Student Edition & Teacher Guide

IMKH

An IM K–12 Math™ Curriculum

iM v.360 by Kendall Hunt

PRINT SAMPLER

GRADE

6

Illustrative Mathematics CERTIFIED ®

An IM K–12 Math™ Curriculum

IM® v.360

All-embracing, all-encompassing, and all-inclusive

IM® v.360, the new version of the IM K-12 Math curriculum has undergone significant upgrades, enhancements, and revisions based upon feedback from school leaders, teachers and students nationwide. This updated version introduces fresh activities, lessons, problems, and titles.

What is different with IM® v.360?

Upgrades to the K–5 curriculum include:

- *NEW!* Language Learning Goals, End of Unit Guidance, Checklist Guidance
- Strengthened representations of diverse cultures
- Revisions to the Course Guide content, Instructional Routines, and blackline masters
- 2 lessons added in Kinder for number writing/sense (previously found in centers but do direct lesson)
- More blackline masters included in SE so teachers don't need to copy and distribute (alleviates lift)
- Reviewing activities that could create stress (especially food/recipes - when scarcity is a real issue in urban districts)

Upgrades to the 6–12 curriculum include:

- NEW! Narrative Structures, Section-level Assessments (Checkpoints), Instructional Goals, and Teacher Reflection Questions
- Embedded guidance for building a classroom community
- Embedded Math Language Routines and revised Instructional Routine language, including for 5 Practices activities
- Revised context and activity launches to invite more students into the mathematics, including more representations of diverse cultures
- Revised lesson contexts to align with the California framework, including environmental literacy enhancements
- Unit Narratives being revised for accuracy, clarity, and length
- More guidance around BLM's which to laminate and reuse
- More blackline masters included in SE so teachers don't need to copy and distribute (alleviates lift)

Illustrative® Mathematics
LEARN MATH FOR LIFE

Kendall Hunt

GRADE 6

Student Edition

UNIT 1

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Book 1
Certified by Illustrative Mathematics®

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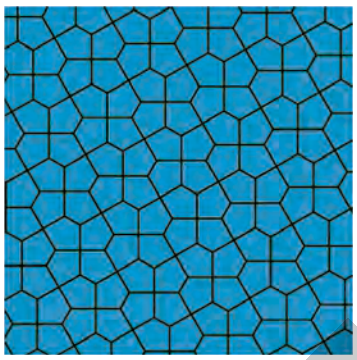
Tiling the Plane

Let’s look at tiling patterns and think about area.

1.1 Which Three Go Together: Tilings

Which three go together? Why do they go together?

A



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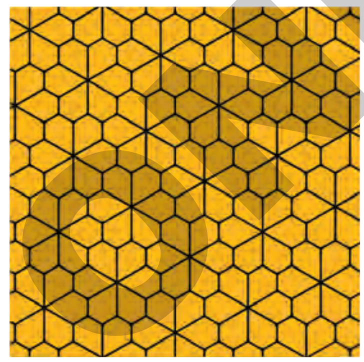

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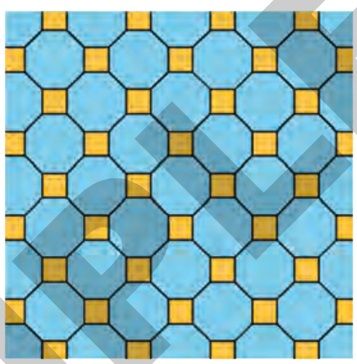

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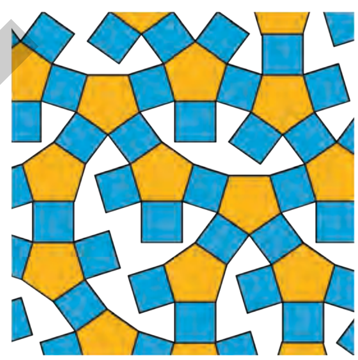

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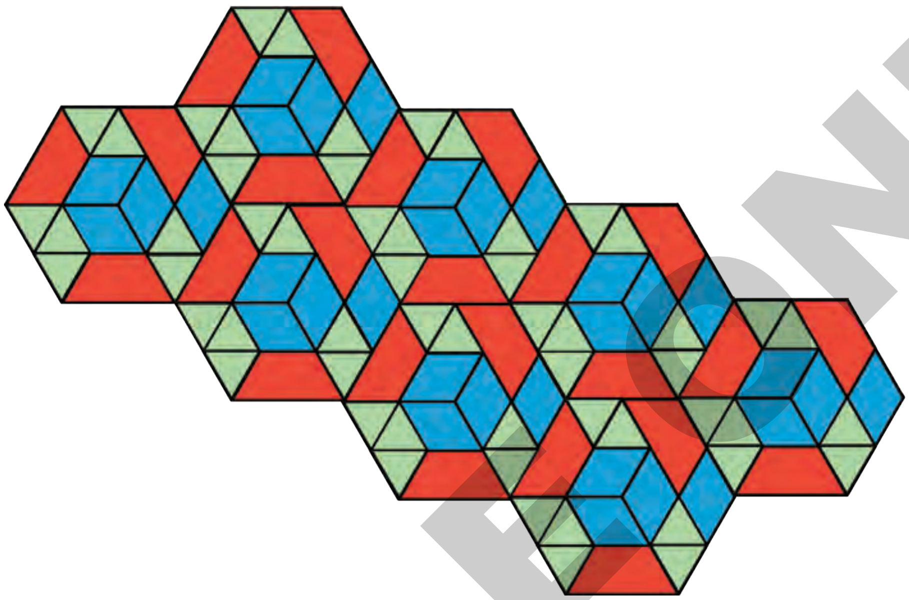
1.2 More Red, Green, or Blue?

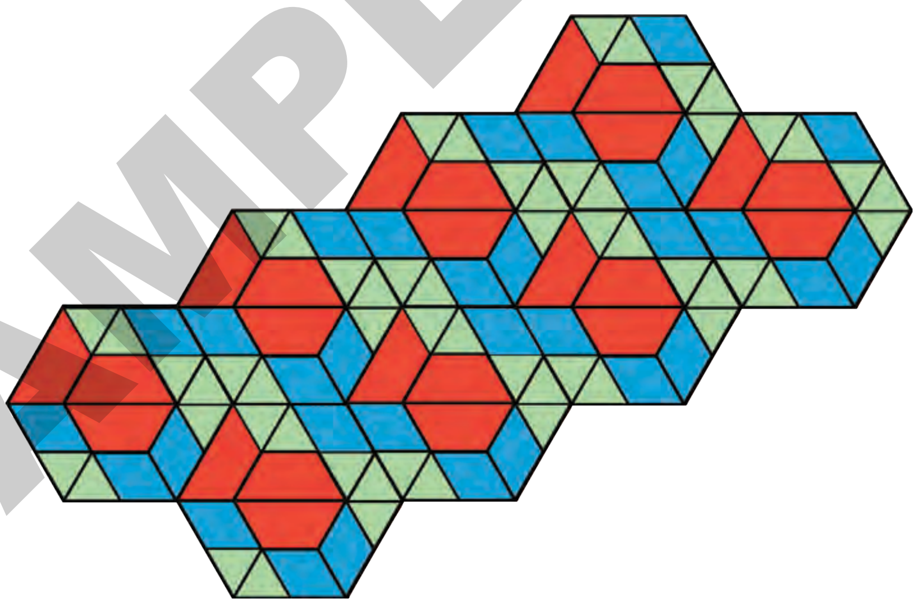
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A



Pattern B



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Are you ready for more?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

Lesson 1 Summary

In this lesson, we learned about *tiling* the plane, which means "covering a two-dimensional **region** with copies of the same shape or shapes such that there are no gaps or overlaps."

Then we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about **area**.

In future lessons, we will continue with this reasoning, and we will continue learning how to use mathematical tools strategically to help us do mathematics.

Glossary

- region

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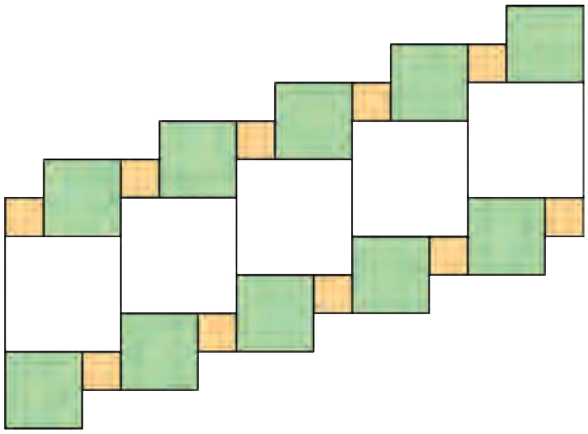
Sec A

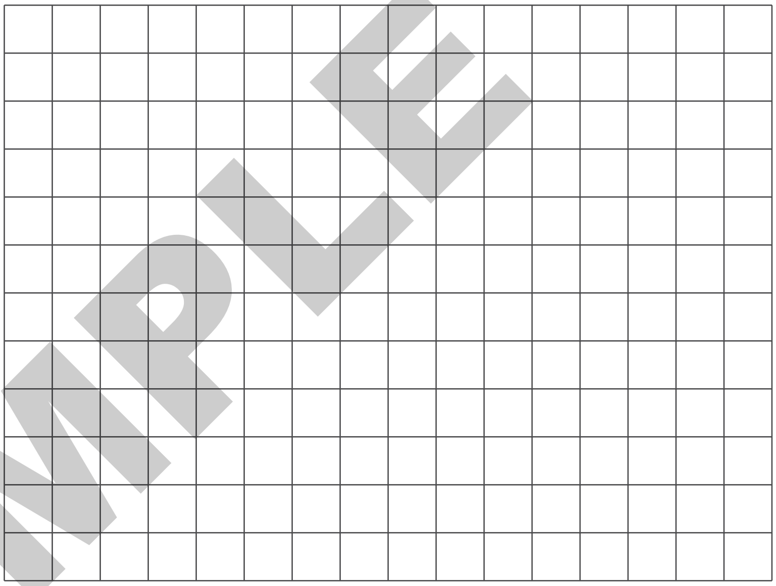
Practice Problems

1 Which square—large, medium, or small—covers more of the plane? Explain your reasoning.



2 from an earlier course

Draw three different quadrilaterals, each with an area of 12 square units.



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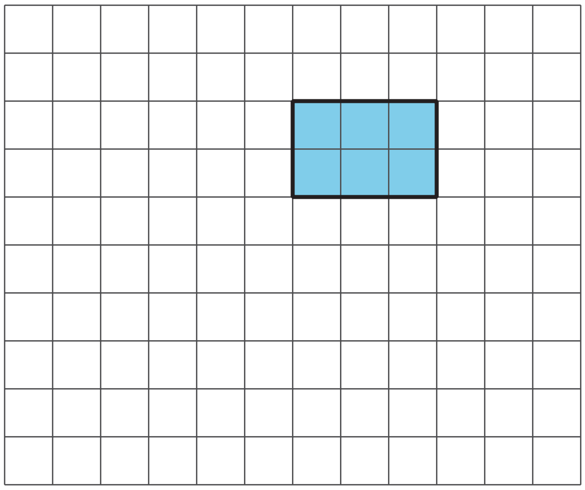
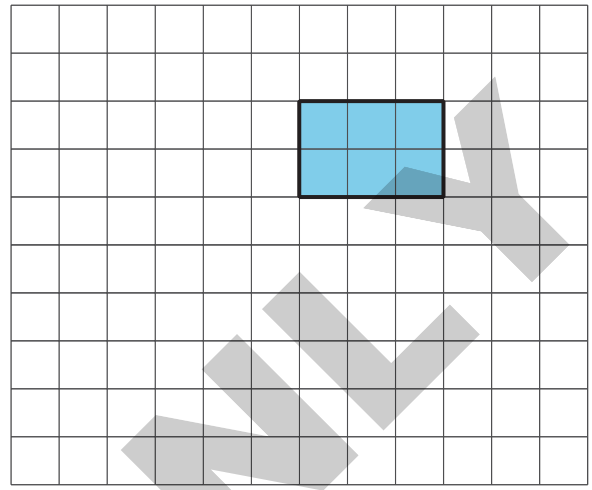
3 Use copies of the rectangle to show how a rectangle could:

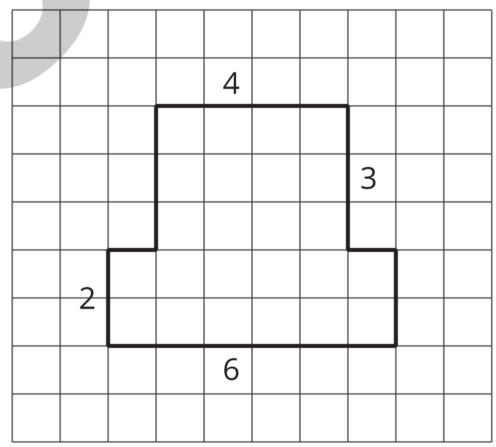
a. Tile the plane.



b. *Not* tile the plane.



4 The area of this shape is 24 square units. Select **all** the statements that are true about the area.



A. The area can be found by counting the number of squares that touch the edge of the shape.

B. It takes 24 grid squares to cover the shape without gaps and overlaps.

C. The area can be found by multiplying the sides lengths that are 6 units and 4 units.

D. The area can be found by counting the grid squares inside the shape.

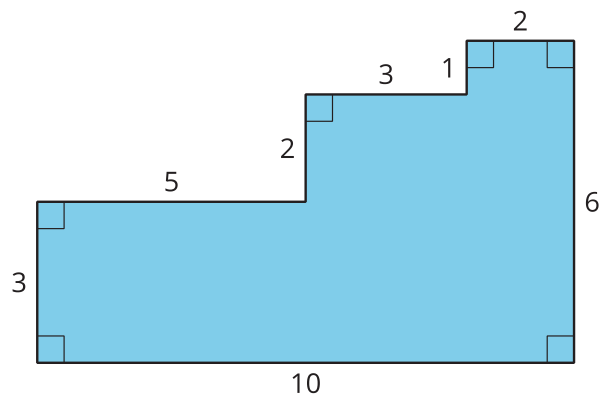
E. The area can be found by adding 4×3 and 6×2.

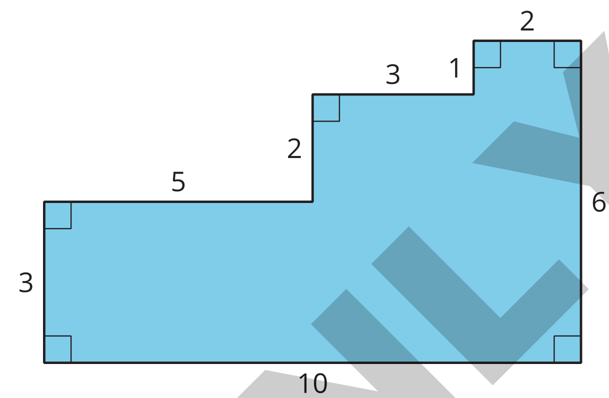
SAMPLE ONLY

Sec A

5 from an earlier course

Here are two copies of the same figure. All angles are right angles. Show two different ways for finding the area of the shaded region.





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Unit 1, Lesson 2

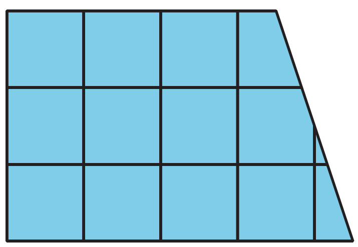
Finding Area by Decomposing and Rearranging

Let's create shapes and find their areas.

2.1 Notice and Wonder: Squares in Shapes

What do you notice? What do you wonder?

A



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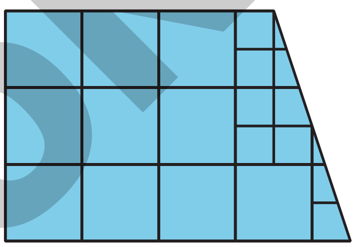

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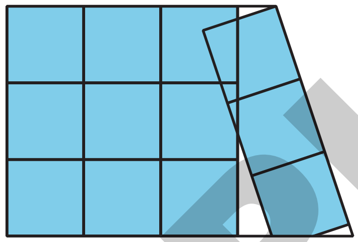

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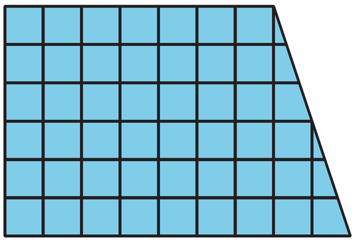

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Sec A

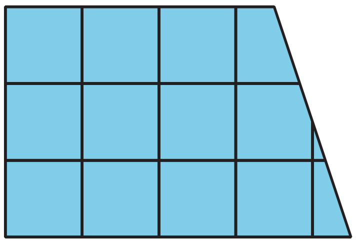
Sec A

2.2 What is Area?

You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.

A



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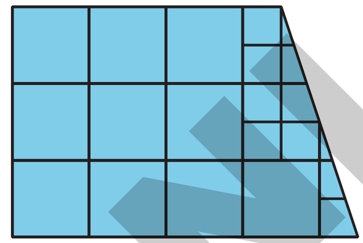

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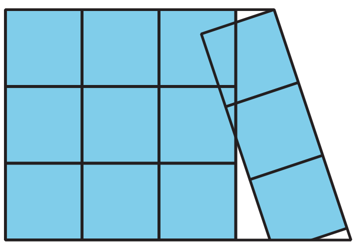

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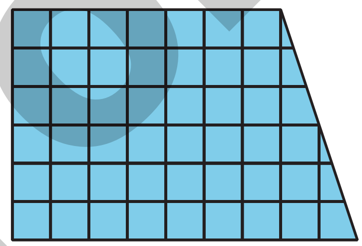

2. Write a definition of "area" that includes all the information that you think is important.

SAMPLE ONLY

2.3 Composing Shapes

Your teacher will give you 1 square and some small, medium, and large right triangles. The area of the square is 1 square unit.

1. Notice that you can put together 2 small triangles to make a square. What is the area of the square composed of 2 small triangles? Be prepared to explain your reasoning.

2. Use your shapes to create a new shape with an area of 1 square unit that is *not* a square. Trace your shape.

3. Use your shapes to create a new shape with an area of 2 square units. Trace your shape.

SAMPLE ONLY

4. Use your shapes to create a *different* shape with an area of 2 square units. Trace your shape.

Sec A

5. Use your shapes to create a new shape with an area of 4 square units. Trace your shape.

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Are you ready for more?

Find a way to use all of your pieces to compose a single large square. What is the area of this large square?

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2.4 Tangram Triangles

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is ________ square units. I know this because . . .

2. The area of the medium triangle is ________ square units. I know this because . . .

3. The area of the large triangle is ________ square units. I know this because . . .

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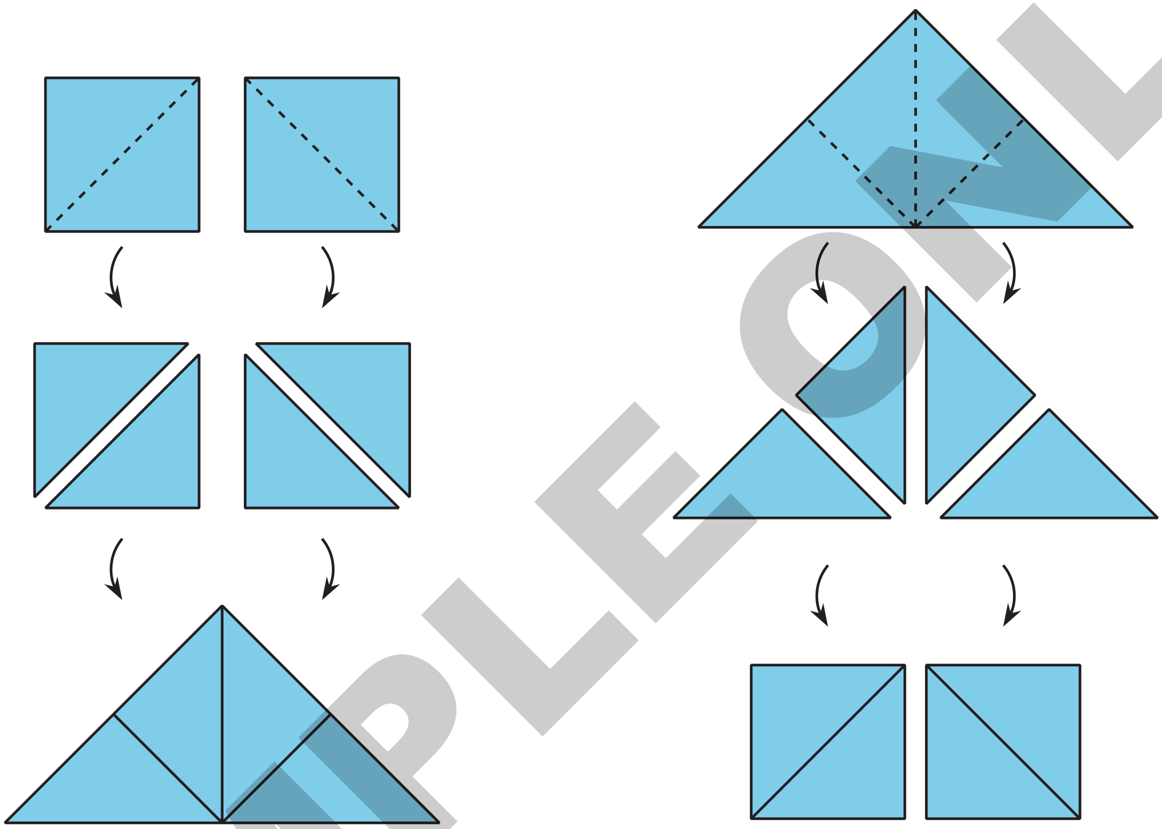
Lesson 2 Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.
2. We can *decompose* a figure (break a figure into pieces) and *rearrange* the pieces (move the pieces around) to find its area.

Here are illustrations of the two principles.



- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So, the large triangle has the *same area* as the 2 squares.
- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The triangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

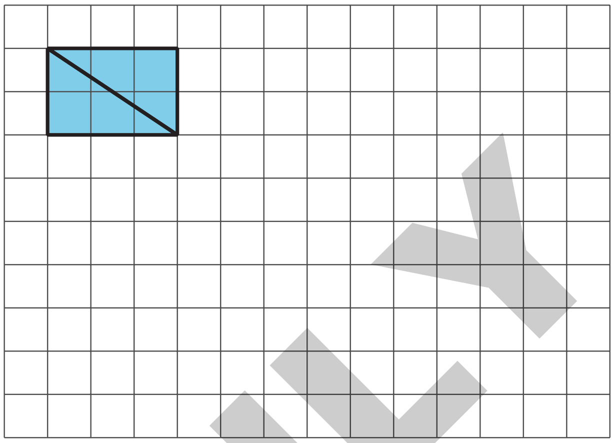
Glossary

- area

SAMPLE ONLY

Practice Problems

1 The diagonal of a rectangle is shown.

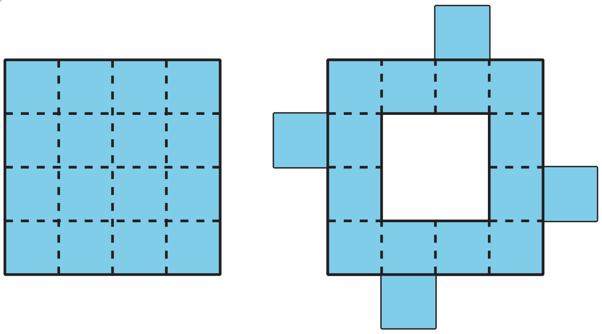


a. Decompose the rectangle along the diagonal, and recompose the two pieces to make a *different* shape.

b. How does the area of this new shape compare to the area of the original rectangle? Explain how you know.

2 Priya decomposed a square into 16 smaller, equal-size squares and then cut out 4 of the small squares and attached them around the outside to make the new figure shown.

How does the area of the new figure compare with that of the original square?



A. The area of the new figure is greater.

B. The two figures have the same area.

C. The area of the original square is greater.

D. We don't know because neither the side length nor the area of the original square is known.

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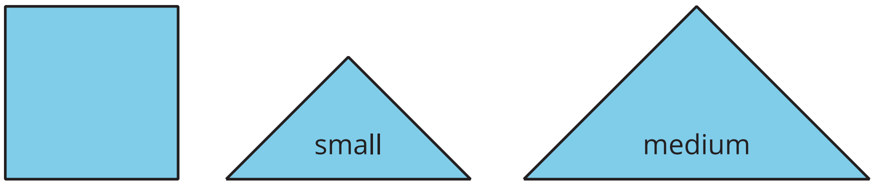
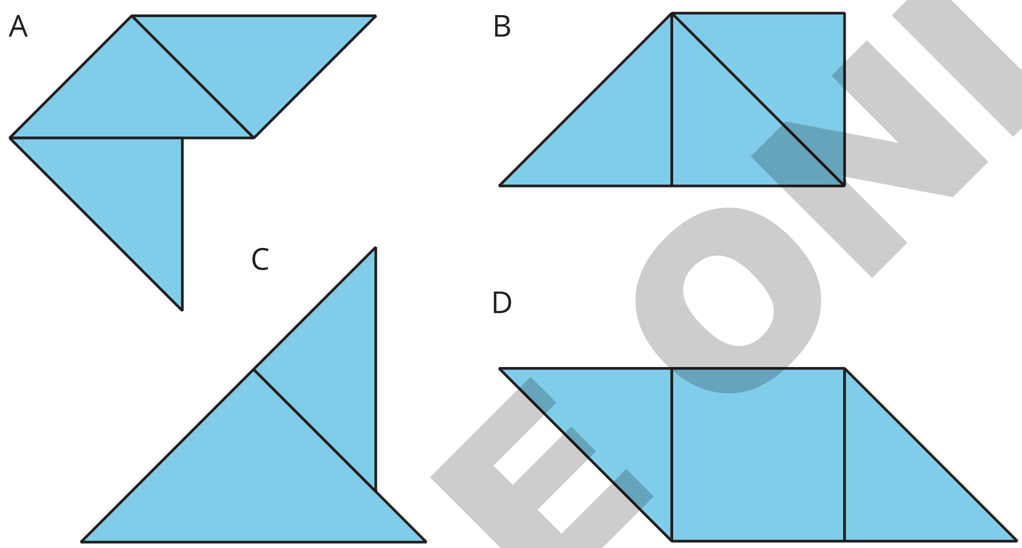
3 The area of the square is 1 square unit. Two small triangles can be put together to make a square or to make a medium triangle.



Which figure also has an area of $1\frac{1}{2}$ square units? Select **all** that apply.



A. Figure A

B. Figure B

C. Figure C

D. Figure D

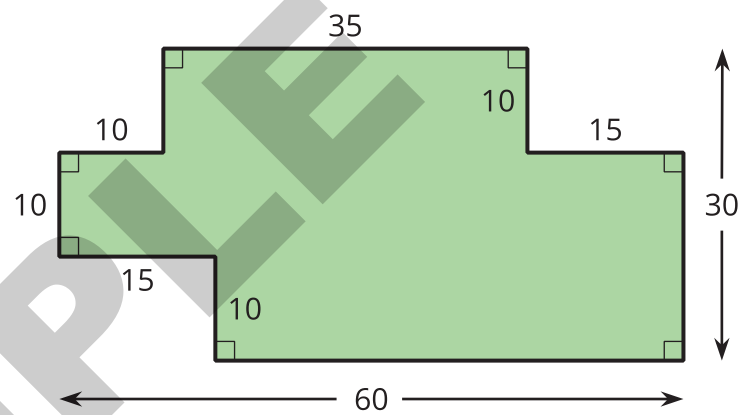
SAMPLE ONLY

4 from an earlier course

The area of a rectangular playground is 78 square meters. If the length of the playground is 13 meters, what is its width?

5 from Unit 1, Lesson 1

A student said, “We can’t find the area of this shaded region because the shape has many different measurements, instead of just a length and a width that we could multiply.”



Explain why the student’s statement about area is incorrect.

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Unit 1, Lesson 3

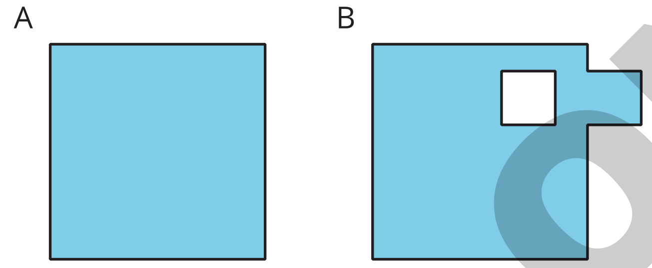
Reasoning to Find Area

Let's decompose and rearrange shapes to find their areas.

Sec A

3.1 Comparing Regions

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.



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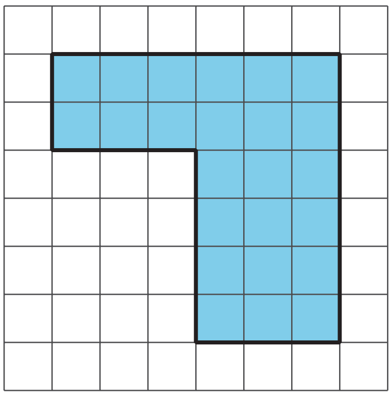
3.2 On the Grid

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.

A



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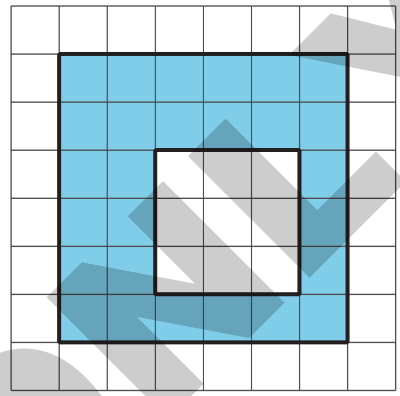

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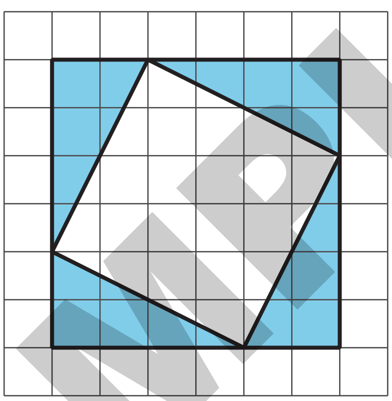

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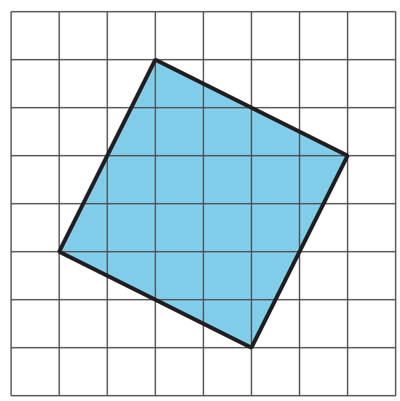

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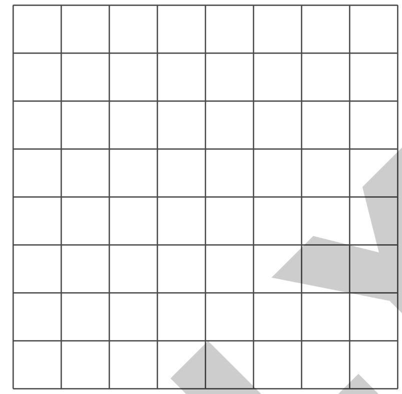
Sec A

Are you ready for more?

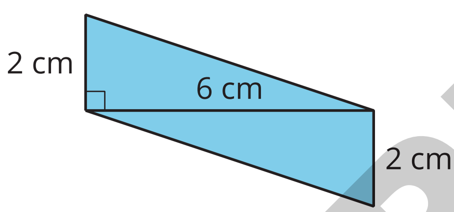
Rearrange the shaded triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.



3.3 Off the Grid

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

E



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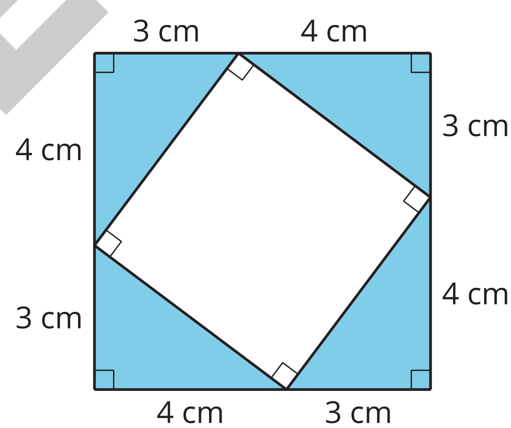

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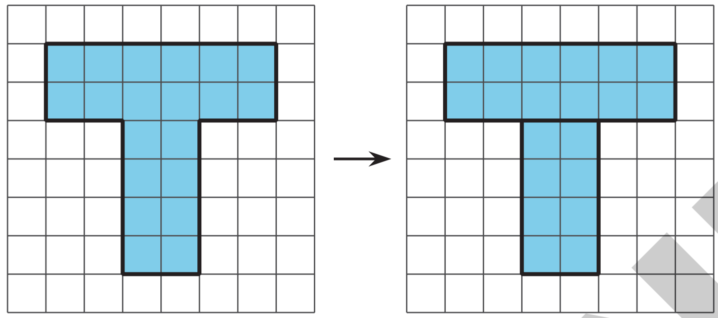
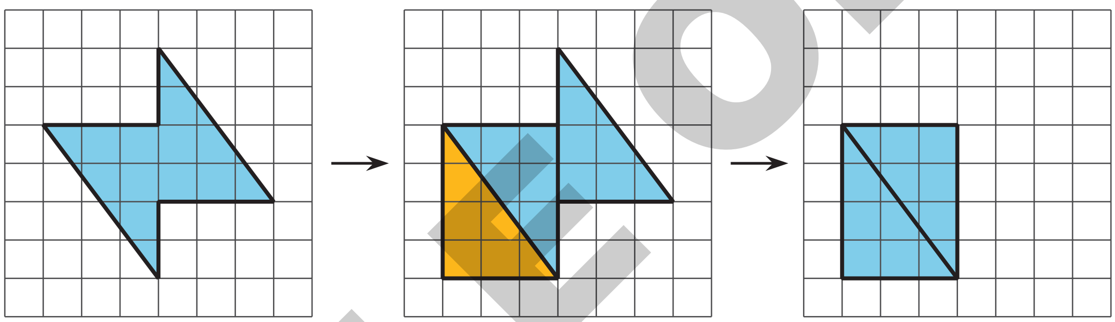
Lesson 3 Summary

There are different strategies we can use to find the area of a region. We can:

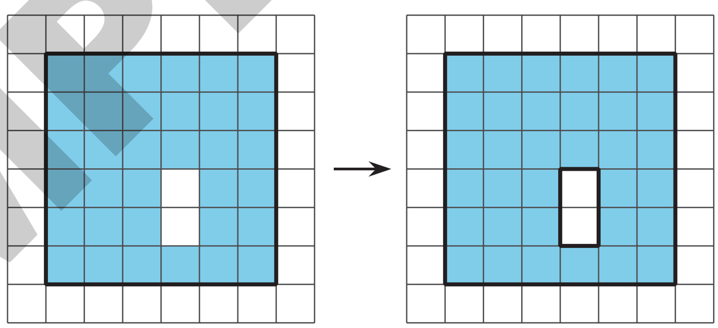
- Decompose it into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.



- Decompose it and rearrange the pieces into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.

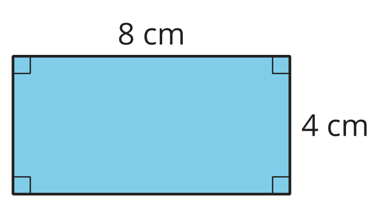


- Consider it as a shape with a missing piece. We calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



The area of a figure is always measured in square units.

When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters. For example, the area of this rectangle is 32 square centimeters.



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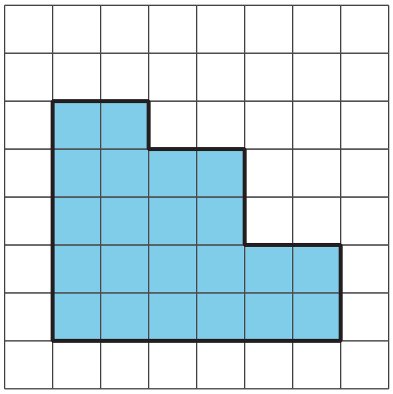
Practice Problems

1 Find the area of each shaded region. Show your reasoning.

A



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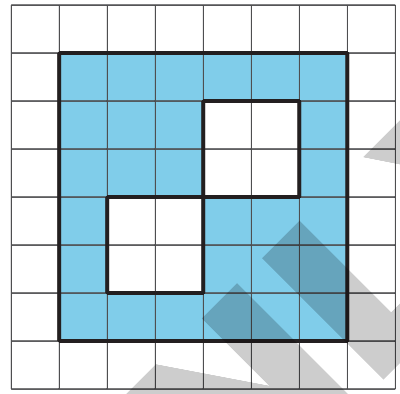

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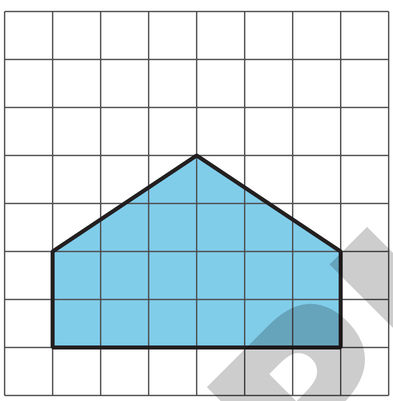
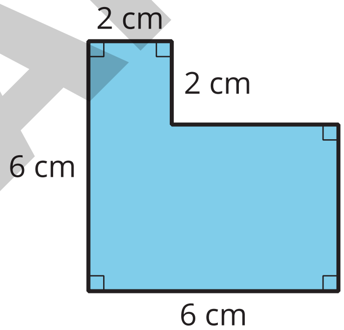


SAMPLE ONLY

2 Find the area of each shaded region. Show or explain your reasoning.

A



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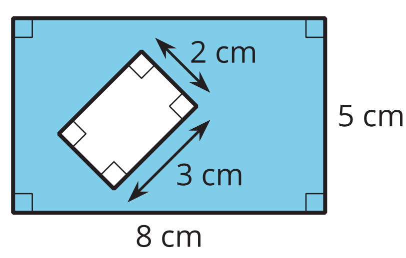

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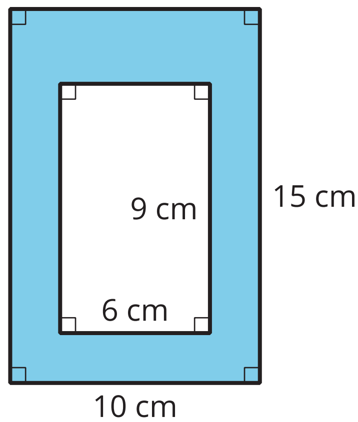
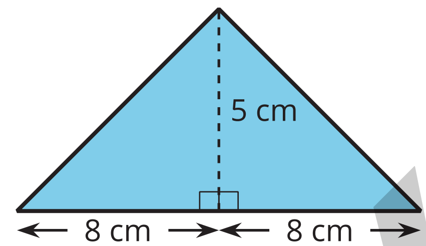
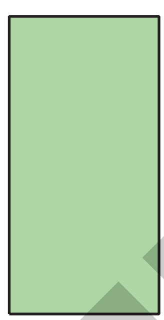


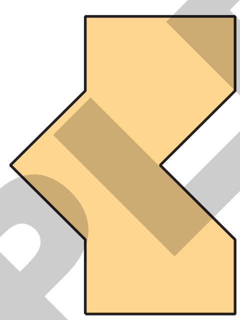
D



3 Two plots of land have very different shapes. Noah said that both plots of land have the same area. Do you agree with Noah? Explain your reasoning.



plot A



plot B

SAMPLE ONLY

4 from Unit 1, Lesson 2

A homeowner is deciding on one size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches.

Tell whether or not you agree with each statement about the tiles. Explain your reasoning.

a. Regardless of the size she chooses, she will need the same number of tiles.

b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.

c. She will need two 2-inch tiles to cover the same area as one 4-inch tile.

d. She will need four 4-inch tiles to cover the same area as one 8-inch tile.

e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

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5 from an earlier course

Find the area of the rectangle with each set of side lengths.

a. 5 in and $\frac{1}{3}$ in

b. 5 in and $\frac{4}{3}$ in

c. $\frac{5}{2}$ in and $\frac{4}{3}$ in

d. $\frac{7}{6}$ in and $\frac{6}{7}$ in

SAMPLE ONLY

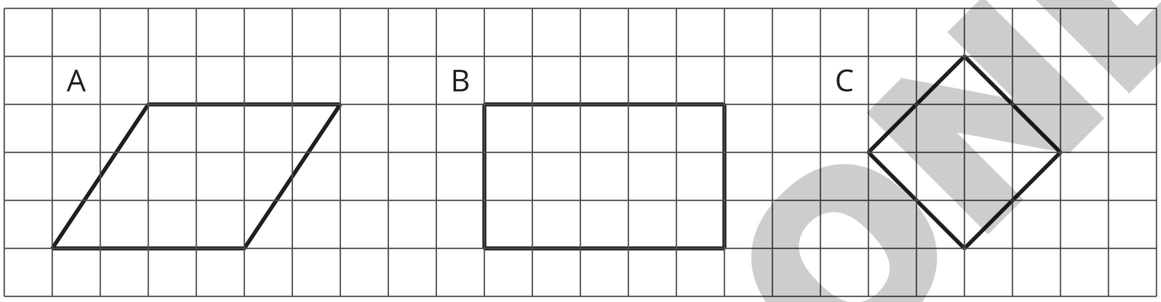
Unit 1, Lesson 4

Parallelograms

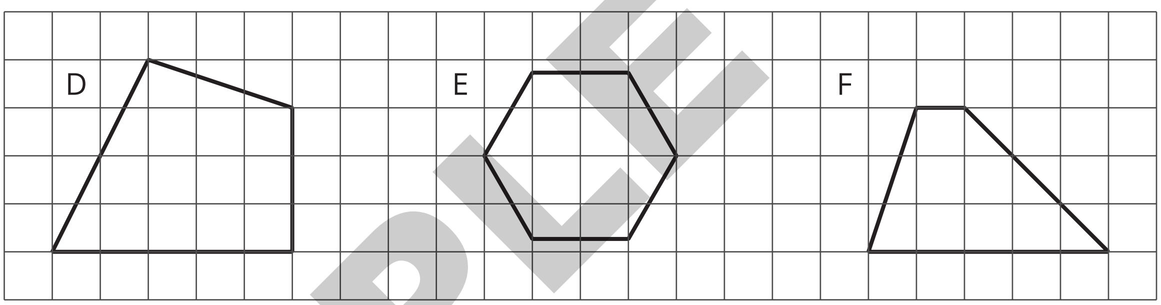
Let's investigate the characteristics and area of parallelograms.

4.1 What are Parallelograms?

Figures A, B, and C are *parallelograms*.



Figures D, E, and F are not parallelograms.



What do you notice about:

1. The number of sides that a parallelogram has?

2. Opposite sides of a parallelogram?

3. Opposite angles of a parallelogram?

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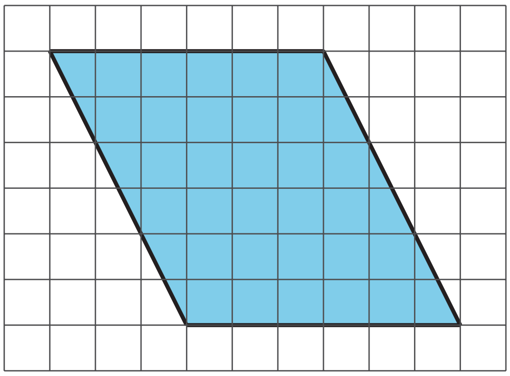
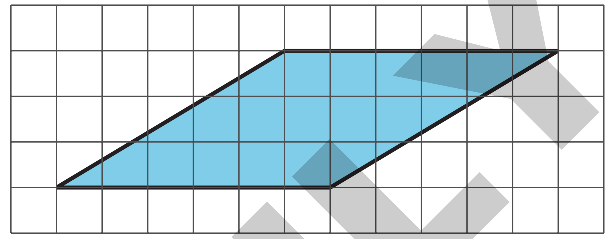
4.2 Area of a Parallelogram

Find the area of each parallelogram. Show your reasoning.

1.

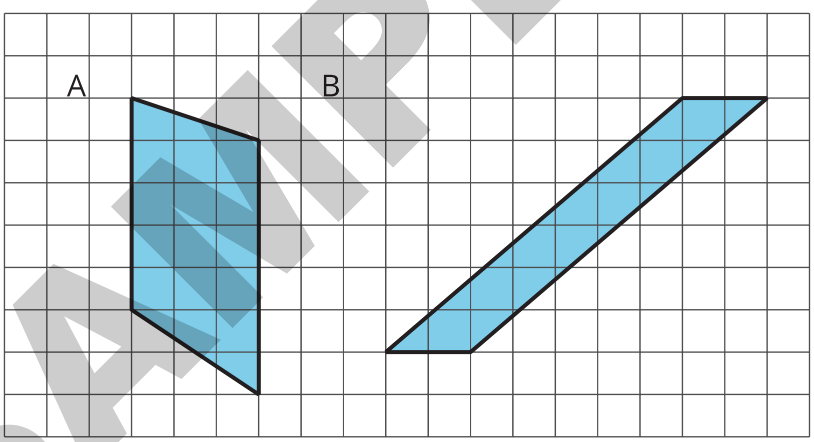


2.

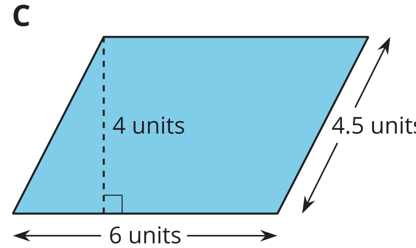


4.3 Lots of Parallelograms

Find the area of each parallelogram. Show your reasoning.



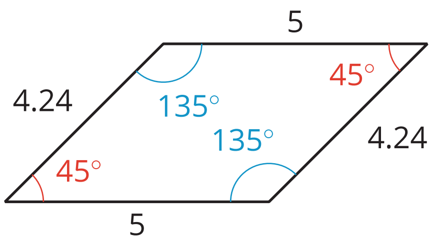
C

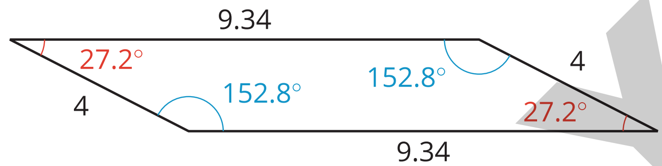


SAMPLE ONLY

Lesson 4 Summary

A *parallelogram* is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. The opposite sides of a parallelogram have the same length, and the opposite angles of a parallelogram have the same measure in degrees.

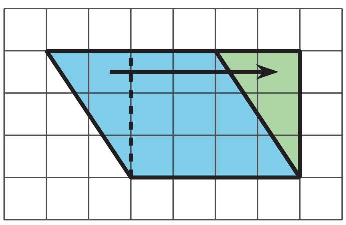


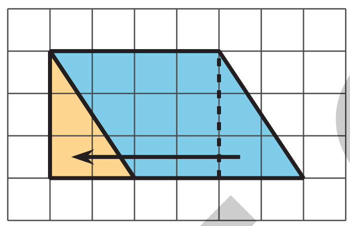


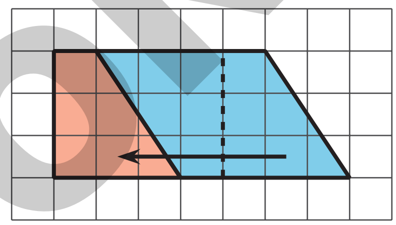
Sec B

There are several strategies for finding the area of a parallelogram.

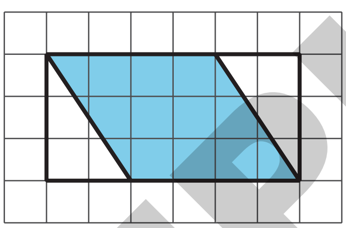
- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways:

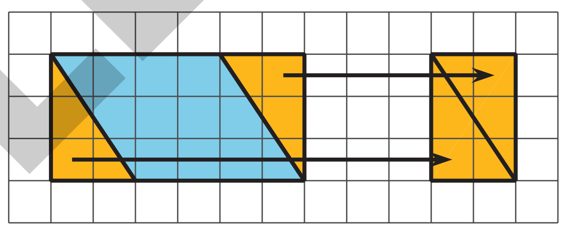




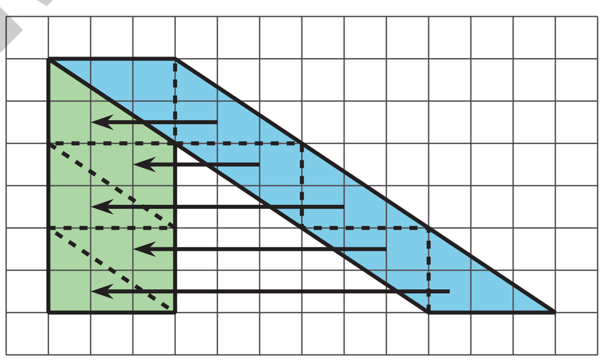


- We can enclose the parallelogram and then subtract the area of the two triangles in the corner.





Both of these ways will work for any parallelogram. However, for some parallelograms the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners.



SAMPLE ONLY

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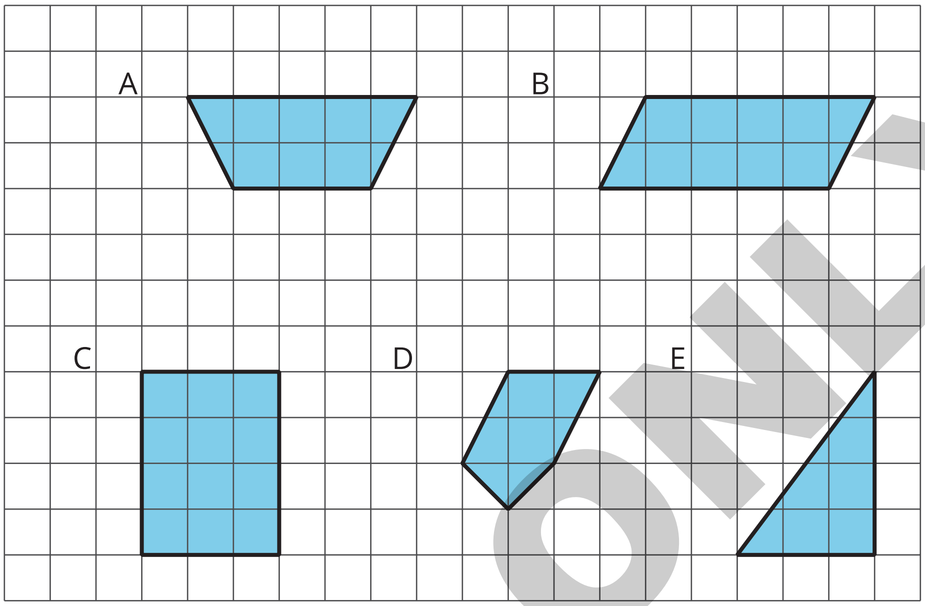
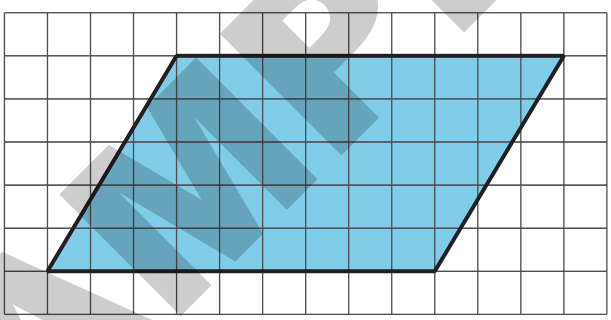
Practice Problems

1 Select **all** of the parallelograms. For each figure that is *not* selected, explain how you know it is not a parallelogram.



2



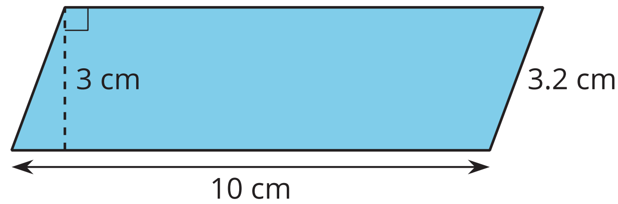
a. Decompose and rearrange this parallelogram to make a rectangle.

b. What is the area of the parallelogram? Explain your reasoning.

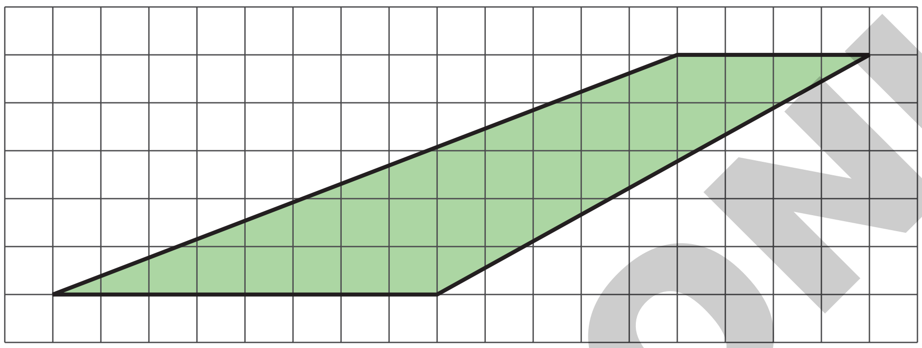
SAMPLE ONLY

Sec B

3 Find the area of the parallelogram.



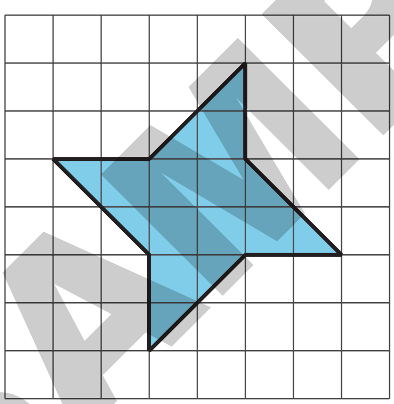
4 Explain why this quadrilateral is *not* a parallelogram.



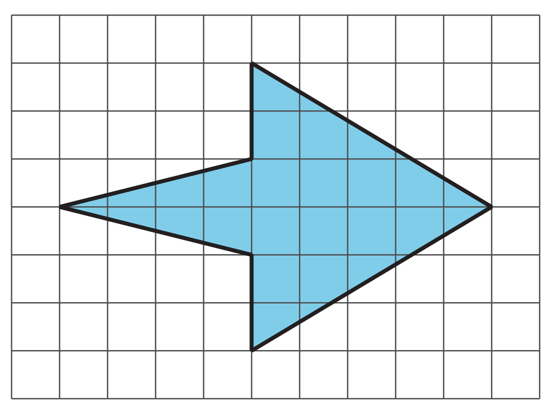
5 from Unit 1, Lesson 3

Find the area of each shape. Show your reasoning.

A



B



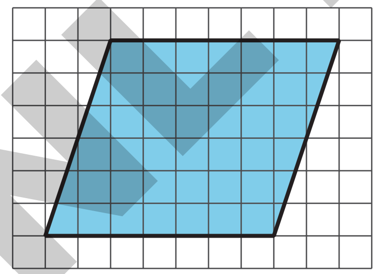
SAMPLE ONLY

Bases and Heights of Parallelograms

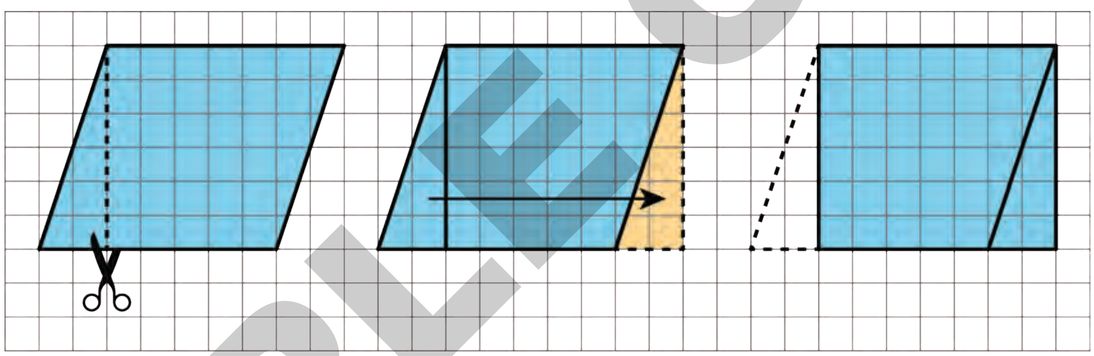
Let's investigate the area of parallelograms some more.

5.1 A Parallelogram and Its Rectangles

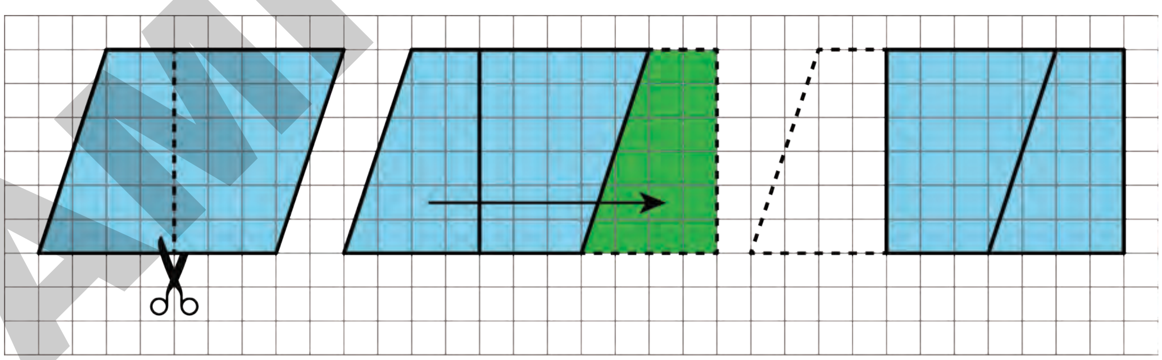
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:



How are the two strategies for finding the area of a parallelogram the same? How they are different?

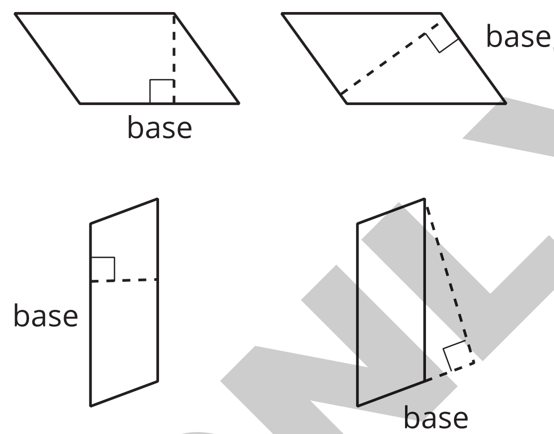
SAMPLE ONLY

5.2 The Right Height?

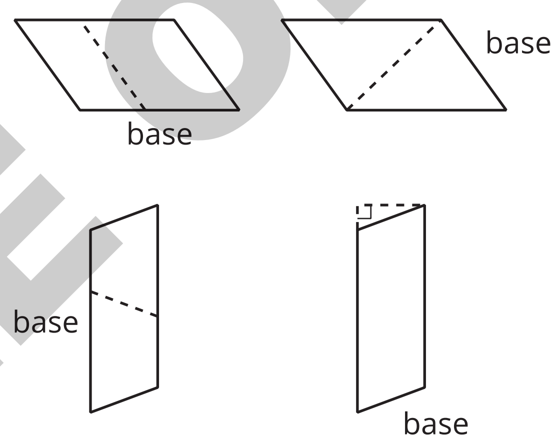
Sec B

Here are some drawings of parallelograms. In each drawing, one side is labeled "**base**."

In the first four drawings, each dashed segment represents a **height** that corresponds to the given base.



In the next four drawings, each dashed segment does not represent a height that corresponds to the given base.



1. Select **all** the statements that are true about bases and heights in a parallelogram.

   A. Only a horizontal side of a parallelogram can be a base.

   B. Any side of a parallelogram can be a base.

   C. A height can be drawn at any angle to the side chosen as the base.

   D. A base and its corresponding height must be perpendicular to each other.

   E. A height can only be drawn inside a parallelogram.

   F. A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.

   G. A base cannot be extended to meet a height.

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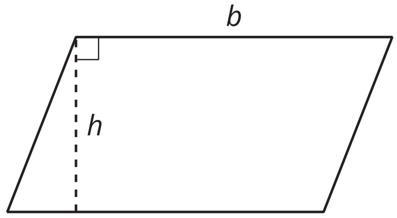
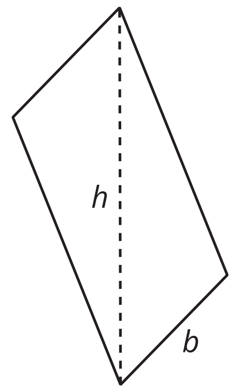
2. Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.

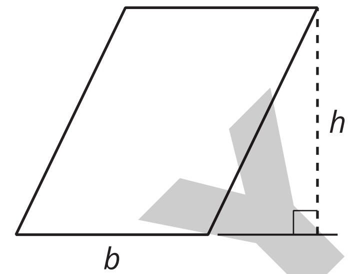
A



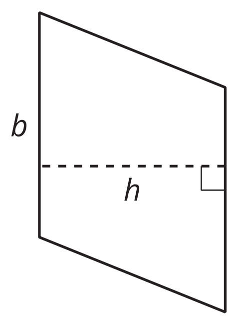
B



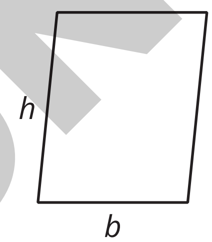
C



D



E

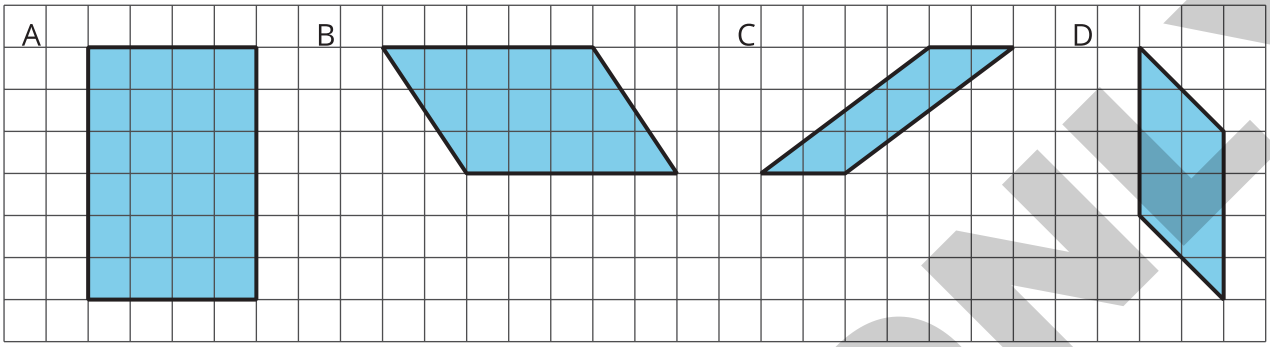


SAMPLE ONLY

5.3 Finding the Formula for Area of Parallelograms

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the parallelogram and record it in the last column of the table.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

In the last row of the table, write an expression for the area of any parallelogram, using b and h .

Are you ready for more?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?

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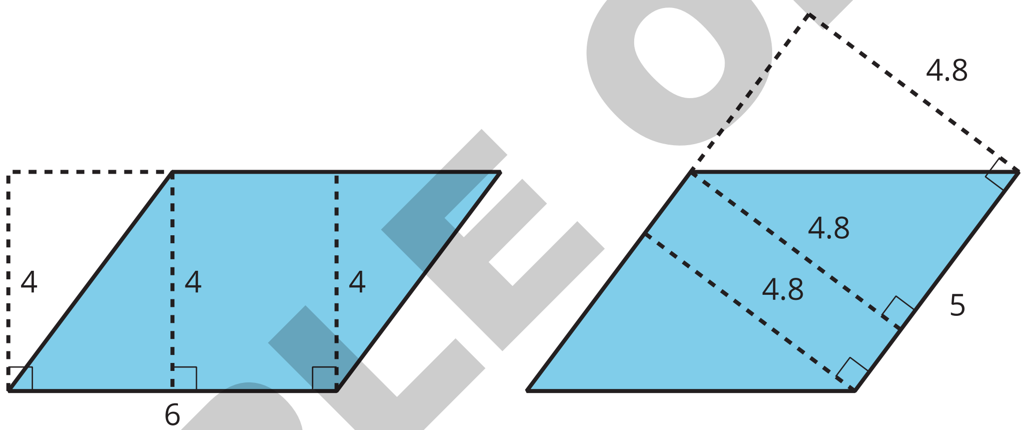
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

Lesson 5 Summary

- We can choose any side of a parallelogram as the **base**. Both the side selected (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many segments that can represent the height!



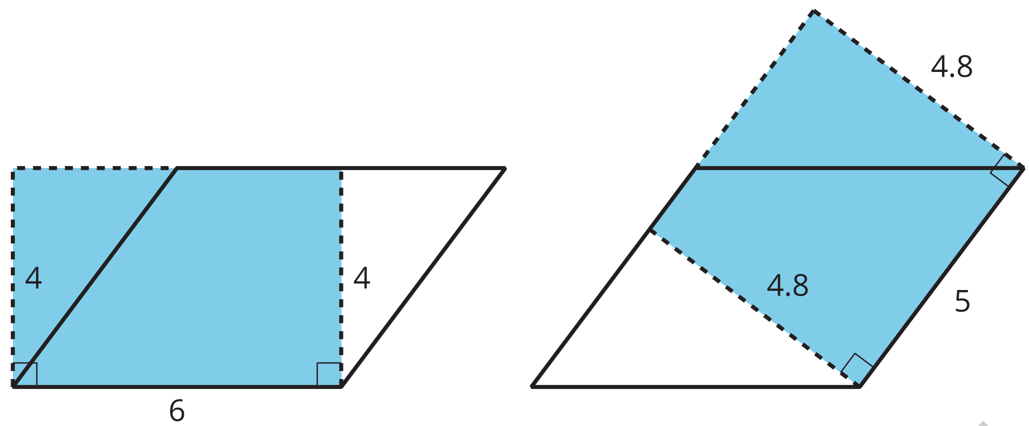
Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its corresponding height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more!

No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check this:

$4 \times 6 = 24$ and $4.8 \times 5 = 24$

We can see why this is true by decomposing and rearranging the parallelograms into rectangles.

SAMPLE ONLY



Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as does the parallelogram.

We often use letters to stand for numbers. If b is base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers.:

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a $\times$ symbol. This is so that we don't get confused about whether $\times$ means multiply, or whether the letter x is standing in for a number.

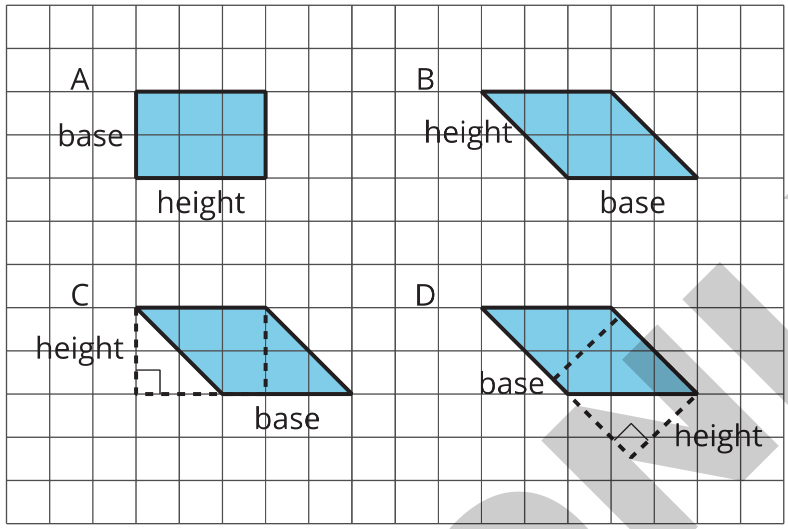
Glossary

- base (of a parallelogram or triangle)
- height (of a parallelogram or triangle)

SAMPLE ONLY

Practice Problems

1 Select **all** parallelograms that have a correct height labeled for the given base.



A. A

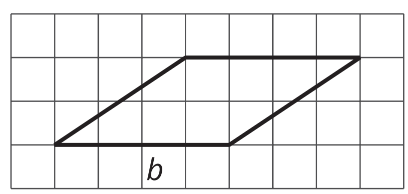
B. B

C. C

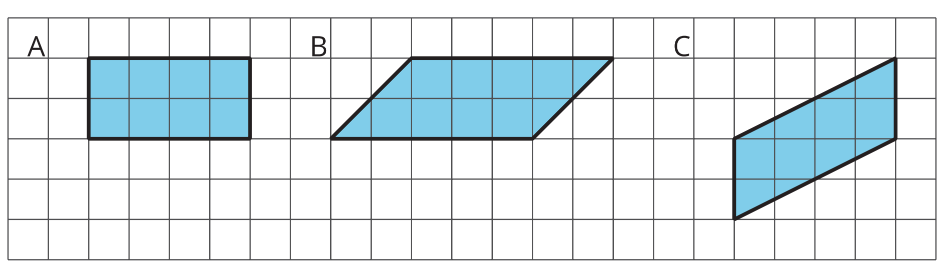
D. D

2 The side labeled b has been chosen as the base for this parallelogram.

Draw a segment showing the height corresponding to that base.

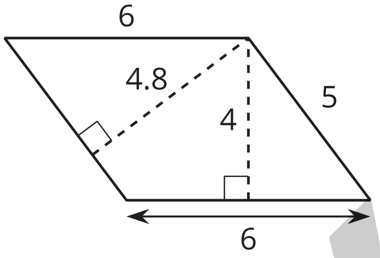


3 Find the area of each parallelogram.



SAMPLE ONLY

4 If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



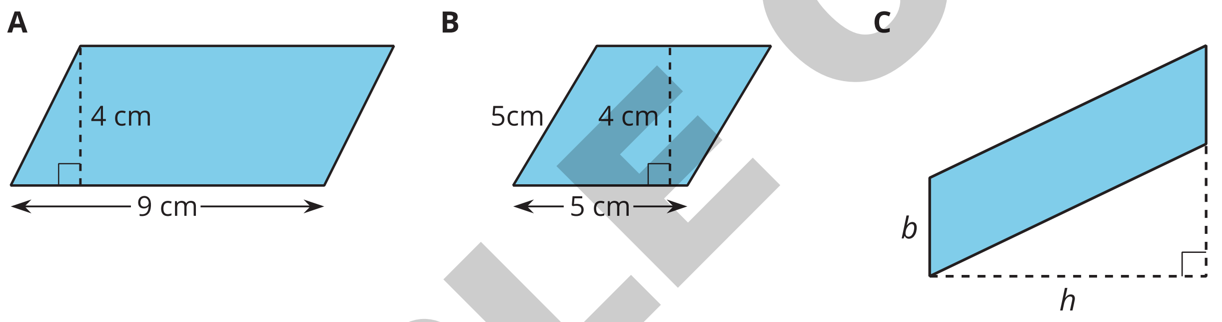
A. 6 units

B. 4.8 units

C. 4 units

D. 5 units

5 Find the area of each parallelogram.



6 from Unit 1, Lesson 4

Do you agree with each of these statements? Explain your reasoning.

a. A parallelogram has six sides.

b. Opposite sides of a parallelogram are parallel.

SAMPLE ONLY

c. A parallelogram can have one pair or two pairs of parallel sides.

d. All sides of a parallelogram have the same length.

e. All angles of a parallelogram have the same measure.

7 from Unit 1, Lesson 2

A square with an area of 1 square meter is decomposed into 9 identical smaller squares. Each smaller square is decomposed into two identical triangles.

a. What is the area, in square meters, of 6 triangles? If you get stuck, consider drawing a diagram.

b. How many triangles are needed to compose a region that is $1\frac{1}{2}$ square meters?

8 from an earlier course

Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

SAMPLE ONLY

Unit 1, Lesson 6

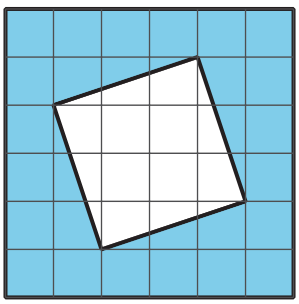
Area of Parallelograms

Let's practice finding the area of parallelograms.

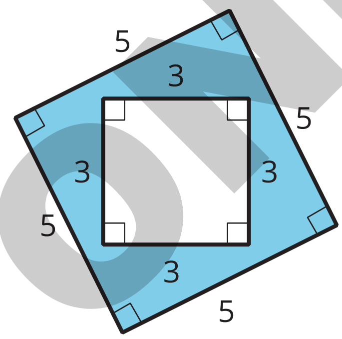
6.1 Which Three Go Together: Squares

Which three go together? Why do they go together?

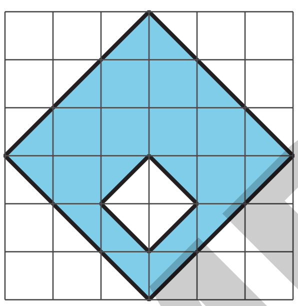
A



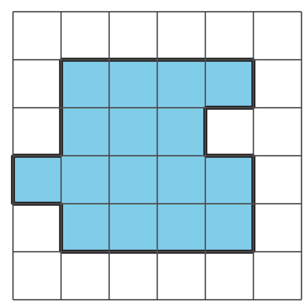
B



C



D



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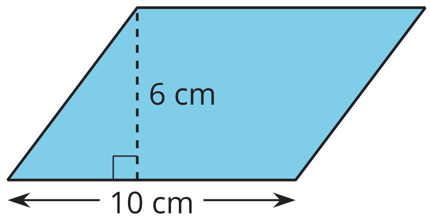
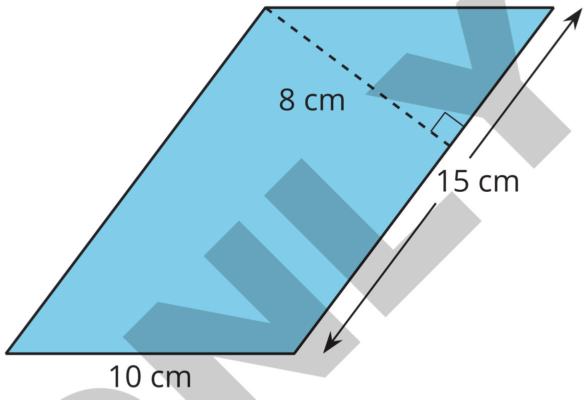
6.2 More Areas of Parallelograms

1. Find the area of each parallelogram. Show your reasoning.

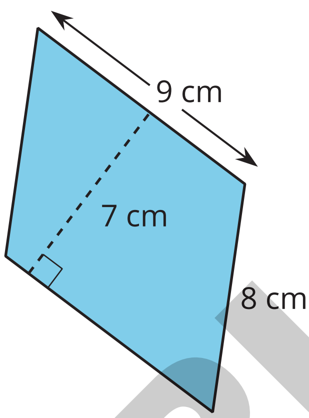
A



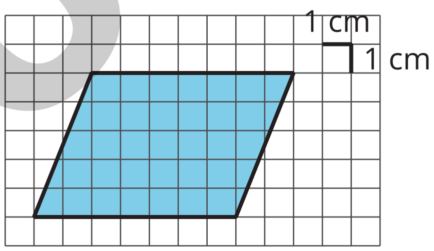
B



C



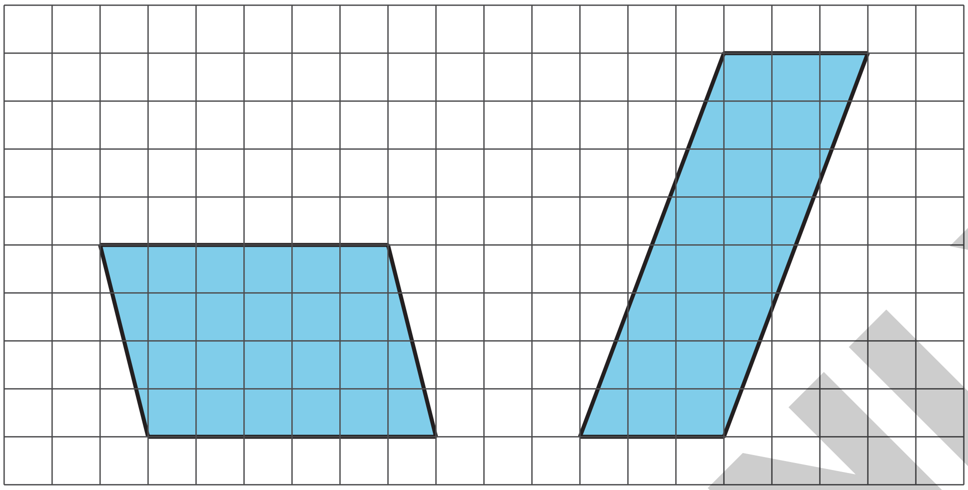
D



2. In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

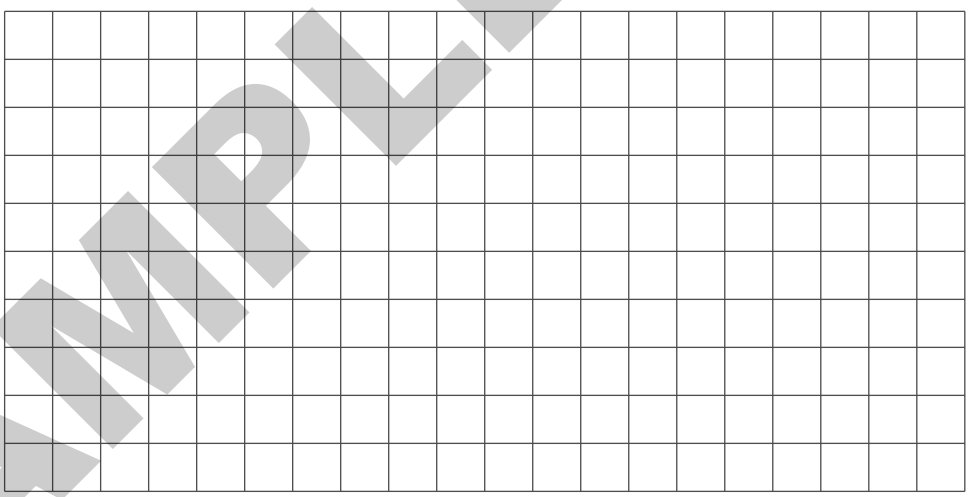
SAMPLE ONLY

3. a. Here are two different parallelograms with the same area. Explain why their areas are equal.



b. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

On the grid, draw two parallelograms that could be P and Q. Explain how you know.



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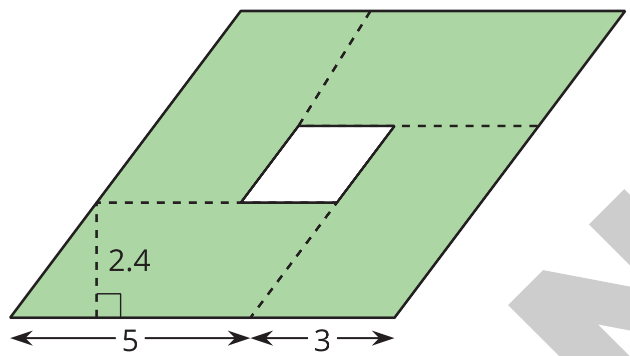
Are you ready for more?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.

What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

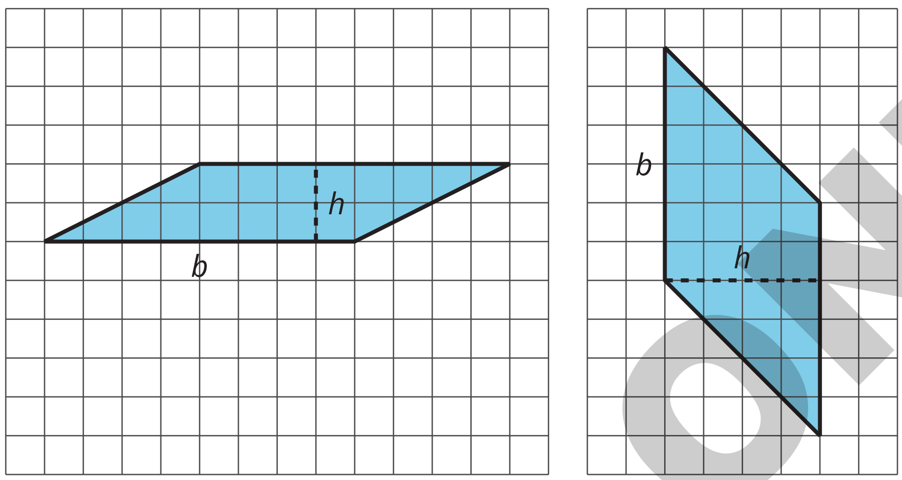


SAMPLE ONLY

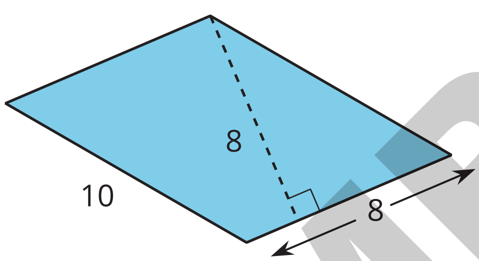
Lesson 6 Summary

Any pair of a base and a corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily found than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

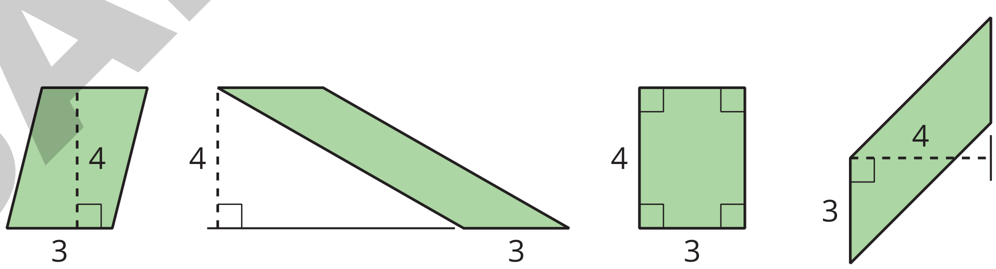


When a parallelogram is *not* drawn on a grid, we can still find its area if we know a base and a corresponding height.



In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are 4 different parallelograms with the same pair of base-height measurements.



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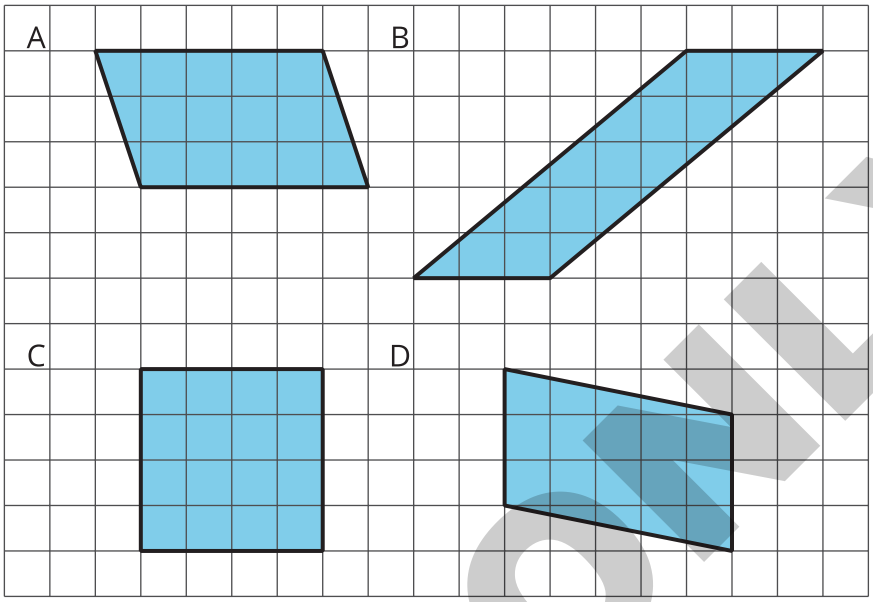
Practice Problems

1 Which three of these parallelograms have the same area as each other?



A. A

B. B

C. C

D. D

2 Which pair of base and height produces the greatest area? All measurements are in centimeters.

A. $b = 4, h = 3.5$

B. $b = 0.8, h = 20$

C. $b = 6, h = 2.25$

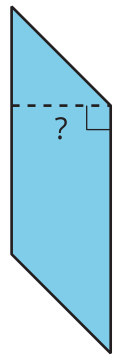
D. $b = 10, h = 1.4$

SAMPLE ONLY

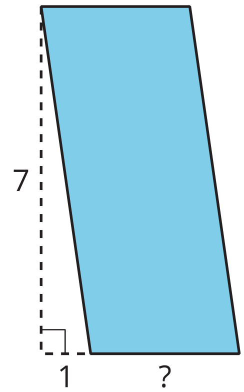
Sec B

3 Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.

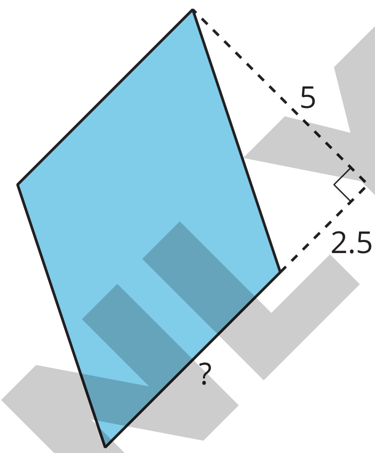
A: 10 square units



B: 21 square units



C: 25 square units



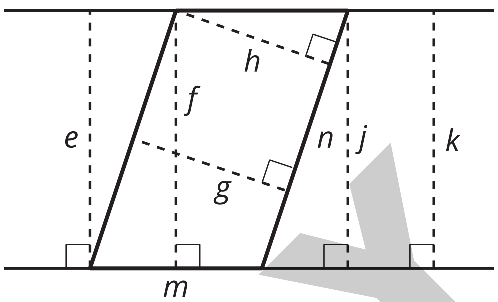
SAMPLE ONLY

4 The Dockland Building in Hamburg, Germany is shaped like a parallelogram.

If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

5 from Unit 1, Lesson 5

Select **all** segments that could represent a corresponding height if the side m is the base.



A. e

B. f

C. g

D. h

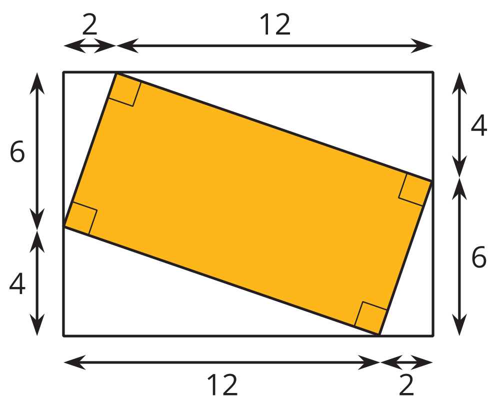
E. j

F. k

G. n

6 from Unit 1, Lesson 3

Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.



SAMPLE ONLY

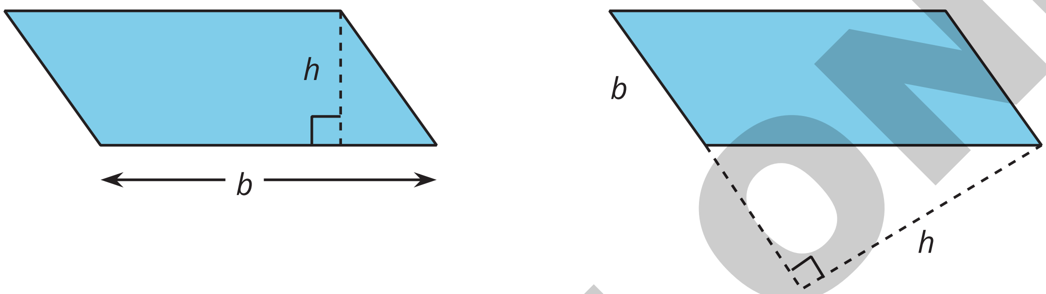
Unit 1, Lesson 7

From Parallelograms to Triangles

Let's compare parallelograms and triangles.

7.1 Same Parallelograms, Different Bases

Here are two copies of a parallelogram. Each copy has one side labeled as the base b and a segment drawn for its corresponding height and labeled h.



1. The base of the parallelogram on the left is 2.4 centimeters; its corresponding height is 1 centimeter. Find its area in square centimeters.

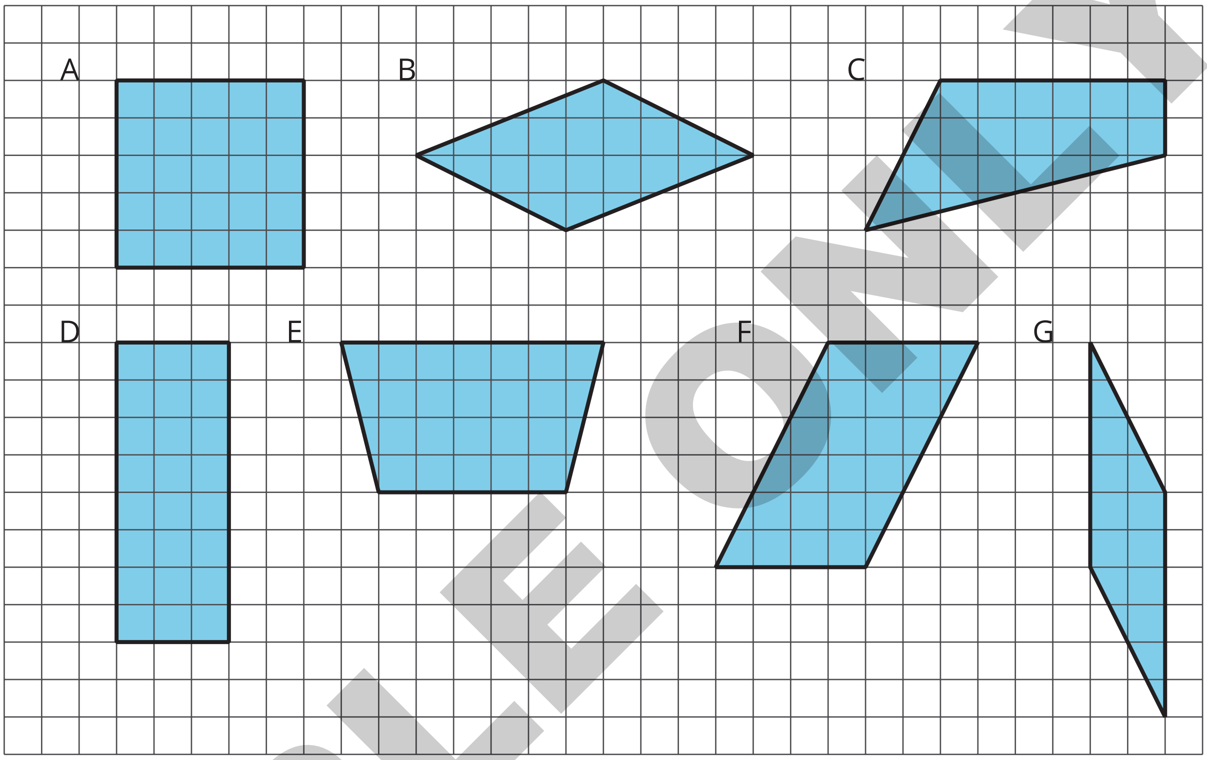
2. The height of the parallelogram on the right is 2 centimeters. How long is the base of that parallelogram? Explain your reasoning.

SAMPLE ONLY

7.2 A Tale of Two Triangles (Part 1)

Two polygons are identical if they match up exactly when placed one on top of the other.

1. Draw *one* line to decompose each polygon into two identical triangles, if possible. Use a straightedge to draw your line.



2. Which quadrilaterals can be decomposed into two identical triangles?

   Pause here for a small-group discussion.

3. Study the quadrilaterals that can, in fact, be decomposed into two identical triangles. What do you notice about them? Write a couple of observations about what these quadrilaterals have in common.

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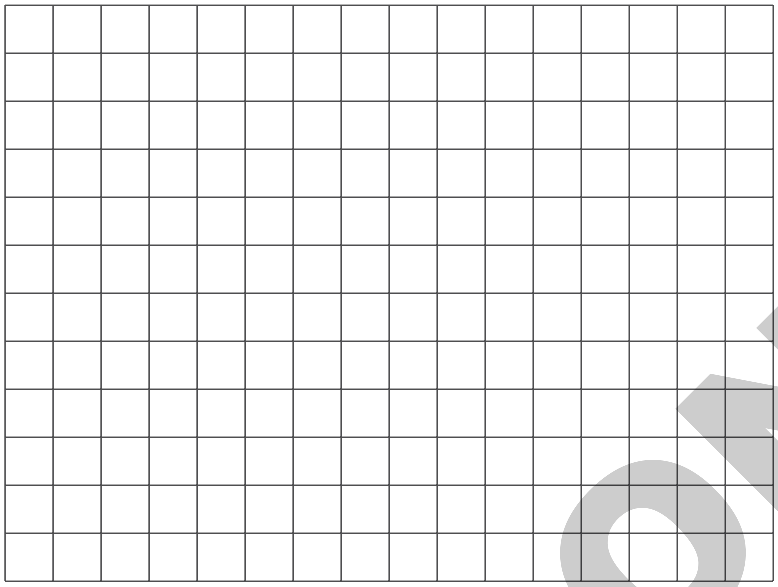
Are you ready for more?

On the grid, draw some other types of quadrilaterals that are not already shown. Try to decompose them into two identical triangles. Can you do it?



Come up with a rule about what must be true about a quadrilateral for it to be decomposed into two identical triangles.

SAMPLE ONLY

7.3 A Tale of Two Triangles (Part 2)

Your teacher will give your group several pairs of triangles. Each group member should take 1 or 2 pairs.

1. a. Which pair(s) of triangles do you have?

   b. Can each pair be composed into a rectangle? A parallelogram?

2. Discuss with your group your responses to the first question. Then, complete each statement with *All*, *Some*, or *None*. Sketch 1 or 2 examples to illustrate each completed statement.

   a. ______________ of these pairs of identical triangles can be composed into a *rectangle*.

   b. ______________ of these pairs of identical triangles can be composed into a *parallelogram*.

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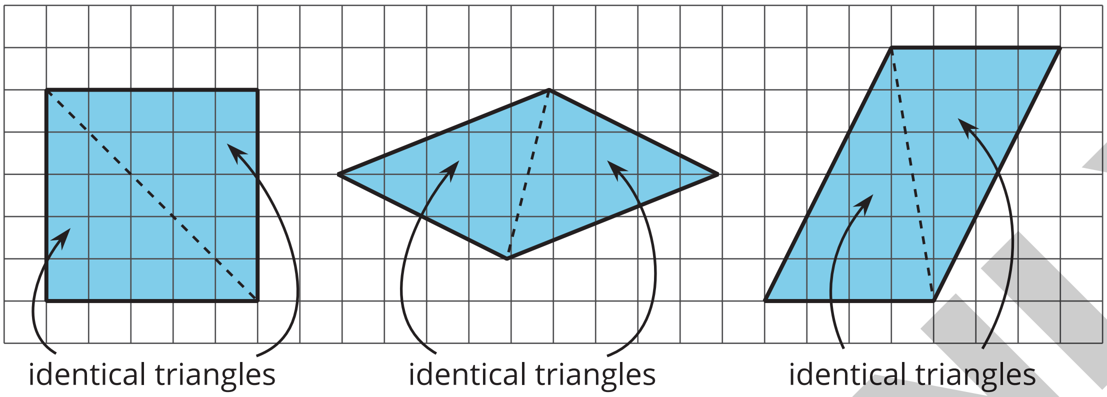
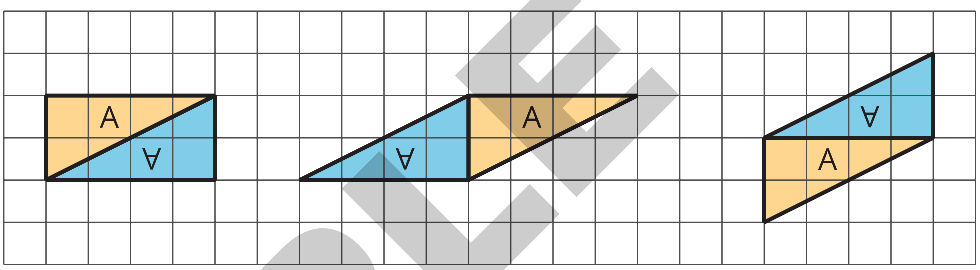
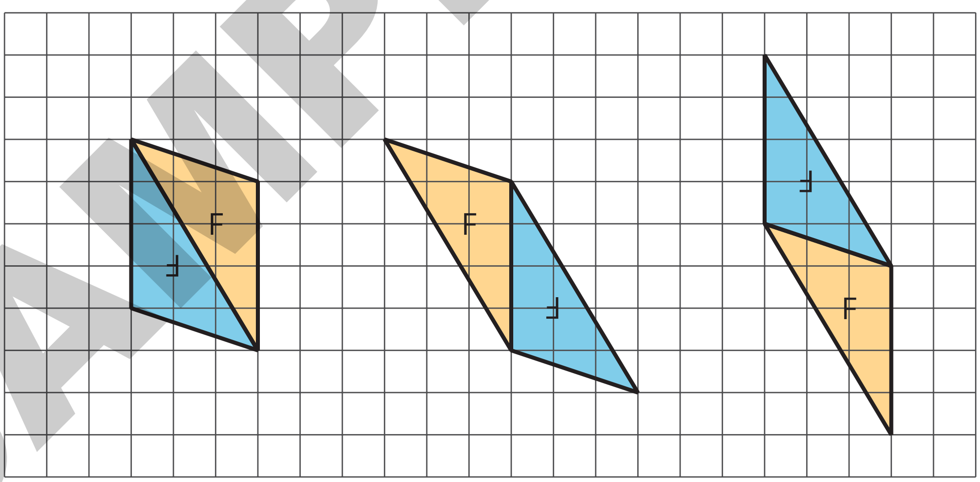
Lesson 7 Summary

A parallelogram can always be decomposed into two identical triangles by a segment that connects opposite vertices.



Going the other way around, two identical copies of a triangle can always be arranged to form a parallelogram, regardless of the type of triangle being used. To produce a parallelogram, we can join a triangle and its copy along any of the three sides that match, so the same pair of triangles can make different parallelograms. Here are examples of how two copies of both Triangle A and Triangle F can be composed into three different parallelograms.





This special relationship between triangles and parallelograms can help us reason about the area of any triangle.

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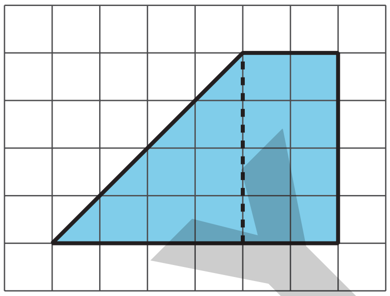
Practice Problems

1 To decompose a quadrilateral into two identical shapes, Clare drew a dashed line as shown in the diagram.

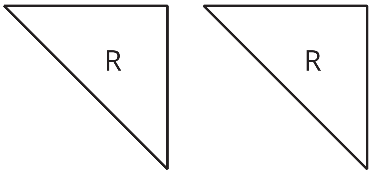


a. She said that the two resulting shapes have the same area. Do you agree? Explain your reasoning.

b. Did Clare partition the figure into two identical shapes? Explain your reasoning.

2 Triangle R is a right triangle. Can we use two copies of Triangle R to compose a parallelogram that is not a square?

If so, explain how or sketch a solution. If not, explain why not.

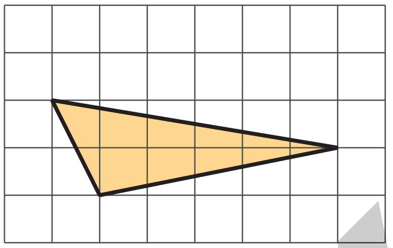


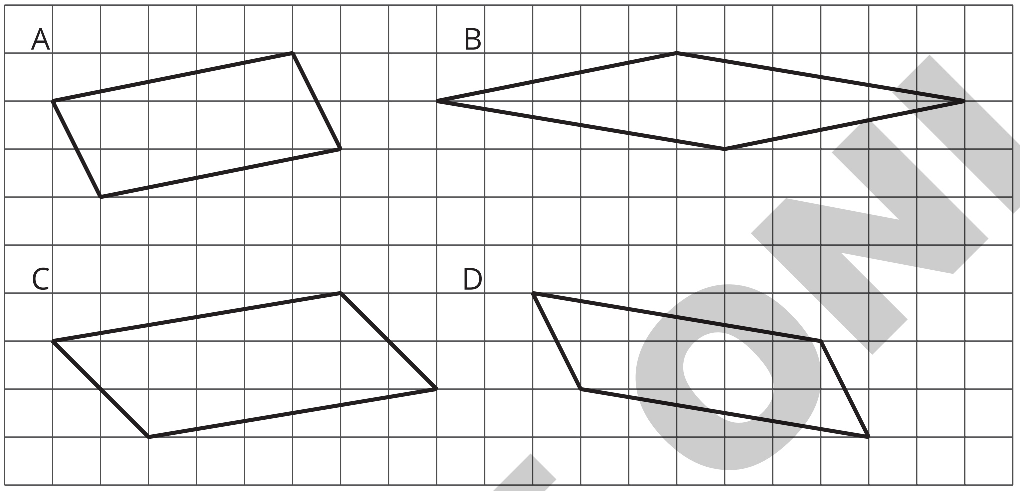
SAMPLE ONLY

Sec C

3 Two copies of this triangle are used to compose a parallelogram.

Which parallelogram *cannot* be a result of the composition? If you get stuck, consider using tracing paper.





A. A

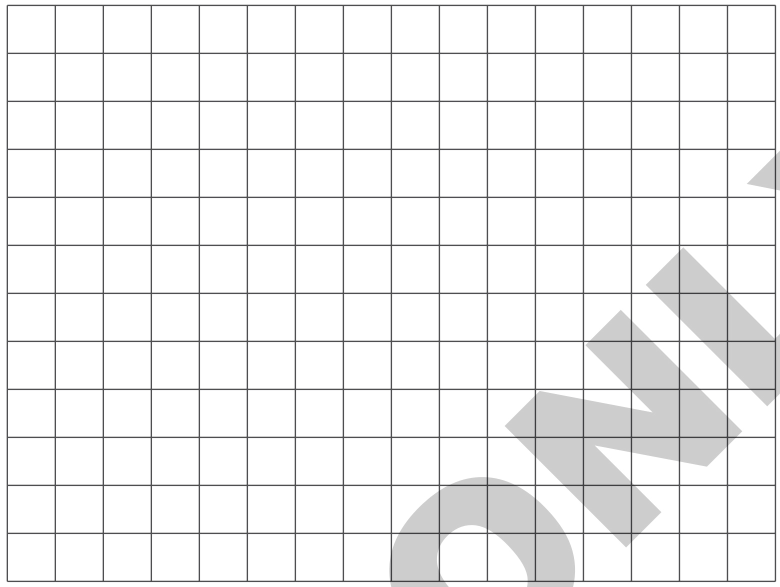
B. B

C. C

D. D

SAMPLE ONLY

4. a. On the grid, draw at least three different quadrilaterals that can each be decomposed into two identical triangles with a single cut (show the cut line). One or more of the quadrilaterals should have non-right angles.



b. Identify the type of each quadrilateral.

5. from Unit 1, Lesson 6

a. A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?

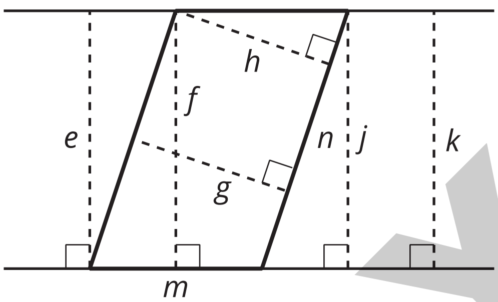
b. A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?

c. A parallelogram has an area of 7 square units. If the height that corresponds to a base is $\frac{1}{4}$ unit, what is the base?

SAMPLE ONLY

6 from Unit 1, Lesson 5

Select **all** the segments that could represent the height if side n is the base.



A. e

B. f

C. g

D. h

E. m

F. n

G. j

H. k

Sec C

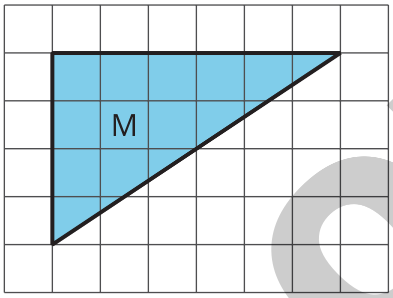
SAMPLE ONLY

Area of Triangles

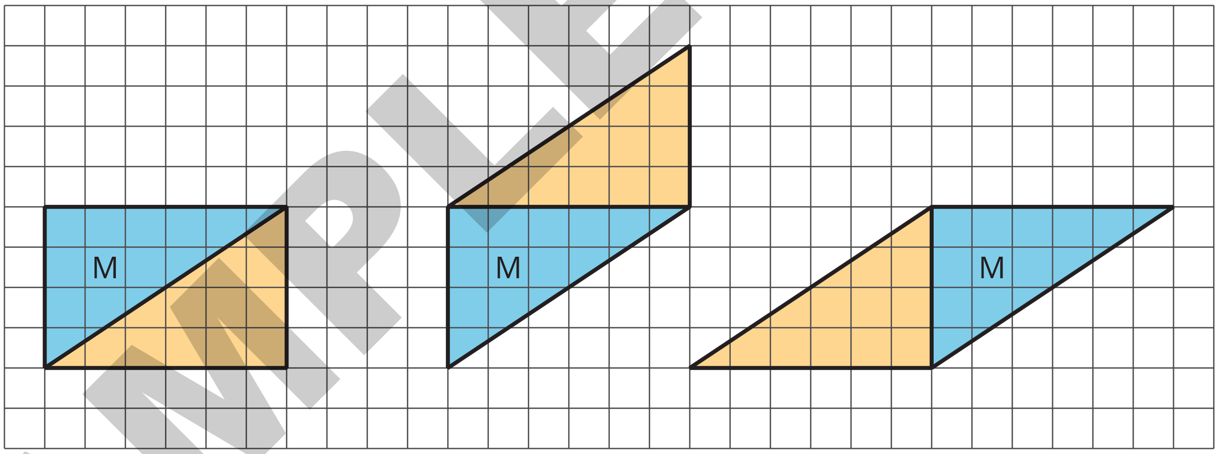
Let's use what we know about parallelograms to find the area of triangles.

8.1 Composing Parallelograms

Here is Triangle M.



Han made a copy of Triangle M and composed three different parallelograms using the original M and the copy, as shown here.



1. For each parallelogram that Han composed, identify a base and a corresponding height, and write the measurements on the drawing.
2. Find the area of each parallelogram that Han composed. Show your reasoning.

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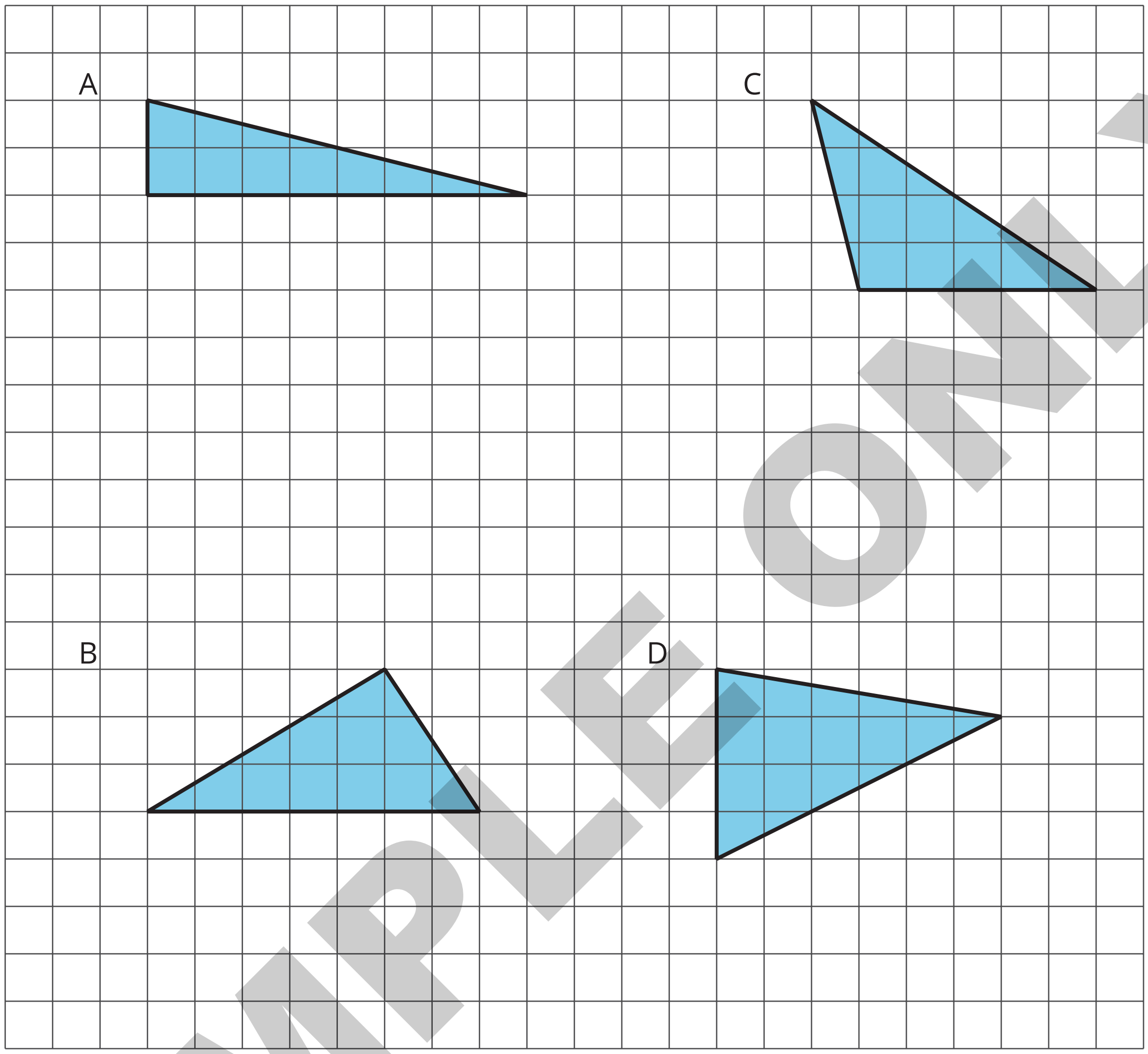
8.2 More Triangles

Find the areas of at least two of these triangles. Show your reasoning.



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8.3 Decomposing a Parallelogram

1. Your teacher will give you two copies of a parallelogram. Glue or tape *one* copy of your parallelogram here and find its area. Show your reasoning.

2. Decompose the second copy of your parallelogram by cutting along the dotted lines. Take *only* the small triangle and the trapezoid, and rearrange these two pieces into a different parallelogram. Glue or tape the newly composed parallelogram on your paper.

3. Find the area of the new parallelogram that you composed. Show your reasoning.

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4. What do you notice about the relationship between the area of this new parallelogram and the original one?

5. How do you think the area of the large triangle compares to that of the new parallelogram: Is it larger, the same, or smaller? Why is that?

6. Glue or tape the remaining large triangle to your paper. Use any part of your work to help you find its area. Show your reasoning.

Sec C

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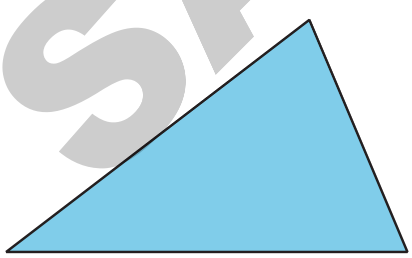
Are you ready for more?

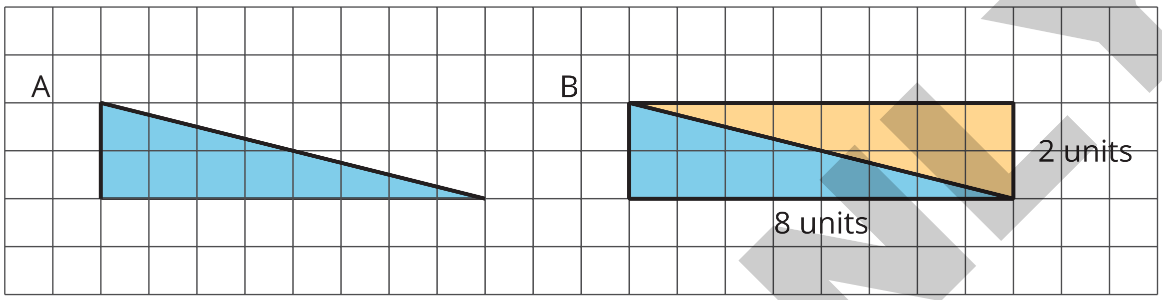
Can you decompose this triangle and rearrange its parts to form a rectangle? Describe how it could be done.



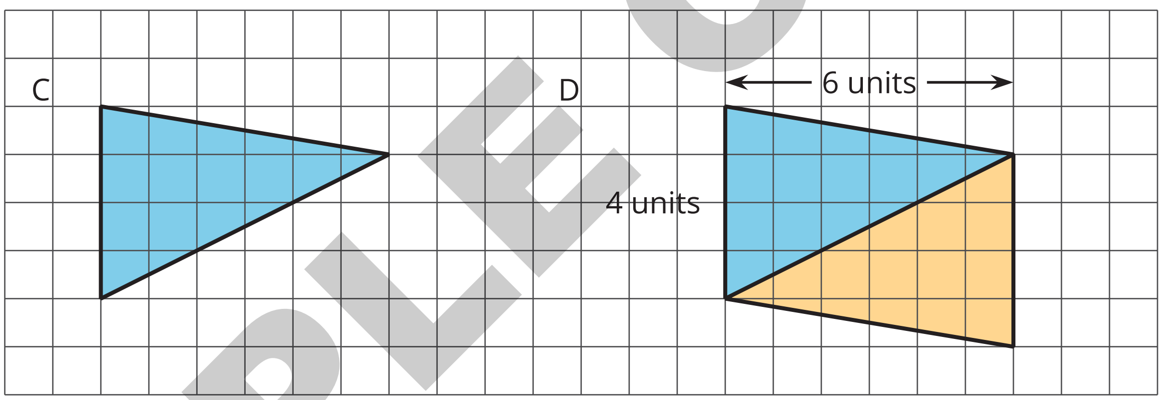
Lesson 8 Summary

We can reason about the area of a triangle by using what we know about parallelograms. Here are three general ways to do this:

- Make a copy of the triangle and join the original and the copy along an edge to create a parallelogram. Because the two triangles have the same area, one copy of the triangle has half the area of that parallelogram.

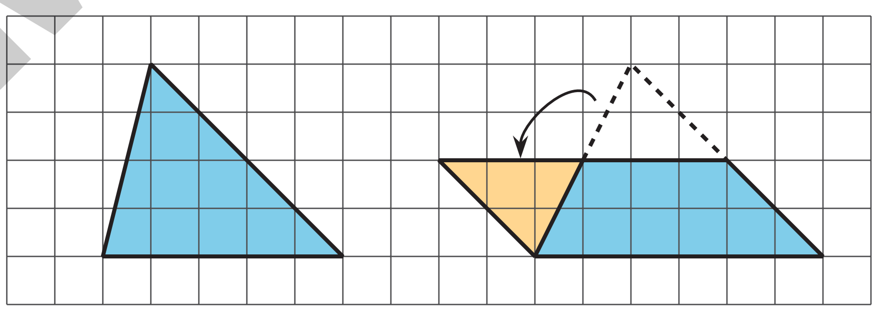


The area of Parallelogram B is 16 square units because the base is 8 units and the height 2 units. The area of Triangle A is half of that, which is 8 square units.



The area of Parallelogram D is 24 square units because the base is 4 units and the height 6 units. The area of Triangle C is half of that, which is 12 square units.

- Decompose the triangle into smaller pieces and compose them into a parallelogram.

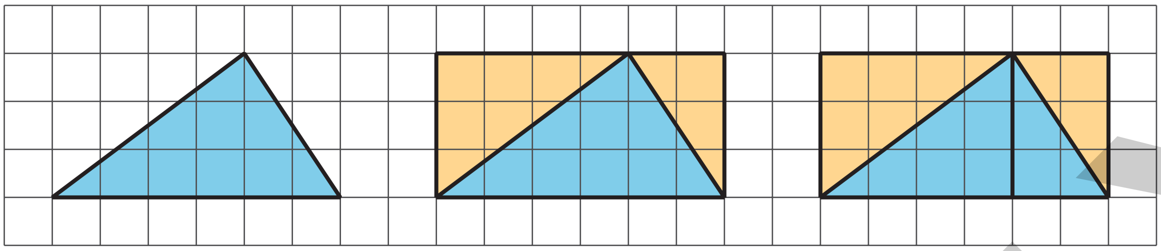


In the new parallelogram, $b = 6$, $h = 2$, and $6 \cdot 2 = 12$, so its area is 12 square units. Because the original triangle and the parallelogram are composed of the same parts, the area of the

SAMPLE ONLY

original triangle is also 12 square units.

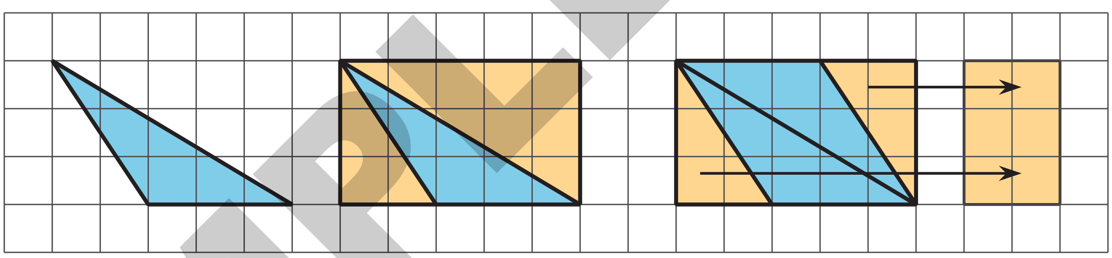
- Draw a rectangle around the triangle. Sometimes the triangle has half of the area of the rectangle.



The large rectangle can be decomposed into smaller rectangles. Each smaller rectangle can be decomposed into two right triangles.

- The rectangle on the left has an area of $4 \cdot 3$, or 12, square units. Each right triangle inside it is 6 square units in area.
- The rectangle on the right has an area of $2 \cdot 3$, or 6, square units. Each right triangle inside it is 3 square units in area.
- The area of the original triangle is the sum of the areas of a large right triangle and a small right triangle: 9 square units.

Sometimes, the triangle is half of what is left of the rectangle after removing two copies of the smaller right triangles.



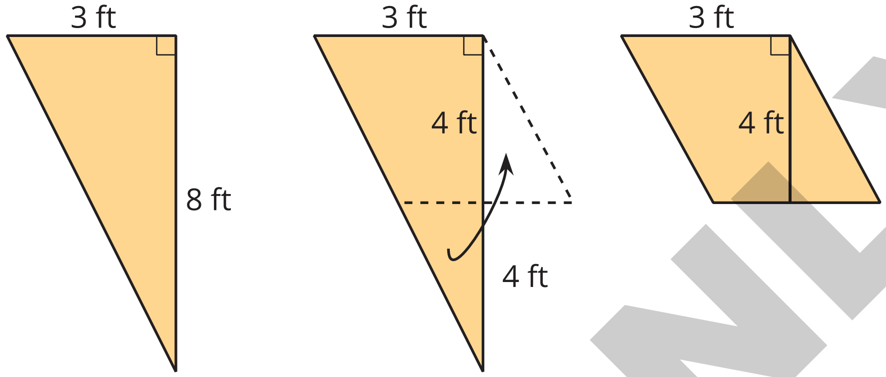
The right triangles being removed can be composed into a small rectangle with area $(2 \cdot 3)$ square units. What is left is a parallelogram with area $5 \cdot 3 - 2 \cdot 3$, which equals $15 - 6$, or 9, square units.

Notice that we can compose the same parallelogram with two copies of the original triangle! The original triangle is half of the parallelogram, so its area is $\frac{1}{2} \cdot 9$, or 4.5, square units.

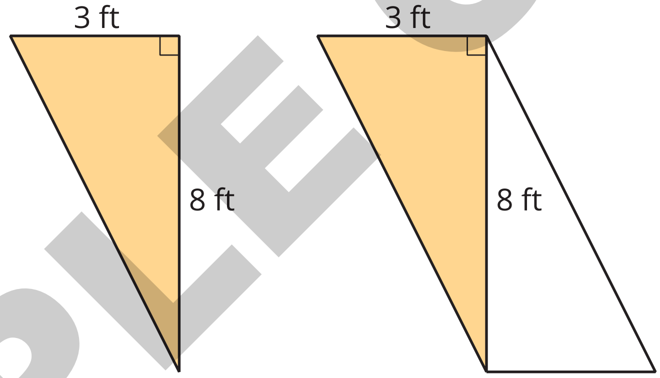
SAMPLE ONLY

Practice Problems

1 To find the area of this right triangle, Diego and Jada used different strategies. Diego drew a line through the midpoints of the two longer sides, which decomposes the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes into a parallelogram.



Jada made a copy of the triangle, rotated it, and lined it up against one side of the original triangle so that the two triangles make a parallelogram.



a. Explain how Diego might use his parallelogram to find the area of the triangle.

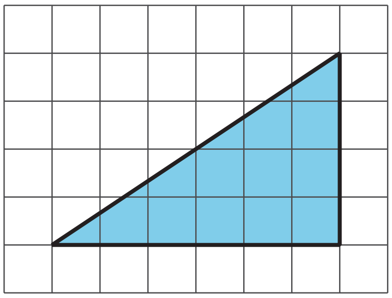
b. Explain how Jada might use her parallelogram to find the area of the triangle.

SAMPLE ONLY

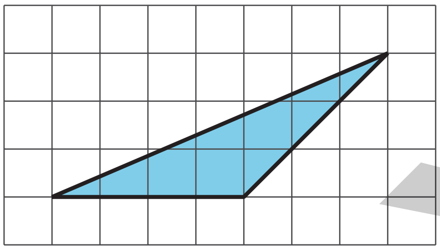
Sec C

2 Find the area of the triangle. Explain or show your reasoning.

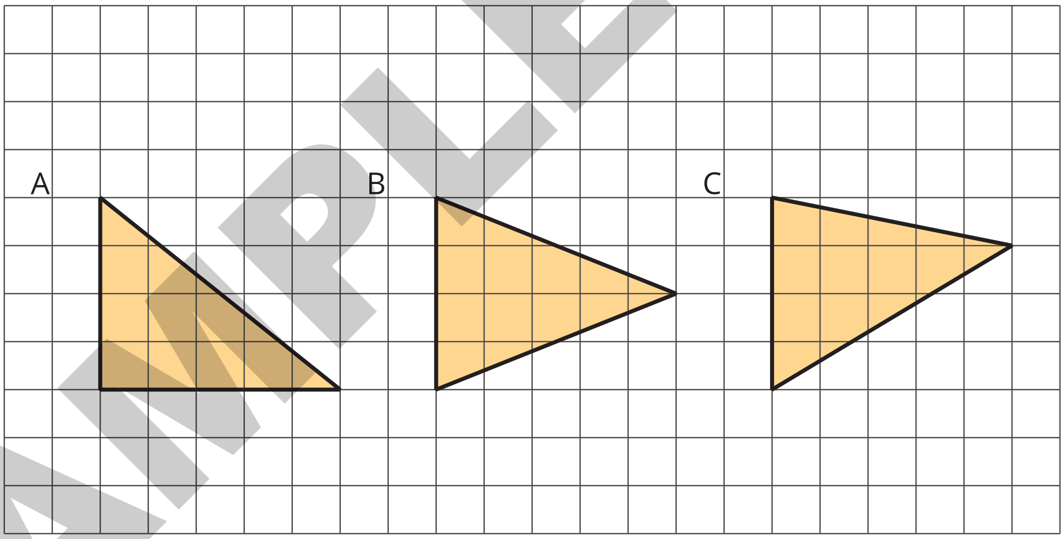
a.



b.



3 Which of the three triangles has the greatest area? Show your reasoning. If you get stuck, try using what you know about the area of parallelograms.



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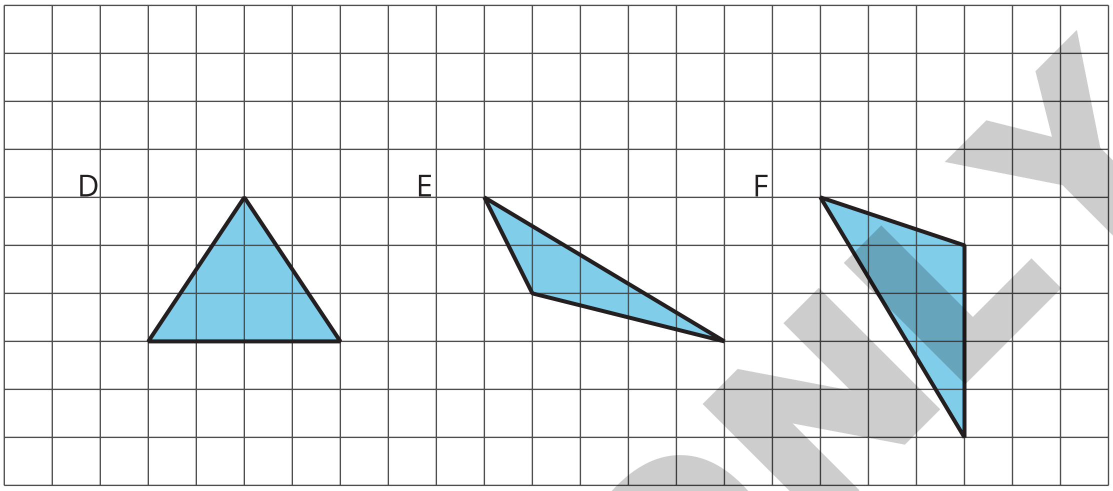
4 from Unit 1, Lesson 7

Draw an identical copy of each triangle such that the two copies together form a parallelogram. If you get stuck, consider using tracing paper.



5 from Unit 1, Lesson 6

a. A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?

b. A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?

c. A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the base?

SAMPLE ONLY

Unit 1, Lesson 9

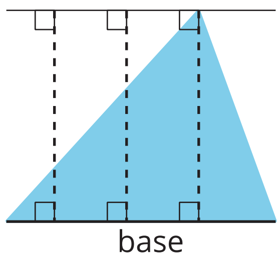
Formula for the Area of a Triangle

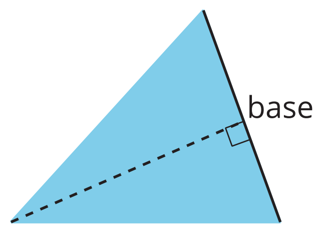
Let's write and use a formula to find the area of a triangle.

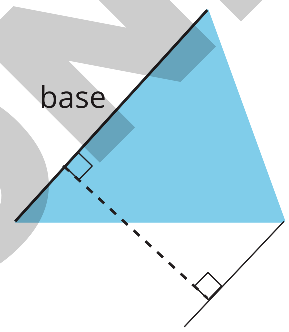
9.1 Bases and Heights of a Triangle

Here are six copies of a triangle. In each copy, one side is labeled *base*.

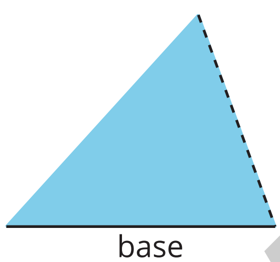
In the first three drawings, the dashed segments represent heights of the triangle.

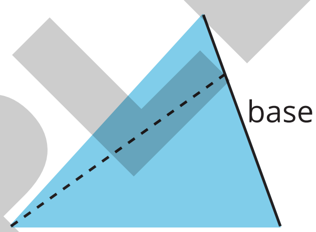


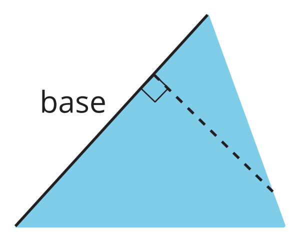




In the next three drawings, the dashed segments do not represent heights of the triangle.







Select **all** the statements that are true about bases and heights in a triangle.

1. Any side of a triangle can be a base.
2. There is only one possible height.
3. A height is always one of the sides of a triangle.
4. A height that corresponds to a base must be drawn at a right angle to the base.
5. Once we choose a base, there is only one segment that represents the corresponding height.
6. A segment representing a height must go through a vertex.

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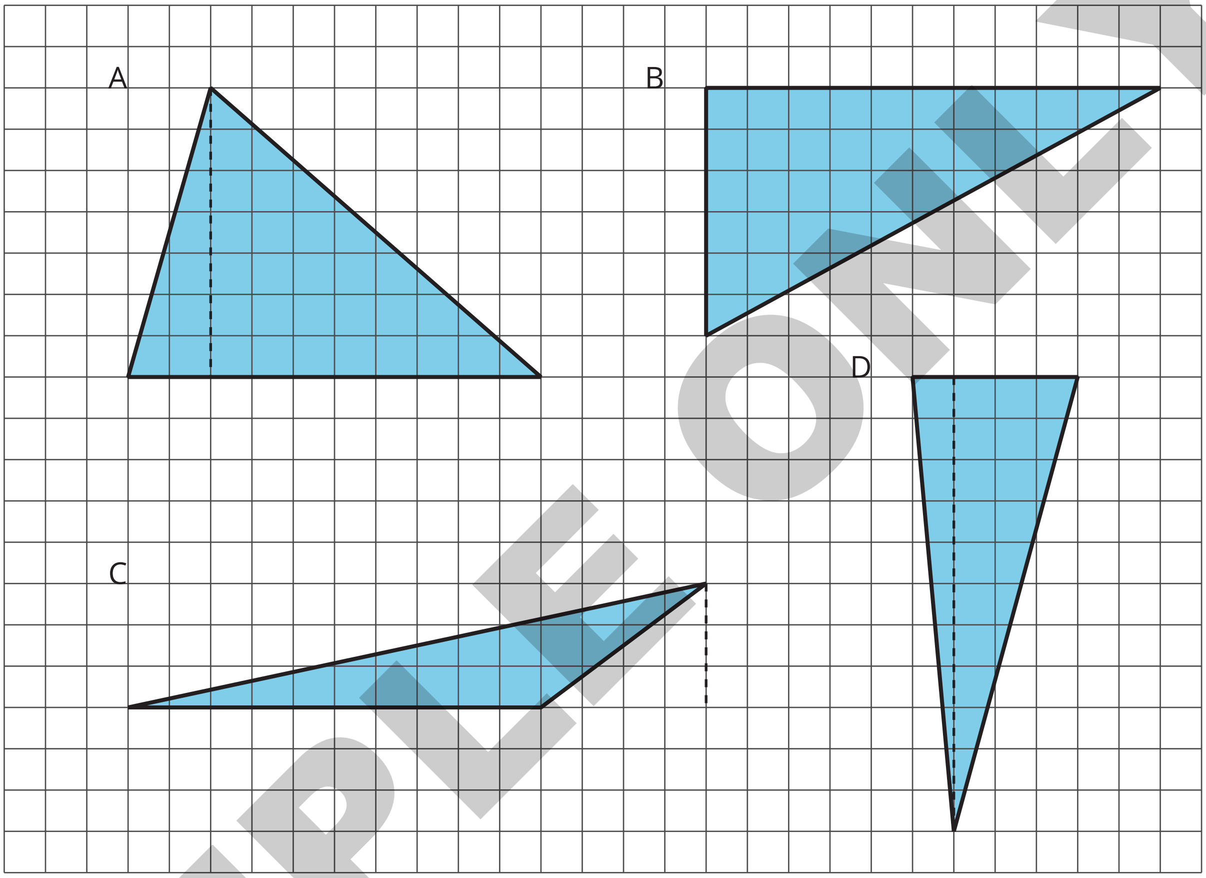
9.2 Finding a Formula for the Area of a Triangle

For each triangle:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the triangle and record it in the last column of the table.



triangle	base (units)	height (units)	area (square units)
A			
B			
C			
D			
any triangle	b	h	

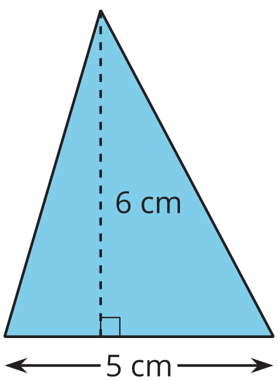
SAMPLE ONLY

In the last row, write an expression for the area of any triangle, using b and h.

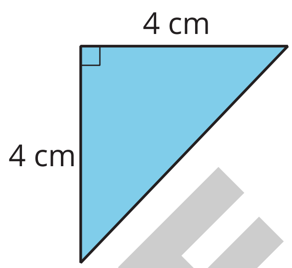
9.3 Applying the Formula for Area of Triangles

For each triangle, circle a base measurement that you can use to find the area of the triangle. Then, find the area of any *three* triangles. Show your reasoning.

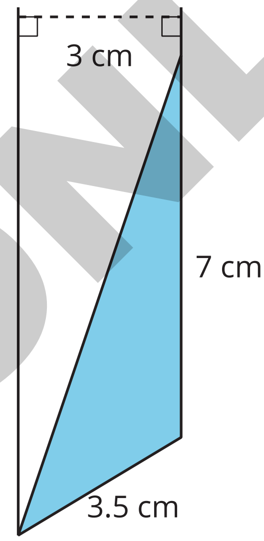
A



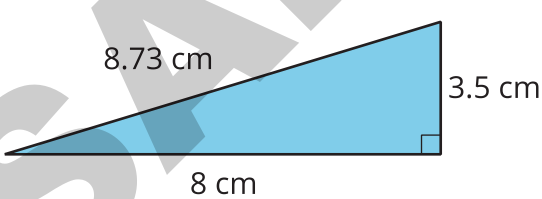
B



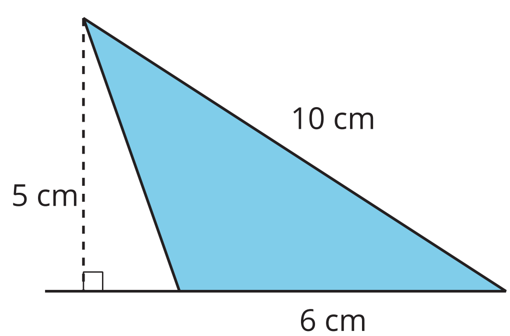
C



D



E



Sec C

SAMPLE ONLY

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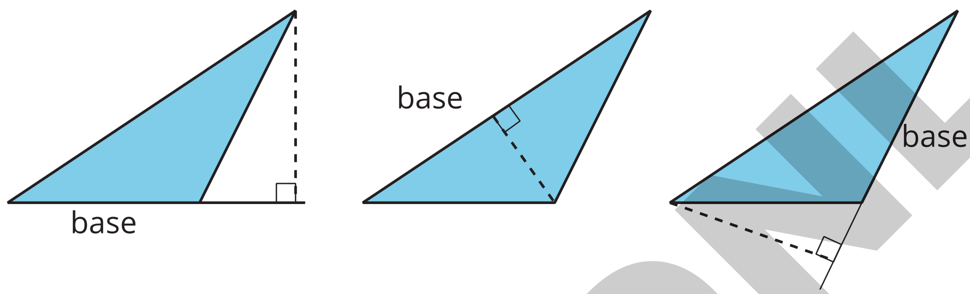
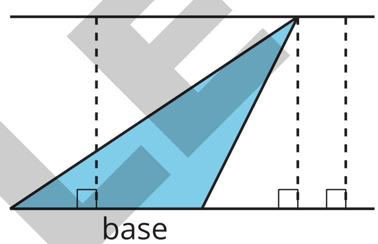
Lesson 9 Summary

- We can choose any of the three sides of a triangle to call the base. The term "base" refers to both the side and its length (the measurement).
- The corresponding height is the length of a perpendicular segment from the base to the vertex opposite it. The **opposite vertex** is the vertex that is *not* an endpoint of the base.

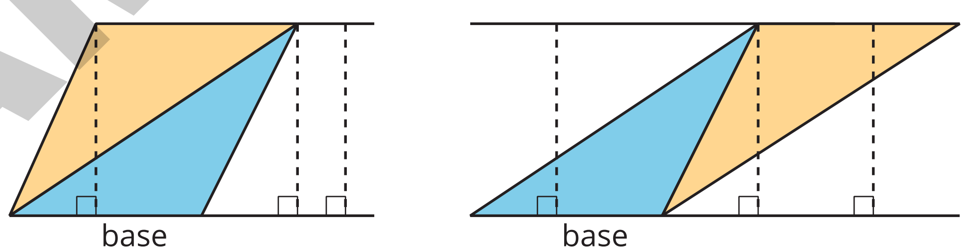
Here are three pairs of bases and heights for the same triangle. The dashed segments in the diagrams represent heights.



A segment showing a height must be drawn at a right angle to the base, but it can be drawn in more than one place. It does not have to go through the opposite vertex, as long as it connects the base and a line that is parallel to the base and goes through the opposite vertex, as shown here.



The base-height pairs in a triangle are closely related to those in a parallelogram. Recall that two copies of a triangle can be composed into one or more parallelograms. Each parallelogram composed of the triangle and its copy shares at least one base with the triangle.



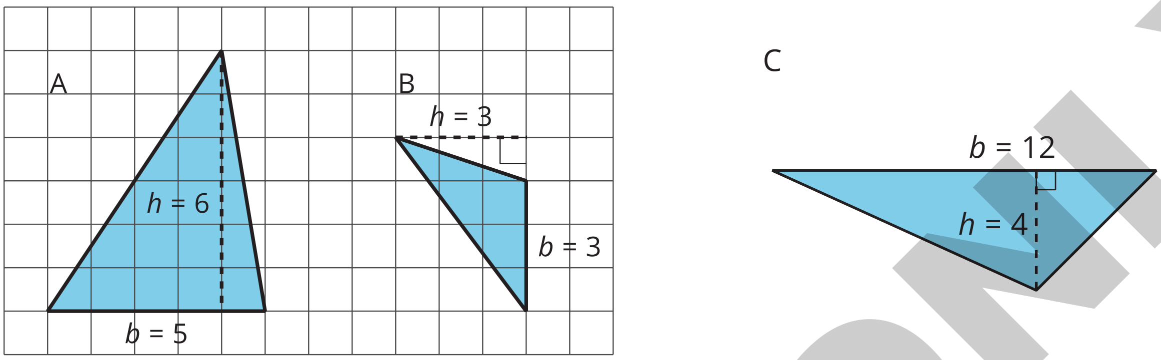
For any base that they share, the corresponding height is also shared, as shown by the dashed segments.

SAMPLE ONLY

We can use the base-height measurements and our knowledge of parallelograms to find the area of any triangle.

- The formula for the area of a parallelogram with base b and height h is $b \cdot h$.
- A triangle takes up half of the area of a parallelogram with the same base and height. We can therefore express the area, A, of a triangle as:

$$A = \frac{1}{2} \cdot b \cdot h$$

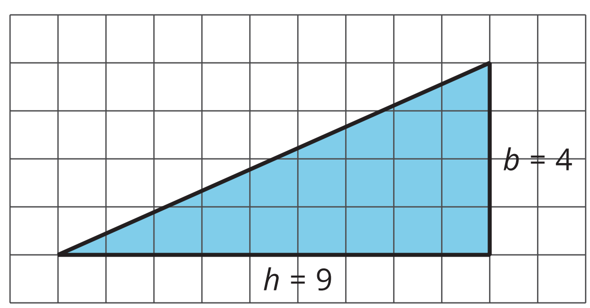


- The area of Triangle A is 15 square units because $\frac{1}{2} \cdot 5 \cdot 6 = 15$.
- The area of Triangle B is 4.5 square units because $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.
- The area of Triangle C is 24 square units because $\frac{1}{2} \cdot 12 \cdot 4 = 24$.

In each case, one side of the triangle is the base but neither of the other sides is the height. This is because the angle between them is not a right angle.

In right triangles, however, the two sides that are perpendicular can be a base and a height.

The area of this triangle is 18 square units whether we use 4 units or 9 units for the base.



Glossary

- opposite vertex

SAMPLE ONLY

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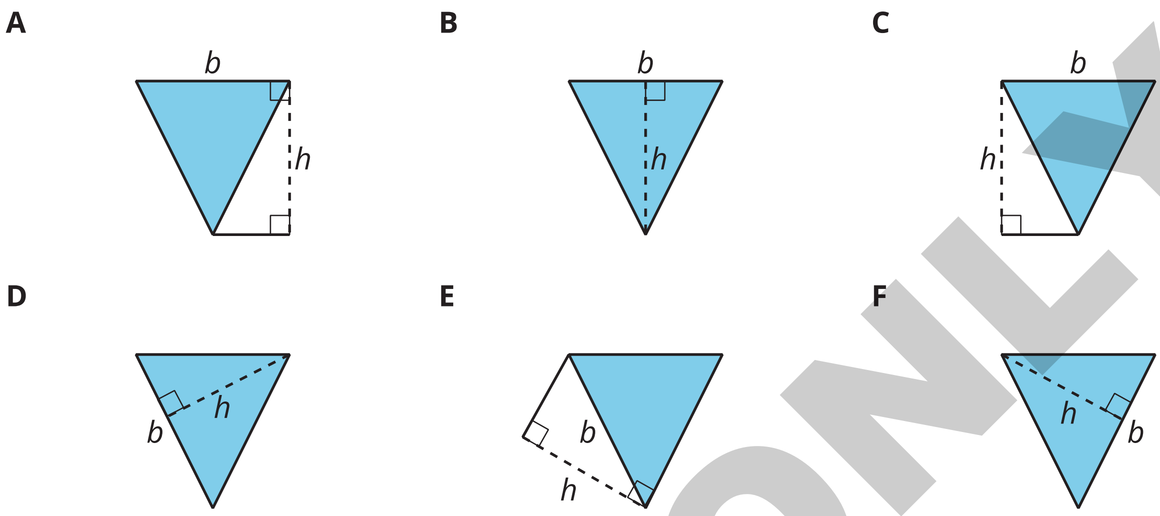
Practice Problems

1 Select **all** drawings in which a corresponding height h for a given base b is correctly identified.



A. A

B. B

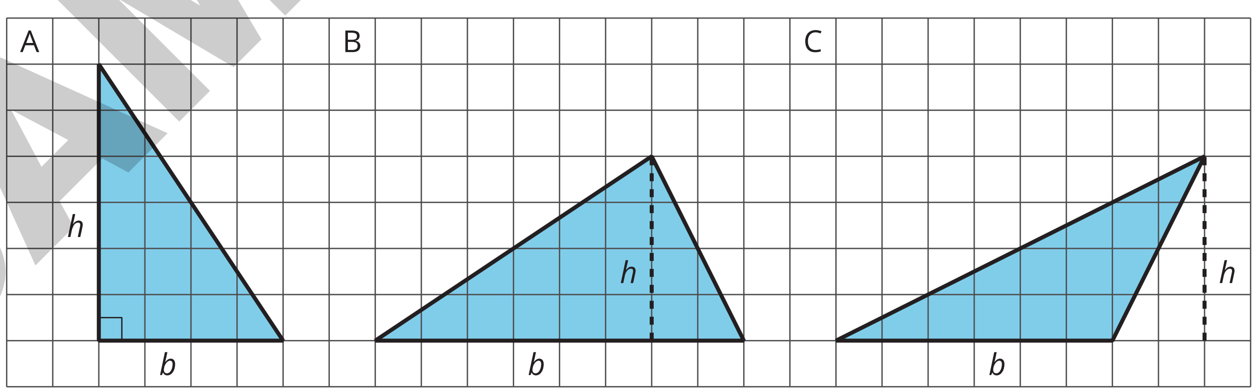
C. C

D. D

E. E

F. F

2 For each triangle, a base and its corresponding height are labeled.



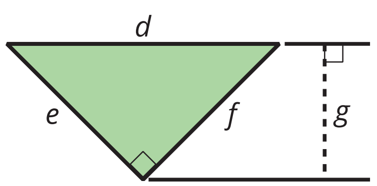
SAMPLE ONLY

a. Find the area of each triangle.

b. How is the area related to the base and its corresponding height?

Sec C

3 Here is a right triangle. Name a corresponding height for each base.



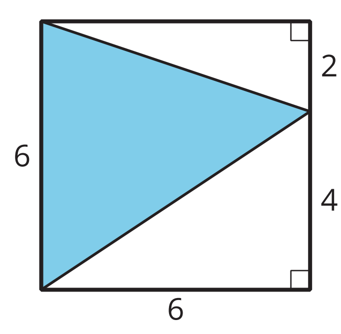
a. Side d

b. Side e

c. Side f

4 from Unit 1, Lesson 8

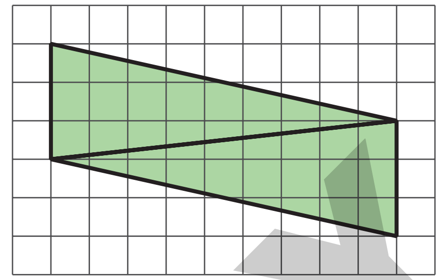
Find the area of the shaded triangle. Show your reasoning.



SAMPLE ONLY

5 from Unit 1, Lesson 7

Andre drew a line connecting two opposite corners of a parallelogram. Select **all** true statements about the triangles created by the line that Andre drew.



A. Each triangle has two sides that are 3 units long.

B. Each triangle has a side that is the same length as the diagonal line.

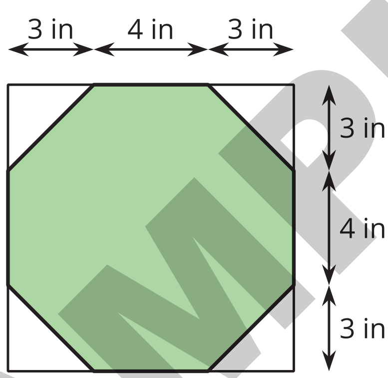
C. Each triangle has one side that is 3 units long.

D. When one triangle is placed on top of the other and their sides are aligned, we will see that one triangle is larger than the other.

E. The two triangles have the same area as each other.

6 from Unit 1, Lesson 3

Here is an octagon. (Note: The diagonal sides of the octagon are *not* 4 inches long.)



a. While estimating the area of the octagon, Lin reasoned that it must be less than 100 square inches. Do you agree? Explain your reasoning.

b. Find the exact area of the octagon. Show your reasoning.

SAMPLE ONLY

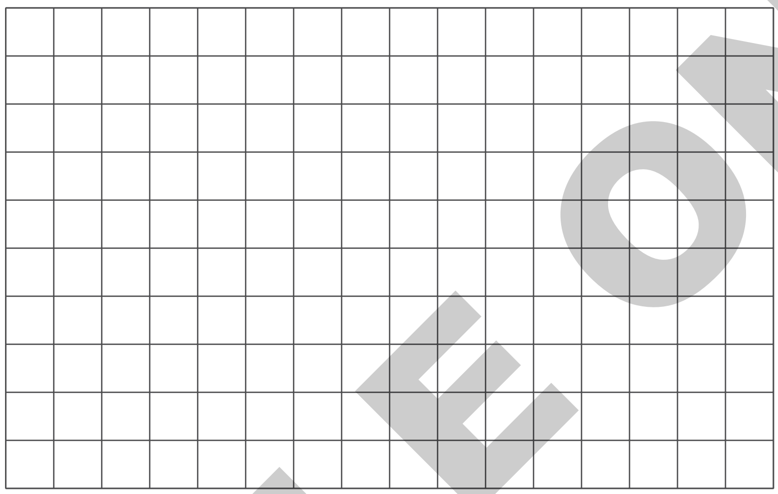
Sec C

Bases and Heights of Triangles

Let’s use different base-height pairs to find the area of a triangle.

10.1 An Area of 12

On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.

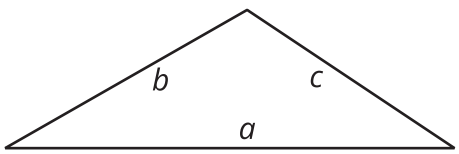


SAMPLE ONLY

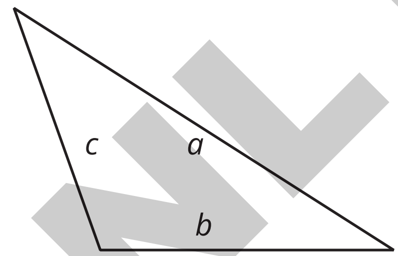
10.2 Hunting for Heights

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

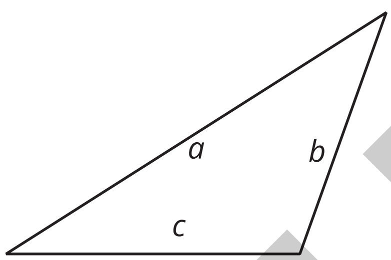
   Use Side a as the base:

   Use Side b as the base:

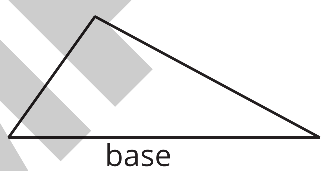
   Use Side c as the base:

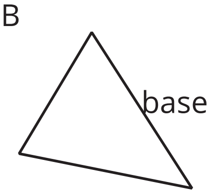
   Pause for your teacher's instructions before moving to the next question.

2. Draw a line segment to show the height for the chosen base in each triangle.

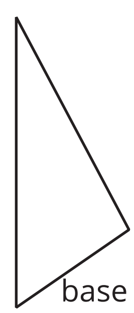
A



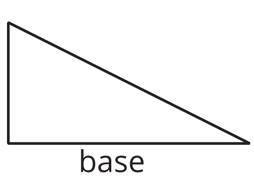
B



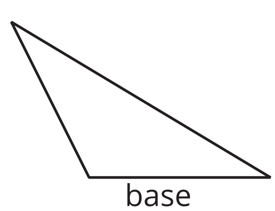
C



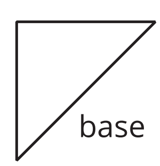
D



E



F

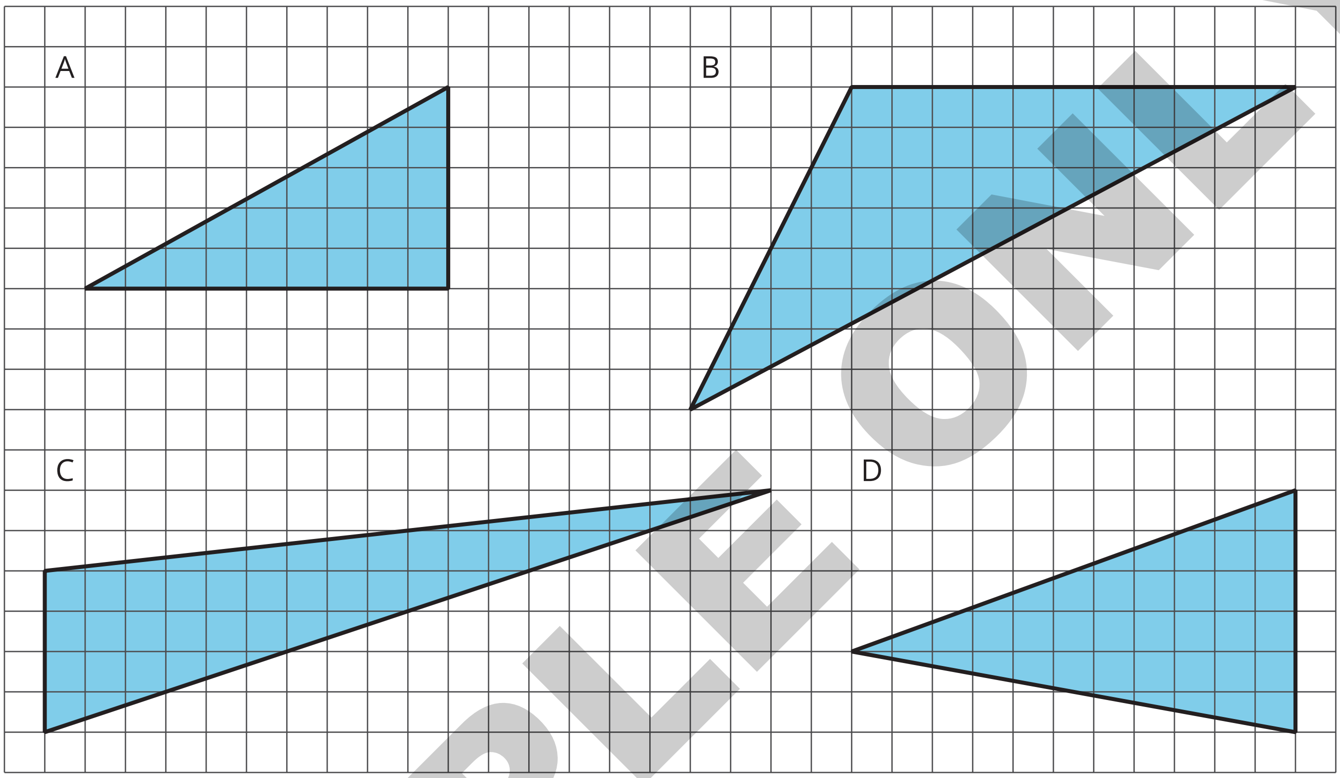


SAMPLE ONLY

10.3 Some Bases Are Better Than Others

For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)



SAMPLE ONLY

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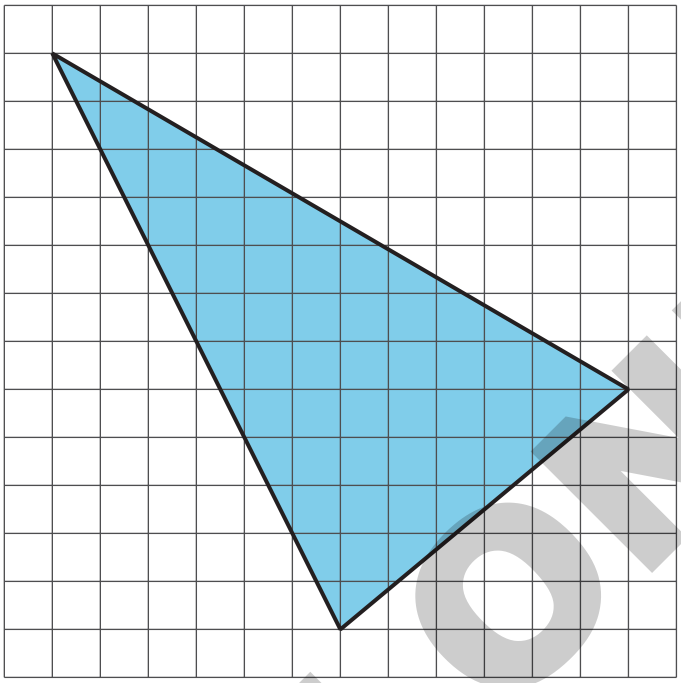
Are you ready for more?

Find the area of this triangle. Show your reasoning.



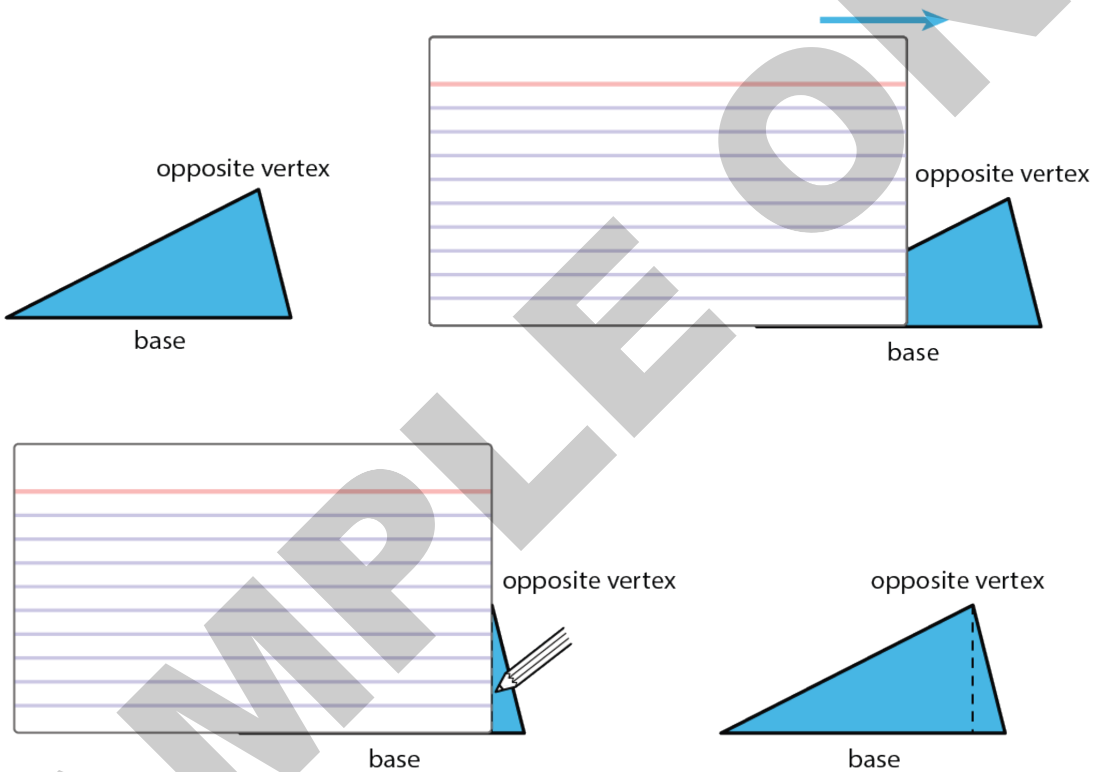
SAMPLE ONLY

Lesson 10 Summary

A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite **vertex**. We can use tools with right angles to help us draw height segments.

An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
2. Line up one **edge** of the index card with that base.
3. Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4. Use the card edge to draw a line from the vertex to the base. That segment represents the height.



Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.

SAMPLE ONLY

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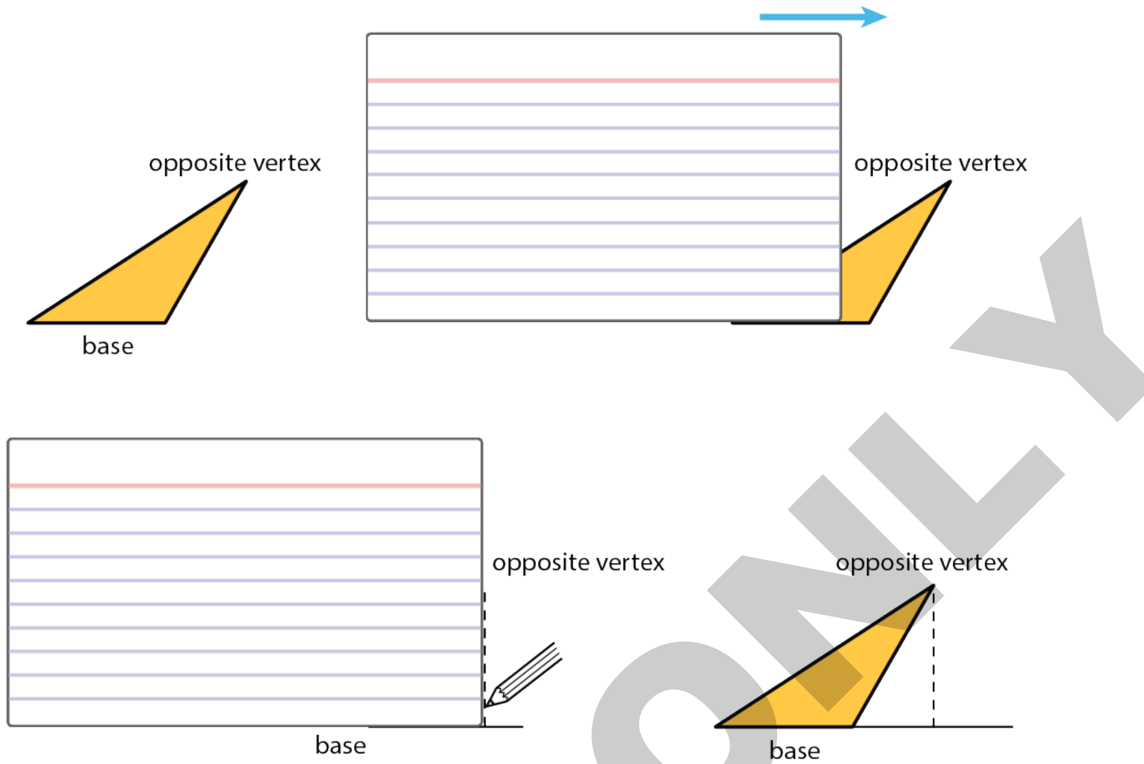
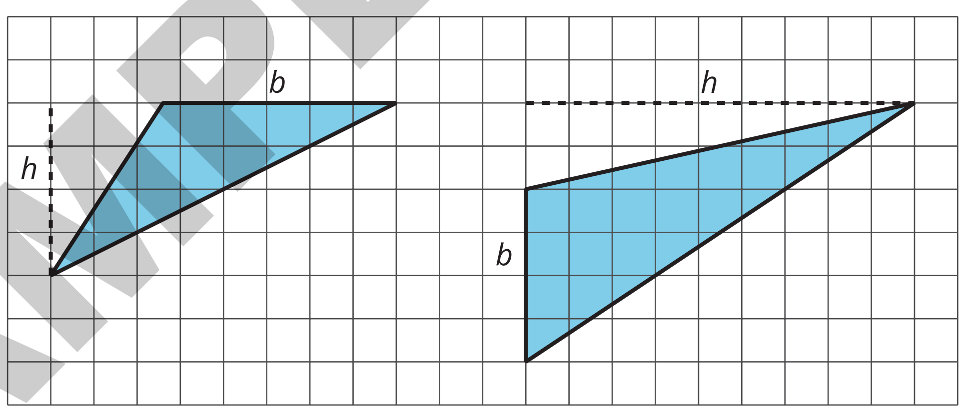


Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically. For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:



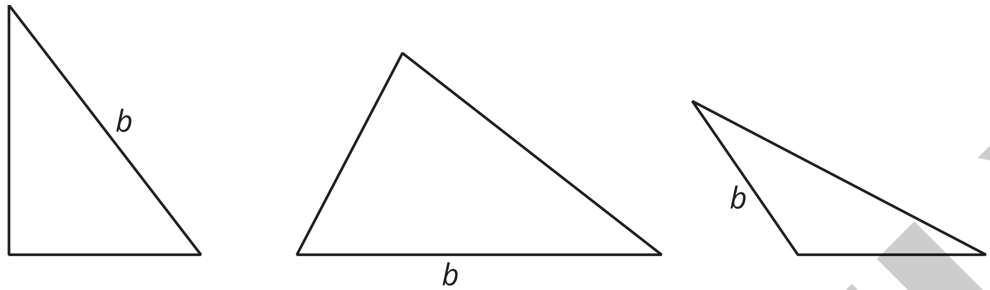
Glossary

- edge
- vertex

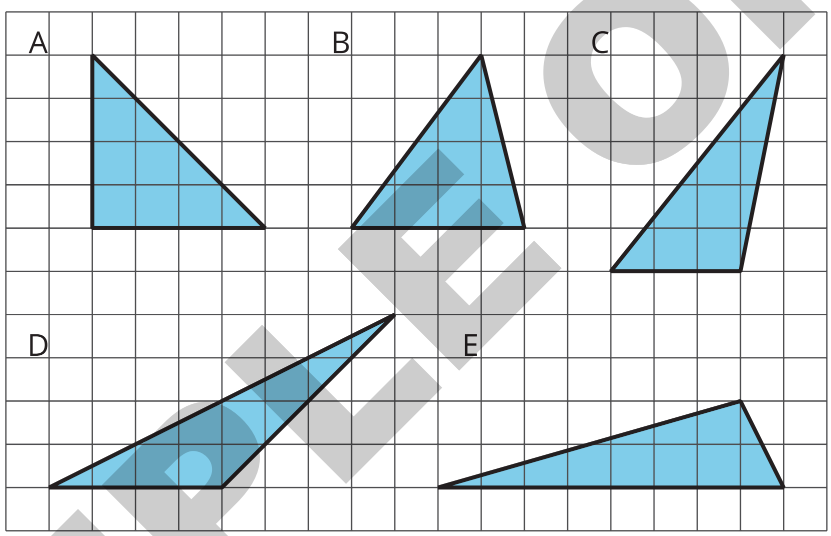
SAMPLE ONLY

Practice Problems

1 For each triangle, a base is labeled b. Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.



2 Select **all** triangles that have an area of 8 square units. Explain how you know.



SAMPLE ONLY

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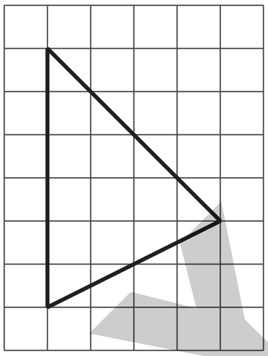
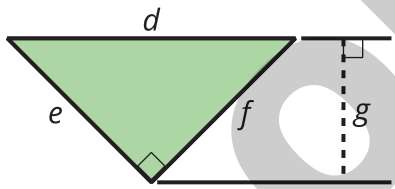
3 Find the area of the triangle. Show your reasoning.

If you get stuck, carefully consider which side of the triangle to use as the base.

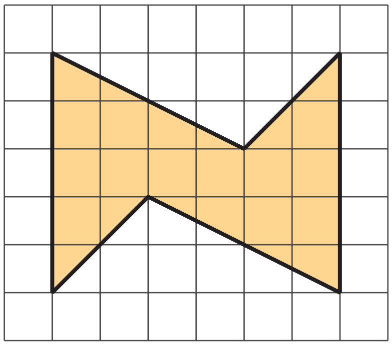


4 Can Side d be the base for this triangle? If so, which length would be the corresponding height? If not, explain why not.



5 from Unit 1, Lesson 3

Find the area of this shape. Show your reasoning.

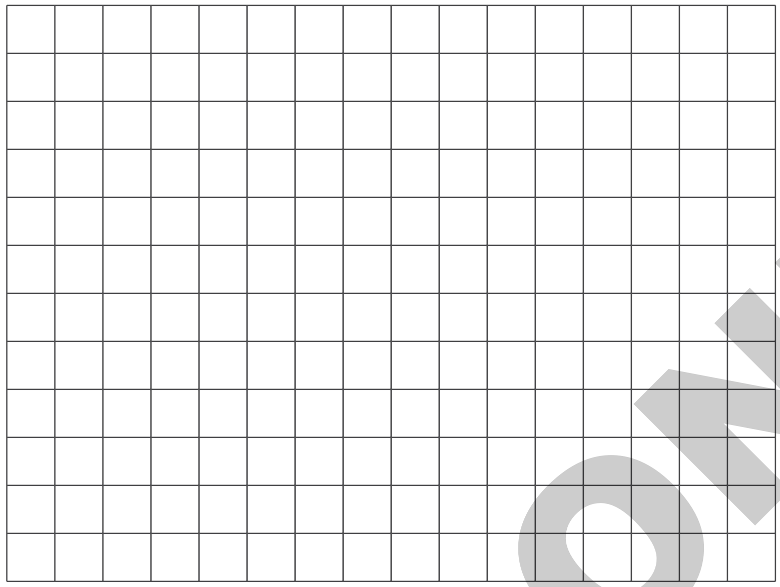


SAMPLE ONLY

Sec C

6 from Unit 1, Lesson 6

On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know that the areas are the same.



Sec C

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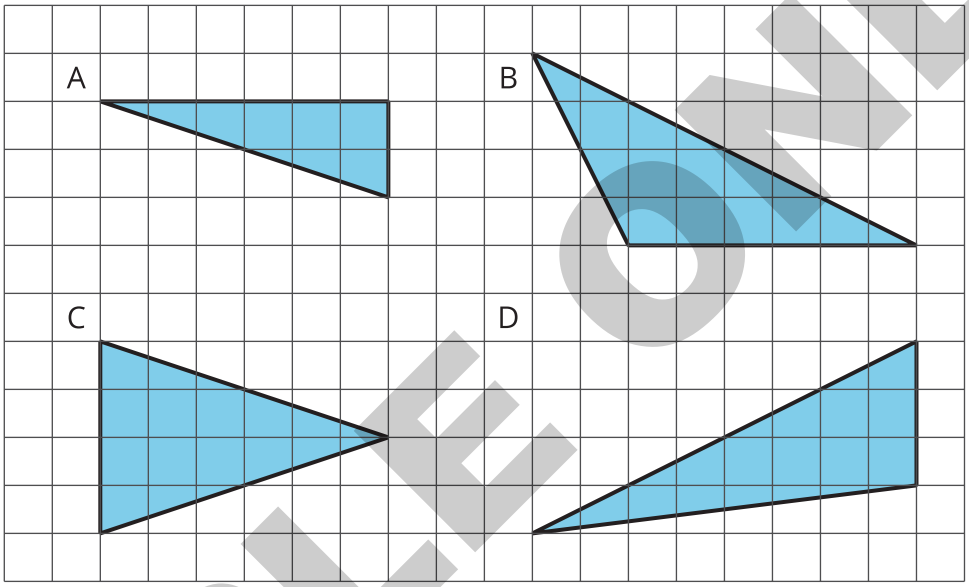
Polygons

Let's investigate polygons and their areas.

11.1 Which Three Go Together: Triangles

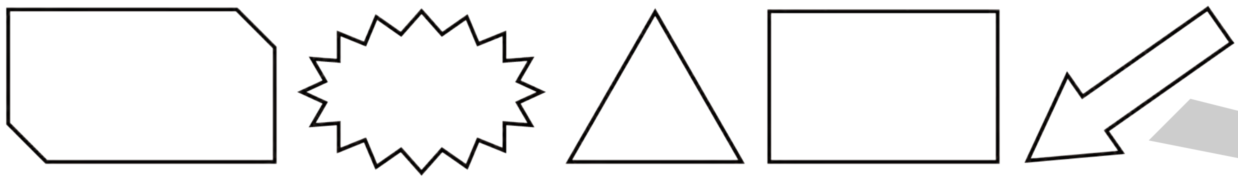
Which three go together? Why do they go together?



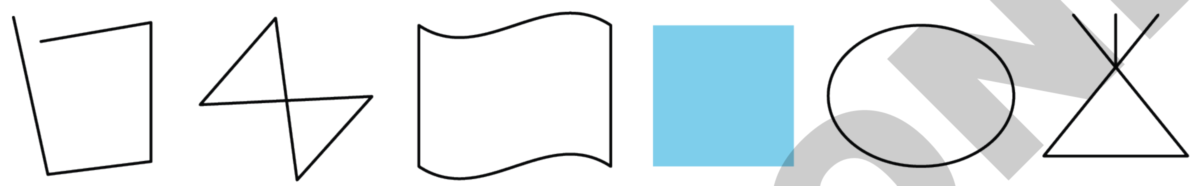
SAMPLE ONLY

11.2 What Are Polygons?

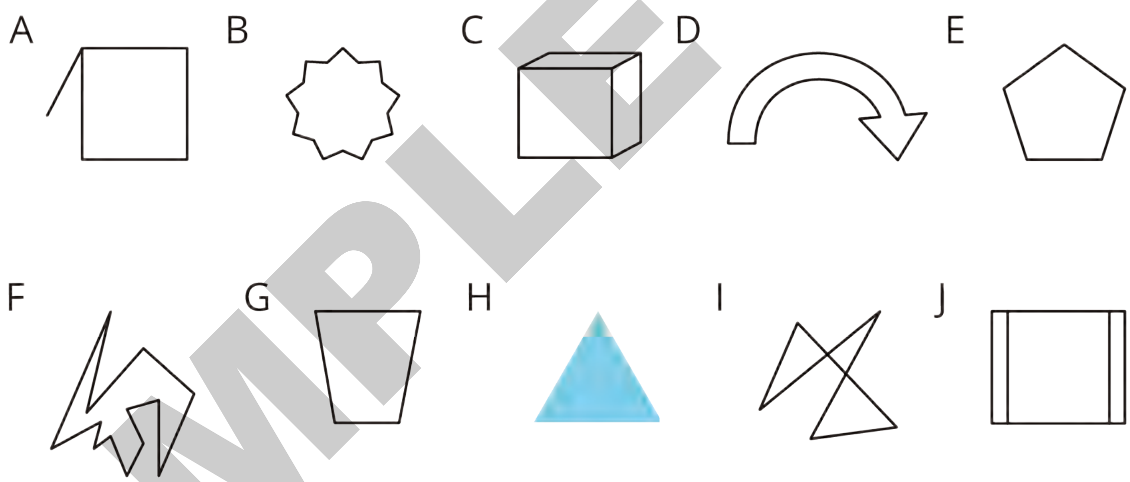
These five figures are **polygons**.



The next six figures are *not* polygons.



1. Circle the figures that are polygons.

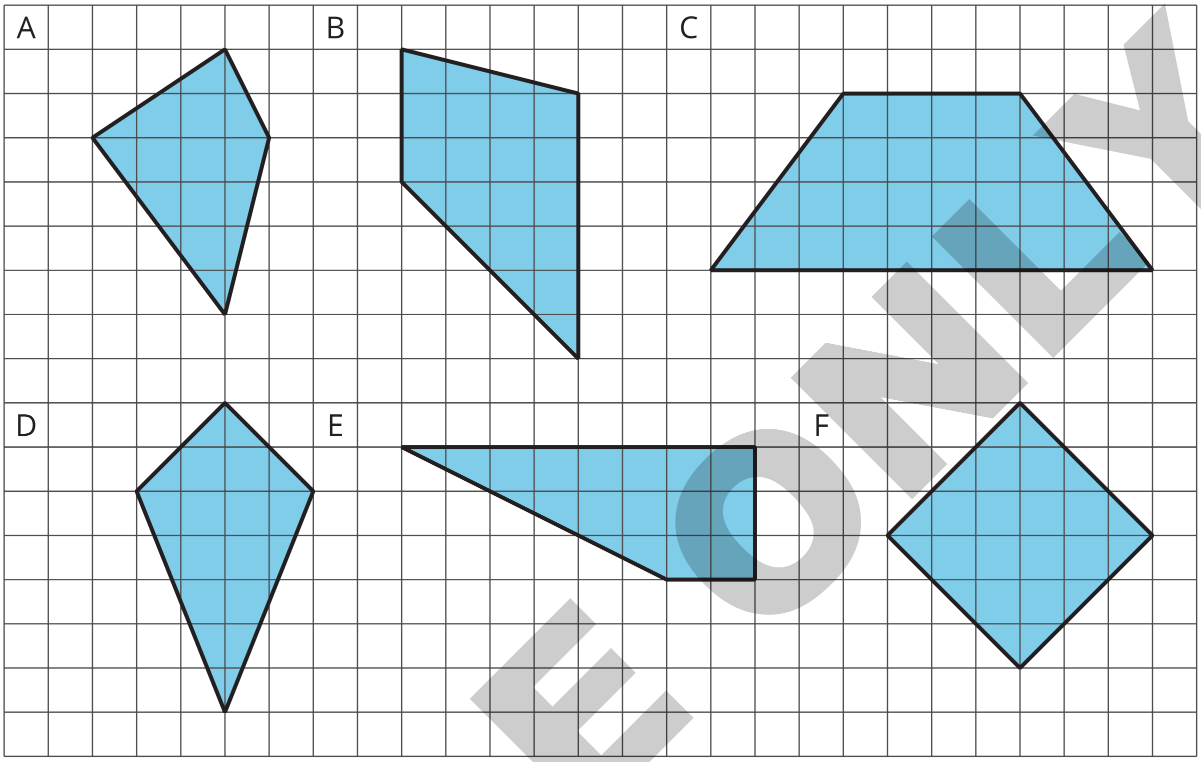


2. What do the figures you circled have in common? What characteristics helped you decide whether a figure was a polygon?

SAMPLE ONLY

11.3 Quadrilateral Strategies

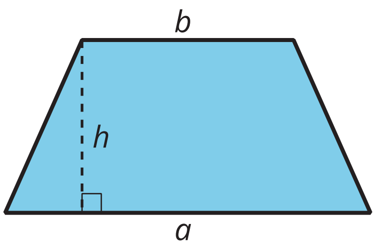
Find the area of *two* quadrilaterals of your choice. Show your reasoning.



SAMPLE ONLY

Are you ready for more?

Here is a trapezoid. a and b represent the lengths of its bottom and top sides. The segment labeled h represents its height; it is perpendicular to both the top and bottom sides.



Apply area-reasoning strategies—decomposing, rearranging, duplicating, etc.—to the trapezoid so that you have one or more shapes with areas that you already know how to find. Use the shapes to help you write a formula for the area of a trapezoid. Show your reasoning.

SAMPLE ONLY

Sec C

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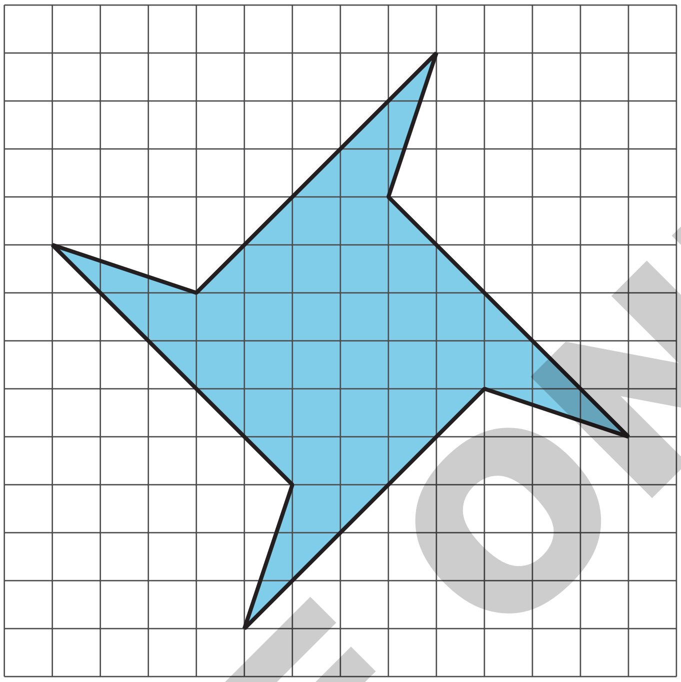
11.4 Pinwheel

Find the area of the shaded region in square units. Show your reasoning.



SAMPLE ONLY

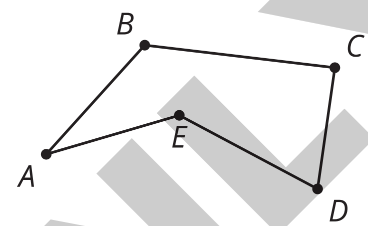
Sec C

Lesson 11 Summary

A **polygon** is a two-dimensional figure composed of straight line segments.

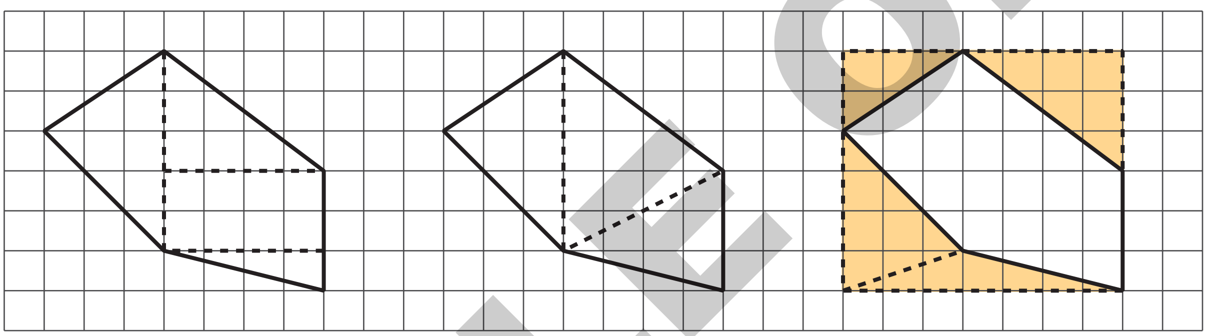
- Each end of a line segment connects to one other line segment. The point where two segments connect is a vertex. The plural of vertex is vertices.
- The segments are called the edges or sides of the polygon. The sides never cross each other. There are always an equal number of vertices and sides.

Here is a polygon with 5 sides. The vertices are labeled $A, B, C, D,$ and E.



A polygon encloses a region. The area of a polygon is the area of the region inside it.

We can find the area of a polygon by decomposing the region inside it into triangles and rectangles.



The first two diagrams show the polygon decomposed into triangles and rectangles. The sum of their areas is the area of the polygon. The last diagram shows the polygon enclosed with a rectangle. Subtracting the areas of the triangles from the area of the rectangle gives us the area of the polygon.

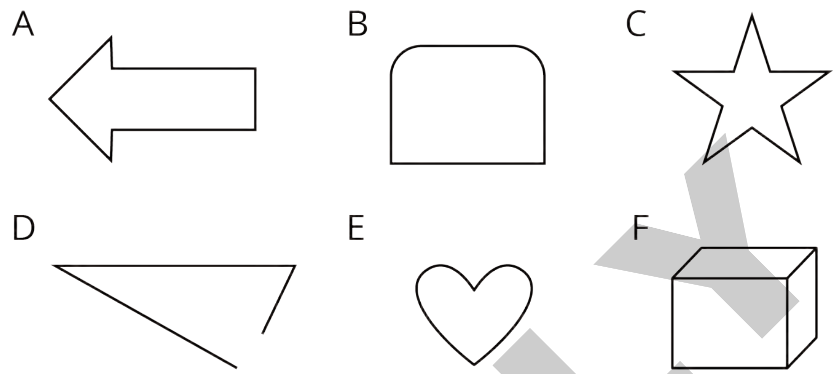
Glossary

- polygon

SAMPLE ONLY

Practice Problems

1 Select **all** the polygons.



A. A

B. B

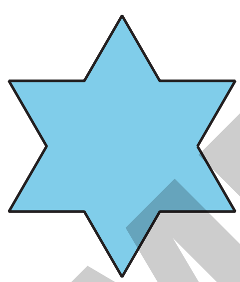
C. C

D. D

E. E

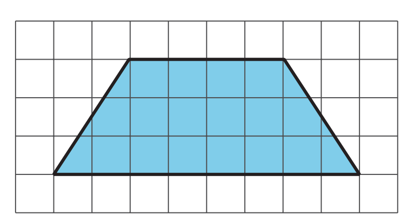
F. F

2



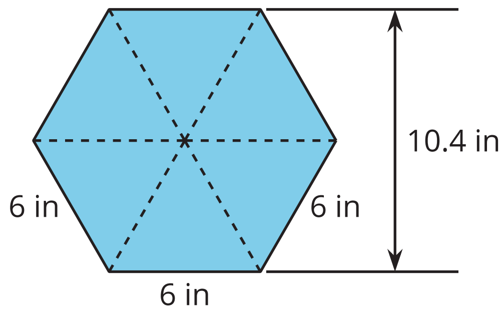
Mark each vertex with a large dot. How many edges and vertices does this polygon have?

3 Find the area of this trapezoid. Explain or show your reasoning.

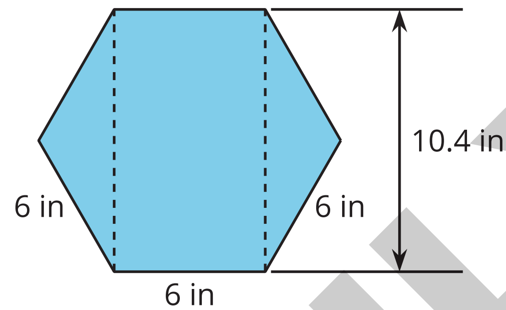


SAMPLE ONLY

4 Lin and Andre used different methods to find the area of a regular hexagon with 6-inch sides. Lin decomposed the hexagon into six identical, equilateral triangles. Andre decomposed the hexagon into a rectangle and two triangles.



Lin's method



Andre's method

Find the area of the hexagon using each person's method. Show your reasoning.

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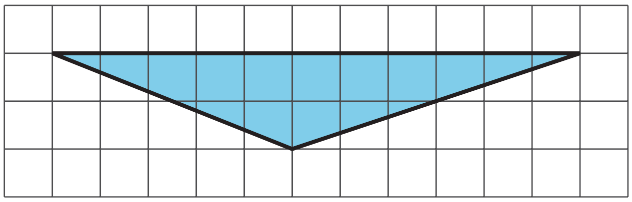
5 from Unit 1, Lesson 9

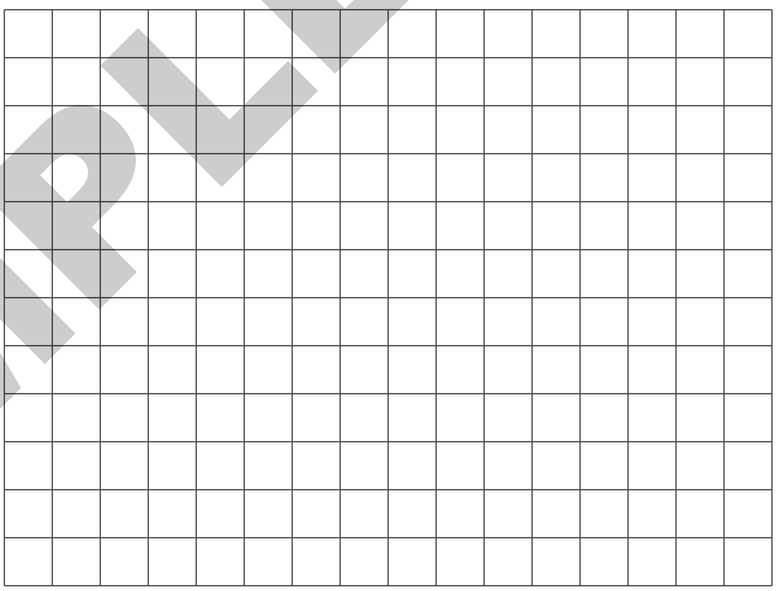
a. Identify a base and a corresponding height that can be used to find the area of this triangle. Label the base b and the corresponding height h.



b. Find the area of the triangle. Show your reasoning.

6 from Unit 1, Lesson 10

On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.



SAMPLE ONLY

What is Surface Area?

Let's cover the surfaces of some three-dimensional objects.

12.1 Covering the Cabinet (Part 1)

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

12.2 Covering the Cabinet (Part 2)

Earlier you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

2. Use the information you have to find the number of sticky notes needed to cover the cabinet. Show your reasoning.

SAMPLE ONLY

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Are you ready for more?

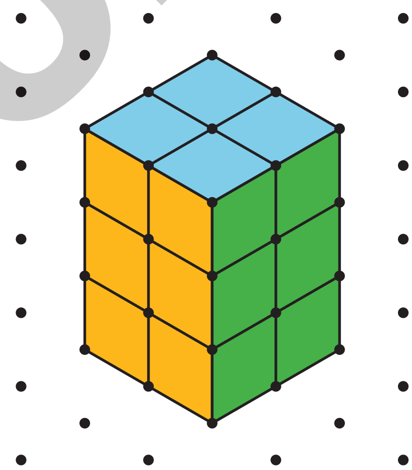
How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

SAMPLE ONLY

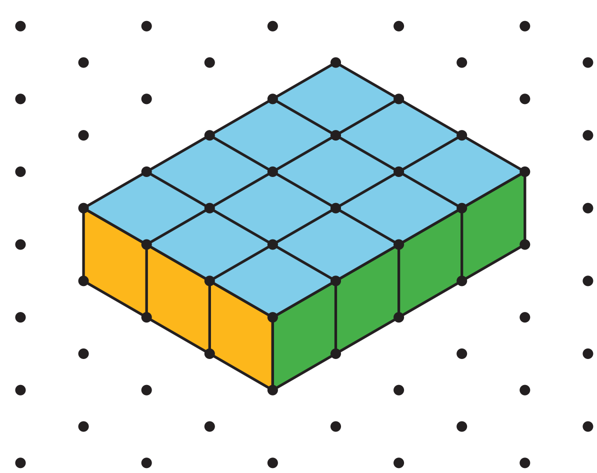
12.3 Prisms Built from Cubes

1. Here is a sketch of a rectangular prism built from 12 cubes. It has six **faces**, but you can see only three of them in the sketch.

   Show that it has a **surface area** of 32 square units.

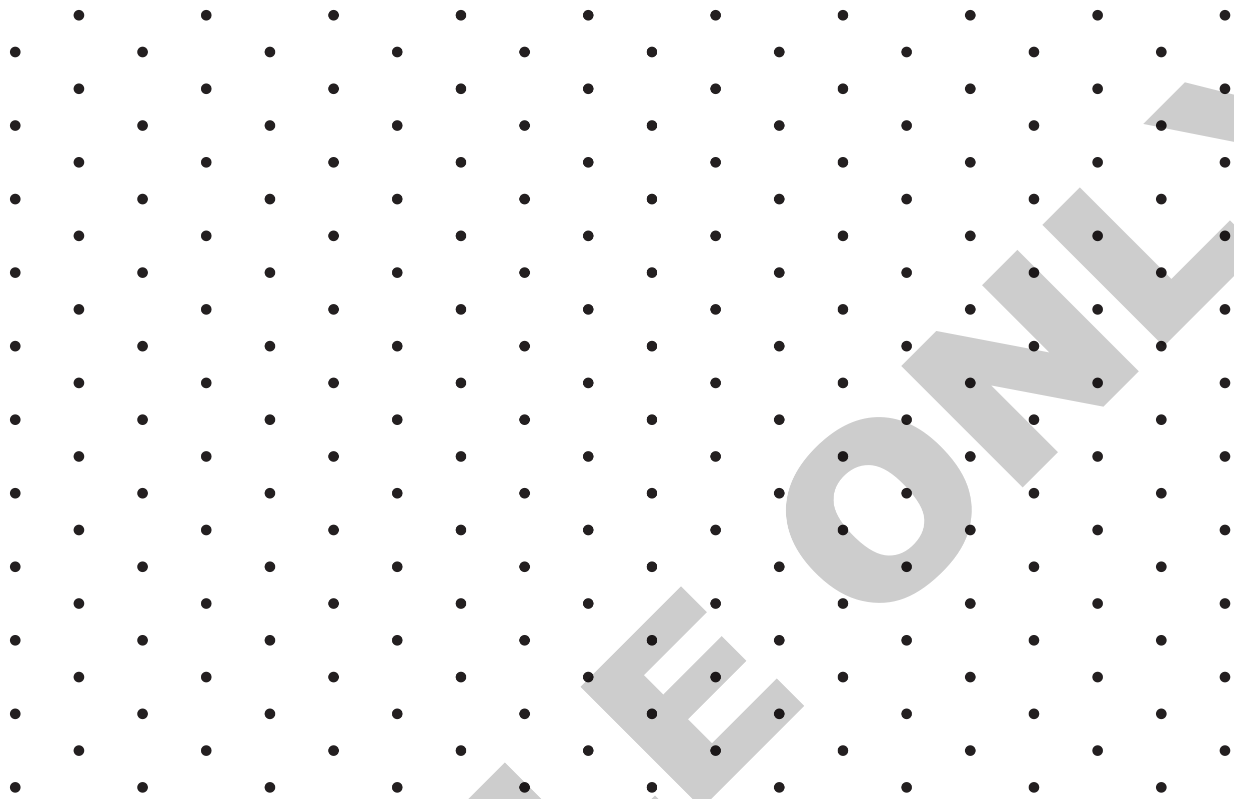


2. Here is a sketch of another rectangular prism built from 12 cubes. What is its surface area? Be prepared to explain or show your reasoning.



Are you ready for more?

Is it possible to use 12 cubes to build a rectangular prism that has a greater surface area than either prism shown earlier? Explain or show your reasoning. You can draw prisms on the dot paper if it helps.

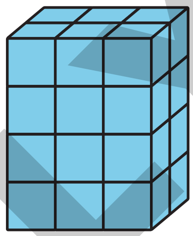


SAMPLE ONLY

Lesson 12 Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.



So the surface area of a rectangular prism that has edge-lengths of 2 cm, 3 cm, and 4 cm has a surface area of

$$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 4) + (3 \cdot 4) + (3 \cdot 4)$$

or 52 square centimeters.

Glossary

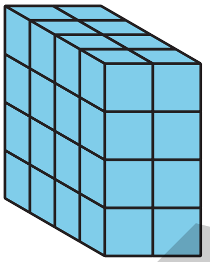
- face
- surface area

SAMPLE ONLY

Sec D

Practice Problems

1 What is the surface area of this rectangular prism?



A. 16 square units

B. 32 square units

C. 48 square units

D. 64 square units

2 Which description can represent the surface area of this trunk?

A. The number of square inches that cover the top of the trunk.

B. The number of square feet that cover all the outside faces of the trunk.

C. The number of square inches of horizontal surface inside the trunk.

D. The number of cubic feet that can be packed inside the trunk.

SAMPLE ONLY

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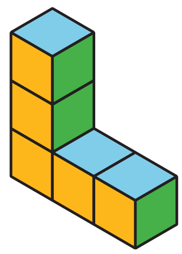
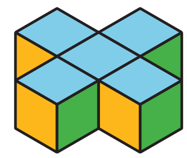
3 Which figure has a greater surface area?

A



B



4 A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

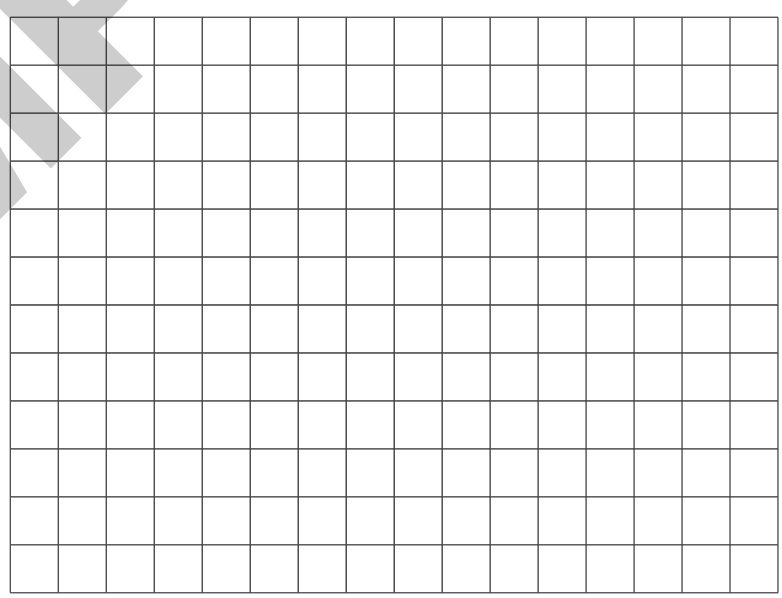
5 from Unit 1, Lesson 9

Draw an example of each of these triangles on the grid.

a. A right triangle with an area of 6 square units.

b. An acute triangle with an area of 6 square units.

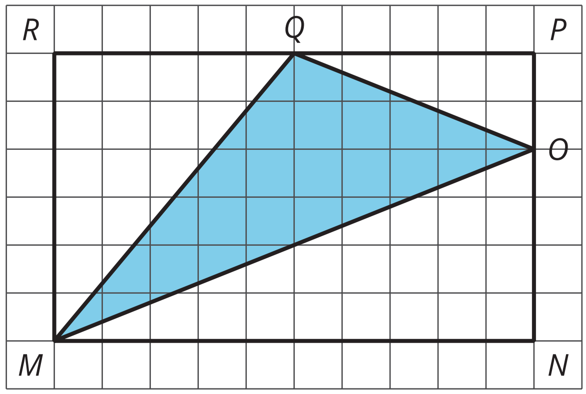
c. An obtuse triangle with an area of 6 square units.



SAMPLE ONLY

6 from Unit 1, Lesson 10

Find the area of triangle MOQ in square units. Show your reasoning.

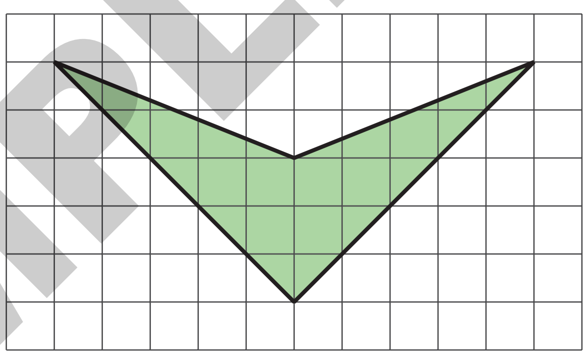


SAMPLE ONLY

Sec D

7 from Unit 1, Lesson 3

Find the area of this shape. Show your reasoning.



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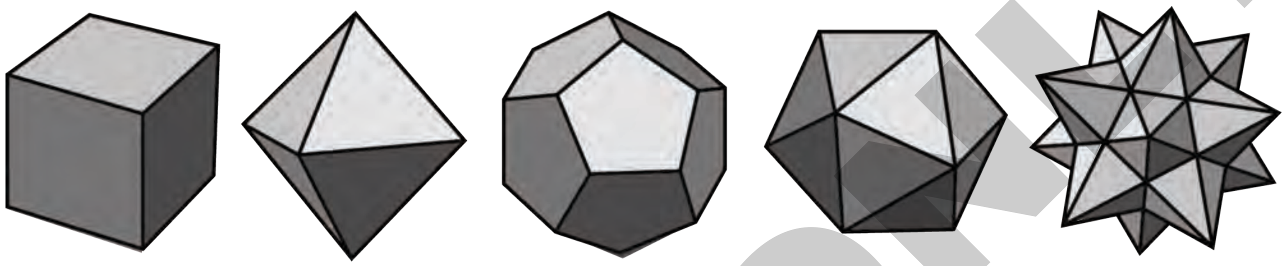
Polyhedra

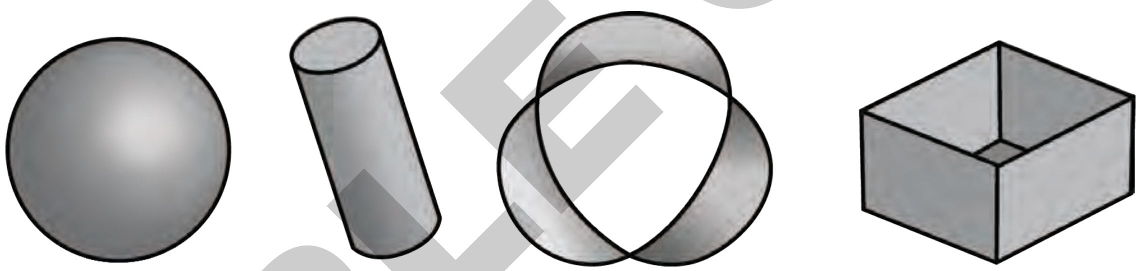
Let's investigate polyhedra.

13.1 What are Polyhedra?



These five drawings represent **polyhedra**.

The next four drawings do *not* represent polyhedra.



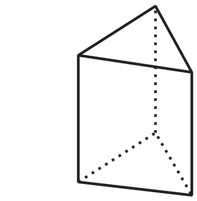
1. Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.
2. What characteristics helped you distinguish the polyhedra from the other figures?

SAMPLE ONLY

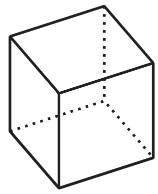
13.2 Prisms and Pyramids

1. Here are some polyhedra called **prisms**.

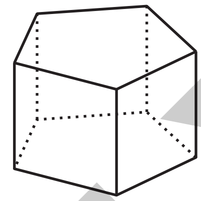
A



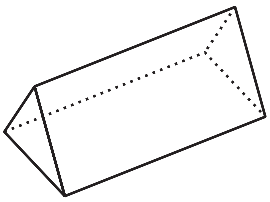
B



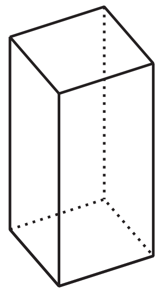
C



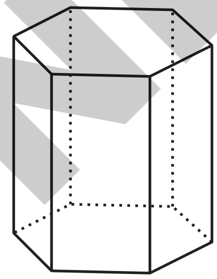
D



E

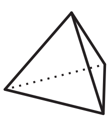


F

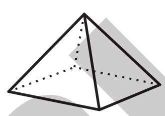


Here are some polyhedra called **pyramids**.

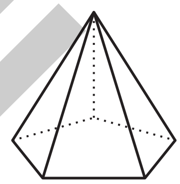
P



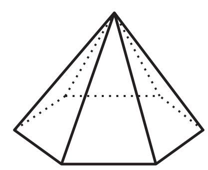
Q



R



S

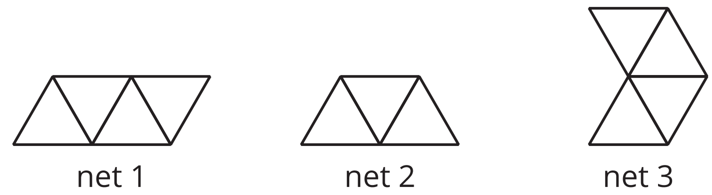


a. Look at the prisms. What are their characteristics?

b. Look at the pyramids. What are their characteristics?

SAMPLE ONLY

2. Which of these **nets** can be folded into Pyramid P? Select all that apply.



3. Your teacher will give your group some polygons and assign a polyhedron.
   a. Decide which polygons are needed to compose your assigned polyhedron. List the polygons and how many of each are needed.
   b. Arrange the cut-outs into a net that, if taped and folded, can be assembled into the polyhedron. Sketch the net. If possible, show a different net for the same polyhedron.

SAMPLE ONLY

Sec D

Are you ready for more?

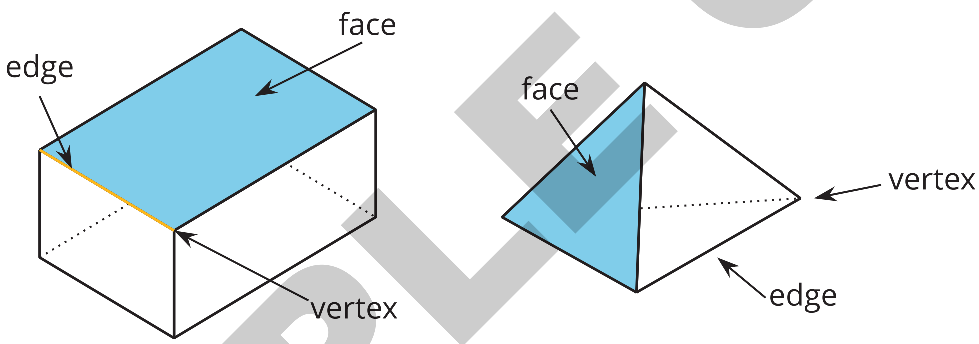
What is the smallest number of faces a polyhedron can possibly have? Explain how you know.

13.3 Assembling Polyhedra

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.
2. How many vertices, edges, and faces are in your polyhedron?

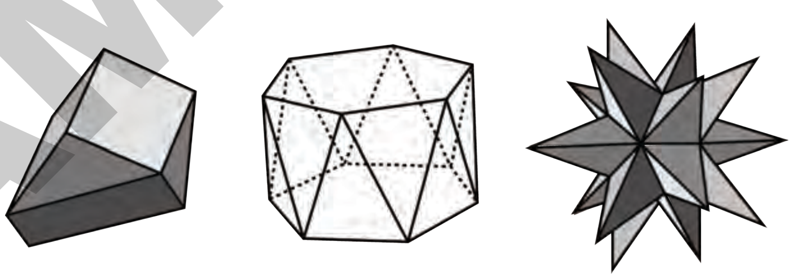
Lesson 13 Summary

A **polyhedron** is a three-dimensional figure composed of faces. Each face is a polygon and meets only one other face along a complete edge. The ends of the edges meet at points that are called vertices.



A polyhedron always encloses a three-dimensional region.

The plural of polyhedron is polyhedra. Here are some drawings of polyhedra:

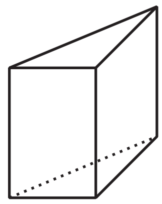


A **prism** is a type of polyhedron with two identical faces that are parallel to each other and that are called **bases**. The bases are connected by a set of rectangles (or sometimes parallelograms that aren't rectangles).

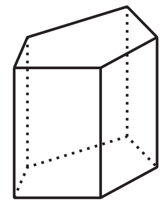
SAMPLE ONLY

A prism is named for the shape of its bases. For example, if the base is a pentagon, then it is called a "pentagonal prism."

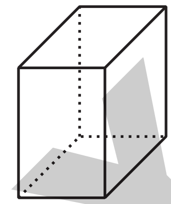
triangular prism



pentagonal prism



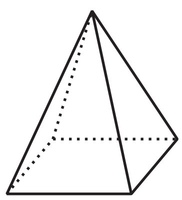
rectangular prism



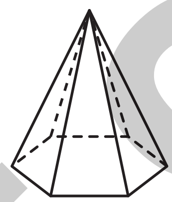
A **pyramid** is a type of polyhedron that has one special face called the base. All of the other faces are triangles that all meet at a single vertex.

A pyramid is named for the shape of its base. For example, if the base is a pentagon, then it is called a "pentagonal pyramid."

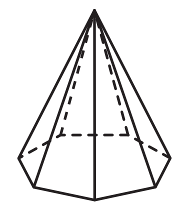
rectangular pyramid



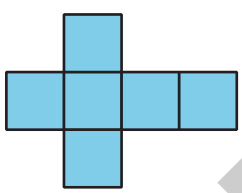
hexagonal pyramid



heptagonal pyramid



A **net** is a two-dimensional representation of a polyhedron. It is composed of polygons that form the faces of a polyhedron.



A cube has 6 square faces, so its net is composed of six squares, as shown here.

A net can be cut out and folded to make a model of the polyhedron.

In a cube, every face shares its edges with 4 other squares. In a net of a cube, not all edges of the squares are joined with another edge. When the net is folded, each of these open edges will join another edge.

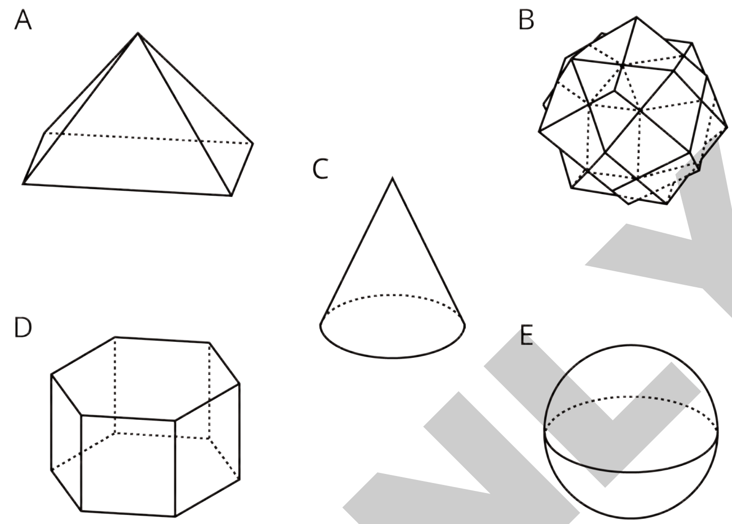
Glossary

- base (of a prism or pyramid)
- net
- polyhedron
- prism
- pyramid

SAMPLE ONLY

Practice Problems

1 Select **all** the polyhedra.



A. A

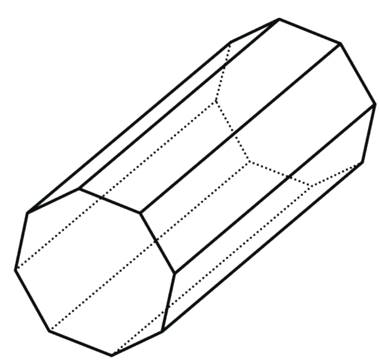
B. B

C. C

D. D

E. E

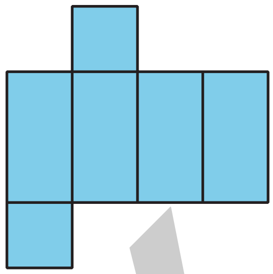
2 a. Is this polyhedron a prism, a pyramid, or neither? Explain how you know.



b. How many faces, edges, and vertices does it have?

SAMPLE ONLY

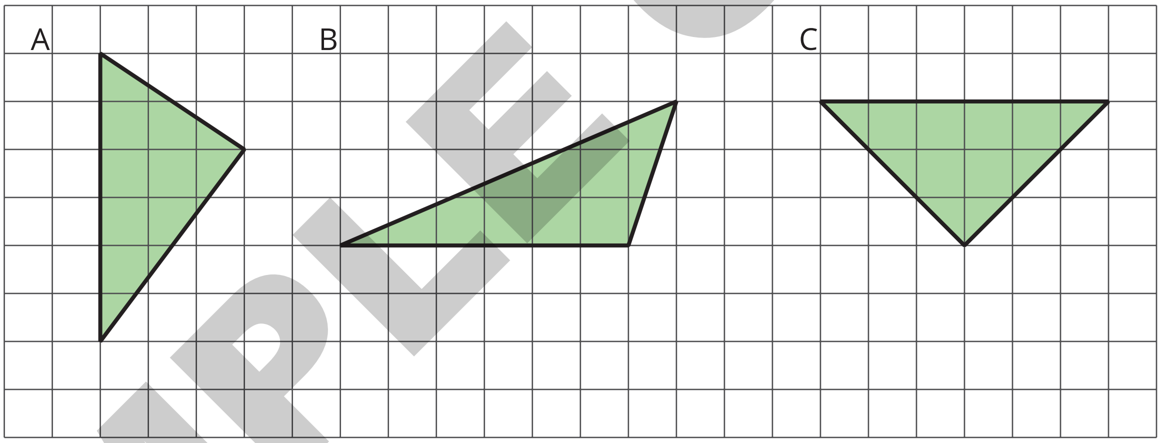
3 Tyler said this net cannot be a net for a square prism because not all the faces are squares.



Do you agree with Tyler? Explain your reasoning.

4 from Unit 1, Lesson 8

Explain why each of these triangles has an area of 9 square units.



SAMPLE ONLY

5 from Unit 1, Lesson 9

a. A parallelogram has a base of 12 meters and a height of 1.5 meters. What is its area?

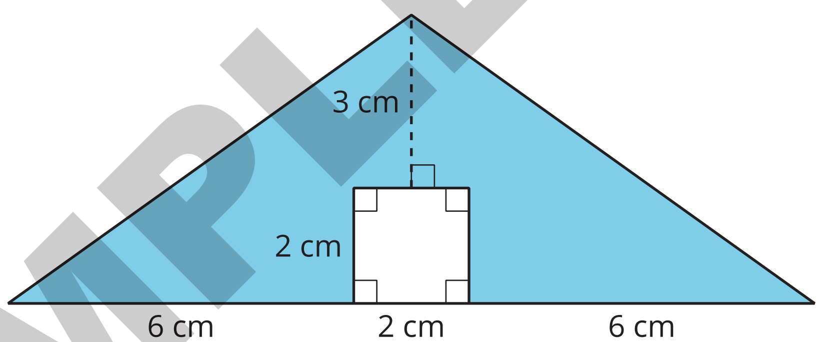
b. A triangle has a base of 16 inches and a height of $\frac{1}{8}$ inches. What is its area?

c. A parallelogram has an area of 28 square feet and a height of 4 feet. What is its base?

d. A triangle has an area of 32 square millimeters and a base of 8 millimeters. What is its height?

6 from Unit 1, Lesson 3

Find the area of the shaded region. Show or explain your reasoning.



SAMPLE ONLY

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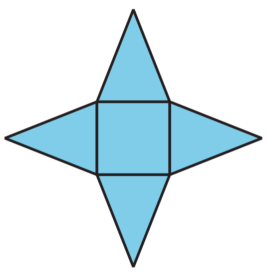
Nets and Surface Area

Let's use nets to find the surface area of polyhedra.

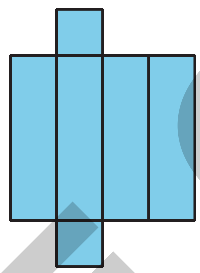
14.1 Matching Nets

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.

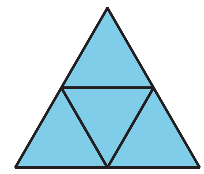
A



B



C



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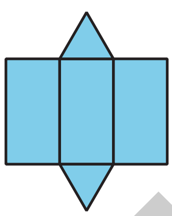

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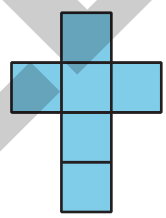
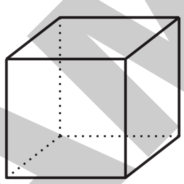


1



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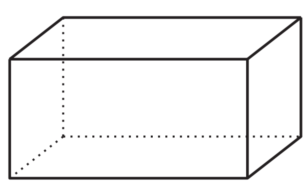

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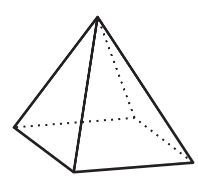

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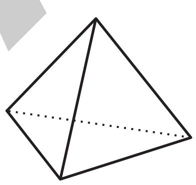

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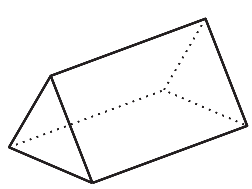
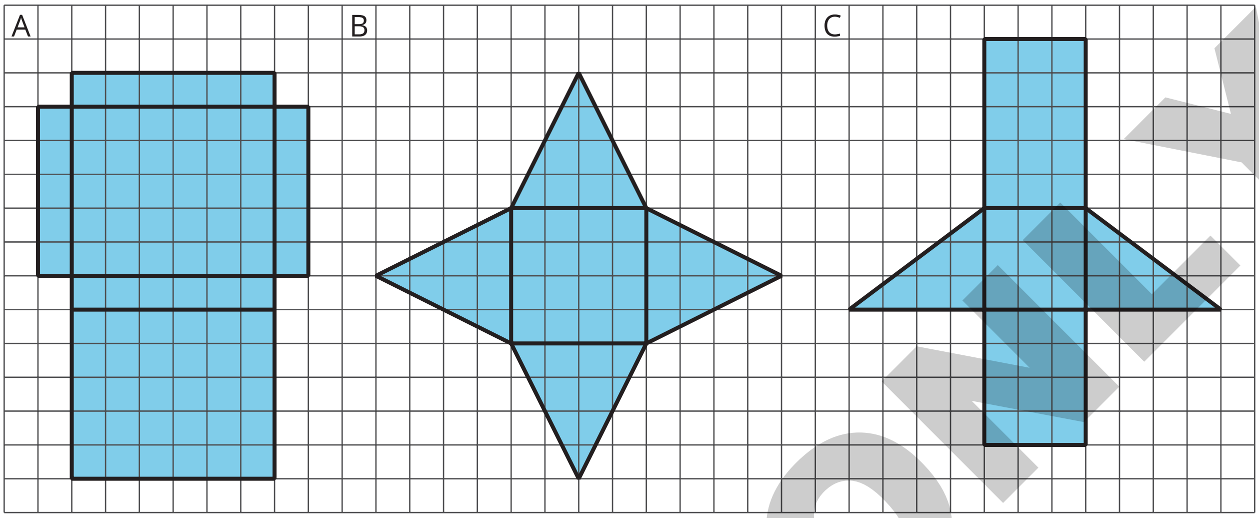


SAMPLE ONLY

14.2 Using Nets to Find Surface Area

1. Name the polyhedron that each net would form when assembled.



2. Your teacher will give you the nets of three polyhedra. Cut out the nets and assemble the three-dimensional shapes.

3. Find the surface area of each polyhedron. Explain or show your reasoning.

SAMPLE ONLY

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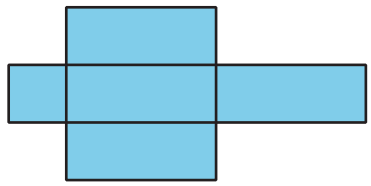
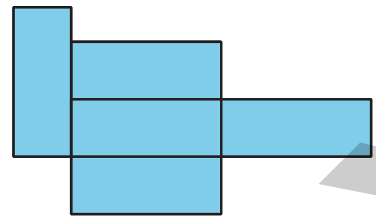
Are you ready for more?

1. For each net, decide if it can be assembled into a rectangular prism.

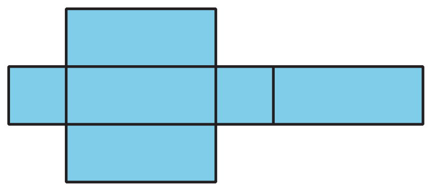
A



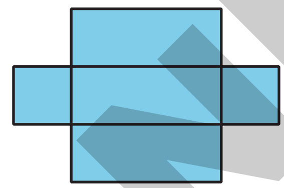
B



C

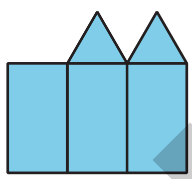


D



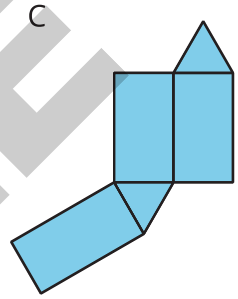
2. For each net, decide if it can be folded into a triangular prism.

A

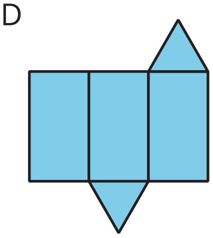


B

C



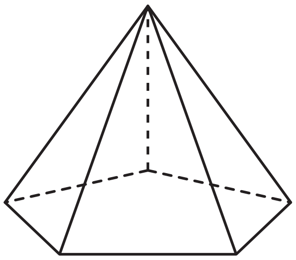
D

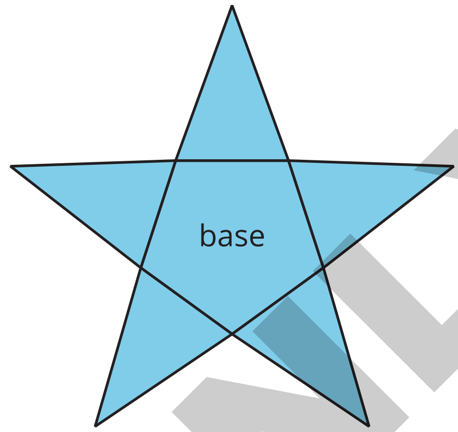


SAMPLE ONLY

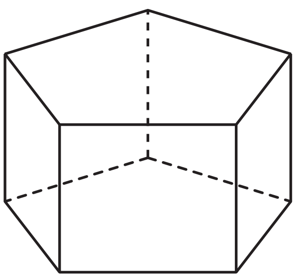
Lesson 14 Summary

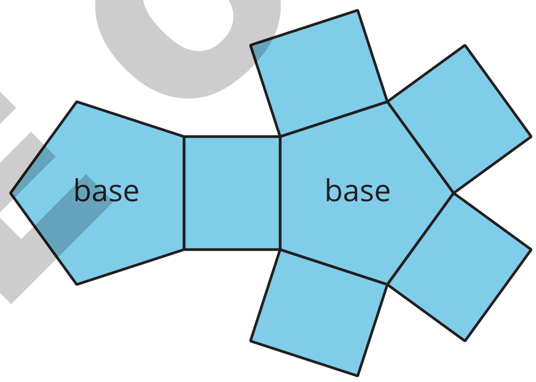
A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.



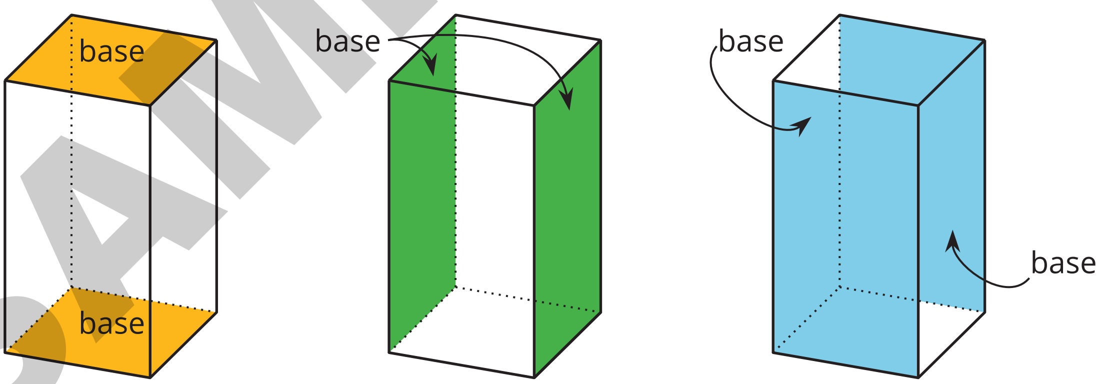


A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.



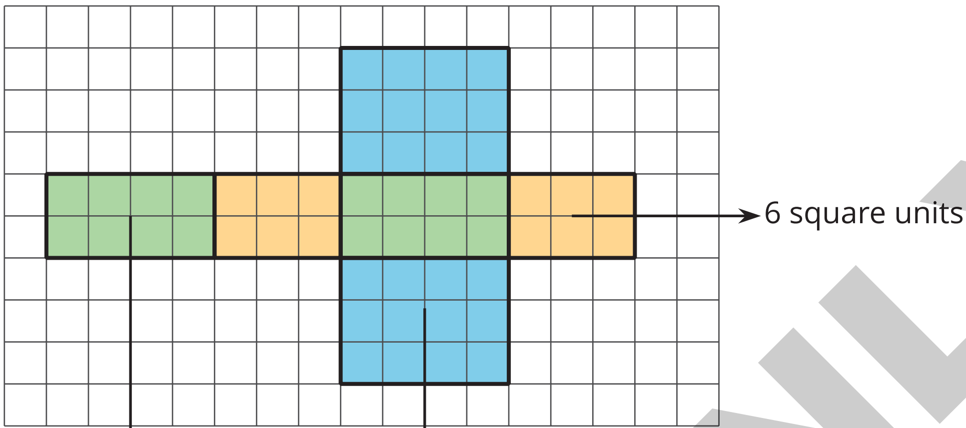


In a rectangular prism, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.



SAMPLE ONLY

Because a net shows all the faces of a polyhedron, we can use it to find its surface area. For instance, the net of a rectangular prism shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.



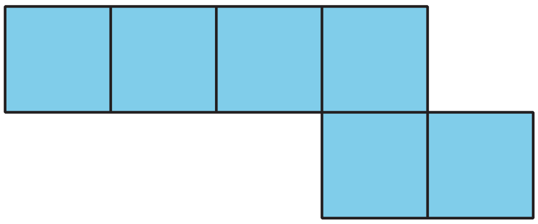
The surface area of the rectangular prism is 52 square units because $8 + 8 + 6 + 6 + 12 + 12 = 52$.

SAMPLE ONLY

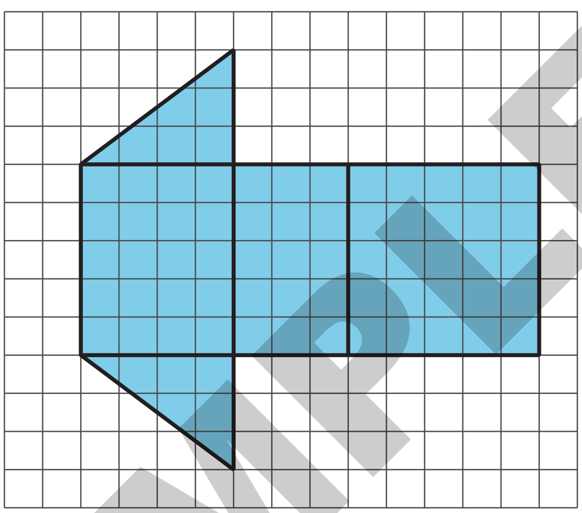
Sec D

Practice Problems

1 Can this net be assembled into a cube? Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.



2 a. What polyhedron can be assembled from this net? Explain how you know.



b. Find the surface area of this polyhedron. Show your reasoning.

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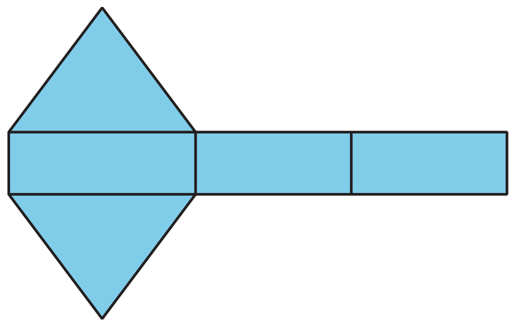
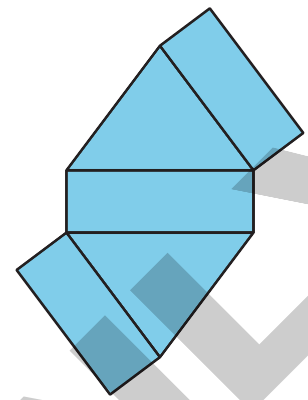
3 Here are two nets. Mai said that both nets can be assembled into the same triangular prism. Do you agree? Explain or show your reasoning.

A



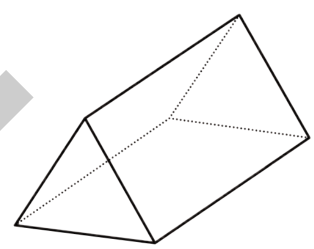
B



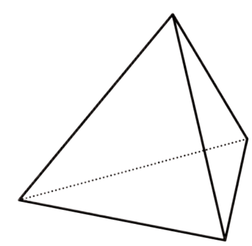
4 from Unit 1, Lesson 13

Here are two three-dimensional figures.

Tell whether each of the following statements describes Figure A, Figure B, both, or neither.



A



B

a. This figure is a polyhedron.

b. This figure has triangular faces.

c. There are more vertices than edges in this figure.

d. This figure has rectangular faces.

e. This figure is a pyramid.

f. There is exactly one face that can be the base for this figure.

g. The base of this figure is a triangle.

h. This figure has two identical and parallel faces that can be the base.

SAMPLE ONLY

5 from Unit 1, Lesson 12

Select **all** units that can be used for surface area.

A. square meters

B. feet

C. centimeters

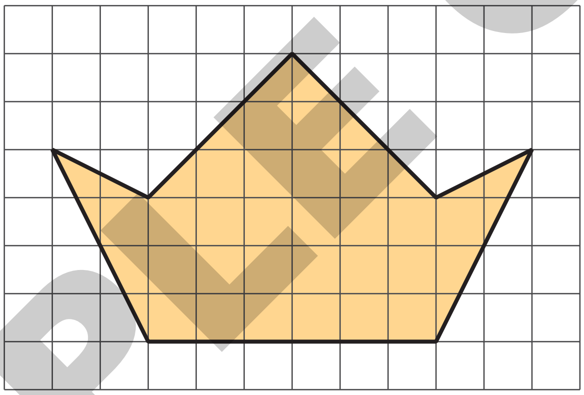
D. cubic inches

E. square inches

F. square feet

6 from Unit 1, Lesson 11

Find the area of this polygon. Show your reasoning.



SAMPLE ONLY

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Unit 1, Lesson 15

More Nets, More Surface Area

Let's draw nets and find the surface area of polyhedra.

15.1 Math Talk: Adjusting a Factor

Find the value of each product mentally.

- $6 \cdot 15$
- $12 \cdot 15$
- $6 \cdot 45$
- $13 \cdot 45$

15.2 Building Prisms and Pyramids

Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

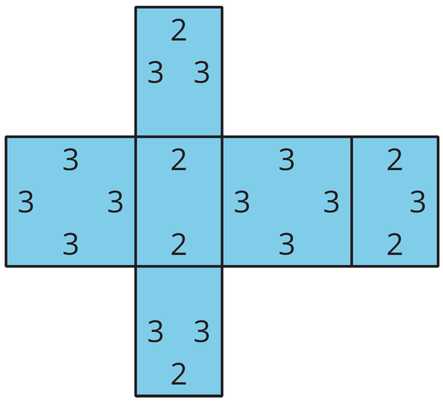
1. What polyhedron do you have?
2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.
3. Label each polygon on the net with a name or number.
4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others.

SAMPLE ONLY

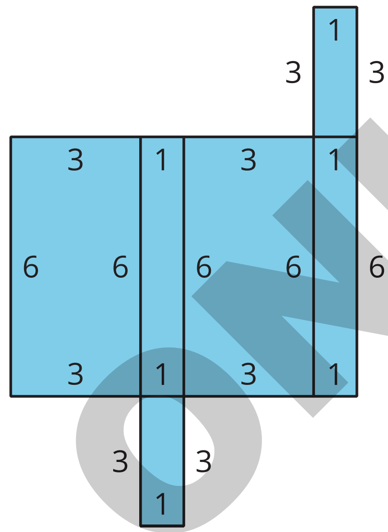
15.3 Comparing Boxes

Here are the nets of three cardboard boxes that are all rectangular prisms. The boxes will be packed with 1-centimeter cubes. All lengths are in centimeters.

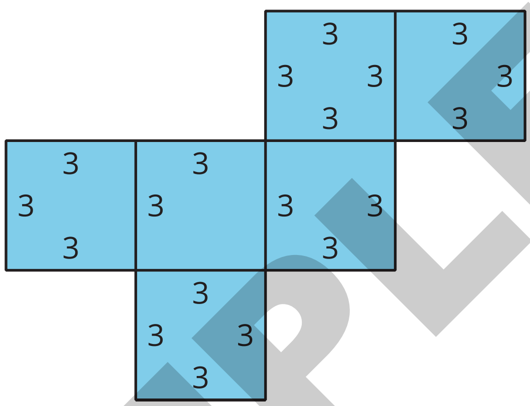
A



B



C



1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.

2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? Show your reasoning.

SAMPLE ONLY

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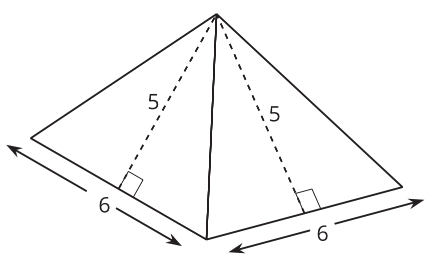
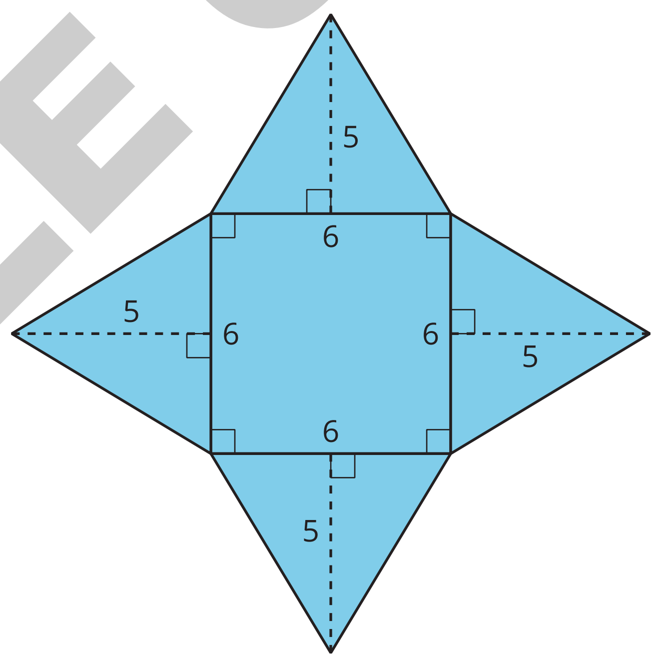
Are you ready for more?

Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

Lesson 15 Summary

A net can help us find the surface area of a polyhedron that has different polygons for its faces. We can find the areas of all polygons in the net and add them.





A square pyramid has a square and 4 triangles for its faces. Its surface area is the sum of the areas of the square base and the 4 triangular faces:

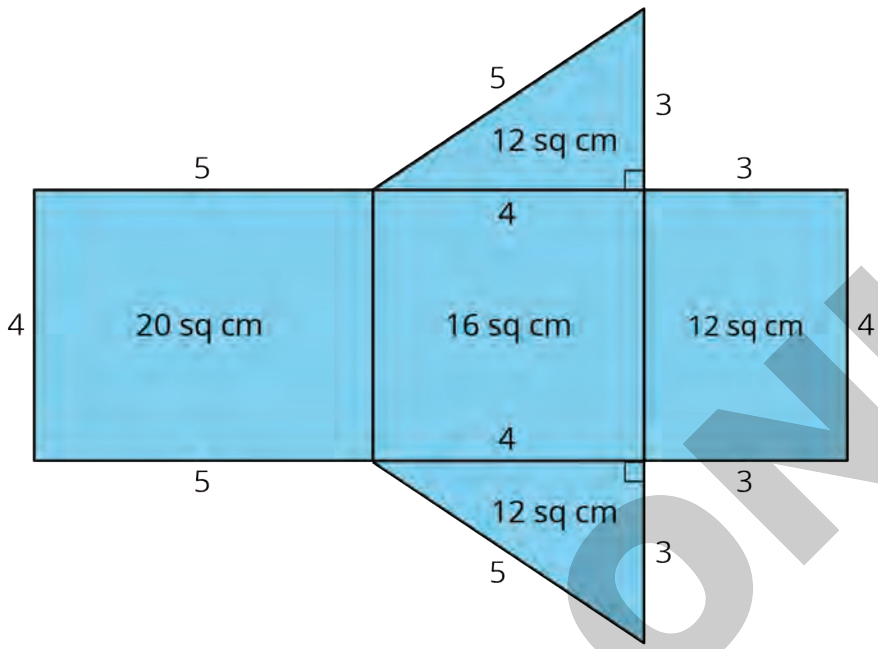
$$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$$

The surface area of this square pyramid is 96 square units.

SAMPLE ONLY

Practice Problems

1 Jada drew a net for a polyhedron and calculated its surface area.



a. What polyhedron can be assembled from this net?

b. Jada made some mistakes in her area calculation. What were the mistakes?

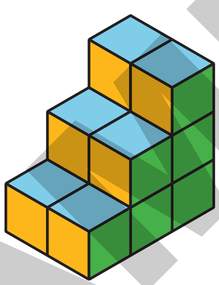
c. Find the surface area of the polyhedron. Show your reasoning.

SAMPLE ONLY

2 A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labeling the edges with their measurements.

3 from Unit 1, Lesson 12

Twelve cubes are stacked to make this figure.



a. What is its surface area?

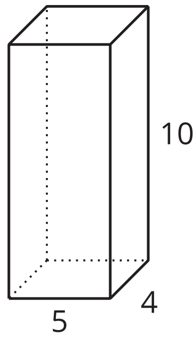
b. How would the surface area change if the top two cubes are removed?

SAMPLE ONLY

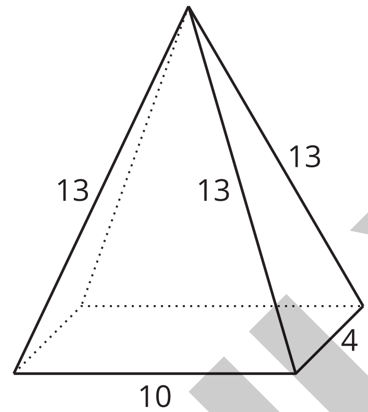
Sec D

4 Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.

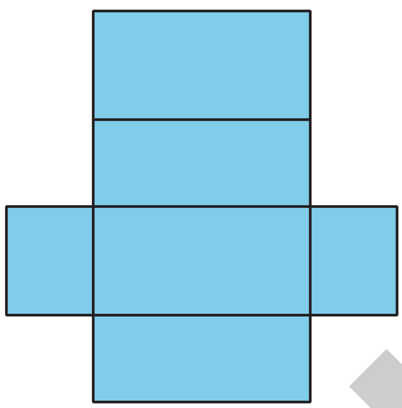
A



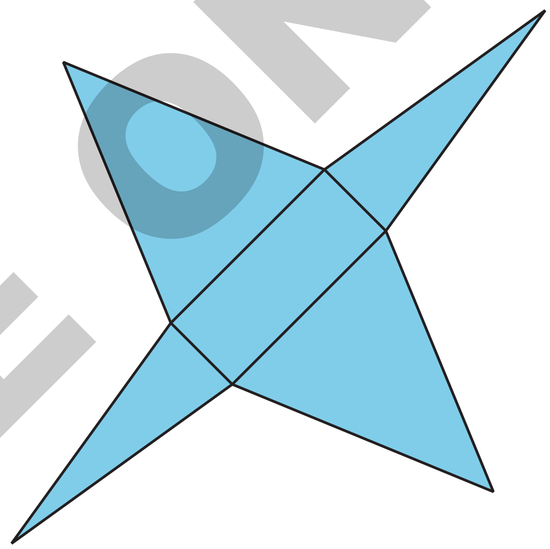
B



A



B



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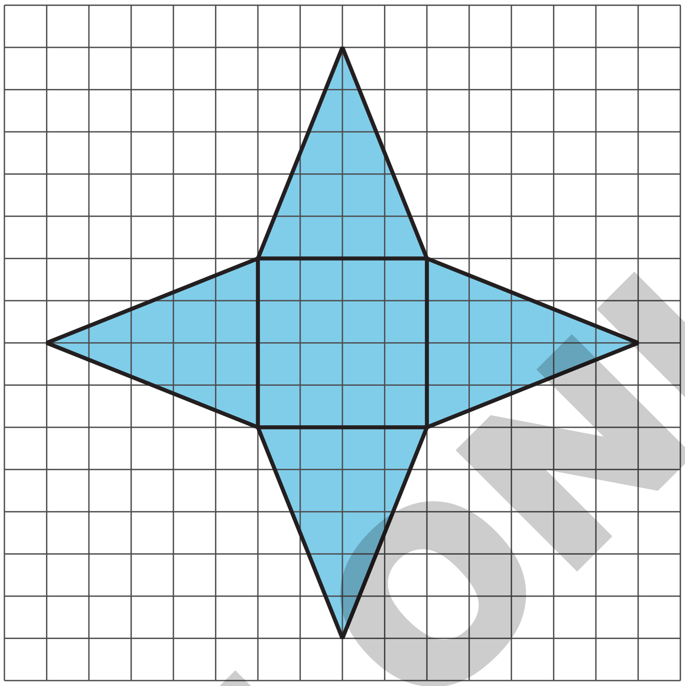
5 from Unit 1, Lesson 14

a. What three-dimensional figure can be assembled from the net?



b. What is the surface area of the figure? (One grid square is 1 square unit.)

SAMPLE ONLY

Unit 1, Lesson 16

Distinguishing Between Surface Area and Volume

Let's contrast surface area and volume.

16.1 Attributes and Their Measures

For each quantity, choose one or more appropriate units of measurement.

For the last two, think of a quantity that could be appropriately measured with the given units.

Quantities

1. Perimeter of a parking lot:
2. Volume of a semi truck:
3. Surface area of a refrigerator:
4. Length of an eyelash:
5. Area of a state:
6. Volume of an ocean:
7. ____________________: miles
8. ____________________: cubic meters

Units

- millimeters (mm)
- feet (ft)
- meters (m)
- square inches (sq in)
- square feet (sq ft)
- square miles (sq mi)
- cubic kilometers (cu km)
- cubic yards (cu yd)

Sec D

16.2 Building with 8 Cubes

Your teacher will give you 16 cubes. Build two different figures using 8 cubes for each.

For each shape, complete these three steps and record your responses on a sticky note:

1. Give it a name or a label (such as Mai's First Shape or Diego's Steps).
2. Find the volume.
3. Find the surface area.

SAMPLE ONLY

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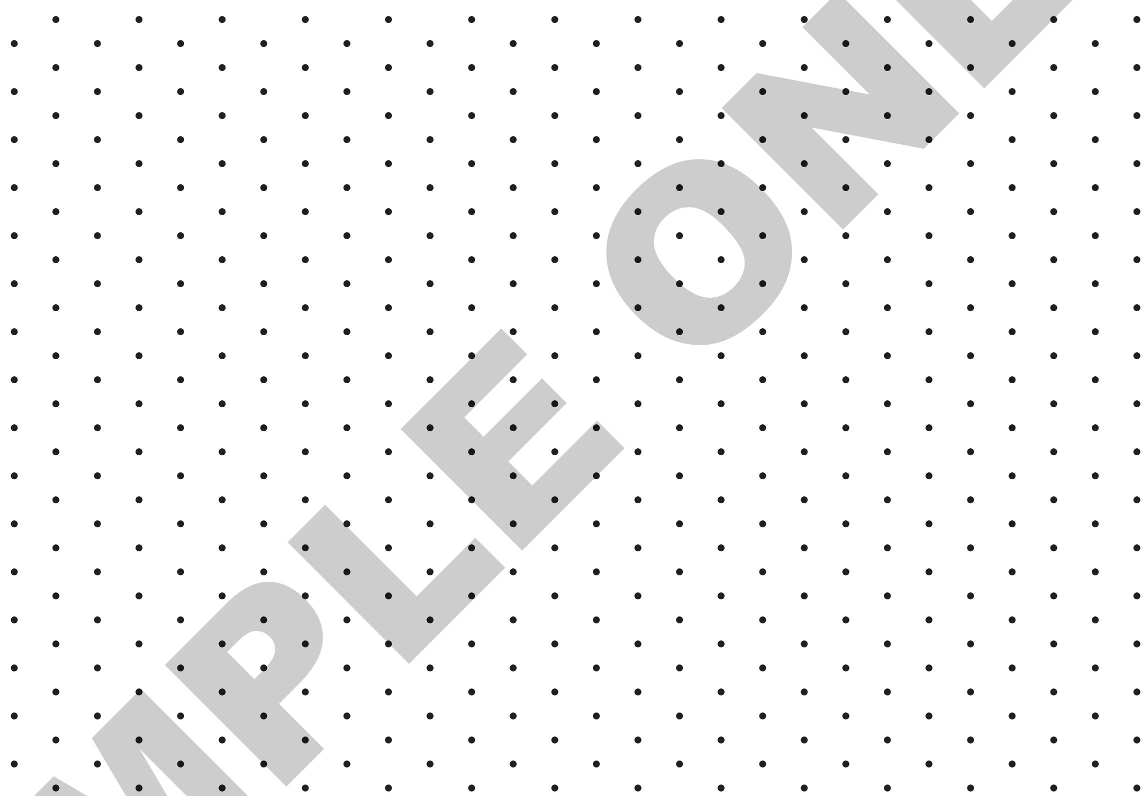
16.3 Comparing Prisms

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1. Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.



2. Analyze the volumes and surface areas of the prisms. What do you notice? Write 1 or 2 observations about them.

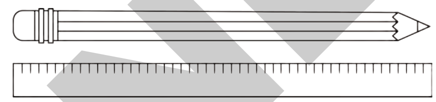
SAMPLE ONLY

Are you ready for more?

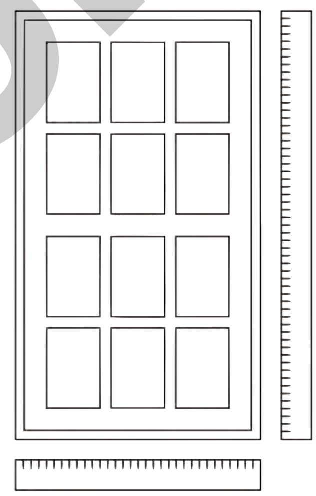
Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Lesson 16 Summary

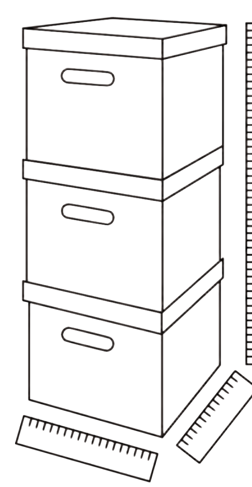
Length is a one-dimensional attribute of a geometric figure. We measure lengths using units like millimeters, centimeters, meters, kilometers, inches, feet, yards, and miles.



Area is a two-dimensional attribute. We measure area in square units. For example, a square that is 1 centimeter on each side has an area of 1 square centimeter.



Volume is a three-dimensional attribute. We measure volume in cubic units. For example, a cube that is 1 kilometer on each side has a volume of 1 cubic kilometer.



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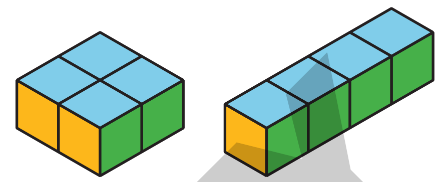
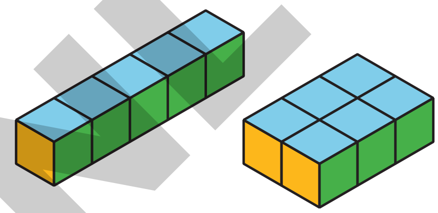
Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

Two figures can have the same volume but different surface areas. For example:



- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.

Similarly, two figures can have the same surface area but different volumes.



- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.

Glossary

- volume

SAMPLE ONLY

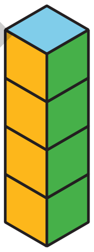
Practice Problems

1 Match each quantity with an appropriate unit of measurement.

A. The surface area of a tissue box
B. The amount of soil in a planter box
C. The area of a parking lot
D. The length of a soccer field
E. The volume of a fish tank

1. Square meters
2. Yards
3. Cubic inches
4. Cubic feet
5. Square centimeters

2 Here is a figure built from snap cubes.



a. Find the volume of the figure in cubic units.

b. Find the surface area of the figure in square units.

c. True or false: If we double the number of cubes being stacked, both the volume and surface area will double. Explain or show how you know.

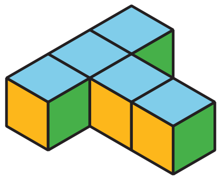
SAMPLE ONLY

3 Lin said, “Two figures with the same volume also have the same surface area.”

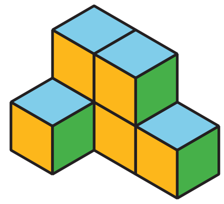
a. Which two figures suggest that her statement is true?

b. Which two figures could show that her statement is *not* true?

A



B



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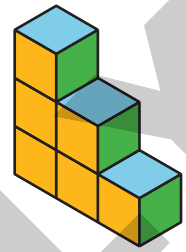

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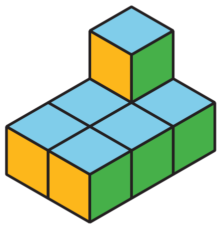
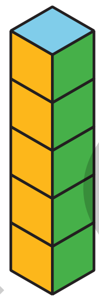


E



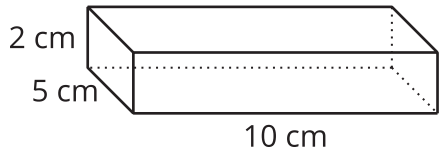
4 from Unit 1, Lesson 11

Draw a pentagon (five-sided polygon) that has an area of 32 square units. Label all relevant sides or segments with their measurements, and show that the area is 32 square units.

SAMPLE ONLY

5 from Unit 1, Lesson 15

a. Draw a net for this rectangular prism.



b. Find the surface area of the rectangular prism.

SAMPLE ONLY

Sec D

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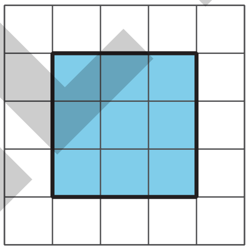
Squares and Cubes

Let's investigate perfect squares and perfect cubes.

17.1 Perfect Squares

1. The number 9 is a "perfect square." Find four numbers that are perfect squares and two numbers that are not perfect squares.



2. A square has side length 7 in. What is its area?

3. The area of a square is 64 sq cm. What is its side length?

17.2 Building with 32 Cubes

Your teacher will give you 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?

2. What is the edge length of the cube you built?

3. What is the area of each face of the built cube? Be prepared to explain your reasoning.

4. What is the volume of the built cube? Be prepared to explain your reasoning.

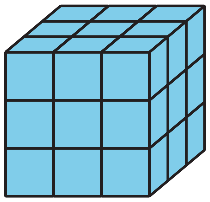
SAMPLE ONLY

Are you ready for more?

Combine your 32 snap cubes with another group's 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Show your reasoning.
4. What is the volume of the built cube? Show your reasoning.

17.3 Perfect Cubes



1. The number 27 is a "perfect cube." Find four other numbers that are perfect cubes and two numbers that are *not* perfect cubes.
2. A cube has a side length of 4 cm. What is its volume?
3. A cube has a side length of 10 inches. What is its volume?
4. A cube has a side length of s units. What is its volume?

SAMPLE ONLY

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17.4 Introducing Exponents

Make sure to include correct units of measure as part of each answer.

1. A square has side length 10 cm. Use an **exponent** to express its area.

2. The area of a square is 7^2 sq in. What is its side length?

3. The area of a square is 81 m^2. Use an exponent to express this area.

4. A cube has edge length 5 in. Use an exponent to express its volume.

5. The volume of a cube is 6^3 cm^3. What is its edge length?

6. A cube has edge length s units. Use an exponent to write an expression for its volume.

Are you ready for more?

The number 15,625 is both a perfect square and a perfect cube. It is a perfect square because it equals 125^2. It is also a perfect cube because it equals 25^3. Find another number that is both a perfect square and a perfect cube. How many of these can you find?

SAMPLE ONLY

Sec E

Lesson 17 Summary

When we multiply two of the same numbers together, such as $5 \cdot 5$, we say that we are *squaring* the number. We can write it like this:

$$5^2$$

The raised 2 in 5^2 is called an **exponent**.

Because $5 \cdot 5 = 25$, we write $5^2 = 25$ and we say, "5 **squared** is 25."

When we multiply three of the same numbers together, such as $4 \cdot 4 \cdot 4$, we say that we are *cubing* the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4 = 64$, we write $4^3 = 64$ and we say, "4 **cubed** is 64."

We also use an exponent for square units and cubic units.

- A square with a side length of 5 inches has an area of 25 in^2.
- A cube with an edge length of 4 cm has a volume of 64 cm^3.

To read 25 in^2, we say "25 square inches," just like before.

The area of a square with a side length of 7 kilometers is 7^2 km^2. The volume of a cube with an edge length of 2 millimeters is 2^3 mm^3.

In general, the area of a square with a side length of s is s^2, and the volume of a cube with an edge length of s is s^3.

Glossary

- cubed
- exponent
- squared

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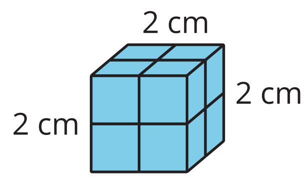
Practice Problems

1 What is the volume of this cube?



2

a. Decide if each number on the list is a perfect square.

16	125
20	144
25	225
100	10,000

b. Write a sentence that explains your reasoning.

SAMPLE ONLY

3

a. Decide if each number on the list is a perfect cube.

1	27
3	64
8	100
9	125

b. Explain what a perfect cube is.

4 a. A square has a side length of 4 cm. What is its area?

b. The area of a square is 49 m^2. What is its side length?

c. A cube has an edge length of 3 in. What is its volume?

5 from Unit 1, Lesson 16

Prism A and Prism B are rectangular prisms.

- Prism A is 3 inches by 2 inches by 1 inch.
- Prism B is 1 inch by 1 inch by 6 inches.

Select **all** statements that are true about the two prisms.

A. They have the same volume.

B. They have the same number of faces.

C. More inch cubes can be packed into Prism A than into Prism B.

D. The two prisms have the same surface area.

E. The surface area of Prism B is greater than that of Prism A.

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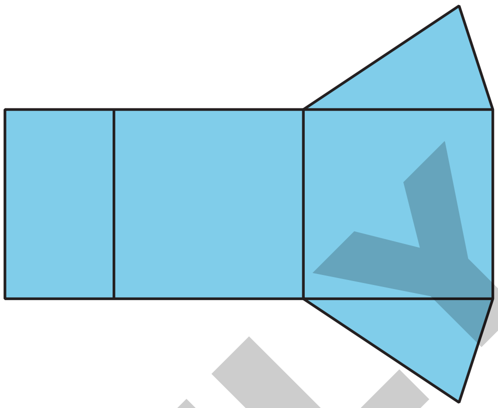
6 from Unit 1, Lesson 14

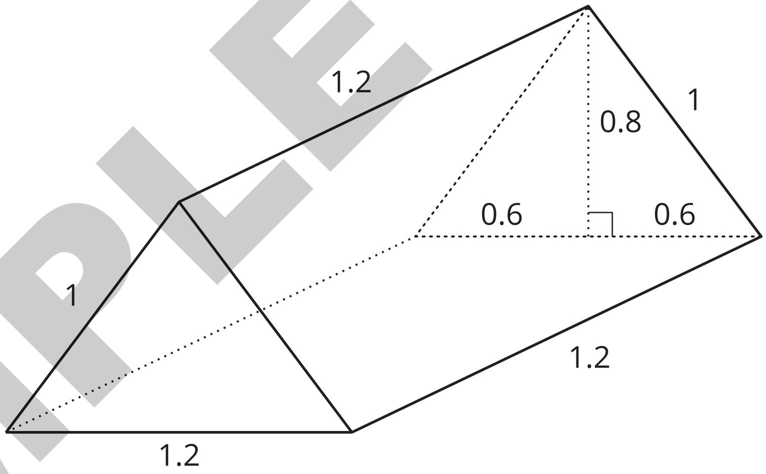
a. What polyhedron can be assembled from this net?



b. What information would you need to find its surface area? Be specific, and label the diagram as needed.

7 from Unit 1, Lesson 15

Find the surface area of this triangular prism. All measurements are in meters.



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Sec E

Surface Area of a Cube

Let's write a formula to find the surface area of a cube.

18.1 Math Talk: Expressions and Their Values

Decide mentally which expression has a greater value.

- $12 + 12 + 12 + 12 + 12$ or $4 \cdot 12$
- $15 \cdot 3$ or 15^3
- 19^2 or $18 \cdot 18$
- $5 \cdot 21^2$ or $(5 \cdot 21) \cdot (5 \cdot 21)$

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18.2 The Net of a Cube

1. A cube has an edge length of 5 units.
    a. Draw a net for this cube on graph paper. Label its sides with measurements.
    b. What is the shape of each face?
    c. What is the area of each face?
    d. What is the surface area of this cube?
    e. What is the volume of this cube?

2. A second cube has an edge length of 17 units.
    a. Sketch a net for this cube. Label its sides with measurements.
    b. Explain why the area of each face of this cube is 17^2 square units.
    c. Write an expression for the surface area, in square units.
    d. Write an expression for the volume, in cubic units.

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18.3 Every Cube in the Whole World

A cube has an edge length of s.

1. Draw a net for the cube.

2. Write an expression for the area of each face. Label each face with its area.
3. Write an expression for the surface area.
4. Write an expression for the volume.

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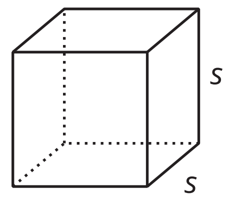
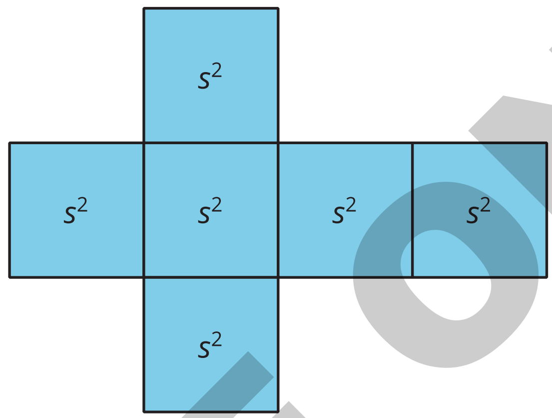
Lesson 18 Summary

The volume of a cube with an edge length of s is s^3.



A cube has 6 faces that are all identical squares. For a cube with an edge length of s, the area of each square face is s^2. This means that the surface area of the cube is $6 \cdot s^2$.



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Sec E

Practice Problems

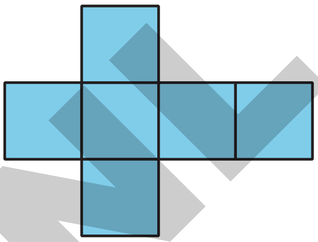
1

a. What is the volume of a cube with an edge length of 8 in?

b. What is the volume of a cube with an edge length of $\frac{1}{3}$ cm?

c. A cube has a volume of 8 ft^3. What is its edge length?

2 What three-dimensional figure can be assembled from this net?



If each square has a side length of 61 cm, write an expression for the surface area and another for the volume of the figure.

3

a. Draw a net for a cube with an edge length of x cm.

b. What is the surface area of this cube?

c. What is the volume of this cube?

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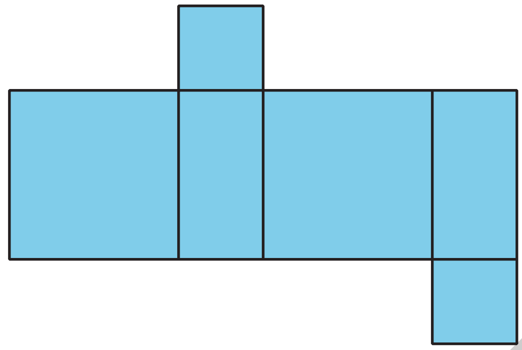
4 from Unit 1, Lesson 14

Here is a net for a rectangular prism that was not drawn accurately.



a. Explain what is wrong with the net.

b. Draw a net that can be assembled into a rectangular prism.

c. Create another net for the same prism.

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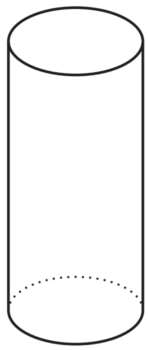
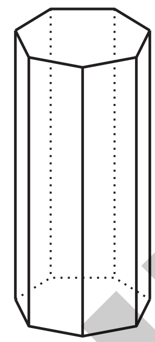
5 from Unit 1, Lesson 13

State whether each figure is a polyhedron. Explain how you know.

A

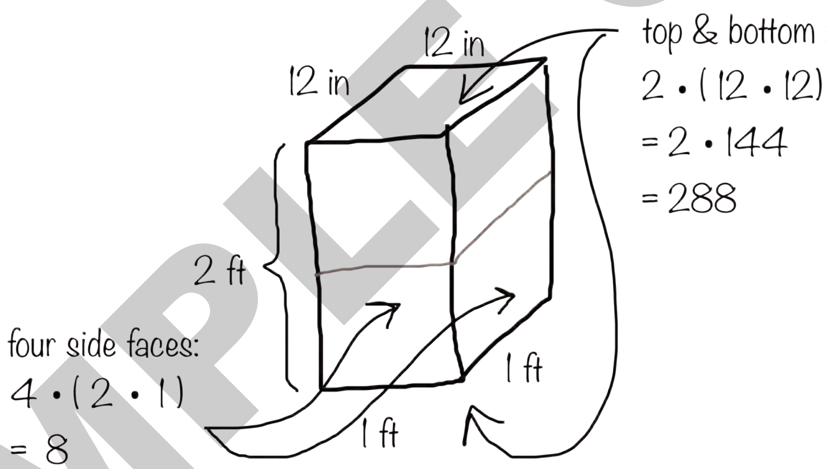


B



6 from Unit 1, Lesson 12

Here is Elena’s work for finding the surface area of a rectangular prism that is 1 foot by 1 foot by 2 feet.



She concluded that the surface area of the prism is 296 square feet. Do you agree with her? Explain your reasoning.

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Sec E

All about Tents

Let's find out how much material is needed to build some tents.

19.1 Notice and Wonder: Structures

What do you notice? What do you wonder?

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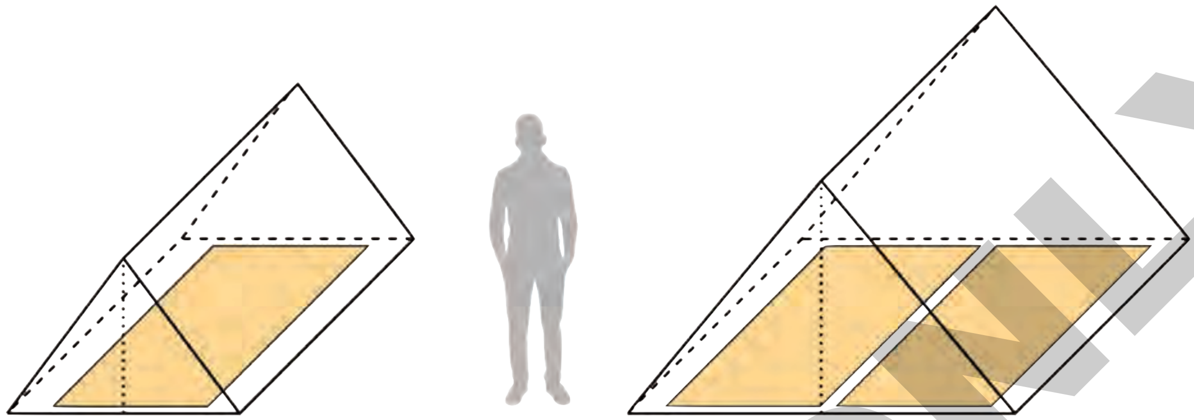
Sec F

19.2 Two Tents

Here is an image of two tents.



1. Record the question that your class is answering.

2. What do you need to know to be able to answer the question? List the information that you need.

3. Use the information that you received to answer the question. Show your reasoning, including any assumptions that you make about the situation. Organize your work so that it can be followed by others.

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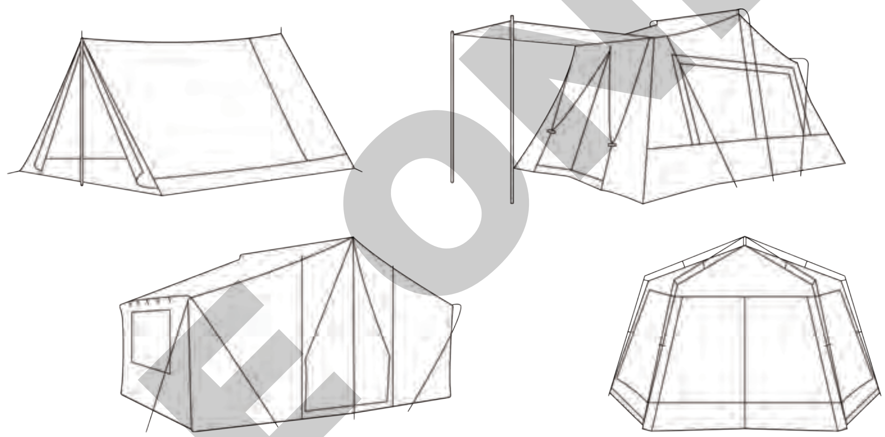
19.3 Tent Design (Part 1)

You are going to design a tent for 4 people. Your design must:

- Include a floor panel.
- Show how the sleeping bags fit inside.
- Be tall enough that people can at least kneel inside the tent.

After creating a design, you will estimate the amount of material needed to build it and show your reasoning.

Sample Tent Styles



Tent Height Specifications

height description	height of tent	notes
sitting height	3 feet	Campers are able to sit, lie, or crawl inside tent.
kneeling height	4 feet	Campers are able to kneel inside tent. Found mainly in 3–4 person tents.
stooping height	5 feet	Campers are able to move around on their feet inside tent, but most campers will not be able to stand upright.
standing height	6 feet	Most adult campers are able to stand upright inside tent.
wheelchair seating height	4.5 feet	Most campers in a wheelchair have enough head clearance.
roaming height	7 feet	Adult campers are able to stand upright and walk around inside tent.

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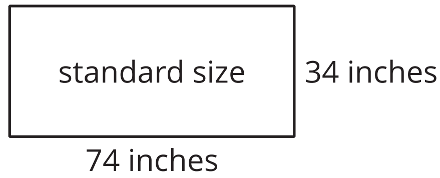
Sleeping Bag Measurements



1. Create your design.
   a. Tent floor: Sketch the shape of the floor panel and the placements of sleeping bags. Think about approximate measurements. How large is this floor panel?
   b. Relevant information: What decisions did you make for your tent? What assumptions did you make?
   c. Overall design: Sketch what the tent would look like. Think about approximate measurements. How high is the tallest point of your tent?
2. Estimate the amount of material needed to build your tent. Your estimate must:
   - Be based on measurements you researched or received.
   - Include sketches that show the parts of the tent and their measurements.
   - Be supported by calculations.

   Organize your work so it can be followed by others. You can use additional paper if you need more space.

19.4 Tent Design (Part 2)

1. Explain your tent design and material estimate to your group. Be sure to explain why you chose this design and how you found your material estimate.
2. Compare the estimated material necessary for each tent in your group. Discuss the following questions:
   - Which tent design used the least material? Why?
   - Which tent design used the most material? Why?

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Learning Targets

Lesson 1 Tiling the Plane

- I can explain the meaning of "area."

Lesson 2 Finding Area by Decomposing and Rearranging

- I can explain how to find the area of a figure that is composed of other shapes.
- I know how to find the area of a figure by decomposing it and rearranging the parts.
- I know what it means for two figures to have the same area.

Lesson 3 Reasoning to Find Area

- I can use different reasoning strategies to find the area of shapes.

Lesson 4 Parallelograms

- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
- I know how to describe the characteristics of a parallelogram using mathematical vocabulary.

Lesson 5 Bases and Heights of Parallelograms

- I can identify pairs of base and height of a parallelogram.
- I can write and explain the formula for the area of a parallelogram.
- I know what the terms "base" and "height" refer to in a parallelogram.

Lesson 6 Area of Parallelograms

- I can use the area formula to find the area of any parallelogram.

Lesson 7 From Parallelograms to Triangles

- I can explain the special relationship between a pair of identical triangles and a parallelogram.

Lesson 8 Area of Triangles

- I can use what I know about parallelograms to reason about the area of triangles.

Lesson 9 Formula for the Area of a Triangle

- I can use the area formula to find the area of any triangle.
- I can write and explain the formula for the area of a triangle.
- I know what the terms “base” and “height” refer to in a triangle.

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Lesson 10 Bases and Heights of Triangles

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson 11 Polygons

- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.
- Puedo usar vocabulario matemático para describir las características de un polígono.

Lesson 12 What is Surface Area?

- I know what the surface area of a three-dimensional object means.

Lesson 13 Polyhedra

- I can describe the features of a polyhedron using mathematical vocabulary.
- I can explain the difference between prisms and pyramids.
- I understand the relationship between a polyhedron and its net.

Lesson 14 Nets and Surface Area

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

Lesson 15 More Nets, More Surface Area

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

Lesson 16 Distinguishing Between Surface Area and Volume

- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

Lesson 17 Squares and Cubes

- I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
- When I know the edge length of a cube, I can find the volume and express it using appropriate units.

Lesson 18 Surface Area of a Cube

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- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson 19 All about Tents

- I can apply what I know about the area of polygons to find the surface area of three-dimensional objects.
- I can use surface area to reason about real-world objects.

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Kendall Hunt

GRADE 6

Teacher Guide

UNIT 1

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Book 1
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UNIT 3: UNIT RATES AND PERCENTAGES

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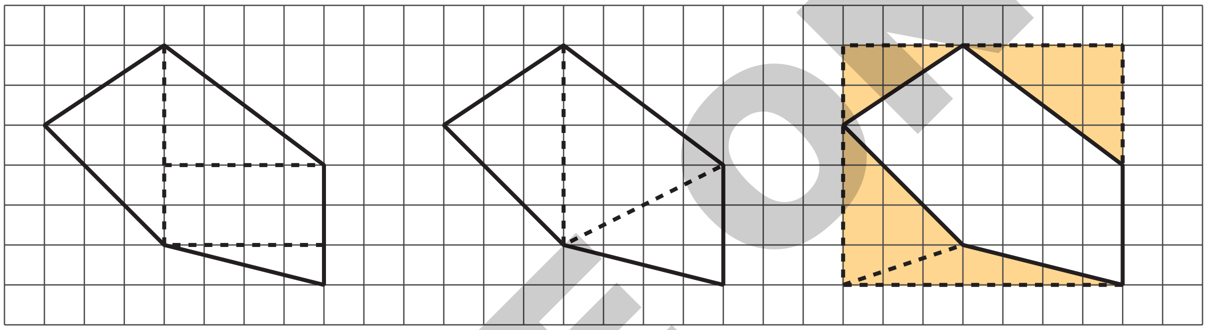
Unit 1: Area and Surface Area

Unit Narrative

In this unit, students reason about areas of polygons and surface areas of polyhedra, building on geometric understandings developed in earlier grades.

In grade 3, students found the area of rectangles with whole-number side lengths. They also found the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding those areas. Students used a formula for the area of rectangles in grade 4 and found the area of rectangles with fractional side lengths in grade 5.

In this unit, students extend their reasoning about area to include shapes that are not composed of rectangles. They use strategies such as decomposing and rearranging to find areas of parallelograms and generalize their process as a formula. Their work with parallelograms then becomes the basis for finding the area of triangles. Students see that other polygons can be decomposed into triangles and use this knowledge to find areas of polygons.



Next, students calculate the surface areas of polyhedra with triangular and rectangular faces. They study, assemble, and draw nets of prisms and pyramids and use nets to determine surface areas. Students also learn to use exponents ^{2} and ^{3} to express surface areas and volumes of cubes and their units.

In many lessons, students engage in geometric work without a context. This design choice is made in recognition of the significant intellectual work of reasoning about area. Later in the unit, students have opportunities to apply their learning in context.

Students will draw on the work here to further study exponents later in grade 6 and to find volumes of prisms and pyramids in grade 7. Their understanding of “two figures that match up exactly” will support their work on congruence and rigid motions in grade 8.

A note about multiplication notation:

Students in grade 6 will be writing algebraic expressions and equations involving the letter x. Because x is easily confused with the “cross” notation for multiplication, $\times$, these materials use the “dot” notation for multiplication. Starting a few lessons into the unit, students will see, for instance, $2 \cdot 3$ instead of 2×3. The notation will be new to many students, so they will need explicit guidance in using it.

A note about tools:

Students are likely to need physical tools to support their reasoning. For instance, they may find that tracing paper is an excellent tool for verifying that figures “match up exactly.” At all times in the unit, each student should have access to a geometry toolkit, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Access to the toolkit also enables students to practice selecting appropriate tools and using them strategically (MP5). In a digitally enhanced classroom, apps and simulations should be considered

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additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Geometric patterns and shapes (Lesson 1).
- Strategies for finding areas of shapes (Lesson 3) and polygons (Lesson 11).
- The characteristics of prisms and pyramids (Lesson 13).
- The measurements and units of 1-, 2-, and 3-dimensional attributes (Lesson 16).
- Representations of area and volume (Lesson 17).

Explain

- How to find areas by composing (Lesson 3).
- Strategies used to find areas of parallelograms (Lesson 4) and triangles (Lesson 8).
- How to determine the area of a triangle using its base and height (Lesson 9).
- Strategies to find surface areas of polyhedra (Lesson 14).

Describe

- Observations about decomposition of parallelograms (Lesson 7).
- Information needed to find the surface area of rectangular prisms (Lesson 12).
- The features of polyhedra and their nets (Lesson 13).
- The features of polyhedra (Lesson 15).
- Relationships among features of a tent and the amount of fabric needed for the tent (Lesson 19).

In addition, students are expected to justify claims about the base, height, or area of shapes; generalize about the features of parallelograms and polygons; interpret relevant information for finding the surface area of rectangular prisms; and represent the measurements and units of 2- and 3-dimensional figures. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

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lesson	new terminology	
	receptive	productive
6.1.1	**area** **region** plane gap overlap	
6.1.2	compose decompose rearrange two-dimensional	
6.1.3	shaded strategy	
6.1.4	parallelogram opposite (sides or angles)	quadrilateral
6.1.5	**base (of a parallelogram or triangle)** **height** corresponding expression represent	
6.1.6	horizontal vertical	
6.1.7	identical	parallelogram
6.1.8	diagram	**base (of a parallelogram or triangle)** **height** **compose** **decompose** rearrange
6.1.9	**opposite vertex**	

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lesson	new terminology	
	receptive	productive
6.1.10	**vertex** **edge**	
6.1.11	**polygon**	horizontal vertical
6.1.12	**face** **surface area**	**area** **region**
6.1.13	**polyhedron** **net** **prism** **pyramid** **base (of a prism or pyramid)** three-dimensional	**polygon** **vertex** **edge** **face**
6.1.15		**prism** **pyramid**
6.1.16	**volume** appropriate quantity	two-dimensional three-dimensional
6.1.17	**squared** **cubed** **exponent** edge length	
6.1.18	value (of an expression)	**squared** **cubed** **net**
6.1.19	estimate description	**surface area** **volume**

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Materials To Gather

- Chart paper
- Demonstration nets with and without flaps
- Geometry toolkits
  For grade 6: Tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

  For grades 7 and 8: Everything listed for grade 6, plus a ruler and a protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

  Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially available "patty paper" is 5 inches by 5 inches and is ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6–8, they are listed as separate Required Material.

- Glue or glue sticks
- Index cards
- Math Community Chart
- Nets of polyhedra
- Pre-assembled or commercially produced polyhedra
- Pre-assembled or commercially produced tangrams
- Rulers
- Scissors
- Snap cubes
- Sticky notes
- Tape

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Materials Needed

Lesson	Materials to Gather	Materials to Copy
Lesson 1	• Chart paper: Activity 1 • Sticky notes: Activity 1 • Geometry toolkits: Activity 2	• 6–12 Blank Math Community Chart (1 copy for every 30 students): Activity 1
Lesson 2	• Geometry toolkits: Lesson • Math Community Chart: Lesson • Pre-assembled or commercially produced tangrams: Lesson • Sticky notes: Lesson • Math Community Chart: Activity 1 • Sticky notes: Activity 1 • Pre-assembled or commercially produced tangrams: Activity 3, Activity 4 • Geometry toolkits: Activity 4	• Composing Shapes Cutouts (1 copy for every 2 students): Activity 3
Lesson 3	• Geometry toolkits: Lesson • Geometry toolkits: Activity 1, Activity 2, Activity 3	• Comparing Regions Handout (1 copy for every 1 students): Activity 1
Lesson 4	• Geometry toolkits: Activity 1, Activity 2, Activity 3	• Area of a Parallelogram Cutouts (1 copy for every 1 students): Activity 2
Lesson 5	• Geometry toolkits: Lesson • Math Community Chart: Lesson • Geometry toolkits: Activity 1, Activity 3	

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Lesson 6	• Geometry toolkits: Activity 2	
Lesson 7	• Geometry toolkits: Activity 1, Activity 2 • Rulers: Activity 2	• A Tale of Two Triangles (Part 2) Cutouts (1 copy for every 3 students): Activity 3
Lesson 8	• Math Community Chart: Lesson • Geometry toolkits: Activity 1, Activity 2 • Math Community Chart: Activity 1 • Sticky notes: Activity 1 • Glue or glue sticks: Activity 3 • Tape: Activity 3	• Decomposing a Parallelogram Cutouts (1 copy for every 4 students): Activity 3
Lesson 9	• Geometry toolkits: Activity 2	
Lesson 10	• Geometry toolkits: Activity 1, Activity 3 • Index cards: Activity 2, Activity 3	
Lesson 11	• Math Community Chart: Activity 1 • Geometry toolkits: Activity 3, Activity 4	• Pinwheel Handout (1 copy for every 4 students): Activity 4
Lesson 12	• Snap cubes: Activity 3	

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Lesson 13	• Chart paper: Activity 1 • Math Community Chart: Activity 1 • Pre-assembled or commercially produced polyhedra: Activity 1 • Nets of polyhedra: Activity 2, Activity 3 • Scissors: Activity 2, Activity 3 • Tape: Activity 2, Activity 3 • Geometry toolkits: Activity 3 • Glue or glue sticks: Activity 3	• Assembling Polyhedra Cutouts (1 copy for every 12 students): Activity 1 • Prisms and Pyramids Cutouts (1 copy for every 4 students): Activity 2 • Assembling Polyhedra Cutouts (1 copy for every 6 students): Activity 3
Lesson 14	• Nets of polyhedra: Activity 1, Activity 2 • Scissors: Activity 1, Activity 2 • Geometry toolkits: Activity 2 • Glue or glue sticks: Activity 2 • Tape: Activity 2	• Matching Nets Cutouts (1 copy for every 2 students): Activity 1 • Using Nets to Find Surface Area Cutouts (1 copy for every 3 students): Activity 2
Lesson 15	• Demonstration nets with and without flaps: Activity 2 • Geometry toolkits: Activity 2 • Glue or glue sticks: Activity 2 • Scissors: Activity 2 • Tape: Activity 2	• Building Prisms and Pyramids Cards (1 copy for every 9 students): Activity 2
Lesson 16	• Snap cubes: Activity 2, Activity 3 • Sticky notes: Activity 2 • Geometry toolkits: Activity 3	
Lesson 17	• Math Community Chart: Activity 1 • Snap cubes: Activity 2	

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Lesson 18	• Math Community Chart: Activity 1 • Geometry toolkits: Activity 2, Activity 3	
Lesson 19	• Geometry toolkits: Activity 3	

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Check Your Readiness (A)

1

Standards

Addressing **3.MD.C.6, 3.MD.C.7.a, 3.MD.C.7.b, 4.MD.A.3**

Narrative

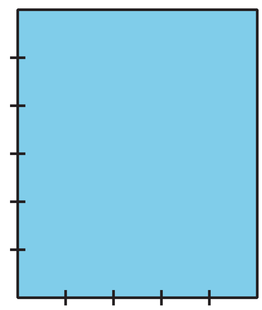
The content assessed in this problem is first encountered in Lesson 1: Tiling the Plane.

This item assesses how students approach finding the area of a rectangle with whole-number side lengths. The purpose of including tick marks is to assist to students who wish to draw a grid of unit squares. Responses that show drawings with the incorrect number of unit squares, irregular rows, or irregular columns may indicate that students have not yet learned to structure two-dimensional space; that is, to see a rectangle with whole-number side lengths as composed of unit squares, or composed of iterated rows or columns of unit squares.

If most students struggle with this item, plan to use the activities in Lesson 1 to support their understanding of area. The Practice Problems in Lesson 1 can be used for extra practice in calculating area. In Lesson 2 they will decompose and rearrange shapes to find their areas. Plan to emphasize tiling and square units in Activity 2 of Lesson 2 if students struggle to make sense of tiling the rectangle with 30 squares to find its area.

Student Task Statement

The rectangle has sides measuring 6 cm and 5 cm. What is the area of this rectangle? Explain your reasoning.



Solution

30 square centimeters. Sample reasoning:

- Use a formula like $l \times w$.
- Draw and count unit squares.
- Think in terms of tiling, but multiply to find the area.

2

Standards

Addressing **3.MD.C.7.d, 5.OA.A.2**

SAMPLE ONLY

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Narrative

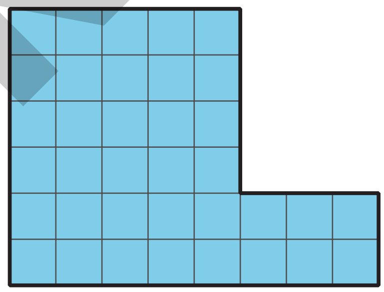
The content assessed in this problem is first encountered in Lesson 2: Finding Area by Decomposing and Rearranging.

This problem assesses two different prerequisite skills. To reason about the area of the shape, students will need to decompose the shape into two rectangles. Interpreting the expressions in the answer choices may also pose a challenge for some students. Note that Choices B and D represent two different methods. Choice B involves enclosing the figure with a larger rectangle and subtracting the extra rectangle. Choice C involves decomposing the figure into smaller rectangles.

If most students struggle with this item, plan to begin this lesson with a few examples of rectilinear figures whose areas can be found by decomposing rectangles. The Practice Problems in Lesson 1 can be used for this. Students will also get many opportunities in the first several lessons of this unit to decompose shapes and compare different ways to decompose the same shape.

Student Task Statement

Select **all** the expressions that give the area of the figure in square units.



A. $(5+6)\times(3+2)$

B. $(8\times 6)-(3\times 4)$

C. $6\times 8\times 2\times 3\times 4\times 5$

D. $(5\times 6)+(3\times 2)$

E. 8×6

Solution

B, D

3

Standards

Addressing 5.NF.B.4.b

Narrative

The content assessed in this problem is first encountered in Lesson 3: Reasoning to Find Area.

SAMPLE ONLY

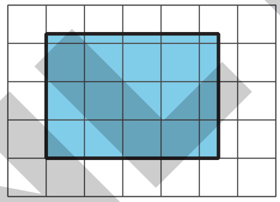
This problem requires students to identify an expression that gives the area of a rectangle that is on a grid and whose side lengths are not whole numbers. Rather than relying on counting, students will need to recognize that the product of the side lengths gives the area of the rectangle and to estimate fractional units visually. In fifth grade, students learned to calculate the area of a rectangle by multiplying fractional side lengths.

If most students struggle with this item, plan to use this problem to draw out and support any misconceptions during the synthesis of Lesson 3 Activity 2. When the idea of multiplying side lengths to find the area of a rectangle comes up, ask students if it can also be done when the side lengths are not whole numbers.

Student Task Statement

Each small square in the graph paper represents 1 square unit.

Which expression is closest to the area of the rectangle, in square units?



A. $5\frac{1}{2} \times 4\frac{1}{4}$

B. $4\frac{1}{2} \times 3\frac{1}{4}$

C. 4×3

D. 5×4

Solution

B

4

Standards

Addressing **4.G.A.1**

Narrative

The content assessed in this problem is first encountered in Lesson 4: Parallelograms.

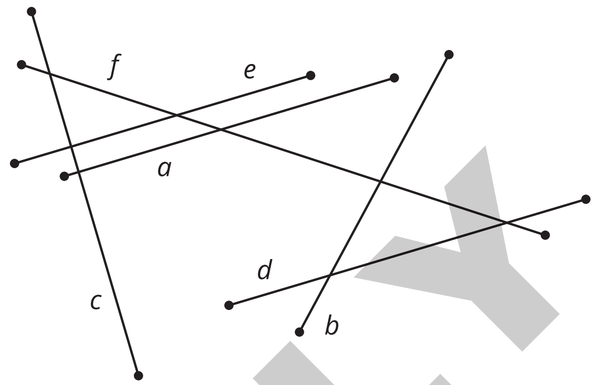
Students will need to be comfortable recognizing parallel lines before beginning their work with parallelograms later in the unit. Some students may correctly select segment e, but not notice segment d since that line is farther away.

If most students struggle with this item, plan to start with Lesson 4 Activity 1 Launch with an emphasis on defining the term "parallel."

SAMPLE ONLY

Student Task Statement

Select **all** the line segments that appear to be parallel to a.



A. Segment a

B. Segment b

C. Segment c

D. Segment d

E. Segment e

F. Segment f

Solution

D, E

5

Standards

Addressing 4.G.A.2

Narrative

The content assessed in this problem is first encountered in Lesson 5: Bases and Heights of Parallelograms.

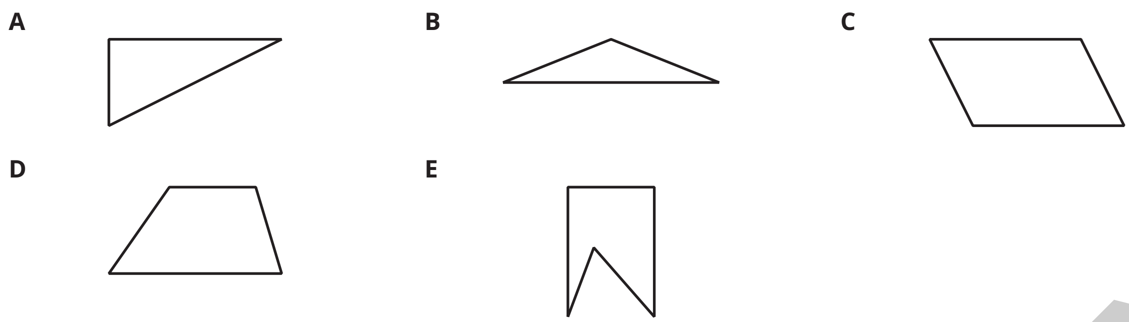
In this unit, students will find the area of parallelograms and triangles by decomposing them into shapes with perpendicular sides and rearranging the pieces. Students will need to be familiar with perpendicular lines in order to make sense of the term "height" of a parallelogram or triangle.

If most students struggle with this item, plan to start Lesson 5 Activity 2 by amplifying the term "perpendicular" for the students. Students may need some visual cues to support this concept.

Student Task Statement

Select **all** the figures that have sides that appear to be perpendicular.

SAMPLE ONLY



A. A

B. B

C. C

D. D

E. E

Solution

A, E

6

Standards

Addressing **5.NBT.A.2**

Narrative

The content assessed in this problem is first encountered in Lesson 17: Squares and Cubes.

Exponential notation is introduced in Lesson 17 of this unit, in the context of calculating surface area and volume of cubes. Students may have prior knowledge of exponents from their work with place value in fifth grade.

If most students struggle with this item, plan to review exponents using this problem as part of the *Launch* into Activity 4. This problem can further emphasize the concept of cubing, as it is described in the Launch and then used throughout the activity.

Student Task Statement

Select **all** the expressions that are equal to 10^3.

A. 30

B. 3,000

C. $1{,}000 \times 3$

D. 10×3

SAMPLE ONLY

E. $10 \times 10 \times 10$

F. 100

G. 1,000

Solution

E, G

7

Standards

Addressing **4.G.A.2**

Narrative

The content assessed in this problem is first encountered in Lesson 3: Reasoning to Find Area.

This problem assesses whether students understand the term "right triangle." This problem will also reveal whether students can picture a 90-degree angle well enough to draw one freehand or to reach for an appropriate tool, like the corner of a piece of paper.

If most students struggle with this item, plan to focus on this concept during Lesson 3 Activity 3, Off the Grid. In the Launch of this activity, include a discussion about the term "right angle" and point out the symbol used to identify it in the shapes. This concept will continue to be reinforced in the next several lessons.

Student Task Statement

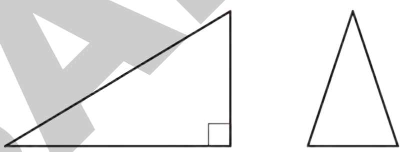
Draw two triangles:

- One that is a right triangle
- One that is *not* a right triangle

Then, explain what makes a triangle a right triangle.

Solution

Sample response:



A right triangle has a right (or 90°) angle.

SAMPLE ONLY

Check Your Readiness (B)

1

Standards

Addressing **3.MD.C.6, 3.MD.C.7.a, 3.MD.C.7.b, 4.MD.A.3**

Narrative

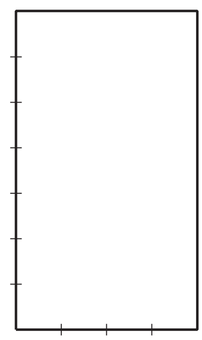
The content assessed in this problem is first encountered in Lesson 1: Tiling the Plane.

This item assesses how students approach finding the area of a rectangle with whole-number side lengths. The purpose of including tick marks is to give assistance to students who wish to draw a grid of unit squares. Responses that show drawings with the incorrect number of unit squares, irregular rows, or irregular columns may indicate that students have not yet learned to structure two dimensional space, that is, to see a rectangle with whole-number side lengths as composed of unit squares, or composed of iterated rows or columns of unit squares.

If most students struggle with this item, plan to use the activities in Lesson 1 to support their understanding of area. The Practice Problems in Lesson 1 can be used for extra practice in calculating area. In Lesson 2, students will decompose and rearrange shapes to find their areas. Plan to emphasize tiling and square units in Activity 2 of Lesson 2 if students struggle to make sense of tiling the rectangle with 30 squares to find its area.

Student Task Statement

The rectangle has sides measuring 7 cm and 4 cm. What is the area of this rectangle? Explain your reasoning.



Solution

28 square centimeters. Sample reasoning:

- Use a formula like $l \cdot w$.
- Draw and count unit squares.
- Think in terms of tiling, but multiply to find the area.

SAMPLE ONLY

2

Standards

Addressing **3.MD.C.7.d, 5.OA.A.2**

Narrative

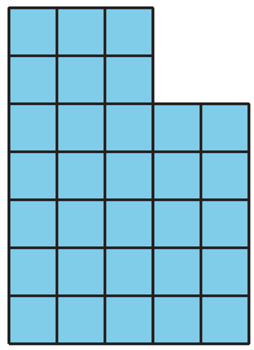
The content assessed in this problem is first encountered in Lesson 2: Finding Area by Decomposing and Rearranging.

This problem assesses two different prerequisite skills. To reason about the area of the shape, students will need to decompose the shape into two rectangles. Interpreting the expressions in the answer choices may also pose a challenge for some students. Note that Choices C and E represent two different methods. Choice C involves decomposing the figure into smaller rectangles. Choice E involves enclosing the figure with a larger rectangle and subtracting the extra rectangle.

If most students struggle with this item, plan to begin this lesson with a few examples of rectilinear figures whose areas can be found by decomposing rectangles. The Practice Problems in Lesson 1 can be used for this. Students will also get many opportunities in the first several lessons of this unit to decompose shapes and compare different ways to decompose the same shape.

Student Task Statement

Select **all** the expressions that give the area of the figure in square units.



A. 7×5

B. $(7 + 5) \times (5 + 2)$

C. $(7 \times 3) + (5 \times 2)$

D. $7 \times 5 \times 5 \times 2 \times 2 \times 3$

E. $(7 \times 5) - (2 \times 2)$

Solution

C, E

SAMPLE ONLY

3

Standards

Addressing **5.NF.B.4.b**

Narrative

The content assessed in this problem is first encountered in Lesson 3: Reasoning to Find Area.

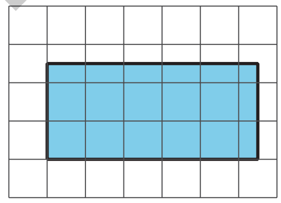
This problem requires students to identify an expression that gives the area of a rectangle that is on a grid and whose side lengths are not whole numbers. Rather than relying on counting, students will need to recognize that the product of the side lengths gives the area of the rectangle and to estimate fractional units visually. In fifth grade, students learned to calculate the area of a rectangle by multiplying fractional side lengths.

If most students struggle with this item, plan to use this problem to draw out and support any misconceptions during the synthesis of Lesson 3 Activity 2. When the idea of multiplying side lengths to find the area of a rectangle comes up, ask students if it can also be done when the side lengths are not whole numbers.

Student Task Statement

Each small square in the graph paper represents 1 square unit.

Which expression is closest to the area of the shaded rectangle, in square units?



A. 6×3

B. 5×2

C. $5\frac{1}{2} \times 2\frac{1}{2}$

D. $6\frac{1}{2} \times 3\frac{1}{4}$

Solution

C

4

Standards

Addressing **4.G.A.1**

Narrative

The content assessed in this problem is first encountered in Lesson 4: Parallelograms.

Students will need to be comfortable recognizing parallel lines before beginning their work with parallelograms

SAMPLE ONLY

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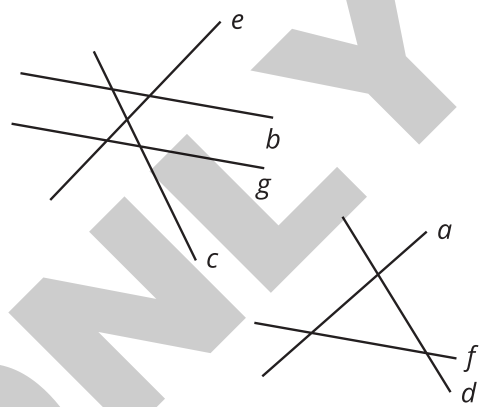
later in the unit. Some students may correctly select segment b as being parallel to segment g, but not notice segment f because that segment is farther away.

If most students struggle with this item, plan to start with Lesson 4 Activity 1 Launch with an emphasis on defining the term “parallel.”

Student Task Statement

Select **all** the line segments that appear to be parallel to g.



A. Segment a

B. Segment b

C. Segment c

D. Segment d

E. Segment e

F. Segment f

Solution

B, F

5

Standards

Addressing **4.G.A.2**

Narrative

The content assessed in this problem is first encountered in Lesson 5: Bases and Heights of Parallelograms.

In this unit, students will find the area of parallelograms and triangles by decomposing them into shapes with perpendicular sides and rearranging the pieces. Students will need to be familiar with perpendicular lines in order to make sense of the term “height” of a parallelogram or triangle.

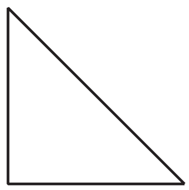
SAMPLE ONLY

If most students struggle with this item, plan to start Lesson 5 Activity 2 by amplifying the term “perpendicular” for the students. Students may need some visual cues to support this concept.

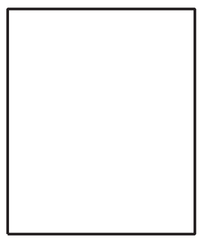
Student Task Statement

Select **all** the figures that have sides that appear to be perpendicular.

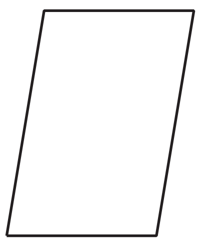
A.



B.

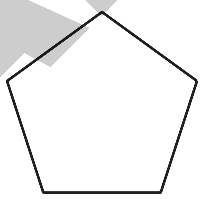


C.



D.

E.



SAMPLE ONLY

Solution

A, B

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6

Standards

Addressing **5.NBT.A.2**

Narrative

The content assessed in this problem is first encountered in Lesson 17: Squares and Cubes.

Exponential notation is introduced in Lesson 17 of this unit, in the context of calculating the surface area and volume of cubes. Students may have prior knowledge of exponents from their work with place value in fifth grade.

Student Task Statement

Select **all** the expressions that are equal to 10^4.

A. 10,000

B. 4,000

C. $1{,}000 \times 4$

D. 10×4

E. $10 \times 10 \times 10 \times 10$

F. 1,000

G. 40

Solution

A, E

7

Standards

Addressing **4.G.A.2**

Narrative

The content assessed in this problem is first encountered in Lesson 3: Reasoning to Find Area.

This problem assesses whether students understand the term “right triangle.” This problem will also reveal whether students can picture a 90-degree angle.

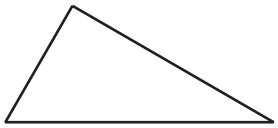
If most students struggle with this item, plan to focus on this concept during Lesson 3 Activity 3, Off the Grid. In the Launch of this activity, include a discussion about the term “right angle” and point out the symbol used to identify it in the shapes. This concept will continue to be reinforced in the next several lessons.

SAMPLE ONLY

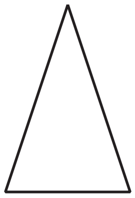
Student Task Statement

Select **all** triangles that appear to be a right triangle.

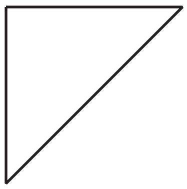
A.



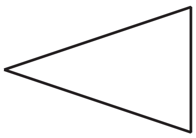
B.



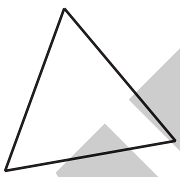
C.



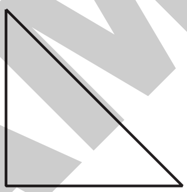
D.



E.



F.



Solution

A, C, F

SAMPLE ONLY

Mid-Unit Assessment (A)

Teacher Instructions

Give this assessment after Lesson 11.

1

Standards

Addressing **6.G.A.1**

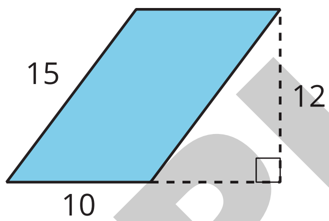
Narrative

Students who select Parallelogram A are likely applying the formula for the area of a triangle, 12bh. Students who fail to select Parallelogram B may be multiplying one base of the parallelogram by the other base, rather than multiplying one of the bases by the height. They may also be confused by the fact that the height is horizontal. Students who select Parallelogram C are picking the parallelogram with a perimeter of 60 units, and may need to revisit the conceptual differences between area and perimeter. Students who select Parallelogram D may be multiplying the two bases together, or they may be multiplying the height by the incorrect base.

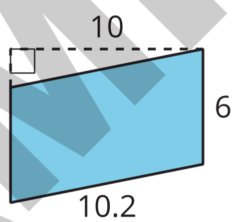
Student Task Statement

Which parallelogram has an area of 60 square units?

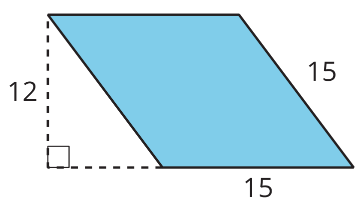
A.



B.

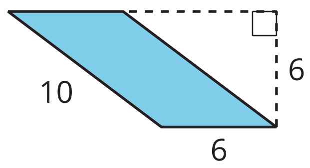


C.



D.

SAMPLE ONLY



Solution

B

2

Standards

Addressing **6.G.A.1**

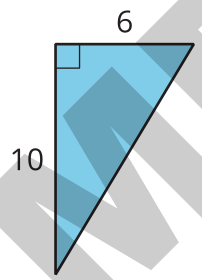
Narrative

Students who select Triangle B may be calculating the perimeter rather than the area. Students who select Triangle D may be treating the side of length 10 as the height. Students who select Triangle E may be multiplying the base and height but may not be multiplying that product by $\frac{1}{2}$. Students who fail to select Triangle A may have a major misconception about the area of a triangle or may be neglecting to divide by 2. Students who fail to select Triangle C may not recognize the external height.

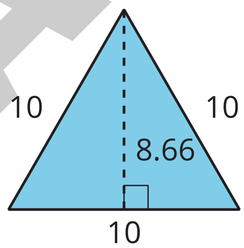
Student Task Statement

Select **all** the triangles that have an area of 30 square units.

A.

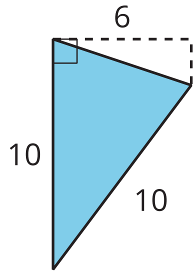


B.

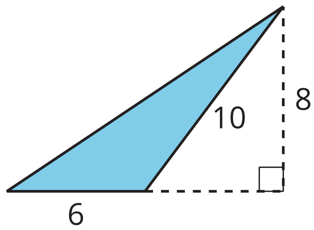


C.

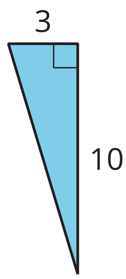
SAMPLE ONLY



D.



E.



Solution

A, C

3

Standards

Addressing **6.G.A.1**

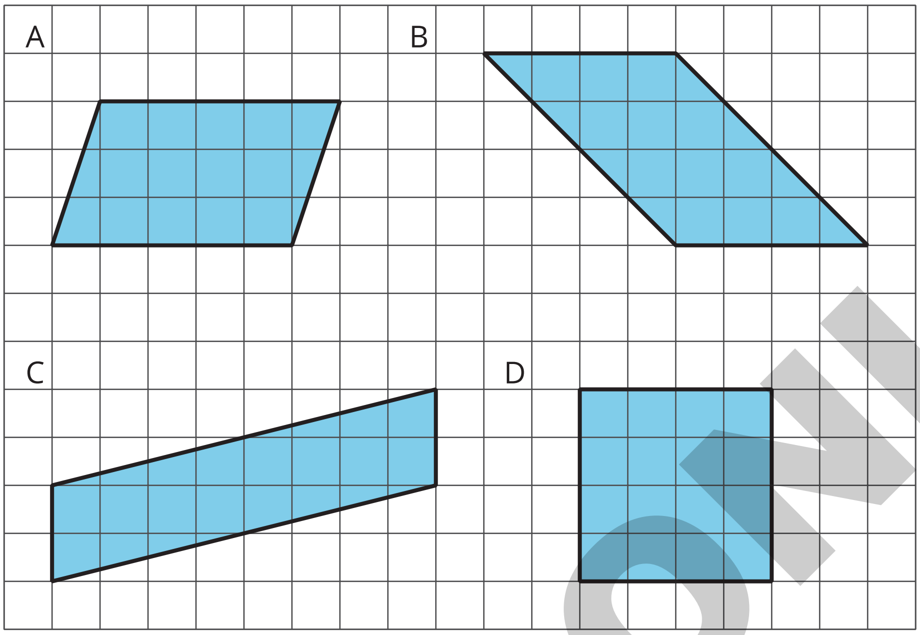
Narrative

Students who select Parallelogram A may be decomposing and rearranging the parallelogram, but miscounting the number of unit squares; the area is 15 square units. Students who fail to select Parallelogram B may be confusing the side length with the height. Students who fail to select Parallelogram C may be using the area formula instead of decomposing and rearranging, and failing to use the vertical side length as the base. Students who fail to select D may think that a square is not a parallelogram.

SAMPLE ONLY

Student Task Statement

Select **all** the parallelograms that have an area of 16 square units.



A. Parallelogram A

B. Parallelogram B

C. Parallelogram C

D. Parallelogram D

Solution

B, C, D

4

Standards

Addressing **6.G.A.1**

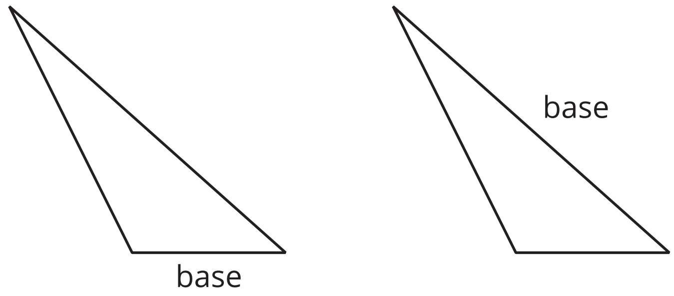
Narrative

Identifying the height for a chosen base is important for calculating the area of a triangle.

Student Task Statement

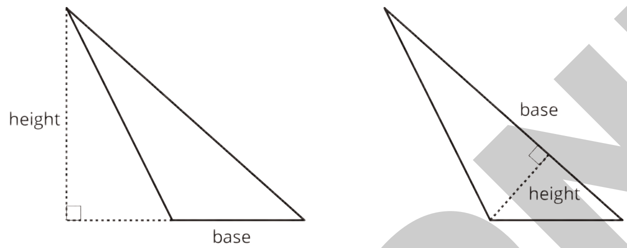
On each triangle, draw a segment to represent the height that corresponds to the given base. Label each height with the word "height."

SAMPLE ONLY



Solution

Sample response:



5

Standards

Addressing **6.G.A.1**

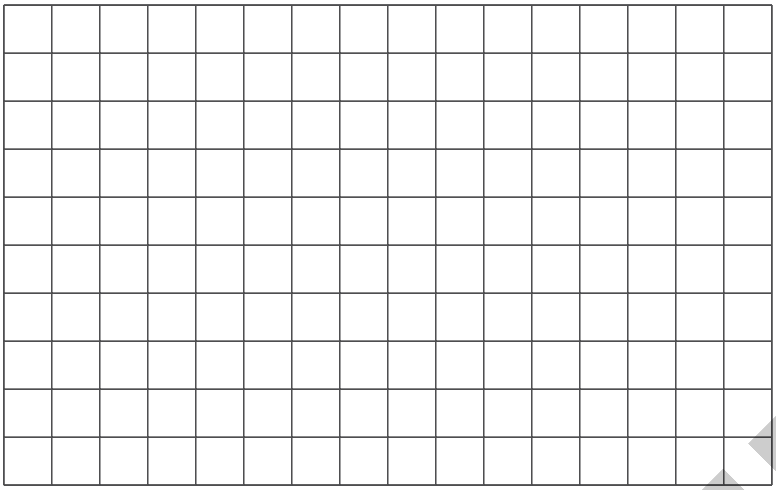
Narrative

This question purposely admits the possibility that students might draw rectangles if they know that a rectangle is a particular kind of parallelogram. The question could be modified to instruct students to draw parallelograms that are not rectangles.

Student Task Statement

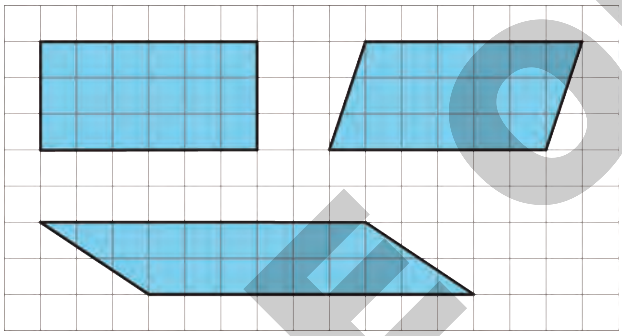
Draw two parallelograms, both with areas of 18 square units. The two parallelograms should not be identical copies of each other.

SAMPLE ONLY



Solution

Sample responses:



Minimal Tier 1 response:

- Work is complete and correct.
- The two parallelograms may have the same base and height as long as they are not congruent.
- Sample: Two parallelograms (rectangles allowed) with base and height lengths that when multiplied equal 18.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Only one parallelogram is drawn. Only one parallelogram has the correct area. An equation such as "$6 \cdot 3 = 18$" is written, but the base or height of the parallelogram is slightly off.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: A length or a side that is not perpendicular to the base is used as the height. Shapes drawn are not parallelograms.

6

Standards

Addressing **6.G.A.1**

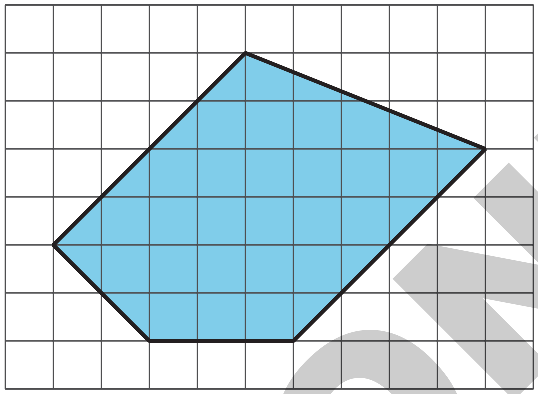
SAMPLE ONLY

Narrative

Watch for students who break the shape into parts in unusual ways. These students may not be recalling well the methods developed in the lessons, and are likely to create parts for which they cannot determine the area.

Student Task Statement

Find the area of the figure. Explain or show your reasoning.



SAMPLE ONLY

Solution

31 square units. Sample reasoning:

- Decompose the figure into parallelograms and triangles with known bases and heights. Find the sum of the areas of the components.
- Enclose the figure in a 9-by-6 rectangle. From 54 square units, subtract the areas of the right triangles that are not part of the figure.

Minimal Tier 1 response:

- Work is complete and correct.
- A diagram is included.
- Acceptable errors: No units are included.
- Sample: A box around the shape has an area of 54. The triangles on the outside have areas of 8, 5, 8, and 2. The area of the box minus the triangles is 31.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: The area of a small number of the partitioned shapes are calculated incorrectly. Addition or subtraction to find the area of the polygon is done incorrectly. The area of one of the partitioned shapes cannot be correctly calculated because the base or height does not lie on a vertical or horizontal line. The partitioned shapes are not shown on the diagram.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: The partitioned shapes are not shapes for which students can find the area. Incorrect area

formulas for triangles or rectangles are used. Calculations show many errors.

7

Standards

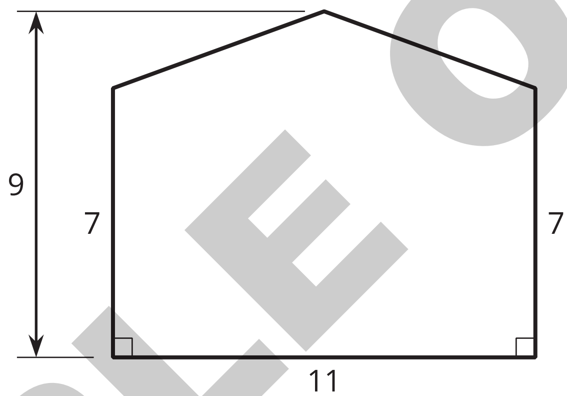
Addressing **6.G.A.1**

Narrative

Students may decompose each wall into a rectangle and one or more triangles, and rearrange the triangles into rectangles. For each wall, students may enclose the given shape with a rectangle and subtract the areas of the triangles that are not part of the wall.

Student Task Statement

The figure is a diagram of a wall. Lengths are given in feet.



A room has two walls that are this shape and size. A painter has a bucket of paint that can cover 160 square feet of wall surface. Is that enough paint to cover both walls? Explain or show your reasoning.

Solution

No, that is not enough paint. Sample reasoning: Each wall is made up of a rectangle that is 11 feet by 7 feet and a triangle with a base of 11 feet and a height of 2 feet.

- The area of one wall is $(11 \cdot 7) + (\frac{1}{2} \cdot 11 \cdot 2)$, or $77 + 11$, which is 88 square feet.
- The area of two walls is $2 \cdot 88$, which is 176 square feet.
- The total area is more than 160 square feet.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample: Not enough, because the combined area of both walls is more than 160 square feet. The area of one wall is the area of a 9-by-11 rectangle reduced by the area of 2 triangles, each with a base of 5.5 feet and a height of 2 feet.
  - Area of rectangle: $9 \cdot 11 = 99$

SAMPLE ONLY

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- Area of 2 triangles: $2 \cdot \frac{(5.5) \cdot 2}{2} = 2 \cdot (5.5) = 11$
- Area of one wall: $99 - 11$ or 88 square units
- Area of two walls: $2 \cdot 88$ or 176 square units

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: Units are omitted. Arithmetic errors are made when calculating area. Acceptable errors: Incorrect total area due to an error in computing a partial area.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: A plausible but flawed strategy is used for decomposing or enclosing the wall. Incorrect area formulas are used with correct decomposition strategy.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: No reasonable strategy is used for decomposing or enclosing the wall.

SAMPLE ONLY

Mid-Unit Assessment (B)

Teacher Instructions

Give this assessment after Lesson 11.

1

Standards

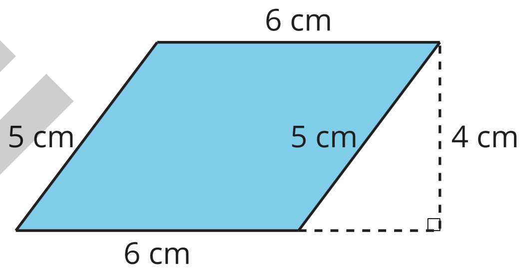
Addressing **6.G.A.1**

Narrative

Students who select Choice A may be multiplying the one side of the parallelogram by the other side, rather than multiplying one of the bases by the height. Students who fail to select Choice B may be confused about which measurements to use to calculate the area of a parallelogram. Students who select Choice C are calculating the perimeter of the parallelogram and may need to revisit the conceptual differences between area and perimeter. Students who select D are likely applying the formula for the area of a triangle.

Student Task Statement

What is the area of this parallelogram?



A. 30 cm^2

B. 24 cm^2

C. 22 cm^2

D. 12 cm^2

Solution

B

SAMPLE ONLY

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2

Standards

Addressing **6.G.A.1**

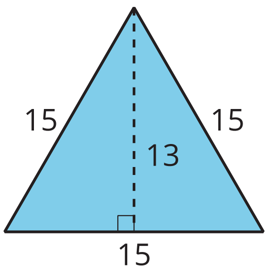
Narrative

Students who select Triangle A are calculating the perimeter rather than the area. Students who select Triangle B are using the side with length 10 as the height. Students who select Triangle C are multiplying the base and the height but are not dividing that product by 2. Students who fail to select D may have a major misconception about the area of a triangle or may have neglected to divide by 2. Students who fail to select E may be using the incorrect base or height to calculate the area.

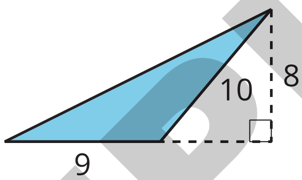
Student Task Statement

Select **all** the triangles that have an area of 45 square units.

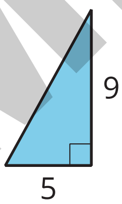
A.



B.

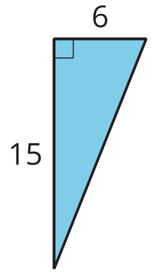


C.

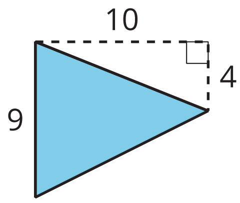


D.

SAMPLE ONLY



E.



Solution

D, E

3 Standards

Addressing **6.G.A.1**

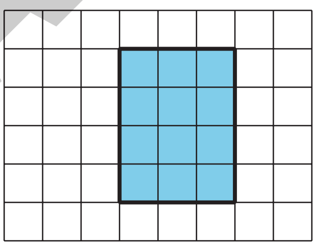
Narrative

Students who fail to select Parallelogram A may think that a rectangle is not a parallelogram. Students who fail to select Parallelogram B may be confusing the side length with the height. Students who select Parallelogram C may be calculating the perimeter instead of the area. Students who select Parallelogram D are not applying the formula for finding the area of a parallelogram.

Student Task Statement

Select **all** the parallelograms that have an area of 12 square units.

A.



B.

SAMPLE ONLY

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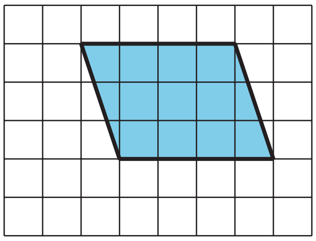
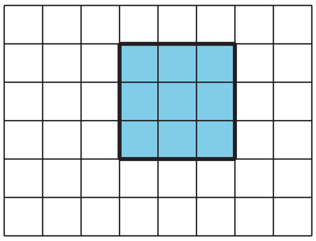
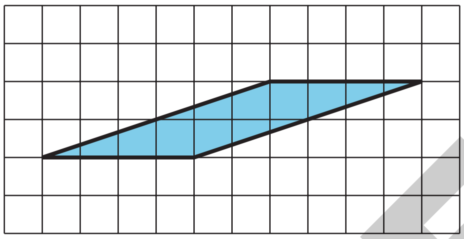


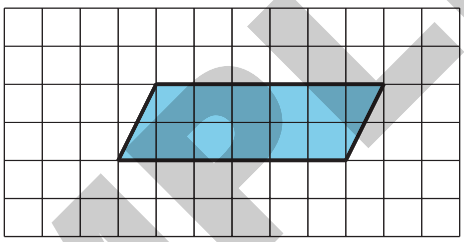
C.



D.



E.



Solution

A, B, E

4

Standards

Addressing **6.G.A.1**

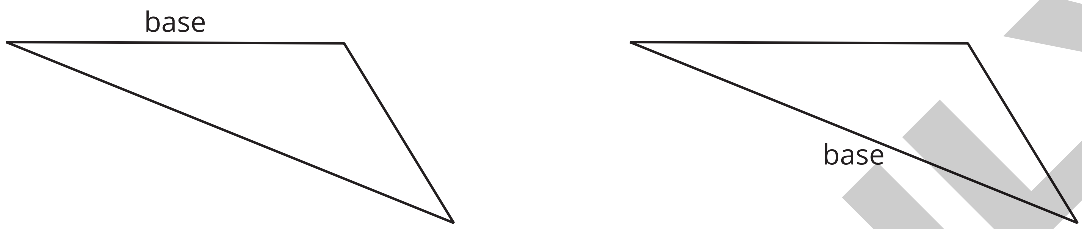
SAMPLE ONLY

Narrative

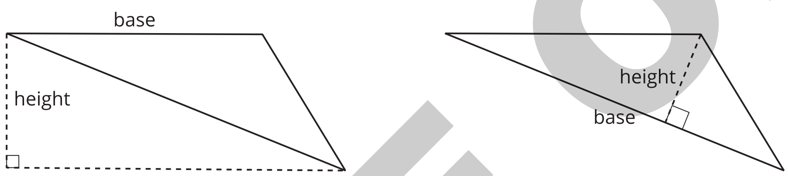
Identifying the height for a chosen base is important for calculating the area of a triangle.

Student Task Statement

On each triangle, draw a segment to represent the height that corresponds to the given base. Label each height with the word "height."



Solution



5

Standards

Addressing **6.G.A.1**

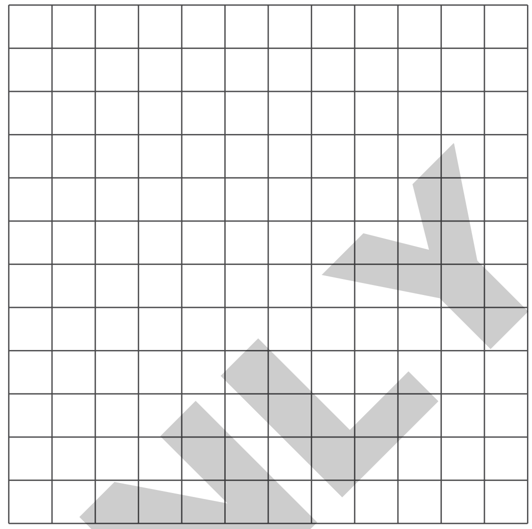
Narrative

This question purposely admits the possibility that students might draw rectangles if they know that a rectangle is a particular kind of parallelogram. The question could be modified to instruct students to draw parallelograms that are not rectangles.

SAMPLE ONLY

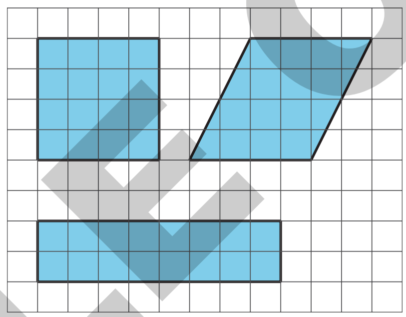
Student Task Statement

Draw two parallelograms, each with an area of 16 square units. The two parallelograms should not be identical copies of each other.



Solution

Sample responses:



Minimal Tier 1 response:

- Work is complete and correct.
- The two parallelograms may have the same base and height as long as they are not congruent.
- Sample: Two parallelograms (rectangles allowed) with base and height lengths that when multiplied equal 16.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Only one parallelogram is drawn. Only one parallelogram has the correct area. An equation such as “$8 \cdot 2 = 16$” is written, but the base or height of the parallelogram is slightly off.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: A length or a side that is not perpendicular to the base is used as the height. Shapes drawn are not parallelograms.

SAMPLE ONLY

6

Standards

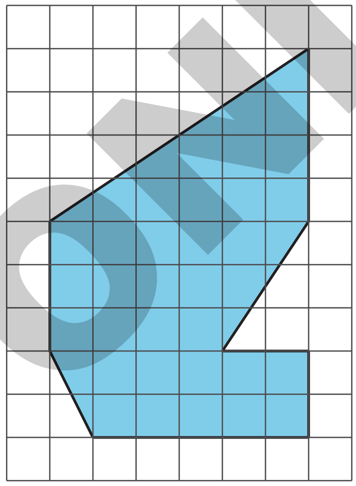
Addressing **6.G.A.1**

Narrative

Watch for students who break the shape into parts in unusual ways. These students may not be recalling the methods developed in the lessons, and may create parts for which they cannot determine the area.

Student Task Statement

Find the area of the figure. Explain or show your reasoning.



Solution

38 square units. Possible strategies:

- Decompose the figure into rectangles and triangles with known bases and heights. Find the sum of the areas of the components.
- Enclose the figure in a 6-by-9 rectangle. From 54 square units, subtract the areas of the right triangles that are not part of the figure.

Minimal Tier 1 response:

- Work is complete and correct.
- A diagram is included.
- Acceptable errors: No units are included.
- Sample: A box around the shape has an area of 54. The triangles on the outside have areas of 12, 3, and 1, respectively. The area of the box minus combined areas of the triangles is 38.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: The area of a small number of the partitioned shapes are calculated incorrectly. Addition or subtraction to find the area of the polygon is done incorrectly. The area of one of the partitioned shapes

SAMPLE ONLY

cannot be correctly calculated because the base or height does not lie on a vertical or horizontal line. The partitioned shapes are not shown on the diagram.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: The partitioned shapes are not shapes for which students can find the area. Incorrect area formulas for triangles or rectangles are used. Calculations show many errors.

7

Standards

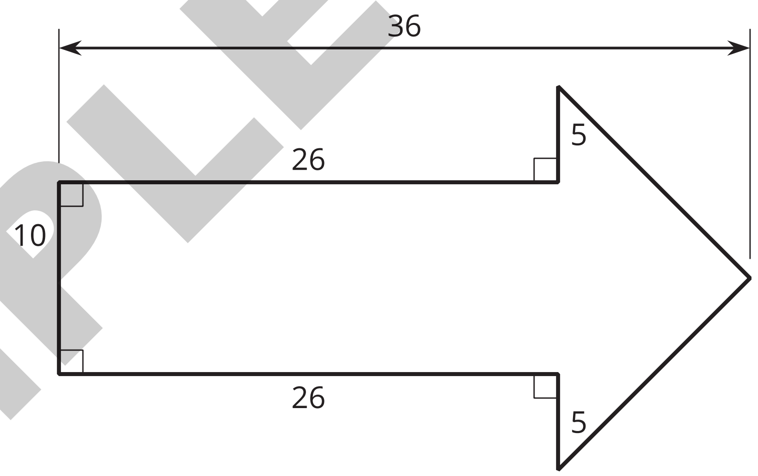
Addressing **6.G.A.1**

Narrative

Strategies for this problem range from decomposing the sign into rectangles and triangles, to enclosing the sign and subtracting the areas of the extra regions. Watch for students who struggle to find the measurements of the tip of the arrow. Others may struggle with doubling the area in the second part.

Student Task Statement

The figure is a diagram of a sign. Lengths are given in inches.



A painter is going to paint the sign on both the front and the back. The painter has a small can of paint that can cover 800 square inches. Is that enough to paint both sides of the sign? Explain or show your reasoning.

Solution

Yes, that is enough paint. Sample reasoning: The sign is composed of a rectangle that is 26 inches by 10 inches and a triangle with a base of 20 inches and a height of $36 - 26$, which is 10 inches.

- The area of the rectangle is 260 square inches because $26 \cdot 10 = 260$.
- The area of the triangle is 100 square inches because $\frac{1}{2} \cdot 20 \cdot 10 = 100$.

SAMPLE ONLY

- The combined area is $260 + 100$ or 360 square inches.
- The area of both faces of the sign (front and back) is twice that amount, $2 \cdot 360$, which is 720 square inches. This is less than 800 square inches.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample: The area of the sign is the area of a rectangle that is 36 inches by 20 inches minus the area of 2 extra rectangles and 2 extra right triangles.
  - Area of the enclosing rectangle: $36 \cdot 20 = 720$, or 720 square inches
  - Area of the 2 extra rectangles: $2 \cdot (5 \cdot 26) = 260$, or 260 square inches
  - Area of the 2 right triangles: $2 \cdot (\frac{1}{2} \cdot 10 \cdot 10) = 100$, or 100 square inches
  - Area of each face of the sign: $720 - 260 - 100$, which is 360 square inches. For both faces, double that to 720 square inches, which is less than 800 square inches.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: Units omitted; arithmetic errors in calculating area. Acceptable errors: Incorrect total area due to an error in computing a partial area.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: Plausible but flawed strategy for partitioning; incorrect area formulas with correct partitioning strategy.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: No reasonable strategy for partitioning.

SAMPLE ONLY

End-of-Unit Assessment (A)

1

Standards

Addressing **6.G.A**

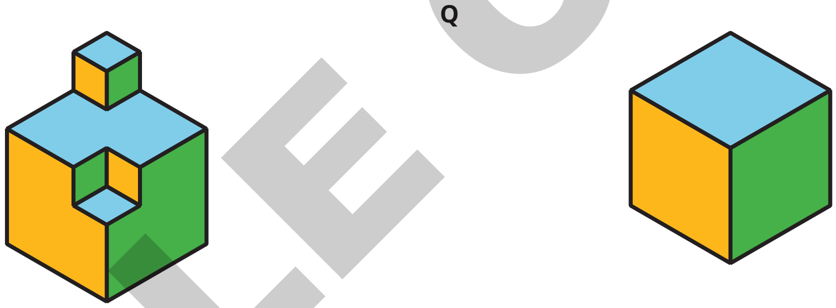
Narrative

Students who select Choice A may think (or may just be guessing) that removing a piece of a solid always decreases its surface area. Students who select Choice B are correct that the large cube has the same surface area before and after the small cube is removed, but they are forgetting that putting the small cube back on top also contributes to the surface area. Students who select Choice D may not believe that the problem can be solved without specific measurements, such as a side length.

Student Task Statement

Polyhedron P is a cube with a corner removed and relocated to the top of the cube. Polyhedron Q is a cube with the same size base as Polyhedron P. How do their surface areas compare?

P **Q**



A. Polyhedron P's surface area is less than Polyhedron Q's surface area.

B. Polyhedron P's surface area is equal to Polyhedron Q's surface area.

C. Polyhedron P's surface area is greater than Polyhedron Q's surface area.

D. There is not enough information given to compare their surface areas.

Solution

C

2

Standards

Addressing **6.G.A.4**

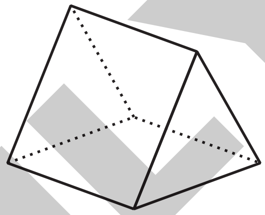
SAMPLE ONLY

Narrative

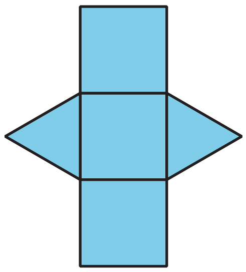
Students who fail to select Choice A or Choice D may be having trouble visualizing how the nets can be folded to form the prism. Students who select Choice B or choice C may not understand that each face of the net corresponds to a face of the prism: There are 2 triangular faces and 3 rectangular faces in the prism, but that is not true of the nets in Choice B or Choice C.

Student Task Statement

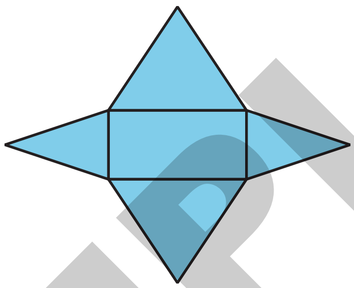
Select **all** of the nets that can be folded and assembled into a triangular prism like this one.



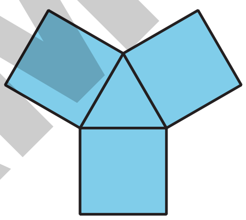
A.



B.



C.



D.

SAMPLE ONLY

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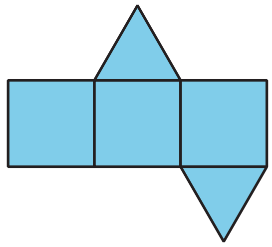
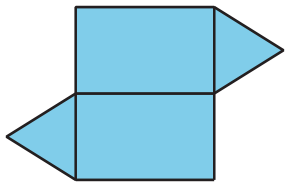


E.



Solution

A, D

3

Standards

Addressing **6.EE.A.1**

Narrative

Students who select Choice A may be thinking of the area of a square. Students who fail to select Choice B may not understand the meaning of the exponent. Students who select Choice C may be thinking of the surface area of a cube. Students who select Choice D may also be thinking of surface area, treating each face of the cube as if it has an area of 8 square inches. Students who select Choice B, but fail to select Choice E, may have memorized that the volume of a cube is the cube of its side length, without thinking about what it means to cube a number.

Student Task Statement

A cube has a side length of 8 inches.

Select **all** the values that represent the cube's volume in cubic inches.

A. 8^2

B. 8^3

C. $6 \cdot 8^2$

D. $6 \cdot 8$

E. $8 \cdot 8 \cdot 8$

SAMPLE ONLY

Solution

B, E

4

Standards

Addressing **6.EE.A.1**

Narrative

This problem has students find the area of a square given a side length, and then find the side length of a square given the area. Watch for students who treat 9 as a side length in the second part of the problem.

Student Task Statement

a. A square has a side length of 9 cm. What is its area?

b. A square has an area of 9 cm^2. What is its side length?

Solution

a. 81 cm^2

b. 3 cm

5

Standards

Addressing **6.EE.A.1**

Narrative

The first two parts of this problem can be solved without direct computation, by comparing factors. For the third part, students are very likely to compute each value and then directly compare the results. Some students will say that the value of 30^2 is larger because 30 is larger than 10.

Student Task Statement

For each pair of expressions, circle the expression with the greater value.

a. 13^2 or 15^2

b. $7 \cdot 6^2$ or 6^3

SAMPLE ONLY

c. 10^3 or 30^2

Solution

a. 15^2

b. $7 \cdot 6^2$

c. 10^3

6

Standards

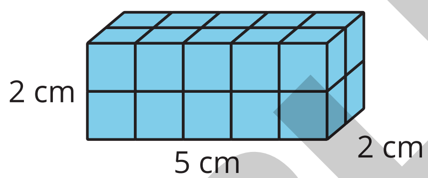
Addressing **6.G.A.4**

Narrative

Students use an understanding of area in rectangles to find the surface area of a rectangular prism.

Student Task Statement

A rectangular prism measures 2 cm by 2 cm by 5 cm. What is its surface area? Explain or show your reasoning.



Solution

48 cm^2. Sample reasoning: The left and right faces each have area of 4 cm^2 $(2 \cdot 2 = 4)$. The top, bottom, front, and back faces each have area of 10 cm^2 $(2 \cdot 5 = 10)$. So the surface area is $(2 \cdot 4) + (4 \cdot 10) = 48$, or 48 cm^2.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: $2 \cdot 2 + 2 \cdot 2 + 5 \cdot 2 + 5 \cdot 2 + 5 \cdot 2 + 5 \cdot 2 = 48$, so 48 cm^2.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work shows the correct surface area but does not include reasoning or units. Work contains arithmetic mistakes but still indicates an intent to add up the areas of the six faces. Response (with work shown) is the sum of the areas of the three visible faces only.

Tier 3 response:

SAMPLE ONLY

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Student calculates a different quantity, such as volume or the area of only one face. Incorrect answer with no work is shown.

7

Standards

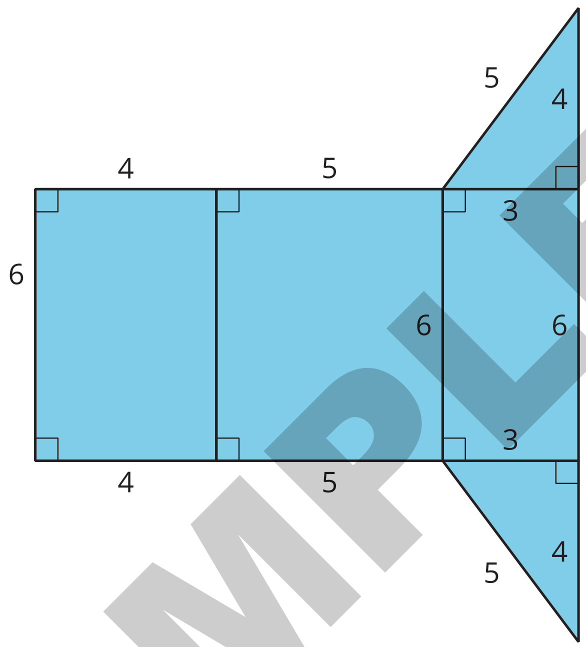
Addressing **6.G.A.1, 6.G.A.4**

Narrative

Students identify the polyhedron associated with a net, then calculate its surface area.

Student Task Statement

Here is a net made of a polyhedron. All measurements are given in centimeters.



a. If the net were folded and assembled, what type of polyhedron would it make?

b. Find the surface area of the polyhedron in square centimeters. Explain or show your reasoning.

Solution

a. Triangular prism

b. 84 cm^2. Sample reasoning: The net is made of two triangles with a base of 4 cm and a height of 3 cm, and one big rectangle with a base of 12 cm and a height of 6 cm. The two triangles can be rearranged and put together to make a rectangle that is 4 cm by 3 cm, which results in an area of 12 cm^2. The area of the big rectangle is $6 \cdot 12$ or 72 cm^2. So the total (surface) area is 84 cm^2.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

SAMPLE ONLY

- Sample:

a. Triangular prism

b. $\frac{1}{2} \cdot 4 \cdot 3 + \frac{1}{2} \cdot 4 \cdot 3 + 4 \cdot 6 + 5 \cdot 6 + 3 \cdot 6 = 84$, so 84 cm^2.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: Arithmetic errors accompany otherwise correct work. The area of one face is missing or incorrect. Units are omitted. Surface area is correct and well-justified but the polyhedron is incorrect yet somewhat reasonable, such as a triangular pyramid or rectangular prism.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: The shape is correctly identified as a triangular prism but work shows little progress on finding the surface area. No reasonable attempt is made at identifying the polyhedron (or an answer like "trapezoid"). Surface area calculations involve serious mistakes, such as incorrect processes for calculating the area of a right triangle. Work includes a significant error in one part but a correct answer in the other part.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Work shows significant omissions or Tier 3 errors across both problem parts.

SAMPLE ONLY

End-of-Unit Assessment (B)

1

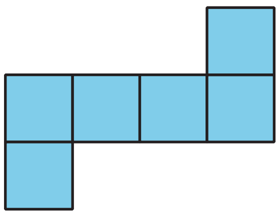
Standards

Addressing 6.G.A.4

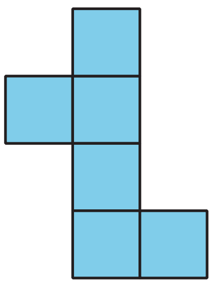
Student Task Statement

Select **all** of the nets that can be folded and assembled into a cube.

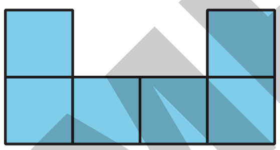
A.



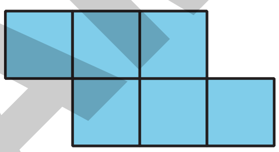
B.



C.

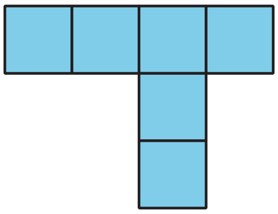


D.



E.

SAMPLE ONLY



Solution

A, B

2

Standards

Addressing **6.G.A**

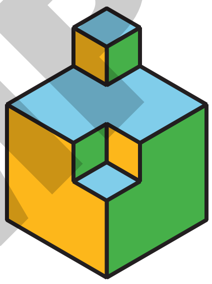
Narrative

Students who select Choice B are correct that the large cube has the same surface area before and after the small cube is removed, but they may be forgetting that putting the small cube back on top also contributes to the surface area. Or, students who select Choice B may be confusing surface area with volume. Students who select Choice D may not believe that the problem can be solved without specific measurements, such as a side length.

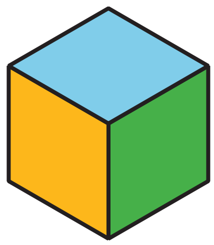
Student Task Statement

Polyhedron P is a cube with a corner removed and relocated to the top of P. Polyhedron Q is a cube with the same size base as Polyhedron P. How do their surface areas compare?

P



Q



A. Q's surface area is less than P's surface area.

B. Q's surface area is equal to P's surface area.

C. Q's surface area is greater than P's surface area.

D. There is not enough information given to compare their surface areas.

SAMPLE ONLY

Solution

A

3

Standards

Addressing 6.EE.A.1

Narrative

Students who select Choice A may be confusing 4^3 with $3 \cdot 4$. Students who select Choice B may be confusing surface area with volume. Students who select Choice D may think that 4 could represent the volume of a single cube.

Student Task Statement

For a cube whose side length is 4 inches, the expression 4^3 could represent . . .

A. The length of 3 cubes lined up, in inches

B. The cube's surface area in square inches

C. The cube's volume in cubic inches

D. The total volume of 3 cubes, in cubic inches

Solution

C

4

Standards

Addressing 6.EE.A.1

Narrative

This problem has students find the side length when the area of square is given, and then find the area of a square when its side length is given.

Student Task Statement

a. A square has an area of 16 cm^2. What is the length of each of its sides?

b. A square has a side length of 8 cm. What is its area?

SAMPLE ONLY

Solution

a. 4 cm

b. 64 cm^2

5

Standards

Addressing **6.EE.A.1**

Narrative

Students will evaluate the expressions involving exponents. Students who fail to correctly rearrange these expressions may not have a clear understanding of the definition of bases and exponents.

Student Task Statement

Here is a list of expressions. Order them from least to greatest.

40^2 8^3 10^3 $7 \cdot 8^2$ 10^4

Solution

$7 \cdot 8^2, 8^3, 10^3, 40^2, 10^4$

6

Standards

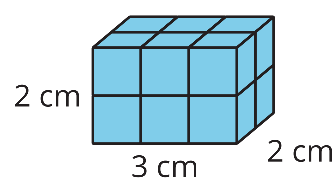
Addressing **6.G.A.4**

Narrative

Students use an understanding of area in rectangles to find the surface area of a rectangular prism.

Student Task Statement

A rectangular prism measures 3 cm by 2 cm by 2 cm. What is its surface area? Explain or show your reasoning.



SAMPLE ONLY

Solution

The surface area is 32 cm^2. Sample reasoning: The left and right faces each have an area of 4 cm^2. The top, bottom, front, and back faces each have an area of 6cm^2. $(2 \cdot 4) + (4 \cdot 6) = 32$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: $2 \cdot 2 + 2 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 = 32$, so 32 cm^2.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work shows the correct surface area but does not include reasoning or units. Work contains arithmetic mistakes but still indicates an intent to add up the areas of the six faces. Response (with work shown) is the sum of the areas of the three visible faces only.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Student calculates a different quantity, such as volume or the area of only one face. Incorrect answer with no work is shown.

7

Standards

Addressing **6.G.A.1, 6.G.A.4**

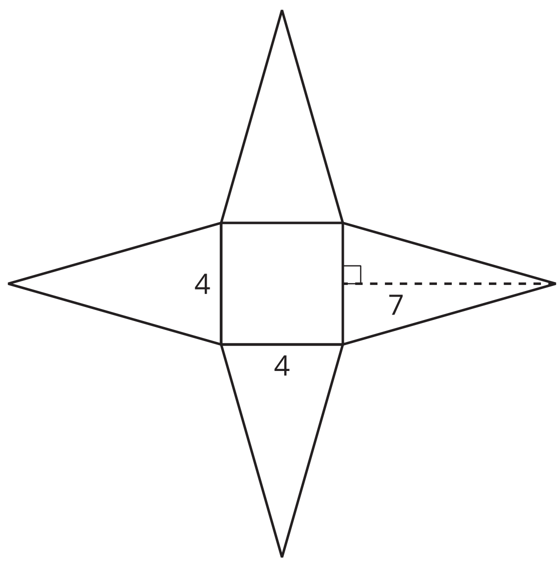
Narrative

Students identify the polyhedron associated with a net and calculate its surface area.

Student Task Statement

Here is a net made of a square and four identical triangles. All measurements are given in centimeters.

SAMPLE ONLY



a. If the net were folded and assembled, what type of polyhedron would it make?

b. What is the surface area of the polyhedron? Explain your reasoning.

Solution

a. Square pyramid

b. The surface area is 72 cm^2. Sample reasoning: The net is made of four triangles, each with a base of 4 cm and a height of 7 cm, as well as a square with side lengths of 4 cm. The four triangles each have an area of 14 cm^2. The square has an area of 16 cm^2. $4 \cdot 14 + 16 = 72$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample: A. Square pyramid B. $(0.5)(4)(7) = 14 \times 4 = 56$, which is the total area of the 4 triangles, and the base has an area of 4 x 4 = 16. Because 56+16=7 , the surface area is 72cm^2.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: Arithmetic errors accompany otherwise correct work. The area of one face is missing or incorrect. Units are omitted. Surface area is correct and well-justified but the polyhedron is incorrect yet somewhat reasonable, such as a triangular pyramid or rectangular prism.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: The shape is correctly identified as a square pyramid but work shows little progress on finding the surface area. No reasonable attempt is made at identifying the polyhedron (or an answer like “squares and triangles”). Surface area calculations involve serious mistakes such as incorrect processes for calculating the area of a triangle. Work includes a significant error in one part but a correct answer in the other part.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Omissions or Tier 3 errors across both problem parts.

SAMPLE ONLY

Section A: Reasoning to Find Area

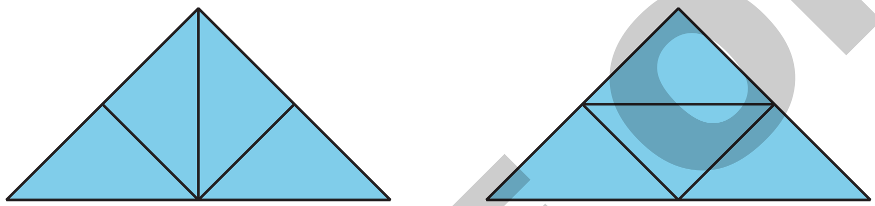
Goals

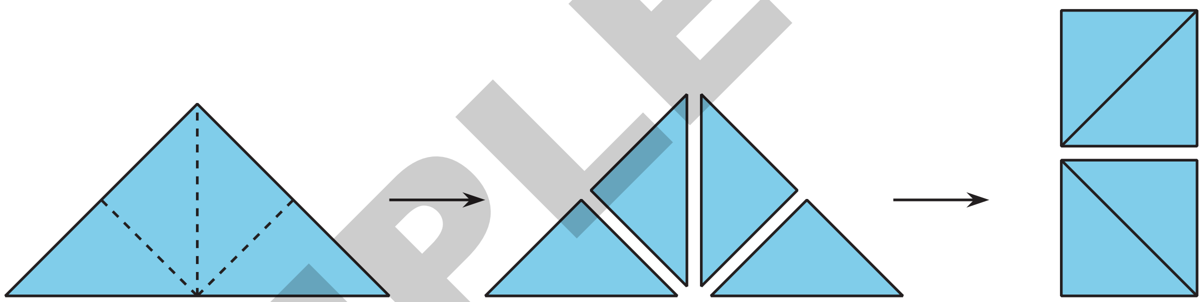
- Find the area of a two-dimensional region with straight boundaries by decomposing, rearranging, subtracting, or enclosing shapes, and explain the solution method.

Section Narrative

In this section, students explore strategies for reasoning about the area of two-dimensional figures. The explorations highlight two principles about area:

- Figures that match exactly have equal areas. If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- Area is additive. If a given figure is decomposed into pieces, then the area of the given figure is the sum of the areas of the pieces. If a figure is composed from pieces that don't overlap, the sum of the areas of the pieces is the area of the figure. Rearranging the pieces doesn't change their areas.





First, students compare the amount of the plane covered by different shapes by thinking about the relationships between the shapes. They also recall their understanding of what area is, refine it, and develop a shared definition.

Next, students use their definition of “area” and strategies, such as composing, decomposing, and rearranging, to reason about areas of tangram shapes. They create figures with certain areas and find the area of given shapes. Finally, students apply these reasoning strategies to find the areas of figures drawn on and off a grid.

A note about terminology:

In Grade 6, the term “congruent” is not used to describe “two figures that match up exactly.” Instead, these materials use “identical,” “identical copies,”or similar terms. . What “identical” means might require clarification (for instance, that it is independent of color and orientation). In Grade 8, students will learn to refer to such figures as “congruent” and to describe congruence in terms of rigid motions (reflections, rotations, and translations).

The term “polygon” is not used with students until it is defined later in the unit.

Teacher Reflection Questions

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- **Math Content and Student Thinking:** This section highlights strategies such as decomposing, rearranging, and enclosing and subtracting for reasoning about area. How will these strategies prepare students to find the area of parallelograms, triangles, and other polygons?
- **Pedagogy:** Which curriculum resources are you using as you plan lessons? In what ways are they helping you use student thinking to drive the learning during the lessons?
- **Access and Equity:** How did you use the *Collect and Display* routine(s) in this section to connect student language to new vocabulary or mathematical ideas?

SAMPLE ONLY

Section A Checkpoint

Sec A

1

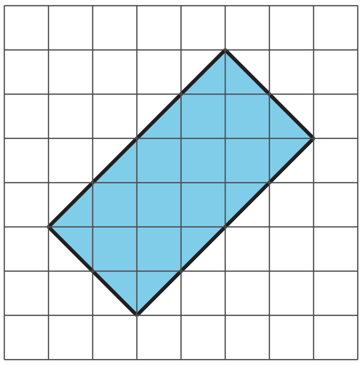
Goals Assessed

- Find the area of a two-dimensional region with straight boundaries by decomposing, rearranging, subtracting, or enclosing shapes, and explain the solution method.

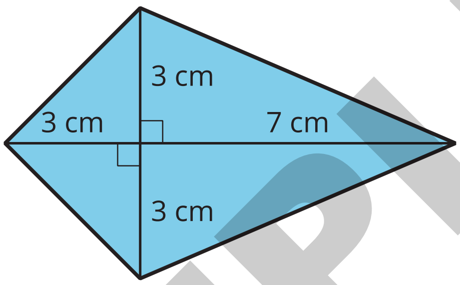
Student Task Statement

Find the area of each shaded region. Explain or show your reasoning.

a.



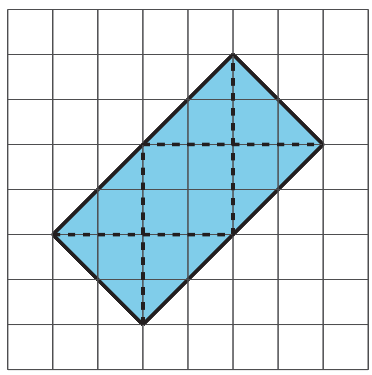
b.



Solution

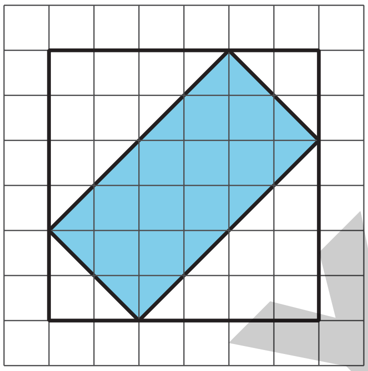
a. The areas is 16 square units. Sample reasoning:

- The rectangle can be decomposed into 1 square of 4 square units and 6 identical right triangles. Two right triangles make a square of 4 square units, so 6 right triangles make 3 squares with a combined area of 12 square units. $4 + 12 = 16$

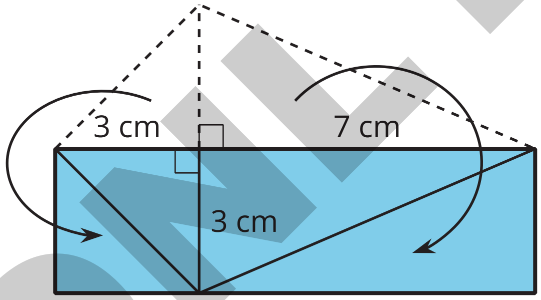


SAMPLE ONLY

- The rectangle can be enclosed by a 6-by-6 square (36 square units). The square creates 2 larger right triangles and 2 smaller ones. The larger triangles can be rearranged to make a 4-by-4 square (16 square units). The smaller triangles make a 2-by-2 square (4 square units). Subtracting the areas of the triangles from the 6-by-6 square gives 16 square units. $36 - 16 - 4 = 16$



b. 30 sq cm. Sample reasoning: The two small triangles can be rearranged into a 3-by-3 square (9 sq cm). The two large triangles can be rearranged into a 3-by-7 rectangle (21 sq cm). The combined area is $9 + 21$, or 30 sq cm.



Responding To Student Thinking

More Chances
Students will have more opportunities to develop this understanding in later lessons. There is no need to slow down or add additional work to review this concept at this time.

SAMPLE ONLY

Unit 1, Lesson 1

Tiling the Plane

Goals

- Compare (orally) areas of the shapes that make up a geometric pattern.
- Comprehend that the word "area" (orally and in writing) refers to how much of the plane a shape covers.

Learning Targets

- I can explain the meaning of "area."

Lesson Narrative

In the first lesson of the course, students compare the amounts of a plane covered by different shapes and recall what they know about **area**. The investigations allow students to experience two important ideas that will be made explicit in the next lesson:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- The area of a **region** does not change when the region is decomposed and rearranged.

At the end of this lesson, students are asked to write their best definition of "area." It is important to let them formulate their definition in their own words. It is especially important to encourage Multilingual learners to use their own words and the words of their peers. In a future lesson, students will revisit the definition of "area" as "the number of square units that cover a region without gaps or overlaps."

While the mathematics that students explore in this lesson is not complicated and offers a low threshold for entry, it does prompt students to make sense of problems and persevere in solving them (MP1). The activities also enable the teacher to begin setting the expectations for mathematical discourse; that is for students to construct logical arguments and listen to the reasoning of others (MP3).

The lesson allows some time for the teacher to begin establishing classroom norms and routines.

A note about terminology:

In these materials, when we talk about a two-dimensional figure, such as a rectangle, triangle, or circle, we usually mean the boundary of the figure (such as the sides of a rectangle), not including the region inside. However, we also use shorthand language such as "the area of a rectangle" to mean the "the area of the region inside the rectangle." The term "shape" could refer to a figure with or without its interior. Although the terms "figure," "region," and "shape" are used without being defined precisely for students, help students understand that sometimes our focus is on the boundary (which in this unit will always be composed of black line segments), and sometimes it is on the region inside (which in this unit will be shown in color and referred to as "the shaded region").

Math Community
This is the first exercise that focuses on the work of building a mathematical community. Students have the opportunity to think about what a mathematical community is and to share their initial thoughts about what it looks like and sounds like to do math together in a community.

Standards

Building On **3.G.A**

Instructional Routines

- 5 Practices

SAMPLE ONLY

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Building Towards **6.G.A.1**

- Which Three Go Together?

Required Materials

Materials To Gather

- Chart paper: Activity 1
- Sticky notes: Activity 1
- Geometry toolkits: Activity 2

Materials To Copy

- 6–12 Blank Math Community Chart (1 copy for every 30 students): Activity 1

Sec A

Required Preparation

Activity 1:

Make a space for students to place their sticky notes at the end of the *Warm-up*. For example, hang a sheet of chart paper on a wall near the door.

Activity 2:

Assemble geometry toolkits. Toolkits include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Prepare pattern blocks (with triangles, rhombuses, and trapezoids), if available.

For the digital version of the activity, acquire devices that can run the applet.

Lesson:

Assemble geometry toolkits. Toolkits include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Student Facing Learning Goals

Let's look at tiling patterns and think about area.

1.1 Which Three Go Together: Tilings

Warm-up

10 mins

Activity Narrative

This is the first *Which Three Go Together* routine in the course. In this routine, students are presented with four items or representations and asked: "Which three go together? Why do they go together?"

Students are given time to identify a set of three items, explain their rationale, and refine their explanation to be more precise or find additional sets. The reasoning here prompts students to notice common mathematical attributes, look for structure (MP7), and attend to precision (MP6), which deepens their awareness of connections across representations.

This *Warm-up* prompts students to compare four geometric patterns. It gives students a reason to use language precisely and allows the teacher to hear how students use terminology in describing geometric characteristics. Comparing the patterns also urges students to think about shapes that cover the plane without gaps and overlaps, which supports future conversations about the meaning of "area."

SAMPLE ONLY

Before students begin their work, consider establishing a small, discreet hand-signal that students can display when they have an answer that they can support with reasoning. This might include a thumbs-up or a certain number of fingers that indicates the number of responses that they have. Using a signal is a quick way to see if students have had enough time to think about the problem. A subtle signal keeps students from being distracted or rushed by seeing hands being raised around the class.

Students may choose to describe the patterns in terms of:

- Colors (blue, green, yellow, white, or no color).
- Size of shapes or other measurements.
- Geometric shapes (squares, pentagons, hexagons, or polygons).
- Relationships of shapes (whether each side of the polygons meets the side of another polygon, what polygon is attached to each side, whether there is a gap between polygons, and so on).

Standards

Building On **3.G.A**

Instructional Routines

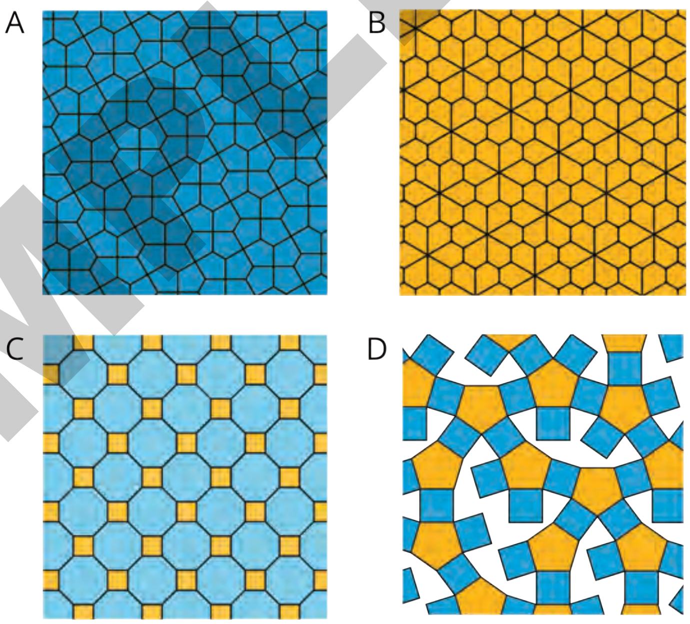
- Which Three Go Together?

Launch

Arrange students in groups of 2–4. Display the four patterns for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three patterns that go together and can explain why. Next, tell students to share their response with their group and then work together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



SAMPLE ONLY

Student Response

Sample responses:

A, B, and C go together because:

- All the shapes have color (or are shaded).
- All the colored shapes meet other colored shapes on all sides. There are no gaps between the shapes.
- They don't have white (or non-shaded) regions.
- The shapes have different side lengths.

A, B, and D go together because:

- They all have pentagons.

A, C, and D go together because:

- They have the color blue.
- They have shapes with one or more right angles.

B, C, and D go together because:

- They have the color yellow.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure that the reasons given are correct.

During the discussion, ask students to explain the meaning of any geometric terminology they use (names of polygons or angles, parts of polygons, "area") and to clarify their reasoning as needed. For example, a student may say that Patterns A, B, and C each have shapes with different side lengths, but all the shapes in Pattern D have the same side lengths. . Ask how they know that is the case, and whether this is true for the white (or non-filled) regions in Pattern D.

Explain to students that covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps is called "tiling" the plane. Patterns A, B, and C are examples of tiling. Tell students that they will explore more tilings in upcoming activities.

Math Community
Tell students that today is the start of planning the type of mathematical community they want to be a part of for this school year. The start of this work will take several weeks as the class gets to know one another, reflects on past classroom experiences, and shares their hopes for the year.

Display and read aloud the question "What do you think it should look like and sound like to do math together as a mathematical community?" Give students 2 minutes of quiet think time and then 1–2 minutes to share with a partner. Ask students to record their thoughts on sticky notes and then place the notes on the sheet of chart paper. Thank students for sharing their thoughts and tell them that the sticky notes will be collected into a class chart and used at the start of the next discussion.

After the lesson is complete, review the sticky notes to identify themes. Make a Math Community Chart to display in the classroom. See the blackline master Blank Math Community Chart for one way to set up this chart. Depending on resources and wall space, this may look like a chart paper hung on the wall, a regular sheet of paper to display using a document camera, or a digital version that can be projected. Add the identified themes from the students' sticky notes to the student section of the "Doing Math" column of the chart.

SAMPLE ONLY

1.2 More Red, Green, or Blue?

25 mins

Activity Narrative

There is a digital version of this activity.

This activity asks students to compare the amounts of the plane covered by two tiling patterns, with the aim of supporting two big ideas of the unit:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- A region can be decomposed and rearranged without changing its area.

Students are likely to notice that in each pattern:

- The same 3 polygons (triangles, rhombuses, and trapezoids) are used as tiles.
- The shapes are arranged without gaps and overlaps, but their arrangements are different.
- A certain set of smaller tiles form a larger hexagon. Each hexagon has 3 trapezoids, 4 rhombuses, and 7 triangles.
- The entire tiling pattern is composed of these hexagons.

Expect students to begin their comparison by counting each shape, either in the entire pattern or in a portion that repeats. For students to effectively compare how much of the plane is covered by each shape, they need to be aware of the relationships between the shapes. For example, two green triangles can be placed on top of a blue rhombus so that they match up exactly. This means that two green triangles cover the same amount of the plane as one blue rhombus covers. (Students don't need to use the word "area" in their explanations. At this point, phrases such as "they match up" or "two triangles make one rhombus" suffice.)

Geometry toolkits are introduced here, giving students an opportunity to use tools strategically (MP5). Students could, for instance, use tracing paper to trace shapes, use a straightedge to extend lines, or use scissors to cut out shapes.

Monitor for the different ways in which students make comparisons. Here are some approaches students may take, from more common to less common:

- Compare the numbers of shapes in the entire pattern—56 green triangles, 32 blue rhombuses, and 24 red trapezoids. This doesn't yet tell us which shape covers more of the plane.
- Compare the numbers of shapes in a larger hexagon—7 green triangles, 4 rhombuses, and 3 trapezoids. There are fewer shapes to count, but these numbers alone also don't tell us which shape covers more of the plane.
- Compare the numbers of shapes and their relationships. Examples: One rhombus matches up exactly with 2 triangles. One trapezoid matches up exactly with 3 triangles. In a larger hexagon, there are 7 green triangles, 4 rhombuses that match up with 8 triangles, and 3 trapezoids that match up with 9 triangles. (Students may also relate the areas of shapes in the entire pattern this way. See student responses for additional examples.)

The routine of Anticipate, Monitor, Select, Sequence, Connect requires a balance of planning and flexibility. The anticipated approaches might not surface in every class, and there may be reason to change the order in which strategies are presented. While monitoring, keep in mind the learning goal and adjust the order to ensure that all students have access to the first idea presented (whether that be a common misconception or a different approach).

In the digital version of the activity, students use an applet to explore the shapes that comprise the pattern. The applet allows students to see the patterns on a triangular grid and to frame the repeating larger hexagons. It also allows students to isolate a larger hexagon and to move and rotate individual shapes within it. These features might help students in quantifying and relating the shapes.

SAMPLE ONLY

Standards

Building Towards **6.G.A.1**

Instructional Routines

- 5 Practices

Launch

Arrange students in groups of 2. Ask one partner to analyze Pattern A and the other to analyze Pattern B. Tell students that their job is to compare the amount of the plane covered by each shape in their pattern.

Before students begin their work, introduce them to the geometry toolkits. Encourage students to consider using one or more tools in the toolkits for help, if needed. If pattern blocks are available, they can be offered as well.

Give students 4–5 minutes of quiet think time. Then ask students to discuss their responses with their partner.

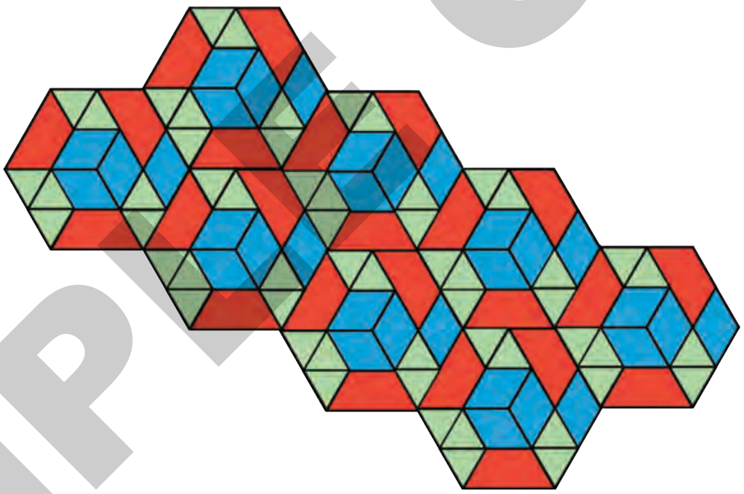
Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

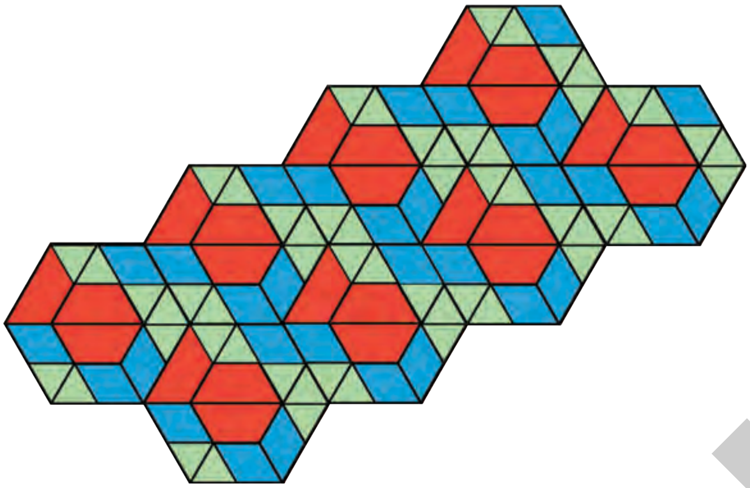
Pattern A



SAMPLE ONLY

Sec A

Pattern B



Student Response

In both Patterns A and B, more of the plane is covered by red trapezoids than by green triangles or blue rhombuses. Sample reasoning:

- Patterns A and B are each made of 56 green triangles, 32 blue rhombuses, and 24 red trapezoids.
    - One blue rhombus covers the same amount of the plane as do 2 green triangles, so 32 blue rhombuses cover the same amount of the plane as do 64 green triangles. This is more than the amount covered by 56 green triangles. So, blue rhombuses cover more of the plane than green triangles cover.
    - One red trapezoid covers the same amount of the plane as do 3 green triangles, so 24 red trapezoids cover the same amount of the plane as do 72 green triangles. This is more than the amount covered by 64 green triangles. So, red trapezoids cover more of the plane than blue rhombuses cover.
- Each pattern is composed of 8 larger hexagons. In each of these hexagons there are 3 red trapezoids, 4 blue rhombuses, and 7 green triangles.
    - Comparing 3 trapezoids and 4 rhombuses: Two red trapezoids can be arranged into a small hexagon. Three rhombuses can also be arranged into the same small hexagon. This means that 2 trapezoids cover the same amount of the plane as do 3 rhombuses. The last red trapezoid covers more of the plane than does the last rhombus. So, red trapezoids cover more of the plane than blue rhombuses cover.
    - Comparing 3 trapezoids and 7 triangles: One trapezoid covers the same amount of the plane as do 3 triangles, so 2 trapezoids cover the same amount of the plane as do 6 triangles. The last trapezoid covers more of the plane than does the last triangle. So, red trapezoids cover more of the plane than green triangles cover.

Building on Student Thinking

Students may say more of the area is covered by the color they see the most in each image, saying, for example, “It just looks like there is more red.” Ask these students if there is a way to prove their observations.

Students may only count the number of green triangles, red trapezoids, and blue rhombuses but not account for the area covered by each shape. If students suggest that the shape with the most pieces in the pattern covers the most amount of the plane, ask them to test their hypothesis. For example, ask, "Do 2 triangles cover more of the plane than 1 trapezoid covers?"

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Students may not recall the terms “trapezoid,” “rhombus,” and “triangle.” Consider reviewing the terms, although they do not need to know the formal definitions to work on the task.

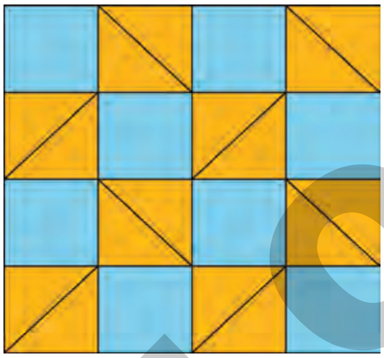
Are You Ready for More?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

Extension Student Response

Sample response:



Activity Synthesis

The purpose of this discussion is to help students connect their comparisons of shapes to the idea of area.

Invite previously selected students to share their answers and explanations. Sequence the discussion of strategies in the order listed in the *Activity Narrative*. Be sure to discuss the idea of comparing shapes by placing them on top of one another and seeing if or how they match. Consider demonstrating this idea using the applet.

Make it explicit that when students are asked, “Which shape covers more of the plane?”, they are being asked to compare the areas covered by the different shapes.

Connect the discussion to the learning goals by asking questions, such as:

- “How does the area of the trapezoid compare to the area of the triangle?” (The area of the trapezoid is three times the area of the triangle.)
- “How does the area of the rhombus compare to the area of the triangle?” (The area of the rhombus is twice the area of the triangle.)
- “Is it possible to compare the area that all of the rhombuses cover in Pattern A to the area that all the triangles cover in Pattern B? If yes, how? If no, why not?” (Yes, we can count the number of rhombuses in A and the number of triangles in B. Because 2 triangles have the same area as does 1 rhombus, divide the number of rhombuses in A by 2. Then compare the result to the number of triangles in B.)

SAMPLE ONLY

Lesson Synthesis

In this lesson, students started to reason about what it means for two shapes to have the same area. They also started to think about tools that can help them do mathematics. Ask students:

- "Draw two shapes that you know do not have the same area. How can you tell?"
- "What are some of the tools in the geometry toolkit and what are they used for?"

Tell students that they will continue to think about area and to learn to use tools strategically when doing mathematics.

1.3

What is Area?

Cool-down

5 mins

The purpose of this cool-down is to check how students are thinking about area after engaging in the activities. While the task prompts students to reflect on the work in this lesson, ideas about area from students' prior work in grades 3–5 may also emerge. Knowing the range of student thinking will help to inform the next day's lesson.

Student Task Statement

Think about your work today, and write your best definition of "area."

Student Response

Sample responses:

- The amount of space inside a two-dimensional shape
- The measurement of the inside of a shape
- The number of square units inside a shape
- The amount of space a shape covers
- The amount of the plane a shape covers

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 1 Summary

In this lesson, we learned about *tiling* the plane, which means "covering a two-dimensional **region** with copies of the same shape or shapes such that there are no gaps or overlaps."

Then we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about **area**.

In future lessons, we will continue with this reasoning, and we will continue learning how to use mathematical

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tools strategically to help us do mathematics.

Glossary

- region

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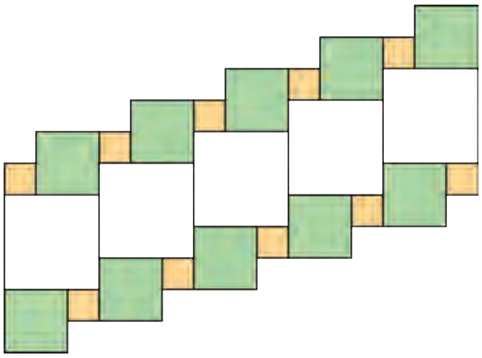
Practice Problems

1 Student Task Statement

Which square—large, medium, or small—covers more of the plane? Explain your reasoning.



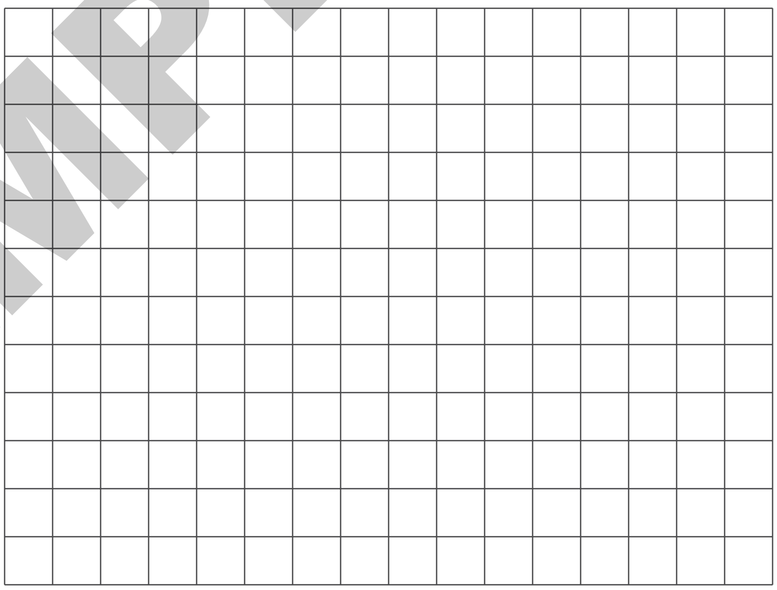
Solution

The large square covers more of the plane. Sample reasoning: A large square can fit exactly 9 small squares. A medium square can fit exactly 4 small squares. There are 5 large squares, which cover the same amount of the plane as do 45 small squares. There are 10 medium squares, which cover the same amount of the plane as do 40 small squares. There are only 10 small squares.

2 from an earlier course

Student Task Statement

Draw three different quadrilaterals, each with an area of 12 square units.



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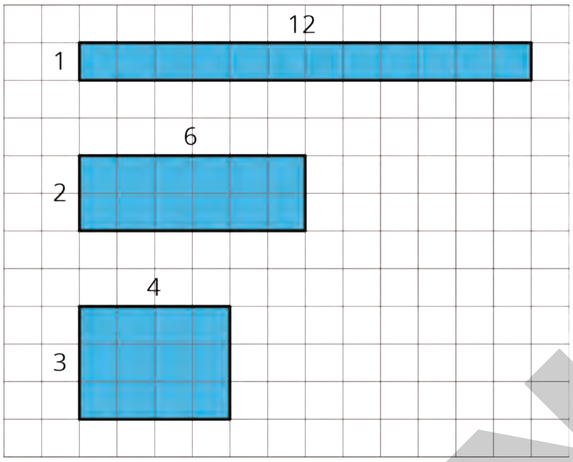
Solution

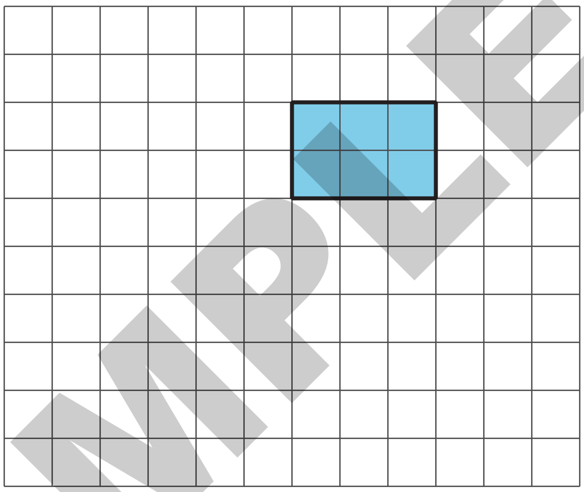
Sample response:



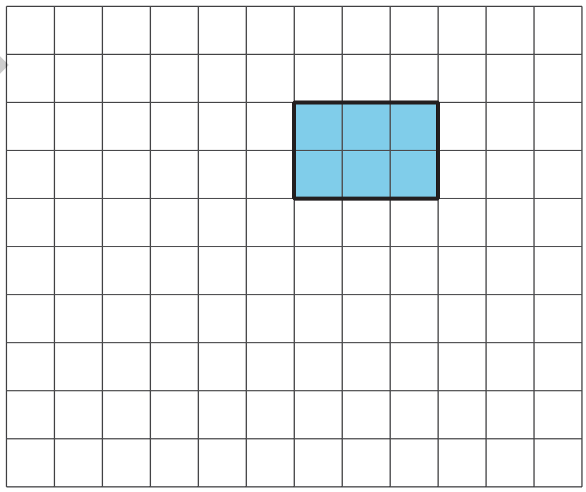
3 Student Task Statement

Use copies of the rectangle to show how a rectangle could:

a. Tile the plane.



b. *Not* tile the plane.



Solution

a. Sample response:

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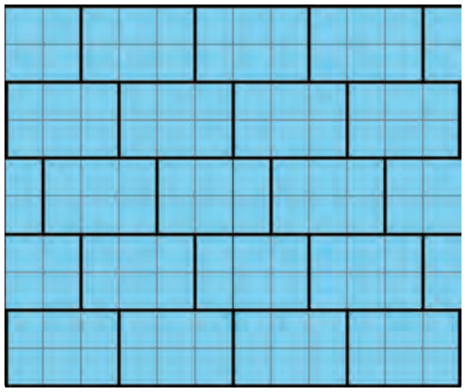
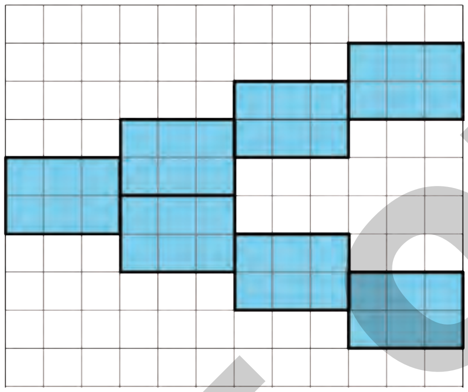
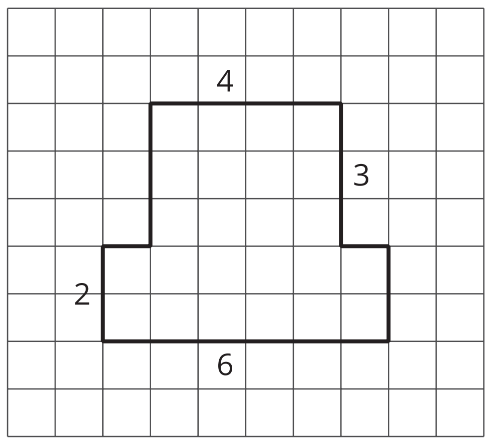


b. Sample response:



4 Student Task Statement

The area of this shape is 24 square units. Select **all** the statements that are true about the area.



A. The area can be found by counting the number of squares that touch the edge of the shape.

B. It takes 24 grid squares to cover the shape without gaps and overlaps.

C. The area can be found by multiplying the sides lengths that are 6 units and 4 units.

D. The area can be found by counting the grid squares inside the shape.

E. The area can be found by adding 4×3 and 6×2.

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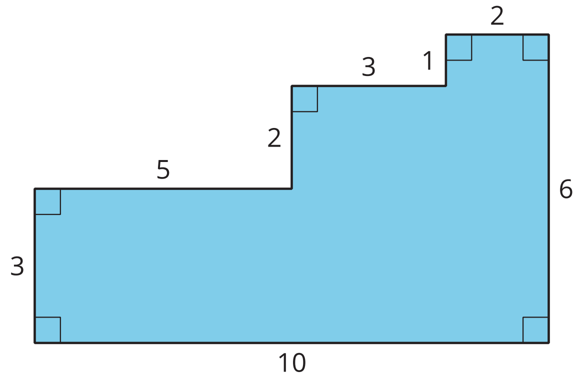
Solution

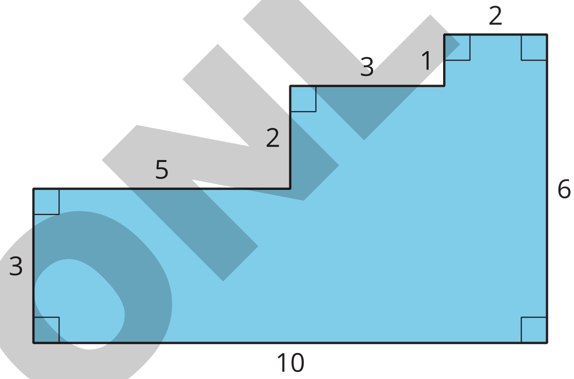
B, D, E

5 from an earlier course

Student Task Statement

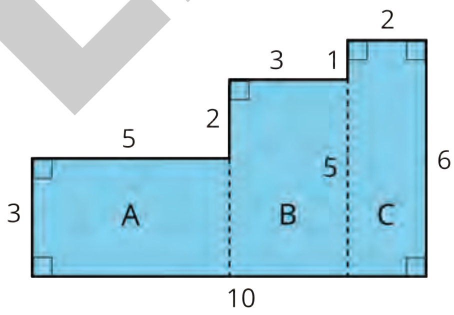
Here are two copies of the same figure. All angles are right angles. Show two different ways for finding the area of the shaded region.





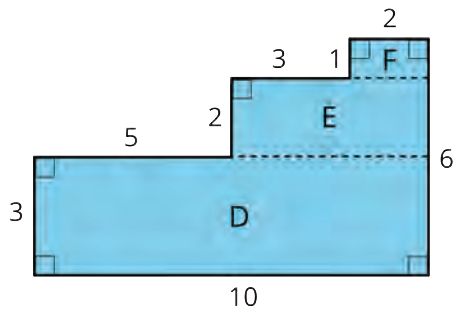
Solution

Sample reasoning:

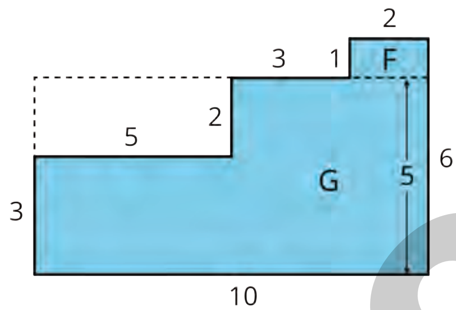


Area of A is 15 square units. Area of B is 15 square units. Area of C is 12 square units. The area of the entire region is $15 + 15 + 12$, or 42 square units.

SAMPLE ONLY



Area of D is 30 square units. Area of E is 10 square units. Area of F is 2 square units. The area of the entire region is $30 + 10 + 2$ or 42 square units.



Area of F is 2 square units. Area of G is the area of the 10-by-5 rectangle subtracted by the area of the 5-by-2 rectangle in the upper left.$(10 \times 5) - (5 \times 2) = 50 - 10 = 40$, so the area of G is 40 square units. The total area is $40 + 2$, or 42 square units.

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Unit 1, Lesson 2

Finding Area by Decomposing and Rearranging

Sec A

Goals

- Calculate the area of a region by decomposing it and rearranging the pieces, and explain (orally and in writing) the solution method.
- Recognize and explain (orally) that if two figures can be placed one on top of the other so that they match up exactly, they must have the same area.
- Show that area is additive by composing polygons with a given area.

Learning Targets

- I can explain how to find the area of a figure that is composed of other shapes.
- I know how to find the area of a figure by decomposing it and rearranging the parts.
- I know what it means for two figures to have the same area.

Lesson Narrative

In this lesson, students learn to reason flexibly about two-dimensional figures to find their **areas**.

Students begin by revisiting the definitions for "area" that they learned in earlier grades. They refine these definitions and arrive at a definition that can be used by the class for the rest of the unit. Along the way, students practice attending to precision (MP6).

Next, students use tangram pieces to explore ways of reasoning about area. They *compose* and rearrange a square and some triangles to form figures of certain areas. Students see that the area of a two-dimensional figure can be determined in multiple ways:

- by composing that figure using smaller pieces with known areas
- by *decomposing* the figure into shapes whose areas we can determine and adding the areas of those shapes
- by decomposing it and rearranging the pieces into a different but familiar shape whose area can be found

In an optional activity, students use these strategies to reason about the area of individual tangram pieces and practice constructing logical arguments to justify their reasoning (MP3).

By the end of the lesson, two key principles about area are made explicit: Figures that match exactly have equal areas, and area is additive (in that the area of a figure is the sum of the areas of all non-overlapping pieces that compose it).

Math Community
In this lesson, students review the themes that arose when they shared their initial thoughts in Exercise 1 about what they think it should look like and sound like to do math together as a community. Students then have a chance to both affirm and add to the ideas that were generated.

Standards

Building On **3.MD.C.5.b**

Instructional Routines

- MLR2: Collect and Display

SAMPLE ONLY

Addressing	**6.G.A.1**
Building Towards	**6.G.A**

- Notice and Wonder

Required Materials

Materials To Gather

- Geometry toolkits: Lesson
- Math Community Chart: Lesson
- Pre-assembled or commercially produced tangrams: Lesson
- Sticky notes: Lesson
- Math Community Chart: Activity 1
- Sticky notes: Activity 1
- Pre-assembled or commercially produced tangrams: Activity 3, Activity 4
- Geometry toolkits: Activity 4

Materials To Copy

- Composing Shapes Cutouts (1 copy for every 2 students): Activity 3

Required Preparation

Activity 3:

For every group of 2 students, prepare 1 set of tangrams that contains 1 square, 4 small triangles, 1 medium triangle, and 2 large triangles. Print and cut out the blackline master (printing on card stock is recommended), or use commercially-available tangrams.

Note that the tangram pieces used here differ from a standard set in that two additional small triangles are used instead of a parallelogram.

For the digital version of the activity, acquire devices that can run the applet.

Activity 4:

For the digital version of the activity, acquire devices that can run the applet.

Lesson:

Make sure that students have access to their geometry toolkits, which should include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Student Facing Learning Goals

Let's create shapes and find their areas.

SAMPLE ONLY

2.1 Notice and Wonder: Squares in Shapes

Warm-up

5 mins

Activity Narrative

This is the first Notice and Wonder activity in this course. Students are shown four drawings and asked: "What do you notice? What do you wonder?"

Students are given time to write down what they notice and wonder about the images and then time to share their thoughts. Their responses are recorded for all to see. Often, the goal is to elicit observations and curiosities about a mathematical idea that students are about to explore. Pondering the two open questions allows students to build interest about and gain entry into an upcoming task.

The purpose of this *Warm-up* is to elicit observations about squares that tile a region, which will be useful when students think about the meaning of "area" later in the lesson. While students may notice and wonder many things about these images, the important discussion points are observations about equal-size squares covering a region completely without gaps or overlaps.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language that they use to describe what they see (MP6). They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Standards

Building On	**3.MD.C.5.b**
Building Towards	**6.G.A**

Instructional Routines

- Notice and Wonder

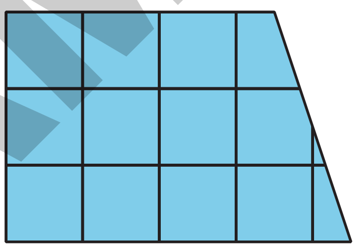
Launch

Arrange students in groups of 2. Display the four images for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?

A



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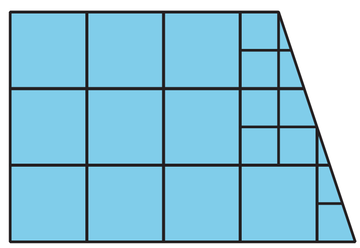
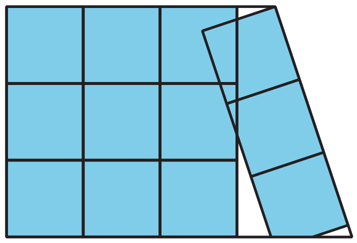
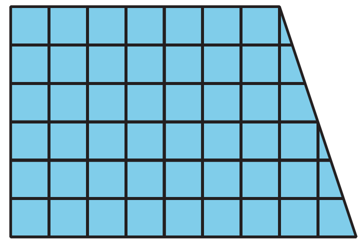


SAMPLE ONLY

C



D



Sec A

Student Response

Students may notice:

- A, B, C, and D are all the same shape.
- All four drawings are filled with squares or parts of squares on the inside.
- A, C, and D are filled with squares that are the same size. The squares in B are of two different sizes.
- In C, there are some gaps between the squares, and some squares overlap.
- There are many more squares in D compared to the other figures.

Students may wonder:

- Why are three squares in C rotated to follow the direction of the slanted side?
- Why is B filled with different-size squares?
- What is the size of each large square and each small square?
- Can the partial squares in A, B, and D be put together to make whole squares?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see, without editing or commentary. If possible, record the relevant reasoning on or near the images. Next, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the gaps and overlaps in C don’t come up during the conversation, ask students to discuss this idea.

Math Community
Display the class Math Community Chart for all to see and explain that the listed “doing math” actions come from the sticky notes students wrote in the first exercise. Give students 1 minute to review the chart. Then invite students to identify something on the chart they agree with and hope for the class or something they feel is missing from the chart and would like to add. Record any additions on the chart. Tell students that the chart will continue to grow and that they can suggest other additions that they think of throughout today’s lesson during the Cool-down.

SAMPLE ONLY

2.2 What is Area?

10 mins

Activity Narrative

In this activity, students recall and refine their prior knowledge of area. They articulate a definition of "area" that can be used for the rest of the unit. This definition of "area" is not new but rather reiterates what students learned in grades 3–5.

In the *Warm-up*, students analyzed four ways that a region was tiled or otherwise fitted with squares. Here, students revisit the same images and decide which arrangements of squares can be used to find the area of the region and why. Students use their analysis to write a definition of "area." In identifying the most important aspects that should be included in the definition, students attend to precision (MP6).

Students' initial definitions may be incomplete. During partner discussions, monitor for students who mention the following aspects so they can share later:

- Plane or two-dimensional region
- Square units
- Covering a region completely without gaps or overlaps

Standards

Building On	**3.MD.C.5.b**
Building Towards	**6.G.A**

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time for the first question and ask them to be ready to explain their decision. Then give partners 3–4 minutes to share their responses and to complete the second question together.

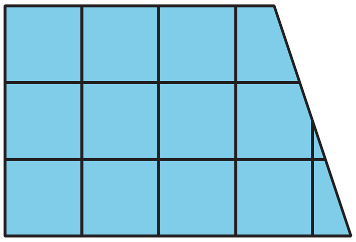
Student Task Statement

You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

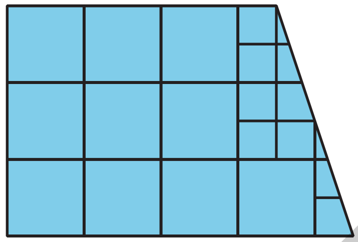
SAMPLE ONLY

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.

A



B



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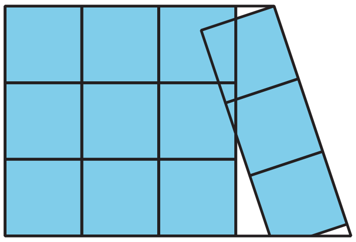
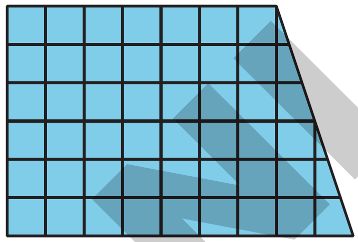


D



2. Write a definition of “area” that includes all the information that you think is important.

Student Response

1. Shapes A and D. Shape B could be considered if the larger squares and the smaller ones are distinguished when determining area.
2. Answers vary, but the working definition should contain all of these components: “The area of a two-dimensional region (in square units) is the number of unit squares that cover the region without gaps or overlaps.”

Building on Student Thinking

Students may focus on how they have typically found the area of a rectangle—by multiplying its side lengths—instead of thinking about what “the area of any region” means. Ask them to consider what the product of the side lengths of a rectangle actually tells us. (For example, if they say that the area of a 5-by-3 rectangle is 15, ask what the 15 means.)

Some students may think that none of the options, including Options A and D, could be used to find the area of the region because they involve partial squares, or because the partial squares do not appear to be familiar fractional parts. Use of benchmark fractions may help students see that the area of a region could be a non-whole number. For example, ask students if the area of a rectangle could be $8\frac{1}{2}$ or $2\frac{1}{4}$ square units.

Activity Synthesis

The purpose of this discussion is to elicit key points to include in a class definition of "area." Invite students to share their responses to the first question and to explain their reasoning. Ask questions such as:

- “What is it about Shapes A and D that can help us find the area?” (The squares are all the same size. They are unit squares.)
- “What is it about Shape C that might make it unhelpful for finding the area?” (The squares overlap and do not cover the entire region, so counting the squares won't give us the area.)
- “If you think Shape B *cannot* be used to find the area, why not?” (We can't just count the number of squares and say that this number is the area because the squares are not all the same size.)

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- "If you think we *can* use Shape B to find the area, how?" (Four small squares make a large square. If we count the number of large squares and the number of small squares separately, we can convert one to the other and find the area in terms of either one of them.)

If time permits, discuss:

- "How are Shapes A and D different?" (Shape A uses larger unit squares and Shape D uses smaller ones. Each size represents a different unit.)
- "Will Shapes A and D give us different areas?" (They will give us the same area, but in different units—for example, square inches and square centimeters.)

Select 2–3 groups previously identified to share their definitions of "area" or what they think should be included in the class definition of "area." The discussion should lead to a class definition that conveys key aspects of area: The area of a two-dimensional region (in square units) is the number of unit squares that cover the region without gaps or overlaps.

Display the class definition and revisit as needed throughout this unit. Tell students that this will be a working definition that can be revised as they continue their work in the unit.

SAMPLE ONLY

2.3 Composing Shapes

20 mins

Activity Narrative

There is a digital version of this activity.

In this activity, students further develop their understanding that area is additive. Students compose tangram pieces—consisting of triangles and a square—into shapes with certain areas. The square serves as a unit square. Because students have only one square, they need to use these principles in their reasoning:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a figure is decomposed and rearranged to compose another figure, then the area of the new figure is the same as the area of the original figure.

Each question in the activity aims to elicit discussions about those two principles. Though they may seem obvious, these principles still need to be stated explicitly (at the end of the lesson) because a more-advanced understanding of the area of complex figures depends on them. As students work, look for those whose reasoning illustrates the principles.

This is the first time Math Language Routine 2: Collect and Display is suggested in this course. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing that students use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. The purpose of this routine is to capture a variety of students' words and phrases—especially including everyday or social language and non-English—in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.

In the digital version of the activity, students use an applet showing 8 tangram pieces to determine the relationships between the areas. Consider using the applet if physical tangram pieces are not available. The applet is adapted from the work of Harry Drew in Geogebra.

Access for English Language Learners

This activity uses the Collect and Display math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Keep students in groups of 2. Give each group the following set of tangram pieces from the blackline master or from commercially available sets. Note that the tangram pieces used here differ from a standard set in that two additional small triangles are used instead of a parallelogram.

- 1 square
- 4 small triangles
- 1 medium triangle
- 2 large triangles

It is important not to give them more than these pieces.

Give students 2–3 minutes of quiet think time to consider the first three questions. Ask them to pause afterward and compare their solutions to their partner's. If partners created the same shape for each question, ask them to create a different shape that has the same given area before moving on. Then ask them to work together to answer the remaining questions.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe their work with the tangram pieces. Display words and phrases such as: "make," "build," "put together," "join," "compose," "break" or "break apart," "decompose," "match up," "move around," and "rearrange."

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards. They can trace the square composed of 2 medium triangles and use it as a reference for 1 square unit.
Supports accessibility for: Memory, Organization

Student Task Statement

Your teacher will give you 1 square and some small, medium, and large right triangles. The area of the square is 1 square unit.

1. Notice that you can put together 2 small triangles to make a square. What is the area of the square composed of 2 small triangles? Be prepared to explain your reasoning.
2. Use your shapes to create a new shape with an area of 1 square unit that is *not* a square. Trace your shape.
3. Use your shapes to create a new shape with an area of 2 square units. Trace your shape.
4. Use your shapes to create a *different* shape with an area of 2 square units. Trace your shape.
5. Use your shapes to create a new shape with an area of 4 square units. Trace your shape.

Student Response

1. The area of the square made from 2 small triangles is 1 square unit because it is identical to the given square with an area of 1 square unit. "Identical" means you can put one on top of the other and they match up exactly.

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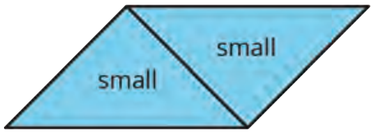
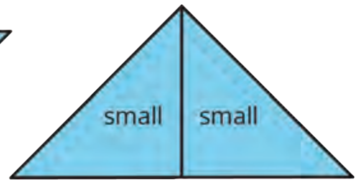
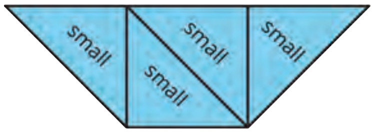
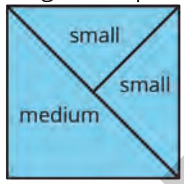
2. Any composite of 2 small triangles.



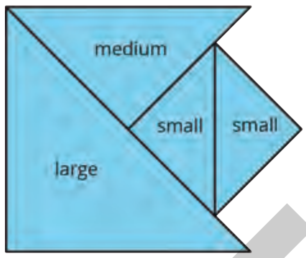


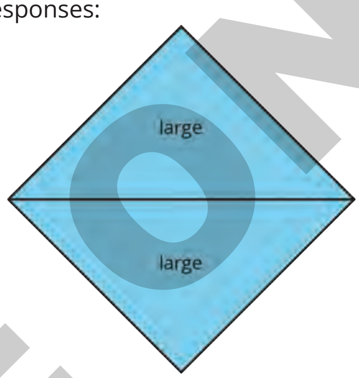
3. Any composite of 4 small triangles, or 2 small triangles and 1 medium triangle. Sample responses:





4. Any composite of 4 small triangles, or 2 small triangles and 1 medium triangle.
5. Any composite with an area of 4 square units. Sample responses:





Building on Student Thinking

Students may consider the area to be the number of pieces in a composition, instead of the number of square units. This confusion may be more likely to arise when the number of pieces is the same as the number of square units, as in the *Are You Ready for More?* Remind students of the meaning of "area," or prompt them to review the definition of "area" discussed in an earlier activity.

Because the two large triangles in the tangram set can be arranged to form a square, students may consider that square to be the square unit rather than the smaller square composed of two small triangles. Ask students to review the task statement and verify the size of the unit square.

Are You Ready for More?

Find a way to use all of your pieces to compose a single large square. What is the area of this large square?

Extension Student Response

The area is 8 square units. Sample response:

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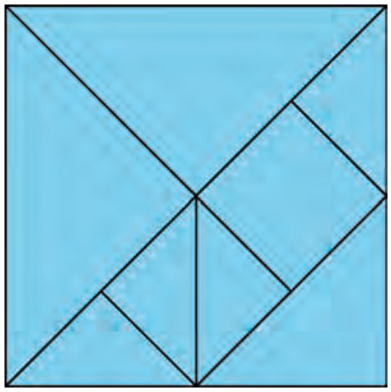


Activity Synthesis

The purpose of the discussion is to make two principles explicit: Two regions have the same area if they match up exactly when superimposed, and the area of a region is preserved even when the region is decomposed and rearranged.

Direct students' attention to the reference created using *Collect and Display*. Ask previously identified students to share the shapes they created and to explain how they knew those shapes have certain areas. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

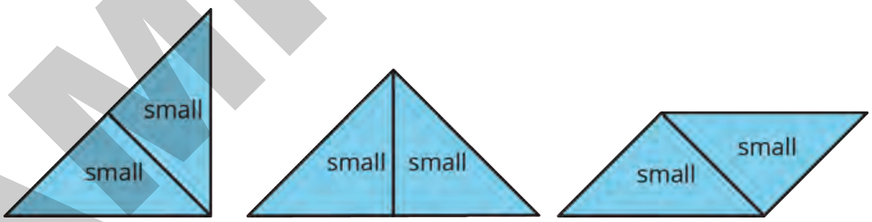
After each student shares an explanation, use the terms "compose," "decompose," and "rearrange" to name these moves. Clarify that to *compose* means to join or put together and *decompose* means to break apart or separate.

Highlight the following points:

- First question: Two small triangles can be composed into a square that matches up exactly with the given square piece. This means that the two squares—the composite and the unit square—have the same area.

Tell students, "If a region can be placed on top of another region so that they match up exactly, then they have the same **area**."

- Second question: Two small triangles can be rearranged to compose a different figure, but the area of that composite is still 1 square unit. The following three shapes—each composed of 2 triangles—have the same area. If we rotate the first figure, it can be placed on top of the second so that they match up exactly. The third figure has a different shape than the other two, but because it is made up of the same 2 triangles, it has the same area.



Emphasize: "If a figure is decomposed and rearranged as a new figure, the area of the new figure is the same as the area of the original figure."

- Third and fourth questions: The composite figures could be formed in several ways—with only the four small triangles, with two small triangles and a medium triangle, or with two small triangles and a square.
- Last question: A large triangle is needed here. To find its area, either compose four smaller triangles into a large triangle, or recognize that the large triangle could be decomposed into four smaller triangles, which can then be composed into 2 unit squares.

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Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following terms and maintain the display for reference throughout the unit: “area,” “compose,” “decompose,” and “rearrange.”
Supports accessibility for: Conceptual processing; Language

2.4 Tangram Triangles

Optional

15 mins

Activity Narrative

There is a digital version of this activity.

In this activity, students reason about the area of each tangram triangle based on the areas of composite shapes that they previously created. Students may have recognized that the area of one small triangle is $\frac{1}{2}$ square unit, the area of one medium triangle is 1 square unit, and the area of one large triangle is 2 square units. Here, they articulate how they know that these conclusions are true, using written words or illustrations that are clearly labeled. They also listen to their partner’s explanations. In doing so, students practice constructing logical arguments and critiquing the reasoning of others (MP3).

As partners discuss, look for two ways of thinking about the area of each assigned triangle: by composing copies of the triangle into a square or a larger triangle, or by decomposing the triangle or the unit square into smaller pieces and rearranging the pieces. Identify at least one student who uses each approach.

In the digital version of the activity, students can use the same applet as in the previous activity to support their reasoning.

This activity uses the Collect and Display math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for English Language Learners

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Keep students in groups of 2. Assign the first and second questions to one partner and the second and third questions to the other partner. Give each group access to the geometry toolkits and the same set of tangram pieces as used in the earlier activity, “Composing Shapes.”

Give students 3–4 minutes of quiet time to find the areas of their assigned triangles and to construct their explanations, followed by a few minutes to share their responses with their partner. Tell students that as one partner explains, the other should listen carefully and either agree or disagree with the explanation. Partners should then come to an agreement about the answers and explanations.

SAMPLE ONLY

Use *Collect and Display* to direct attention to words collected and displayed from the "Composing Shapes" activity. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Sec A

Student Task Statement

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is ________ square units. I know this because . . .
2. The area of the medium triangle is ________ square units. I know this because . . .
3. The area of the large triangle is ________ square units. I know this because . . .

Student Response

1. $\frac{1}{2}$ square unit. Sample reasoning:
    - Two small triangles can be put together to make a square, which has an area of 1 square unit. Because this composite shape matches the unit square exactly, their areas must be equal. This means that the area of each small triangle is half the area of the unit square.
    - A square can be decomposed into exactly two small triangles. So, the area of each small triangle must be half of the area of the square.
2. 1 square unit. Sample reasoning:
    - Two small triangles can be put together to make one medium triangle. Two triangles can also be put together to make a square with an area of 1 square unit. Because two small triangles make either a medium triangle or a square, the area of the medium triangle must be 1 square unit.
    - One medium triangle can be decomposed into two small triangles. These can be rearranged into a square whose area is 1 square unit, so the area of the medium triangle is also 1 square unit.
3. 2 square units. Sample reasoning:
    - Two medium triangles can be arranged into one large triangle. Because the area of the medium triangle is 1 square unit, a figure that is composed of two of them has area 2 square units.
    - A large triangle can be decomposed into 4 small triangles, which can in turn be rearranged into two squares. The combined area of the two squares is 2 square units.

Building on Student Thinking

If students initially have trouble determining the areas of the shapes, ask how they reasoned about areas in the previous activity. Show examples of composed and decomposed shapes that form 1 square unit to which students can refer.

Activity Synthesis

After partners share and agree on the correct areas and explanations, discuss with the class:

- "Did you and your partner use the same strategy to find the area of each triangle?"
- "How were your explanations similar? How were they different?"

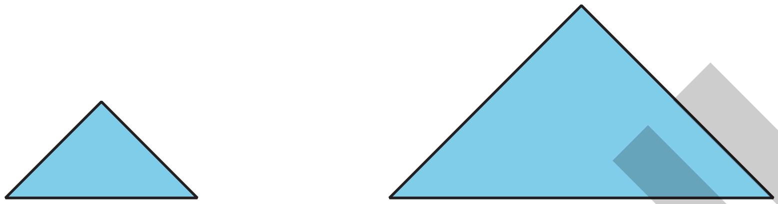
Select two previously identified students to share their explanations: One who reasoned in terms of *composing* copies of the assigned triangle into another shape, and one who reasoned in terms of *decomposing* the triangle or the unit

SAMPLE ONLY

square into smaller pieces and *rearranging* them. If these strategies are not brought up by students, be sure to make them explicit at the end of the lesson.

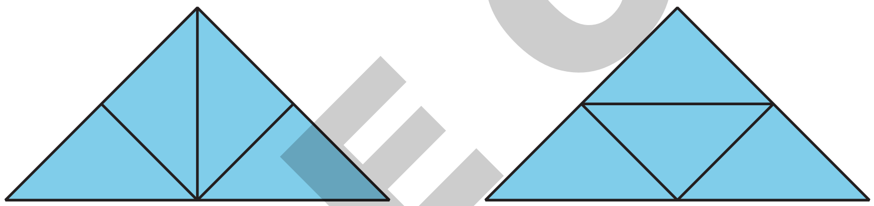
Lesson Synthesis

To highlight the key principles for reasoning about area, reiterate the strategies used in this lesson for comparing areas. Present an example: “Suppose we know the area of a small triangle. How can it help us find the area of a large triangle?”

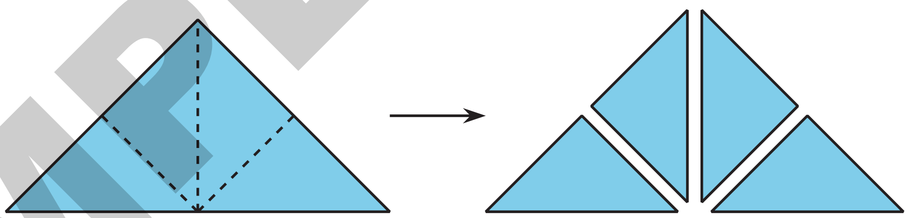


Demonstrate the following strategies (using tangram pieces, if possible):

- We can *compose* a large triangle from 4 small triangles. If we place a large triangle on top of the 4 small triangles and they match up exactly, we know that the area of the large triangle is equal to the combined area of 4 small triangles.



- We can *decompose* the large triangle into 4 small triangles. Again, we can reason that the area of one large triangle is equal to the combined area of 4 small triangles.



Give another example: “Suppose we don't know the area of a small triangle, but we do know the area of a square that is composed of 2 small triangles. How can we find the area of a large triangle?”

Demonstrate another strategy:

- We can decompose the large triangle into 4 small triangles and then *rearrange* them into 2 squares. We can reason that the area of the large triangle is equal to the combined area of 2 squares.

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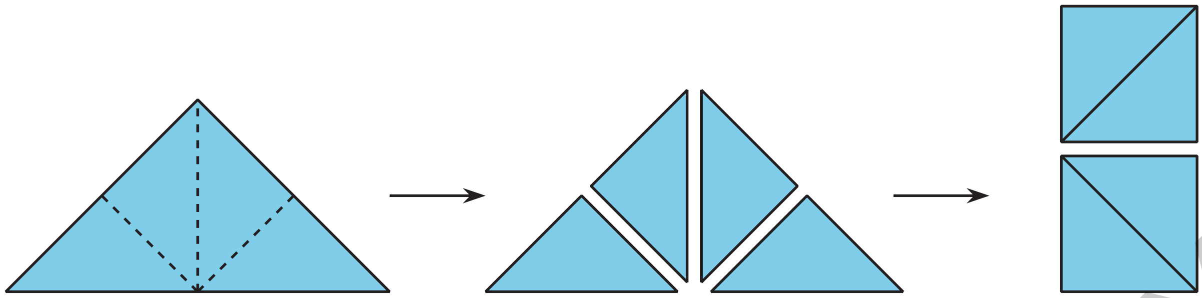

Sec A

Consider asking students:

- "Two small triangles can be arranged to match up exactly with a square. What does that tell us about their areas?" (The 2 triangles and the square have the same area.)
- "Two small triangles can also be arranged into a medium triangle. What does that tell us about the area of the medium triangle?" (It is the sum of the areas of the 2 small triangles. It is twice the area of a small triangle. It is the same as the area of a square.)

2.5 Tangram Rectangle

Cool-down

5 mins

Standards

Addressing **6.G.A.1**

Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "Is there anything that you would like to add to the student 'Doing Math' section of the chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use the themes to add to or revise the student section of the Math Community Chart before Exercise 3.

Give students access to the tangram shapes and geometry toolkits. Tell students that this figure is composed of two small right triangles, two medium right triangles, and a square, just like the ones they used earlier.

Note that students might not, at first, see the "square in the middle" as a square, or they might think of it a diamond (which would have unequal angles). Make sure that everyone understands that square-ness does not depend on how we turn the paper: A square is a rectangle (with all four angles being right angles) that has 4 equal sides.

Student Task Statement

The square in the middle has an area of 1 square unit. What is the area of the entire rectangle in square units? Explain your reasoning.

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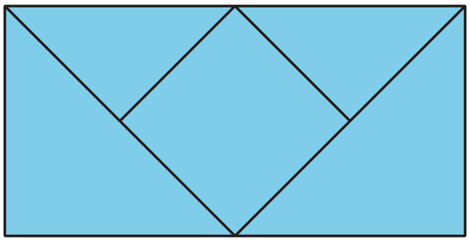

Student Response

4 square units. Sample reasoning:

- Put together the two small triangles to make a square. Its area is 1 square unit. Decompose each medium triangle into two small triangles that can be arranged as a square. Each of these squares has an area of 1 square unit. Together with the square in the middle, the sum of the areas of these pieces is 4 square units.
- A small triangle has an area of $\frac{1}{2}$ square unit, and a medium triangle has an area of 1 square unit. $1 + 1 + 1 + \frac{1}{2} + \frac{1}{2} = 4$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 2 Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.
2. We can *decompose* a figure (break a figure into pieces) and *rearrange* the pieces (move the pieces around) to find its area.

Here are illustrations of the two principles.

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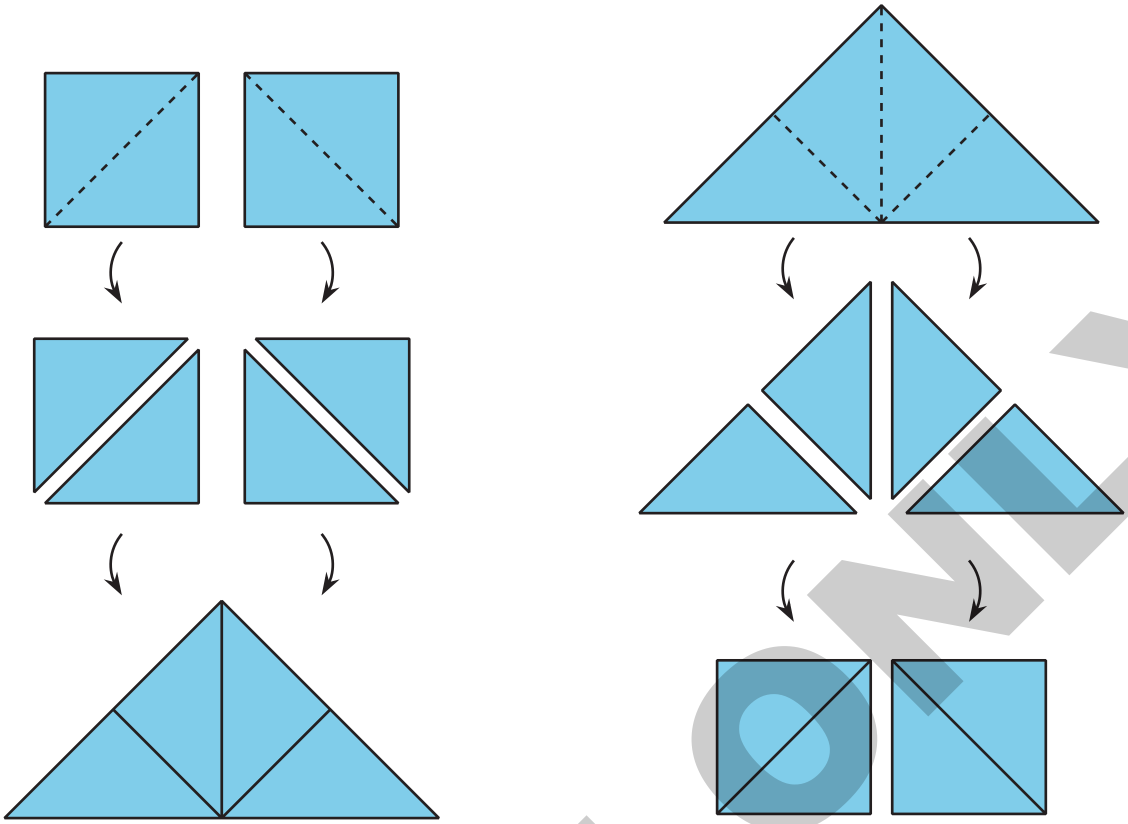

- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So, the large triangle has the *same area* as the 2 squares.
- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The triangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

Glossary

- area

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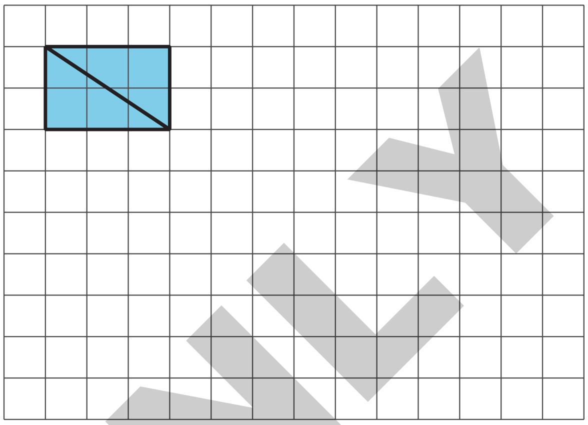
Practice Problems

1 Student Task Statement

The diagonal of a rectangle is shown.

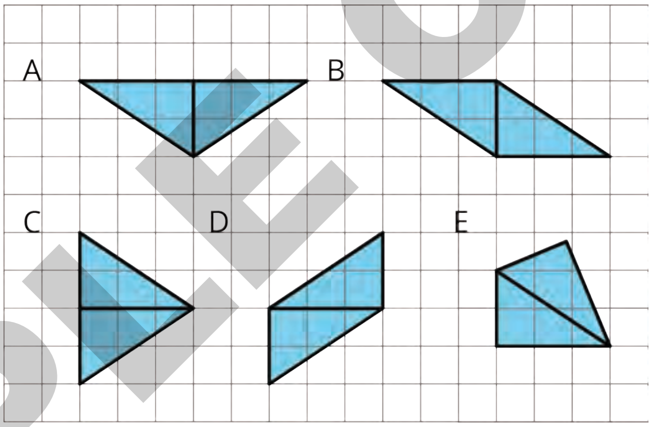


a. Decompose the rectangle along the diagonal, and recompose the two pieces to make a *different* shape.

b. How does the area of this new shape compare to the area of the original rectangle? Explain how you know.

Solution

a. Sample responses:



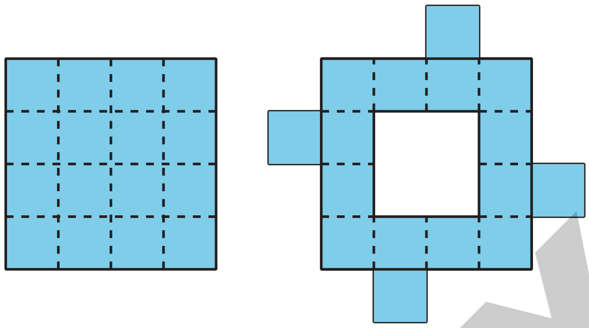
b. The areas are the same. Sample reasoning: All of the shapes are composed of two copies of the same triangle.

2 Student Task Statement

Priya decomposed a square into 16 smaller, equal-size squares and then cut out 4 of the small squares and attached them around the outside to make the new figure shown.

SAMPLE ONLY

How does the area of the new figure compare with that of the original square?



A. The area of the new figure is greater.

B. The two figures have the same area.

C. The area of the original square is greater.

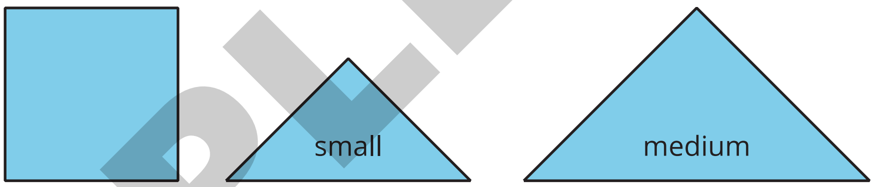
D. We don't know because neither the side length nor the area of the original square is known.

Solution

B

3 Student Task Statement

The area of the square is 1 square unit. Two small triangles can be put together to make a square or to make a medium triangle.



Which figure also has an area of $1\frac{1}{2}$ square units? Select **all** that apply.

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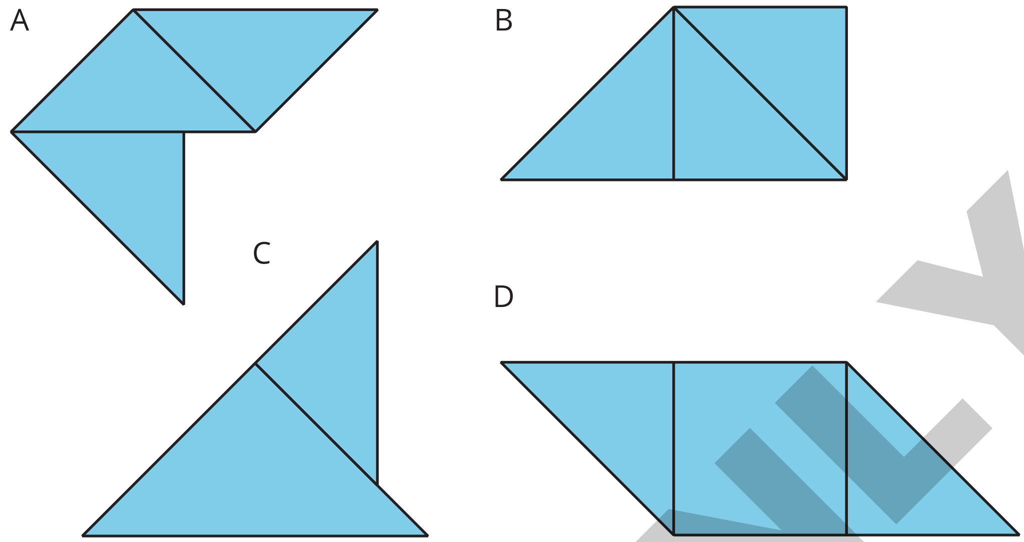

A. Figure A

B. Figure B

C. Figure C

D. Figure D

Solution

A, B, C

4 from an earlier course

Student Task Statement

The area of a rectangular playground is 78 square meters. If the length of the playground is 13 meters, what is its width?

Solution

6 meters

5 from Unit 1, Lesson 1

Student Task Statement

A student said, "We can't find the area of this shaded region because the shape has many different measurements, instead of just a length and a width that we could multiply."

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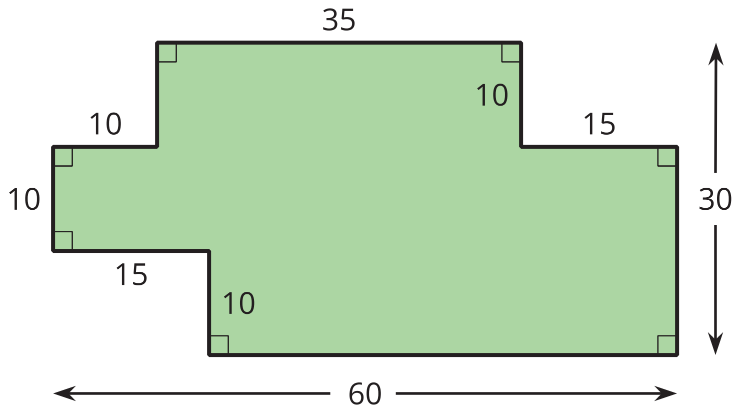

Explain why the student’s statement about area is incorrect.

Solution

Sample response: Area measures how many unit squares cover a region without gaps or overlaps. We multiply a length and a width when finding the area of a rectangle because that product tells us the number of unit squares in it. We can still find the area of a shape as shown, but first we will need to break it apart into rectangles whose areas we can find and then add them to find the total area. We can also enclose the 30-by-60 region with a rectangle, find its area, and subtract the areas of the unshaded portions.

SAMPLE ONLY

Reasoning to Find Area

Goals

- Compare and contrast (orally) different strategies for calculating the area of a polygon.
- Find the area of a polygon by decomposing, rearranging, subtracting or enclosing shapes, and explain (orally and in writing) the solution method.
- Include appropriate units (in spoken and written language) when stating the area of a polygon.

Learning Targets

- I can use different reasoning strategies to find the area of shapes.

Lesson Narrative

This lesson prompts students to find areas of figures—first on a grid and then without a grid. This work reiterates the two key principles about area: that two figures that match exactly when placed one on top of the other have the same area, and that the area of a figure is the sum of the areas of the non-overlapping pieces that compose it.

Students continue to use strategies from earlier explorations to find area, namely:

- Decompose a figure into shapes whose areas they can calculate.
- Decompose and rearrange it into shapes whose areas they can calculate.

The given figures in this lesson allow students to see that they can also:

- consider a figure as a familiar shape with one or more missing pieces, calculate the area of the shape, and then subtract the areas of the missing pieces
- enclose a figure with a shape whose area they can calculate and subtract the area of extra pieces created by the enclosure

For now, rectangles are the only shapes whose areas students know how to calculate, but the strategies will become more powerful as students' repertoires grow.

As students consider strategies for finding areas and use them, they practice looking for and making use of structure (MP7). In explaining their thinking, students practice constructing logical arguments (MP3).

A note about notation:

Starting in this lesson, consider using the "dot" notation instead of the "cross" notation when recording students' solutions. Explain that the $\cdot$ symbol and the $\times$ symbol both represent multiplication. Doing so familiarizes students with the use of the notation before they see it in student-facing materials.

Standards

Building On	**3.MD.C.7.d**
Addressing	**6.G.A.1**

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

SAMPLE ONLY

Required Materials

Materials To Gather

- Geometry toolkits: Lesson
- Geometry toolkits: Activity 1, Activity 2, Activity 3

Materials To Copy

- Comparing Regions Handout (1 copy for every 1 students): Activity 1

Required Preparation

Activity 1:

Prepare several copies of the pair of figures on the blackline master, in case students propose cutting them out to compare the areas.

Lesson:

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Student Facing Learning Goals

Let's decompose and rearrange shapes to find their areas.

SAMPLE ONLY

3.1 Comparing Regions

Warm-up

5 mins

Activity Narrative

This activity prompts students to use reasoning strategies from earlier lessons to compare the areas of two figures. It is also an opportunity to use (or introduce) tracing paper as a way to illustrate *decomposing* and *rearranging* a figure.

During the activity, look for students who are able to explain or show how they know that the areas are equal. Some students may simply look at the figures and say, with no justification, that they have the same area. Urge them to think of a way to show that their conclusion is true.

Standards

Building On **3.MD.C.7.d**

Launch

Give students access to their geometry toolkits, and allow for 2 minutes of quiet think time. Ask them to be ready to support their answer, and remind them to use the tools at their disposal. Have copies of the blackline master ready for students who propose cutting the figures out for comparison or as a way to differentiate the activity.

Student Task Statement

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.

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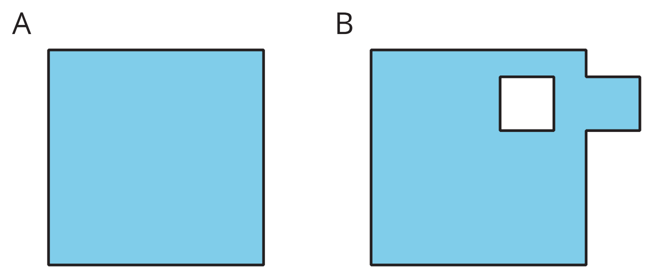

Student Response

The areas are equal. Sample reasoning:

- Measuring: Measure the side lengths of the small square-shaped hole and the small shaded square on the side of Figure B. They have the same side lengths, so their areas are equal. This means the square on the side fills the hole on the inside.. Measure the side lengths of the large shaded square in Figure A and then in Figure B. They have the same side lengths, so their areas are equal.
- Using scissors: Cut off the little square on the side of Figure B, and use it to fill the hole inside Figure B. The result is a square that matches up exactly with Figure A.
- Using tracing paper: Trace the boundary of the little square on the side of Figure B and move the tracing paper over the unshaded hole. Doing this shows that the little shaded square is exactly the same size as the hole. Movingthat little shaded square to fill the unshaded hole creates a big shaded square. If the boundary of that big shaded square is traced and the drawing is placed over Figure A, it would up exactly with Figure A.

Building on Student Thinking

Students may interpret the area of Figure B as the entire region inside the outer boundary including the unfilled square. Clarify that we want to compare the areas of only the shaded parts of Figure B and Figure A.

Activity Synthesis

Start the discussion by asking students to indicate which of the three possible responses—area of Figure A is greater, area of Figure B is greater, or the areas are equal—they choose.

Select previously identified students to share their explanations. If no student mentioned using tracing paper, demonstrate the following:

- *Decomposing and rearranging Figure B*: Place a piece of tracing paper over Figure B. Draw the boundary of the small side square, making a dotted auxiliary line to show its separation from the large square. Move the tracing paper so that the boundary of the small square matches up exactly with the boundary of the square-shaped hole in Figure B. Draw the boundary of the large square. Explain that the small square matches up exactly with the hole, so we know the small, shaded square and the hole have equal area.
- *Matching the two figures*: Move the tracing paper over Figure A so that the boundary of the rearranged Figure B matches up exactly with that of Figure A. Say, “When two figures that are overlaid one on top of another match up exactly, their areas are equal.”

Highlight the strategies and principles that are central to this unit. Tell students, “We just decomposed and rearranged Figure B so that it matches up exactly with Figure A. When two figures that are overlaid one on top of another match up exactly, we can say that their areas are equal.”

SAMPLE ONLY

3.2 On the Grid

20 mins

Activity Narrative

This activity elicits different strategies for reasoning about finding the area of regions:

- decomposing
- decomposing and rearranging
- subtracting
- enclosing and subtracting

The figures are on a grid, which reinforces the meaning of area and supports students in quantifying the square units. Students may start by counting squares, as they had done in earlier grades, but the figures have been chosen to encourage other approaches.

Monitor for students who use at least two different strategies for finding the area of each figure (one strategy as shown in the student responses and at least one other). Here are some approaches students might take for each figure:

- Figure A can be easily decomposed into rectangles.
- Figure B can be decomposed into rectangles. Or, more efficiently, it can be seen as a square with a missing piece. The area of the unshaded inner square can be subtracted from the area of the larger square.
- Figure C can also be seen as having a missing piece, but subtracting the area of the unshaded shape does not work because the side lengths of the inner square are unknown. Instead, the shaded triangles can be decomposed and rearranged into rectangles.
- Figure D can be decomposed and rearranged into rectangles. It can also be viewed as the inner square of Figure C.

Standards

Addressing **6.G.A.1**

Instructional Routines

- 5 Practices

Launch

Tell students that they will find the areas of various figures on a grid. To encourage students to use a more grade-appropriate strategy for finding areas, first show them a strategy from earlier grades. As a class, find the area of Figure A by counting the squares aloud, one by one. Confirm that there are 24 square units. Then ask students to think about other ways to find the area of Figure A (or other figures) besides counting each square.

Arrange students in groups of 2. Ask one partner to start with Figures A and C, and the other with B and D. Give students 4–5 minutes of quiet think time and access to their geometry toolkits. Then give them a few minutes to share their responses with their partner. Emphasize that as one partner explains, the other should listen carefully and see if they agree or disagree with the answer and explanations.

For each figure, select at least 2 students with different strategies, such as those described in the Activity Narrative, to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every

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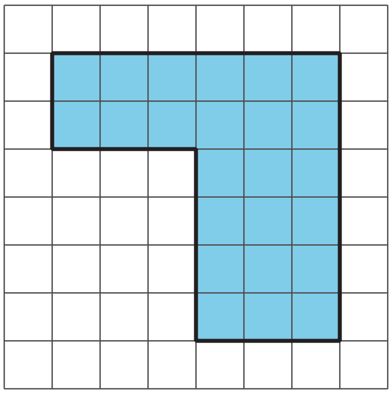
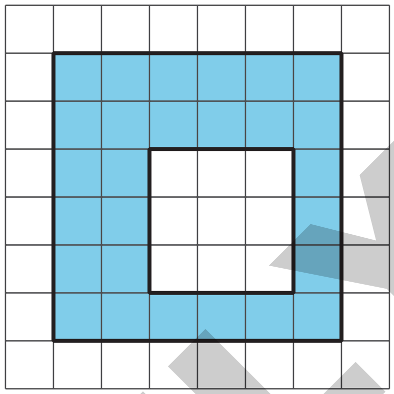
square. Be prepared to explain your reasoning.

A



B



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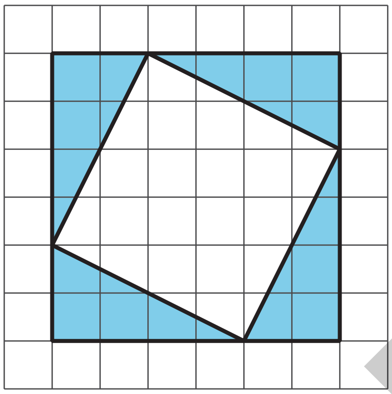
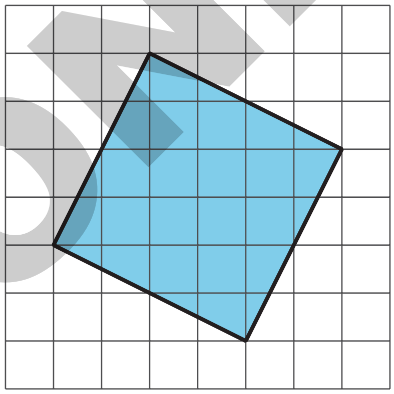
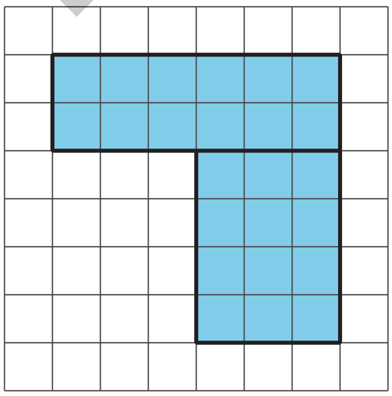


D



Student Response

A: 24 square units. Sample reasoning: Decompose the figure into rectangles. One way is shown here.
$(2 \cdot 6) + (4 \cdot 3) = 24$



B: 27 square units. Sample reasoning:

- Decompose the figure into four rectangles.
- Subtract the area of the inner square from the larger square. $(6 \cdot 6) - (3 \cdot 3) = 27$

C: 16 square units. Sample reasoning:

- Decompose into right triangles and rearrange into rectangles. $(2 \cdot 4) + (4 \cdot 2) = 16$

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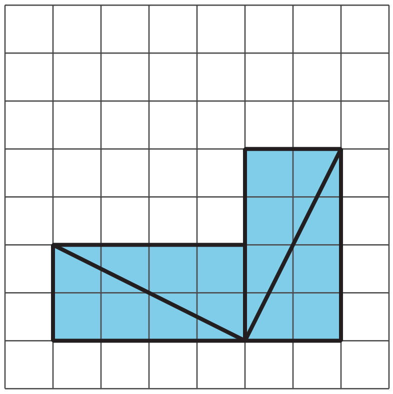
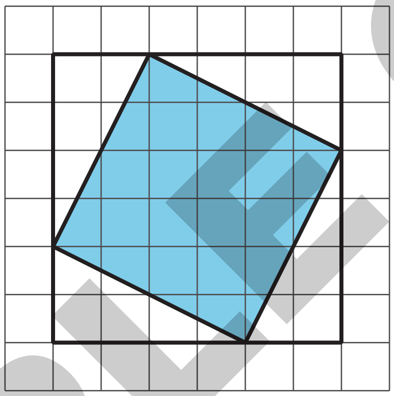


- Find the area of Figure D first, and then subtract it from the 6-by-6 square.

D: 20 square units. Sample reasoning:

- Decompose the shaded square into four right triangles and a 2-by-2 square. (See *Student Response for Are You Ready for More?*) Rearrange the right triangles into two rectangles that are each 2 units by 4 units, with a combined area of 16 square units. Adding the area of the small square (4 square units) gives a total of 20 square units.
- Notice that the shaded square is the inner square of Figure C, enclose it in a square as in Figure C, and subtract the areas of the four right triangles (or the area of Figure C) from the area of the enclosing square. $(6 \cdot 6) - 16 = 20$



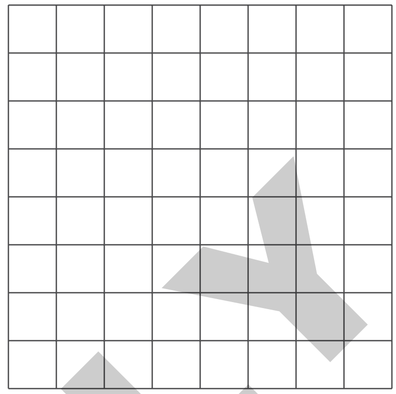
Building on Student Thinking

Some students may count both complete and partial grid squares instead of looking for ways to decompose and rearrange larger shapes. Ask them if they can think of a way to find the area by decomposing and rearranging larger pieces. The discussion at the end, during which everyone sees a variety of strategies, is especially important for these students.

SAMPLE ONLY

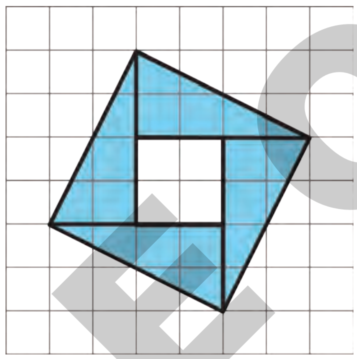
Are You Ready for More?

Rearrange the shaded triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.



Extension Student Response

Sample response: The triangles fit inside the square, with a smaller 2-by-2 square in the center.



Activity Synthesis

The purpose of this discussion is to highlight a few key strategies for finding area. Invite previously selected students to share their work. Before sharing begins, ask students to notice similarities and differences in the strategies and be ready to explain them. Sequence the discussion of the strategies for each figure as shown here, starting with decomposing.

- Figure A:
    - decomposing
    - enclosing and subtracting
- Figure B:
    - decomposing
    - subtracting
- Figure C:
    - decomposing and rearranging
    - subtracting (after finding the area of the inner square, which is Figure D)
- Figure D:
    - decomposing and rearranging

SAMPLE ONLY

◦ enclosing and subtracting

As students share their strategies, record and display their moves on each figure for all to see (or ask the presenters to do so).

After each student shares, name the strategy and ask if anyone else reasoned the same way. If one of these strategies does not appear in students' work, illustrate it for the class.

Connect the different responses to the learning goals by asking questions such as:

- "How are the strategies used to find the areas of figures A and D alike?" (We can find the areas by decomposing the figures.)
- "How are the strategies used to find the areas of figures A and D different?" (After decomposing A, we can find the area of each piece as they are. After decomposing D, we need to rearrange the pieces before finding their areas.)
- "After decomposing figures C and D, what shapes were the pieces rearranged into?" (rectangles) "Why might that be?" (We know how to find the area of rectangles.)

Highlight that there are multiple ways to find the area of each figure and that decomposing, with or without rearranging, is generally a useful strategy.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, create a display that includes multiple copies of each figure. As students describe their strategies, use color and annotation to scribe their thinking so that it is visible for all students. Label each figure with the strategy described (decomposing, rearranging, subtracting, or enclosing).
Supports accessibility for: Visual-Spatial Processing

3.3 Off the Grid

10 mins

Activity Narrative

In this activity, students apply the strategies they have learned to find the areas of figures, but now the figures are *not* on a grid.

- Figure E can easily be decomposed and rearranged into a rectangle.
- Figure F can be decomposed and rearranged into rectangles (as was done with Figure C in the previous activity). Students cannot use the strategy of subtracting the area of the inner square from that of the outer square because the side lengths of the inner square are unknown.

As students discuss their approaches in groups, support students in naming the strategies and by asking clarifying questions. Monitor for students who observed that the same strategies for reasoning about area can be applied both on and off the grid.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

SAMPLE ONLY

Launch

Tell students that they will now find areas of figures that are not on a grid. Give students access to their geometry toolkits. Remind students that if the side lengths of a rectangle are given in a particular unit (such as meters), then the area is given in square units (such as square meters).

Allow for 3–4 minutes of quiet time to find the areas of the two figures. Then arrange students into groups of 4 and give them 2–3 minutes to compare their responses and strategies. Ask students to discuss the following questions, displayed for all to see:

- "What units did you use for each area? Why?"
- "How are your strategies the same? How are they different?"
- "Which strategies are similar to the ones you used when finding the areas of figures on a grid?"

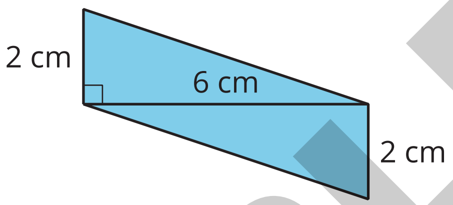
Access for English Language Learners

MLR8 Discussion Supports. Invite students to repeat their reasoning using mathematical language: "Can you say that again, using the words 'compose,' 'decompose,' or 'rearrange' in your explanation?"
Advances: Speaking, Representing

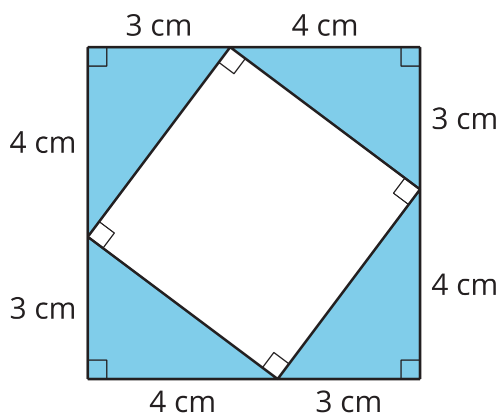
Student Task Statement

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

E



F



Student Response

Figure E: 12 square centimeters. Sample reasoning: Decompose the two triangles and rearrange them to form a rectangle with side lengths of 6 centimeters and 2 centimeters.

Figure F: 24 square centimeters. Sample reasoning: Decompose the triangles and rearrange them to form two rectangles with side lengths of 4 centimeters and 3 centimeters.

Building on Student Thinking

For Figure F, students may estimate the side lengths of the inner square so that its area could be subtracted from that of the outer square. They may struggle to see how the triangles could be rearranged. Suggest that they use tracing paper to help them in their thinking.

SAMPLE ONLY

Students might not be familiar with the symbols that indicate right angles and might think these symbols indicate square units. Remind them that those symbols indicate 90 degree angles.

Activity Synthesis

Briefly discuss the question, "Which strategies are similar to the ones you used in an earlier activity to find the areas of figures on a grid?"

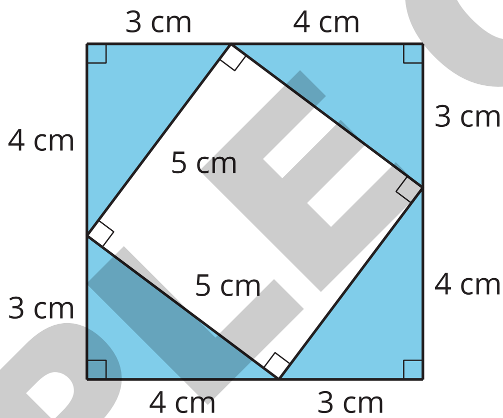
Select 1–2 previously identified students to share their observations—that in both activities they decomposed and rearranged figures to find their area. (Some students may have also enclosed Figure A and subtracted the areas of unshaded triangles after rearranging them into a rectangle.)

Emphasize that these strategies for finding area can be used whether the measurements are indicated by a grid or given directly (without a grid).

If students did not include appropriate area units for figures E and F, remind them that the side lengths of these figures are given in centimeters, so their areas are in square centimeters.

If time permits, ask students:

- "For Figure F, can we use the strategy of subtracting the area of the inner square to find the area of the shaded region? Why or why not?" (No, because we don't know the side lengths of the inner square or its area.)
- Display or sketch Figure F with the sides of the inner square labeled "5 cm."



"Can we use subtraction to find the area of the shaded region now? Why or why not? (Yes, because we know the side lengths of the outer and inner squares. We can find the area of the smaller square, 25 square centimeters, and subtract it from the area of the larger square, 49 square centimeters, which gives 24 square centimeters.)

Lesson Synthesis

This lesson was all about identifying strategies for finding area and applying them to various figures. Students reasoned about the area of a figure on and off a grid by:

- decomposing it into familiar shapes
- decomposing it and rearranging the pieces into familiar shapes
- considering it as a shape with missing pieces and subtracting the areas of the missing pieces from the area of the shape
- enclosing a figure with a shape whose area can be calculated and subtracting the area of extra pieces created by

SAMPLE ONLY

the enclosure

Ask students to go back through this lesson's activities and find problems in which each of these strategies was used.

Tell students we will have many more opportunities to use these strategies in upcoming lessons.

3.4 Maritime Flag

Cool-down

5 mins

This task does not explicitly ask students to state area units because one purpose of the task is to assess if students understand what units are appropriate given the information presented.

Standards

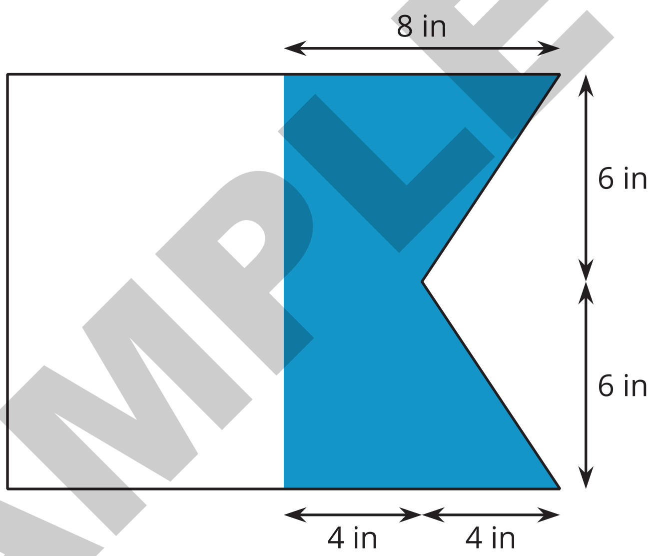
Addressing **6.G.A.1**

Launch

Give students access to their geometry toolkits.

Student Task Statement

A maritime flag is shown. What is the area of the shaded part of the flag? Explain or show your reasoning.



Student Response

72 square inches. Sample reasoning: If we draw a line down the middle of the shaded area, we would have a 4 inch-by-12 inch rectangle on the left and two right triangles. The 4-by-12 rectangle has an area of 48 square inches. The two triangles on the right can be composed into a 4 inch-by-6 inch rectangle, so their combined area is 24 square inches. $48 + 24 = 72$

SAMPLE ONLY

Responding To Student Thinking

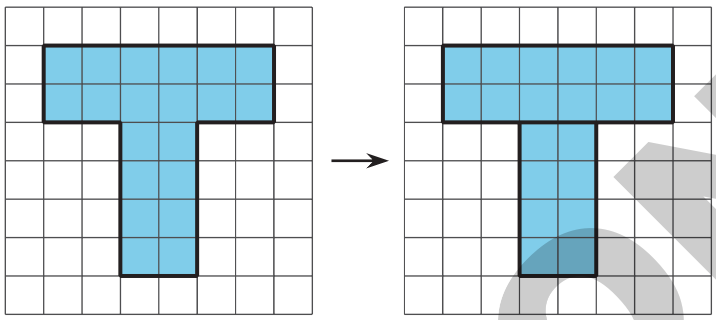
More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

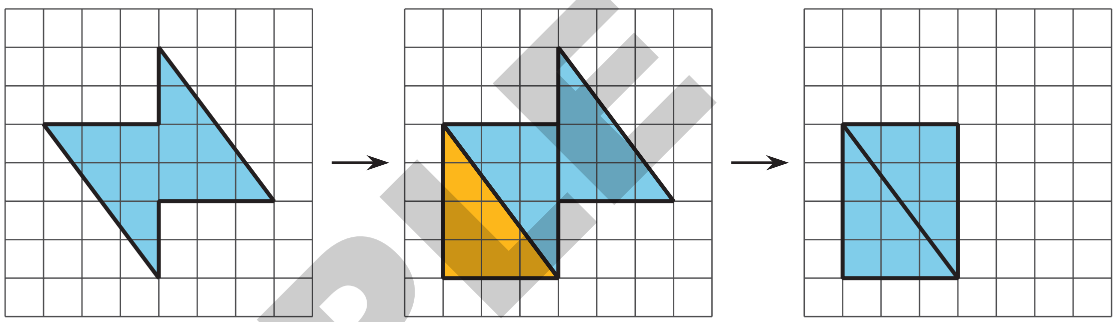
Lesson 3 Summary

There are different strategies we can use to find the area of a region. We can:

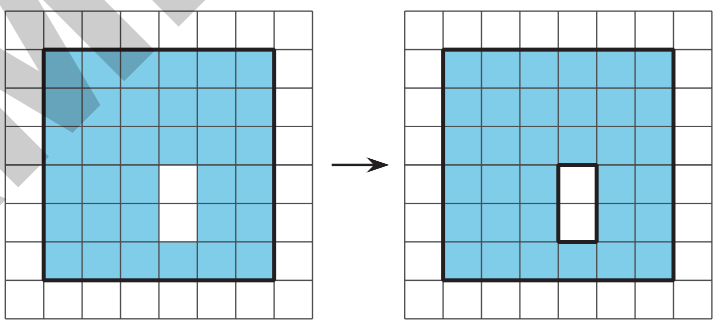
- Decompose it into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.



- Decompose it and rearrange the pieces into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.



- Consider it as a shape with a missing piece. We calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



The area of a figure is always measured in square units.

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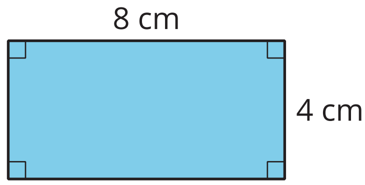
When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters. For example, the area of this rectangle is 32 square centimeters.



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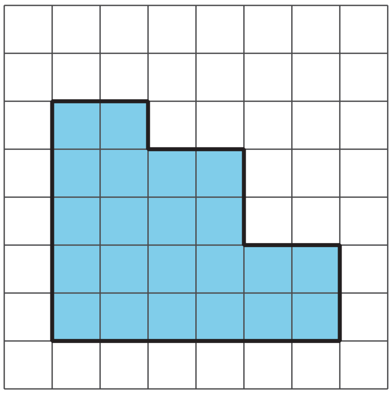
Practice Problems

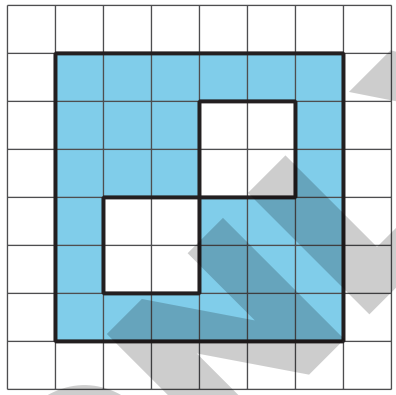
1 Student Task Statement

Find the area of each shaded region. Show your reasoning.

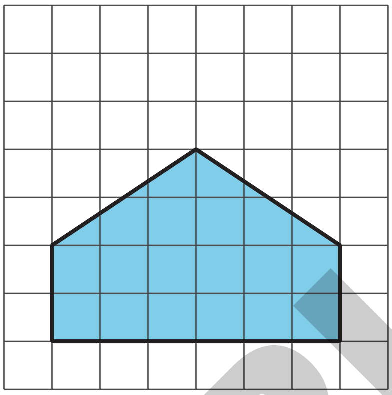
A



B



C



Solution

A: 22 square units. Sample reasoning: The shaded region can be partitioned into rectangles. One way to do this is shown. Rectangle 1 is 2 units by 5 units, so its area is 10 square units. Rectangle 2 is 2 units by 4 units, so its area is 8 square units. The area of Rectangle 3 is 4 square units. The total shaded area is 22 square units, since $10 + 8 + 4 = 22$.

B: 28 square units. Sample reasoning: The outer square is 6 units by 6 units, so its area is 36 square units. There are two smaller squares inside. Square 1 and Square 2 have been removed. Each small square has an area of 4 square units. To get the shaded area, compute $36 - 4 - 4$, which equals 28.

C: 18 square units. Sample reasoning: The region can be decomposed to form a 2-by-6 rectangle and two right triangles that when rearranged form a 2-by-3 rectangle. $(2 \cdot 6) + (2 \cdot 3) = 18$.

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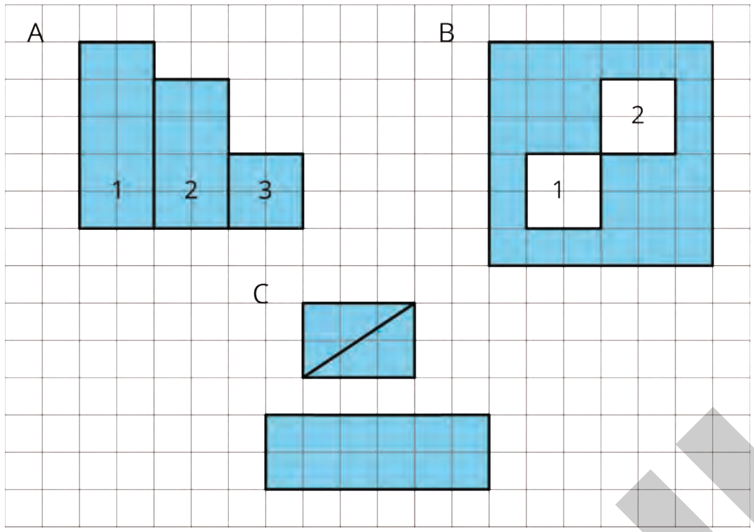
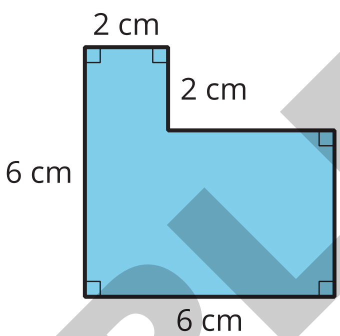


Sec A

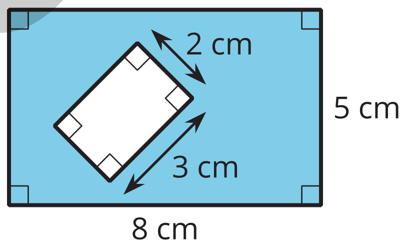
2 Student Task Statement

Find the area of each shaded region. Show or explain your reasoning.

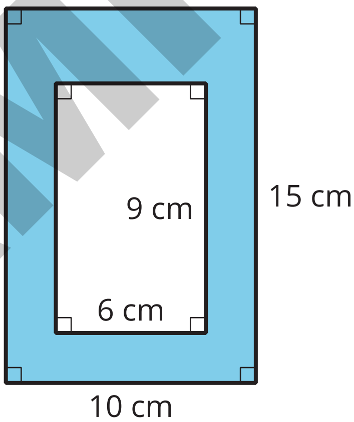
A



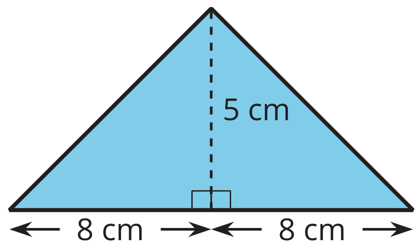
B



C



D



SAMPLE ONLY

Solution

A: 28 sq cm. Sample reasoning: A horizontal cut partitions the region into a 2 cm-by-2 cm square (4 sq cm) and a 4 cm-by-6 cm rectangle (24 sq cm).

B: 34 sq cm. Sample reasoning: The outer rectangle has an area of 40 sq cm, while the inner rectangle has an area of 6 sq cm. $40 - 6 = 34$

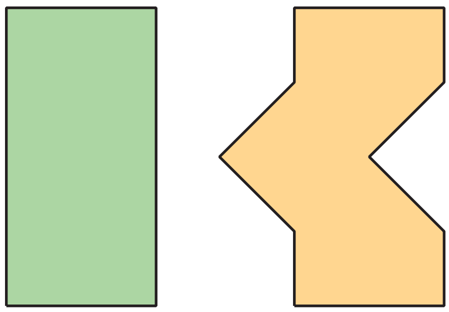
C: 96 sq cm. Sample reasoning: The outer rectangle has an area of 150 sq cm, while the inner rectangle has an area of 54 sq cm. $150 - 54 = 96$

D: 40 sq cm. Sample reasoning: The two right triangles can be put together to make a 5 cm-by-8 cm rectangle.

3

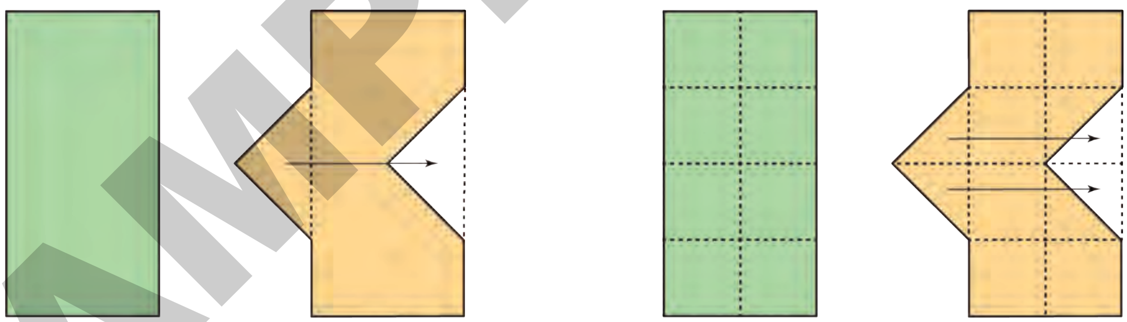
Student Task Statement

Two plots of land have very different shapes. Noah said that both plots of land have the same area. Do you agree with Noah? Explain your reasoning.


plot A plot B

Solution

Agree. Sample reasoning: The triangular shape that juts out from the left side of plot B can be cut off and moved to the right side of plot B. The resulting shape is a rectangle that matches exactly with the shape of plot A. We can use tracing paper to verify. Sample diagrams:


plot A plot B plot A plot B

SAMPLE ONLY

4 from Unit 1, Lesson 2

Student Task Statement

A homeowner is deciding on one size of tiles to use to fully tile a rectangular wall in her bathroom that is 80

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inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches.

Tell whether or not you agree with each statement about the tiles. Explain your reasoning.

a. Regardless of the size she chooses, she will need the same number of tiles.

b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.

c. She will need two 2-inch tiles to cover the same area as one 4-inch tile.

d. She will need four 4-inch tiles to cover the same area as one 8-inch tile.

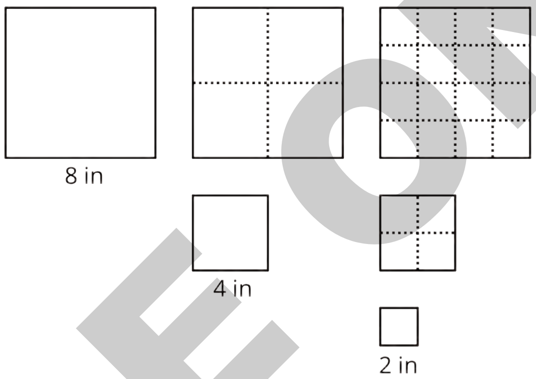
e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

Solution

a. Disagree. Sample reasoning: She will need fewer of the larger tiles and more of the smaller tiles.

b. Agree. Sample reasoning: The region being covered does not change regardless of what tiles she chooses.

c. Disagree. Sample reasoning: She will need four 2-inch tiles to cover the same area as one 4-inch tile.



d. Agree. Sample reasoning: Two rows of two 4-inch tiles cover the same area as one 8-inch tile.

e. Disagree. Sample reasoning: Because one 8-inch tile covers the same area as four 4-inch tiles, she will need $\frac{1}{16}$ as many 8-inch tiles as she would with 2-inch tiles.

5 from an earlier course

Student Task Statement

Find the area of the rectangle with each set of side lengths.

a. 5 in and $\frac{1}{3}$ in

b. 5 in and $\frac{4}{3}$ in

c. $\frac{5}{2}$ in and $\frac{4}{3}$ in

d. $\frac{7}{6}$ in and $\frac{6}{7}$ in

Solution

a. $\frac{5}{3}$ square inches

SAMPLE ONLY

b. $\frac{20}{3}$ square inches

c. $\frac{10}{3}$ square inches

d. 1 square inch

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Section B: Parallelograms

Goals

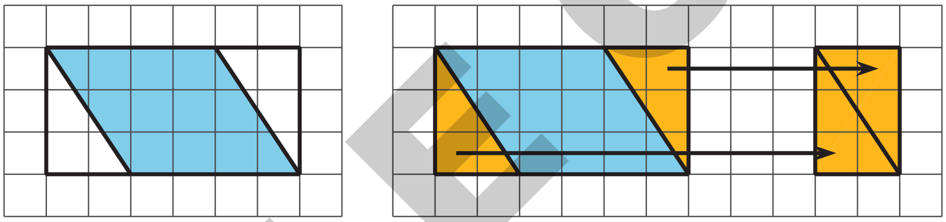
- Apply the formula for area of a parallelogram to find the area, the length of a base, or the height.
- Understand bases and heights in a parallelogram and recognize base-height pairs to use to calculate its area.

Section Narrative

In this section, students reason about areas of parallelograms. They learn about bases and heights and analyze the measurements that can be used to find the area of any parallelogram.

First, students use strategies they learned earlier in the unit to find the areas of given parallelograms. They see that one way to find the area of a parallelogram is by decomposing it and rearranging the pieces into a rectangle. Another way is to enclose it in a rectangle and subtract the areas of the extra pieces.

Along the way, students notice regularity in both the process of finding area (that it can be done using one or more related rectangles) and in the measurements that are useful for finding area (that they are side lengths of the related rectangles). Students make sense of these lengths as "bases" and "heights" of a parallelogram and learn to identify them. Then, students generalize the process of using bases and heights to find areas, express it as a formula, and use the formula to find the area of any parallelogram and to solve problems.



Teacher Reflection Questions

- **Math Content and Student Thinking:** What did you learn about students' understanding of area as they reasoned about areas of parallelograms in this section? What connections did they make between the area of a parallelogram and the area of a rectangle?
- **Pedagogy:** Reflect on how comfortable your students are asking questions to you and to each other. What can you do to encourage students to ask questions?
- **Access and Equity:** In Lesson 6, Activity 2: More Areas of Parallelograms, some sentence frames are offered as a support to encourage peer interaction and discourse. Which of your students could this accommodation support and in what ways?

SAMPLE ONLY

Section B Checkpoint

1

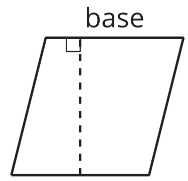
Goals Assessed

- Understand bases and heights in a parallelogram and recognize base-height pairs to use to calculate its area.

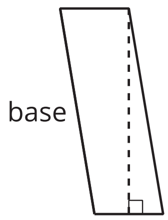
Student Task Statement

Select all parallelograms that show a base and its corresponding height (as a dashed segment).

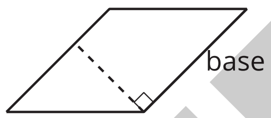
A.



B.



C.



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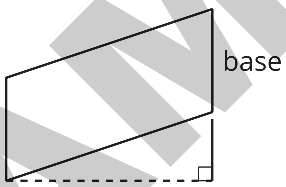
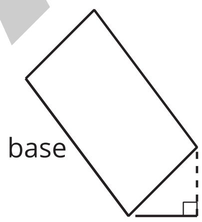


E.



SAMPLE ONLY

Solution

A, C, D

Responding To Student Thinking

Points to Emphasize
If students struggle with finding base-height pairs for parallelograms, highlight these attributes when opportunities arise over the next several lessons. For example, urge students identify at least one base and a corresponding height for each parallelogram they compose in this activity:

Grade 6, Unit 1, Lesson 7, Activity 3 A Tale of Two Triangles (Part 2)
Press Pause
If most students struggle with identifying base-height pairs in triangles and parallelograms, make time to examine related work in the referenced section. The Course Guide provides additional ideas for revisiting earlier work.

Grade 6, Unit 1, Section C Triangles and Other Polygons

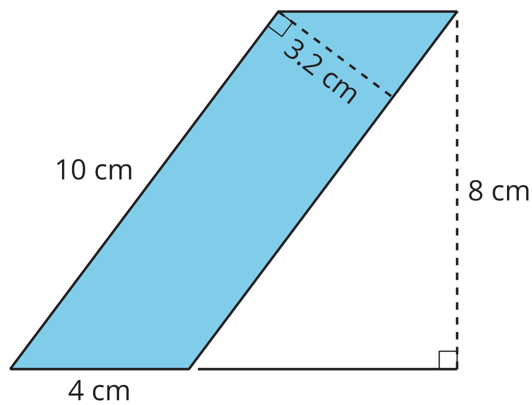
2

Goals Assessed

- Apply the formula for area of a parallelogram to find the area, the length of a base, or the height.
- Understand bases and heights in a parallelogram and recognize base-height pairs to use to calculate its area.

Student Task Statement

Find the area of the parallelogram. Explain or show your reasoning.



Solution

32 sq cm. Sample reasoning:

- If the side that is 4 cm is the base, then its corresponding height is 8 cm. $4 \cdot 8 = 32$
- If the side that is 10 cm is the base, then its corresponding height is 3.2 cm. $10 \cdot (3.2) = 32$
- The parallelogram can be decomposed along the dashed line that is 3.2 cm long. Rearranging the pieces

SAMPLE ONLY

makes a rectangle that is 10 cm by 3.2 cm. $10 \cdot (3.2) = 32$

Responding To Student Thinking

Points to Emphasize
If students struggle with finding the area of a parallelogram, discuss ways of doing so when opportunities arise over the next several lessons. For example, ask students to explain to a partner how to find the area of one of the parallelograms in this activity:

Grade 6, Unit 1, Lesson 8, Activity 1 Composing Parallelograms
Press Pause
If most students struggle with finding the area of a triangle using given base-height pairs or by identifying those measurements first, make time to revisit these concepts. For example, plan to do the referenced optional activity about finding a base and a corresponding height that would facilitate area calculation. The Course Guide provides additional ideas for revisiting earlier work.

Grade 6, Unit 1, Lesson 10, Activity 3 Some Bases Are Better Than Others

3

Goals Assessed

- Apply the formula for area of a parallelogram to find the area, the length of a base, or the height.

Student Task Statement

A parallelogram has an area of 60 square inches and a base that is 5 inches long. How long is the corresponding height?

Solution

12 inches

Responding To Student Thinking

Points to Emphasize
If students struggle with finding un unknown base or height of a parallelogram when the area is known, emphasize the relationship between those quantities when opportunities arise over the next several lessons. For example, highlight the connections between finding area and finding a missing base or height (given two other measurements) in this practice problem:

Grade 6, Unit 1, Lesson 8, Practice Problem 5
Points to Emphasize
If students struggle to find the area of polygons, discuss ways to decompose polygons into triangles and parallelograms when opportunities arise in the next section. For example, provide access to colored pencils and ask students to color code the decomposed regions as they work on these practice problems:

Grade 6, Unit 1, Lesson 13, Practice Problem 6

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Parallelograms

Goals

- Compare and contrast (orally) different strategies for determining the area of a parallelogram.
- Describe (orally and in writing) observations about the opposite side and opposite angles of parallelograms.
- Explain (orally and in writing) how to find the area of a parallelogram by rearranging its parts or by enclosing it in a rectangle.

Learning Targets

- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
- I know how to describe the characteristics of a parallelogram using mathematical vocabulary.

Lesson Narrative

In this lesson, students recall the defining attributes of parallelograms and other properties that follow from that definition. Students then use reasoning strategies from earlier work to find areas of parallelograms.

One approach for finding the area of a parallelogram is to decompose the parallelogram and rearrange the pieces into a rectangle. Another is to enclose the parallelogram in a rectangle of the same height and then subtract the area of the extra regions—two right triangles that can be rearranged into a rectangle.

By working with various parallelograms, students begin to see that the shape of certain parallelograms may encourage the use of certain strategies. For instance, a parallelogram that is narrow and stretched out may be cumbersome to decompose and rearrange. Enclosing it in a rectangle and subtracting the areas of the two extra pieces might be preferable.

Through repeated reasoning, students begin to see regularity (MP8): parallelograms have related rectangles that can be used to find their area. Students also describe the process of finding the area of a parallelogram more generally, which prepares them to express that process as a formula.

A *note about notation*:

When recording students' solutions and reasoning in this lesson, consider using the "dot" notation instead of the "cross" notation to indicate multiplication. Explain that the symbol and the symbol both represent multiplication. Doing so familiarizes students with the use of the notation before they see it in student-facing materials.

Standards

Building On	**4.G.A.2, 5.G.B**
Addressing	**6.G.A.1**

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials To Gather

- Geometry toolkits: Activity 1, Activity 2, Activity 3

Materials To Copy

- Area of a Parallelogram Cutouts (1 copy for every 1 students): Activity 2

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Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's investigate the characteristics and area of parallelograms.

4.1 What are Parallelograms?

Warm-up

5 mins

Activity Narrative

In this activity, students examine examples and non-examples of parallelograms and identify their defining characteristics. Students recall that a parallelogram is a quadrilateral whose opposite sides are parallel. They observe other properties that follow from that definition—that opposite sides of a parallelogram have the same length and opposite angles have the same measure.

Standards

Building On **4.G.A.2, 5.G.B**

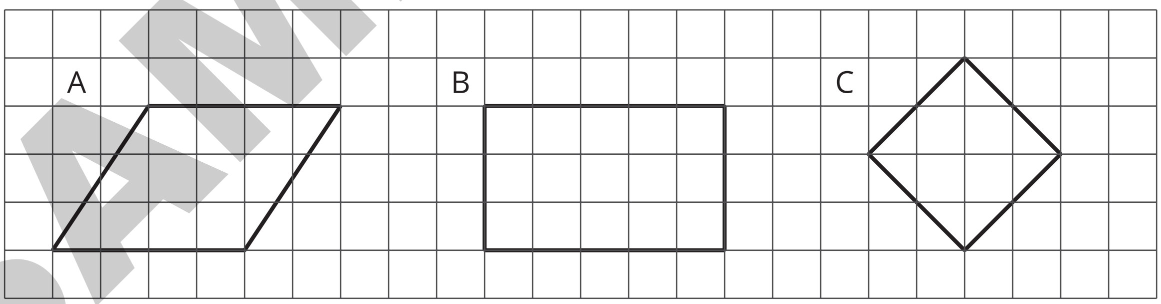
Launch

Display the images of Figures A–F for all to see. Tell students that Figures A, B, and C are parallelograms and Figures D, E, and F are not parallelograms.

Arrange students into groups of 2 and provide access to geometry toolkits. Give students 1–2 minutes of quiet think time to complete the task. Afterward, give them a minute to discuss their answers and observations with their partner.

Student Task Statement

Figures A, B, and C are *parallelograms*.



Figures D, E, and F are not parallelograms.

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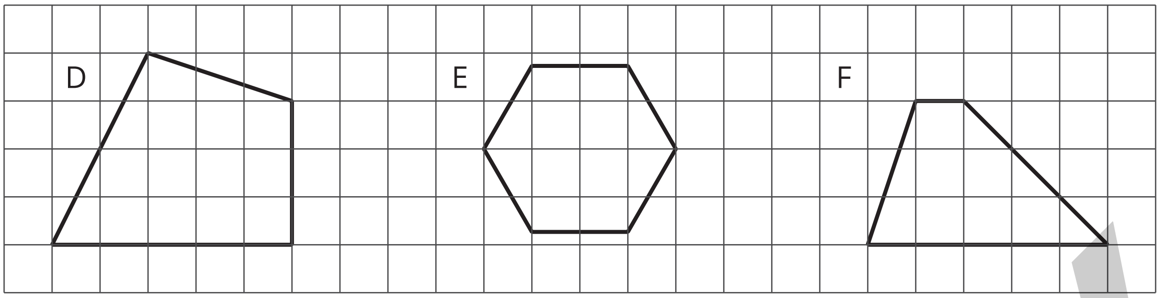

What do you notice about:

1. The number of sides that a parallelogram has?
2. Opposite sides of a parallelogram?
3. Opposite angles of a parallelogram?

Student Response

1. Parallelograms have four sides.
2. Opposite sides of parallelograms are parallel and have the same length.
3. Opposite angles of parallelograms have the same measure.

Building on Student Thinking

Students may not realize that Figure C isa square or relate Figure C to the other parallelograms because of its orientation. Encourage students to use patty paper or another tool in the geometry toolkit to help them compare the characteristics of Figure C to those of Figures A and B.

Activity Synthesis

Ask a few students to share their responses to the questions. After each response, ask students to indicate whether they agree. If a student disagrees, discuss the disagreement. Record the agreed-upon responses for all to see and highlight these characteristics of *parallelograms*:

- A parallelogram is a quadrilateral (a polygon with four sides).
- Both pairs of its opposite sides are parallel.
- Its opposite sides have the same length.
- Its opposite angles have the same measure.

Students may wonder how to know if two non-horizontal or non-vertical sides of a figure are parallel. Explain that because parallel lines never intersect, the length of any perpendicular line segments between them are the same length. Consider demonstrating how to use an index card to check this in Figures A and C.

Tell students that for now we will just take these characteristics of parallelograms as facts. Later they will learn some ways to prove that these characteristics are always true.

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15 mins

4.2 Area of a Parallelogram

Activity Narrative

There is a digital version of this activity.

In this activity, students explore different methods of decomposing a parallelogram and rearranging the pieces to find its area. Here are two key approaches for finding the area of parallelograms:

- Decompose the parallelogram, rearrange the parts into a rectangle, and multiply the length of the base by the length of a side to find the area.
- Enclose the parallelogram in a rectangle and subtract the combined area of the extra regions.

Presenting the parallelograms on a grid makes it easier for students to see that the area does not change as they decompose and rearrange the pieces. This investigation lays a foundation for later reasoning about the area of triangles and other polygons.

Monitor for students who use the two key approaches and whose reasoning can highlight the usefulness of using a related rectangle to find the area of a parallelogram.

Some students may begin by counting squares but that strategy is not reinforced here. Encourage students to listen for and try more sophisticated, grade-appropriate methods shared during the class discussion.

In the digital version of the activity, students use an applet with some given rectangles and right triangles to visualize their reasoning (decomposition, rearrangement, and enclosure). The digital version may be helpful for visualizing the rearrangement of pieces to form rectangles.

Standards

Addressing **6.G.A.1**

Launch

Arrange students in groups of 2–4. Ask students to find the area of the parallelograms using recently learned strategies and tools. A blackline master with a larger version of the parallelograms is provided. Make copies of the blackline master available in case students wish to reason by cutting the parallelograms.

Give students 5 minutes of quiet think time and access to their toolkits. Ask them to share their strategies with their group afterward.

To encourage students to be mindful of their strategies and to create the foundation for the whole-class discussion, display and read aloud the following reflection questions before students begin working.

- "Why did you decompose the parallelogram the way you did?"
- "Why did you rearrange the pieces the way you did?"

Student Task Statement

Find the area of each parallelogram. Show your reasoning.

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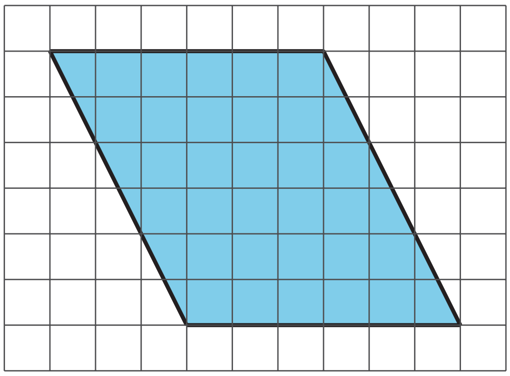
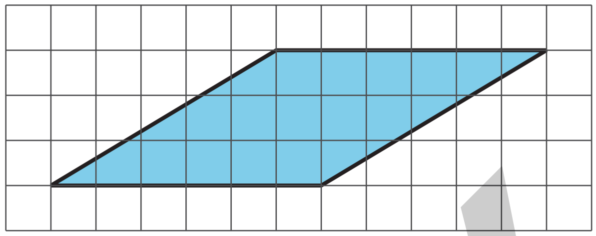
1.

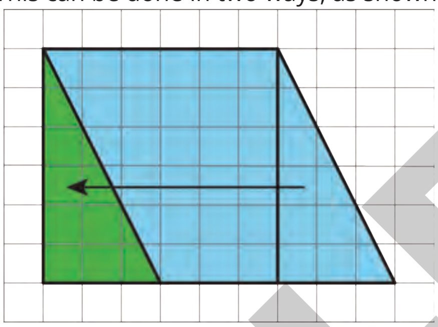


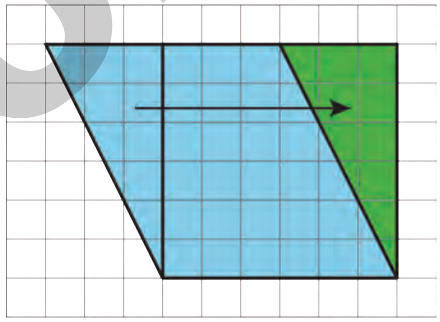
2.



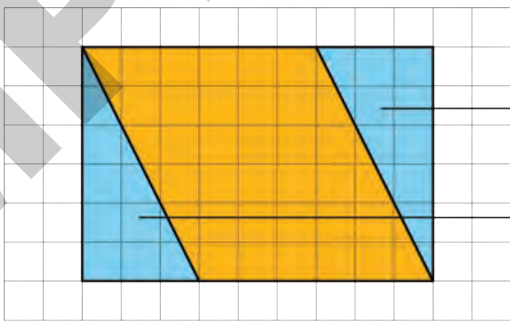
Student Response

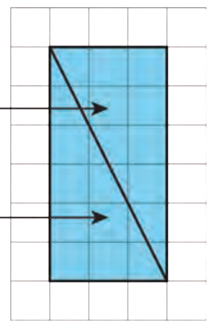
1. 36 square units. Sample reasoning:
   - Decompose the parallelogram to create a right triangle and move it to form a rectangle that is 6 units by 6 units. (This can be done in two ways, as shown.) $6 \cdot 6 = 36$





   - Enclose the parallelogram within a rectangle and subtract the extra pieces. To subtract the area of the two right triangles, students may count the squares, or put them together to form a rectangle. $(6 \cdot 9) - (6 \cdot 3) = 36$.





2. 18 square units. Reasoning should involve decomposition and rearrangement, or other area reasoning strategies.

Activity Synthesis

Invite selected students to share how they found the area of the first parallelogram. Begin with students who decomposed the parallelogram in different ways. Follow with students who enclosed the parallelogram and rearranged the extra right triangles. (For students using the digital version, begin with those who reasoned with the smaller

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rectangles in the applet. Follow with students who reasoned with the largest rectangle.)

As students share, display and list the strategies for all to see. Restate them in terms of decomposing, rearranging, and enclosing, as needed. The list will serve as a reference for upcoming work. If one of the key strategies is not mentioned, illustrate it and add it to the list.

Use the reflection questions in the launch to help highlight the usefulness of rectangles in finding the area of parallelograms. Consider using the applet to illustrate this point, https://ggbm.at/kj5DcRvn.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, color code the diagrams that display each strategy used by students to find the area of the parallelograms. Label each diagram with the strategy shown (decomposing, rearranging, or enclosing).
Supports accessibility for: Visual-Spatial Processing

15 mins

4.3 Lots of Parallelograms

Activity Narrative

While there is not one correct way to find the area of a parallelogram, each parallelogram here is designed to elicit a particular strategy. Monitor for students who use these two main strategies:

- Decomposing and rearranging the parallelograms into a rectangle. Parallelograms A and C encourage this strategy.
- Enclosing the parallelogram and subtracting the area of the extra pieces. Parallelogram B is not as easy to decompose and rearrange (though some students are likely to try that approach first) and may prompt this strategy.

The presence of a grid for Parallelograms A and B and its absence for Parallelogram C allow students to reason concretely and abstractly, respectively, about the measurements that they need to find the area (MP2).

With repeated reasoning, students may begin to see regularity in the segments and measurements that tend to be useful in finding area (MP8). Students formalize this awareness in an upcoming lesson. Highlighting the segments and measurements without referring to bases or heights can help support students' future work.

This is the first time that Math Language Routine 7: Compare and Connect is suggested in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations, and they record their method for all to see. They then compare and identify correspondences across strategies by means of a teacher-led gallery walk with commentary or by teacher think-aloud (such as "I notice . . . I wonder . . ."). A typical discussion prompt is "What is the same and what is different?", comparing their own strategy to the strategies of others. The purpose of this routine is to allow students to make sense of mathematical strategies and, through constructive conversations, develop awareness of the language used as they compare, contrast, and connect other ways of thinking to their own.

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

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Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR7: Compare and Connect

Launch

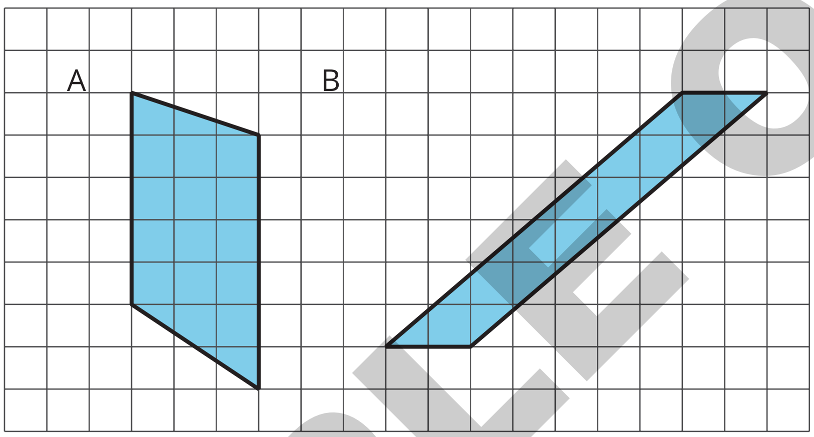
Keep students in groups of 2–4 and ask them to find the areas of several more parallelograms. Give them 5–7 minutes of quiet think time, followed by a couple of minutes to share their strategies with their groups. Ask students to attempt at least the first two questions individually before discussing with their group. Provide access to geometry toolkits.

Encourage students to think, as they work through the problems, about which measurements of the parallelogram seem to be helpful for finding its area.

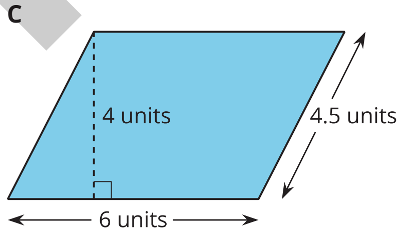
Select students who used each strategy described in the *Activity Narrative* to share later. Try to elicit both key mathematical ideas and a variety of student responses, especially from students who haven't shared recently.

Student Task Statement

Find the area of each parallelogram. Show your reasoning.

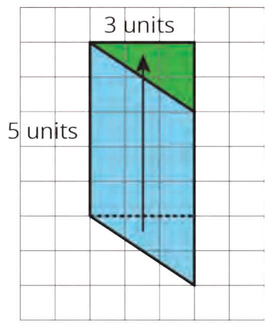


C



Student Response

A: 15 square units. Sample reasoning: Decompose and rearrange the pieces to form a rectangle and multiply the base and side lengths of the rectangle to find the area. $5 \cdot 3 = 15$



B: 12 square units. Sample reasoning: Enclose the parallelogram with a rectangle and subtract the area of the extra pieces. The area of the extra pieces is found by rearranging the triangles to form a rectangle. $6 \cdot 9 - 6 \cdot 7 = 12$

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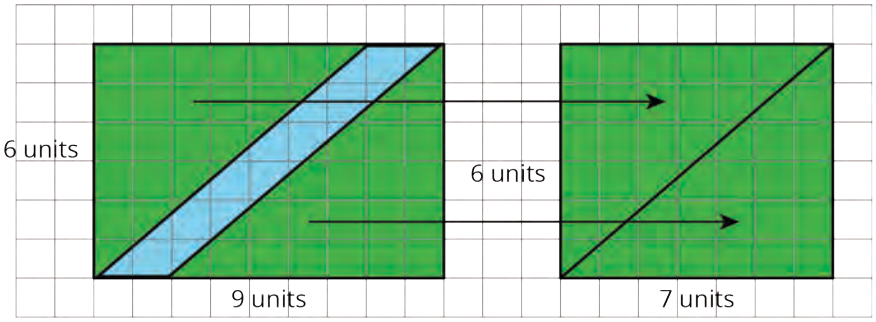
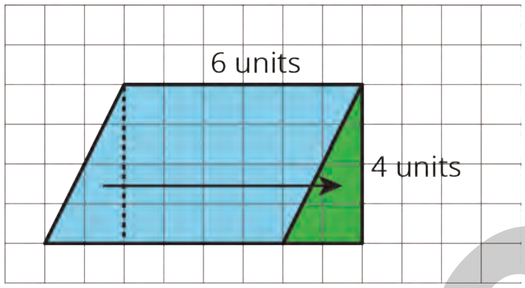


C: 24 square units. Sample reasoning: Decompose the parallelogram and rearrange into a rectangle. Multiply the base and height lengths of the rectangle. $6 \cdot 4 = 24$



Sec B

Building on Student Thinking

Some students may think that it is not possible to decompose and rearrange Parallelogram A because it has a pair of vertical sides instead of a pair of horizontal sides. Suggest that those students rotate their paper 90 degrees and back again to help them see that they could still use the same reasoning strategy regardless of the orientation. Also, they may find it helpful to first reason about area with the parallelogram rotated 90 degrees and then rotate it back to its original orientation.

Some students may spend time unsucessfully trying to decompose Parallelogram B into parts and rearrange the parts into a rectangle. Draw their attention to the list of strategies used in an earlier activity and urge them to try a different strateg**y.**

Activity Synthesis

The goal of this discussion is to highlight strategies and measurements that may be useful for finding the area of parallelograms with different characteristics or when given different information.

For each parallelogram, display 2 or 3 approaches used by previously selected students and share their strategies with everyone. If time allows, invite students to briefly describe their work. If an important strategy is not mentioned, bring it up and illustrate it. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Ask students:

- "How are the strategies the same? How are they different?"
- "Did anyone solve the problem the same way, but would explain it differently?"

After hearing from students about all parallelograms, consider asking the following questions. Provide access to tracing paper or a copy of the drawing in case students wish to verify their thinking by physically cutting and rearranging pieces.

- "Which strategy—decomposing and rearranging, or enclosing and subtracting—seems more practical for finding the area of a shape similar to Parallelogram B? Why?" (Enclosing and subtracting, because it can be done in fewer

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steps. Decomposing the figure into small pieces could get confusing and lead to errors.)

- “If you decomposed Parallelogram C into a right triangle and another shape, how do you know that the cut-out piece actually fits on the other side, given that there's no grid to use?” (The two opposite sides of a parallelogram are parallel, so the longest side of the right triangle that is rearranged would match up perfectly with the segment on the other side.)
- “Three measurements are shown for Parallelogram C. Which ones did you use? Which ones did you not use? Why and why not?” (The 4 units and 6 units are side lengths of a rectangle that has the same area as that of the parallelogram. If we decompose the parallelogram with a vertical cut and move the piece on the left to the right to make a rectangle, the 4.5-unit length is no longer relevant.)

Lesson Synthesis

Revisit the definition of a parallelogram: A parallelogram has four sides. The opposite sides of a parallelogram are parallel.

Briefly revisit the last task, displaying for all to see the multiple strategies that students used. Point out that in some cases, students chose to decompose and rearrange parts, and in others they chose to enclose the parallelogram with a rectangle and subtract the area of the extra pieces from the area of the rectangle. Consider asking students: “What was it about each parallelogram that led you to choose a certain strategy?”

To help students make connections across strategies and generalize their observations, discuss questions such as:

- “When you decomposed and rearranged the parallelogram into another shape, did the area change?” (No.)
- “Why is it helpful to use a rectangle?” (We know how to find the area of a rectangle. We can multiply two adjacent side lengths—a base and a height.)
- “For those of you who enclosed the parallelogram with a rectangle, how did the two right triangles help you?” (They can be combined into a rectangle, whose area we can find and subtract from the area of the large rectangle.)
- “Which measurements or lengths were useful for finding the area of the parallelogram?" (One side length of the parallelogram and the length of a perpendicular segment between that side and the opposite side.) “Which lengths did you not use?” (The other side length.)

4.4 How Would You Find the Area?

Cool-down

5 mins

This activity sets the stage for the next lesson, which formalizes how to find the area of any parallelogram. Notice the strategies that students are currently using to help make connections to the algebraic expression $b \cdot h$ that they will see in the next lesson.

Standards

Addressing **6.G.A.1**

Student Task Statement

How would you find the area of this parallelogram? Describe your strategy.

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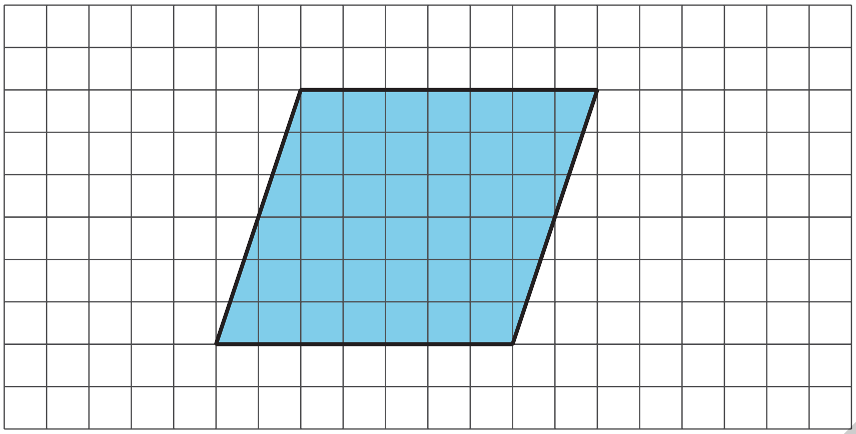


Student Response

Sample responses:

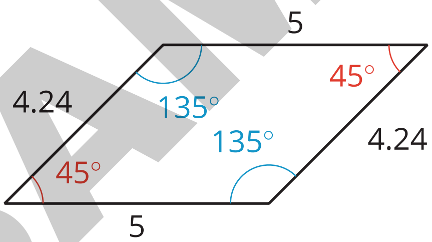
- Decompose a triangle from one side of the parallelogram and move it to the other side to make a rectangle. Multiply the base and side (height) lengths of the rectangle.
- Draw a rectangle that just fits around the parallelogram, multiply the bottom length of that rectangle by its side length to find the area of the rectangle, and then subtract the combined area of the triangles that do not belong to the parallelogram.
- Count how many squares are across the bottom of the parallelogram and how many squares tall it is and multiply them.

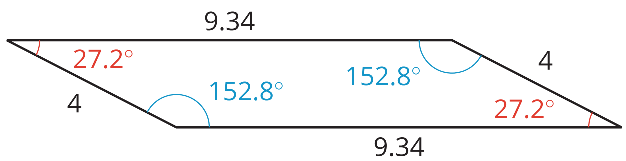
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 4 Summary

A *parallelogram* is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. The opposite sides of a parallelogram have the same length, and the opposite angles of a parallelogram have the same measure in degrees.





There are several strategies for finding the area of a parallelogram.

- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways:

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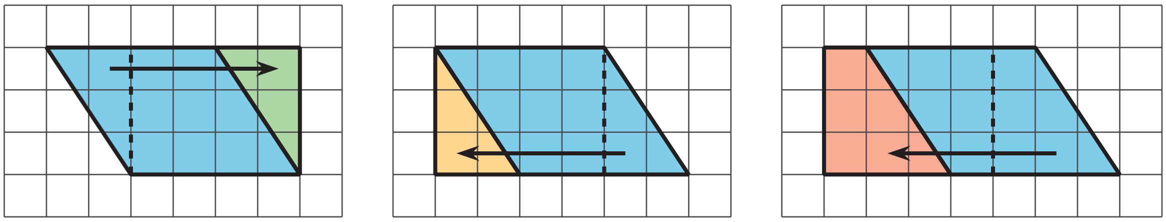
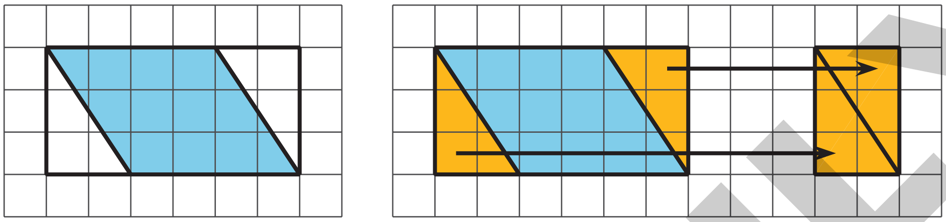
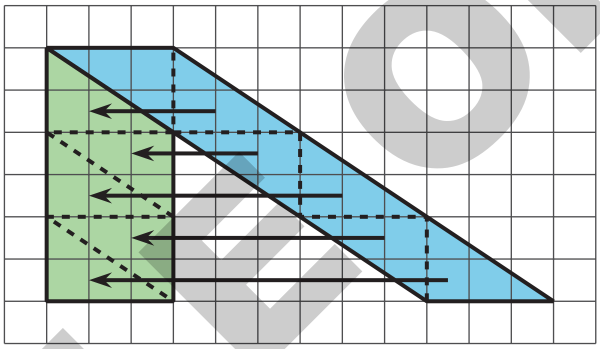


- We can enclose the parallelogram and then subtract the area of the two triangles in the corner.



Both of these ways will work for any parallelogram. However, for some parallelograms the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners.



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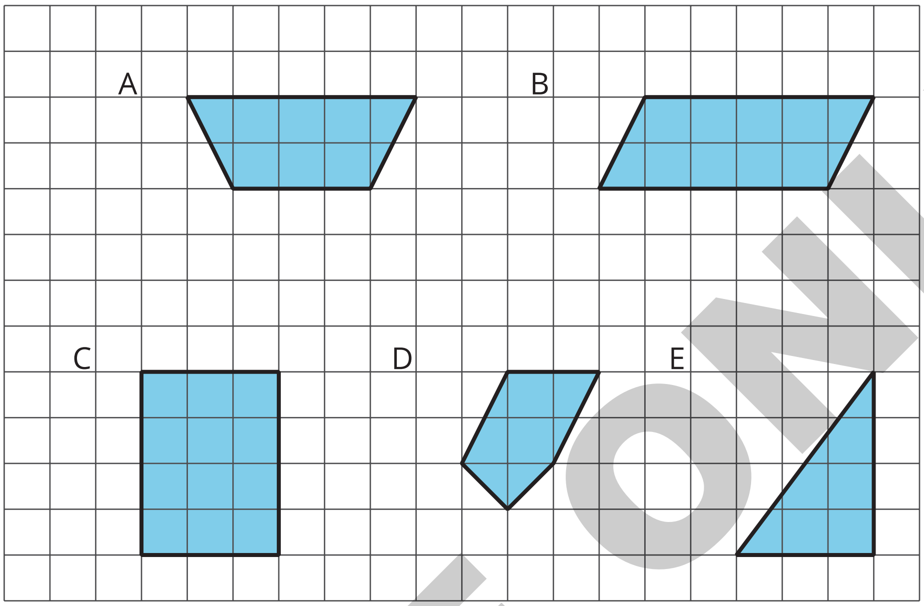
Sec B

Practice Problems

Sec B

1 Student Task Statement

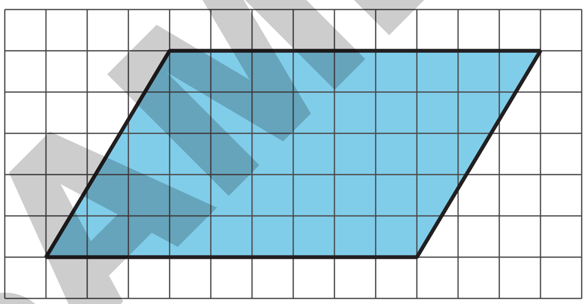
Select **all** of the parallelograms. For each figure that is *not* selected, explain how you know it is not a parallelogram.



Solution

B and C are parallelograms (C is also a rectangle). A is a trapezoid (two opposite sides are not parallel and two are not the same length), D is a pentagon, and E is a (right) triangle.

2 Student Task Statement



a. Decompose and rearrange this parallelogram to make a rectangle.

b. What is the area of the parallelogram? Explain your reasoning.

Solution

a. Sample response: The diagram shows that we get a rectangle that is 5 units by 9 units by decomposing and rearranging.

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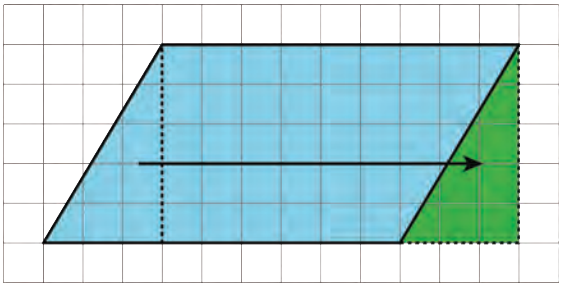

b. 45 square units. Sample reasoning: The area of the parallelogram is the same as the area of the rectangle, which is 45 square units.

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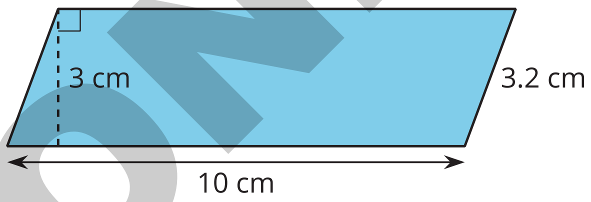
Sec B

3 Student Task Statement

Find the area of the parallelogram.

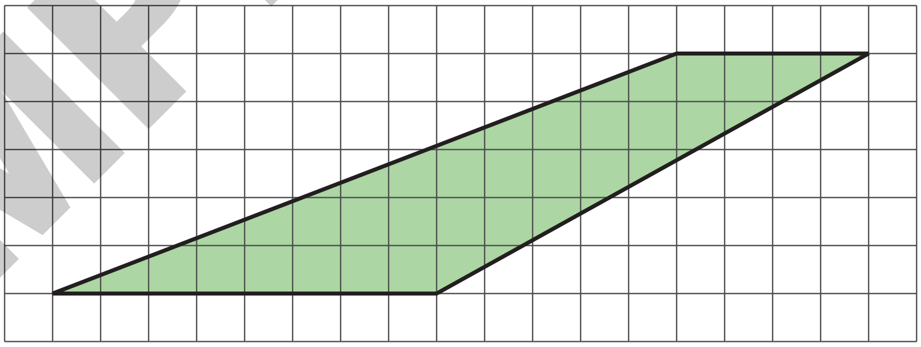


Solution

30 sq cm

4 Student Task Statement

Explain why this quadrilateral is *not* a parallelogram.



Solution

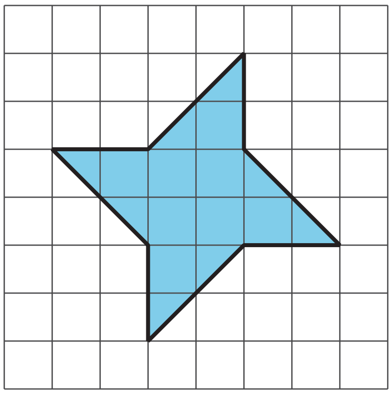
Sample response: Both pairs of opposite sides would need to be parallel and the same length, and that is not true. Also, the opposite angles are not equal.

5 from Unit 1, Lesson 3

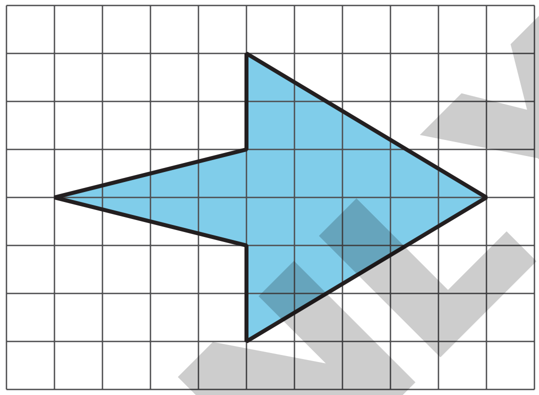
Student Task Statement

Find the area of each shape. Show your reasoning.

A



B



Solution

A: 12 square units. Sample reasoning: The shape can be decomposed into a square in the middle and 4 right triangles. The area of the square is 4 square units. Putting 2 triangles together also make a square with an area of 4 square units, so 4 triangles make 8 square unit. $4 + 8 = 12$

B: 19 square units. Sample reasoning: The shape can be decomposed into 2 triangles by using a vertical cut. The triangle on the left can be decomposed and rearranged into a rectangle that is 1-by-4 rectangle, which is 4 square units in area. The one on the right can be rearranged into a 3-by-5 rectangle, which is 15 square units in area. $4 + 15 = 19$

SAMPLE ONLY

Bases and Heights of Parallelograms

Goals

- Comprehend that the terms “base” and “height” refer to one side of a parallelogram and the perpendicular distance between that side and the opposite side.
- Generalize (orally) a process for finding the area of a parallelogram, using the length of a base and the corresponding height.
- Identify a base and the corresponding height for a parallelogram, and understand that there are two different base-height pairs for any parallelogram.

Learning Targets

- I can identify pairs of base and height of a parallelogram.
- I can write and explain the formula for the area of a parallelogram.
- I know what the terms "base" and "height" refer to in a parallelogram.

SAMPLE ONLY

Lesson Narrative

In this lesson, students learn about **bases** and **heights** of a parallelogram and generalize the process for finding the area of a parallelogram.

Students begin by comparing two strategies for finding the area of a parallelogram. This comparison sets the stage for seeing a rectangle that is associated with a parallelogram and for understanding bases and heights.

Next, students examine examples and non-examples of base-height pairs to make sense of the terms “base” and “height” in a parallelogram. They also practice identifying the corresponding height for a given base.

Then, students identify both a base and its corresponding height for given parallelograms and find their areas. Through repeated reasoning, students notice regularity in the process of finding the area of a parallelogram and express it as a formula in terms of base and height (MP8).

A note about terminology:

The terms “base” and “height” are potentially confusing because they are sometimes used to refer to particular line segments, and sometimes to the length of a line segment or the distance between parallel lines. Furthermore, there are always two base-height pairs for any parallelogram, so asking for the base and the height is not, technically, a well-posed question. Instead, asking for a base and its corresponding height is more appropriate. In these materials, the words “base” and “height” mean the numbers unless it is clear from the context that it means a segment and that there is no potential confusion.

A note about notation:

In this lesson, students see the “dot” notation for multiplication in their materials for the first time. If needed, reiterate that both the $\cdot$ symbol and the $\times$ symbol represent multiplication.

Math Community
Today’s community building centers on the teacher sharing their draft commitments as part of the mathematical community. At the end of the lesson, students are invited to suggest additions to the teacher sections of the chart

Standards

Addressing **6.EE.A.2.a, 6.EE.A.2.c, 6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Geometry toolkits: Lesson
- Math Community Chart: Lesson
- Geometry toolkits: Activity 1, Activity 3

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

In the “Doing Math” teacher section of the Math Community Chart, add 2–5 commitments you have for what your teaching practice “looks like” and “sounds like” this year

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let’s investigate the area of parallelograms some more.

5.1 A Parallelogram and Its Rectangles

Warm-up

10 mins

Activity Narrative

There is a digital version of this activity.

In this *Warm-up*, students compare and contrast two ways of decomposing and rearranging a parallelogram on a grid such that its area can be found. This work allows students to practice communicating their observations and prompts them to notice features of a parallelogram that are useful for finding area—a base and its corresponding height.

The flow of key ideas—to be uncovered during discussion and gradually throughout the lesson—is as follows:

- There are multiple ways to decompose a parallelogram (with one cut) and rearrange it into a rectangle whose area we can determine.
- The cut can be made in different places, but to compose a rectangle, the cut has to be at a *right angle* to two opposite sides of the parallelogram.
- The length of one side of this newly composed rectangle is the same as the length of one side of the parallelogram. We use the term **base** to refer to this side.
- The length of the other side of the rectangle is the length of the cut we made to the parallelogram. We call this segment a **height** that corresponds to the chosen base.

SAMPLE ONLY

- We use these two lengths to determine the area of the rectangle, and thus also the area of the parallelogram.

As students work and discuss, identify those who recognize that both Elena and Tyler decomposed the parallelogram by making a cut that is perpendicular to one side and then rearranged the pieces into a rectangle. Ask them to share their observations later. Be sure to leave enough time to discuss the first four key ideas as a class.

In the digital version of the warm-up, students use applets to animate the moves that Elena and Tyler made (decomposing and rearranging) to find the area of the parallelogram.

Standards

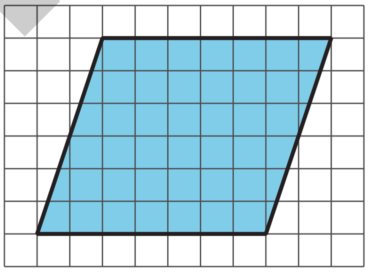
Addressing **6.G.A.1**

Launch

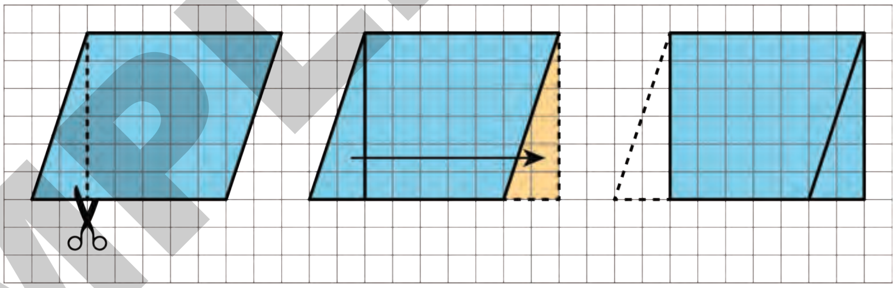
Arrange students in groups of 2. Give students 2 minutes of quiet think time and access to geometry toolkits. Ask them to share their responses with a partner afterward.

Student Task Statement

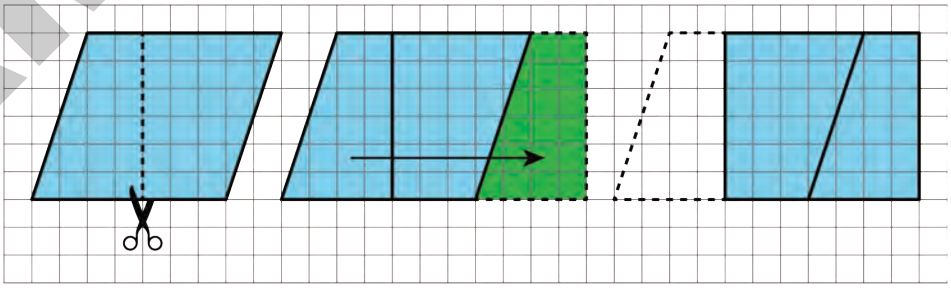
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:



How are the two strategies for finding the area of a parallelogram the same? How they are different?

SAMPLE ONLY

Student Response

Sample responses:

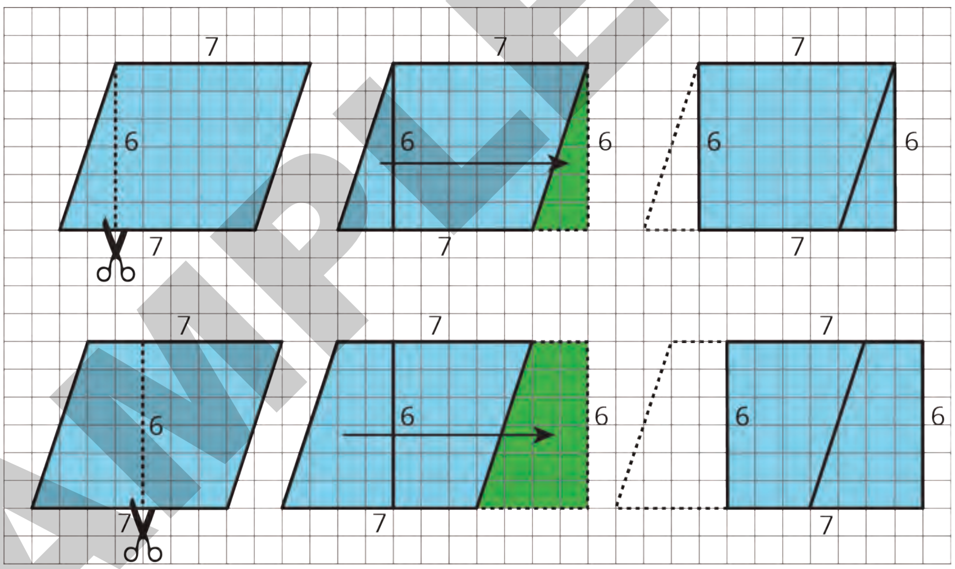
- Similar: They both cut off a piece from the left of the parallelogram and moved it over to the right to make a rectangle. The rectangles they made are identical.
- Different: They cut the parallelogram at different places. Elena cut a right triangle from the left side and Tyler cut off a trapezoid. The rectangles they made are not in the same place.

Activity Synthesis

Select previously identified students to share what was the same and what was different about Elena's and Tyler's methods.

If not already mentioned by students, highlight the following points on how Elena's and Tyler's approaches are the same, though do not expect students to use the language as written here. Clarify each point by gesturing, pointing, and annotating the images.

- The rectangles are identical. They have the same side lengths. (Label the side lengths of the rectangles.)
- The cuts were made in different places, but the length of the cuts was the same. (Label the lengths along the vertical cuts.)
- The horizontal sides of the parallelogram have the same length as the horizontal sides of the rectangle. (Point out how both segments have the same length.)
- The length of each cut is the distance between the two horizontal sides of the parallelogram. It is also the vertical side length of the rectangle. (Point out how that distance stays the same across the horizontal length of the parallelogram.)



Begin to connect the observations to the terms "**base**" and "**height**." For example, explain:

- "The two measurements that we see here have special names. The length of one side of the parallelogram—which is also the length of one side of the rectangle—is called a *base*. The length of the vertical cut segment—which is also the length of the vertical side of the rectangle—is called a *height* that corresponds to that base."
- "Here, the side of the parallelogram that is 7 units long is also called a base. In other words, the word 'base' is used for both the segment and the measurement."

Tell students that we will explore bases and heights of a parallelogram in this lesson.

SAMPLE ONLY

Math Community

After the *Warm-up*, display the Math Community Chart with the “doing math” actions added to the teacher section for all to see. Give students 1 minute to review. Then share 2–3 key points from the teacher section and your reasoning for adding them. For example,

- If “questioning vs. telling,” a shared reason could focus on your belief that students are capable mathematical thinkers and your desire to understand how students are making meaning of the mathematics.
- If “listening,” a shared reason could be that sometimes you want to sit quietly with a group just to listen and hear student thinking and not because you think the group needs help or is off-track.

After sharing, tell students that they will have the opportunity to suggest additions to the teacher section during the *Cool-down*.

SAMPLE ONLY

5.2 The Right Height?

15 mins

Activity Narrative

There is a digital version of this activity.

In this activity, students further develop their understanding of bases and heights of parallelograms by studying examples and non-examples and by analyzing statements. The goal is for students to see that in a parallelogram:

- The term "base" refers to the length of one side and "height" to the length of a perpendicular segment between that side and the opposite side.
- Any side of a parallelogram can be a base.
- There are always two base-height pairs for a given parallelogram.

In the digital version of *Are You Ready for More?*, students use an applet to create a dynamic parallelogram with a height displayed. The applet allows students to see placements of height segments in a variety of parallelograms and when any side is chosen as a base. Consider allowing students to use the applet to check their responses to the last question (about whether the bases and heights in parallelograms A–E are correctly labeled) and to gain additional insights about base-height pairs.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

Launch

Display the examples and non-examples of bases and heights for all to see. Read aloud the first paragraph of the activity and the description of each set of images. Give students a minute to observe the images. Then tell students to use the examples and non-examples to determine what is true about bases and heights in a parallelogram.

Arrange students in groups of 2. Give students 4–5 minutes to complete the first question with their partner. Ask them to pause for a class discussion after the first question. Select a student or a group to make a case for whether each statement is true or false. If one or more students disagree, ask them to explain their reasoning and discuss to reach a consensus. Before moving on to the next question, be sure students record the verified true statements so that they can be used as a reference later.

Give students 3 minutes of quiet time to answer the second question and another 2–3 minutes to share their responses with a partner. Ask them to focus partner conversations on the following questions, displayed for all to see:

- How do you know the parallelogram is labeled correctly or incorrectly?
- Is there another way a base and height could be labeled on this parallelogram?

After answering the questions, students with digital access can explore the applet ggbm.at/UnfbrN96 and use it to verify their responses and further their understanding of bases and heights.

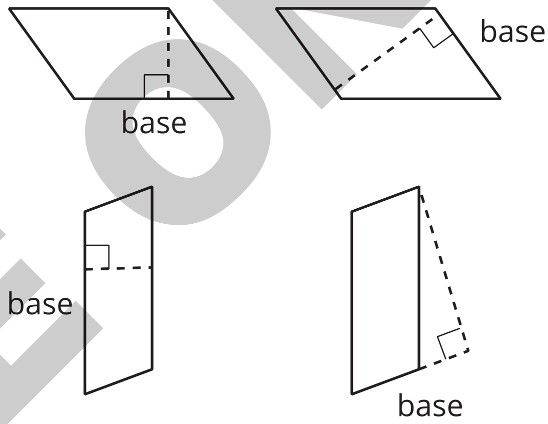
Access for English Language Learners

MLR8 Discussion Supports. Pair gestures with verbal directions to clarify the meaning of any unfamiliar terms such as "dashed," "horizontal," "opposite," or "perpendicular." *Advances: Listening, Representing*

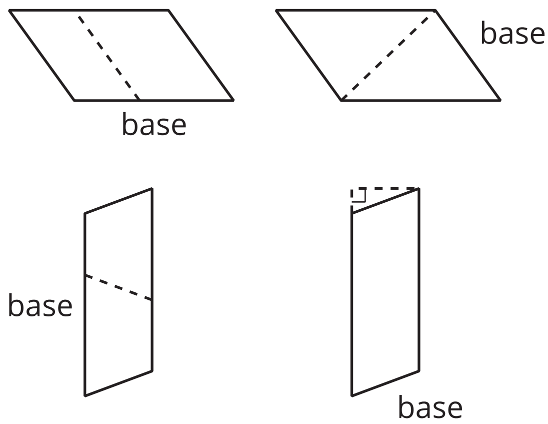
Student Task Statement

Here are some drawings of parallelograms. In each drawing, one side is labeled "**base**."

In the first four drawings, each dashed segment represents a **height** that corresponds to the given base.



In the next four drawings, each dashed segment does not represent a height that corresponds to the given base.



1. Select **all** the statements that are true about bases and heights in a parallelogram.
   A. Only a horizontal side of a parallelogram can be a base.
   B. Any side of a parallelogram can be a base.
   C. A height can be drawn at any angle to the side chosen as the base.
   D. A base and its corresponding height must be perpendicular to each other.

SAMPLE ONLY

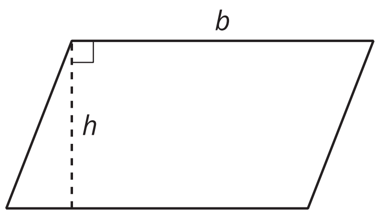
E. A height can only be drawn inside a parallelogram.

F. A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.

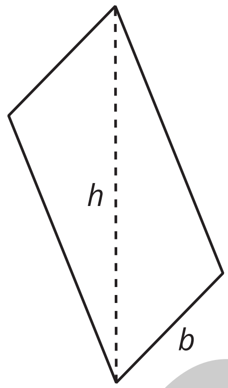
G. A base cannot be extended to meet a height.

2. Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.

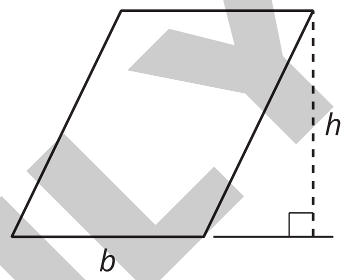
A



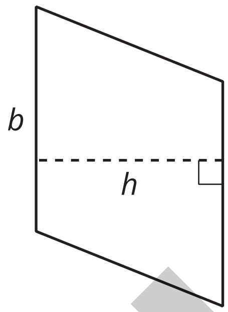
B



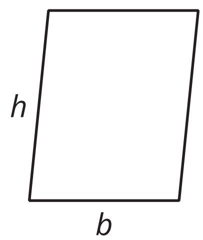
C



D



E



SAMPLE ONLY

Student Response

1. Statements B, D, and F are true.
2. A, C, and D are correct. Sample reasoning: B and E are not correct because in each, the segment labeled with an h is not perpendicular to the side labeled with a b.

Building on Student Thinking

Students might say that Parallelogram E is correctly labeled because the labeled sides remind them of the labeled length and width of a rectangle. Ask students to revisit the true statements about base-height pairs and see if those conditions are met in Parallelogram E.

Activity Synthesis

Ask the class to give a quick agree-or-disagree signal on whether each figure in the last question is labeled correctly. After getting the responses for each figure, ask a student to explain how they know it is correct or incorrect.

If a parallelogram is incorrectly labeled, ask students where a correct height could be. If it is correctly labeled, ask students if there is another base and height that could be labeled on this parallelogram.

Before moving forward in this lesson, be sure that students understand which parallelograms are labeled correctly and emphasize the following points:

- We can choose any side of a parallelogram as a base.
- To find the height that corresponds to that base, draw a segment that joins the base and its opposite side. That segment has to be perpendicular to both the base and the opposite side.

Consider using the applet ggbm.at/UnfbrN96 to further illustrate possible base-height pairs and reinforce students' understanding of them.

Access for Students with Disabilities

Representation: Internalize Comprehension. Invite students to identify which details were most important in finding base-height pairs to solve the problem. Display the sentence frame, "The next time I need to find the base and height of a parallelogram, I will look for . . ."
Supports accessibility for: Language

5.3 Finding the Formula for Area of Parallelograms

10 mins

Activity Narrative

In this activity, students find the area of more parallelograms, generalize the process, and write an expression for finding the area of any parallelogram. To do so, they apply what they learned in previous lessons about base-height pairs in parallelograms and about strategies for reasoning about area.

As students discuss their work, monitor conversations for any disagreements between partners. Support them by asking clarifying questions:

- "How did you choose a base? How can you be sure that is the height?"
- "How did you find the area? Why did you choose that strategy for this parallelogram?"
- "Is there another way to find the area and to check your answer?"

Standards

Addressing **6.EE.A.2.a, 6.G.A.1**

Launch

Keep students in groups of 2. Give students access to their geometry toolkits and 4–5 minutes of partner work time to complete the table. Ask them to be prepared to share their reasoning. Encourage students to use their work from earlier activities (on bases and heights) as a reference.

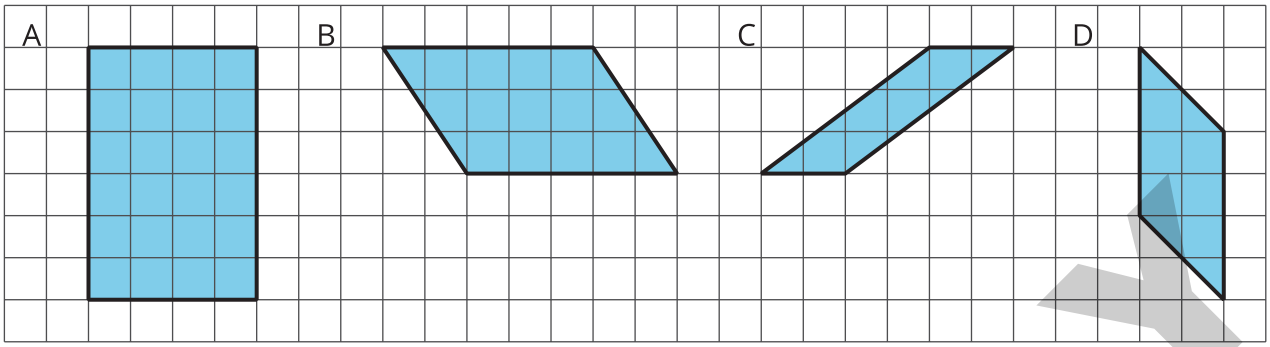
Student Task Statement

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table.

SAMPLE ONLY

- Find the area of the parallelogram and record it in the last column of the table.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

In the last row of the table, write an expression for the area of any parallelogram, using b and h .

Student Response

While there are two possible base-height pairs, these are the easiest ones for students to use given the orientation of each parallelogram on the grid.

parallelogram	base (units)	height (units)	area (square units)
A	6 (or 4)	4 (or 6)	24
B	5	3	15
C	2	3	6
D	4	2	8
any parallelogram	b	h	$b \cdot h$

Building on Student Thinking

Finding a height segment outside of the parallelogram may still be unfamiliar to students. Have examples from the "The Right Height?" section visible so they can serve as a reference in finding heights.

Students may say that the base of Parallelogram D cannot be determined because, as displayed, it does not have a horizontal side. Remind students that in an earlier activity we learned that any side of a parallelogram could be a base and that rotating our paper can help us see this. Ask students to see if there is a side whose length can be determined.

SAMPLE ONLY

Are You Ready for More?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

Extension Student Response

1. The area doubles, triples, is multiplied by 100.
2. The area quadruples, is 9 times the original area, is 10,000 times the original area.

Activity Synthesis

Display the parallelograms and the table for all to see. Select a few students to share the correct answers for each parallelogram. As students share, highlight the base-height pairs on each parallelogram and record the responses in the table. Although only one base-height pair is named for each parallelogram, reiterate that there is another pair. Show the second pair on the diagram or ask students to point it out.

After the first four rows of the table are completed, discuss the expression in the last row. Ask students:

- "How did you figure out the expression for the area for any parallelogram?" (The areas of Parallelograms A–D are each the product of base and height.)
- "Suppose you decompose a parallelogram with a cut and rearrange it into a rectangle. Does this expression for finding area still work? Why or why not?" (Yes. One side of the rectangle will have the same length as the base of the parallelogram. The height of the parallelogram is also the height of the rectangle—both are perpendicular to the base.)

Be sure everyone has the correct expression for finding the area of a parallelogram by the end of the discussion.

Lesson Synthesis

In this lesson, students identified a *base* and a corresponding *height* in a parallelogram, and then wrote an algebraic expression for finding the area of any parallelogram. Consider asking students:

- "How do you identify the base of a parallelogram?**"**(Any side can be a base. Sometimes one side is preferable over another because its length is known or easy to know.)
- "Once we have chosen a base, how can we identify a height that corresponds to it?" (Identify a perpendicular segment that connects that base and the opposite side; find the length of that segment.)
- "In how many ways can we identify a base and a height for a given parallelogram?" (There are two possible bases. For each base, many possible segments can represent the corresponding height.)
- "What is the relationship between the base and height of a parallelogram and its area?" (The area is the product of base and height.)

If time permits, ask students: "Do you think this expression will always work?" Students are not expected to prove their answer here. Speculation is expected at this point. The question is intended to prompt students to think of other differently-shaped parallelograms beyond the four shown here.

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5.4

Parallelograms S and T

Cool-down

5 mins

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

Launch

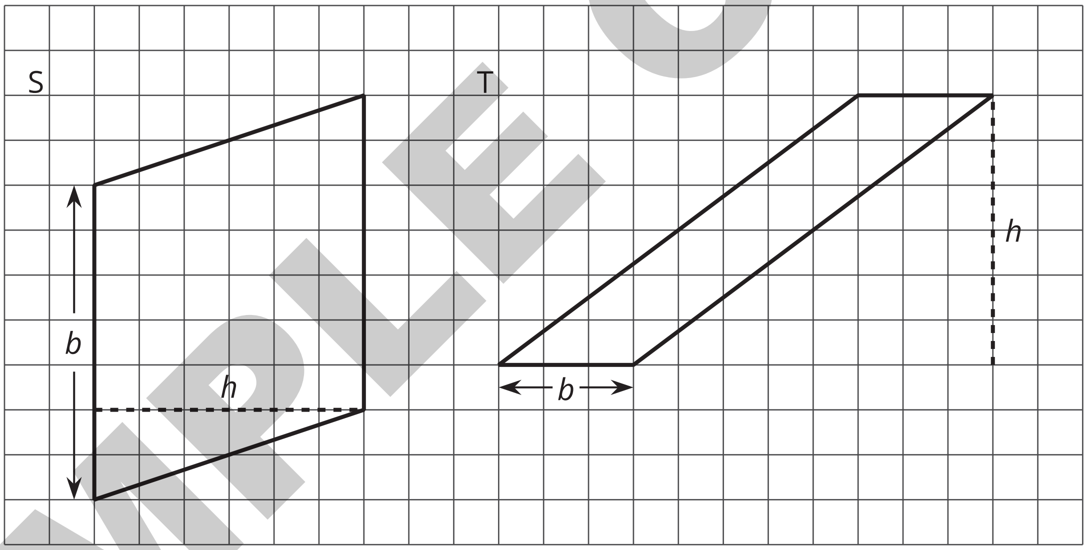
Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "What additions would you make to the teacher 'Doing Math' section of the Math Community Chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the teacher "Doing Math" section of the Math Community Chart before Exercise 4.

Student Task Statement

Parallelograms S and T are each labeled with a base and a corresponding height.



1. What are the values of b and h for each parallelogram?
   - Parallelogram S: b = ________, h = ________
   - Parallelogram T: b = ________, h = ________
2. Use the values of b and h to find the area of each parallelogram.
   - Area of Parallelogram S:
   - Area of Parallelogram T:

Student Response

1. - Parallelogram S: $b = 7$, $h = 6$

SAMPLE ONLY

- Parallelogram T: $b = 3$, $h = 6$

2. - Area of Parallelogram S: 42 square units. $7 \cdot 6 = 42$
   - Area of Parallelogram T: 18 square units. $3 \cdot 6 = 18$

Responding To Student Thinking

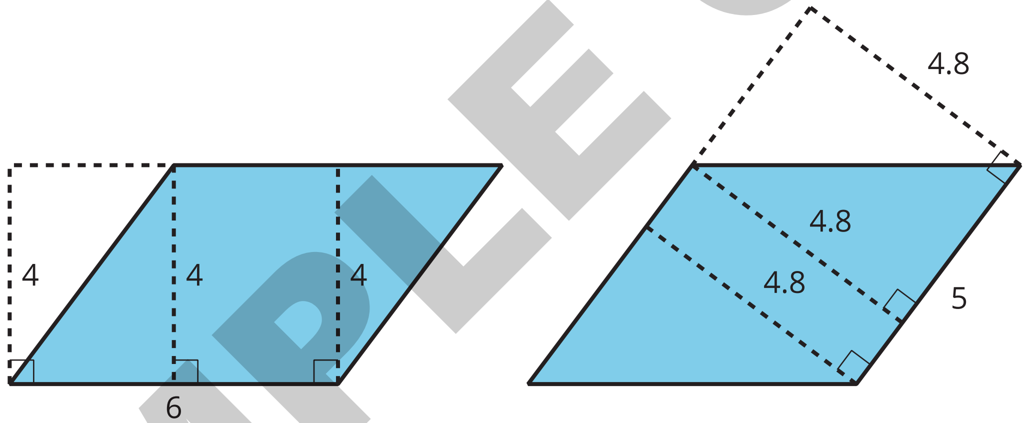
Points to Emphasize

If students struggle with finding the areas of the parallelograms, highlight ways to decompose and rearrange a parallelogram into a rectangle with a known base and height. For example, in this activity, emphasize that each parallelogram can be decomposed and rearranged into a rectangle with the same pair of base and height measurements:

Grade 6, Unit 1, Lesson 6, Activity 2 More Areas of Parallelograms

Lesson 5 Summary

- We can choose any side of a parallelogram as the **base**. Both the side selected (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many segments that can represent the height!



Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its corresponding height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more!

No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check this:

$4 \times 6 = 24$ and $4.8 \times 5 = 24$

We can see why this is true by decomposing and rearranging the parallelograms into rectangles.

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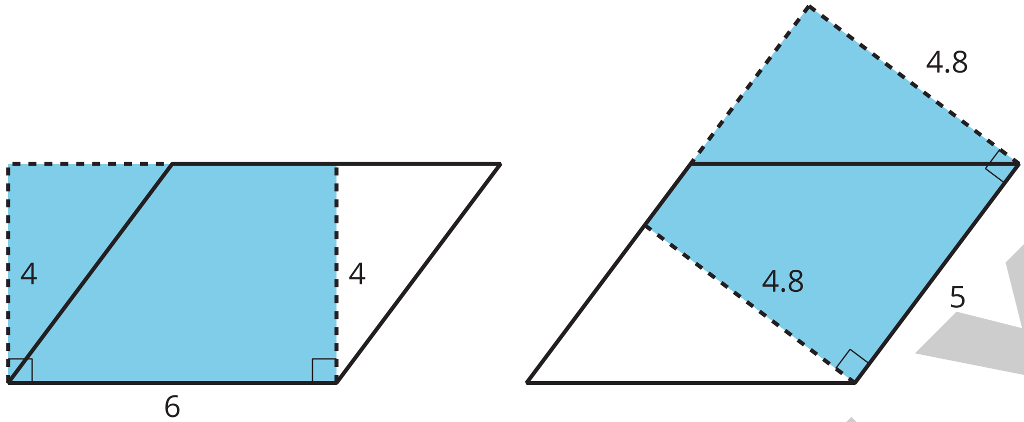


Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as does the parallelogram.

We often use letters to stand for numbers. If b is base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers.:

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a × symbol. This is so that we don't get confused about whether × means multiply, or whether the letter x is standing in for a number.

Glossary

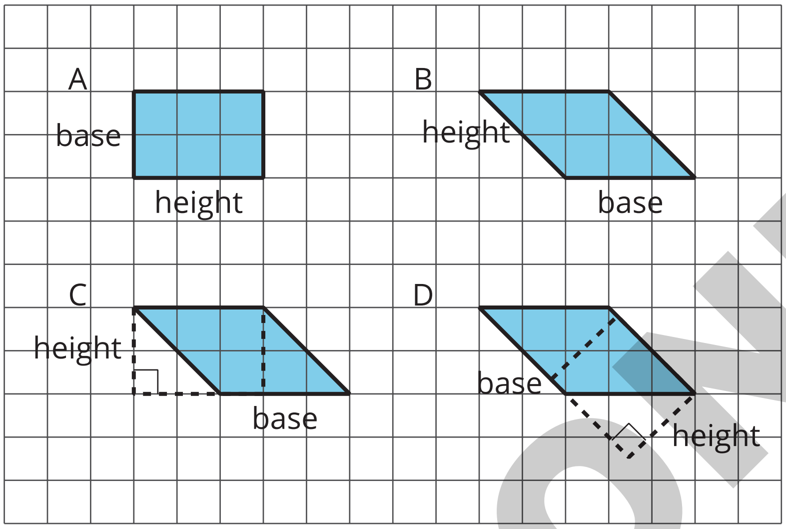
- base (of a parallelogram or triangle)
- height (of a parallelogram or triangle)

SAMPLE ONLY

Practice Problems

1 Student Task Statement

Select **all** parallelograms that have a correct height labeled for the given base.



A. A

B. B

C. C

D. D

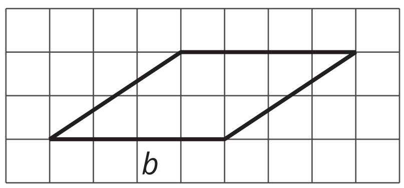
Solution

A, C, D

2 Student Task Statement

The side labeled b has been chosen as the base for this parallelogram.

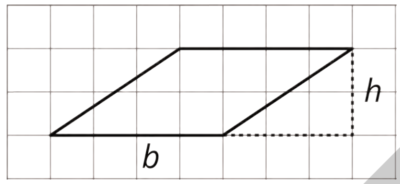
Draw a segment showing the height corresponding to that base.



SAMPLE ONLY

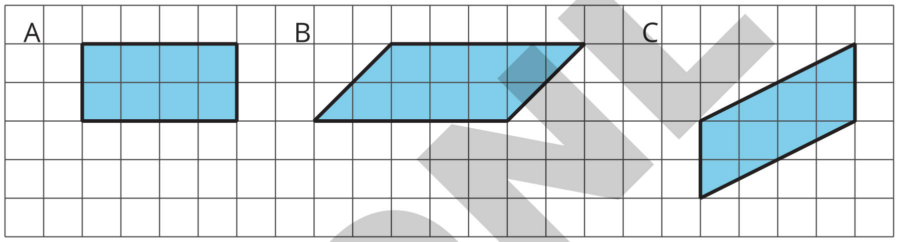
Solution

The height can be any segment perpendicular to the base that joins the line containing the base to the line containing the side opposite the base. Sample response:



3 Student Task Statement

Find the area of each parallelogram.



Solution

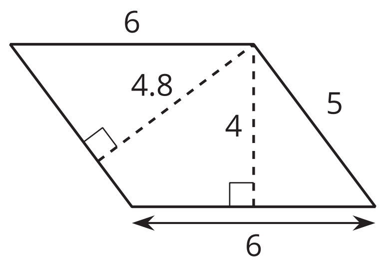
A: 8 square units. This is a 2-by-4 rectangle.

B: 10 square units. The horizontal side is 5 units long and can be the base. The height for this base is 2 units.

C: 8 square units. The vertical side can be used as the base. The base is 2 units, and the height for this base is 4 units.

4 Student Task Statement

If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



A. 6 units

B. 4.8 units

C. 4 units

D. 5 units

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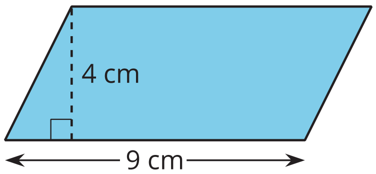
Solution

C

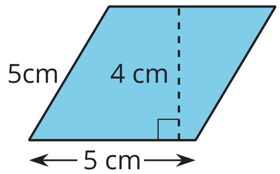
5 Student Task Statement

Find the area of each parallelogram.

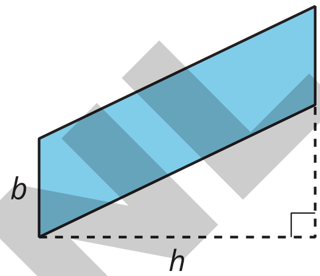
A



B



C



Solution

A: 36 sq cm. The base is 9 cm, and the height for that base is 4 cm.

B: 20 sq cm. The base is 5 cm, and the height for this base is 4 cm.

C: bh. The base is b, and the corresponding height is h.

6

from Unit 1, Lesson 4

Student Task Statement

Do you agree with each of these statements? Explain your reasoning.

a. A parallelogram has six sides.

b. Opposite sides of a parallelogram are parallel.

c. A parallelogram can have one pair or two pairs of parallel sides.

d. All sides of a parallelogram have the same length.

e. All angles of a parallelogram have the same measure.

Solution

a. Disagree. Sample reasoning: A parallelogram is a quadrilateral.

b. Agree. Sample reasoning: By definition, opposite sides of a parallelogram are parallel.

c. Disagree. Sample reasoning: By definition, a parallelogram has two pairs of parallel sides.

d. Disagree. Sample reasoning: Sometimes all sides of a parallelogram have the same length, but not always. Opposite sides of a parallelogram always have the same length.

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e. Disagree. Sample reasoning: Sometimes all angles of a parallelogram have the same measure (when the parallelogram is a rectangle), but not always. Opposite angles of a parallelogram have the same measure.

7 from Unit 1, Lesson 2

Student Task Statement

A square with an area of 1 square meter is decomposed into 9 identical smaller squares. Each smaller square is decomposed into two identical triangles.

a. What is the area, in square meters, of 6 triangles? If you get stuck, consider drawing a diagram.

b. How many triangles are needed to compose a region that is $1\frac{1}{2}$ square meters?

Solution

a. $\frac{6}{18}$ or $\frac{1}{3}$ square meter.

b. 27 triangles. It takes 18 triangles to make an area of 1 square meter and 9 triangles to make an area of $\frac{1}{2}$ square meter. $18 + 9 = 27$.

8 from an earlier course

Student Task Statement

Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

Solution

The square is larger. Sample reasoning: Its area is $2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2}$, which is $\frac{25}{4}$ or $6\frac{1}{4}$ square inches. The rectangle has an area of $5\frac{1}{4}$ square inches because $7 \times \frac{3}{4} = \frac{21}{4}$.

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Area of Parallelograms

Goals

- Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.
- Apply the formula for the area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.
- Choose which measurements to use for calculating the area of a parallelogram when more than one base or height measurement is given, and explain (orally and in writing) the choice.

Learning Targets

- I can use the area formula to find the area of any parallelogram.

Lesson Narrative

This lesson allows students to practice using the formula for the area of parallelograms, and to choose the measurements to use as a base and its corresponding height.

The parallelograms are shown both on and off a grid. As students work to find the areas, they begin to see that some measurements are more helpful than others. For example, if a parallelogram on a grid has a vertical or horizontal side, and one of those sides is used as a base, then both the base and the height can be relatively easily determined.. As students think about which measurements to use, they practice looking for and making use of structure (MP7).

Students also see that parallelograms with the same base and the same height have the same area because the products of those two numbers are equal, even if the parallelograms look very different.

Standards

Building On **3.OA.A**

Addressing **6.EE.A.2.c, 6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports
- Take Turns
- Which Three Go Together?

Required Materials

Materials To Gather

- Geometry toolkits: Activity 2

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

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Student Facing Learning Goals

Let's practice finding the area of parallelograms.

6.1 Which Three Go Together: Squares

Warm-up

5 mins

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four figures with shaded regions. The figures are similar to the ones for which students found the area earlier in the unit. In making comparisons, students have a reason to use language precisely (MP6). The activity also enables the teacher to hear how students talk about attributes of figures and two-dimensional regions.

While students may or may not think to compare the four areas, they are likely to notice regions that can be decomposed, rearranged, and subtracted. They may also notice that figures that look different can have the same area. These observations will support students in reasoning about the areas of parallelograms and other polygons later.

Standards

Building On **3.OA.A**

Instructional Routines

- Which Three Go Together?

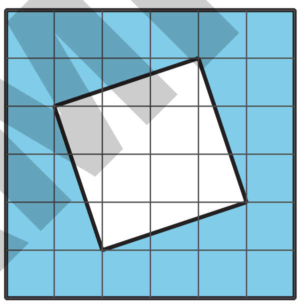
Launch

Arrange students in groups of 2–4. Display the four figures for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three figures that go together and can explain why. Next, tell each student to share their response with their group and then together find as many sets of three as they can.

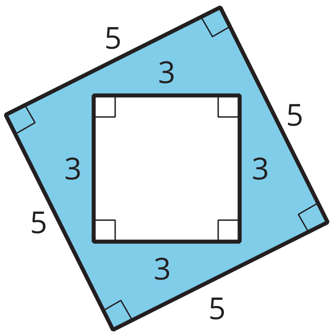
Student Task Statement

Which three go together? Why do they go together?

A



B



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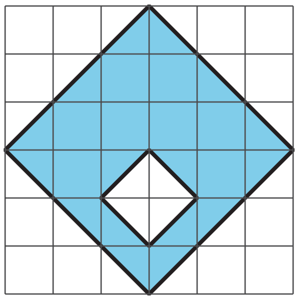
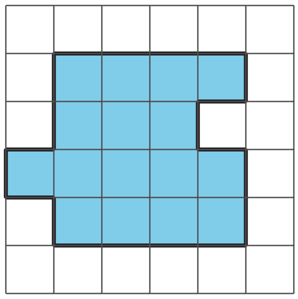
C



D



Student Response

Sample responses:

A ,B, and C go together because:

- They all show two squares of different sizes.
- There is a small square inside a larger square. The smaller square is not shaded.
- There is at least one square that is tilted (or that has no horizontal or vertical sides).

A, B, and D go together because:

- They have shapes with vertical and horizontal sides.
- We can tell the side lengths of the outer shape.

A, C, and D go together because:

- The shapes are on a grid.
- The sides are not labeled with their length.

B, C, and D go together because:

- The area of the shaded region is 16 square units.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, ask students to clarify their reasoning as needed. For example, a student may claim that each of the Figures A, B, and C has a smaller square removed from a larger square. Ask how they know that the smaller unshaded rectangles in Figures A and C are squares.

If no students mentioned the areas of the shaded regions, ask them if and how the areas could be compared. As needed, reiterate strategies for reasoning about area and the idea that different shapes can have the same area.

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25 mins

6.2 More Areas of Parallelograms

Activity Narrative

There is a digital version of this activity.

This activity allows students to practice finding and reasoning about the area of various parallelograms—on and off a grid. Students make sense of the measurements and relationships in the given figures and identify an appropriate pair of base-height measurements to use (the length of a side and that of a segment that is perpendicular to that side). Students learn to recognize that two parallelograms with the same base-height measurements (or with different base-height measurements but the same product) have the same area.

As they work individually, notice how students determine base-height pairs to use. As they work in groups, listen to their discussions and identify those who can explain how they found the area of each of the parallelograms.

In the digital version of the activity, students use an applet to create two parallelograms with the same area.

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Arrange students in groups of 4. Give each student access to their geometry toolkit and 5 minutes of quiet time to find the areas of the parallelograms in the first question. Then, assign each student one parallelogram (A, B, C or D). Ask the students to take turns explaining to the group how they found the area of their assigned parallelogram. Explain that while one group member explains, the others should listen and make sure they agree. If they don't agree, they should discuss their thinking and work to reach an agreement before moving to the next parallelogram.

Afterward, give students another 5–7 minutes of quiet work time to complete the rest of the activity.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example:

- "First, I ________ because ________."
- "Then, I ________."
- "I noticed ________, so I ________."
- "How did you get ________?"

Supports accessibility for: Language, Social-Emotional Functioning

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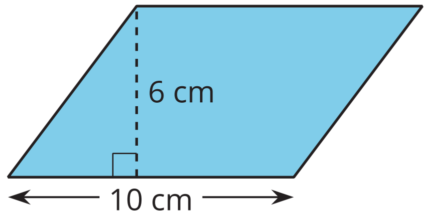
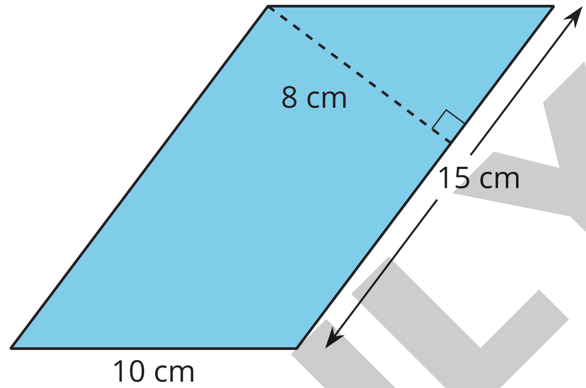
Student Task Statement

1. Find the area of each parallelogram. Show your reasoning.

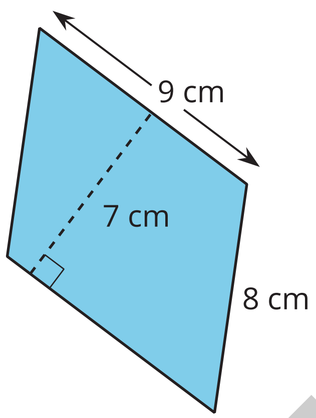
A



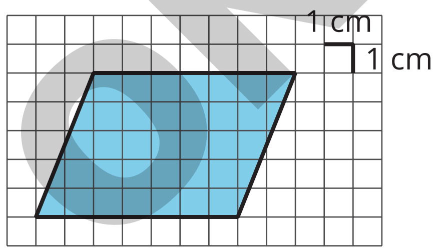
B



C

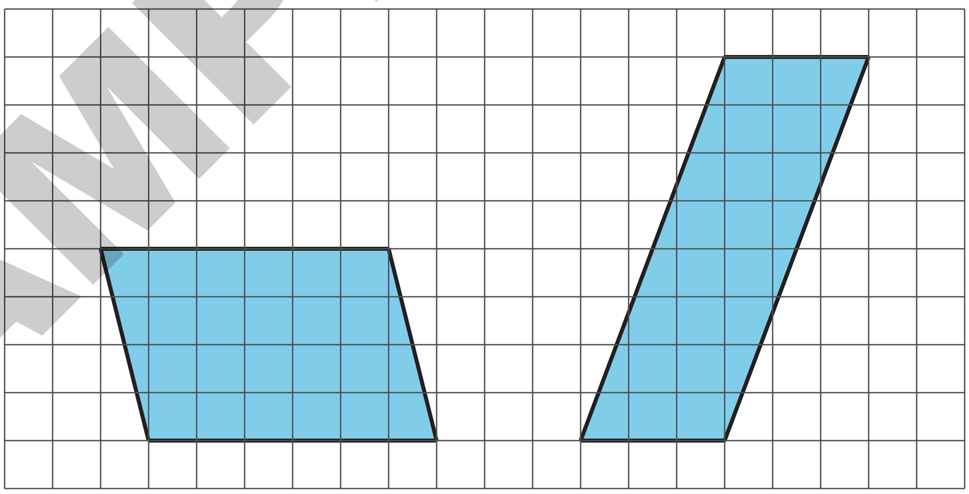


D



2. In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

3. a. Here are two different parallelograms with the same area. Explain why their areas are equal.



   b. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

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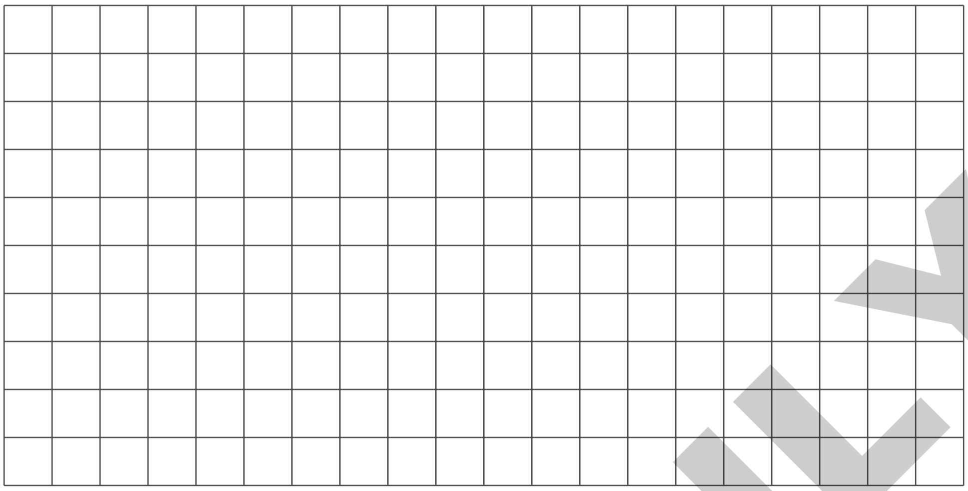
On the grid, draw two parallelograms that could be P and Q. Explain how you know.



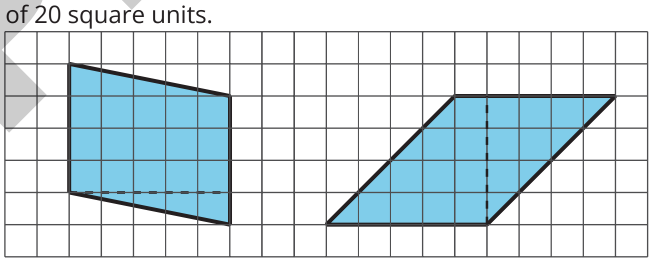
Sec B

Student Response

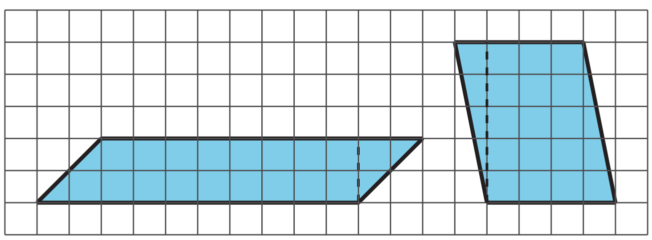
1. A: $10 \cdot 6 = 60$ square centimeters
   B: $15 \cdot 8 = 120$ square centimeters
   C: $9 \cdot 7 = 63$ square centimeters
   D: $7 \cdot 5 = 35$ square centimeters
2. 12 centimeters. Sample reasoning: We found the area of the parallelogram to be 120 square centimeters. If the side that is 10 centimeters is the base, then $10 \cdot h$ must equal 120, so the height must be $120 \div 10$, or 12, centimeters.
3. a. Sample response: They both have an area of 24 square units. The first parallelogram can be decomposed and rearranged into a rectangle that is 6 units by 4 units. The second one can be rearranged into a rectangle that is 3 units by 8 units.

   b. Sample responses:

   - One parallelogram has a base of 5 units and a height of 4 units. The other has a base that is 4 units and a height that is 5 units. They can both be decomposed and rearranged into a rectangle that is 4 units by 5 units, which has an area of 20 square units.



   - One parallelogram has a base that is 10 units and a height that is 2 units. The other has a base that is 4 units and a height that is 5 units. They could be P and Q because $10 \cdot 2$ and $4 \cdot 5$ both give 20, which is the area.



SAMPLE ONLY

- Two parallelograms with equal base and equal height but with different orientations or with the pair of parallel bases positioned differently. They could be P and Q because multiplying the base and height of each parallelogram gives 20, which is the area.

Building on Student Thinking

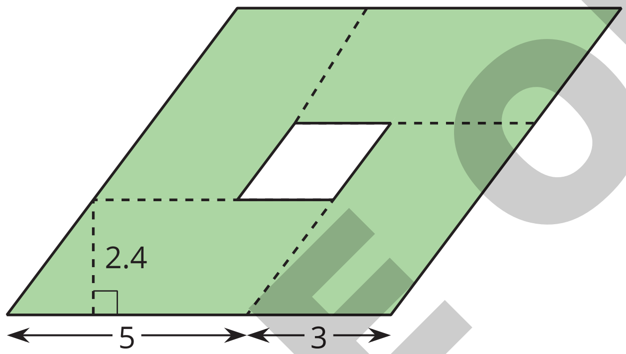
Some students may continue to use visual reasoning strategies (decomposition, rearranging, enclosing, and subtracting) to find the area of parallelograms. This is fine at this stage, but to help them gradually transition toward abstract reasoning, encourage them to try solving one problem both ways—using visual reasoning and using their generalization about bases and heights from an earlier lesson. They can start with one method and use the other to check their work.

Are You Ready for More?

Sec B

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.

What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.



Extension Student Response

3.2 square inches. Sample reasoning:

- The area of one shaded parallelogram is 12 square inches, because one base is 5 inches and its corresponding height is 2.4 inches ($5 \cdot (2.4) = 12$). This means the corresponding height for the side that is 3 inches is 4 inches ($3 \cdot 4 = 12$). The height of the small parallelogram is the difference between 4 inches and 2.4 inches, which is 1.6 inches. The horizontal side of the unshaded parallelogram, which can be a base, is 2 inches ($5 - 3 = 2$). The area of the unshaded parallelogram is therefore $2 \cdot (1.6)$, or 3.2, square inches.
- The base of the overall parallelogram is 8 inches ($5 + 3 = 8$). Its height is 6.4 inches ($4 + 2.4 = 6.4$). Its area is therefore $8 \cdot (6.4)$, or 51.2, square inches. The area of the four shaded parallelograms is $4 \cdot 12$, or 48, square inches. The area of the unshaded region is therefore $51.2 - 48$, or 3.2, square inches.

Activity Synthesis

Use whole-class discussion to highlight three important points:

- We need base and height information to calculate the area of a parallelogram, so we generally look for the length of one side and the length of a perpendicular segment that connects that side to the opposite side. Other measurements may not be as useful.
- A parallelogram has two pairs of base and corresponding height. Both pairs produce the same area, so the product

SAMPLE ONLY

of one pair of numbers should equal the product of the other pair.

- Two parallelograms with different pairs of base and corresponding height can have the same area, as long as their products are equal. A 3-by-6 rectangle and a parallelogram with a base of 1 and a height of 18 will have the same area because $3 \cdot 6 = 1 \cdot 18$.

To highlight the first point, consider asking:

- "The parallelograms in the first question show multiple measurements. How did you know which ones would help you find the area?"
- "Which pieces of information in Parallelograms B and C were not needed? Why not?"

To highlight the second point, select 1 or 2 students to share how they found the missing height in the second question. Emphasize that the product of 815 and that of 10 and the unknown h must be equal because both give us the area of the same parallelogram.

To highlight the last point, invite a few students to share their pair of parallelograms and how they know that the areas are equal. If not made explicit in students' explanations, stress that the base-height pairs must have the same product. Consider displaying the applet for all to see and using it to facilitate this discussion.

Access for English Language Learners

MLR8 Discussion Supports. For each explanation that is shared about creating two parallelograms of equal area, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Lesson Synthesis

In this lesson, students used the formula for area to practice finding the area of various parallelograms. Discuss with students:

- "When a parallelogram is shown on a grid, how do we know which side to choose for a base? Can we use any side?" (It is helpful to use a horizontal or a vertical side as a base because it would be easier to tell the length of that side and of its corresponding height.)
- "Off a grid, how do we know which measurements can help us find the area of a parallelogram?" (We need the length of one side of the parallelogram and of a perpendicular segment that connects that side to the opposite side.)
- "Do parallelograms that have the same area always look the same?" (No.) "Can you show an example?"
- "Do parallelograms that have the same base and height always look the same?" (No.) "Can you show an example?"
- "How can we draw two different parallelograms with the same area?" (We can find any two pairs of base-height lengths that have the same product. We can also use the same pair of numbers but draw the parallelograms differently.)

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6.3

One More Parallelogram

Cool-down

5 mins

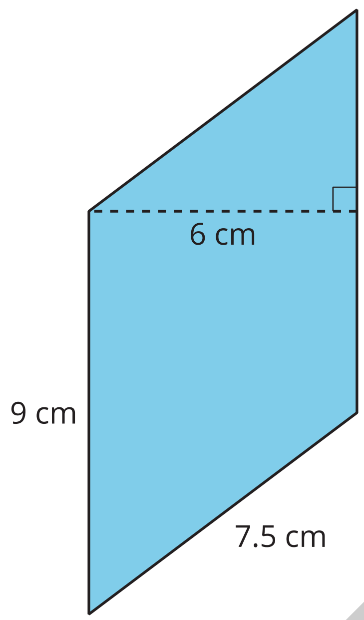
Standards

Addressing **6.G.A.1**

Launch

Access to geometry toolkits.

Student Task Statement



1. Find the area of the parallelogram. Explain or show your reasoning.
2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

Student Response

1. 54 sq cm. Sample reasoning: A base is 9 cm and its corresponding height is 6 cm. $9 \cdot 6 = 54$.
2. The 7.5 cm length was not used. Sample reasoning:
   - If the side that is 7.5 cm was used to find area, we would need the length of a perpendicular segment between that side and the opposite side as its corresponding height. We don't have that information.
   - The parallelogram can be decomposed and rearranged into a rectangle by cutting it along the horizontal line and moving the right triangle to the bottom side. Doing this means the side that is 7.5 cm is no longer relevant. The rectangle is 6 cm by 9 cm; we can use those side lengths to find area.

Responding To Student Thinking

Points to Emphasize

If students struggle with finding the area of a parallelogram that is not on a grid, integrate discussions about how to find the area of a parallelogram more generally. For example, when students find the areas of parallelograms in the practice problems of the lesson referred to here, encourage them to decompose and rearrange the paralellograms into

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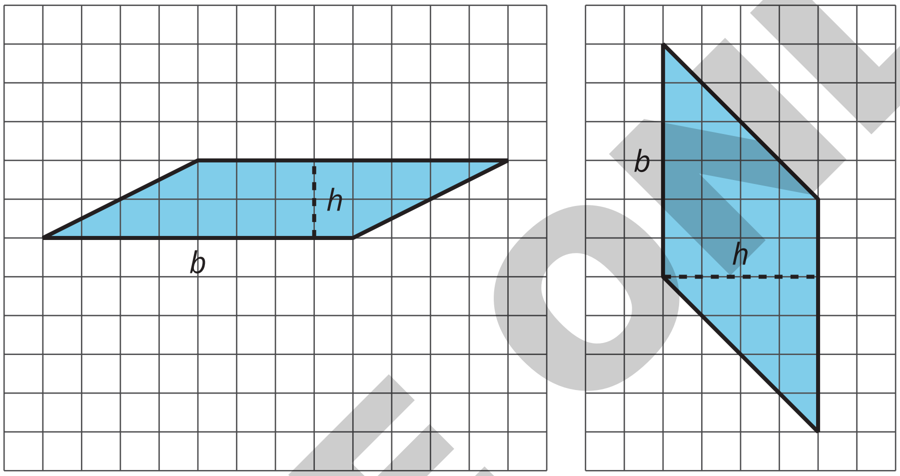
rectangles, reason about the bases and heights, and notice regularity in the process.

Grade 6, Unit 1, Lesson 7 From Parallelograms to Triangles

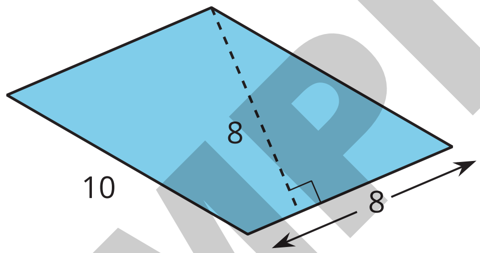
Lesson 6 Summary

Any pair of a base and a corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily found than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

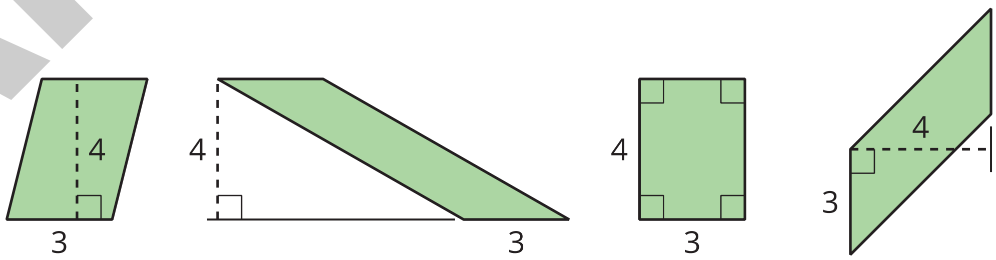


When a parallelogram is *not* drawn on a grid, we can still find its area if we know a base and a corresponding height.



In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are 4 different parallelograms with the same pair of base-height measurements.



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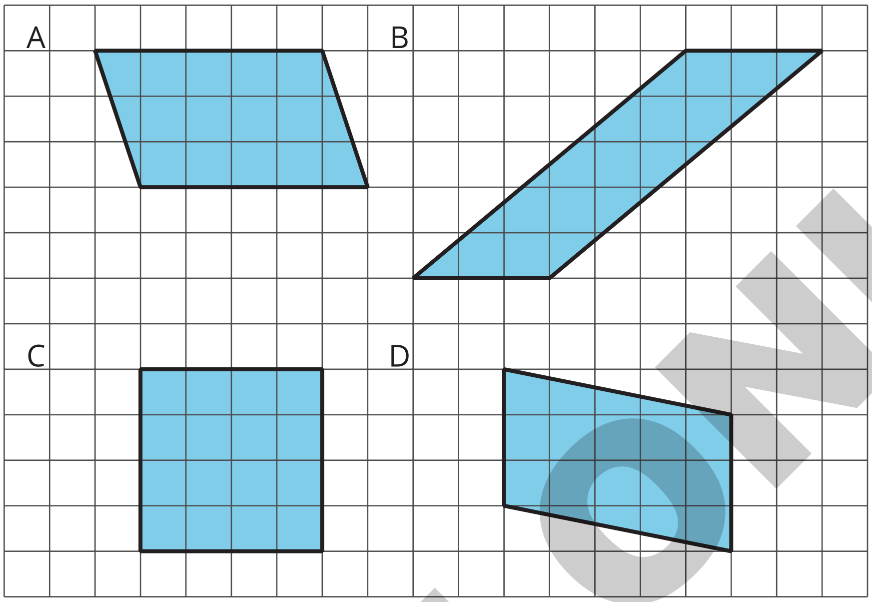
Practice Problems

1 Student Task Statement

Which three of these parallelograms have the same area as each other?



A. A

B. B

C. C

D. D

Solution

A, B, D

2 Student Task Statement

Which pair of base and height produces the greatest area? All measurements are in centimeters.

A. $b = 4$, $h = 3.5$

B. $b = 0.8$, $h = 20$

C. $b = 6$, $h = 2.25$

D. $b = 10$, $h = 1.4$

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Solution

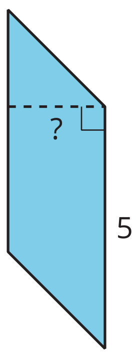
B

3

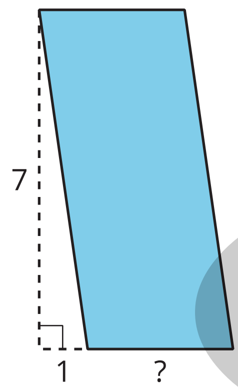
Student Task Statement

Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.

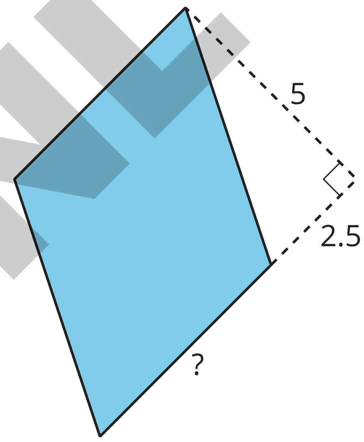
A: 10 square units



B: 21 square units



C: 25 square units



Solution

A: 2 units

B: 3 units

C: 5 units

4

Student Task Statement

The Dockland Building in Hamburg, Germany is shaped like a parallelogram.

If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

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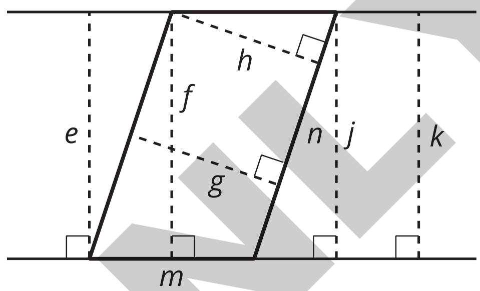
Solution

4,730 square meters ($86 \cdot 55 = 4,730$).

5 from Unit 1, Lesson 5

Student Task Statement

Select **all** segments that could represent a corresponding height if the side m is the base.



A. e

B. f

C. g

D. h

E. j

F. k

G. n

Solution

A, B, E, F

Sec B

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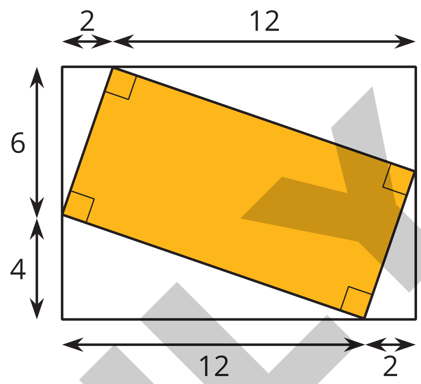
6 from Unit 1, Lesson 3

Student Task Statement

Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.



Solution

80 square centimeters. Sample reasoning: The area of the large rectangle is 140 square centimeters, because $14 \cdot 10 = 140$. The areas of the small, unshaded right triangles are each 6 square centimeters, because $6 \cdot 2 \div 2 = 6$. The areas of the larger, unshaded right triangles are each 24 square centimeters, because $4 \cdot 12 \div 2 = 24$. Subtracting the areas of the four unshaded right triangles from the area of the large rectangle gives 80: $140 - 6 - 6 - 24 - 24 = 80$.

SAMPLE ONLY

Section C: Triangles and Other Polygons

Goals

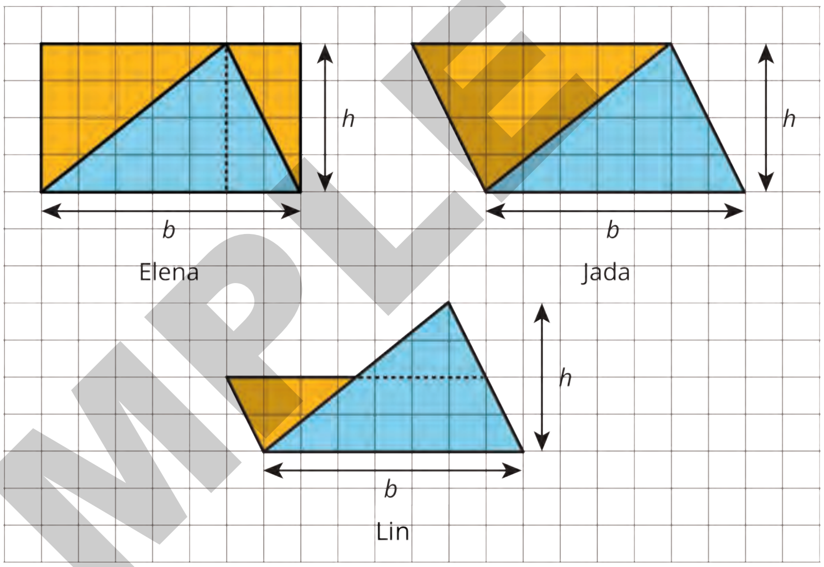
- Find the area of a polygon by decomposing it into parallelograms and triangles.
- Understand bases and heights in a triangle and recognize base-height pairs to use to find the area of a triangle.
- Understand why the process of finding the area of a triangle can be abstracted as $\frac{1}{2} \cdot b \cdot h$ (or equivalent) and apply the formula to find the area of a triangle.

Section Narrative

In this section, students explore ways to find areas of triangles, generalize their observations as a formula, and use the formula to find the area of any triangle. They also apply their insights regarding triangles and parallelograms to find areas of other polygons.

Students begin by investigating the relationship between triangles and parallelograms. They see that a parallelogram can always be decomposed into two identical triangles. Likewise, two copies of any triangle can be composed into a parallelogram.

Next, students learn that triangles also have bases and heights, which correspond to those in a related parallelogram. They observe that the area of a triangle is half that of a related parallelogram that shares the same base and height. Students generalize their observations with the expression $\frac{1}{2} \cdot b \cdot h$ and use it to solve problems.



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Teacher Reflection Questions

- **Math Content and Student Thinking:** What connections are important for students to make between finding the area of a parallelogram and finding the area of a triangle? What about connections between finding the area of a triangle and the area of a polygon?
- **Pedagogy:** How have you used the formative assessment data from cool-downs to adjust instruction? How did those adjustments affect student learning?
- **Access and Equity:** What makes someone want to persist through a difficult math problem? In what ways are you making assumptions about which of your students are resilient, motivated, or persistent with their mathematics?

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Section C Checkpoint

1

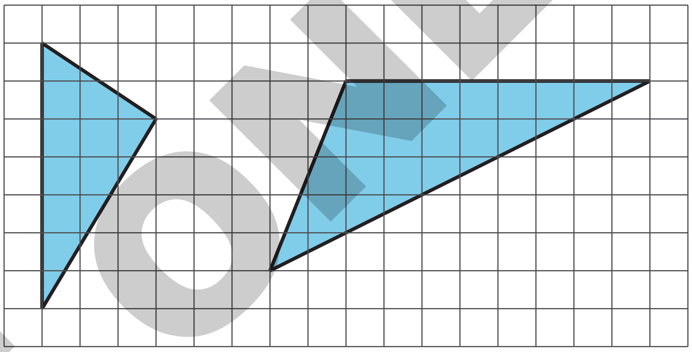
Goals Assessed

- Understand bases and heights in a triangle and recognize base-height pairs to use to find the area of a triangle.

Student Task Statement

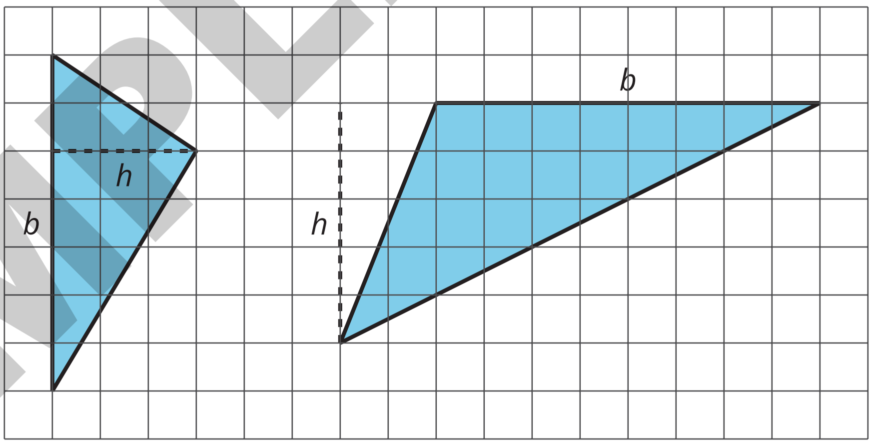
Identify a base and a height that you can use to find the area of each triangle. (You don't have to actually find the areas.)

- Label each base with "b."
- Draw a segment for each height and label it with "h."



Solution

Sample response:



SAMPLE ONLY

2

Goals Assessed

- Understand bases and heights in a triangle and recognize base-height pairs to use to find the area of a

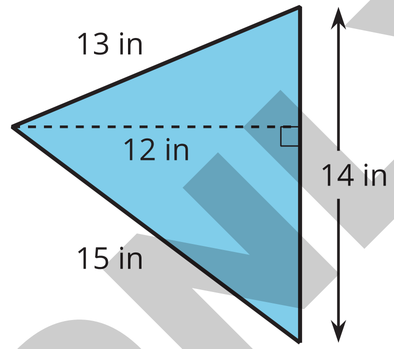
Sec C

triangle.

- Understand why the process of finding the area of a triangle can be abstracted as $\frac{1}{2} \cdot b \cdot h$ (or equivalent) and apply the formula to find the area of a triangle.

Student Task Statement

Find the area of the triangle. Explain or show your reasoning.



Sec C

Solution

84 square inches. Sample reasoning: If the side that is 14 inches long is the base, its corresponding height is 12 inches. $\frac{1}{2} \cdot 14 \cdot 12 = \frac{1}{2} \cdot 168 = 84$

3

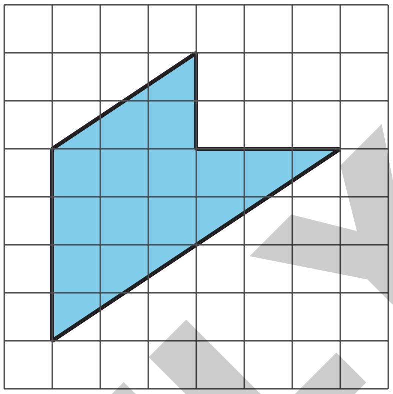
Goals Assessed

- Find the area of a polygon by decomposing it into parallelograms and triangles.
- Understand bases and heights in a triangle and recognize base-height pairs to use to find the area of a triangle.
- Understand why the process of finding the area of a triangle can be abstracted as $\frac{1}{2} \cdot b \cdot h$ (or equivalent) and apply the formula to find the area of a triangle.

SAMPLE ONLY

Student Task Statement

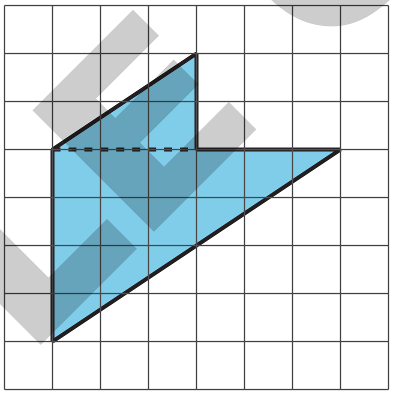
Find the area of the shaded polygon in square units. Show your reasoning.



Solution

15 square units. Sample reasoning:

- The polygon can be decomposed into two right triangles.
- The area of the small triangle is half of a 3-by-2 rectangle, which is 3 square units. The area of the large triangle is half of a 6-by-4 rectangle, which is 12 square units. $3 + 12 = 15$



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Unit 1, Lesson 7

From Parallelograms to Triangles

Goals

- Describe (orally and in writing) ways in which two identical triangles can be composed, i.e., into a parallelogram or into a rectangle.
- Show how any parallelogram can be decomposed into two identical triangles by drawing a diagonal, and generalize (in writing) that this property applies to all parallelograms, but not all quadrilaterals.

Learning Targets

- I can explain the special relationship between a pair of identical triangles and a parallelogram.

Lesson Narrative

This lesson prepares students to apply what they know about the area of parallelograms to reason about the area of triangles.

Highlighting the relationship between triangles and parallelograms is a key goal of this lesson. The activities make use of both the idea of *decomposition* (of a quadrilateral into triangles) and *composition* (of two triangles into a quadrilateral). The two-way study is designed to help students view and reason about the area of a triangle differently and to look for structure (MP7). Students see that a parallelogram can always be decomposed into two identical triangles, and that any two identical triangles can always be composed into a parallelogram.

Because a lot happens in this lesson and timing might be tight, it is important to both prepare all the materials and consider grouping arrangements in advance.

Standards

Addressing	**6.G.A.1**
Building Towards	**6.G.A.1**

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials To Gather

- Geometry toolkits: Activity 1, Activity 2
- Rulers: Activity 2

Materials To Copy

- A Tale of Two Triangles (Part 2) Cutouts (1 copy for every 3 students): Activity 3

Required Preparation

Activity 3:

Print pairs of triangles from the blackline master. If students are cutting out the triangles, use the first page only. If the triangles are to be pre-cut by the teacher, print the second and third pages. Prepare enough sets so that each group of 3–4 students has a complete set of 6 pairs of triangles labeled P, Q, R, S, T, and U (2 copies of each).

For the digital version of the activity, acquire devices that can run the applet.

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Student Facing Learning Goals

Let's compare parallelograms and triangles.

7.1 Same Parallelograms, Different Bases

5 mins

Warm-up

Activity Narrative

This *Warm-up* reinforces students' understanding of bases and heights in a parallelogram. In previous lessons, students calculated areas of parallelograms using bases and heights. They have also determined possible bases and heights of a parallelogram given a whole-number area. They saw, for instance, that finding possible bases and corresponding heights of a parallelogram with an area of 20 square units means finding two numbers with a product of 20. Students extend that work here by working with decimal side lengths and area.

As students work, notice students who understand that the two identical parallelograms have equal area and who use that understanding to find the unknown base. Ask them to share later.

Standards

Addressing **6.G.A.1**

Launch

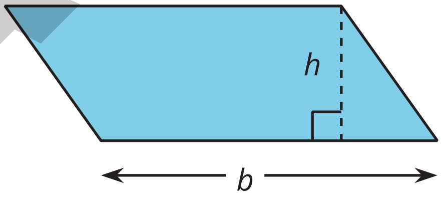
Give students 2 minutes of quiet work time and access to their geometry toolkits.

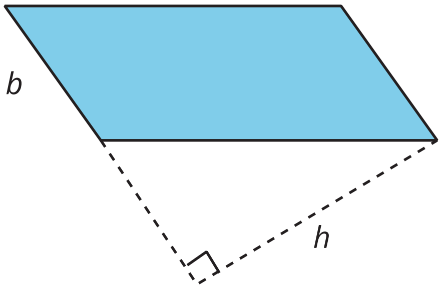
Students should be adequately familiar with bases and heights to begin the warm-up. If needed, however, briefly review the relationship between a pair of base and height in a parallelogram, using questions such as:

- "Can we use any side of a parallelogram as a base?" (Yes.)
- "Is the height always the length of one of the sides of the parallelogram?" (No.)
- "Once we have identified a base, how do we identify a height?" (It can be any segment that is perpendicular to the base and goes from the base to the opposite side.)
- "Can a height segment be drawn outside of a parallelogram?" (Yes.)

Student Task Statement

Here are two copies of a parallelogram. Each copy has one side labeled as the base b and a segment drawn for its corresponding height and labeled h.





SAMPLE ONLY

1. The base of the parallelogram on the left is 2.4 centimeters; its corresponding height is 1 centimeter. Find its area in square centimeters.
2. The height of the parallelogram on the right is 2 centimeters. How long is the base of that parallelogram? Explain your reasoning.

Student Response

1. 2.4 square centimeters. $(2.4) \cdot 1 = 2.4$
2. 1.2 centimeters. Sample reasoning: The area of the second parallelogram is also 2.4 square centimeters. Since the base and height must multiply to the same area of 2.4, the base must be 1.2 centimeters because $(1.2) \cdot 2 = 2.4$.

Building on Student Thinking

Some students may not know how to begin answering the questions because measurements are not shown on the diagrams. Ask students to label the parallelograms based on the information in the task statement.

Students may say that there is not enough information to answer the second question because only one piece of information is known (the height). Ask them what additional information might be needed. Prompt them to revisit the task statement and see what it says about the two parallelograms. Ask what they know about the areas of two figures that are identical.

Sec C

Students may know what to do to find the unknown base in the second question but be unsure how to divide a number containing a decimal. Ask them to explain how they would reason about it if the area were a whole number. If they understand that they need to divide the area by 2 (because the height is 2 cm and the area is 2.4 sq cm), encourage them to reason in terms of multiplication, for instance by asking, "2 times what number is 2.4?" Or, urge them to consider dividing using fractions, for instance, by seeing 2.4 as $2\frac{4}{10}$ or $\frac{24}{10}$. Ask, "what is 24 tenths divided by 2?"

Activity Synthesis

Select 1–2 previously identified students to share their responses. If not already explained by students, emphasize that we know the parallelograms have the same area because they are identical, which means that when one is placed on top of the other they would match up exactly.

Before moving on, ask students: "How can we verify that the height we found is correct, or that the two pairs of bases and heights produce the same area?" (We can multiply the values of each pair and see if they both produce 2.4.)

15 mins

7.2 A Tale of Two Triangles (Part 1)

Activity Narrative

In this activity, students are given various quadrilaterals and asked to decompose each into two identical triangles by drawing a line segment. They observe the kinds of quadrilaterals for which this is possible. To check whether the two triangles in a quadrilateral are identical, students trace one triangle on tracing paper and then rotate it to match the other triangle. The process prepares students to see any triangle as occupying half of a parallelogram, and consequently, as having one half of its area.

To generalize about quadrilaterals that can be decomposed into identical triangles, students need to analyze the

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features of the given shapes and look for structure (MP7).

There are a number of geometric observations in this unit that must be taken for granted at this point in students' study of mathematics. This is one of those instances. Students have seen examples of only a parallelogram being decomposable into two copies of the same triangle, or have verified this conjecture through only physical experimentation, but for the time being it can be considered a fact. Starting in grade 8, they will begin to prove some of the observations they have previously taken to be true.

Standards

Building Towards **6.G.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 3–4. Give students access to geometry toolkits and 2 minutes of quiet think time for the first two questions. Then, ask them to share their drawings with their group and discuss how they drew their lines. If group members disagree on whether a quadrilateral can be decomposed into two identical triangles, they should note the disagreement, but it is not necessary to come to an agreement. They will soon have a chance to verify their responses.

Next, ask students to use tracing paper to check that the pairs of triangles that they believe to be identical are indeed so. (If placed on top of one another, two triangles that are identical will match up exactly.) Tell students to divide the checking work among the members of their group to optimize time.

Though students have worked with tracing paper earlier in the unit, some may not recall how to use it to check the congruence of two shapes, so some explicit guidance might be needed. Encourage students to work carefully and precisely. A straightedge can be used in tracing but is not essential and may get in the way.

Once students finish checking the triangles in their list and verify that the triangles are identical (or correct their initial response), ask them to answer the last question.

Access for English Language Learners

MLR2 Collect and Display. Collect the language students use to characterize the quadrilaterals they decomposed and the resulting triangles. Display words or phrases such as: “parallelograms,” “two pairs of parallel sides,” “two pairs of equal sides,” and “the triangles match up exactly.” During the synthesis, invite students to suggest ways to update the display: “What are some other words or phrases we should include?” Invite students to borrow language from the display as needed.
Advances: Conversing, Representing

Access for Students with Disabilities

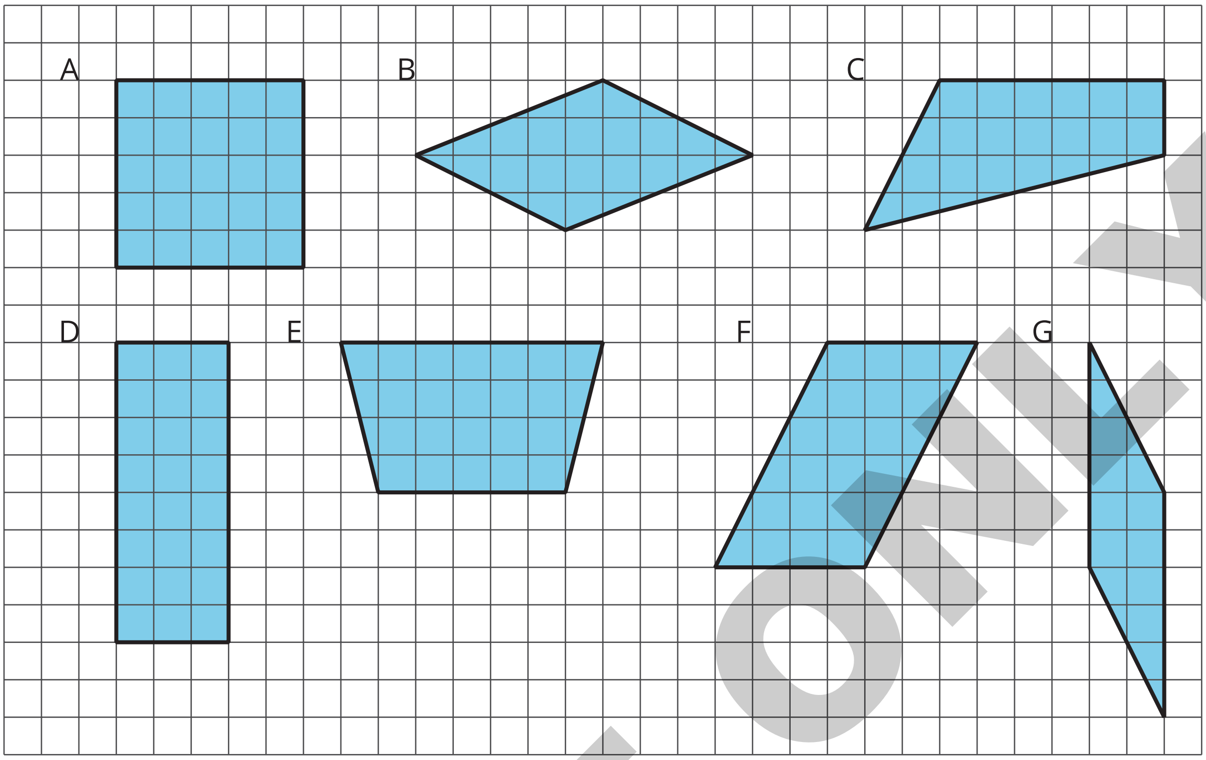
Action and Expression: Provide Access for Physical Action. Give students who need support with fine-motor skills the option of representing the situation in the activity kinesthetically on a larger scale. For example, enlarge the figures using a copier or make new copies on plain or graph-ruled chart paper. Ask students to draw or show how they would decompose the quadrilaterals into two congruent triangles.
Supports accessibility for: Fine Motor Skills, Visual-Spatial Processing

Student Task Statement

Two polygons are identical if they match up exactly when placed one on top of the other.

SAMPLE ONLY

1. Draw *one* line to decompose each polygon into two identical triangles, if possible. Use a straightedge to draw your line.



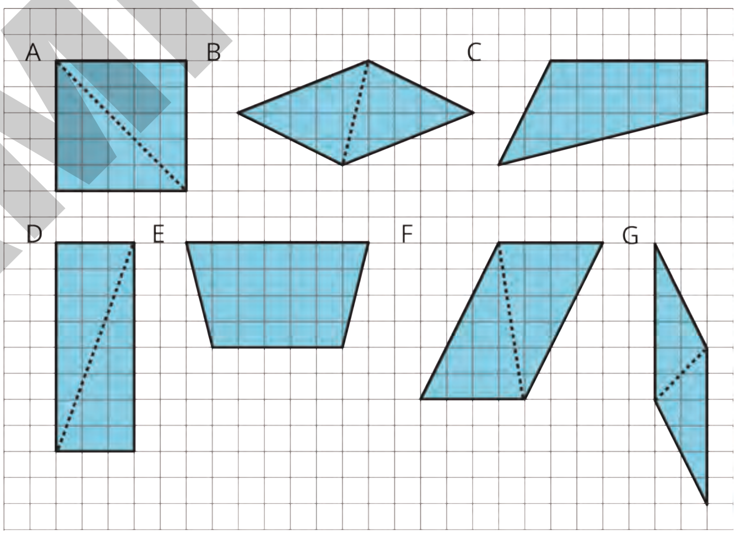
2. Which quadrilaterals can be decomposed into two identical triangles?

   Pause here for a small-group discussion.

3. Study the quadrilaterals that can, in fact, be decomposed into two identical triangles. What do you notice about them? Write a couple of observations about what these quadrilaterals have in common.

Student Response

1. Sample response:



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2. Quadrilaterals A, B, D, F, and G can be decomposed into two identical triangles.
3. Sample responses:
    - They have two pairs of parallel sides and each pair has equal length.
    - They are all parallelograms.
    - The triangles are formed by drawing a diagonal connecting opposite vertices.
    - Some triangles are right triangles, some are acute, and some are obtuse.
    - For some quadrilaterals, there is more than one way to decompose it into two identical triangles.

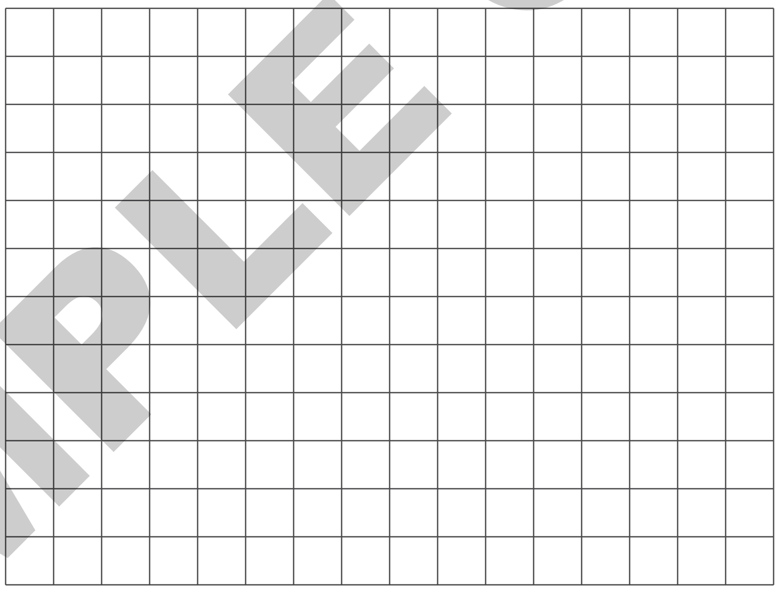
Building on Student Thinking

It may not occur to students to rotate triangles to check congruence. If so, tell students that we still consider two triangles identical even when one needs to be rotated to match the other.

Students may incorrectly generalize that Figure E can be decomposed into two identical triangles because it shares some common characteristics with Figures A, B, and D: It has two sides that are parallel, two sides that are the same length, and two pairs of equal-size angles. Remind students to use the tools at their disposal to verify their thinking.

Are You Ready for More?

On the grid, draw some other types of quadrilaterals that are not already shown. Try to decompose them into two identical triangles. Can you do it?



Come up with a rule about what must be true about a quadrilateral for it to be decomposed into two identical triangles.

Extension Student Response

Answers vary.

SAMPLE ONLY

Activity Synthesis

The discussion should serve two goals: to highlight how quadrilaterals can be decomposed into triangles and to help students make generalizations about the types of quadrilaterals that can be decomposed into two identical triangles. Consider these questions:

- "How did you decompose the quadrilaterals into two triangles?" (Connect opposite vertices by drawing a diagonal.)
- "Did the strategy of drawing a diagonal work for decomposing all quadrilaterals into two triangles?" (Yes) "Are all of the resulting triangles identical?" (No)
- "What is it about C and E that they cannot be decomposed into two identical triangles?" (They don't have equal sides or equal angles. Their opposite sides are not parallel.)
- "What do A, B, and D have that C and E do not?" (A, B, and D have two pairs of parallel sides that are of equal lengths. They are parallelograms.)

Ask students to complete this sentence starter: "For a quadrilateral to be decomposable into two identical triangles it must be (or must have) . . ."

If time permits, discuss how students verified the congruence of the two triangles.

- "How did you check if the triangles are identical? Did you simply stack the traced triangle or did you do something more specific?" (They may notice that it is necessary to rotate one triangle—or to reflect one triangle twice—before the triangles could be matched up.)
- "Did anyone use another way to check for congruence?" (Students may also think in terms of the parts or composition of each triangle. For example, they might say that both triangles have all the same side lengths and have a right angle.)

Sec C

7.3 A Tale of Two Triangles (Part 2)

15 mins

Activity Narrative

There is a digital version of this activity.

In this activity, students compose quadrilaterals using pairs of identical triangles. Previously, students saw that a triangle can be seen as half of a familiar quadrilateral. This activity prompts them to think the other way—to see that two identical triangles of any kind can always be joined to produce a parallelogram. Both explorations prepare students to make connections between the area of a triangle and that of a parallelogram in a subsequent lesson.

A key understanding to uncover here is that two identical copies of a triangle can be joined along any corresponding side to compose a parallelogram. This means more than one parallelogram can be formed by the same pair of triangles.

As students work, look for different compositions of the same pair of triangles. Select students who use different approaches to share.

When manipulating the cutouts, students are likely to notice that right triangles can be composed into rectangles (and sometimes squares) and that non-right triangles produce parallelograms that are not rectangles.

As before, students make generalizations here that they don't yet have the tools to justify. This is appropriate at this stage. Later in their study of mathematics, they will learn to verify what they now take as facts.

In the digital version of the activity, students use an applet to compose pairs of triangles into quadrilaterals. The applet allows students to move and rotate the triangles.

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Standards

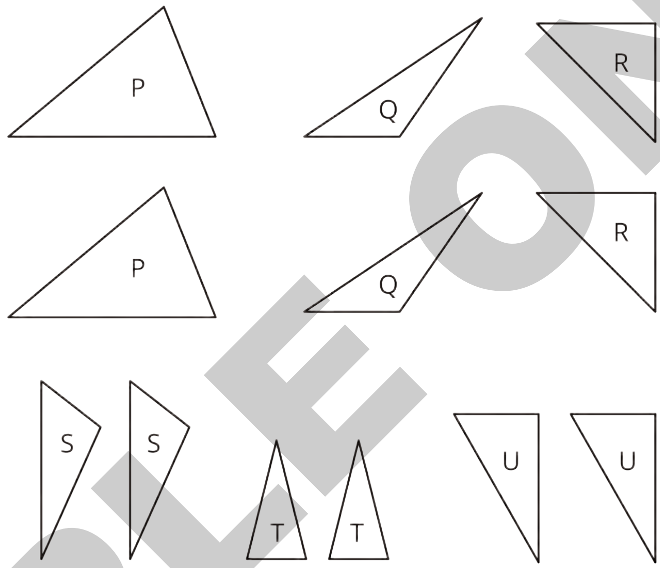
Building Towards **6.G.A.1**

Launch

Keep students in the same groups. Give each group one set (6 pairs) of triangles labeled P, Q, R, S, T, and U from the blackline master and access to scissors if the triangles are not pre-cut. Instruct each group member to take 1–2 pairs of triangles.

Remind students that in the previous activity they saw that certain types of quadrilaterals can be decomposed into two identical triangles. Explain that they will now see if it is possible to compose quadrilaterals out of two identical triangles and, if so, to find out what types of quadrilaterals would result.

Give students 1–2 minutes of quiet work time for the first question. Then, give them 5 minutes to discuss their responses and answer the second question with their group.



Student Task Statement

Your teacher will give your group several pairs of triangles. Each group member should take 1 or 2 pairs.

1. a. Which pair(s) of triangles do you have?

   b. Can each pair be composed into a rectangle? A parallelogram?

2. Discuss with your group your responses to the first question. Then, complete each statement with *All*, *Some*, or *None*. Sketch 1 or 2 examples to illustrate each completed statement.

   a. ______________ of these pairs of identical triangles can be composed into a *rectangle*.

   b. ______________ of these pairs of identical triangles can be composed into a *parallelogram*.

Student Response

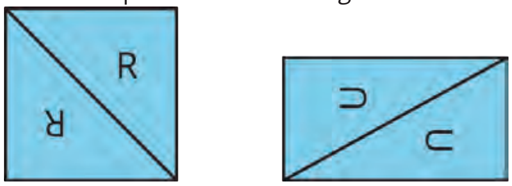
1. a. Answers vary.

   b. Sample responses:

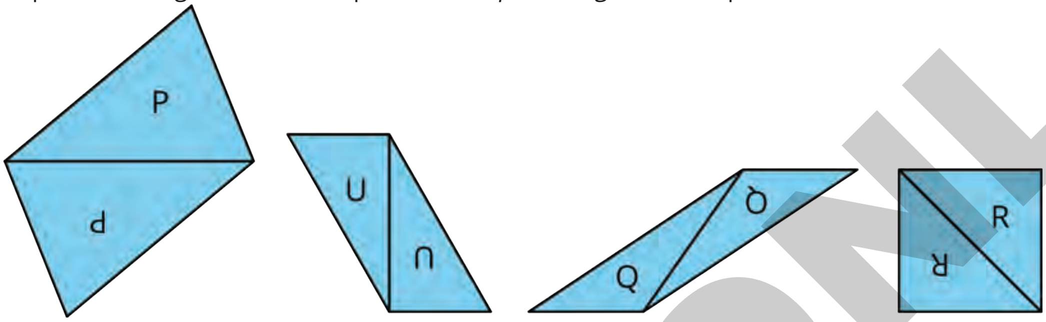
SAMPLE ONLY

- Yes for triangles R and U, no for the rest
- Yes for all triangles

2. a. *Some* of these pairs of triangles can be composed into a *rectangle*.



   b. *All* of these pairs of triangles can be composed into a *parallelogram*. Examples:



Sec C

Building on Student Thinking

Students may draw incorrect conclusions if certain pieces of their triangles are turned over (to face down), or if it did not occur to them that the pieces could be moved. Ask them to try manipulating the pieces in different ways.

Seeing that two copies of a triangle can always be composed into a parallelogram, students might mistakenly conclude that any two copies of a triangle can *only* be composed into a parallelogram (that is, no other quadrilaterals can be formed from joining two identical triangles). Showing a counterexample may be a simple way to help students see that this is not the case.

Activity Synthesis

The focus of this discussion would be to clarify whether or not two copies of each triangle can be composed into a rectangle or a parallelogram, and to highlight the different ways in which two triangles could be composed into a parallelogram.

Ask a few students who composed different parallelograms from the same pair of triangles to share. Invite the class to notice how these students ended up with different parallelograms. To help them see that a triangle can be joined along any side of its copy to produce a parallelogram, ask questions such as:

- “Here is one way of composing Triangles S into a parallelogram. Did anyone else do it this way? Did anyone obtain a parallelogram in a different way?”
- “How many different parallelograms can be created with any two copies of a triangle? Why?” (3 ways, because there are 3 sides along which the triangles could be joined.)
- “What kinds of triangles can be used to compose a rectangle? How?” (Right triangles, by joining two copies along the side opposite the right angle.)
- “What kinds of triangles can be used to compose a parallelogram? How?” (Any triangle, by joining two copies along any sides with the same length.)

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Lesson Synthesis

Display and revisit representative works from the two main activities. Draw out key observations about the special connections between triangles and parallelograms.

First, students tried to decompose or break apart quadrilaterals into two identical triangles. Consider asking students:

- "What strategy allowed us to do that?" (Drawing a segment connecting opposite vertices.)
- "Which types of quadrilaterals could always be decomposed into two identical triangles?" (Parallelograms.)
- "Can quadrilaterals that are not parallelograms be decomposed into triangles?" (Yes, but the resulting triangles may not be identical.)

Then, students explored the relationship between triangles and quadrilaterals the other way around: by composing quadrilaterals from pairs of identical triangles. Consider asking students:

- "What types of quadrilaterals were you able to compose with a pair of identical triangles?" (Parallelograms—some of them are rectangles.)
- "Does it matter which triangles were used?" (No. Any two copies of a triangle could be composed into a parallelogram.)
- "Was there a particular side along which the two triangles must be joined to form a parallelogram?" (No. Any of the three sides could be used as long as the sides match.)

Emphasize that two identical copies of a triangle can be combined to make a parallelogram. This is true for any triangle. The reverse is also true: any parallelogram can be split into two identical triangles.

Tell students that in grade 8 they will acquire some tools to prove these observations. For now, they will take the special relationships between triangles and parallelograms as a fact and use those relationships to find the area of any triangle.

7.4 A Tale of Two Triangles (Part 3)

Cool-down

5 mins

Standards

Building Towards **6.G.A.1**

Launch

Give students access to their geometry toolkits if needed.

Student Task Statement

1. Here are some quadrilaterals.

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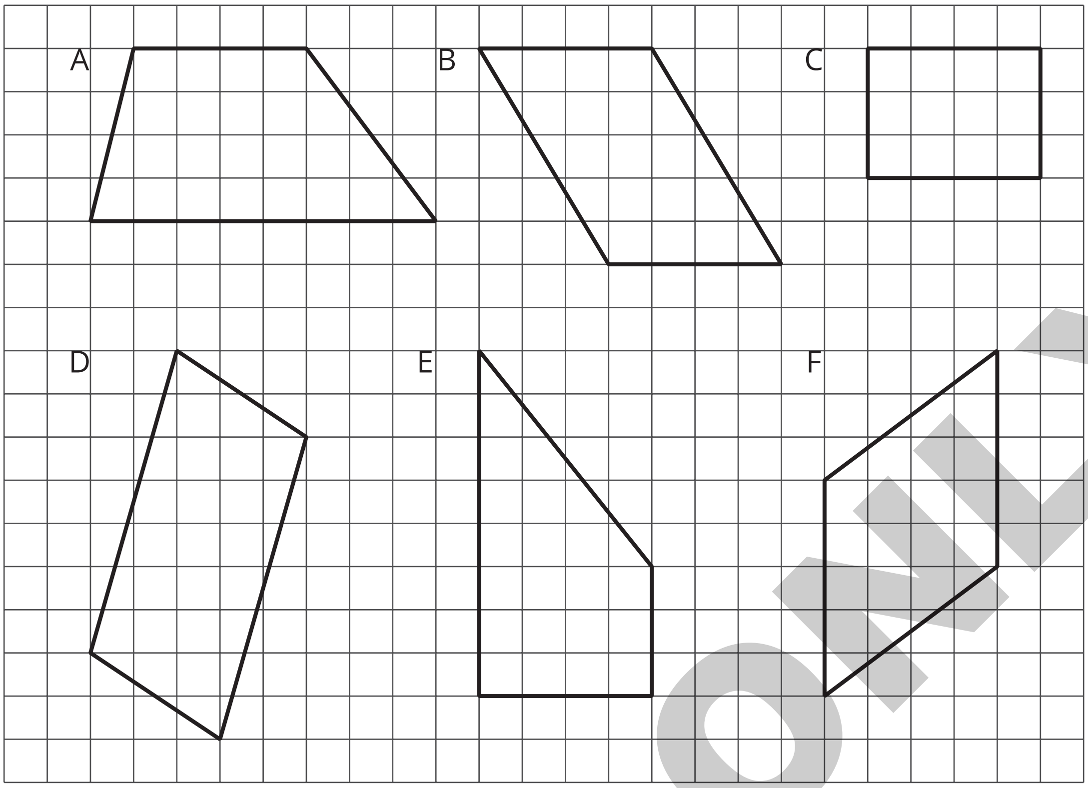
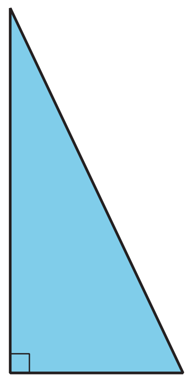


a. Circle all quadrilaterals that you think can be decomposed into two identical triangles using only one line.

b. What characteristics do the quadrilaterals that you circled have in common?

2. Here is a right triangle. Show or briefly describe how two copies of it can be composed into a parallelogram.



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Student Response

1. a. Quadrilaterals B, C, D, and F should be circled.
   b. They all have two pairs of parallel sides. They are all parallelograms.
2. Sample response: Joining two copies of the triangle along a side that is the same length (for instance, the shortest side of one and the shortest side of the other) would make a parallelogram. (Three parallelograms are possible, since there are three sides at which the triangles could be joined. One of the parallelograms is a rectangle.)

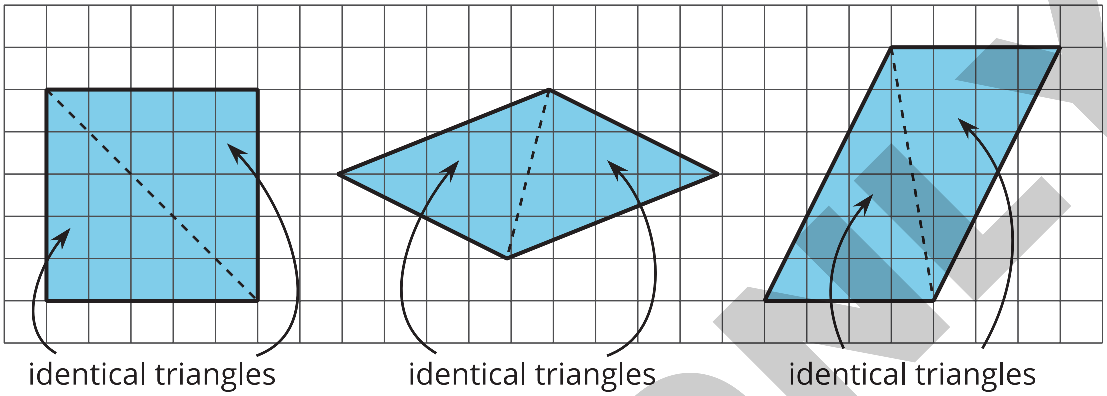
Responding To Student Thinking

More Chances

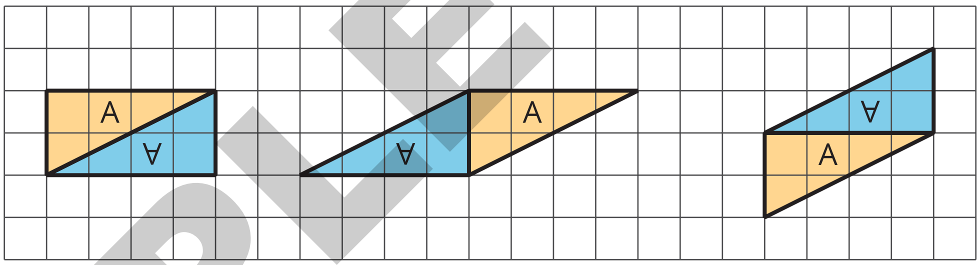
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

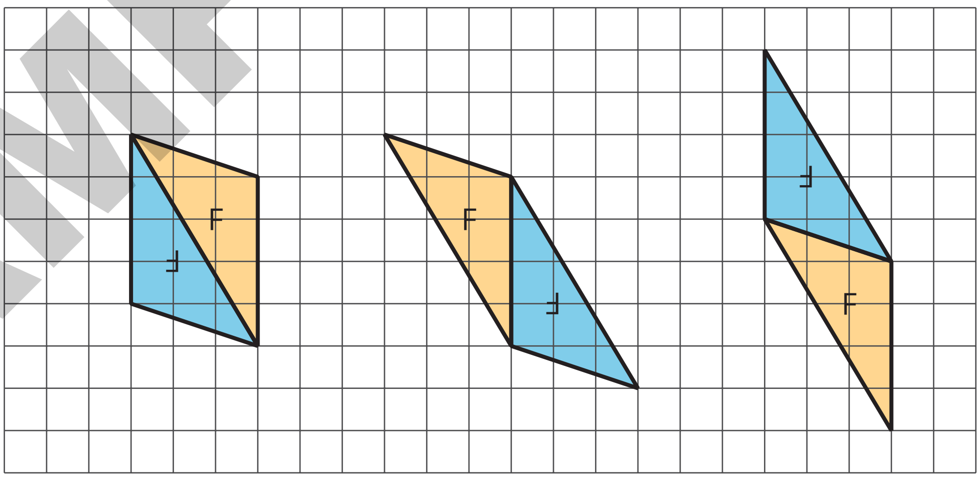
Lesson 7 Summary

A parallelogram can always be decomposed into two identical triangles by a segment that connects opposite vertices.



Going the other way around, two identical copies of a triangle can always be arranged to form a parallelogram, regardless of the type of triangle being used. To produce a parallelogram, we can join a triangle and its copy along any of the three sides that match, so the same pair of triangles can make different parallelograms. Here are examples of how two copies of both Triangle A and Triangle F can be composed into three different parallelograms.





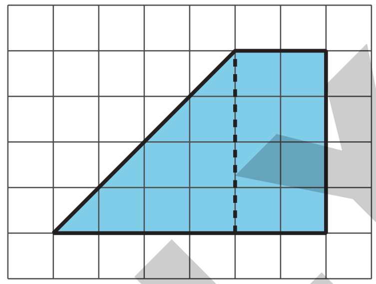
This special relationship between triangles and parallelograms can help us reason about the area of any triangle.

SAMPLE ONLY

Practice Problems

1 Student Task Statement

To decompose a quadrilateral into two identical shapes, Clare drew a dashed line as shown in the diagram.



a. She said that the two resulting shapes have the same area. Do you agree? Explain your reasoning.

b. Did Clare partition the figure into two identical shapes? Explain your reasoning.

Solution

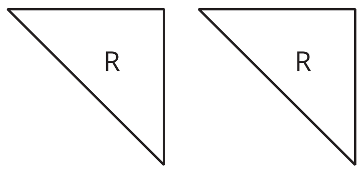
a. Agree. Sample reasoning: The rectangle is 2 units by 4 units, so it has an area of 8 square units. The triangle is half of a 4-by-4 square, so its area is also 8 square units.

b. No. Sample reasoning: Although the shapes have the same area, they are not identical shapes—one is a rectangle and the other is a triangle.

2 Student Task Statement

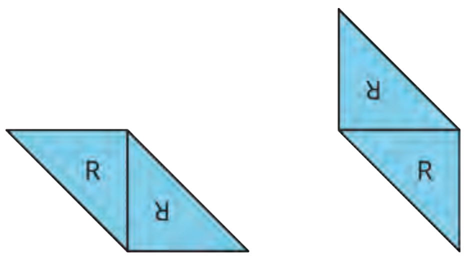
Triangle R is a right triangle. Can we use two copies of Triangle R to compose a parallelogram that is not a square?

If so, explain how or sketch a solution. If not, explain why not.



Solution

Yes. Sample reasoning: We can use two right triangles R to compose a parallelogram that is not a square by joining them along one of the shorter sides (the sides that make the right angle).

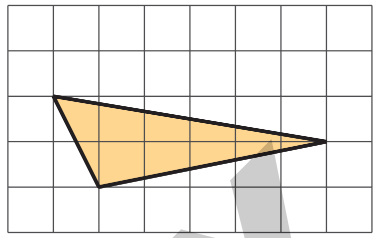


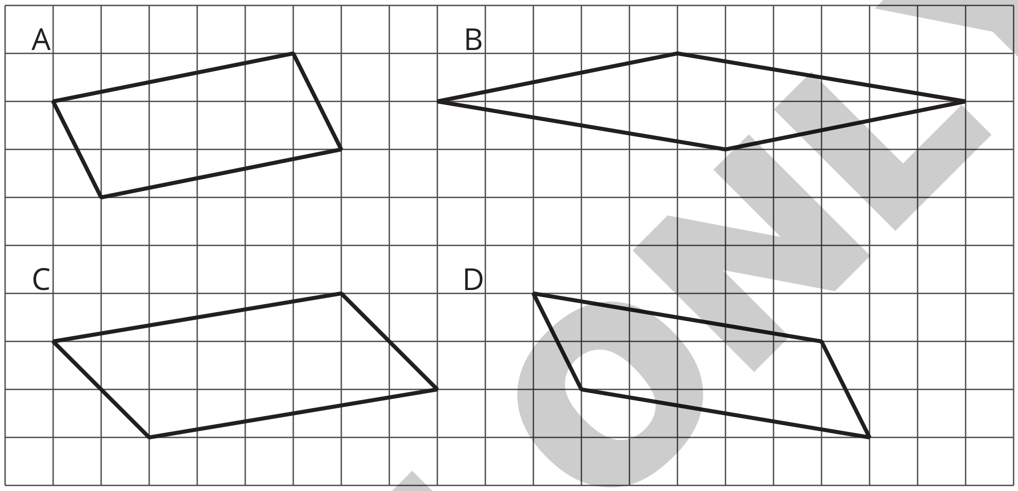
SAMPLE ONLY

3 Student Task Statement

Two copies of this triangle are used to compose a parallelogram.

Which parallelogram *cannot* be a result of the composition? If you get stuck, consider using tracing paper.





A. A

B. B

C. C

D. D

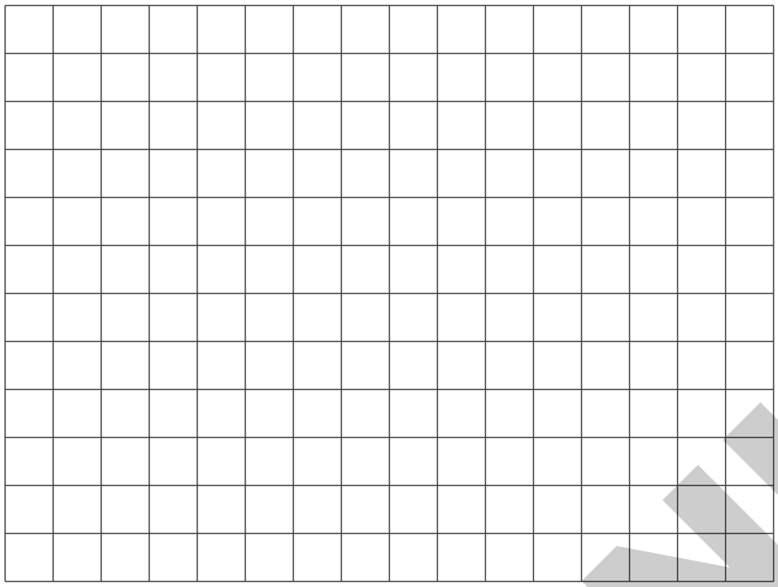
Solution

C

4 Student Task Statement

a. On the grid, draw at least three different quadrilaterals that can each be decomposed into two identical triangles with a single cut (show the cut line). One or more of the quadrilaterals should have non-right angles.

SAMPLE ONLY



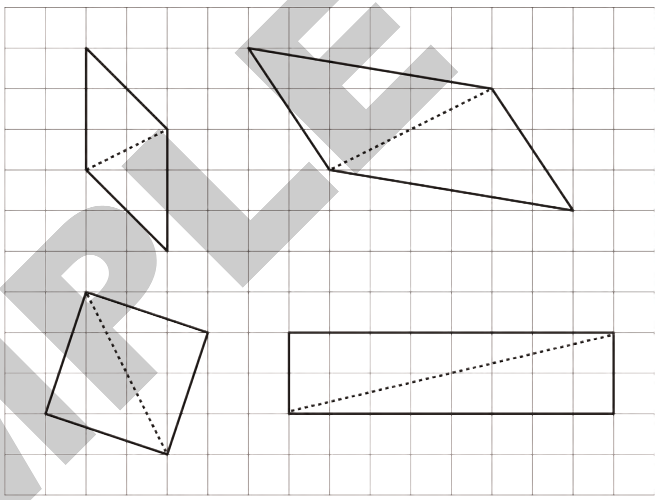
b. Identify the type of each quadrilateral.

Sec C

Solution

Sample responses:

a.



b. The top two are parallelograms. The bottom left one is a square. The bottom right one is a rectangle. (All of them are parallelograms.)

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5 from Unit 1, Lesson 6

Student Task Statement

a. A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?

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b. A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?

c. A parallelogram has an area of 7 square units. If the height that corresponds to a base is $\frac{1}{4}$ unit, what is the base?

Solution

a. $\frac{18}{3}$ square units (or equivalent)

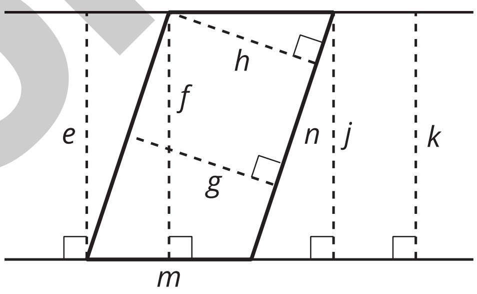
b. $\frac{12}{9}$ units (or equivalent)

c. 28 units

6 from Unit 1, Lesson 5

Student Task Statement

Select **all** the segments that could represent the height if side n is the base.



A. e

B. f

C. g

D. h

E. m

F. n

G. j

H. k

Solution

C, D

SAMPLE ONLY

Area of Triangles

Goals

- Draw a diagram to show that the area of a triangle is half the area of an associated parallelogram.
- Explain (orally and in writing) strategies for using the base and height of an associated parallelogram to determine the area of a triangle.

Learning Targets

- I can use what I know about parallelograms to reason about the area of triangles.

Lesson Narrative

In this lesson, students reason about areas of triangles. They see that they can find the area of a triangle by applying strategies such as decomposing and rearranging, or enclosing and subtracting. They can also use the relationship between parallelograms and triangles.

Students observe that the area of a triangle is half of the area of a parallelogram that shares the same base as the triangle and has the same height. Students arrive at this observation by:

- Recalling that two copies of a triangle can be composed into a parallelogram
- Reasoning about one or more rectangles that have the same height as the triangle.

An optional activity is included to help students make another, related observation: that a triangle can be decomposed and rearranged into a parallelogram that shares the same base but is half the triangle's height.

In making these observations and applying them to find the areas of triangles on and off a grid, students practice looking for and making use of structure (MP7).

A note about terminology:

At this point, students have not yet learned about bases and height in a triangle. They are not expected to use these terms when referring to measurements used to find the area of a triangle or when describing the connections between a triangle and a related parallelogram. While students may use the terms intuitively, the meanings of a triangle's base and height will be formalized in the next lesson.

Math Community
Today, students use sticky notes to document actions in the "Doing Math" sections of the Math Community Chart that they see or hear throughout the lesson. During the *Lesson Synthesis,* students share what they noticed, and then they suggest additions for the chart as part of the *Cool-down.* The work today continues to build a foundation for developing math community norms in a later exercise and is the start of students identifying strengths in the actions of their peers.

Standards

Addressing **6.G.A.1**

Instructional Routines

- 5 Practices
- MLR7: Compare and Connect
- Notice and Wonder

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Required Materials

Materials To Gather

- Math Community Chart: Lesson
- Geometry toolkits: Activity 1, Activity 2
- Math Community Chart: Activity 1
- Sticky notes: Activity 1
- Glue or glue sticks: Activity 3
- Tape: Activity 3

Materials To Copy

- Decomposing a Parallelogram Cutouts (1 copy for every 4 students): Activity 3

Required Preparation

Activity 3:

Each copy of the blackline master contains two copies of each of Parallelograms A, B, C, and D. Prepare enough copies so that each student receives two copies of a parallelogram.

Students need access to tape *or* glue; it is not necessary to have both.

Student Facing Learning Goals

Let's use what we know about parallelograms to find the area of triangles.

8.1 Composing Parallelograms

Warm-up

10 mins

Activity Narrative

This *Warm-up* has two aims: to solidify what students learned about the relationship between triangles and parallelograms and to connect their new insights back to the concept of area.

Students are given a right triangle and the three parallelograms that can be composed from two copies of the triangle. Though students are not asked to find the area of the triangle, they may make some important observations along the way. They are likely to see that:

- The triangle covers half of the region of each parallelogram.
- The base-height measurements for each parallelogram involve the numbers 6 and 4, which are the lengths of two sides of the triangle.
- All parallelograms have the same area of 24 square units.

These observations enable them to reason that the area of the triangle is half of the area of a parallelogram (in this case, any of the three parallelograms can be used to find the area of the triangle). In upcoming work, students will test and extend this awareness, generalizing it to help them find the area of any triangle.

Standards

Addressing **6.G.A.1**

Instructional Routines

- Notice and Wonder

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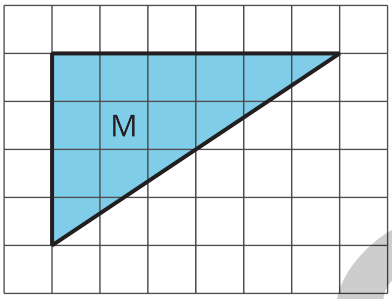
Launch

Arrange students in groups of 2. Display the image of the three parallelograms for all to see. Ask students to think of at least one thing that they notice and at least one thing that they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things that they notice and wonder.

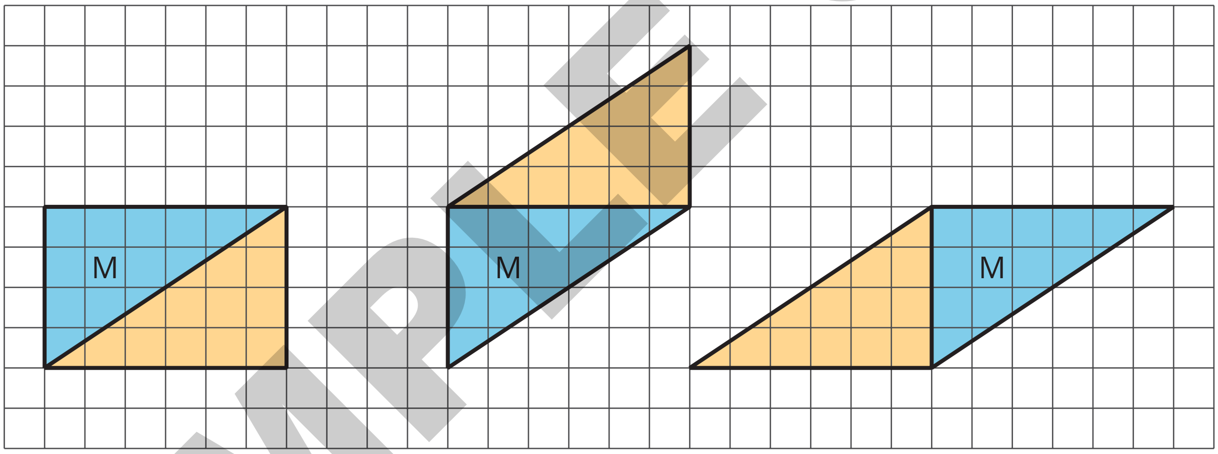
Give students 2–3 minutes of quiet time to complete the activity and access to their geometry toolkits. Follow with a whole-class discussion.

Student Task Statement

Here is Triangle M.



Han made a copy of Triangle M and composed three different parallelograms using the original M and the copy, as shown here.



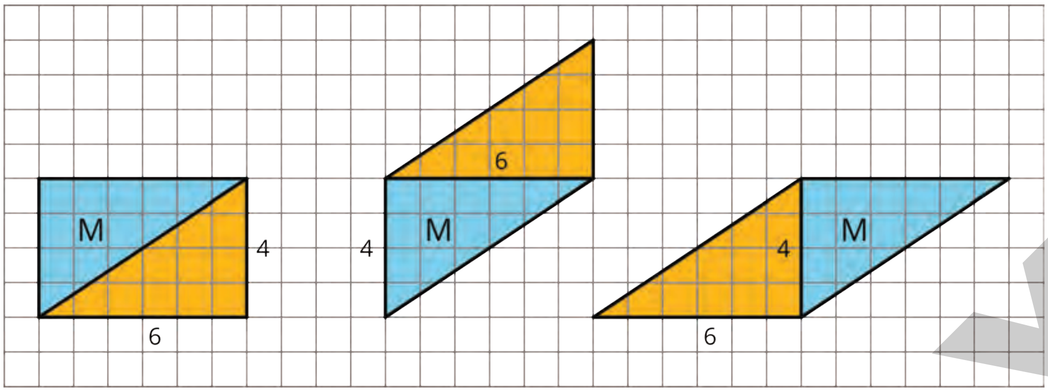
1. For each parallelogram that Han composed, identify a base and a corresponding height, and write the measurements on the drawing.
2. Find the area of each parallelogram that Han composed. Show your reasoning.

Student Response

1. First parallelogram: $b = 6$ and $h = 4$, second parallelogram: $b = 4$ and $h = 6$, third parallelogram: $b = 6$ and $h = 4$

SAMPLE ONLY

Sec C



2. The area of each parallelogram is 24 square units. Sample reasoning: The base and height measurements for the parallelograms are 4 units and 6 units, or 6 units and 4 units. $4 \cdot 6 = 24$ and $6 \cdot 4 = 24$.

Building on Student Thinking

When identifying bases and heights of the parallelograms, some students may choose a non-horizontal or non-vertical side as a base and struggle to find its length and the length of its corresponding height. Ask them to see if there's another side that could serve as a base and has a length that can be more easily determined. Clarify that we can use the grid to measure a length only if the segment is parallel to the grid lines.

Students may not immediately recall that squares and rectangles are also parallelograms. Prompt them to recall the defining characteristics of parallelograms, by asking: "What makes a figure a parallelogram? What are its characteristics?"

Activity Synthesis

Ask one student to identify the base, height, and area of each parallelogram, as well as how they reasoned about the area. If not already answered by students in their explanations, discuss the following questions:

- "Why do all the pictured parallelograms have the same area even though they all have different shapes?" (They are composed of the same parts—two copies of the same right triangles. They have the same pair of numbers for their base and height. They all can be decomposed and rearranged into a 6-by-4 rectangle.)
- "What do you notice about the bases and heights of the parallelograms?" (They are the same pair of numbers.)
- "How are the base-height measurements related to the right triangle?" (They are the lengths of two sides of the right triangles.)
- "Can we find the area of the triangle? How?" (Yes, the area of the triangle is 12 square units because it is half of the area of every parallelogram, which is 24 square units.)

Math Community
After the *Warm-up*, display the revised Math Community Chart created from student responses in Exercise 3. Tell students that today they are going to monitor for two things:

- "Doing Math" actions from the chart that they see or hear happening.
- "Doing Math" actions that they see or hear that they think should be added to the chart.

Provide sticky notes for students to record what they see and hear during the lesson.

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8.2 More Triangles

25 mins

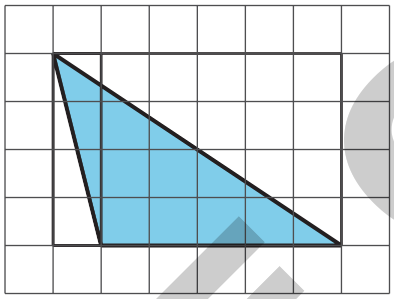
Activity Narrative

In this activity, students apply what they have learned to find the area of various triangles. They use reasoning strategies and tools that make sense to them. Students are not expected to use a formal procedure or to make a general argument. They will think about general arguments in an upcoming lesson.

Monitor for different ways of reasoning about the area. Here are some paths that students may take, from more elaborate to more direct:

- Draw two smaller rectangles that decompose the given triangle into two right triangles. Find the area of each rectangle and take half of its area. Add the areas of the two right triangles. (This is likely used for B and D.)

  For Triangle C, some students may choose to draw two rectangles around and on the triangle (as shown here), find half of the area of each rectangle, and *subtract* one area from the other.



- Enclose the triangle with one rectangle, find the area of the rectangle, and take half of that area. (This is likely used for right triangle A.)
- Duplicate the triangle to form a parallelogram, find the area of the parallelogram, and take half of its area. (This is likely used with any triangle.)

Standards

Addressing **6.G.A.1**

Instructional Routines

- 5 Practices

Launch

Tell students that they will now apply their observations from the past few activities to find the area of several triangles.

Arrange students in groups of 2–3. Give students 6–8 minutes of quiet work time and a few more minutes to discuss their work with a partner. Ask them to confer with their group only after each person has attempted to find the area of at least two triangles. Provide access to their geometry toolkits (especially tracing paper).

Select students who reasoned in different ways to share later.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for finding the areas of the triangles before they begin. Students can speak quietly to themselves, or share with a partner. *Supports accessibility for: Organization, Conceptual Processing, Language*

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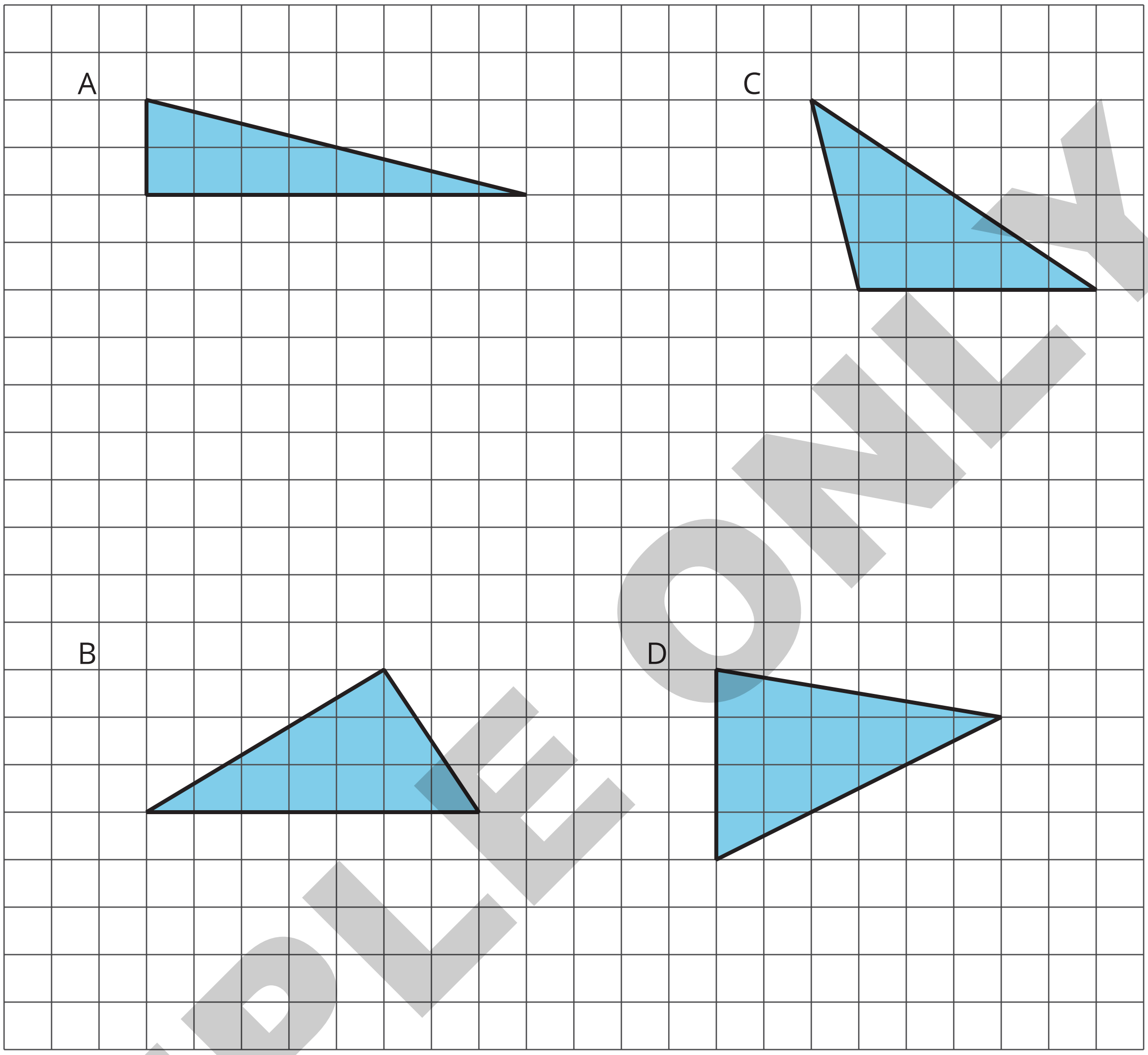
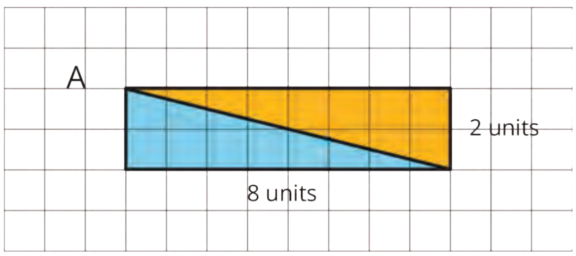
Student Task Statement

Find the areas of at least two of these triangles. Show your reasoning.



Student Response

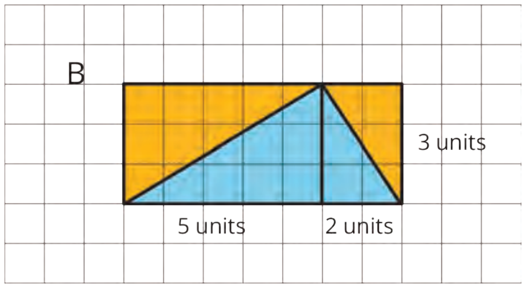
A: 8 square units. Sample reasoning: $8 \cdot 2 = 16$, so the area of the rectangle is 16 square units. The area of the triangle is half of that of the rectangle, so it is 8 square units.



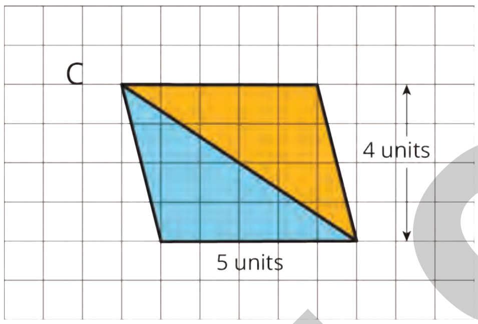
B: 10.5 square units. Sample reasoning: $5 \cdot 3 = 15$, so the area of the left rectangle is 15 square units. The area of the left triangle is then 7.5 square units. $2 \cdot 3 = 6$, so the area of the right rectangle is 6 square units and the area of the

SAMPLE ONLY

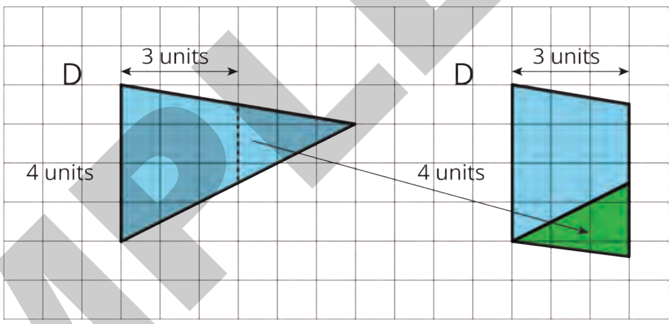
right triangle is 3 square units. The sum of the areas of the small triangles which make up the large triangle is $7.5 + 3 = 10.5$, so the large triangle has an area of 10.5 square units.



C: 10 square units. Sample reasoning: If we make a copy of the triangle, rotate it, and join them along the longest side we would get a parallelogram. The base length is 5 units and the height is 4 units, so the area of the parallelogram is 20 square units. The area of the triangle is half of that area, so it is 10 square units.



D: 12 square units. Sample reasoning: Decompose the triangle into a trapezoid and a small triangle by drawing a vertical line 3 units from the left side. Rotate the small triangle to line up with the bottom side of the trapezoid to create a parallelogram. To get the area of that parallelogram: $4 \cdot 3 = 12$.



Building on Student Thinking

At this point students should not be counting squares to determine area. If students are still using this approach, steer them in the direction of recently learned strategies (decomposing, rearranging, enclosing, or duplicating).

Students may not recognize that the vertical side of Triangle D could be the base and try to measure the lengths of the other sides. If so, remind them that any side of a triangle can be the base.

Activity Synthesis

The goal of this discussion is for students to see a wide range of ways to reason about the area of triangles.

Invite previously selected students to share their approach and display their reasoning for all to see. Sequence the

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presentations as shown in the *Activity Narrative*—starting with the most elaborate (most likely a strategy that involves enclosing a triangle) and moving toward the most direct (most likely duplicating the triangle to compose a parallelogram).

Connect the different responses to the learning goals by asking questions such as:

- “Did anyone else reason the same way?”
- “Did anyone else draw the same diagram but think about the problem differently?”
- “Can this strategy be used on another triangle in this set? Which one?”
- “Is there a triangle for which this strategy would *not* be helpful? Which one, and why not?”

8.3 Decomposing a Parallelogram

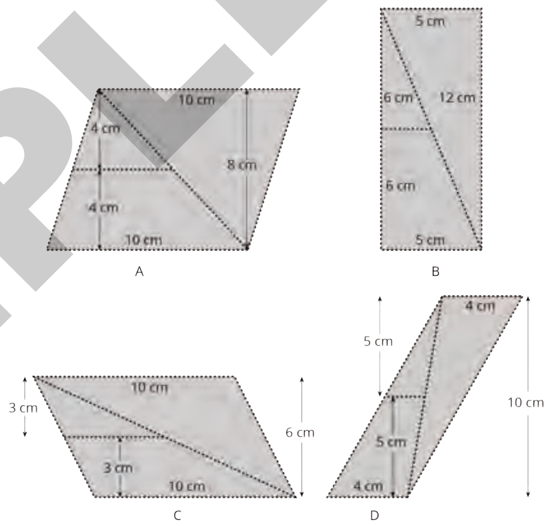
Optional

25 mins

Activity Narrative

This activity offers one more lens for thinking about the relationship between triangles and parallelograms. The reasoning here supports students in generalizing the process of finding the area of a triangle in a future lesson. Previously, students duplicated triangles to compose parallelograms. Here they see that a different set of parallelograms can be created from a triangle, not by duplicating it, but by decomposing it.

Students are assigned a parallelogram to be cut into two congruent triangles. They take one triangle and decompose it into smaller pieces by cutting along a line that goes through the midpoints of two sides. They then use these pieces—a trapezoid and a small triangle—to compose a new parallelogram and reason about its area. Two parallelograms can be created. Monitor for students who create different parallelograms from the same pieces.



Students notice that the height of this new parallelogram is half of the height of the original parallelogram, and the area is also half of the area of the original parallelogram. Because the new parallelogram is composed of the same parts as the remaining (or uncut) large triangle, the area of the triangle is also half of that of the original parallelogram. This reasoning provides another way to understand the formula for the area of triangles.

Of the four given parallelograms, Parallelogram B is likely the most manageable for students. When decomposed, its pieces (each with a right angle) resemble those seen in earlier work on parallelograms. Consider this when assigning

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parallelograms to students.

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR7: Compare and Connect

Launch

Tell students that they will investigate another way in which triangles and parallelograms are related. Arrange students in groups of 2–4. Assign a different parallelogram from the blackline master to each student in the group. Give each student two copies of the parallelogram and access to a pair of scissors and some tape or glue.

Each parallelogram shows some measurements and dotted lines for cutting. For the first question, students who have Parallelograms C and D should *not* cut off the measurements shown outside of the figures.

Give students 10 minutes to complete the activity, followed by a few minutes to discuss their work (especially the last three questions). Ask students who finish early to find someone with the same original parallelogram and compare their work.

Select work from students who composed different parallelograms from the same cut-up pieces to share later.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, reveal only one question at a time, pausing to check for understanding before moving on.
Supports accessibility for: Organization, Attention

Student Task Statement

1. Your teacher will give you two copies of a parallelogram. Glue or tape *one* copy of your parallelogram here and find its area. Show your reasoning.
2. Decompose the second copy of your parallelogram by cutting along the dotted lines. Take *only* the small triangle and the trapezoid, and rearrange these two pieces into a different parallelogram. Glue or tape the newly composed parallelogram on your paper.
3. Find the area of the new parallelogram that you composed. Show your reasoning.
4. What do you notice about the relationship between the area of this new parallelogram and the original one?
5. How do you think the area of the large triangle compares to that of the new parallelogram: Is it larger, the same, or smaller? Why is that?
6. Glue or tape the remaining large triangle to your paper. Use any part of your work to help you find its area. Show your reasoning.

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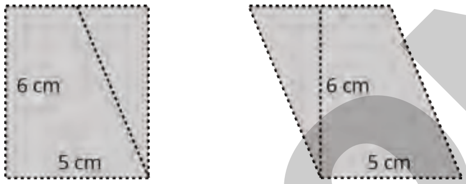
Student Response

Parallelogram A:

1. 80 sq cm. $10 \cdot 8 = 80$
2. 

3. 40 sq cm. $10 \cdot 4 = 40$

Parallelogram B:

1. $5 \cdot 12 = 60$
2. 

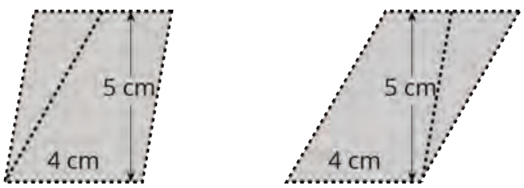
3. 30 sq cm. $5 \cdot 6 = 30$

Parallelogram C:

1. 60 sq cm. $10 \cdot 6 = 60$
2. 

3. 30 sq cm. $10 \cdot 3 = 30$

Parallelogram D:

1. 40 sq cm. $4 \cdot 10 = 40$
2. 

3. 20 sq cm. $4 \cdot 5 = 20$

All parallelograms:

1. The area of the new parallelogram is half the area of the original one.
2. Sample responses:
    - The area of the large triangle is the same as that of the new parallelogram. I know that because the trapezoid and little triangle together can be arranged into a triangle that is identical to the large triangle.

SAMPLE ONLY

- The new parallelogram and the large triangle have the same area since they are two halves of the original parallelogram.

3. Sample responses:
    - The large triangle in Parallelogram A has an area of 40 sq cm since that is the area of the new parallelogram.
    - The large triangle in Parallelogram D has an area of 20 sq cm since it is half of the original parallelogram, which has an area of 40 sq cm.

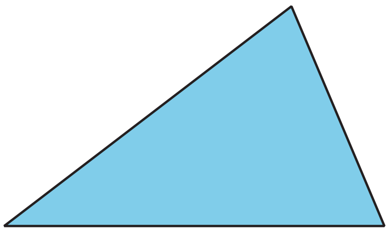
Building on Student Thinking

Students may struggle to form a new parallelogram because the two composing pieces are not both facing up (either the triangle or the trapezoid is facing down). Tell them that the shaded side of the cut-outs should face up.

Students may struggle to use the appropriate measurements needed to find the area of the parallelogram in the first question. They may multiply more numbers than necessary because the measurements are given. If this happens, remind them that only two measurements (base and height) are needed to determine the area of a parallelogram.

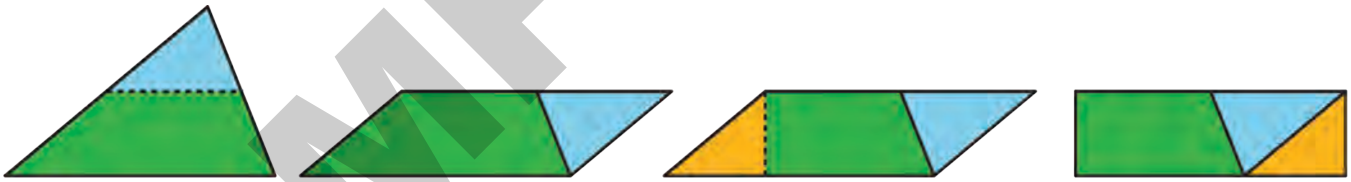
Are You Ready for More?

Can you decompose this triangle and rearrange its parts to form a rectangle? Describe how it could be done.



Extension Student Response

Sample response: Cutting the triangle at half its height, parallel to the base, creates a parallelogram, then another cut helps create a rectangle.



Activity Synthesis

The goal of this discussion is to enable students to further explain the relationship between the area of a triangle and a related parallelogram with the same base and height.

Display 2–3 approaches from previously selected students for all to see: Two different parallelograms that could be composed from the trapezoid and small triangle cut out from each Parallelogram A, B, C, and D. Use *Compare and Connect* to help students compare, contrast, and connect the area of each original parallelogram and the areas of the shapes that compose it. Here are some questions for discussion:

- "What do the new parallelograms have in common?" (They are shorter and smaller than the original parallelogram. They are all composed of a trapezoid and a triangle. Each parallelogram in the new pair has the same area.)

SAMPLE ONLY

- "How does the area of each new parallelogram compare to the area of the original parallelogram? How do you know?" (It is half the area of the original. The original and new parallelograms have a base that is the same length, but the height of the new parallelogram is half of that of the original.)
- "How does the area of each new parallelogram compare to the area of the large triangle? How do you know?" (They are equal. Each new parallelogram is made of two shapes that can be rearranged to match the large triangle exactly.)

Reiterate that we can establish two things about the area of each newly composed parallelogram:

- It is half of the area of the original parallelogram.
- It is equal to the area of the larger triangle.

As a result, we know that the area of a large triangle (formed by decomposing the parallelogram into two equal triangles) is also half of the area of the original parallelogram.

Lesson Synthesis

In this lesson, students practiced using what they know about parallelograms to reason about areas of triangles. They duplicated a triangle to make a parallelogram, decomposed and rearranged a triangle into a parallelogram, or enclosed a triangle with one or more rectangles.

To reiterate the connections between the areas of triangles and parallelograms, consider asking students:

- "What can we say about the area of a triangle and that of a parallelogram with the same height?" (The area of the triangle is half of the area of the related parallelogram.)
- "In the second activity, we cut a triangle along a line that goes through the midpoints of two sides and rearranged the pieces into a parallelogram. What did we notice about the area and the height of the resulting parallelogram?" (It has the same area as the original triangle but half its height.)
- "How might we start finding the area of any triangle, in general?" (Start by finding the area of a related parallelogram whose base is also a side of the triangle.)

Math Community
Invite 2–3 students to share what "Doing Math" actions they noticed. Record and display their responses for all to see, such as by adding check marks to any already listed items or adding new items near the chart for the class to consider adding. Next, give students 1–2 minutes with a partner to discuss any changes or revisions they think the chart needs. Tell students they can suggest revisions during the *Cool-down*.

8.4 An Area of 14

Cool-down

5 mins

Students have explored several ways to reason about the area of a triangle. This cool-down prompts them to articulate at least one way to do so. Not all methods will be equally intuitive or clear to them. In writing a commentary about at least one approach, students can show what makes sense to them at this point.

Standards

Addressing **6.G.A.1**

SAMPLE ONLY

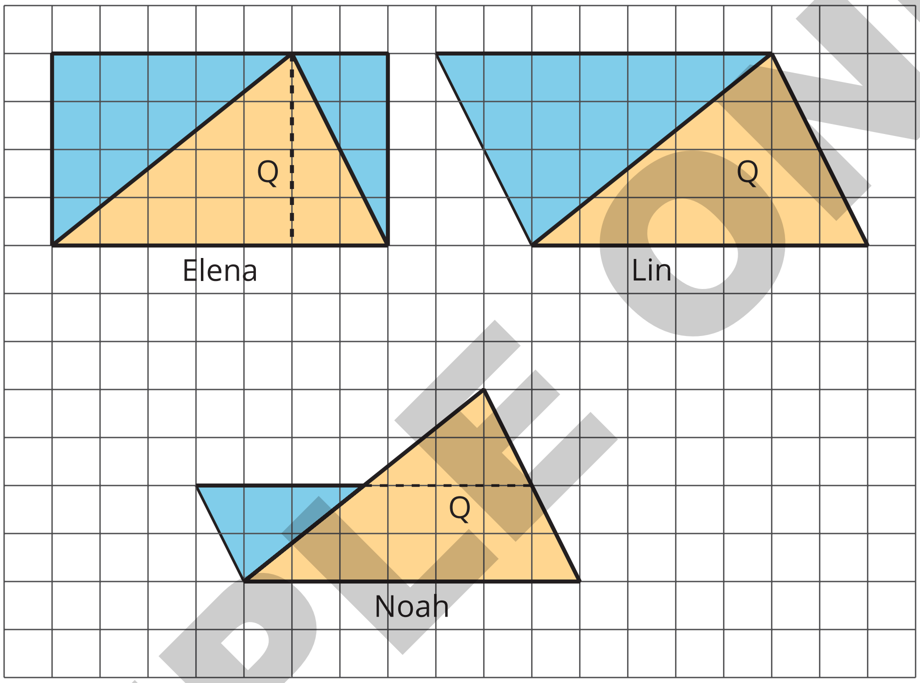
Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "What additions or revisions would you make to the Math Community Chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the Math Community Chart before Exercise 5.

Student Task Statement

Elena, Lin, and Noah all found the area of Triangle Q to be 14 square units but reasoned about it differently, as shown in the diagrams. Explain *at least one* student's way of thinking and why his or her answer is correct.



Student Response

Sample responses:

- Elena drew two rectangles that decomposed the triangle into two right triangles. She found the area of each right triangle to be half of the area of its enclosing rectangle. This means that the area of the original triangle is the sum of half of the area of the rectangle on the left and half of the rectangle on the right. Half of $(4 \cdot 5)$ plus half of $(4 \cdot 2)$ is $10 + 4$, so the area is 14 square units.
- Lin saw it as half of a parallelogram with the base of 7 units and height of 4 units (and thus an area of 28 square units). Half of 28 is 14.
- Noah decomposed the triangle by cutting it at half of the triangle's height, turning the top triangle around, and joining it with the bottom trapezoid to make a parallelogram. He then calculated the area of that parallelogram, which has the same base length but half the height of the triangle. $7 \cdot 2 = 14$, so the area is 14 square units.

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Responding To Student Thinking

Points to Emphasize

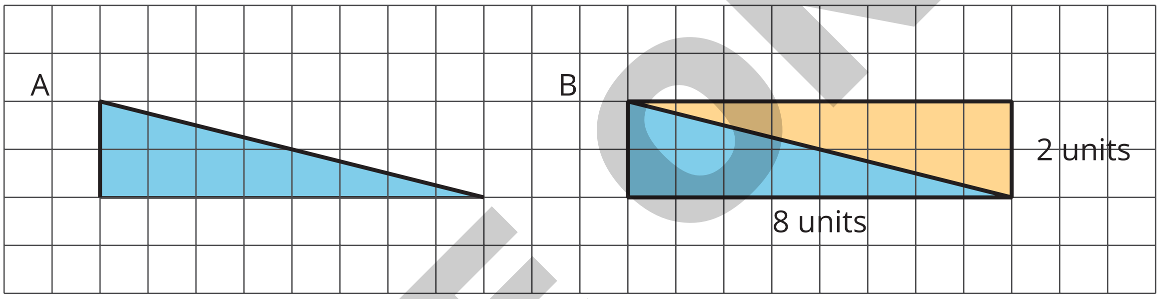
If students struggle with explaining the area of the triangle, give students opportunities to justify the areas of triangles over the next several lessons. For example, in this activity, after students find the areas of Triangles A–D and before discussing a general formula for area, encourage students to verbally explain the area of one or more triangles:

Grade 6, Unit 1, Lesson 9, Activity 2 Finding a Formula for the Area of a Triangle

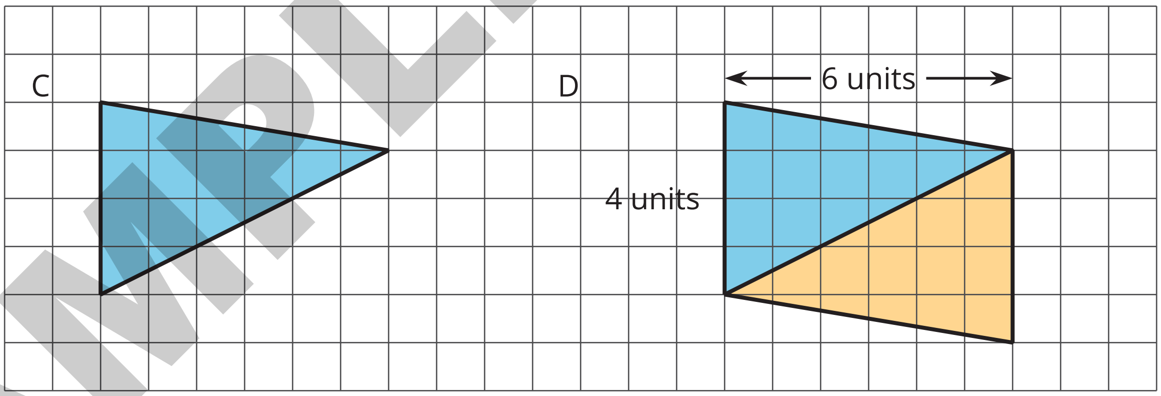
Lesson 8 Summary

We can reason about the area of a triangle by using what we know about parallelograms. Here are three general ways to do this:

- Make a copy of the triangle and join the original and the copy along an edge to create a parallelogram. Because the two triangles have the same area, one copy of the triangle has half the area of that parallelogram.



The area of Parallelogram B is 16 square units because the base is 8 units and the height 2 units. The area of Triangle A is half of that, which is 8 square units.



The area of Parallelogram D is 24 square units because the base is 4 units and the height 6 units. The area of Triangle C is half of that, which is 12 square units.

- Decompose the triangle into smaller pieces and compose them into a parallelogram.

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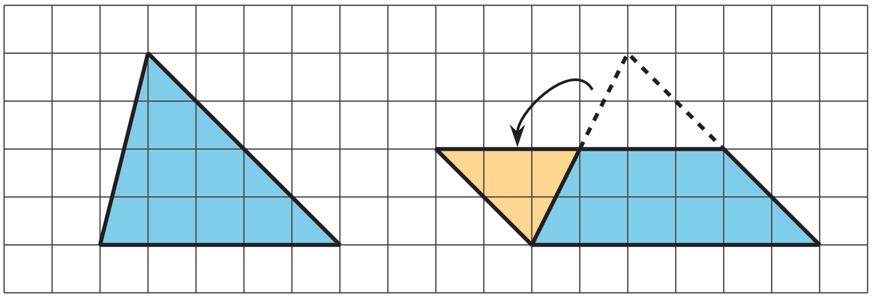
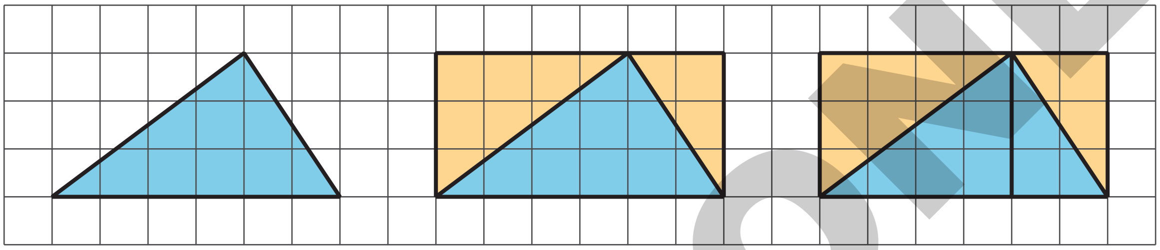
Sec C



In the new parallelogram, $b = 6$, $h = 2$, and $6 \cdot 2 = 12$, so its area is 12 square units. Because the original triangle and the parallelogram are composed of the same parts, the area of the original triangle is also 12 square units.

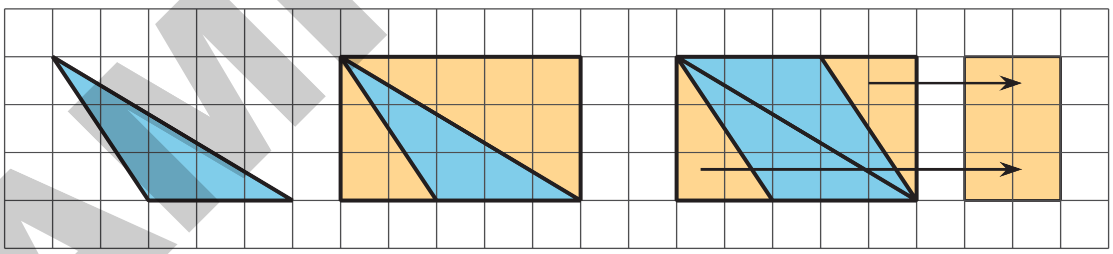
- Draw a rectangle around the triangle. Sometimes the triangle has half of the area of the rectangle.



The large rectangle can be decomposed into smaller rectangles. Each smaller rectangle can be decomposed into two right triangles.

- The rectangle on the left has an area of $4 \cdot 3$, or 12, square units. Each right triangle inside it is 6 square units in area.
- The rectangle on the right has an area of $2 \cdot 3$, or 6, square units. Each right triangle inside it is 3 square units in area.
- The area of the original triangle is the sum of the areas of a large right triangle and a small right triangle: 9 square units.

Sometimes, the triangle is half of what is left of the rectangle after removing two copies of the smaller right triangles.



The right triangles being removed can be composed into a small rectangle with area $(2 \cdot 3)$ square units. What is left is a parallelogram with area $5 \cdot 3 - 2 \cdot 3$, which equals $15 - 6$, or 9, square units.

Notice that we can compose the same parallelogram with two copies of the original triangle! The original triangle is half of the parallelogram, so its area is $\frac{1}{2} \cdot 9$, or 4.5, square units.

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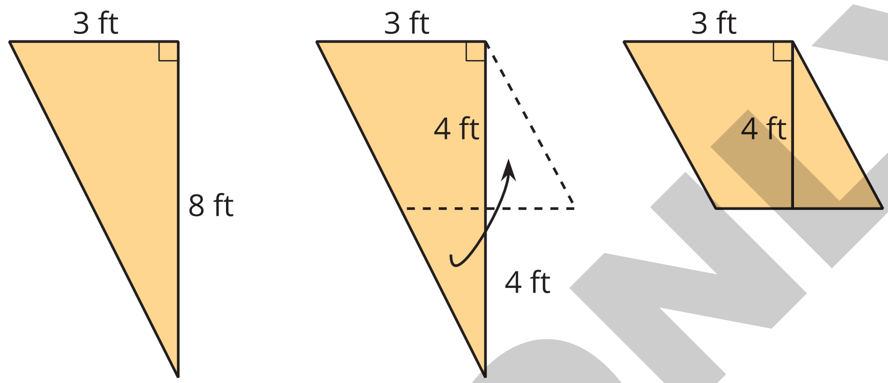
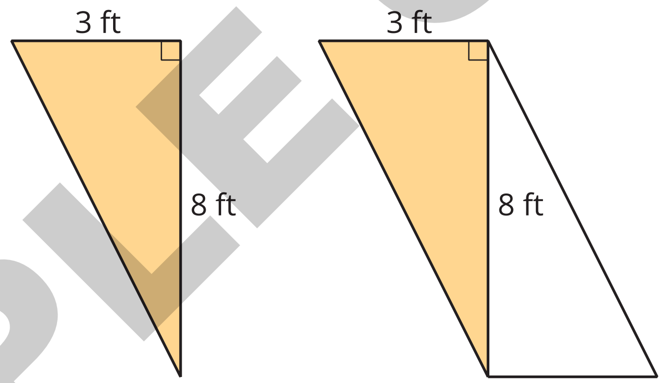
Practice Problems

1 Student Task Statement

To find the area of this right triangle, Diego and Jada used different strategies. Diego drew a line through the midpoints of the two longer sides, which decomposes the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes into a parallelogram.



Jada made a copy of the triangle, rotated it, and lined it up against one side of the original triangle so that the two triangles make a parallelogram.



a. Explain how Diego might use his parallelogram to find the area of the triangle.

b. Explain how Jada might use her parallelogram to find the area of the triangle.

Solution

Sample responses:

a. Diego's parallelogram has a base of 3 feet and a height of 4 feet, so its area is 12 square feet. Because the original right triangle and the parallelogram are composed of the same parts, they have the same area. The area of the triangle is also 12 square feet.

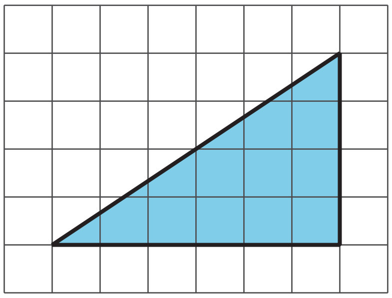
b. Jada's parallelogram has a base of 3 feet and a height of 8 feet, so its area is 24 square feet. Because it is composed of two copies of the right triangle, she could divide 24 by 2 to find the area of the triangle. $24 \div 2 = 12$ or 12 square feet.

SAMPLE ONLY

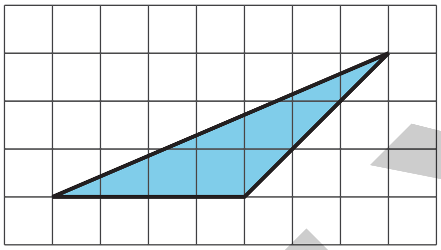
2 Student Task Statement

Find the area of the triangle. Explain or show your reasoning.

a.

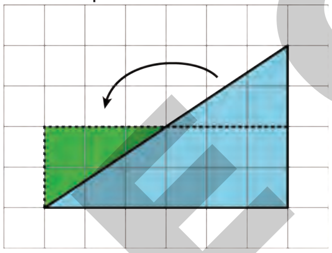


b.



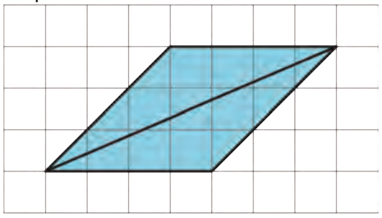
Solution

a. 12 square units. Sample reasoning: Make a horizontal cut, and rearrange the pieces to make a rectangle. The rectangle is 2 units by 6 units, so its area is 12 square units.



b. 6 square units. Sample reasoning:

- Duplicate the triangle, and rearrange the pieces to make a parallelogram. The parallelogram has a base of 4 units and a height of 3 units, so its area is 12 square units. Since the parallelogram's area is double the triangle's, the triangle's area is 6 square units.



- Decompose the triangle with a cut line half-way between the base and the opposite vertex. Rearrange the smaller triangle to form a parallelogram. This parallelogram has a horizontal base of length 4 units and a height of 1.5 units, so its area is 6 square units. That means the area of the original triangle is 6 square units.

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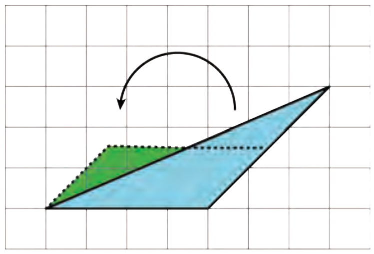
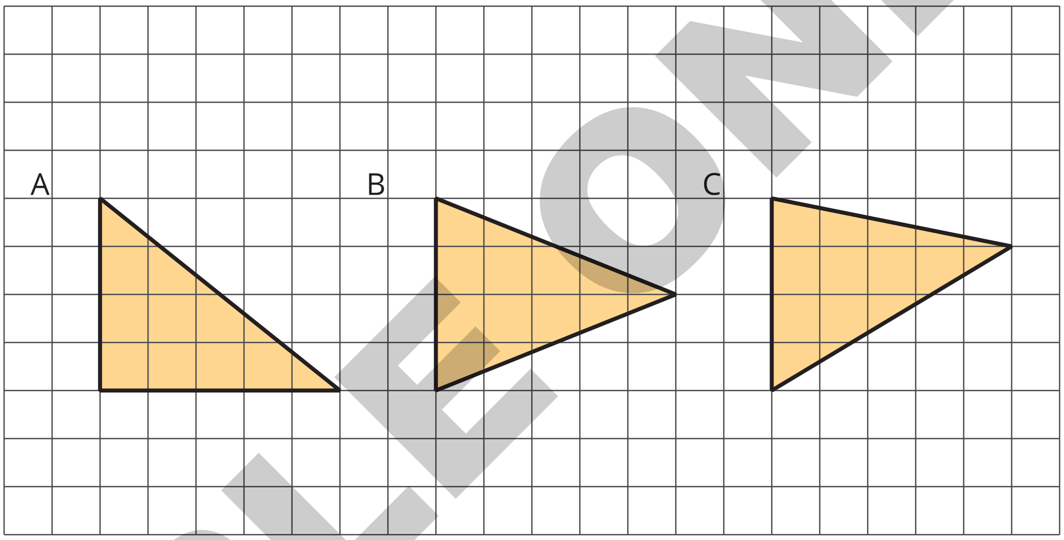


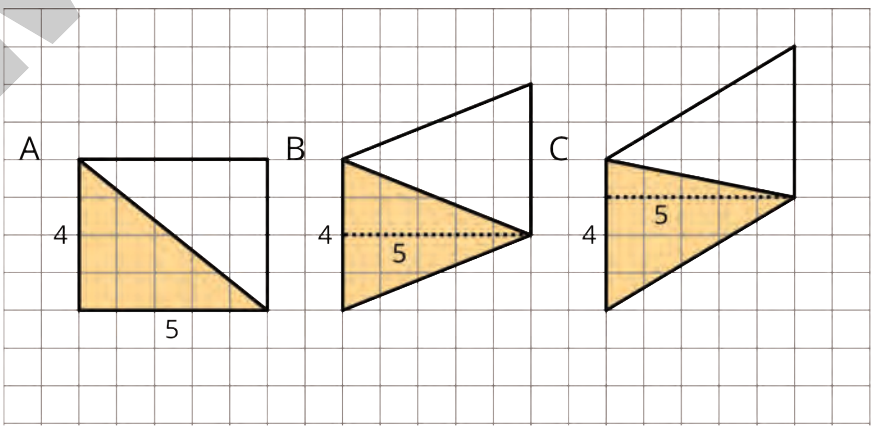
3 **Student Task Statement**

Which of the three triangles has the greatest area? Show your reasoning. If you get stuck, try using what you know about the area of parallelograms.



Solution

All three triangles have the same area of 10 square units. Sample reasoning: Two identical copies of each triangle can be composed into a parallelogram with a base of 5 units and a corresponding height of 4 units, which means an area of 20 square units. The area of each triangle is half of that of the parallelogram. $\frac{1}{2} \cdot 20 = 10$.



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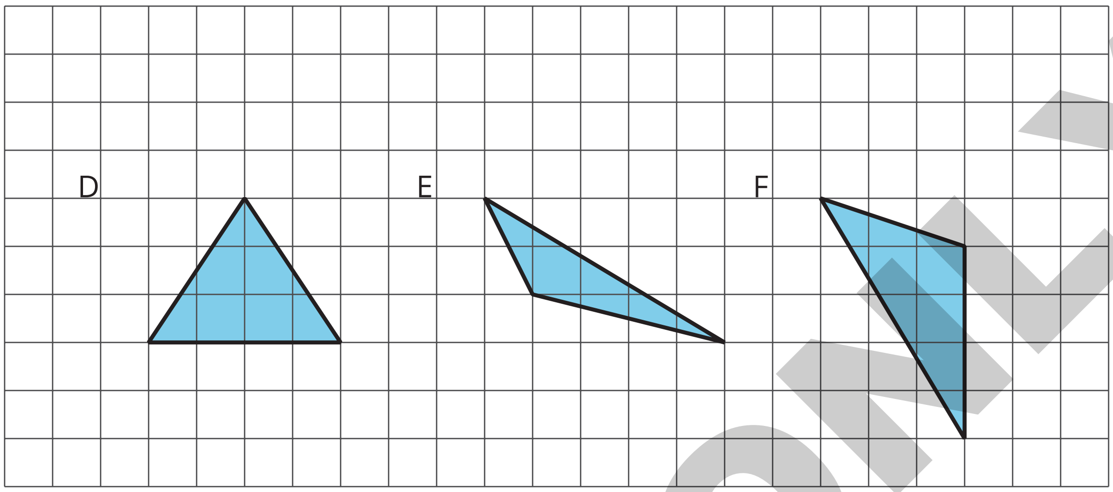
4 from Unit 1, Lesson 7

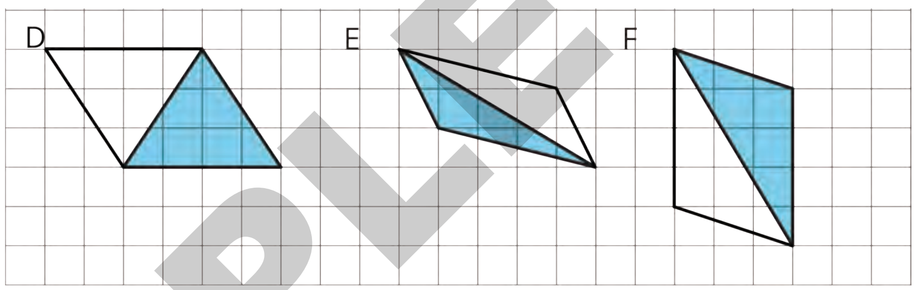
Student Task Statement

Draw an identical copy of each triangle such that the two copies together form a parallelogram. If you get stuck, consider using tracing paper.



Solution

Sample response:



5 from Unit 1, Lesson 6

Student Task Statement

a. A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?

b. A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?

c. A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the base?

Solution

a. 7 square units

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b. 0.6 units

c. 5.1 units

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Sec C

Unit 1, Lesson 9

Formula for the Area of a Triangle

Goals

- Compare, contrast, and critique (orally) different strategies for determining the area of a triangle.
- Generalize a process for finding the area of a triangle, and justify (orally and in writing) why this can be abstracted as $\frac{1}{2} \cdot b \cdot h$.
- Recognize that any side of a triangle can be considered its base, choose a side to use as the base when calculating the area of a triangle, and identify the corresponding height.

Learning Targets

- I can use the area formula to find the area of any triangle.
- I can write and explain the formula for the area of a triangle.
- I know what the terms “base” and “height” refer to in a triangle.

Sec C

Lesson Narrative

In this lesson, students begin to reason about areas of triangles more methodically: by generalizing their observations up to this point and expressing the area of a triangle in terms of its base and height.

Students first learn about bases and heights in a triangle by studying examples and counterexamples. They see that any side of a triangle can be its base, as is the case for parallelograms. They also learn that the height that corresponds to a chosen base is the length of a perpendicular segment that connects the base to the **opposite vertex**.

Next, they identify base-height measurements of triangles and use them to determine area. Then, students look for a pattern in their reasoning to help them write a formula for finding the area of any triangle (MP8). They also have a chance to build an informal argument about why the formula works for any triangle (MP3).

SAMPLE ONLY

Standards

Addressing	6.EE.A.2.a, 6.EE.A.2.c, 6.G.A.1
Building Towards	6.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Geometry toolkits: Activity 2

Student Facing Learning Goals

Let’s write and use a formula to find the area of a triangle.

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9.1

Bases and Heights of a Triangle

Warm-up

10 mins

Activity Narrative

In this activity, students think about the meaning of base and height in a triangle by studying examples and non-examples. Then, they examine some statements about bases and heights and determine if the statements are true. The goal is for students to see that in a triangle:

- Any side can be a base.
- A segment that represents a height must be drawn at a right angle to the base, but can be drawn in more than one place. The length of this perpendicular segment is the distance between the base and the vertex opposite it.
- A triangle can have three possible bases, each with a corresponding height.

Monitor for students who notice similarities between the bases and heights in a triangle and those in a parallelogram. Ask them to share their observations later.

As students justify how they know whether the given statements are true and consider others' justifications, they practice constructing logical reasoning and critiquing the reasoning of others (MP3).

Students will have many opportunities to make sense of bases and heights in this lesson and an upcoming one, so they do not need to know how to draw a height correctly at this point.

Standards

Building Towards **6.G.A.1**

Launch

Remind students that recently they looked at bases and heights of parallelograms. Tell students that they will now examine bases and heights of triangles.

Display the examples and non-examples of bases and heights for all to see. Read aloud the first paragraph of the activity and the description of each set of images. Give students a minute to observe the images. Then, tell students to use the examples and non-examples to determine what is true about bases and heights in a triangle.

Arrange students in groups of 2. Give them 2–3 minutes of quiet think time and then a minute to discuss their responses with a partner.

Student Task Statement

Here are six copies of a triangle. In each copy, one side is labeled *base*.

In the first three drawings, the dashed segments represent heights of the triangle.

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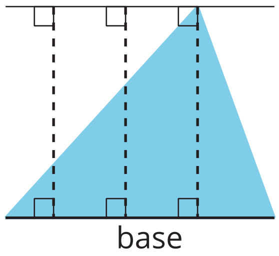
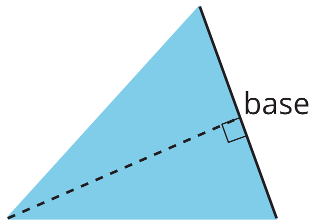
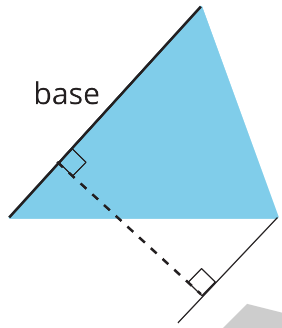
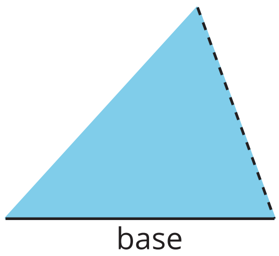
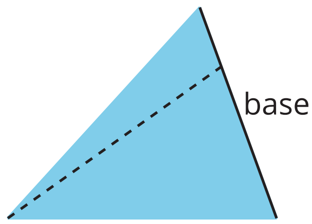


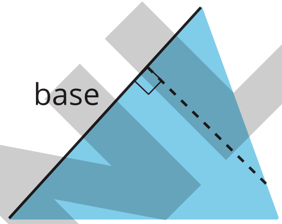




In the next three drawings, the dashed segments do not represent heights of the triangle.







Select **all** the statements that are true about bases and heights in a triangle.

1. Any side of a triangle can be a base.
2. There is only one possible height.
3. A height is always one of the sides of a triangle.
4. A height that corresponds to a base must be drawn at a right angle to the base.
5. Once we choose a base, there is only one segment that represents the corresponding height.
6. A segment representing a height must go through a vertex.

Student Response

Only statements 1 and 4 are true.

Building on Student Thinking

If students are unsure how to interpret the diagrams, ask them to point out parts of the diagrams that might be unclear. Clarify as needed.

Students may not remember from their experience with parallelograms that a height needs to be perpendicular to a base. Consider posting a diagram of a parallelogram—with its base and height labeled—in a visible place in the room so that it can serve as a reference.

Activity Synthesis

For each statement, ask students to indicate whether they think it is true. For each statement, ask one or two students to explain how they know. Encourage students to use the examples and counterexamples to support their argument (for instance, "The last statement is not true because the examples show dashed segments or heights that do not go through a vertex."). Make sure that students agree about each statement before moving on. Display the true statements for all to see.

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Students should see that only Statements 1 and 4 are true—that any side of a triangle can be a base, and a segment for the corresponding height must be drawn at a right angle to the base. What is missing—an important gap to fill during discussion—is the length of any segment representing a height.

Ask students, "How long should a segment that shows a height be? If we draw a perpendicular line from the base, where do we stop?" Solicit some ideas from students. Then, highlight the following:

- The length of each perpendicular segment is the shortest distance between the base and its opposite vertex. The **opposite vertex** is the vertex that is not an endpoint of the base. (Point out the opposite vertex for each base.)
- The segment representing height does not have to be drawn through the vertex (although that would be a natural place to draw it). It does need to maintain that distance between the base and the opposite vertex.

If any students noticed connections between the bases and heights in a triangle with those in a related parallelogram, invite them to share their observations. Otherwise, draw students' attention to it. Consider duplicating a triangle that shows a base and a height (by tracing on patty paper or creating a paper cutout). Use the original and the copy to compose a parallelogram. Ask students: "Suppose we choose the same side as the base of both the parallelogram and the triangle. What do you notice about the height of each shape?" (The two shapes have the same height.)

9.2 Finding a Formula for the Area of a Triangle

15 mins

Activity Narrative

In this activity, students generalize a formula for the area of triangles. They build on their observations about the relationship between the area of a triangle and the area of a parallelogram with the same base and height.

Students first find the areas of several triangles given base and height measurements. They notice regularity in repeated reasoning and arrive at an expression for finding the area of any triangle (MP8). Students might write $b \cdot h \div 2$, $b \cdot h \cdot \frac{1}{2}$, or another equivalent expression.

At the end of the activity, consider giving students a chance to reason more abstractly and think about why the expression $b \cdot h \div 2$ would hold true for all triangles. See the activity synthesis for prompts and diagrams that support such reasoning.

Standards

Addressing **6.EE.A.2.a, 6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2–3. Explain that they will now find the area of some triangles using what they know about base-height pairs in triangles and the relationships between triangles and parallelograms.

Give students 5 minutes to complete the activity. Provide access to geometry toolkits, especially tracing paper. Ask them to find the area of at least two triangles independently before discussing with their partner(s).

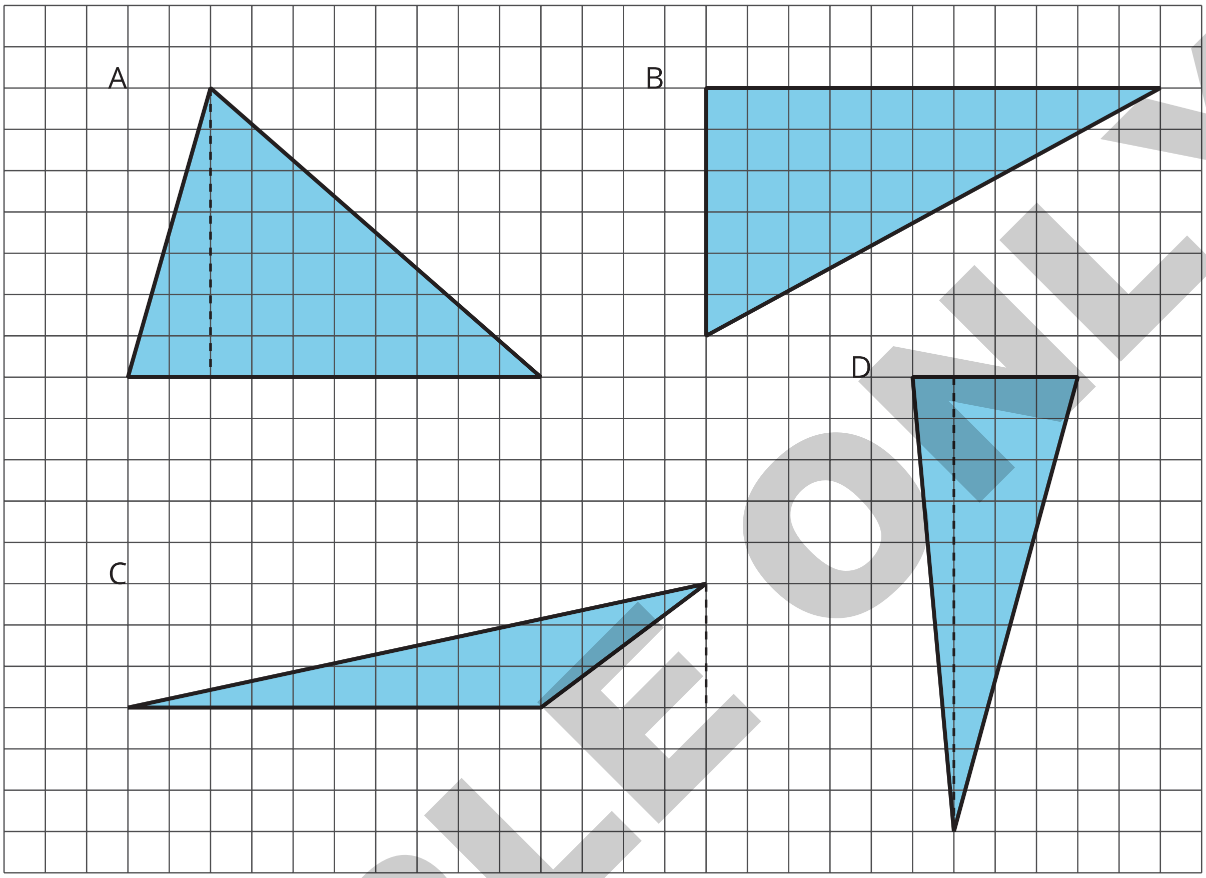
If needed, remind students how they reasoned about the area of triangles in the previous lesson (such as by composing a parallelogram, enclosing a triangle with one or more rectangles, and so on.). Encourage them to refer to their previous work and to use tracing paper as needed.

SAMPLE ONLY

Student Task Statement

For each triangle:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the triangle and record it in the last column of the table.



triangle	base (units)	height (units)	area (square units)
A			
B			
C			
D			
any triangle	b	h	

In the last row, write an expression for the area of any triangle, using b and h.

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Student Response

1.

triangle	base (units)	height (units)	area (square units)
A	10	7	35
B	11 (or 6)	6 (or 11)	33
C	10	3	15
D	4	11	22
any triangle	b	h	$b \cdot h \div 2$ (or equivalent)

2. Sample responses:
   - We can make a parallelogram from any triangle using the same base and height. The triangle will be half of the parallelogram. The area of a parallelogram is the length of the base times the length of the height, so the area of the triangle will be $b \cdot h \div 2$.
   - I can cut off the top half of a triangle and rotate it to make a parallelogram. That parallelogram has a base of b and a height that is half of the original triangle, which is $\frac{1}{2} \cdot h$, so its area is $b \cdot \frac{1}{2} \cdot h$. Since the parallelogram is just the triangle rearranged, the area of the triangle is also $\frac{1}{2}b \cdot h$.

Sec C

Building on Student Thinking

Students may not be inclined to write an expression using the variables b and h and instead replace the variables with numbers of their choice. Ask them to reflect on what they did with the numbers for the first four triangles. Then, encourage them to write the same operations but using the letters b and h rather than numbers.

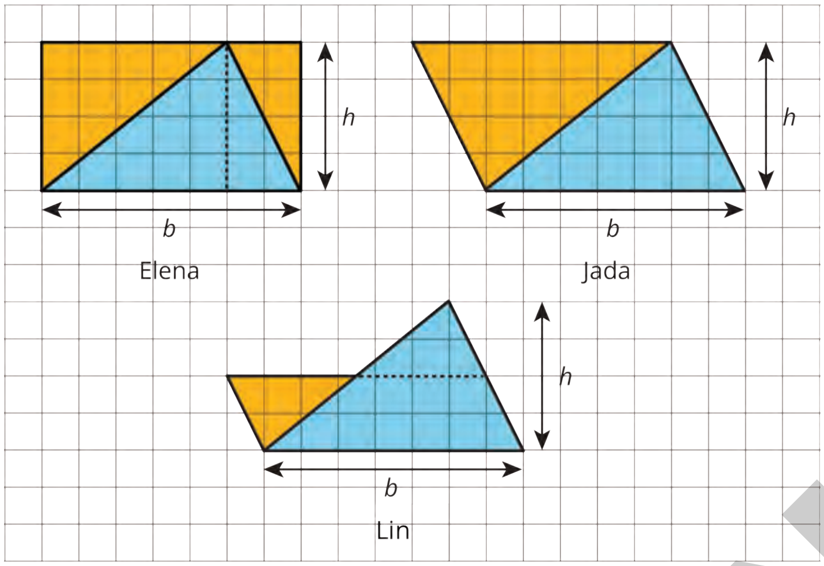
Activity Synthesis

Select a few students to share their expression for finding the area of any triangle. Record each expression for all to see.

To give students a chance to reason logically and deductively about their expression, ask, "Can you explain why this expression is true for *any* triangle?"

Display the following diagrams for all to see. Give students a minute to observe the diagrams. Ask them to choose one that makes sense to them and use that diagram to explain or show in writing that the expression $b \cdot h \div 2$ works for finding the area of any triangle. (Consider giving each student an index card or a sheet of paper on which to write their reasoning so that their responses could be collected, if desired.)

SAMPLE ONLY



When dealing only with the variables b and h and no numbers, students are likely to find Jada's and Lin's diagrams more intuitive to explain. Those choosing to use Elena's diagram are likely to suggest moving one of the extra triangles and joining it with the other to form a non-rectangular parallelogram with an area of $b \cdot h$. Expect students to be less comfortable reasoning in abstract terms than in concrete terms. Prepare to support them in piecing together a logical argument using only variables.

If time permits, select students who used different diagrams to share their explanation, starting with the most commonly used diagram (most likely Jada's). Ask other students to support, refine, or disagree with their arguments. If time is limited, consider collecting students' written responses now and discussing them in an upcoming lesson.

Access for English Language Learners

MLR8 Discussion Supports. Provide students with the opportunity to rehearse with a partner their explanation for why the expression $b \cdot h \div 2$ works for finding the area of any triangle before they share with the whole class or explain it in writing.
Advances: Speaking. Writing

Access for Students with Disabilities

Representation: Develop Language and Symbols. Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of finding the area of a triangle. Terms may include: opposite vertex, base (of a triangle), height (of a triangle), and formula for finding area.
Supports accessibility for: Conceptual Processing, Language

9.3 Applying the Formula for Area of Triangles

10 mins

Activity Narrative

In this activity, students apply the expression they previously generated to find the areas of various triangles. Each

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diagram is labeled with two or three measurements. Before calculating, students think about which lengths can be used to find the area of each triangle.

As students work, monitor for students who choose different bases for Triangles B and D. Later, invite them to contribute to the discussion about finding the areas of right triangles.

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

Launch

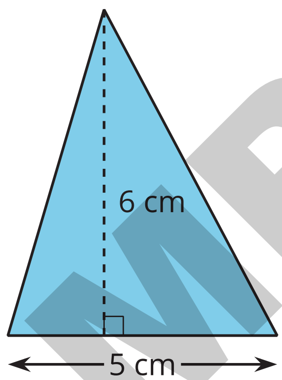
Explain to students that they will now practice using their expression to find the area of triangles without a grid. For each triangle, ask students to be prepared to explain which measurement they chose for the base and which one for the corresponding height and why.

Keep students in groups of 2–4. Give students 3 minutes of quiet think time and 1 minute to discuss their responses with their group.

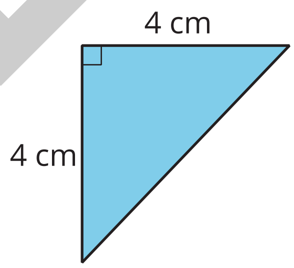
Student Task Statement

For each triangle, circle a base measurement that you can use to find the area of the triangle. Then, find the area of any *three* triangles. Show your reasoning.

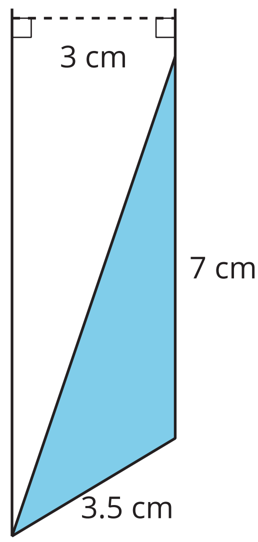
A



B



C



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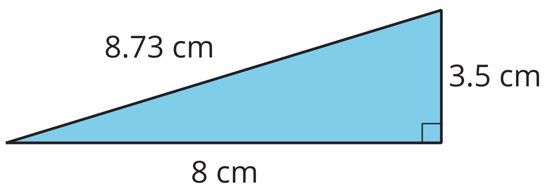
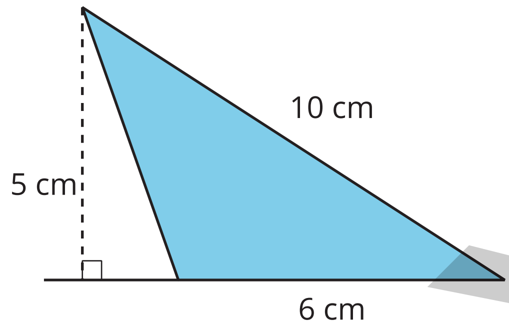
D

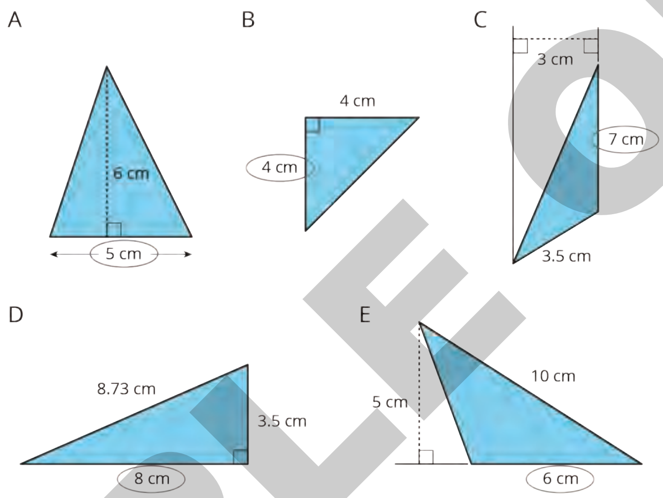


E



Student Response

In B either of the given pair of measurements can be the base.



Triangle A: 15 square cm, $b = 5$, $h = 6$, $A = 5 \cdot 6 \div 2 = 15$

Triangle B: 8 square cm, $b = 4$, $h = 4$, $A = 4 \cdot 4 \div 2 = 8$

Triangle C: 10.5 square cm, $b = 7$, $h = 3$, $A = 7 \cdot 3 \div 2 = 10.5$

Triangle D: 14 square cm, $b = 8$, $h = 3.5$, $A = 8 \cdot (3.5) \div 2 = 14$

Triangle E: 15 square cm, $b = 6$, $h = 5$, $A = 6 \cdot 5 \div 2 = 15$

Building on Student Thinking

The extra measurement in Triangles C, D, and E may confuse some students. If they are unsure how to decide which measurement to use, ask what they learned must be true about a base and a corresponding height in a triangle. Urge them to review the work from the *Warm-up* activity.

Some students may omit the step of dividing by two when finding the area of a triangle. Remind them of the idea that a triangle takes up half as much space as does a parallelogram with the same base and height, post images for reference, or point out the importance of the $\frac{1}{2}$ in the formula for the area of a triangle.

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Activity Synthesis

The aim of this whole-class discussion is to deepen students' awareness of the base and height of triangles. Discuss questions such as:

- "For Triangle A, can we say that the 6-cm segment is the base and the 5-cm segment is the height? Why or why not?" (No, the base of a triangle is one of its sides.)
- "For Triangle C, can the 3.5-cm segment serve as the base? Why or why not?" (Yes, it is a side of the triangle.) "What about the 3-cm segment?" (No, that segment is not a side of the triangle.)
- "More than two measurements are given for Triangles C, D, and E. Which ones are helpful for finding area?" (We need a base and a corresponding height, which means the length of one side of the triangle and the length of a perpendicular segment between that side and the opposite vertex.)

Lesson Synthesis

In this lesson, students learned that, similar to the area of a parallelogram, the area of a triangle can also be determined using base and height measurements. Discuss with students:

- "How do we locate the base of a triangle? How many possible bases are there?" (Any side of a triangle can be a base. There are 3 possible bases.)
- "How do we locate the height once we know the base?" (Find the length of a perpendicular segment that connects the base and its opposite vertex.)
- "Can both the base and height be sides of the triangle? (Yes) "When is that possible?" (In a right triangle, both the base and height can be the sides of the triangle.)
- "In the last activity, Triangles C, D, and E each have more than two given measurements. Which ones are helpful for finding area? (The length of one side of the triangle and the length of a perpendicular segment between that side and the opposite vertex.)

Next, discuss the formula for finding the area of a triangle. Consider asking students:

- "What expression works for finding the area of a triangle?" ($\frac{1}{2} \cdot b \cdot h$ or $\frac{b \cdot h}{2}$)
- "Can you explain briefly why this expression or formula works?" (The area of a triangle is always half of the area of a related parallelogram that shares the same base and height.)

9.4 Two More Triangles

Cool-down

5 mins

Students apply what they learned about the area formula and about the base and height of a triangle in this *Cool-down*. Multiple measurements are given, so students need to be attentive in choosing the right pair of measurements that would allow them to calculate the area.

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

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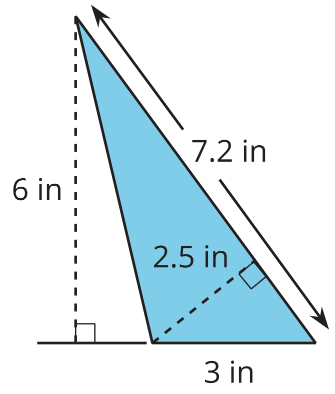
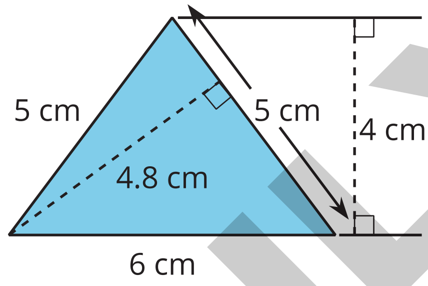
Student Task Statement

For each triangle, identify a base and a corresponding height. Use them to find the area. Show your reasoning.





Student Response

Triangle A: 9 sq in. Sample reasoning:

- $b = 3$, $h = 6$, area: 9 sq in, $\frac{1}{2} \cdot 3 \cdot 6 = 9$
- $b = 7.2$, $h = 2.5$, area: 9 sq in, $\frac{1}{2} \cdot (7.2) \cdot (2.5) = 9$

Triangle B: 12 sq in. Sample reasoning:

- $b = 6$, $h = 4$, area: 12 sq cm, $\frac{1}{2} \cdot 6 \cdot 4 = 12$
- $b = 5$, $h = 4.8$, area: 12 sq cm, $\frac{1}{2} \cdot 5 \cdot (4.8) = 12$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 9 Summary

- We can choose any of the three sides of a triangle to call the base. The term "base" refers to both the side and its length (the measurement).
- The corresponding height is the length of a perpendicular segment from the base to the vertex opposite it. The **opposite vertex** is the vertex that is *not* an endpoint of the base.

Here are three pairs of bases and heights for the same triangle. The dashed segments in the diagrams represent heights.

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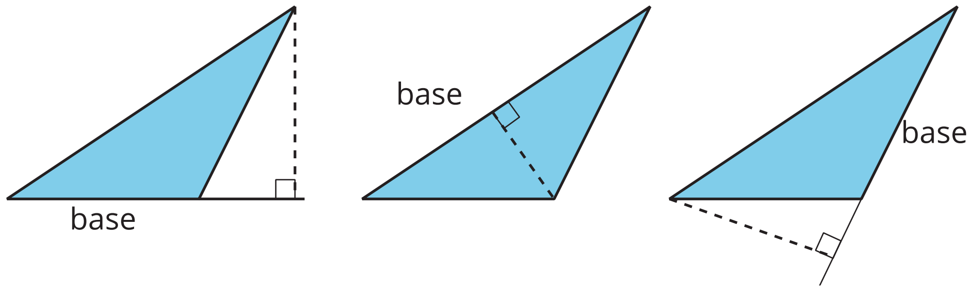
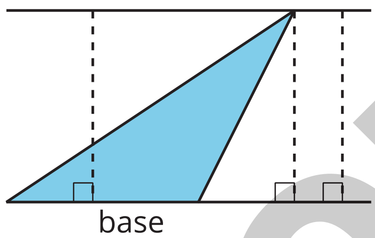
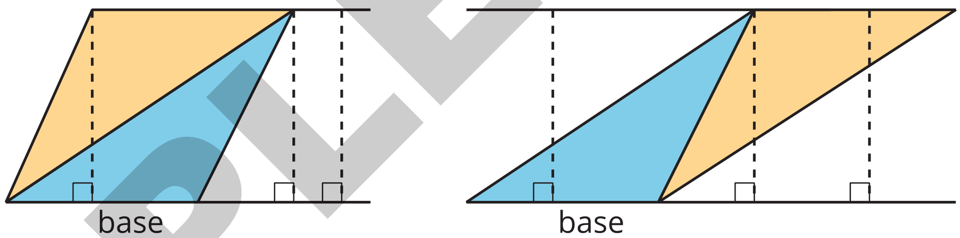


A segment showing a height must be drawn at a right angle to the base, but it can be drawn in more than one place. It does not have to go through the opposite vertex, as long as it connects the base and a line that is parallel to the base and goes through the opposite vertex, as shown here.



The base-height pairs in a triangle are closely related to those in a parallelogram. Recall that two copies of a triangle can be composed into one or more parallelograms. Each parallelogram composed of the triangle and its copy shares at least one base with the triangle.



For any base that they share, the corresponding height is also shared, as shown by the dashed segments.

We can use the base-height measurements and our knowledge of parallelograms to find the area of any triangle.

- The formula for the area of a parallelogram with base b and height h is $b \cdot h$.
- A triangle takes up half of the area of a parallelogram with the same base and height. We can therefore express the area, A, of a triangle as:

$$A = \frac{1}{2} \cdot b \cdot h$$

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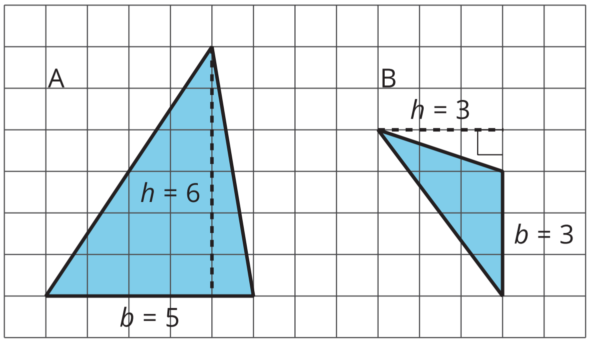
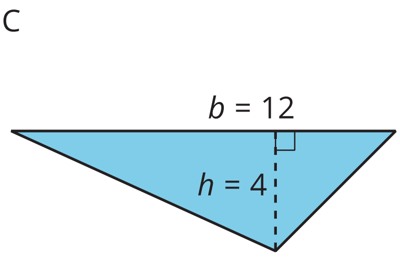


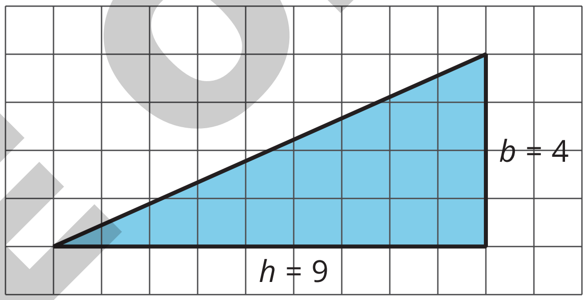


- The area of Triangle A is 15 square units because $\frac{1}{2} \cdot 5 \cdot 6 = 15$.
- The area of Triangle B is 4.5 square units because $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.
- The area of Triangle C is 24 square units because $\frac{1}{2} \cdot 12 \cdot 4 = 24$.

In each case, one side of the triangle is the base but neither of the other sides is the height. This is because the angle between them is not a right angle.

In right triangles, however, the two sides that are perpendicular can be a base and a height.

The area of this triangle is 18 square units whether we use 4 units or 9 units for the base.



Glossary

- opposite vertex

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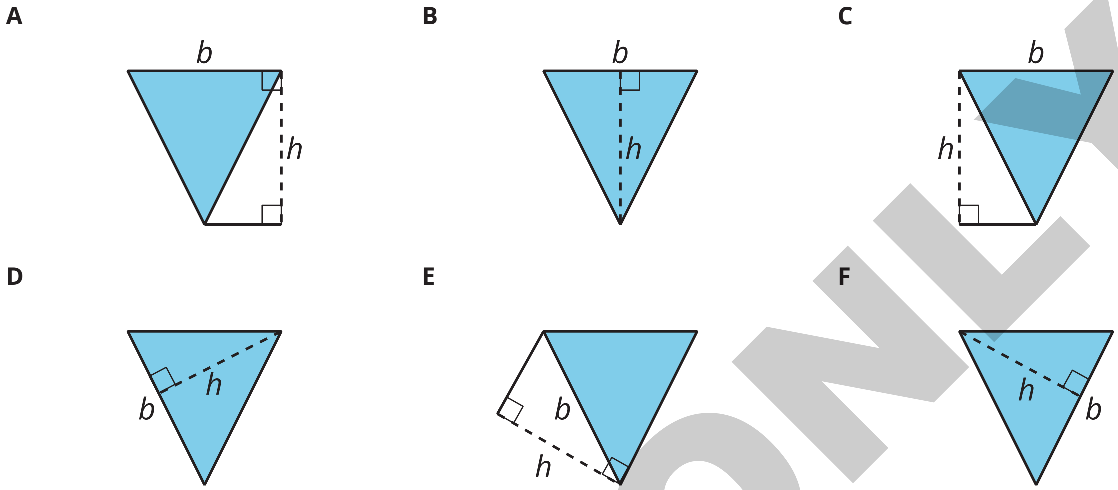
Sec C

Practice Problems

1 Student Task Statement

Select **all** drawings in which a corresponding height h for a given base b is correctly identified.



A. A

B. B

C. C

D. D

E. E

F. F

Solution

A, B, D, F

2 Student Task Statement

For each triangle, a base and its corresponding height are labeled.

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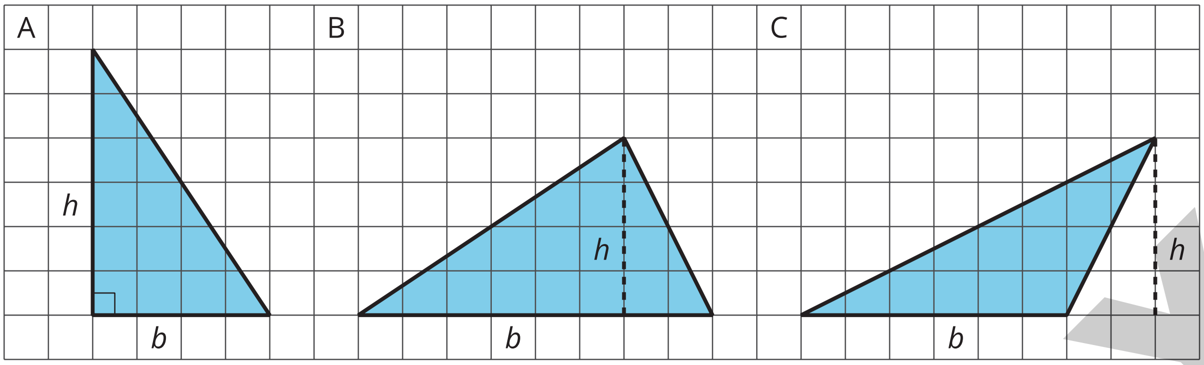
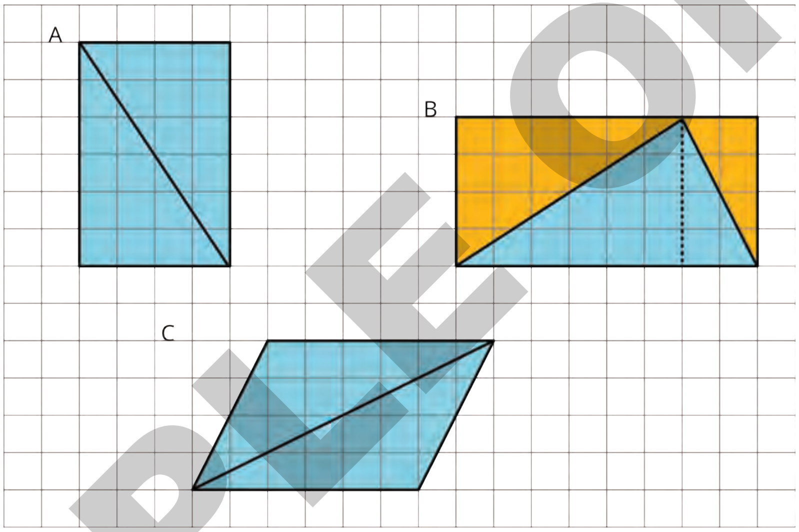


a. Find the area of each triangle.

b. How is the area related to the base and its corresponding height?

Solution

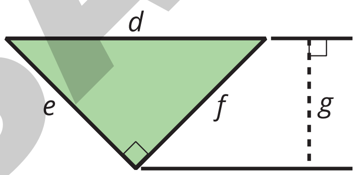
a. Triangle A: 12 square units, Triangle B: 16 square units, Triangle C: 12 square units



b. In each case, the area of the triangle, in square units, is half of the base times its corresponding height, $\frac{b \cdot h}{2}$.

3 Student Task Statement

Here is a right triangle. Name a corresponding height for each base.



a. Side d

b. Side e

c. Side f

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Solution

a. Segment g

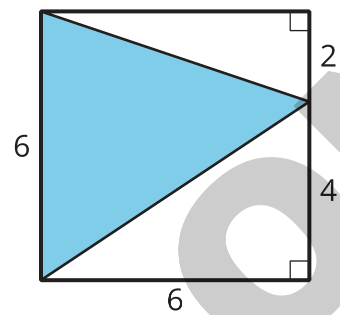
b. Side f

c. Side e

4 from Unit 1, Lesson 8

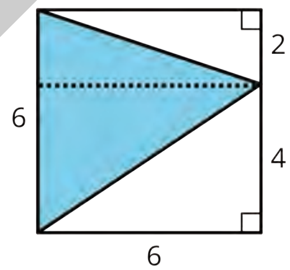
Student Task Statement

Find the area of the shaded triangle. Show your reasoning.



Solution

18 square units. Sample reasoning: Decomposing the triangle with a horizontal line forms two rectangles and splits the triangle into two smaller triangles. The top triangle is half of the top rectangle, so its area is $\frac{1}{2} \cdot 6 \cdot 2 = 6$. The bottom triangle is half of the bottom rectangle, so its area is $\frac{1}{2} \cdot 6 \cdot 4 = 12$. The area of the original triangle is $6 + 12$ or 18 square units.

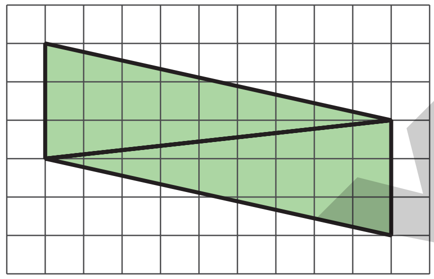


SAMPLE ONLY

5 from Unit 1, Lesson 7

Student Task Statement

Andre drew a line connecting two opposite corners of a parallelogram. Select **all** true statements about the triangles created by the line that Andre drew.



A. Each triangle has two sides that are 3 units long.

B. Each triangle has a side that is the same length as the diagonal line.

C. Each triangle has one side that is 3 units long.

D. When one triangle is placed on top of the other and their sides are aligned, we will see that one triangle is larger than the other.

E. The two triangles have the same area as each other.

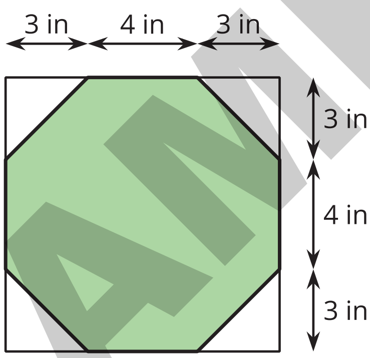
Solution

B, C, E

6 from Unit 1, Lesson 3

Student Task Statement

Here is an octagon. (Note: The diagonal sides of the octagon are *not* 4 inches long.)



a. While estimating the area of the octagon, Lin reasoned that it must be less than 100 square inches. Do you agree? Explain your reasoning.

b. Find the exact area of the octagon. Show your reasoning.

Solution

a. Yes. Sample reasoning: The octagon fits in a square that is 10 inches by 10 inches, but with four corners of the square removed. The square has an area of 100 square inches, so the area of the octagon must be less than that.

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b. 82 square inches. Sample reasoning: A 10-inch-by-10-inch square that encloses the octagon has an area of 100 square inches. Two corner triangles compose a 3 inch-by-3 inch square, so their combined area is 9 square inches. $100 - 2(3 \cdot 3) = 100 - 18 = 82$.

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Unit 1, Lesson 10

Bases and Heights of Triangles

Goals

- Draw and label the height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.
- Evaluate (orally) the usefulness of different base-height pairs for finding the area of a given triangle.

Learning Targets

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson Narrative

In this lesson, students further their ability to identify and work with bases and heights in a triangle by:

- Drawing a triangle with a given area (reversing the reasoning process they have done so far).
- Learning to draw (not just to recognize) a segment to show the corresponding height for any given base.

The lesson also includes an optional activity, in which students learn to choose appropriate base-height pairs to enable area calculations. Because there are three possible pairs of bases and heights in any triangle, some care is needed in identifying the right combination of measurements. Some base-height pairs may be more practical or efficient to use than others, so it helps to be strategic in choosing a side to use as a base.

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Geometry toolkits: Activity 1, Activity 3
- Index cards: Activity 2, Activity 3

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

Activity 2:

Each student especially needs an index card.

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Student Facing Learning Goals

Let's use different base-height pairs to find the area of a triangle.

10.1 An Area of 12

Warm-up

10 mins

Activity Narrative

There is a digital version of this activity.

In this *Warm-up*, students are given an area measure and are asked to create several triangles with that area. This work involves reversing the reasoning process used in previous lessons, in which students were given triangles with measurements and asked to find the area.

Students are likely to gravitate toward right triangles first (or to halve rectangles that have factors of 12 as their side lengths). This is a natural and productive starting point. Prompting students to create non-right triangles encourages them to apply their understanding of the area of non-right parallelograms.

As students work alone and discuss with partners, notice the strategies they use to draw their triangles and to verify their areas. Identify a few students with different strategies and, later, ask them to share.

In the digital version of the activity, students use an applet to draw triangles on a grid and reason about their area. The applet allows students to adjust the vertices of line segments, measure lengths, and make annotations.

Standards

Addressing **6.G.A.1**

Launch

Arrange students in groups of 2. Provide access to geometry toolkits. Give students 2–3 minutes of quiet think time and 2 minutes to share their drawings with their partner afterwards. Encourage students to refer to previous work as needed. If students finish their first drawing early, tell them to draw a different triangle with the same area.

During partner discussion, each partner should convince the other that the triangle drawn is indeed 12 square units.

Student Task Statement

On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.

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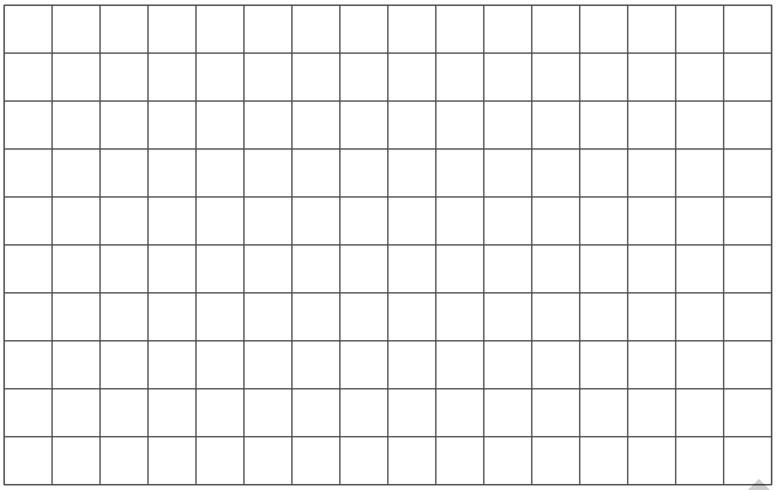
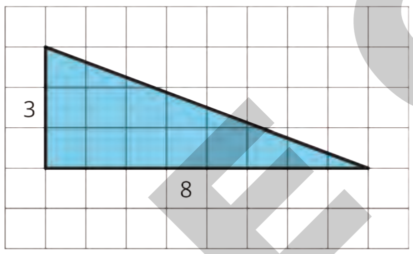


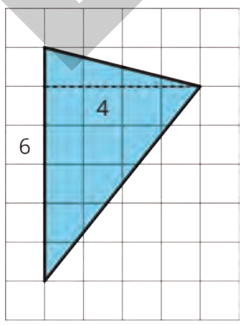
Student Response

Sample responses:

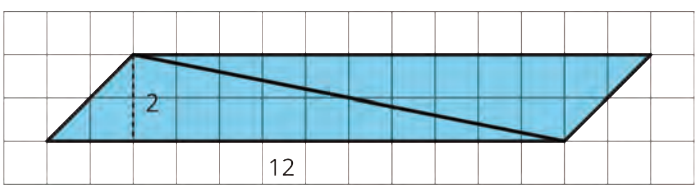
- This right triangle has a base of 8 units and a height of 3 units. The area is half of $3 \cdot 8$ or half of 24, which is 12.



- This triangle has a side of 6 units. This can be the base. Draw a height segment that is perpendicular to the base and is 4 units long. The area of the triangle is $b \cdot h \div 2$, so it is $6 \cdot 4 \div 2$, which is 12.



- Draw a parallelogram with a base of 12 and a height of 2, and then draw a diagonal line to create two identical triangles. Each of the triangles has an area of 12 because it is half of a parallelogram with an area of 24.



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Building on Student Thinking

If students have trouble getting started, ask:

- "Can you draw a quadrilateral with an area of 12?"
- "Can you use what you know about parallelograms to help you?"
- "Can you use any of the area strategies—decomposing, rearranging, enclosing, subtracting—to arrive at an area of 12?"

Students who start by drawing rectangles and other parallelograms may use factors of 12, instead of factors of 24, for the base and height. If this happens, ask them what the area of the their quadrilateral is and how it relates to the triangle they are trying to draw.

Activity Synthesis

Invite a few students to share their drawings and ways of reasoning with the class. For each drawing shared, ask the creator for the base and height and record them for all to see. Ask the class:

- "Did anyone else draw an identical triangle?"
- "Did anyone draw a different triangle but with the same base and height measurements?"

To reinforce the relationship between base, height, and area, discuss:

- "Which might be a better way to draw a triangle: by starting with the base measurement or with the height? Why?"
- "Can you name other base-height pairs that would produce an area of 12 square units without drawing? How?"

10.2 Hunting for Heights

25 mins

Activity Narrative

Students may be able to recognize a measurement that can be used for height when they see it, but identifying and drawing an appropriate segment is more challenging. This activity, and the demonstration needed to launch it, gives students a concrete strategy for identifying a height accurately. When students use a strategy of drawing an auxiliary line to solve problems, they are looking for and making use of structure (MP7). Explicit instruction, as in this activity, is often needed before students can be expected to use this strategy spontaneously.

This is the first time Math Language Routine 3: *Critique, Correct, Clarify* is suggested in this course. In this routine, students are given a "first draft" statement or response to a question that is intentionally unclear, incorrect, or incomplete. Students analyze and improve the written work by first identifying what parts of the writing need clarification, correction, or details, and then writing a second draft (individually or with a partner). Finally, the teacher scribes as a selected second draft is read aloud by its author(s), and the whole class is invited to help edit this "third draft" by clarifying meaning and adding details to make the writing as convincing as possible to everyone in the room. Typical prompts are: "Is anything unclear?" and "Are there any reasoning errors?". The purpose of this routine is to engage students in analyzing mathematical writing and reasoning that is not their own, and to solidify their knowledge and use of language.

Access for English Language Learners

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as

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students critique and revise mathematical arguments.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR3: Critique, Correct, Clarify

Launch

Explain to students that they will try to draw a height that corresponds to each side of a triangle. Arrange students in groups of 2. Give each student an index card and 1–2 minutes to complete the first question. Remind them that there is more than one correct way to draw the corresponding height for a base. Ask them to pause after the first question. As students work, notice how students are using the index cards (if at all).

Afterward, solicit a few quick comments on the exploration. Ask questions such as:

- “How did you know where to draw the segments?”
- “How did you draw them?”
- “Why were you given index cards? How might they help?”

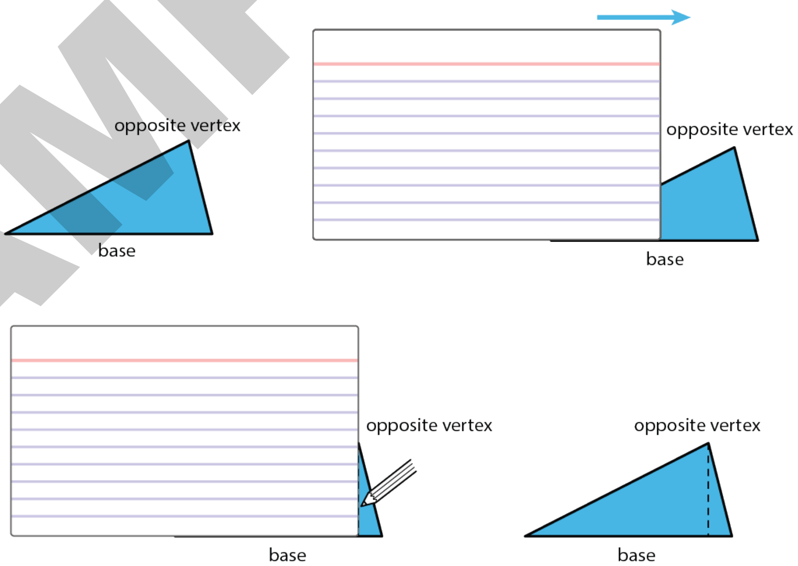
Explain that you will now demonstrate a way to draw heights effectively. (If any students used the index card correctly, acknowledge that they were on the right track.)

Remind students that any line we draw to show the height of a triangle must be drawn *perpendicular* to the base. Having a tool with a right angle and with straight edges can help us make sure the line we draw is both straight and perpendicular to the base. This is what the index card is for.

Ask: “How do we know where to stop this line we are drawing? How long should it be?”

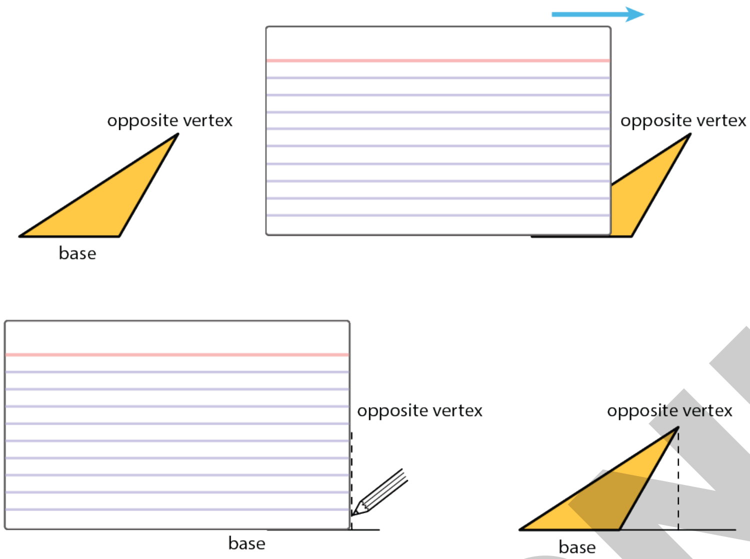
Explain that the easiest way is to draw the line so it would pass through the **vertex** opposite the chosen base. Draw or display a triangle for all to see. Demonstrate the following.

- Choose one side of the triangle as the base. Identify the opposite vertex.
- Line up one **edge** of the index card with that base.
- Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
- Use that edge to draw a line segment from that vertex to the base. The measure of that segment is the height.



SAMPLE ONLY

Ask: "What if the opposite vertex is not directly over the base?" Explain that sometimes we need to extend the line of the base. Demonstrate the process.



Demonstrate the process with another example in which the card needs to slide from right to left (for example, by rotating the obtuse triangle above clockwise). Left-handed students may find this particularly helpful.

Prompt students to use this method to check the heights they drew in response to the first question, revise the drawings if they were incorrect, and share their revisions with their partners. Circulate, and support students as they draw. Those who finish verifying the heights in the first question can move on to complete the rest of the activity with their partners.

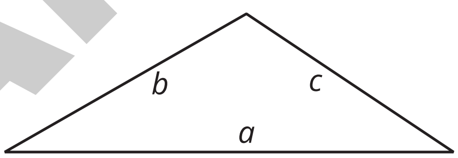
Access for Students with Disabilities

Representation: Access for Perception. Students may benefit from watching the demonstration more than once. Consider pulling a small group of students aside for additional demonstration and clarification.
Supports accessibility for: Language, Attention

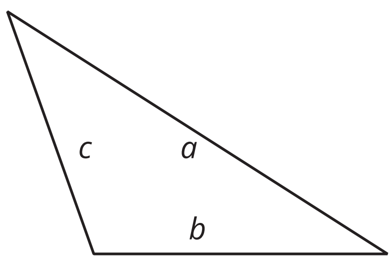
Student Task Statement

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

   Use Side a as the base:

   Use Side b as the base:

   Use Side c as the base:

Pause for your teacher's instructions before moving

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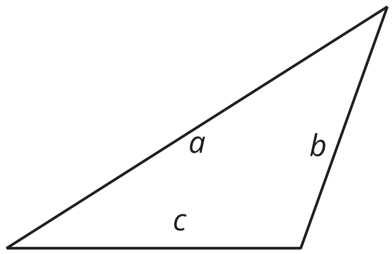
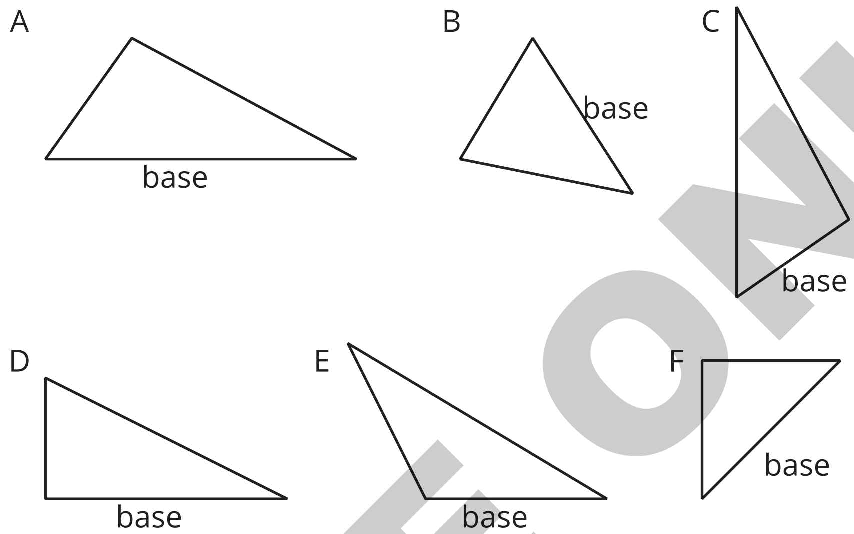


to the next question.

2. Draw a line segment to show the height for the chosen base in each triangle.



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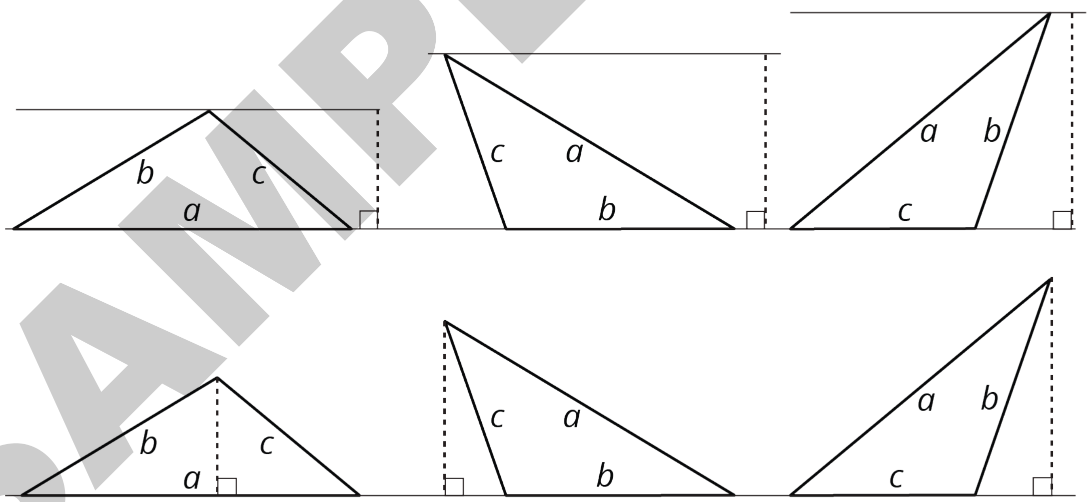
Student Response

1. Sample responses:



2.

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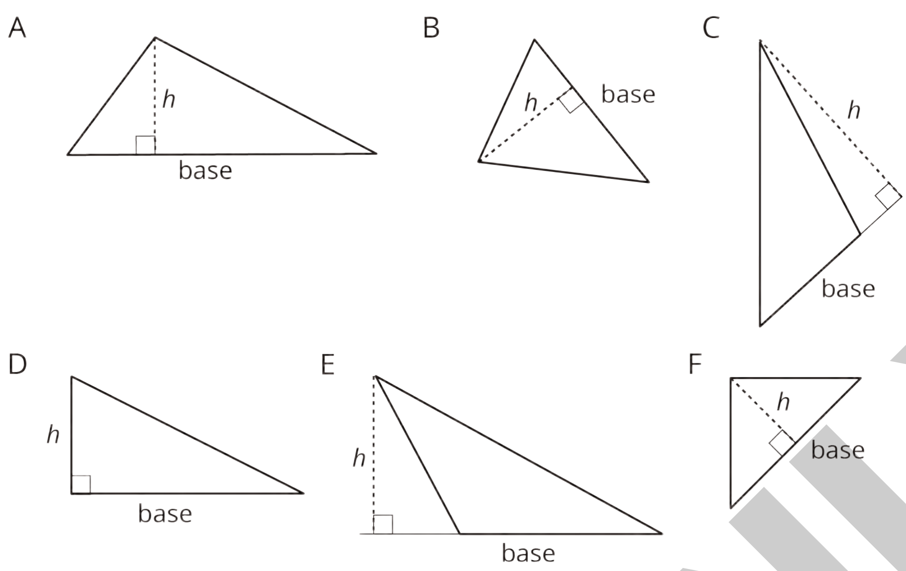


Building on Student Thinking

Some students may use the index card simply as a straightedge and therefore draw heights that are not perpendicular to the given base. Remind them that a height needs to be perpendicular (at a right angle) to the base. If necessary, demonstrate again how the corner of the index card can be used to draw the height at a right angle to the base.

Students may mistakenly think that a base must be a horizontal side of a triangle (or one closest to being horizontal) and a height must be drawn inside of the triangle. Point to some examples from earlier work to remind students that neither is true. Remind them to align their index card to the side labeled "base."

Some students may find it awkward to draw height segments when the base is not horizontal. Encourage students to rotate their paper as needed to make drawing easier.

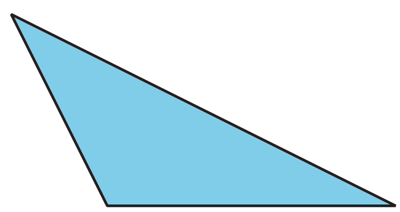
Activity Synthesis

If time permits, consider selecting one student to share the height drawing for each triangle, or display the solutions for all to see.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response about drawing a height segment of a triangle by correcting errors, clarifying meaning, and adding details.

- Display the following triangle and first draft:

  "To draw a height segment for a triangle like this one, we would always need to extend the base, no matter which side it is. "

- Ask, "What parts of this response are unclear, incorrect, or incomplete?" As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.
- Give students 2–3 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft

SAMPLE ONLY

even more clear and more convincing.

The goal of the discussion is to highlight that:

- A segment that represents height needs to be drawn perpendicular to the side chosen as the base.
- The perpendicular segment needs to connect the base and the vertex opposite the base.
- If this can't be done, we can first extend the base. Then, we can draw a perpendicular segment that connects the extension and the opposite vertex.
- Alternatively, we can draw a line that is parallel to the base and goes through the opposite vertex. Then, we can draw a perpendicular segment that connects the base and that line.

10.3 Some Bases Are Better Than Others

Optional

15 mins

Activity Narrative

This activity allows students to practice identifying the base and height of triangles and using them to find areas.

Because there are no directions on which base or height to use, and because not all sides would enable them to calculate area easily, students need to think structurally and choose strategically. All triangles in the problems have either a vertical or a horizontal side. Choosing such a side as the base makes it easier to identify the corresponding height.

In some cases, students may opt to use a combination of area-reasoning strategies rather than finding the base and height of the shaded triangles and applying the formula. For instance, they may enclose a shaded triangle with a rectangle and subtract the areas of extra triangles (with or without using the formula on those extra triangles). Notice students who use such strategies so they can share later.

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Explain that they will now practice locating or drawing heights and using them to find the area of triangles. Give students 8–10 minutes of quiet think time and time to share their responses with a partner afterward. Provide access to their geometry toolkits (especially index cards).

If time is limited, consider asking students to find the area of two or three triangles instead of all four.

Student Task Statement

For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)

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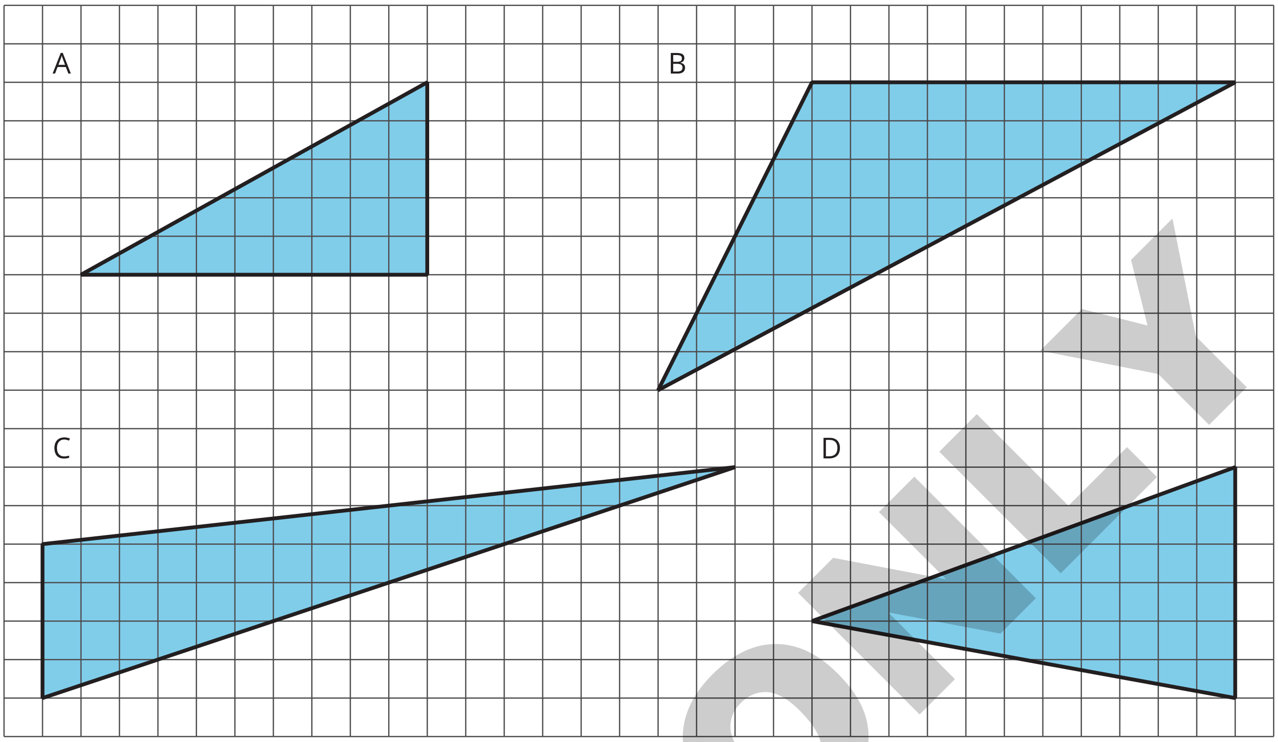
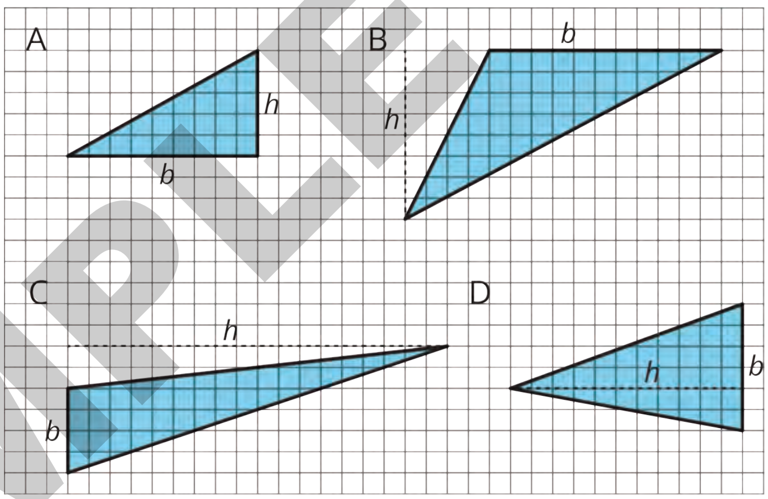


Student Response



Triangle A: $b = 9$ and $h = 5$, $9 \cdot 5 \div 2 = 22.5$, area: 22.5 square units

Triangle B: $b = 11$ and $h = 8$, $11 \cdot 8 \div 2 = 44$, area: 44 square units

Triangle C: $b = 4$ and $h = 18$, $4 \cdot 18 \div 2 = 36$, area: 36 square units.

Triangle D: $b = 6$ and $h = 11$, $6 \cdot 11 \div 2 = 33$, area: 33 square units.

Building on Student Thinking

Students may think that a vertical side of a triangle is the height regardless of the segment used as the base. If this happens, have them use an index card as a straightedge to check if the two segments they are using as base and height

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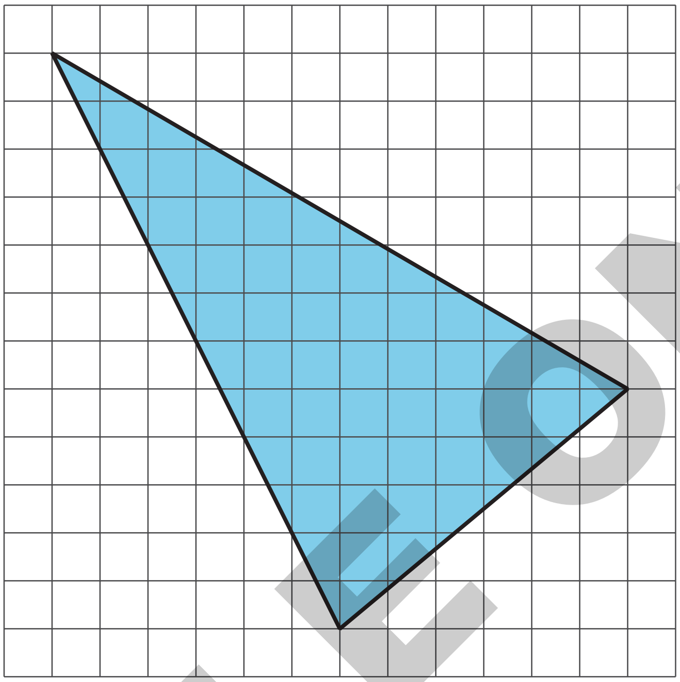
Sec C

are perpendicular.

Some students may not immediately see that choosing a side that is either vertical or horizontal would enable them to find the corresponding height very easily. They may choose a non-vertical or non-horizontal side and not take advantage of the grid. Ask if a different side might make it easier to determine the base-height lengths without having to measure.

Are You Ready for More?

Find the area of this triangle. Show your reasoning.



Extension Student Response

51, since we can enclose the given triangle in a square that has an area of 144 ($12 \cdot 12 = 144$), then subtract away the area from right triangles in each corner.

Activity Synthesis

Focus the whole-class discussion on how students went about identifying bases and heights. Discuss:

- “Which side did you choose as the base for Triangle B? Triangle C? Why?”
- “Aside from choosing a vertical or horizontal side as the base, is there another way to find the area of the shaded triangles without using their bases and heights?” (Invite a couple of students who use the enclose-and-subtract method to find the area of Triangles B, C, or D to share.)
- “Which strategy do you prefer or do you think is more efficient?”
- “Can you think of an example where it might be preferable to find the base and height of the triangle of interest?” (Students may point to any of the triangles in the task.)
- “Can you think of an example where it might be preferable to enclose the triangle of interest and subtract other areas?” (Students may point to the triangle shown in "Are you ready for more?", where none of the shaded triangle's sides are horizontal or vertical.)

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Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to help students produce statements that describe the strategies they use to identify bases and heights, and to find area. For example, “For triangle ___, I chose side ___ as the base because . . .” or “The next time I need to find the area of a triangle, the strategy I will use is _____, because . . . .”
Advances: Speaking, Conversing

Lesson Synthesis

In this lesson, students looked closely at the heights of a triangle. They located or drew a height using any side of a triangle as the base. They also considered which pair of base and height to use to find area. Discuss with students:

- "What must we remember about the relationship between a base of a triangle and its corresponding height?" (The height must be perpendicular to the base.)
- "What tools might help us draw a height segment? What is it about an index card or a ruler that helps us?" (A tool with straight edges and a right angle can help us draw perpendicular segments.)
- "Every triangle has multiple base-height pairs. Does it matter which side we choose as the base? How do we decide?" (For the base, we need a side with a known length. For the height, we need a segment that is perpendicular to that base and whose length we can determine.)

10.4 Stretched Sideways

Cool-down

5 mins

Standards

Addressing **6.G.A.1**

Launch

Provide access to geometry toolkits.

Student Task Statement

1. For each triangle, draw a height segment that corresponds to the given base, and label it h. Use an index card if needed.

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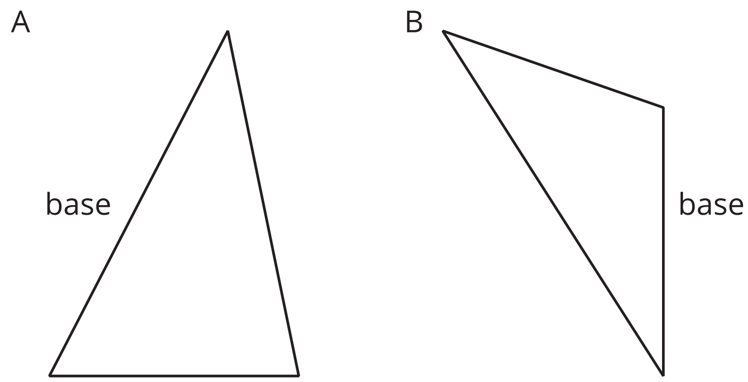
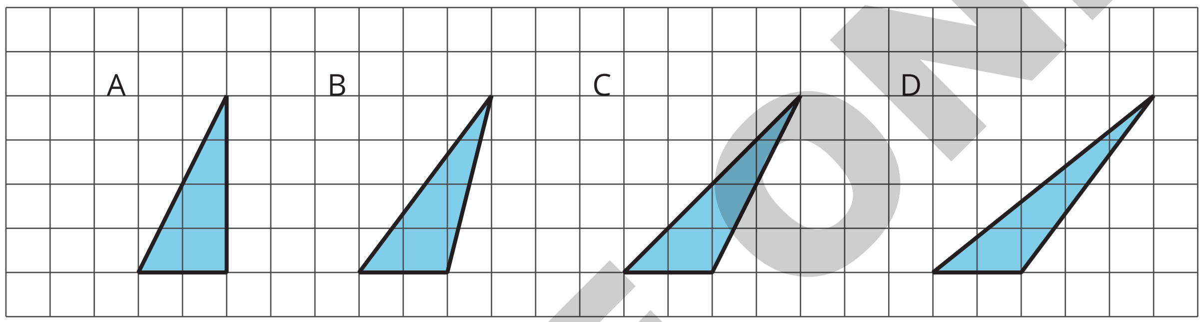
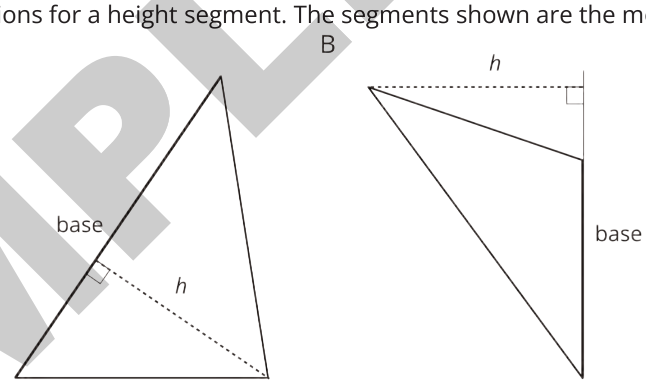


2. Which triangle has the greatest area? The least area? Explain your reasoning.



Student Response

1. There are many possible locations for a height segment. The segments shown are the most straightforward.



2. All of the triangles have the same area: 4 square units. Sample reasoning: They all have a base of 2 units and a height of 4 units.

Responding To Student Thinking

Points to Emphasize
If students struggle with identifying bases and corresponding heights or with calculating areas, revisit this idea when opportunities arise over the next several lessons. For example, these practice problems require students to identify the bases and corresponding heights of triangles and to calculate their areas:

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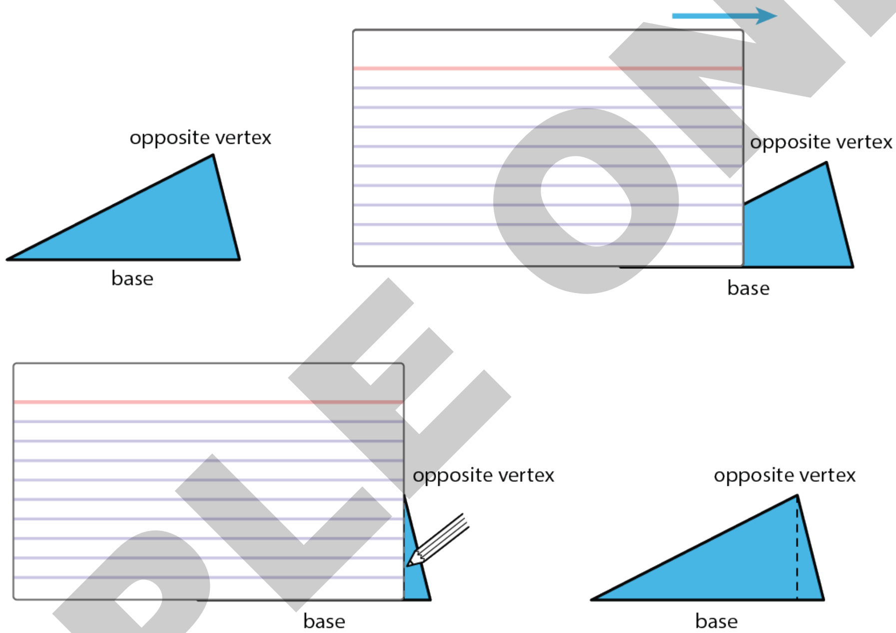
Grade 6, Unit 1, Lesson 11, Practice Problem 5
Grade 6, Unit 1, Lesson 11, Practice Problem 6

Lesson 10 Summary

A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite **vertex**. We can use tools with right angles to help us draw height segments.

An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
2. Line up one **edge** of the index card with that base.
3. Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4. Use the card edge to draw a line from the vertex to the base. That segment represents the height.



Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.

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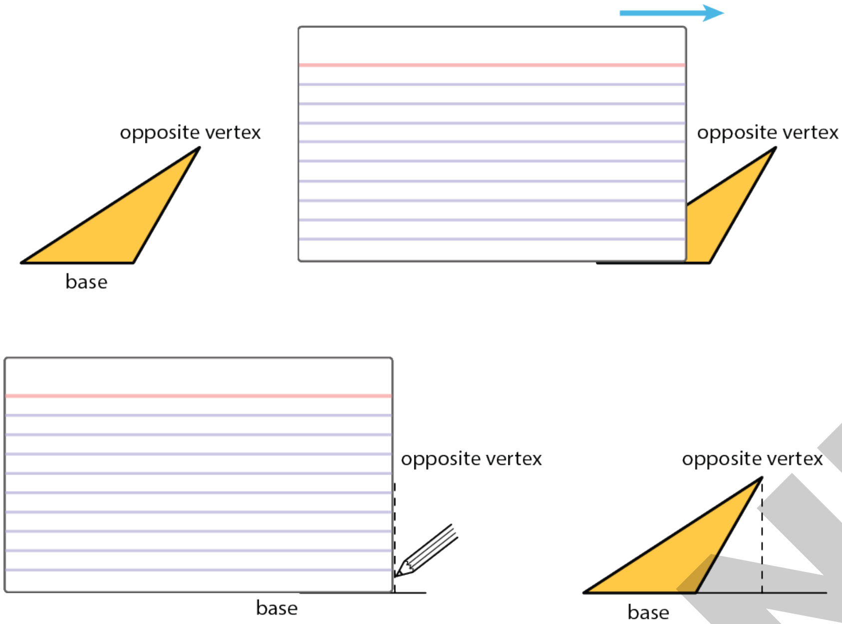
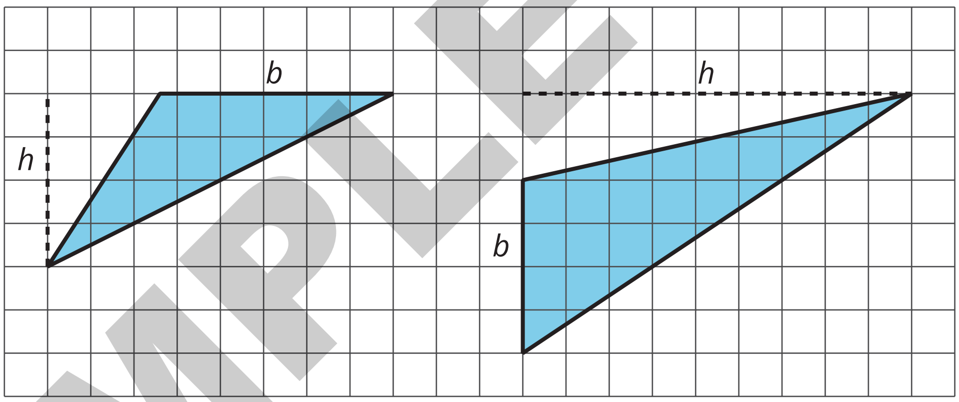


Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically. For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:



Glossary

- edge
- vertex

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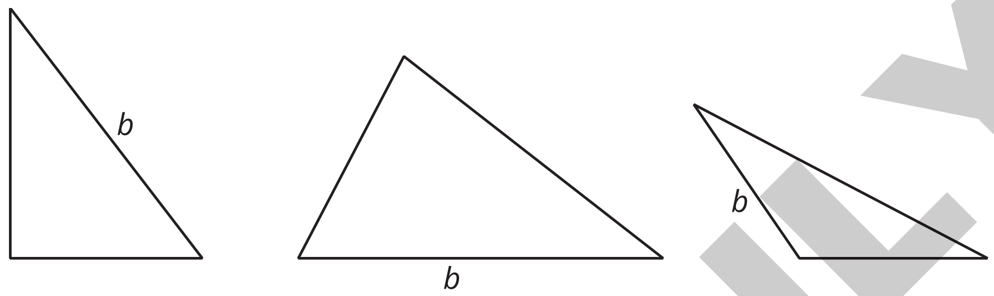
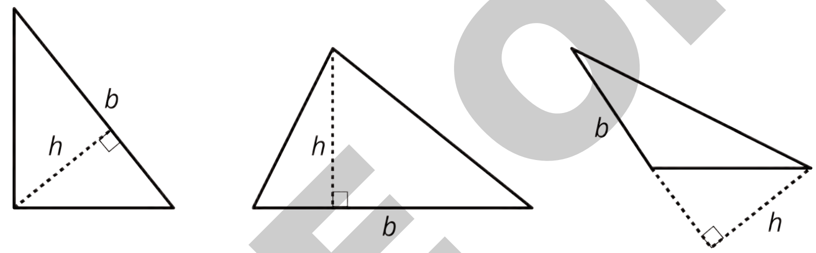
Practice Problems

1 Student Task Statement

For each triangle, a base is labeled *b*. Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.

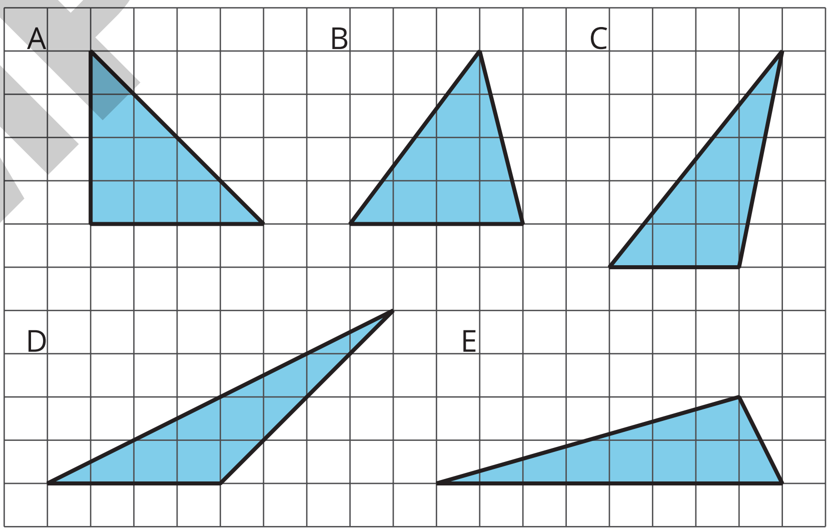


Solution



2 Student Task Statement

Select **all** triangles that have an area of 8 square units. Explain how you know.



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Sec C

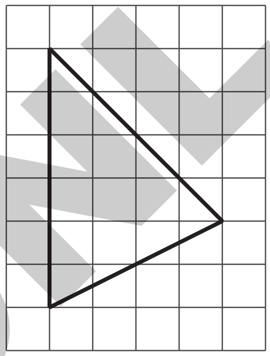
Solution

A, B, D, and E. Sample reasoning: Triangles A, B, and D all have a horizontal base of 4 units and a height of 4 units. $\frac{4 \cdot 4}{2} = 8$, so the area of each is 8 square units. Triangle C has a horizontal base of 3 units and a height of 5 units, so its area is 7.5 square units. Triangle E has a horizontal base of 8 units and a height of 2 units, so its area is 8 square units, since $\frac{8 \cdot 2}{2} = 8$.

3 Student Task Statement

Find the area of the triangle. Show your reasoning.

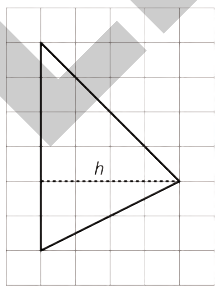
If you get stuck, carefully consider which side of the triangle to use as the base.



Sec C

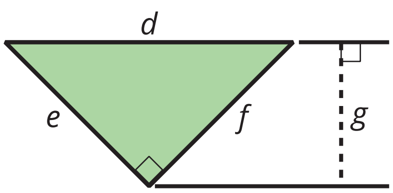
Solution

12 square units. Sample reasoning: The vertical side is 6 units long, and that side can be used as the base. The corresponding height, shown in the diagram, is 4 units. So the area is 12 square units. Another method is to surround the triangle with a rectangle then subtract the parts that are not in the triangle.



4 Student Task Statement

Can Side d be the base for this triangle? If so, which length would be the corresponding height? If not, explain why not.



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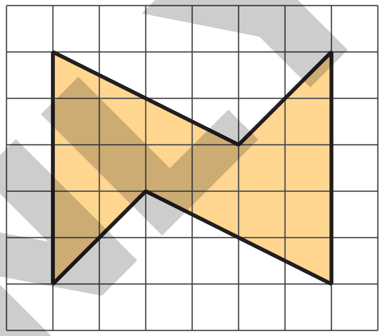
Solution

Yes, Side d can be the base, because it is a side of the triangle. The corresponding height is g.

5 from Unit 1, Lesson 3

Student Task Statement

Find the area of this shape. Show your reasoning.



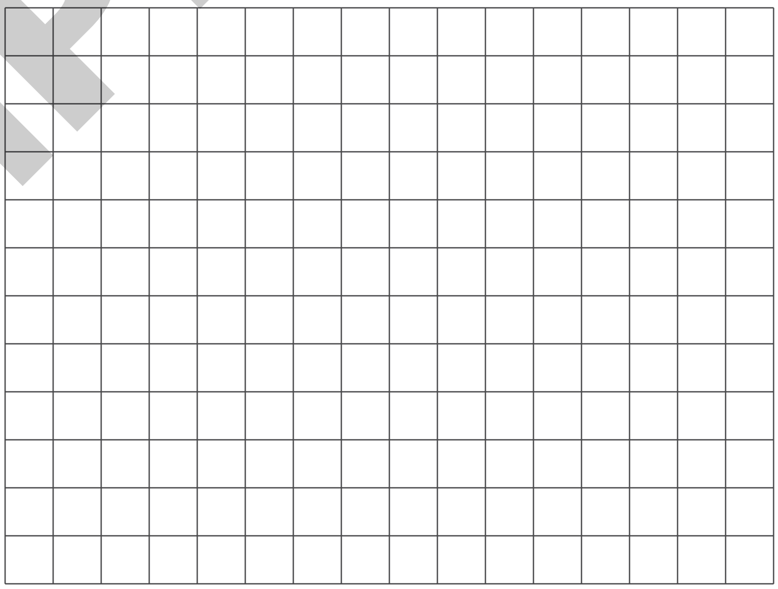
Solution

18 square units. Reasoning varies.

6 from Unit 1, Lesson 6

Student Task Statement

On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know that the areas are the same.



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Solution

Answers vary.

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Polygons

Goals

- Compare and contrast (orally) different strategies for finding the area of a polygon.
- Describe (orally and in writing) the defining characteristics of polygons.
- Find the area of a polygon, by decomposing it into rectangles and triangles, and present the solution method (using words and other representations).

Learning Targets

- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.
- Puedo usar vocabulario matemático para describir las características de un polígono.

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Lesson Narrative

In this lesson, students explore the defining characteristics of **polygons**. Then, they find the areas of polygons by decomposing the regions into triangles or parallelograms.

Students have worked with polygons in earlier grades and throughout this unit without having a formal term for this category of shapes. This lesson prompts them to examine examples and non-examples of polygons and write a definition for a polygon. There are many different accurate definitions for a polygon. The goal is not to find the most succinct definition possible, but to articulate the defining characteristics of a polygon in a way that makes sense to students.

Then, students reason about the areas of quadrilaterals on a grid. The lesson also includes an optional activity that involves finding the area of a polygon in the shape of a pinwheel. The activity is an opportunity for students to apply familiar reasoning strategies to find the area of a more complex figure—a polygon with 8 sides.

The work here allows students to see that the area of a polygon can be found by decomposing it into triangles. In observing and using this fact, students look for and make use of structure (MP7).

Math Community

Today's math community building time has two goals. The first is for students to make a personal connection to the math actions chart and to share on their *Cool-down* the math action that is most important to them. The second is to introduce the idea that the math actions that students have identified will be used to create norms for their mathematical community in upcoming lessons.

Standards

Building On	**4.G.A.2, 5.G.B**
Addressing	**6.G.A.1**
Building Towards	**6.G.A.1**

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Which Three Go Together?

Required Materials

Materials To Gather

- Math Community Chart: Activity 1
- Geometry toolkits: Activity 3, Activity 4

Materials To Copy

- Pinwheel Handout (1 copy for every 4 students): Activity 4

Required Preparation

Activity 4:

For this optional activity, if larger paper (and a photocopier that can accommodate it) is available, it would be helpful to have larger-format copies of the blackline master

Student Facing Learning Goals

Let's investigate polygons and their areas.

Sec C

11.1 Which Three Go Together: Triangles

Warm-up

5 mins

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare features of triangles. In making comparisons, students have a reason to use language precisely (MP6). The activity also enables the teacher to hear how students talk about characteristics of triangles and their area.

Students may describe the differences in the triangles in terms of:

- The angles (acute, right, or obtuse).
- The orientation of sides (vertical, horizontal).
- The side likely to be chosen as a base.
- The length of base or height.
- The area.

Standards

Building On **4.G.A.2, 5.G.B**

Instructional Routines

- Which Three Go Together?

Launch

Arrange students in groups of 2–4. Display the triangles for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three triangles that go together and can explain why. Next, tell each student to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

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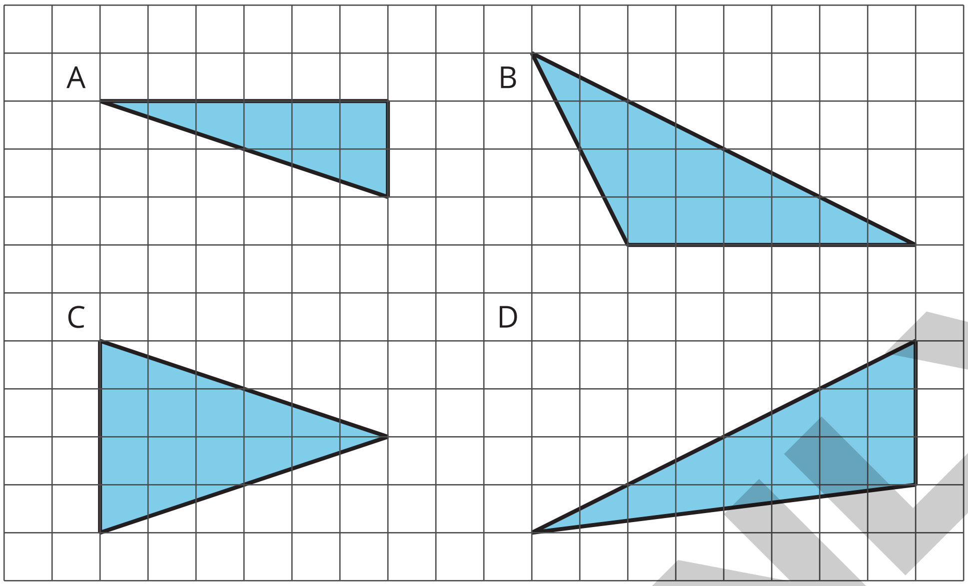

Student Response

Sample responses:

A ,B, and C go together because:

- They have a base or a height that is 6 units long.
- They have a side that slants down from left to right.

A, B, and D go together because:

- All their sides have different lengths.
- All their angles have different measures.

A, C, and D go together because:

- They all have a side that is vertical.
- We could find their area by choosing the vertical side as a base.

B, C, and D go together because:

- They are not right triangles (or have no right angle).
- They all have an area of 12 square units.
- They have just one side that would be easy to use as the base (where we can tell the corresponding height from the grid).

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use (such as "vertical," "horizontal," "right angle," "base," and "height") and to clarify their reasoning. Consider asking:

SAMPLE ONLY

- "How do you know . . . ?"
- "What do you mean by . . . ?"
- "Can you say that in another way?"

Math Community

After the *Warm-up*, display the revisions to the class Math Community Chart that were made from student suggestions in an earlier exercise. Tell students that over the next few exercises, this chart will help the class decide on community norms—how they as a class hope to work and interact together over the year. To get ready for making those decisions, students are invited at the end of today's lesson to share which "Doing Math" action on the chart is most important to them personally.

11.2 What Are Polygons?

15 mins

Activity Narrative

In this activity, students examine examples and non-examples of polygons and identify the defining characteristics of a polygon.

Developing a useful and complete definition of a polygon is harder than it seems. A formal definition is often very wordy or hard to parse. Polygons are often referred to as "closed" figures, but if used, this term needs to be defined, as the everyday meaning of "closed" is different from its meaning in a geometric context.

This activity prompts students to develop a working definition of polygon that makes sense to them, but that also captures all of the necessary aspects that makes a figure a polygon. Here are some important characteristics of a polygon.

- It is composed of line segments. Line segments are always straight.
- Each line segment meets one and only one other line segment at each end.
- The line segments never cross each other except at the end points.
- It is two-dimensional.

One consequence of the definition of a polygon is that there are always as many vertices as edges. Students may observe this and want to include it in their definition, although technically it is a result of the definition rather than a defining feature.

As students work, monitor for both correct and incorrect definitions of a polygon. Listen for clear and correct descriptions as well as common but inaccurate descriptions (so they can be discussed and refined later).

Standards

Building On	**5.G.B**
Building Towards	**6.G.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2–4. Give students 3–4 minutes of quiet think time. Afterward, ask them to share their responses with their group and complete the second question together. If there is a disagreement about whether a figure is a polygon, ask them to discuss each point of view and try to come to an agreement. Follow with a whole-class

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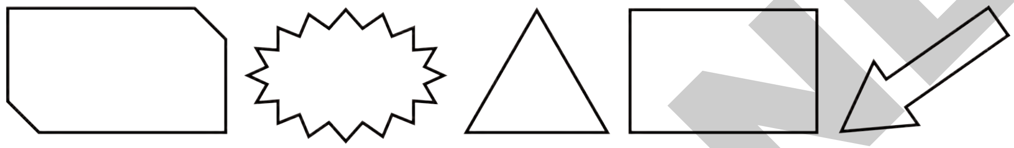
discussion.

Access for English Language Learners

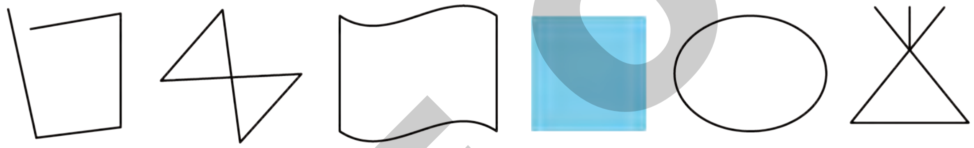
MLR2 Collect and Display. Circulate, and listen for and collect the language that students use as they discuss characteristics of polygons. On a visible display, record words and phrases such as “straight sides,” “edges,” “vertex (vertices),” or “intersect.” Invite students to borrow language from the display as needed and update it throughout the lesson.
Advances: Conversing, Reading

Student Task Statement

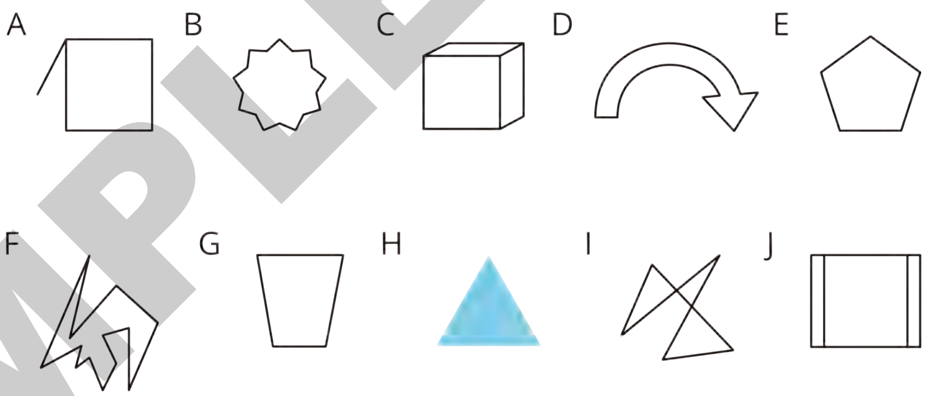
These five figures are **polygons**.



The next six figures are *not* polygons.



1. Circle the figures that are polygons.



2. What do the figures you circled have in common? What characteristics helped you decide whether a figure was a polygon?

Student Response

1. B, E, F, and G are polygons.
2. Answers vary. Characteristics that the polygons have in common: They are two-dimensional, composed of line segments that never cross each other, and each line segment meets one and only one other line segment at each end.

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Building on Student Thinking

Students may think that Figures C and I are polygons because they can see several triangles or quadrilaterals in each figure. Ask students to look closely at the examples and non-examples and see if there is a figure composed of multiple triangles or quadrilaterals, and if so, to see in which group it belongs.

Activity Synthesis

Display the figures in the first question for all to see. For each figure, ask at least one student to explain why they think it is or is not a polygon. (It is fine if students' explanations are not precise at this point.) Then, circle the figures that are polygons on the visual display.

Next, ask students to share their ideas about the characteristics of polygons. Record them for all to see. For each one, ask the class if they agree or disagree. If they generally agree, ask if there is anything they would add or elaborate on to make the description clearer or more precise. If they disagree, ask for an explanation.

Make the key characteristics of a polygon explicit: It is a two-dimensional figure, it is composed of line segments, the line segments meet only one other line segment at each end, and the line segments cross another line segment only at the endpoints. If one of these key characteristics is not mentioned by students, bring it up and revisit it at the end of the lesson.

Tell students we call the line segments in a polygon the "edges" or "sides," and we call the points where the edges meet the "vertices." Point to the sides and vertices in a few of the identified polygons.

If time permits, point out that polygons always enclose a region, but the region is not technically part of the polygon. When we talk about finding the area of a polygon, we are in fact finding the area of the region it encloses. So “the area of a triangle,” for example, is really shorthand for “area of the region enclosed by the triangle.”

11.3 Quadrilateral Strategies

15 mins

Activity Narrative

This activity has several aims. It prompts students to apply what they learned to find the area of quadrilaterals that are not parallelograms, encourages them to plan before jumping into a problem, and urges them to reflect on the merits of different methods.

Students begin by thinking about the moves they would make to find the area of a quadrilateral and by explaining their preference to their partners. They then consider and discuss the different strategies taken by other students. Along the way, they may notice that some strategies are more direct or efficient than others. Students reflect on these strategies and use their insights to plan the work of finding the area of polygons in this activity and beyond.

Note that it is unnecessary for students to take the most efficient path. It is more important that they choose an approach that makes sense to them but have the chance to see the pros and cons of various approaches.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

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Launch

Ask students to recall the definition of "quadrilateral" from earlier grades, or tell students that a quadrilateral is a polygon with 4 sides. Tell students that we will now think about how to find the area of quadrilaterals.

Arrange students in groups of 4. Display the image of Quadrilaterals A–F for all to see. Direct their attention to Quadrilateral D.

Give students a minute of quiet time to think about the first 2–3 moves they would make to find the area of Quadrilateral D. Offer some sentence starters: "First, I would . . . Next I would . . ., and then I would . . ." Encourage them to show their moves on the diagram in their material. Emphasize that we are interested only in the plan for finding area and not in the area itself, so no calculation is expected. Then, give them 1–2 minutes to share their moves with their group.

Ask students to indicate what their first planned move is. Does their very first move involve:

- Decomposing the quadrilateral?
- Enclosing the quadrilateral?
- Another move?

Ask the students whose first move is to decompose the figure:

- “How many pieces will result from the decomposition? 2 pieces? 3 pieces? 4 pieces? More?”
- “What is the next move? Rearrange? Duplicate a piece? Calculate the area of a piece? Something else?”

Ask the students whose first move is to enclose the figure:

- “How many rectangles will you create? 1 rectangle? 2 rectangles? More?”
- “What is the next move? Rearrange the extra pieces? Calculate the area of an extra piece? Something else?”

For each sequence that students mentioned, draw a quick diagram to illustrate it for all to see.

Once students have a chance to see a variety of approaches, ask students to revisit their sequence of moves. Give students 1–2 minutes to think about the pros and cons of their original plan, and if there was another strategy that they found productive. Invite a few students to share their reflections.

Then, give students access to their geometry toolkits and quiet time to complete the activity. Ask students to keep in mind the merits of the different strategies they have seen as they plan their work.

Student Task Statement

Find the area of *two* quadrilaterals of your choice. Show your reasoning.

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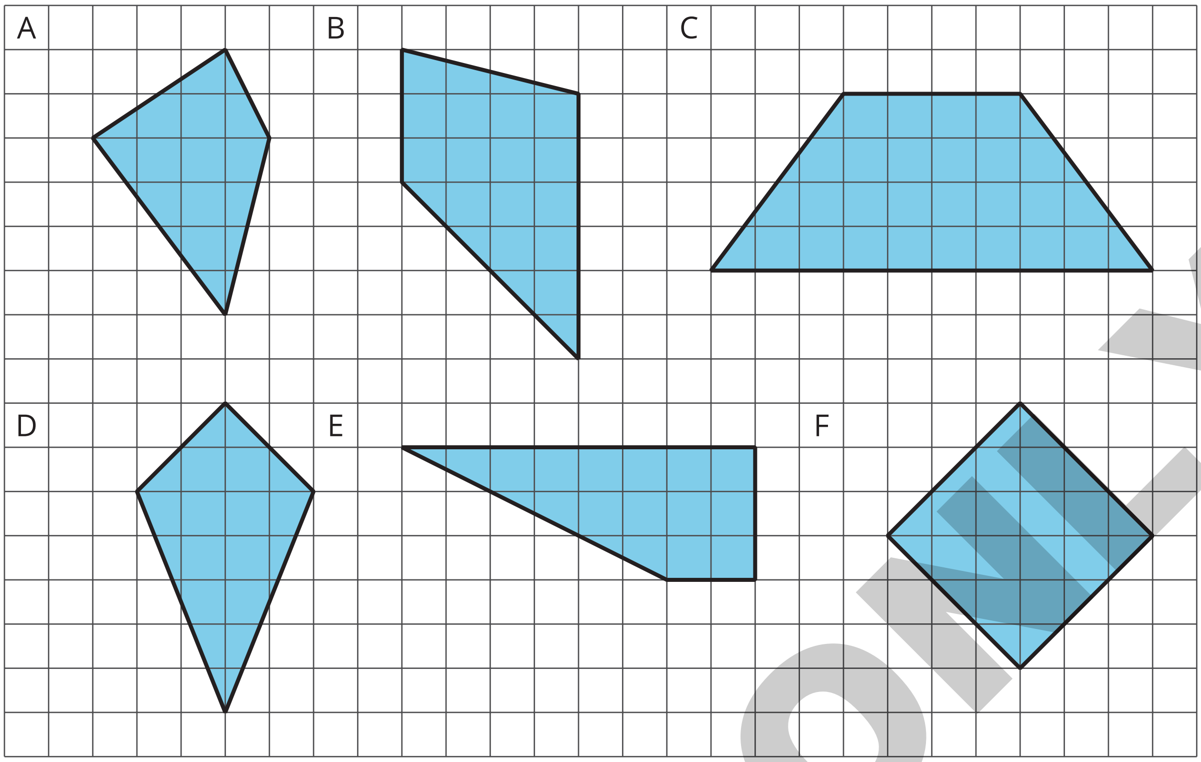


Student Response

Figure A: 12 square units

Figure B: 18 square units

Figure C: 28 square units

Figure D: 14 square units

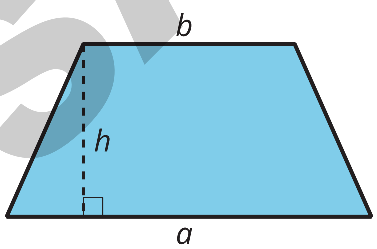
Figure E: 15 square units

Figure F: 18 square units

Reasoning varies. Students could decompose the quadrilateral into parallelograms and triangles to find the area, decompose and rearrange the pieces into a shape of which they can easily find the area, or enclose the figure in a rectangle and subtract the area of the extra pieces

Are You Ready for More?

Here is a trapezoid. a and b represent the lengths of its bottom and top sides. The segment labeled h represents its height; it is perpendicular to both the top and bottom sides.



Apply area-reasoning strategies—decomposing, rearranging, duplicating, etc.—to the trapezoid so that you have one or more shapes with areas that you already know how to find. Use the shapes to help you write a formula for the area of a trapezoid. Show your reasoning.

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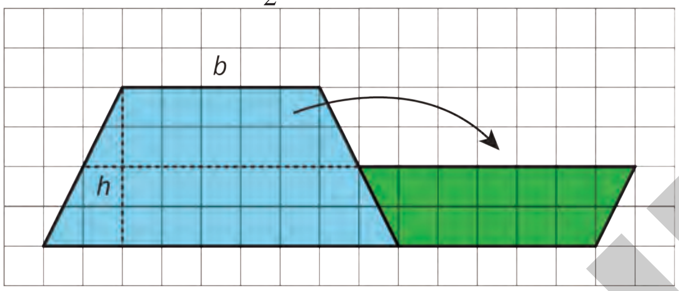
Extension Student Response

Sample responses:

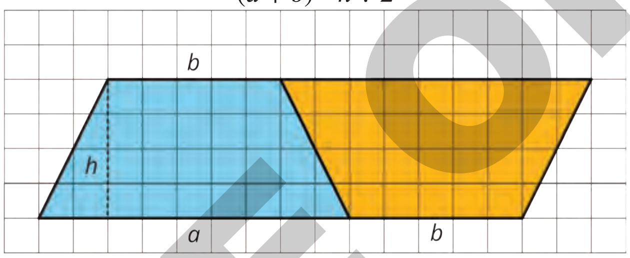
- Cut the trapezoid in half horizontally, rotate the top piece, and attach it to the bottom piece. Add the top and bottom side lengths and multiply that by half of the original height. The result is a parallelogram.

$$\frac{1}{2} \cdot h \cdot (a + b)$$

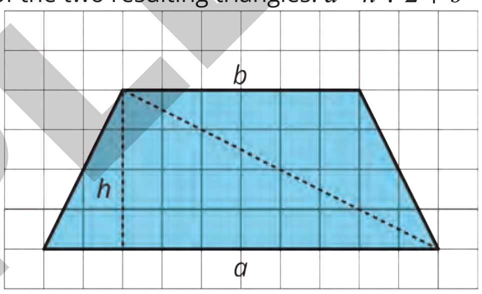


- Place an identical but rotated copy of the same trapezoid next to the original to make a parallelogram. Then, find the area of the parallelogram and divide that by 2.

$$(a + b) \cdot h \div 2$$



- Draw a diagonal and add the areas of the two resulting triangles. $a \cdot h \div 2 + b \cdot h \div 2$



Activity Synthesis

To conclude the activity, ask students to choose one quadrilateral they worked on (other than D) and tell their group the first couple of moves they made for finding its area and why. Encourage other group members to listen carefully, check that the reasoning is valid, and offer feedback.

Students may have noticed that all the approaches involved decomposing one or more regions into triangles, rectangles, or both. If not mentioned by students, point this out. Emphasize that we can decompose any polygon into triangles and rectangles to find its area.

Access for English Language Learners

MLR8 Discussion Supports. Students who are working toward verbal output may benefit from access to mini-

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whiteboards, sticky notes, or spare paper to write down and show to their group the first couple of moves they made to find the area of their quadrilateral.
Advances: Writing, Representing

11.4

Pinwheel

Optional

30 mins

Activity Narrative

In this activity, students determine the area of an unfamiliar polygon and think about various ways for doing so. The task prepares students to find the areas of other unfamiliar shapes in real-world contexts. It also reinforces the practice of sense-making, planning, and persevering when solving a problem (MP1). Students reason independently before discussing and recording their strategies in groups.

Because the shape of the polygon is more complex than what students may have seen so far, expect students to experiment with one or more strategies. Consider preparing extra copies of the blackline master for students to use, if needed.

As students work, monitor for those who:

- Decompose the pinwheel into triangles and find the areas.
- Decompose the pinwheel into rectangles and triangles, rearrange the pieces into parallelograms (right or non-right), and find the areas.
- Enclose the pinwheel with one large square or several smaller rectangles, decompose the extra regions into triangles and rectangles, find the areas of the extra pieces, and subtract them from the area(s) of the enclosing rectangle(s).

Make note of the variations and complexities in how students obtain shapes whose areas can be found. If there is limited variation in strategies, look for different ways of recording the same strategy.

Students have opportunities to notice and make use of the structure of the pinwheel in their reasoning (MP7). For instance, the pinwheel can be decomposed into four identical pieces (or sets of pieces). Also, enclosing the pinwheel with a square creates four extra regions that are identical.

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 4. Give students access to their geometry toolkits and 5 minutes of quiet time to plan an approach for finding the area of the pinwheel. Then, ask them to share their plan with their group.

The group then decides on one or more strategies to pursue, works together to find the area, and creates a visual display of the strategy (or strategies) used. Give each group one or more copies of the blackline master for the visual

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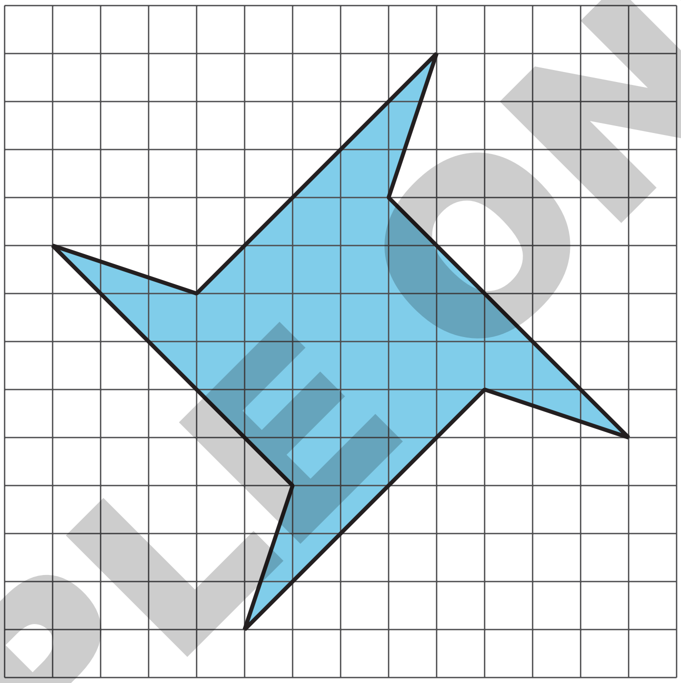
display. Encourage students to include details that will help others interpret their thinking. For example, specific language, use of different colors, shading, arrows, labels, notes, diagrams, or drawings.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, remind students that triangles can be decomposed, rearranged, enclosed, or duplicated to determine area.
Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Student Task Statement

Find the area of the shaded region in square units. Show your reasoning.



Student Response

40 square units. Sample reasoning:

- There are 4 identical sets of two different triangles. The area of each triangle can be found by enclosing it in a rectangle and subtracting the areas of right triangles, or by using the formula for the area of a triangle.
  $4 \cdot \frac{1}{2}(3 \cdot 4 + 2 \cdot 4) = 40$

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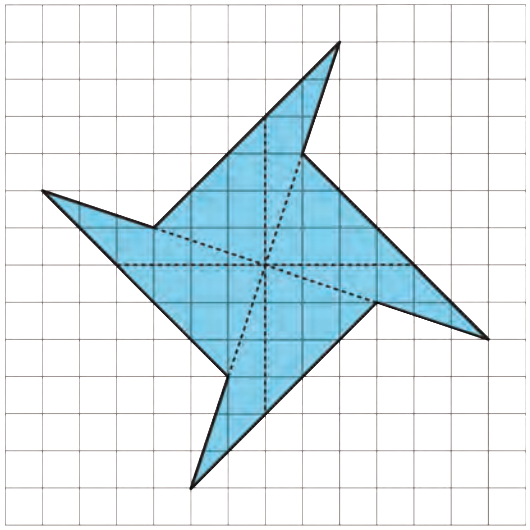
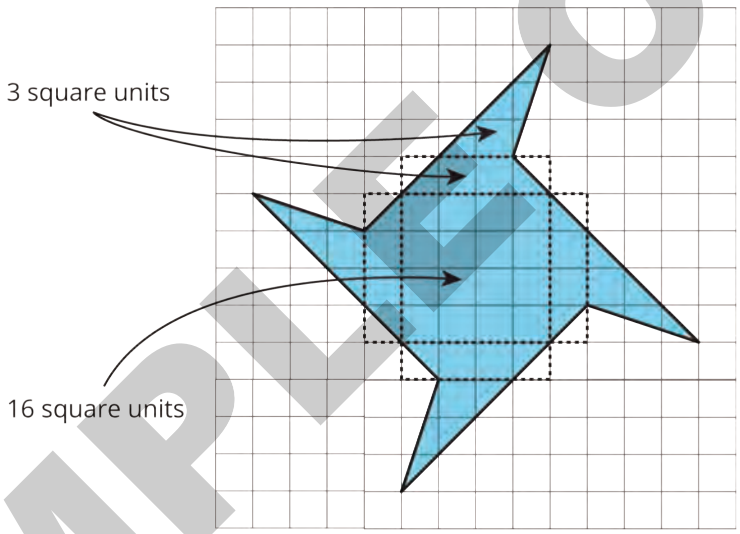


- The middle of the pinwheel is a square with the area of 16 square units. Around the square are 4 identical trapezoids. The area of each trapezoid can be found by enclosing it with a rectangle and subtracting the area of the extra pieces, or by decomposing and rearranging the pieces into a rectangle with an area of 3 square units. The pointy parts of the pinwheel are triangles, each with a base of 2 units and a height of 3 units, so the area of each is 3 square units.



Building on Student Thinking

Students who overlay a rotated square over the figure such that the four pins are shown as four right triangles may use incorrect side lengths for the square or the triangles (for instance, assuming that one of the side lengths is 2 units instead of a little less than 3 units). Help them see, by measuring one, that the diagonal of a unit square is longer than its side length.

Activity Synthesis

Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Give students time to visit one another’s visual display. Consider displaying the following questions for students to discuss as they investigate others' work:

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- "Did this group find the same area as our group? If not, why?"
- "How is their strategy like our strategy?"
- "How is their strategy different from ours?"

During the whole-class discussion, ask students:

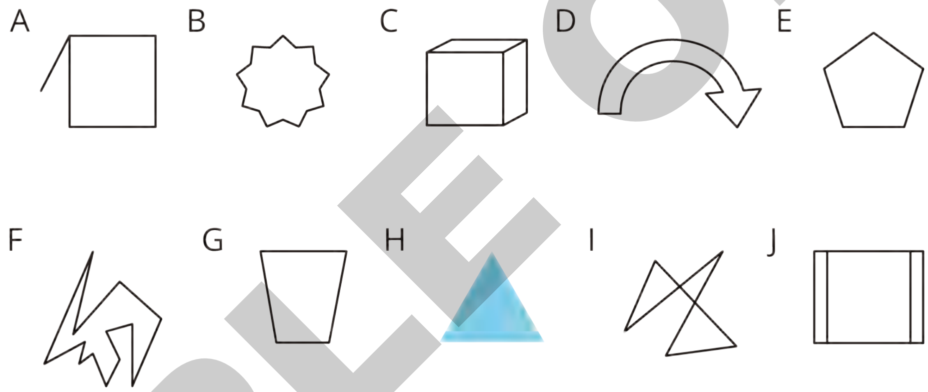
- "What did the strategies have in common? How were they different?"
- "Are there any benefits or drawbacks to one representation compared to another?"

Highlight similarities in students' work in broader terms, as outlined in the activity narrative. Reinforce that all approaches involve decomposing a polygon into triangles and rectangles to find area.

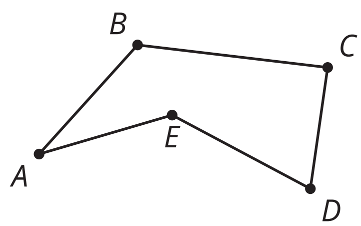
Lesson Synthesis

To review the defining characteristics of a polygon, return to the image in the *"What are Polygons?"* activity and display the list of defining features generated by students in that activity.

Revisit each figure that is *not* a polygon and ask students to explain why it is not a polygon. Encourage students to use their list to support their explanations, as well as to suggest revisions to their working definition.



Here is a polygon with 5 sides.



Ask students:

- "How do we know this figure is a polygon?" (It is composed of line segments. Each segment meets only one other segment at each end. The segments do not cross one another. It is two-dimensional.)
- "What does it mean to find the area of this polygon?" (It means finding the area of the region inside it.)
- "How can we find the area of this polygon?" (We can decompose the region inside it into triangles and rectangles.)

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11.5

Triangulation

Cool-down

5 mins

This *Cool-down* assesses students' understanding of the defining characteristics of a polygon and the ways it can be decomposed.

Standards

Addressing **6.G.A.1**

Launch

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community-building question "Which 'Doing Math' action is most important to you, and why?" Ask students to respond to the question after completing the *Cool-down*.

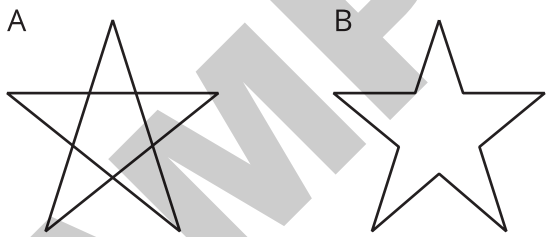
After collecting the *Cool-downs*, review student responses to the community-building question. Use the responses to draft a student norm and a teacher norm to use as an example in Exercise 6. For example, if "sharing ideas" is a common choice for students, a possible norm is "We listen as others share their ideas."

For the teacher norms section, if "questioning vs. telling" from the "Doing Math" section is key for your teaching practice, then one way to express that as a norm is "Ask questions first to make sure I understand how someone is thinking."

Give students access to their geometry toolkits. Tell students that they need to show only how the area could be found; they do not have to actually calculate the area.

Student Task Statement

1. Here are two five-pointed stars. A student said, "Both figures A and B are polygons. They are both composed of line segments and are two-dimensional. Neither have curves." Do you agree with the statement? Explain your reasoning.



2. Here is a five-sided polygon. Describe or show the strategy you would use to find its area. Mark up and label the diagram to show your reasoning so that it can be followed by others. (It is not necessary to actually calculate the area.)

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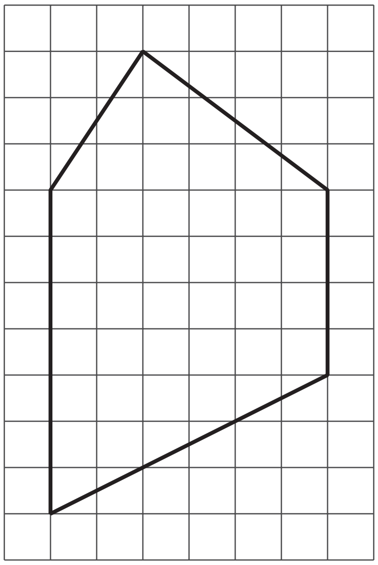
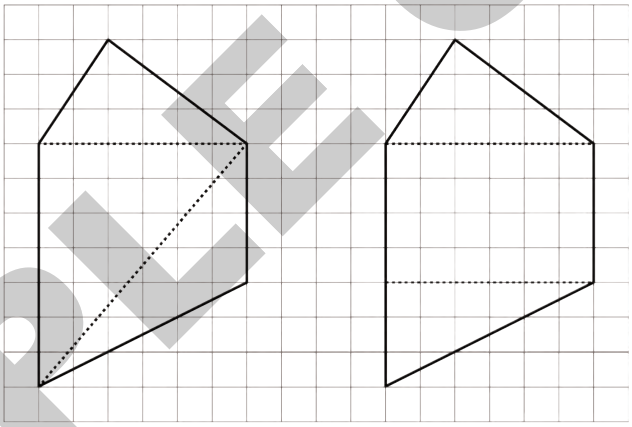


Student Response

1. Disagree. Only Figure B is a polygon. Sample reasoning: Every segment in Figure A meets or crosses more than two segments at its ends, so it is not a polygon. Each segment in Figure B meets only one other segment at each end.
2. Sample responses:



   - The polygon can be decomposed into three triangles: one with a base of 6 units and a height of 3, a second one with a base of 7 and a height of 6, and a third with a base of 4 and a height of 6. All areas can be calculated using the area formula.
   - The polygon can be decomposed into two triangles and a rectangle. One triangle has a base of 6 and a height 3, and the second has a base of 6 and a height of 3. Their areas can be calculated with the area formula. The rectangle is 6 by 4, so its area is the product of 6 and 4.

Responding To Student Thinking

Points to Emphasize
If students struggle with decomposing a polygon into parallelograms and triangles whose areas can be calculated, integrate discussions about different ways to find the area of a polygon. For example, ask students to analyze Lin's and Andre's ways of decomposing the hexagon in this practice problem and explain how its area can be found using each method:

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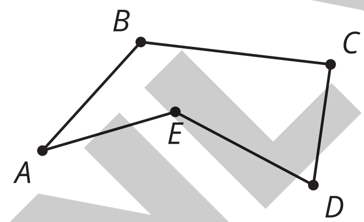
Lesson 11 Summary

A **polygon** is a two-dimensional figure composed of straight line segments.

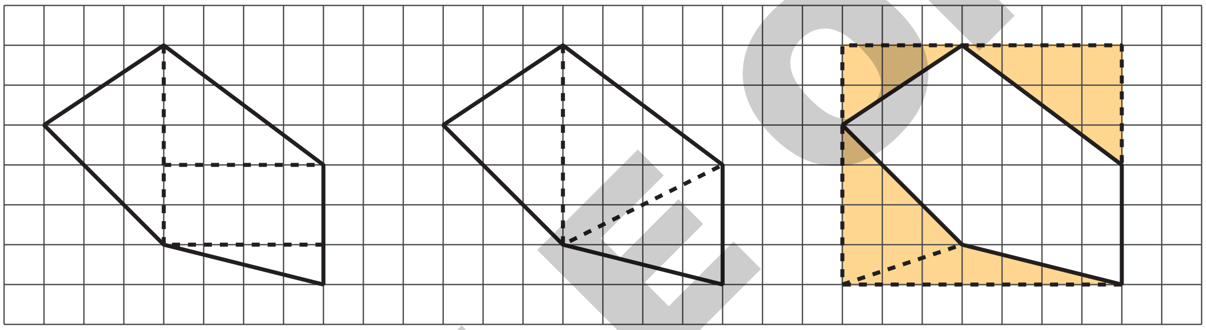
- Each end of a line segment connects to one other line segment. The point where two segments connect is a vertex. The plural of vertex is vertices.
- The segments are called the edges or sides of the polygon. The sides never cross each other. There are always an equal number of vertices and sides.

Here is a polygon with 5 sides. The vertices are labeled A, B, C, D, and E.

A polygon encloses a region. The area of a polygon is the area of the region inside it.



We can find the area of a polygon by decomposing the region inside it into triangles and rectangles.



The first two diagrams show the polygon decomposed into triangles and rectangles. The sum of their areas is the area of the polygon. The last diagram shows the polygon enclosed with a rectangle. Subtracting the areas of the triangles from the area of the rectangle gives us the area of the polygon.

Glossary

- polygon

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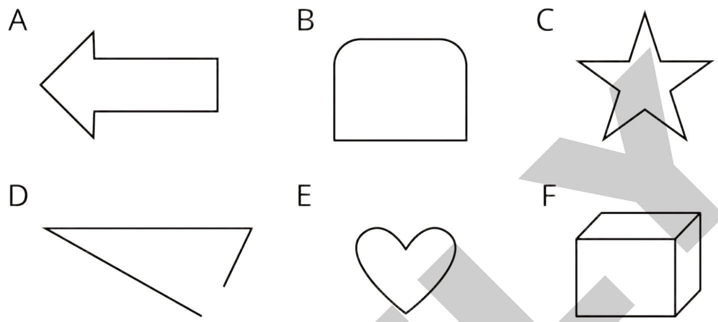
Practice Problems

1 Student Task Statement

Select **all** the polygons.



A. A

B. B

C. C

D. D

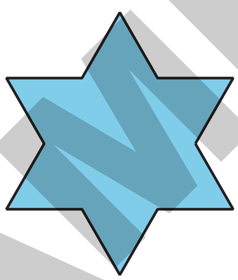
E. E

F. F

Solution

A, C

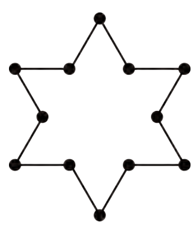
2 Student Task Statement



Mark each vertex with a large dot. How many edges and vertices does this polygon have?

Solution

12 edges and
12 vertices



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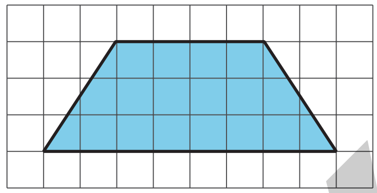
3 Student Task Statement

Find the area of this trapezoid. Explain or show your reasoning.

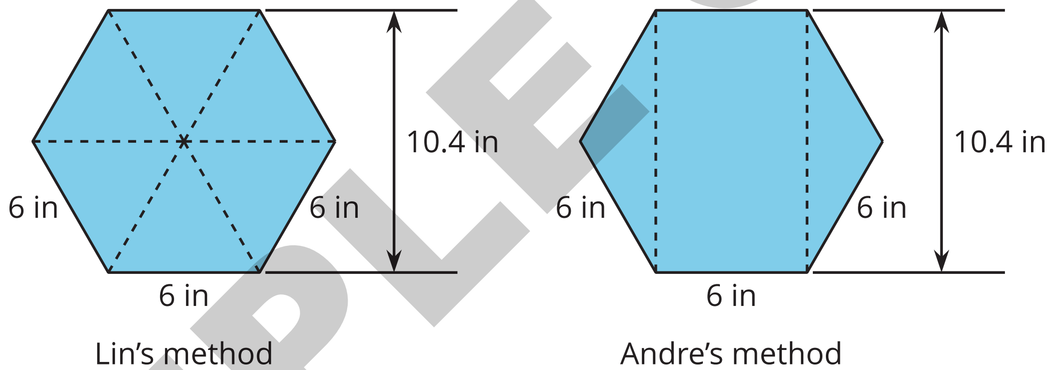


Solution

18 square units. Sample reasoning: Enclose the trapezoid inside a 3-unit-by-8-unit rectangle. The area of the rectangle is 24 square units because $8 \cdot 3 = 24$. The area of each unshaded triangle within the rectangle is 3 square units because $(2 \cdot 3) \div 2 = 3$. The sum of areas of the two triangles is 6 square units. $24 - 6 = 18$, so the area of the trapezoid is 18 square units.

4 Student Task Statement

Lin and Andre used different methods to find the area of a regular hexagon with 6-inch sides. Lin decomposed the hexagon into six identical, equilateral triangles. Andre decomposed the hexagon into a rectangle and two triangles.



Find the area of the hexagon using each person's method. Show your reasoning.

Solution

93.6 square inches. Sample reasoning:

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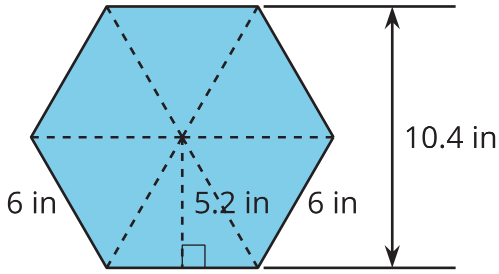
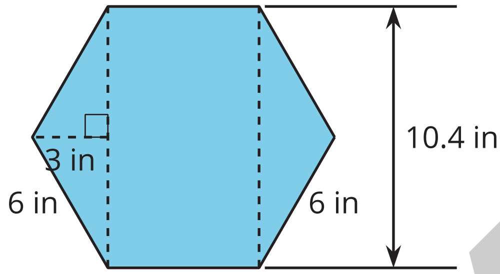


Lin's method



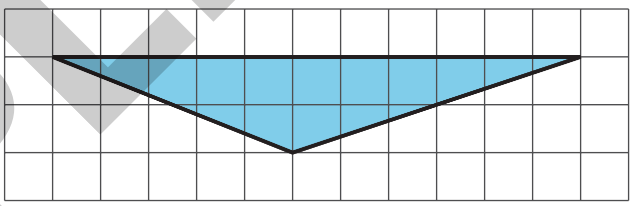
Andre's method

- The height of each triangle in Lin's diagram is half of 10.4 inches or 5.2 inches. The area of each triangle is 15.6 square inches. $\frac{1}{2} \cdot 6 \cdot (5.2) = 15.6$. The hexagon is composed of 6 triangles, so its area is $6 \cdot (15.6)$ or 93.6 square inches.
- The rectangle in Andre's diagram is $(10.4) \cdot 6$ or 62.4 square inches. Each triangle has a base of 10.4 inches and a height of 3 inches. (The horizontal distance across the middle of the hexagon is composed of two 6-inch segments. The vertical line that Andre drew cuts one 6-inch segment in half, so the segment on one side is 3 inches long.) The area of each triangle is $\frac{1}{2} \cdot 10.4 \cdot 3$ or 15.6 square inches. The area of the hexagon is therefore $62.4 + 15.6 + 15.6$ or 93.6 square inches.

5 from Unit 1, Lesson 9

Student Task Statement

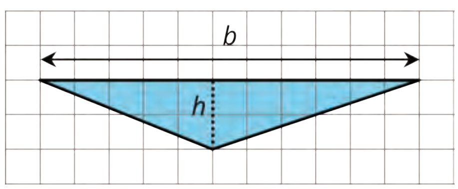
a. Identify a base and a corresponding height that can be used to find the area of this triangle. Label the base b and the corresponding height h.



b. Find the area of the triangle. Show your reasoning.

Solution

a.

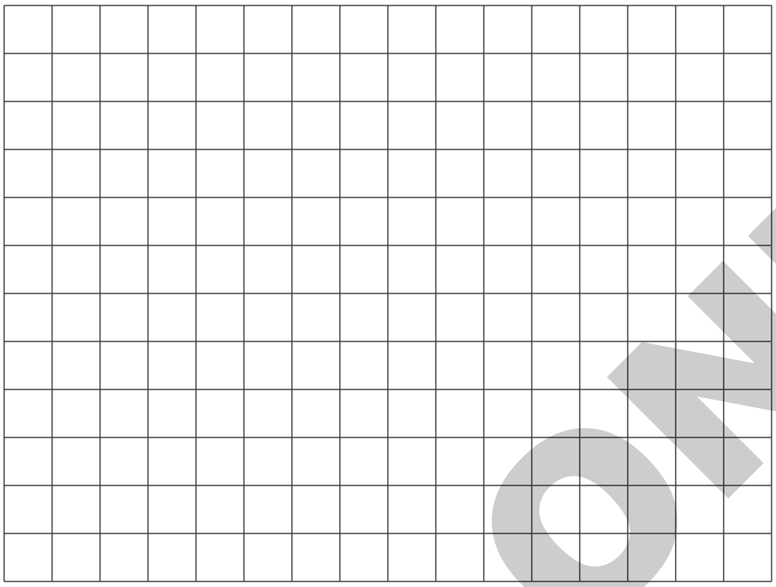


b. 11 square units. $\frac{1}{2} \cdot 11 \cdot 2 = 11$.

SAMPLE ONLY

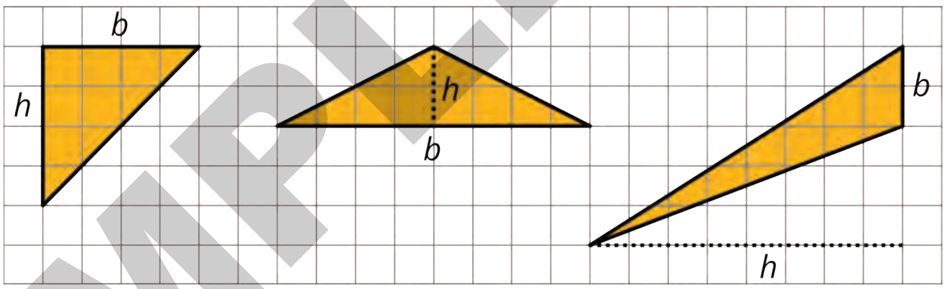
Student Task Statement

On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.



Solution

Drawings should show triangles with a base and a height that multiply to be 24 square units (that is, each triangle is half of a parallelogram with an area of 24 square units). Sample responses:



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Section D: Surface Area

Goals

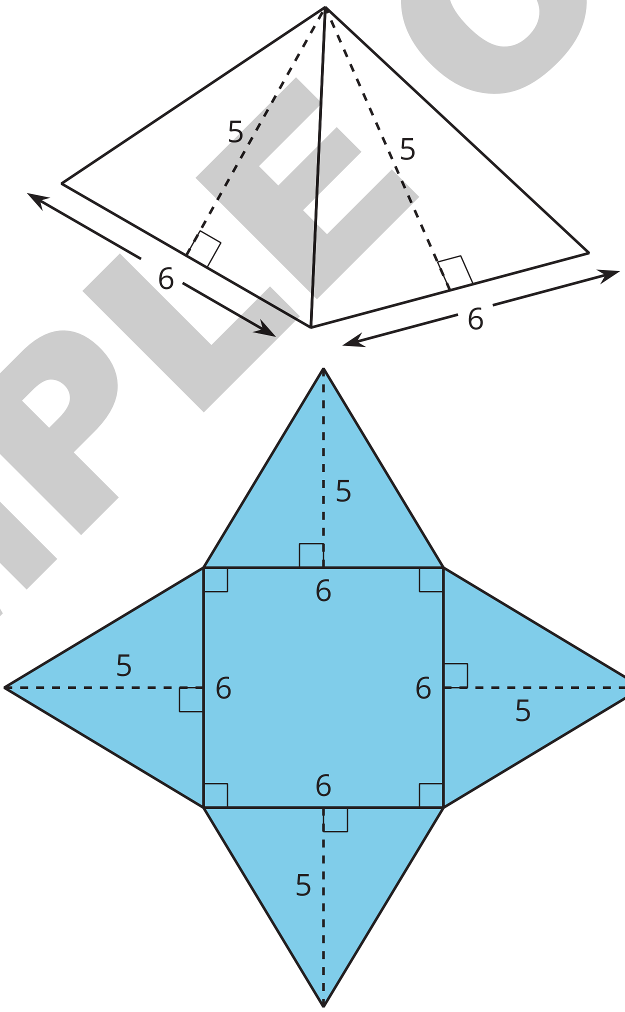
- Identify or create a net that represents a prism or pyramid.
- Use a net to calculate the surface area of a prism or pyramid and explain the solution method.

Section Narrative

This section introduces students to polyhedra and surface area. Students apply their knowledge about areas of polygons to create nets and find surface areas of three-dimensional figures.

First, students learn that surface area is the number of unit squares that covers all the faces of a three-dimensional figure, without gaps or overlaps. They reason about the surface areas of rectangular prisms and figures built from unit cubes.

Then, students explore the characteristics of a polyhedron and develop a working definition for one. They learn that a net is a two-dimensional representation of a polyhedron. Students assemble prisms and pyramids from nets and also visualize the polyhedron that can be assembled from a given net. Later, students create nets for given prisms or pyramids and use the nets to calculate the surface areas.



SAMPLE ONLY

Sec D

The section includes an optional lesson to help reinforce students' understanding of surface area and volume as distinct attributes of three-dimensional figures.

Teacher Reflection Questions

- **Math Content and Student Thinking:** In this section, how did students see the connections between a polyhedron and its net, or between a three-dimensional figure and a two-dimensional representation of it? What helped them make connections between the representations?
- **Pedagogy:** Reflect on the descriptors that students contributed in the "sounds like" and "looks like" categories in your Math Community Chart. As the year progresses, what new descriptors do you hope that students might add based on their experiences in your classroom?
- **Access and Equity:** Identify who has been sharing their ideas in class lately. Make a note of students whose ideas have not been shared and look for an opportunity for them to share their thinking in tomorrow's lesson.

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Section D Checkpoint

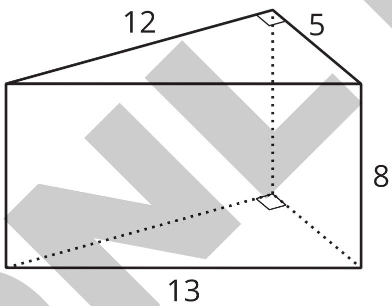
1

Goals Assessed

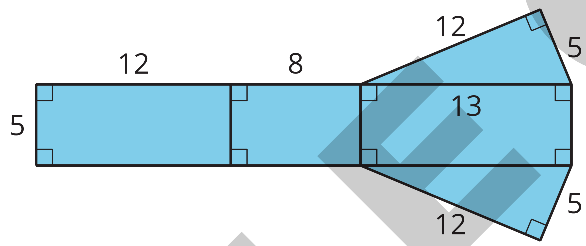
- Identify or create a net that represents a prism or pyramid.

Student Task Statement

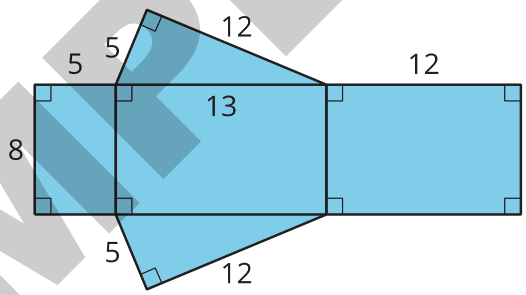
Select all nets that can be assembled into this triangular prism.



A.



B.



C.

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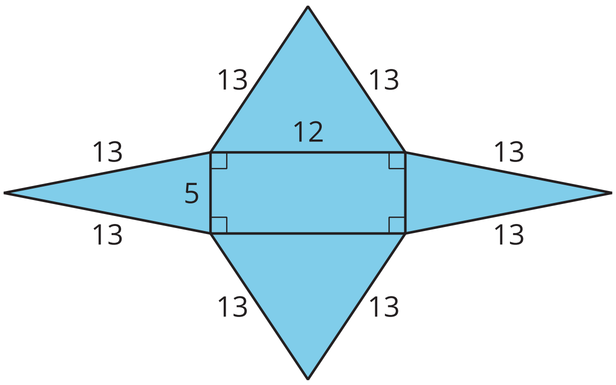
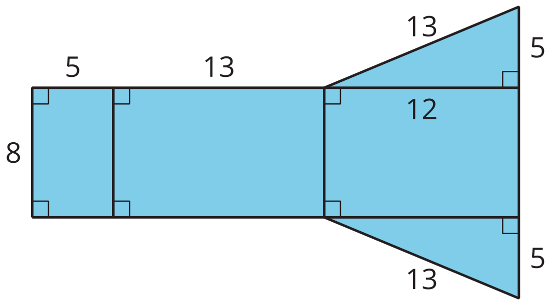
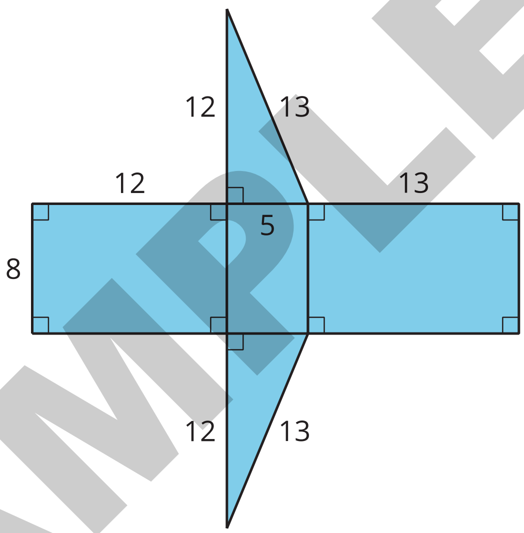
Sec D



D.



E.



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Solution

B, D, E

Responding To Student Thinking

Points to Emphasize

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If students struggle to connect polyhedra and their nets, discuss ways to relate the two representations. Do this when opportunities arise over the next several lessons. For example, instruct students to draw at least two nets: one net that would create a cube and one that would not create a cube. As they work on this activity, ask them to explain their reasoning:

Grade 6, Unit 1, Lesson 18, Activity 2 The Net of a Cube

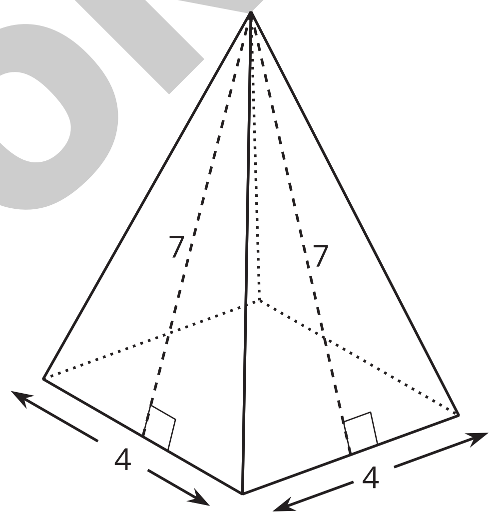
2

Goals Assessed

- Identify or create a net that represents a prism or pyramid.
- Use a net to calculate the surface area of a prism or pyramid and explain the solution method.

Student Task Statement

a. Sketch a net for this square pyramid and label the known lengths.



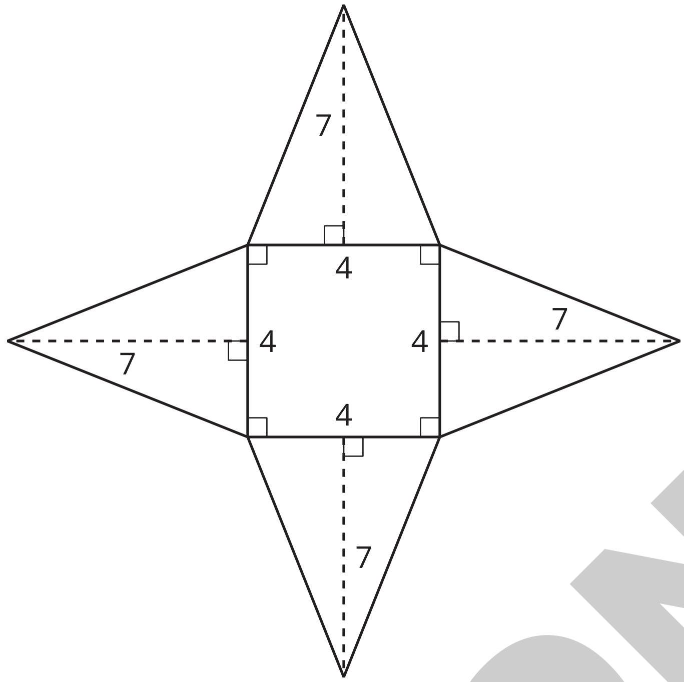
b. Find the surface area of the pyramid in square units. Show your reasoning.

Solution

a. Sample response:

SAMPLE ONLY

Sec D



b. 72 square units. Sample reasoning: The square base is $4 \cdot 4$ or 16 square units. The area of each triangle is $\frac{1}{2} \cdot 4 \cdot 7$, which is 14 square units. There are 4 triangles, so the surface area is: $16 + (4 \cdot 14)$, which is $16 + 56$ or 72.

Sec D

Responding To Student Thinking

Points to Emphasize

If students struggle to find the surface area, emphasize ways to find the area of each face of a polyhedron and to organize the calculations systematically. For example, when students work on the indicated practice problem, ask them to identify all of its faces and then to determine (and, possibly, record) the shape and known measurements of each face. Next, ask them to come up with a strategy for finding the area of each face and the total surface area of the prism. Ask them to repeat those steps for Prism B.

Grade 6, Unit 1, Lesson 17, Practice Problem 5

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What is Surface Area?

Goals

- Calculate the surface area of a rectangular prism and explain (orally and in writing) the solution method.
- Comprehend that the term "surface area" (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object.

Learning Targets

- I know what the surface area of a three-dimensional object means.

Lesson Narrative

This lesson introduces students to the concept of **surface area**. They use what they learned about the area of rectangles to find the surface areas of prisms with rectangular **faces**.

Students begin by exploring surface area in concrete terms, by thinking about the number of square sticky notes it would take to cover a filing cabinet. First, they make an estimate, and then they think about what information is needed to calculate the actual number of sticky notes. Because no techniques are given, students need to make sense of the problem and persevere in solving it (MP1). As they analyze the situation, think about the problem in context, and consider the mathematics strategies that they know, students practice modeling with mathematics (MP4).

Students learn that "surface area" (in square units) is the number of unit squares it takes to cover all the faces of a three-dimensional figure without gaps or overlaps.

Later in the lesson, students determine the surface area of rectangular prisms that are built from snap cubes.

Standards

Addressing	**6.G.A.4**
Building Towards	**6.G.A.4**

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder
- Poll the Class

Required Materials

Materials To Gather

- Snap cubes: Activity 3

Required Preparation

Activity 3:

Prepare 12 cubes per student and extra copies of isometric dot paper.

Lesson:

Build several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes for the *Cool-down*.

SAMPLE ONLY

Student Facing Learning Goals

Let’s cover the surfaces of some three-dimensional objects.

12.1 Covering the Cabinet (Part 1)

Warm-up

5 mins

Activity Narrative

This activity prepares students to think about surface area, which they explore in this lesson and in upcoming lessons. Students watch a video of a cabinet being gradually tiled with non-overlapping sticky notes. The cabinet was left only partially tiled, which raises the question of the number of sticky notes it takes to cover the entire rectangular prism. Students estimate the answer to this question.

This activity was inspired by Andrew Stadel. Media used with permission. http://www.estimation180.com/filecabinet.

Standards

Building Towards **6.G.A.4**

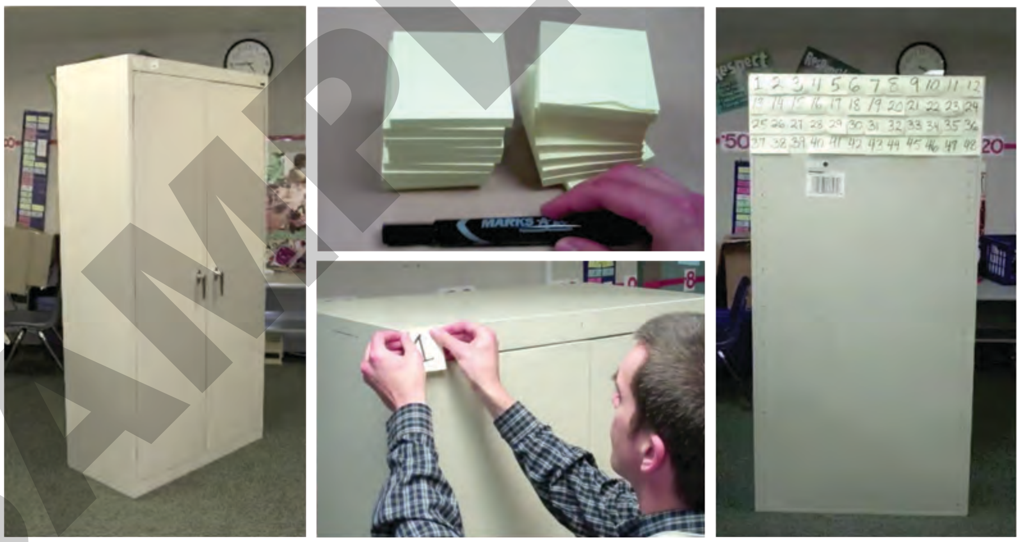
Instructional Routines

- Notice and Wonder
- Poll the Class

Launch

Arrange students in groups of 2. Show the video of a teacher beginning to cover a large cabinet with sticky notes or display the following still images for all to see. Before starting the video or displaying the image, ask students to be prepared to share one thing they notice and one thing they wonder.

Video 'File Cabinet - Act 1' available here: https://player.vimeo.com/video/304136534.



Give students a minute to share their observation and question with a partner. Invite a few students to share their questions with the class. If the question, “How many sticky notes would it take to cover the entire cabinet?”, is not mentioned, ask if anyone wondered how many sticky notes it would take to cover the entire cabinet.

Give students a minute to make an estimate.

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Student Task Statement

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

Student Response

Estimates vary. The actual number of sticky notes is 935. Good estimates are in the 800–1,200 range.

Activity Synthesis

Poll the class for students' estimates, and record them for all to see. Invite a couple of students to share how they made their estimate. Explain to students that they will now think about how to answer this question.

12.2 Covering the Cabinet (Part 2)

20 mins

Activity Narrative

After making an estimate of the number of sticky notes on the cabinet in the *Warm-up*, students now brainstorm ways to find that number more accurately. They then go about calculating an answer. The activity prompts students to transfer their understanding of the area of polygons to find the surface area of a three-dimensional object.

Students learn that the surface area of a three-dimensional figure is the total area of all its faces. Because the area of a region is the number of square units it takes to cover the region without gaps and overlaps, surface area can be thought of as the number of square units that are needed to cover all sides of an object without gaps and overlaps. The square sticky notes illustrate this idea in a concrete way.

As students work, notice the various approaches that they take to determine the number of sticky notes needed to tile the faces of the cabinet (excluding the bottom). Identify students with different strategies to share later.

Standards

Addressing **6.G.A.4**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–4. Give students 1 minute of quiet time to think about the first question and another minute to share their responses with their group. Ask students to pause afterward.

Select some students to share how they might figure out the number of sticky notes and what information they would need. Students may ask for some measurements:

- The measurements of the cabinet in units of sticky notes: 24 by 12 by 6 sticky notes.
- The measurements of the cabinet in inches or centimeters: Tell students that you don't have that information and prompt them to think of another piece of information that they could use.
- The measurements of each sticky note: 3 inches by 3 inches.

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If no students mention needing the edge measurements of the cabinet in terms of sticky notes, let them begin working on the second question and provide the information when they realize that it is needed. Give students 8–10 minutes for the second question.

Student Task Statement

Earlier you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?
2. Use the information you have to find the number of sticky notes needed to cover the cabinet. Show your reasoning.

Student Response

1. Find the area of each side of the cabinet, excluding the bottom, and add them together. Needed information: Measurements of the cabinet edge lengths in sticky notes.
2. Reasoning may be a combination of the following two strategies:
    - Multiply the number of sticky notes along each edge of each side. Add all of the products.
    - Multiply the edge lengths of each side of the cabinet to find the area of each side. Add all of the areas.

Building on Student Thinking

Students may treat all sides as if they were congruent rectangles. That is, they find the area of the front of the cabinet and then just multiply by 5, or act as if the top is the only side that is not congruent to the others. If there is a real cabinet (or any other large object in the shape of a rectangular prism) in the classroom, consider showing students that only the sides opposite each other can be presumed to be identical.

Students may neglect the fact that the bottom of the cabinet will not be covered. Point out that the bottom is inaccessible because of the floor.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

Extension Student Response

Two cabinets: 1,582 sticky notes. Three cabinets: 2,229 sticky notes. Twenty cabinets: 13,228 sticky notes.

Activity Synthesis

Invite previously identified students or groups to share their answer and strategy. On a visual display, record each answer and each distinct process for determining the surface area (that is, multiplying the side lengths of each rectangular face and adding up the products). After each presentation, poll the class on whether others had the same answer or process.

Play the video that reveals the actual number of sticky notes needed to cover the cabinet. If students' answers vary from that shown on the video, discuss possible reasons for the differences. (For example, students may not have

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accounted for the cabinet's door handles. Some may have made a calculation error.)

Tell students that the question they have been trying to answer is one about the surface area of the cabinet. Explain that the **surface area** of a three-dimensional figure is the total area of all its surfaces. We call the flat surfaces on a three-dimensional figure its **faces**.

The surface area of a rectangular prism would then be the combined area of all six of its faces. In the context of this problem, we excluded the bottom face because it is sitting on the ground and will not be tiled with sticky notes. Discuss:

- "What unit of measurement are we using to represent the surface area of the cabinet?" (Square sticky notes)
- " Would the surface area, in terms of the number of square units, change if we used larger or smaller sticky notes? How?" (Yes, if we use larger sticky notes, we would need fewer. If we use smaller ones, we would need more.)

Access for English Language Learners

MLR7 Compare and Connect. After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask:

- "What did the approaches have in common? How were they different?"
- "In each approach, where do we see the number of sticky notes for opposite faces of the cabinet?" (Some students may find the number of sticky notes on each of the five faces of the cabinet and add them. Others may see that opposite faces are congruent and only find the number of sticky notes on three faces and double the number for two of them.)
- "Why did the different approaches lead to the same outcome?"

Advances: Representing, Conversing

10 mins

12.3 Prisms Built from Cubes

Activity Narrative

There is a digital version of this activity.

In this activity, students reason about the surface area of rectangular prisms built from cubes. To verify and find surface area, students may count the number of square units on the visible faces and double the number of squares on each face (or double the total number of squares on the three visible faces). They may also use the squares to determine the edge lengths of the prisms, multiply them to find the area of each face, and then combine the areas.

In *Are You Ready for More?*, students are prompted to build a different prism from 12 cubes, draw it on isometric dot paper, and find its surface area. If physical cubes aren't available, consider using the digital version, in which students can use an applet to build a prism.

Standards

Addressing **6.G.A.4**

Launch

Display the image of the first prism in the activity and read the first question aloud. Remind students that we refer to the

SAMPLE ONLY

flat surfaces of a three-dimensional figure as "faces." Tell students that in this activity, we call the area of each face of a single cube, "1 square unit." Point to a single square on the displayed image to clarify 1 square unit on the prism.

Give students 4–5 minutes of quiet work time to complete the activity.

Give 12 cubes to each student who opts to do the extension.. If students are using snap cubes, tell them that we will pretend that all of the faces are completely smooth and not to worry about the "innies and outies" of the snap cubes. Consider doing a quick demonstration on how to draw a simple prism on isometric dot paper. (Start with one cube and then add a cube in each dimension.)

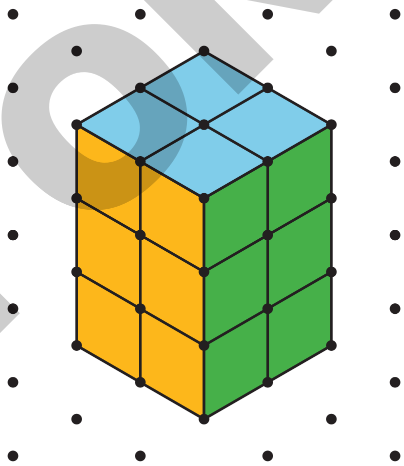
Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: Students can share their learning orally using virtual or concrete manipulatives such as snap cubes.
Supports accessibility for: Language, Fine Motor Skills

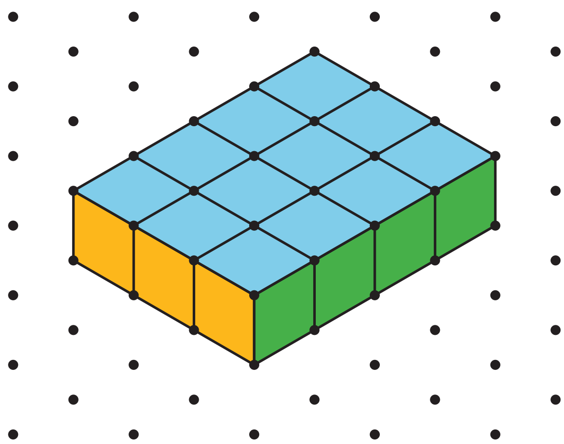
Student Task Statement

1. Here is a sketch of a rectangular prism built from 12 cubes. It has six **faces**, but you can see only three of them in the sketch.

   Show that it has a **surface area** of 32 square units.



2. Here is a sketch of another rectangular prism built from 12 cubes. What is its surface area? Be prepared to explain or show your reasoning.



Student Response

1. Sample response: There are 2 faces with 4 square units each ($2 \cdot 4 = 8$) and 4 faces with 6 square units each ($4 \cdot 6 = 24$). $8 + 24 = 32$

SAMPLE ONLY

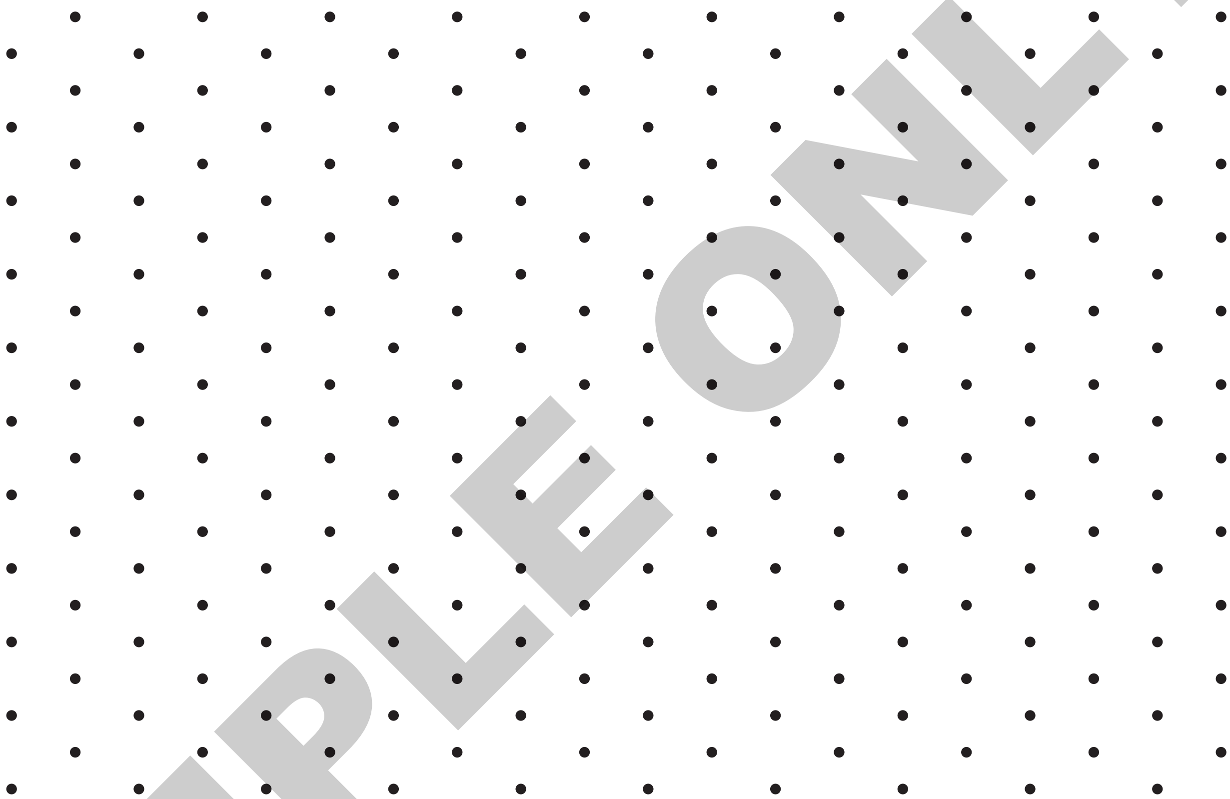
2. 38 square units

Building on Student Thinking

Students may count the faces of the individual snap cubes rather than faces of the completed prism. Help them understand that the faces are the visible ones on the outside of the figure.

Are You Ready for More?

Is it possible to use 12 cubes to build a rectangular prism that has a greater surface area than either prism shown earlier? Explain or show your reasoning. You can draw prisms on the dot paper if it helps.



Extension Student Response

Yes. Drawings vary, but all prisms should have one edge length that is 1 unit.

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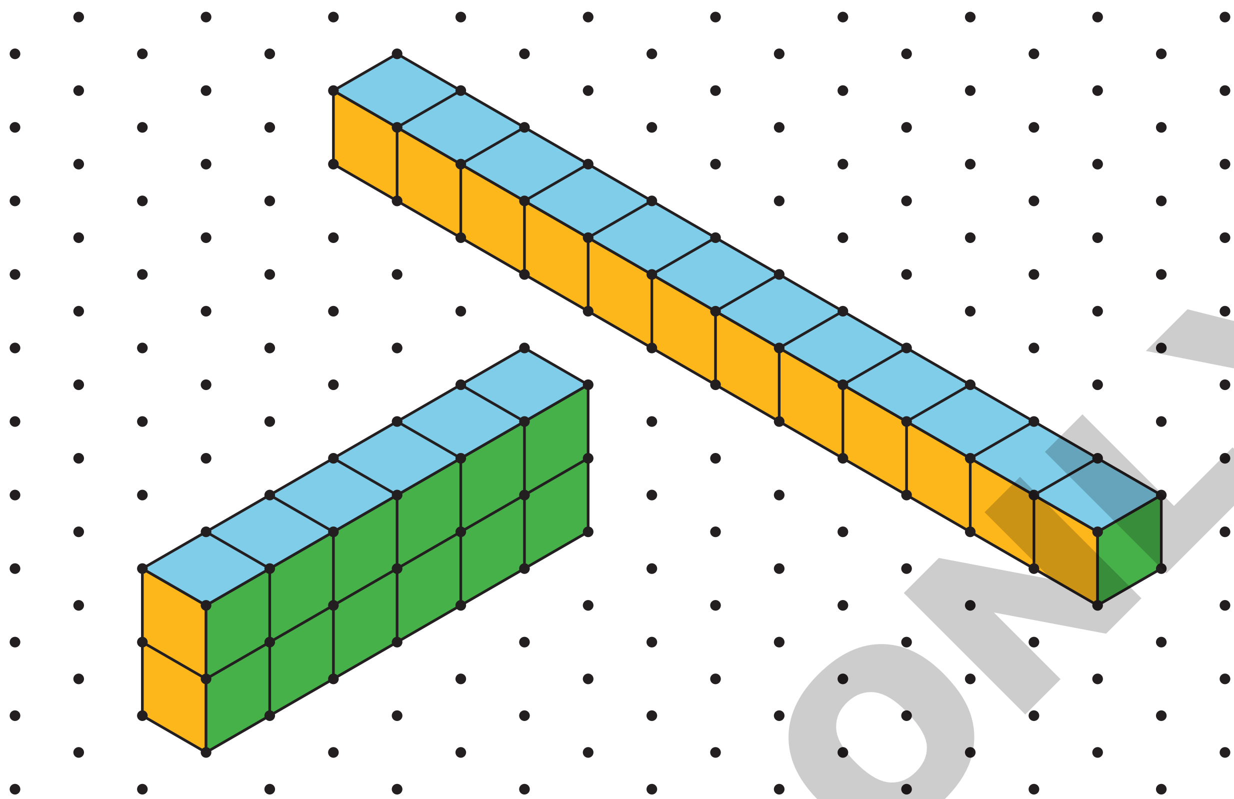

Sample reasoning:

- A prism that is 12 units by 1 unit by 1 unit has a surface area of 50 square units. $(4 \cdot 12) + (2 \cdot 1) = 48 + 2 = 50$
- A prism that is 6 units by 2 units by 1 unit has a surface area of 40 square units. $(2 \cdot 12) + (2 \cdot 6) + (2 \cdot 2) = 24 + 12 + 4 = 40$

Activity Synthesis

Select 1 or 2 students to share how they know the surface area of the first prism is 32 square units. Use students' explanations to highlight the meaning of surface area. Emphasize that the areas of all the faces need to be accounted for, including those we cannot see when looking at a two-dimensional drawing.

Select 1 or 2 students to briefly share their reasoning about the area of the second prism.

Point out that, in this activity, each face of their prism is a rectangle. We can find the area of each rectangle (by multiplying its base by its corresponding height) and then add the areas of all the faces to figure out the surface area. Explain that later, when we encounter non-rectangular prisms, we can likewise reason about the area of each face. We can find the areas of faces that are not rectangles the way we reasoned about the area of polygons earlier in the unit.

Lesson Synthesis

In this lesson, students found the surface areas of a cabinet and of rectangular prisms built out of cubes. Discuss with students:

- "What does it mean to find the surface area of a three-dimensional figure?" (It means finding the number of unit squares that cover the entire surface of the object without gaps or overlaps.)

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- "How can we find the number of unit squares that cover the entire surface of an object?" (We can count them, or we can find the area of each face of the object and add the areas of all faces.)
- "How are finding surface area and finding area alike? How are they different?" (They both involve finding the number of unit squares that cover a region entirely without gaps and overlaps. Both have to do with two-dimensional regions. Finding area involves a single polygon. Finding surface area means finding the sum of the areas of multiple polygons (faces) of which a three-dimensional figure is composed.)

12.4 A Snap Cube Prism

Cool-down

5 mins

Standards

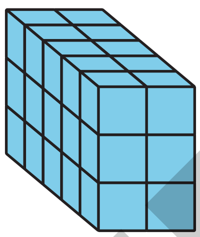
Addressing **6.G.A.4**

Launch

Prepare several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes. Display one for all to see and pass the rest around for students to examine, if needed.

Student Task Statement

A rectangular prism is 3 units high, 2 units wide, and 5 units long. What is its surface area in square units? Explain or show your reasoning.



Student Response

62 square units. Sample reasoning: $2 \cdot [(3 \cdot 5) + (2 \cdot 5) + (2 \cdot 3)] = 62$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

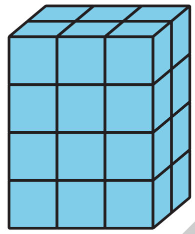
Lesson 12 Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.

SAMPLE ONLY

- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.



So the surface area of a rectangular prism that has edge-lengths of 2 cm, 3 cm, and 4 cm has a surface area of

$$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 4) + (3 \cdot 4) + (3 \cdot 4)$$

or 52 square centimeters.

Glossary

- face
- surface area

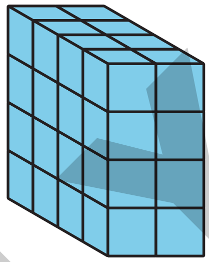
Sec D

SAMPLE ONLY

Practice Problems

1 Student Task Statement

What is the surface area of this rectangular prism?



A. 16 square units

B. 32 square units

C. 48 square units

D. 64 square units

Solution

D

Sec D

2 Student Task Statement

Which description can represent the surface area of this trunk?

A. The number of square inches that cover the top of the trunk.

B. The number of square feet that cover all the outside faces of the trunk.

C. The number of square inches of horizontal surface inside the trunk.

D. The number of cubic feet that can be packed inside the trunk.

Solution

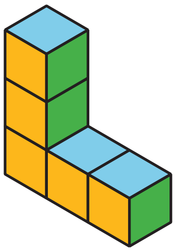
B

SAMPLE ONLY

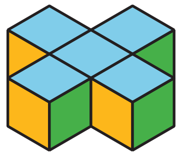
3 Student Task Statement

Which figure has a greater surface area?

A



B



Solution

Figure A and Figure B have the same surface area of 22 square units.

4 Student Task Statement

A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

Solution

88 square units. Sample reasoning: Two faces are 4 units by 2 units, amounting to 16 square units. Two faces are 4 units by 6 units, amounting to 48 square units. Two faces are 2 units by 6 units, amounting to 24 square units. $16 + 48 + 24 = 88$.

5 from Unit 1, Lesson 9

Student Task Statement

Draw an example of each of these triangles on the grid.

a. A right triangle with an area of 6 square units.

b. An acute triangle with an area of 6 square units.

c. An obtuse triangle with an area of 6 square units.

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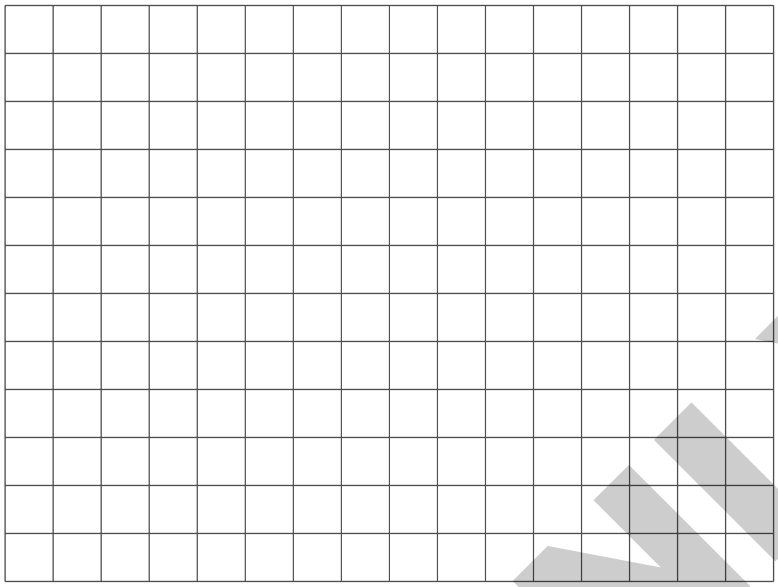
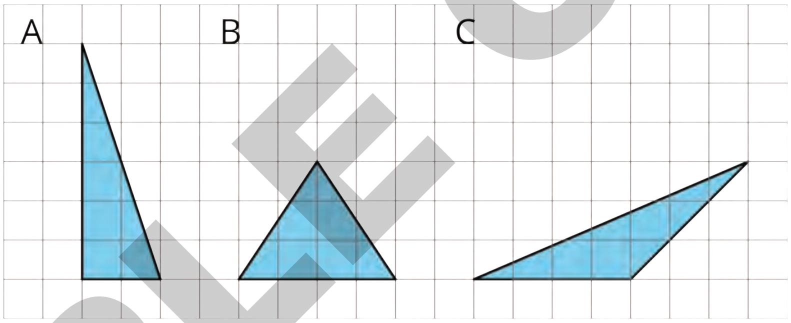


Solution

Sample response:



6 from Unit 1, Lesson 10

Student Task Statement

Find the area of triangle MOQ in square units. Show your reasoning.

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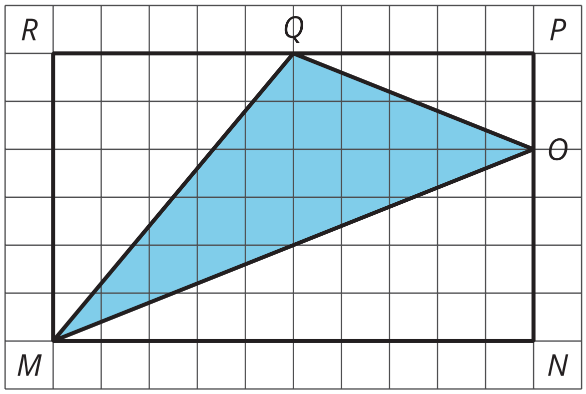


Solution

20 square units. Sample reasoning: The area of triangle MOQ can be found by subtracting the areas of the three right triangles from the area of rectangle $MNPR$.

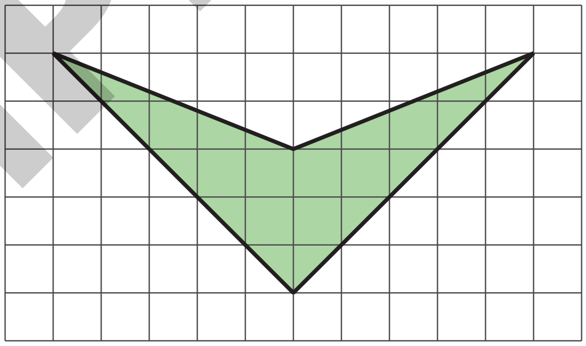
- The area of rectangle $MNPR$ is $10 \cdot 6$ or 60 square units.
- The area of triangle QRM is $\frac{1}{2} \cdot 6 \cdot 5$ or 15 square units.
- The area of triangle MNO is $\frac{1}{2} \cdot 10 \cdot 4$ or 20 square units.
- The area of triangle OPQ is $\frac{1}{2} \cdot 2 \cdot 5$ or 5 square units. $60 - (15 + 20 + 5) = 20$.

Sec D

7 from Unit 1, Lesson 3

Student Task Statement

Find the area of this shape. Show your reasoning.



Solution

15 square units. Sample reasoning:

- The shape can be decomposed into two identical triangles with a vertical cut down the middle. Each triangle has base 3 units and height 5 units, so its area is $\frac{1}{2} \cdot 3 \cdot 5$, or 7.5 square units. $2 \cdot (7.5) = 15$.
- The shape can be decomposed into two identical triangles and rearranged into a parallelogram with a base

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of 3 units and a height of 5 units. $3 \cdot 5 = 15$.

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Unit 1, Lesson 13

Polyhedra

Goals

- Compare and contrast (orally and in writing) features of prisms and pyramids.
- Comprehend and use the words “face”, “edge”, “vertex”, and “base” to describe polyhedra (in spoken and written language).
- Understand that the word “net” refers to a two-dimensional figure that can be assembled into a polyhedron, and create a net for a given polyhedron.

Learning Targets

- I can describe the features of a polyhedron using mathematical vocabulary.
- I can explain the difference between prisms and pyramids.
- I understand the relationship between a polyhedron and its net.

Lesson Narrative

In this lesson, students learn about polyhedra and their **nets**. They also study **prisms** and **pyramids** as types of **polyhedra** with certain defining features.

Students begin by identifying the defining characteristics of polyhedra. They learn or review terminology such as faces, edges, and vertices as they develop a working definition of polyhedra.

Next, students explore the defining characteristics of prisms and pyramids. They consider the polygons that constitute the faces of a given prism or pyramid and how to arrange them into nets that can be assembled into the given polyhedron.

As students analyze polyhedra, prisms, and pyramids for defining characteristics and use their observations to distinguish these figures, they practice looking for and making use of structure (MP7). In communicating the geometric attributes that they see, students practice attending to precision (MP6).

An optional activity is included to give students an opportunity to assemble a net into a polyhedron and identify the number of vertices, edges, and faces.

A note about polygons and polyhedra:

Here are some important aspects of polygons:

- They are made out of line segments called edges.
- Edges meet at a vertex.
- The edges meet only at vertices.
- Polygons always enclose a two-dimensional region.

Here is an analogous way to characterize polyhedra:

- They are made out of filled-in polygons called faces.
- Faces meet at an edge.
- The faces meet only at edges.
- Polyhedra always enclose a three-dimensional region.

Students do not need to memorize a formal definition of a polyhedron, but recognizing its defining characteristics can help students make sense of nets and surface area.

Math Community
In today’s activities, students are introduced to the idea of math norms as expectations that help everyone in the room

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feel safe, comfortable, and productive doing math together. Students then consider what norms would connect and support the math actions that the class recorded so far in the Math Community Chart.

Standards

Addressing	**6.G.A.4**
Building Towards	**6.G.A.4**

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials To Gather

- Chart paper: Activity 1
- Math Community Chart: Activity 1
- Pre-assembled or commercially produced polyhedra: Activity 1
- Nets of polyhedra: Activity 2, Activity 3
- Scissors: Activity 2, Activity 3
- Tape: Activity 2, Activity 3
- Geometry toolkits: Activity 3
- Glue or glue sticks: Activity 3

Materials To Copy

- Assembling Polyhedra Cutouts (1 copy for every 12 students): Activity 1
- Prisms and Pyramids Cutouts (1 copy for every 4 students): Activity 2
- Assembling Polyhedra Cutouts (1 copy for every 6 students): Activity 3

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Required Preparation

Activity 1:

Assemble collections of geometric figures that each contains at least 2 familiar polyhedra, 2 unfamiliar polyhedra, and 2 non-polyhedra. Prepare one collection for each group of 3–4 students. If pre-made polyhedra are unavailable, assemble some from the nets in the blackline master.

Activity 2:

Print and pre-cut the nets and polygons in the blackline master. Prepare 1 set per group of 3–4 students, along with tape to join the polygons into a net.

Activity 3:

Print the nets from the same blackline master as the one used for the *Warm-up*. For each student, prepare 2 copies of one net and tape or glue to assemble the net.

Student Facing Learning Goals

Let’s investigate polyhedra.

13.1

What are Polyhedra?

Warm-up

10 mins

Activity Narrative

In this *Warm-up*, students analyze examples and counterexamples of polyhedra, observe their defining characteristics, and use their insights to sort objects into polyhedra and non-polyhedra. They then start developing a working definition of "polyhedron."

Prepare physical examples of polyhedra and non-polyhedra for students to sort. These examples should be geometric figures rather than real-world objects such as shoe boxes or canisters. If such figures are not available, make some ahead of time using the nets in the blackline master.

As students work and discuss, notice those who can articulate defining features of a polyhedron and invite them to share later.

Standards

Building Towards **6.G.A.4**

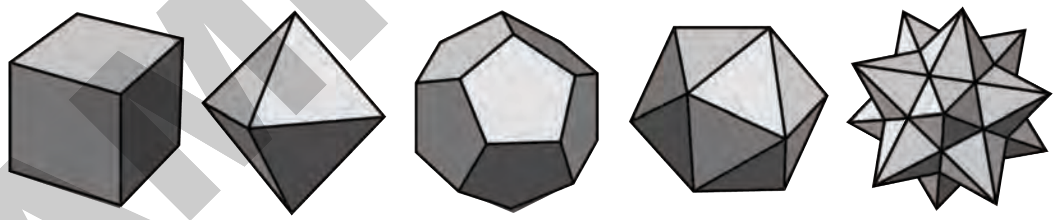
Launch

Arrange students in groups of 3–4. Give students 1 minute of quiet time to study the examples and non-examples in the task statement. Ask them to be ready to share at least one thing that they notice and one thing that they wonder. Give the class a minute to share some of their observations and questions.

Next, give each group a physical set of three-dimensional figures. The set should include some familiar polyhedra, some unfamiliar ones, and some non-polyhedra.

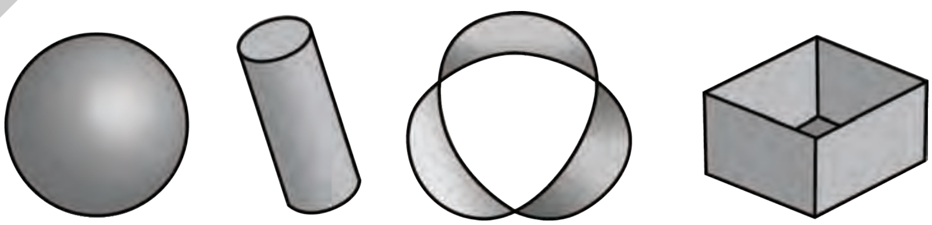
Ask groups to sort the figures into polyhedra and non-polyhedra (the first question). If members disagree about whether a figure is a polyhedron, prompt them to discuss the disagreements with their group. When the group has come to an agreement, give them 2–3 minutes of quiet time to complete the second question.

Student Task Statement



These five drawings represent **polyhedra**.

The next four drawings do *not* represent polyhedra.



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1. Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.
2. What characteristics helped you distinguish the polyhedra from the other figures?

Student Response

1. No response required.
2. Sample responses:
    - Polyhedra are made from polygons.
    - Polyhedra don’t have any unattached edges.
    - Non-polyhedra sometimes have curved or round surfaces.
    - Some non-polyhedra have a face that is not a polygon.

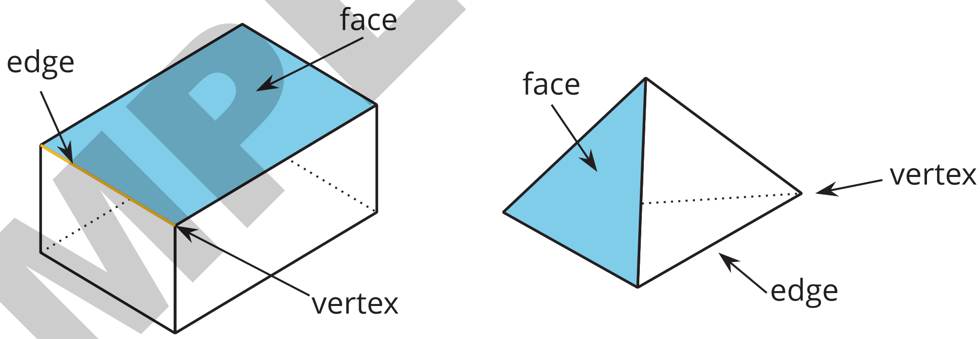
Activity Synthesis

Invite students to share what they see as characteristics of polyhedra. Record their responses for all to see. For each one, ask the class if they agree or disagree. If they generally agree, ask if there is anything they would add or elaborate on to make the description clearer or more precise. If they disagree, ask for an explanation or a counterexample.

Students will have a chance to refine their definition of polyhedra later in the lesson—after exploring prisms and pyramids and learning about nets, so it is not important to compile a complete or precise set of descriptions or features.

Use a sample polyhedron or a diagram as shown here to introduce or reinforce the terminology surrounding polyhedra**.**

- The polygons that make up a polyhedron are called "faces."
- The places where the sides of the faces meet are called edges.
- The “corners” are called vertices. (Clarify that the singular form is "vertex" and the plural form is "vertices.")



Math Community
At the end of the *Warm-up*, display the Math Community Chart. Tell students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Using the Math Community Chart, offer an example of how the “Doing Math” actions can be used to create norms. For example, "In the last exercise, many of you said that our math community sounds like ‘sharing ideas.’ A norm that supports that is 'We listen as others share their ideas.’ For a teacher norm, ‘questioning vs telling’ is very important to me, so a norm to support that is ‘Ask questions first to make sure I understand how someone is thinking.’”

Invite students to reflect on both individual and group actions. Ask, “As we work together in our mathematical community, what norms, or expectations, should we keep in mind?” Give 1–2 minutes of quiet think time and then invite as many students as time allows to share either their own norm suggestion or to “+1” another student’s suggestion.

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Record student thinking in the student and teacher "Norms" sections on the Math Community Chart.

Conclude the discussion by telling students that what they made today is only a first draft of math community norms and that they can suggest other additions during the *Cool-down*. Throughout the year, students will revise, add, or remove norms based on those that are and are not supporting the community.

13.2 Prisms and Pyramids

25 mins

Activity Narrative

This activity serves two goals: to uncover the defining features of **prisms** and **pyramids** as well as to introduce **nets** as two-dimensional representations of polyhedra.

Students first analyze prisms and pyramids and try to define their characteristics. Next, they learn about nets and think about the polygons needed to compose the nets of given prisms and pyramids. They then use their experience with the nets of prisms and pyramids to sharpen and refine their definitions of these polyhedra.

As students discuss the features of prisms and pyramids, encourage them to use the terms face, edge, and vertex (vertices) in their descriptions.

Standards

Addressing **6.G.A.4**

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 3–4. Tell students that there are 3 questions in this activity and that the class will pause for a discussion after responding to each question and before moving on to the next one.

Display images of the prisms and pyramids in the activity, or display and pass around physical representations of those polyhedra, if available. Tell students that Polyhedra A–F are all prisms and Polyhedra P–S are all pyramids.

For the first question:

- Give students 2–3 minutes of quiet think time and 1–2 minutes to discuss their observations in their groups.
- Solicit students' ideas about characteristics that distinguish prisms and pyramids. Record students' responses in two columns—one for prisms and the other for pyramids. It is not important that the lists are complete at this point.

Next, tell students that we are going to use nets to better understand prisms and pyramids. Explain that a **net** is a two-dimensional representation of a polyhedron.

Display a cube assembled for the *Warm-up*, as well as a cutout of an unfolded net (consider removing the flaps). Demonstrate how the net with squares could be folded and assembled into a cube, or use this book of digital applets, https://ggbm.at/rcu3Ka3j, created in GeoGebra by the GeoGebra DocuTeam. Point out how the number and the shape of the faces on the cube correspond to the number and the shape of the polygons in the net.

For the second question:

- Give students 1 minute of quiet think time and 1 minute to discuss their response in their groups.
- Tell students that they will now verify their answer. Give each group one of the three nets from the first three pages

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of the blackline master. Ask them to try to assemble a triangular pyramid from their net.

- Invite groups to share with the class whether it can be done. Discuss why Net 3 cannot be assembled into Pyramid P (two of the triangles would overlap).

For the last question, tell students that they will create a net of another prism or pyramid:

- Assign each group a prism or a pyramid from the task statement (except for Prism B and Pyramid P).
- Give each group a set of pre-cut polygons from the last two pages of the blackline master.
- Tell students to choose the right kind and number of polygons that make up their polyhedron. Then, arrange the polygons so that, when taped and folded, the arrangement is a net and could be assembled into their prism or pyramid. Encourage them to think of more than one net, if possible.

Access for English Language Learners

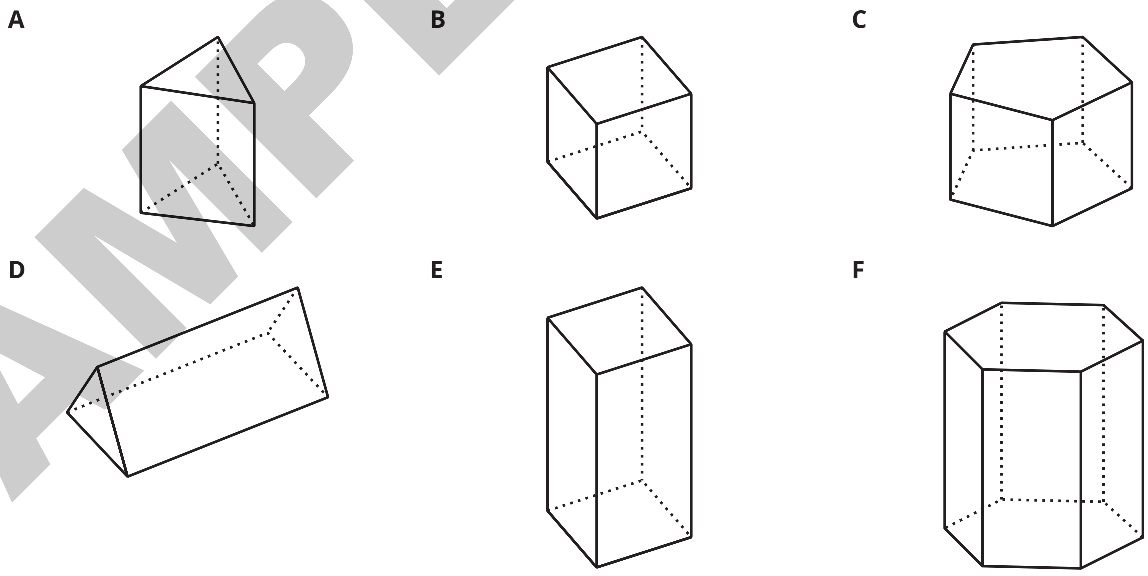
MLR2 Collect and Display. Circulate, and listen for and collect the language that students use as they talk about characteristics of prisms and pyramids. On a visible display, record words and phrases such as: “faces,” “edges,” “vertex,” “parallel,” “rectangular (or triangular) faces,” and names of polygons. Invite students to borrow language from the display as needed and update it throughout the lesson.
Advances: Speaking, Conversing

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide prisms and pyramids for students to view or manipulate. These hands-on models help students identify characteristics of polyhedra and support net building.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Student Task Statement

1. Here are some polyhedra called **prisms**.



Here are some polyhedra called **pyramids**.

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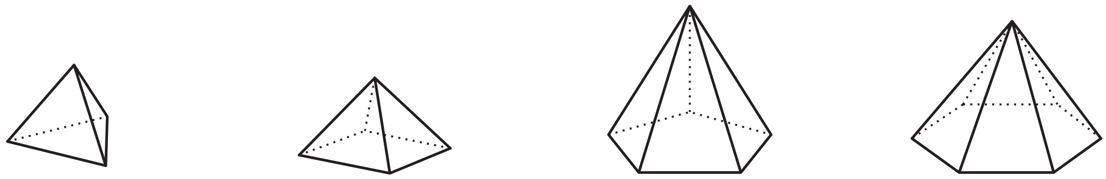
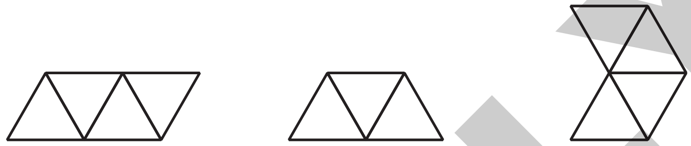
P Q R S



a. Look at the prisms. What are their characteristics?

b. Look at the pyramids. What are their characteristics?

2. Which of these **nets** can be folded into Pyramid P? Select all that apply.



net 1 net 2 net 3

3. Your teacher will give your group some polygons and assign a polyhedron.

   a. Decide which polygons are needed to compose your assigned polyhedron. List the polygons and how many of each are needed.

   b. Arrange the cut-outs into a net that, if taped and folded, can be assembled into the polyhedron. Sketch the net. If possible, show a different net for the same polyhedron.

Student Response

1. Sample responses:
   a. A prism has rectangular faces. Some of the faces are parallel to one another. A prism may have two faces that are not rectangles.
   b. A pyramid has a triangle for all or all but one of its faces. That one face might be a different polygon, and all triangles share an edge with it. All triangles also meet at a single vertex.
2. Net 1 can be assembled into Pyramid P, but Nets 2 and 3 cannot.
3. a. Sample responses:
      - Prism A: 2 triangles, 3 squares
      - Prism C: 2 pentagons, 5 rectangles
      - Prism D: 2 triangles, 3 rectangles
      - Prism E: 2 squares, 4 rectangles
      - Prism F: 2 hexagons, 6 rectangles
      - Pyramid Q: 1 square, 4 triangles
      - Pyramid R: 1 pentagon, 5 triangles
      - Pyramid S: 1 hexagon, 6 triangles

   b. Drawings vary.

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Are You Ready for More?

What is the smallest number of faces a polyhedron can possibly have? Explain how you know.

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Extension Student Response

Four faces (a triangular pyramid, also known as a tetrahedron). Sample reasoning: The triangle is the polygon with the fewest number of sides. If there is one triangular face, there must be another polygon attached at each edge of it, so it is impossible to use fewer than four faces. It is possible to create a polyhedron with four triangular faces. (Pyramid P in this activity is an example.)

Activity Synthesis

Select groups to share their arrangements of polygons. If time permits and if possible, have students tape their polygons and fold the net to verify that it could be assembled into the intended polyhedron. Discuss:

- "What do the nets of prisms have in common?" (They all have rectangles. They have a pair of polygons that may not be rectangles.)
- "What do the nets of pyramids have in common?" (They all have triangles. They have one polygon that may not be a triangle.)
- "Is there only one possible net for a prism or a pyramid?" (No, the polygons can be arranged in different ways and still be assembled into the same prism or pyramid.)

Explain the following points about prisms and pyramids:

- A prism has two parallel, identical faces called **bases** and a set of rectangles connecting the bases.
- Prisms are named for the shape of the bases. For example, if the base of a prism is a pentagon, then the prism is called a "pentagonal prism."
- A pyramid has one face called the **base** that can be any polygon and a set of faces that are all triangles. Each edge of the base is shared with an edge of a triangle. All of these triangles meet at a single vertex.
- Pyramids are named for the shape of their base. For example, if the base of a pyramid is a square, then the pyramid is called a "square pyramid."

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13.3 Assembling Polyhedra

Optional

20 mins

Activity Narrative

This optional activity gives students the experience of assembling polyhedra from nets. Counting the vertices, edges, and faces of a polyhedra helps to reinforce their understanding of the vocabulary. You will need the same blackline master as the one provided for the *Warm-up*.

Students are likely to need assistance in assembling their polyhedra. Circulate, and support students as needed.

Standards

Addressing **6.G.A.4**

Launch

Tell the class that they are going to assemble polyhedra from nets. Point out that the net has shaded and unshaded polygons. Display an example and explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are "flaps" to make it easier to glue or tape the polygons together; they will get tucked behind the shaded polygons and are not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold up the net and attach the various polygons together. A straightedge can be very helpful for making the creases.

Give each student two copies of a net so they can compare the assembled version with the unfolded net. Provide access to geometry toolkits and glue or tape. Ask students to build their figures and complete the question, and then to discuss their responses with another student who has the same polyhedron.

Student Task Statement

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.
2. How many vertices, edges, and faces are in your polyhedron?

Student Response

1. No answer required.
2. Sample responses:
    - A: 6 vertices, 9 edges, 5 faces
    - B: 8 vertices, 12 edges, 6 faces
    - C: 8 vertices, 12 edges, 6 faces
    - D: 8 vertices, 12 edges, 6 faces
    - E: 4 vertices, 6 edges, 4 faces
    - F: 5 vertices, 8 edges, 5 faces
    - G: 6 vertices, 10 edges, 6 faces
    - H: 8 vertices, 14 edges, 8 faces
    - J: 10 vertices, 20 edges, 12 faces
    - K: 9 vertices, 15 edges, 8 faces

Building on Student Thinking

Students may have trouble getting an accurate count of faces, edges, and vertices. Suggest that they set the figure on the table and then separately count the amount on the top, bottom, and lateral sides of the figure. Or recommend that they label each face with a number or a name and keep track of the parts associated with each face, taking care not to double count edges and vertices.

Activity Synthesis

After students have conferred with another student and agreed on the number of vertices, edges, and faces of their polyhedron, tell the class they will now share their completed polyhedra and the unfolded version of the net with the class. Consider either asking students to pass their two items around, or to leave their the polyhedra and nets displayed while students circulate around the room to view others' work.

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Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to pre-cut nets to reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time. Supports accessibility for: Fine Motor Skills, Organization, Visual-Spatial Processing

Lesson Synthesis

Review the features of prisms and pyramids by selecting 1 or 2 polyhedra used in the *Warm-up*. Ask students to explain (using the terminology they learned, if possible) why each one is or is not a prism or a pyramid. If it is a prism or pyramid, ask students to name it.

Revisit the working definition of polyhedra generated earlier in the lesson and ask students to see if or how it might be refined. Ask if there is anything they should add, remove, or adjust given their work with prisms, pyramids, and nets.

Highlight the following points about polyhedra. Ask students to illustrate each point using a figure or a net.

- A "polyhedron" is a three-dimensional figure built from filled-in polygons. We call the polygons "faces." (The plural of polyhedron is polyhedra.)
- Every "edge" of a polygon meets another polygon along a complete edge.
- Each polygon meets one and only one polygon on each of the edges.
- The polygons enclose a three-dimensional region.

Consider displaying in a visible place the key ideas from the students' list and from this discussion so that they key ideas can serve as a reference later.

13.4 Three-Dimensional Shapes

Cool-down

5 mins

Standards

Addressing **6.G.A.4**

Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the norms question "Which norm has not already been listed that you'd like to add to our chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms question. Use that information to add to the initial draft of the "Norms" sections of the Math Community Chart.

Student Task Statement

1. Write your best definition or description of a polyhedron. If possible, use the terms you learned in this lesson.

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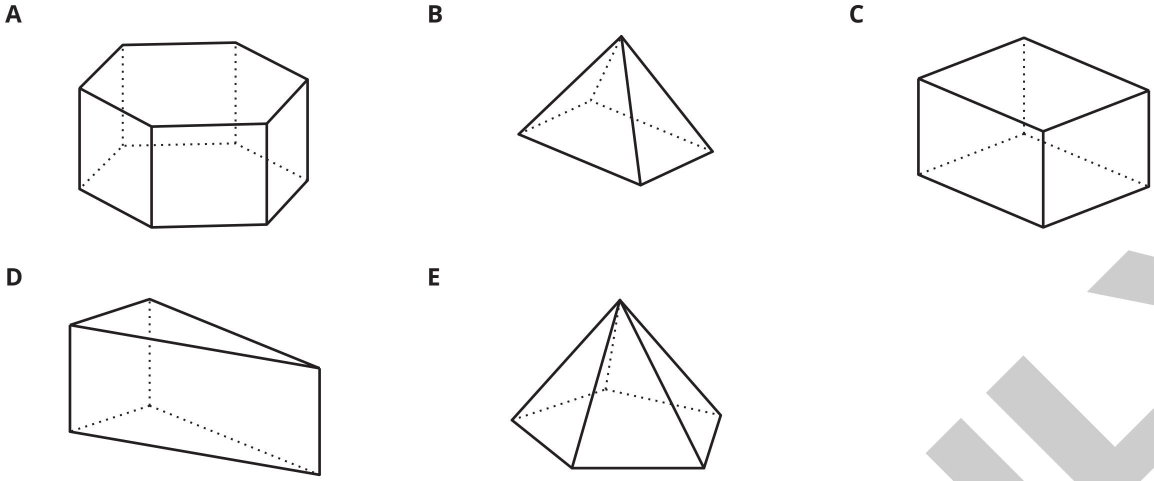
2. Which of these five polyhedra are prisms? Which are pyramids?



Student Response

1. Answers might include one or more of these elements: A polyhedron is a three-dimensional figure made from faces that are filled-in polygons. Each face meets one and only one other face along a complete edge. The points where edges meet are called vertices.
2. A, C, and D are prisms. B and E are pyramids.

Sec D

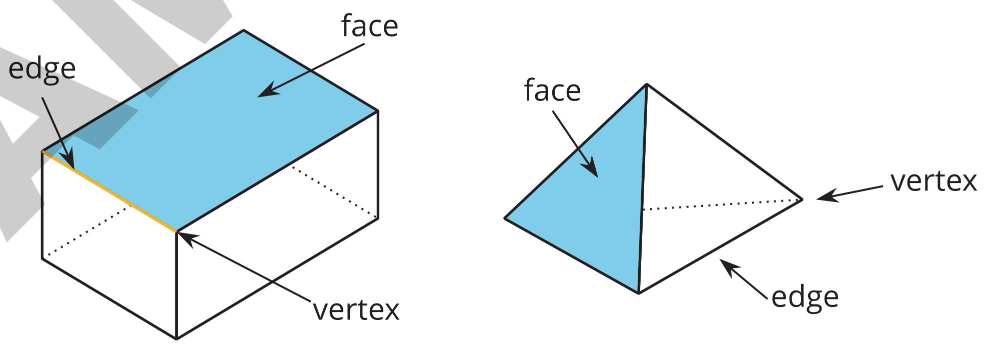
Responding To Student Thinking

Points to Emphasize
If students struggle with identifying polyhedra, revisit this idea when opportunities arise over the next several lessons. For example, in this activity, emphasize the properties of polyhedra when students are asked to match nets with their respective polyhedra:

Grade 6, Unit 1, Lesson 14, Activity 1 Matching Nets

Lesson 13 Summary

A **polyhedron** is a three-dimensional figure composed of faces. Each face is a polygon and meets only one other face along a complete edge. The ends of the edges meet at points that are called vertices.



A polyhedron always encloses a three-dimensional region.

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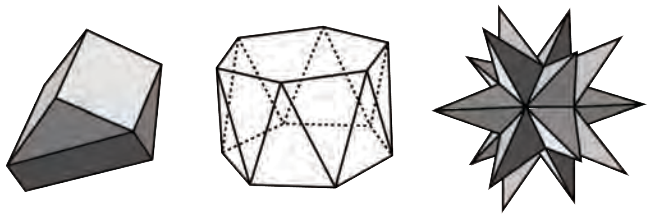
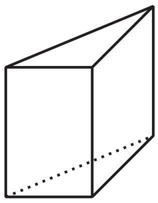
The plural of polyhedron is polyhedra. Here are some drawings of polyhedra:



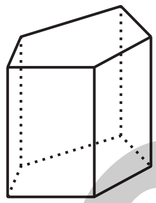
A **prism** is a type of polyhedron with two identical faces that are parallel to each other and that are called **bases**. The bases are connected by a set of rectangles (or sometimes parallelograms that aren't rectangles).

A prism is named for the shape of its bases. For example, if the base is a pentagon, then it is called a "pentagonal prism."

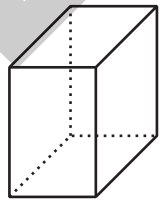
triangular prism



pentagonal prism



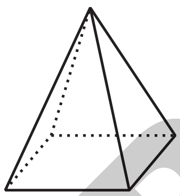
rectangular prism



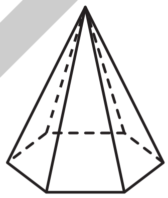
A **pyramid** is a type of polyhedron that has one special face called the base. All of the other faces are triangles that all meet at a single vertex.

A pyramid is named for the shape of its base. For example, if the base is a pentagon, then it is called a "pentagonal pyramid."

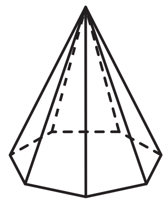
rectangular pyramid



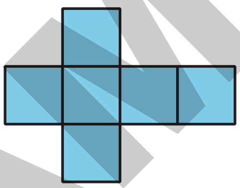
hexagonal pyramid



heptagonal pyramid



A **net** is a two-dimensional representation of a polyhedron. It is composed of polygons that form the faces of a polyhedron.



A cube has 6 square faces, so its net is composed of six squares, as shown here.

A net can be cut out and folded to make a model of the polyhedron.

In a cube, every face shares its edges with 4 other squares. In a net of a cube, not all edges of the squares are joined with another edge. When the net is folded, each of these open edges will join another edge.

Glossary

- base (of a prism or pyramid)
- net
- polyhedron

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- prism
- pyramid

Sec D

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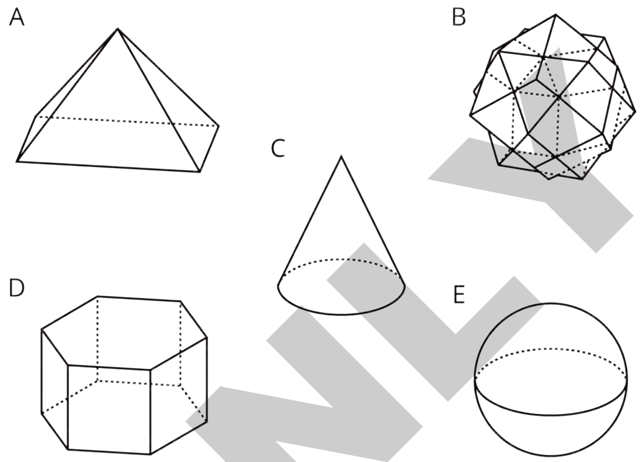
Practice Problems

1 Student Task Statement

Select **all** the polyhedra.



A. A

B. B

C. C

D. D

E. E

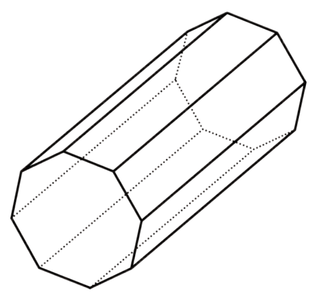
Solution

A, B, D

2 Student Task Statement

a. Is this polyhedron a prism, a pyramid, or neither? Explain how you know.

b. How many faces, edges, and vertices does it have?



Solution

a. Prism. Sample reasoning: It has two parallel octagonal bases that match up exactly.

b. 10 faces, 24 edges, 16 vertices

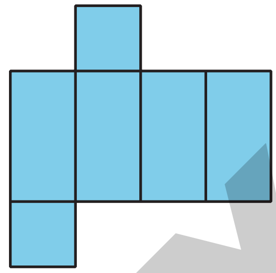
SAMPLE ONLY

3

Student Task Statement

Tyler said this net cannot be a net for a square prism because not all the faces are squares.

Do you agree with Tyler? Explain your reasoning.



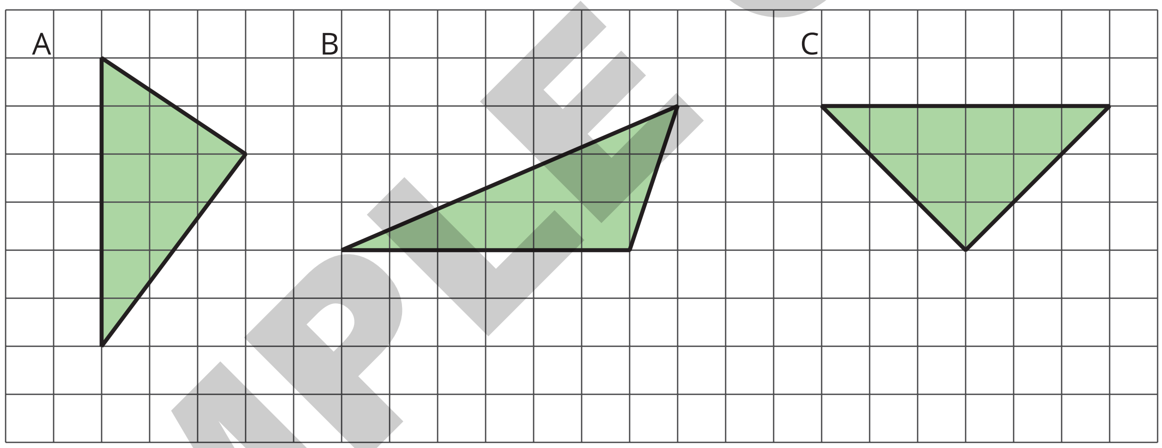
Solution

Disagree. Sample reasoning: A square prism must have two bases that are squares, but the other faces can be non-square rectangles. There are two squares in the net, and the net can be folded into a square prism.

4 from Unit 1, Lesson 8

Student Task Statement

Explain why each of these triangles has an area of 9 square units.



Solution

Sample reasoning: Each triangle is half of a parallelogram with an area of 18 square units (with a base of 6 units and a height of 3 units), as shown in these diagrams.

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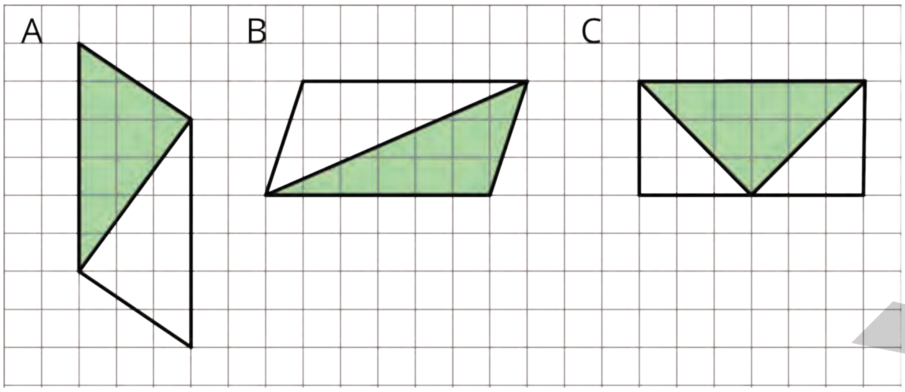


5 from Unit 1, Lesson 9

Student Task Statement

a. A parallelogram has a base of 12 meters and a height of 1.5 meters. What is its area?

b. A triangle has a base of 16 inches and a height of $\frac{1}{8}$ inches. What is its area?

c. A parallelogram has an area of 28 square feet and a height of 4 feet. What is its base?

d. A triangle has an area of 32 square millimeters and a base of 8 millimeters. What is its height?

Solution

a. 18 square meters

b. 1 square inch

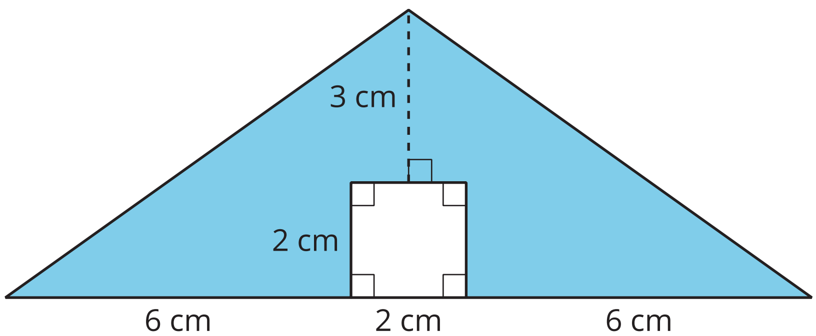
c. 7 feet

d. 8 millimeters

6 from Unit 1, Lesson 3

Student Task Statement

Find the area of the shaded region. Show or explain your reasoning.



SAMPLE ONLY

Sec D

Solution

31 sq cm. Sample reasoning: The two right triangles can be put together to make a 7 cm-by-5 cm rectangle whose area is 35 sq cm. However, a 2 cm-by-2 cm square is removed. The shaded area is 31 sq cm.

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Unit 1, Lesson 14

Nets and Surface Area

Goals

- Match polyhedra with their nets and justify (orally) that they match.
- Use a net with gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.
- Visualize and identify the polyhedron that can be assembled from a given net.

Learning Targets

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

Lesson Narrative

This lesson extends students' understanding of polyhedra, their nets, and their surface area.

Students begin by matching polyhedra and their nets. Then, they work in groups to assemble two prisms and a pyramid from given nets and use nets to find the surface area of each polyhedron. To support students in reasoning about area, the nets are presented with a grid.

As students calculate surface area and consider how to account for all the faces of a polyhedron, they have opportunities to look for and make use of structure, in both the arrangement and measurements of the polygons that compose the polyhedron (MP7).

Standards

Addressing **6.G.A.4**

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Nets of polyhedra: Activity 1, Activity 2
- Scissors: Activity 1, Activity 2
- Geometry toolkits: Activity 2
- Glue or glue sticks: Activity 2
- Tape: Activity 2

Materials To Copy

- Matching Nets Cutouts (1 copy for every 2 students): Activity 1
- Using Nets to Find Surface Area Cutouts (1 copy for every 3 students): Activity 2

Required Preparation

Activity 1:

Prepare physical copies of the nets in the *Warm-up*, in case needed to support students with visualization. The blackline

SAMPLE ONLY

master contains a larger version of these nets.

Activity 2:

Make copies of the nets in the blackline master. Prepare one set of 3 nets (A, B, and C) and provide some glue or tape for each group of 3 students.

Student Facing Learning Goals

Let’s use nets to find the surface area of polyhedra.

14.1 Matching Nets

Warm-up

10 mins

Activity Narrative

This *Warm-up* prompts students to match nets to polyhedra. It invites them to think about the polygons that make up a polyhedron and to mentally manipulate nets, which helps develop their visualization skills.

Standards

Addressing **6.G.A.4**

Launch

Give students 3 minutes of quiet think time to match nets to polyhedra and then another 2 minutes to discuss their response and reasoning with a partner. Encourage students to use the terminology that they learned in prior lessons.

To support students who need more time or help in visualization, prepare physical models of the polyhedra and copies of the nets from the blackline master. Pre-cut the nets or have scissors available so that students can assemble the nets and test their ideas.

Access for English Language Learners

MLR2 Collect and Display. Direct attention to words collected and displayed from the previous lesson. Invite students to borrow language from the display as needed when explaining how they know a net and a polyhedron go together. Update the display throughout the lesson.
Advances: Conversing, Reading

Student Task Statement

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.

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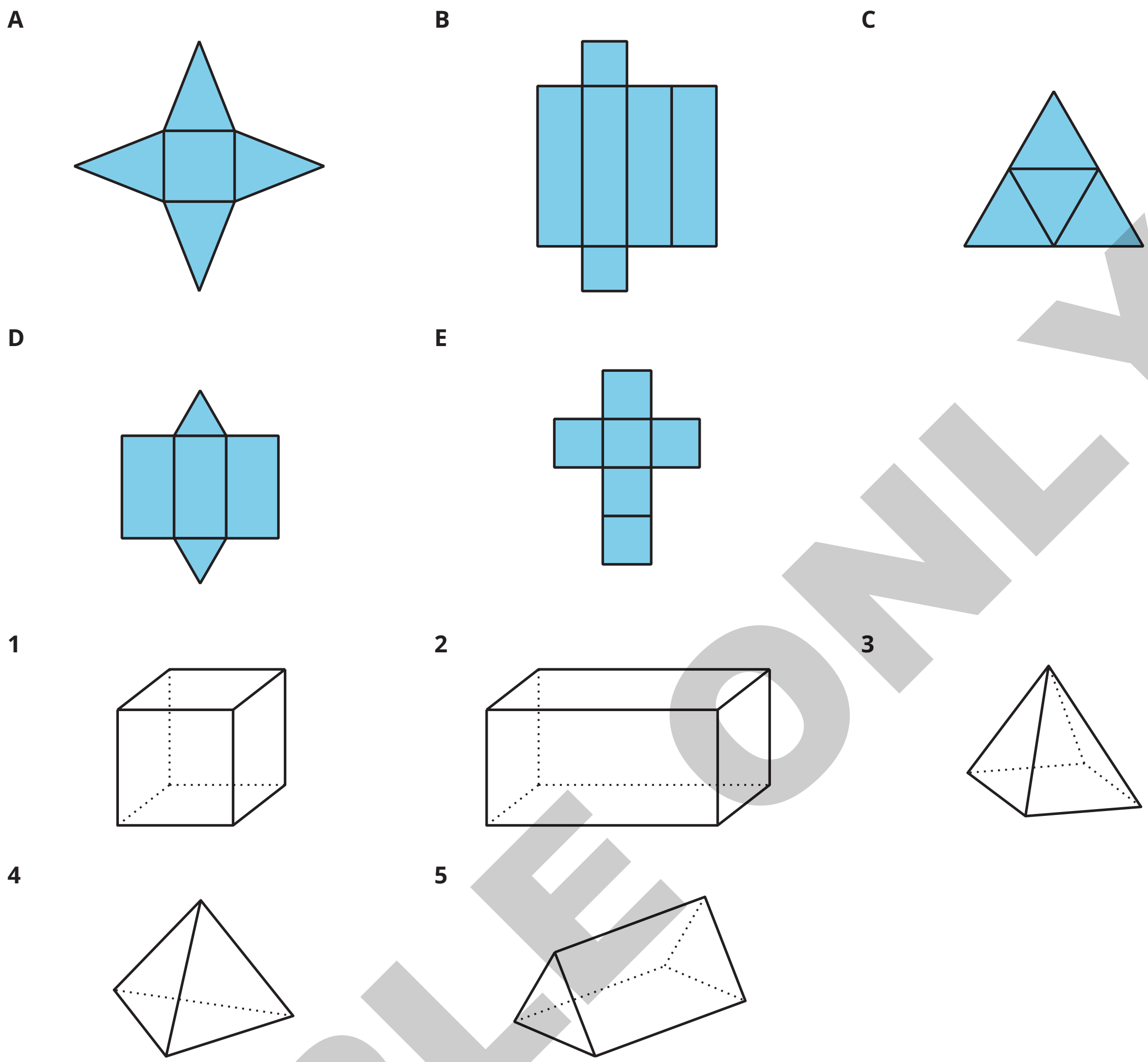

Student Response

Students are likely to give similar reasoning for each figure as in the given sample reasoning.

Net A is a square pyramid (3). Sample reasoning: It has five faces: one square and four triangles, just like the square pyramid.

Net B is a rectangular prism (2).

Net C is a triangular pyramid (4).

Net D is a triangular prism (5).

Net E is a cube or square prism (1).

Building on Student Thinking

If students have trouble distinguishing between Figures A, C, and D, remind them that prisms and pyramids can both contain faces that are triangles. In a pyramid, all triangular faces that are not the base meet at a one vertex and have shared edges. In a prism, there can be a triangular base, but the other faces are quadrilaterals.

SAMPLE ONLY

Activity Synthesis

Invite a few students to share their matching decisions and reasoning with the class. Ask students: "What clues did you use to help you match? How did you check if you were right?" If there is not unanimous agreement on any of the nets, ask students with differing opinions to explain their reasoning. Discuss to come to an agreement.

14.2 Using Nets to Find Surface Area

25 mins

Activity Narrative

In this activity, students cut and assemble nets into polyhedra. They use nets to find surface area, applying what they learned earlier about areas of triangles and parallelograms. The presence of a grid supports students in their calculations. It also reinforces the idea of area as the number of unit squares in a region and the connection between area and surface area.

As they find surface area, students have an opportunity to look for and make use of structure (MP7). For instance, they may identify multiple copies of the same polygon and find the combined area with a single calculation. They may also group rectangles with a common side length into a larger rectangle and find the area of the latter.

Monitor for the different approaches that students take. Here are some likely approaches, starting from less systematic to more systematic:

- Find the area of each face separately and add the areas.
- Identify unique shapes, find their areas, multiply the area of each unique shape by how many there are, and add the areas.
- Rearrange and group two or more faces (for instance, rearrange a pair of triangles into a rectangle or a parallelogram) before finding and adding the areas.

Also monitor for how students organize their work, such as whether they record the measurements of each face, label the shapes in the net and the associated calculations, and account for all the faces of their polyhedron. Encourage students with disorganized or scattered work to take a more methodical approach.

Standards

Addressing **6.G.A.4**

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

Launch

Arrange students in groups of 3. Give each group one of each net (A, B, and C), tape, and access to their geometry toolkits (especially scissors). Explain to students that they will cut some nets, assemble them into polyhedra, and calculate their surface areas. Remind students that the surface area of a three-dimensional figure is the sum of the areas of all of its faces. Ask students to complete the first question before cutting anything.

Point out that the net has shaded and unshaded polygons. Explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are "flaps" to make it easier to glue or tape the polygons together. They will get tucked behind the shaded polygons and are not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold up the net and attach the various polygons together. A straightedge

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can be very helpful for making the creases.

Tell students that it is easy to miss or double-count the area of a face when finding surface area. Ask them to think carefully about how to record their calculations to ensure that all faces are accounted for, correct measurements are used, and errors are minimized.

When students have completed their calculations, ask them to compare and discuss their work with another student who has the same polyhedron.

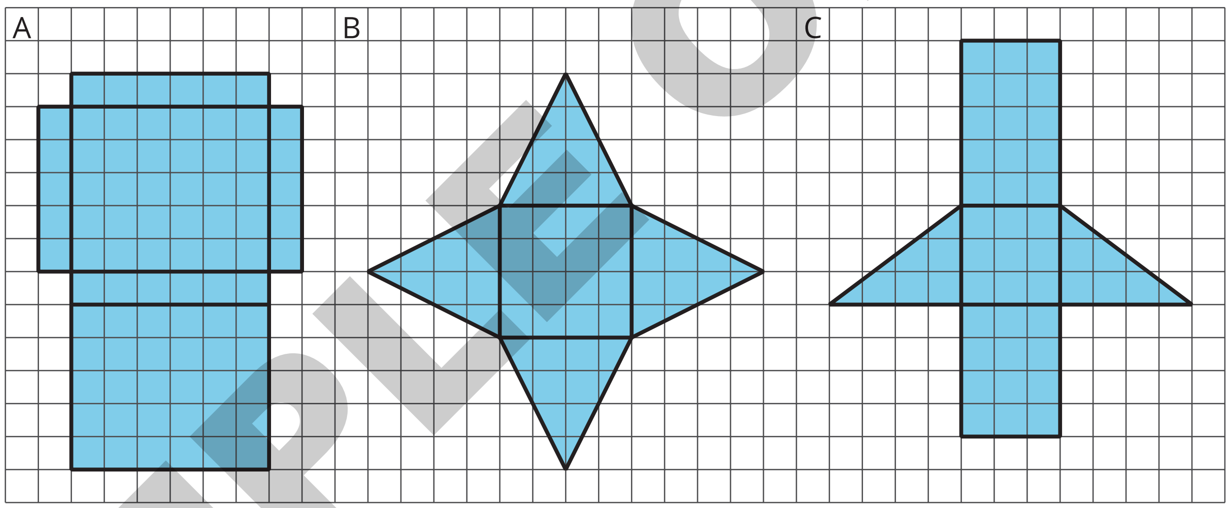
For whole-class discussion, select 1 or 2 polyhedra whose surface area is found using different approaches, including different ways to keep track of calculations. Select students who use different strategies to share later.

Access for Students with Disabilities

Representation: Access for Perception. Provide access to pre-cut nets and the polyhedra. Ask students to identify correspondences between the faces of the polyhedra and regions of each net.
Supports accessibility for: Visual-Spatial Processing, Organization

Student Task Statement

1. Name the polyhedron that each net would form when assembled.



2. Your teacher will give you the nets of three polyhedra. Cut out the nets and assemble the three-dimensional shapes.
3. Find the surface area of each polyhedron. Explain or show your reasoning.

Student Response

1. A: rectangular prism, B: square pyramid, C: triangular prism
2. No answer required.
3. Sample responses:
    - A: The surface area is 82 square units. $2(6 \cdot 1) + 2(5 \cdot 1) + 2(6 \cdot 5) = 82$
    - B: The surface area is 48 square units. $(4 \cdot 4) + 4(\frac{1}{2} \cdot 4 \cdot 4) = 48$
    - C: The surface area is 48 square units. $(3 \cdot 5) + (3 \cdot 3) + (3 \cdot 4) + 2(\frac{1}{2} \cdot 3 \cdot 4) = 48$

Sec D

SAMPLE ONLY

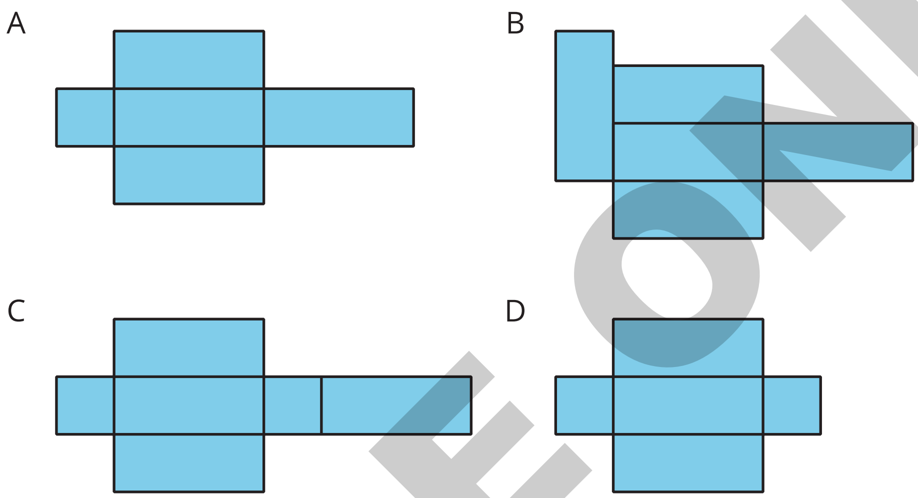
- C: The combined area of the three rectangular faces is 36 square units. $3 \cdot 12 = 36$. The combined area of the two right triangles is 12 square units. $2(\frac{1}{2} \cdot 3 \cdot 4) = 12$. The surface area is 48 square units because $36 + 12 = 48$.

Building on Student Thinking

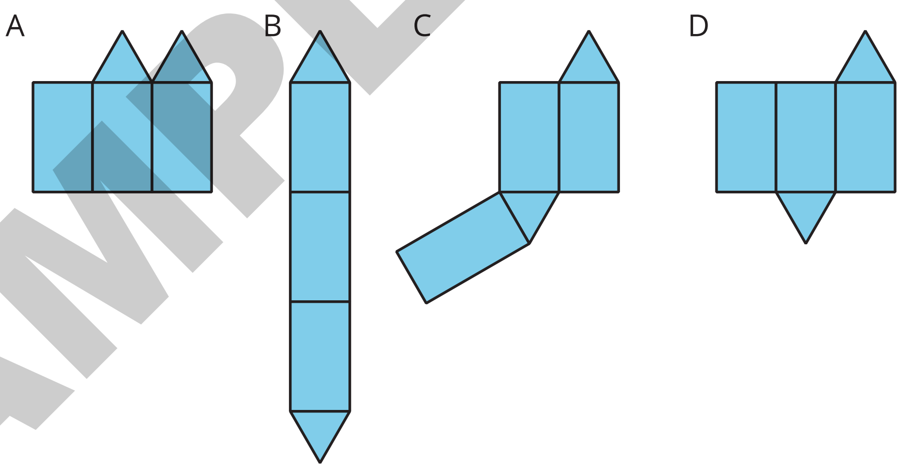
If students do not identify the specific type of prism or pyramid, remind them that they should also name each figure by the shape of its base.

Are You Ready for More?

1. For each net, decide if it can be assembled into a rectangular prism.



2. For each net, decide if it can be folded into a triangular prism.



Extension Student Response

1. Only C can be folded into a rectangular prism.
2. C and D can be folded into triangular prisms.

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Activity Synthesis

Invite previously selected students to share their responses and reasoning. For each polyhedron, sequence the discussion of the strategies in the order listed in the activity narrative. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- "Before cutting out and assembling your net, how did you know what polyhedron it would create?"
- "We saw more than one strategy for finding the surface area of a polyhedron [letter]. How are the strategies alike? How are they different?"
- "How can we make sure that we've included the areas of all the faces of the polyhedron?"

If methods for organizing the work—such as listing measurements and labeling polygons and computations—are not mentioned, bring them up. If needed, demonstrate these organizational methods and discuss their benefits.

Connect the approaches used here to those used earlier in the unit by asking students: "How is finding the surface area of a polyhedron like finding the area of a polygon? How is it different?"

Highlight that even though polyhedra are three-dimensional figures, surface area is a two-dimensional measure, so the same strategies we used to find area—such as decomposing and rearranging—still apply here.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Lesson Synthesis

In this lesson, students matched nets to polyhedra, assembled polyhedra from nets, and used nets to find surface area. Discuss with students:

- "How do we use a net to find surface area?" (We calculate the area of each polygon on the net and add all the areas.)
- "How is finding surface area using a net different from finding surface area by looking at a picture of a polyhedron—as we had done with the filing cabinet, or by studying the actual object—as we had done with the snap cubes?" (A net allows us to see all the faces of a polyhedron at once. When working from a picture or drawing, we need to visualize the hidden faces. Working with an actual polyhedron could help, but again we are not looking at all the faces at once; we have to rotate the object and might miss or double-count a face.)
- "When using a net, how do we keep track of the calculations or make sure that all faces are accounted for?" (We can label all the polygons and the calculations.)
- "Are there ways to simplify the calculations? Or is it necessary to find the area of each polygon one at a time?" (Sometimes we can simplify the process by combining polygons and finding the area of the combined region—for example, finding the area of a group of rectangles with the same side length. If there are several polygons that are identical, we can find the area of one polygon and multiply it by the number of identical polygons in the net.)

SAMPLE ONLY

14.3 Unfolded

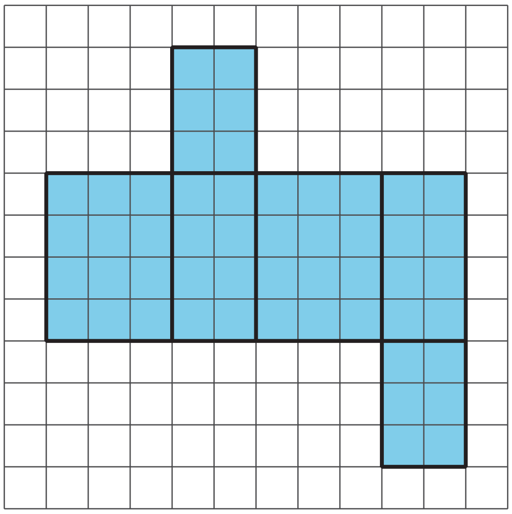
Cool-down

5 mins

Standards

Addressing **6.G.A.4**

Student Task Statement



1. What kind of polyhedron can be assembled from this net?
2. Find the surface area (in square units) of the polyhedron. Show your reasoning.

Student Response

1. A rectangular prism
2. 52 square units. Sample reasoning: $2(3 \cdot 4) + 2(2 \cdot 4) + 2(2 \cdot 3) = 52$

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 14 Summary

A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.

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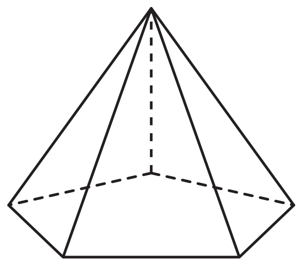
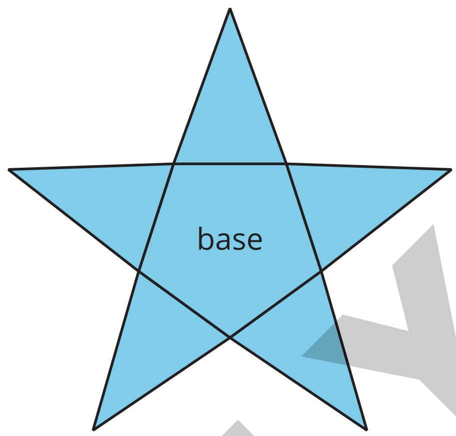
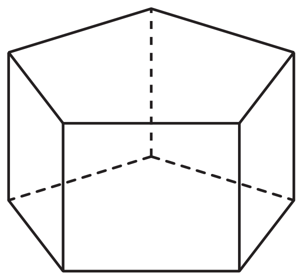
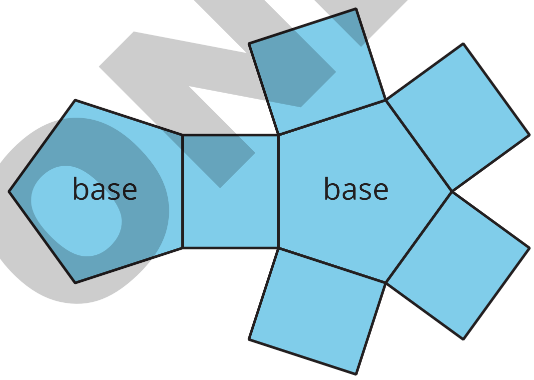


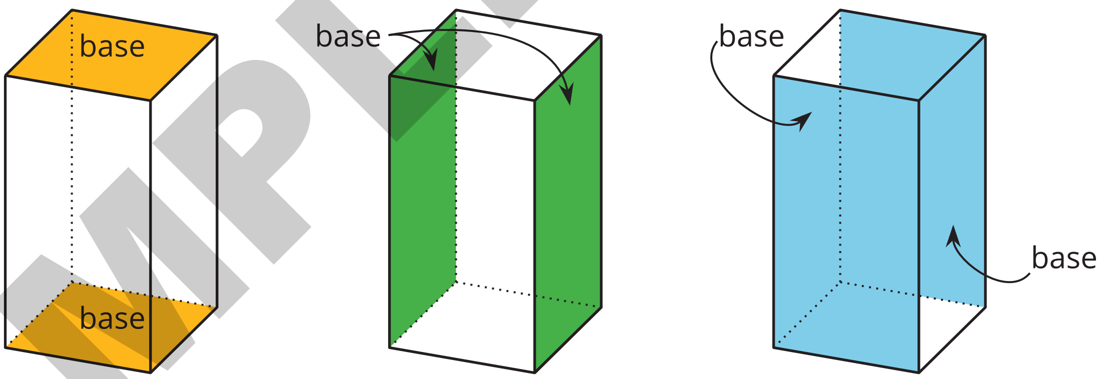


A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.





In a rectangular prism, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.



Because a net shows all the faces of a polyhedron, we can use it to find its surface area. For instance, the net of a rectangular prism shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.

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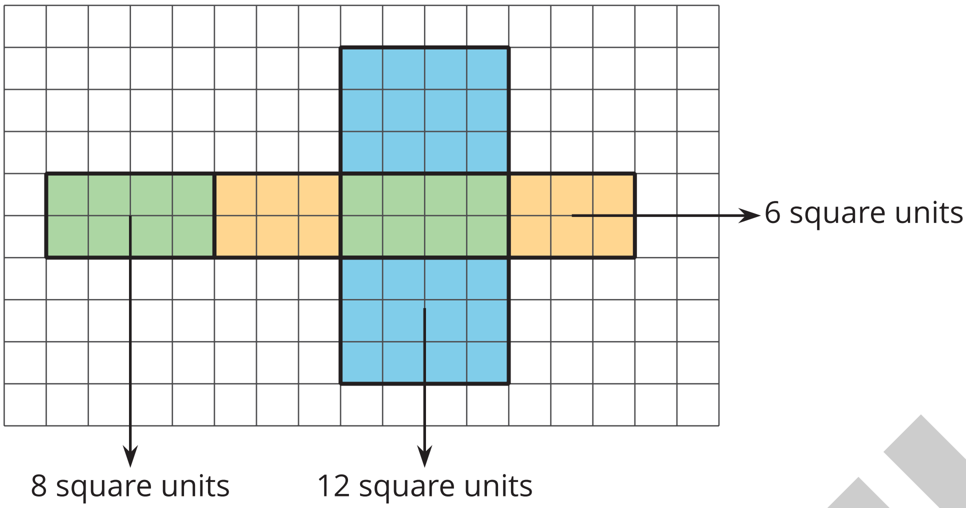


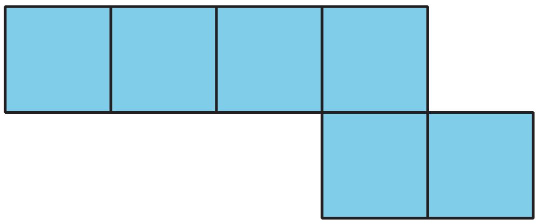
The surface area of the rectangular prism is 52 square units because $8 + 8 + 6 + 6 + 12 + 12 = 52$.

SAMPLE ONLY

Practice Problems

1 Student Task Statement

Can this net be assembled into a cube? Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.

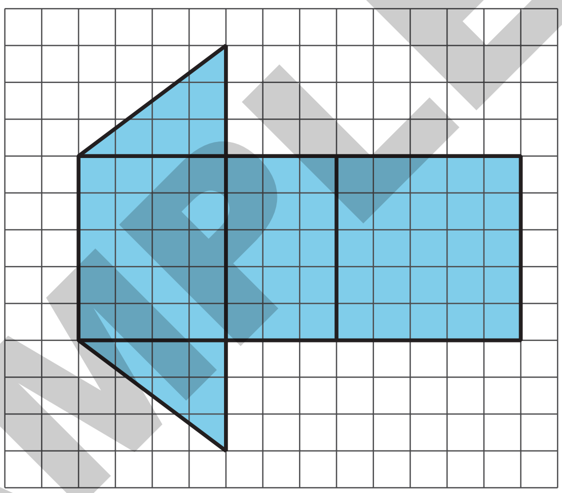


Solution

No. Sample reasoning: The four squares placed side by side can only be folded in one way to meet up with one another, making a cube without a top and bottom. One of the remaining two squares can be folded to make the top or bottom, but the other one cannot be used.

2 Student Task Statement

a. What polyhedron can be assembled from this net? Explain how you know.



b. Find the surface area of this polyhedron. Show your reasoning.

Solution

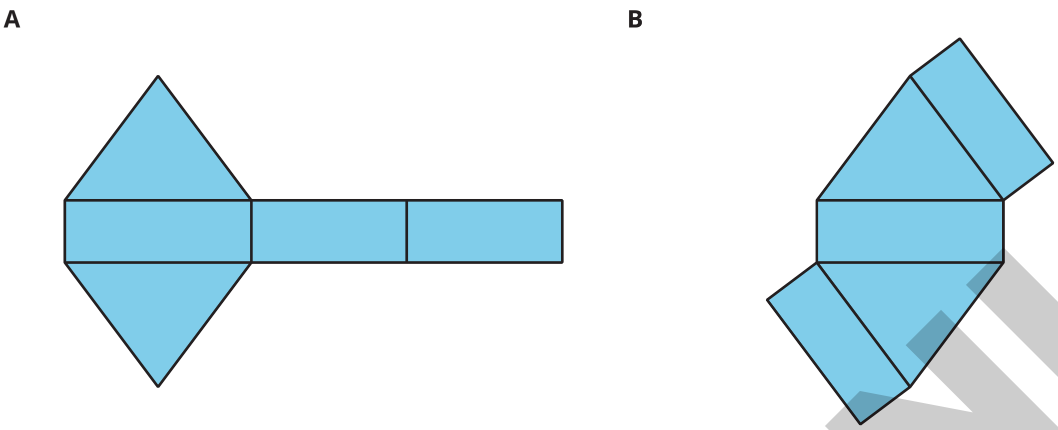
a. A triangular prism. Sample reasoning: There are two identical triangles that are the bases. The rest of the faces are rectangles.

b. 72 square units. Sample reasoning: The area of the three rectangles are 20, 15, and 25 square units. The area of the two triangles are $2(\frac{1}{2} \cdot 4 \cdot 3)$ or 12 square units. $20 + 15 + 25 + 2(6) = 72$

SAMPLE ONLY

3 ## Student Task Statement

Here are two nets. Mai said that both nets can be assembled into the same triangular prism. Do you agree? Explain or show your reasoning.



Solution

Agree. Sample reasoning: Both nets are composed of the same set of polygons. The positions of the one rectangular face are different, but when assembled, that face will meet the same edge of three other polygons.

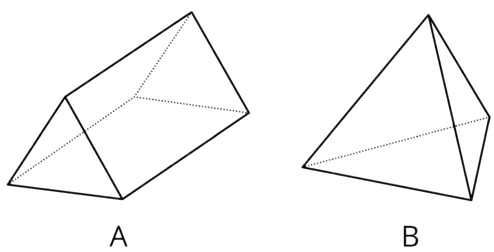
Sec D

4 from Unit 1, Lesson 13

Student Task Statement

Here are two three-dimensional figures.

Tell whether each of the following statements describes Figure A, Figure B, both, or neither.



a. This figure is a polyhedron.
b. This figure has triangular faces.
c. There are more vertices than edges in this figure.
d. This figure has rectangular faces.
e. This figure is a pyramid.
f. There is exactly one face that can be the base for this figure.
g. The base of this figure is a triangle.
h. This figure has two identical and parallel faces that can be the base.

SAMPLE ONLY

Solution

a. Both
b. Both
c. Neither
d. Figure A
e. Figure B
f. Neither
g. Both
h. Figure A

5 from Unit 1, Lesson 12

Student Task Statement

Select **all** units that can be used for surface area.

A. square meters
B. feet
C. centimeters
D. cubic inches
E. square inches
F. square feet

Solution

A, E, F

6 from Unit 1, Lesson 11

Student Task Statement

Find the area of this polygon. Show your reasoning.

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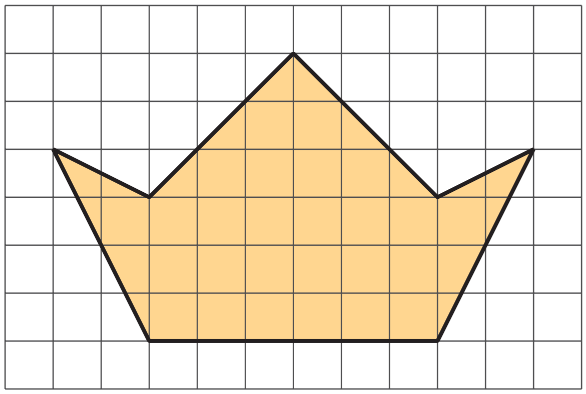
Sec D



Solution

33 square units. Sample reasoning: The figure can be decomposed into two triangles with a base of 3 units and a height of 2 units (area: $2 \cdot (\frac{1}{2} \cdot 3 \cdot 2) = 6$), a triangle with a base of 6 units and a height of 3 units (area: $\frac{1}{2} \cdot 6 \cdot 3 = 9$), and a rectangle that is 6 units by 3 units (area: $6 \cdot 3 = 18$). The total area is $6 + 9 + 18$, which is 33.

SAMPLE ONLY

Sec D

More Nets, More Surface Area

Goals

- Draw and assemble a net for the prism or pyramid shown in a given drawing.
- Interpret (using words and other representations) two-dimensional representations of prisms and pyramids.
- Use a net without gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.

Learning Targets

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

Lesson Narrative

This lesson further develops students' ability to visualize the relationship between nets and polyhedra and their capacity to reason about surface area.

Previously, students identified polygons that make up the faces of a polyhedron and arranged them into a net. They also assembled given nets into polyhedra. In this lesson, students reason about the nets of polyhedra with less scaffolding. They practice mentally unfolding three-dimensional shapes, drawing two-dimensional nets, and using nets to calculate surface area.

As students coordinate edge lengths and arrangements of polygons in drawings of three-dimensional figures to those in the corresponding nets, they practice reasoning concretely and abstractly (MP2).

In an optional activity, students practice visualizing prisms that could be assembled from given nets (shown without a grid) and then compare and contrast their surface areas and volumes.

Standards

Building On **4.NBT.B.5, 5.MD.C.5**

Addressing **6.G.A.2, 6.G.A.4**

Instructional Routines

- Math Talk
- MLR7: Compare and Connect

Required Materials

Materials To Gather

- Demonstration nets with and without flaps: Activity 2
- Geometry toolkits: Activity 2
- Glue or glue sticks: Activity 2
- Scissors: Activity 2
- Tape: Activity 2

Materials To Copy

- Building Prisms and Pyramids Cards (1 copy for every 9 students): Activity 2

SAMPLE ONLY

Required Preparation

Activity 2:

Copy and cut the blackline master. Make one copy for every 9 students, so that each student gets one drawing of a polyhedron. Consider assignments of polyhedra in advance.

Student Facing Learning Goals

Let’s draw nets and find the surface area of polyhedra.

15.1 Math Talk: Adjusting a Factor

Warm-up

5 mins

Activity Narrative

This is the first *Math Talk* activity in the course. See the launch for extended instructions for facilitating this activity successfully.

This *Math Talk* focuses on multiplication of two whole numbers. It encourages students to observe the impact of adjusting a factor and to rely on the structure of base-ten numbers and the properties of operations to find products (MP7).

Each expression is designed to elicit slightly different reasoning. In explaining their strategies, students need to be precise in their word choice and use of language (MP6). While many ways of reasoning may emerge, it may not be feasible to discuss every strategy. Consider gathering only 2–3 different strategies per expression. As students explain their strategies, ask them how the factors impacted their approach.

Standards

Building On **4.NBT.B.5**

Addressing **6.G.A.2, 6.G.A.4**

Instructional Routines

- Math Talk

Launch

This is the first time students do the *Math Talk* instructional routine in this course, so it is important to explain how it works before starting.

Explain that a *Math Talk* has four problems, revealed one at a time. For each problem, students have a minute to quietly think and are to give a signal when they have an answer and a strategy. The teacher then selects students to share different strategies (likely 2 or 3, given limited time), and might ask questions such as "Who thought about it in a different way?" The teacher then records the responses for all to see, and might ask clarification questions about the strategies before revealing the next problem.

Consider establishing a small, discreet hand signal that students can display when they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.

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- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each product mentally.

- $6 \cdot 15$
- $12 \cdot 15$
- $6 \cdot 45$
- $13 \cdot 45$

Student Response

- 90. Sample reasoning: $(6 \cdot 10) + (6 \cdot 5) = 90$
- 180. Sample reasoning: Since the 6 from the first question doubled to 12, and the 15 stayed the same, the product doubles to 180. This is because there are twice as many groups of 15 as in the first question.
- 270. Sample reasoning: Since the 6 is the same as the in the first question, and the 15 tripled to 45, the product triples to 270. This is because the number of groups stayed the same, but the amount in each group got three times as large.
- 585. Sample reasoning: Since the 45 is the same as the previous question, we can double the 6 and the product to get 540. We need one more group of 45, and $540 + 45 = 585$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity

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to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

15.2 Building Prisms and Pyramids

30 mins

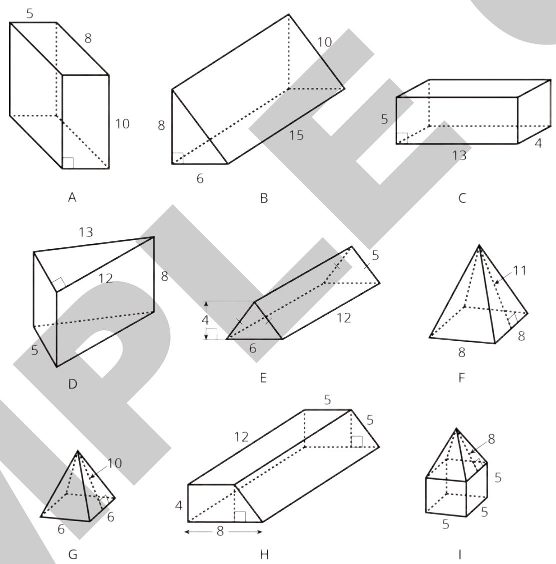
Activity Narrative

Previously, students used a given net of a polyhedron to find its surface area. Here they use a given polyhedron to draw a net and then calculate its surface area.

Use the provided polyhedra to differentiate the work for students with varying degrees of visualization skills. Rectangular prisms (A and C), triangular prisms (B and D), and square pyramids (F and G) can be managed by most students. Triangular Prism E requires a little more interpretive work

(in that the measurements of some sides may not be immediately apparent to students). Trapezoidal Prism H and Polyhedron I (a composite of a cube and a square pyramid) require additional interpretation and reasoning. The work here offers opportunities for students to make sense of problems and persevere in solving them (MP1).

As students work, remind them of the organizational strategies discussed in previous lessons, such as labeling polygons, showing measurements on the net, and so on.



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Standards

Addressing **6.G.A.4**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–3. Give each student in the group a different polyhedron from the blackline master and access to their geometry toolkits. Students need graph paper and a straightedge from their toolkits.

Explain to students that they will draw a net, find its surface area, and have their work reviewed by a peer. Give students 4–5 minutes of quiet time to draw their net on graph paper and then 2–3 minutes to share their net with their group and get feedback. When the group is sure that each net makes sense and all polygons of each polyhedron are accounted for, students can proceed and use the net to help calculate surface area.

If time permits, prompt students to cut and assemble their net into a polyhedron. Demonstrate how to add flaps to their net to accommodate gluing or taping. There should be as many flaps as there are edges in the polyhedron. (Remind students that this is different from the number of edges in the polygons of the net.)

Student Task Statement

Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have?
2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.
3. Label each polygon on the net with a name or number.
4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others.

Student Response

Answers vary depending on the polyhedron received.

1. Sample responses:
    - A and C are rectangular prisms.
    - B, D and E are triangular prisms.
    - F and G are square pyramids.
    - H is a trapezoidal prism.
    - I is a composite of a cube and a square pyramid.
2. Sample responses:
    - A and C should have 6 rectangles.
    - B, D, and E should have 5 polygons: 2 right triangles and 3 rectangles.
    - F and G should have 5 polygons: 1 square and 4 triangles.
    - H should have 6 polygons: 2 trapezoids and 4 rectangles.
    - I should have 9 polygons: 5 squares and 4 triangles.
3. Answers vary.
4. Sample responses:
    - A: 340 square units. $2(5 \cdot 8) + 2(5 \cdot 10) + 2(8 \cdot 10) = 340$
    - B: 408 square units. $2(\frac{1}{2} \cdot 6 \cdot 8) + (6 \cdot 15) + (8 \cdot 15) + (10 \cdot 15) = 408$
    - C: 274 square units. $2(13 \cdot 4) + 2(13 \cdot 5) + 2(4 \cdot 5) = 274$
    - D: 300 square units. $2(\frac{1}{2} \cdot 5 \cdot 12) + (5 \cdot 8) + (12 \cdot 8) + (13 \cdot 8) = 300$
    - E: 216 square units. $2(\frac{1}{2} \cdot 6 \cdot 4) + (6 \cdot 12) + 2(5 \cdot 12) = 216$
    - F: 240 square units. $4(\frac{1}{2} \cdot 8 \cdot 11) + (8 \cdot 8) = 240$
    - G: 156 square units. $4(\frac{1}{2} \cdot 6 \cdot 10) + (6 \cdot 6) = 156$

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- H: 316 square units. The trapezoidal base can be decomposed into a 5-by-4 rectangle and a right triangle with a base of 3 units and a height of 4. $2(5 \cdot 4) + 2(\frac{1}{2} \cdot 3 \cdot 4) + (8 \cdot 12) + 2(5 \cdot 12) + (4 \cdot 12) = 316$
- I: 205 square units. $5(5 \cdot 5) + 4(\frac{1}{2} \cdot 5 \cdot 8) = 205$

Building on Student Thinking

Students may know what polygons make up the net of a polyhedron but arrange them incorrectly on the net (for instance, allowing the faces to overlap instead of meeting at shared edges, orienting the faces incorrectly, or placing them in the wrong places). Suggest that students label some faces of the polyhedron drawing and transfer the adjacencies they see to the net. If needed, demonstrate the reasoning, for instance: "Face 1 and Face 5 both share the edge that is 7 units long, so I can draw them as two attached rectangles sharing a side that is 7 units long."

It may not occur to students to draw each face of the polyhedron to scale. Remind them to use the grid squares on their graph paper as units of measurement.

If a net is inaccurate, this becomes more evident when it is being folded. This may help students see which parts need to be adjusted and decide the best locations for the flaps. Reassure students that a few drafts of a net may be necessary before all the details are worked out, and encourage them to persevere.

Activity Synthesis

Ask students who finish their calculation to find another person in the class who has the same polyhedron and discuss the following questions (displayed for all to see):

- "Do your calculations match? Should they?"
- "Do your nets result in the same polyhedra? Should they?"
- "Do your models match the picture you were given? Why or why not?"

If time is limited, consider having the answer key posted somewhere in the classroom so students can quickly check their surface area calculations.

Reconvene briefly for a whole-class discussion. Invite students to reflect on the process of drawing a net and finding surface area based on a picture of a polyhedron. Ask questions such as:

- "How did you know that your net shows all the faces of your polyhedron?"
- "How did you know where to put each polygon or how to arrange all polygons so that, if folded, they can be assembled into the polyhedron in the drawing?"
- "How did the net help you find surface area?"

Access for English Language Learners

MLR7 Compare and Connect. Invite students to prepare a visual display that shows their net drawings and surface area calculations. Encourage students to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students:

- "What did the nets and calculations have in common? How were they different?"
- "What kinds of additional details or language helped you understand the displays?"

Advances: Representing, Conversing

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15.3 Comparing Boxes

Optional

15 mins

Activity Narrative

In this activity, students compare the surface areas and volumes of three rectangular prisms given nets that are not on a grid. To do this, they need to be able to visualize the three-dimensional forms that the two-dimensional nets would take when folded.

In grade 5, students had learned to distinguish area and volume as measuring different attributes. This activity clarifies and reinforces that distinction.

Standards

Building On	**5.MD.C.5**
Addressing	**6.G.A.4**

Launch

Keep students in the same groups of 2–3. Tell students that this activity involves working with both volume and surface area. To refresh students' understanding of volume from grade 5, ask students:

- "When we find the volume of a prism, what are we measuring?"
- "How is volume different from surface area?"
- "How might we find the volume of a rectangular prism?"

Reiterate that volume measures the number of unit cubes that can be packed into a three-dimensional shape and that we can find the number of unit cubes in a rectangular prism by multiplying the side lengths (length, width, and height) of a prism.

Give students 1–2 minutes to read the task statement and questions. Ask them to think about how they might go about answering each question and to be prepared to share their ideas. Give students a minute to discuss their ideas with their group. Then, ask groups to collaborate: Each member should perform the calculations for one prism (A, B, or C). Give students 5–7 minutes of quiet time to find the surface area and volume for their prism and then additional time to compare their results and answer the questions.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide students with a graphic organizer such as a two-column table to record measurements and calculations of surface area and volume. A table with space for students to record their calculations for volume and surface area for Boxes A, B, and C will help them keep their thinking organized. *Supports accessibility for: Visual-Spatial Processing, Organization*

Student Task Statement

Here are the nets of three cardboard boxes that are all rectangular prisms. The boxes will be packed with 1-centimeter cubes. All lengths are in centimeters.

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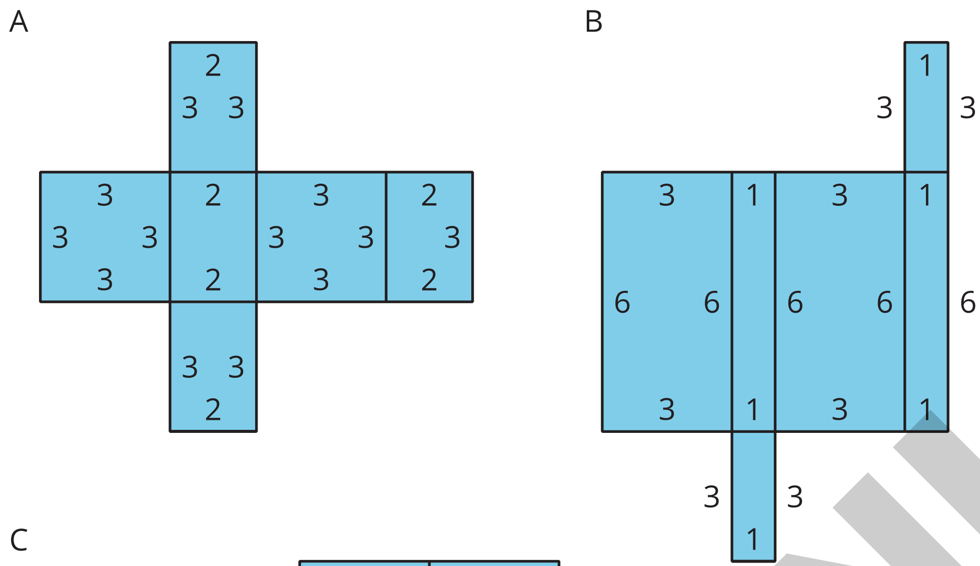

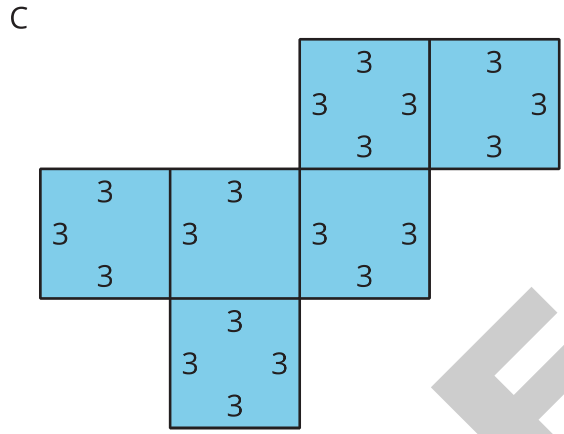

1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.
2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? Show your reasoning.

Student Response

1. Box A uses the least cardboard. Sample reasoning: The surface area of A is 42 square centimeters. A: $4(2 \cdot 3) + 2(3 \cdot 3) = 42$. The surface area of B and C is 54 square centimeters. B: $2(3 \cdot 6) + 2(3 \cdot 1) + 2(6 \cdot 1) = 54$. C: $6(3 \cdot 3) = 54$. Boxes B and C require the same amount of cardboard, both more cardboard than A.
2. Box C fits the most 1-centimeter cubes. Sample reasoning: The volume of A and B is 18 cubic centimeters. A: $3 \cdot 2 \cdot 3 = 18$. B: $6 \cdot 1 \cdot 3 = 18$. The volume of C is 27 cubic centimeter. C: $3 \cdot 3 \cdot 3 = 27$. A and B fit the same number of cubes, but fewer than C.

Building on Student Thinking

Students should have little trouble finding areas of rectangles but may have trouble keeping track of pairs of measurements to multiply and end up making calculation errors. Suggest that they label each polygon in the net and the corresponding written work and double-check their calculations to minimize such errors.

If students struggle to find the volume of their prism using information on a net, suggest that they sketch the prism that can be assembled from the net and label the edges of the prism.

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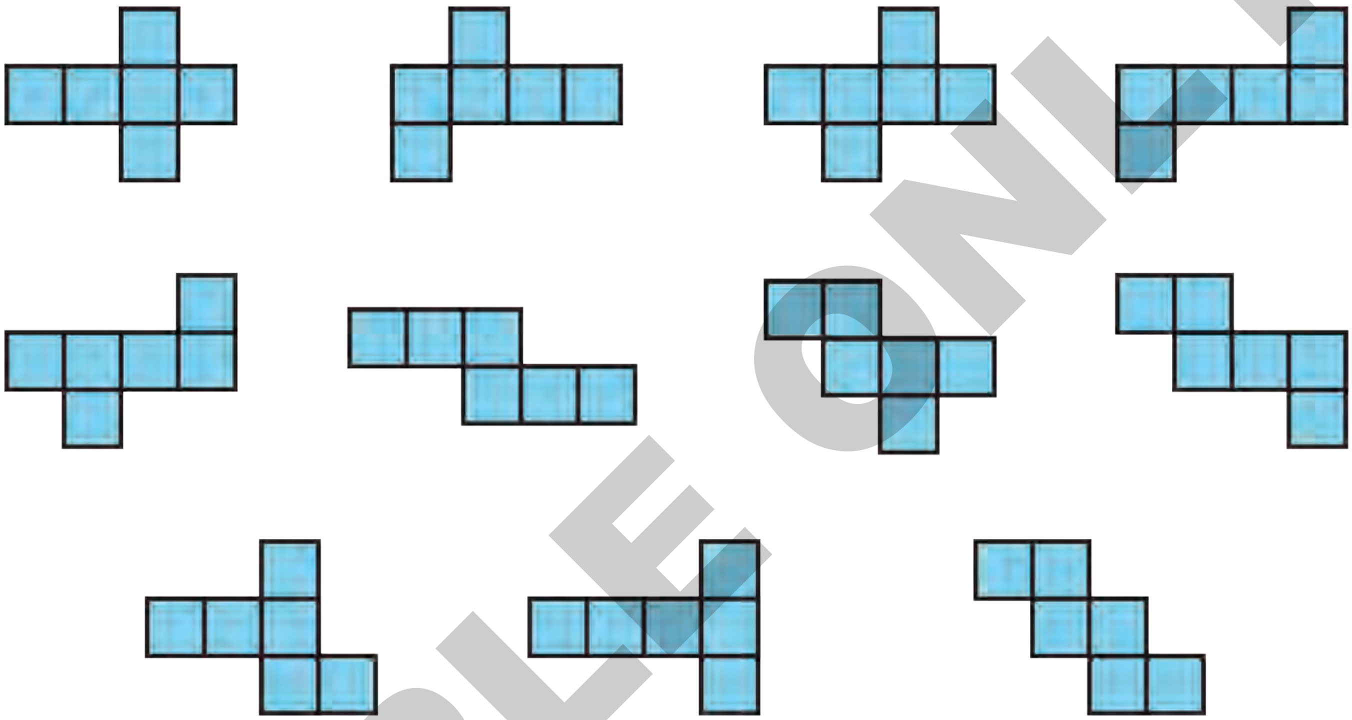
Students may need a reminder that area is measured in square units and that volume is measured in cubic units.

Are You Ready for More?

Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

Extension Student Response

There are 11 different nets for a cube. Any other net would be congruent to one of these.



Activity Synthesis

students who worked on the same prism if they agree or disagree about the calcualation. Record and display the results for all to see.

Invite students to share a few quick observations about the relationship between the surface areas and volumes for these three prisms, or between the amounts of material needed to build the boxes and the number of cubes that they can contain. Discuss questions such as:

- "If these prisms are boxes, which prism—B or C—would take more material to build? Which can fit more unit cubes?" (Prisms B and C would likely take the same amount of material to build because their surface areas are the same. Prism C has a greater volume than does Prism B, so it can fit more unit cubes.)
- "Which prism—A or B—would take more material to build? Which can fit more unit cubes?" (A and B can fit the same number of unit cubes but, B would require more material to build.)
- "If two prisms have the same surface area, would they also have the same volume? How do you know?" (No, Prisms A, B, and C are examples of how two figures with the same volume may not have the same surface area, and vice versa.)

Students will gain more insights into these ideas as they explore squares, cubes, and exponents in upcoming lessons. If

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students could benefit from additional work on distinguishing area and volume as different measures, do the optional lesson "Distinguishing Between Surface Area and Volume."

Lesson Synthesis

To highlight some key points from the lesson, display a picture of a prism or a pyramid and a drawing of its net. Discuss these questions:

- "Can you find the surface area of a simple prism or pyramid from a picture, if all the necessary measurements are given?"
- "Can you find the surface area from a net, if all the measurements are given?"
- "Which might be more helpful for calculating surface area—a picture of a polyhedron or a net?" (If the polyhedron is simple—such as a cube or a square pyramid—and does not involve hidden faces with different measurements or require a lot of visualizing, either a picture or a net can work. Otherwise, a net may be more helpful because we can see all of the faces at once and can find the area of each polygon more easily. A net may also help us keep track of our calculations and notice missing or extra areas.)

15.4 Surface Area of a Triangular Prism

5 mins

Cool-down

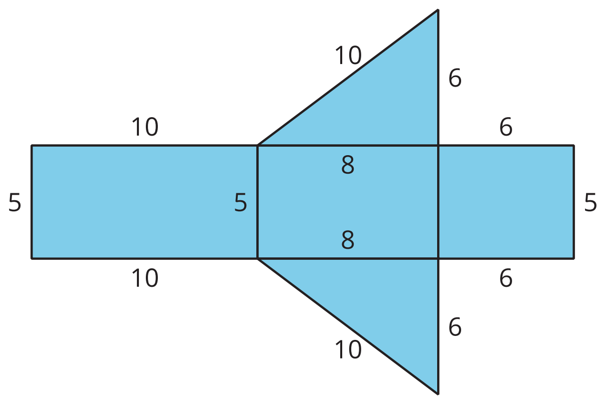
Sec D

Standards

Addressing **6.G.A.4**

Student Task Statement

1. In this net, the two triangles are right triangles. All quadrilaterals are rectangles. What is its surface area in square units? Show your reasoning.



2. If the net is assembled, which of the following polyhedra would it make?

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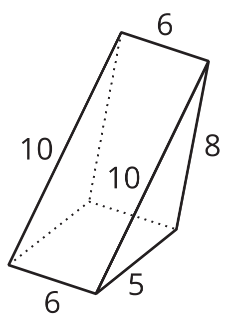
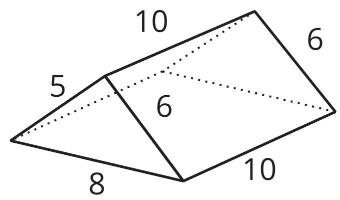
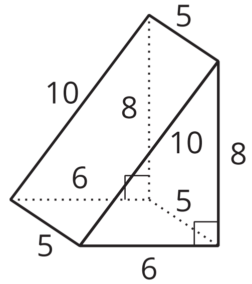
A



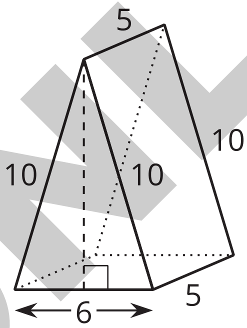
B



C



D



Student Response

1. 168 square units. Sample reasoning: There are two triangular faces with area of 24 square units each. $\frac{1}{2} \cdot 6 \cdot 8 = 24$. There is a rectangular face with area of 50 square units. $10 \cdot 5 = 50$. There is one rectangular face with area of 40 square units. $5 \cdot 8 = 40$. There is one rectangular face with area $5 \cdot 6 = 30$ square units. $2 \cdot 24 + 50 + 40 + 30 = 168$
2. Prism C

Responding To Student Thinking

Points to Emphasize
If students struggle with relating polyhedra and their nets, discuss the connections between different representations of a polyhedron when opportunities arise in the next lesson. For example, review this practice problem, and discuss how students know the side lengths of the polygons on each net:

Grade 6, Unit 1, Lesson 15, Practice Problem 4

Lesson 15 Summary

A net can help us find the surface area of a polyhedron that has different polygons for its faces. We can find the areas of all polygons in the net and add them.

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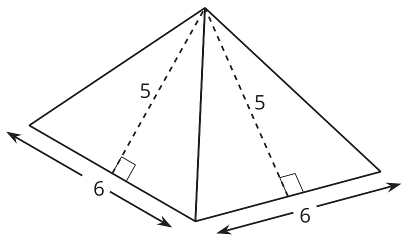

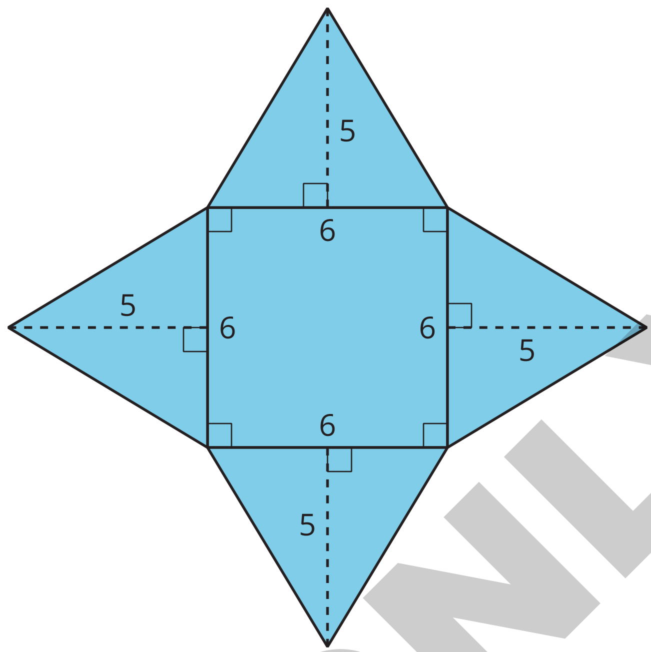

A square pyramid has a square and 4 triangles for its faces. Its surface area is the sum of the areas of the square base and the 4 triangular faces:

$$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$$

The surface area of this square pyramid is 96 square units.

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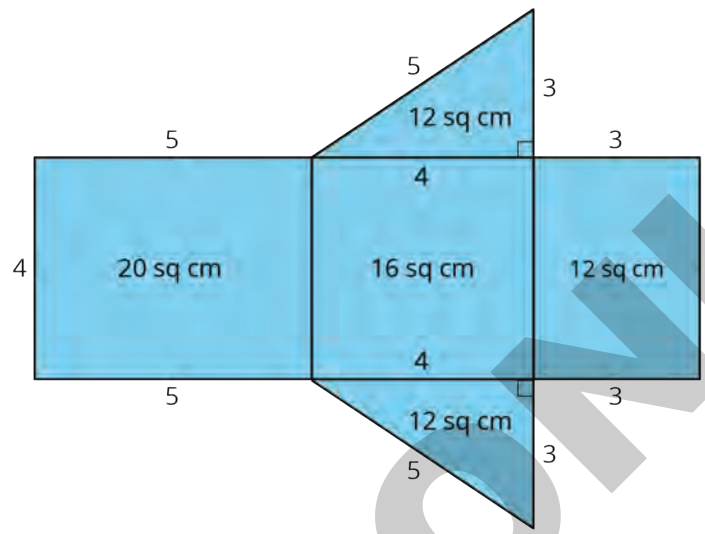
Practice Problems

1 Student Task Statement

Jada drew a net for a polyhedron and calculated its surface area.



a. What polyhedron can be assembled from this net?

b. Jada made some mistakes in her area calculation. What were the mistakes?

c. Find the surface area of the polyhedron. Show your reasoning.

Solution

a. Triangular prism

b. She calculated the areas of the two triangular faces incorrectly. The right triangles have a base of 4 cm and a height of 3 cm, so the area of each should be $\frac{1}{2} \cdot 4 \cdot 3$, or 6 sq cm. Jada wrote "12 sq cm" for the area of each triangle.

c. 60 sq cm. Sample reasoning: The triangular faces should be 6 sq cm each, so the surface area is $20 + 16 + 12 + 6 + 6$, or 60.

2 Student Task Statement

A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labeling the edges with their measurements.

Solution

272 square inches. Sample reasoning:

- The top and bottom faces are 2 inches by 8 inches each, so their combined area is $2(2 \cdot 8)$ or 32 square inches.

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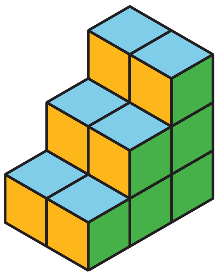
- The front and back faces are 8 inches by 12 inches each, so their combined area is $2(8 \cdot 12)$ or 192 square inches.
- The side faces are 2 inches by 12 inches each, so their combined area is $2(2 \cdot 12)$ or 48 square inches.
- The surface area is $32 + 192 + 48$ or 272 square inches.

3 from Unit 1, Lesson 12

Student Task Statement

Twelve cubes are stacked to make this figure.



a. What is its surface area?

b. How would the surface area change if the top two cubes are removed?

Solution

a. 36 square units

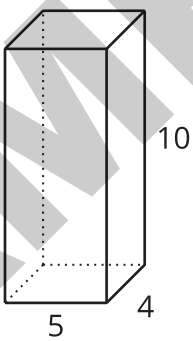
b. The surface area would decrease by 6 square units.

4

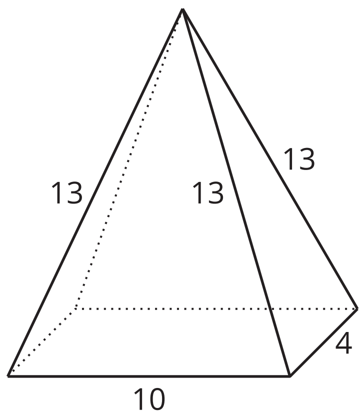
Student Task Statement

Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.

A



B



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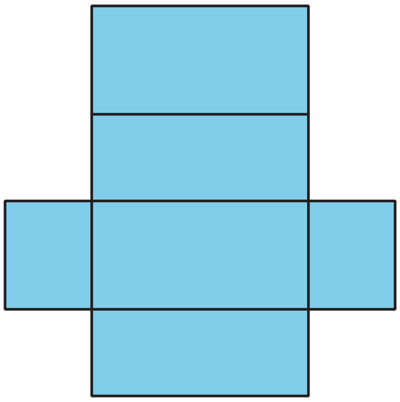
A



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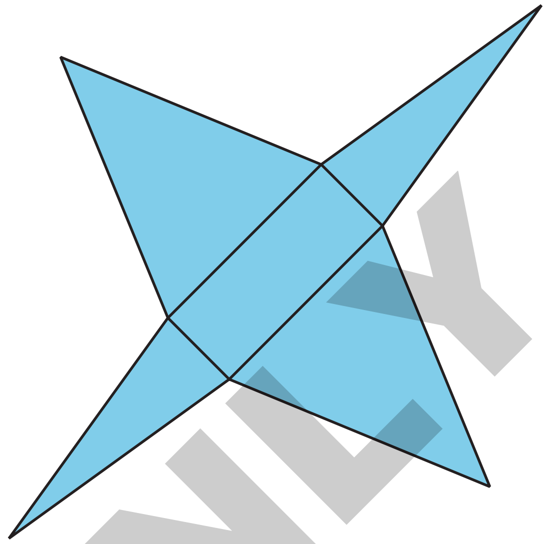
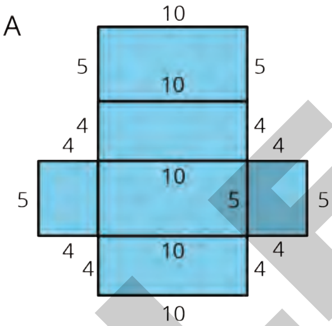
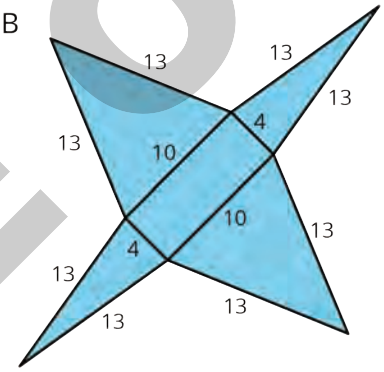


Solution

A



B



5 from Unit 1, Lesson 14

Student Task Statement

a. What three-dimensional figure can be assembled from the net?

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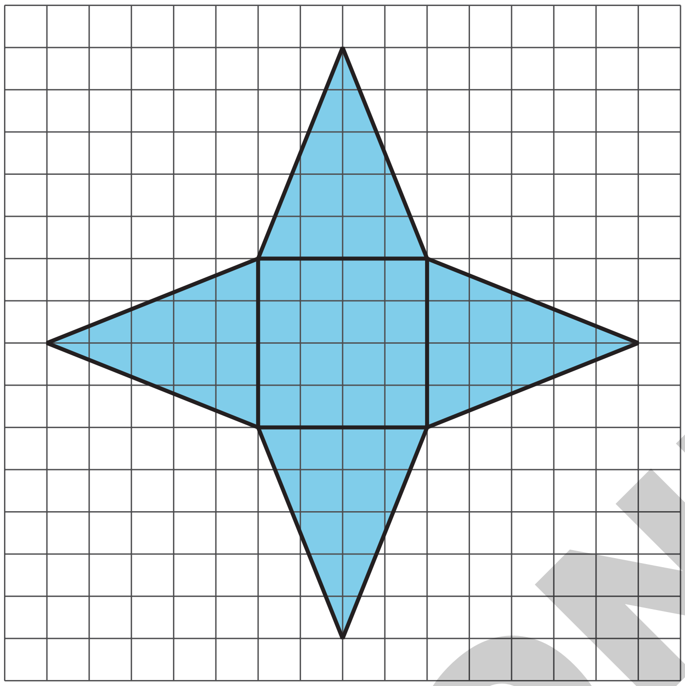

b. What is the surface area of the figure? (One grid square is 1 square unit.)

Sec D

Solution

a. Square pyramid

b. 56 square units. The area of the base is 16 square units. Each triangular face has a base of 4 units and a height of 5 units. This means each triangular face has an area of 10 square units. The total surface area is 56 square units, because $16 + 10 + 10 + 10 + 10 = 56$.

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Distinguishing Between Surface Area and Volume

Goals

- Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
- Describe (orally and in writing) shapes built out of cubes, including observations about their surface area and volume.
- Determine the surface area and volume of shapes made out of cubes.

Learning Targets

- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

Lesson Narrative

This optional lesson reminds students of the distinctions between measures of one-, two-, and three-dimensional attributes. It reinforces the idea that length is a one-dimensional attribute of geometric figures, surface area is a two-dimensional attribute, and **volume** is a three-dimensional attribute.

Students take a closer look at the distinction between surface area and volume. They build or draw representations of polyhedra and then calculate both the surface area and volume. In doing so, students see that different three-dimensional figures can have the same volume but different surface areas, and vice versa. This is analogous to the fact that two-dimensional figures can have the same area but different perimeters, and vice versa. Throughout the lesson, students practice attending to precision (MP6) as they consider the geometric attributes being studied and the corresponding units of measurement.

Standards

Building On	**3.MD.C.5, 4.MD.A.1, 5.MD.C, 5.MD.C.3.b, 5.MD.C.4, 5.MD.C.5.a**
Addressing	**6.G.A.4**

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Snap cubes: Activity 2, Activity 3
- Sticky notes: Activity 2
- Geometry toolkits: Activity 3

SAMPLE ONLY

Required Preparation

Activity 1:

Prepare solutions to the first question of the *1-2-3 Dimensional Attributes* activity on a large visual display.

Activity 2:

Prepare sets of 16 snap cubes and two sticky notes for each student.

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's contrast surface area and volume.

16.1 Attributes and Their Measures

Warm-up

10 mins

Activity Narrative

This activity strengthens students' awareness of one-, two-, and three-dimensional attributes and the units commonly used to measure them. Students decide on the units based on the attributes being measured and on the size of the units and how appropriate they would be for describing given quantities.

As students work, select a few students to share their responses to the last two questions of the activity (on the quantities that could be measured in miles and in cubic meters).

Standards

Building On **3.MD.C.5, 4.MD.A.1, 5.MD.C**

Launch

Consider a quick review of metric and standard units of measurement before students begin work. Include some concrete examples that could help illustrate the size of each unit.

Then, pick an object in the classroom for which surface area and volume could be measured (for example, a desk). Ask students, "What units might we use to measure the surface area of the desktop? What units might we use to measure the **volume** of a drawer?"

Clarify the relative sizes of the different units that come up in the conversation. For instance, discuss how a meter is a little over three feet, a yard is three feet, a kilometer is about two-thirds of a mile, a millimeter is one tenth of a centimeter, and so on.

Give students 4–5 minutes of quiet think time and then a couple of minutes to share their responses with a partner. Prepare to display the answers to the first six questions for all to see.

Student Task Statement

For each quantity, choose one or more appropriate units of measurement.

SAMPLE ONLY

For the last two, think of a quantity that could be appropriately measured with the given units.

Quantities

1. Perimeter of a parking lot:
2. Volume of a semi truck:
3. Surface area of a refrigerator:
4. Length of an eyelash:
5. Area of a state:
6. Volume of an ocean:
7. ____________________: miles
8. ____________________: cubic meters

Units

- millimeters (mm)
- feet (ft)
- meters (m)
- square inches (sq in)
- square feet (sq ft)
- square miles (sq mi)
- cubic kilometers (cu km)
- cubic yards (cu yd)

Student Response

1. Meters, feet
2. Cubic yards
3. Square inches, square feet
4. Millimeters
5. Square miles
6. Cubic kilometers, cubic yards
7. Sample responses: distance between home and school, length of a river
8. Sample responses: volume of a room, volume of a swimming pool

Building on Student Thinking

Depending on the students' familiarity with metric and standard units, there may be some confusion about the size of each unit. Consider displaying measuring tools or a reference sheet that shows concrete examples of items measured in different-sized units.

Activity Synthesis

Display the solutions to the first six questions for all to see and to use for checking. Then, select a few students to share their responses to the last two questions.

Ask students what they notice about the units for area and the units for volume. If not already mentioned by students, highlight that area is always measured in square units and volume in cubic units.

SAMPLE ONLY

16.2 Building with 8 Cubes

Optional

25 mins

Activity Narrative

There is a digital version of this activity.

This activity clarifies the distinction between volume and surface area and illustrates that two polyhedra can have the same volume but different surface areas.

Students build figures using two sets of 8 cubes and determine their volumes and surface areas. Because all of the designs are made of the same number of cubes, they all have the same volume. Students then examine all of the designs and discuss what distinguishes figures with smaller surface areas from those with greater ones.

As students work, monitor the range of surface areas for the figures that students built.

This activity works best when each student has access to snap cubes. If physical cubes are not available, consider using the digital version of the activity. In the digital version, students use an applet that has 16 cubes with which to build the two figures.

Standards

Building On **5.MD.C.3.b, 5.MD.C.4**

Addressing **6.G.A.4**

Instructional Routines

- MLR8: Discussion Supports

Launch

Give each student (or group of 2 students) 16 snap cubes and two sticky notes. Explain that their job is to design and build two figures—using 8 cubes for each—and find the volume and surface area of each figure. Ask them to give each figure a name or a label and then record the name, surface area, and volume of each figure on a sticky note.

Give students 8–10 minutes of work time. Select several students whose designs collectively represent a range of surface areas to share their work later.

Student Task Statement

Your teacher will give you 16 cubes. Build two different figures using 8 cubes for each.

For each shape, complete these three steps and record your responses on a sticky note:

1. Give it a name or a label (such as Mai's First Shape or Diego's Steps).
2. Find the volume.
3. Find the surface area.

Student Response

Sample response:

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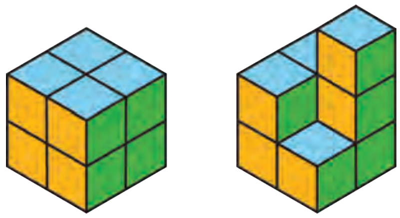


1. Answers vary.
2. They both have a volume of 8 cubic units.
3. The first has a surface area of 24 square units. The second has a surface area of 28 square units. (The smallest possible surface area for an 8-cube construction is 24 square units, and the largest is 34 square units.)

Building on Student Thinking

Even though students are dealing with only 8 cubes at a time, they may make counting errors by inadvertently omitting or double-counting squares or faces. This is especially likely for designs that are non-prisms. Encourage students to think of a systematic way to track the number of square units they are counting.

Some students may associate volume only with prisms and claim that the volume of non-prism designs cannot be determined. Remind them of the definition of volume.

Activity Synthesis

Ask all students to display their designs and their sticky notes and give students a couple of minutes to circulate and view one another's work.

Then, ask previously identified students to arrange their designs in the order of their surface area, from least to greatest, and display their designs for all to see. Record the information about the designs in a table, in the same sequence. Display the table for all to see. Here is an example.

shapes	volume	surface area
Andre's cube	8	24
Lin's steps	8	28
Jada's first shape	8	28
. . .	. . .	. . .
Noah's tower	8	34

Give students a minute to examine the designs and the information in the table. Then, discuss the following questions:

- "What do all of the shapes have in common?" (They have the same volume.)
- "Why are all the volumes the same?" (Volume measures the number of unit cubes that can be packed into a figure. All the designs are built using the same number of cubes.)
- "Why do some shapes have larger surface areas than others? What do shapes with larger surface areas look like?" (The cubes are more spread out and have more of their faces exposed.)
- "What about those with smaller surface areas?" (They are more compact and have more of their faces hidden or shared with another cube.)

SAMPLE ONLY

- “Is it possible to build a shape with a different volume? How?” (Yes, but it would involve using fewer or more cubes.)

If students have trouble visualizing how surface area changes when the design changes, demonstrate the following:

- Make a cube made of 8 smaller cubes. Point to one cube and ask how many of its faces are exposed (3).
- Pop that cube off and move it to another place.
- Point out that, in the “hole” left by the cube that was moved, 3 previously interior faces now contribute to the surface area. At the same time, the relocated cube now has 5 faces exposed.

Access for English Language Learners

MLR8 Discussion Supports. To help students describe and explain their comparisons, provide language that students can use. Demonstrate contrasting words such as “spread out” and “compact,” “exposed” and “hidden,” or “visible” and “covered” using one or more cube designs. Display sentence frames to support students with their explanations, for example: “The surface area of Shape _____ is larger (or smaller) than the surface area of Shape _____ because . . . .”
Advances: Speaking, Representing

16.3 Comparing Prisms

Optional

20 mins

Activity Narrative

Previously, students studied shapes with the same volume but different surface areas. Here they see that it is also possible for shapes to have the same surface area but different volumes. Students think about how the appearance of these shapes might compare visually.

Students are given the side lengths of three rectangular prisms and asked to find the surface area and the volume of each. Some students can visualize these, but others may need to draw nets, sketch the figures on isometric grid paper, or build physical prisms. Prepare cubes for students to use. Each of the three prisms can be built with 15 or fewer cubes, but 40 cubes are needed to build all three simultaneously. (If the cubes are not centimeter cubes, ask students to treat them as if the edge length of each cube was 1 cm.)

As students work, look out for errors in students’ calculations in the first question, which will affect the observations they make in the second question. Select a few students who notice that the volumes of the prisms are all different but the surface areas are the same.

Access for English Language Learners

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Standards

Building On	**5.MD.C.5.a**
Addressing	**6.G.A.4**

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports

SAMPLE ONLY

Launch

Arrange students in groups of 2. Provide access to snap cubes and geometry toolkits. Give students 6–7 minutes of quiet think time and then 2–3 minutes to discuss their responses with their partner. Ask partners to agree upon one key observation to share with the whole class.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Use this routine to help students describe what they noticed about the volume and surface areas of their prisms. Students should take turns stating an observation and their reasoning with their partner. Invite Partner A to begin with this sentence frame: "I noticed ____, so . . ." Invite the listener, Partner B, to press for additional details referring to specific features of the prisms. Students should switch roles until they have listed all observations.
Design Principle(s): Support sense-making; Cultivate conversation

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to show thinking using snap cubes or blocks.
Supports accessibility for: Language, Visual-Spatial Processing

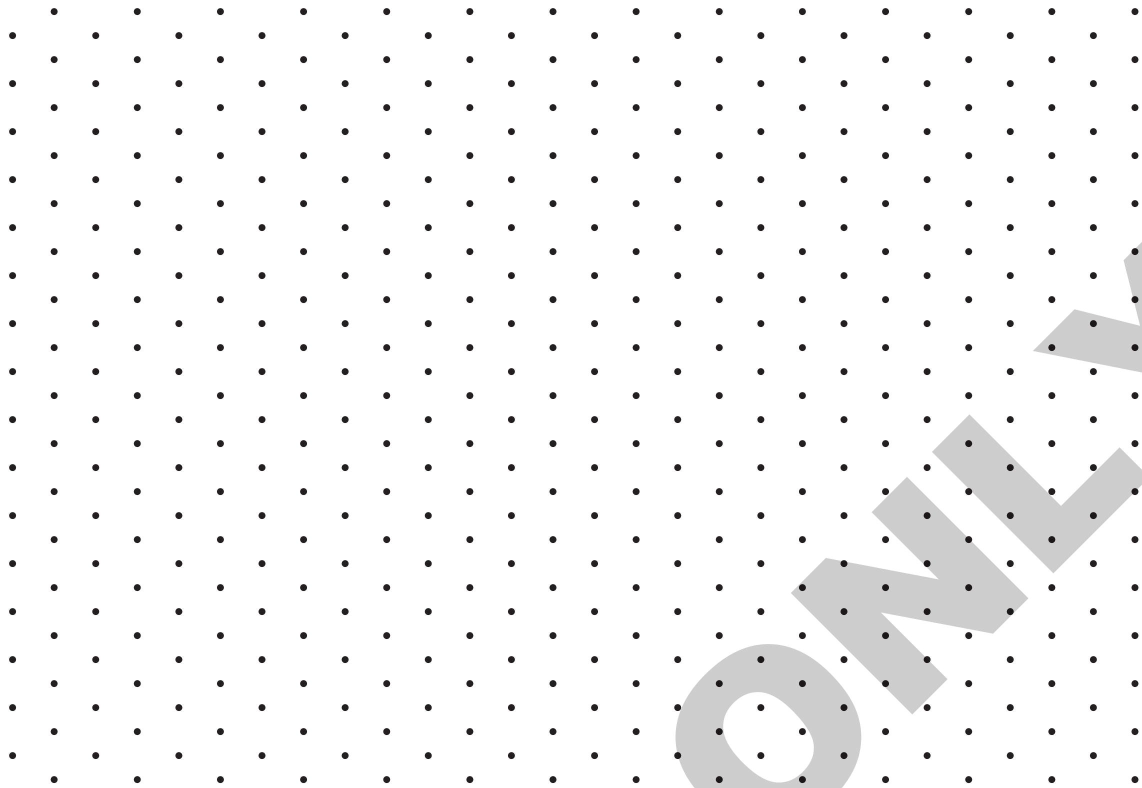
Student Task Statement

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1. Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.

SAMPLE ONLY



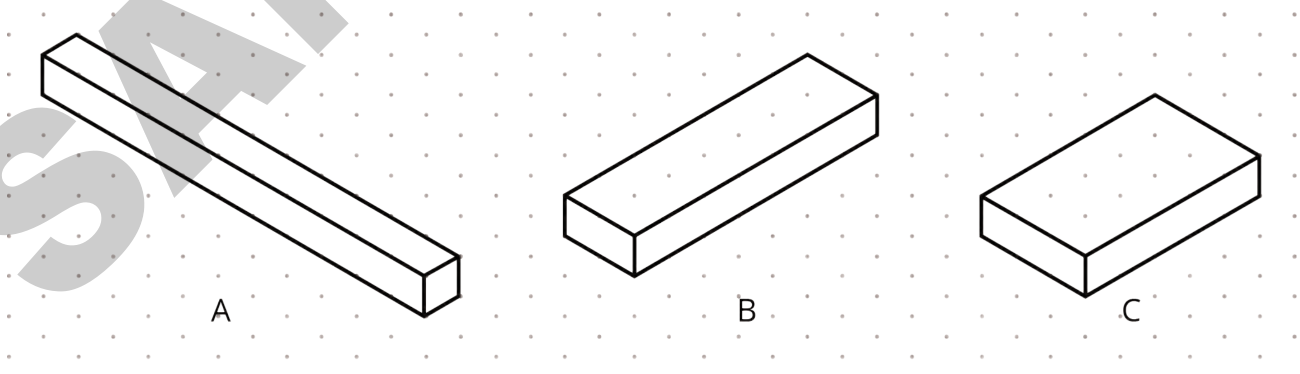
Sec D

2. Analyze the volumes and surface areas of the prisms. What do you notice? Write 1 or 2 observations about them.

Student Response

1. Surface areas:
   - Prism A: $4(11 \cdot 1) + 2(1 \cdot 1) = 46$ square centimeters
   - Prism B: $2(7 \cdot 2) + 2(7 \cdot 1) + 2(2 \cdot 1) = 46$ square centimeters
   - Prism C: $2(5 \cdot 3) + 2(5 \cdot 1) + 2(3 \cdot 1) = 46$ square centimeters

   Volumes:
   - Prism A: 11 cubic centimeters ($11 \cdot 1 \cdot 1 = 11$)
   - Prism B: 14 cubic centimeters ($7 \cdot 2 \cdot 1 = 14$)
   - Prism C: 15 cubic centimeters ($5 \cdot 3 \cdot 1 = 15$)



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2. Sample responses:

    - The surface areas of the prisms are all the same, but the volumes are all different.
    - The polygons that make up the faces of each prism are different-sized rectangles, but their areas all add up to the same total of square centimeters.
    - Prism C can fit the most centimeter cubes, but because the cubes would fit together in a compact way, some of the cubes would only have two square centimeters of exposed faces.
    - Prism A can fit the fewest centimeter cubes, but because the cubes would be more spread out, more of their faces would be exposed.

Building on Student Thinking

Students may miss or double-count one or more faces of the prisms and miscalculate surface areas. Encourage students to be systematic in their calculations and to use organizational strategies that they learned when they used nets to find surface areas.

Students may need reminders to use square units for area and cubic units for volume.

Are You Ready for More?

Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Extension Student Response

Sample responses:

- A prism that is 4 units by 5 units by 1 unit and one that is 2 units by 9 units by 1 unit have the same surface area but different volumes.
- Generate examples by finding different pairs of factors of the same number and subtracting 1 from each factor. However, there are other ways. For example, $60 = 6 \cdot 10$ and $60 = 5 \cdot 12$. The 5-by-9-by-1 and 4-by-11-by-1 prisms have the same surface areas but different volumes.

Activity Synthesis

The goal of this discussion is to highlight volume and surface area as distinct measurements of a polyhedron.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written statement about volume and surface area by correcting errors, clarifying meaning, and adding details.

- Display an image of Prisms A, B, and C for all to see.

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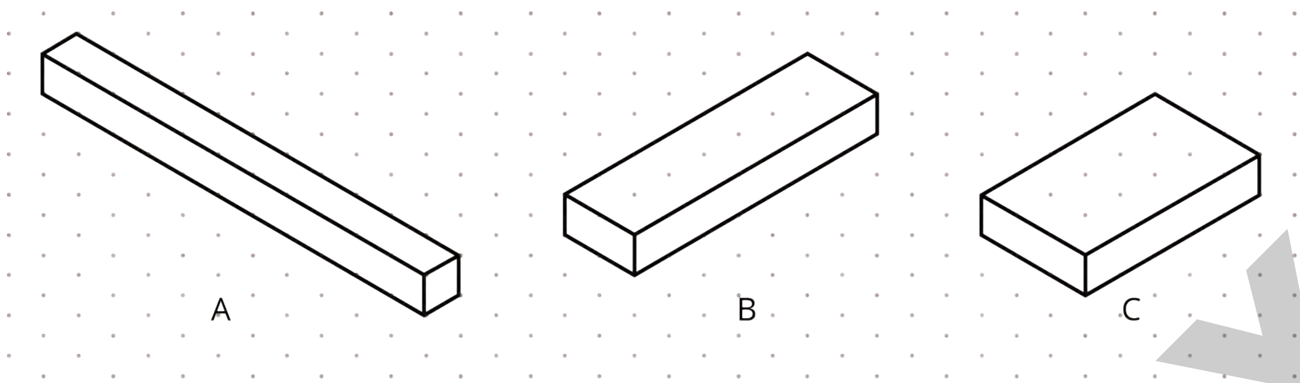

- Display this first draft:
  Prism C has a greater volume than do Prisms A and B, so Prism C also has a greater surface area than the other two prisms.
- Ask, "What parts of this statement are unclear, incorrect, or incomplete?" As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.
- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, and then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

If needed, consider referring to the filing cabinet activity in an earlier lesson to help students conceptualize the idea of figures with different volumes having the same surface area. The number of square sticky notes needed to cover all of the faces of the filing cabinet was its surface area. If we use all of those square notes (no more, no less) to completely cover (without overlapping sticky notes) a cabinet that has a different volume, we can say that the two pieces of furniture have the same surface area and different volumes.

Lesson Synthesis

In this lesson, students refreshed their memory of measures of one-, two-, and three-dimensional attributes. Reiterate that length is a one-dimensional attribute of geometric figures, area is a two-dimensional attribute, and volume is a three-dimensional attribute. Revisit a few examples of units for length, area, and volume.

To reiterate the distinctions between surface areas and volumes of polyhedra, consider asking students:

- "How could two figures that both have a volume of 4 cubic units have different surface areas?" (Surface area and volume measure different attributes of a three-dimensional shape.)
- "How is surface area different from volume?" (Surface area is a two-dimensional attribute. It measures how many unit squares cover all the faces of a figure. Volume is a three-dimensional attribute. It measures how many unit cubes fill the figure.)
- "Are the two measures related? Does a greater volume necessarily mean a greater surface area, and vice versa?" (No, one measure does not affect the other. A figure that has a greater volume than another may not necessarily have a greater surface area.)

A note about materials for an upcoming unit:

For the first lesson in the unit on ratios, students will need to bring in a collection of 10–20 small, inexpensive objects. Examples include rocks, seashells, erasers, stickers, bottle caps, bread clips, or hair clips. Inform students about this and remind them in the days leading to that lesson. If any students are concerned about having enough items to bring,

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brainstorm possibilities with them or suggest making a collection using paper or other materials available at school.

16.4 Same Surface Area, Different Volumes

Cool-down

5 mins

Standards

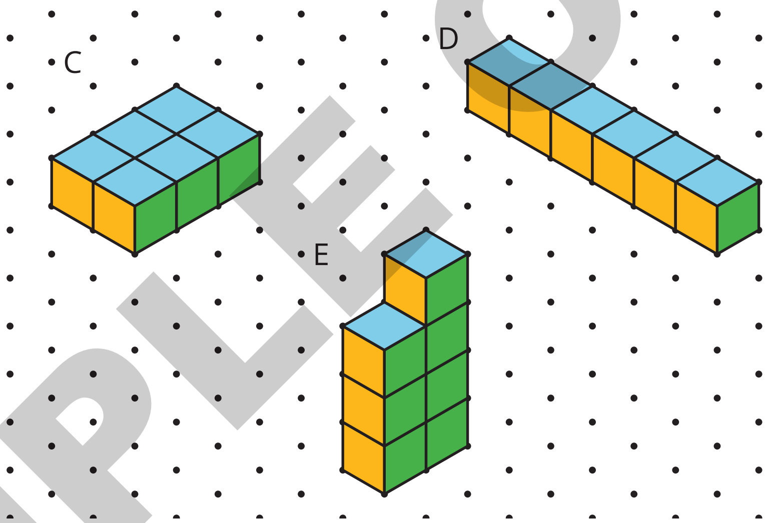
Addressing **6.G.A.4**

Launch

Encourage students to refer to the class list of observations from the previous activity

Student Task Statement

Choose two figures that have the same surface area but different volumes. Show your reasoning.



Student Response

Figures D and E both have a surface area of 26 square units, but D has a volume of 6 cubic units, and E has a volume of 7 cubic units.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

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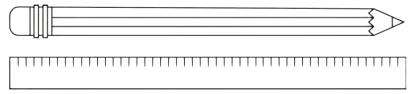
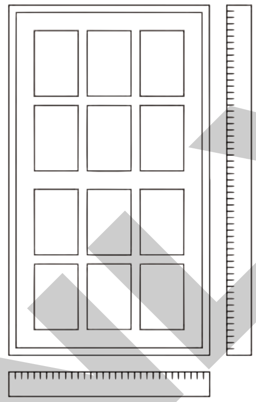
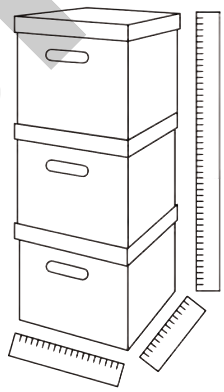
Lesson 16 Summary



Length is a one-dimensional attribute of a geometric figure. We measure lengths using units like millimeters, centimeters, meters, kilometers, inches, feet, yards, and miles.



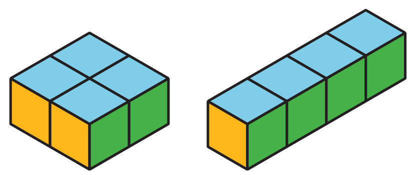
Area is a two-dimensional attribute. We measure area in square units. For example, a square that is 1 centimeter on each side has an area of 1 square centimeter.



Volume is a three-dimensional attribute. We measure volume in cubic units. For example, a cube that is 1 kilometer on each side has a volume of 1 cubic kilometer.

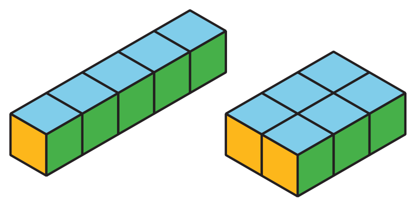
Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

Two figures can have the same volume but different surface areas. For example:



- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.

Similarly, two figures can have the same surface area but different volumes.



- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.

Glossary

- volume

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Practice Problems

1 Student Task Statement

Match each quantity with an appropriate unit of measurement.

A. The surface area of a tissue box
B. The amount of soil in a planter box
C. The area of a parking lot
D. The length of a soccer field
E. The volume of a fish tank

1. Square meters
2. Yards
3. Cubic inches
4. Cubic feet
5. Square centimeters

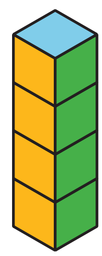
Solution

- A matches 5
- B matches 3
- C matches 1
- D matches 2
- E matches 4

SAMPLE ONLY

2 Student Task Statement

Here is a figure built from snap cubes.



a. Find the volume of the figure in cubic units.
b. Find the surface area of the figure in square units.
c. True or false: If we double the number of cubes being stacked, both the volume and surface area will double. Explain or show how you know.

Solution

a. 4 cubic units. $(1 \cdot 1 \cdot 4) = 4$
b. 18 square units. $(4 \cdot 4) + (2 \cdot 1) = 18$
c. False. Sample reasoning: The volume will double to 8 cubic units, but the surface area will not. Only the side

faces will double in area, to $(4 \cdot 8)$ or 32 square units, but the top and bottom faces will not double, so the surface area will be 34, not 36, square units.

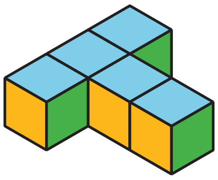
3 Student Task Statement

Lin said, "Two figures with the same volume also have the same surface area."

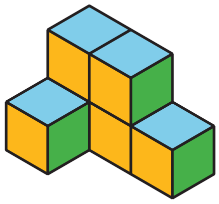
a. Which two figures suggest that her statement is true?

b. Which two figures could show that her statement is *not* true?

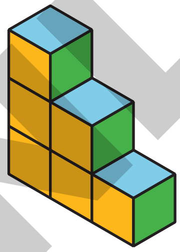
A



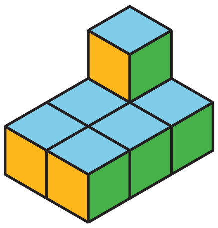
B



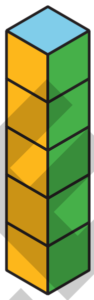
C



D



E



Solution

a. B and C

b. A and B, or A and C

4

from Unit 1, Lesson 11

Student Task Statement

Draw a pentagon (five-sided polygon) that has an area of 32 square units. Label all relevant sides or segments with their measurements, and show that the area is 32 square units.

Solution

Sample responses:

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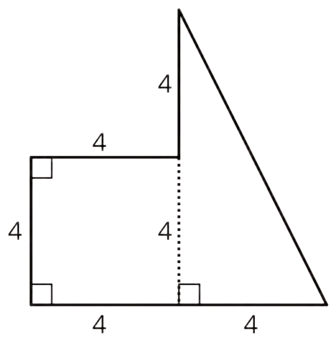
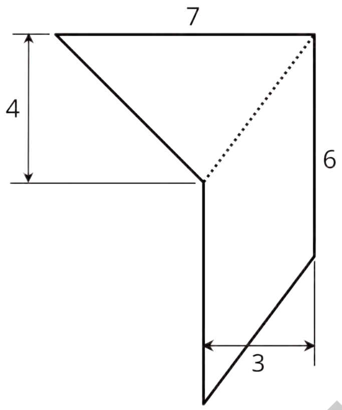


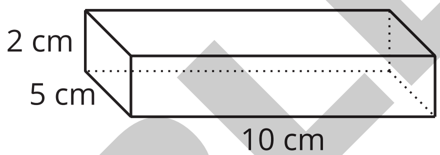


- The first pentagon is composed of a square and a right triangle. The square has an area of 16 square units. The triangle has a base of 4 and a height of 8, so its area is 16 square units. The combined area is $16 + 16$ or 32 square units.
- The second pentagon is composed of a parallelogram with a base of 6 and a height of 3, and a triangle with a base of 7 and a height of 4. The area of the parallelogram is $6 \cdot 3$ or 18 square units. The area of the triangle is $\frac{1}{2} \cdot 7 \cdot 4$ or 14 square units. The combined area is $18 + 14$ or 32 square units.

5 from Unit 1, Lesson 15

Student Task Statement

a. Draw a net for this rectangular prism.



b. Find the surface area of the rectangular prism.

Solution

a. Sample response:

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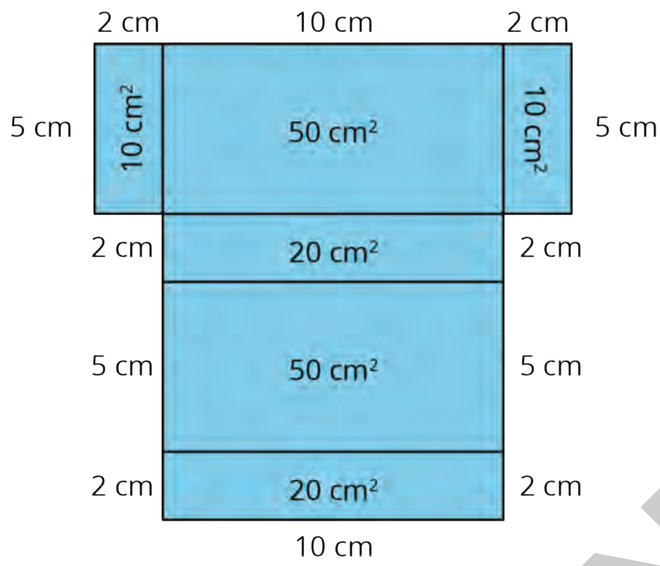

b. 160 square units. (There are two faces with an area of 50 square cm, two faces with an area of 20 square cm, and two faces with an area of 10 square cm.)

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Section E: Squares and Cubes

Goals

- Interpret and write expressions with exponents ^{2} and ^{3} to represent the area of a square or the volume of a cube.
- Write expressions, with or without exponents, to represent the surface area of a given cube.

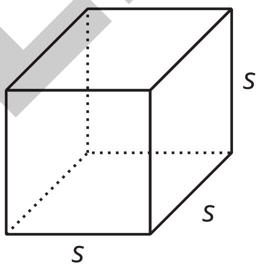
Section Narrative

In this short section, students learn to use exponents to express areas of squares and surface areas and volumes of cubes.

Students first explore perfect squares as areas of squares and perfect cubes as volumes of cubes with whole-number edge lengths. Next, they learn that the exponents ^{2} and ^{3} can be used to express the multiplication of edge lengths of these figures. For example, the area of a square with a side length of 5 units is 5^2 square units and the volume of a cube with an edge length of 5 units is 5^3 cubic units.

The exponents ^{2} and ^{3} can also be used to express square units and cubic units. The area of a square with 5-inch sides is 5^2 inch^2 and the volume of a cube with 5-cm edges is 5^3 cm^3. Students practice using and interpreting this new notation in the context of squares and cubes.

area = s^2



volume = s^3

Then, students draw nets of cubes with numerical edge lengths and write expressions for the surface areas, using an exponent to express repeated multiplication of a value. Finally, they use an exponent to write a formula for the surface area of any cube.

Teacher Reflection Questions

- **Math Content and Student Thinking:** In grade 5, students used exponents to express powers of 10. How does this experience compare and contrast to using the exponents of 2 and 3 in the geometric context as they work with surface area and volume of polyhedra?
- **Pedagogy:** Think about a recent activity that went longer than planned. If you could go back and teach it again, would you make the same choices?

SAMPLE ONLY

- **Access and Equity:** Think about who volunteers to share their thinking with the class. Are the same students always volunteering, while some students never offer to share? What can you do to help the class understand the value of hearing the ideas of every mathematician?

SAMPLE ONLY

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Section E Checkpoint

1

Goals Assessed

- Interpret and write expressions with exponents ^{2} and ^{3} to represent the area of a square or the volume of a cube.

Student Task Statement

A square has a side length of 15 centimeters.

Select **all** expressions that represent the area of the square in square centimeters:

A. 15^3

B. $15 \cdot 15$

C. $15 + 15$

D. $15 \cdot 2$

E. 15^2

Solution

B, E

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

2

Goals Assessed

- Interpret and write expressions with exponents ^{2} and ^{3} to represent the area of a square or the volume of a cube.

Student Task Statement

A cube has a volume of 2^3 cubic inches.

Select **all** statements that are true about the cube:

A. The volume of the cube is 8 cubic inches.

SAMPLE ONLY

B. The edge length of the cube is 3 inches.

C. The expression $3 \cdot 2$ also represents its volume in cubic inches.

D. The edge length of the cube is 2 inches.

E. The volume of the cube is 6 cubic inches.

Solution

A, D

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

3

Goals Assessed

- Write expressions, with or without exponents, to represent the surface area of a cube.

Student Task Statement

A cube has an edge length of 47 units. Write an expression to represent its surface area in square units.

Solution

Sample responses:

- $6 \cdot 47^2$
- $6 \cdot (47 \cdot 47)$
- $47^2 + 47^2 + 47^2 + 47^2 + 47^2 + 47^2$
- $(47 \cdot 47) + (47 \cdot 47) + (47 \cdot 47) + (47 \cdot 47) + (47 \cdot 47) + (47 \cdot 47)$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

SAMPLE ONLY

Unit 1, Lesson 17

Squares and Cubes

Goals

- Generalize a process for finding the volume of a cube, and justify (orally) why this can be abstracted as s^3.
- Include appropriate units (orally and in writing) when reporting lengths, areas, and volumes, e.g., cm, cm^2, cm^3.
- Interpret and write expressions with exponents ^{2} and ^{3} to represent the area of a square or the volume of a cube.

Learning Targets

- I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
- When I know the edge length of a cube, I can find the volume and express it using appropriate units.

Lesson Narrative

In this lesson, students learn about perfect squares and perfect cubes. They see that these names come from the areas of squares and the volumes of cubes with whole-number side lengths. Students are introduced to **exponents** in this context.

Students learn to use the exponent ^{2} to express multiplication of two side lengths of a square and the exponent ^{3} to express multiplication of three edge lengths of a cube. They learn that the words **squared** and **cubed** can be used to describe expressions with exponents ^{2} and ^{3} and see the geometric motivation for this terminology. (The term "exponent" is deliberately not defined more generally at this time. Students will work with exponents in more depth in a later unit.)

Throughout the lesson, students attend to precision (MP6) as they think about the units for length, area, and volume. To write the formula for the volume of a cube, students also look for and express regularity in repeated reasoning (MP8).

Math Community
Today's activity is for students to individually reflect on the norms generated so far. During the *Cool-down*, students provide feedback on the norms, sharing those they agree with and those they feel need revision or removal. These suggestions will inform the next version of the classroom norms.

Standards

Building On	**4.MD.A.3, 5.MD.C.5.a**
Addressing	**6.EE.A, 6.EE.A.1**
Building Towards	**6.EE.A.1**

Instructional Routines

- 5 Practices
- MLR2: Collect and Display

Required Materials

Materials To Gather

- Math Community Chart: Activity 1
- Snap cubes: Activity 2

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Required Preparation

Activity 2:

Prepare sets of 32 snap cubes for each group of 2 students.

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's investigate perfect squares and perfect cubes.

17.1 Perfect Squares

Warm-up

5 mins

Activity Narrative

This activity introduces the concept of "perfect squares." It also includes opportunities to practice using units of measurement, which offers insights about students' knowledge from preceding lessons.

Some students may benefit from using physical tiles to reason about perfect squares. Provide access to square tiles, if available.

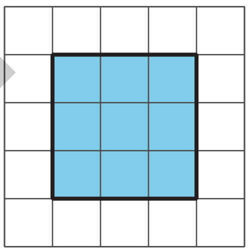
As students work, notice whether they use appropriate units for the questions about area.

Standards

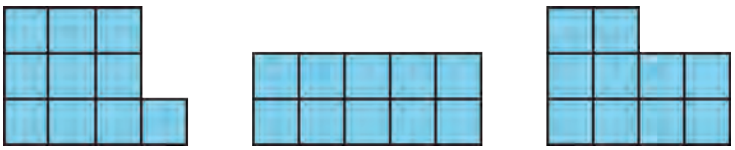
Building On **4.MD.A.3**

Launch

Tell students, "Some numbers are called 'perfect squares.' For example, 9 is a perfect square. Nine copies of a small square can be arranged into a large square." Display a square like this for all to see:



Explain that 10, however, is not a perfect square. Display images such as shown here, emphasizing that 10 small squares can not be arranged into a large square (the way 9 small squares can).



Tell students that in this warm-up they will find more numbers that are perfect squares. Give students 2 minutes of quiet think time to complete the activity.

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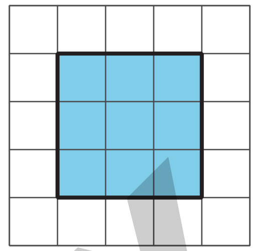
Student Task Statement



1. The number 9 is a "perfect square." Find four numbers that are perfect squares and two numbers that are not perfect squares.
2. A square has side length 7 in. What is its area?
3. The area of a square is 64 sq cm. What is its side length?

Student Response

1. Sample response: Perfect squares: 9, 25, 4, 49, 100. Not perfect squares: $\frac{1}{2}$, 2, 3, 10.
2. 49 square inches
3. 8 centimeters

Building on Student Thinking

If students do not recall what the abbreviations km, cm, and sq stand for, provide that information.

Students may divide 64 by 2 for the third question. If students are having trouble with this, ask them to check by working backward i.e., by multiplying the side lengths to see if the product yields the given area measure.

Activity Synthesis

Invite students to share the examples and non-examples they found for perfect squares. Solicit some ideas on how they decided if a number is or is not a perfect square.

If a student asks about 0 being a perfect square, wait until the end of the lesson, when the exponent notation is introduced. 0 *is* a perfect square because $0^2 = 0$.

Briefly discuss students' responses to the last two questions, the last one in particular. If not already uncovered in discussion, highlight that because the area of a square is found by multiplying side lengths to each other, finding the side lengths of a square with a known area means figuring out if that area measure is a product of two of the same number.

Math Community
Display the Math Community Chart. Remind students that norms are agreements that everyone in the class shares responsibility for, so it is important that everyone understands the intent of each norm and can agree with it. Tell students that today's *Cool-down* includes a question asking for feedback on the drafted norms. This feedback will help identify which norms the class currently agrees with and which norms need revising or removing.

17.2 Building with 32 Cubes

Optional

15 mins

Activity Narrative

There is a digital version of this activity.

SAMPLE ONLY

In this activity, students revisit the meaning of volume and how to find it by building the largest cube possible from 32 snap cubes. Students also become familiar with two perfect cubes, 27 and 64, before the next activity introduces this term.

The activity prompts students to recall their work from grade 5—that the volume of a rectangular prism can be calculated in two different ways: by counting unit cubes that can be packed into the prism, and by multiplying the edge lengths of the prism.

Monitor the different approaches students take to find the volume of the built cube. Here are some likely ways, from less efficient to more efficient:

- Counting all of the snap cubes individually
- Counting the number of snap cubes per layer and then multiplying that by the number of layers
- Multiplying three edge lengths

This activity works best when each student has access to snap cubes. If physical cubes are not available, consider using the digital version of the activity. In the digital version, students use an applet that has 32 cubes with which to build a prism. For students who finish early, another applet with 64 cubes can be found in *Are You Ready for More?*

Standards

Building On **5.MD.C.5.a**

Instructional Routines

- 5 Practices

Launch

Arrange students in groups of 2. Give 32 snap cubes to each group. If centimeter cubes are available, have students work in centimeters instead of the generic units listed here. Give students 8–10 minutes to build the largest cube they can and answer the questions.

For groups who finish early, consider asking them to combine their cubes and build the largest single cube they can with 64 cubes. Then, ask them to answer the same four questions as in the activity.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student approaches, especially from students who haven't shared recently.

Student Task Statement

Your teacher will give you 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Be prepared to explain your reasoning.
4. What is the volume of the built cube? Be prepared to explain your reasoning.

Student Response

1. 27
2. 3 units
3. 9 square units
4. 27 cubic units

SAMPLE ONLY

Building on Student Thinking

Students may neglect to write units for length or area and may need a reminder to do so.

When determining area, students may multiply a side by two instead of squaring it. When determining volume, they may multiply a side by three instead of cubing it. If this happens, ask them to count individual squares so that they can see that there is an error in their reasoning.

Are You Ready for More?

Combine your 32 snap cubes with another group's 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Show your reasoning.
4. What is the volume of the built cube? Show your reasoning.

Extension Student Response

1. 64 cubes
2. 4 units
3. 16 square units. Sample reasoning: $4 \cdot 4 = 16$
4. 64 cubic units. Sample reasoning: $4 \cdot 4 \cdot 4 = 16$

Activity Synthesis

Focus the whole-class discussion on the ways in which students found the volumes of the two cubes they built. Invite previously selected students to share their strategies. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*, starting from the less efficient (counting individual cubes) to the most efficient (multiplying side lengths).

Connect the last strategy to the learning goals by emphasizing how the side length of a cube determines its volume. Specifically, highlight that the number 27 is $3 \cdot 3 \cdot 3$. If any group built a cube with 64 snap cubes, point out that the number 64 is $4 \cdot 4 \cdot 4$. These observations prepare students to think about perfect cubes in the next activity and about a general expression for the volume of a cube later in the lesson.

10 mins

17.3 Perfect Cubes

Activity Narrative

In this activity, students think about examples and non-examples of perfect cubes and find the volumes of cubes given their edge lengths. They see that the edge length of a cube determines its volume, notice the numerical expressions that can be written when calculating volumes, and write a general expression for finding the volume of a cube (MP8).

Some students may be unsure about writing the answer to the last question symbolically because it involves a variable.

SAMPLE ONLY

Those students may prefer to write a verbal explanation. This is fine, because in an upcoming lesson they will learn to use exponential notation with numbers and variables.

Access for English Language Learners

This activity uses the Collect and Display math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

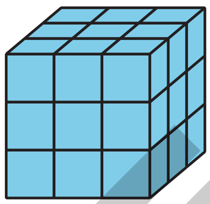
Building On	**5.MD.C.5.a**
Addressing	**6.EE.A**
Building Towards	**6.EE.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Tell students, "Some numbers are called 'perfect cubes.' For example, 27 is a perfect cube." Display a cube like this for all to see:



Arrange students in groups of 2. Give students a few minutes of quiet think time, and another minute to discuss their responses with their partner.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe how they reason about the volume of a cube. Display phrases such as: "a number times itself and times itself again," "multiply a number by itself twice," "multiply three of the same number," "multiply three edge lengths," "a perfect square times the side length of the square," "perfect cube," and "not a perfect cube." Also collect the language that students use to describe how they know if a number is a perfect cube. Display phrases such as: "I can't make a cube that has a volume of _____" or "There is no number I can multiply by itself twice to get _____."

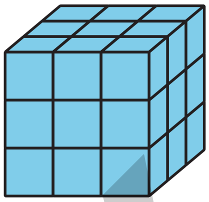
Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to snap cubes or blocks, or dot paper to help students analyze examples and non-examples of perfect cubes, paying close attention to the relationships between side lengths and volume..
Supports accessibility for: Visual-Spatial Processing, Organization

SAMPLE ONLY

Student Task Statement

1. The number 27 is a "perfect cube." Find four other numbers that are perfect cubes and two numbers that are *not* perfect cubes.
2. A cube has a side length of 4 cm. What is its volume?
3. A cube has a side length of 10 inches. What is its volume?
4. A cube has a side length of s units. What is its volume?



Student Response

1. Sample responses: Cubes: 1, 8, 64, 125, 216, 1,000. Non-cubes: 2, 3, 4.
2. $4 \cdot 4 \cdot 4$ or 64 cubic cm
3. $10 \cdot 10 \cdot 10$ or 1,000 cubic inches
4. $s \cdot s \cdot s$ cubic units

Building on Student Thinking

Watch for students who use square units instead of cubic units. Remind them that volume is a measure of the space inside the cube and is measured in cubic units.

Students may multiply by 3 when finding the volume of a cube instead of multiplying three edge lengths (which happen to be the same number). Likewise, they may think a perfect cube is a number times 3. Suggest that they sketch or build a cube with that edge length and count the number of unit cubes. Or ask them to think about how to find the volume of a prism when the edge lengths are different (for instance, a prism that is 1 unit by 2 units by 3 units).

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they thought about the first question and decided if a number is or is not a perfect cube. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Highlight the idea that multiplying three edge lengths allows us to determine volume efficiently, and that determining if a number is a perfect cube involves thinking about whether it is a product of three of the same number.

If a student asks about 0 being a perfect cube, wait until the end of the lesson, when exponent notation is introduced. 0 *is* a perfect cube because $0^3 = 0$.

Make sure that students see the answers to the last three questions written as expressions:

$$4 \cdot 4 \cdot 4$$

,

$$10 \cdot 10 \cdot 10$$

, and

$$s \cdot s \cdot s$$

SAMPLE ONLY

Sec E

17.4 Introducing Exponents

15 mins

Activity Narrative

This activity introduces students to the exponents "2" and "3" and the language that we use to talk about them. Students use and interpret this notation in the context of both geometric squares and their areas, and geometric cubes and their volumes. Students are likely to have seen exponent notation for 10^3 in their work on place values in grade 5. That experience would be helpful but is not necessary.

Note that the term “exponent” is deliberately *not* defined more generally at this time. Students will work with exponents in more depth in a later unit.

As students work, observe how they approach the last two questions. Identify a couple of students who approach the fourth question differently so they can share later. Also notice whether students include appropriate units, written using exponents, in their answers.

Standards

Addressing **6.EE.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Ask students if they have seen an expression such as 10^3 before. Tell students that in this expression, the 3 is called an **exponent**. Explain the use of the exponents "2" and "3":

- “When we multiply two of the same number together, such as $5 \cdot 5$, we say that we are ‘squaring’ the number and can write the expression as 5^2. The raised 2 in 5^2 is called an ‘exponent.’”
- “Because 5^2 is 25, we can write $5^2 = 25$ and say, ‘5 **squared** is 25. We can also say that 25 is a ‘perfect square.’”
- “When we multiply three of the same number together, such as $4 \cdot 4 \cdot 4$, we say that we are 'cubing' the number. We can write it as 4^3. The raised 3 in 4^3 is called an ‘exponent.’”
- “Because 4^3 is 64, we can write $4^3 = 64$ and say, ‘4 **cubed** is 64.’ We also say that 64 is a ‘perfect cube.’”

Explain that we can also use exponents as a shorthand for the units used for area and volume:

- A square with side length 5 inches has an area of 25 square inches, which we can write as 25 in^2.
- A cube with an edge length 4 centimeters has a volume of 64 cubic centimeters, which we can write as 64 cm^3.

Ask students to read a few areas and volumes in different units (for instance, 100 ft^2 is read “100 square feet” and 125 yd^3 is read “125 cubic yards”).
Keep students in groups of 2. Give students 3–4 minutes of quiet time to complete the activity and a minute to discuss their response with their partner. Ask partners to note any disagreements so they can be discussed.

Use Collect and Display to direct attention to words collected and displayed from an earlier activity. Collect the language students use when discussing their responses to the questions about volume. Display words and phrases such as: “perfect square,” “squaring” or “squared,” "an exponent of 2,” “perfect cube,” “cubing _____,” "_____ cubed is ____,” “an exponent of 3.”

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Student Task Statement

Make sure to include correct units of measure as part of each answer.

1. A square has side length 10 cm. Use an **exponent** to express its area.
2. The area of a square is 7^2 sq in. What is its side length?
3. The area of a square is 81 m^2. Use an exponent to express this area.
4. A cube has edge length 5 in. Use an exponent to express its volume.
5. The volume of a cube is 6^3 cm^3. What is its edge length?
6. A cube has edge length s units. Use an exponent to write an expression for its volume.

Student Response

1. 10^2 cm^2
2. 7 inches
3. 9^2 m^2
4. 5^3 in^3
5. 6 cm
6. s^3 $units^3$ or s^3 cubic units

Building on Student Thinking

Upon seeing the expression 6^3, some students may neglect to interpret the question, automatically calculate, and conclude that the edge length is 216 cm. Ask them to check their answer by finding the volume of a cube with edge length 216 cm.

Are You Ready for More?

The number 15,625 is both a perfect square and a perfect cube. It is a perfect square because it equals 125^2. It is also a perfect cube because it equals 25^3. Find another number that is both a perfect square and a perfect cube. How many of these can you find?

Extension Student Response

The smallest examples are 0, 1, 64, 729, and 4,096.

Activity Synthesis

Direct students' attention to the reference created using Collect and Display. Ask partners to share disagreements in their responses, if any. Then, focus the whole-class discussion on the last two questions. Select a couple of previously identified students to share their interpretations of the question about a cube with an edge length of 5 inches. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Highlight that a cube with a volume of 6^3 cubic units has an edge length of 6 units, because we know there are $6 \cdot 6 \cdot 6$ unit cubes in a cube with that edge length.

SAMPLE ONLY

In other words, we can express the volume of a cube using a number (216), a product of three numbers ($6 \cdot 6 \cdot 6$), or an expression that uses exponent (6^3). This idea can be extended to all cubes. The volume of a cube with edge length s is:

$$s \cdot s \cdot s$$

, or

$$s^3$$

. Students will have more opportunities to generalize the expressions for the volume of a cube in the next lesson.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Use this routine before the whole-class discussion of the last two questions. Display an incorrect response for students to consider. For example, "If the volume of a cube is 6^3 cm^3, then the edge length is 216 cm because $6 \cdot 6 \cdot 6$ is 216." Ask students to identify the error, critique the reasoning, and write a correct explanation. Listen for students who clarify that the volume of a cube can be represented as an exponent or a value and that neither of these represent the edge length. Invite students to share their critiques and corrected explanations with the class. Listen for and amplify the language that students use to describe ways to generalize the relationship between the three representations of volume: a number, a product of three numbers, or an expression with an exponent. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they clarify their understanding of exponents. *Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

Access for Students with Disabilities

Representation: Develop Language and Symbols. Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of exponents in the context of geometry. Terms may include: "squared," "cubed," "exponent," and "base."

Lesson Synthesis

Review the language and notation for squaring and cubing a number. Remind students that we use this notation for square units and cubic units, too.

- When we multiply two of the same number, such as $10 \cdot 10$, we say that we are "squaring" the number. We write, for example, $10^2 = 100$, and say, "Ten squared is one hundred."
- When we multiply three of the same number, such as $10 \cdot 10 \cdot 10$, we say that we are "cubing" the number. We write, for example, $10^3 = 1{,}000$, and say, "Ten cubed is one thousand."
- Exponents are used to write square units and cubic units. The area of a square with a side length of 7 km is 49 km^2. The volume of a cube with an edge length of 2 millimeters is 8 mm^3.

A note about materials for an upcoming unit:

For the first lesson in the unit on ratios, students will need to bring in a personal collection of 10–50 small objects. Examples include rocks, seashells, trading cards, or coins. Inform or remind students about this.

SAMPLE ONLY

17.5 Exponent Expressions

Cool-down

5 mins

Standards

Addressing **6.EE.A.1**

Launch

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- What norm(s) should stay the way they are?
- What norm(s) do you think should be made more clear? How?
- What norms are missing that you would add?
- What norm(s) should be removed?

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet. Make sure students know they can make suggestions for both student and teacher norms.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about.

Student Task Statement

1. Which is larger, 5^2 or 3^3?
2. A cube has an edge length of 21 cm. Use an exponent to express its volume.

Student Response

1. $3^3 = 27$ and $5^2 = 25$, so 3^3 is larger than 5^2.
2. 21^3 cm^3 or 21^3 cubic centimeters

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 17 Summary

When we multiply two of the same numbers together, such as $5 \cdot 5$, we say that we are *squaring* the number. We can write it like this:

$$5^2$$

The raised 2 in 5^2 is called an **exponent**.

SAMPLE ONLY

Because $5 \cdot 5 = 25$, we write $5^2 = 25$ and we say, "5 **squared** is 25."

When we multiply three of the same numbers together, such as $4 \cdot 4 \cdot 4$, we say that we are *cubing* the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4 = 64$, we write $4^3 = 64$ and we say, "4 **cubed** is 64."

We also use an exponent for square units and cubic units.

- A square with a side length of 5 inches has an area of 25 in^2.
- A cube with an edge length of 4 cm has a volume of 64 cm^3.

To read 25 in^2, we say "25 square inches," just like before.

The area of a square with a side length of 7 kilometers is 7^2 km^2. The volume of a cube with an edge length of 2 millimeters is 2^3 mm^3.

In general, the area of a square with a side length of s is s^2, and the volume of a cube with an edge length of s is s^3.

Glossary

- cubed
- exponent
- squared

Sec E

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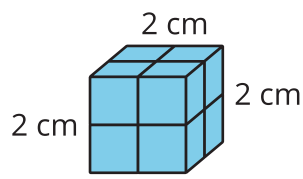
Practice Problems

1 Student Task Statement

What is the volume of this cube?



Solution

8 cu cm $(2 \cdot 2 \cdot 2 = 8)$

2 Student Task Statement

a. Decide if each number on the list is a perfect square.

16	125
20	144
25	225
100	10,000

b. Write a sentence that explains your reasoning.

Solution

a. All of these numbers, except 20 and 125, are perfect squares.
b. Sample response: Perfect squares can be found by multiplying a whole number by itself.

3 Student Task Statement

a. Decide if each number on the list is a perfect cube.

1	27
3	64
8	100
9	125

b. Explain what a perfect cube is.

SAMPLE ONLY

Solution

a. All of the numbers except 3, 9, and 100 are perfect cubes.

b. Sample response: Perfect cubes can be found by using a whole number as a factor three times.

4

Student Task Statement

a. A square has a side length of 4 cm. What is its area?

b. The area of a square is 49 m^2. What is its side length?

c. A cube has an edge length of 3 in. What is its volume?

Solution

a. 16 cm^2

b. 7 m

c. 27 in^3

5 from Unit 1, Lesson 16

Student Task Statement

Prism A and Prism B are rectangular prisms.

- Prism A is 3 inches by 2 inches by 1 inch.
- Prism B is 1 inch by 1 inch by 6 inches.

Select **all** statements that are true about the two prisms.

A. They have the same volume.

B. They have the same number of faces.

C. More inch cubes can be packed into Prism A than into Prism B.

D. The two prisms have the same surface area.

E. The surface area of Prism B is greater than that of Prism A.

Solution

A, B, E

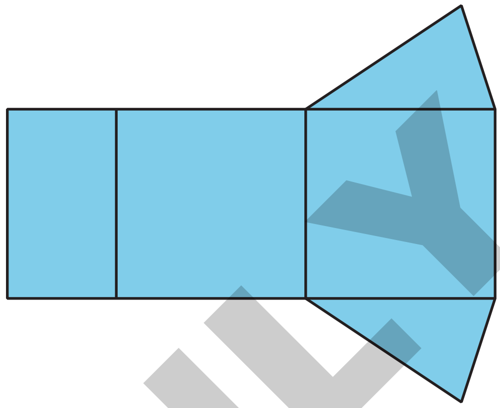
SAMPLE ONLY

6 from Unit 1, Lesson 14

Student Task Statement

a. What polyhedron can be assembled from this net?

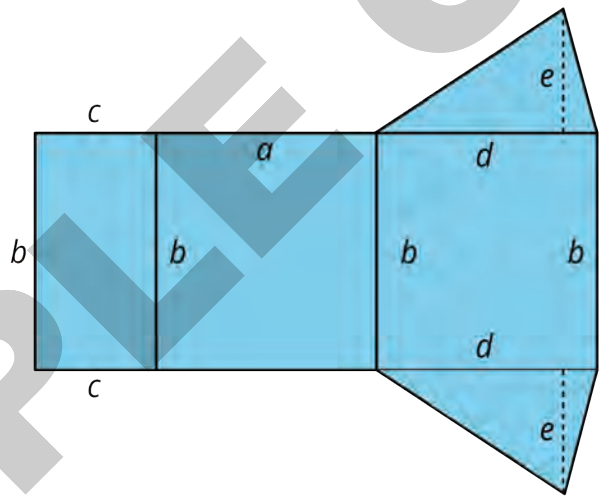
b. What information would you need to find its surface area? Be specific, and label the diagram as needed.



Solution

a. Triangular prism

b. Length and width of each rectangular face (as shown in the diagram), as well as the height of the triangular faces



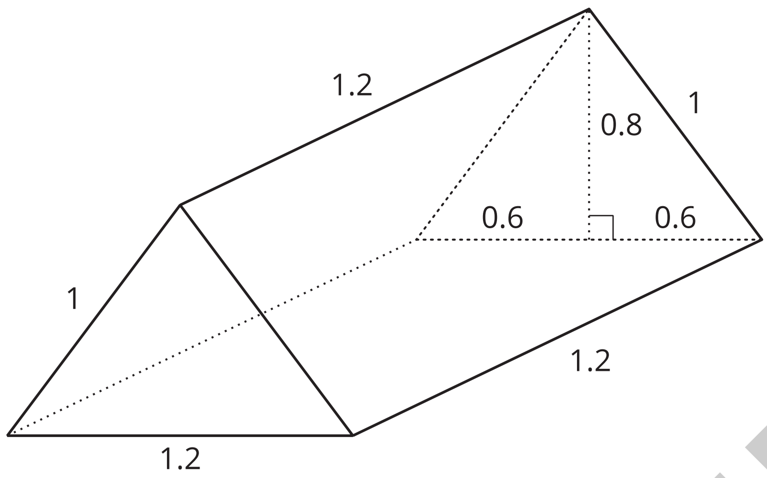
7 from Unit 1, Lesson 15

Student Task Statement

Find the surface area of this triangular prism. All measurements are in meters.

SAMPLE ONLY

Sec E



Solution

4.8 square meters. Sample reasoning:

- There are two triangular faces with an area of 0.48 square meters each. $\frac{1}{2} \cdot (1.2) \cdot (0.8) = 0.48$
- There are two rectangular faces with area of 1.2 square meters each. $1 \cdot (1.2) = 1.2$
- There is one rectangular face with an area of $(1.2) \cdot (1.2) = 1.44$ square meters.
- $2 \cdot (0.48) + 2 \cdot (1.2) + (1.44) = 4.8$, or 4.8 square meters.

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Unit 1, Lesson 18

Surface Area of a Cube

Goals

- Generalize a process for finding the surface area of a cube, and justify (orally) why this can be abstracted as $6 \cdot s^2$.
- Interpret (orally) expressions that include repeated addition, multiplication, repeated multiplication, or exponents.
- Write expressions, with or without exponents, to represent the surface area of a given cube.

Learning Targets

- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson Narrative

In this lesson, students practice using the exponents "2" and "3 to express products that describe surface areas and volumes of cubes. They also use exponents to write square units and cubic units.

Students begin by writing numerical expressions to represent the surface areas and volumes of cubes with whole-number side lengths. Then, they generalize their observations to write variable expressions for the surface area and volume of any cube.

As they write numerical expressions, students practice looking for and making use of structure (MP7). They also practice looking for and expressing regularity in repeated reasoning (MP8) to write formulas for the surface area and the volume of a cube.

Math Community
The goal of today's exercise is to use the suggestions from the previous exercise to revise the "Norms" sections of the Math Community Chart and to invite students to reflect on one norm that will be a strength for them. Both activities begin to build shared accountability for and investment in the classroom norms.

Standards

Addressing	**6.EE.A.1, 6.EE.A.2.a, 6.G.A.4**
Building Towards	**6.EE.A.2.b**

Instructional Routines

- 5 Practices
- Math Talk
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Math Community Chart: Activity 1
- Geometry toolkits: Activity 2, Activity 3

Student Facing Learning Goals

Let's write a formula to find the surface area of a cube.

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18.1 Math Talk: Expressions and Their Values

5 mins

Warm-up

Activity Narrative

This Math Talk focuses on the meaning of numbers and symbols in expressions. It encourages students to relate repeated addition to multiplication and to relate repeated multiplication to exponents. The numbers are selected to discourage students from computing the values of the expressions. Instead, they prompt students to rely on what they know about operations and exponents to mentally make comparisons. The understanding elicited here will be helpful later in the lesson when students write expressions to represent the surface area and volume of cubes.

To decide, without calculations, which of two expressions has the greater value, students need to look for and make use of structure (MP7). In explaining their reasoning, students need to be precise in their word choice and use of language (MP6).

Standards

Addressing **6.EE.A.1**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Decide mentally which expression has a greater value.

- $12 + 12 + 12 + 12 + 12$ or $4 \cdot 12$
- $15 \cdot 3$ or 15^3
- 19^2 or $18 \cdot 18$
- $5 \cdot 21^2$ or $(5 \cdot 21) \cdot (5 \cdot 21)$

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Student Response

- $12 + 12 + 12 + 12 + 12$. Sample reasoning: This expression is equivalent to $5 \cdot 12$, which is greater than $4 \cdot 12$.
- 15^3. Sample reasoning: 15^3 is $15 \cdot 15 \cdot 15$, so it is greater than $15 \cdot 3$, which is $15 + 15 + 15$.
- 19^2. Sample reasoning: 19^2 is $19 \cdot 19$, which is greater than $18 \cdot 18$.
- $(5 \cdot 21) \cdot (5 \cdot 21)$. Sample reasoning: $5 \cdot 21$ is more than 100, so multiplying this number by itself will give a number greater than 10,000. Squaring 21 gives a number that is a little more than 400. Multiplying that number by 5 gives a product that is more than 2,000 but not anywhere near 10,000.

Building on Student Thinking

When given an expression with an exponent, students may misinterpret the base and the exponent as factors and multiply the two numbers. Remind them about the meaning of the exponent notation. For example, show that $5 \cdot 3 =$ 15, which is much smaller than $5 \cdot 5 \cdot 5$, which equals 125.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

To support students in upcoming work, highlight the following ideas if they are not already mentioned by students:

- We can express repeated addition with multiplication. $5 \cdot 12$ is a more concise way to write $12 + 12 + 12 + 12 + 12$.
- We can express repeated multiplication with an exponent. 15^3 is a more concise way to write $15 \cdot 15 \cdot 15$.
- The parentheses in the last expression tells us that it is the value of $5 \cdot 21$, not just one of the numbers, that is being squared.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Math Community
Display the Math Community Chart and a list of 2–5 revisions suggested by the class in the previous exercise for all to see. Remind students that norms are agreements that everyone in the class shares responsibility for, so everyone needs to understand and agree to work on upholding the norms. Briefly discuss any revisions and make changes to the "Norms" sections of the chart as the class agrees. Depending on the level of agreement or disagreement, it may not be possible to discuss all suggested revisions at this time. If that happens, plan to discuss the remaining suggestions over the next few lessons.

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Tell students that the class now has an initial list of norms or "hopes" for how the classroom math community will work together throughout the school year. This list is just a start, and over the year it will be revised and improved as students in the class learn more about each other and about themselves and math learners.

18.2 The Net of a Cube

20 mins

Activity Narrative

This activity serves two goals. One goal is to allow students to practice drawing a net and finding the surface area and volume of a cube. The other goal is to encourage students to write expressions to represent the surface area and volume of a cube.

In the first set of questions, the edge length of the cube is a small number (5), enabling students to compute the surface area and volume numerically. Use students' work here to check that they are drawing a net correctly. In the next set of questions, the edge length of the cube is a larger number (17), making computation more cumbersome and prompting students to build expressions rather than evaluating them.

As students work on the questions about a cube with a 17-inch edge length, monitor for students who write different expressions for the second question. Here are some expressions they may write, from longer (more expanded) to shorter (more succinct):

- Products, such as $17 \cdot 17$, or $17 \cdot 17 \cdot 17$
- Sums of products, such as $(17 \cdot 17) + (17 \cdot 17) + \ldots$
- Combination of like terms, such as $6 \cdot (17 \cdot 17)$
- Exponents, such as $17^2 + 17^2 + \ldots$, or 17^3
- Completed calculations, such as 1,734 or 4,913

A note about notation:
In a later unit, students will learn that $5 \cdot x$ means the same as $5x$. At this point, expect them to write $6 \cdot 17^2$ instead of $6(17^2)$. It is not critical that they understand that a number placed next to a variable (or a number placed next to an expression in parentheses) are being multiplied.

Standards

Addressing	**6.EE.A.1, 6.G.A.4**
Building Towards	**6.EE.A.2.b**

Instructional Routines

- 5 Practices

Launch

Arrange students in groups of 2. Give students access to their geometry toolkits. Tell students to use graph paper to draw a net of the first cube and to try to answer the questions without using a calculator. Give students 8–10 minutes of quiet work time followed by 1–2 minutes to share their responses with their partner.

Select students with different expressions, such as those described in the activity narrative, to share later.

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Access for Students with Disabilities

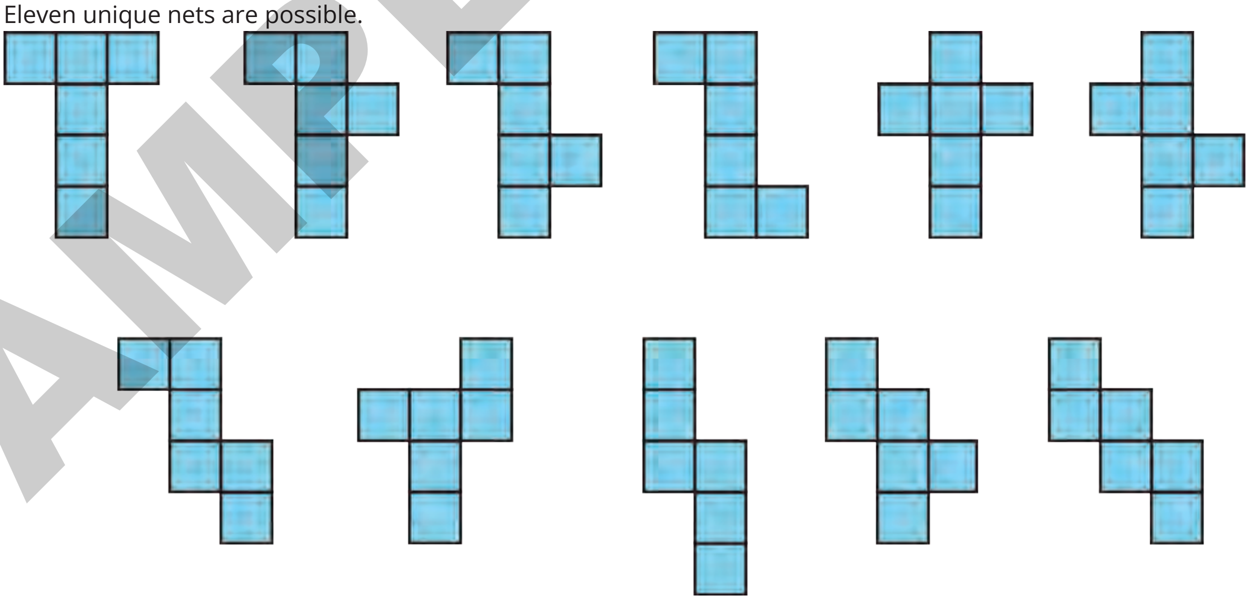
Representation: Develop Language and Symbols. Activate or supply background knowledge. To help students recall the meanings of the terms “net,” “surface area,” and “volume,” ask, “How is a net related to a three-dimensional shape?”, “What parts of a shape contribute to its surface area?”, or “How are volume and surface area different?”
Supports accessibility for: Memory, Language

Student Task Statement

1. A cube has an edge length of 5 units.
    a. Draw a net for this cube on graph paper. Label its sides with measurements.
    b. What is the shape of each face?
    c. What is the area of each face?
    d. What is the surface area of this cube?
    e. What is the volume of this cube?
2. A second cube has an edge length of 17 units.
    a. Sketch a net for this cube. Label its sides with measurements.
    b. Explain why the area of each face of this cube is 17^2 square units.
    c. Write an expression for the surface area, in square units.
    d. Write an expression for the volume, in cubic units.

Student Response

1. For the cube that has an edge length of 5:
    a. Eleven unique nets are possible.



    b. A square
    c. 25 square units

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d. 150 square units

e. 125 cubic units

2. For the cube that has an edge length of 17:

 a. Drawing should be one of the 11 nets shown in the previous problem.

 b. Sample response: The side length of each square face is 17 units, so its area is $17 \cdot 17$, or 17^2 square units.

 c. $6 \cdot 17^2$ (or equivalent)

 d. 17^3 (or equivalent)

Building on Student Thinking

Students might think the surface area is $(17 \cdot 17)^6$. Prompt students to write down how they would compute surface area step by step, before trying to encapsulate their steps in an expression. Dissuade students from using calculators in the last two problems and assure them that building an expression does not require extensive computation.

Students may think that refraining from using a calculator means performing all calculations—including those of larger numbers—on paper or mentally, especially if they are unclear about the meaning of the term "expression." Ask them to refer to the expressions in the *Warm-up*, or share examples of expressions in a few different forms, to help them see how surface area and volume can be expressed without computation.

Activity Synthesis

The purpose of this discussion is to emphasize that exponents can be used to concisely express calculations for surface area and volume. Briefly discuss the answers to the questions about the cube with a 5-inch edge length. Then, focus the discussion on the larger cube with a 17-inch edge length.

Invite previously selected students to share their expressions for the last two questions. Sequence the responses in the following order to help students see how the expressions $6 \cdot 17^2$ and 17^3 come about. If any expressions are missing but are needed to illustrate the idea of writing succinct expressions, add them to the lists. Refer to parts of expressions using terms such as "sum," "product," and "factor" to support students in using mathematical terms when working with expressions.

Surface area:

- $(17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17)$
- $17^2 + 17^2 + 17^2 + 17^2 + 17^2 + 17^2$
- $6 \cdot (17 \cdot 17)$
- $6 \cdot (17^2)$
- $6 \cdot (289)$
- 1,734

Volume:

- $17 \cdot 17 \cdot 17$
- 17^3
- 4,913

Connect the expressions to the learning goals by asking questions such as:

- "In each expression, where do you see the area of one face of the cube?" ($17 \cdot 17$, 17^2, 289)

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- “In one expression, the number 17 is written twelve times. In another expression, it is written six times. How do they both represent the same surface area?” (They both show the sum of the areas of the six square faces. The second expression uses the exponent 2 to represent the product of two 17s.)
- “Some expressions show addition but others don’t. How do they all represent the surface area?” (Multiplication is used for repeated addition. Adding six of the same expression is the same as multiplying it by 6.)
- “Which expression doesn't require a lot of computation and is the most efficient way to represent the surface area of the cube?” ($6 \cdot 17^2$)
- “Which expression doesn't require a lot of computation and is the most efficient way to represent the volume of the cube?” (17^3)

To further encourage students to see regularity in the expressions, consider offering another example, for instance: “Suppose the edge length of a cube is 38 cm. How can we express its surface area and volume?” ($6 \cdot (38^2)$ cm^2 and 38^3 cm^3, respectively) Using a large number for the edge length will discourage calculation and prompt students to focus on building an expression and using exponents.

If time permits, consider directing students’ attention to the units of measurements. Remind students that, rather than writing $6 \cdot (17^2)$ square units, we can write $6 \cdot (17^2)$ units^2, and instead of 17^3 cubic units, we can write 17^3 units^3. Unit notations will appear again later in the course, so it can also be reinforced later.

10 mins

18.3 Every Cube in the Whole World

Activity Narrative

In this activity, students develop the formulas for the surface area and the volume of a cube in terms of a variable edge length s.

Students should be encouraged to refer to their work in the preceding activity as much as possible and to generalize from it. As before, monitor for different ways of writing expressions for surface area and volume. The expressions are likely to be in the following forms, listed from longer (more expanded) to shorter (more succinct):

- Products, such as $s \cdot s$ or $s \cdot s \cdot s$
- Sums of products, such as $(s \cdot s) + (s \cdot s) + \ldots$
- Combination of like terms, such as $6 \cdot (s \cdot s)$
- Expressions with exponents, such as $s^2 + s^2 + \ldots$, or s^3

As they write and discuss expressions for volume and surface area, students practice looking for and making use of structure (MP7) and seeing regularity through repeated reasoning (MP8).

Standards

Addressing	**6.EE.A.2.a, 6.G.A.4**
Building Towards	**6.EE.A.2.b**

Instructional Routines

- 5 Practices

Launch

Give students access to their geometry toolkits and 7–8 minutes of quiet think time. Tell students that they will be

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answering the same questions as before, but with a variable for the side length. Encourage them to use the work they did earlier to help them here.

Select students who wrote different expressions to share their work later.

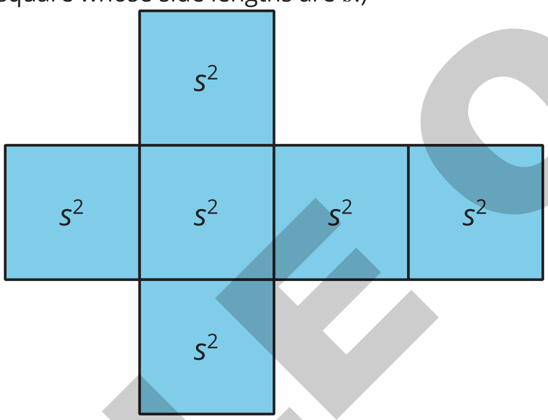
Student Task Statement

A cube has an edge length of s.

1. Draw a net for the cube.
2. Write an expression for the area of each face. Label each face with its area.
3. Write an expression for the surface area.
4. Write an expression for the volume.

Student Response

1. Sample response: (Each face is a square whose side lengths are s.)



2. The area of each face is s^2.
3. The surface area is $6 \cdot s^2$.
4. The volume is s^3.

Building on Student Thinking

If students are unclear or unsure about using the variable s, explain that we are looking for an expression that would work for any edge length, and that a variable, such as s, can represent any number. The s could be replaced with any edge length in finding surface area and volume.

To connect students' work to earlier examples, point to the cube with edge length 17 units from the previous activity. Ask: "If you wrote the surface area as $6 \cdot 17^2$ before, what should it be now?"

As students work, encourage those who may be more comfortable using multiplication symbols to instead use exponents whenever possible.

Activity Synthesis

To support students in seeing structure and generalizing, discuss the problems in as similar a fashion as was done in the earlier activity involving a cube with an edge length of 17 units.

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Ask previously selected students to share their responses with the class. If possible, sequence the responses in the following order (as shown in the activity narrative) to help students see how the expressions $6 \cdot s^2$ and s^3 come about. If any expressions are missing but are needed to illustrate the idea of writing succinct expressions, add them to the lists. Refer to parts of expressions using terms such as “sum,” “product,” and “factor” to support students in using mathematical terms when working with expressions.

Surface area:

- $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$
- $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$
- $6(s \cdot s)$
- $6 \cdot (s^2)$ or $6 \cdot s^2$

Volume

- $s \cdot s \cdot s$
- s^3

If students have trouble understanding where the most concise expression of surface area comes from, refer back to the example involving a numerical side length (a cube with an edge length of 17 units).

Present the surface area as $6 \cdot s^2$.

Lesson Synthesis

In this lesson, students wrote expressions for the volume and surface area of a cube. Consider asking students:

- "A cube has an edge length of s. Why does the expression s^3 describe its volume?" (The volume is $s \cdot s \cdot s$, which can be written as s^3
- "Why does the expression $6 \cdot s^2$ describe its surface area?" (A cube has 6 faces that are all identical squares. Each face has an area of $s \cdot s$ or s^2, so the total area for all the faces is 6 times s^2.).

A note about materials for an upcoming unit:

For the first lesson on the unit on ratios, students will need to bring in a personal collection of 10–50 small objects. Examples include rocks, seashells, trading cards, or coins. Inform or remind students about this.

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18.4 From Volume to Surface Area

Cool-down

5 mins

Standards

Addressing **6.EE.A.1, 6.G.A.4**

Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the question “What is one of our classroom norms that is a strength for you? Why?” Tell students that as a culmination to establishing the initial list of mathematical

community norms, they are now asked to share one norm they think will be a strength for them. To help students understand what the question is asking, share a personal example. For example, "I think that 'Ask clarifying questions' is a norm that is a strength for me because I am good at asking questions when I don't think I understand how someone else is thinking about a problem. Instead of just telling you what I think you should do, I make sure to ask questions until I understand what YOU are doing."

Display these prompts for all to see:

- One of our classroom norms that will be a strength for me is ________.
- I think this will be a strength for me because ________.

Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify which norms students feel more confident about and which norms were not listed as strengths by many students. In some cases, students may not think a norm is a strength because they are not sure what that norm looks like or sounds like. So, focus on identifying those norms in the class when they happen.

For example, during group work students ask a quiet group member which representation they prefer, and that student shares a third representation that the group had not even considered. Asking the quiet student illustrates a norm like "we invite others into the math." Pointing out that action when it happens helps students understand the norm and see how it can benefit the math thinking of the entire group. This understanding and appreciation can promote the use of that norm in the math community.

Student Task Statement

1. A cube has an edge length of 11 inches. Write an expression for its volume and an expression for its surface area.
2. A cube has a volume of 7^3 cubic centimeters. What is its surface area?

Student Response

1. Volume: 11^3 or $11 \cdot 11 \cdot 11$. Surface area: $6 \cdot (11 \cdot 11)$ (or equivalent).
2. 294 square centimeters. $6 \cdot 7^2 = 294$

Sec E

Responding To Student Thinking

Points to Emphasize
If students struggle with interpreting or writing expressions for the surface area or the volume of a cube, as opportunities arise, highlight the distinctions between these geometric attributes and ways to quantify them. For example, encourage students to draw a sketch, label the edge lengths, and write expressions for the volume and surface area of each cube described in this practice problem:

Grade 6, Unit 1, Lesson 18, Practice Problem 1

Lesson 18 Summary

The volume of a cube with an edge length of s is s^3.

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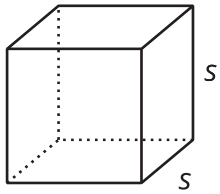
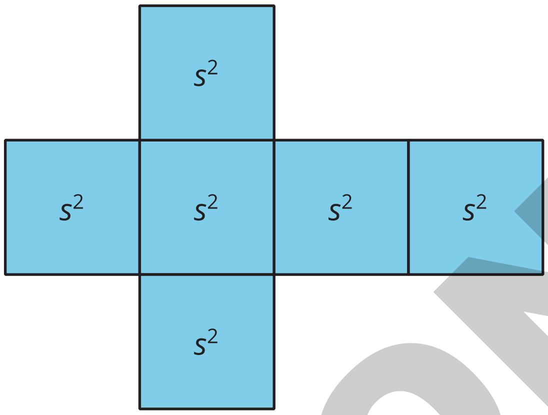


A cube has 6 faces that are all identical squares. For a cube with an edge length of s, the area of each square face is s^2. This means that the surface area of the cube is $6 \cdot s^2$.



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Sec E

Practice Problems

1 Student Task Statement

a. What is the volume of a cube with an edge length of 8 in?

b. What is the volume of a cube with an edge length of $\frac{1}{3}$ cm?

c. A cube has a volume of 8 ft^3. What is its edge length?

Solution

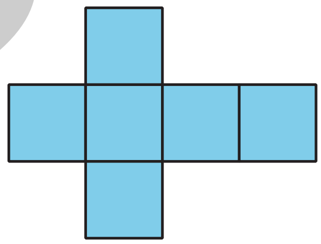
a. 512 cu in ($8 \cdot 8 \cdot 8 = 512$)

b. $\frac{1}{27}$ cu cm ($\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$)

c. 2 ft ($2 \cdot 2 \cdot 2 = 8$)

2 Student Task Statement

What three-dimensional figure can be assembled from this net?



If each square has a side length of 61 cm, write an expression for the surface area and another for the volume of the figure.

Solution

a. Cube

b. Surface area: $6 \cdot 61^2$ sq cm. Volume: 61^3 cu cm

3 Student Task Statement

a. Draw a net for a cube with an edge length of x cm.

b. What is the surface area of this cube?

c. What is the volume of this cube?

Solution

a.

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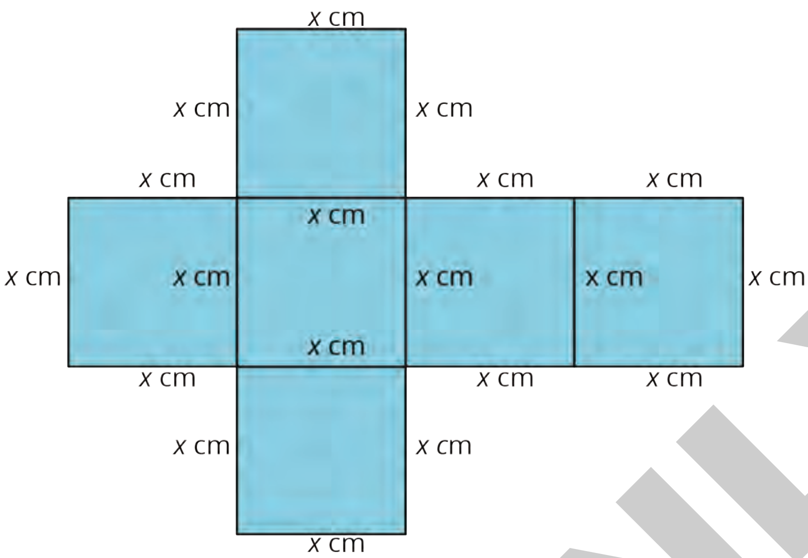


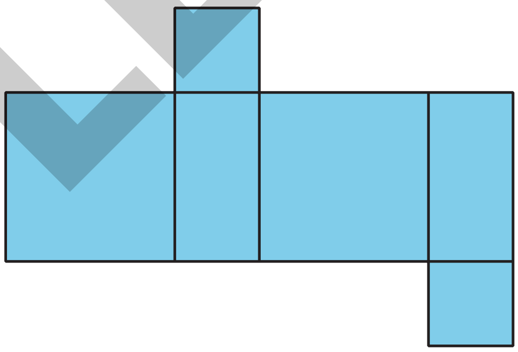
b. $6x^2$ sq cm (or equivalent)

c. $x \cdot x \cdot x$ cu cm (or equivalent)

4 from Unit 1, Lesson 14

Student Task Statement

Here is a net for a rectangular prism that was not drawn accurately.



a. Explain what is wrong with the net.

b. Draw a net that can be assembled into a rectangular prism.

c. Create another net for the same prism.

Solution

a. When the shape is folded, the two small squares are not the right size to close the three-dimensional figure. The small squares can be replaced with rectangles as in the picture, or the large squares can be the same size and shape as the two (non-square) rectangles in the net.

b. Sample response:

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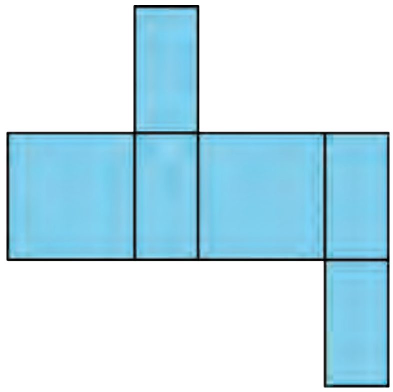
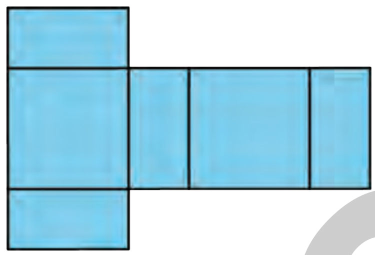
Sec E



c. Sample response:

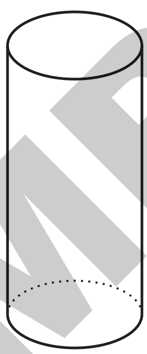


5 from Unit 1, Lesson 13

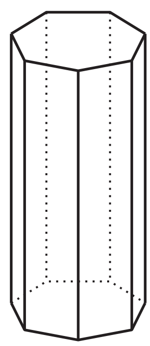
Student Task Statement

State whether each figure is a polyhedron. Explain how you know.

A



B



Solution

Figure A is not a polyhedron. Sample reasoning: It has a curved surface and there are faces that are not polygons.

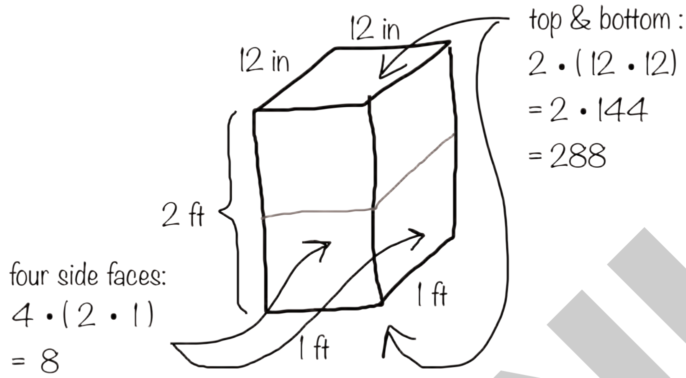
Figure B is a polyhedron. Sample reasoning: It is composed of polygons and each side of every polygon joins a side of another polygon.

SAMPLE ONLY

6 from Unit 1, Lesson 12

Student Task Statement

Here is Elena's work for finding the surface area of a rectangular prism that is 1 foot by 1 foot by 2 feet.



She concluded that the surface area of the prism is 296 square feet. Do you agree with her? Explain your reasoning.

Solution

Disagree. Sample reasoning: Elena calculated the area of the top and bottom faces in square inches but the area of the side faces in square feet. The combined area of the top and bottom faces is 2 square feet, so the correct surface area is 10 square feet.

SAMPLE ONLY

Sec E

Section F: Let's Put it to Work

Section Narrative

In this final section, students have the opportunity to apply their thinking from throughout the unit. As this is a short section followed by an End-of-Unit Assessment, there are no section goals or checkpoint questions. The lesson in this section is optional because it offers additional opportunities to practice standards that are not a focus of the grade.

Teacher Reflection Questions

- **Math Content and Student Thinking:** Reflect on work that revealed the depth of students' understanding about area and surface area. Where did you see evidence of flexible reasoning, structural thinking, or deep understanding?

SAMPLE ONLY

All about Tents

Goals

- Apply understanding of surface area to estimate the amount of material in a tent, and explain (orally and in writing) the estimation strategy.
- Compare and contrast (orally) different tent designs.
- Interpret information (presented in writing and through other representations) about tents and sleeping bags.

Learning Targets

- I can apply what I know about the area of polygons to find the surface area of three-dimensional objects.
- I can use surface area to reason about real-world objects.

Lesson Narrative

In this culminating lesson, students apply what they learned in this unit to solve problems about surface area in context.

Students begin by looking at different examples of tents, analyzing two simple tents of different sizes, and reasoning about the amount of material needed to construct these tents. Then, they can create their own tent design and estimate how much material is needed for the tent. They need to ensure that their design meets specified parameters and their estimate is backed by sound reasoning and calculations. Finally, students present, compare, and reflect on their design solutions and estimates. They consider the impact of design decisions on the surface areas of their tents.

The activities in the lesson prompt students to model a situation with the mathematics they know, make assumptions, and plan a path to solve a problem (MP4) and to make a logical argument to support their reasoning (MP3).

Depending on instructional choices made, this lesson could take one or more class meetings. The warm-up activity and the first activity about two camping tents can be completed in a typical class period. The last two activities (designing a tent and presenting the design) are optional and may take another class period or more, depending on the instructional decisions made, such as:

- Whether students use the provided information about tents and sleeping bags or perform additional research.
- Expectations regarding drafting, revising, and the final product.
- How students' work is ultimately shared with the class (not at all, informally, or with formal presentations).

A note about context:

While experience with camping or tents is not necessary for comparing surface areas in the *Two Tents* activity, it might affect students' readiness to design a tent in the subsequent activity. Consider showing additional images, videos, or an actual camping tent to orient students as needed. If the camping-tent context is anticipated to be challenging or sensitive (such as for students who have experienced housing insecurity or displacement), consider adapting the design task to be about a different structure or object. A tent for a party, a booth for a fair, a birdhouse or house for a pet, and packaging for a toy are some examples.

Standards

Addressing **6.G.A.1, 6.G.A.4**

Instructional Routines

- MLR5: Co-Craft Questions

SAMPLE ONLY

Building Towards **6.G.A.1, 6.G.A.4**

- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials To Gather

- Geometry toolkits: Activity 3

Student Facing Learning Goals

Let's find out how much material is needed to build some tents.

19.1 Notice and Wonder: Structures

Warm-up

5 mins

Activity Narrative

The purpose of this *Warm-up* is to familiarize students with the context of tents, which will be useful when students investigate the amount of material needed to construct tents later in the lesson. While students may notice and wonder many things about these images, the important discussion points are that many structures use fabric (or another flexible material) for cover or enclosure. These structures can be designed in different shapes and sizes and to accommodate different purposes or numbers of occupants.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language that they use to describe what they see (MP6). They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Standards

Building Towards **6.G.A.1, 6.G.A.4**

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Display the images for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?

SAMPLE ONLY






Student Response

Students may notice:

- These are all pictures of tents or shelters built with fabric or a flexible material.
- Most images show outdoor tents. One picture shows an indoor tent or a fort.
- The tents are of different kinds, shapes, and sizes.
- Some tents are held up by sticks or posts. Others are held up by frames and strings.

Students may wonder:

- How many people can fit in each tent?
- What are the tents used for? Why are they put up where they are?
- How many blankets are needed to make the indoor fort? What are the strings tied to?
- How is the teepee built? How much fabric does it take?

SAMPLE ONLY

Activity Synthesis

Record and display their responses for all to see, without editing or commentary. If possible, record the relevant reasoning on or near the images. Next, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the idea of structures that use a flexible material does not come up during the conversation, invite students to discuss this idea. Relate it to students' experience by asking questions such as:

- "Have you seen a tent?"
- "Have you been inside a tent or a structure that uses fabric or another flexible material for cover?"
- "Have you built one?"

Invite students to reflect on the size of the structure, how it was built, the functions it served, and the experience inside or underneath it.

Tell students that they will look at some tent designs and the amount of materials needed to build them.

19.2 Two Tents

35 mins

Activity Narrative

The purpose of this activity is for students to apply their understanding of area and surface area to solve a problem about tents. It also familiarizes students with some considerations that are important in tent design, preparing students to design their own tent in the next activity.

Students are presented with an image of two tents of the same design but of different sizes. While the problem situation is fairly well defined, students are not initially given a problem to solve or any measurements. Instead, they have an opportunity to think about mathematical questions that could be asked, consider the information necessary to answer one of the questions, and make assumptions or estimates if needed. As students make sense of a situation, consider relevant quantities in context, and think about how to solve a problem with the mathematics that they know, they practice aspects of mathematical modeling (MP4).

As students work to solve the chosen problem, monitor for different assumptions students make about the tents (such as whether there is a floor panel inside the tent or whether the tents have an open side). Also monitor for various ways in which students reason about the surface areas (including their choice of units of measurement) and communicate their reasoning.

This is the first time Math Language Routine 5: Co-Craft Questions is suggested in this course. In this routine, students are given a context or situation, often in the form of a problem stem (for example, a story, image, video, or graph) with or without numerical values. Students develop mathematical questions that can be asked about the situation. A typical prompt is: "What mathematical questions could you ask about this situation?" The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce answers, and to develop students' awareness of the language used in mathematics problems.

Access for English Language Learners

This activity uses the Co-Craft Questions math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

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Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Introduce the context of camping tents. Tell students that camping tents come in many shapes and sizes. Tent designs can also vary quite a bit, from simple to elaborate. Use Co-Craft Questions to give students an opportunity to familiarize themselves with the context, and to practice producing the language of mathematical questions.

- Display only the image of the two tents. Ask students, “What mathematical questions could you ask about this situation?”
- Give students 1–2 minutes to write a list of mathematical questions that could be asked about the tents before comparing questions with a partner.

  As partners discuss, support students in using conversation and collaboration skills to generate and refine their questions, for instance, by revoicing a question, seeking clarity, or referring to their written notes. Listen for how students talk in context about the characteristics and measurements of the triangular prisms.
- Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask, “What do these questions have in common? How are they different?” Listen for and amplify language related to the learning goal, such as “area of each face of the tent,” “ floor area inside the tent,” “surface area of the tent,” and “the amount of material needed to build each tent.”

If students asked questions related to the amount of material needed to build both tents, choose or adapt one of those questions for the class to answer. Otherwise, select and pose one of the following questions:

1. “About how many square feet of material is needed to build each tent?”
2. “How much more material is needed to build a two-person tent than a one-person tent?”
3. “Does it take twice as much material to build a two-person tent than to build a one-person tent? How do you know?”

Give students a minute to record the selected question and 2–3 minutes to discuss with their partner what they need to know to answer the question. Then, ask them to list the information needed.

Next, invite students to ask for information, and then provide the following measurements as requested. For any additional information, ask students to make estimates or assumptions, or to compute what can be computed.

- Each sleeping bag or mat measures 34 inches by 74 inches.
- The person in the image is about 70 inches tall.
- The edge lengths of the tents in inches and in feet are as shown:

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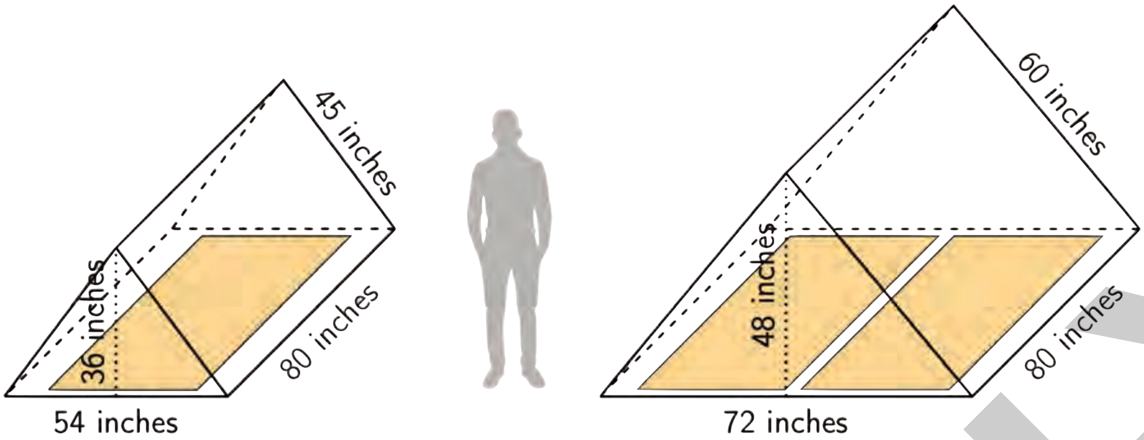
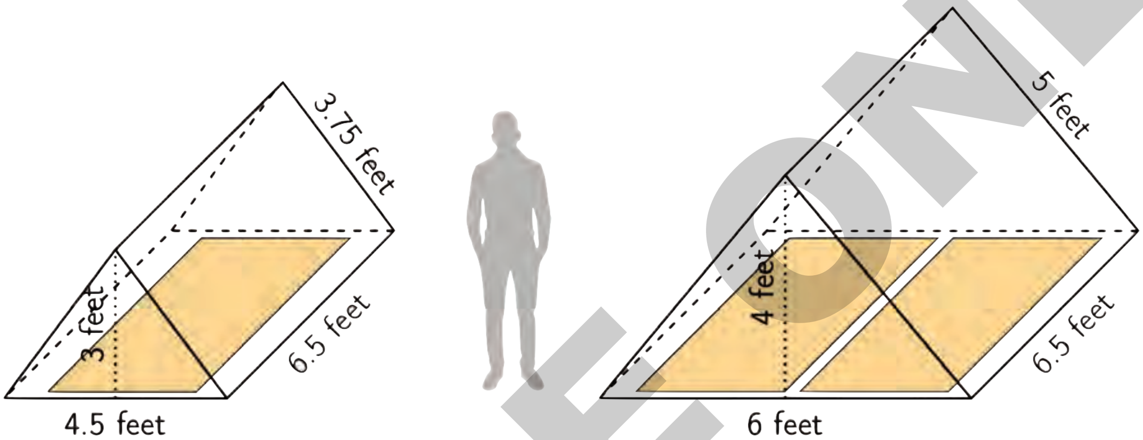
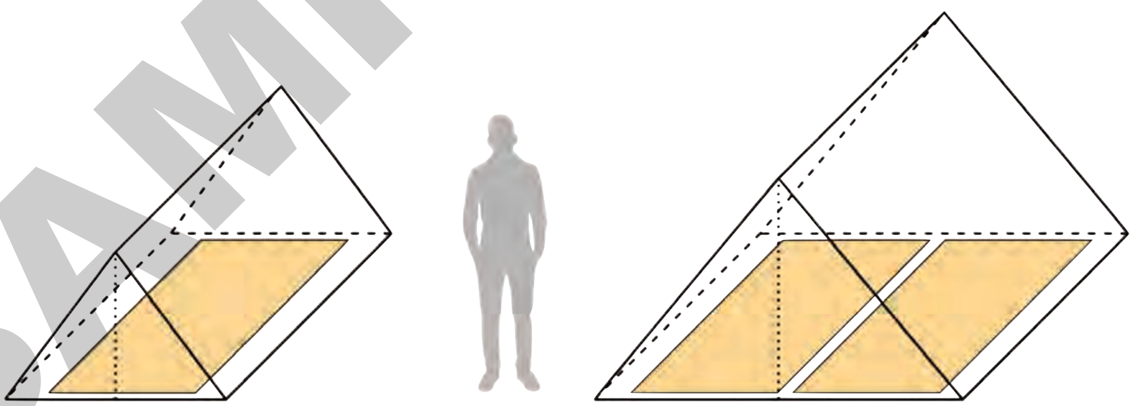




Give students 12–15 minutes to complete the activity individually or with their partner. Provide access to calculators.

Student Task Statement

Here is an image of two tents.



1. Record the question that your class is answering.
2. What do you need to know to be able to answer the question? List the information that you need.

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3. Use the information that you received to answer the question. Show your reasoning, including any assumptions that you make about the situation. Organize your work so that it can be followed by others.

Student Response

1. Questions vary based on the teacher's selection.
2. Sample responses:
    - The length and width of the floor of each tent
    - The height of each tent
    - The size of each sleeping bag or mat
    - The size of the gaps between the sleeping bag and the edges of the tent
    - Whether the floor is covered
    - Whether both triangular ends of the tent are covered
3. All sample responses are based on these assumptions:
    - The tent material covers the floor and both front and back triangular faces.
    - The calculation doesn't include extra material for stitching the pieces together or reinforcing the edges.

    Sample response for Question A (about how much material is needed for both tents): 259 square feet.

    - The surface area of the smaller tent is 91.5 square feet.
        - Left and right panels: $2 \cdot (3.75) \cdot (6.5) = 2 \cdot (24.375) = 48.75$
        - Front and back panels: $2 \cdot \frac{1}{2} \cdot (4.5) \cdot 3 = 2 \cdot (6.75) = 13.5$
        - Floor panel: $(4.5) \cdot (6.5) = 29.25$
        - Total: $48.75 + 13.5 + 29.25 = 91.5$
    - The surface area of the larger tent is 128 square feet.
        - It has 3 rectangular faces that make a big rectangle of 11 feet by 6.5 feet, so its area is $11 \cdot (6.5)$ or 104 square feet.
        - Each triangular face has a base of 6 feet and a height of 4 feet, so the area for 2 triangles is $2 \cdot \frac{1}{2} \cdot 6 \cdot 4$ or 24 square feet.
        - $104 + 24 = 128$
    - The surface area of both tents: $91.5 + 141 = 232.5$

    Sample response for Question B (about how much more material is needed for the two-person tent): 76 square feet ($167.5 – 91.5 = 76$).

    Sample response for Question C (about whether a two-person tent requires twice as much material): No, because twice 91.5 is 183, and a two-person tent requires only 167.5 square feet of material.

Activity Synthesis

Select students or groups who made different assumptions about the two tents to share their thinking. Then, discuss how the differences in assumptions affect their answers to the question.

Next, select those who reasoned about surface area in different ways to briefly present their solutions. Consider asking students:

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- “How did you account for the areas of all the faces of each tent? How did you organize your calculations?”
- “What units of measurement did you use? Why did you choose to use that unit?”
- “How is working in inches different from working in feet?”

The goal of the discussion is for students to see that different assumptions about a situation may affect the solutions to problems about that situation, and that different reasoning strategies may affect the problem-solving process. For example, calculating surface area in inches involves working with large numbers and doing so in feet involves working with decimals.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “To find the surface area of each tent I . . .”, “The amount of material needed for both tents is _____ because . . .”
Supports accessibility for: Language, Organization

19.3 Tent Design (Part 1)

Optional

45 mins

Activity Narrative

In this activity, students apply the concepts and skills from this unit to design a camping tent. They learn about the design problem, consider the parameters for the design, and ask clarifying questions. Then, they make decisions about their tent, create a design and necessary representations of it, and estimate the amount of material needed to construct it.

This work prompts students to draw on the reasoning strategies developed in this unit, such as:

- Decomposing polygons and rearranging the parts to find area.
- Drawing and labeling a net to account for all the faces of a polyhedron.
- Using formulas to facilitate area calculation.
- Calculating areas with appropriate degree of precision.
- Including appropriate units of measurement.

In creating their tent design, students may need to estimate lengths that cannot be computed exactly given their current mathematical knowledge. For instance, the length of a slanted edge of a roof panel may be calculated using the Pythagorean Theorem but students are not expected to do so at this point. To support students in making reasonable estimates, encourage them to make comparisons. Consider asking questions such as:

- “If the horizontal side of this right triangle is 5 feet and the vertical side is 3 feet, would the slanted side be longer or shorter than 5 feet? About how much longer?”
- “About how many times as long as this 3-foot-long side is that side?”

As students make assumptions and decisions for their design and apply the mathematics that they know to solve a problem, they practice aspects of mathematical modeling (MP4).

Students also have an opportunity to choose tools strategically (MP5) as they develop and represent a three-dimensional object. Some students may find it useful to think in two-dimensional terms and start by drawing a net.

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Others may wish to build a physical model of their design or to use a digital drawing tool. Students should be encouraged to consider the tools at their disposal and choose those that would enable them to complete the task.

Given the open-ended nature and high cognitive demand of a design activity, students may benefit from seeing a sample final product along with the intermediate steps. Consider sharing an example of completed work (such as the provided sample student response) to help students understand the expected outcome.

Standards

Addressing **6.G.A.1, 6.G.A.4**

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Read aloud the introduction to the design task or invite a couple of students to do so. Give students 2 minutes to discuss with their partner their understanding of the task. Then, invite students to ask any clarifying questions.

Make sure students understand the parameters of the tent design:

- The tent needs to accommodate sleeping bags for 4 people.
- The tent needs to be a minimum height.
- There is not one right design.

Next, give partners 10 minutes to look more closely at the given information (sample tent designs, tent specifications, and sleeping bag information) and to discuss their ideas. The sample tent styles are provided for inspiration and reference, but students are not limited to them. If desired or if needed, allow students to perform additional research on tent styles. Students designing a wheelchair-accessible tent, for instance, may want to account for minimum clearances for the width and height of the tent's opening.

Before students begin working, make sure that they understand that their estimate for the amount of material needed should:

- Be based on given or researched facts.
- Reflect the decisions made about their tent (for example, that it can accommodate standing height).
- Be supported by sketches and calculations.

Provide access to blank paper, geometry toolkits, and calculators. (Note that a scale drawing is not an expectation, because scale factor is a grade 7 standard.)

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students in stating their assumptions or decisions about their tent, for example:

- "My tent will have . . . (or will be . . .)"
- "I would like my tent to have . . . (or to be . . .)"
- "I would like the campers to have . . . (or to be able to . . .)"

Advances: Speaking, Writing

Access for Students with Disabilities

Engagement: Internalize Self-Regulation. Provide a project checklist that chunks the various steps of the project

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into a set of manageable tasks.
Supports accessibility for: Organization; Attention

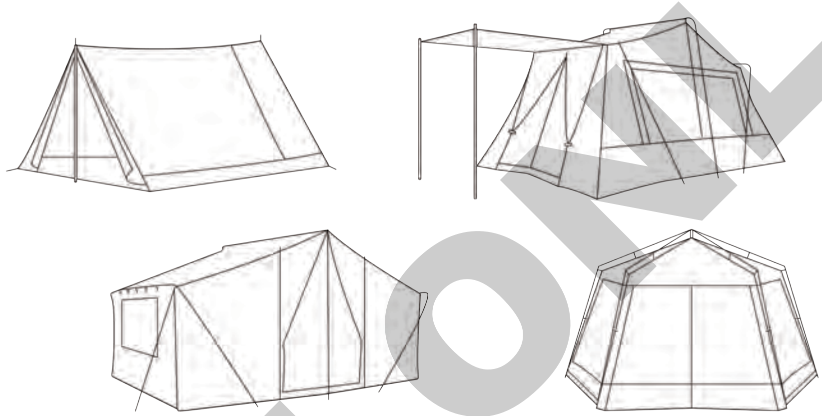
Student Task Statement

You are going to design a tent for 4 people. Your design must:

- Include a floor panel.
- Show how the sleeping bags fit inside.
- Be tall enough that people can at least kneel inside the tent.

After creating a design, you will estimate the amount of material needed to build it and show your reasoning.

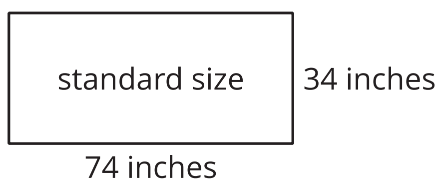
Sample Tent Styles



Tent Height Specifications

height description	height of tent	notes
sitting height	3 feet	Campers are able to sit, lie, or crawl inside tent.
kneeling height	4 feet	Campers are able to kneel inside tent. Found mainly in 3–4 person tents.
stooping height	5 feet	Campers are able to move around on their feet inside tent, but most campers will not be able to stand upright.
standing height	6 feet	Most adult campers are able to stand upright inside tent.
wheelchair seating height	4.5 feet	Most campers in a wheelchair have enough head clearance.
roaming height	7 feet	Adult campers are able to stand upright and walk around inside tent.

Sleeping Bag Measurements



1. Create your design.

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a. Tent floor: Sketch the shape of the floor panel and the placements of sleeping bags. Think about approximate measurements. How large is this floor panel?

b. Relevant information: What decisions did you make for your tent? What assumptions did you make?

c. Overall design: Sketch what the tent would look like. Think about approximate measurements. How high is the tallest point of your tent?

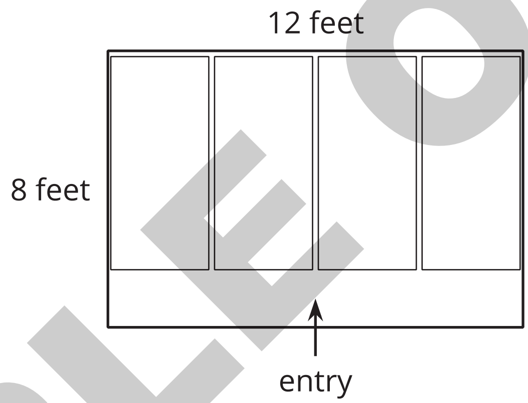
2. Estimate the amount of material needed to build your tent. Your estimate must:
   - Be based on measurements you researched or received.
   - Include sketches that show the parts of the tent and their measurements.
   - Be supported by calculations.

   Organize your work so it can be followed by others. You can use additional paper if you need more space.

Student Response

Sample response:

1. Tent floor:



2. Relevant information:
   - Campers will have at least stooping height at the shorter sides of the tent but will be able to stand toward the center of the tent.
   - There will be some space to put campers' bags.
3. Overall design:

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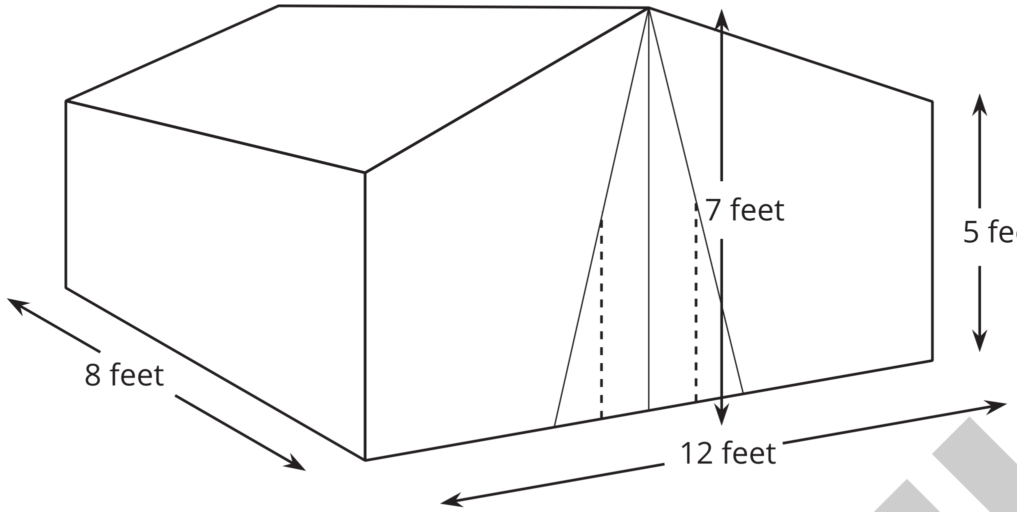
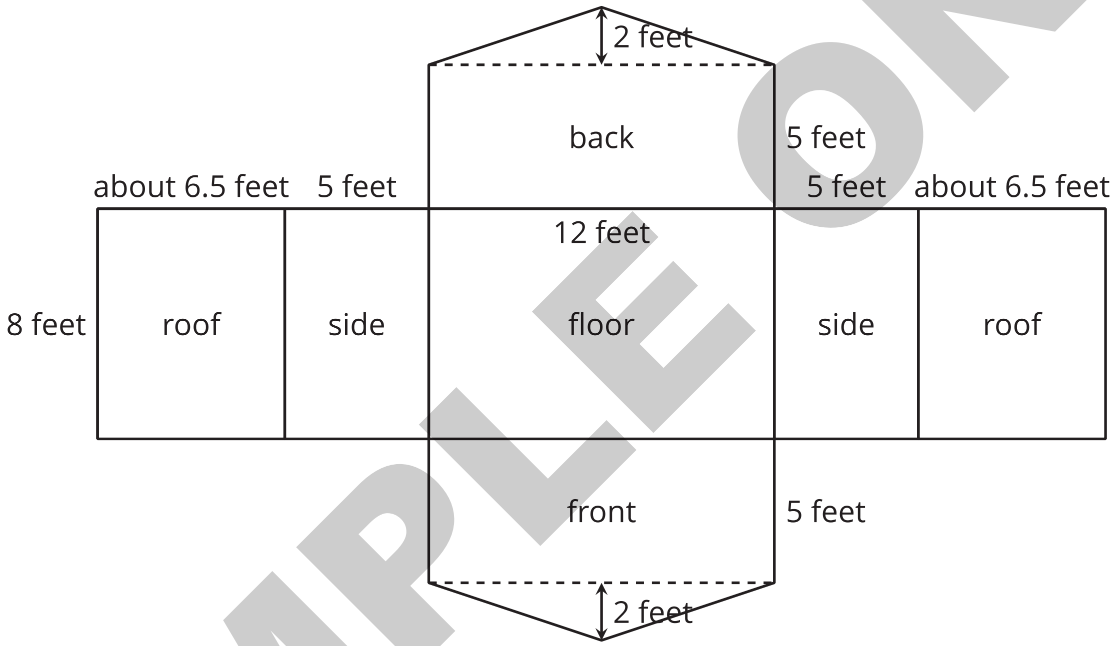


4. Estimate of material needed: About 424 square feet



- Floor panel: 96 square feet. $12 \cdot 8 = 96$
- Side panels: 80 square feet. The two rectangles are 8 feet by 5 feet or 40 square feet each. $2 \cdot 40 = 80$
- Roof panels: About 104 square feet. The slanted length is about 6.5 feet, so the roof panels are two 6.5-by-8 rectangles with 52 square feet of area in each.
- Front and back panels: 144 square feet. Each panel can be decomposed into a rectangle that is 12 feet by 5 feet (area 60 square feet) and a triangle with a base of 12 and a height of 2 (area 12 square feet), so its area is 72 square feet. The area of both panels are $2 \cdot 72$, which is 144.
- Surface area of tent: $96 + 80 + 104 + 144 = 424$

Activity Synthesis

Students will share and reflect on their designs in the next activity. Before moving on, engage them in a brief whole-class

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discussion. Invite students to share some of the things that they considered and decisions that they made as they were designing their tent. Also invite them to share some challenges that they encountered when trying to estimate the necessary amount of material.

19.4 Tent Design (Part 2)

15 mins

Optional

Activity Narrative

This activity gives students a chance to explain and reflect on their work. Students share drawings of their tent design, an estimate of the amount of material needed, and the justification. Then, students compare their creations with their peers' creations and discuss not only the amount of material required, but also the effects that different designs have on that amount.

As students discuss in groups, notice how they reason about and communicate their work. Monitor for whether they:

- Provide justification for their measurements and choices.
- Include drawings that reflect their decisions and show relevant measurements and labels.
- Explain clearly their process of calculating surface area.

Also notice how students compare the amounts of material needed for the different designs in their group. Monitor for students who recognize how design decisions (such as tent types, tent measurements, and arrangements of sleeping bags) affect the amount of material needed.

Standards

Addressing **6.G.A.1, 6.G.A.4**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Tell students that they will now reflect on and discuss their tents with their group. Ask students to take turns sharing their work, allowing 3–4 minutes per person, and then compare the amounts of material needed.

Access for English Language Learners

MLR8 Discussion Supports. Encourage students to begin group discussions by reading their written responses aloud. If time allows, invite students to revise or add to their responses based on the conversation that follows. *Advances: Conversing, Speaking*

Student Task Statement

1. Explain your tent design and material estimate to your group. Be sure to explain why you chose this design and how you found your material estimate.
2. Compare the estimated material necessary for each tent in your group. Discuss the following questions:
    - Which tent design used the least material? Why?
    - Which tent design used the most material? Why?

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Student Response

No written response required.

Activity Synthesis

Much of the discussion will take place within the groups. Once groups have had an opportunity to share their designs, reconvene as a class. Consider displaying tent designs that used the most and the least amount of material and discussing with students:

- "All the tents were designed to accommodate 4 people and a minimum height inside the tent. How did the tents end up using very different amounts of material?"
- "What design decisions impact the amount of material needed?"
- "Can you give examples of decisions that had a major impact? What about decisions that had a lesser impact?"
- "When calculating the surface area of your tent, what reasoning strategies from this unit did you find useful?"

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide sentence frames to support student explanations. Display sentence frames such as: "We chose our tent design because . . .", "This tent design uses the least/most fabric because . . . ."
Supports accessibility for: Language; Organization

Lesson Synthesis

This culminating lesson could be wrapped up in a number of ways, depending on which activities students completed, the time available, and your goals and expectations.

Consider inviting students to reflect on their journey through the unit and on the connections between the work in earlier lessons and application problems such as those solved in this lesson. Discuss questions such as:

- "You started reasoning about area very early in the unit. What reasoning strategies that you used then continued to be useful when working on your tent designs?" (decomposing a region and rearranging the parts into shapes whose area we know how to find, adding the areas of individual parts, finding the area of a larger region and subtracting the areas of extra parts)
- "Can you give examples of when you applied something you learned in the unit to make your reasoning more efficient or more reliable?" (using formulas to find the areas of parallelograms and triangles, combining several shapes into a larger shape and computing the area once instead of finding the areas of individual pieces and adding them, using a net to make sure that all faces are included in a surface area calculation)
- "We used the math ideas from this unit to find the number of squares to cover a file cabinet, the number of square units in walls or floors, and the amount of material needed to build tents. In what other situations might we apply our knowledge of area and surface area?"

A note about materials for an upcoming unit:

For the first lesson in the unit on ratios, students will need to bring in a personal collection of 10–50 small objects. Examples include rocks, seashells, trading cards, or coins. Inform or remind students about this.

SAMPLE ONLY

Learning Targets

Lesson 1 Tiling the Plane

- I can explain the meaning of "area."

Lesson 2 Finding Area by Decomposing and Rearranging

- I can explain how to find the area of a figure that is composed of other shapes.
- I know how to find the area of a figure by decomposing it and rearranging the parts.
- I know what it means for two figures to have the same area.

Lesson 3 Reasoning to Find Area

- I can use different reasoning strategies to find the area of shapes.

Lesson 4 Parallelograms

- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
- I know how to describe the characteristics of a parallelogram using mathematical vocabulary.

Lesson 5 Bases and Heights of Parallelograms

- I can identify pairs of base and height of a parallelogram.
- I can write and explain the formula for the area of a parallelogram.
- I know what the terms "base" and "height" refer to in a parallelogram.

Lesson 6 Area of Parallelograms

- I can use the area formula to find the area of any parallelogram.

Lesson 7 From Parallelograms to Triangles

- I can explain the special relationship between a pair of identical triangles and a parallelogram.

Lesson 8 Area of Triangles

- I can use what I know about parallelograms to reason about the area of triangles.

Lesson 9 Formula for the Area of a Triangle

- I can use the area formula to find the area of any triangle.
- I can write and explain the formula for the area of a triangle.
- I know what the terms “base” and “height” refer to in a triangle.

Lesson 10 Bases and Heights of Triangles

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson 11 Polygons

- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.
- Puedo usar vocabulario matemático para describir las características de un polígono.

Lesson 12 What is Surface Area?

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- I know what the surface area of a three-dimensional object means.

Lesson 13 Polyhedra

- I can describe the features of a polyhedron using mathematical vocabulary.
- I can explain the difference between prisms and pyramids.
- I understand the relationship between a polyhedron and its net.

Lesson 14 Nets and Surface Area

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

Lesson 15 More Nets, More Surface Area

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

Lesson 16 Distinguishing Between Surface Area and Volume

- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

Lesson 17 Squares and Cubes

- I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
- When I know the edge length of a cube, I can find the volume and express it using appropriate units.

Lesson 18 Surface Area of a Cube

- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson 19 All about Tents

- I can apply what I know about the area of polygons to find the surface area of three-dimensional objects.
- I can use surface area to reason about real-world objects.

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