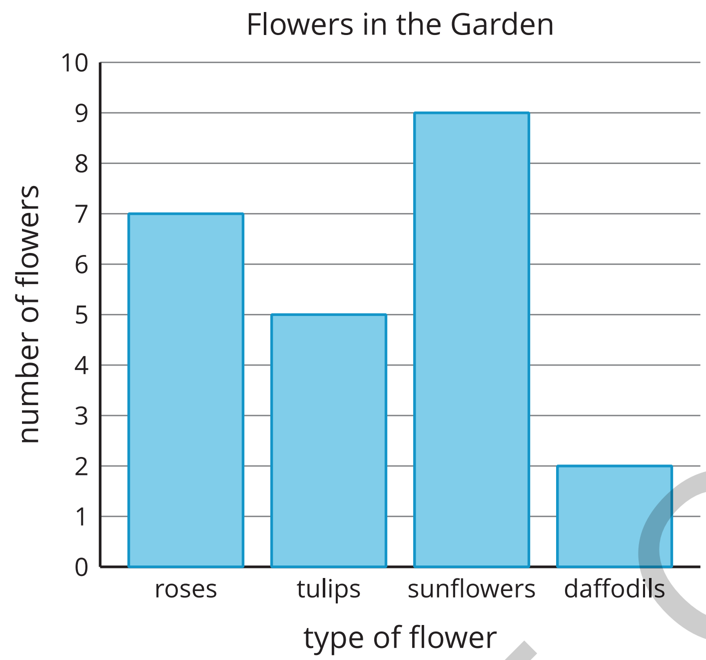
b.



6 from Unit 1, Lesson 1

Sec A

The graph shows some information about flowers in the garden.



a. Write 1 fact you know based on the data shown in the graph.

____

____

b. Write 2 questions that can be answered by reading the graph.

____

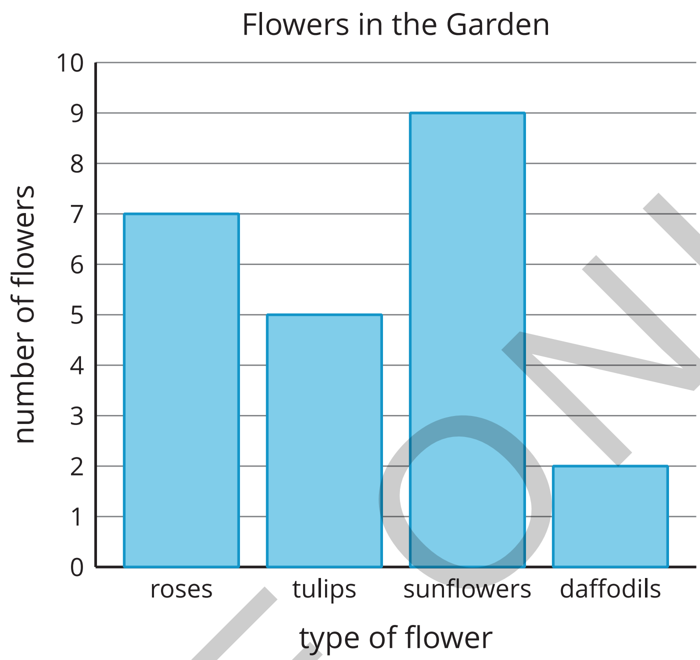
____

____

____

7 from Unit 1, Lesson 2

The bar graph shows the numbers of different types of flowers in the garden. Use the graph to answer the questions.

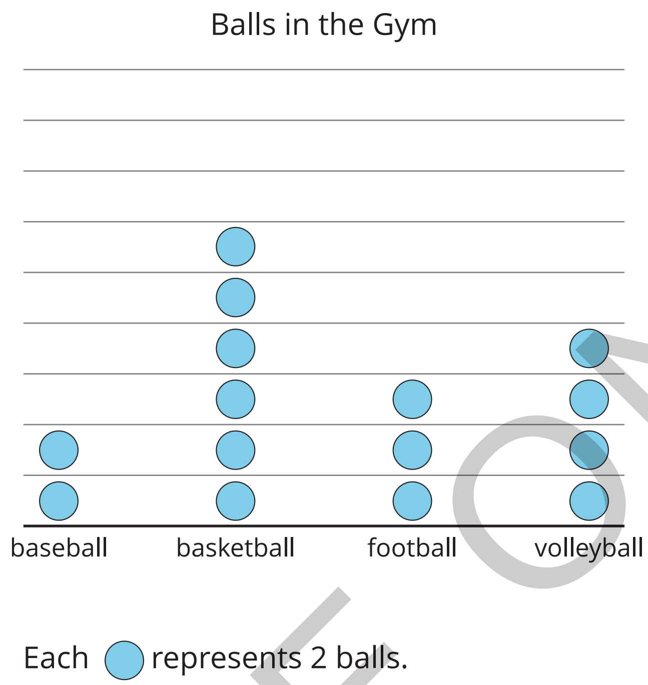


a. How many flowers are represented on the graph?

b. How many tulips, sunflowers, and daffodils are in the garden altogether?

8 from Unit 1, Lesson 3

This picture graph shows the numbers of different types of balls in the gym. Use the graph to answer the questions.



a. How many basketballs are in the gym?

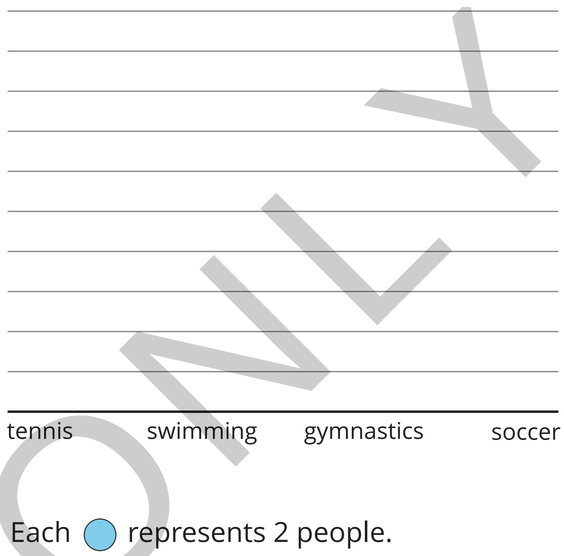
b. How many more basketballs are there than footballs?

c. Write 1 other question that can be answered by reading the graph.

9 from Unit 1, Lesson 4

The table shows the favorite sports of some students. Use the table to complete the scaled picture graph.

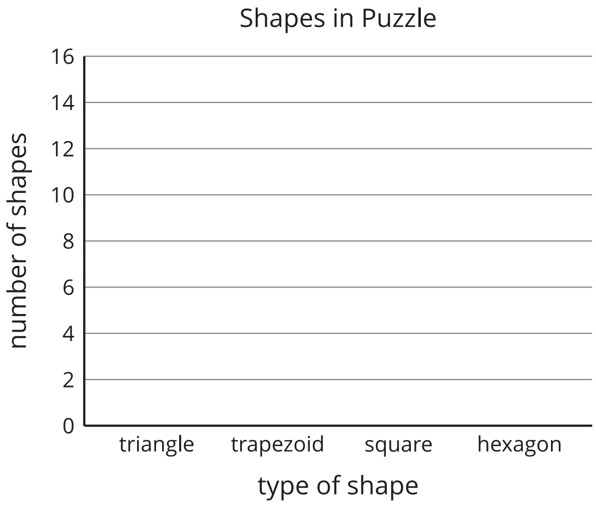
sport	number
tennis	6
swimming	6
gymnastics	4
soccer	8



10 from Unit 1, Lesson 5

The table shows the numbers of different shapes in a pattern block puzzle. Use it to complete the scaled bar graph.

shape	number
triangle	13
trapezoid	10
square	9
hexagon	15



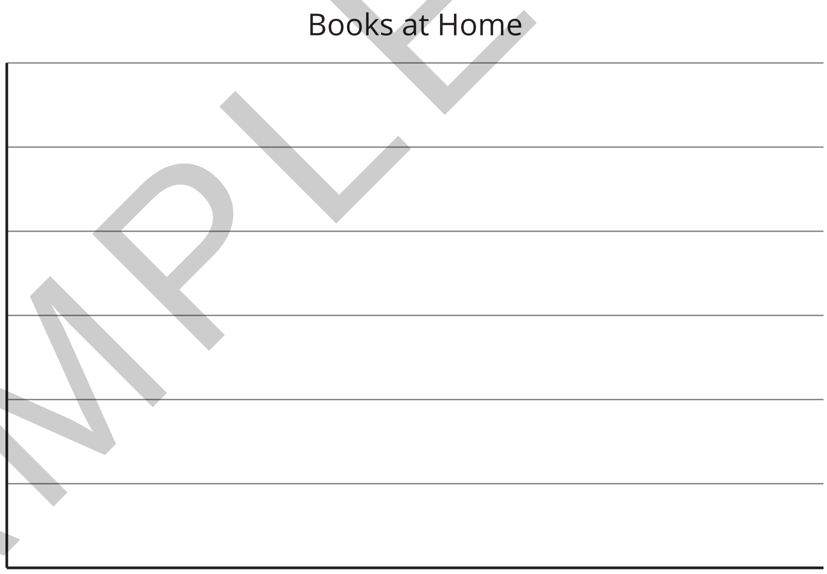
Sec A

11 from Unit 1, Lesson 6

The table shows the numbers of books some students have at home.

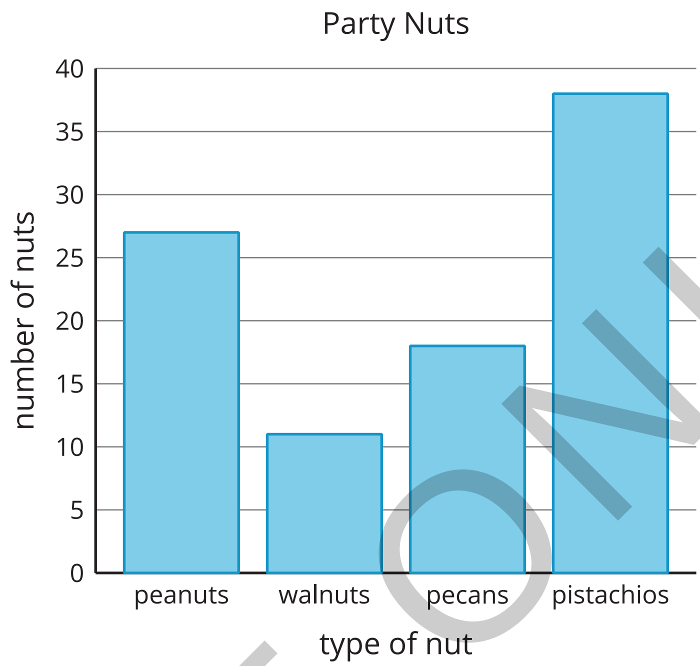
Use the information from the table to create a scaled bar graph.

books	number
Elena	25
Andre	9
Tyler	16
Clare	21



12 from Unit 1, Lesson 7

The bar graph shows the numbers of different kinds of nuts in a bowl.

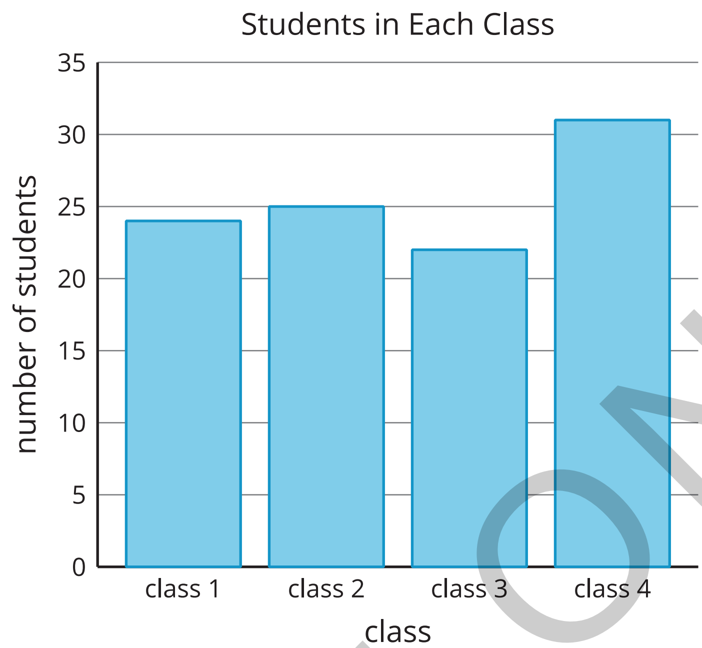


a. How many more pistachios are there than walnuts? Explain or show your reasoning.

b. How many fewer pecans are there than peanuts? Explain or show your reasoning.

13 from Unit 1, Lesson 8

The bar graph shows how many students are in each class.



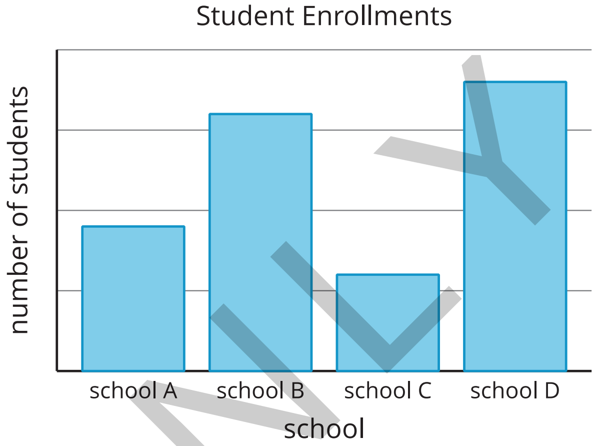
a. How many students are in the 4 classes altogether? Explain or show your reasoning.

b. How many fewer students are in Class 1 than in Class 4? Explain or show your reasoning.

14 Exploration

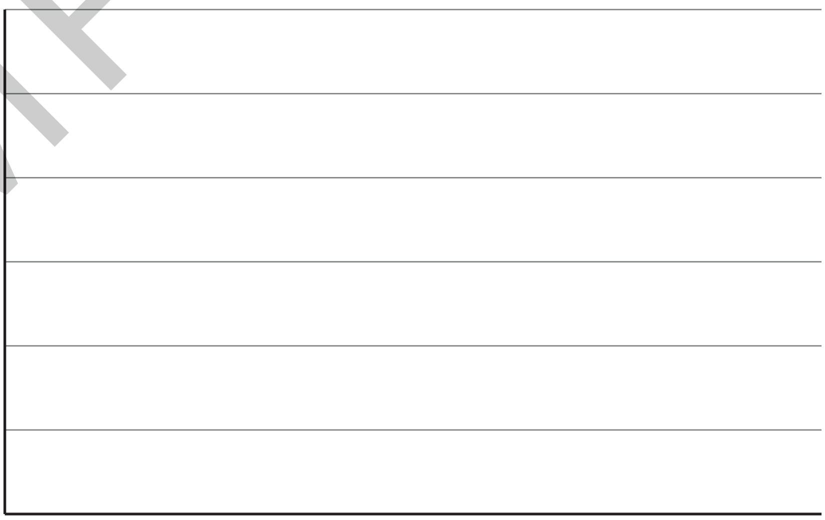
The bar graph shows data about 1,000 students who attend 4 schools.

What is the approximate scale for the bar graph? Explain or show your reasoning.



15 Exploration

Collect data of interest to you and represent the data on a bar graph. You may use the bar graph template if you wish.



Unit 1, Lesson 9

Multiplication for Equal Groups

Let's work with equal groups of things.

Sec B

Warm-up

Number Talk: More Addition

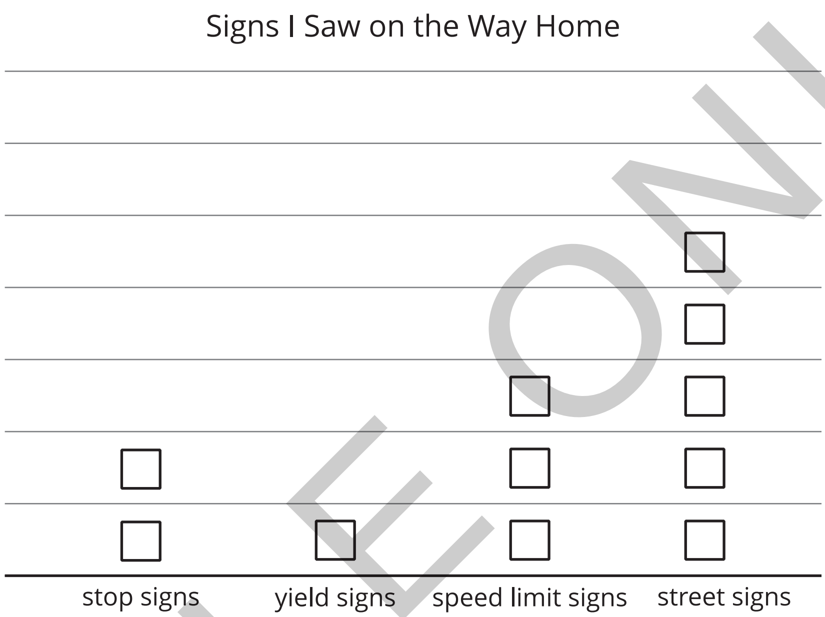
Find the value of each expression mentally.

- $40 + 35$
- $45 + 35$
- $45 + 36$
- $34 + 58$

Activity 1

From Scaled Graphs to Equal Groups

Elena collected data about signs she saw on the way home. The data is shown in this picture graph:



Each □ represents 2 signs.

1. Represent the number of speed limit signs Elena saw on the way home.

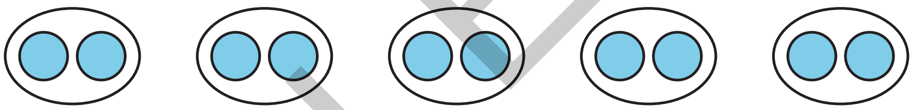
2. Which statement describes how the graph represents the number of speed limit signs Elena saw? Explain your reasoning.

   A. There are 3 pictures, and each picture represents 1 speed limit sign.

   B. There are 3 pictures, and each picture represents 2 speed limit signs.

   C. There are 2 pictures, and each picture represents 2 speed limit signs.

3. How could this drawing represent the street signs Elena saw on the way home?



Activity 2

Situations with Equal Groups

Represent each situation.

1. There are 4 people wearing shoes. Each person is wearing 2 shoes.

2. There are 2 boxes of markers. Each box has 10 markers.

3. There are 3 basketball teams. Each team has 5 players.

Unit 1, Lesson 10

Situations, Drawings, and Diagrams, Oh My!

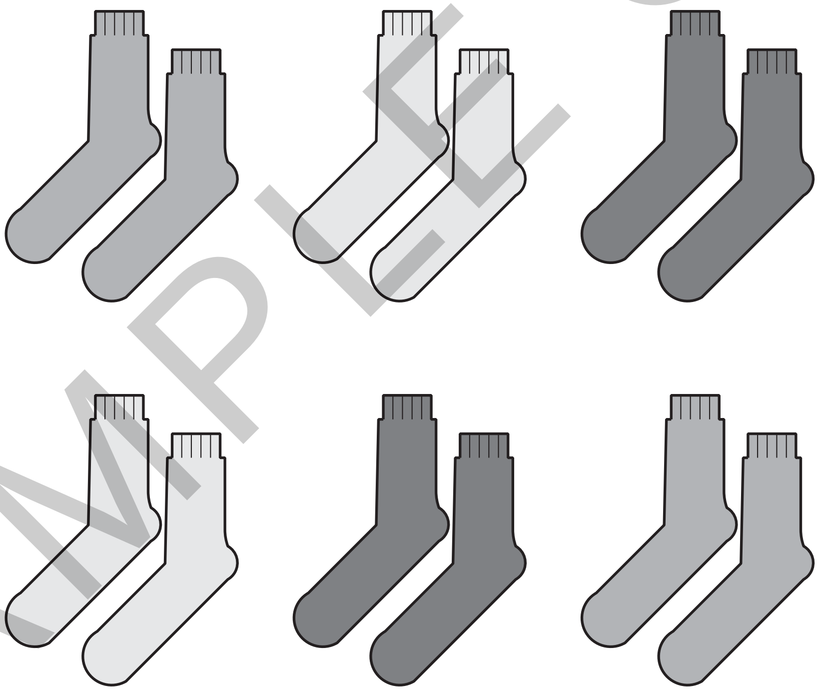
Let's represent equal groups.

Sec B

Warm-up

Notice and Wonder: Socks

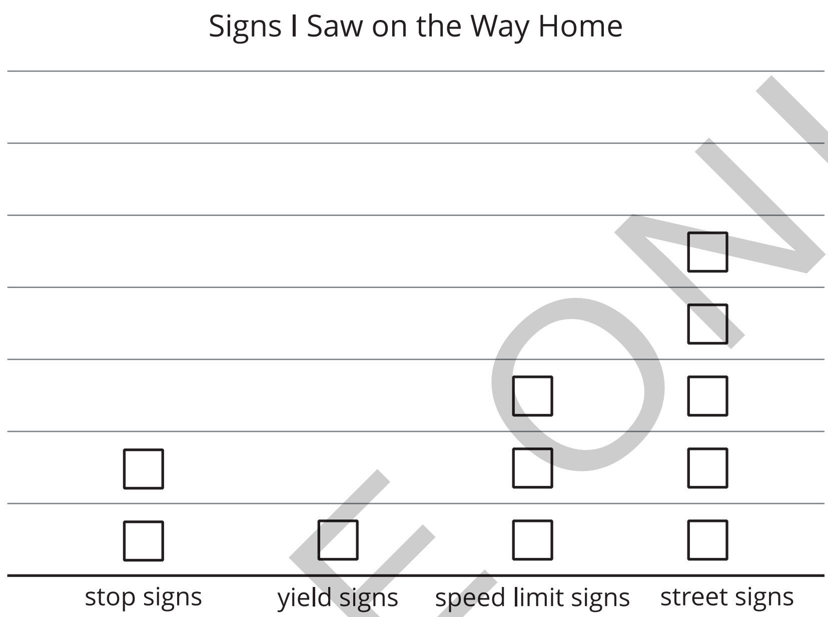
What do you notice? What do you wonder?



Activity 1

Scaled Picture Graph to Diagram

The graph shows the number of signs Elena saw on the way home.



Each ☐ represents 2 signs.

1. How does the diagram represent the speed limit signs that Elena saw?

2	2	2

2. Represent the data from another category in the graph with your own drawing or diagram.

Sec B

Activity 2

Card Sort: Equal Groups

Your teacher will give you a set of cards that show situations, drawings, and diagrams.

1. Find the cards that match. Be ready to explain your reasoning.

2. Create a drawing or diagram for each situation.

   a. There are 4 bags. Each bag has 2 strawberries.

   b. There are 4 hands. Each hand has 5 fingers.

Sec B

Unit 1, Lesson 11

Multiplication Expressions

Let’s write multiplication expressions.

Sec B

Activity 1

Multiplication Expression Match

Your teacher will give you a card showing a situation, a drawing, or a diagram.

Match it to 1 of the expressions posted around the room. Be prepared to explain your reasoning.

Activity 2

Expressions to Drawings and Diagrams

1. Create a drawing or diagram for each expression. Explain your reasoning.

   a. 5×2

   b. 3×4

Sec B

c. 3×10

Sec B

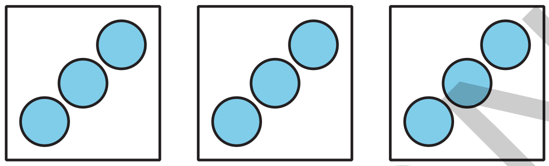
2. Write your own expression and matching diagram. Explain your reasoning.

Activity 3

Write Multiplication Expressions

Write a multiplication expression to match each situation, drawing, or diagram. Explain your reasoning.

1.



2.

3. There were 2 packs of water. Each pack had 6 bottles of water.

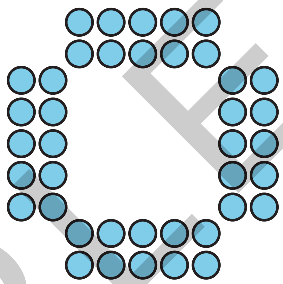
Represent and Solve Multiplication Problems

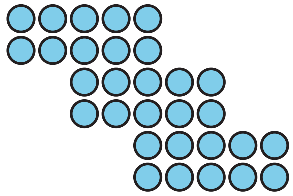
Let’s represent and solve problems involving equal groups.

Warm-up

How Many Do You See: Lots of Dots

How many do you see? How do you see them?





Activity 1

Tyler’s Boxes

Tyler has 3 boxes. He has 5 baseballs in each box. How many baseballs does he have altogether? Show your thinking using diagrams, symbols, or other representations.

Sec B

Activity 2

Solve Equal Groups Problems

Solve each problem. Show your thinking using diagrams, symbols, or other representations.

1. There are 4 soccer fields. Two teams are on each field. How many teams are there altogether?

2. There are 7 windows. Each window has 2 pieces of glass. How many pieces of glass are there in the windows?

3. Jada has 5 bags. Each bag has 10 earrings. How many earrings does Jada have?

4. Kiran has 4 boxes. Each box has 5 pencils in it. How many pencils does Kiran have?

5. Andre has 3 bags of carrots. Each bag has 10 carrots. How many carrots does Andre have?

Multiplication Equations

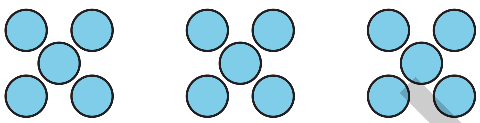
Let’s learn about multiplication equations.

Warm-up

Which Three Go Together: Representations

Which 3 go together?

A



B

3×5

C

$2 \times 5 = 10$

D

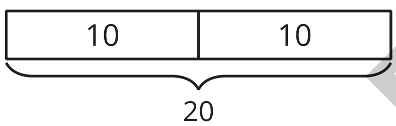
$7 + 8 = 15$

Activity 1

Multiplication Equation Match

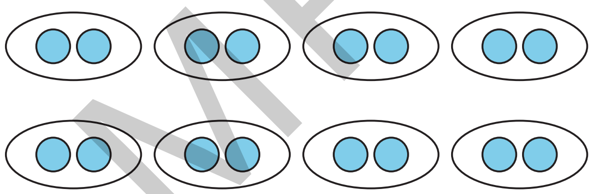
Find an equation from the list that can represent each situation, drawing, or diagram. Record the equation. Be prepared to explain your reasoning.

- $3 \times 5 = 15$
- $4 \times 10 = 40$
- $2 \times 10 = 20$
- $10 = 5 \times 2$
- $30 = 6 \times 5$
- $4 \times 2 = 8$
- $16 = 8 \times 2$
- $4 \times 5 = 20$
- $50 = 5 \times 10$

1. 

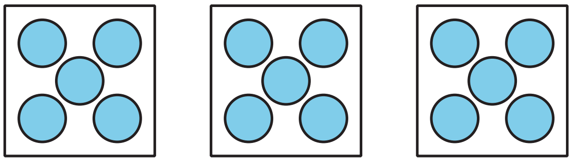
____

2. Andre has 5 pairs of socks. ______________________

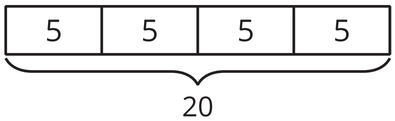
3. 
____

4. There are 6 hands on the table.
Each hand has 5 fingers. ______________________

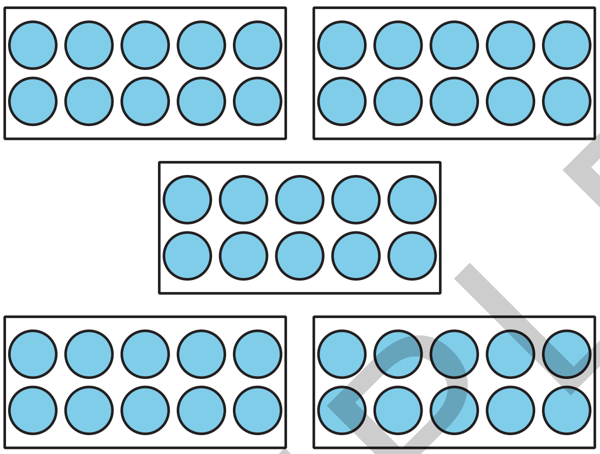
5.



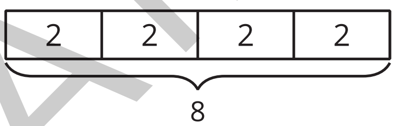
6.



7.



8.



9.

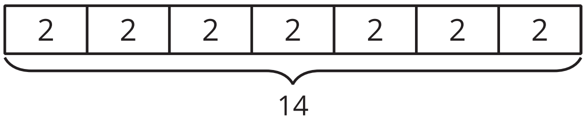
There are 4 boxes of markers.
Each box has 10 markers.

Activity 2

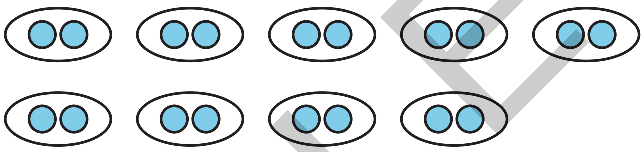
Write Multiplication Equations

Write a multiplication equation that represents each situation, drawing, or diagram. Be prepared to explain your reasoning.

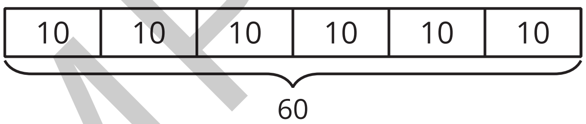
1. A package has 6 pairs of socks. ______________
2. ______________



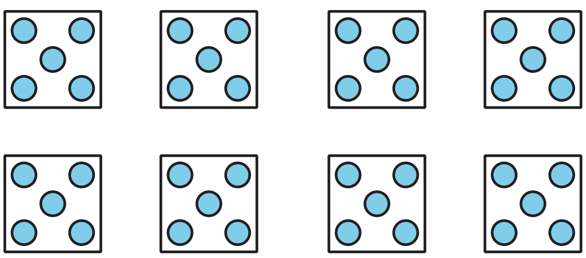
3. Diego has 7 sections in his notebook. Each section has 10 pages. ______________
4. ______________



5. ______________



6. Elena has 4 bags of oranges. Each bag has 5 oranges in it. ______________
7. ______________



Write and Solve Equations with Unknowns

Let's work with equations with unknown numbers.

Warm-up

Number Talk: Fives

Find the value of each expression mentally.

- 1×5
- 2×5
- 3×5
- 4×5

Activity 1

Card Sort: Unknown Numbers

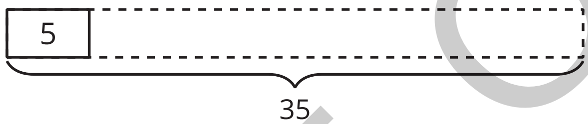
Your teacher will give you a set of cards. Match each equation to a situation or diagram. Be ready to explain your reasoning.

Activity 2

Write Equations with an Unknown Number

- Write a multiplication equation to represent each diagram or situation. Use a symbol for the unknown. Be prepared to share your reasoning.
- Find the number that makes each equation true. Rewrite the equation with the solution.

1.

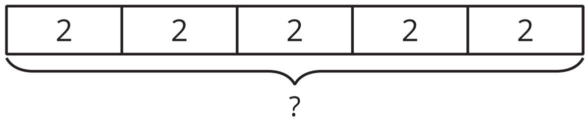


- equation with symbol: ____________________
- equation with solution: ____________________

2. Jada has some packs of sports cards. Each pack has 5 cards. If Jada has 45 cards, how many packs of cards does she have?

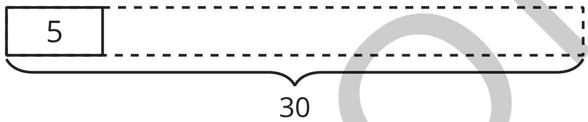
- equation with symbol: ____________________
- equation with solution: ____________________

3.



- equation with symbol: ______________________
- equation with solution: ______________________

4.



- equation with symbol: ______________________
- equation with solution: ______________________

5. The school has 6 bags. Each bag has 10 basketballs in it. How many basketballs does the school have?

- equation with symbol: ______________________
- equation with solution: ______________________

Unit 1, Lesson 15

More Factors, More Problems

Let's solve more multiplication problems.

Warm-up

Sec B

Number Talk: Tens

Find the value of each expression mentally.

- 1×10
- 2×10
- 3×10
- 4×10

Activity 1

Represent Situations with Equations

For each problem:

- Write a multiplication equation with a symbol for the unknown to represent the situation.
- Find the number that makes the equation true. Show your reasoning.

1. There are 15 bottles of paint. Han placed 5 bottles of paint on each table. How many tables have paint on them?
   a. equation:
   b. solution:

2. Lin's class has 6 tables. Each table has 2 bags of clay. How many bags of clay does the class have?
   a. equation:
   b. solution:

3. Han's class has 60 markers. There are 10 markers in a pack. How many packs of markers does the class have?
   a. equation:
   b. solution:

Activity 2

Multiplication Mashup

Solve each problem. Explain or show your reasoning.

1. Clare has 16 socks. She puts them in piles of 2. How many piles can she make?

2. Diego has 8 piles of socks. Each pile has 2 socks. How many socks does Diego have?

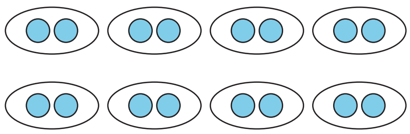
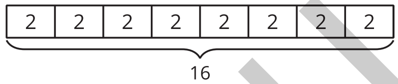
3. Andre has 16 socks. He puts them in 8 groups that are the same size. How many socks are in each group?

4. A store has 9 boxes. Each box has 5 shirts. How many shirts are there?

5. There are 80 sweaters in piles on a shelf. Each pile has 8 sweaters. How many piles of sweaters are on the shelf?

Section B Summary

We learned about equal groups. We created drawings and diagrams to represent situations that involve equal groups.

Situation	Drawing	Diagram
Diego has 8 piles of socks. Each pile has 2 socks.		

We wrote multiplication **expressions** and **equations** to represent equal groups.

Expression	Equation
8×2	$8 \times 2 = 16$

We learned that the numbers that are multiplied are called **factors** and the number that is the result of multiplying is called a **product**. In the equation $8 \times 2 = 16$, the numbers 8 and 2 are the factors and 16 is the product.

Practice Problems

10 Problems

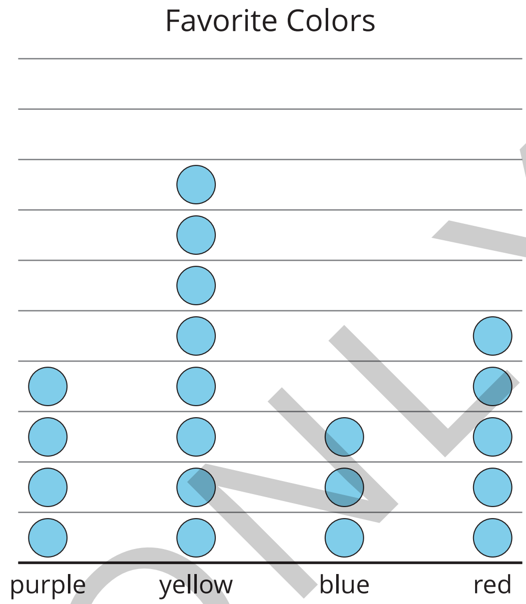
1 from Unit 1, Lesson 9

There are 6 tennis courts. There are 2 players on each tennis court.

Create a drawing or diagram to represent the tennis players. Then find how many players are on the tennis courts. Explain or show your reasoning.

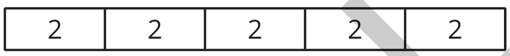
2 from Unit 1, Lesson 10

The picture graph shows the favorite colors of some people.

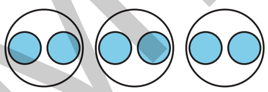


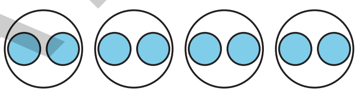
Each ● represents 2 people.

Match each diagram or drawing to the number of people who chose each color.

A. 

B. 

C. 

D. 

1. red
2. blue
3. purple
4. yellow

3 from Unit 1, Lesson 11

Create a drawing or diagram to represent the expression 4×3.

4 from Unit 1, Lesson 12

There are 4 stacks of books on the table. Each stack has 5 books. How many books are on the table? Explain or show your reasoning.

Sec B

5 from Unit 1, Lesson 13

There are 6 basketball teams in the gym. There are 5 people on each team. How many people are on the basketball teams in the gym?

a. Write a multiplication equation with a symbol for the unknown to represent the situation.

b. Find the number that makes the equation true. Show your reasoning.

6 from Unit 1, Lesson 14

Write a multiplication equation for the situation. Use ? for the unknown. Find the number that makes the equation true.

There are 4 soccer teams. Each soccer team has 10 players. How many players are there altogether?

7 from Unit 1, Lesson 15

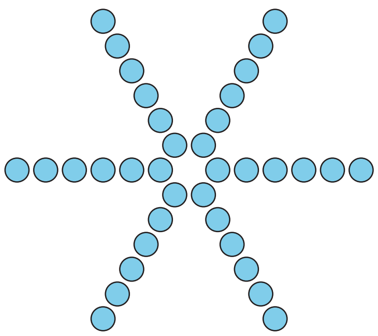
Solve each problem. Explain or show your reasoning.

a. There are 7 flowers. Each flower has 5 petals. How many petals are there?

b. There are 50 petals on some flowers. Each flower has 5 petals. How many flowers are there?

8 Exploration

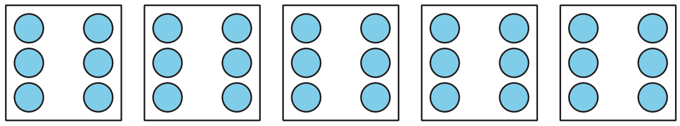
Write an expression for the number of circles in the image. Then find the number of circles.



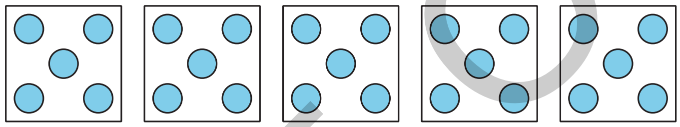
9 Exploration

For each image, determine if there is an even or odd number of circles. Explain or show your reasoning.

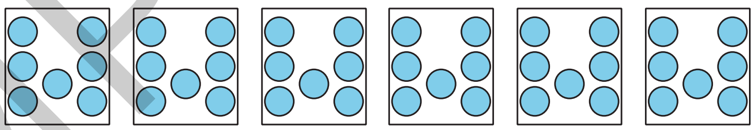
a.



b.



c.



10 Exploration

Look outdoors or in your school or home to find some equal groups of objects.

a. Describe the objects.

b. Create a drawing to represent the objects.

c. Write an equation showing how many objects there are.

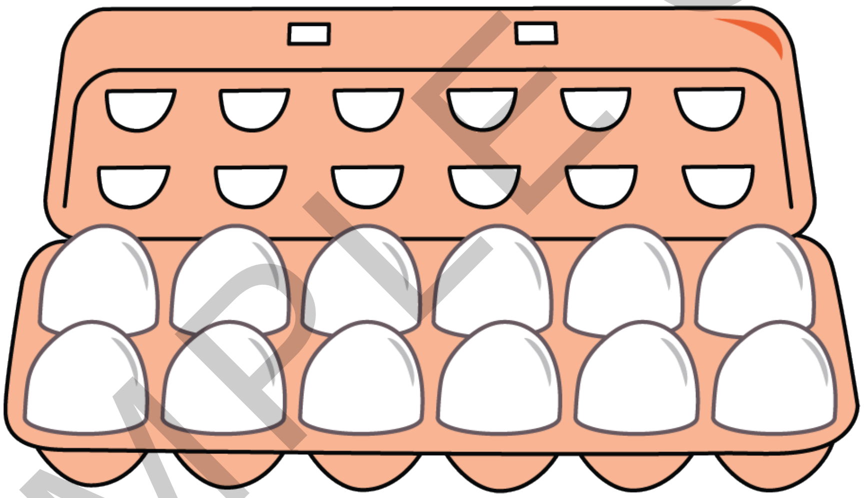
Arrange Objects into Arrays

Let's make some arrays.

Warm-up

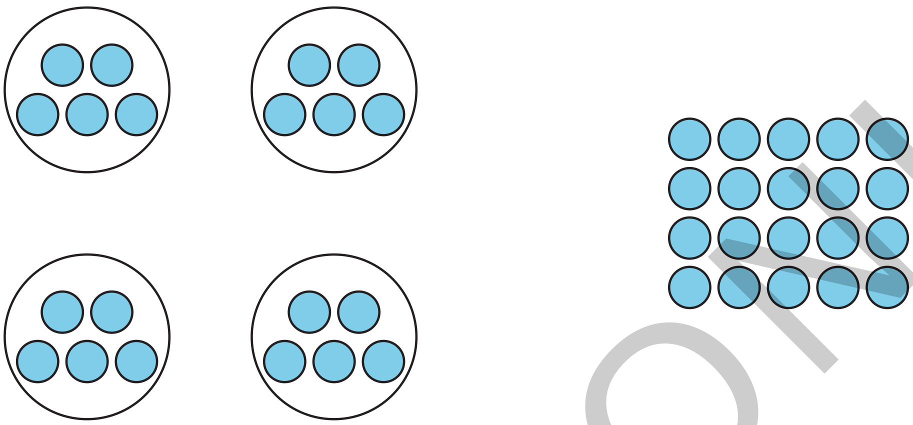
Notice and Wonder: Eggs

What do you notice? What do you wonder?



Activity 1

Compare Equal Groups and Arrays



1. How does arranging the dots into an array affect how you see the number?

2. Noah says he sees equal groups in the drawing with 4 circles and 5 dots in each circle, but he says there are no equal groups in the array. Do you agree with Noah? Explain your reasoning.

Activity 2

Arrange into Arrays

1. Use cubes to make 6 groups of 5.
    - Arrange them into an array.
    - Explain or show how the array is related to equal groups.

Sec C

2. Count out 20 cubes.
    - Arrange them into as many arrays as you can.
    - Explain or show how each array is related to equal groups.

3. Count out 24 cubes.

    ◦ Arrange them into as many arrays as you can.
    ◦ Explain or show how each array is related to equal groups.

Unit 1, Lesson 17

Match and Draw Arrays

Let's match arrays to equal groups and draw arrays.

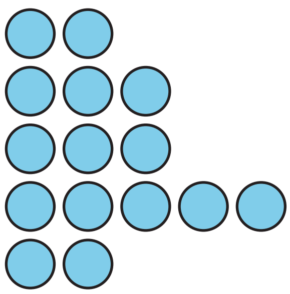
Warm-up

Which Three Go Together: Arrangements

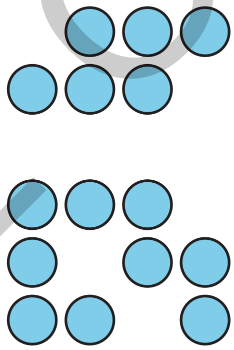
Sec C

Which 3 go together?

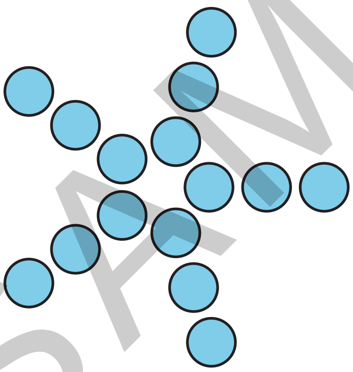
A



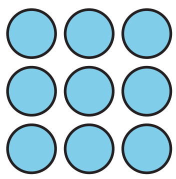
B



C



D



Activity 1

Card Sort: Arrays

Your teacher will give you a set of cards.

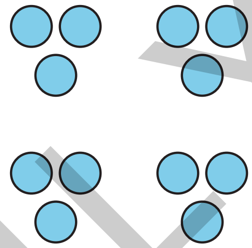
1. Match each drawing of equal groups to an array. Be ready to explain your reasoning.
2. Choose a match you and your partner made. Write down how you know the drawing matches the array.

Activity 2

Draw Arrays

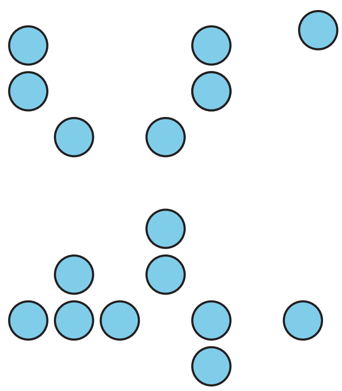
Sec C

1. a. Draw 1 way the dots could be rearranged into an array.



b. Explain or show how the array is related to multiplication.

2. a. Draw ways that the dots could be arranged into arrays. Draw as many ways as you can.



b. Explain or show how each array is related to multiplication.

Unit 1, Lesson 18

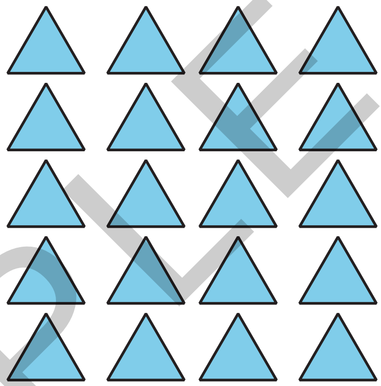
Represent Arrays with Expressions

Let's represent situations with arrays and expressions.

Warm-up

How Many Do You See: An Array of Shapes

How many do you see? How do you see them?



Activity 1

Represent Array Situations

1. Use objects or drawings to represent each situation with an array.

   a. There are 3 rows of chairs. Each row has 5 chairs.

   b. There are 4 rows of cars. Each row has 5 cars in it.

   c. There are 2 rows of eggs. Each row has 6 eggs.

d. There are 2 teams of students lined up. Each team has 10 students.

2. Write a multiplication expression to represent each situation.

   a. ____________________

   b. ____________________

   c. ____________________

   d. ____________________

Activity 2

Connect Arrays to Expressions

Draw an array for each multiplication expression. Be prepared to share your reasoning.

1. 2×3

2. 5×2

3. 4×4

Unit 1, Lesson 19

Solve Problems Involving Arrays

Let's solve problems involving arrays.

Warm-up

Number Talk: One Less Group

Sec C

Find the value of each expression mentally.

- 10×2
- 9×2
- 8×2
- 7×2

Activity 1

Array of Colors

There are 7 rows. Each row has 5 crayons. How many crayons are there?

1. Solve this problem. Explain or show your reasoning.

2. Represent the situation with an array and a multiplication equation with a symbol for the unknown.

Activity 2

Tyler's Trees

For each problem:

- Write a multiplication equation with a symbol for the unknown to represent the situation.
- Solve the problem. Show your reasoning.

Sec C

1. A field of coconut trees in Mexico has 5 rows of trees. Each row has 9 trees. How many trees are there?

2. Tyler wants to plant coconut trees in a community garden in Florida. He will plant 2 rows of 4 trees. How many trees will Tyler plant?

The Commutative Property

Let's learn about the commutative property.

Warm-up

Number Talk: Subtraction

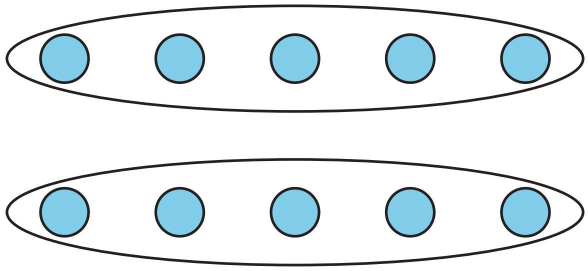
Find the value of each expression mentally.

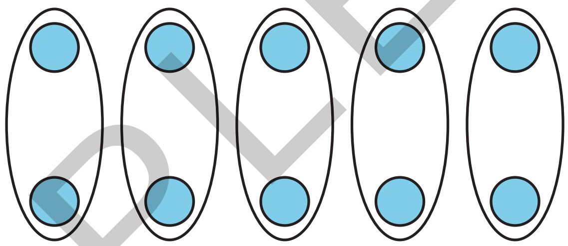
- $70 - 10$
- $68 - 10$
- $70 - 12$
- $68 - 12$

Activity 1

Learn More about Multiplication

What do you notice? What do you wonder?





Sec C

Image A

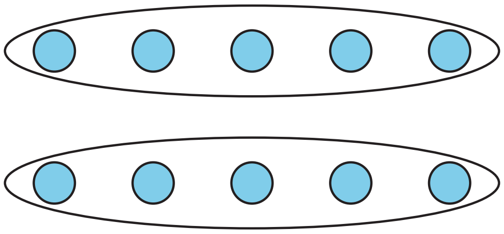
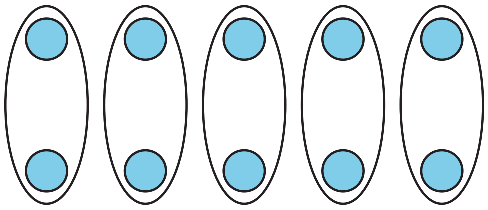


Image B



1. a. Write a description of a situation for each array.

Image A

____

____

Image B

____

____

b. How are the situations alike? How are they different?

____

____

____

____

2. a. Write a multiplication equation for each situation.

Image A

Image B

b. How does your equation connect to the situation and array?

Image A

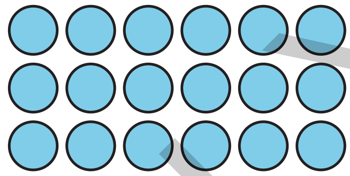
Image B

Sec C

Activity 2

Revisit Arrays

1. Write 2 multiplication equations that represent the array.



____

____

2. Explain why both equations can represent the array.

____

____

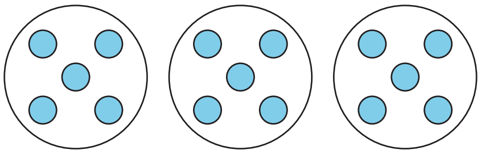
____

____

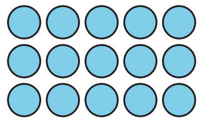
Section C Summary

We learned how equal groups are related to **arrays** and how to represent arrays with multiplication expressions and equations.

Equal groups:



Array:



Expression:

3×5

Equation:

$3 \times 5 = 15$

We also learned that we can multiply numbers in any order and get the same product.

$3 \times 5 = 15$ $\qquad$ $5 \times 3 = 15$ $\qquad$ $3 \times 5 = 5 \times 3$

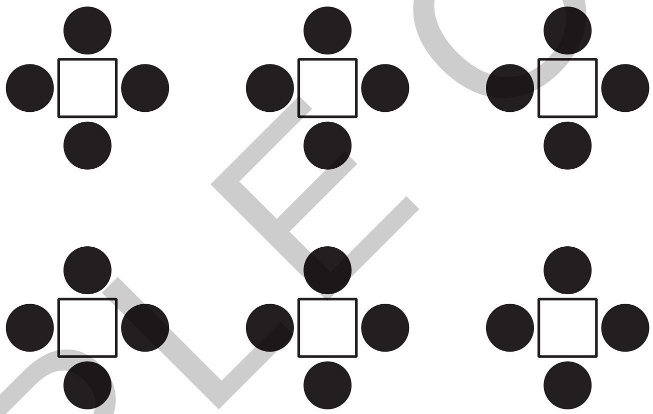
Game Night Seating Plan

Let's plan a game night.

Warm-up

Notice and Wonder: Squares and Circles

What do you notice? What do you wonder?



Activity 1

Game Night

Your club is planning a game night.

Guests can play 1 of 4 games that require a different number of players:

- Game A - 2 players
- Game B - 4 players
- Game C - 5 players
- Game D - 10 players

The game room has 16 identical square tables. One person can sit on each side of the table.

1. Make a seating plan that shows a table arrangement so that each guest can play 1 of the games and all the tables are used.

2. Make a poster that includes:

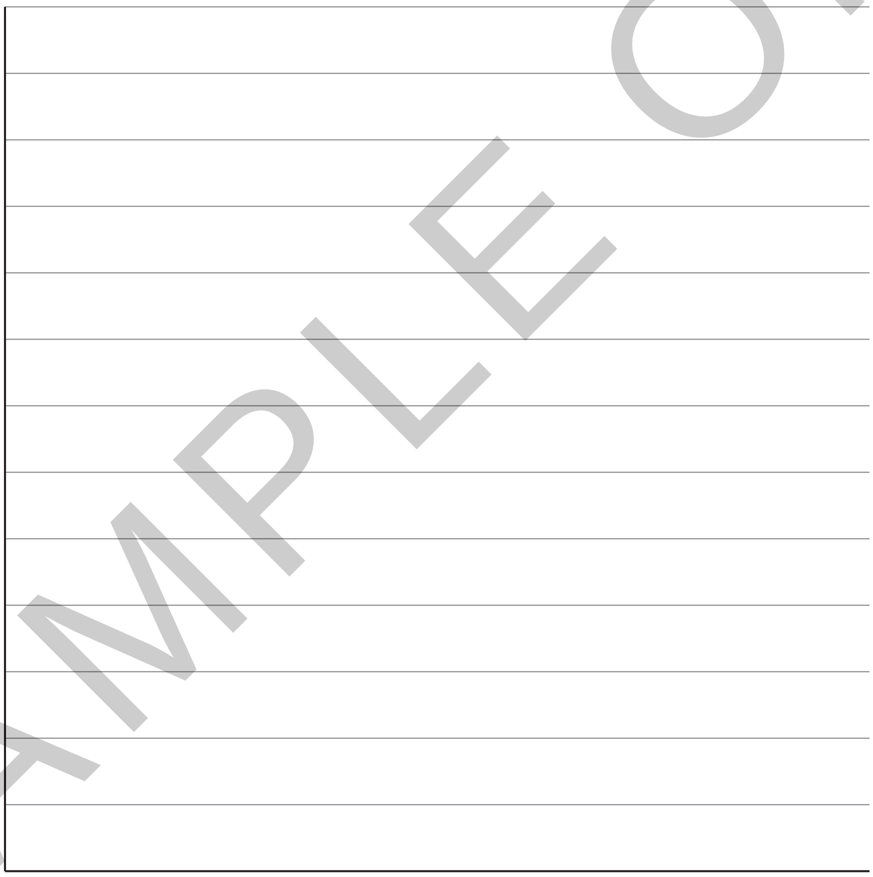
    a. a seating chart
    b. an explanation about how you decided on your seating plan
    c. how many people can play games in the room with your seating plan

Activity 2

Game Night on a Graph

Make a scaled bar graph that shows the number of guests that can play each of the games A, B, C, and D. Be sure to include:

- a title and other labels
- a scale that counts by a number other than 1

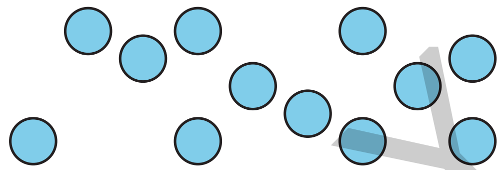


Practice Problems

7 Problems

1 from Unit 1, Lesson 17

Rearrange the circles to make an array in 2 different ways.



2 from Unit 1, Lesson 18

There are 4 rows of water bottles in the box. There are 5 bottles in each row.

a. Draw an array to represent the situation.

b. Write a multiplication expression to represent the number of bottles.

3 from Unit 1, Lesson 19

There are 5 rows of chairs in the room. There are 4 chairs in each row. How many chairs are in the room?

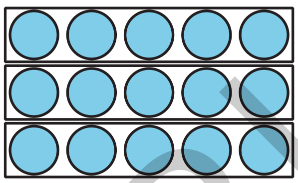
a. Write a multiplication equation to represent the situation. Use a symbol for the unknown.

b. Find the value that makes your equation true.

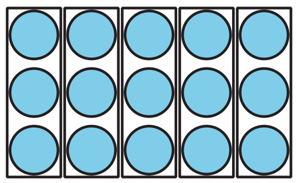
4 from Unit 1, Lesson 20

a. Write a multiplication equation that represents each array.

A



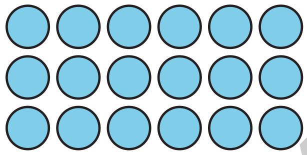
B



b. How are the arrays alike? How are they different?

5 from Unit 1, Lesson 16

a. Explain or show 2 different ways that you see equal groups in the array.



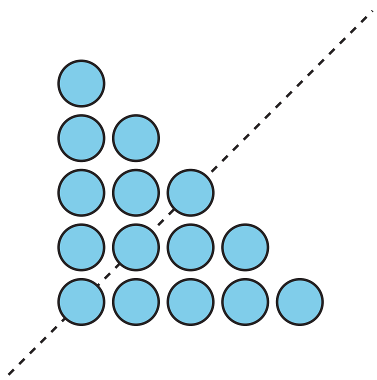
b. Rearrange the dots to make a different array.

Sec C

6 Exploration

Andre says that there are an odd number of circles in this picture.

Do you agree with Andre? Explain or show your reasoning.



7 Exploration

Find a collection of objects in the classroom or at home that is arranged in an array.

a. Describe the objects.

b. Create a drawing of the objects.

c. Write an equation showing how many objects there are.

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Kendall Hunt

GRADE 3

Teacher Guide

UNIT

1

Book 1
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Table of Contents

UNIT 2: AREA AND MULTIPLICATION

Unit 1: Introducing Multiplication

Goals

- Students represent and solve multiplication problems through the context of scaled picture and bar graphs and equal-group situations.

Narrative

In this unit, students interpret and represent data on scaled picture graphs and scaled bar graphs. Then they learn the concept of multiplication.

This is the first of four units that focus on multiplication. In this unit, students explore scaled picture graphs and bar graphs as an entry point for learning about equal-size groups and multiplication.

In grade 2, students analyzed picture graphs in which one picture represented one object and bar graphs that were scaled by single units. Here, students encounter picture graphs in which each picture represents more than one object and bar graphs that are scaled by 2, 5, or 10 units. The idea that one picture can represent multiple objects helps to introduce the idea of equal-size groups.

Students learn that multiplication can mean finding the total number of objects in a groups of b objects each, and can be represented by $a \times b$. They then relate the idea of equal groups and the expression $a \times b$ to the rows and columns of an array. In working with arrays, students begin to notice the commutative property of multiplication.

In all cases, students make sense of the meaning of multiplication expressions before finding their value and before writing equations that relate two factors and a product.

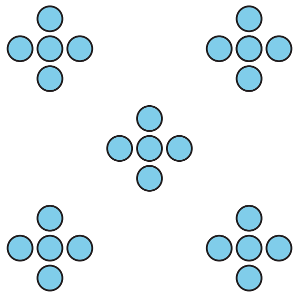
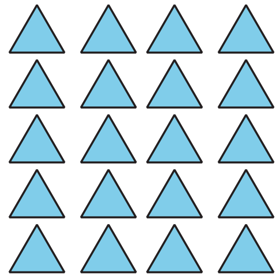
Later in the unit, students see situations in which the total number of objects is known but either the number of groups or the size of each group is not known. Problems with a missing factor offer students a preview to division.

Throughout the unit, students should have access to connecting cubes or counters, as they may choose to use such tools to represent and solve problems.

Throughout The Unit

Students work toward fluency in multiplying by 2, 5, and 10. The *How Many Do You See?* routine is used to encourage students to look for equal groups. It prompts students to subitize a group of dots as one unit, see the iterations of the groups, and skip-count to say the total number of dots they see in the image. This routine progresses from dots to drawings of equal groups to array formations.

Here is a sampling of the *How Many Do You See?* warm-ups in this unit.

lesson 4	lesson 18
	

Number Talks are likewise designed to help students build fluency with equal groups and multiplication expressions. The sequence of expressions encourages students to relate multiplication to skip-counting. For example, in the sequence 1×10, 2×10, 3×10, 4×10, students can discover that the products increase in the same way as in skip-counting by 10. Some *Number Talks* elicit students' understanding of addition and subtraction within 100 in preparation for the work in an upcoming unit.

Here is a sampling of the Number Talk warm-ups in the unit.

lesson 5	lesson 15	lesson 19	lesson 20
$2+2+2+2$	1×10	10×2	$70-10$
$2+2+2+2+2+2+2+2$	2×10	9×2	$68-10$
$5+5+5+5$	3×10	8×2	$70-12$
$5+5+5+5+5+5+5+5$	4×10	7×2	$68-12$

Materials Needed

Lesson	Materials to Gather	Materials to Copy
Lesson 1	• Chart paper: Lesson	
Lesson 2	• Math community poster: Lesson • Sticky notes: Activity 1	
Lesson 3	• Math community poster: Lesson	
Lesson 4	• Math community poster: Lesson	
Lesson 5	• Math community poster: Lesson • Materials from a previous lesson: Activity 2	
Lesson 6	• Math community poster: Lesson	
Lesson 7	• Materials from a previous lesson: Activity 1	
Lesson 8		
Lesson 9	• Connecting cubes or counters: Activity 1, Activity 2	

Lesson 10		• Card Sort Equal Groups Cards (1 copy for every 2 students): Activity 2
Lesson 11	• Materials from a previous lesson: Activity 1	
Lesson 12		
Lesson 13		
Lesson 14		• Card Sort Unknown Numbers Cards (1 copy for every 2 students): Activity 1
Lesson 15		
Lesson 16	• Connecting cubes: Activity 2	
Lesson 17	• Connecting cubes or counters: Activity 2	• Card Sort Arrays Cards (1 copy for every 2 students): Activity 1
Lesson 18	• Connecting cubes or counters: Activity 1	
Lesson 19		
Lesson 20		

Lesson 21	• Connecting cubes or counters: Activity 1 • Inch tiles: Activity 1 • Tools for creating a display: Activity 1	• Centimeter Grid Paper - Standard (1 copy for every 2 students): Activity 1 • Centimeter Grid Paper - Standard (1 copy for every 2 students): Activity 2

Section A: Interpret and Represent Data in Scaled Graphs

Standards

Building On	**2.MD.D, 2.MD.D.10, 2.NBT.B.5, 2.OA.C.3, 2.OA.C.4**
Addressing	**3.MD.B, 3.MD.B.3**
Building Towards	**3.MD.B.3**

Goals

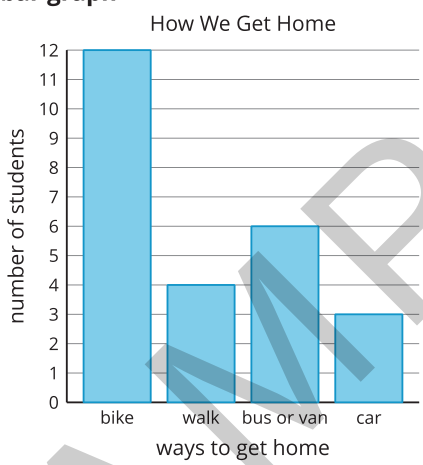
- Interpret scaled picture and bar graphs.
- Represent data using scaled picture and bar graphs.
- Solve one- and two-step story problems using addition and subtraction.

Narrative

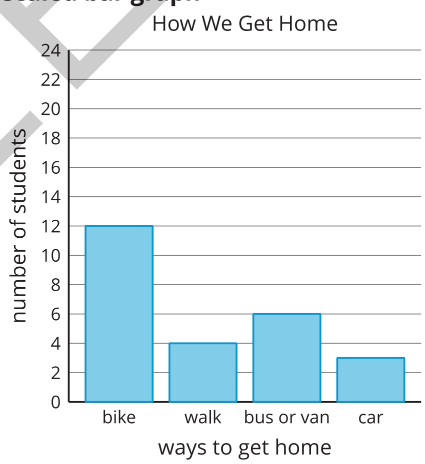
In this section, students interpret and draw scaled picture graphs and scaled bar graphs to represent data. This builds on their grade 2 experiences with data representation and with skip-counting by 2, 5, and 10.

Students see that each picture in a picture graph and each line or increment in a bar graph can represent more than one object. They work with familiar number scales of 2, 5, and 10.

bar graph



scaled bar graph



Students use the information in scaled bar graphs to solve one- and two-step “how many more?” and “how many fewer?” problems within 100. This work allows teachers to formatively assess students’ fluency with addition and subtraction within 100, a grade 2 expectation..

Suggested Centers

Lesson 1

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Lesson 2

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Lesson 3

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Lesson 4

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Lesson 5

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Lesson 6

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Lesson 7

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Lesson 8

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Section A Checkpoint

1

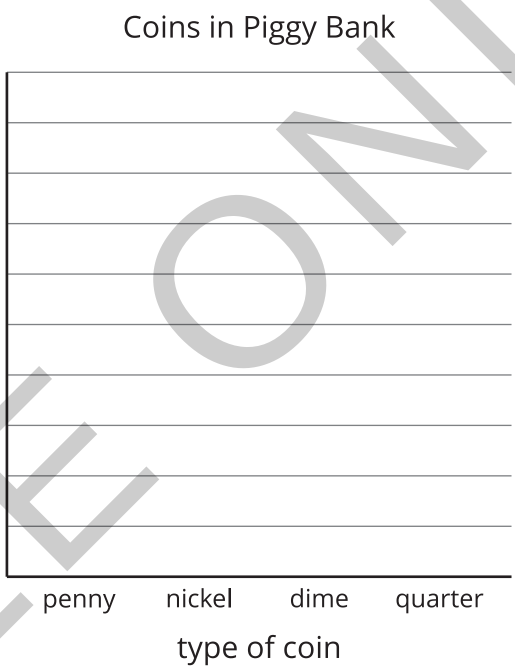
Goals Assessed

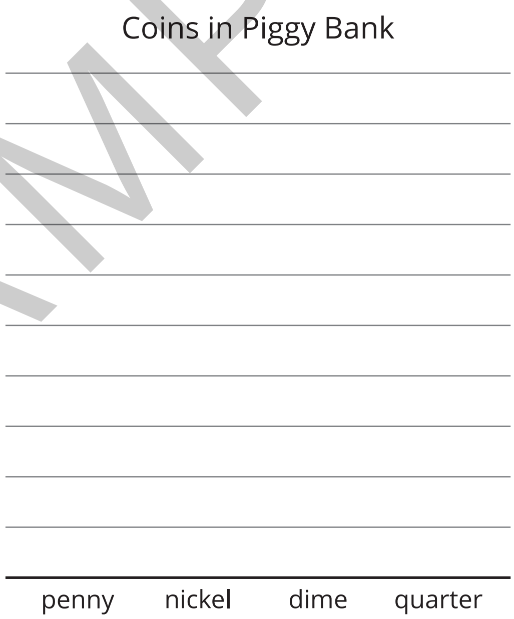
- Represent data using scaled picture and bar graphs.

Student Task Statement

The table shows the number of different coins in a piggy bank. Create a scaled picture graph and a scaled bar graph to represent the data. Use a scale of 2.

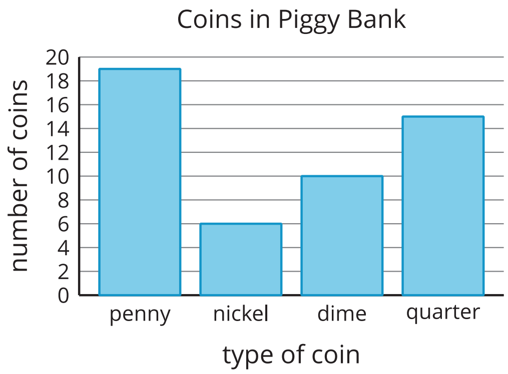
coin	number
penny	19
nickel	6
dime	10
quarter	15

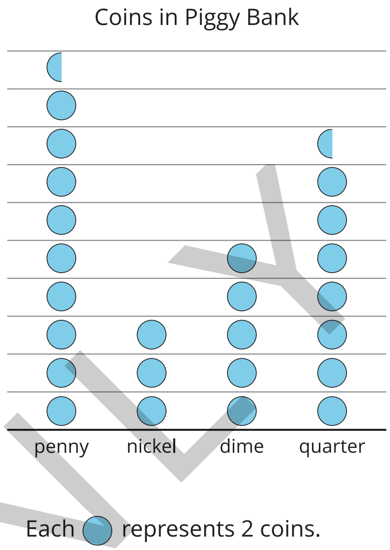




Solution

Sample responses:





Responding To Student Thinking

Students choose 5 or 10 as the scale, but they do not yet represent the numbers with reasonable accuracy on the scale. For example, they may stop at the nearest multiple of 5 for each quantity in the bar graph or only draw whole pictures for data that are not multiples of the scale in the picture graph.

Students represent the quantities accurately, but they do not match the numbers to the appropriate categories.

Invite selected students to continue to play *Sort and Display*, Stage 3 during Center Time in the next section. As students play, ask them to explain how they chose their scale. Before they graph a number that will not line up on a tick mark on their scale, ask students to explain how high they expect to draw the bar. For example, if the student uses a scale of 5 and is graphing 37, consider asking, “If this tick mark represents 35, where would 37 be? How do you know?”

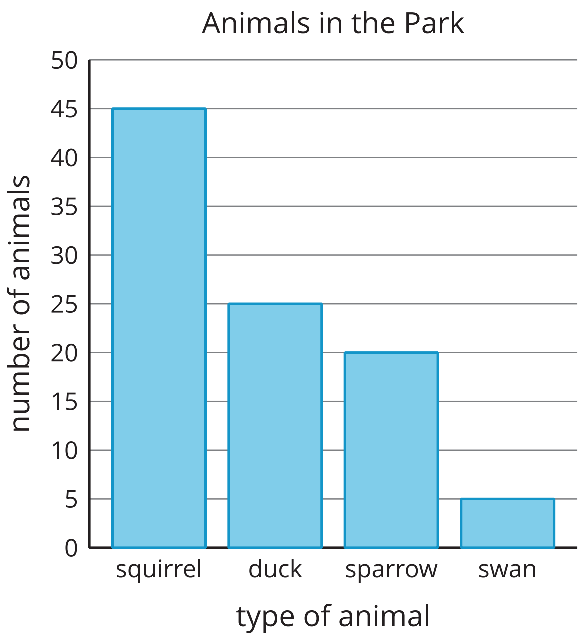
2

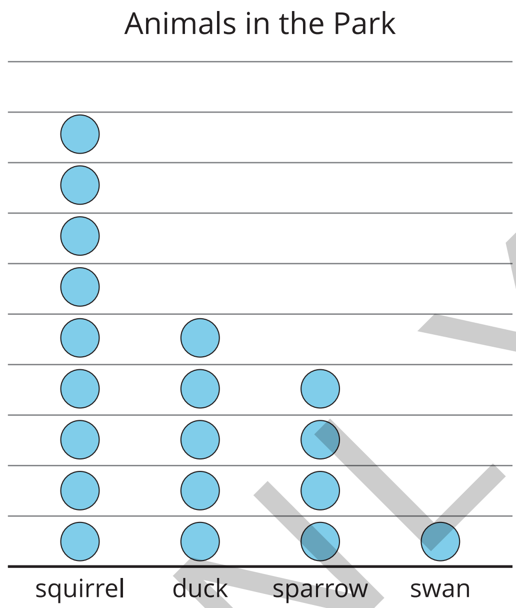
Goals Assessed

- Interpret scaled picture and bar graphs.
- Solve one- and two-step story problems using addition and subtraction.

Student Task Statement

The bar graph and picture graph both show the same data—the number of different animals seen in a park.





Each ◯ represents 5 animals.

a. How many squirrels and sparrows were seen altogether? Explain or show your reasoning.

b. How many more squirrels were seen than sparrows and swans combined? Explain or show your reasoning.

Solution

a. 65. Sample response: $45 + 20 = 65$

b. 20. Sample response: $20 + 5 = 25, 45 - 25 = 20$

Responding To Student Thinking

Students show they can interpret and solve the Add To question, but they do not yet solve the two-step Compare problem. For example, they only find the difference between squirrels and sparrows, or they add all three quantities.

Before an upcoming lesson, use the *Three Reads* routine to revisit the problem. Invite students to compare this conversation about the problem to their work and identify ways to revise their answers. For additional practice, if students play *Sort and Display* in the next section, invite them to use the question frame "How many more ___ are there than ___ and ___ combined?"

Practice Problems

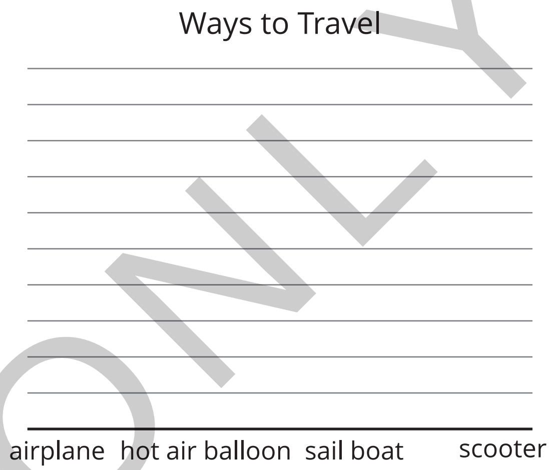
15 Problems

1 Pre-unit

Student Task Statement

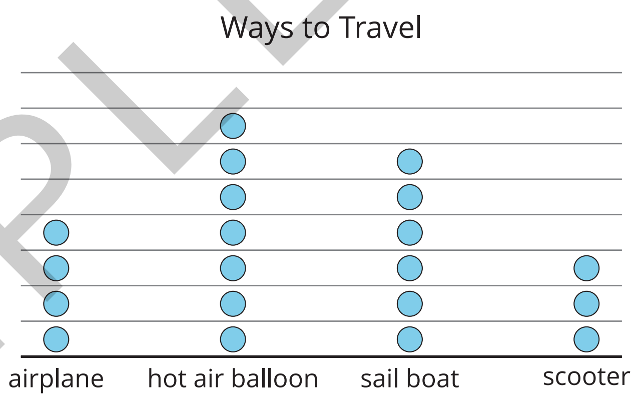
The table shows how a group of students chose between 4 ways they would most like to travel. Use the table to complete the picture graph.

way to travel	number of students
airplane	4
hot air balloon	7
sail boat	6
scooter	3



Solution

Sample response:



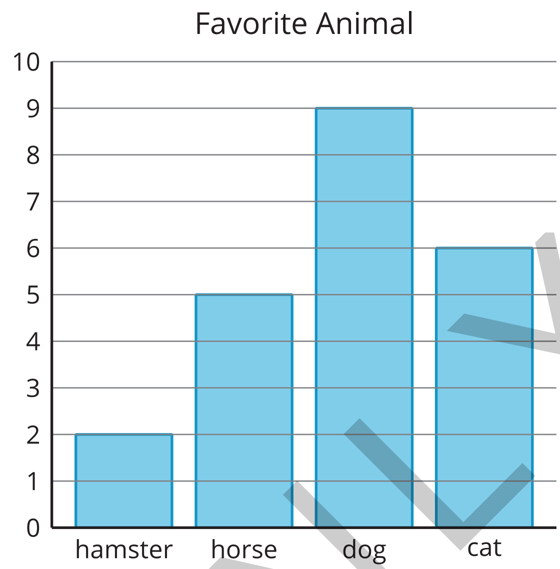
2 Pre-unit

Student Task Statement

The bar graph shows students’ choices for favorite animals.

a. How many students recorded their favorite animal?

b. How many fewer students chose hamsters than dogs?



Solution

a. 22 students

b. 7 fewer students

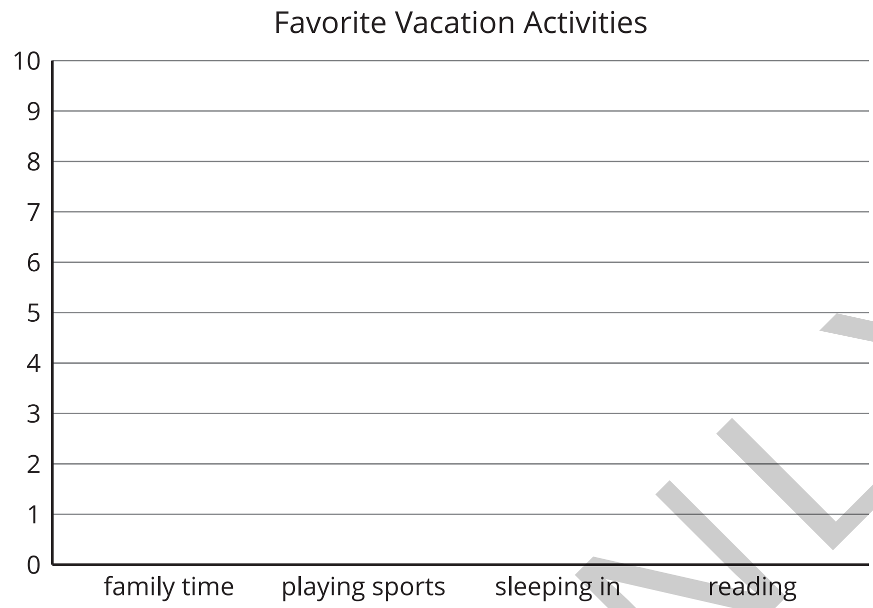
3 Pre-unit

Student Task Statement

The table shows the favorite summer vacation activity for a group of students.

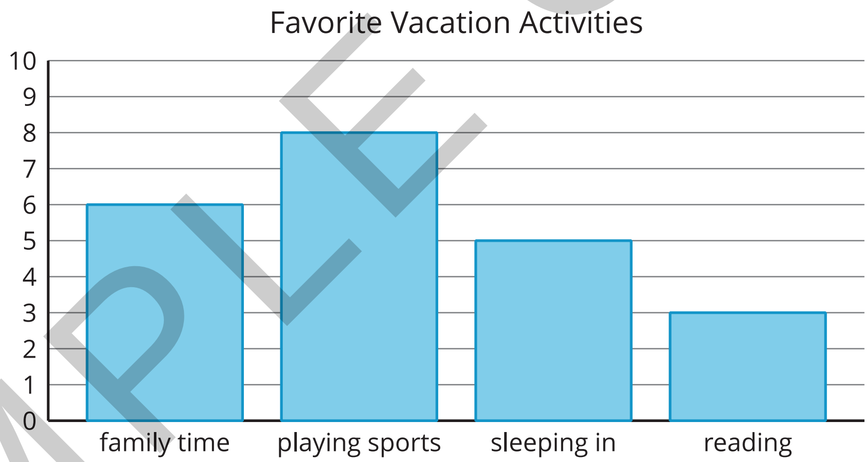
vacation activities	number of students
family time	6
playing sports	8
sleeping in	5
reading	3

Use the table to complete the bar graph.



Solution

Sample response:



4 Pre-unit

Student Task Statement

Find each sum or difference. Show your reasoning.

a. $25 + 62$

b. $37 - 9$

c. $24 + 47$

d. $84 - 59$

Solution

a. 87. Sample response: I combined the tens and the ones.

b. 28. Sample response: $37 - 7 = 30$, $30 - 2 = 28$

c. 71. Sample response: $20 + 40 = 60$, $60 + 7 + 3 + 1 = 71$

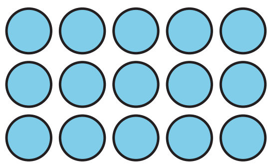
d. 25. Sample response: 84 is 7 tens and 14 ones. $70 - 50 = 20$, $14 - 9 = 5$, $20 + 5 = 25$

5 Pre-unit

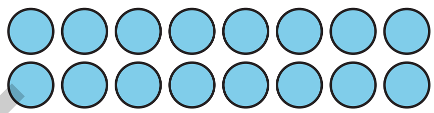
Student Task Statement

How many objects are in each array? Explain or show your reasoning.

a.



b.



Solution

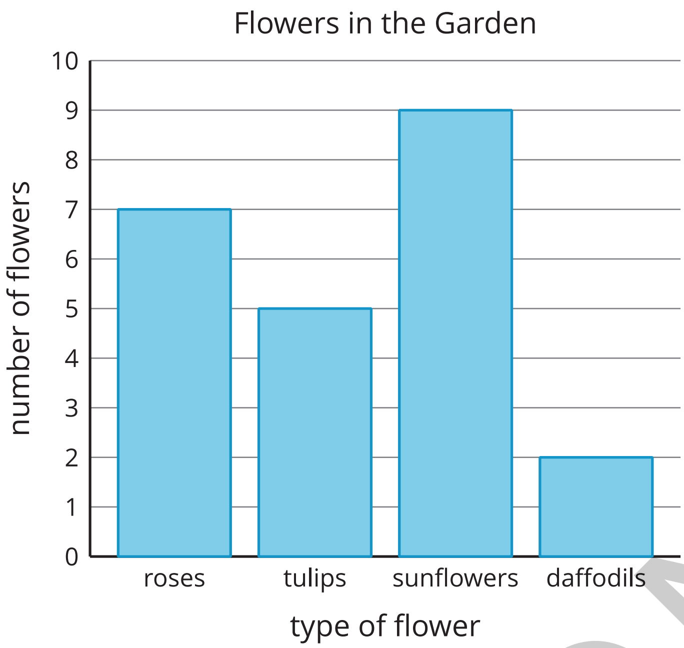
a. 15 objects. Sample response: There are 5, 10, 15.

b. 16 objects. Sample response: There are 8 and 8 more, so that's 16.

6 from Unit 1, Lesson 1

Student Task Statement

The graph shows some information about flowers in the garden.



a. Write 1 fact you know based on the data shown in the graph.

b. Write 2 questions that can be answered by reading the graph.

Solution

Sample responses:

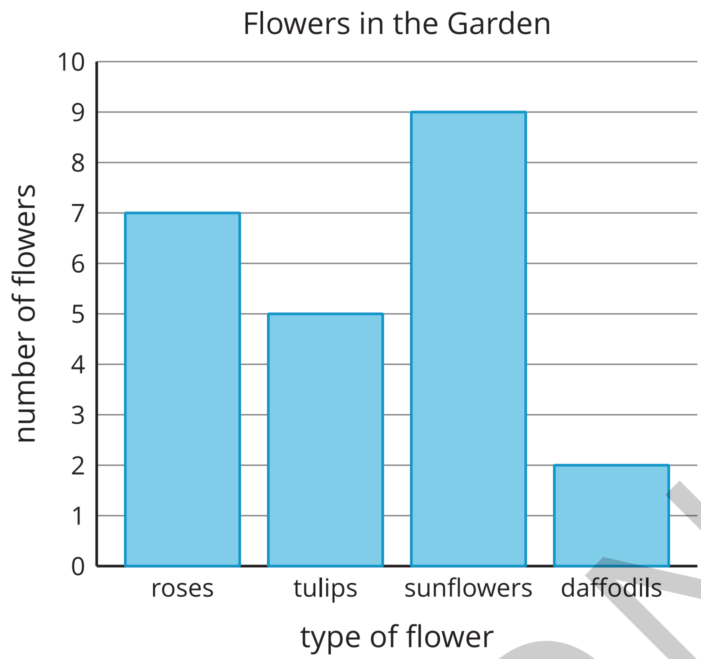
a. The graph shows the number of different flowers. There are more sunflowers than roses. There are fewer daffodils than any of the other flowers.

b. How many more sunflowers are there than daffodils? How many roses and tulips are there together?

7 from Unit 1, Lesson 2

Student Task Statement

The bar graph shows the numbers of different types of flowers in the garden. Use the graph to answer the questions.



a. How many flowers are represented on the graph?

b. How many tulips, sunflowers, and daffodils are in the garden altogether?

Solution

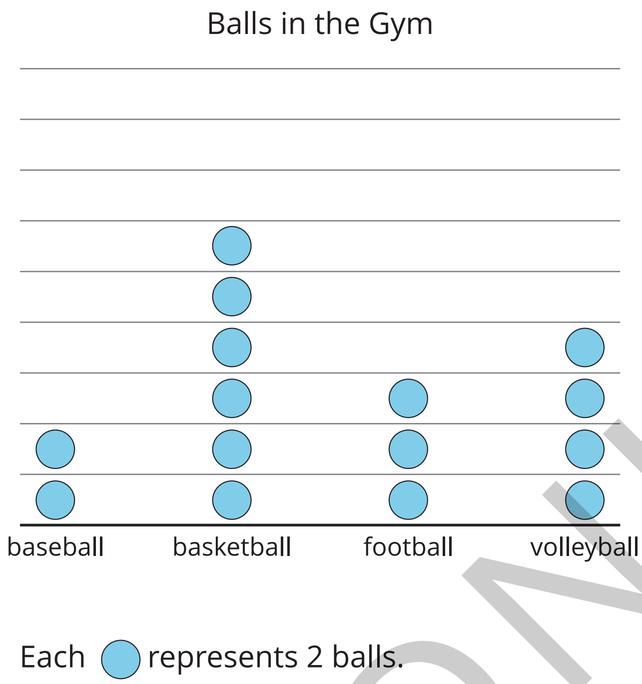
a. 23 flowers

b. 16 flowers

8 from Unit 1, Lesson 3

Student Task Statement

This picture graph shows the numbers of different types of balls in the gym. Use the graph to answer the questions.



a. How many basketballs are in the gym?

b. How many more basketballs are there than footballs?

c. Write 1 other question that can be answered by reading the graph.

Solution

a. 12 basketballs

b. 6 more basketballs

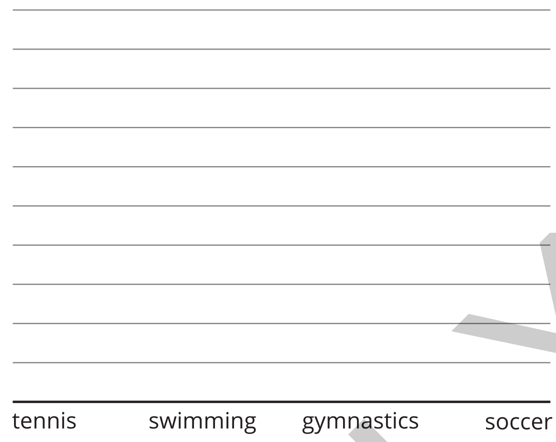
c. Sample response: How many volleyballs are in the gym?

9 from Unit 1, Lesson 4

Student Task Statement

The table shows the favorite sports of some students. Use the table to complete the scaled picture graph.

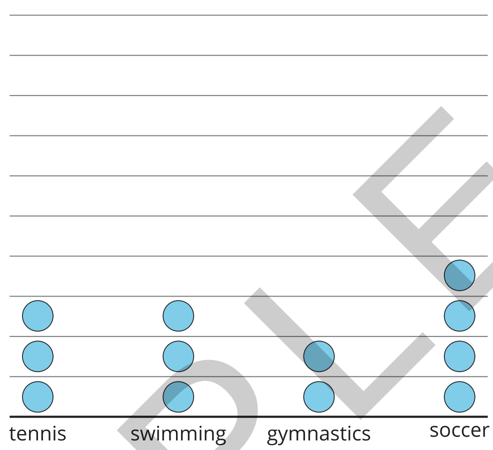
sport	number
tennis	6
swimming	6
gymnastics	4
soccer	8



Each ● represents 2 people.

Solution

Favorite Sports



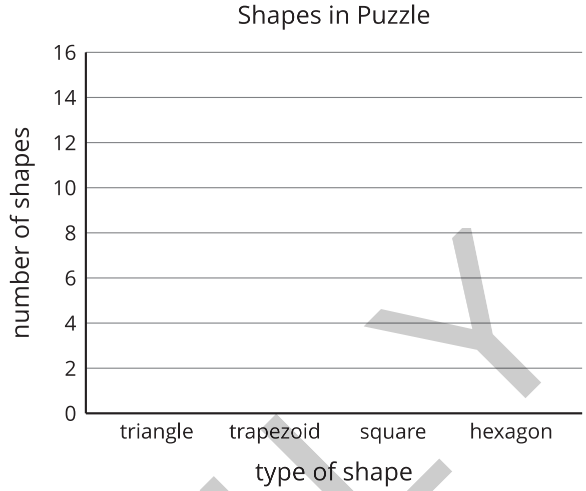
Each ● represents 2 people.

10 from Unit 1, Lesson 5

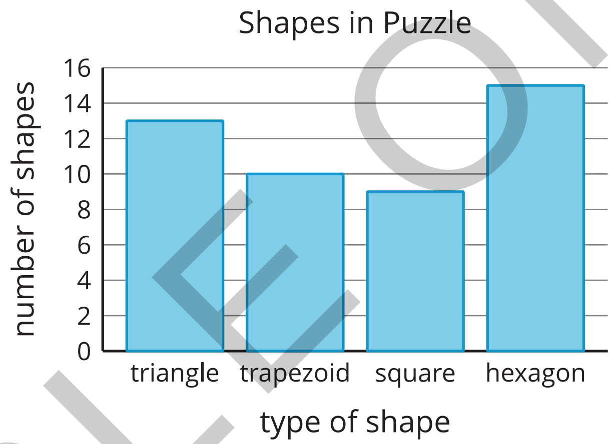
Student Task Statement

The table shows the numbers of different shapes in a pattern block puzzle. Use it to complete the scaled bar graph.

shape	number
triangle	13
trapezoid	10
square	9
hexagon	15



Solution



11 from Unit 1, Lesson 6

Student Task Statement

The table shows the numbers of books some students have at home.

Use the information from the table to create a scaled bar graph.

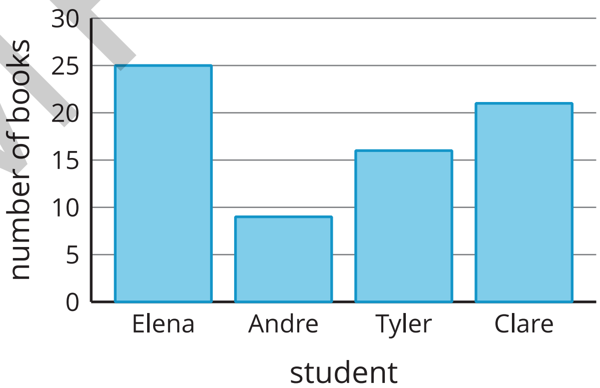
books	number
Elena	25
Andre	9
Tyler	16
Clare	21

Books at Home

Solution

Sample response:

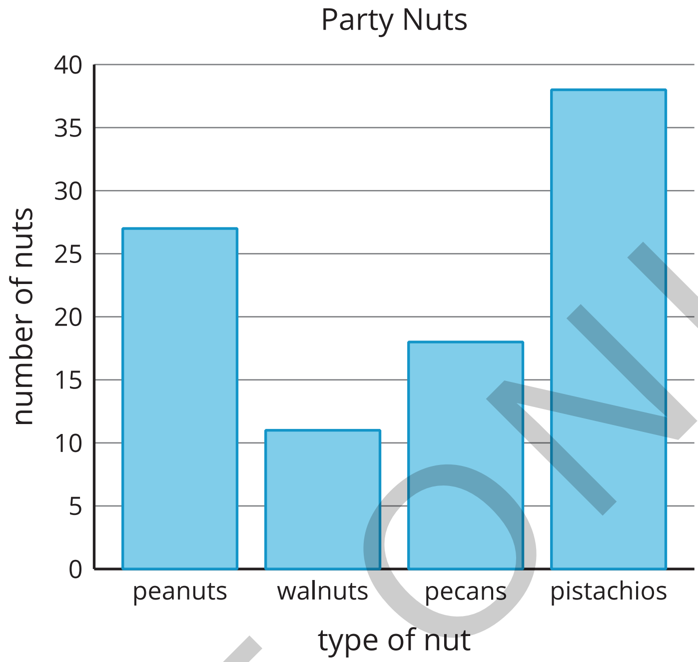
Books at Home



12 from Unit 1, Lesson 7

Student Task Statement

The bar graph shows the numbers of different kinds of nuts in a bowl.



a. How many more pistachios are there than walnuts? Explain or show your reasoning.

b. How many fewer pecans are there than peanuts? Explain or show your reasoning.

Solution

Student answers may differ by 1 or 2 because they may read the number of each type of nut slightly differently from the graph.

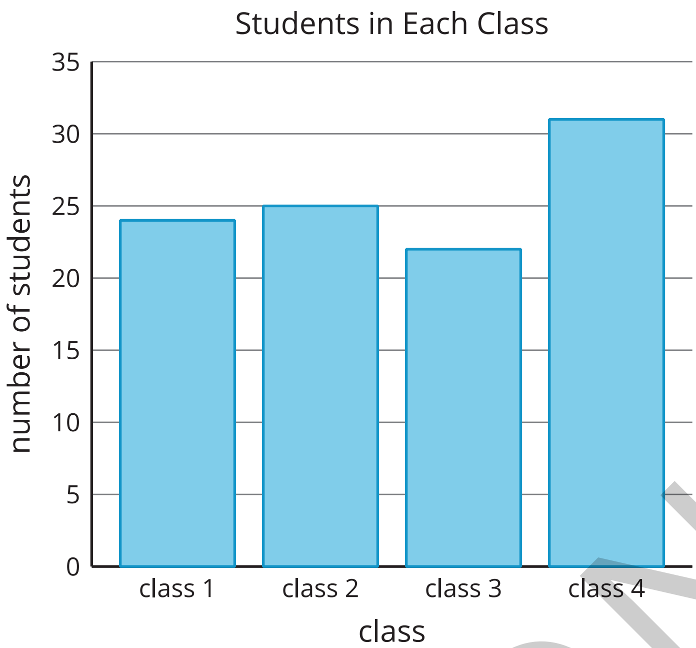
a. 27 more pistachios. Sample response: $38 - 11 = 27$

b. 9 fewer pecans. Sample response: $27 - 18 = 9$

13 from Unit 1, Lesson 8

Student Task Statement

The bar graph shows how many students are in each class.



a. How many students are in the 4 classes altogether? Explain or show your reasoning.

b. How many fewer students are in Class 1 than in Class 4? Explain or show your reasoning.

Solution

Student answers may differ slightly because they may read the number of students in each class slightly differently from the graph.

a. 102 students. Sample response: Class 1 has 24 students, class 2 has 25 students, class 3 has 22 students and class 4 has 31 students. The total number is $24 + 25 + 22 + 31$, which is 102 students.

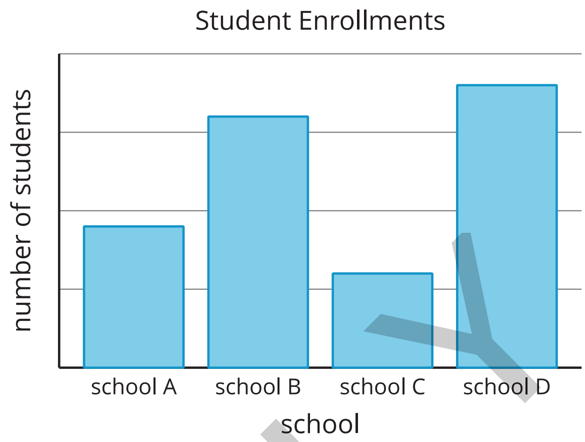
b. 7 fewer students. Sample response: $31 - 24 = 7$

14 Exploration

Student Task Statement

The bar graph shows data about 1,000 students who attend 4 schools.

What is the approximate scale for the bar graph? Explain or show your reasoning.



Solution

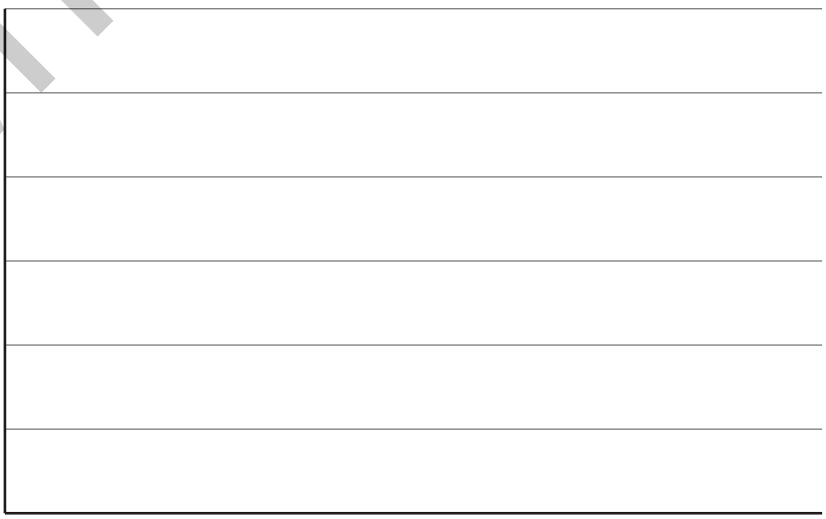
Sample responses:

- The bars for the students cover8 full units of the scale and some extra that cover about 2 more units in all. So, the 1,000 students cover about 10 units of the scale. 1,000 is 10 hundreds, so a scale of 100 works.
- I tried a scale of 10, but then there were between 10 and 30 students at each school, so that is too small. Then I tried a scale of 50, but it still was not enough, as the biggest school had fewer than 200 students. A scale of 100 looked like it worked out about right.

15 Exploration

Student Task Statement

Collect data of interest to you and represent the data on a bar graph. You may use the bar graph template if you wish.



Solution

Answers vary.

Unit 1, Lesson 1

Make Sense of Data

Standards

Building On	**2.MD.D, 2.MD.D.10**
Building Towards	**3.MD.B.3**

Instructional Routines

- Notice and Wonder

Goals

- Comprehend (in spoken language) the meaning of the terms "key" and "scale."
- Interpret picture graphs and bar graphs and ask (orally and in writing) questions that can be answered by the data in a graph.

Student Facing Learning Goals

Let's read and ask questions about data.

Lesson Purpose

The purpose of this lesson is to elicit students' prior understandings of single-unit scale picture graphs and bar graphs in preparation for upcoming work with scaled bar graphs.

Narrative

In grade 2, students learned how to draw and label single-unit scale bar graphs and picture graphs and used categorical data presented in graphs to solve simple problems. In this lesson, students revisit the structure of picture graphs and bar graphs, the features of graphs that help communicate information clearly, and the information they can learn by analyzing a graph. Students learn that a **key** is the part of a picture graph that tells what each picture represents. They also learn that the **scale** is the part of a bar graph that tells what number each bar represents. Students contextualize and make sense of the data based on the title, the given values, and their own experiences (MP2).

Math Community

In the *Lesson Synthesis*, students discuss what it means to be a part of a mathematical community. Prepare a Math Community poster by drawing a two-column chart as shown. Note that there are sections for students and the teacher to emphasize that both parties are responsible for the way math is done in the classroom. In this lesson, students add their ideas to the "Doing Math" column. In upcoming lessons, students will add to and revise these ideas, including drafting classroom goals and expectations for the "Norms" column. Keep the poster displayed in the classroom.

Math Community	
Doing Math	*Norms*
Students	Students
Teacher	Teacher

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Chart paper: Lesson

Required Preparation

Create a Math Community poster as described in the *Lesson Narrative*.

Lesson Timeline

Warm-up	15 mins
Activity 1	10 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

Today's lesson provided an opportunity to learn from your students. How were you able to incorporate your students' lived experience into the lesson?

Warm-up

15 mins

Notice and Wonder: Graphs

Standards

Building On **2.MD.D.10**

Instructional Routines

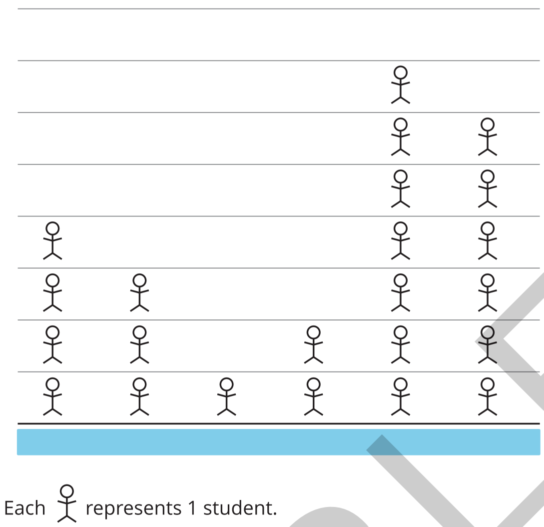
- Notice and Wonder

The purpose of this *Warm-up* is to elicit students' prior understandings about categorical data representations, which will be useful when students engage with single-unit scale picture and bar graphs in later activities. While students may notice and wonder many things about this graph, it is important to pay attention to the ways in which students make sense of a picture graph, the questions they have about the categorical data, and the contexts that make sense for the categorical data shown. This is the first time students experience the *Notice and Wonder* routine in IM Grade 3. Students should be familiar with this routine from a previous grade. However, they may benefit from a brief review of the steps involved.

For all *Warm-up* routines, consider establishing a small, discreet hand signal that students can display to indicate they have an answer they can support with reasoning. Signals might include a thumbs-up or a certain number of fingers that tells the number of responses they have. Using signals is a quick way to see if students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class. Since this is the first *Warm-up* of the year, 5 additional minutes have been allocated to help establish the structure of a routine.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Students may notice:

- It looks like a picture graph.
- There is no title.
- The categories are hidden.
- There are stick people in each column.

Students may wonder:

- What is this graph about?
- What do the stick people represent?
- Why are there so many stick people in the last two columns?
- Where is the title?

Launch

- Groups of 2
- Display the graph.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- “What situations could the graph represent?” (favorite day of the week, favorite type of food, types of animals that people saw in the park)

Activity 1

Picture Time

10 mins

Sec A

Standards

Building On 2.MD.D.10

Building Towards 3.MD.B.3

The purpose of this activity is to elicit students' prior understandings about essential parts of a picture graph. The graph in this activity is the same as the one in the *Warm-up*, but it now includes a title. Students are encouraged to consider what categories could be in the graph. Students contextualize and make sense of the data based on the title, the given values, and their own experiences (MP2). This is an opportunity for students to connect their lived experience to the mathematics, which supports the development of their math identities.

Access for Students with Disabilities

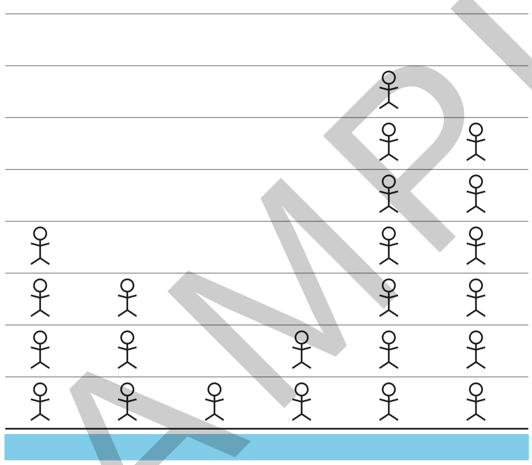
Representation: Develop Language and Symbols. Activate or supply background knowledge to help students recall the terms "picture graph" and "key." Ask, "Why do we call this graph a picture graph?" or "What kind of information does a key show?"
Supports accessibility for: Memory, Language

Student Task Statement

What could the categories be for this picture graph?

Be prepared to explain your reasoning.

How We Get Home



Each represents 1 student.

Student Response

Sample responses:

- Car, because that's how I get home from school.

Launch

- Groups of 2
- Display the graph.
- "What is different about this graph from the first graph that we discussed?" (It has a title. We know what the graph is about.)
- 30 seconds: quiet think time
- Share responses.
- "The title of the graph helps us make sense of the data shown in the graph."
- As needed, remind students that *data* is a collection of facts, such as observations, numbers, or measurements.
- "How do you and other students in our community get home from school?"
- Share responses.

Activity

- "This is a picture graph that represents how students get home from school. A **picture graph** shows how many in each group or category using pictures of the objects or symbols. Picture graphs have **keys** that tell what each picture represents."

- Bus, because some students in our class take the bus home.

- "What could the categories be in this picture graph? Be prepared to explain your reasoning."
- As needed, remind students that a *category* is a label that tells how things in a group are alike.
- 2–3 minutes: partner work time

Activity Synthesis

- Display the graph.
- Invite students to share possible categories for the graph.
- Consider asking: "How many categories will there be for this graph? How can you tell?"

Activity 2

20 mins

Picture Graphs and Bar Graphs

Standards

Building On	**2.MD.D.10**
Building Towards	**3.MD.B.3**

The purpose of this activity is to prepare students for work with scaled bar graphs in upcoming lessons. Now that students have reasoned about the parts of a picture graph, they look at how picture graphs and bar graphs are alike and how they are different. Students use the information presented on the axes of the bar graph to read the graph, interpret the categorical data presented in the graphs, and generate questions that can be answered using the graphs.

Access for English Language Learners

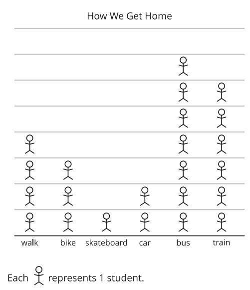
MLR8 Discussion Supports. Synthesis: For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

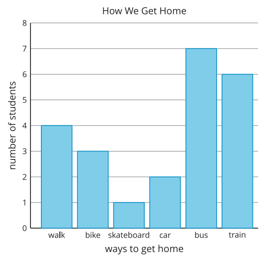
Student Task Statement

A group of students were asked, "How do you get home?" Their responses are shown in a picture graph and a bar graph.

Launch

- Groups of 2
- Display the picture graph and the **bar graph**.
- "The second image is a bar graph. A bar graph shows how many in each group or category using the length of rectangles."
- Point to the numbers along the left side of the graph. "This is the graph's scale. The **scale** is a number line that shows the length of each rectangle in a bar graph. The length represents how many in each





1. How are the 2 graphs alike? How are they different?
2. What can you learn about how students get home based on the graphs?
3. Write 2 questions that can be answered by reading the graphs.

Student Response

1. Sample responses:
    - They both have a title at the top and categories at the bottom.
    - The picture graph has pictures to show the number in each category, but the bar graph uses bars.
    - The bar graph has labels on the bottom and the side to help you know what the bars mean.
    - The picture graph has a key, but the bar graph has the numbers on the side of the graph.
2. Sample responses:
    - 7 students take a bus home.
    - More students get home by bus or train than any other way.
3. Sample responses:
    - How many students walk home?
    - How many more students take a bus home than ride a bike?

Advancing Student Thinking

If students write questions that can't be answered with the graphs, consider asking:

- "How did you come up with your question?"
- "How could we come up with a question that could be answered with the graph?"

category."

- "How are these two graphs alike? How are they different?"
- 2 minute: partner discussion
- Share and display responses.

Activity

- "What could you learn from the graphs about how students get home? Write two questions that the graphs could answer."
- 7–10 minutes: partner work time

Activity Synthesis

- Display the graphs.
- "What can we learn about how students get home from school based on the complete graphs?"
- "What questions could you ask about how students get home from school based on the graphs?"

Lesson Synthesis

"What did we learn about picture graphs and bar graphs today?" (Bar graphs and picture graphs show data. In a picture graph, a picture represents an object or a person. In a bar graph, the rectangles and the scale tell how many objects or people. We can ask and answer questions about the data in graphs.)

If these ideas do not arise, consider asking the following questions:

- "What parts of graphs help us communicate the data in the graph with others?"
- "How are picture graphs and bar graphs the same? How are they different?"

Math Community

Display the Math Community poster.

"Today we read bar graphs and picture graphs and asked questions about the data in the graphs. What does it look and sound like to do math together as a mathematical community? What was I doing? What were you doing?" (We talked to each other and to the teacher. We had quiet time to think. We shared our ideas. We thought about the math ideas and words we knew. We listened to each other share ideas. You were writing down our answers. You were waiting until we gave the answers.)

Record responses in the "Doing Math" column of the poster.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Cool-down

5 mins

Describe and Ask

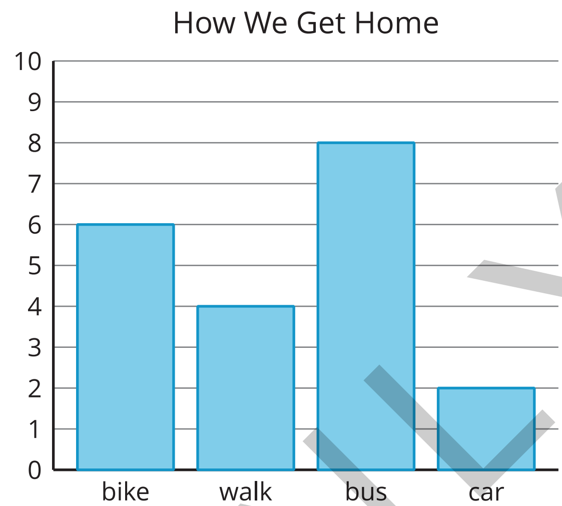
Standards

Building On	**2.MD.D**
Building Towards	**3.MD.B.3**

Student Task Statement

A group of students were asked, "How do you get home from school each day?"

Their responses are shown in this bar graph:



Based on the data shown on the graph:

1. Write one fact you learned about how the students get home.
2. Write one question you could ask about how the students get home from school.

Student Response

Sample responses:

1. More students take the bus home than ride bikes. Eight students take the bus home. Two students ride in a car.
2. How many more students take the bus than walk home? How many students ride their bikes home?

Responding To Student Thinking

Students do not read and interpret the bar graph accurately, or students write questions that can't be answered with the graph.	Next Day Supports Use the *Launch* of the next day's first activity to have students interpret the picture graph and generate questions that could be answered with the picture graph.
The work of this lesson builds from the categorical data concepts developed in a prior unit.	Prior Unit Support Grade 2, Unit 1, Section B Ways to Represent Data

Unit 1, Lesson 2

Represent Data and Solve Problems

Sec A

Standards

Building On	**2.MD.D.10, 2.OA.C.3**
Addressing	**3.MD.B, 3.MD.B.3**
Building Towards	**3.MD.B.3**

Instructional Routines

- How Many Do You See?

Goals

- Explain (orally) how to use the features of a graph to solve one- and two-step Compare problems within 20.
- Represent data using picture graphs and bar graphs.

Student Facing Learning Goals

Let's create graphs and answer questions.

Lesson Purpose

The purpose of this lesson is for students to solve one- and two-step problems about data represented in bar graphs.

Narrative

Students solved one-step problems about data in grade 2. In this lesson, students first create a picture graph and a bar graph that represent how they get home from school. Then they solve one- and two- step "how many more?" and "how many fewer?" problems using data presented in a bar graph.

Consider launching the lesson with a read-a-loud of *Last Stop* on *Market Stree*t by Matt de la Peña. This book's story about a boy's bus trip with his grandmother connects well to the context of travel that is used in this and other lessons in this unit. The book also addresses race, class, and other topics that can encourage conversations and allow students and the teacher to learn more about one another.

Math Community

Tell students they will have a chance to revise their math community ideas at the end of this lesson. As they work today, students should think about actions that may be missing from the current "Doing Math" column of the poster.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Math community poster: Lesson
- Sticky notes: Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

Think about who participated in math class today. What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Warm-up

10 mins

How Many Do You See: Dots in Groups

Standards

Building On **2.OA.C.3**

Instructional Routines

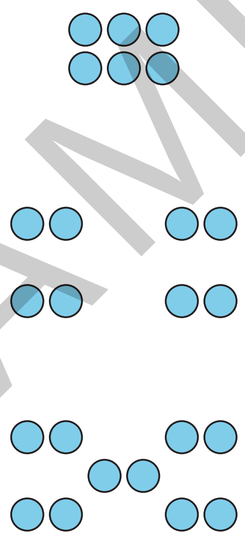
- How Many Do You See?

The purpose of this *How Many Do You See?* is for students to subitize or use grouping strategies to describe the number of dots they see. Students also make connections between the images to determine the number of dots. Grouping strategies and skip-counting by 2, 5, and 10 offer a review of grade 2 work and build toward multiplication in future lessons. In the *Synthesis*, students revisit the language of "how many more?" to prepare them to use data from a bar graph to solve "how many more?" problems throughout this lesson.

This is the first time students experience the *How Many Do You See?* routine in IM Grade 3. Students should be familiar with this routine from a previous grade. However, they may benefit from a brief review of the steps involved.

Student Task Statement

How many do you see? How do you see them?



Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the first image.
- 30 seconds: quiet think time

Activity

- Display the first image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Activity Synthesis

- "How many more dots were in the third image than in the second image? What equation matches your

Student Response

Sample responses:

- 6: I see 2 rows of 3.
- 8: I see 2 groups of 4.
- 10: I see 5 groups of 2.

thinking?" (There were 2 more dots in the third image. $8 + 2 = 10$.)

Activity 1

15 mins

How We Get Home

Standards

Building On **2.MD.D.10**

The purpose of this activity is for students to create a bar graph that includes features that help communicate the data clearly. A class picture graph is created, and students make a bar graph using that data. During the *Activity Synthesis*, focus attention on similarities and differences between picture and bar graphs.

When you create the blank "ways to get home" picture graph for the *Launch*, feel free to adjust the categories based on how your students get home from school. When students label their graphs, including a title, categories, a key for a picture graph, and scale numbers for a bar graph, they are communicating clearly and precisely (MP6).

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Required Materials

Materials To Gather

- Sticky notes: Activity 1

Required Preparation

- Create a visual display with a blank picture graph that will be large enough to fit a column of sticky notes in each category.

Student Task Statement

1. Follow your teacher's instructions to represent the class data in a picture graph.
2. Represent the same data in a bar graph.

Launch

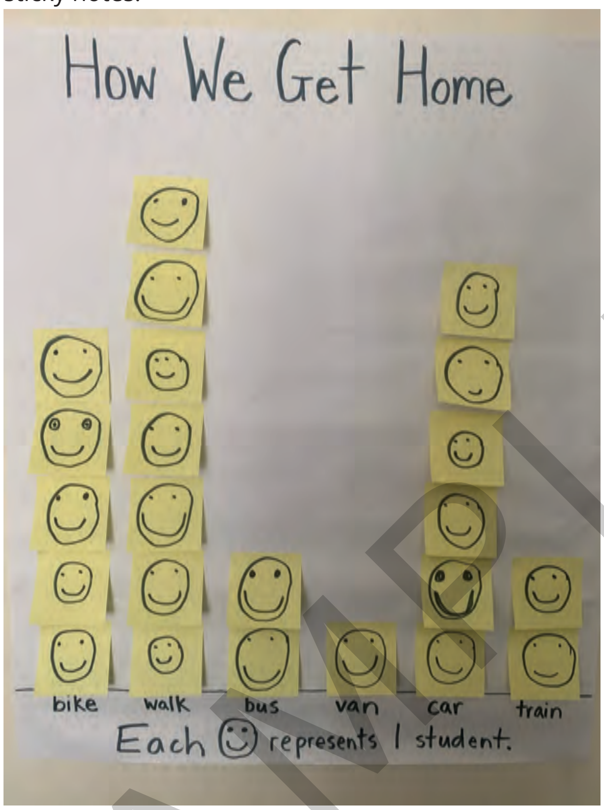
- Groups of 2
- Display a blank picture graph with labels along the bottom axis for bike, walk, bus, van, car, and train.
- Give each student a small sticky note.
- Have each student draw a smiley face on their sticky

Sec A

Student Response

Sample responses:

1. Class picture graph created on chart paper with sticky notes:



2.

note.

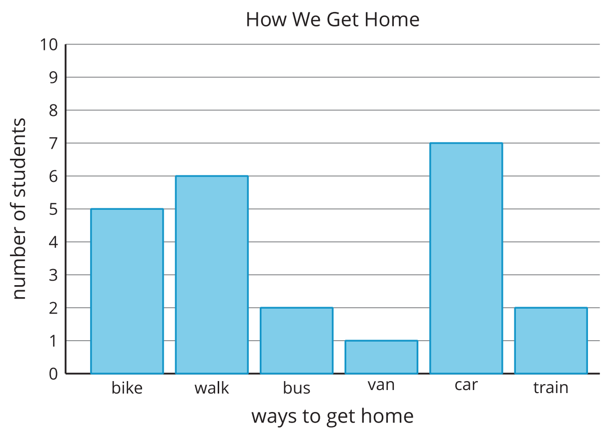
- “Put your smiley face on the graph based on how you get home from school.”
- “What needs to be added to the class picture graph to communicate the data clearly?” (A title. A key so we know what each smiley face represents.)
- Facilitate the addition of a title and key to the class picture graph.

Activity

- “Represent the data shown in the class picture graph on a bar graph with your partner. Make sure to include the parts of the graph that will help someone else read it.”
- 5–7 minutes: partner work time

Activity Synthesis

- “How are our picture graph and bar graph alike?” (They both show the same data. They have the same categories.)
- “How are our picture graph and bar graph different?” (The picture graph has a key, but the bar graph has a scale. For the picture graph, we have to count each picture, but in the bar graph, we can use the numbers on the side to tell how many.)



Advancing Student Thinking

If students create bar graphs that do not match the data in the class picture graph, consider asking:

- “Tell me about how you made your bar graph.”
- “How could we use the data in the class picture graph to help make the bar graph?”

Activity 2

20 mins

Questions about a Bar Graph

Standards

Addressing	**3.MD.B**
Building Towards	**3.MD.B.3**

The purpose of this activity is for students to answer one- and two-step “how many more?” questions using data represented in a bar graph. Students decide if statements about the data shown in the bar graph from the previous activity are true or false and then answer questions about the data. When students use expression or equations or describe adding or subtracting to find how many more or less, they show they can decontextualize and recontextualize the data to make sense of and solve the problems (MP2). The teacher will generate the questions students answer in this task from the class graph.

Access for Students with Disabilities

Representation: Access for Perception. Read the directions and statements aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language

Student Task Statement

1. Decide if each statement about how our class gets home is true or false. Use the graph to explain your reasoning to your partner.

Launch

- Groups of 2

a. More students walk than go home any other way.

b. More students ride home on a bus than in a car.

c. Fewer students walk home than ride their bikes.

d. More students walk or ride their bikes than ride in a van.

2. Fill in the blanks as directed by your teacher. Then answer each question.

a. "How many more students

_______________ than

_______________?"

b. "How many more students

_______________ or

_______________ than

_______________?"

Student Response

Sample responses based on sample class data shown in the previous activity:

1. 1. False. The "car" category has the longest bar, so more students go home in a car than any other way.

   2. False. The "bus" bar is shorter than the "car" bar, so fewer students ride home on a bus than in a car.

   3. False. The "bar for "walk" is longer than the bar for "bike," so more students walk than ride bikes.

   4. True. The "van" category has the shortest bar.

2.

   a. "How many more students bike than ride in a bus?" 3 more students

   b. "How many more students ride in a car or van than walk? 2 more students

Activity

- "Now you're going to use your bar graph to decide if statements are true or false."
- 1–2 minutes: independent work time
- 3–5 minutes: partner discussion
- As students work, decide which categories will go in the question stems for the next problem.
- Consider providing these sentence stems if students need support explaining their reasoning:
  - "I knew the statement was false because . . . ."
  - "I knew the statement was true because . . . ."
- "How did you know if each statement was true or false?"
- Share responses.
- Guide the whole class to fill in the blanks in the questions using the previously identified categories.
- "Use the data in your bar graph to answer the questions."
- 3–5 minutes: partner work time

Activity Synthesis

- Ask students to share their responses to each Compare problem.
- "Do you have any lingering questions about how to answer these questions from the bar graph?"
- Consider asking: "What equation matches your thinking?"

Advancing Student Thinking

If students find differences that do not match the data in the graph, consider asking:

- "How did you answer the questions?"
- "How could you use the graphs to answer the questions?"

Lesson Synthesis

Display a student-created bar graph.

Generate a few questions for students to answer about how they get home using the bar graph. For example, you might ask:

- "How many more students ______________ than ______________?"
- "How many fewer students ______________ than______________?"
- "How many more students ______________ or ______________ than ______________?"

Invite students to answer the questions and share their reasoning.

Math Community
After the *Cool-down*, give students 2–3 minutes to discuss in small groups any revisions to the "Doing Math" actions listed on the Math Community poster. . Share ideas as a whole group and record any revisions.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Cool-down

5 mins

Questions About a Bar Graph

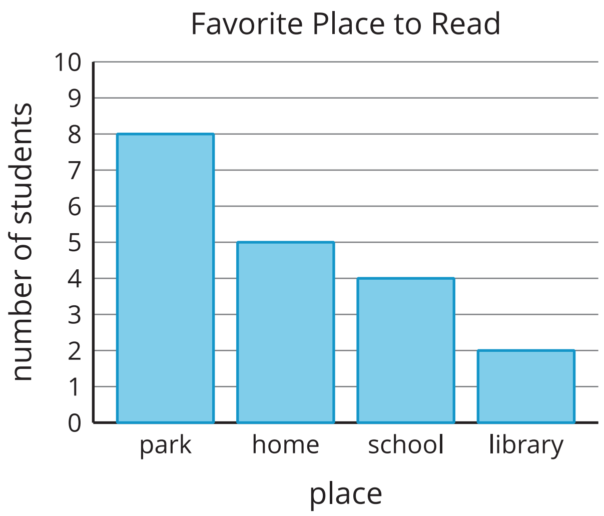
Standards

Addressing	**3.MD.B.3**
Building Towards	**3.MD.B.3**

Student Task Statement

A group of students were asked, "Where is your favorite place to read?"

Their responses are shown in this bar graph:



1. How many more students chose the park than home as their favorite place to read?

2. True or false: More students chose the school or library than the park. Explain or show your reasoning.

Student Response

1. 3 more students
2. False. Sample response: 6 students $(4 + 2)$ chose the school or library, and 8 students chose the park.

Sec A

Responding To Student Thinking

Students count lines on the bar graph to find the number represented by a bar instead of finding the scale number that matches the top of the bar.Students count lines on the bar graph to find the number represented by a bar instead of finding the scale number that matches the top of the bar.

Next Day Supports
During the *Launch* of the next day's activity, have students discuss how the scale on the bar graph can be used to determine the number of people or objects in each category.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 1, Section B Ways to Represent Data

Unit 1, Lesson 3

Scaled Picture Graphs

Standards

Building On	**2.NBT.B.5**
Addressing	**3.MD.B**
Building Towards	**3.MD.B.3**

Instructional Routines

- Number Talk

Goals

- Comprehend (in spoken language) the meaning of the term “scaled picture graph.”
- Interpret scaled picture graphs and ask questions (orally and in writing) about the data.

Student Facing Learning Goals

Let’s explore scaled picture graphs.

Lesson Purpose

The purpose of this lesson is for students to read and answer questions about scaled picture graphs.

Narrative

In previous lessons, students reviewed how to create and interpret single-unit scale picture graphs. In this lesson, students learn that a **scaled picture graph** is a picture graph where each picture represents an amount other than 1. Students read, interpret, and answer questions about scaled picture graphs with a scale of 2 and 5. They also generate questions that can be answered by these graphs.

Math Community

Tell students that, at the end of the lesson, they will be asked to identify specific actions from their “Doing Math” list that they personally experienced.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Math community poster: Lesson

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins

Teacher Reflection Questions

In this lesson, students make sense of scaling a picture graph by a number other than 1. How does this support the work that students will do with multiplication later in this unit?

Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Sec A

Warm-up

10 mins

Number Talk: Addition

Standards

Building On **2.NBT.B.5**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for adding within 100. These understandings help students develop fluency and will be helpful later in this lesson when students need to add to find the total number of students represented in a picture graph. When students use strategies based on place value to add, they look for and make use of structure (MP7).

This is the first time students experience the *Number Talk* routine in IM Grade 3. Students should be familiar with this routine from a previous grade. However, they may benefit from a brief review of the steps involved.

Student Task Statement

Find the value of each expression mentally.

- $50 + 10$
- $50 + 12$
- $60 + 13$
- $65 + 13$

Student Response

- 60: 5 tens and 1 ten make 6 tens, which is 60.
- 62: It's just like the first one, but there are 2 ones, so it is 62.
- 73: It's like the second problem, but there's 1 more ten and 1 more one. So each digit goes up by 1 and that's 73.
- 78: There are 7 tens and 8 ones, so it's 78.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "How was place value helpful as you added these numbers?" (I was able to use tens and ones to help me find the sum.)
- Consider asking:
    - "Who can restate ____'s reasoning in a different way?"
    - "Did anyone use the same strategy but would explain it differently?"
    - "Did anyone approach the problem in a different way?"

- "Does anyone want to add on to _____'s strategy?"

Activity 1

So Many Responses

15 mins
PLC Activity

Standards

Addressing **3.MD.B**

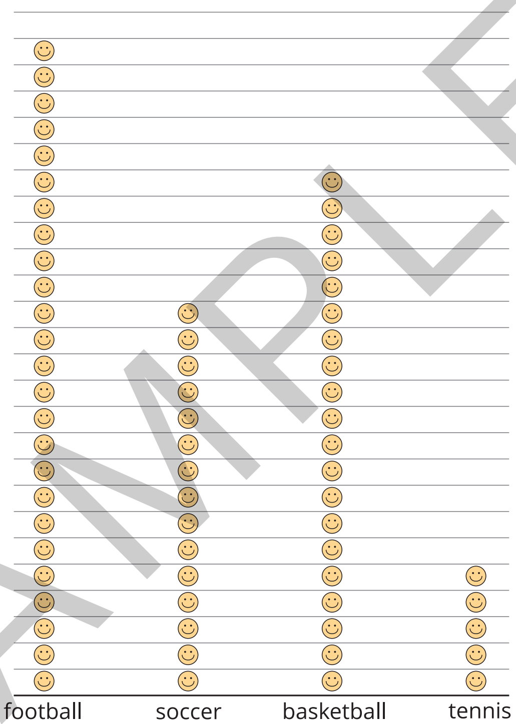
Building Towards **3.MD.B.3**

The purpose of this activity is for students to read a scaled picture graph. A scale of 5 is used to encourage skip-counting because students skip-counted by 5 in grade 2. The questions in the task focus on the structure of a scaled picture graph and the strategies students can use to read them.

Student Task Statement

1. A group of students were asked, "Which of these 4 sports is your favorite?" Their responses are shown in this picture graph:

Favorite Sports



Each ☺ represents 1 student.

How many students are represented in the graph? ______________

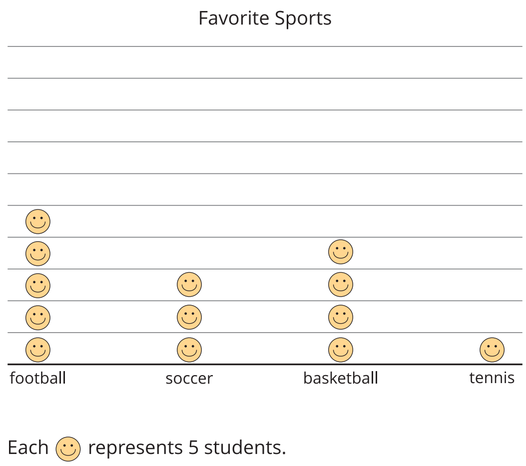
Launch

- Groups of 2
- "What is your favorite sport or activity outside of school?"
- Share responses.
- Display the first image of the single-unit scale picture graph.
- "What do you notice? What do you wonder?" (The graph is about students' favorite sports. There are a lot of smiley faces. Each smiley face represents 1 student. It takes a lot of time to count the students in each category. How many student responses are shown in the whole graph? How could we make the graph take up less space?)
- 1 minute: quiet think time
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity

- "Work with your partner to find how many students are represented in the graph."
- 3–5 minutes: partner work time
- Monitor for students who group the smiley faces by 2, 5, or 10 to make them easier to count.
- Have students who grouped the smiley faces share their strategies for how they found the total number

2. The students' responses are also shown in this picture graph:



How is counting the total number of students in this graph different from counting the total number of students in the first graph?

of students represented in the graph.

- If no students use this strategy ask, "How could grouping the pictures in the graph make them easier to count?" (We could circle tens so we could count by ten. It would be easier to keep track of your count than by counting by 1.)
- Display the second image of the scaled picture graph.
- "How could we count the total number of students in this graph?"
- 2 minutes: partner work time

Activity Synthesis

- "In a graph where there's a lot of data, we can adjust the scale so each picture represents more than 1 object. When each picture represents something other than 1, we say that it's a **scaled picture graph**. The key tells us that in this graph, each smiley face represents 5 students."

Student Response

1. 65 students
2. Sample response: We count by 5 to find the total instead of counting by 1. We are able to count to find the total a lot faster.

Advancing Student Thinking

If students count the students in the scaled picture graph and get a total other than 65, consider asking:

- "How did you find the total number of students represented in the graph?"
- "How could you use counting by 5 to find the total number of students represented in the graph?"

Activity 2

20 mins

Questions about Scaled Picture Graphs

Standards

Addressing	**3.MD.B**
Building Towards	**3.MD.B.3**

The purpose of this activity is for students to interpret a scaled picture graph and write questions that can be asked based on the data represented in a scaled picture graph.

Access for English Language Learners

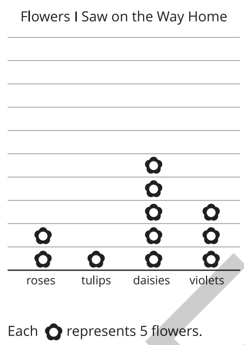
MLR8 Discussion Supports. Use multimodal examples to show the meaning of a symbol. Use verbal descriptions along with gestures, drawings, or concrete objects to show how each flower on the graph is a symbol that represents 5 flowers that were seen on the way home.
Advances: Listening, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis*:* Invite students to identify which details were needed to solve the problem. Display the sentence frame, "The next time I read a scaled picture graph I will pay attention to . . ."
Supports accessibility for: Conceptual Processing

Student Task Statement

1. Andre collects data about 4 types of flowers he sees on the way home. The data is shown in this picture graph:



   a. How many of each type of flower did Andre see on the way home?

      roses ____ tulips ____ daisies ____ violets ____

   b. Write 2 questions that can be answered by reading the graph.

2. A group of students were asked, "Which is your favorite type of book?" Their responses are shown in this picture graph:

Launch

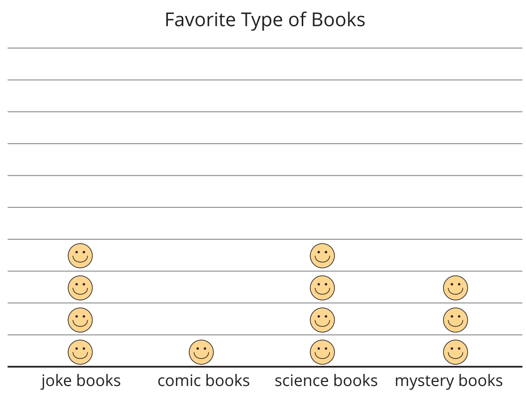
- Groups of 2
- Display the graphs for all to see.
- "What are some strategies you could use to read the graphs?" (In the Flowers I Saw on the Way Home graph I could count by 5 to find the number for each category because each picture represents 5 flowers.)
- 1 minute: quiet think time
- Share and record responses.

Activity

- "Now, we're going to answer some questions about the scaled picture graphs. You will also have a chance to write your own question that can be asked based on each graph."
- 8–10 minutes: partner work time
- If there is time, have groups trade books and answer each other's questions.

Activity Synthesis

- Invite students to share responses to the questions they answered and explain their reasoning.
- Consider asking:
  - "How did you use the key to answer the questions?" (I counted by the number that each picture represents—by 5 for the flowers and by 2 for the students.)
- Share a variety of student written questions.
- Consider asking:



Each ☺ represents 2 students.

a. How many students chose each type of book? How do you know?

b. Write 2 questions that can be answered by reading the graph.

- ◦ "How did you know your question could be answered with the graph?" (The data needed to answer the question is shown in the graph.)
- ◦ If time allows, ask, "What questions cannot be answered by this graph?" (How many students' favorite type of book is graphic novels?)

Student Response

1. a. 10 roses, 5 tulips, 25 daisies, 15 violets
   b. Sample responses: How many more roses did Andre see than tulips? How many fewer violets did Andre see than daisies?
2. a. 8 chose joke books, 2 chose comic books, 8 chose science books, and 6 chose mystery books. Sample response: Each smiley face represents 2 students, so to find the number in each category I counted by 2.
   b. Sample responses: How many more students chose science books than mystery books? How many students chose joke books or comic books?

Advancing Student Thinking

If students answer questions about the graph with numbers that don't match the graph, consider asking:

- "How did you answer the questions about the graph?"
- "What does the key tell us about each picture in the graph?"

Lesson Synthesis

Display the images of the two "Favorite Sports" graphs.

"Today we learned about scaled picture graphs. Why would we make a scaled picture graph?" (When there is a lot of data to represent, it is faster to use a scale greater than 1.)

“How is reading scaled picture graphs different from reading picture graphs that have a scale of 1?” (In a scaled picture graph, each picture doesn’t represent 1 thing, so you need to use the key. In a scaled picture graph, you can count by a number greater than 1 to find the total in each category.)

Math Community

After the *Cool-down*, ask students to individually reflect on the questions “Which ‘Doing Math’ action did you feel was most important in your work today? Why?” Students can write their responses on the bottom of their *Cool-down* paper, on a separate sheet of paper, or in a math journal.

Collect and read their responses after class. These responses will offer insight into how students feel about their own mathematical work and how you make personal connections to the norms students will create during Days 4–6.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Cool-down

5 mins

Birds in the Park

Standards

Addressing **3.MD.B**

Student Task Statement

Jada collects data about 4 types of birds she sees on her way home.

The data is shown in this picture graph:

Based in the data on the graph:

1. How many sparrows did Jada see on the way home?
2. Write one question you could ask about the birds Jada saw on the way home.

Student Response

1. 10 sparrows
2. Sample responses: How many birds did Jada see on the way home? How many blue jays and cardinals did Jada see on the way home?

Responding To Student Thinking

Students say 5 sparrows are represented in the graph.

Next Day Supports
Use the next day's *Warm-up* to practice counting by 5, and discuss how such counting could be used to read a scaled picture graph.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 1, Section B Ways to Represent Data

Unit 1, Lesson 4

Create Scaled Picture Graphs

Sec A

Standards

Addressing	**3.MD.B, 3.MD.B.3**
Building Towards	**3.MD.B.3**

Instructional Routines

- How Many Do You See?

Goals

- Represent data using a scaled picture graph.

Student Facing Learning Goals

Let's make a scaled picture graph.

Lesson Purpose

The purpose of this lesson is for students to create a scaled picture graph to represent categorical data.

Narrative

In a previous lesson, students interpreted and answered questions about scaled picture graphs. In this lesson, they gather and organize data about ways that students would like to travel and represent the data in a scaled picture graph with a scale of 2. Students make sense of how to represent a single student on a scaled picture graph that has a scale of 2.

Math Community

Before the lesson, explain to students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Offer an example, such as: "It may help us share our ideas as a whole class if we have the norm 'Listen as others share their ideas.'" Tell students to think about norms that help everyone do math as they work today. Explain that you will record these norms during the *Lesson Synthesis*.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Math community poster: Lesson

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins

Teacher Reflection Questions

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Synthesis Estimate	10 mins
Cool-down	5 mins

Warm-up

10 mins

How Many Do You See: More Groups of Dots

Standards

Building Towards **3.MD.B.3**

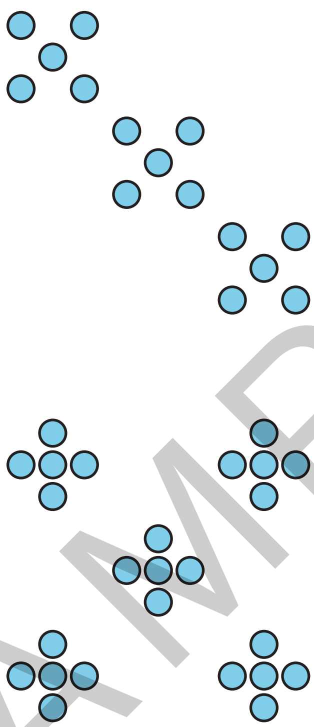
Instructional Routines

- How Many Do You See?

The purpose of this activity is for students to subitize or use grouping strategies to describe the number of dots they see. Although the dots have been deliberately grouped by 5 to elicit counting by 5 as a strategy, students may see 2 groups of 5 as 10. Grouping strategies and skip-counting by 2, 5, and 10 offer a review of grade 2 work and build toward multiplication in future lessons.

Student Task Statement

How many do you see? How do you see them?



Student Response

Sample response:

- 15: I saw 3 groups of 5.
- 25: I saw 5 groups of 5.

Launch

- Groups of 2
- “How many do you see? How do you see them?”
- Flash the first image.
- 30 seconds: quiet think time

Activity

- Display the first image.
- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Activity Synthesis

- “What pattern did you see first, and how did this help you figure out the total?” (I saw that the dots were in groups of 5. This helped me because I know how to count by 5.)
- Consider asking:
  - “Did anyone see the dots the same way but would explain it differently?”
  - “Does anyone want to add an observation to the way _____ saw the dots?”

Activity 1

15 mins

Ways to Travel

Standards

Addressing **3.MD.B**

Building Towards **3.MD.B.3**

The purpose of this activity is for students to gather and organize categorical data about their classmates' preferred way to travel.

To make the data collection process faster, students collect their responses within their group first, and then each group shares their responses to make a class data list. As an alternative, students can collect all of the data on their own. In that case, students' names can be pre-printed in a table for them, or they could only write the person's first name and way of travel abbreviation (given in the *Task Statement*) in the table.

In the *Activity Synthesis*, students analyze the advantages and disadvantages of showing data in a list or table like this. Then the responses are counted and displayed for each category. This will make it easier for students to use the data to create a scaled picture graph data in the next activity.

Student Task Statement

How would you like to travel?

- car (C)
- train (T)
- boat (B)
- balloon (Bal)
- plane (P)
- helicopter (H)

student's name	way of traveling

Student Response

Answers vary.

Launch

- Groups of 4

Activity

- "Today you will survey your classmates. You will start with your group and then we will collect the group data as a class."
- Instruct students to record their group's 4 responses in their book's table.
- 1–2 minutes: small-group work time
- Create a large table on chart paper with a row for each student. Ask groups to take turns reporting their responses and record them in the table.
- 5 minutes: whole-class work time

Activity Synthesis

- "What's helpful about having this data in the form of a list? What's not helpful?" (We know the way each person would like to travel. It's hard to see how many people would like to travel each way.)
- Create a two-column table with six rows. Label the rows with the travel choices in the survey. Lead students to use the class data list to count how many students chose each travel choice and record the numbers in the table. Instruct students to make their

own copy of this table to use in the next lesson.

Activity 2

20 mins

Create a Scaled Picture Graph

Sec A

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to apply understandings from previous lessons to create a picture graph with a scale of 2 to display the categorical data they gathered. Students are guided to use a scale of 2 but can choose their own symbol. Depending on the data, students may need to use a half symbol in order to represent an odd number of students choosing a specific method of travel. This idea is discussed in the *Activity Synthesis.*

Students will use their scaled picture graphs again in the next lesson.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: When students compare graphs, display the following sentence frames: "The symbol I chose to represent _____ is _____, because . . . ." "One way our graphs are the same is . . . ." "One way our graphs are different is . . . ."
Advances: Speaking, Representing

Access for Students with Disabilities

*Representation: Internalize Comprehension.*Invite students to begin by creating a physical model of a picture graph. Provide access to physical objects, such as connecting cubes, that students can use to represent each person, and then organize into groups of 2.
Supports accessibility for: Visual-spatial processing, Conceptual processing

Student Task Statement

Represent the class survey data in a scaled picture graph. Have each picture represent 2 students.

Launch

- Groups of 2
- "How can we represent our survey data in a picture graph without having to draw a picture for each student in our class?" (We can make each symbol represent more than one student so we don't have to draw as much.)

Activity

- a
- 10 minutes: independent work time
- Circulate as students work:
  - Encourage them to include a title, category labels, and a key.

- Pay attention to how students are grouping by 2.
- Support students with questions they may have (especially around representing odd number amounts).

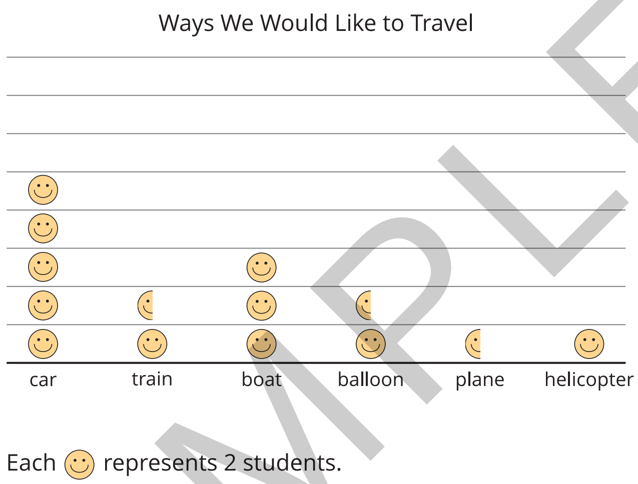
- “Compare your graph with your partner.”
- 2 minutes: partner discussion
- Monitor for a graph that uses a half picture to show an odd number of students in one of the categories to share during the *Activity Synthesis*.

Activity Synthesis

- Display selected student work.
- “How does this graph represent the survey data from our class?”
- “How did _____ represent the number of students who picked a way of travel when it was an odd number?”
- “What questions do you have about creating a scaled picture graph?” (Could a face represent 3 students or 5 students? Can you use whatever picture you want to represent 2 students?)

Student Response

Students’ graphs match the class data. Sample response:



Advancing Student Thinking

If students choose symbols that are time-consuming to draw, consider asking:

- “How did you choose the symbol to use on your graph?”
- “How could you make your symbol easier to draw?”

Lesson Synthesis

Display a scaled picture graph from today's lesson. "What if 2 more students chose to travel by balloon? How could we represent that on this graph?" (Add 1 more picture in that category.)

"What if 1 more student chose to travel by car? How could we represent that on this graph?" (Add half of the picture in that category.)

Sec A

Math Community
Ask students to reflect on both individual and group actions while considering the question "What norms, or expectations, were we mindful of as we did math together in our math community?"

Record and display their responses in the "Norms" column of the chart on the Math Community poster.

Suggested Centers

- Sort and Display (1–3), Stage 2: Picture or Bar Graphs (Supporting)
- Capture Squares (1–5) , Stage 3: Add within 20 (Supporting)

Cool-down

5 mins

Complete the Picture Graph

Standards

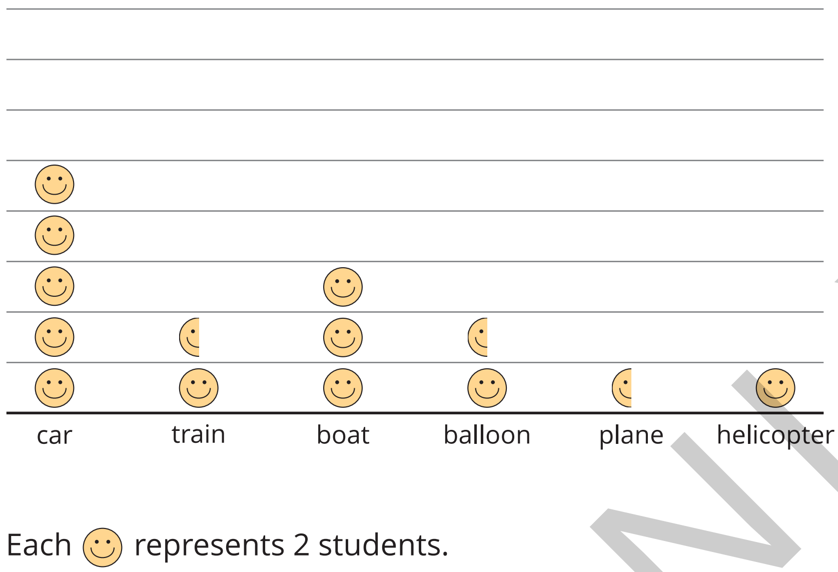
Addressing **3.MD.B.3**

Student Task Statement

A group of students were asked, "Which way would you like to travel?"

Their responses are shown in this picture graph:

Ways We Would Like to Travel



Four students were absent when this data was collected. They would like to travel by plane.

Add their data to the graph.

Student Response

Students draw two more smiley faces in the plane column.

Responding To Student Thinking

Students draw 4 smiley faces on the graph instead of 2.

Next Day Supports
Before the *Warm-up*, invite students to work in small groups to discuss a correct response to this *Cool-down*.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 1, Section B Ways to Represent Data

Unit 1, Lesson 5

Represent Data in Scaled Bar Graphs

Sec A

Standards

Building On	**2.OA.C.4**
Addressing	**3.MD.B, 3.MD.B.3**
Building Towards	**3.MD.B.3**

Instructional Routines

- Number Talk

Goals

- Compare and contrast (orally and in writing) bar graphs with different scales.
- Comprehend (in spoken language) the meaning of the term "scaled bar graph."
- Represent data using a scaled bar graph.

Student Facing Learning Goals

Let's make a scaled bar graph.

Lesson Purpose

The purpose of this lesson is for students to create a scaled bar graph.

Narrative

In a previous lesson, students collected categorical class data and learned how to create a scaled picture graph. Students now make connections between scaled picture graphs and scaled bar graphs and expand the idea of a scale that is more than one to bar graphs. In this lesson, students choose a scale of 2 or 5 for their bar graph.

Math Community

Tell students they will have a chance to revise their math community ideas at the end of this lesson. As they work today, students should think about norms that may be missing from the current list

Access For Students with Disabilities

- Engagement

Access For English Learners

- MLR7

Required Materials

Materials To Gather

- Math community poster: Lesson
- Materials from a previous lesson: Activity 2

Lesson Timeline

Warm-up	10 mins

Teacher Reflection Questions

Based on students' prior work with scaled picture graphs, what strategy did you anticipate today? What strategy did

Activity 1	10 mins
Activity 2	25 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

you not anticipate?

Warm-up

10 mins

Number Talk: Twos and Fives

Standards

Building On **2.OA.C.4**

Building Towards **3.MD.B.3**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies students have for counting by 2 and 5. These understandings help students develop fluency and will be used later in this lesson when students will use a scale of 2 or 5 to create bar graphs.

When students notice that the number of equal addends are doubled in the second expression, they are looking for and making sense of structure (MP7). When they notice that the pattern repeats in the second pair of expressions and use the pattern to find the value of the sum, they are also looking for and expressing regularity in repeated reasoning (MP8).

Student Task Statement

Find the value of each expression mentally.

- $2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$

Student Response

- 8: There were four 2s, so I counted by 2 four times: 2, 4, 6, 8.
- 16: I knew that four 2s was 8, and now there are 2 groups of 8, so I doubled 8 to get 16.
- 20: I knew that 2 fives is 10 and there are 4 fives, so I doubled 10.
- 40: I knew that four 5s was 20, and now there are 2 groups of 20, so I doubled 20 to get 40.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "How did the first two expressions help you solve the third and fourth expressions?" (I noticed there are double the number of 2s or 5s, so I can just double the sum.)
- Consider asking:
  - "Who can restate ____'s reasoning in a different way?"
  - "Did anyone use the same strategy but would

explain it differently?”
    - “Did anyone approach the problem in a different way?”
    - “Does anyone want to add on to _____’s strategy?”

Activity 1

Compare Bar Graphs

10 mins

Standards

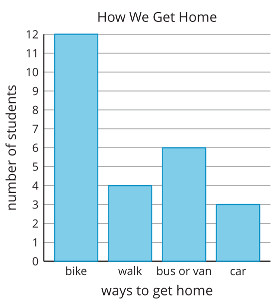
Addressing **3.MD.B**

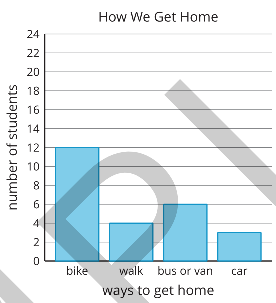
Building Towards **3.MD.B.3**

The purpose of this activity is to introduce students to a scaled bar graph. Students consider a single-unit scale bar graph next to a bar graph with a scale of 2, both representing the same set of categorical data. Students discuss similarities and differences between a single-unit scale bar graph and a bar graph with a scale of 2.

Student Task Statement

All the students in a class were asked, “How do you get home from school?” Their responses are shown in these 2 bar graphs:





Discuss with your partner:

- How are the 2 graphs alike?
- How are they different?

Student Response

Sample responses:

- They both have bars and the same title, categories, and labels.
- One counts by 1 for the scale, and the other counts by 2.
- The 12 on the first graph goes higher than the 12 in

Launch

- Groups of 2

Activity

- Display the images.
- “How are these bar graphs alike? How are they different?”
- 1 minute: quiet think time
- 4 minutes: partner discussion

Activity Synthesis

- “What was the same? What was different?”
- Share and record responses.
- If it doesn't come up, elicit the idea that the scale on the second graph counts by 2.
- “When each jump on the scale is some number other than 1, we say that it’s a **scaled bar graph**.”
- “Why would it be helpful to make a scaled bar graph?” (If you didn't want to count one by one. If you wanted to show larger numbers on your graph.)

the second graph.

Activity 2

25 mins

Create a Scaled Bar Graph

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to create a scaled bar graph. Pairs of students graph the same data, but one uses a scale of 2 and the other uses a scale of 5. Students then compare their graphs and discuss how the different scales affect how the graphs look and how accurately they can tell the exact number each bar represents (MP6). In the *Activity Synthesis*, students discuss how they represented an odd number of students with a scale of 2 and a number of students that was not a multiple of 5 with a scale of 5. These questions should be adjusted based on the class data.

Access for English Language Learners

MLR7 Compare and Connect. Synthesis: Give students time to study the student work displayed with both scales. During the whole-class discussion, ask students, “What do the graphs have in common?” “How are they different?” “Why do the different graphs lead to the same outcome?”
Advances: Representing, Speaking

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after they have represented one method of travel on a graph.
Supports accessibility for: Organization, Attention

Required Materials

Materials To Gather

- Materials from a previous lesson: Activity 2

Required Preparation

- Each student needs the picture graph they created in the previous lesson.

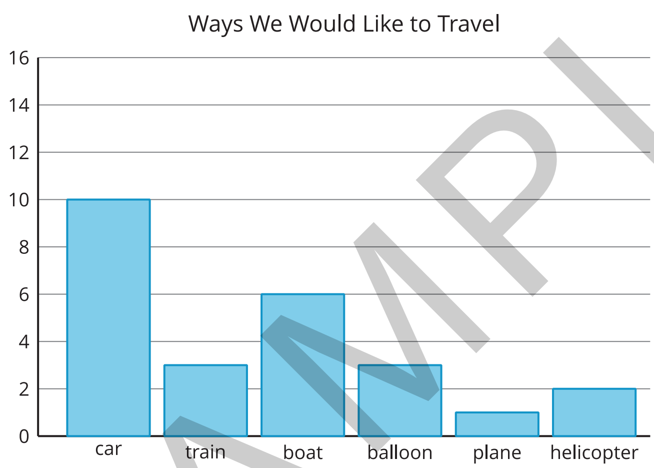
Student Task Statement

Represent the data we collected earlier about travel choices in a **scaled bar graph.** Use the graph with a scale of 2 or the graph with a scale of 5. If you have time, you can make 2 graphs. Be sure to label your title and categories.

Launch

- Groups of 2
- Make sure each student has their scaled picture graph from the previous lesson.

Student Response



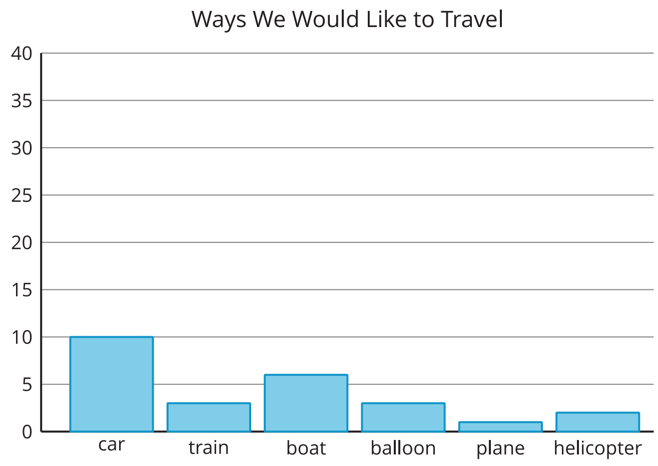
Students' graphs match the class data. Sample responses:

Activity

- "Today, we will represent the data we collected yesterday about ways we would like to travel in a scaled bar graph."
- "Decide with your partner which of you will use a scale of 2 and which will use a scale of 5. If you have time, you can try making a graph using the other scale, too. ."
- 10 minutes: partner work time
- Consider asking, "How did your scaled picture graph help you make your scaled bar graph?"
- Instruct partners to compare their graphs and discuss how accurately they can tell the exact number each bar represents in each scale.
- 2 minutes: partner discussion time.

Activity Synthesis

- Display student work with both scales.
- "How did you represent a way of travel that didn't land right on the numbers in the scale?" (It was between the 2 numbers, so I had to make a guess about where the bar should stop.)
- "What differences do you notice when the graph is with a scale of 2 and when the graph is with a scale of 5?" (The bars look taller when the scale is 2. It's hard to tell what number some bars represent when the scale is 5. It was easier to count up to larger amounts when the scale was 5.)



Advancing Student Thinking

If students draw the top of the bar in a location that doesn't correspond with the data, consider asking:

- "How did you decide where the top of the bar would end?"
- "How could you use counting by 2 (or 5) to help you decide where the top of the bar should end?"

Lesson Synthesis

Display a scaled bar graph from the lesson.

"We've been learning about how to make scaled bar graphs. If you were going to help a friend create a scaled bar graph, what advice would you give them?" (I would tell them that the scale goes up by a number other than 1. They should look at the number of people or objects in each category and think about whether those numbers are easy to count by 2 or 5 or some other number.)

Be sure to highlight ideas about using scales of 2 or 5.

Math Community
After the *Cool-down*, give students 2–3 minutes to discuss in small groups any revisions to the list of norms. Share ideas as a whole group and record any revisions.

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Cool-down

5 mins

Complete a Scaled Bar Graph

Standards

Addressing **3.MD.B.3**

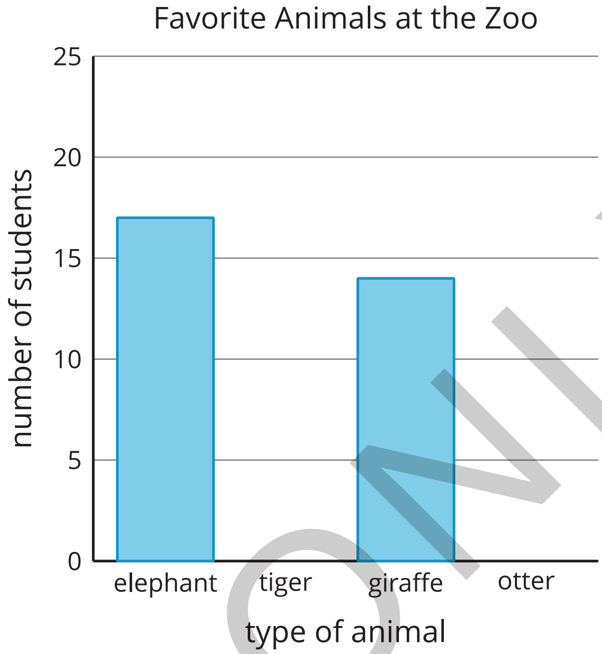
Student Task Statement

Students were asked, "Which is your favorite animal at the zoo?"

Their responses are shown in this table:

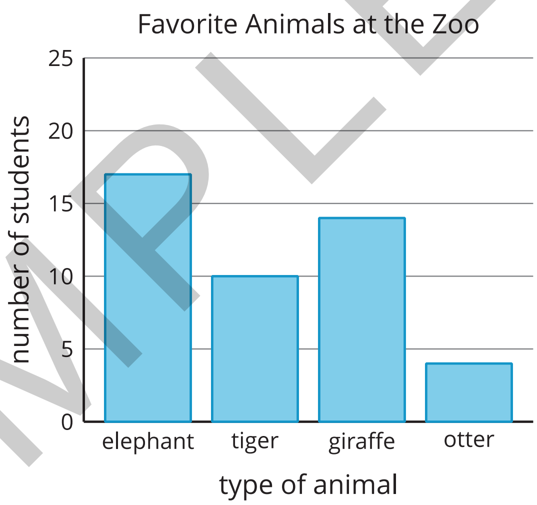
animal	number of students
elephant	17
tiger	10
giraffe	14
otter	4

Use the data in the table to complete the scaled bar graph.



Student Response

Student bar graphs show 10 students chose tigers and 4 students chose otters.



Responding To Student Thinking

Students draw bars that use a scale of 1 instead of a scale of 5.

Next Day Supports
Use the *Launch* of the next day's activity to discuss the difference between a bar graph that has a scale of 1 and a scaled bar graph.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 1, Section B Ways to Represent Data

Sec A

Unit 1, Lesson 6

Choose a Scale

Standards

Addressing	**3.MD.B.3**
Building Towards	**3.MD.B.3**

Instructional Routines

- Notice and Wonder

Goals

- Represent data using a scaled bar graph and explain (orally) how a scale was chosen.

Student Facing Learning Goals

Let's choose a scale for our bar graph.

Lesson Purpose

The purpose of this lesson is for students to consider the advantages and disadvantages of various bar graph scales.

Narrative

In previous lessons, students created scaled picture graphs and bar graphs with a given scale of 2 or 5. This lesson extends this work to allow students to choose and create the scale for their bar graph and reflect on the advantages or disadvantages of their choices. Through the work of the lesson, students notice that they can choose a scale based on the numbers in the data set and that the scale can make a graph easier or more difficult to read (MP6).

Math Community
Tell students they will reflect on their identified norms at the end of this lesson.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Math community poster: Lesson

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

In tomorrow's lesson, students solve one- and two-step "how many more?" and "how many fewer?" problems using data presented in scaled bar graphs. Based on the work you have seen students doing in previous lessons, what strategies do you anticipate each student will use to solve these problems? How will you encourage each student to share their understandings and listen to one another's strategies?

Warm-up

10 mins

Notice and Wonder: Bar Graph Scales

Standards

Building Towards **3.MD.B.3**

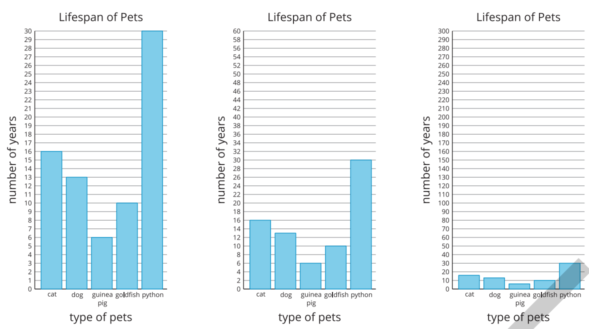
Instructional Routines

- Notice and Wonder

The purpose of this *Warm-up* is to elicit the idea that adjusting the scale changes the size of the bars in a bar graph and can make it easier or more difficult to interpret. While students may notice and wonder many things about these graphs, the different scales in the bar graphs are the most important discussion points.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Students may notice:

- The data is the same.
- The jumps on the scales are different. The highest number on the graphs is different.
- It's harder to tell the number for guinea pigs on the last graph.

Students may wonder:

- Why are the jumps on the scales different?
- Why are the lines the same on each graph?

Launch

- Groups of 2
- Display the graphs.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- "How are the three graphs different?" (They show the same data, but the bars are different heights. They have different scales.)

Activity 1

20 mins

Represent Pattern Blocks

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to analyze a scale and create a scaled bar graph. Students consider a large collection of pattern blocks and decide which scale will work best to represent the categorical data. They consider three students' ideas, choose a scale of 2, 5, or 10, and create a scaled bar graph to represent the categorical data. Students must justify why they agree that a particular scale would be best.

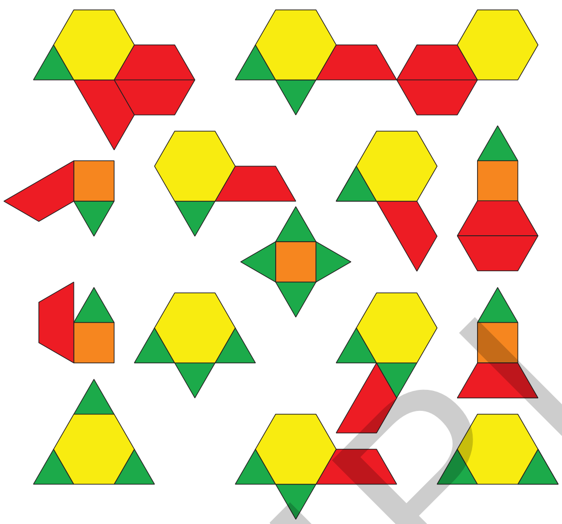
During the activity and whole-class discussion, students share their thinking and have opportunities to listen to and critique the reasoning of their peers (MP3). Providing a variety of scales for students to choose from allows for discussion about the benefits of using larger scales for larger groups of objects and about how the scale affects reading and interpreting data in a graph.

Access for Students with Disabilities

Representation: Access for Perception. Provide access to pattern blocks to model the collection of pattern blocks in the *Task Statement.*
Supports accessibility for: Organization, Visual-Spatial Processing

Student Task Statement

Here is a collection of pattern blocks.



Mai, Noah, and Priya want to make a bar graph to represent the number of triangles, squares, trapezoids, and hexagons in the collection.

- Mai says the scale of the bar graph should be 2.
- Noah says the scale of the bar graph should be 5.
- Priya says the scale of the bar graph should be 10.

1. Who do you agree with? Explain your reasoning.
2. Use the scale that you chose to create a scaled bar graph to represent the collection.

Launch

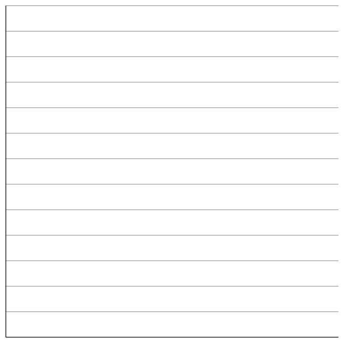
- Groups of 2
- Display the image.
- "Take a minute to consider these pattern blocks and think about how you could represent them in a scaled bar graph."
- 30 seconds: quiet think time

Activity

- "Now answer the question about organizing and representing the pattern blocks in a bar graph with your partner. Be prepared to justify your choice of scale. Then use your chosen scale to make your bar graph."
- 12 minutes: partner work
- Monitor for students who used each of the scales to create their bar graph.

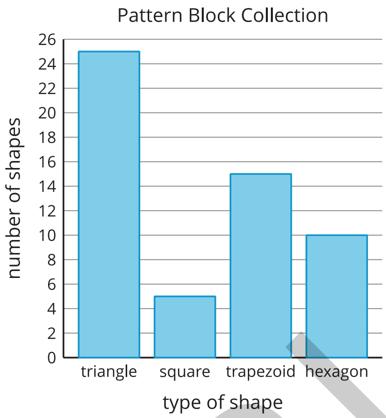
Activity Synthesis

- Display selected student work showing each of the scales.
- "What scale did you use for your bar graph? Why did you choose that scale?" (I used a scale of 5 because each amount can be counted by 5. I used a scale of 10 so I don't have to make as many marks on the scale.)



Student Response

1. Sample response: I agreed with Mai because there are a lot of blocks and counting by 2 is easy. My partner agreed with Priya because it would be really fast to count by 10.
2. Sample response:



Advancing Student Thinking

If students choose a scale of 2, consider asking:

- “How did you choose the scale to use for your graph?”
- “What is the greatest number you need to represent? How can you check to see if it is possible to represent it with the scale you chose?

Activity 2

15 mins

Represent More Data in a Scaled Bar Graph

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to represent data in a scaled bar graph. In this activity, the categorical data is presented in a table. Students choose a scale and make a scaled bar graph of the categorical data. Students have prior experience with scales of 2, 5, and 10, and are not directed to a specific scale in this activity.

However, a scale of 2 cannot be used for this data with the given graph outline because there are only enough rows to label to 26 and the greatest data value is 40. (See the *Advanced Student Thinking* if students try to use a scale of 2.) Due to the larger numbers, it is likely that students will choose a scale of 5 or 10. If students struggle to get started, you could suggest a scale of 5 or 10. In the whole-class discussion, students share how their choice of scale affected their graph.

Students will use their scaled bar graphs again in the next lesson.

Access for English Language Learners

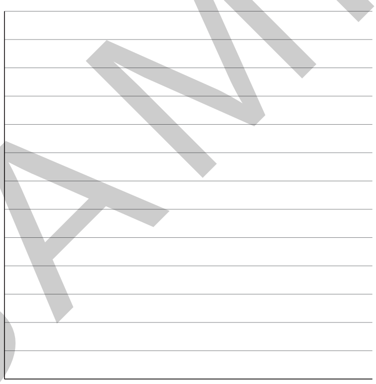
MLR8 Discussion Supports. During small-group discussion, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . . ." Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

Student Task Statement

All the third-grade students at a school were asked, "What is your favorite season?" Their responses are shown in this table.

favorite season of the year	winter	spring	summer	fall
number of students	24	13	40	22

Use the data from the table to create a scaled bar graph.



Launch

- Groups of 4
- "What is your favorite season of the year?"
- 30 seconds: quiet think time
- Share responses.
- "We are going to make a scaled bar graph to represent some third-grade students' favorite seasons of the year."

Activity

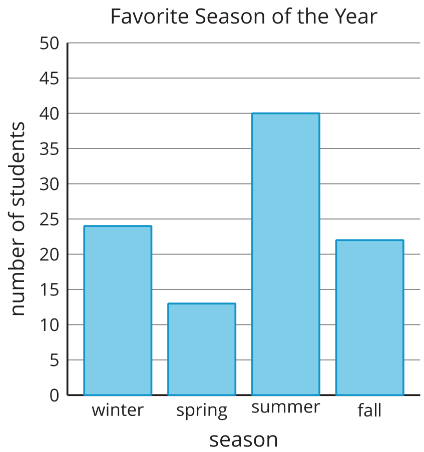
- "Represent the data shown in the table in a scaled bar graph. Think about a scale that makes sense with the numbers of students."
- 5–7 minutes: independent work time
- "Share your graphs with your small-group. Discuss the scales you chose to use."
- 2–3 minutes: small-group discussion

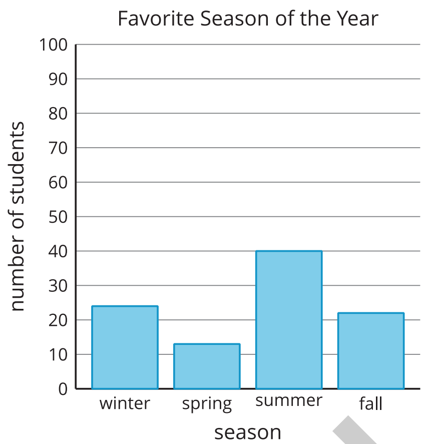
Activity Synthesis

- "How did the scale you chose for your graph affect how your graph looked in the end?" (Certain scales make it easier or more difficult to read the data. For example, with a scale of 10, it might be more difficult to read the exact values from the graph.)

Student Response

Sample responses:





Lesson Synthesis

Display several bar graphs from today's lesson.

"What did you learn today that will help you make decisions about how to create scaled bar graphs in the future?" (You can pick scales that match the data. If there's mostly larger numbers, you might pick a scale like 5 or 10. The scale can help make the graph easier to read.)

Math Community

After the *Cool-down*, ask students to individually reflect on the questions: "Which one of the norms did you feel was most important in your work today? Why?" Students can write their responses on the bottom of their *Cool-down* paper, on a separate sheet of paper, or in a math journal.

Tell students that as their math community works together over the course of the year, they will continually add to and revise their "Doing Math" and "Norms" actions and expectations.

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Cool-down

5 mins

Reflection on Bar Graphs and Scale

Sec A

Standards

Addressing **3.MD.B.3**

Student Task Statement

1. How did you decide on the scale for your graph in the last activity?
2. What was the most important thing you learned today that will help when you make your next scaled bar graph?

Student Response

1. Sample response: I chose a scale of 5 so I would have fewer numbers to write on my scale.
2. Sample response: I learned to think about the numbers in my data to help me choose a scale.

Responding To Student Thinking

Students have responses they'd like to share with a partner.

Next Day Supports
Before the first activity, pair students to discuss their responses.

The work of this lesson builds from the categorical data concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 1, Section B Ways to Represent Data

Answer Questions about Scaled Bar Graphs

Standards

Building On	**2.OA.C.3**
Addressing	**3.MD.B.3**
Building Towards	**3.MD.B.3**

Instructional Routines

- How Many Do You See?

Goals

- Explain (orally) strategies used to solve one-step Compare problems within 100 based on data presented in scaled bar graphs.
- Interpret scaled bar graphs and ask (in writing) questions that can be answered by the data in a scaled bar graph.

Student Facing Learning Goals

Let's solve problems based on data represented in bar graphs.

Lesson Purpose

The purpose of this lesson is for students to solve one-step "how many more?" and "how many fewer?" problems based on data presented in a scaled bar graph.

Narrative

In grade 2, students solved simple Put Together, Take Apart, and Compare problems using data represented in a single-unit scaled bar graph.

In this lesson, students solve one-step Compare problems using data represented in scaled bar graphs.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Materials from a previous lesson: Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins

Teacher Reflection Questions

Think about a time you recently made a mistake during math class. How did you leverage your mistake to show students that mistakes are just learning in process?

Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Warm-up

10 mins

How Many Do You See: Groups of Dots

Standards

Building On **2.OA.C.3**

Building Towards **3.MD.B.3**

Instructional Routines

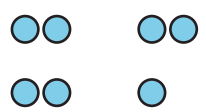
- How Many Do You See?

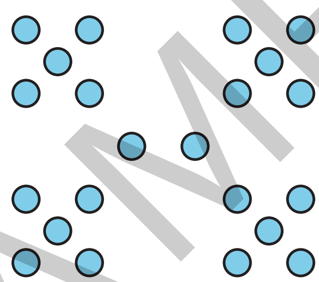
The purpose of this *How Many Do You See?* is for students to subitize or use grouping strategies to describe the images they see.

When students notice that some of the dots are in equal groups and skip-count to find the total number of dots, they are looking for and making use of structure (MP7).

Student Task Statement

How many do you see? How do you see them?





Student Response

Sample responses:

- 7: I see 3 groups of 2 and 1 more.
- 22: I counted by 5 for the 4 groups of 5 to get 20, then added 2 more for the dots in the middle.

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the first image.
- 30 seconds: quiet think time

Activity

- Display the first mage.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Activity Synthesis

- "How did thinking about groups help you find the total number of dots?" (Some of the dots were in groups and some were not. I used skip-counting to count the groups that were the same size, then added on the rest of the dots.)
- Consider asking:
    - "Who can restate the way _____ saw the dots in different words?"

◦ "Did anyone see the dots the same way but would explain it differently?"

Activity 1

15 mins

Questions about Favorite Season

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to use data presented in scaled bar graphs to solve one-step "how many more?" and "how many fewer?" problems. In a previous lesson, students created scaled bar graphs to show data about favorite seasons. Answering questions about a familiar graph prepares students to answer questions about a new graph in the next activity.

This activity provides an opportunity for formative assessment of students' addition and subtraction methods. In grade 2, students were expected to fluently add and subtract within 100.

Required Materials

Materials To Gather

- Materials from a previous lesson: Activity 1

Required Preparation

Students will need their Favorite Season of the Year graphs from the previous lesson.

Student Task Statement

Use your Favorite Season bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many students are represented in the graph?
2. How many students chose spring or fall as their favorite season?
3. How many more students chose summer than winter?
4. How many fewer students chose spring than fall?

Student Response

1. 99 students. Sample response: $24 + 13 = 37$

Launch

- Groups of 2
- Make sure that students have their Favorite Season of the Year graphs from a previous lesson.
- "Take a minute to look over the questions you'll answer using the Favorite Season graph from a previous lesson."
- 1 minute: quiet think time

Activity

- "Work with your partner to answer the questions."
- 7–10 minutes: partner work time
- Monitor for different strategies students use to add or subtract, particularly strategies that use tens and ones.

$37 + 40 + 77$
$77 + 22 = 99$

2. 35 students. Sample response:
   $13 + 22 = ?$
   $13 + 2 = 15$
   $15 + 20 = 35$
3. 16 more students. Sample response:
   $40 - 24$
   $40 - 20 = 20$
   $20 - 4 = 16$
4. 9 fewer students. Sample response:
   $13 + ? = 22$
   $13 + 7 = 20$
   $20 + 2 = 22$
   $7 + 2 = 9$

Activity Synthesis

- Have students share responses to the last 2 questions.
- "How are these questions the same? How are they different?" (They both ask about the difference between 2 categories. The first question uses "more", and the second question uses "fewer.")
- As students share, use this as an opportunity to highlight addition and subtraction strategies in which students use tens and ones.

Advancing Student Thinking

If students find the sum of the quantities in the Compare problems or do not find a value, consider asking:

- "What is this problem about?"
- "How would you describe two categories in the graph using the phrase, 'more than' or 'fewer than'?"

Activity 2

20 mins

Questions about Insects in the Garden

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to use data presented in scaled bar graphs to solve one-step "how many more?" and "how many fewer?" problems. The graph in the previous activity was familiar to students since they had created it in a previous lesson, but the graph used in this activity is new to students. Because the graph has a scale of 10, students need to estimate values that do not show an exact multiple of 10. As a result, answers may vary slightly. Accept all answers that align to reasonable estimates.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: Involve both students in sharing their response with the whole class. While one student speaks, invite the other student to follow along and point to the corresponding parts of the bar graph on the display.
Advances: Speaking, Representing, Listening

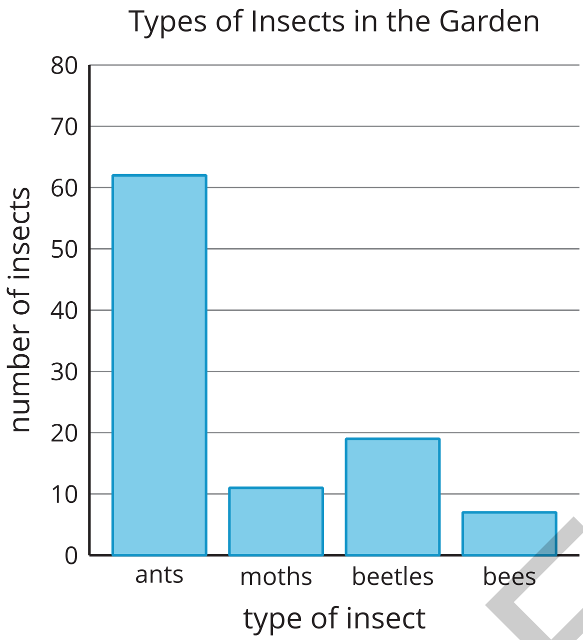
Access for Students with Disabilities

Representation: Internalize Comprehension. Begin by asking, "Does this situation remind anyone of something we

have seen, read, or done before?"
Supports accessibility for: Social-Emotional Functioning

Student Task Statement

Data was collected to see how many of the 4 types of insect were in a garden. The data is shown in this bar graph:



Use the bar graph to answer the questions. Show your thinking using expressions or equations.

1. How many insects were in the garden?
2. How many more ants were in the garden than bees?
3. How many fewer moths were there than ants?
4. Work with your partner to write 2 other questions that can be answered by reading the graph.
5. Trade with another group and answer each other's questions.

Launch

- Groups of 2
- "Look at the scaled bar graph and tell your partner one thing you notice."
- 1 minute: partner discussion

Activity

- "Work with your partner to use the data in the bar graph to complete the first four problems."
- 7–10 minutes: partner work time
- Monitor for different strategies students use to add or subtract, particularly strategies that use tens and ones.
- "Now, trade the questions you wrote for the fourth question with another group and answer their questions."
- 2–3 minutes: partner work time

Activity Synthesis

- Invite students to share responses for the first three problems. Be sure to share a variety of reasonable estimates for the values in each category to make sure students know it's okay if they don't know the exact value for sure.
- Ask 2–3 groups to share a question they wrote and ask the whole class to answer it. As students share, use this as an opportunity to highlight addition and subtraction strategies in which students use tens and ones.

Student Response

1. Sample response: 99 insects
   $62 + 11 + 19 + 7 = ?$
   $11 + 19 = 30$
   $30 + 62 = 92$

$92 + 7 = 99$

2. Sample response: 55 more. $62 - 7$ is $62 - 2 - 5$, which is 55.
3. Sample response: 51 fewer. Sample response: $62 - 11 = 51$
4. Sample response: How many of the insects were bees or beetles?
5. Sample response: 26 of the insects were bees or beetles. $7 + 19 = 6 + 1 + 19$, which is 26.

Advancing Student Thinking

If students find sums for Compare problems or do not find a solution, consider asking:

- “How did you start solving this problem?”
- “How could you use the information from the graph to solve the problem?”

Lesson Synthesis

Display the bar graph from the last activity in today's lesson.

"What were some strategies that were helpful today as you answered questions about the data represented in bar graphs?” (I used the bar graph to get the numbers before I added or subtracted. I used the bar graph to find the answers by looking at one of the bars and counting up to the other bar in the problem.)

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Cool-down

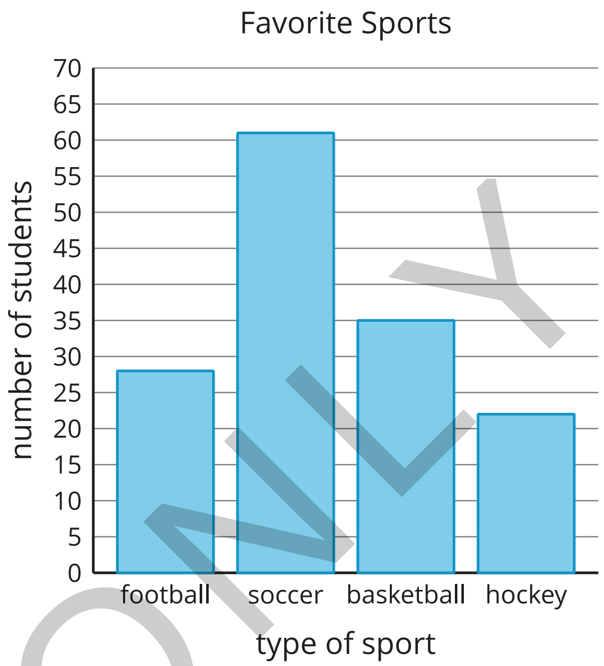
5 mins

Favorite Sports

Student Task Statement

A group of students were asked, "Which is your favorite sport?"

Their responses are shown in this bar graph:



Use the graph to answer the questions.

1. How many more students chose soccer than football? Show your thinking using expressions or equations.
2. How many fewer students chose hockey than basketball? Show your thinking using expressions or equations.

Student Response

Answers may vary by 1 or 2 because students may read some of the data slightly differently for bars that represent numbers that are not multiples of 5.

1. 33 students. Sample response: $28 + 2 = 30$, $30 + 31 = 61$, and $2 + 31 = 33$
2. 13 students. Sample response: $35 - 22 = 13$

Responding To Student Thinking

Students show they understand which categories to compare, but they use numbers that are not reasonable based on the scale of the graph.

Next Day Supports
Use the *Launch* of the next day's activity to brainstorm tips for reading a scaled bar graph.

The work of this lesson builds from the addition and subtraction concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 2, Section C Represent and Solve Story Problems

Unit 1, Lesson 8

More Questions about Scaled Bar Graphs

Sec A

Standards

Building On	**2.OA.C.4**
Addressing	**3.MD.B.3**
Building Towards	**3.MD.B.3**

Instructional Routines

- MLR6 Three Reads
- Number Talk

Goals

- Explain (orally) strategies for solving one- and two-step Compare problems.
- Interpret (orally) a two-step Compare problem based on data presented in scaled bar graphs.

Student Facing Learning Goals

Let's solve problems using data shown on bar graphs.

Lesson Purpose

The purpose of this lesson is for students to solve one- and two-step "how many more?" and "how many fewer?" problems based on data presented in a scaled bar graph.

Narrative

In this lesson, students continue to interpret graphs that represent quantities that are not exact multiples of the scale and may require students to estimate values. As a result, answers may vary slightly. Accept all answers that align to reasonable estimates.

This lesson introduces the *Math Language Routine 6: Three Reads* (MLR 6) to support students in making sense of situations and solving problems

This lesson has a *Student Section Summary*.

Access For Students with Disabilities

- Engagement

Access For English Learners

- MLR8

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

Who did math today in class and how do you know? Identify the norms or routines that allowed those students to engage in mathematics. How can you adjust these norms and routines so all students do math tomorrow?

Warm-up

10 mins

Number Talk: Repeated Addition

Standards

Building On **2.OA.C.4**

Building Towards **3.MD.B.3**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for adding groups of 2 and groups of 5. These understandings help students develop fluency and will be helpful later in this lesson when students need to use data in scaled bar graphs to solve one- and two-step “how many more?” and “how many fewer?” problems. Students use the structure of the expressions and repeated reasoning when they use methods based on skip-counting by 2 or 5 or counting on 2 or 5 from a previous known value (MP7, MP8).

Student Task Statement

Find the value of each expression mentally.

- $2 + 2 + 2 + 2 + 2$
- $2 + 2 + 2 + 2 + 2 + 2$
- $5 + 5 + 5 + 5 + 5 + 5$
- $5 + 5 + 5 + 5 + 5 + 5 + 5$

Student Response

- 10: I skip-counted by 2 five times: 2, 4, 6, 8, 10.
- 12: It is 1 more 2 than the first problem, $10 + 2 = 10$.
- 30: I counted by 5.
- 35: It is 1 more 5 than the problem before this.

Launch

- Display the first expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- “What is a strategy that you hadn’t used before that you might use in the future?” (Think about 1 more group if I already know part of the problem or skip-count.)
- Consider asking:
    - “Who can restate _____’s reasoning in a different way?”
    - “Did anyone use the same strategy but would explain it differently?”
    - “Did anyone solve the problem in a different way?”
    - “Do you agree or disagree? Why?”

Activity 1

New School Year

20 mins

Standards

Addressing **3.MD.B.3**

Instructional Routines

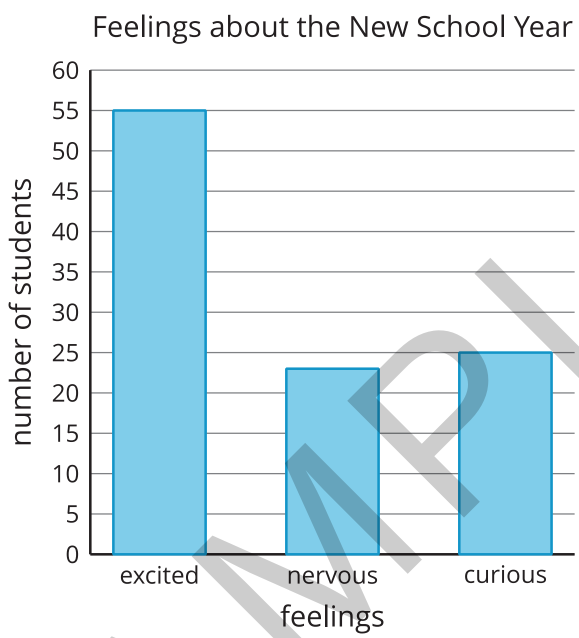
- MLR6 Three Reads

The purpose of this activity is to introduce the *Three Reads* math language routine and to solve a two-step "how many fewer?" problem using data presented in a scaled bar graph. The *Three Reads* routine prompts students to read a problem three times, each for a different purpose. This process supports students in making sense of the problem (MP1).

Because the graph in this activity uses a scale of 5, students may read the data value for nervous as 22, 23, or 25, even though the exact value is 23. For this reason, students' answers may vary by 1 or 2.

Student Task Statement

A group of students were asked, "Which way do you feel about the new school year?" Their responses are shown in this bar graph:



How many more students are excited about the new school year than are nervous or curious?

Student Response

Sample response: 7 students

Launch

- Groups of 2

MLR6 Three Reads

- "Keep your books closed."
- Display only the graph, without revealing the question.
- "We are going to read this graph 3 times."
- 1st read: "Take a moment to read the data displayed by this graph"
- "What is this graph about?" (It's about how students are feeling about the new year.)
- 1 minute: partner discussion
- Listen for and clarify any questions about the context.
- 2nd read: "Read and interpret the graph a second time. What quantities are represented? What can be counted or measured in this situation?" (The numbers of students who chose each feeling are represented. We can count the number of students in each category.)
- 30 seconds: quiet think time
- 2 minutes: partner discussion
- Record quantities on a display for all to see.
- Reveal the question.
- 3rd read: Read the question aloud.
- "What are some strategies we can use to solve this problem?" (We can use the bar graph to find the numbers. We can add to find how many students are

nervous or curious. We can subtract how many students are nervous or curious from the number of students that are excited.)

- 30 seconds: quiet think time
- 1–2 minutes: partner discussion

Activity

- 5–7 minutes: partner work time
- Monitor for students who:
  - Add $25 + 23$ and then subtract $55 - 48$.
  - Subtract $55 - 23$ and then subtract $32 - 25$.
  - Subtract $55 - 25$ and then subtract $30 - 23$.

Activity Synthesis

- Have 1–2 selected students share their solution and strategy.
- Display the steps of *Three Reads* routine and keep displayed for the next activity.
- "How did reading the problem three times help you make sense of the problem?" (I was more comfortable with the problem each time I read it. It made more sense every time I read the problem. I noticed different details each time I read it.)

Activity 2

15 mins

Use Bar Graphs to Solve Problems

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to practice the *Three Reads* math language routine on their own and use data presented in a scaled bar graph to solve a two-step "how many more?" problem. Because the graph has a scale of 10, students need to estimate values that do not show an exact multiple of 10. As a result, answers may vary slightly. Accept all answers that align to reasonable estimates.

The *Three Reads* routine has students read a problem three times for different purposes. This process helps students make sense of the problem and persevere in solving it (MP1).

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

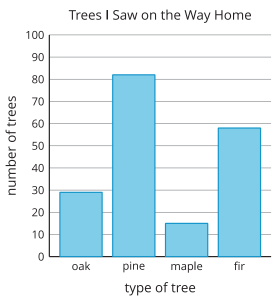
Advances: Listening, Speaking

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to decide which problem to start with or decide the order to complete the task.
Supports accessibility for: Social-Emotional Functioning

Student Task Statement

The bar graph shows how many of the 4 types of trees Clare saw on the way home. Use the graph to answer the questions. Show your thinking using expressions or equations.



1. How many more pine trees did Clare see than fir trees?
2. How many more pine trees did Clare see than oak or maple trees?
3. How many fewer oak trees did Clare see than pine trees?
4. How many fewer maple or oak trees did Clare see than fir trees?

Student Response

1. Sample response: 24 more
   - $82 - 20 = 62$
     $62 - 4 = 58$
   - $58 + 2 = 60$
     $60 + 20 = 80$
     $80 + 2 = 82$
     $2 + 20 + 2 = 24$
2. Sample responses: 38 more
   - $20 + 10 = 30$
     $9 + 5 = 14$
     $30 + 14 = 44$
     $82 - 40 = 42$
     $42 - 4 = 38$
     $29 + 15 = 29 + 1 + 14 = 44$
     $82 - 40 = 42$

Launch

- Groups of 2
- "What types of plants do you see on your way home from school?" (trees, bushes, flowers, vines)
- Share responses.

Activity

- Display the graph.
- "Work independently to use the graph to solve the problems. You can use the *Three Reads* routine if it's helpful to you."
- 7–10 minutes: independent work time
- Monitor for students who solve the one-step problems by:
  - Using the bars on the graph to count up to or back from one number to another.
  - Using equations to show adding or subtracting by place value.
- Monitor for students who solve thetwo-step problems by:
  - Adding two numbers then subtracting the sum from another number.
  - Subtracting a number, then subtracting again from the difference.

Activity Synthesis

- Invite 1–2 students to share the method, including any expressions or equations they used to solve the first problem.
- "How are these methods the same? How are they different?" (Each way found the same number of trees. Some people subtracted the number of pine trees from fir trees. Some counted up from the number of pine trees to the number of fir trees.)
- Invite 2–3 students to share the expressions or

$42 - 4 = 38$

3. Sample response: 53 fewer
   - $82 - 20 = 62$
     $62 - 2 = 60$
     $60 - 7 = 53$
   - $29 + 1 = 30$
     $30 + 50 = 80$
     $80 + 2 = 82$
     $1 + 50 + 2$
4. Sample response: 14 fewer
   - $20 + 10 = 30$
     $9 + 5 = 14$
     $30 + 14 = 44$
     $58 - 44 = 14$
   - $15 + 29 = 15 + 30 - 1 = 45 + 1 = 44$
     $58 - 44 = 38$

equations they used to solve the last problem.

- "How are these methods the same? How are they different?" (Each way found the same number of trees. Each way used subtraction. Some people added the maple and oak trees together and then subtracted it from the number of fir trees. Some people subtracted the number of maple trees from fir trees and then subtracted the number of oak trees.)

Advancing Student Thinking

If students find the sum of all categories or don't find a solution to the problems, consider asking:

- "How did you start solving this problem?"
- "How could you use the information from the graph to solve the problem?"

Lesson Synthesis

Display equations from the first problem in the last activity.

"How did you use what you know about tens and ones to solve the problems?" (Some questions I saw I could just count by 10 on the graph because the scale was 10. We didn't subtract all at once. We subtracted the tens, then the ones. We were thinking about how to get to the next ten to make adding the tens easier.)

"In the future, how could you use the *Three Reads* strategy on your own, without a partner?" (I can first read a problem to figure out what it's about. Then read it again to look for what can be measured or counted. Then read it a third time to think about strategies I could use to solve the problem.)

Suggested Centers

- Sort and Display (1–3), Stage 3: Scaled Graphs (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

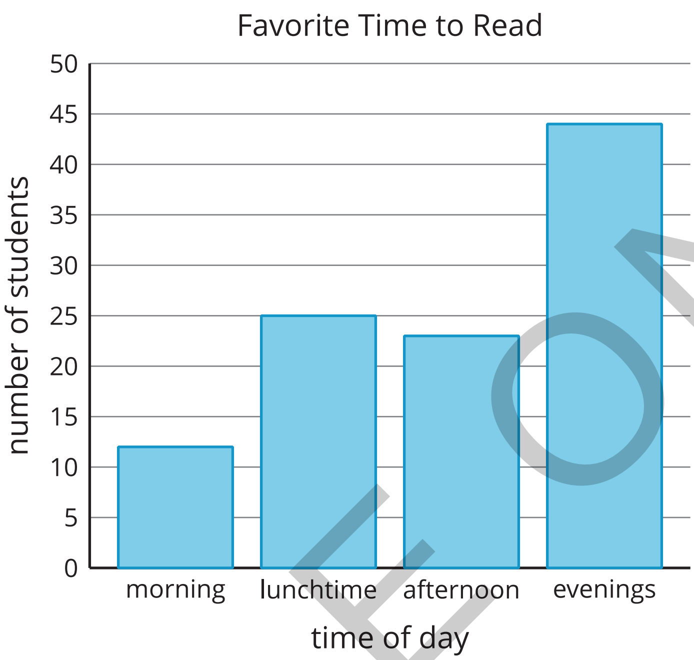
Cool-down

5 mins

Reading Time

Student Task Statement

A group of students were asked, “Which is your favorite time to read?” Their responses are shown in this bar graph:



Use the graph to answer the questions.

1. How many fewer students chose morning than afternoon? Show your thinking using expressions or equations.
2. How many more students chose evening than morning or lunchtime? Show your thinking using expressions or equations.

Student Response

1. Sample response: 12 fewer students. $23 - 11 = 12$
2. Sample response: 7 more students

- $12 + 25 = 37$
- $44 - 37 = ?$
- $44 - 30 = 14$
- $14 - 7 = 7$

Responding To Student Thinking

Students find sums rather than the differences.

Next Day Supports

In the two-step Compare problem, students only find the difference between evening and morning (or evening and lunchtime).

Before the *Warm-up*, pass back the *Cool-down* and have students work in small groups to make corrections.

The work of this lesson builds from the addition and subtraction concepts developed in a prior unit.

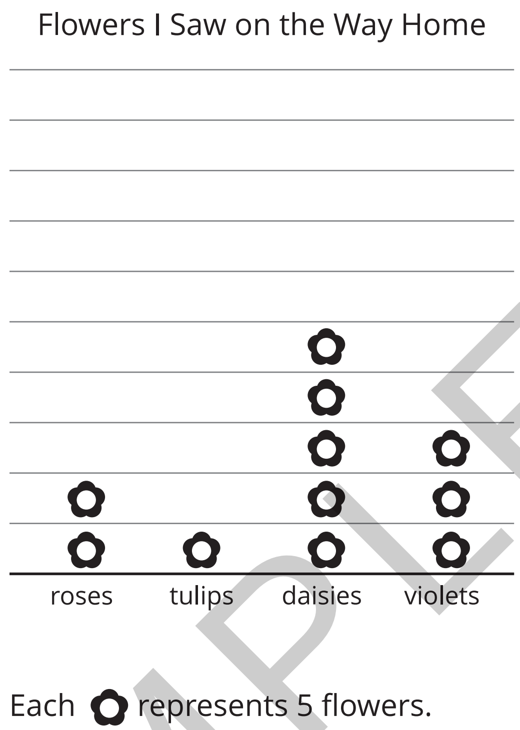
Prior Unit Support
Grade 2, Unit 2, Section C Represent and Solve Story Problems

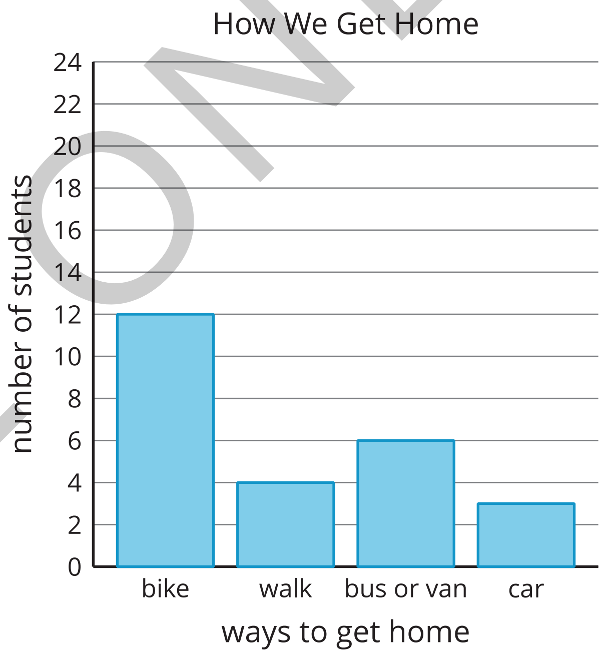
Section A Summary

We created **scaled picture graphs** and **scaled bar graphs**.

The **key** tells what each picture represents in a picture graph.

The **scale** tells what number each bar represents in a bar graph.





We asked and answered questions about data represented in the graphs.

- How many more daisies were seen than violets?
- How many fewer students walk home than bike home?
- How many more students bike home than walk or ride in a car?

Section B: From Graphs to Multiplication

Standards

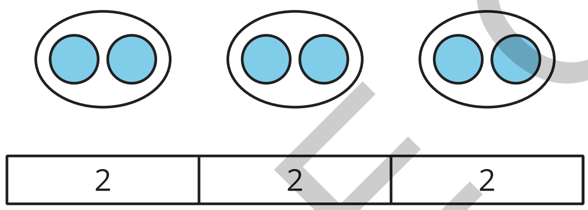
Building On	**2.NBT.B.5**
Addressing	**3.OA.A, 3.OA.A.1, 3.OA.A.3, 3.OA.A.4, 3.OA.C.7, 3.OA.D.9**
Building Towards	**3.OA.A.1**

Goals

- Represent and solve multiplication problems involving equal groups.
- Understand multiplication in terms of equal groups.

Narrative

In this section, students make sense of multiplication in terms of equal groups of objects. They use discrete drawings and tape diagrams that show equal groups to represent multiplication. Students then relate these representations to expressions, such as 3×2, interpreting them to mean “3 groups of 2.”



Note that expressions of the form $a \times b$ could be interpreted to mean a groups of b or b groups of a. Because we tend to say “___ groups of ___” when referring to equal groups, however, in these materials we write multiplication expressions in that order:

$$\text{number of groups} \times \text{size of each group}$$

It is not necessary for students to use this convention as long as they can explain what each number in their expression represents.

At the end of the section, students write equations to represent multiplication situations and find unknown products or factors. In reasoning about the latter, students begin to make sense of the relationship between multiplication and division, without formally using the language of division..

Suggested Centers

Lesson 9
- Capture Squares (1–5) , Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Lesson 10
- Capture Squares (1–5) , Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Lesson 11

- Capture Squares (1–5) , Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Lesson 12

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Lesson 13

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Lesson 14

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 8: Add within 1,000, with Composing (Supporting)

Lesson 15

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 8: Add within 1,000, with Composing (Supporting)

Section B Checkpoint

1

Goals Assessed

- Understand multiplication in terms of equal groups.

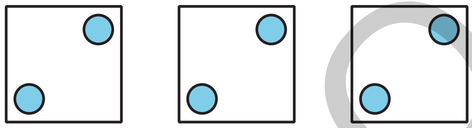
Student Task Statement

Jada has 2 bowls. Each bowl has 3 apples. Select **all** representations of Jada's apples.

A.

3	3

B.



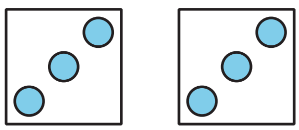
C.

6	6	6

D. 2×3

E. $2 + 3$

F.



Solution

A, D, F

Responding To Student Thinking

Students show they may understand that the situation involves equal groups, but they choose representations (for example, B or C) that do not match the number or size of the groups in the situation.

As students match equal-groups drawings and arrays in the next section, listen for the ways they describe how they see equal groups in each representation. Emphasize the language students use to describe the number of groups and the size of each group.

Students match diagrams to the situation, but they do not yet match a multiplication expression.

Throughout the next section, invite selected students to make connections between drawings of equal groups, arrays, expressions, and situations. Emphasize the language students use to describe the number of groups and the size of each group.

2

Goals Assessed

- Represent and solve multiplication problems involving equal groups.

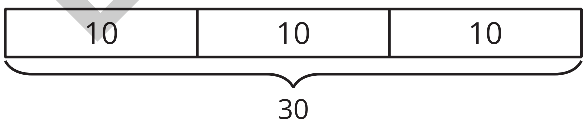
Student Task Statement

a. There are 3 bunches of grapes. Each bunch has 10 grapes. How many grapes are there? Explain or show your reasoning.

b. There are 30 people in some cars. Each car has 5 people in it. How many cars are there? Explain or show your reasoning.

Solution

a. 30 grapes. Sample responses:
   - $3 \times 10 = 30$
   -
   - $10 \times 3 = 30$

   -

b. 6 cars. Sample response: I counted 5, 10, 15, 20, 25, 30 and that's 6 times to get to 30. So, there are 6 cars.

Responding To Student Thinking

Students show they may not yet reason about equal groups to solve problems. For example, they may add the given numbers.

Students show they can solve an Unknown Product problem, but they do not yet solve a Number of Groups Unknown problem.

During Center Time in the next section, invite selected students to revisit the problems using the *Three Reads* routine. Ask students to represent the situation in a way that makes sense to them. Emphasize the way students describe whether they need to find the number of groups, how many are in each group, or the total number. As needed, ask students to compare the structure of the two problems.

Practice Problems

10 Problems

1

from Unit 1, Lesson 9

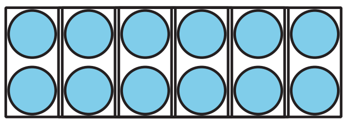
Student Task Statement

There are 6 tennis courts. There are 2 players on each tennis court.

Create a drawing or diagram to represent the tennis players. Then find how many players are on the tennis courts. Explain or show your reasoning.

Solution

12 players. Sample response:



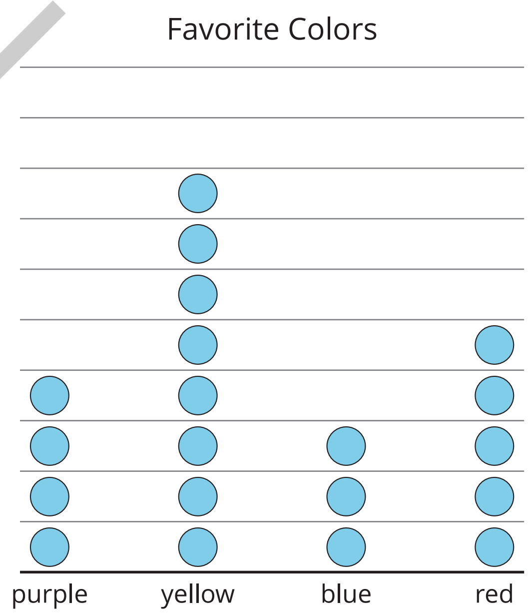
$2 + 2 + 2 + 2 + 2 + 2 = 12$

2

from Unit 1, Lesson 10

Student Task Statement

The picture graph shows the favorite colors of some people.

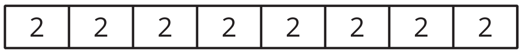


Each ◯ represents 2 people.

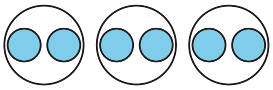
Match each diagram or drawing to the number of people who chose each color.

A.

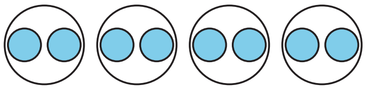
B.



C.



D.



1. red
2. blue
3. purple
4. yellow

Solution

- A matches 1
- B matches 4
- C matches 2
- D matches 3

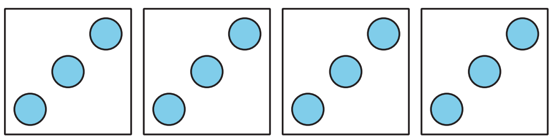
3 from Unit 1, Lesson 11

Student Task Statement

Create a drawing or diagram to represent the expression 4×3.

Solution

Sample response:



4 from Unit 1, Lesson 12

Student Task Statement

There are 4 stacks of books on the table. Each stack has 5 books. How many books are on the table? Explain or show your reasoning.

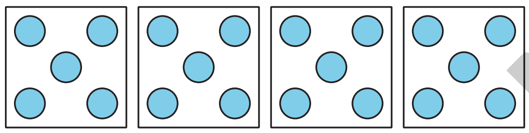
Solution

There are 20 books. Sample responses:

- $5 + 5 + 5 + 5$
-

5	5	5	5

-



5 from Unit 1, Lesson 13

Student Task Statement

There are 6 basketball teams in the gym. There are 5 people on each team. How many people are on the basketball teams in the gym?

a. Write a multiplication equation with a symbol for the unknown to represent the situation.

b. Find the number that makes the equation true. Show your reasoning.

Solution

a. $6 \times 5 = ?$ or $5 \times 6 = ?$

b. 30. Sample response: I made a diagram and added the numbers.

5	5	5	5	5	5

6 from Unit 1, Lesson 14

Student Task Statement

Write a multiplication equation for the situation. Use ? for the unknown. Find the number that makes the equation true.

There are 4 soccer teams. Each soccer team has 10 players. How many players are there altogether?

Solution

$4 \times 10 = ?$ or $10 \times 4 = ?$ 40 players. Sample response: I counted by 10 like 10, 20, 30, 40.

7 from Unit 1, Lesson 15

Student Task Statement

Solve each problem. Explain or show your reasoning.

a. There are 7 flowers. Each flower has 5 petals. How many petals are there?

b. There are 50 petals on some flowers. Each flower has 5 petals. How many flowers are there?

Solution

a. 35 petals. Sample response:

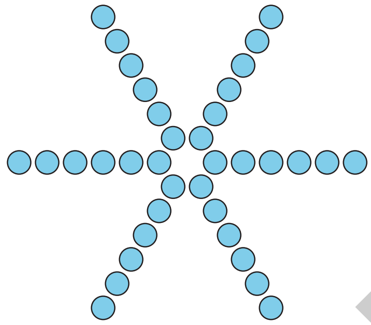
5	5	5	5	5	5	5

b. 10 flowers. Sample response: I drew groups of 5 until I reached 50. There were 10 groups, so there are 10 flowers.

8 Exploration

Student Task Statement

Write an expression for the number of circles in the image. Then find the number of circles.



Sec B

Solution

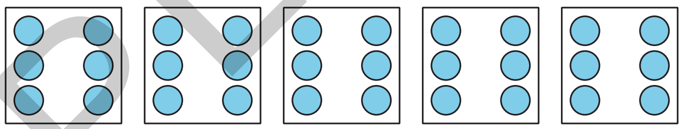
36 circles. Sample response: 6×6. There are 6 groups of circles, coming out from the center, and there are 6 circles in each group so there are 6×6 circles. That’s 36 circles total.

9 Exploration

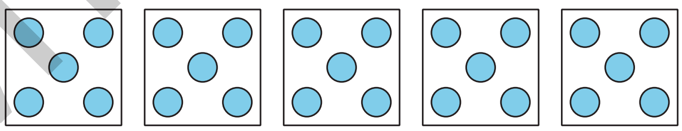
Student Task Statement

For each image, determine if there is an even or odd number of circles. Explain or show your reasoning.

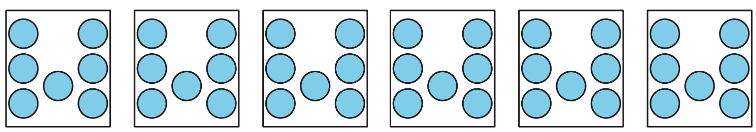
a.



b.



c.



Solution

a. There are an even number of circles. Sample response: Each rectangle has 3 pairs of circles so all of the circles come in pairs.

b. There are an odd number of circles. Sample response: I can pair off the first 2 groups of 5 and the second two groups of 5. Then there are 2 more pairs of circles with 1 left over.

c. There are an even number of circles. Sample response: There are 3 pairs of rectangles and the circles inside of them can be grouped into pairs.

10 Exploration

Student Task Statement

Look outdoors or in your school or home to find some equal groups of objects.

a. Describe the objects.

b. Create a drawing to represent the objects.

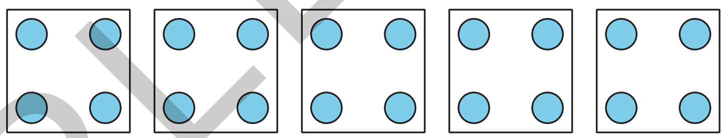
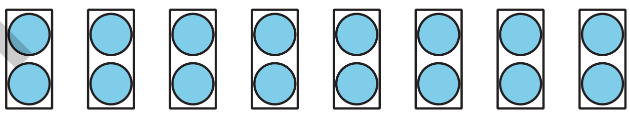
c. Write an equation showing how many objects there are.

Solution

a. Sample responses:

- There are 5 tables in the classroom. There are 4 chairs at each table.
- There are 8 pairs of socks in my dresser.

b. Sample responses:

- 
- 

c. Sample responses:

- $5 \times 4 = 20$ or $4 \times 5 = 20$
- $8 \times 2 = 16$ or $2 \times 8 = 16$

Multiplication for Equal Groups

Standards

Building On	**2.NBT.B.5**
Addressing	**3.OA.A, 3.OA.A.1**
Building Towards	**3.OA.A.1**

Instructional Routines

- Number Talk

Goals

- Describe (orally) how a drawing or diagram represents equal groups.
- Represent a situation involving equal groups with drawings or diagrams.

Student Facing Learning Goals

Let's work with equal groups of things.

Lesson Purpose

The purpose of this lesson is for students to use scaled picture graphs as an introduction to **multiplication** as a way to express equal groups.

Narrative

Scaled picture graphs provide an equal grouping context that naturally elicits multiplication. Multiplication expressions aren't introduced in this lesson so that students spend more time with concrete representations of multiplication before being introduced to the more abstract representation. The next few lessons focus on the meaning and representations of multiplication, not the product. While students may want to go right to finding the product, it is important to focus on the meaning of multiplication as equal groups and the ways in which it can be represented in the discussions.

Throughout this section, make connecting cubes or counters available to students who need them.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins

Teacher Reflection Questions

How did students' work with scaled picture graphs and bar graphs set up the introduction of multiplication in today's lesson?

Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Warm-up

10 mins

Number Talk: More Addition

Standards

Building On **2.NBT.B.5**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for addition within 100. It also provides an opportunity to observe student strategies as they work toward becoming fluent in addition within 1,000.

When students use strategies based on place value to add, they look for and make use of structure (MP7).

Student Task Statement

Find the value of each expression mentally.

- $40 + 35$
- $45 + 35$
- $45 + 36$
- $34 + 58$

Student Response

- 75: There are 7 tens and 5 ones, which is 75.
- 80: There are 7 tens and 5 ones plus 5 makes another ten, which would be 80.
- 81: It's like the one right before it, but there's 1 more, which would make 81.
- 92: There are 8 tens and 12 ones. If I break apart the 12 into 1 ten and 2 ones, then I have 9 tens and 2 ones, which is 92.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "How did you compose new tens as you solved these problems?" (In the second problem I composed a ten from the 2 fives. In the third problem I composed a new ten from the 5 ones and the 6 ones and still had 1 one leftover.)
- Consider asking:
  - "Who can restate ______ 's reasoning in a different way?"
  - "Did anyone use the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"

◦ “Does anyone want to add on to ___'s strategy?” 15 mins

Activity 1

From Scaled Graphs to Equal Groups

Standards

Addressing 3.OA.A

Building Towards 3.OA.A.1

The purpose of this activity is for students to connect scaled picture graphs to situations involving equal groups. The scale of the picture graph will be used to help students think about a category of the graph as a situation involving equal groups.

The *Launch* is an opportunity for students to share their experiences and ask questions about the graph to ensure each student has access to the context. If it is helpful, display a few images of different types of signs students may see in their community.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were important or most useful to solve the problem. Display the sentence frame: “The next time I read a scaled picture graph, I will pay attention to . . . .“
Supports accessibility for: Visual-Spatial Processing

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 1

Required Preparation

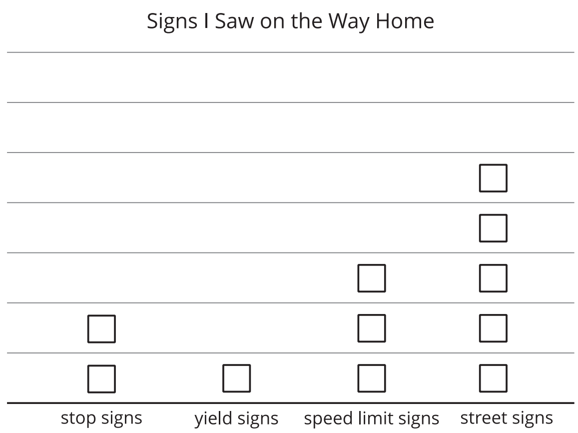
Each student needs 20 connecting cubes or counters.

Student Task Statement

Elena collected data about signs she saw on the way home. The data is shown in this picture graph:

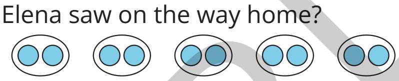
Launch

- Groups of 2
- Give students access to connecting cubes or counters.
- “We're going to look at a scaled picture graph about signs that Elena saw on the way home. What types of signs do you see in the community?” (stop signs, speed limit signs, street signs, billboards)
- 30 seconds: quiet think time
- 1 minute: partner discussion
- Share responses.
- Display the graph.



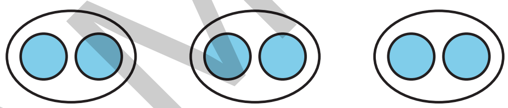
Each ☐ represents 2 signs.

1. Represent the number of speed limit signs Elena saw on the way home.
2. Which statement describes how the graph represents the number of speed limit signs Elena saw? Explain your reasoning.
   A. There are 3 pictures, and each picture represents 1 speed limit sign.
   B. There are 3 pictures, and each picture represents 2 speed limit signs.
   C. There are 2 pictures, and each picture represents 2 speed limit signs.
3. How could this drawing represent the street signs Elena saw on the way home?



Student Response

1. Sample response:



2. B. Sample response: B describes 6 signs and that's how many speed limit signs Elena saw on the way home. Also, there were 3 pictures in the graph and each picture represents 2 signs.
3. Sample responses: There are 5 pictures in the graph, and there are 5 groups in the drawing. Each picture represents 2 signs, and there are 2 dots in each group. The graph shows 10 street signs, and the drawing shows 10 dots.

Activity

- "Work independently to represent the number of speed limit signs that Elena saw on the way home."
- 1 minute: independent work time.
- Monitor for students who create drawings of equal groups similar to the one shown in the last problem to display during the *Activity Synthesis.*
- 1 minute: partner discussion
- "Work with your partner to complete the next problem."
- 1 minute: quiet think time
- 2 minutes: partner discussion
- "How did you know which statement described the speed limit signs that Elena saw on the way home?" (There were 3 pictures on the graph. Each picture represents 2 signs.)
- Share responses.
- "Take a few minutes to complete the last problem on your own."
- 3 minutes: independent work time
- "Share your responses with your partner."
- 1 minute: partner discussion

Activity Synthesis

- Display a student-created drawing of equal groups that represents the speed limit signs that Elena saw on the way home and the drawing in the last problem.
- "These drawings show equal groups. How do these drawings represent data in the picture graph?" (The drawings show groups of 2, and the picture graph shows the data in groups of 2 because each picture represents 2 signs.)

Activity 2

20 mins

Situations with Equal Groups

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to represent situations involving equal groups in a way that makes sense to them. Have connecting cubes available for students to use to represent the situation, if they would like. Students may also draw a picture. One partner could use the objects while one draws, and then they could switch for each problem. The focus of the discussion is on the important quantities of each situation and how students used their representation to model each quantity (MP4).

In the *Launch*, it may be helpful to ask students to tell their partner a quick story or ask any questions about the focus of each of the three contexts to ensure each student has access. It may also be helpful to display images for students to reference.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: Involve both partners in sharing their response with the whole class. While one student speaks, invite the other student to follow along and point to where the numbers are in their representations.
Advances: Representing, Listening

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 2

Required Preparation

Each student needs 20 connecting cubes or counters.

Student Task Statement

Represent each situation.

1. There are 4 people wearing shoes. Each person is wearing 2 shoes.
2. There are 2 boxes of markers. Each box has 10 markers.
3. There are 3 basketball teams. Each team has 5 players.

Student Response

Sample responses:

1. ◦ 4 groups of 2 connecting cubes

Launch

- Groups of 2
- Give students access to connecting cubes or counters.
- "What are some places you see groups of 2 in the community? Groups of 5? Groups of 10?" (Shoes, socks, and wings come in groups of 2. Hands have 5 fingers. Flowers can have 5 petals. Markers come in packs of 10. Ten people on a bus.)
- 30 seconds: quiet think time
- 1 minute: partner discussion
- Share and record responses.
- Choose a student-generated example with small

- A drawing of 4 groups with 2 dots in each group

2.
   - 2 groups of 10 counters
   - A drawing of 2 groups with 10 dots in each group
3.
   - 3 groups of 5 connecting cubes
   - A drawing of 3 groups with 5 tallies in each group

numbers or display this situation: "There are 3 flowers. Each flower has 5 petals."

- "How could you represent this situation?"
- 30 seconds: quiet think time
- 1–2 minutes: partner work time
- Share and record responses. Focus on how the representation connects to the problem.
- Consider asking:
  - "How did you represent the 3 flowers?"
  - "How did you represent the 5 petals on each flower?"
  - "Did someone represent this differently?"

Activity

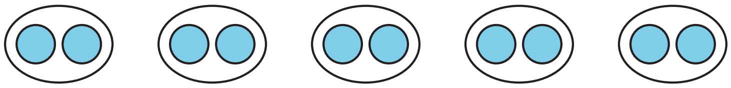
- "Now you are going to represent some more situations involving equal groups with your partner."
- 5–7 minutes: partner work time
- If some students finish earlier than others, encourage them to write their own situation and trade with their partner.

Activity Synthesis

- Ask 2–3 students to share their work for each problem. Be sure to share a variety of different representations.
- For each, ask how the numbers in the situation are represented in their work.
- "How do the representations help you picture the situation?" (I can pretend the objects are the things in the story like the shoes. The drawing is like a picture of what's happening in the story.)

Lesson Synthesis

Display a representation of equal groups from the lesson.



"The situations we looked at today were all **multiplication.** Multiplication is how we represent the total number of objects when we have a certain number of equal groups. For example, in this picture, we would say we have 5 groups of 2."

“Describe a situation with equal groups that you could represent as multiplication.” (Packs of pencils, bins or baskets with the same number of things in each one, pairs of shoes, rows of seats on the bus.)

Suggested Centers

- Capture Squares (1–5) , Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–3) , Stage 6: Add within 100, with Composing (Supporting)

Cool-down

5 mins

Represent Equal Groups

Standards

Addressing **3.OA.A**

Student Task Statement

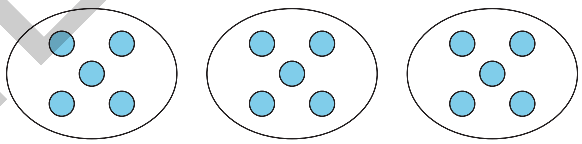
Jada has 3 bags. Each bag has 5 bracelets in it.

Represent the situation.

Student Response

Sample responses:

- Students make 3 groups of 5 counters or 3 groups of 5 connecting cubes.
- Students create a drawing of 3 groups with 5 items in each.



Responding To Student Thinking

Students indicate that they mixed up the number of groups and the number of objects in each group by making 5 groups of 3.

Next Day Supports
Use the next day's *Warm-up* to discuss how to represent a situation involving equal groups, differentiating between the number of groups and the number of objects in each group.

The work of this lesson builds from the equal-group concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 8, Section A Odd and Even

Unit 1, Lesson 10

Situations, Drawings, and Diagrams, Oh My!

Sec B

Standards

Addressing	**3.OA.A, 3.OA.A.1**
Building Towards	**3.OA.A.1**

Instructional Routines

- Card Sort
- Notice and Wonder

Goals

- Comprehend the term “multiplication” (in spoken language) refers to a way to represent the total number of objects when you have a certain number of equal groups.
- Interpret a situation with equal groups and represent it with a drawing or diagram.
- Match (orally and in writing) situations, tape diagrams, and drawings that represent the same equal groups.

Student Facing Learning Goals

Let’s represent equal groups.

Lesson Purpose

The purpose of this lesson is for students to connect situations involving equal groups to tape diagrams.

Narrative

This lesson introduces tape diagrams as a way to represent equal groups and multiplication, building on students' work with scaled picture graphs and discrete drawings of equal groups. Students deepen their understanding of multiplication as they connect tape diagrams to situations that involve equal groups. They are then introduced to multiplication **expressions** as a way to represent the quantities and situations encountered in the lesson. This happens at the end of the lesson, so students work with other representations of multiplication before they learn about abstract symbols that represent multiplication.

Access For Students with Disabilities

- Engagement

Access For English Learners

- MLR8

Required Materials

Materials To Copy

- Card Sort Equal Groups Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

How did connecting different representations of multiplication during the card sort support students in developing their understanding of multiplication?

Warm-up

10 mins

Notice and Wonder: Socks

Standards

Addressing **3.OA.A.1**

Instructional Routines

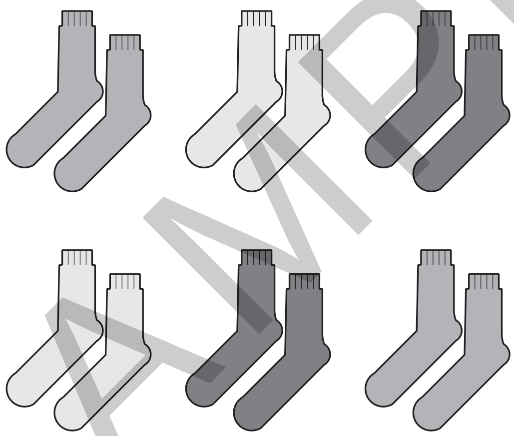
- Notice and Wonder

The purpose of this *Warm-up* is to elicit different strategies for counting objects arranged in groups of 2, which will be useful when students multiply by 2 in a later activity. While students may notice and wonder many things about these images, flexible ways of seeing the groups and strategies for finding the total number of objects are the important discussion points.

When students see the socks are grouped by 2 and use that to find the total, they are looking for and making use of structure (MP7).

Student Task Statement

What do you notice? What do you wonder?



Student Response

Students may notice:

- The socks are in pairs.

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- "How does this problem relate to what we know about multiplication?" (There are equal groups of socks, we can say there are 6 groups of 2.)

- There are 12 socks.
- You could count by 2 to find the total number of socks.

Students may wonder:

- How many socks are there?
- How many pairs of socks are there?
- Why are there 6 pairs of socks?

Activity 1

15 mins

Scaled Picture Graph to Diagram

Standards

Addressing	**3.OA.A**
Building Towards	**3.OA.A.1**

The purpose of this activity is for students to build on the work they have done with scaled picture graphs to use the tape diagram as a new representation of multiplication. The scale of the picture graph will be used to help students think about a category of the graph as a situation involving equal groups.

To add movement to this activity, students could find someone in the class who represented a different category than they did or represented the same category in a different way. When they find a person, they can describe what is the same and what is different about their representations.

Access for English Language Learners

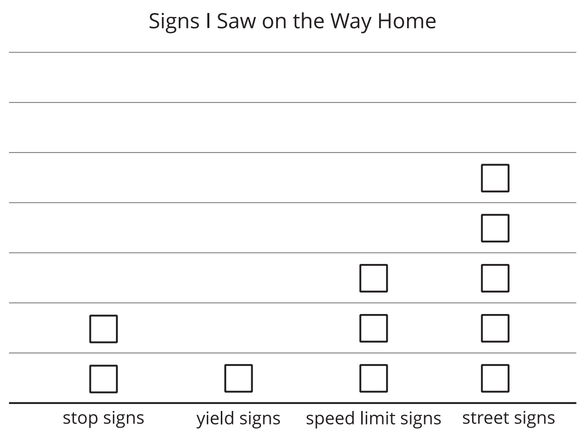
MLR8 Discussion Supports. Synthesis: When students compare the diagram and the scaled picture graph, display sentence frames to support whole-class discussion: "_____ and _____ are the same because . . . ." "_____ and _____ are different because . . . ."
Advances: Speaking, Representing

Student Task Statement

The graph shows the number of signs Elena saw on the way home.

Launch

- Groups of 2
- Display the picture graph and tape diagram.
- "What do you notice? What do you wonder?" (The picture graph key says each square represents 2 signs. The diagram shows 3 groups of 2. I wonder whether they represent the same thing.)
- 1 minute: quiet think time
- 1 minute: partner discussion time
- "How does the diagram show the speed limit signs that Elena saw on the way home?"



Each ☐ represents 2 signs.

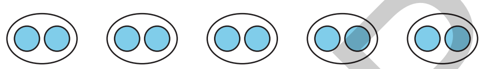
1. How does the diagram represent the speed limit signs that Elena saw?

2	2	2

2. Represent the data from another category in the graph with your own drawing or diagram.

Student Response

1. Sample response: There are 3 pictures (squares) in the graph and 3 sections of the diagram. The graph tells us that each picture (square) represents 2 signs. In the diagram, each section is labeled with a 2.
2. Sample responses for street signs:



2	2	2	2	2

- Share responses.

Activity

- "Now independently represent the data from another category in the graph with your own drawing or diagram."
- 1–2 minutes: independent work time
- "Share how you represented the data in your drawing or diagram with your partner."
- 2–3 minutes: partner discussion
- Monitor for students who create a tape diagram to represent one of the other categories to use during the *Activity Synthesis*.

Activity Synthesis

- Have students share different ways they represented a category in the graph.
- Display a student created tape diagram or make a quick sketch of one to represent the street signs Elena saw on the way home.
- "Which category does this tape diagram represent? How do you know?"
- "How is the diagram the same as the scaled picture graph?" (Each picture and each part of the tape diagram represents 2 signs.)
- "How is the diagram different than the scaled picture graph?" (In the graph, you have to read the key to know that each picture shows two signs. In the diagram, each part is labeled with a 2.)

Activity 2

Card Sort: Equal Groups

20 mins
PLC Activity

Standards

Addressing	3.OA.A
Building Towards	3.OA.A.1

Instructional Routines

- Card Sort

The purpose of this activity is for students to connect situations involving equal groups to drawings and tape diagrams. This sorting task gives students opportunities to analyze situations and diagrams closely and make connections (MP2, MP7). Students explain why two cards match and have opportunities to critique and question their peers' reasoning (MP3). When explaining, students have opportunities to revise their language to make their explanations more precise and clear (MP6). After sorting and describing their sort, students notice that all of the representations reinforce the meaning of multiplication as a way to express equal groups.

Students will spend all of the next lesson working with multiplication expressions. Keep the equal groups cards for the next lesson.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Attention, Organization

Required Materials

Materials To Copy

- Card Sort Equal Groups Cards (1 copy for every 2 students): Activity 2

Required Preparation

- Create a set of cards from the blackline master for each group of 2.

Student Task Statement

Your teacher will give you a set of cards that show situations, drawings, and diagrams.

1. Find the cards that match. Be ready to explain your reasoning.
2. Create a drawing or diagram for each situation.
    a. There are 4 bags. Each bag has 2 strawberries.
    b. There are 4 hands. Each hand has 5 fingers.

Student Response

1. Matches:
    - A, F, L
    - B, H, K
    - C, I
    - D, G
    - E, J
    Sample response: A and F go together because

Launch

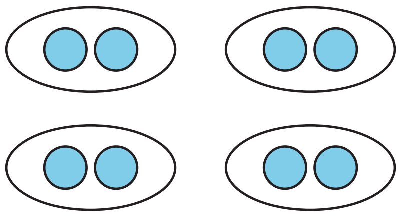
- Groups of 2
- Give each group a set of cards

Activity

- "This set of cards includes situations, drawings, and diagrams. Find the cards that match. Work with your partner to explain your reasoning."
- 5 minutes: partner work time
- Monitor for students who:
    - Connect the ways the features of the drawings or diagrams represent the story.
    - Connect the ways each representation shows the same number of equal groups and the same size of each group.
- Invite 1–2 previously selected students to share the matches they made and how they know those cards go together.
- Listen for the language students use to describe their match. If students only reference the numbers that

the 3 big ovals are like the 3 bags in the situation and the 5 blue circles are like the footballs. L goes with them too because the 3 rectangles are like the bags and the number 5 in each rectangle shows the number of footballs.

2. a. Sample response:



b. Sample response:

5	5	5	5

match, consider asking:

- "What do you mean when you say _____?"
- "How could you use the words 'equal groups' to explain?"

- "Now you're going to create a drawing or diagram to represent two different situations."
- 3–5 minutes: independent work time
- Monitor for the students who draw equal groups and students who draw a tape diagram.

Activity Synthesis

- For each situation in the last 2 problems, display 2–3 student representations: at least one drawing of equal groups and one tape diagram. Leave them displayed.
- "What do all these representations have in common?" (They all show equal groups. They all have groups. Each group has the same amount.)
- "Where are the 4 bags and 2 strawberries in each drawing or diagram?"
- "Where are the 4 hands and 5 fingers in each drawing or diagram?"

Advancing Student Thinking

If students create representations that do not match the number of groups or size of the groups in the situations, consider asking:

- "How did you represent the situation?"
- "How could you show the groups in the situation? How could you show the objects in each group?"

Lesson Synthesis

Display the tape diagram.

5	5	5	5

"Today's lesson was all about multiplication. How can a diagram show multiplication?" (The number of parts in the diagram can show the number of groups, and the same number written in each part shows how many in each group. This diagram has 4 parts, so there are 4 groups, and the 5 written in each part shows there are 5 in each group. The diagram shows 4 groups of 5.)

Display:

4 groups and 5 in each group

$$4 \times 5$$

"You may remember that an **expression** has at least 2 numbers and at least one math operation. A multiplication expression is how we represent the number of groups and the number in each group in a situation. For example, the multiplication expression 4×5 would represent this diagram because we have 4 groups and 5 in each group." Point to the 4 and 5 in the diagram and the expression as you explain.

"The symbol in the middle of the expression is the multiplication symbol. 4×5 can be read as '4 groups of 5.'"

Suggested Centers

- Capture Squares (1–5) , Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Cool-down

5 mins

Boxes of Shirts

Standards

Addressing **3.OA.A**

Student Task Statement

The store has 4 boxes. Each box has 10 shirts in it.

Does this diagram match the situation? Explain your reasoning.

10	10	10	10

Student Response

Yes. Sample response: The 4 parts represent the 4 boxes, and the 10 in each part represents the 10 shirts in each box.

Responding To Student Thinking

Students answer no and explain that the diagram should have 10 parts with 4 in each part.

Next Day Supports
During the *Launch* of the next day's activity, have students discuss a matching situation and diagram, such as cards E and J.

The work of this lesson builds from the equal-groups concepts developed in a prior unit.

Prior Unit Support
Grade 2, Unit 8, Section A Odd and Even

Multiplication Expressions

Standards

Addressing **3.OA.A.1**

Instructional Routines

- Choral Count

Goals

- Comprehend the notation '×' can be used to create an expression to represent a number of equal groups.
- Match (orally) situations, drawings, tape diagrams, and expressions that represent the same equal groups.
- Represent situations involving equal groups with multiplication expressions.

Student Facing Learning Goals

Let's write multiplication expressions.

Lesson Purpose

The purpose of this lesson is for students to use multiplication expressions to represent equal groups.

Narrative

In previous lessons, students represented situations involving equal groups with drawings and tape diagrams. Students were also introduced to representing equal groups as an expression. In this lesson, students connect the structure of drawings, tape diagrams, and multiplication situations to the structure of multiplication expressions (MP7). Students create diagrams and drawings to represent multiplication expressions and ultimately write their own expressions to represent drawings, diagrams, and situations (MP2).

When generating multiplication expressions, consider using the convention of the number of groups as the first factor and the size of the groups as the second factor. However, it is not necessary for students to write the factors in this order. It is important that students connect their expressions to the corresponding situations and representations. They should be able to correctly explain what each factor represents in their expressions. If students ask questions about the idea of commutativity, consider recording the questions publicly for future investigation.

To allow time for students to focus on the meaning of multiplication, it is not an expectation that students find the product of each expression in this lesson. In subsequent lessons, students will work on strategies for finding the product. If students mention the product in today's lesson, it is okay to note that, but try to maintain focus on the connections between the expressions and the diagrams.

Access For Students with Disabilities

- Action and Expression

Access For English Learners

- MLR2

Required Materials

Materials To Gather

- Materials from a previous lesson: Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	10 mins
Activity 2	15 mins
Activity 3	10 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

What did you say, do, or ask during the *Lesson Synthesis* that helped students be clear on the learning of the day? How did understanding the *Cool-down* of the lesson before you started teaching today help you synthesize that learning?

Warm-up

10 mins

Choral Count: Twos and Fives

Standards

Addressing **3.OA.A.1**

Instructional Routines

- Choral Count

The purpose of this *Choral Count* is for students to practice counting by 5 and 2 and notice patterns in the count. These understandings help students begin to develop fluency and will be helpful later in this lesson when students write multiplication expressions. When students notice patterns in the count, such as the digit in the ones place alternates between 0 and 5 when counting by 5, they look for and express regularity in repeated reasoning (MP8).

This is the first time students experience the *Choral Count* routine in grade 3. Students should be familiar with this routine from a previous grade. However, they may benefit from a brief review of the steps involved.

Student Response

0	0
5	2
10	4
15	6
20	8
25	10
30	12
35	14
40	16
45	18
50	20

Sample responses:

In the fives column:

- The digit in the ones place alternates 0 and 5.
- The digit in the tens place changes after every 2 counts.

In the twos column:

- The digit in the ones place counts 0, 2, 4, 6, 8, then repeats.
- The digit in the tens place changes after 5 counts.

Launch

- "Count by 5, starting at 0."
- Record as students count. See *Student Responses* for recording structure.
- Stop counting and recording at 50.

Activity

- "What patterns do you see?"
- 1–2 minutes: quiet think time
- Record responses.
- Repeat activity. Count by 2, starting at 0 and stopping at 20.

Activity Synthesis

- "How could some of the patterns help you with counting by these numbers?" (I know that the next count by 5 should end in 5. I know that the next count by 2 should have a 2 in the ones place.)
- Consider asking:
  - "Who can restate the pattern in different words?"
  - "Does anyone want to add an observation on why that pattern is happening here?"
  - "Do you agree or disagree? Why?"

Activity 1

Multiplication Expression Match

10 mins

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to match drawings, tape diagrams, and situations to multiplication expressions (MP2). Students build on their understanding of how the structure of drawings, tape diagrams, and multiplication situations show equal groups and connect this to the structure of a multiplication expression (MP7). This will be helpful later in the lesson when students create drawings or diagrams to match expressions and write expressions that represent drawings, diagrams, and situations.

Required Materials

Materials To Gather

- Materials from a previous lesson: Activity 1

Required Preparation

- Each group of 2 needs 1 card from the card sort in the previous lesson.
- Post these expressions around the room:
    - 3×5
    - 4×3
    - 3×2
    - 2×10
    - 3×10

Student Task Statement

Your teacher will give you a card showing a situation, a drawing, or a diagram.

Match it to 1 of the expressions posted around the room. Be prepared to explain your reasoning.

Student Response

- A, F, and L: 3×5
- B, H, and K: 4×3
- C and I: 3×2
- D and G: 2×10
- E and J: 3×10

Launch

- Groups of 2
- Give each group 1 card from the blackline master.

Activity

- "Work with your partner to find the expression that matches your card. Then discuss how you know the expression matches your card."
- 2 minutes: partner work time

Activity Synthesis

- Have students standing near each expression share how they know their card matches the expression.
- Consider asking:
    - "Where do you see each number in the expression on your card?"

Activity 2

15 mins

Expressions to Drawings and Diagrams

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to demonstrate a conceptual understanding of multiplication expressions by creating drawings of equal groups or tape diagrams that match expressions. Drawings of equal groups and tape diagrams are familiar representations to students from previous lessons and using them now supports students as they make sense of multiplication expressions.

Three expressions are given. Consider providing support to students for the first expression and then allowing them to try the second and third expressions on their own. To keep things simple and allow ideas about commutativity to develop over time, display student responses using the convention of groups as the first factor and the size of the groups as the second factor. The *Activity Synthesis* focuses on analyzing the second expression, 3×4.

If there is time, and you want to include more movement, students can share their own expressions and matching diagrams in a Gallery Walk.

Access for English Language Learners

MLR2 Collect and Display. Collect the language students use to describe the diagrams for each of the expressions. Display words and phrases, such as “5 groups of 2,” “there are 5 groups, and 2 in each group,” and “there are 5 equal groups.” During the *Activity Synthesis*, invite students to suggest ways to update the display and to borrow language from the display as needed.
Advances: Conversing, Reading

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to a variety of tools, such as mini-whiteboards and counters.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Task Statement

1. Create a drawing or diagram for each expression. Explain your reasoning.
   a. 5×2
   b. 3×4
   c. 3×10
2. Write your own expression and matching diagram. Explain your reasoning.

Student Response

1. Sample responses:
   a. I showed 5 groups of 2.

2	2	2	2	2

   b. There are 3 groups and 4 in each group.

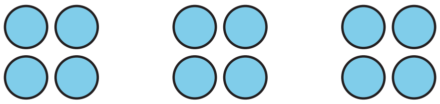
Launch

- Groups of 2
- Review key understandings of multiplication:
  - “Multiplication is how we express equal groups.”
  - “We use the multiplication symbol to create an expression like 5×10 which represents the total number of objects in ‘5 groups of 10’ or ‘5 tens.’”
- Display expressions.
- “Think about the drawings or diagrams you could make for these expressions.”
- 30 seconds: quiet think time

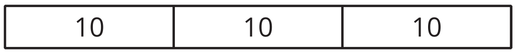
Activity

- “Work with your partner to create a drawing or diagram for each expression. Then write your own expression and matching diagram. Explain your reasoning.”
- 5–7 minutes: partner work time
- Monitor for student-created drawings and tape diagrams for 3×4 to share during the Activity

Synthesis.



c. It is 3 groups of 10.



2. Answers vary.

Activity Synthesis

- For the expression 3×4, display 2 different representations side by side (one drawing of equal groups and one tape diagram).
- "How are they the same? How are they different?"
- If time, consider asking:
  - "How would the diagram change if the expression was 5×4?" (There would be 5 parts instead of 3.)
  - "How would the diagram change if the expression was 3×5?" (Each part would be labeled 5 instead of 4.)

Activity 3

10 mins

Write Multiplication Expressions

Standards

Addressing **3.OA.A.1**

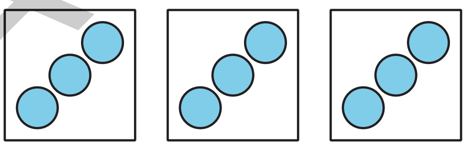
The purpose of this activity is for students to write expressions to represent drawings of equal groups, tape diagrams, and multiplication situations. As students work, continually ask how each number in the expression represents part of the drawing, diagram, or situation.

If students finish early, ask them to find something in the room they can represent with a multiplication expression. Have them record what they represented and their expression.

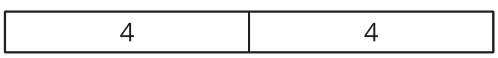
Student Task Statement

Write a multiplication expression to match each situation, drawing, or diagram. Explain your reasoning.

1.



2.



Launch

- Groups of 2
- "Now you are going to write multiplication expressions to represent a drawing, a diagram, and a situation. Take a minute to look them over before you begin working."
- 1 minute: quiet think time

Activity

- "Work with your partner to write a multiplication expression to match each representation. Explain your reasoning."
- 3–5 minutes: partner work

3. There were 2 packs of water. Each pack had 6 bottles of water.

Student Response

1. 3×3. Sample response: It's 3 groups of 3.
2. 2×4 or 4×2. Sample response: There are 2 parts, and each part has 4 in it.
3. 2×6 or 6×2: Sample response: There are 2 groups with 6 in each group.

Activity Synthesis

- Share responses.
- "Why does each of the representations show multiplication?" (They all show groups where there is the same number of things in each group.)

Sec B

Advancing Student Thinking

If a student writes a multiplication expression that doesn't match the given representation, consider asking:

- "How could you describe the equal groups in this drawing/diagram/situation?"
- "How could you turn your statement into a multiplication expression?"

Lesson Synthesis

Display a multiplication expression from the first activity and its matching drawing of equal groups, tape diagram, and situation. "We've learned about different ways to represent multiplication. Share something that you learned today about multiplication with your partner." (Multiplication can be shown with drawings of equal groups or with diagrams. Multiplication can be real-world situations that involve equal groups. Multiplication can be expressed using the multiplication symbol ($\times$). An expression like 4×5 means the total number of objects in 4 groups of 5.)

Suggested Centers

- Capture Squares (1–5) , Stage 4: Subtract within 20 (Supporting)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Cool-down

5 mins

Write an Expression

Standards

Addressing **3.OA.A.1**

Student Task Statement

There were 6 envelopes. Each envelope had 2 notes in it.

Write a multiplication expression to represent the situation. Explain or show your reasoning. Create a drawing or diagram if it's helpful.

Student Response

6×2 or 2×6. Sample response: The 6 represents the 6 envelopes and the 2 represents the 2 notes in each envelope.

Responding To Student Thinking

Students draw 2 groups of 6.

Next Day Supports
Use the next day's *Warm-up* for students to practice differentiating the number of groups in the image from the number of dots in each group.

Unit 1, Lesson 12

Represent and Solve Multiplication Problems

Standards

Addressing **3.OA.A.1, 3.OA.A.3**

Instructional Routines

- How Many Do You See?
- MLR5 Co-Craft Questions

Goals

- Represent and solve multiplication problems.

Student Facing Learning Goals

Let's represent and solve problems involving equal groups.

Sec B

Lesson Purpose

The purpose of this lesson is for students to represent and solve multiplication problems.

Narrative

In previous lessons, students learned different ways to represent situations involving equal groups with drawings, tape diagrams, and expressions. The purpose of this lesson is for students to solve problems involving equal groups with a representation of their choice.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

What strategy did each student seem most comfortable using to find products today?

Warm-up

10 mins

How Many Do You See: Lots of Dots

Standards

Addressing **3.OA.A.1**

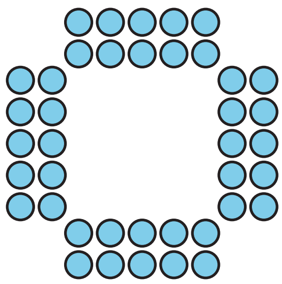
Instructional Routines

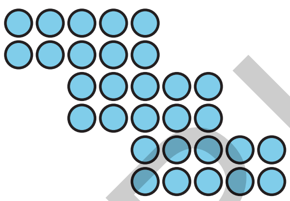
- How Many Do You See?

The purpose of this *Warm-up* is for students to subitize or use grouping strategies to describe the images they see. When students decompose the images into groups of 10 to count efficiently, they are looking for and making use of structure (MP7). Students may need these images displayed for a longer amount of time in order to see the structure.

Student Task Statement

How many do you see? How do you see them?





Student Response

Sample responses:

- 40: I counted by 5 (or 2 or 10) to 40.
- 30: I saw that each step had 10, so I counted by 10 to 30.

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the first image.
- 30 seconds: quiet think time

Activity

- Display the first image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses. Use multiplication expressions when students share explanations involving equal groups.
- Repeat for the second image.

Activity Synthesis

- "What pattern was helpful in finding the total number of dots?"
- Consider asking:
    - "Who can restate the way ____ saw the dots in different words?"
    - "Did anyone see the dots the same way but would explain it differently?"
    - "Does anyone want to add an observation to the way _____ saw the dots?"

Activity 1

Tyler's Boxes

15 mins

Standards

Addressing **3.OA.A.3**

Instructional Routines

- MLR5 Co-Craft Questions

The purpose of this activity is for students to use the *Co-Craft Questions* math language routine to make sense of a multiplication situation before solving. Students are first asked to generate questions they could ask about part of a problem. Then, students are given the full problem and asked to solve it. The activity concludes with students reflecting on the representations they used. In this activity, students will need to see the full problem to solve. Before the lesson, record the problem and have it hidden until the appropriate time in the lesson or write it for all to see at that point during the activity.

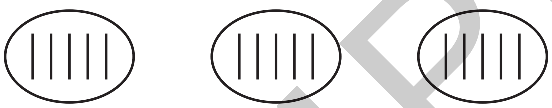
This activity uses *MLR5 Co-Craft Questions.* Advances: writing, reading, representing.

Student Task Statement

Tyler has 3 boxes. He has 5 baseballs in each box. How many baseballs does he have altogether? Show your thinking using diagrams, symbols, or other representations.

Student Response

15 baseballs. Sample responses:

- 
- Tyler has 15 baseballs because I counted by 5.
- Tyler has 15 baseballs. 3×5 or 5×3

Launch

- Groups of 2

MLR5 Co-Craft Questions

- Display only the first part of the problem stem, "Tyler has 3 boxes." Do not reveal the question.
- "Write a list of mathematical questions that could be asked about this situation." (What's in the boxes? How many things are in the boxes? How many things does he have altogether?)
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "What do these questions have in common? How are they different?"
- Reveal the task (students open books), and invite additional connections.

Activity

- "Think about how you'll solve the problem."
- 30 seconds: quiet think time
- "Now solve the problem."
- 2–3 minutes: partner work time

Activity Synthesis

- Display student work with different representations

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of the problem one at a time (drawings of equal groups, tape diagrams, and expressions). If no student writes an expression, write one for students to analyze.

- "How does each representation help us see what's happening in the problem?"

Activity 2

20 mins

Solve Equal Groups Problems

Standards

Addressing **3.OA.A.3**

Sec B

The purpose of this activity is for students to use what they've learned about multiplication to solve and represent situations that involve equal groups. Students now have experience with multiple representations and have had the opportunity to choose which representation is most helpful to represent multiplication situations.

The Launch is an opportunity for students to share their experiences and ask questions about the objects to ensure each student has access to the context. If it is helpful, display images of the objects for students to reference.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: As students describe their representations of the problems, use gestures to emphasize the number of groups and how many are in each group. For example, trace with your finger around each group, and point to each object in each group to show how many there are.
Advances: Listening, Representing

Access for Students with Disabilities

Representation: Access for Perception. Provide access to connecting cubes. Ask students to identify correspondences between the concrete and visual representation used.
Supports accessibility for: Conceptual Processing

Student Task Statement

Solve each problem. Show your thinking using diagrams, symbols, or other representations.

1. There are 4 soccer fields. Two teams are on each field. How many teams are there altogether?
2. There are 7 windows. Each window has 2 pieces of glass. How many pieces of glass are there in the windows?
3. Jada has 5 bags. Each bag has 10 earrings. How many earrings does Jada have?

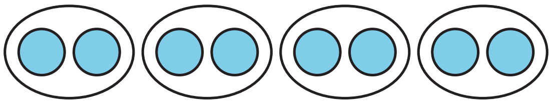
Launch

- Groups of 2
- Write the list of objects ("teams, earrings, pencils, pieces of glass, carrots") on a display for all students to see.
- "Take a minute to read this list. When you are done, discuss the objects you know or have questions about."
- 3 minutes: partner discussion
- Share a few responses.

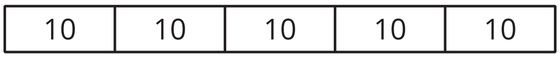
4. Kiran has 4 boxes. Each box has 5 pencils in it. How many pencils does Kiran have?
5. Andre has 3 bags of carrots. Each bag has 10 carrots. How many carrots does Andre have?

Student Response

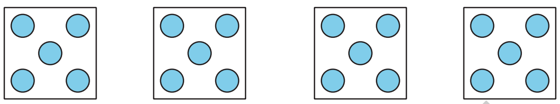
1. 8 teams. Sample response:



   I used the groups to count the dots. Each dot represents a team.
2. 14 pieces of glass. Sample response: 7×2. I counted by 2 seven times because each window had 2 pieces of glass.
3. 50 earrings. Sample response:



   I used a diagram to show 5 groups of 10, then counted by 10 to find the total.
4. 20 pencils. Sample response:



   I used a drawing to show 4 boxes with 5 in each box.
5. 30 carrots. Sample response: I counted the 3 bags of 10 like 10, 20, 30.

- "Now we're going to represent and solve more problems with equal groups. Take a couple of minutes to begin working on the problems before you work with your partner."
- 2–3 minutes: independent work time

Activity

- "Work with your partner to solve each problem."
- 5–7 minutes: partner work
- Identify students who use different representations, such as drawings of equal groups or tape diagrams, as they solve the problems.

Activity Synthesis

- For each problem, display different representations, one at a time.
- "How does this representation help us see what's happening in the problem?" (We can see there are 4 groups for the 4 fields and 2 dots in each group for the 2 teams on each field. The diagram is split into 5 parts for the 5 bags, and each part has a 10 in it for the number of earrings.)
- "How could each representation help us solve the problem?" (Counting the dots. Counting by 10.)
- If needed, "What expression could we write to represent this situation?"

Advancing Student Thinking

If students add or subtract instead of multiply to solve the problems, consider asking:

- "Tell me about how you solved this problem."
- "How does the problem involve equal groups?"

Lesson Synthesis

Display samples of student work with different representations (drawings of equal groups, tape diagrams, and expressions).

"Which representation did you find most helpful today and why?" (Drawings of equal groups helped me see what was happening in the problem. Diagrams helped me understand the problem, but I didn't have to draw all the dots.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Cool-down

5 mins

Ducks in a Pond

Standards

Addressing **3.OA.A.3**

Student Task Statement

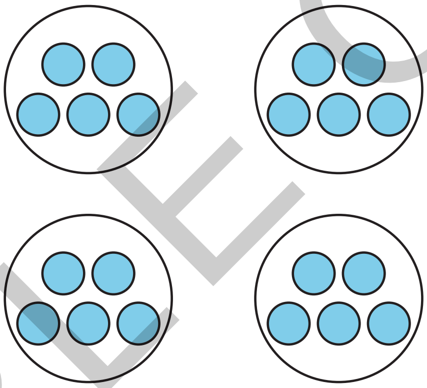
There are 4 ponds. Each pond has 5 ducks. How many ducks are there altogether?

Use diagrams, symbols, or other ways to show your thinking.

Student Response

20 ducks. Sample responses:

- 4×5
- 5, 10, 15, 20



Responding To Student Thinking

Students add or subtract the numbers in the problem instead of multiplying.

Next Day Supports
During the *Launch* of the next day's activity, have students discuss what the situations have in common that make them multiplication problems.

Sec B

Unit 1, Lesson 13

Multiplication Equations

Sec B

Standards

Addressing **3.OA.A.1, 3.OA.A.3**

Instructional Routines

- Which Three Go Together?

Goals

- Comprehend (in spoken language) the meaning of the terms “factor” and “product.”
- Match (orally) multiplication equations with situations, diagrams, and drawings.
- Represent multiplication situations and diagrams with an equation.

Student Facing Learning Goals

Let’s learn about multiplication equations.

Lesson Purpose

The purpose of this lesson is for students to relate multiplication equations to situations and diagrams and to write equations.

Narrative

In previous lessons students represented situations and diagrams with multiplication expressions. In this lesson, students learn the meaning of **factor** and **product**. Students do not have to use the vocabulary in this lesson as they will continue to have opportunities to do so throughout the year. In future lessons, students will represent situations and diagrams with equations that use a symbol for the unknown number, which may be either a factor or the product.

Consider continuing to use the convention of groups as the first factor and the size of the groups as the second factor when writing equations. However, it is not necessary for students to write the factors in this order. It is important that students connect their equations to the corresponding situations and representations (MP2). They should be able to correctly explain what each factor represents in their equations. If students ask questions about the idea of commutativity, consider recording the questions publicly for future investigation.

Access For Students with Disabilities

- Engagement

Access For English Learners

- MLR7

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins

Teacher Reflection Questions

How were the terms “factor” and “product” helpful as students began working with multiplication equations?

Cool-down	5 mins

Warm-up

10 mins

Which Three Go Together: Representations

Standards

Addressing **3.OA.A.1, 3.OA.A.3**

Instructional Routines

- Which Three Go Together?

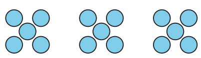
This *Warm-up* prompts students to carefully analyze and compare features of expressions and equations. When students compare the drawing, expression, and equations, they must use language precisely to describe how the representations are alike (MP6). Listen to the language students use to describe the different characteristics of the multiplication expression and equation. After the new terms "factor" and "product" are introduced in the *Synthesis*, connect students' descriptions to those terms.

This is the first time students experience the *Which Three Go Together?* routine in IM Grade 3. Students should be familiar with this routine from a previous grade. However, they may benefit from a brief review of the steps involved.

Student Task Statement

Which 3 go together?

A



B

3×5

C

$2 \times 5 = 10$

D

$7 + 8 = 15$

Student Response

Sample responses:

A, B, and C go together because:
- They show equal groups.
- They represent multiplication.
- They show groups of 5.

A, B, and D go together because:
- They represent a total of 15.

A, C, and D go together because:
- They show the total.

B, C, and D go together because:
- They use numbers in expressions or equations.

Launch

- Groups of 2
- Display the image, expression, and equations.
- "Pick 3 that go together. Be ready to share why they go together."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Activity Synthesis

- "How is C different from the other ways we've represented equal groups before?" (It has an equal sign. It's an equation.)
- "C is a multiplication **equation** because it contains a multiplication symbol and the equal sign."
- "There are words that help us talk about different parts of the multiplication equation. The **factors** are the numbers being multiplied. The **product** is the result of multiplying some numbers. In the equation in C, the numbers 2 and 5 are the factors. The product is 10. Keep these words in mind today as we

work with other multiplication equations." 20 mins

Activity 1

Multiplication Equation Match

Standards

Addressing **3.OA.A.1, 3.OA.A.3**

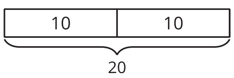
The purpose of this activity is for students to match multiplication equations to situations and representations. Students make explicit connections between the factors and the number of groups and the number of objects in each group and between the product and the total number of objects. These connections are discussed explicitly during the *Activity Synthesis*. When students make explicit connections between multiplication situations and equations, they are reasoning abstractly and quantitatively (MP2).

Sec B

Student Task Statement

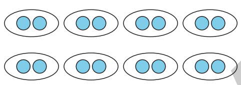
Find an equation from the list that can represent each situation, drawing, or diagram. Record the equation. Be prepared to explain your reasoning.

- $3 \times 5 = 15$
- $4 \times 10 = 40$
- $2 \times 10 = 20$
- $10 = 5 \times 2$
- $30 = 6 \times 5$
- $4 \times 2 = 8$
- $16 = 8 \times 2$
- $4 \times 5 = 20$
- $50 = 5 \times 10$

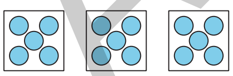
1.


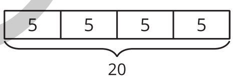
____

2. Andre has 5 pairs of socks. ______________

3.

____

4. There are 6 hands on the table. Each hand has 5 fingers. ______________

5.

____

6.


____

7.

Launch

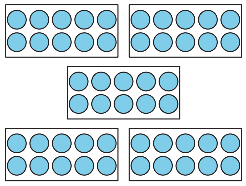
- Groups of 2 and 4
- "Think about how you might match these equations to a situation or diagram."
- 1 minute: quiet think time

Activity

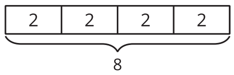
- "Take turns finding a situation or diagram that matches each equation. Explain your reasoning to your partner."
- 5–7 minutes: partner discussion
- Monitor for students who make direct connections between each factor representing the number in each group or the number of groups and the product representing the total number of objects to share during the *Activity Synthesis*.
- "Get together with another group to discuss the matches you made."
- 3–5 minutes: small-group discussion

Activity Synthesis

- "Were there any matches you disagreed on? How did you come to an agreement?" (We went back and recounted the dots together.)
- For the $2 \times 10 = 20$, $30 = 6 \times 5$, and $50 = 5 \times 10$ matches ask, "How does the equation represent the situation (or diagram)?" (The 2 represents the 2 parts in the diagram. The 5 represents the 5 fingers on each hand. The 50 represents how many dots were in the groups altogether.)



8.



9.

There are 4 boxes of markers. Each box has 10 markers.

Student Response

1. $2 \times 10 = 20$
2. $10 = 5 \times 2$
3. $16 = 8 \times 2$
4. $30 = 6 \times 5$
5. $3 \times 5 = 15$
6. $4 \times 5 = 20$
7. $50 = 5 \times 10$
8. $4 \times 2 = 8$
9. $4 \times 10 = 40$

Activity 2

15 mins

Write Multiplication Equations

Standards

Addressing **3.OA.A.1, 3.OA.A.3**

The purpose of this activity is for students to write equations that match situations and diagrams. Students use what they learned in the last activity to use multiplication equations to represent situations and diagrams. In the *Lesson Synthesis*, use the words "factor" and "product" to help students connect the vocabulary to the concepts.

Access for English Language Learners

MLR7 Compare and Connect. Synthesis: Invite groups to prepare a visual display that shows their reasoning for one of the equations using details, such as different colors, arrows, labels, diagrams or drawings. Give students time to investigate each others' work. Ask, "Which details or language helped you understand the displays?" "Did

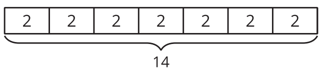
anyone create the same equation, but would explain it differently?"
Advances: Representing, Conversing

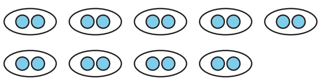
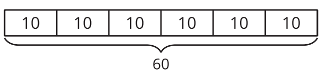
Access for Students with Disabilities

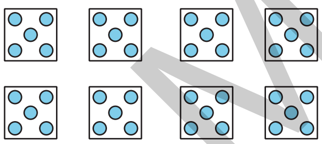
Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to decide which problem to start with or decide the order to complete the task.
Supports accessibility for: Social-Emotional Functioning

Student Task Statement

Write a multiplication equation that represents each situation, drawing, or diagram. Be prepared to explain your reasoning.

1. A package has 6 pairs of socks. ____________
2. 

____
3. Diego has 7 sections in his notebook. Each section has 10 pages. ____________
4. 
____
5. 

____
6. Elena has 4 bags of oranges. Each bag has 5 oranges in it. ____________
7. 
____

Launch

- Groups of 2

Activity

- "Work with your partner to write an equation that represents each situation, drawing, and diagram."
- 5–7 minutes: partner work time
- Monitor for students who can justify the equations they wrote by explaining the meaning of the factors and products in their equations.

Activity Synthesis

- For each problem, invite a student to share their equation. Consider asking:
  - "How does this equation make sense for this situation, drawing, or diagram?"
  - "What parts of the situation, drawing, or diagram were especially helpful as you wrote the equation?"

Student Response

1. $6 \times 2 = 12$ or $2 \times 6 = 12$. Sample response: There are 6 groups of socks with 2 socks in each group. This makes 12 socks.
2. $7 \times 2 = 14$ or $2 \times 7 = 14$. Sample response: There are 7 parts, and each part has 2 in it. If you count by 2 seven times, you get 14.
3. $7 \times 10 = 70$ or $10 \times 7 = 70$. Sample response: There

are 7 sections, and each section has 10 pages, so there are 70 pages altogether.

4. $9 \times 2 = 18$ or $2 \times 9 = 18$. Sample response: There are 9 circles, and each circle has 2 dots in it. This is a total of 18 dots.
5. $6 \times 10 = 60$ or $10 \times 6 = 60$. Sample response: There are 6 parts with 10 in each part. This makes 60.
6. $4 \times 5 = 20$ or $5 \times 4 = 20$. Sample response: There are 4 bags, and each bag has 5 oranges. This is 20 oranges.
7. $8 \times 5 = 40$ or $5 \times 8 = 40$. Sample response: There are 40 dots because there are 8 squares and each has 5 dots.

Lesson Synthesis

Display:

Expression: 3×5
Equation: $3 \times 5 = 15$

"Today we learned about equations and how we can use them to represent multiplication. In this equation, 3 and 5 are the factors and 15 is the product."

"How are multiplication expressions and equations alike?" (They both use the multiplication symbol. They both have factors.)

"How are multiplication expressions and equations different?" (Equations have an equal sign and a product. Multiplication equations have numbers on both sides of the equal sign.)

"When would each be helpful?" (Expressions are helpful when you want to describe a situation with equal groups. Equations are helpful if you are trying to find the total number of objects, or the product.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 7: Add within 1,000, without Composing (Supporting)

Cool-down

5 mins

Match the Equation

Standards

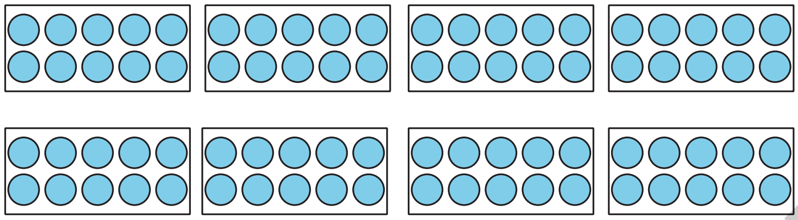
Addressing **3.OA.A.1**

Student Task Statement

Select **all** the drawings, diagrams, and situations that could represent the equation.

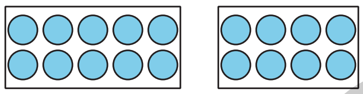
$$80 = 8 \times 10$$

A.

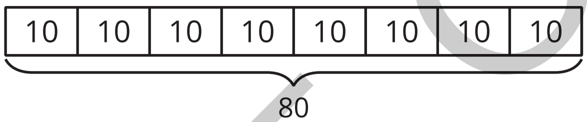


B. Andre has 8 boxes. Each box has 10 cars in it. He has 80 cars altogether.

C.

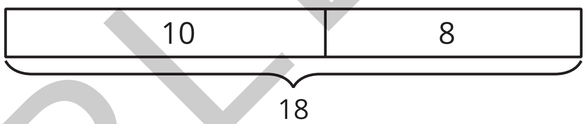


D.



E. Andre had 8 boxes. Then, he found 10 more boxes. How many boxes does Andre have?

F.



Student Response

A, B, and D

Responding To Student Thinking

Students choose drawings, diagrams, or situations that represent addition instead of multiplication.

Next Day Supports
Use the *Launch* of the next day's activity to discuss how problems that involve multiplication are different from problems that involve addition.

Unit 1, Lesson 14

Write and Solve Equations with Unknowns

Standards

Addressing **3.OA.A.1, 3.OA.A.3, 3.OA.A.4, 3.OA.C.7, 3.OA.D.9**

Instructional Routines

- Card Sort
- Number Talk

Goals

- Explain (orally) strategies for finding an unknown factor or product.
- Match (orally) multiplication equations with a symbol for an unknown product or factor with situations and diagrams.
- Represent a multiplication situation or diagram using an equation with a symbol for the unknown product or factor.

Student Facing Learning Goals

Let's work with equations with unknown numbers.

Lesson Purpose

The purpose of this lesson is for students to relate equations to and write equations for multiplication situations and diagrams using a symbol for the unknown number.

Narrative

Students have worked with addition and subtraction equations with a symbol to represent the unknown number in grades 1 and 2. Students build on that work and the work with multiplication equations in the previous lesson as they encounter multiplication equations that have a symbol for the unknown number for the first time.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Copy

- Card Sort Unknown Numbers Cards (1 copy for every 2 students): Activity 1

Lesson Timeline

Warm-up	10 mins

Teacher Reflection Questions

How do tape diagrams help students make sense of equations in which the unknown number is in different

Activity 1	15 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

positions?

Warm-up

10 mins

Number Talk: Fives

Standards

Addressing **3.OA.C.7, 3.OA.D.9**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for multiplying by 5. These understandings help students develop fluency and will be helpful later in this lesson when students represent and solve a problem involving groups of 5.

When students reason why the product increases by 5 as one factor increases by 1, they are looking for and expressing regularity in the expressions (MP8).

Student Task Statement

Find the value of each expression mentally.

- 1×5
- 2×5
- 3×5
- 4×5

Student Response

- 5: It's only 1 group of 5.
- 10: I added 5 more to the first answer.
- 15: I added $5 + 5 + 5$.
- 20: I counted by 5 four times.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "What pattern do you see as you look at the expressions and their values? Why is that happening?" (The factor that isn't 5 goes up by 1 each time. The value of the product increases by 5 each time because we add another group of 5.)
- As needed, record student thinking using equal-groups drawings to help all students visualize the pattern.
- Consider asking:
  - "Did anyone notice a different pattern?"

- ◦ “Did anyone notice the same pattern but would explain it differently?”

Activity 1

15 mins

Card Sort: Unknown Numbers

Standards

Addressing **3.OA.A.1, 3.OA.A.4**

Instructional Routines

- Card Sort

The purpose of this activity is for students to relate equations to multiplication situations and diagrams using a symbol for the unknown number. This sorting task gives students opportunities to analyze these equations, situations, and diagrams closely and make connections (MP2, MP7). Students explain their matches to their peers and revise their language for precision and clarity when they describe how the numbers and symbols in the equations match the representations (MP3, MP6). In the *Activity Synthesis*, students explain the meaning of the factors and products and what a symbol in an equation represents.

Access for English Language Learners

MLR8 Discussion Supports. Invite students to take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: “I noticed ___ , so I matched . . . .” Encourage students to challenge each other when they disagree.
Advances: Conversing, Representing

Required Materials

Materials To Copy

- Card Sort Unknown Numbers Cards (1 copy for every 2 students): Activity 1

Required Preparation

- Create a set of cards from the blackline master for each group of 2.

Student Task Statement

Your teacher will give you a set of cards. Match each equation to a situation or diagram. Be ready to explain your reasoning.

Student Response

Matches:

- A, L
- B, H

Launch

- Groups of 2
- Display: $4 \times 5 = ?$
- “What might this equation mean?” (There are 4 groups of 5. There's a number in the equation that we don't know. We don't know the total.)
- 2 minutes: partner discussion
- Share responses.
- “Different symbols can be used to represent the unknown number in an equation. Some that are

- C, E
- D, F
- G, K
- I, J

Sample response: A and L go together because there are 3 groups that together make 6, but we don't know the size of each group.

common are question marks, blank spaces, and boxes."

- "For example, in the equation $80 = 8 \times 10$, if we didn't know the product we could write $? = 8 \times 10$." Display these equations as you explain.
- "If we didn't know one of the factors, what is an equation you could write using a symbol for the unknown number?" ($80 = \underline{\hspace{2em}} \times 10, 80 = ? \times 10$)
- Give each group a set of cards.

Activity

- "This set of cards includes equations, situations, and diagrams. Match each equation to a situation or diagram. Work with your partner to explain your reasoning."
- 5–7 minutes: partner work time
- Monitor for students who explain the meaning of the factors and the product, specifically that the symbol is for an unknown number that represents the unknown amount in the situation or diagram.

Activity Synthesis

- Invite previously selected students to share how they decided which cards go together.
- Choose 1–2 equations (at least one with a missing factor) and ask:
  - "What does each number in the equation represent?"
  - "What does each question mark (or blank or box) represent?"
  - "How can we figure out which number goes in the blank to make the equation true?"
- Listen for the language students use to describe their matches and the equations, diagrams, and situations clearly and precisely. As needed, ask:
  - "What do you mean when you say _____?"
  - "Was the unknown the product or one of the factors? Explain."
  - "What did the unknown factor represent? Explain."
- Highlight the terms "factor" and "product."

Activity 2

20 mins

Write Equations with an Unknown Number

Standards

Addressing **3.OA.A.3, 3.OA.A.4**

The purpose of this activity is for students to write equations for multiplication situations and diagrams using a symbol for the unknown number. When students write an equation to represent a situation, including a symbol for the unknown number, they model a situation with mathematics (MP4).

Students find an unknown factor or unknown product in multiplication problems. In this task, the unknown factor diagrams and situations only include the "how many groups?" problem type and the factors 2, 5, and 10. This sets up students to skip-count to find the unknown number.

This problem type will be revisited extensively in future lessons and will be related to division. It is not necessary to make the connection to division now. In the *Activity Synthesis*, students explain how the equations they wrote represent the diagram or situation.

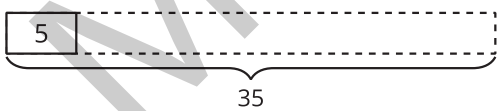
Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were important or most useful to pay attention to. Display the sentence frame, "The next time I write an equation with an unknown number, I will . . . ."
Supports accessibility for: Visual-Spatial Processing

Student Task Statement

- Write a multiplication equation to represent each diagram or situation. Use a symbol for the unknown. Be prepared to share your reasoning.
- Find the number that makes each equation true. Rewrite the equation with the solution.

1.



   - equation with symbol:
     ____
   - equation with solution:
     ____

2. Jada has some packs of sports cards. Each pack has 5 cards. If Jada has 45 cards, how many packs of cards does she have?

Launch

- Groups of 2

Activity

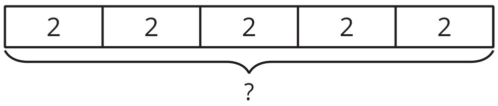
- "Now you will practice writing your own equations with a symbol for the unknown."
- 2–3 minutes: independent work time
- "Share your equations with your partner. Discuss how you know each equation matches the diagram or situation."
- 2–3 minutes: partner discussion
- Have a whole-class discussion focused on how the equations match the different representations.
- Consider asking:
  - "How did you use the representations to write an equation with a symbol for the unknown?" (I looked for what was missing in the diagram. I thought about the situation to figure out if it was the number in each group, the number of groups, or the total that was unknown.)

- equation with symbol:

  ____

- equation with solution:

  ____

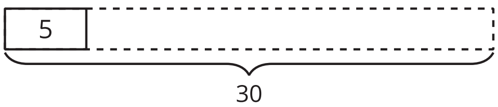
3. 

   - equation with symbol:

     ____

   - equation with solution:

     ____

4. 

   - equation with symbol:

     ____

   - equation with solution:

     ____

5. The school has 6 bags. Each bag has 10 basketballs in it. How many basketballs does the school have?
   - equation with symbol:

     ____

   - equation with solution:

     ____

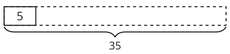
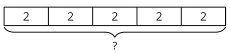
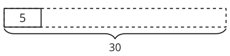
- "Now find the unknown number in each equation and write a new equation that includes the solution."
- 3–5 minutes: partner work time

Activity Synthesis

- "What strategies did you use to find the unknown numbers?" (I counted by 2 to get the total. I counted by different numbers until I found a number that gave me the product.)
- "How did each equation change as you found the unknown number?" (The symbol was replaced with the number.)

Sec B

Student Response

diagram or situation	equation with symbol	equation with solution
1. 	$? \times 5 = 35$ $5 \times ? = 35$	$7 \times 5 = 35$ $5 \times 7 = 35$
2. Jada has some packs of sports cards. Each pack has 5 cards. If Jada has 45 cards, how many packs of cards does she have?	$? \times 5 = 45$ $5 \times ? = 45$	$9 \times 5 = 45$ $5 \times 9 = 45$
3. 	$5 \times 2 = ?$ $2 \times 5 = ?$	$5 \times 2 = 10$ $2 \times 5 = 10$
4. 	$? \times 5 = 30$ $5 \times ? = 30$	$6 \times 5 = 30$ $5 \times 6 = 30$
5. The school has 6 bags. Each bag has 10 basketballs in it. How many basketballs does the school have?	$6 \times 10 = ?$ $10 \times 6 = ?$	$6 \times 10 = 60$ $10 \times 6 = 60$

Advancing Student Thinking

If students write an equation without a symbol for the unknown number, consider asking:

- “How does your equation represent the diagram (or situation)?”
- “How could you show the number that was unknown with a symbol?”

Lesson Synthesis

Display:

$$6 \times 5 = ?$$
$$6 \times ? = 30$$
$$? \times 5 = 30$$

"Today we found the unknown number in multiplication equations."

"How was finding an unknown factor different from finding an unknown product?" (If we didn't know the product, we could skip-count by the number the right number of times, like 5, 10, 15, 20, 25, 30. If we didn't know one of the factors, we might have to skip-count by the number enough times to get to the product. If we didn't know one of the factors, we might not know what to skip-count by, just the number of times to count.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 8: Add within 1,000, with Composing (Supporting)

Cool-down

5 mins

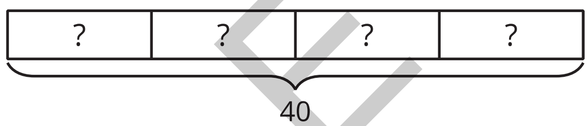
Unknown and a Number

Standards

Addressing 3.OA.A.1, 3.OA.A.4

Student Task Statement

1. Write an equation to match the diagram. Use a symbol for the unknown.



2. Find the number that makes the equation true. Rewrite the equation with that number. Explain your reasoning.

Student Response

1. $4 \times ? = 40$ or $? \times 4 = 40$
2. $4 \times 10 = 40$ or $10 \times 4 = 40$. Sample response: If I count by ten 4 times, I get 40. So, the unknown number is 10.

Responding To Student Thinking

Students write a number for the unknown that doesn't make the equation true.

Next Day Supports
Use the *Launch* of the next day's activity to brainstorm ways to find the unknown number in a multiplication equation.

Unit 1, Lesson 15

More Factors, More Problems

Standards

Addressing **3.OA.A.3, 3.OA.A.4, 3.OA.C.7, 3.OA.D.9**

Instructional Routines

- Number Talk

Goals

- Represent and solve multiplication problems.

Student Facing Learning Goals

Let's solve more multiplication problems.

Sec B

Lesson Purpose

The purpose of this lesson is for students to solve multiplication problems.

Narrative

Students write equations with a symbol for the unknown to represent multiplication problems and then solve the problems. As in the previous lesson, some problems are unknown factor problems, which students do not relate to division until a future unit. Students put together what they have learned about drawings, diagrams, expressions, and equations to solve multiplication problems.

Access For Students with Disabilities

- Engagement

Access For English Learners

- MLR8

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Warm-up

10 mins

Number Talk: Tens

Standards

Addressing **3.OA.C.7, 3.OA.D.9**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for multiplication by 10. These understandings help students develop fluency and will be helpful later in this lesson when students need to be able to represent and solve a problem involving groups of 10.

When students reason why the product increases by 10 as one factor increases by 1, they look for and express regularity in the expressions (MP8).

Sec B

Student Task Statement

Find the value of each expression mentally.

- 1×10
- 2×10
- 3×10
- 4×10

Student Response

- 10: It's only 1 group of 10.
- 20: I added 10 more to the first answer.
- 30: I added $10 + 10 + 10$.
- 40: I counted by 10 four times.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression

Activity Synthesis

- "What pattern do you see as you look at the expressions and their values? Why is that happening?" (The factor that isn't 10 goes up by 1 each time. The value of the product increases by 10 each time because I have 1 more group of 10.)
- As needed, record student thinking using equal-groups drawings or tape diagrams to help all students visualize the pattern.
- Consider asking:
  - "Did anyone notice a different pattern?"
  - "Did anyone notice the same pattern but would explain it differently?"

Activity 1

15 mins

Represent Situations with Equations

Standards

Addressing **3.OA.A.3, 3.OA.A.4**

The purpose of this activity is for students to represent a situation with a multiplication equation that has a symbol for the unknown and find the number that makes the equation true. Students are able to use an earlier representation to help them solve the problem, however some students may just write the equation and skip-count to find the product.

Either is okay.

In the *Activity Synthesis*, share different ways students represented the problem in addition to the equation. If students used repeated addition, avoid saying "Multiplication is repeated addition," because while repeated addition is one way to find the product, it is not the meaning of multiplication.

To add movement to this activity, students can work in groups of 4 to make a poster for one of the problems. After each group is done, they can do a Gallery Walk to look for things that are the same or different in the posters.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk.
Supports accessibility for: Attention, Memory

Student Task Statement

For each problem:

- Write a multiplication equation with a symbol for the unknown to represent the situation.
- Find the number that makes the equation true. Show your reasoning.

1. There are 15 bottles of paint. Han placed 5 bottles of paint on each table. How many tables have paint on them?
    a. equation:
    b. solution:
2. Lin's class has 6 tables. Each table has 2 bags of clay. How many bags of clay does the class have?
    a. equation:
    b. solution:
3. Han's class has 60 markers. There are 10 markers in a pack. How many packs of markers does the class have?
    a. equation:
    b. solution:

Student Response

1. a. $? \times 5 = 15$ or $5 \times ? = 15$
   b. $3 \times 5 = 15$ or $5 \times 3 = 15$

   Sample response:

Launch

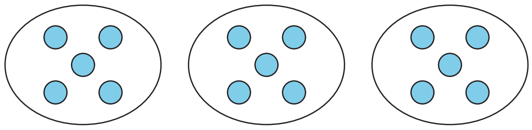
- Groups of 2
- "We are going to solve some problems about equal groups that you may see when you are preparing to do an art project with your class."
- "What are some equal groups that you might see when a class is working on an art project?"
- Share responses.
- "Think about how you could represent these problems in a way that could help you write an equation with an unknown number for each problem."
- 1 minute: quiet think time
- 1 minute: partner discussion

Activity

- "Now, independently work on these problems."
- 5–7 minutes: independent work time
- As you circulate, consider asking:
    - "How could you represent this situation?"
    - "What information is missing from the situation?"

Activity Synthesis

- Display samples of student work for each problem next to each other, including a drawing of equal groups and a tape diagram.
- "Where do we see the parts of the problem in the drawing and the diagram?" (The number of objects in each group are the dots in the drawing, but the

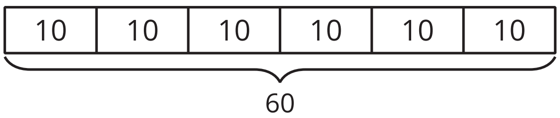


2. a. $6 \times 2 =$ ______ or $2 \times 6 =$ ______
   b. $6 \times 2 = 12$ or $2 \times 6 = 12$

   Sample response: I counted 2, 4, 6, 8, 10, 12.

3. a. $? \times 10 = 60$ or $10 \times ? = 60$
   b. $6 \times 10 = 60$ or $10 \times 6 = 60$

   Sample response:



number is written in each part of the diagram.)

- "How did you use the factors in each equation to find the product?" (The factors told me how many groups there were and how many were in each group.)
- "How are drawings and diagrams useful for finding the solution to the problem?" (You can count the dots in the drawing. The diagram can be used to count by 10.)

Activity 2

Multiplication Mashup

20 mins

Standards

Addressing **3.OA.A.3**

The purpose of this activity is for students to practice solving multiplication problems in which the unknown amount can be the number of groups, the number in each group, or the total. The first three problems have the unknown in each of those locations. The sequence of these problems, the context, and the use of the same factors and product encourages students to use a known fact to find the unknown factor in the "how many in each group?" problem. Students will make the connection between this problem type and division in a future unit. Students are able to choose the representation they use to represent and solve the problems.

Access for English Language Learners

MLR8 Discussion Supports. Monitor and clarify any questions about the context. As students look over the problems, ask, "Are there any words that are unfamiliar or that you have questions about?"
Advances: Reading, Representing

Student Task Statement

Solve each problem. Explain or show your reasoning.

1. Clare has 16 socks. She puts them in piles of 2. How many piles can she make?
2. Diego has 8 piles of socks. Each pile has 2

Launch

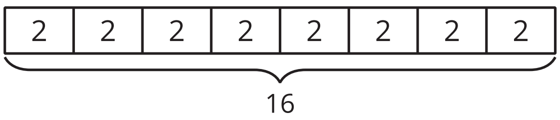
- Groups of 2
- "Take a minute to look over these problems. What representations or strategies might be helpful to you as you solve these problems?"
- 1 minute: quiet think time

socks. How many socks does Diego have?

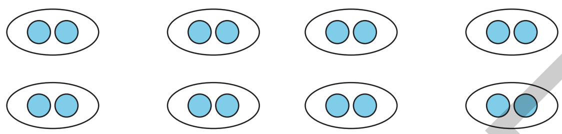
3. Andre has 16 socks. He puts them in 8 groups that are the same size. How many socks are in each group?
4. A store has 9 boxes. Each box has 5 shirts. How many shirts are there?
5. There are 80 sweaters in piles on a shelf. Each pile has 8 sweaters. How many piles of sweaters are on the shelf?

Student Response

1. 8 piles. Sample response:



2. 16 socks. Sample response: It's 8 groups of 2. I counted the piles of 2 like 2, 4, 6, 8, 10, 12, 14, 16.
3. 2 socks. Sample response:



4. 45 shirts. Sample response: $5 + 5 = 10$, $10 + 10 = 20$, $20 + 20 = 40$. That's 8 boxes. One more is $40 + 5 = 45$.
5. 10 piles. Sample response: $80 = ? \times 8$. I know 10×8 is 80. So, 10 groups of 8 must be 80.

- Share and record responses.

Activity

- "Work with your partner to solve each problem."
- 8–10 minutes: partner work time
- Circulate and consider the following questions to focus students on the structure of the situations:
  - "What information is missing in the situation?"
  - "How could you represent this situation?"

Activity Synthesis

- Share student work for each problem and ask students to explain their reasoning. Be sure to share a variety of strategies and representations.
- As students share, consider asking the class:
  - "Why does this strategy make sense?"
  - "Why does this representation make sense?"
  - "Did anyone solve this problem in a different way?"
  - "What do you notice is the same about the representations and strategies that we are using to solve these problems?"

Lesson Synthesis

"Today we solved multiplication problems using any strategy or representation that we wanted."

"What strategy or representation do you find most helpful when you are solving these types of problems? Why?" (I like to draw equal groups so I can see how many groups there are and how many are in each group. I think a diagram is nice to draw because I don't have to draw all the things, but I can still see the groups. I like to use an equation, so I can see where the unknown number is.)

"What are the most important things to remember when you are solving multiplication problems?" (There are always groups that are the same size. We could be looking for the number of groups, how many things are in each group, or the total number of things in all the groups.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Addition and Subtraction (1–3) , Stage 8: Add within 1,000, with Composing (Supporting)

Cool-down

5 mins

Solve the Problem

Standards

Addressing **3.OA.A.3**

Sec B

Student Task Statement

Solve each problem. Explain or show your reasoning.

1. There are 4 boxes. Each box has 10 toys. How many toys are there?
2. Elena has 10 socks. She puts them in piles of 2. How many piles does she make?

Student Response

1. 40 toys. Sample response:

10	10	10	10

2. 5 piles. Sample response: $10 = ? \times 2$. I know that 5×2 is 10.

Responding To Student Thinking

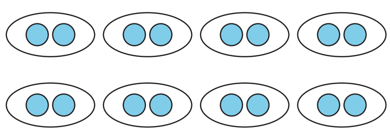
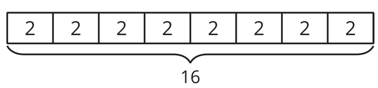
Students find solutions other than 40 toys and 5 piles.

Students do not clearly show how they found the solution or do not show a solution.

Next Day Supports
Before the *Warm-up*, pass back the *Cool-down* and invite students to work in small groups to make corrections.

Section B Summary

We learned about equal groups. We created drawings and diagrams to represent situations that involve equal groups.

Situation	Drawing	Diagram
Diego has 8 piles of socks. Each pile has 2 socks.		

We wrote multiplication **expressions** and **equations** to represent equal groups.

Expression

Equation

8×2 $8 \times 2 = 16$

We learned that the numbers that are multiplied are called **factors** and the number that is the result of multiplying is called a **product**. In the equation $8 \times 2 = 16$, the numbers 8 and 2 are the factors and 16 is the product.

Section C: Represent Multiplication with Arrays and the Commutative Property

Standards

Building On	**2.NBT.B.5, 2.OA.C.4**
Addressing	**3.MD.B.3, 3.OA.A, 3.OA.A.1, 3.OA.A.3, 3.OA.B.5, 3.OA.C.7, 3.OA.D.9**
Building Towards	**3.NBT.A.2, 3.OA.A.1**

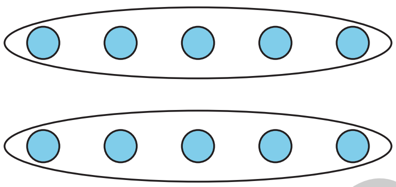
Goals

- Represent and solve multiplication problems involving arrays.

Narrative

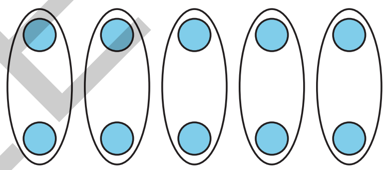
In this section, students relate the idea of equal groups to the structure of an array, a representation introduced in grade 2.

Students see that the rows and columns of an array represent equal groups. The number of rows (or columns), the number of items in each row (or column), and the total number of objects in an array can therefore be represented with a multiplication equation. The equations may involve an unknown value, be it one of the factors or the product. As students reason about arrays, they also notice that multiplication is commutative.



2 rows with 5 in each row is 10 total.

$2 \times 5 = 10$ or $5 \times 2 = 10$



5 columns with 2 in each column is 10 total.

$5 \times 2 = 10$ or $2 \times 5 = 10$

Suggested Centers

Lesson 16
- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Lesson 17
- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Lesson 18
- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)

- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Lesson 19
- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Lesson 20
- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Lesson 21
- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Section C Checkpoint

1

Goals Assessed

- Represent and solve multiplication problems involving arrays.

Student Task Statement

Draw an array that represents the expression 3×4. Explain or show your reasoning.

Solution

Sample responses:

- Students draw an array with 3 rows and 4 columns.
- There are 3 rows of 4 dots, so that's 3×4 dots.
- Students draw an array with 3 columns and 4 rows.
- There are 4 rows of 3 dots, so that's 3×4 dots.

Responding To Student Thinking

Students do not yet draw an array to represent a given expression.

Throughout the next unit, invite students to explain how expressions and equations match arrays, tiled rectangles, and area diagrams. Emphasize the language students use to relate each factor to the rows and columns or length and width of the diagram.

2

Goals Assessed

- Represent and solve multiplication problems involving arrays.

Student Task Statement

There are 5 rows of chairs in the room. Each row has 7 chairs. How many chairs are there?

a. Draw an array to represent the situation.
b. Write an equation to represent the situation. Use a symbol for the unknown.
c. Solve the problem. Explain or show your reasoning.

Solution

a. Students draw a 5 by 7 array or 7 by 5 array.

b. $5 \times 7 = ?$ or $7 \times 5 = ?$

c. 35 chairs. Sample response:
$7 + 7 = 14$
$14 + 14 = 28$
$28 + 7 = 35$

Responding To Student Thinking

Students show they may understand the structure of an array, but their array does not match the situation.

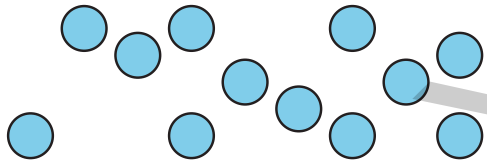
Before a lesson in the next unit, display sample student work from this item that shows different arrays. Invite students to compare the arrays. Reread the situation and ask students to explain which array(s) match the situation and why.

Practice Problems

1 from Unit 1, Lesson 17

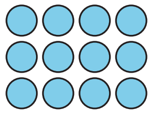
Student Task Statement

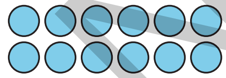
Rearrange the circles to make an array in 2 different ways.



Solution

Sample response:





Sec C

2 from Unit 1, Lesson 18

Student Task Statement

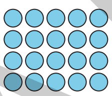
There are 4 rows of water bottles in the box. There are 5 bottles in each row.

a. Draw an array to represent the situation.

b. Write a multiplication expression to represent the number of bottles.

Solution

Sample responses: a. 4×5 or 5×4



a.

3 from Unit 1, Lesson 19

Student Task Statement

There are 5 rows of chairs in the room. There are 4 chairs in each row. How many chairs are in the room?

a. Write a multiplication equation to represent the situation.
Use a symbol for the unknown.

b. Find the value that makes your equation true.

Solution

a. $5 \times 4 = ?$ or $4 \times 5 = ?$

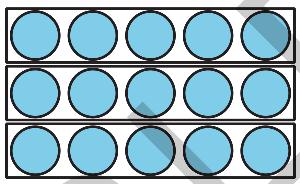
b. $5 \times 4 = 20$ or $4 \times 5 = 20$ There are 20 chairs in the room.

4 from Unit 1, Lesson 20

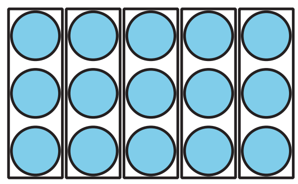
Student Task Statement

a. Write a multiplication equation that represents each array.

A



B



b. How are the arrays alike? How are they different?

Solution

a. Sample response:

A. $3 \times 5 = 15$ or $5 \times 3 = 15$

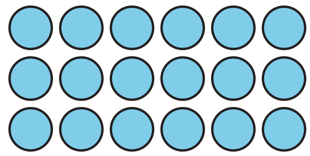
B. $5 \times 3 = 15$ or $3 \times 5 = 15$

b. Sample response: Both arrays have 15 circles in 3 rows and 5 columns. The first one shows the 3 rows of 5, and the second one shows the 5 columns of 3.

5 from Unit 1, Lesson 16

Student Task Statement

a. Explain or show 2 different ways that you see equal groups in the array.

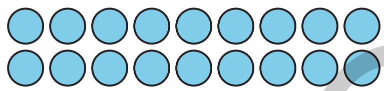


b. Rearrange the dots to make a different array.

Solution

a. Sample response:
   - There are 3 rows, and each row has 6 dots.
   - There are 6 columns, and each column has 3 dots.

b. Sample response:

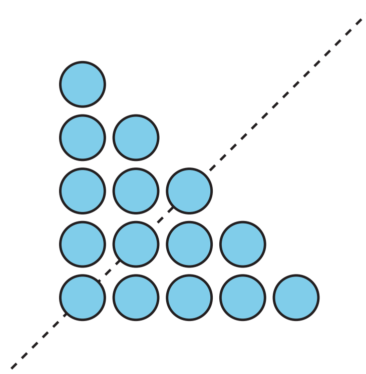


6 Exploration

Student Task Statement

Andre says that there are an odd number of circles in this picture.

Do you agree with Andre? Explain or show your reasoning.



Solution

Sample response: Yes. The circles that are not on the dotted line can be put in pairs: 4 pairs for the first row and column and 2 pairs for the second row and column. The line goes through 3 more circles, which is 1 pair and 1 leftover circle.

7 Exploration

Student Task Statement

Find a collection of objects in the classroom or at home that is arranged in an array.

a. Describe the objects.

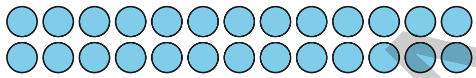
b. Create a drawing of the objects.

c. Write an equation showing how many objects there are.

Solution

Sample response:

a. There are 13 rows of seats on the bus. Two kids sit in each row.



b.

c. $2 \times 13 = 26$ or $13 \times 2 = 26$

Sec C

Unit 1, Lesson 16

Arrange Objects into Arrays

Standards

Building On	**2.OA.C.4**
Addressing	**3.OA.A.1**
Building Towards	**3.OA.A.1**

Instructional Routines

- Notice and Wonder

Goals

- Create arrays and describe (orally) the relationship between objects arranged in an array and equal groups of objects.

Student Facing Learning Goals

Let's make some arrays.

Sec C

Lesson Purpose

The purpose of this lesson is for students to describe arrays and arrange objects into arrays.

Narrative

In grade 2, students were introduced to arrays and related them to addition expressions and equations. In this lesson, students deepen their understanding of arrays as they arrange physical objects and relate arrays to multiplication and equal groups. Students use connecting cubes to represent arrays in Activity 2 and in the *Cool-Down*.

When working with array situations, students may see the equal groups in an array in either the rows or the columns. For example, when representing 3 rows of 5 chairs, they may create a 3 by 5 array or a 5 by 3 array. This is fine, as long as students can correctly describe where the "3 rows of 5 chairs" are represented in their array. Students will learn about commutativity in the last lesson in this section, so if questions about commutativity arise, record them publicly for discussion in that lesson.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Connecting cubes: Activity 2

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins

Teacher Reflection Questions

In previous grades, students saw examples of arrays, such as 10-frames, and counted objects arranged in arrays. How does that previous experience support their work with equal groups in arrays in this lesson?

Synthesis Estimate	10 mins
Cool-down	5 mins

Warm-up

10 mins

Notice and Wonder: Eggs

Standards

Building On **2.OA.C.4**

Building Towards **3.OA.A.1**

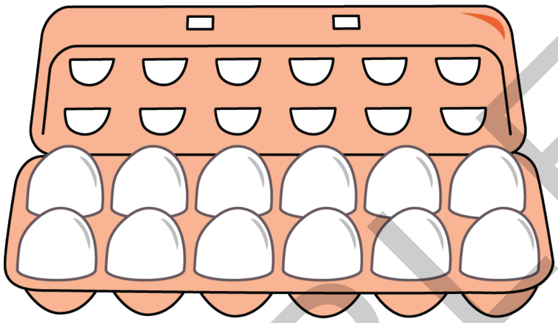
Instructional Routines

- Notice and Wonder

The purpose of this *Warm-up* is to elicit ideas students have about objects arranged in an array, which will be useful when students arrange equal groups into arrays in a later activity. While students may notice and wonder many things about this image, ideas around arrangement and equal groups are the important discussion points. When students notice the arrangement of the eggs, they look for and make use of structure (MP7).

Student Task Statement

What do you notice? What do you wonder?



Student Response

Students may notice:

- There are 12 eggs.
- There are 2 groups of 6.
- I see 6 groups of 2.

Students may wonder:

- How many eggs would be in two cartons?
- Are there other equal groups that I don't see?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- "How does having the eggs in a carton help you see equal groups?" (I can see how they could be split into equal groups. I can see 6 eggs in each row. I can see 6 groups of 2.)
- "The eggs are arranged in an array. An **array** is an arrangement of objects in rows and columns. Each column must contain the same number of objects as the other columns, and each row must have the same number of objects as the other rows."

Sec C

Activity 1

15 mins

Compare Equal Groups and Arrays

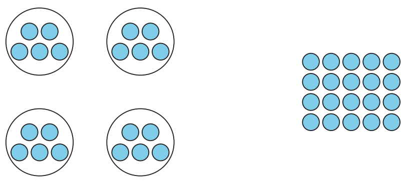
Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to describe an array as an arrangement of objects into rows with an equal number of objects in each row and into columns with an equal number in each column. This will be helpful in the next activity when students arrange objects into arrays and describe arrays in terms of multiplication.

When students decide whether or not they agree with Noah about seeing equal groups in the array and explain their reasoning, they construct a viable argument and critique the reasoning of others (MP3).

Student Task Statement



1. How does arranging the dots into an array affect how you see the number?
2. Noah says he sees equal groups in the drawing with 4 circles and 5 dots in each circle, but he says there are no equal groups in the array. Do you agree with Noah? Explain your reasoning.

Student Response

1. Sample response: The array allows you to see different numbers inside the 20, like 2 groups of 10 or 5 groups of 4.
2. Sample response: No, there are 5 dots in each row, so each row is an equal group.

Launch

- Groups of 2
- Display the images.
- "Consider these drawings. On the left is a drawing showing equal groups. On the right is an array. How are the drawings alike? How are they different?" (They both have 20 dots. They both have groups of 5. The dots are arranged differently. The second group has the dots in rows. The first drawing has the dots in circles.)
- 1–2 minutes: quiet think time
- Share and record responses.

Activity

- "Work with your partner to describe how arranging the dots into an array affects how you see the number."
- 2–3 minutes: partner work time
- Share responses.
- "Read what Noah says about the dots and decide whether you agree or disagree. Be ready to share your reasoning."
- 1 minute: independent work time
- "Now discuss whether you agree or disagree with Noah with your partner."
- 2–3 minutes: partner discussion

Activity Synthesis

- "How are arrays related to equal groups?" (There are the same number of dots in each row, so the rows are equal groups. There are the same number of dots in each column, so the columns are equal groups.)

Activity 2

20 mins

Arrange into Arrays

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to build arrays with physical objects and describe the arrays in terms of multiplication. Students focus on where equal groups can be seen in arrays. Students will write expressions and equations to represent arrays in future lessons. In the activity, students are asked to create different arrays with 24 cubes. It is not an expectation of grade 3 for students to find all the arrays for a given number.

When students notice that the rows or columns in an array have the same number of objects and relate this to equal groups, they look for and make use of structure (MP7).

Keep connecting cubes out for the *Cool-down*.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: Create a visual display of the various arrays created by the students. As students discuss each array, annotate the display with the language used, such as “array,” “rows,” “columns,” and “equal groups.”
Advances: Speaking, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify what is important or most useful to pay attention to. Display the sentence frame, “The next time I arrange objects in an array, I will remember to . . . .“
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Memory

Required Materials

Materials To Gather

- Connecting cubes: Activity 2

Required Preparation

- Each group of 2 needs 60 cubes.

Student Task Statement

1. Use cubes to make 6 groups of 5.
   - Arrange them into an array.
   - Explain or show how the array is related to equal groups.
2. Count out 20 cubes.
   - Arrange them into as many arrays as you can.
   - Explain or show how each array is related to equal groups.
3. Count out 24 cubes.
   - Arrange them into as many arrays as you can.
   - Explain or show how each array is related to equal groups.

Sec C

Student Response

1. Students may arrange the cubes into a 5 by 6, 6 by 5, 3 by 10, 10 by 3, 15 by 2, 2 by 15, 1 by 30, or 30 by 1 array.

   Sample response: The array is related to multiplication because there are 5 in each row, which is like having equal groups, and that is what multiplication represents.
2. Students may arrange the cubes into a 1 by 10, 10 by 1, 2 by 10, 10 by 2, 5 by 4, or 4 by 5 array.

   Sample response: The array is related to multiplication because I see 2 cubes in each row. The rows are like equal groups.
3. Students may arrange the cubes into a 4 by 6, 6 by 4, 3 by 8, 8 by 3, 2 by 12, 12 by 2, 1 by 24 or 24 by 1 array.

   Sample response: The array is related to multiplication because I see 4 cubes in each column. The columns are like equal groups.

Launch

- Groups of 2
- Give each group of 2 students at least 60 connecting cubes.

Activity

- 7–10 minutes: partner work time
- For the last two problems, make sure students know that they need to use all 20 or all 24 cubes in each array.
- Monitor for students who relate the same number of objects in each row and column in an array to how multiplication expresses equal groups of objects.

Activity Synthesis

- Have students share different arrays they made for the last problem.
- "Why were you able to create different arrays with 24 objects?" (There are many ways to put 24 things into equal groups.)

Lesson Synthesis

"Today we arranged objects into arrays and described how arrays are related to equal groups."

“What did you think about when you arranged cubes into arrays?” (I dealt the cubes into equal groups and then arranged them into an array. I thought about equal groups I could use to make the total and made each group a row with the same number of things in each row. I arranged them until there were the same number of cubes in each row and the same number of cubes in each column.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Cool-down

5 mins

Array Arrangement

Standards

Addressing **3.OA.A.1**

Student Task Statement

Arrange 12 cubes into an array.

Explain or show how the array is related to multiplication.

Student Response

Sample response: It’s related to multiplication because there are 4 cubes in each row, which is like 3 equal groups.

Responding To Student Thinking

Students aren’t sure how to arrange cubes into an array.	Next Day Supports Use the next day's *Warm-up* to brainstorm tips for arranging objects into an array.
The work of this lesson builds from the equal-group concepts developed in a prior unit.	Prior Unit Support Grade 2, Unit 8, Section B Rectangular Arrays

Unit 1, Lesson 17

Match and Draw Arrays

Standards

Addressing	**3.OA.A.1**
Building Towards	**3.OA.A.1**

Instructional Routines

- Card Sort
- MLR1 Stronger and Clearer Each Time
- Which Three Go Together?

Goals

- Draw arrays to represent equal groups and describe (orally and in writing) the relationship between the arrays and multiplication.
- Match (orally) arrays and drawings that represent the same equal groups.

Student Facing Learning Goals

Let's match arrays to equal groups and draw arrays.

Lesson Purpose

The purpose of this lesson is for students to relate arrays to drawings of equal groups and to describe arrays in terms of multiplication.

Narrative

Students first match arrays to drawings of equal groups. Then they redraw drawings of equal groups as arrays. The work of this lesson connects to upcoming lessons when students represent arrays with expressions and equations.

Make connecting cubes or counters available to students who need them. Keep collecting ideas that arise about commutativity.

Access For Students with Disabilities

- Engagement

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 2

Materials To Copy

- Card Sort Arrays Cards (1 copy for every 2 students): Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins

Teacher Reflection Questions

What question do you wish you had asked today? When and why should you have asked it?

Synthesis Estimate	10 mins
Cool-down	5 mins

Warm-up

10 mins

Which Three Go Together: Arrangements

Standards

Building Towards **3.OA.A.1**

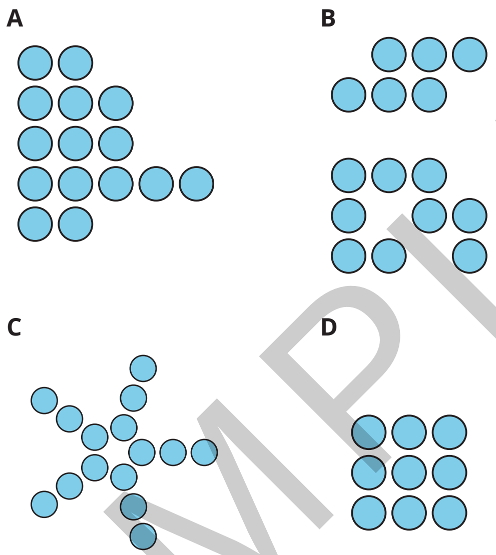
Instructional Routines

- Which Three Go Together?

The purpose of this *Warm-up* is for students to compare four arrangements of dots to elicit the attributes, or structure, of an array. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. During the discussion, ask students to explain the meaning of any terminology they use, such as “rows,” “columns,” “groups,” and “array.”

Student Task Statement

Which 3 go together?



Student Response

Sample responses:

A, B, and C go together because:
- They have 15 dots.

A, B, and D go together because:
- They are organized in rows and columns.

A, C, and D go together because:
- The equal groups of dots are right next to each other

Launch

- Groups of 2
- Display the image.
- “Pick 3 that go together. Be ready to share why they go together.”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 2–3 minutes: partner discussion
- Share and record responses.

Activity Synthesis

- “How would you rearrange each group of dots to make an array?" (For A, I would rearrange the 5 dots on the right side to make a column of 5 dots. For B, I’d push all the dots over to the left so they line up in columns and move the top 2 rows closer to the bottom 3 rows. For C, I’d rearrange the groups of 3 to make rows or columns. I wouldn’t have to do anything for D because it is an array.)”

or touching.

B, C, and D go together because:

- They show equal groups of 3 in straight lines — horizontal (rows) or diagonal lines.

Activity 1

20 mins

Card Sort: Arrays

Standards

Addressing **3.OA.A.1**

Instructional Routines

- Card Sort
- MLR1 Stronger and Clearer Each Time

The purpose of this activity is for students to relate drawings of equal groups to arrays. Specifically, students identify arrays that have the same number of objects in each row or column as each drawing has in each group. In some arrays, the equal groups in the drawing are represented as rows, and in some, they are represented in columns. Students have the opportunity to explain the connections they see between the drawings and arrays, receive feedback from their peers, and revise their explanation for precision and clarity (MP3, MP6). This will be useful in future lessons when students record multiplication expressions and equations to represent arrays.

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: reading, writing.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Attention, Organization

Required Materials

Materials To Copy

- Card Sort Arrays Cards (1 copy for every 2 students): Activity 1

Required Preparation

- Create a set of cards from the blackline master for each group of 2 or 4 students.

Student Task Statement

Your teacher will give you a set of cards.

1. Match each drawing of equal groups to an array. Be ready to explain your reasoning.
2. Choose a match you and your partner made.

Launch

- Groups of 2 or 4
- Give each group a set of cards.

Write down how you know the drawing matches the array.

Student Response

1. Matches:
    - A, J
    - B, K
    - C, G
    - D, H
    - E, F
    - I, L
2. Sample response: B and K match because the array has 2 dots in each row and the drawing has 2 dots in each group. Also, the drawing has 7 groups, and the array has 7 rows. They both have 14 dots.

Activity

- "This set of cards includes drawings of equal groups and arrays. Match each drawing to an array. Work with your group to explain your reasoning."
- 8 minutes: small group work time
- "Independently choose a match you and your group made. Write down how you know that the drawing matches the array."
- 2 minutes: independent work time

MLR1 Stronger and Clearer Each Time

- Instruct students to find a partner in their group.
- "Share your response to why your cards match with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time

Activity Synthesis

- Invite 2–3 students to share how they decided which cards go together.
- "Did your group agree on the matches? What did you look for to decide two cards were matches?" (Yes, we looked for equal groups that had the same number of dots in a group as one of the rows or columns in the array.)
- Listen for language students use to describe their matches and the structure of the arrays. As needed, ask:
    - "What do you mean by _____?"
    - "What else could we call _____?"
    - "How could you use 'equal groups' to explain your match?"
- Highlight the use of terms like "row," "column," and "equal groups."

Activity 2

Draw Arrays

15 mins

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to draw arrays from given arrangements of dots. Students draw an array from dots in equal groups to reinforce the definition of an array and then draw as many arrays as they can from 16 randomly placed dots. Having cubes or counters for students to physically rearrange would be helpful in this activity.

Access for English Language Learners

MLR8 Discussion Supports. To support partner discussion, display the following sentence frames: “This array matches the diagram because . . . .” “This array shows multiplication because . . . .”
Advances: Conversing, Representing

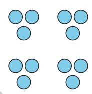
Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 2

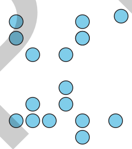
Student Task Statement

1. a. Draw 1 way the dots could be rearranged into an array.



   b. Explain or show how the array is related to multiplication.

2. a. Draw ways that the dots could be arranged into arrays. Draw as many ways as you can.



   b. Explain or show how each array is related to multiplication.

Student Response

1. Sample responses:
   a. 4 by 3 array, 3 by 4 array, 6 by 2 array, 2 by 6 array
   b. The array shows multiplication because each row has 3 dots, which is an equal group.
2. Sample responses:
   a. 2 by 8 array, 8 by 2 array, 4 by 4 array

Launch

- Groups of 2
- Give students access to connecting cubes or counters.

Activity

- “Work independently to draw a way that the dots in the first problem could be arranged into an array.”
- 2 minutes: independent work time
- “Discuss how you arranged your dots and how the array is related to multiplication with your partner.”
- 1 minute: partner discussion
- “How did you rearrange the dots to make an array?” (Since there were 3 in each group, I put 3 dots in each row. I saw 2 groups of 6, so I made 2 rows of 6.)
- Consider asking:
  - “Did anyone create a different array?”
- “Now you are going to make as many arrays as you can from 16 dots.”
- 2–3 minutes: independent work time
- “Share how you rearranged the dots into arrays with

b. The array shows multiplication because each column has 2 dots, which is an equal group.

your partner. See if together you can come up with any other arrays.”

- 3–5 minutes: partner work time

Activity Synthesis

- “What kinds of equal groups did you make from 16 dots? How can you see the equal groups in the arrays you made?” (I can make 2 groups of 8. I drew it as 2 rows of 8 dots.)

Lesson Synthesis

“Today we made drawings that showed how groups of dots could be rearranged into arrays.”

“What do you need to think about when you draw an array?” (Make sure all the rows have the same number of dots and all the columns have the same number of dots. Make the number of groups the number of columns or rows in the array and then draw how many are in each group in each column or row. All the dots have to be used.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Cool-down

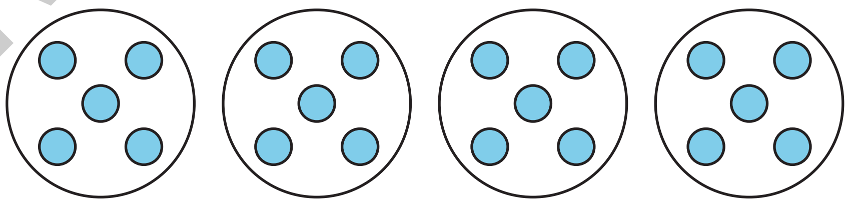
5 mins

Draw and Describe

Standards

Addressing **3.OA.A.1**

Student Task Statement



1. Redraw the equal groups as an array.
2. Describe how the drawing and the array are related.

Student Response

1. Students create a 4×5, 5×4, 2×10, or 10×2 array.
2. Sample response: There are 5 dots in each group and there are 5 dots in each row. There are 4 groups and there are 4 rows. They both have 20 dots.

Responding To Student Thinking

Students do not describe how the rows and columns of the array are connected to the equal groups in the drawing.	Next Day Supports Use the next day's *Warm-up* to have students discuss how they see equal groups in the drawing and in the array.
The work of this lesson builds from the equal-group concepts developed in a prior unit.	Prior Unit Support Grade 2, Unit 8, Section B Rectangular Arrays

Unit 1, Lesson 18

Represent Arrays with Expressions

Standards

Addressing **3.OA.A.1**

Instructional Routines

- How Many Do You See?

Goals

- Represent multiplication situations using arrays and multiplication expressions.

Student Facing Learning Goals

Let’s represent situations with arrays and expressions.

Lesson Purpose

The purpose of this lesson is for students to represent multiplication situations with arrays and multiplication expressions.

Narrative

In a previous lesson, students arranged objects into arrays and described the arrays in terms of equal groups. In this lesson, students write expressions to represent arrays to further connect arrays and multiplication (MP2).

As students connect arrays to expressions, they may write 3×5 or 5×3 to represent 3 rows of 5 chairs. This is fine as long as students can correctly describe where the “3 rows of 5 chairs” is represented in their array or expression. Keep collecting ideas that arise about commutativity.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR2

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

In an upcoming lesson, students will learn about the commutative property of multiplication. What do you notice in their work from today’s lesson that you might leverage in that future lesson?

Sec C

Warm-up

10 mins

How Many Do You See: An Array of Shapes

Standards

Addressing **3.OA.A.1**

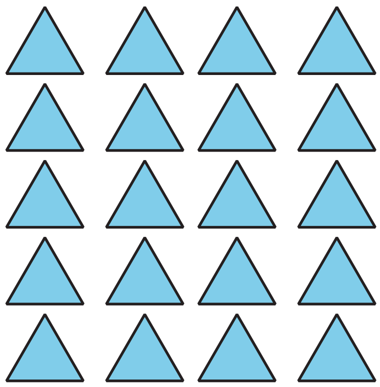
Instructional Routines

- How Many Do You See?

The purpose of this *How Many Do You See?* is for students to subitize or use grouping strategies to describe the images they see. When students use the structure of the array to figure out how many objects are shown, they look for and make use of structure (MP7).

Student Task Statement

How many do you see? How do you see them?



Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Student Response

Sample responses:

- 20: I saw 4 groups of 5 or 5 groups of 4.
- 20: I see 2 groups of 10 or 10 groups of 2.

Activity Synthesis

- "How did seeing equal groups help you know how many triangles there were in the array?"
- Consider asking:
  - "Who can restate the way _____ saw the triangles in different words?"
  - "Did anyone see the triangles the same way but would explain it differently?"

Activity 1

20 mins

Represent Array Situations

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to represent equal-groups situations with arrays and multiplication expressions. Students should have the option to use math tools, such as counters or connecting cubes, to create the arrays before they draw them. Connecting situations, arrays, and expressions reinforces the idea that multiplication can be used to express the total number of objects in equal groups (MP2).

Access for English Language Learners

MLR2 Collect and Display. Amplify language used to describe arrays. On a visible display, record words, phrases and expressions, such as "row," "column," "each," "for every," "3 by 5," "5 by 3," "5×3," and "3×5." Include diagrams and annotations. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Representing, Reading, Speaking

Access for Students with Disabilities

Representation: Internalize Comprehension. Make connections between representations visible. Invite students to use gestures or drawings as they verbally describe correspondences between their arrays and expressions. For example, "The 4 in my expression 4×5, shows the number of rows, and the 5 shows that there are 5 cars in each row."
Supports accessibility for: Conceptual Processing, Memory

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 1

Required Preparation

- Each group of 2 will need 20 connecting cubes or counters.

Student Task Statement

1. Use objects or drawings to represent each situation with an array.
    a. There are 3 rows of chairs. Each row has 5 chairs.
    b. There are 4 rows of cars. Each row has 5 cars in it.
    c. There are 2 rows of eggs. Each row has 6 eggs.
    d. There are 2 teams of students lined up. Each team has 10 students.
2. Write a multiplication expression to represent each situation.
    a. ____________________
    b. ____________________
    c. ____________________

Launch

- Groups of 2
- Give students access to connecting cubes or counters.
- Display and read the first situation.
- "Take a minute to represent this situation with an array. You can use drawings or objects."
- 2 minutes: independent work time
- "Discuss your ideas with your partner."
- 2 minutes: partner discussion
- Share responses. Emphasize ways students used equal groups to create their arrays.

Activity

- "Work with your partner to represent the next three

d. ______________________

Student Response

1. a. 3 by 5 or 5 by 3 array drawn or built
   b. 4 by 5 or 5 by 4 array drawn or built
   c. 2 by 6 or 6 by 2 array drawn or built
   d. 2 by 10 or 10 by 2 array drawn or built
2. a. 3×5 or 5×3
   b. 4×5 or 5×4
   c. 2×6 or 6×2
   d. 2×10 or 10×2

situations with an array. Be prepared to share how you see equal groups in your array."

- If students build an array, tell them to also draw it in the space provided so they can refer to the array when they write their expressions.
- 5 minutes: partner work time
- Have students share an array for problems 2–4. Try to show both drawings and arrays made of objects.
- "How do you see equal groups in your arrays?" (The rows can show the number of groups, and the number in each group is the number in each row.)
- Display the first situation again.
- "Let's revisit the first situation. What multiplication expression would represent this situation?"
- 30 seconds: quiet think time
- Share responses. Emphasize how students use equal groups to write the expression.
- "Work with your partner to write multiplication expressions for the other three situations."
- 2 minutes: partner work time

Activity Synthesis

- "How did you use equal groups to write your multiplication expression?" (I thought about how many groups were in the situation and then how many things were in each group. For example, I know that 2 teams of 10 students is 2 groups of 10, so I can write 2×10.)

Activity 2

Connect Arrays to Expressions

15 mins

Standards

Addressing **3.OA.A.1**

The purpose of this activity is for students to apply their knowledge from previous activities to draw arrays that match multiplication expressions. Have connecting cubes or counters available for students who need them. In the Launch, students use themselves to make an array for the expression 4×6. Feel free to adjust this expression to better fit the number of students in your class.

Student Task Statement

Draw an array for each multiplication expression. Be prepared to share your reasoning.

1. 2×3
2. 5×2
3. 4×4

Student Response

1. 3 by 2 or 2 by 3 array
2. 2 by 5 or 5 by 2 array
3. 4 by 4 array

Launch

- Groups of 2
- Display 4×6.
- "Work together as a class to use yourselves to create an array for this multiplication expression."
- If the number of students does not exactly match the product, ask extra students to monitor the array and be prepared to explain where they see parts of the expression in the array.
- 3–5 minutes: whole-class work time

Activity

- "Work with your partner to draw an array for each multiplication expression. Be ready to share connections you notice between the multiplication expressions and arrays."
- 2–3 minutes: partner work time

Activity Synthesis

- Share an array or two for each expression.
- "What connections did you and your partner see between the multiplication expressions and arrays?" (The factors tell us how many things are in each row and column. For 2×3, we drew an array with 2 columns that have 3 things in each column. For 2×3, we drew an array with 2 rows that have 3 things in each row.)

Advancing Student Thinking

If students draw equal groups that are not arrays, consider asking:

- "How did you use the expression to create your drawing?"
- "How could we rearrange your drawing into an array?"

Lesson Synthesis

Display a situation from the first activity, and an array and an expression that represents the situation.

"We learned that multiplication is how we express the total number of objects in equal groups."

"How did your knowledge of equal groups help you create arrays and write expressions for multiplication situations?" (I thought about how many groups there were and drew the groups as each row [or column]. Then the number of groups tells me how many rows [or columns] there are. The array and the expression represent the total number of objects in the problem.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Cool-down

5 mins

Array Situation

Standards

Addressing **3.OA.A.1**

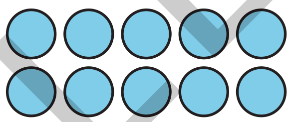
Student Task Statement

There are 2 rows of plants. Each row has 5 plants.

1. Draw an array to represent the situation.
2. Write a multiplication expression to represent the situation.

Student Response

1. Sample response:



2. 2×5 or 5×2

Responding To Student Thinking

Students write a multiplication expression that does not match the array they created, or they do not write an expression.	Next Day Supports Use the *Launch* of the next day's activity to have students discuss how to write an expression that represents an array.
The work of this lesson builds from the equal-group concepts developed in a prior unit.	Prior Unit Support Grade 2, Unit 8, Section B Rectangular Arrays

Unit 1, Lesson 19

Solve Problems Involving Arrays

Standards

Addressing **3.OA.A.1, 3.OA.A.3, 3.OA.C.7, 3.OA.D.9**

Instructional Routines

- MLR5 Co-Craft Questions
- Number Talk

Goals

- Explain (orally) a strategy for representing and solving multiplication problems involving arrays.
- Represent multiplication problems involving arrays using equations with a symbol for the unknown.

Student Facing Learning Goals

Let's solve problems involving arrays.

Lesson Purpose

The purpose of this lesson is for students to represent an array situation using an equation with a symbol for the unknown number and solve problems.

Narrative

In previous lessons, students represented equal-groups situations using arrays and multiplication expressions, with an emphasis on equal groups. Equal groups continue to be emphasized in this lesson, as students learn that finding the product in a multiplication equation gives the total number of objects in the related array.

As students connect arrays to equations, they may write $3 \times 5 = 15$ or $5 \times 3 = 15$ to represent 3 rows of 5 chairs. This is fine as long as students can correctly describe where the "3 rows of 5 chairs" is represented in their array or equation. Keep collecting ideas that arise about commutativity.

Access For Students with Disabilities

- Action and Expression

Access For English Learners

- MLR8

Lesson Timeline

Warm-up	10 mins
Activity 1	25 mins
Activity 2	10 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

Who has been sharing their ideas in class lately? Make a note of students whose ideas have not been shared and look for an opportunity for them to share their thinking in tomorrow's lesson.

Warm-up

10 mins

Number Talk: One Less Group

Standards

Addressing **3.OA.C.7, 3.OA.D.9**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have about equal groups in multiplication expressions and to see a pattern as one factor is decreased. These understandings help students develop fluency and will be helpful later in this lesson when students will need to use multiplication to answer questions about array situations.

When students notice that the product decreases by a group of 2 as the number being multiplied by 2 decreases by 1, they look for and express regularity in repeated reasoning (MP8).

Student Task Statement

Find the value of each expression mentally.

- 10×2
- 9×2
- 8×2
- 7×2

Student Response

- 20: 2 groups of 10 is 20.
- 18: I counted by 2 (2, 4, 6, 8, 10, 12, 14, 16, 18).
- 16: I counted by 2, or it's one group of 2 less than the last equation.
- 14: I counted by 2, or it's one group of 2 less than the last equation.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "What pattern do you see as you look at the expressions and their values? Why is that happening?" (The factor that isn't 2 goes down by 1 each time. The value of the product decreases by 2 because I have one less group of 2.)
- As needed, record student thinking using equal-groups drawings or arrays to help all students visualize the pattern.
- Consider asking:
  - "Did anyone notice a different pattern"?
  - "Did anyone notice the same pattern but would explain it differently?"

Activity 1

25 mins

Array of Colors

Standards

Addressing **3.OA.A.1, 3.OA.A.3**

Instructional Routines

- MLR5 Co-Craft Questions

The purpose of this activity is for students to use the *Co-Craft* math language routine to write questions that can be asked about array situations and to relate array situations to equations. Students should be encouraged to use whatever strategy or representation feels appropriate to them. Given their prior experiences, they may represent the situation with an array and skip-count or consider equal groups in other ways to find the total.

This activity uses *MLR5 Co-Craft Questions. Advances*: writing, reading, representing.

Student Task Statement

There are 7 rows. Each row has 5 crayons. How many crayons are there?

1. Solve this problem. Explain or show your reasoning.
2. Represent the situation with an array and a multiplication equation with a symbol for the unknown.

Student Response

1. 35 crayons. Sample response: I skip-counted by 5.
2. A 7 by 5 array or a 5 by 7 array. $7 \times 5 = ?$ or $5 \times 7 = ?$

Launch

- Groups of 2

MLR5 Co-craft Questions

- "Keep your books closed."
- Display: "There are 7 rows."
- "Write a list of mathematical questions that could be asked about this situation." (What's in the rows? How many things are in each row? How many things are there altogether?)
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "What do these questions have in common? How are they different?" (They all have to do with rows. They have to do with more detail about the rows, like what's in the rows and how many are in each row.)
- Reveal the task (students open books), and invite additional connections.

Activity

- "Complete the first problem by solving this problem in any way that makes sense to you."
- 3–4 minutes: independent work time
- 1–2 minutes: partner discussion
- Share a variety of student representations and solution strategies.
- "Think about the situation we have been considering. How could you represent the situation using an

equation with a symbol for the unknown?"

- 2 minutes: quiet think time
- "Share your equation with your partner. Together, rewrite the equation with the solution you found in place of the symbol."
- 2 minutes: partner work

Activity Synthesis

- "What equation(s) did you write?"
- Display an equation with a symbol and one with the solution.
- "How does each part of the equation connect to the situation?" (The 7 is the number of rows. The 5 is the number of crayons in each row. The 35 represents the total number of crayons, but it was a question mark when we didn't know how many there were.)

Activity 2

10 mins

Tyler's Trees

Standards

Addressing **3.OA.A.3**

The purpose of this activity is for students to write an equation with a symbol for the unknown to represent an array situation. Then they answer the question about the situation. Encourage students to use whatever strategy or representation feels appropriate to them. Given their prior experiences, they might represent the situation with an array and skip-count or consider equal groups in other ways to find the total.

In the *Launch*, it may be helpful to ask students to tell their partner a quick story or ask any questions about the context of the first problem. To ensure all students have access, it may also be helpful to display images for students to reference about coconut trees or Mexico.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: If necessary, invite students to repeat their question using mathematical language. For example, "Can you say that again, using the word array?"
Advances: Listening, Speaking

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Activity: Invite students to show thinking using connecting cubes or counters.
Supports accessibility for: Conceptual Processing

Student Task Statement

For each problem:

- Write a multiplication equation with a symbol for the unknown to represent the situation.
- Solve the problem. Show your reasoning.

1. A field of coconut trees in Mexico has 5 rows of trees. Each row has 9 trees. How many trees are there?
2. Tyler wants to plant coconut trees in a community garden in Florida. He will plant 2 rows of 4 trees. How many trees will Tyler plant?

Student Response

1. $5 \times 9 =$ ______ or $9 \times 5 =$ ______, 45 trees. Sample response: I counted by five 9 times. 5, 10, 15, 20, 25, 30, 35, 40, 45
2. $2 \times 4 = ?$ or $4 \times 2 = ?$, 8 trees. Sample response: I know that 2 groups of 4 is 8.

Launch

- Groups of 2
- Display the image.
- "Coconut trees are grown as a crop in warm climates and have lots of uses. Sometimes they are grown in rows. What are some other crops that are grown in rows?" (corn, strawberries, carrots)

Activity

- "Now you are going to practice what we just learned about solving array situations and writing an equation with a symbol for the unknown."
- 5–7 minutes: independent work time
- As you circulate, consider asking:
  - "How does each number or symbol in your equation connect to the situation?"
  - "How are you using equal groups to find the solution to the problem?"
- "Share your strategy with your partner. Ask any questions you have about your partner's ideas."
- 2 minutes: partner discussion

Activity Synthesis

- "What questions do you still have about solving array problems or writing an equation with a symbol for the unknown?"

Lesson Synthesis

Display the information from the first problem in Activity 2.

A field of coconut trees in Mexico has 5 rows of trees. Each row has 9 trees. How many trees are there?

$5 \times 9 = ?$

$5 \times 9 = 45$

"How do each of these equations represent the array situation?" (In both equations, 5×9 represents the 5 rows of trees with 9 trees in each row. The first equation uses =? to show the question, "How many trees are there?" The second equation uses = 45 to show the answer because 5 rows of 9 trees equals 45 trees in all.)

Consider asking: "What does the question mark in the first equation represent in the situation? What does the 45 in the second equation represent in the situation?"

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Cool-down

5 mins

Clare's Cards

Standards

Addressing **3.OA.A.3**

Student Task Statement

Clare has 3 rows of baseball cards. Each row has 10 cards. How many cards does she have?

1. Write an equation with a symbol for the unknown number to represent the situation.
2. Find the number that makes the equation true. Explain or show your reasoning.

Sec C

Student Response

1. Sample response: 3×10 = ? or 10×3 = ?
2. 30. Sample response: 30 because 3 rows of 10 is 10, 20, 30.

Responding To Student Thinking

Students write an equation that doesn't use a symbol for the unknown number.

Next Day Supports
Before the *Warm-up*, have students discuss how they can represent the problem with an equation before they know the solution, and how they can represent the problem with an equation once they know the solution.

Unit 1, Lesson 20

The Commutative Property

Standards

Building On	**2.NBT.B.5**
Addressing	**3.OA.B.5**
Building Towards	**3.NBT.A.2**

Instructional Routines

- MLR1 Stronger and Clearer Each Time
- Number Talk

Goals

- Compare and contrast (orally) situations involving equal groups of rows and situations involving an equal group of columns.
- Represent an array with different multiplication equations and explain (orally) how they each represent the array.

Student Facing Learning Goals

Let's learn about the commutative property.

Lesson Purpose

The purpose of this lesson is for students to describe the commutative property of multiplication using arrays.

Narrative

In previous lessons, students used drawings of equal groups and arrays to represent multiplication situations. They also connected multiplication expressions and equations to these representations. In this lesson, students are introduced to the commutative property. Students will notice that the same product can be represented by different situations, arrays, or equations. Re-organizing the arrays or reversing the order of the factors in a multiplication expression does not change the total number of objects. It is important that students connect their equations to the corresponding situations and representations. They should be able to correctly explain what each factor and the product represents in their equations.

Note that students are not expected to use the name of the property. They should, however, be able to rely on their conceptual understanding of multiplication to explain why the product does not change when the order of the factors changes.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins

Teacher Reflection Questions

What part of the lesson went really well today in terms of students learning? What did you do that made that part go well?

Sec C

Synthesis Estimate	10 mins
Cool-down	5 mins

Warm-up

10 mins

Number Talk: Subtraction

Standards

Building On **2.NBT.B.5**

Building Towards **3.NBT.A.2**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for subtracting within 100. It also provides an opportunity to observe student strategies as they work toward becoming fluent in addition within 1,000.

When students use strategies based on place value to subtract, they look for and make use of structure (MP7).

Sec C

Student Task Statement

Find the value of each expression mentally.

- 70 – 10
- 68 – 10
- 70 – 12
- 68 – 12

Student Response

- 60: Seven tens minus 1 ten would be 6 tens, which is 60.
- 58: It's just like the first one, but the number you're subtracting from is 2 less, so the difference would be 2 less.
- 58: This is like the first problem, but you're subtracting 2 more from 70, so instead of 60, it would be 58.
- 56: It's like the first problem, but you're starting with 2 less and taking 2 more, so it would be 4 less than the first answer. I subtracted tens from tens and ones from ones and ended up with 5 tens and 6 ones.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "How was place value helpful as you found the difference in these problems?" (I subtracted the tens first then adjusted my answer each time. I subtracted tens from tens and ones from ones.)
- Consider asking:
  - "Who can restate ____'s reasoning in a different way?"
  - "Did anyone use the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to ____'s strategy?"

Activity 1

Learn More about Multiplication

20 mins
PLC Activity

Standards

Addressing **3.OA.B.5**

The purpose of this activity is to introduce the commutative property. Students write array situations for a pair of arrays and discuss similarities and differences. While the situations will have the same total number of objects, how the objects are grouped should be different. Then students write equations to go with the arrays and situations, and make connections between the representations (MP2). Students notice that, while the order of the factors in the multiplication equation changes, the product does not change (MP7).

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: Create a visual display of the equations and corresponding arrays. As students describe their connections between the equations and the situations, annotate the display to illustrate the connections. For example, below each number, write either “rows,” “columns,” or “total.”
Advances: Speaking, Representing

Access for Students with Disabilities

Representation: Access for Perception. Students may benefit from the opportunity to observe a demonstration that shows the grouping of dots in the arrays. For example, prepare a display of Image A and Image B showing only the dots. Then, invite students to watch as you circle the groups accordingly.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Task Statement

What do you notice? What do you wonder?

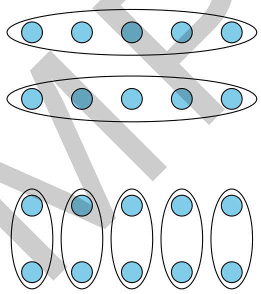
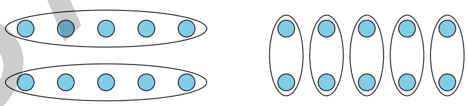


Image A Image B



1. a. Write a description of a situation for each array.

   Image A

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?” (Both groups of dots are arranged as arrays. They both have 10 dots. One array has groups of 2. One array has groups of 5. Why are the dots grouped differently? Are the arrays the same?)
- 1 minute: quiet think time
- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Activity

- “Let’s consider these two arrays in more detail. Write an array situation for each array.”
- 3–5 minutes: independent work time

Image B

b. How are the situations alike? How are they different?

2. a. Write a multiplication equation for each situation.

Image A Image B

b. How does your equation connect to the situation and array?

Image A

Image B

Student Response

Sample responses:

1. a. There are 2 rows of students. Each row has 5 students. There are 5 groups of students. Each group has 2 students. or There are 2 bags with 5 apples in each bag. There are 5 bags with 2 apples in each bag.
   b. The first situation is 2 rows of 5, but the second situation is 5 columns of 2. The situations both describe 10 objects.
2. a. $2 \times 5 = 10$ and $5 \times 2 = 10$
   b. The 5 and the 2 are switched in the second equation just like the rows and columns are switched. The product is the same.

- "Share your situations with your partner. Together, consider how the situations you wrote for the first array are the same and different from the situations you wrote for the second array."
- 3 minutes: partner work time
- "What was the same and what was different about the situations you wrote?"
- Share responses.
- "Now, write an equation for each situation you just came up with to match the arrays."
- 1 minute: independent work time
- "Share your equations with your partner and discuss how each number in your equations connects to your situations and the array."
- 2 minutes: partner discussion

Activity Synthesis

- "Let's write down the equations we came up with."
- Display $2 \times 5 = 10$ and $5 \times 2 = 10$.
- "How do each of the numbers in the equations connect to the situation you wrote?" (5 is the number of rows in one situation, but it's how many students are in each row in the other situation. 2 is the number of groups in one situation, but it's how many are in each group in the other situation. 10 is the total number of objects in both situations.)

Activity 2

15 mins

Revisit Arrays

Standards

Addressing **3.OA.B.5**

Instructional Routines

- MLR1 Stronger and Clearer Each Time

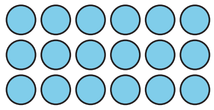
The purpose of this activity is to reinforce the idea of the commutative property. In this activity, students write two equations to match an array to show again that reversing the order of the factors does not change the product. If students do not immediately see how they might write different equations for the array, encourage them to consider different ways of grouping the dots in the array, similar to the previous activity. Students use the vocabulary they have learned for describing arrays and multiplication to explain why both equations match an array with their partner. The *Stronger and Clearer Each Time* math language routine allows students to receive feedback and revise their explanation for clarity (MP3, MP6).

If students finish early, consider drawing another array. Have students write two equations for the array and consider how they can think of the rows or columns as equal groups.

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: reading, writing.

Student Task Statement

1. Write 2 multiplication equations that represent the array.



   ____

   ____

2. Explain why both equations can represent the array.

Student Response

1. $3 \times 6 = 18$ and $6 \times 3 = 18$
2. Sample response: Both equations can represent the same array because we could think about the array showing 6 groups of 3 or 3 groups of 6. If we think of the columns as groups of 3, there are 6 of the groups. If we think about the rows as groups of 6, there are 3 of the groups. In either case, the array has a total of 18 dots, so both equations have a product of 18.

Launch

- Groups of 2
- "Talk to your partner about equations that could represent this array."
- 1 minute: partner discussion

Activity

- "Write two equations for this array. If it helps you, you can imagine grouping the dots as in the previous activity. Then write down why both equations can represent the array."
- 5 minutes: independent work time

Activity Synthesis

- "Let's write down the equations we came up with."
- Display the equations $3 \times 6 = 18$ and $6 \times 3 = 18$.

MLR1 Stronger and Clearer Each Time

- "Share why both equations can represent the array with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion.
- Repeat with 2–3 different partners.
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time
- Have students share the revisions they made to their initial draft.

Lesson Synthesis

Display a 3 by 4 array and the equations $3 \times 4 = 12$ and $4 \times 3 = 12$.

"What did we learn from thinking about arrays and seeing pairs of equations like this today?" (The order of the factors does not change the product or the total number of objects in the array or situation. Connecting the numbers in your equations to arrays and situations helps clarify what each number means.)

Display $3 \times 4 = 4 \times 3$.

"Since $3 \times 4 = 12$ and $4 \times 3 = 12$, we can write $3 \times 4 = 4 \times 3$."

"The idea that we can multiply two numbers in any order and get the same product is called the commutative property."

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

Cool-down

5 mins

Multiplication Reflection

Standards

Addressing **3.OA.B.5**

Student Task Statement

Summarize what you learned about multiplication today.

Student Response

Sample response: If we switch the order of the numbers we're multiplying, we get the same product.

Responding To Student Thinking

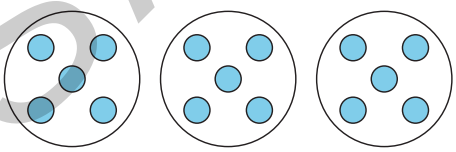
Students have responses they'd like to share with a partner.

Next Day Supports
Before the first activity, pair students to discuss their responses.

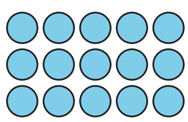
Section C Summary

We learned how equal groups are related to **arrays** and how to represent arrays with multiplication expressions and equations.

Equal groups:



Array:



Expression:

3×5

Equation:

$3 \times 5 = 15$

We also learned that we can multiply numbers in any order and get the same product.

$3 \times 5 = 15$ $\quad$ $5 \times 3 = 15$ $\quad$ $3 \times 5 = 5 \times 3$

Unit 1, Lesson 21

Game Night Seating Plan

Standards

Addressing **3.MD.B.3, 3.OA.A, 3.OA.A.3**

Instructional Routines

- Notice and Wonder

Goals

- Present (using words and other representations) a representation of a real-world problem involving equal groups.
- Represent data using a scaled bar graph.

Student Facing Learning Goals

Let's plan a game night.

Lesson Purpose

Sec C

The purpose of this lesson is for students to use their understanding of equal groups to solve a design problem.

Narrative

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling (MP4).

In previous lessons, students created scaled bar graphs and solved problems involving equal groups. In this lesson, they use these skills as they make a seating arrangement.

Students first examine a diagram showing equal groups and consider the situations it could represent. After learning that the diagram represents a seating chart, they consider the information needed to set up the seating arrangement for a game night. Students then plan a seating arrangement given some constraints—the total number of game tables and a combination of games that each involve a certain number of players—and create a display to present their solution. Finally, students create a scaled bar graph to represent the number of players that can play each game in their seating solution.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 1
- Inch tiles: Activity 1
- Tools for creating a display: Activity 1

Materials To Copy

- Centimeter Grid Paper - Standard (1 copy for every 2 students): Activity 1
- Centimeter Grid Paper - Standard (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	10 mins
Activity 1	25 mins
Activity 2	15 mins
Synthesis Estimate	10 mins

Teacher Reflection Questions

How did the modeling task support collaboration between students?

Warm-up

10 mins

Notice and Wonder: Squares and Circles

Standards

Addressing **3.OA.A**

Instructional Routines

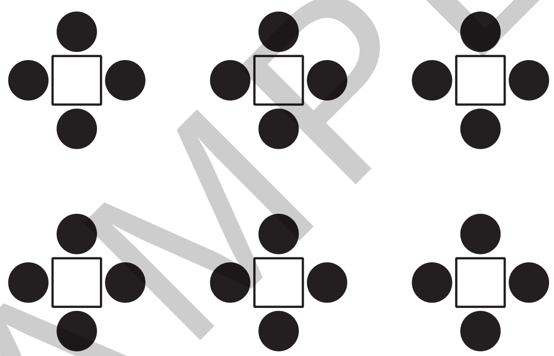
- Notice and Wonder

The purpose of this *Warm-up* is to elicit ideas about equal groups in seating arrangements and about variables that might be important when solving a real-world problem. These ideas will be useful when students plan the seating arrangement for a game night in a later activity. While students may notice and wonder many things about this image, observations about equal groups are the important discussion points.

During the *Synthesis* students learn that the image represents a seating chart and consider information they may need if they were in charge of planning the seating arrangement for a game night. When students identify information they need to know to answer a question, they model with mathematics (MP4).

Student Task Statement

What do you notice? What do you wonder?



Student Response

Students may notice:

- There are 6 squares.
- There are 4 circles around each square.
- There are 24 circles.

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- "Somebody drew the diagram to illustrate a real-world situation. What could that situation be?" (table arrangement, gameboard, sports arrangement)
- "The image shows table arrangements for a game night."
- "If you were in charge of planning the seating

- The squares are arranged like an array with 2 rows and 3 squares in each row.

Students may wonder:

- Why are the circles around the squares?
- Does something go inside the squares?
- What situation could this represent?
- Do the squares represent tables? Do the circles represent chairs?
- Is this a game board?

arrangement for a game night, what questions would you need to ask to have enough information to plan?" (How many tables are there? How many people can play each game? How many people can sit at each table? How many chairs are there? How many guests are going to the game night? How big are the tables?)

- Share and record responses. Keep this visible for the next activity.
- "In the next activity, you will create a seating chart for a game night. You will get answers to some of these questions. For other questions, you will need to make some decisions."

Activity 1

25 mins

Game Night

Standards

Addressing **3.MD.B.3, 3.OA.A.3**

The purpose of this activity is for students to plan a seating arrangement. Students are only given the information of the number of players required for each game and the total number of tables. The numbers 2, 4, 5, and 10 have been chosen to reflect the multiplication work students have done in previous lessons. Students make their own decisions about other aspects of the scenario before planning their seating arrangement. They also choose how to represent their seating arrangement.

Students may want answers from the teacher before making the arrangement. Encourage them to make their own assumptions as long as it does not contradict the given information. When students consider assumptions about information not given in a situation, they model with mathematics (MP4).

Access for English Language Learners

MLR8 Discussion Supports. Clarify any questions about the context. Give students 1–2 minutes to read and make sense of the task. Ask, "Are there any words that are unfamiliar or that you have questions about?"
Advances: Reading, Representing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Differentiate the degree of difficulty or complexity. Some students may benefit from the opportunity to complete the task with fewer game types.
Supports accessibility for: Organization, Attention

Required Materials

Materials To Gather

- Connecting cubes or counters: Activity 1
- Inch tiles: Activity 1
- Tools for creating a display: Activity 1

Materials To Copy

- Centimeter Grid Paper - Standard (1 copy for every 2 students): Activity 1

Student Task Statement

Your club is planning a game night.

Guests can play 1 of 4 games that require a different number of players:

- Game A - 2 players
- Game B - 4 players
- Game C - 5 players
- Game D - 10 players

The game room has 16 identical square tables. One person can sit on each side of the table.

1. Make a seating plan that shows a table arrangement so that each guest can play 1 of the games and all the tables are used.
2. Make a poster that includes:
    a. a seating chart
    b. an explanation about how you decided on your seating plan
    c. how many people can play games in the room with your seating plan

Student Response

Sample response:

Launch

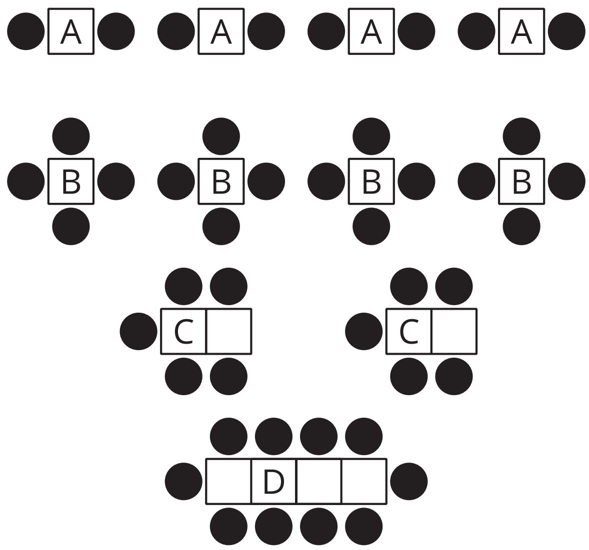
- Groups of 2 or 4
- Give each group tools for creating a visual display and access to inch tiles, grid paper, and connecting cubes or counters.
- "The first part of the task answers some of our questions, such as the number of people needed for each game and the total number of tables. You can decide the information that is not given. In the poster you make, include the information you assumed and explain what new information you got as a result. Also, include how many people can play games in the room with your seating plan."
- If needed, point out that the square tables can be pushed together to make other shapes.

Activity

- 20 minutes: small-group work time
- Monitor for:
    - Groups that describe their assumptions and explain how their assumptions impacted the arrangement. For example, if they want to set up 6 games of Game A, then they need space for 12 people.
    - Groups that make assumptions about the total number of people.

Activity Synthesis

- Invite previously selected students to display their posters for all to see.
- "What does this arrangement tell us about the situation?" (It shows us how many of each game are played. It shows us how many people can play each game. It shows us how many people can play games in the room if it's set up like this.)
- "What multiplication expression represents the



number of people that can play Game A? B? C? D?" (For Game A, 4×2 because there are 4 tables and each table has 2 players. 4×4 represents Game B because there are 4 tables with 4 players at each table. For Game C, 2×5 because there are 2 tables with 5 players at each table. Game D would be 1×10 because there's 1 table with 10 players.)

We made sure that there were tables for each game. We used 4 tables for Game D, which left us 12 tables. We decided to use 4 tables for each of the other games. So, 8 people could play Game A, 16 people could play game B, 10 people could play Game C, and 10 people could play Game D. We added $8 + 16 + 10 + 10$ to find out that 44 people could play games in the room if we used this seating chart.

Advancing Student Thinking

If students find it challenging to make decisions about unknown information, consider asking:

- "Tell me about how you've designed your seating chart so far."
- "Is there information given in the problem for what you're choosing? What are some choices you have about _____? How would it affect your seating chart if you _____?"

Activity 2

15 mins

Game Night on a Graph

Standards

Addressing **3.MD.B.3**

The purpose of this activity is for students to represent their game night plans on a scaled bar graph. In the *Activity Synthesis*, students consider how their graphs communicate information about their game night plans.

Required Materials

Materials To Copy

- Centimeter Grid Paper - Standard (1 copy for every 2 students): Activity 2

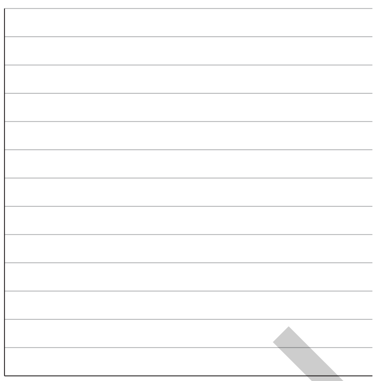
Required Preparation

- Each student needs a sheet of grid paper.

Student Task Statement

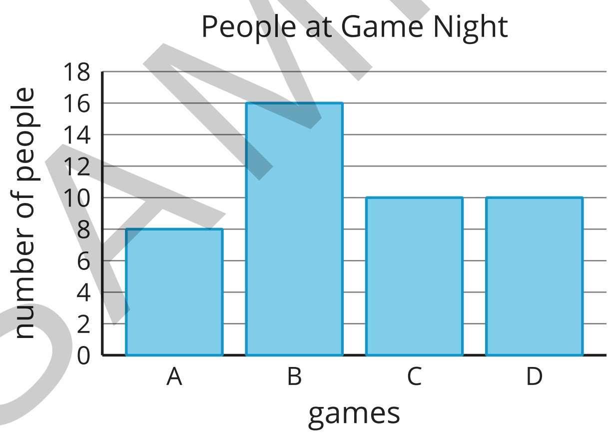
Make a scaled bar graph that shows the number of guests that can play each of the games A, B, C, and D. Be sure to include:

- a title and other labels
- a scale that counts by a number other than 1



Student Response

Sample response:



Launch

- Groups of 2 or 4
- Give each student grid paper.
- "Now you're going to make a scaled bar graph that shows how many people can play each game with your room arrangement."
- "Discuss what scale your groups will use for your graph."
- 1 minute: small-group discussion
- "You can work with your group, but everyone in your group will make their own graph. You can also choose a different scale than the rest of your group."

Activity

- 5–7 minutes: small-group work time

Activity Synthesis

- Display graphs that use different scales.
- "How did choosing different scales affect the graphs?" (Some of the graphs have shorter bars because each jump on the graph is larger. Some of the graphs are easier to read than others.)
- "What information does this bar graph give us about your game night plan?" (We can see the different games at the game night and the number of people playing each game. We can find the total number of people if we add them all up. We can see that the Game B tables would have room for the most guests.)

Lesson Synthesis

"Today, we made seating arrangements based on some given information and other things we decided."

"What decisions did you make today as you solved this problem? How did the decisions affect your plan for the game night?" (We decided to have 2 groups play Game D. This only left 8 tables for the rest of the games. We decided to try to fit as many guests as we could, so we ended up with a lot of tables used for Game B.)

Suggested Centers

- Capture Squares (1–5) , Stage 5: Multiply with 2, 5, and 10 (Addressing)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (Addressing)

End-of-Unit Assessment

1 Standards

Addressing **3.MD.B.3**

Narrative

Students choose a scale for a scaled bar graph and make the graph. Given the provided graphing space, students have 2 choices for a scale. They can use a scale of 5 and then the largest bar goes almost to the top, or they can use a scale of 10, decreasing the height of the bars. Students could choose a scale of 2 if they extend the given grid by hand. This will influence their answer for the first question but should be considered as fully correct.

Student Task Statement

The table shows the favorite seasons of some students.

favorite season	fall	winter	spring	summer
number of students	36	23	29	48

a. Which of the scales, 2, 5, or 10, will work to make a scaled bar graph to represent the data on the grid?

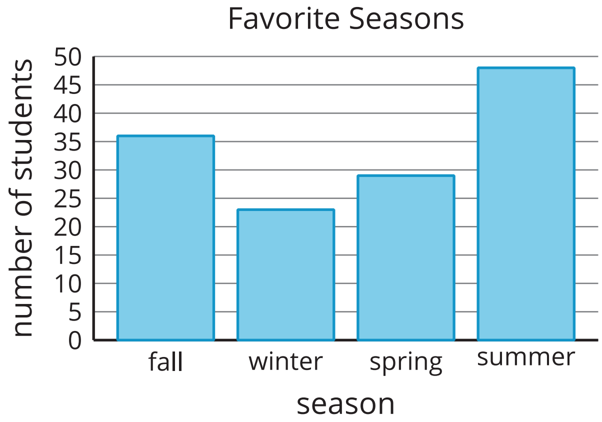
b. Create a scaled bar graph to represent the data. Use 2, 5, or 10 for your scale.

c. Explain how you chose your scale.

Solution

a. Sample response: There are 10 lines. If I make each one represent 2 students, that only goes up to 20 students and I can't show the data. With a scale of 5, it goes up to 50 and I can show all of the numbers. I can also show all of the numbers with a scale of 10 because then it would go up to 100 students.

b. Sample response:



c. Sample response: I decided to choose 5 for a scale because all of the data fit and I was able to draw the bars pretty accurately.

2

Standards

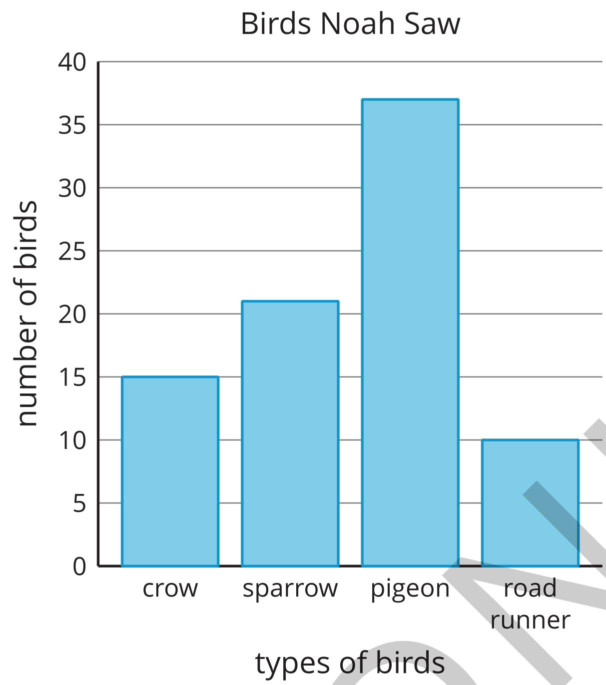
Addressing **3.MD.B.3**

Narrative

Students read a scaled bar graph and answer questions about the data. Students may select A (and not select B or C) if they do not read the scale on the graph. Students may select D or F if they confuse “fewer” and “more.”

Student Task Statement

The bar graph shows the kinds of birds that Noah saw one day.



Select **all** true statements.

A. Noah saw 3 crows.

B. Noah saw 37 pigeons.

C. Noah saw 22 more pigeons than crows.

D. Noah saw fewer sparrows than road runners.

E. Noah saw fewer crows than pigeons.

F. Noah saw 5 fewer crows than road runners.

Solution

B, C, E

3

Standards

Addressing **3.OA.A.1**

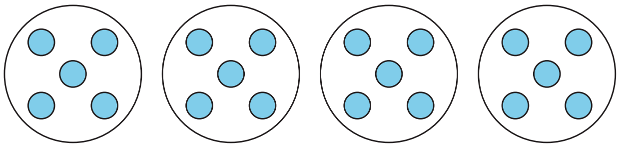
Narrative

Students write multiplication expressions to represent the number of dots in different images. These include an array and an equal groups image. In each case, students may write the order of the factors in two different ways. Students could possibly see the diagrams differently. For example, they could write 2×10 for the first image if they group pairs of 5 dots. This is not likely, but if students write a multiplication expression whose value is 20 for the first image or 30 for the second image,, they may understand the meaning of multiplication, but they may view the images differently.

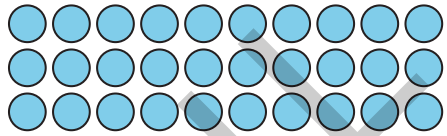
Student Task Statement

Write a multiplication expression that could represent the number of dots in each drawing.

A



B



Solution

Sample responses:

A: 4×5 or 5×4

B: 3×10 or 10×3

4

Standards

Addressing **3.OA.A.3**

Narrative

Students solve a problem about an equal-groups situation. Students may solve the problem using an expression or equation, or they may make a drawing. Listed in increasing order of abstraction, possible drawings include:

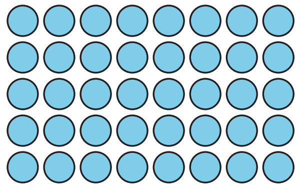
- A drawing of bags with rubber bands.
- An array.
- A tape diagram.

Student Task Statement

Elena has 5 bags. Each bag has 8 rubber bands. How many rubber bands does Elena have? Explain or show your reasoning.

Solution

40 rubber bands. Sample response: There are 5 rows of 8 circles and 40 circles total.



5

Standards

Addressing **3.OA.A.3**

Narrative

This item assesses an understanding of equal-groups situations. Students may choose to use equal-groups drawings or arrays to represent and solve the problem. The numbers are deliberately chosen to foster mental calculation, so many students may choose to show their reasoning by writing $3 \times 10 = 30$ only. Students who do not answer this question correctly may need further review of multiplication. Students who select A or B are likely performing the wrong operation with the numbers 3 and 10. Response C probably indicates incomplete work.

Student Task Statement

There are 3 soccer teams on the field. Each team has 10 players. How many soccer players are on the field altogether? Explain or show your reasoning.

A. 7

B. 13

C. 20

D. 30

Solution

D

6

Standards

Addressing **3.OA.A.3**

Narrative

Students interpret an array situation with an unknown number of columns as a multiplication equation and then find the value that makes the equation true. The numbers are friendly so that students can solve the equation by inspection or by calculating. Students may also draw an array of dots and stop when they reach 16 total dots.

Student Task Statement

Kiran has 16 cards. He arranges the cards in 2 rows. Each row has the same number of cards.

a. Explain how the equation $2 \times ? = 16$ relates to Kiran's cards.

b. How many cards are in each row? Explain how you know.

Solution

a. Sample response: If ? is the number of cards in each row, then $2 \times ? = 16$ because 16 is how many cards he has altogether and 2 is the number of rows.

b. 8 cards. Sample response: I put 16 dots in 2 equal rows and there are 2, 4, 6, 8, 10, 12, 14, 16 dots. It takes 8 dots in each row to get 16 total.

7

Standards

Addressing **3.OA.A.4**

Narrative

Students find an unknown in a multiplication equation using a way of reasoning that makes sense to them. Fluency with these facts is a yearlong progression, so any strategy for solving a multiplication equation at this point of the year is okay.

Student Task Statement

Find the number that makes each equation true.

a. $4 \times 5 =$ ________

b. $2 \times 6 =$ ________

c. $3 \times 4 =$ ________

d. $5 \times$ ________ $= 35$

e. ________ $\times 10 = 40$

Solution

a. 20

b. 12

c. 12

d. 7

e. 4

End-of-Unit Assessment Guidance

Grade 3, Unit 1

Observation	Next Unit Supports	Standard (assessment item(s))
Students show they are counting by 1 or not yet making reasonable estimates when interpreting the scale.	During the next unit, invite students to play *Sort and Display*, Stage 3. As students answer their partner's questions using the graphs, invite them to share how they used the scale. As needed, invite students to describe the scale and describe the value of a bar whose height falls between two tick marks.	3.MD.B.3 (1, 2)
Students are not yet able to explain why they chose their scale.	Encourage students to play *Sort and Display*, Stage 3 during Center Choice Time. As students create their graphs, ask them to describe how they determine the scale. As needed, invite students to consider what the graph would look like if they chose a scale of 2, 5, or 10.	
Students select some comparison statements that do not match the data.	Before students solve two-step story problems in Unit 3 Section D, invite them to make comparison statements about data involving "more" and "fewer." Consider using graphs from Unit 1 and the graphs students create as they play *Sort and Display* to generate comparison statements.	
Students write equations that do not match the images.	Before an upcoming lesson, invite students to work with a partner to explain the equations they wrote for drawings and diagrams, including any diagrams they created when finding unknown values on this assessment. Emphasize the ways students describe how their equations match the features of the drawing or diagram.	3.OA.A.1 (3)
Students create drawings, diagrams, expressions, or equations that do not match the equal-groups situations.	Throughout the next unit, invite students to explain how they see equal groups in arrays, tiled diagrams, and gridded area diagrams. Emphasize the ways students identify the equal groups and count the number of equal groups.	3.OA.A.3 (4, 5, 6)
Students do not yet explain how an equation with an unknown matches the array story.	Throughout the next unit, invite students to explain how equations match arrays, tiled rectangles, and area diagrams. Emphasize the language students use to relate each factor to the rows and columns or length and width of the diagram.	

Students show they may be adding factors rather than multiplying.	Encourage students to play *Capture Squares*, Stage 5 during Center Choice Time in the next unit. As students play, ask them to explain their thinking and any diagrams they create.	3.OA.A.4 (7)
Students show they think about equal groups when both factors are known, but they do not yet have a strategy for what to do when one factor is not known.	Encourage students to play *Five in a Row: Multiplication* during Center Choice Time in the next unit. As students play, choose a product on the board and ask them to explain whether or not it will be possible to cover it in their next turn.	

Center: Sort and Display (1–3)

Narrative

Students sort a collection and represent their data.

Stage 2: Picture or Bar Graphs

Lesson

Supporting

- Grade3.1.A1
- Grade3.1.A2
- Grade3.1.A3
- Grade3.1.A4

Narrative

Students work together to sort 20–30 objects into three or four categories, and then each student makes a picture graph or a bar graph that shows how they sorted. Provide students with an interesting collection of objects, such as:

- Pattern blocks.
- Connecting cubes.
- Counters.
- A combination of pattern blocks, cubes, and counters.
- Books.

Students take turns showing their representation to their partner and asking two questions that can be answered based on the representation.

Standards

Addressing **2.MD.D.10**

Required Materials

Materials To Gather

- Collections of up to 20 small objects

Materials To Copy

- Sort and Display Stage 2 Recording Sheet

Additional Information

Create collections of 20–30 objects with up to 3 attributes by which to sort.

Stage 3: Scaled Graphs

Lesson

Addressing

- Grade3.1.A5
- Grade3.1.A6
- Grade3.1.A7
- Grade3.1.A8

Narrative

Students work together to sort 40–100 objects into three to five categories, and then each student makes a scaled picture graph or bar graph that shows how they sorted. Provide students with an interesting collection of objects, such as:

- Pattern blocks.
- Connecting cubes.
- Counters.
- A combination of pattern blocks, cubes, and counters.
- Books.

Students take turns showing their representation to their partner and asking two questions that can be answered based on the representation.

Standards

Addressing **3.MD.B.3**

Required Materials

Materials To Gather

- Collections of up to 20 small objects

Materials To Copy

- Sort and Display Stage 3 Recording Sheet

Additional Information

Create collections of 40–100 objects with up to 5 attributes by which to sort.

Center: Five in a Row: Addition and Subtraction (1–3)

Narrative

Students take turns generating numbers and placing counters on a gameboard. The first partner to have five counters in a row wins.

Stage 6: Add within 100, with Composing

Lesson

Supporting

- Grade3.1.A5
- Grade3.1.A6
- Grade3.1.A7
- Grade3.1.A8
- Grade3.1.B9

Narrative

Students choose the counter color each will use. One student places a paper clip on the gameboard to cover one number in each gray row. They add the numbers and place their color counter on the sum. Their partner moves one of the paper clips to a different number in the same row, adds the numbers, and places their color counter on the sum. If the sum is already covered on the gameboard, they move the same paper clip to a different number. Students take turns until a player gets five counters in a row or the board is filled.

Two gameboards are provided. On one board students add a one-digit number and a two-digit number. On the other board, they add two 2-digit numbers.

Standards

Addressing **1.NBT.C.4, 2.NBT.B.5**

Required Materials

Materials To Gather

- Paper clips
- Two-color counters

Materials To Copy

- Five in a Row Addition and Subtraction Stage 6 Gameboards

Additional Information

Each group of 2 students needs 25 counters and 2 paper clips.

Stage 7: Add within 1,000, without Composing

Lesson

Supporting

- Grade3.1.B10
- Grade3.1.B11
- Grade3.1.B12
- Grade3.1.B13

Narrative

Students choose the counter color each will use. One student places a paper clip on the gameboard to cover one number in each gray row. They add the numbers and place their color counter on the sum. Their partner moves one of the paper clips to a different number in the same row, adds the numbers, and places their color counter on the sum. If the sum is already covered on the gameboard, they move the same paper clip to a different number. Students take turns until a player gets five counters in a row or the board is filled.

Standards

Addressing **2.NBT.B.7**

Required Materials

Materials To Gather

- Paper clips
- Two-color counters

Materials To Copy

- Five in a Row Addition and Subtraction Stage 7 Gameboard

Additional Information

Each group of 2 students needs 25 counters and 2 paper clips.

Stage 8: Add within 1,000, with Composing

Lesson

Supporting

- Grade3.1.B14
- Grade3.1.B15

Narrative

Students choose the counter color each will use. One student places a paper clip on the gameboard to cover one number in each gray row. They add the numbers and place their color counter on the sum. Their partner moves one of the paper clips to a different number in the same row, adds the numbers, and places their color counter on the sum. If the sum is already covered on the gameboard, they move the same paper clip to a different number. Students take turns until a player gets five counters in a row or the board is filled.

Standards

Addressing **2.NBT.B.7**

Required Materials

Materials To Gather

- Paper clips
- Two-color counters

Materials To Copy

- Five in a Row Addition and Subtraction Stage 8 Gameboard

Additional Information

Each group of 2 students needs 25 counters and 2 paper clips.

Center: Capture Squares (1–5)

Narrative

Students generate a number and connect two dots that are adjacent to the number. If that line closes the square, they capture the square and shade it in with their color. The first player to shade in three squares is the winner.

Stage 3: Add within 20

Lesson

Supporting

- Grade3.1.A1
- Grade3.1.A2
- Grade3.1.A3
- Grade3.1.A4

Narrative

Each student chooses to write in a different color. They take turns spinning the spinner (6–10), choosing a number card (0–10), and finding the sum. They connect two dots that are adjacent to that number on the gameboard. If the line they draw closes the square, they capture the square and shade it in with their color. If they cannot draw a line, they spin and choose a card again. The player to shade in three squares first is the winner. The spinner includes a wild space, where students can choose their own number.

To use the spinner, place the pencil point through one end of the paper clip and onto the center dot of the spinner. Then flick the paper clip to spin around the pencil point. The first bend of the paper clip can be straightened to make a longer dial for the spinner.

Standards

Addressing **1.OA.C.6, 2.OA.B.2**

Required Materials

Materials To Gather

- Colored pencils or crayons
- Paper clips

Materials To Copy

- Number Cards 0–10
- Capture Squares Stage 3 Gameboard
- Capture Squares Stage 3 Spinner

Stage 4: Subtract within 20

Lesson

Supporting

- Grade3.1.B9
- Grade3.1.B10
- Grade3.1.B11

Narrative

Each student chooses to write in a different color. They take turns spinning the spinner (16–20), choosing a number card (0–10), and finding the difference. They connect two dots that are adjacent to that number on the gameboard. If the line they draw closes the square, they capture the square and shade it in with their color. If they cannot draw a line, they spin and choose a card again. The first player to shade in three squares is the winner. The spinner includes a wild space, where students can choose their own number.

Standards

Addressing **1.OA.C.6, 2.OA.B.2**

Required Materials

Materials To Gather

- Colored pencils or crayons
- Paper clips

Materials To Copy

- Number Cards 0–10
- Capture Squares Stage 4 Gameboard
- Capture Squares Stage 4 Spinner

Stage 5: Multiply with 2, 5, and 10

Lesson

Addressing

- Grade3.1.B12
- Grade3.1.B13
- Grade3.1.B14
- Grade3.1.B15
- Grade3.1.C16
- Grade3.1.C17
- Grade3.1.C18
- Grade3.1.C19
- Grade3.1.C20

- Grade3.1.C21

Narrative

Each student chooses to write in a different color. They take turns rolling a number cube, spinning the spinner, and finding the product. They connect two dots that are adjacent to that number on the gameboard. If the line they draw closes the square, they capture the square and shade it in with their color. If they cannot draw a line, they roll and spin again. The first player to shade in three squares is the winner. The spinner has numbers 2, 5, and 10 and a wild space, where students can choose their own number.

Standards

Addressing **3.OA.C.7**

Required Materials

Materials To Gather

- Colored pencils or crayons
- Number cubes
- Paper clips

Materials To Copy

- Capture Squares Stage 5 Spinner
- Capture Squares Stage 5 Gameboard

Additional Information

Each group of 2 students needs 1 number cube.

Center: Five in a Row: Multiplication (3–5)

Narrative

Students take turns generating numbers and placing counters on a board. The first partner to have five counters in a row wins.

Stage 1: Factors 1–5 and 10

Lesson

Addressing

- Grade3.1.C16
- Grade3.1.C17
- Grade3.1.C18
- Grade3.1.C19
- Grade3.1.C20
- Grade3.1.C21

Narrative

Students multiply using factors of 1–5 and 10. They choose the counter color each will use. One student places a paper clip on each of two numbers from the gray row on the gameboard. They multiply the numbers and place their color counter on the product. Their partner moves one of the paper clips to a different number, multiplies the numbers, and places their color counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter. If the product is already covered on the gameboard, they move the same paper clip to a different number. The game ends when a player gets five counters in a row or the board is filled.

Standards

Addressing **3.OA.C.7**

Required Materials

Materials To Gather

- Paper clips
- Two-color counters

Materials To Copy

- Five in a Row Multiplication Stage 1 Gameboard

Additional Information

Each group of 2 students needs 25 two-color counters and 2 paper clips.