Card Game Capers

Big Mathematical Ideas

Lesson 2 continues the exploration of the place- and face-value of digits, as well as regrouping, using a game format. Students record values of the numbers using charts and tables to decide if all possible combinations have been explored. As three cards are drawn by the teacher, students decide where to put two of the numerals based on whether they want a number that has the greatest or least value. Students are encouraged to create generalized rules based on the place-value and regrouping discoveries they observed playing the card game.

Objectives

- Students will generalize place-value understandings to add and subtract multi-digit numbers
- Students will recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to the right.
- Students will identify arithmetic patterns (including patterns in the addition or multiplication table) and explain them using the properties of operations.
- Students will fluently add and subtract multi-digit whole numbers using the standard algorithm.

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Materials

Students

- "Possible Two-Digit Numbers Record Keeping Sheet" (Student Mathematician's Journal p. 11)
- "Mathematician's Journal Think Deeply About..." (Student Mathematician's Journal pp. 13–15)

Teacher

- Blackline master "Number Tiles" (p. 91)
- Blackline master "Hint Cards" (p. 93)
- Blackline master "Think Beyond Cards" (p. 94)

Supplies

- One set of ten cards labeled for each pair of students with the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 (You may cut these out from the blackline master "Number Tiles.")
- One set of $6\frac{1}{2}$" by $4\frac{1}{2}$" ten cards labeled with numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 for teacher

Mathematical Language

- **Greater than** – more than, showing a relationship where one number is more than another (>)
- **Less than** – not as great, showing a relationship where one number is not as great as another (<)

Initiate (1 day)

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Show students the cards that you have numbered from 0–9. Tell them that you are going to play a game where you will choose three cards from the deck without looking. Each student is to write the greatest possible two-digit number using just two of the digits that you have chosen. There is just one catch, however. The digits will be drawn one at a time without looking, and students must place each digit before you draw the next number from the deck. Before you draw any digits, each student should draw two blanks on a sheet of paper for the two-digit number and one for the discard: (Number ____ ____ ; Discard ____). To reinforce the need to think carefully about their choices, have the students use pens or markers to record their decisions.

Number ____ ____

Discard ____

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When you draw the first card, each student should either write the digit on the first or second blank or decide to not use the digit. If students decide not to use the first digit, they should write it next to "Discard." They must put each of the next two digits on one of the two blanks for the two-digit number. Students may not change their minds once a number is recorded. After all three digits have been recorded, determine who has recorded the greatest (or least) number. Choose one of these students with the greatest (or least) number to be the next one to draw the three digits for the game.

ONLINE RESOURCE
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Once students have played the game a few times as a whole class, put them in groups of three to play the same game again and discuss their strategies. One player in each group should turn over the cards and the other two players should try to get the greatest possible number. After each round, the player with the greatest possible number gets one point and becomes the person to turn over the cards for the next round.

Investigate (1 day)

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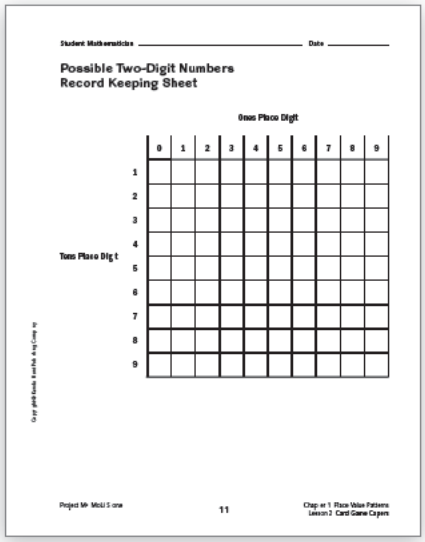
After the students have played the game for a few rounds, ask them how many different two-digit numbers are possible if the first digit cannot be a zero. Encourage students to work individually to record all the possibilities. The results of their work forms the basis for a classroom discussion of the first Think Deeply question.


Student Mathematician's Journal p. 11

Students may wish to use the number cards to try to determine all the possibilities. If you see students who are randomly writing down two-digit numbers, you might give them one of the Hint Cards that suggests they order the information in an organized list or chart. You might remind them of how they kept track of all the ways to make 47¢ from the previous lesson. It is powerful when students see a problem-solving strategy used across contexts. It forces them to focus on the mathematics of the problem, not the context. If they continue to have difficulties after this hint, you might want students to use the "Possible Two-Digit Numbers Record Keeping Sheet" found in the Student Mathematician's Journal, p. 11, to help them organize their thinking and record all the possibilities.

There are 81 possibilities. You have nine choices for each digit for a total of 9×9 or 81 different numbers. The following chart shows the possibilities.

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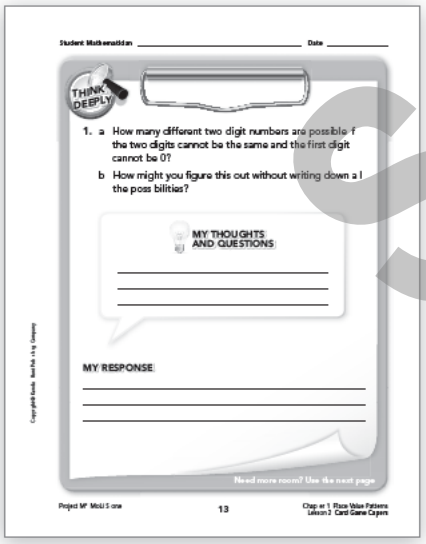
Tens-Place Digit	Ones-Place Digit: 0	1	2	3	4	5	6	7	8	9
1	10	–	12	13	14	15	16	17	18	19
2	20	21	–	23	24	25	26	27	28	29
3	30	31	32	–	34	35	36	37	38	39
4	40	41	42	43	–	45	46	47	48	49
5	50	51	52	53	54	–	56	57	58	59
6	60	61	62	63	64	65	–	67	68	69
7	70	71	72	73	74	75	76	–	78	79
8	80	81	82	83	84	85	86	87	–	89
9	90	91	92	93	94	95	96	97	98	–

Note that the diagonal is blank since the numbers are not replaced in the deck of cards to choose from and therefore numbers such as 11, 22 and 33 are not possible since no digit can appear twice in a number. Also, there are no numbers with a zero in the tens place.

Mathematical Communication ($\frac{1}{2}$ day)



Student Mathematician's Journal p. 13

1. a. How many different two-digit numbers are possible if the two digits cannot be the same and the first digit cannot be 0?

   b. How might you figure this out without writing down all the possibilities?

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- How many two-digit numbers are in between 10–99?
- There are 9 two-digit numbers that repeat the same digit.
- Are there any patterns that could help make this problem easier?

Using the Math Messaging Board

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Discuss the first Think Deeply question as a class. Remind the students to use the Classroom Discussions Rights and Obligations to help create a productive discussion and also use the Math Messaging Board to model and record the thinking involved in answering the Think Deeply questions. The goal of the Math Messaging Board is to help students learn how to clearly explain their reasoning using appropriate mathematical vocabulary while the class discussion helps students organize their thoughts before writing their own responses.

The first Think Deeply Question invites students to look for patterns and generalize rules for finding how many different two-digit numbers there are that do not use repeating digits and do not start with a zero. Use the Math Messaging Board to record the class discussion. A class discussion might reveal some confusion about the number of possible numbers. Start by using partner talk to have students think about and explain what they have written in the *My Thoughts and Questions* section of their Student Mathematician's Journal. As a class, discuss the big ideas with the students and record this as the Talk About It topic on the Math Messaging Board. Record students' ideas as phrases under Keep A Record. Include any misconceptions presented as ideas and discuss these. Finally, in the discussion below, there is confusion caused by how they count the groups of ten and whether or not they count the double digits (i.e., 11, 22, 33). The teacher discusses and records the class conclusion in the Wrap It Up section and can provide links to skills by linking the list and multiplication strategies.

A classroom discussion might reveal some confusion about the number of possible numbers. In the example below, the students make some good generalizations. However, there is confusion caused by how they count the groups of ten (do they start or end with a factor of ten?) and whether or not they count the double digits (i.e., 11, 22, 33).

Teacher: You have been playing Card Game Capers and making different numbers. How many different two-digit numbers are possible? Carlos?

Carlos: I think there are a lot of numbers close to 100.

Teacher: Is that an estimate or did you actually determine that there were about 100?

Carlos: Well, we didn't exactly find 100 numbers but I think there are about 100 because 10 times 10 equals 100.

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Teacher: Carlos thinks there are about 100 numbers because 10 times 10 equals 100. Would someone else in Carlos's group explain why you think you should multiply 10 times 10? Sophie?

Sophie: We started by writing all the numbers you could have, like 20, 21, 22, 23, 24, 25, 26, 27, 28, 29. There are 10 numbers in the 20s so then we just thought there would be 10 numbers in the 30s, 40s and so on so we went 10 times 10.

Teacher: James, would you please repeat what Sophie just said?

James: I didn't really understand it. Could you repeat it again?

Sophie: Well think of the first 10 numbers you can get, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. There are 10 of them. Then in the teens there are 10 numbers, and in the twenties there are 10 numbers so we just went 10 times 10.

Teacher: James, can you give it a try?

James: Okay. I think she is saying that because there are 10 numbers you can write out that you can just multiply the 10 by the number of groups. So 10 times 10 is 100.

Teacher: Kelly?

Kelly: I thought we couldn't use the same number twice – like 22 or 77? I think there are less than 100 numbers.

Teacher: Talk to your partner about what Kelly has just suggested. Can we make numbers with the same digits like 22 and 77? And if not , how will this change the number of possible numbers?

The teacher continues the discussion above, making sure that students consider the fact that the two digits cannot be the same and how that will affect the total number of numbers. She also has other students share their strategies of listing all the numbers in an ordered way. This enables the teacher to link the list strategy to the multiplication strategy.

What to Look for in Responses

- Students should begin to look for patterns and ways to extend them rather than writing out all possible combinations.

Possible Difficulties

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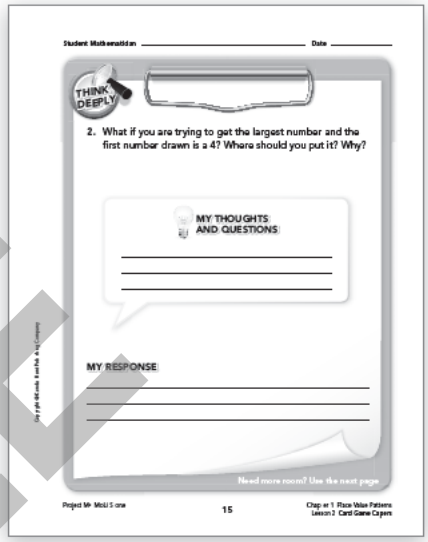
- Some students may ignore the conditions stated in the question and create single digit numbers (i.e., 1, 2, 3...) or double-digit numbers with repeating digits (i.e., 11, 22, 33...).

2. What if you are trying to get the largest number and the first number drawn is a 4? Where should you put it? Why?

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- If I draw a 4 and I am trying to make the largest number, how many combinations can I make with a tens place number greater than 4?
- A 4 in the tens place is worth 4 tens or 40.
- The face value of 4 is 4. What digits between 0–9 have a greater face value than 4?


Student Mathematician's Journal p. 15

What to Look for in Responses

- An understanding of the face value of a digit, i.e., that 7 represents a greater quantity than 4.
- An understanding of place or positional value of a digit, i.e., a 7 in the tens position represents a quantity of 7×10 and a 7 in the ones position represents a quantity of 7×1.
- An understanding that if a 4 is the first card drawn there are 5 ways to get a greater face-value card and 4 ways for a lesser one.

Possible Difficulties

- Students may confuse the difference between numbers and digits. If this occurs consider using the analogy of words and letters: words are made using letters, numbers are made using digits.

After the students have had a chance to play a few rounds in their small groups, ask each group to discuss their strategies. You might give the groups the first Think Deeply question about drawing a 4 on the first card to discuss. Encourage them to look at their lists of possible numbers as they discuss their strategies. Note that if the four is drawn first, there are more remaining numbers greater than 4 (i.e., 5, 6, 7, 8 and 9) than there are remaining numbers

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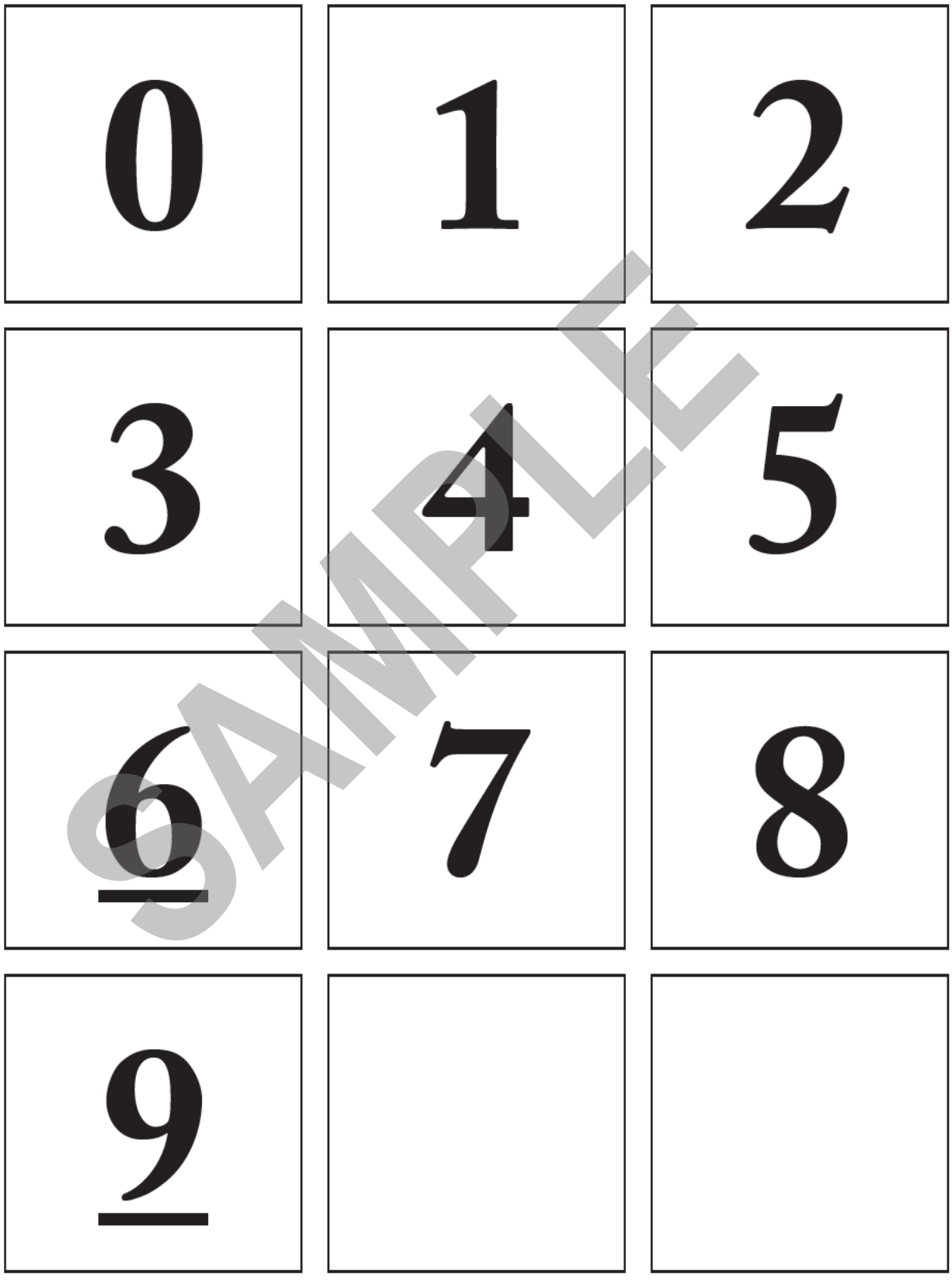
smaller than 4 (i.e., 0, 1, 2 and 3). That would mean that the best strategy for getting the greatest number might be to put the 4 in the second spot, but the best strategy for the least number might be to put it in the discard pile or the second spot.

The Think Beyond questions are designed for students who have mastered the basic concepts of forming the greatest and least possible two-digit numbers and who can make organized lists or charts to determine all the possibilities. In these questions, students are asked to expand to three digits with and without repeating digits, and to determine whether a third digit changes the strategies for playing the game.

In a game with three digits, the best strategy is probably to put the 4 in the middle if it is the first number drawn since it is likely that in the remaining three numbers, there will be a number greater than 4 as well as a number less than 4. An exhaustive (complete) list of all the possibilities for a three-digit number without 0 for the first digit and without any repeats of digits shows there are nine choices for the first digit, nine choices for the second digit (since zero is now a choice, but the first digit is not) and only eight choices for the third digit (not allowing either of the first two digits to be repeated). That would mean that there are $9 \times 9 \times 8$ or 648 different possibilities. Students should not be expected to list all these, but rather to use patterns and reasoning to determine them. Students might say that there are 81 ways to get the first two numbers since those would be the same as the two-digit original problem and that each of these 81 choices would have eight different possibilities for the third number. Note that if repeated digits are allowed, and if 0 is allowed as the first number, there are 1,000 possibilities (starting with 000 and going to 999). This is the same as taking $10 \times 10 \times 10$.

The remaining Think Beyond Cards should be used as ideas for additional journal writing. Students who need extra time with the concepts might be asked to list all two-digit numbers that allow repetition of the digits. Students who readily grasp the concepts with two digits should be given one of the Think Beyond Cards to write about. These journal observations might be used to begin the lesson for the following day.



HINT CARDS

Can you organize the information in a list or chart?

Look for patterns.

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HINT CARDS

Which is greater — 49 or 94?

How do you know if one number is greater than or less than another number?

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HINT CARDS

Compare your list to a classmate's to see if you have found all the possible two-digit numbers.

How will you know if they are all there?

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HINT CARDS

Can you use the cards to find more numbers?

Look for patterns.

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THINK BEYOND

1. What if you play the Largest Number Game by drawing four cards with the goal of writing down the largest three-digit number? (You are allowed to discard one number.) Does this change your strategy of where to place the 4 if that is the first number drawn?

Why or why not?

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THINK BEYOND

2. What if you play the Smallest Number Game by drawing four cards with the goal of writing down the smallest three-digit number? (You are allowed to discard one number.) Does this change your strategy of where to place the 4 if that is the first number drawn?

Why or why not?

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THINK BEYOND

3. How many different three-digit numbers are there if you cannot repeat any digits?

What patterns might you use to figure this out?

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THINK BEYOND

4. How many different three-digit numbers are there if you can repeat any digits?

What patterns might you use to figure this out?

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A QUICK LOOK

NOTES

Chapter 1: Lesson 2
Card Game Capers

Objectives:

- Students will generalize place-value understandings to add and subtract multi-digit numbers
- Students will recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to the right.
- Students will identify arithmetic patterns (including patterns in the addition or multiplication table) and explain them using the properties of operations.
- Students will fluently add and subtract multi-digit whole numbers using the standard algorithm.

Initiate: (1 day)

1. Tell students they will play a game with cards that have been numbered from 0–9.
2. Discuss the directions for the game with students. *(Students will draw two blanks on a sheet of paper to represent a two-digit number and one discard blank to serve as the discard spot. Three cards will be drawn, one at a time, from a pile. As each card is drawn and the number is called out, students must write the selected digit in one of the three blanks on their papers. The object of the game is to create the largest two-digit number possible. Students will not know in advance what digits will be drawn, and they must write each numeral down before the next one is selected. They are not allowed to switch digits once they have placed them in a given blank on their papers.)*
3. Once students have played a few rounds of the game together as a class, put students into groups of three to play the same game again. After each round, the person with the largest number in the group will get to turn over the cards while the other two members of the group play the game. The person who creates the largest number in the round will score 1 point.
4. Have students discuss and share strategies for playing the game.

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5. Assign the first Think Deeply question as a formative assessment. This question asks students to consider where to place the digit 4 when it is the first card drawn in playing the largest number game. Students may work individually on the question in class or it may be assigned for homework.
6. If time allows, you may want to vary the game by having students try to create the smallest two-digit number possible. Have a class discussion afterwards focusing on how strategies might have changed playing this version of the game.

Investigate: (1 day)

1. Ask students how many two-digit numbers are possible using the digits 0–9. Encourage them to work individually to record all the possibilities. **(Note: When drawing digits from the deck, the cards are not replaced. Therefore, numbers such as 11, 22, 33, etc. are not possible. Also, numbers that have zero in the tens place such as 09, 04, 02 should not be considered as an option since it is a mathematical convention that they be written as single-digit numerals.)**
2. Offer Hint Cards or suggest using an organized list for those students who seem to be randomly writing down two-digit numbers.
3. If students have difficulty with organization, suggest they use the "Possible Two-Digit Numbers Record Keeping Sheet" in the Student Mathematician's Journal.
4. Have students share their results with the class. Focus the class discussion on the strategies students used to determine all possible two-digit numbers.

Mathematical Communication: ($\frac{1}{2}$ day)

Assign and discuss the Think Deeply questions.

- The first Think Deeply question asks students to determine how many two-digit numbers are possible if the two digits are not the same. Students are encouraged to explain how they might figure this out without writing down all the possibilities. Assign students to work individually on this question, and then share their responses with the class.
- As stated above, the second Think Deeply question may be assigned after students have completed playing the "Largest Number Game." If assigned as homework, begin the next class with a discussion of student responses.