Understanding and Using Variables

LESSON 1.1 Variables

Suggested Pacing: 3 Days

In this lesson, students first differentiate among equations, expressions and inequalities by sorting Algebra Cards. They then learn about different uses of variables and how the set of numbers being used affects the solution.

LESSON OBJECTIVES

- Students will identify the difference between expressions, equations and inequalities.
- Students will describe the different meanings of variables in expressions, equations and inequalities.
- Students will graph inequalities on a number line.

DAY 1	MATERIALS*	ESSENTIAL *ON YOUR OWN* QUESTIONS
Algebraic Statements	**In Class** ■ Lesson Guide 1.1: Algebra Cards ■ Lesson Guide 1.1: *Sets of Numbers* ■ Lesson Guide 1.1A: *Understanding Variables* ■ Lesson Guide 1.1: *Number Lines*	Questions 1–5, 16, 17
DAY 2	**MATERIALS***	**ESSENTIAL *ON YOUR OWN* QUESTIONS**
Variables Representing a Specific Value Variables Representing Related Varying Quantities		Questions 8, 10–12, 14, 18
DAY 3	**MATERIALS***	**ESSENTIAL *ON YOUR OWN* QUESTIONS**
Variables Representing Many Values Variables in Expressions		Questions 6, 7, 9, 13, 19–20

* The Think Like a Mathematician Daily Record Sheet should be used daily

MATHEMATICALLY SPEAKING

- constant
- evaluate (an expression)
- inequality
- solve (an equation)
- equation
- expression
- solution
- variable

SECTION 1 THINKING LIKE A MATHEMATICIAN

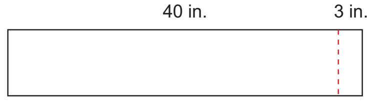
Understanding and Using Variables

Algebra is a powerful tool for understanding the world. You can represent ideas and relationships using symbols, tables and graphs. In this section you will learn about variables and investigate how they are used to model real situations. You may also learn some unusual facts about how people spend their free time!

LESSON 1.1 Variables

Start It Off

Marcelino was asked to find the area of a 43 in. by 8 in. rectangle using mental math. He drew this picture.



1. Marcelino found the areas of the two smaller rectangles and added them together. Why is this a useful application of the "mental math" method?
2. Which of these statements best represents his method? Why?

A.	**B.**	**C.**
8 • 43	8 • 43	8 • 43
= 8(40 + 3)	= 8(40 + 3)	= 8(40 + 3)
= 320 + 3	= 40 + 24	= 320 + 24
= 323	= 64	= 344

3. Marcelino's method is an example of the distributive property. Write down two things you know about the distributive property.

Start It Off

It is important that students see that the area is the same regardless of whether Marcelino multiplies 43 by 8 or breaks one factor (43) into an equivalent sum (40 + 3) and then multiplies each addend by the other factor (8 • 40 + 8 • 3). Discuss Questions 2 and 3 with the whole class. Students need to see how multiplying both "parts" of 43 by 8 results in the same area and recognize its symbolic representation. Write on the board all their ideas about the distributive property.

1. It is easier to multiply 8 by 40 and 8 by 3 and then sum the products than to multiply 8 by 43.
2. **C.** Both the 40 and the 3 are multiplied by 8.
3. Answers will vary.

DAY 1 TEACHING THE LESSON

Algebraic Statements

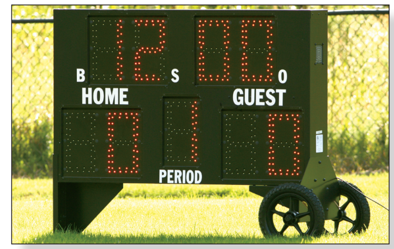
Talk Moves Distribute envelopes with Algebra Cards to pairs or small groups of students. Ask students to spread the cards out so they can see each card. Have partners discuss what they notice and then have a few students share with the whole group. Ask:

- ***Look at the algebraic statements on these cards. What do you notice about them?***
- ***Do each of these have letters? Do each of these have numbers?***
- ***What operations are used on these cards?***
- ***Are any cards similar to one another? In what way are they similar?***

Differentiation

Think Differently: Students commonly misinterpret the variable in an expression like 7*m* as a label (7 meters or 7 minutes) rather than as the product of a number and a variable. You may wish to highlight cards C (7*m*), M (35 = 7*g*) and I (*A* = *lw*) and discuss with students how multiplication can be indicated: with a "×" as in 7 × *m*, with a "•" as in 7 • *m*, or by placing letters or numbers next to each other such as 7*m* and 3(4). Point out how parentheses must be used to indicate multiplication of two numbers, 3(4) = 12, so it won't be confused with a two-digit number (34).

Are you interested in sports? Most students have a favorite sport they enjoy watching or playing. Mathematical symbols such as variables, operations and numbers can be used to represent and describe many situations associated with sports.



Algebraic Statements

Equations, expressions and inequalities are the building blocks of algebra. Examine the cards below.

A $4 \cdot 9 = 6 \cdot 6$	**B** $C = \pi d$	**C** $7m$
D $23 - x > 13$	**E** $y = -2x$	**F** $a + b = b + a$
G $36 + 8$	**H** $y = x - 1$	**I** $A = lw$
J $5 + 9 = n + 6$	**K** $14 - y$	**L** $a \cdot \frac{1}{a} = 1, a \neq 0$
M $35 = 7g$	**N** $5(3 + m) = 15 + 5m$	**O** $6 + s = 10$
P $4x - x = 3x$	**Q** $7x + 2y$	**R** $x < -4$

Have pairs complete Question 1. Bring the group together to discuss. Focus particularly on Question 1c. Ask:

- ***What are the main differences between equations and expressions?*** An equation is a statement that two expressions or quantities are equivalent. Equations have equal signs. An expression is a collection of numbers/variables and operations that represent a quantity. An equation states that two expressions are equal.
- ***Are there any cards that do not show either an equation or an expression?*** Two cards are neither equations nor expressions but are inequalities (23 − *x* > 13 and x < −4). Inequalities represent relationships between expressions using <, > , ≤ or ≥ symbols. Asking this question and having students explain and debate will highlight important differences between equations, expressions and inequalities.

Students may distinguish between equations and expressions by indicating that equations can be "solved" and expressions cannot. This would be a good opportunity to discuss the difference between evaluating, simplifying and solving. Expressions can be simplified. For instance, 7*x* − *x* can be simplified to 6*x* and with Card G, 36 + 8 can be simplified to 44. This can be confusing for students because they may think they have "solved" 36 + 8 by finding the sum of these numbers. Since equations consist of two expressions, they also can be simplified by combining terms. Variable expressions can also

MATHEMATICALLY SPEAKING
- equation
- inequality
- expression
- constant
- variable

1. a) Take a set of the cards and sort them into three groups—equations, inequalities and expressions.
   b) How are the groups the same?
   c) What do you think are the most important differences between expressions, equations and inequalities?
   d) Write a definition for each of the terms: equation, expression and inequality.
2. a) Now sort the equation cards into different groups that you select.
   b) Describe the cards in each group and explain why you put certain cards together.
   c) Write two additional cards to add to each sorted group.

Many equations, inequalities and expressions have letters in them. Some letters are constants. A constant is a symbol that represents exactly one quantity. Every time the constant letter or symbol is used, the same value is substituted. The Greek letter π, called "pi," is an example of a constant. The value of π does not change; it is always the number equal to the circumference of a circle divided by the diameter of that circle.

Other letters, or symbols such as ❑ and Δ, are called variables. In equations, expressions and inequalities, one or more quantities can be substituted for a variable. In this lesson, you will learn about the different types of variables.

Variables Representing a Specific Value

MATHEMATICALLY SPEAKING
- solve (an equation)
- solution

Some variables represent a specific value. When you are asked to "solve an equation," you are to find the value of the variable that makes that equation true. This value is known as the solution. When variables represent specific values, there is often only one variable or letter in the equation.

be evaluated. For instance, with Card K, $14 - y$ could be *evaluated* if we substitute a value for y. In algebra, variable expressions are not "solved." Equations are solved. Namely, students are able to determine the value of the variable in the equation. For example, "solve for x" might be the direction line for $7 + x = -2$.

Have students continue with Question 2. Encourage students to look for different ways to sort the equations into two groups. As students sort the cards, circulate and note the ways they are sorting and the language they are using to describe the symbol strings. This is a good opportunity to informally assess students. Do they know and use the word variable? Do they recognize that some of the cards show mathematical formulas or properties? Following the sorting activity in Question 2, you might want students to read the last paragraphs in this section about variables and constants.

1. a) Equations
   A. $4 \cdot 9 = 6 \cdot 6$
   B. $C = \pi d$
   E. $y = -2x$
   F. $a + b = b + a$
   H. $y = x - 1$
   I. $A = lw$
   J. $5 + 9 = n + 6$
   L. $a \cdot \frac{1}{a} = 1$
   M. $35 = 7g$
   N. $5(3 + m) = 15 + 5m$
   O. $6 + s = 10$
   P. $4x - x = 3x$

   Inequalities
   D. $23 - x > 13$
   R. $x < -4$

   Expressions
   C. $7m$
   G. $36 + 8$
   K. $14 - y$
   Q. $7x + 2y$

   b) All groups have letters and/or numbers combined with at least one operation.
   c) Equations show that two quantities are equivalent, while expressions alone do not. Variable equations can be solved, while variable expressions can be evaluated. Inequalities express a relationship between two expressions such as one is greater than the other.
2. a) Students will sort equations in various ways. Possible ways include: equations with one variable, equations with two variables, equations that are formulas, equations that are properties, equations with no variables, equations with an isolated variable on one side of the equal sign.

Example 1

Major League Baseball teams play 162 games every season. In 2008, the San Diego Padres won 63 games. How many games did they lose?

Let n represent the number of games the Padres lost.

Equation: $63 + n = 162$

Solution: $n = 99$ The Padres lost 99 games in the 2008 season.

Verify solution: $63 + 99 = 162$

When asked to solve equations and find a specific value for a variable, we usually assume that the solution is a real number. But sometimes there are restrictions on which set of numbers can be used. This can affect the solutions.

Let's Review

Natural Numbers ($\mathbb{N}$)	{1, 2, 3, 4, 5, ….}
Whole Numbers	{0, 1, 2, 3, 4, 5, ….}
Integers ($\mathbb{Z}$)	{…−3, −2, −1, 0, 1, 2, 3, …}
Rational Numbers ($\mathbb{Q}$)	Any number that can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$
Real Numbers ($\mathbb{R}$)	Any number that can be represented on a number line

3. Find the solution to each equation below. Note which set of numbers can be used.

a) $5x = -15$ x belongs to the set of integers.

b) $5x = -15$ x belongs to the set of natural numbers.

c) $2x = 13$ x belongs to the set of real numbers.

d) $2x = 13$ x belongs to the set of integers.

Mathematicians use symbols to record information using the least number of words. Rather than write out the name of the set of numbers, they use its abbreviation. Another common abbreviation is the symbol, $\in$, which means "is an element of" or "is a member of."

$x \in \mathbb{Z}$ x is an element of the set of integers.

$x \in \mathbb{N}$ x is an element of the set of natural numbers.

$x \in \mathbb{R}$ x is an element of the set of real numbers.

b) Answers will vary. Sample response:

Equations that are formulas: $A = lw$, $C = \pi d$. Both of these equations are measurement formulas.

c) Answers will vary.

Summarize Day 1

Bring students together to share how they sorted the equations. Select students to discuss the different ways they sorted the cards, such as those mentioned in the answer to Question 2a. Make a list of the attributes students used along with the equations that have those attributes. Encourage students to consider if they agree with their classmates' lists and ask them to create other equations that fit those categories. You may also wish to present pairs of equations and ask students to describe why the equations could be grouped together. Some pairs to use include:

- Card M ($35 = 7g$) and Card I ($A = lw$) (Both show a product and two factors.)
- Card P ($4x - x = 3x$) and Card N ($5(3 + m) = 15 + 5m$) (When the expressions on each side of the equal sign are simplified, you get the same expression on each side.)

Day 2 Teaching the Lesson

Begin by projecting several of the Algebra Cards. Bring students' attention to the variables, and ask:

- ***We noticed that some of these equations and expressions have letters. What are these letters called?*** variables
- ***What do the variables mean?*** Most likely students will indicate that a variable is a symbol that represents an unknown quantity.
- ***Have you ever heard of a constant value? What is a constant?*** A constant represents exactly one quantity.

4. Solve for n.

   a) $\frac{2}{3}n = -\frac{1}{4}$ $\quad n \in \mathbb{Z}$

   b) $\frac{2}{3}n = 2$ $\quad n \in \mathbb{N}$

   c) $\frac{2}{3}n = -\frac{1}{4}$ $\quad n \in \mathbb{R}$

5. Write an equation with a variable to represent each situation. Define the variable. Then, solve the equation. Be sure to check your answer by substituting the value back into the equation.

   a) The total of two teams' scores for a basketball game was 176 points. One team scored 97 points. What was the other team's score?

   b) How much time does it take a bicyclist to finish a 75-mile race if she averages 20 miles per hour?

   c) Pole-vaulting is an athletic field event. Homer can pole-vault 2.5 times as high as Sally. If he can pole-vault 15 feet, how high can Sally pole-vault?

Variables Representing Related Varying Quantities

Some equations have two or more variables. If the value of one variable changes when the value of another variable changes, then there is a mathematical relationship that links the two variables. The variables represent related varying quantities. When two or more variables are mathematically related, there will be many different combinations of values for the variables that will make the equation true.

Differentiation

Think Differently: Some students may benefit from reading the text aloud with their partner or by rereading parts to aid in comprehending the text. You may wish to use Lesson Guide 1.1A: *Understanding Variables* and have students record information in the chart about the different meanings of *variable*. Accommodation Guides (Lesson Guides that include an A after the number) are designed to provide additional support for those students experiencing difficulty with a lesson activity. English language learners may benefit from highlighting vocabulary words in the *Understanding Variables* table and linking the words with the actual examples (e.g., inequality, $50 \geq y$).

Variables Representing a Specific Value

Mention that today students are going to learn that variables can have different meanings. Have students read the last two paragraphs about variables in the Algebraic Statements section, if you haven't done so, and the text from Variables Representing a Specific Value through the example. Have students explain the parts of the example to each other to reinforce their understanding. Students may not have yet learned that in order to check a solution, they should substitute the value of the variable back into the original equation to see if it makes the equation true.

Review the different sets of numbers that were introduced in the unit *Let's Be Rational: Focusing on Fractions, Decimals and Integers*. Then introduce students to the symbols used to indicate the different sets of numbers. Use a transparency of Lesson Guide 1.1: *Sets of Numbers*. Note there is no standard symbol for the set of whole numbers.

$\mathbb{N}$ set of natural numbers {1, 2, 3, 4, 5, . . .}

$\mathbb{Z}$ set of integers {. . . , −3, −2, −1, 0, 1, 2, 3, . . .}

$\mathbb{Q}$ set of rational numbers {numbers in the form $\frac{a}{b}$, a and $b \in \mathbb{Z}$, $b \neq 0$}

$\mathbb{R}$ set of real numbers {all rational and irrational numbers}

Then have students work in pairs on Questions 3–5. Discuss as a class Questions 3 and 4. Whenever solving equations, mathematicians need to know the set of numbers that is being used. If the set used is only the integers, then the equation $5x = 2$ has no solution because there are no integer values for x that make the equation true. However, if the set used is rational numbers or real numbers, then the solution is $x = \frac{2}{5}$. Unless otherwise indicated, students should assume that the real numbers are being used when solving equations.

Variables Representing Related Varying Quantities

Have students read the text under Variables Representing Related Varying Quantities and answer Question 6. Ask:

- ***Explain the baseball ticket example in your own words.***
- ***Identify cards from the Algebra Deck where the variables represent related varying quantities. How can you tell?*** Cards B, E, H and I. There are two or more variables on opposite sides of the equal sign and as the value of one variable changes, the value(s) of the other variable changes.

Then start the section, Variables Representing Many Values, and discuss the first part about properties. Ask students to work with a partner and list as many properties as they can, using variables to generalize the property (e.g., $a + b = b + a$ instead of $3 + 7 = 7 + 3$). Do not worry if students do not have complete knowledge of properties, as this activity is a preview for a lesson later in the unit on this topic. Have pairs

Talk Moves

of students present a property, explain it, and explain why the variable(s) represents many values. Students may give identities that are mathematically true statements, such as $a + b - b = a$, but are not technically properties. This is fine as long as the variables in the equations can represent many values.

Finally, have students answer Question 7 before summarizing the material.

Example 2

Ticket prices for many baseball games depend on the opponent and the day of the week. If a ticket to the Chicago White Sox costs $15 for a specific Monday game, we can write an equation for finding the cost of any number of tickets to that game: $C = 15n$. In this case, n represents the number of tickets purchased and C represents their total cost in dollars.

Number of Tickets (n)	Cost in Dollars (C)
0	0
1	15
2	30
5	75
8	120

variable → C = 15n ← variable

$$C = 15n$$

As the number of tickets, n, increases, the cost in dollars of the tickets, C, increases.

Since there are two variables in this equation, a solution must give a value for both n and C. These two values are often listed as an ordered pair, (n, C). The ordered pairs (0, 0), (1, 15), (2, 30), (5, 75) and (8, 120) are some, but not all, of the solutions to this equation.

6. **a)** What do the variables n and C represent?

b) Why are these variables referred to as "varying quantities"?

c) What is the mathematical relationship between these quantities?

d) List five other pairs of values that make the equation $C = 15n$ true. How many pairs of values are in the solution?

Variables Representing Many Values

You may have seen variables used in the statement of mathematical properties. For example, the commutative property of addition states that $a + b = b + a$, for all real numbers a and b. It is important to realize that a and b are not mathematically related. In other words, changing the value of a does not change the value of b. Rather, a and b are being used as "generalized numbers." The statement $a + b = b + a$ is true for any real numbers a and b.

Variables in mathematical properties generalize important ideas. The variables can represent many values and are not related. Examine the equations below. These equations are true for all values of the variable(s). The variables can equal any real numbers. Not all of these statements are properties.

$a + a = 2a$ $\quad$ $a + b - b = a$

$a + b = b + a$ $\quad$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

7. Write three different equations that use variables as "generalized numbers." Are each of the equations you wrote a property?

An inequality is one type of mathematical sentence that expresses a relationship between numbers and variables. Variables in inequalities usually represent a set of values. This in turn means that there usually are many solutions to an inequality.

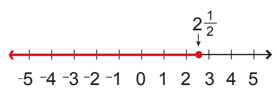
Example 3

At one time Major League Soccer players had a cap on their salaries of $2 million. This restriction can be written using the inequality $x \leq 2{,}000{,}000$, where x is the salary in whole numbers of dollars.

$x \leq 2{,}000{,}000$ $\quad$ $x \in$ whole numbers

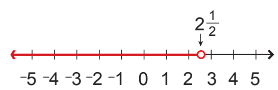
The solution for x is the set of whole numbers between 0 and 2,000,000, including 0 and 2,000,000. That is, salaries can be from $0 through $2 million.

A variable in an inequality often represents many values. The solution to an inequality can be graphed on a number line. When a solid point is used on a graph, it means that the point is a solution. When an open point or circle is used, it means that the point does not satisfy the stated relationship and is not a solution. A ray on the number line includes all possible points along the ray. In the following examples, $x \in \mathbb{R}$.



$x \leq 2\frac{1}{2}$

$2\frac{1}{2}$ is in the set of values that make this statement true.



$x < 2\frac{1}{2}$

$2\frac{1}{2}$ is not in the set of values that make this statement true.

Summarize Day 2

Display cards M, H and F from the Algebra Deck and lead a short discussion about each card, asking the following questions:

$35 = 7g$

- ***What does the g mean in this equation?*** It represents a number.
- ***Could g have lots of different values? Why or why not?*** not if we want the equation to be a true statement
- ***Without actually solving for g, what can we say about g?*** g is a factor of 35; $g < 35$
- ***What set of numbers are we using when thinking about this variable?*** While the value of g is an integer, we usually assume that we are using the real numbers when solving equations.

$y = x - 1$

- ***What does the y mean? The x?*** They represent numbers.
- ***How are x and y related? Do we know which has a greater value? Why or why not?*** x will always be greater than y since $y = x - 1$.
- ***What happens to the value of y if the value of x increases? How do you know?*** As x increases, y increases and as x decreases, y decreases.
- ***Could y or x have many different values? Why or why not?*** Yes, there are many pairs of related values that make the equation true.
- ***How would the set of numbers used for x and y affect the solution?*** The set of numbers used affects the values that can be substituted for x and consequently affects the value of $(x - 1)$ or y. If the set of integers is used, then $x \neq \frac{1}{2}$.

$a + b = b + a$

- ***What types of numbers could be used to represent the variable a? The variable b?*** real numbers or any of its subsets
- ***Can we tell how a and b are related? Do we know from this equation if a = b? Can a = b?*** No, a and b are not related in that one doesn't change as the other changes; no; yes.

Number line graphs look different depending on the set of numbers used.

Example 4

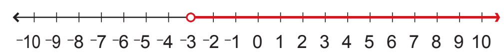
How does the set of numbers used for the variable affect the graph of the inequality?

If the variable is a member of the integers, then the set of values that makes the inequality true will be integers. Thus, only integers are marked on the number line.

Graph $x > -3 \quad x \in \mathbb{Z}$

If the variable is a member of the real numbers, then the set of values that makes the inequality true will be real numbers. A solid arrow is used to show that all fraction and decimal values make the inequality true.

Graph $x > -3 \quad x \in \mathbb{R}$



8. List some numbers that satisfy the following conditions. Then graph the solution to these inequalities on a number line. Note the sets of numbers used.

a) $x \geq 1 \quad x \in \mathbb{R}$

b) $x < -4 \quad x \in \mathbb{Z}$

c) $23 - x > 13 \quad x \in \mathbb{Z}$

d) How would the graph of Part b change if x were a real number? A whole number?

Think Beyond

e) $x < -4$ or $x \geq 1 \quad x \in \mathbb{Z}$

Variables in Expressions

MATHEMATICALLY SPEAKING
- evaluate

Unlike an equation, an expression cannot be true or false. So, you cannot "solve" an expression. However, if you are given specific values for the variables, you can "evaluate" the expression. This means you substitute the values of the variables into the expression and simplify.

- ***Could a and b have many different values? Why or why not?*** Yes, because it is a property.

3. a) $x = -3$

b) There is no solution for x in the set of natural numbers.

c) $x = 6.5$, which is a rational number

d) There is no solution for x in the set of integers.

4. a) no solution in the set of integers

b) $n = 3$

c) $n = -\frac{3}{8}$

5. a) $p = 176 - 97$ or $97 + p = 176$, where p is the number of points scored by the other team; $p = 79$; use the set of positive integers for points scored.

b) $t = 75$ mi ÷ 20 mi/hr where t is the time in hours; 3.75 hours, or 3 hours 45 minutes; use the set of positive rational numbers for time to complete race.

c) $15 = 2.5d$, where d equals the number of feet Sally can pole-vault; $d = 6$ feet; use the set of real numbers for height of jump.

6. a) n represents the number of tickets and C represents the cost in dollars for n tickets.

b) They are called varying quantities because n and C can take on different values. They are related since as n changes, so does C.

c) The cost in dollars, C, is 15 times the number of tickets, n.

d) Answers will vary. Possible answers include (3, 45), (4, 60), (6, 90), (10, 150), (20, 300).

7. Answers will vary. Possible response: $a \cdot b = b \cdot a$, where a and b are elements of the real numbers. This is the commutative property of multiplication.

DAY 3 TEACHING THE LESSON

Variables Representing Many Values

Take a few minutes at the start of class to quickly review the three interpretations of variables studied so far. Then introduce a new interpretation from this section, when variables are part of inequalities. Review Example 3 and explain that "a cap on a salary" means that there is a maximum amount for the salary. Next show students Card R and ask:

$x < {}^{-}4$

- ***How is an inequality different from an equation?***
- ***Compare x = −4 to x < −4. How does the value of the variable change?***
- ***What other types of algebraic statements have variables that represent many values?***

Introduce graphing one-variable equations or inequalities on a number line. The solution to most inequalities, such as $x \leq 2{,}000{,}000$, is a set of numbers rather than a single number. Students may need to list possible solutions in order to fully grasp the size of a solution that is a set. Have students examine the two graphs, $x \leq 2\frac{1}{2}$ and $x < 2\frac{1}{2}$, as you explain when to use open or closed circles.

Next discuss how the set of numbers used affects the solution set by graphing the inequality $x > {}^{-}3$. If x is a member of the integers, then only points will be marked on the number line. But if x belongs to the real numbers, then we use solid rays to indicate that all values are part of the solution.

Students may question why x belongs to the real numbers and not the rational numbers when drawing a solid ray on the number line. Yes, the rational numbers include all fractions and some decimals (those that terminate or repeat), but the rationals do not include *all* decimals. When we draw a solid line between integers, we are saying *all* fraction and decimal values between the two integers are included, so we use the set that includes both types of numbers, the real numbers. It is impossible to show a number line graph that only shows the rational numbers since there are an infinite number of irrationals on the number line. Ask students:

- ***What does the open circle represent on the graph of x > ⁻3?*** ⁻3 is not included
- ***How would we show x ≥ ⁻3 where x belongs to the real numbers?*** a solid dot at ⁻3, solid ray going to the right

Pass out copies of Lesson Guide 1.1: *Number Lines* for students to graph the inequalities in Question 8. Discuss the solutions to any of the inequalities. You may wish to skip Question 8e.

Variables in Expressions

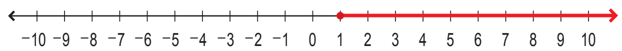
Introduce the final section in this lesson, Variables in Expressions. Have students read and report back on the how variables are interpreted in expressions. Ask questions about Algebra Card C:

7m

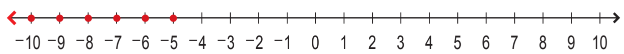
- ***What do we know about m? What does the m mean in this expression?***
- ***Could m have many different values?***
- ***Can we tell from this expression how 7 and m are related?*** No, only that the value of the expression is the product of *m* and 7.

Question 9 will be used in the *Wrap It Up* discussion. Ask students to complete these questions independently and then compare solutions and their reasons with a partner. You can collect informal assessment data on students' understanding by listening to their responses as they share with a partner.

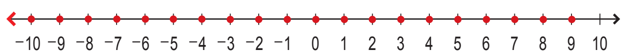
8. a) $x \geq 1$ where x belongs to the set of real numbers. Possible answers: 2, $3\frac{1}{2}$, 5.5, 7



b) $x < -4$ where x is an integer.
Possible answers: −5, −8, −20

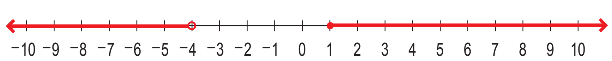


c) $23 - x > 13$ where x is an integer.
Possible answers: 9, 2, −8



d) If x were a real number in Part b, there would be no dots, but instead a solid ray from an open dot at −4 going to the left on the number line. If x were a member of the set of whole numbers in Part b, there would be no dots or rays on the number line as the inequality $x < -4$ has no solutions in the whole numbers.

e) **Think Beyond** $x < -4$ or $x \geq 1$ where x is a real number.



9. a)

Group 1: one specific value
J. $5 + 9 = n + 6$
M. $35 = 7g$
O. $6 + s = 10$

Group 2: related varying quantities
B. $C = \pi d$
E. $y = -2x$
H. $y = x - 1$
I. $A = lw$

Group 3: many values
D. $23 - x > 13$
F. $a + b = b + a$
L. $a \cdot 1/a = 1, a \neq 0$
N. $5(3 + m) = 15 + 5m$
P. $4x - x = 3x$
Q. $7x + 2y$
R. $x < -4$

Group 4: none of these meanings
A. $4 \cdot 9 = 6 \cdot 6$
C. $7m$
G. $36 + 8$
K. $14 - y$

10.

C. $7m = -28$ when $m = -4$

K. $14 - y = 15$ when $y = -1$

Q. $7x + 2y = 1\frac{1}{2}$ when $x = \frac{1}{2}$ and $y = -1$

Wrap It Up

Bring students together to share their responses to Question 9. You may wish to have students fill out Lesson Guide 1.1A: *Understanding Variables*. Then make a large class chart on the board or on the overhead (see sample chart below). Have students talk about their solutions and reasoning for Question 9.

Provide some new algebraic statements such as those listed below. Have pairs talk about the meaning of the variables in each statement and the category in which each should be placed and why.

$12 < y$

$\frac{1}{2}x - 44$

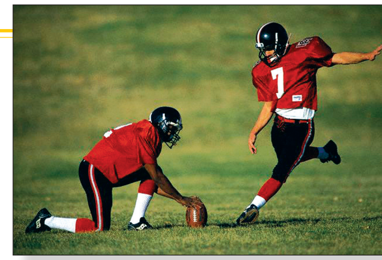
$c = 3a + 20$

$45y + 30y$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Example 5

In football, a team gets 3 points for every field goal. You can represent the points obtained by field goals with the expression $3m$, where m stands for the number of field goals. You can evaluate this expression for specific values of the variable.



To find the number of points scored in four field goals, substitute 4 for m.

$3m = 3(4) = 12$

The value of an expression can change. If there are five field goals, you substitute 5 for m. The expression now is equivalent to $3 \cdot 5$, and its value is 15.

9. Examine the set of cards from earlier in the lesson. Sort the cards with variables into the following groups:

Group 1: The variables each represent one specific value.

Group 2: The variables represent related varying quantities.

Group 3: The variables represent many values.

Group 4: Cards that don't fit into Groups 1–3

10. Use cards C, K and Q from the set you used earlier in the lesson. Evaluate these cards when $m = -4$, $y = -1$, and $x = \frac{1}{2}$.

Wrap It Up

Variables can be used in different ways. Explain these uses and how variables can be used in algebra.

MATHEMATICALLY SPEAKING

- constant
- equation
- evaluate (an expression)
- expression
- inequality
- solution
- solve (an equation)
- variable

Reflect

Use these questions to help you reflect on the lesson and plan for future instruction.

- Do students understand the differences among equations, expressions and inequalities and when each is used?
- Are students able to analyze an expression, equation and inequality and describe the meaning of the variable?
- Can students describe the different interpretations of variables?
- Can students graph inequalities on the number line?

Variables	My Notes	Examples
Variables that Represent a Specific Value	• only one value makes equation true • often only one variable in equation • solve equation to find the one specific value of the variable	$P = 176 - 97$ $88 + n = 162$
Variables that Represent Related Varying Quantities	• equations have 2+ variables • variables related—if value of one changes, value of other changes • replacing variables with different pairs of values gets you true statements	$C = 15n$ $A = lw$
Variables that Represent Many Values	• there are many values that make equation true • change in one variable doesn't mean change in another	$a + 0 = a$ $x \leq 2{,}000{,}000$

On Your Own

1. **Write About It** Answers will vary. Sample response:

Variables may be used to represent specific values. This means there is usually one value that makes an equation true (but there can be more than one specific value, e.g., $x^2 = 4$ has two specific values). Variables may also be used to represent related varying quantities. This means the equation has two or more variables and these variables are related in some way. When the value of one of the variables changes, the value of the other changes as well. Finally, variables can be used to represent many values. This means that the variable represents many unknown values. In these situations there are many numbers that make an equation true. Variables in inequalities are usually equal to a set of values.

2. No, expressions cannot be "solved." There is no equation so we cannot find the value of the variable or variables that makes the equation true. Expressions can be evaluated.

3.

Group 1: 2 variables

Group 2: formula

Group 3: no variables

Group 4: 1 variable

LESSON 1.1 SECTION 1

On Your Own

Write About It

1. Explain the three ways that variables are used in equations and inequalities.
2. Is it possible to "solve" an expression? Why or why not?
3. Kayla sorted the algebra cards in the following way. Describe her sorting plan.

Group 1

$7x + 2y$
$a + b = b + a$
$y = -2x$
$y = x - 1$

Group 2

$A = lw$
$C = \pi d$

Group 3

$36 + 8$
$4 \cdot 9 = 6 \cdot 6$

Group 4

$5(3 + m) = 15 + 5m$
$23 - x > 13$
$7m$
$a \cdot \frac{1}{a} = 1, a \neq 0$
$6 + s = 10$
$35 = 7g$
$5 + 9 = n + 6$
$4x - x = 3x$
$14 - y$
$x < -4$

4. Evaluate the following expressions first for $y = -6$ and then for $y = \frac{1}{3}$.
   a) $14 - y$ b) $7y$ c) $3y + 2y$ d) $6\left(y + \frac{1}{2}\right)$
5. You may have classified some of the equations in the card activity as properties and others as formulas. Give your own example of a property and of a formula.

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4. a) $14 - (-6) = 20$

$14 - \left(\frac{1}{3}\right) = 13\frac{2}{3}$

b) $7(-6) = -42$

$7\left(\frac{1}{3}\right) = \frac{7}{3} = 2\frac{1}{3}$

c) $3(-6) + 2(-6) = -18 + (-12) = -30$

$3\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right) = 1 + \frac{2}{3} = 1\frac{2}{3}$

d) $6\left((-6) + \frac{1}{2}\right) = 6\left(-5\frac{1}{2}\right) = -33$

$6\left(\frac{1}{3} + \frac{1}{2}\right) = 6\left(\frac{5}{6}\right) = 5$

6. Create an example for each of the situations below. State the set of numbers you are using.

   a) an equation with variables that represent related varying quantities

   b) a property with one or more variables that represent many values

   c) an equation with one or more variables that represent specific values

   d) an inequality with a variable that represents a set of numbers

7. A new student in your class has never learned about variables. Tell him what he needs to know about variables in order for him to make sense of their use in equations and expressions. Identify the values the variables can assume.

8. Match the phrases to the mathematical expressions.

a) 2 more than x	i) $\frac{1}{2}x$
b) 2 times the value of x	ii) $5 + 2x$
c) the value of x reduced by 2	iii) $x + 2$
d) 2 times the value of x plus 5	iv) $x - 2$
e) half of x	v) $2x$

9. Graph each inequality on a number line.

   Hint
   See page 157

   a) $-3 < x \quad x \in \mathbb{Z}$

   b) $x \leq 2 \quad x \in \mathbb{Z}$

   c) $4.5 > x \quad x \in \mathbb{R}$

   d) $x \geq -\frac{1}{2} \quad x \in \mathbb{R}$

10. Calvin raised $150 for charity. He plans to divide it equally between n charities so that each group receives D dollars.

   a) Write an equation that shows how Calvin plans to distribute the money.

   b) How are the variables used in this equation?

   c) Describe values for n that make sense in this situation.

5. Answers will vary. Sample response:

Properties: $a + b = b + a$, $a + (b + c) = (a + b) + c$;

Formulas: $A = \pi r^2$, $D = rt$

6. Answers will vary. Sample responses:

a) $y = 4x$, where x and y are real numbers; y is found by multiplying 4 by x. Since x can be any number, y changes depending on the value of x.

b) $5(x + 2) = 5x + 10$. This shows the distributive property. The variable x can take on any real number value and this equation will be true.

c) $x + 2 = 4$, where x is an integer; x represents only one solution, $x = 2$, which will make this equation true.

d) $x \geq -2$ for all integers; x is the set of integer values that are equal to and greater than -2, such as $-2, -1, 0, 1, \ldots$.

7. Answers will vary. Sample response: In equations and expressions that have letters, the letters are variables that represent numerical values. Sometimes a variable can be a specific unknown quantity. For example, in the equation $8 = x - 7$, there is one value of x that makes the equation true (15). Sometimes variables in equations mean a varying quantity. For example, in the equation $x = 5y + 3$, the value of x varies depending on the value of y.

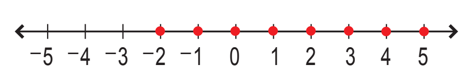
8. **a)** iii

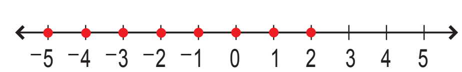
b) v

c) iv

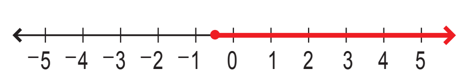
d) ii

e) i

9. **a)**


b)


c)


d)


10. a) $D = 150 \div n$. D is the number of dollars to be donated to each charity; n is the number of charities.

b) The variables are related varying quantities. As the value of n changes, the value of D will change.

c) The variable n must be an integer, and cannot be zero or a negative number because Calvin plans to donate to at least one charity and he cannot donate to a negative number of charities. Some students might state that n cannot equal 1 because Calvin states that he plans to give to charities (plural). Some students might also state that the monetary system limits the possible values of D.

11. $T = a + 2b + 3c$
a = number of 1-pt baskets
b = number of 2-pt baskets
c = number of 3-pt baskets
T = Total Score

LESSON 1.1 SECTION 1

On Your Own

11. In a basketball game, a basket made on a free throw is worth 1 point, baskets made from inside the 3-point line are worth 2 points, and those made from outside the 3-point line are worth 3 points. Use variables to write an equation for the total score of one team for each game. Explain what each of your variables represents.

12. How are the ages of your family members related to you? Let x represent your age today in years.

a) Write two different expressions using x that represent the ages of two people in your family.

Hint
See page 157

b) Evaluate each expression for $x = 5$. What does the value of each expression mean?

13. Write the following expressions using variables.

a) 7 less than n

b) 9 times y

c) the sum of x and 5, divided by 2

d) 12 more than s

e) the product of 5 and w decreased by $^-4$

f) k times the sum of a and b

14. There are many types of mathematical statements in algebra. How are they similar and how are they different? Give examples.

Think Beyond

15. In mathematics, the Greek letter π represents a constant. Are there other letters or symbols mathematicians use to represent constants? Investigate this question using the Internet and write a report for the class.

12. a) Answers will vary. Sample response:
x = my age
Mom's age = $x + 25$
Sister's age = $x - 3$
Grandfather's age = $x + 55$

b) Answers will vary. Sample response:
when $x = 5$

Mom's age = 5 + 25	Mom was 30 when I was 5.
Sister's age = 5 − 3	My sister was 2 when I was 5.
Grandfather's age = 5 + 55	My grandfather was 60 when I was 5.

13. a) $n - 7$

b) $9y$

c) $\frac{x + 5}{2}$ or $(x + 5) \div 2$

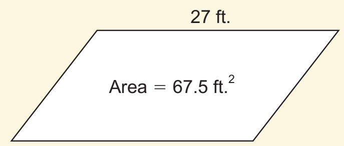
d) $s + 12$

e) $5w - (^-4)$

f) $k(a + b)$

Think Back

16. What is the height of this parallelogram?



A. 2.5 ft.
B. 2.5 ft.²
C. 1,822.5 ft.
D. 1,822.5 ft.²

17. Evaluate: $17 + (-28) \div (-2) - 45$.

18. Solve: $\frac{2}{3}s = 14$. Show your work.

19. Is the following equation always, sometimes or never true? △ and ○ represent real numbers. Explain your answer.

$$△ \bullet ○ = -△ \bullet -○$$

20. If a recipe uses 114 chocolate chips to make 12 giant cookies, what is the average number of chocolate chips per giant cookie?

14. Answers may vary. Sample response:

There are three different types of mathematical statements in algebra: equations, expressions and inequalities. Statements with an equal sign are equations, $y = -2x$; those with an inequality sign are inequalities, $x < 4$; and those without either sign are expressions, $14 - y$. All have numbers and letters, but equations show that mathematical expressions are equivalent; $45 - x = 10$, $3a + 40 = 5a + 10$ and $C = \pi d$ are all equations. Expressions do not show equivalence; $5y$ and $9 - 3x$ are expressions. Equations and inequalities show a relationship between values. The relationship can be one of equality or inequality.

15. Think Beyond Answers will vary. Students may discover the use of the letter *i* for imaginary numbers or the symbol *e*, which represents the base of the natural logarithms, with a value of approximately 2.718281828 … .

Think Back

16. **A.** 2.5 ft.

17. $17 + (-28) \div (-2) - 45 = -14$

18. $\frac{2}{3}s = 14$ or $\frac{2}{3}s = 14$

$s = 14 \div \frac{2}{3}$ $\quad\left(\frac{3}{2}\right)\frac{2}{3}s = 14\left(\frac{3}{2}\right)$

$s = 14 \bullet \frac{3}{2}$ $\quad s = 21$

$s = 21$

19. The equation is always true because multiplying each of the two factors by -1 is the same as multiplying the product by 1, which doesn't change the product.

20. 9.5 chips

ℕ set of natural numbers {1, 2, 3, 4, 5, . . .}

ℤ set of integers {. . . , −3, −2, −1, 0, 1, 2, 3, . . .}

ℚ set of rational numbers {numbers in the form $\frac{a}{b}$, a and $b \in \mathbb{Z}$, $b \neq 0$}

ℝ set of real numbers {all rational and irrational numbers}

There is no standard symbol for the set of whole numbers.

set of whole numbers {0, 1, 2, 3, 4, 5, …}

Lesson Guide 1.1A *Understanding Variables*

Meaning of the Variables	My Notes	Examples
Variables that Represent a Specific Value		
Variables that Represent Related Varying Quantities		
Variables that Represent Many Values		

Lesson Guide 1.1 *Algebra Cards*

A. $4 \bullet 9 = 6 \bullet 6$	**B.** $C = \pi d$
C. $7m$	**D.** $23 - x > 13$
E. $y = -2x$	**F.** $a + b = b + a$
G. $36 + 8$	**H.** $y = x - 1$
I. $A = lw$	**J.** $5 + 9 = n + 6$
K. $14 - y$	**L.** $a \bullet \frac{1}{a} = 1, a \neq 0$
M. $35 = 7g$	**N.** $5(3 + m) = 15 + 5m$
O. $6 + s = 10$	**P.** $4x - x = 3x$
Q. $7x + 2y$	**R.** $x < -4$

Lesson Guide 1.1 *Number Lines*