SECTION 1 THINKING LIKE A MATHEMATICIAN

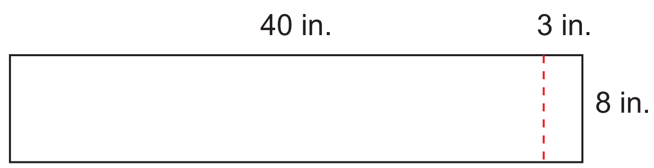
Understanding and Using Variables

Algebra is a powerful tool for understanding the world. You can represent ideas and relationships using symbols, tables and graphs. In this section you will learn about variables and investigate how they are used to model real situations. You may also learn some unusual facts about how people spend their free time!

LESSON 1.1 Variables

Start It Off

Marcelino was asked to find the area of a 43 in. by 8 in. rectangle using mental math. He drew this picture.



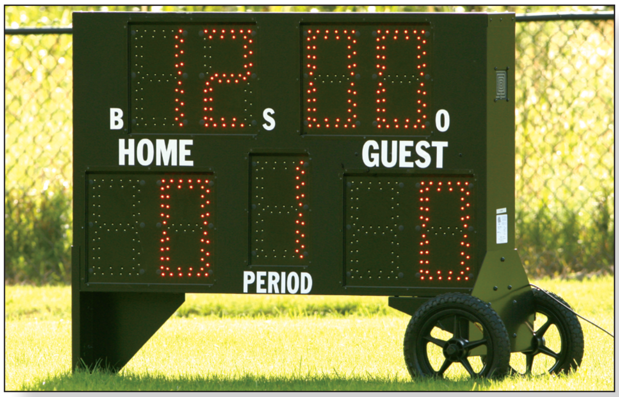
1. Marcelino found the areas of the two smaller rectangles and added them together. Why is this a useful application of the "mental math" method?

2. Which of these statements best represents his method? Why?

A.	B.	C.
$8 \cdot 43$	$8 \cdot 43$	$8 \cdot 43$
$= 8(40 + 3)$	$= 8(40 + 3)$	$= 8(40 + 3)$
$= 320 + 3$	$= 40 + 24$	$= 320 + 24$
$= 323$	$= 64$	$= 344$

3. Marcelino's method is an example of the distributive property. Write down two things you know about the distributive property.

Are you interested in sports? Most students have a favorite sport they enjoy watching or playing. Mathematical symbols such as variables, operations and numbers can be used to represent and describe many situations associated with sports.



Algebraic Statements

Equations, expressions and inequalities are the building blocks of algebra. Examine the cards below.

A $4 \cdot 9 = 6 \cdot 6$	**B** $C = \pi d$	**C** $7m$
D $23 - x > 13$	**E** $y = {}^{-}2x$	**F** $a + b = b + a$
G $36 + 8$	**H** $y = x - 1$	**I** $A = lw$
J $5 + 9 = n + 6$	**K** $14 - y$	**L** $a \cdot \frac{1}{a} = 1, a \neq 0$
M $35 = 7g$	**N** $5(3 + m) = 15 + 5m$	**O** $6 + s = 10$
P $4x - x = 3x$	**Q** $7x + 2y$	**R** $x < {}^{-}4$

MATHEMATICALLY SPEAKING

- equation
- inequality
- expression
- constant
- variable

1. a) Take a set of the cards and sort them into three groups—equations, inequalities and expressions.
   b) How are the groups the same?
   c) What do you think are the most important differences between expressions, equations and inequalities?
   d) Write a definition for each of the terms: equation, expression and inequality.
2. a) Now sort the equation cards into different groups that you select.
   b) Describe the cards in each group and explain why you put certain cards together.
   c) Write two additional cards to add to each sorted group.

Many equations, inequalities and expressions have letters in them. Some letters are constants. A constant is a symbol that represents exactly one quantity. Every time the constant letter or symbol is used, the same value is substituted. The Greek letter π, called "pi," is an example of a constant. The value of π does not change; it is always the number equal to the circumference of a circle divided by the diameter of that circle.

Other letters, or symbols such as ❑ and Δ, are called variables. In equations, expressions and inequalities, one or more quantities can be substituted for a variable. In this lesson, you will learn about the different types of variables.

Variables Representing a Specific Value

MATHEMATICALLY SPEAKING

- solve (an equation)
- solution

Some variables represent a specific value. When you are asked to "solve an equation," you are to find the value of the variable that makes that equation true. This value is known as the solution. When variables represent specific values, there is often only one variable or letter in the equation.

Example 1

Major League Baseball teams play 162 games every season. In 2008, the San Diego Padres won 63 games. How many games did they lose?

Let n represent the number of games the Padres lost.

Equation: $63 + n = 162$

Solution: $n = 99$ The Padres lost 99 games in the 2008 season.

Verify solution: $63 + 99 = 162$

When asked to solve equations and find a specific value for a variable, we usually assume that the solution is a real number. But sometimes there are restrictions on which set of numbers can be used. This can affect the solutions.

Let's Review

Natural Numbers ($\mathbb{N}$)	$\{1, 2, 3, 4, 5, \ldots\}$
Whole Numbers	$\{0, 1, 2, 3, 4, 5, \ldots\}$
Integers ($\mathbb{Z}$)	$\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational Numbers ($\mathbb{Q}$)	Any number that can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$
Real Numbers ($\mathbb{R}$)	Any number that can be represented on a number line

3. Find the solution to each equation below. Note which set of numbers can be used.

a) $5x = -15$ x belongs to the set of integers.

b) $5x = -15$ x belongs to the set of natural numbers.

c) $2x = 13$ x belongs to the set of real numbers.

d) $2x = 13$ x belongs to the set of integers.

Mathematicians use symbols to record information using the least number of words. Rather than write out the name of the set of numbers, they use its abbreviation. Another common abbreviation is the symbol, $\in$, which means "is an element of" or "is a member of."

$x \in \mathbb{Z}$ x is an element of the set of integers.

$x \in \mathbb{N}$ x is an element of the set of natural numbers.

$x \in \mathbb{R}$ x is an element of the set of real numbers.

4. Solve for n.

a) $\frac{2}{3}n = -\frac{1}{4}$ $\quad n \in \mathbb{Z}$

b) $\frac{2}{3}n = 2$ $\quad n \in \mathbb{N}$

c) $\frac{2}{3}n = -\frac{1}{4}$ $\quad n \in \mathbb{R}$

5. Write an equation with a variable to represent each situation. Define the variable. Then, solve the equation. Be sure to check your answer by substituting the value back into the equation.

a) The total of two teams' scores for a basketball game was 176 points. One team scored 97 points. What was the other team's score?

b) How much time does it take a bicyclist to finish a 75-mile race if she averages 20 miles per hour?

c) Pole-vaulting is an athletic field event. Homer can pole-vault 2.5 times as high as Sally. If he can pole-vault 15 feet, how high can Sally pole-vault?

Variables Representing Related Varying Quantities

Some equations have two or more variables. If the value of one variable changes when the value of another variable changes, then there is a mathematical relationship that links the two variables. The variables represent related varying quantities. When two or more variables are mathematically related, there will be many different combinations of values for the variables that will make the equation true.

Example 2

Ticket prices for many baseball games depend on the opponent and the day of the week. If a ticket to the Chicago White Sox costs \$15 for a specific Monday game, we can write an equation for finding the cost of any number of tickets to that game: $C = 15n$. In this case, n represents the number of tickets purchased and C represents their total cost in dollars.

Number of Tickets (n)	Cost in Dollars (C)
0	0
1	15
2	30
5	75
8	120

variable ↘ $C = 15n$ ↙ **variable**

As the number of tickets, n, increases, the cost in dollars of the tickets, C, increases.

Since there are two variables in this equation, a solution must give a value for both n and C. These two values are often listed as an ordered pair, (n, C). The ordered pairs (0, 0), (1, 15), (2, 30), (5, 75) and (8, 120) are some, but not all, of the solutions to this equation.

6. **a)** What do the variables n and C represent?

b) Why are these variables referred to as “varying quantities”?

c) What is the mathematical relationship between these quantities?

d) List five other pairs of values that make the equation $C = 15n$ true. How many pairs of values are in the solution?

Variables Representing Many Values

You may have seen variables used in the statement of mathematical properties. For example, the commutative property of addition states that $a + b = b + a$, for all real numbers a and b. It is important to realize that a and b are not mathematically related. In other words, changing the value of a does not change the value of b. Rather, a and b are being used as “generalized numbers.” The statement $a + b = b + a$ is true for any real numbers a and b.

Variables in mathematical properties generalize important ideas. The variables can represent many values and are not related. Examine the equations below. These equations are true for all values of the variable(s). The variables can equal any real numbers. Not all of these statements are properties.

$$a + a = 2a \qquad\qquad a + b - b = a$$

$$a + b = b + a \qquad\qquad a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

7. Write three different equations that use variables as "generalized numbers." Are each of the equations you wrote a property?

An inequality is one type of mathematical sentence that expresses a relationship between numbers and variables. Variables in inequalities usually represent a set of values. This in turn means that there usually are many solutions to an inequality.

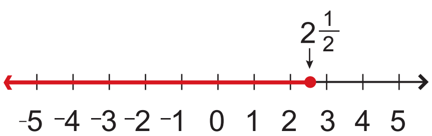
Example 3

At one time Major League Soccer players had a cap on their salaries of $2 million. This restriction can be written using the inequality $x \leq 2{,}000{,}000$, where x is the salary in whole numbers of dollars.

$x \leq 2{,}000{,}000$ $\qquad\qquad$ $x \in$ whole numbers

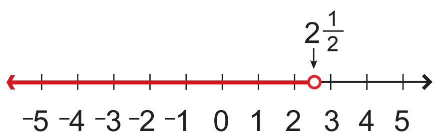
The solution for x is the set of whole numbers between 0 and 2,000,000, including 0 and 2,000,000. That is, salaries can be from $0 through $2 million.

A variable in an inequality often represents many values. The solution to an inequality can be graphed on a number line. When a solid point is used on a graph, it means that the point is a solution. When an open point or circle is used, it means that the point does not satisfy the stated relationship and is not a solution. A ray on the number line includes all possible points along the ray. In the following examples, $x \in \mathbb{R}$.



$x \leq 2\frac{1}{2}$

$2\frac{1}{2}$ is in the set of values that make this statement true.



$x < 2\frac{1}{2}$

$2\frac{1}{2}$ is not in the set of values that make this statement true.

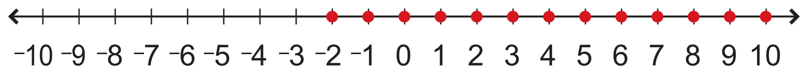
Number line graphs look different depending on the set of numbers used.

Example 4

How does the set of numbers used for the variable affect the graph of the inequality?

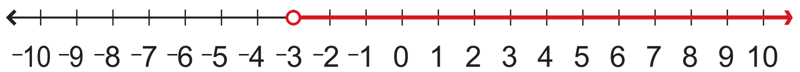
If the variable is a member of the integers, then the set of values that makes the inequality true will be integers. Thus, only integers are marked on the number line.

Graph $x > {}^-3 \quad x \in \mathbb{Z}$



If the variable is a member of the real numbers, then the set of values that makes the inequality true will be real numbers. A solid arrow is used to show that all fraction and decimal values make the inequality true.

Graph $x > {}^-3 \quad x \in \mathbb{R}$



8. List some numbers that satisfy the following conditions. Then graph the solution to these inequalities on a number line. Note the sets of numbers used.

a) $x \geq 1 \quad x \in \mathbb{R}$

b) $x < {}^-4 \quad x \in \mathbb{Z}$

c) $23 - x > 13 \quad x \in \mathbb{Z}$

d) How would the graph of Part b change if x were a real number? A whole number?

Think Beyond

e) $x < {}^-4$ or $x \geq 1 \quad x \in \mathbb{Z}$

Variables in Expressions

MATHEMATICALLY SPEAKING
- evaluate

Unlike an equation, an expression cannot be true or false. So, you cannot "solve" an expression. However, if you are given specific values for the variables, you can "evaluate" the expression. This means you substitute the values of the variables into the expression and simplify.

Example 5

In football, a team gets 3 points for every field goal. You can represent the points obtained by field goals with the expression $3m$, where m stands for the number of field goals. You can evaluate this expression for specific values of the variable.

To find the number of points scored in four field goals, substitute 4 for m.

$3m = 3(4) = 12$

The value of an expression can change. If there are five field goals, you substitute 5 for m. The expression now is equivalent to 3 • 5, and its value is 15.

9. Examine the set of cards from earlier in the lesson. Sort the cards with variables into the following groups:

Group 1: The variables each represent one specific value.

Group 2: The variables represent related varying quantities.

Group 3: The variables represent many values.

Group 4: Cards that don't fit into Groups 1–3

10. Use cards C, K and Q from the set you used earlier in the lesson. Evaluate these cards when $m = -4$, $y = -1$, and $x = \frac{1}{2}$.

Wrap It Up

Variables can be used in different ways. Explain these uses and how variables can be used in algebra.

MATHEMATICALLY SPEAKING

- constant
- equation
- evaluate (an expression)
- expression
- inequality
- solution
- solve (an equation)
- variable

LESSON 1.1

SECTION 1

On Your Own

Write About It

1. Explain the three ways that variables are used in equations and inequalities.

2. Is it possible to "solve" an expression? Why or why not?

3. Kayla sorted the algebra cards in the following way. Describe her sorting plan.

Group 1

$7x + 2y$
$a + b = b + a$
$y = {}^{-}2x$
$y = x - 1$

Group 2

$A = lw$
$C = \pi d$

Group 3

$36 + 8$
$4 \cdot 9 = 6 \cdot 6$

Group 4

$5(3 + m) = 15 + 5m$
$23 - x > 13$
$7m$
$a \cdot \frac{1}{a} = 1, a \neq 0$
$6 + s = 10$
$35 = 7g$
$5 + 9 = n + 6$
$4x - x = 3x$
$14 - y$
$x < {}^{-}4$

4. Evaluate the following expressions first for $y = {}^{-}6$ and then for $y = \frac{1}{3}$.

   a) $14 - y$ **b)** $7y$ **c)** $3y + 2y$ **d)** $6\left(y + \frac{1}{2}\right)$

5. You may have classified some of the equations in the card activity as properties and others as formulas. Give your own example of a property and of a formula.

6. Create an example for each of the situations below. State the set of numbers you are using.

a) an equation with variables that represent related varying quantities

b) a property with one or more variables that represent many values

c) an equation with one or more variables that represent specific values

d) an inequality with a variable that represents a set of numbers

7. A new student in your class has never learned about variables. Tell him what he needs to know about variables in order for him to make sense of their use in equations and expressions. Identify the values the variables can assume.

8. Match the phrases to the mathematical expressions.

a) 2 more than x	i) $\frac{1}{2}x$
b) 2 times the value of x	ii) $5 + 2x$
c) the value of x reduced by 2	iii) $x + 2$
d) 2 times the value of x plus 5	iv) $x - 2$
e) half of x	v) $2x$

9. Graph each inequality on a number line.

Hint
See page 157

a) $^{-}3 < x \quad x \in \mathbb{Z}$

b) $x \leq 2 \quad x \in \mathbb{Z}$

c) $4.5 > x \quad x \in \mathbb{R}$

d) $x \geq -\frac{1}{2} \quad x \in \mathbb{R}$

10. Calvin raised $150 for charity. He plans to divide it equally between n charities so that each group receives D dollars.

a) Write an equation that shows how Calvin plans to distribute the money.

b) How are the variables used in this equation?

c) Describe values for n that make sense in this situation.

LESSON 1.1

SECTION 1

On Your Own

11. In a basketball game, a basket made on a free throw is worth 1 point, baskets made from inside the 3-point line are worth 2 points, and those made from outside the 3-point line are worth 3 points. Use variables to write an equation for the total score of one team for each game. Explain what each of your variables represents.

12. How are the ages of your family members related to you? Let x represent your age today in years.

a) Write two different expressions using x that represent the ages of two people in your family.

Hint
See page 157

b) Evaluate each expression for $x = 5$. What does the value of each expression mean?

13. Write the following expressions using variables.

a) 7 less than n

b) 9 times y

c) the sum of x and 5, divided by 2

d) 12 more than s

e) the product of 5 and w decreased by $^{-}4$

f) k times the sum of a and b

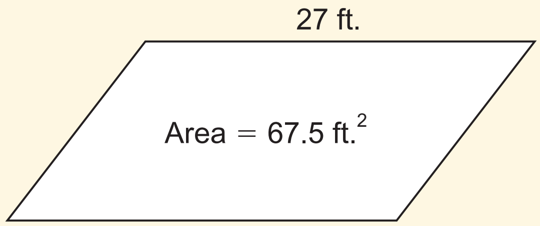
14. There are many types of mathematical statements in algebra. How are they similar and how are they different? Give examples.

Think Beyond

15. In mathematics, the Greek letter π represents a constant. Are there other letters or symbols mathematicians use to represent constants? Investigate this question using the Internet and write a report for the class.

Think Back

16. What is the height of this parallelogram?



A. 2.5 ft.

B. 2.5 $ft.^2$

C. 1,822.5 ft.

D. 1,822.5 $ft.^2$

17. Evaluate: $17 + (-28) \div (-2) - 45$.

18. Solve: $\frac{2}{3}s = 14$. Show your work.

19. Is the following equation always, sometimes or never true? $\triangle$ and $\bigcirc$ represent real numbers. Explain your answer.

$$\triangle \cdot \bigcirc = -\triangle \cdot -\bigcirc$$

20. If a recipe uses 114 chocolate chips to make 12 giant cookies, what is the average number of chocolate chips per giant cookie?