SECTION 1

THINKING LIKE A MATHEMATICIAN

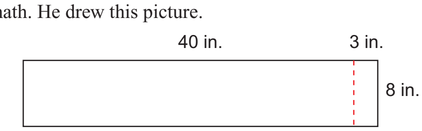
Understanding and Using Variables

Algebra is a powerful tool for understanding the world. You can represent ideas and relationships using symbols, tables and graphs. In this section you will learn about variables and investigate how they are used to model real situations. You may also learn some unusual facts about how people spend their free time!

LESSON 1.1 Variables

Start It Off

Marcelino was asked to find the area of a 43 in. by 8 in. rectangle using mental math. He drew this picture.



1. Marcelino found the areas of the two smaller rectangles and added them together. Why is this a useful application of the "mental math" method?
2. Which of these statements best represents his method? Why?

A.	B.	C.
$8 \cdot 43$	$8 \cdot 43$	$8 \cdot 43$
$= 8(40 + 3)$	$= 8(40 + 3)$	$= 8(40 + 3)$
$= 320 + 3$	$= 40 + 24$	$= 320 + 24$
$= 323$	$= 64$	$= 344$

3. Marcelino's method is an example of the distributive property. Write down two things you know about the distributive property.

Start It Off

It is important that students see that the area is the same regardless of whether Marcelino multiplies 43 by 8 or breaks one factor (43) into an equivalent sum (40 + 3) and then multiplies each addend by the other factor ($8 \cdot 40 + 8 \cdot 3$). Discuss Questions 2 and 3 with the whole class. Students need to see how multiplying both "parts" of 43 by 8 results in the same area and recognize its symbolic representation. Write on the board all their ideas about the distributive property.

1. It is easier to multiply 8 by 40 and 8 by 3 and then sum the products than to multiply 8 by 43.
2. **C.** Both the 40 and the 3 are multiplied by 8.
3. Answers will vary.

DAY 1 TEACHING THE LESSON

Algebraic Statements

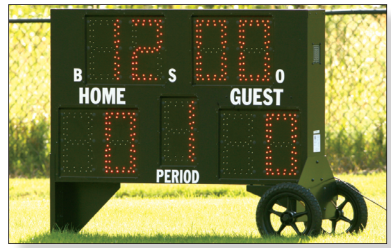
Talk Moves Distribute envelopes with Algebra Cards to pairs or small groups of students. Ask students to spread the cards out so they can see each card. Have partners discuss what they notice and then have a few students share with the whole group. Ask:

- ***Look at the algebraic statements on these cards. What do you notice about them?***
- ***Do each of these have letters? Do each of these have numbers?***
- ***What operations are used on these cards?***
- ***Are any cards similar to one another? In what way are they similar?***

Differentiation

Think Differently: Students commonly misinterpret the variable in an expression like $7m$ as a label (7 meters or 7 minutes) rather than as the product of a number and a variable. You may wish to highlight cards C ($7m$), M ($35 = 7g$) and I ($A = lw$) and discuss with students how multiplication can be indicated: with a "$\times$" as in $7 \times m$, with a "$\bullet$" as in $7 \bullet m$, or by placing letters or numbers next to each other such as $7m$ and 3(4). Point out how parentheses must be used to indicate multiplication of two numbers, $3(4) = 12$, so it won't be confused with a two-digit number (34).

Are you interested in sports? Most students have a favorite sport they enjoy watching or playing. Mathematical symbols such as variables, operations and numbers can be used to represent and describe many situations associated with sports.



Algebraic Statements

Equations, expressions and inequalities are the building blocks of algebra. Examine the cards below.

A $4 \bullet 9 = 6 \bullet 6$	**B** $C = \pi d$	**C** $7m$
D $23 - x > 13$	**E** $y = -2x$	**F** $a + b = b + a$
G $36 + 8$	**H** $y = x - 1$	**I** $A = lw$
J $5 + 9 = n + 6$	**K** $14 - y$	**L** $a \bullet \frac{1}{a} = 1, a \neq 0$
M $35 = 7g$	**N** $5(3 + m) = 15 + 5m$	**O** $6 + s = 10$
P $4x - x = 3x$	**Q** $7x + 2y$	**R** $x \leq -4$

Have pairs complete Question 1. Bring the group together to discuss. Focus particularly on Question 1c. Ask:

- ***What are the main differences between equations and expressions?*** An equation is a statement that two expressions or quantities are equivalent. Equations have equal signs. An expression is a collection of numbers/variables and operations that represent a quantity. An equation states that two expressions are equal.
- ***Are there any cards that do not show either an equation or an expression?*** Two cards are neither equations nor expressions but are inequalities ($23 - x > 13$ and $x < -4$). Inequalities represent relationships between expressions using $<$, $>$, $\leq$ or $\geq$ symbols. Asking this question and having students explain and debate will highlight important differences between equations, expressions and inequalities.

Students may distinguish between equations and expressions by indicating that equations can be "solved" and expressions cannot. This would be a good opportunity to discuss the difference between evaluating, simplifying and solving. Expressions can be simplified. For instance, $7x - x$ can be simplified to $6x$ and with Card G, $36 + 8$ can be simplified to 44. This can be confusing for students because they may think they have "solved" $36 + 8$ by finding the sum of these numbers. Since equations consist of two expressions, they also can be simplified by combining terms. Variable expressions can also

MATHEMATICALLY SPEAKING

- equation
- inequality
- expression
- constant
- variable

1. a) Take a set of the cards and sort them into three groups—equations, inequalities and expressions.
   b) How are the groups the same?
   c) What do you think are the most important differences between expressions, equations and inequalities?
   d) Write a definition for each of the terms: equation, expression and inequality.
2. a) Now sort the equation cards into different groups that you select.
   b) Describe the cards in each group and explain why you put certain cards together.
   c) Write two additional cards to add to each sorted group.

Many equations, inequalities and expressions have letters in them. Some letters are constants. A constant is a symbol that represents exactly one quantity. Every time the constant letter or symbol is used, the same value is substituted. The Greek letter π, called "pi," is an example of a constant. The value of π does not change; it is always the number equal to the circumference of a circle divided by the diameter of that circle.

Other letters, or symbols such as ❏ and Δ, are called variables. In equations, expressions and inequalities, one or more quantities can be substituted for a variable. In this lesson, you will learn about the different types of variables.

Variables Representing a Specific Value

MATHEMATICALLY SPEAKING

- solve (an equation)
- solution

Some variables represent a specific value. When you are asked to "solve an equation," you are to find the value of the variable that makes that equation true. This value is known as the solution. When variables represent specific values, there is often only one variable or letter in the equation.

mathematical formulas or properties? Following the sorting activity in Question 2, you might want students to read the last paragraphs in this section about variables and constants.

be evaluated. For instance, with Card K, $14 - y$ could be *evaluated* if we substitute a value for y. In algebra, variable expressions are not "solved." Equations are solved. Namely, students are able to determine the value of the variable in the equation. For example, "solve for x" might be the direction line for $7 + x = {}^-2$.

Have students continue with Question 2. Encourage students to look for different ways to sort the equations into two groups. As students sort the cards, circulate and note the ways they are sorting and the language they are using to describe the symbol strings. This is a good opportunity to informally assess students. Do they know and use the word variable? Do they recognize that some of the cards show

1. a) Equations

A. $4 \cdot 9 = 6 \cdot 6$
B. $C = \pi d$
E. $y = {}^-2x$
F. $a + b = b + a$
H. $y = x - 1$
I. $A = lw$
J. $5 + 9 = n + 6$
L. $a \cdot \frac{1}{a} = 1$
M. $35 = 7g$
N. $5(3 + m) = 15 + 5m$
O. $6 + s = 10$
P. $4x - x = 3x$

Inequalities

D. $23 - x > 13$
R. $x < {}^-4$

Expressions

C. $7m$
G. $36 + 8$
K. $14 - y$
Q. $7x + 2y$

b) All groups have letters and/or numbers combined with at least one operation.

c) Equations show that two quantities are equivalent, while expressions alone do not. Variable equations can be solved, while variable expressions can be evaluated. Inequalities express a relationship between two expressions such as one is greater than the other.

2. a) Students will sort equations in various ways. Possible ways include: equations with one variable, equations with two variables, equations that are formulas, equations that are properties, equations with no variables, equations with an isolated variable on one side of the equal sign.

b) Answers will vary. Sample response:

Equations that are formulas: $A = lw$, $C = \pi d$. Both of these equations are measurement formulas.

c) Answers will vary.

Summarize Day 1

Bring students together to share how they sorted the equations. Select students to discuss the different ways they sorted the cards, such as those mentioned in the answer to Question 2a. Make a list of the attributes students used along with the equations that have those attributes. Encourage students to consider if they agree with their classmates' lists and ask them to create other equations that fit those categories. You may also wish to present pairs of equations and ask students to describe why the equations could be grouped together. Some pairs to use include:

- Card M ($35 = 7g$) and Card I ($A = lw$) (Both show a product and two factors.)
- Card P ($4x - x = 3x$) and Card N ($5(3 + m) = 15 + 5m$) (When the expressions on each side of the equal sign are simplified, you get the same expression on each side.)

Example 1

Major League Baseball teams play 162 games every season. In 2008, the San Diego Padres won 63 games. How many games did they lose?

Let n represent the number of games the Padres lost.

Equation: $63 + n = 162$

Solution: $n = 99$ The Padres lost 99 games in the 2008 season.

Verify solution: $63 + 99 = 162$

When asked to solve equations and find a specific value for a variable, we usually assume that the solution is a real number. But sometimes there are restrictions on which set of numbers can be used. This can affect the solutions.

Let's Review

Natural Numbers ($\mathbb{N}$)	{1, 2, 3, 4, 5, ….}
Whole Numbers	{0, 1, 2, 3, 4, 5, ….}
Integers ($\mathbb{Z}$)	{…−3, −2, −1, 0, 1, 2, 3, …}
Rational Numbers ($\mathbb{Q}$)	Any number that can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$
Real Numbers ($\mathbb{R}$)	Any number that can be represented on a number line

3. Find the solution to each equation below. Note which set of numbers can be used.

a) $5x = -15$ x belongs to the set of integers.

b) $5x = -15$ x belongs to the set of natural numbers.

c) $2x = 13$ x belongs to the set of real numbers.

d) $2x = 13$ x belongs to the set of integers.

Mathematicians use symbols to record information using the least number of words. Rather than write out the name of the set of numbers, they use its abbreviation. Another common abbreviation is the symbol, ϵ, which means "is an element of" or "is a member of."

$x \in \mathbb{Z}$ x is an element of the set of integers.

$x \in \mathbb{N}$ x is an element of the set of natural numbers.

$x \in \mathbb{R}$ x is an element of the set of real numbers.

DAY 2 TEACHING THE LESSON

Begin by projecting several of the Algebra Cards. Bring students' attention to the variables, and ask:

- ***We noticed that some of these equations and expressions have letters. What are these letters called?*** variables
- ***What do the variables mean?*** Most likely students will indicate that a variable is a symbol that represents an unknown quantity.
- ***Have you ever heard of a constant value? What is a constant?*** A constant represents exactly one quantity.

4. Solve for n.

a) $\frac{2}{3}n = -\frac{1}{4} \qquad x \in \mathbb{Z}$

b) $\frac{2}{3}n = 2x \qquad x \in \mathbb{N}$

c) $\frac{2}{3}n = -\frac{1}{4}n \qquad x \in \mathbb{R}$

5. Write an equation with a variable to represent each situation. Define the variable. Then, solve the equation. Be sure to check your answer by substituting the value back into the equation.

a) The total of two teams' scores for a basketball game was 176 points. One team scored 97 points. What was the other team's score?

b) How much time does it take a bicyclist to finish a 75-mile race if she averages 20 miles per hour?

c) Pole-vaulting is an athletic field event. Homer can pole-vault 2.5 times as high as Sally. If he can pole-vault 15 feet, how high can Sally pole-vault?

Variables Representing Related Varying Quantities

Some equations have two or more variables. If the value of one variable changes when the value of another variable changes, then there is a mathematical relationship that links the two variables. The variables represent related varying quantities. When two or more variables are mathematically related, there will be many different combinations of values for the variables that will make the equation true.

Differentiation

Think Differently: Some students may benefit from reading the text aloud with their partner or by rereading parts to aid in comprehending the text. You may wish to use Lesson Guide 1.1A: *Understanding Variables* and have students record information in the chart about the different meanings of *variable*. Accommodation Guides (Lesson Guides that include an A after the number) are designed to provide additional support for those students experiencing difficulty with a lesson activity. English language learners may benefit from highlighting vocabulary words in the *Understanding Variables* table and linking the words with the actual examples (e.g., inequality, $50 \geq y$).

Variables Representing a Specific Value

Mention that today students are going to learn that variables can have different meanings. Have students read the last two paragraphs about variables in the Algebraic Statements section, if you haven't done so, and the text from Variables Representing a Specific Value through the example. Have students explain the parts of the example to each other to reinforce their understanding. Students may not have yet learned that in order to check a solution, they should substitute the value of the variable back into the original equation to see if it makes the equation true.

Review the different sets of numbers that were introduced in the unit *Let's Be Rational: Focusing on Fractions, Decimals and Integers*. Then introduce students to the symbols used to indicate the different sets of numbers. Use a transparency of Lesson Guide 1.1: *Sets of Numbers*. Note there is no standard symbol for the set of whole numbers.

$\mathbb{N}$ set of natural numbers {1, 2, 3, 4, 5, . . .}

$\mathbb{Z}$ set of integers {. . . , −3, −2, −1, 0, 1, 2, 3, . . .}

$\mathbb{Q}$ set of rational numbers {numbers in the form $\frac{a}{b}$, a and $b \in \mathbb{Z}$, $b \neq 0$}

$\mathbb{R}$ set of real numbers {all rational and irrational numbers}

Section 1: Understanding and Using Variables • Lesson 1.1 5

Lesson Guide 1.1 *Algebra Cards*

A. $4 \bullet 9 = 6 \bullet 6$	**B.** $C = \pi d$
C. $7m$	**D.** $23 - x > 13$
E. $y = -2x$	**F.** $a + b = b + a$
G. $36 + 8$	**H.** $y = x - 1$
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