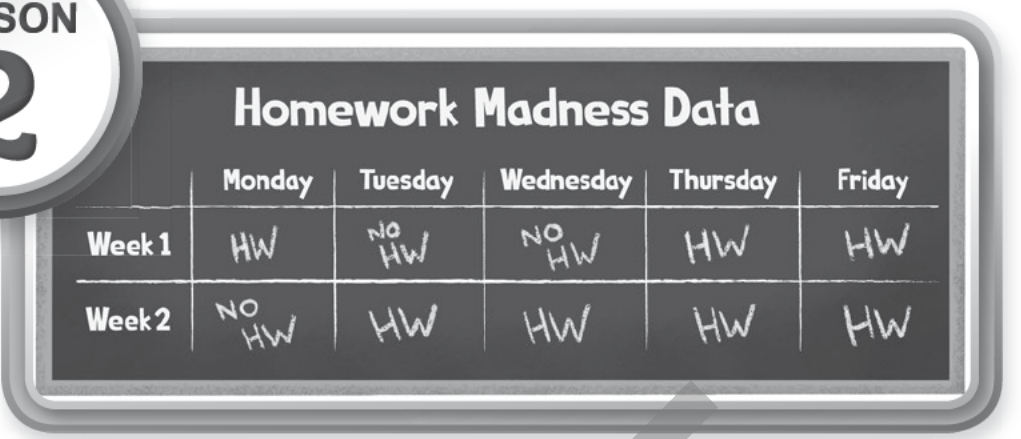
LESSON 2



Homework Madness

Big Mathematical Ideas

Many events do not have an equal probability of occurring. These situations are referred to as unequally-likely events. The probabilities of unequally-likely events can be estimated by collecting data. If data are collected from an experiment and the experimental probabilities are all different, this may indicate such a situation. In unequally-likely situations, one event is more likely to occur than one or more other events. Note that experimental probability may not always approximate the theoretical probability. For instance, in this case, it is possible for the experimental probabilities to not reveal that the situation is an unequally-likely one, when, in fact, it theoretically is. Since theoretical probability is deliberately not introduced until the next lesson, students may have to conduct more experiments to better approximate the theoretical probability and uncover that the situation is, in fact, unequally likely to happen.

Objectives

- Students will learn that many events are not equally likely (unequally likely).
- Students will conduct an experiment to determine if events are equally likely.
- Students will practice expressing experimental results as fractions.
- Students will be introduced to the Law of Large Numbers.

NOTES

Materials

Students

- "Homework Madness" (Student Mathematician's Journal pp. 13–14)
- "Mathematician's Journal Think Deeply About…" (Student Mathematician's Journal pp. 15–17)

Teacher

- Blackline master "Hint Cards" (p. 101)
- Blackline master "Think Beyond Cards" (p. 102)

Supplies

- Small paper bag containing one red, two green, four blue and one yellow chip
- Two different colored blocks, chips or markers (four of each color; one set for every three students)
- Small paper bags or containers to hold blocks, chips or markers (two bags for every three students)
- Coins (one for every three students)

Mathematical Language

- **Law of Large Numbers** – states that experimental probabilities collected over many trials will predict the theoretical probabilities only when a very large number of trials are conducted. The greater the number of trials, the closer the experimental probability is to the theoretical probability.
- **Trial** – one round of an experiment
- **Unequally likely (not equally likely)** – two or more outcomes that do not have the same chance of occurring or that do not have the same probability. For example, in a bag that holds three cubes, two that are green and one that is yellow, the probability of randomly selecting a green cube is $\frac{2}{3}$ and the probability of selecting a yellow cube is $\frac{1}{3}$.

Initiate ($\frac{1}{2}$ day)

NOTES

In Lesson 1 students learned about experimental probability and equally likely events. This lesson continues students' exploration of experimental probability and introduces situations where the events are not equally likely.

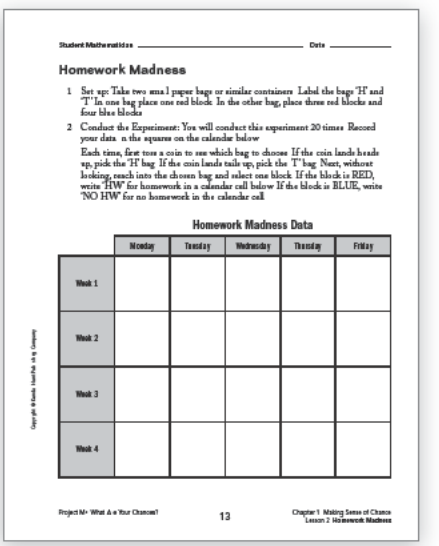
Revisit the experiment from Lesson 1 in which students drew chips out of a bag. Show a bag to students and place one red chip, two green chips, four blue chips and one yellow chip in it. Ask students if each of the four colors is equally likely to be drawn. As a class, conduct 40 trials and record the outcomes on the board. Most likely the number drawn of each color chip will not be equally distributed. The blue chips will occur about $\frac{1}{2}$ of the time, the green chips about $\frac{1}{4}$ of the time and the red and yellow chips each about $\frac{1}{8}$ of the time. (If necessary, more trials can be conducted to have the experimental probabilities be unequal and/or get closer to the theoretical probability.) Introduce the concept of unequally likely events — two or more events that do not have the same chance of occurring — and have students record a definition in their Student Mathematician's Journals.

Investigate ($1\frac{1}{2}$ days)

Homework Madness

Introduce an exploration of unequally likely events by telling the class that you recently learned about a fifth-grade teacher, Mr. Ko, who assigned math homework in a very unusual way. He had two bags, one labeled with an 'H' for heads and the other with a 'T' for tails. In Bag H was one red block, and in Bag T were three red blocks and four blue blocks. The blocks were identical except for their colors. Every day one of the bags was randomly selected by flipping a fair coin, and a block was randomly chosen from that bag. If the block was red, Mr. Ko assigned homework. If the block was blue, no homework was assigned. List the color and number of blocks in each bag on the board. Then ask students what they think of this arrangement. Tell them that they are going to collect data using blocks and bags to see what might happen in Mr. Ko's classroom next month.

In this investigation, students work in pairs to collect data. Have them place blocks into two bags following the directions on "Homework Madness" in their Student Mathematician's Journals. Prior to beginning the activity, ask students to predict the number of days there will be homework for an entire month of school or



Student Mathematician's Journal p. 13

ONLINE RESOURCE
Homework Madness

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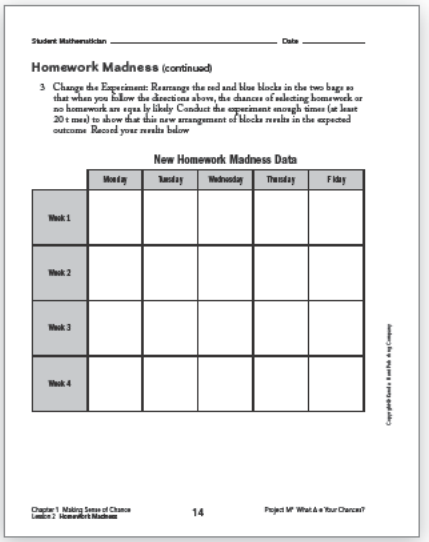
20 days. Next, ask them to predict the probability of the students receiving homework using probability words such as "certain," "equally likely" or "unlikely" and also using fractions such as $\frac{1}{3}$, $\frac{3}{4}$ or $\frac{7}{8}$. Make sure students explain their choice of words and fractions. Instruct students to follow the directions on the worksheet for how to randomly select the blocks.

Once everyone has collected a month of data, bring the class together. First, ask students about their results. Some groups of students might be surprised that almost every night was a homework night whereas other groups had many no homework evenings! Based on the results of a few groups of students, ask the class to predict if they conducted the experiment another 20 times, what the number of homework versus non-homework nights would be. Explain to students that when we conduct any experiment, the results are not certain and we only can predict what will happen. However, conducting the experiment just a few times, or trials, may not be enough to make good predictions. For example, if you only selected a block twice and both times a no homework block (blue) was chosen, you might conclude that many nights in a month would be homework free. However, if we look at our data we can see that on the majority of the nights there is homework. Thus, the more trials we do, the more likely it is that we can use the results to make more accurate predictions.



Student Mathematician _____ Date _____

Homework Madness (continued)

3. Change the Experiment: Rearrange the red and blue blocks in the two bags so that when you follow the directions above, the chances of selecting homework or no homework are equa ly likely. Conduct the experiment enough times (at least 20 t mes) to show that this new arrangement of blocks results in the expected outcome. Record your results below.

New Homework Madness Data

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1					
Week 2					
Week 3					
Week 4					

Chapter 1 Making Sense of Chance
Lesson 2 Homework Madness

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Project M³ What Are Your Chances?

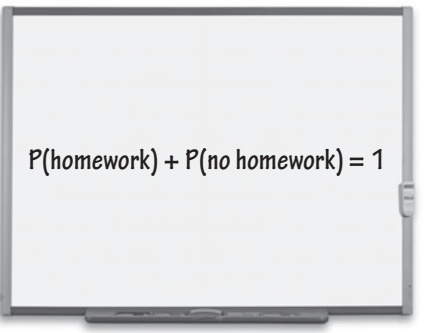
Student Mathematician's Journal p. 14

Build a table on the board that can be used to compile all of the data. Record the information from each group of students in the class as in the example below. At the bottom of the table, total all of the columns so that you now have the results of a much larger number of trials. It should be clear with so much data that homework or no homework are not equally likely events.

Homework	No Homework	Total Number of Trials
15	5	20
17	3	20
…	…	…
Totals 125	55	180

Determine the experimental probability of having homework and not having homework and write each of these as a fraction comparing the number of occurrences to the total number of trials. Thus, if for 125 out of 180 trials a red block was chosen, indicating homework, then the experimental probability would

be $\frac{125}{180}$, or $\frac{25}{36}$. Show students that we can record this symbolically as P(homework) $= \frac{125}{180}$ or $\frac{25}{36}$. Tell them that we want to simplify the fraction to simplest terms, if possible, and ask why this is useful (i.e., fractions in simplest terms are easier to interpret). The probability of no homework, P(no homework), is $\frac{55}{180}$, or $\frac{11}{36}$. Notice that the two probabilities are not close in value, indicating that the two events are not equally likely to occur. Point out to students that the probability of homework plus the probability of no homework is 1 ($\frac{125}{180} + \frac{25}{36} = 1$), and ask them to conjecture why it is always the case that probabilities of an experiment add up to 1. One way to think about this question is that if you account for all possibilities, then the event is certain to happen which is a probability of 1.



Law of Large Numbers

Whereas the Homework Madness activity is designed so that $\frac{5}{7}$ of the time homework is assigned, your experimental results may not tally to exactly $\frac{5}{7}$ ($\frac{125}{180}$ or $\frac{25}{36}$ is close to $\frac{5}{7}$ but not equivalent). However, the experimental probability will be close to $\frac{5}{7}$ if there are a large number of trials. An important law in probability is known as the Law of Large Numbers. It states that the greater the number of trials, the closer the experimental probability is to the theoretical probability. In particular, we can only accurately predict what might happen in a probabilistic situation when a large number of trials or events are examined. Even though your students have not yet learned about theoretical probability (discussed in Lesson 3), introduce the concept of the Law of Large Numbers, or the idea that in order to make accurate predictions about what might happen in a probabilistic situation, we need a lot of data or trials!

Revisit the phrase "unequally likely events" and ask students what they think it means in this situation. Most students will understand that homework occurred more often than no homework, but they may still be thinking in terms of numbers of occurrences rather than comparing the probabilities that can be represented by fractions. Be sure that students are clear that unequally likely events do not have the same probabilities and one event will happen more often than others. However, because of the random nature of events, an unlikely event can occur and may sometimes occur more often than expected when only a small number of trials are conducted.

Finally, have students distribute the four red and four blue blocks into the two bags in a way that they think there will be an equal chance of homework and no homework when a block is drawn (e.g., the chances of picking a red or a blue block are the

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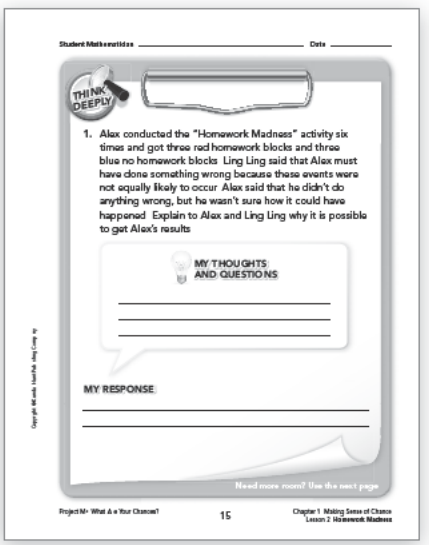
same). It is not expected that students will analyze the situation but rather that they will use intuition and common sense to think about how to distribute the blocks between the two bags. Then students will gather data again to see if their placement of blocks was successful. There are many ways to change the probability — one example is to place the four red blocks in one bag and the four blue blocks in the other bag. Another possibility is to place two red and two blue blocks into each bag.

Mathematical Communication (1 day)


Student Mathematician
Date
THINK DEEPLY
1. Alex conducted the "Homework Madness" activity six times and got three red homework blocks and three blue no homework blocks. Ling Ling said that Alex must have done something wrong because these events were not equally likely to occur. Alex said that he didn't do anything wrong, but he wasn't sure how it could have happened. Explain to Alex and Ling Ling why it is possible to get Alex's results.
MY THOUGHTS AND QUESTIONS
MY RESPONSE
Need more room? Use the next page.
Project M³: What Are Your Chances?
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Chapter 1: Making Sense of Chance
Lesson 2: Homework Madness

Student Mathematician's Journal p. 15

1. Alex conducted the "Homework Madness" activity six times and got three red homework blocks and three blue no homework blocks. Ling Ling said that Alex must have done something wrong because these events were not equally likely to occur. Alex said that he didn't do anything wrong, but he wasn't sure how it could have happened. Explain to Alex and Ling Ling why it is possible to get Alex's results.

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- This doesn't seem right. If you pick the bag marked H, you will always get homework.
- Heads or Tails are equally likely so you should get Heads about half the time and Tails about half the time.
- If you get Tails you can get either a red or a blue block. So it seems like overall there are more chances to get a red block.
- Maybe he put the wrong number of blocks in the bag.

Using the Math Messaging Board

Students have a great deal of difficulty understanding the concept of randomness. They expect that events will follow a regular pattern, even with small samples. For example, students often predict a fair coin flipped six times to land heads, tails, heads, tails, heads, and tails. Since there is a $\frac{1}{2}$ chance of each outcome, students expect that heads and tails will alternate. Likewise if events are not equally likely to occur, students are surprised when they occur in any regular pattern. It is only after observing many situations in which the unexpected occurs and discussing these

situations that students start to internalize the idea of randomness and the effects of the Law of Large Numbers.

Start by using partner talk to have students discuss the questions or thoughts they have written in *My Thoughts and Questions* and to think about what the Think Deeply question is asking them to do. As a class, discuss the focus of the Think Deeply question and record this as the topic under Talk About It on the Math Messaging Board. Record students' ideas as phrases under Keep A Record. Include any misconceptions presented as ideas and discuss these. Finally, discuss and record the class conclusion in the Wrap It Up section.

A class discussion might be similar to the following:

Teacher: Now that you've had some time to think about the first Think Deeply question, do you think Alex could get the outcomes he did?

Hilary: I don't think he could get three homeworks and three no homeworks since we found out that it was more likely we'd pick a red block. [Teacher records this idea under Keep a Record.]

Teacher: Does anyone agree or disagree with Hilary's idea? Stephie?

Stephie: I agree with Hilary. Alex must have done something to make that occur. [Teacher adds this idea to Keep A Record.]

Agree/ Disagree and Why?

Wilson: I disagree. Remember when we did the experiment, one group got a lot of no homeworks. I think this can happen with small numbers. [Teacher adds this idea to Keep A Record.]

Teacher: Wilson, what do you mean "with small numbers"? Can you add on to what you are saying?

Adding On

Wilson: Like, if Alex had 100 trials, I don't think he'd end up with 50 homeworks and 50 no homeworks, but sometimes with small numbers of trials you get funny results.

Teacher: Can someone repeat what Wilson is saying? Hilary?

Repeat/ Rephrase

Hilary: I think Wilson means that you can't predict what will happen when you just do a few trials.

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Revoicing

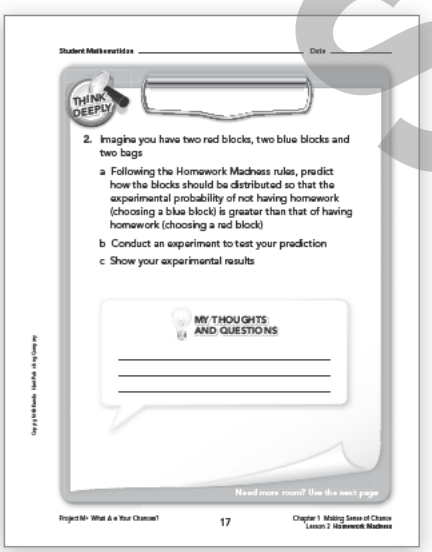
Teacher: Right. Random outcomes can occur with a small number of trials, but as soon as you have about 30 to 40 trials, you'll start to get a better sense of the likelihood of something occurring. As Wilson said, if Alex had selected a block from one of the bags a lot of times, we know from our class data that homework would occur a lot more often than no homework. [Teacher then writes Hilary's summary of Wilson's ideas under Wrap It Up.]

What to Look for in Responses

- Students need to explain that when you only have a small number of trials (six), the results can be quite random and the unexpected can happen, such as getting three "no homework" nights. The results from a small number of trials are not very useful for making predictions. If Alex and Ling Ling had picked blocks 50 times, they probably would have had quite different results.
- Students may compare and contrast equally likely and not equally likely events. If these two events were equally likely, there would be a $\frac{1}{2}$ chance of each event occurring.

Possible Difficulties

- Students may think that the chances of getting homework and no homework are equally likely since Alex selected three occurrences of each.
- Students may not understand the concept of the Law of Large Numbers and assume that a small number of trials is as predictive as a large number of trials.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THINK DEEPLY

2. Imagine you have two red blocks, two blue blocks and two bags.
   a. Following the Homework Madness rules, predict how the blocks should be distributed so that the experimental probability of not having homework (choosing a blue block) is greater than that of having homework (choosing a red block).
   b. Conduct an experiment to test your prediction.
   c. Show your experimental results.

NOTES

Prior to class discussion, provide ample time for students to reflect on the question to understand what is being asked. They should jot down ideas under *My Thoughts and Questions* in their Student Mathematician's Journal. Initial reflections might include the following:

- Heads or Tails are equally likely.
- I can't put two blue in one bag and two red in another. I think that would end up with them being equally likely to be chosen.
- Maybe I can try a blue alone in one bag since it would always be picked, just like the red one in our Homework Madness game.

What to Look for in Responses

- Students distribute the blocks between the two bags so that no homework occurs more often than homework. For example, placing one blue block in one bag and one blue and two red blocks in the other bag will give an advantage to "no homework." Since there are two bags that are being chosen randomly, the probability of choosing either one is $\frac{1}{2}$. (Each bag is listed three of the six times in the diagram below.) Bag H only has one blue block, so any time you select it there will be no homework. Bag T has three blocks, so the probability of selecting each of them is $\frac{1}{3}$. Therefore, the P(no homework) $= \frac{4}{6}$ or $\frac{2}{3}$ and the P(homework) $= \frac{2}{6}$ or $\frac{1}{3}$. Students are not expected to calculate these probabilities at this point. Rather, they should conduct 30–40 trials to show that the probability of getting no homework is greater than the probability of getting homework.

Choice of Bag	Choice of Block	Homework?
H	Blue	No
H	Blue	No
H	Blue	No
T	Blue	No
T	Red	Yes
T	Red	Yes

NOTES

- Students should show a table of experimental data from conducting the experiment. They also should give the experimental probabilities. It is expected that students will solve this problem using a guess-and-check strategy of distributing the blocks and then checking the outcomes.
- Students should use proper notation to represent the probabilities, such as P(no homework) or P(NH).

Possible Difficulties

- Students may still be unclear on the concept of equally likely and not equally likely events.
- Students may not be able to decide how to place the red and blue blocks into the two bags so that the experimental probability of selecting a blue block is favored.
- Students may not fully understand the Law of Large Numbers and, as a result, do not conduct enough trials to determine an appropriate experimental probability or one that closely represents the theoretical probability. They should conduct about 30–40 trials. Encourage students to group their data to get an even better approximation of the theoretical probability.
- Students may not be able to explain why they made specific choices on the placement of the colors — they just guessed and their guess worked! If this is the case, be sure that students understand the concept of unequally likely events but do not spend time trying to analyze this problem.
- Students need many experiences with experimental probability and determining whether or not events are equally likely to occur.

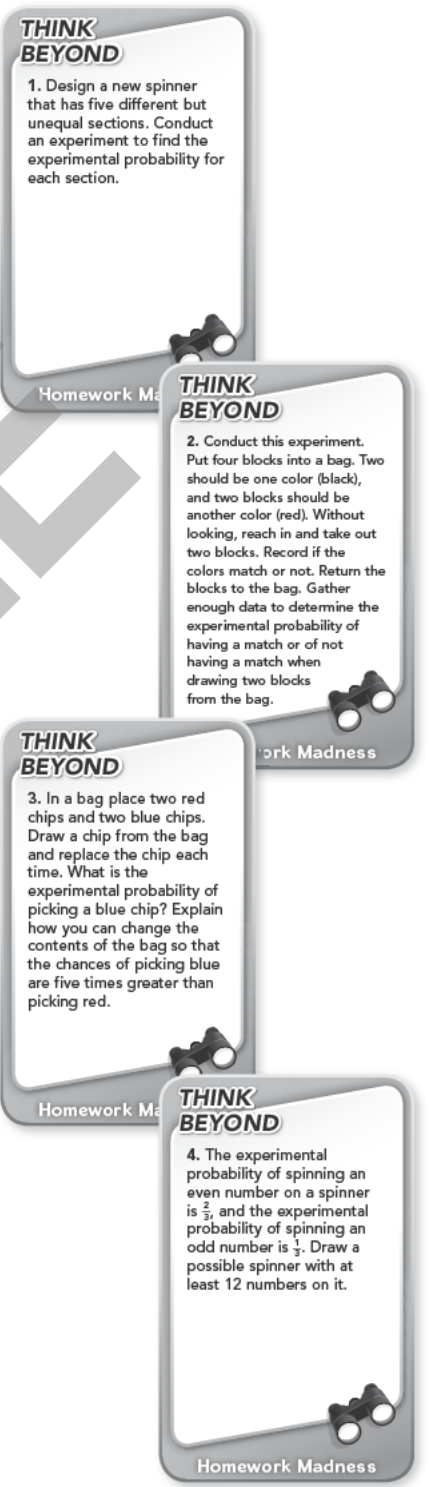
NOTES

The first Think Beyond Card asks students to construct a spinner with five sections on which the probabilities are not equally likely. They then gather data to determine the experimental probabilities. There are many different ways to design a spinner. Just make sure the sections of the spinner are different sizes.

The second Think Beyond Card asks students to investigate a game of chance. They place two black and two red chips or blocks into a bag. Students are to reach into the bag without looking and take out two of the chips. They are to conduct an experiment to see if it is more likely that the two chips are the same color or not. Chances are that their experimental probability will indicate that it is more likely that the chips will not match in color (P(no match) $= \frac{2}{3}$, P(match) $= \frac{1}{3}$).

The third Think Beyond Card presents a situation where two red and two blue chips are placed in a bag, and students are asked to determine the experimental probability of selecting a red chip. If students conduct about 40 trials, the data will suggest that the probability of selecting either color is $\frac{1}{2}$. Students then are asked to consider ways to change the experimental probabilities. One way to make blue much more likely to occur is to have five blue chips for every one red chip in the bag.

The final Think Beyond Card asks students to design a spinner where the experimental probability of landing on an even number is $\frac{2}{3}$ and the experimental probability of landing on an odd number is $\frac{1}{3}$. Students need to divide the spinner into at least 12 equal sections and put even numbers in $\frac{2}{3}$ of the sections. For example, if they divide the spinner into 12 equal sections, they need to put even numbers in eight of the sections since $\frac{8}{12}$ is equivalent to $\frac{2}{3}$. Remind students that they need to conduct sufficient trials, or approximately 30–40 trials, any time they conduct an experiment.



Student Mathematician: _____ Date: _____

Homework Madness

ANSWER SHEET

1. Set-up: Take two small paper bags or similar containers. Label the bags 'H' and 'T.' In one bag place one red block. In the other bag, place three red blocks and four blue blocks.
2. Conduct the Experiment: You will conduct this experiment 20 times. Record your data in the squares on the calendar below.

   Each time, first toss a coin to see which bag to choose. If the coin lands heads up, pick the 'H' bag. If the coin lands tails up, pick the 'T' bag. Next, without looking, reach into the chosen bag and select one block. If the block is RED, write 'HW' for homework in a calendar cell below. If the block is BLUE, write 'NO HW' for no homework in the calendar cell.

   Answers will vary. Make sure students are conducting the experiment as described.

3. Change the Experiment: Rearrange the red and blue blocks in the two bags so that when you follow the directions above, the chances of selecting homework or no homework are equally likely. Conduct the experiment enough times (at least 20 times) to show that this new arrangement of blocks results in the expected outcome. Record your results below.

   Answers will vary. One possible solution is to place all of the red blocks in one bag and all of the blue blocks in another bag. Another solution is to have two of each color in each bag.