Illustrative Mathematics

ACCELERATED 7

Unit 1

ACCELERATED
SAMPLE BOOK
STUDENT

Kendall Hunt

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Rigid Transformations

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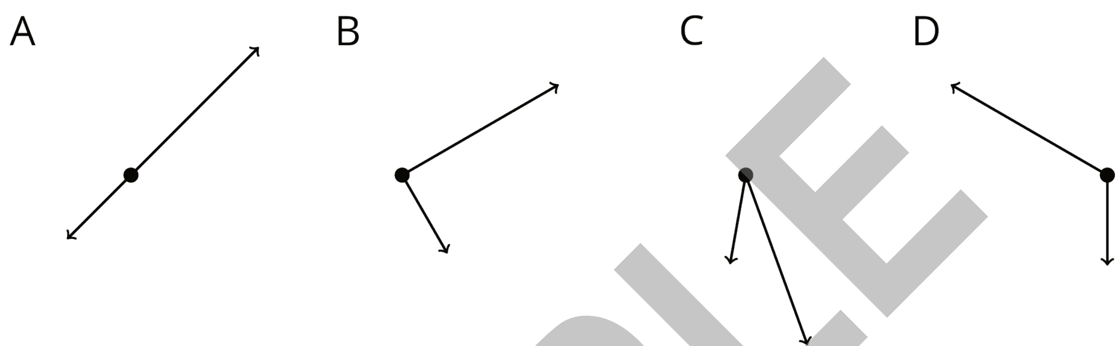
Lesson 1: Moving in the Plane

Let’s describe ways figures can move in the plane.

1.1: Which One Doesn’t Belong: Diagrams

Which one doesn’t belong?



1.2: Triangle Square Dance

Your teacher will give you three pictures. Each shows a different set of dance moves.

1. Arrange the three pictures so you and your partner can both see them right way up. Choose who will start the game.
    - The starting player mentally chooses A, B, or C and describes the dance to the other player.
    - The other player identifies which dance is being talked about: A, B, or C.
2. After one round, trade roles. When you have described all three dances, come to an agreement on the words you use to describe the moves in each dance.
3. With your partner, write a description of the moves in each dance.

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Are you ready for more?

We could think of each dance as a new dance by running it in reverse, starting in the 6th frame and working backwards to the first.

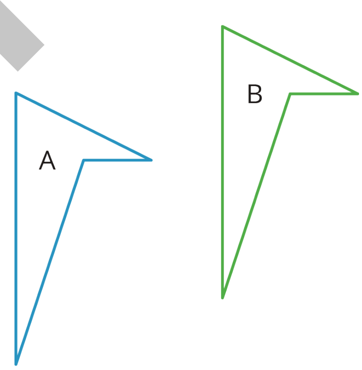
1. Pick a dance and describe in words one of these reversed dances.

2. How do the directions for running your dance in the forward direction and the reverse direction compare?

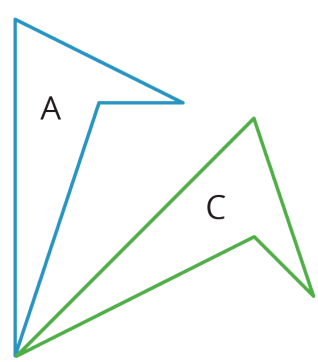
Lesson 1 Summary

Here are two ways for changing the position of a figure in a plane without changing its shape or size:

- Sliding or shifting the figure without turning it. Shifting Figure A to the right and up puts it in the position of Figure B.



- Turning or rotating the figure around a point. Figure A is rotated around the bottom **vertex** to create Figure C.

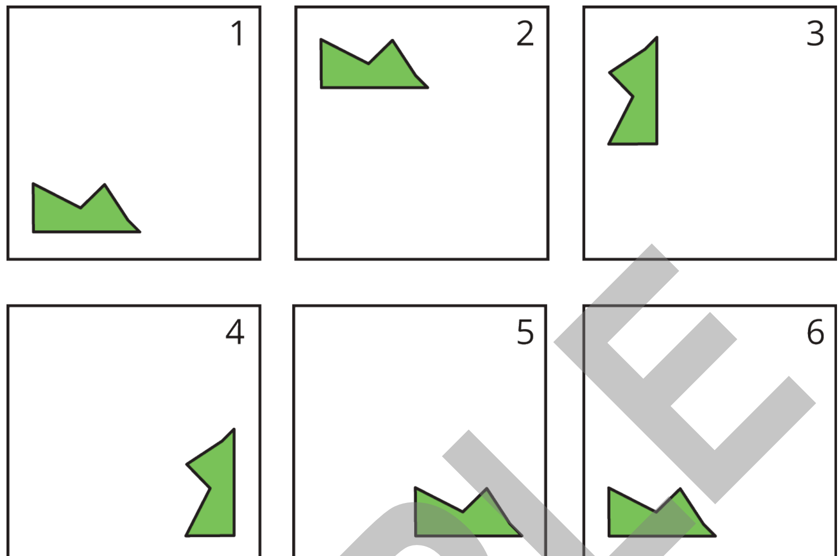


Glossary

- vertex

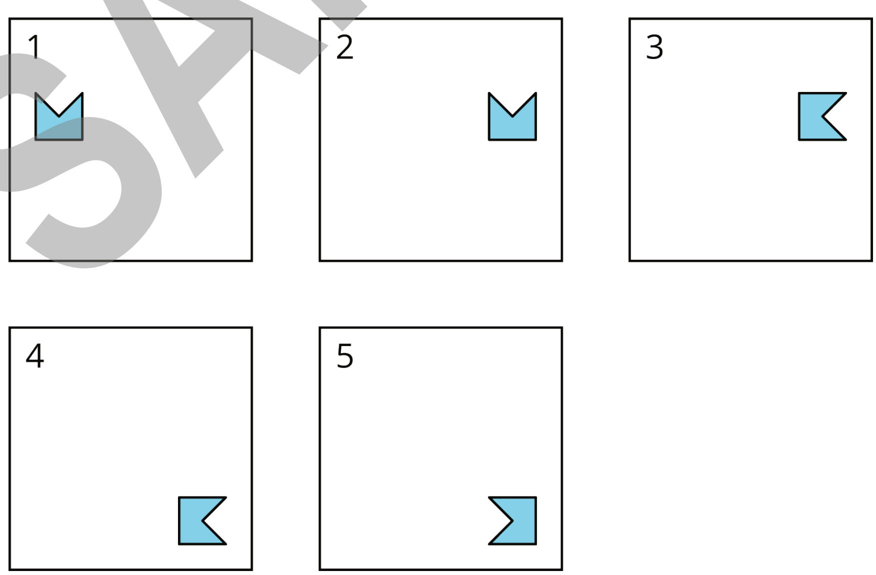
Lesson 1 Practice Problems

1. The six frames show a shape's different positions.



Describe how the shape moves to get from its position in each frame to the next.

2. These five frames show a shape's different positions.



Describe how the shape moves to get from its position in each frame to the next.

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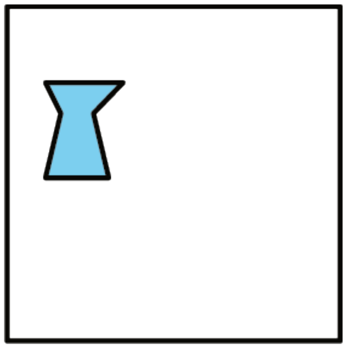
3. Diego started with this shape.



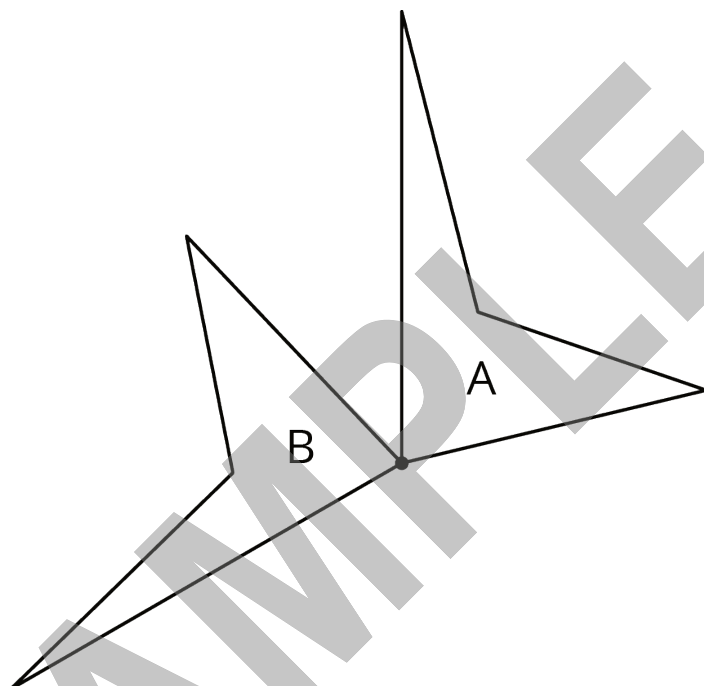
Diego moves the shape down, turns it 90 degrees clockwise, then moves the shape to the right. Draw the location of the shape after each move.

Lesson 2: Naming the Moves

Let's be more precise about describing moves of figures in the plane.

2.1: A Pair of Quadrilaterals

Quadrilateral A can be rotated into the position of Quadrilateral B.



Estimate the angle of rotation.

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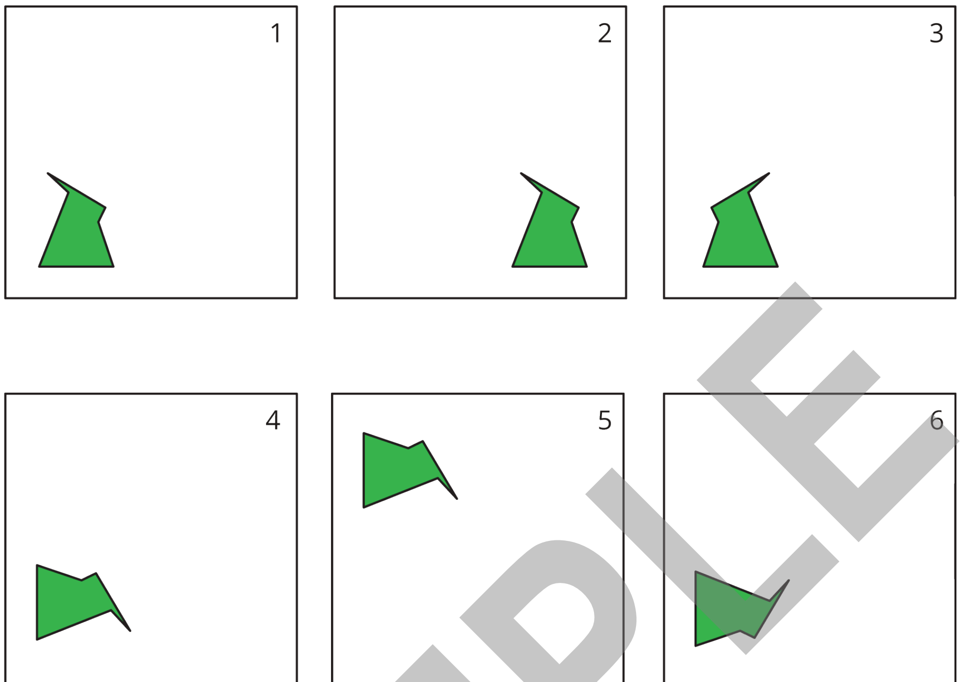
2.2: How Did You Make That Move?

Here is another set of dance moves.



1. Describe each move or say if it is a new move.

   a. Frame 1 to Frame 2.

   b. Frame 2 to Frame 3.

   c. Frame 3 to Frame 4.

   d. Frame 4 to Frame 5.

   e. Frame 5 to Frame 6.

2. How would you describe the new move?

2.3: Card Sort: Move

Your teacher will give you a set of cards. Sort the cards into categories according to the type of move they show. Be prepared to describe each category and why it is different from the others.

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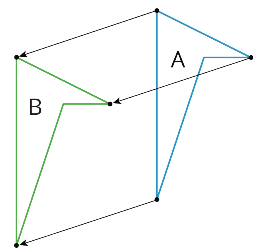
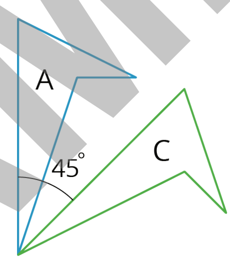
Lesson 2 Summary

Here are the moves we have learned about so far:

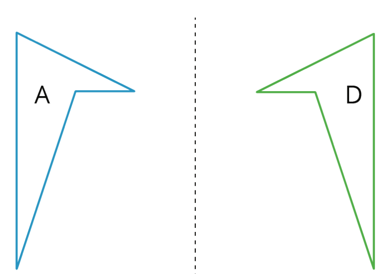
- A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.



- A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.



- A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.



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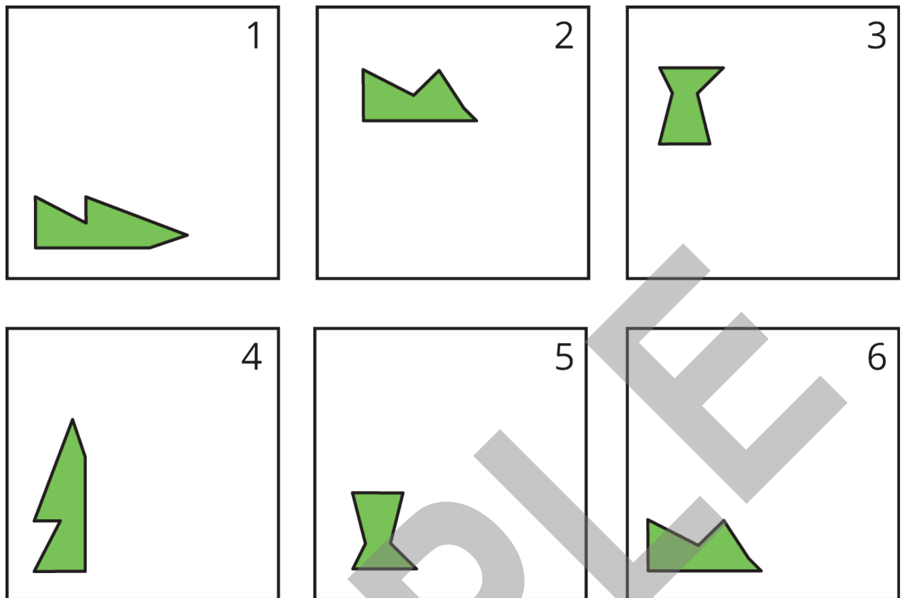
We use the word *image* to describe the new figure created by moving the original figure. If one point on the original figure moves to another point on the new figure, we call them *corresponding* points.

Glossary

- clockwise
- counterclockwise
- reflection
- rotation
- translation

Lesson 2 Practice Problems

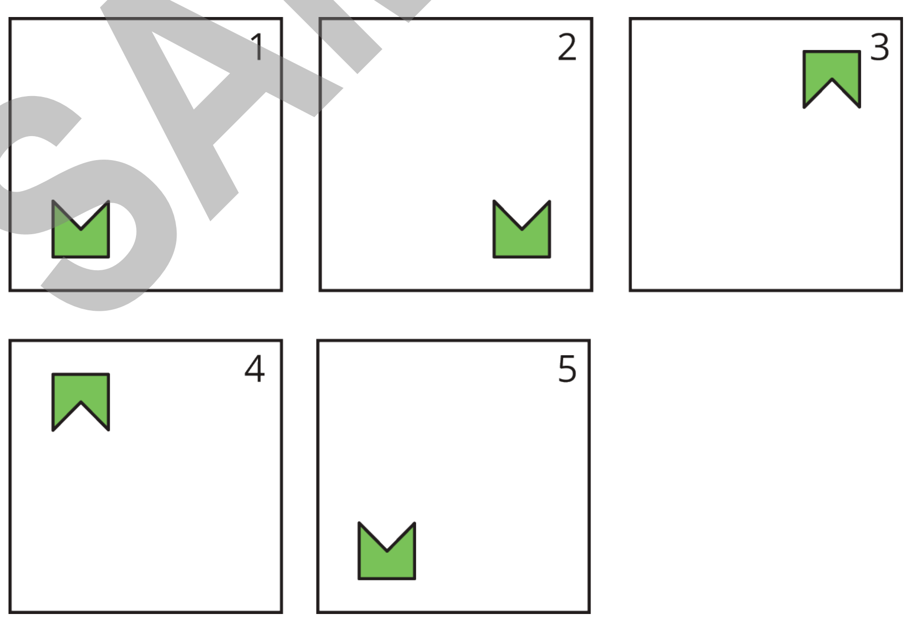
1. Each of the six cards shows a shape.



a. Which pair of cards shows a shape and its image after a rotation?

b. Which pair of cards shows a shape and its image after a reflection?

2. The five frames show a shape's different positions.



Describe how the shape moves to get from its position in each frame to the next.

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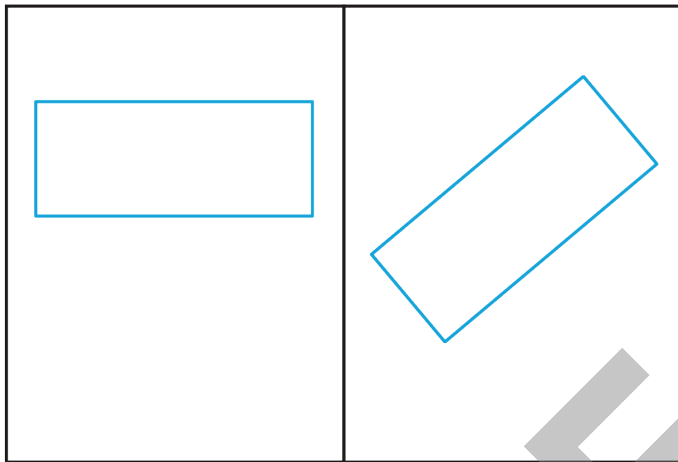
3. The rectangle seen in Frame 1 is rotated to a new position, seen in Frame 2.



Select **all** the ways the rectangle could have been rotated to get from Frame 1 to Frame 2.

A. 40 degrees clockwise

B. 40 degrees counterclockwise

C. 90 degrees clockwise

D. 90 degrees counterclockwise

E. 140 degrees clockwise

F. 140 degrees counterclockwise

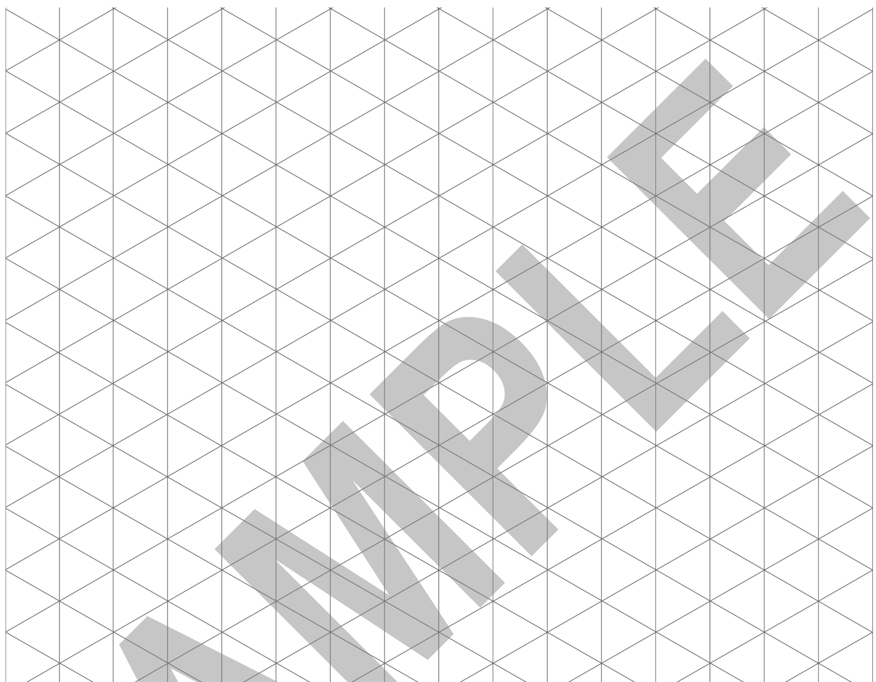
(From Unit 1, Lesson 1.)

Lesson 3: Making the Moves

Let's draw and describe translations, rotations, and reflections.

3.1: Notice and Wonder: The Isometric Grid

What do you notice? What do you wonder?



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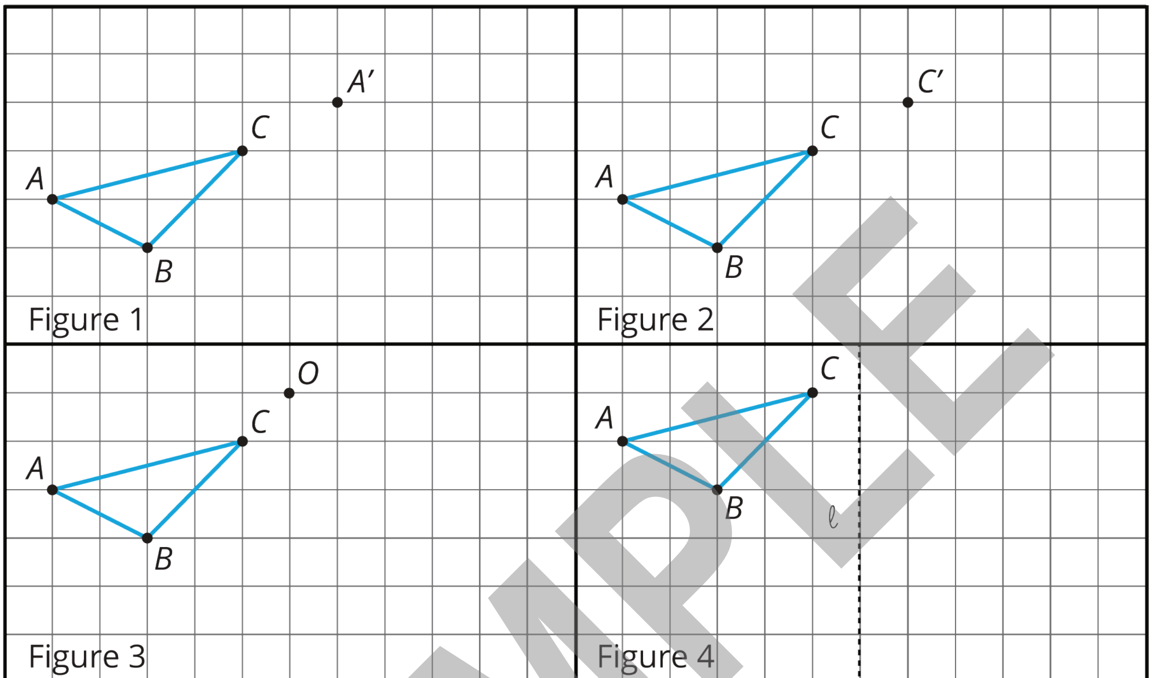
3.2: Transformation Information

Your teacher will give you tracing paper to carry out the moves specified. Use A', B', C', and D' to indicate vertices in the new figure that correspond to the points A, B, C, and D in the original figure.

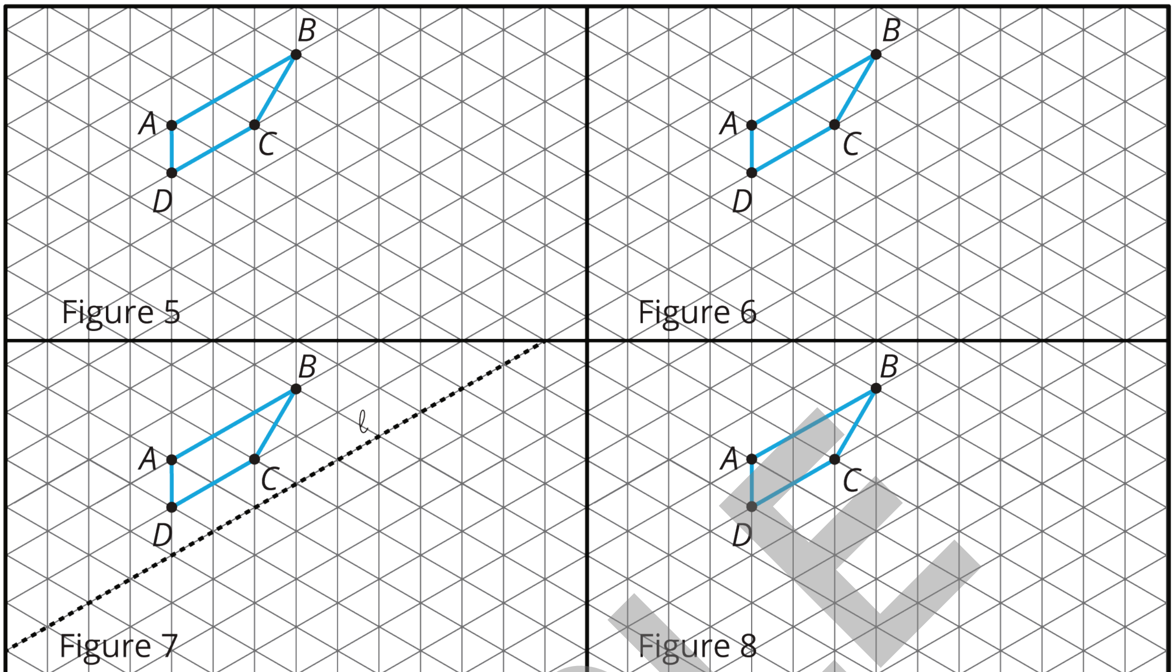


1. In Figure 1, translate triangle ABC so that A goes to A'.

2. In Figure 2, translate triangle ABC so that C goes to C'.

3. In Figure 3, rotate triangle ABC 90° counterclockwise using center O.

4. In Figure 4, reflect triangle ABC using line ℓ.



5. In Figure 5, rotate quadrilateral $ABCD$ 60° counterclockwise using center B.

6. In Figure 6, rotate quadrilateral $ABCD$ 60° clockwise using center C.

7. In Figure 7, reflect quadrilateral $ABCD$ using line ℓ.

8. In Figure 8, translate quadrilateral $ABCD$ so that A goes to C.

Are you ready for more?

The effects of each move can be "undone" by using another move. For example, to undo the effect of translating 3 units to the right, we could translate 3 units to the left. What move undoes each of the following moves?

1. Translate 3 units up

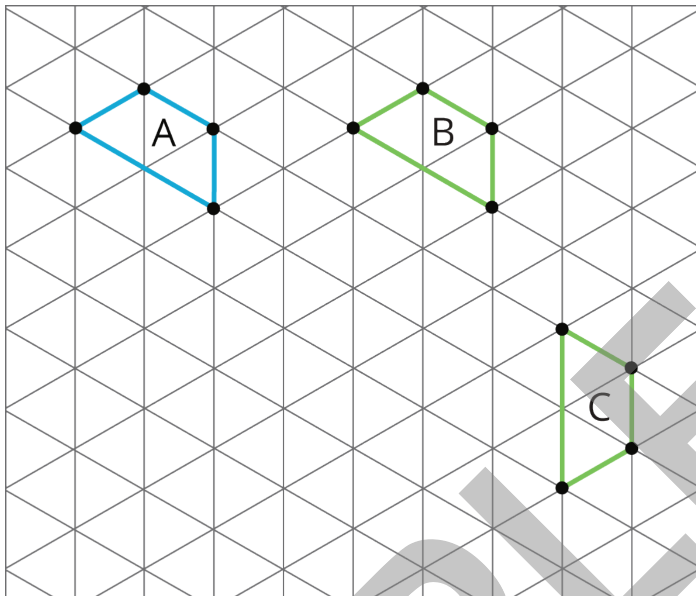
2. Translate 1 unit up and 1 unit to the left

3. Rotate 30 degrees clockwise around a point P

4. Reflect across a line ℓ

3.3: A to B to C

Here are some figures on an isometric grid.



1. Name a transformation that takes Figure A to Figure B. Name a transformation that takes Figure B to Figure C.

2. What is one **sequence of transformations** that takes Figure A to Figure C? Explain how you know.

Are you ready for more?

Experiment with some other ways to take Figure A to Figure C. For example, can you do it with. . .

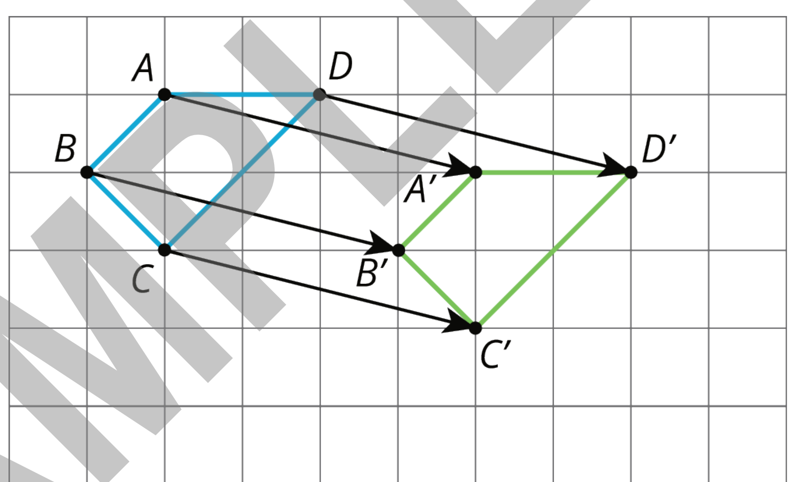
- No rotations?
- No reflections?
- No translations?

Lesson 3 Summary

A move or combination of moves is called a **transformation**. When we do 1 or more moves in a row, we often call that a **sequence of transformations**. When a figure is on a grid, we can use the grid to describe a transformation. We use the word **image** to describe the figure after a transformation. To distinguish the original figure from its image, points in the image are sometimes labeled with the same letters as the original figure, but with the symbol $'$ attached, as in A' (pronounced "A prime") is the image of A after a transformation.

- A translation can be described by two points. If a translation moves point A to point A', it moves the entire figure the same distance and direction as the distance and direction from A to A'. The distance and direction of a translation can be shown by an arrow.

  For example, here is a translation of quadrilateral $ABCD$ that moves A to A'.

- A rotation can be described by an angle and a center. The direction of the angle can be clockwise or counterclockwise.

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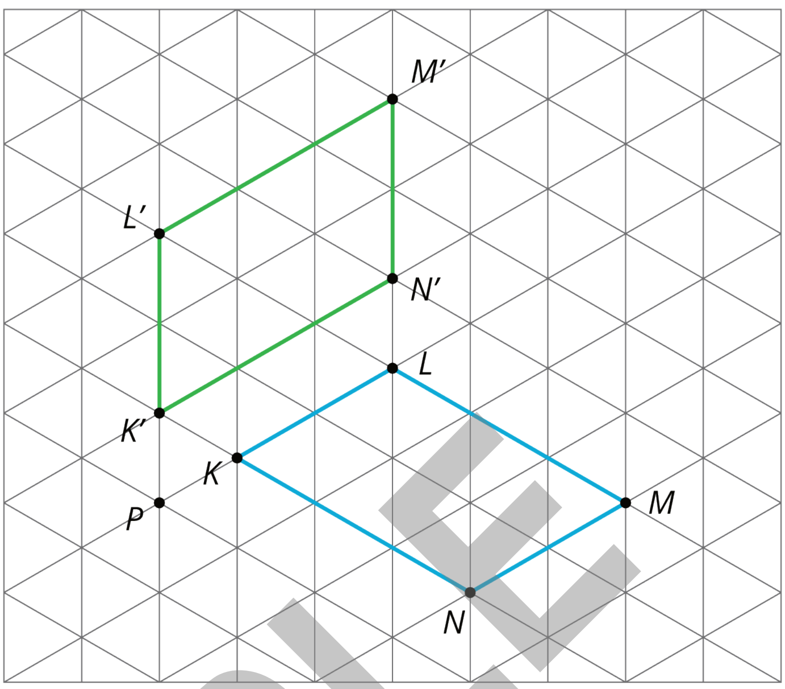
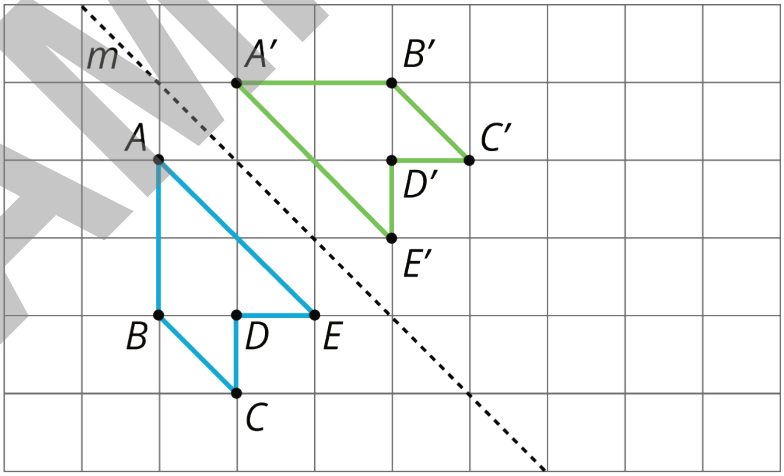
For example, quadrilateral $KLMN$ is rotated 60 degrees counterclockwise using center P. This type of grid is called an **isometric grid**. The isometric grid is made up of equilateral triangles. The angles in the triangles each measure 60 degrees, making the isometric grid convenient for showing rotations of 60 degrees.



- A reflection can be described by a line of reflection (the "mirror"). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.

  For example, pentagon $ABCDE$ is reflected across line m.



Glossary

- image
- sequence of transformations
- transformation

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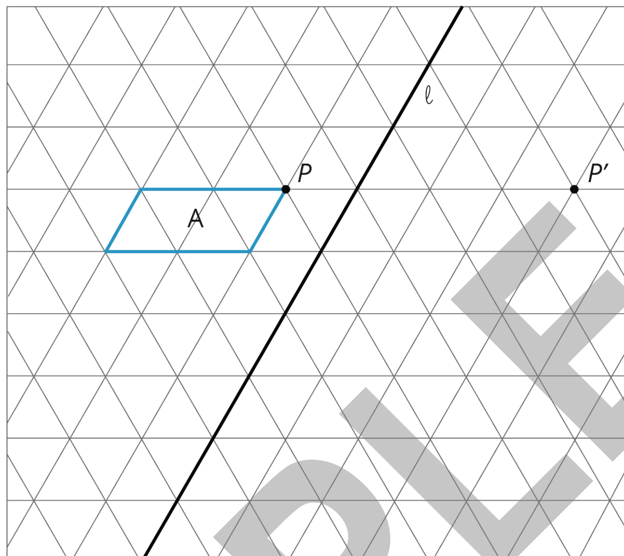
Lesson 3 Practice Problems

1. Apply each transformation described to Figure A. If you get stuck, try using tracing paper.

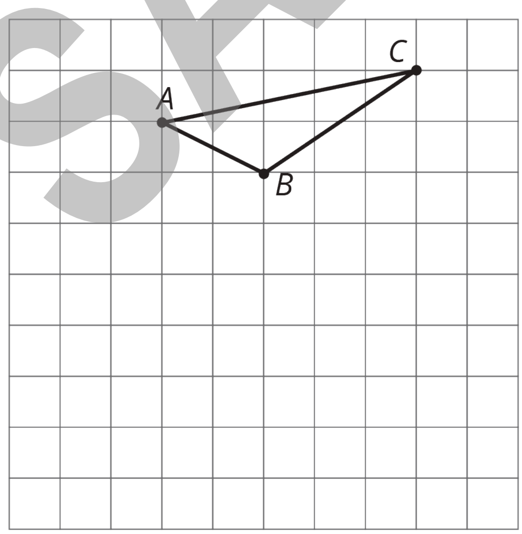


   a. A translation which takes P to P'

   b. A counterclockwise rotation of A, using center P, of 60 degrees

   c. A reflection of A across line ℓ

2. Here is triangle ABC drawn on a grid.



On the grid, draw a rotation of triangle ABC, a translation of triangle ABC, and a reflection of triangle ABC. Describe clearly how each was done.

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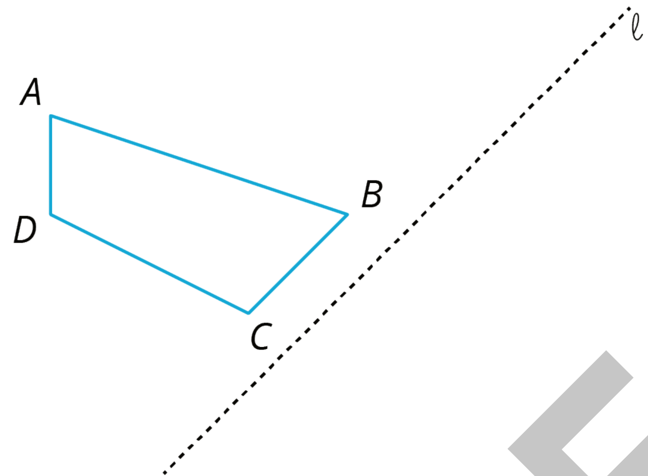
3. Here is quadrilateral $ABCD$ and line ℓ.



Draw the image of quadrilateral $ABCD$ after reflecting it across line ℓ.

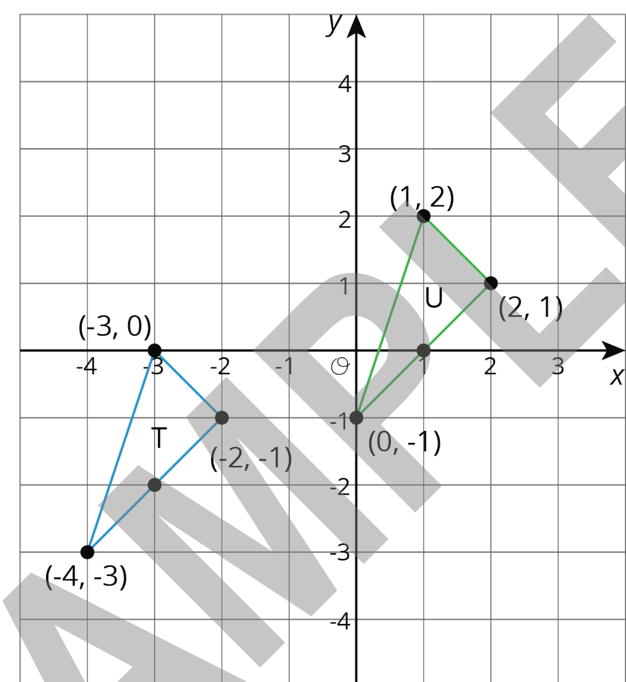
(From Unit 1, Lesson 2.)

Lesson 4: Coordinate Moves

Let's transform some figures and see what happens to the coordinates of points.

4.1: Translating Coordinates

Select all of the translations that take Triangle T to Triangle U. There may be more than one correct answer.



1. Translate $(-3, 0)$ to $(1, 2)$.

2. Translate $(2, 1)$ to $(-2, -1)$.

3. Translate $(-4, -3)$ to $(0, -1)$.

4. Translate $(1, 2)$ to $(2, 1)$.

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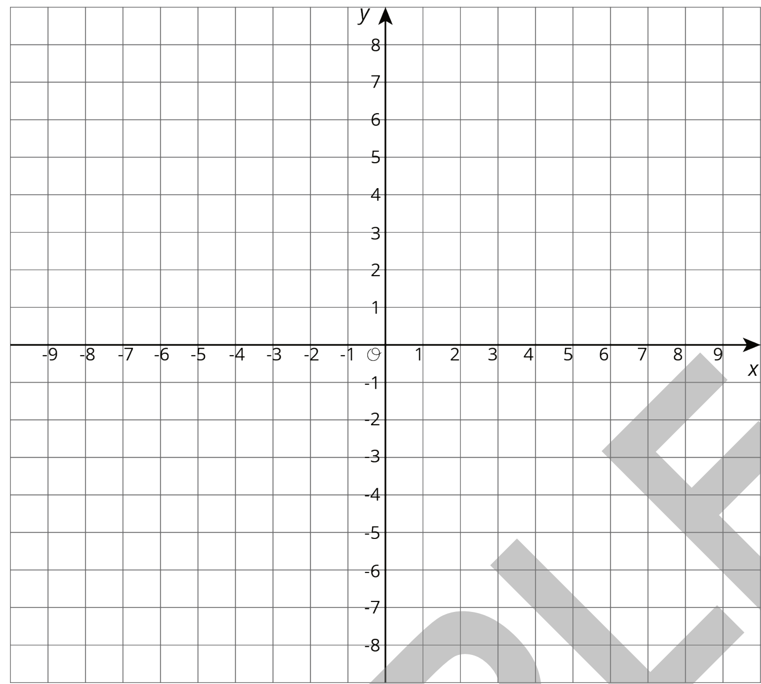
4.2: Reflecting Points on the Coordinate Plane



1. Here is a list of points

   $A = (0.5, 4)$ $B = (\text{-}4, 5)$ $C = (7, \text{-}2)$ $D = (6, 0)$ $E = (0, \text{-}3)$

   On the **coordinate plane**:

   a. Plot each point and label each with its coordinates.

   b. Using the x-axis as the line of reflection, plot the image of each point.

   c. Label the image of each point with its coordinates.

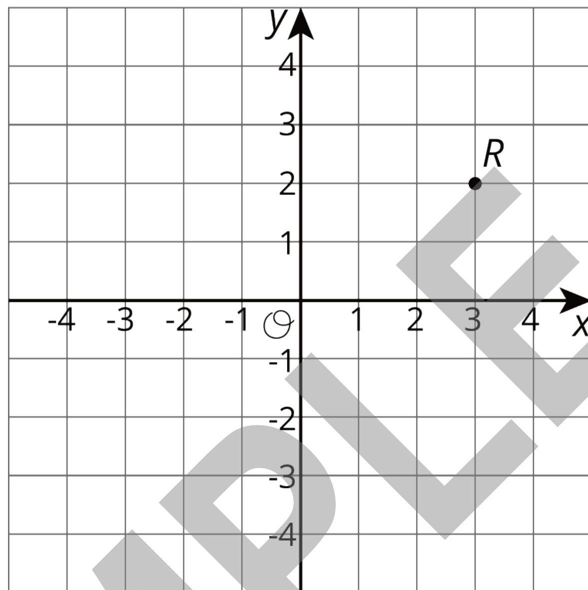
   d. Include a label using a letter. For example, the image of point A should be labeled A'.

2. If the point $(13, 10)$ were reflected using the x-axis as the line of reflection, what would be the coordinates of the image? What about $(13, \text{-}20)$? $(13, 570)$? Explain how you know.

3. The point R has coordinates $(3, 2)$.

   a. Without graphing, predict the coordinates of the image of point R if point R were reflected using the y-axis as the line of reflection.

   b. Check your answer by finding the image of R on the graph.



   c. Label the image of point R as R'.

   d. What are the coordinates of R'?

4. Suppose you reflect a point using the y-axis as line of reflection. How would you describe its image?

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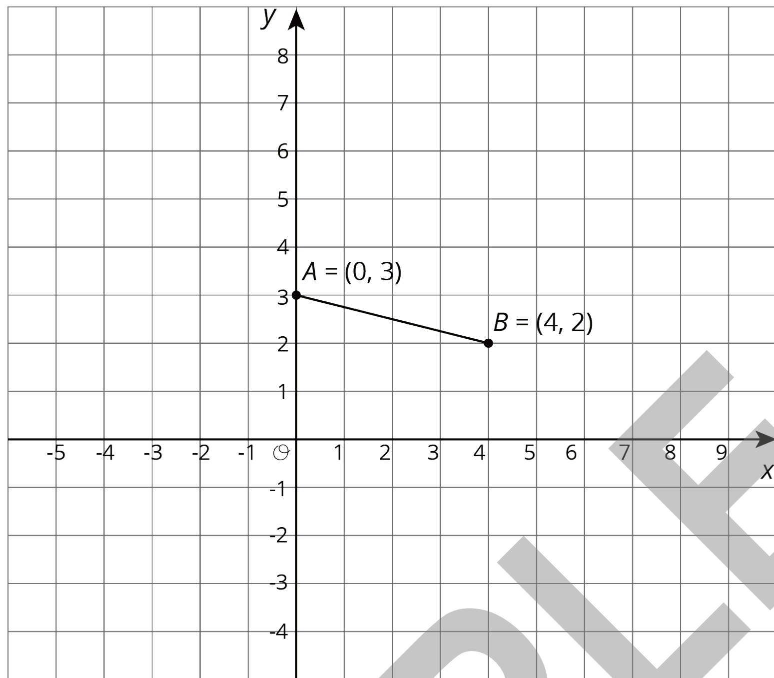
4.3: Transformations of a Segment



Apply each of the following transformations to segment AB.

1. Rotate segment AB 90 degrees counterclockwise around center B. Label the image of A as C. What are the coordinates of C?

2. Rotate segment AB 90 degrees counterclockwise around center A. Label the image of B as D. What are the coordinates of D?

3. Rotate segment AB 90 degrees clockwise around $(0, 0)$. Label the image of A as E and the image of B as F. What are the coordinates of E and F?

4. Compare the two 90-degree counterclockwise rotations of segment AB. What is the same about the images of these rotations? What is different?

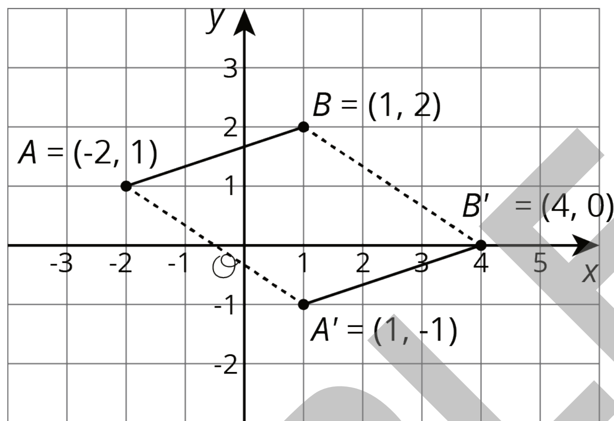
Are you ready for more?

Suppose EF and GH are line segments of the same length. Describe a sequence of transformations that moves EF to GH.

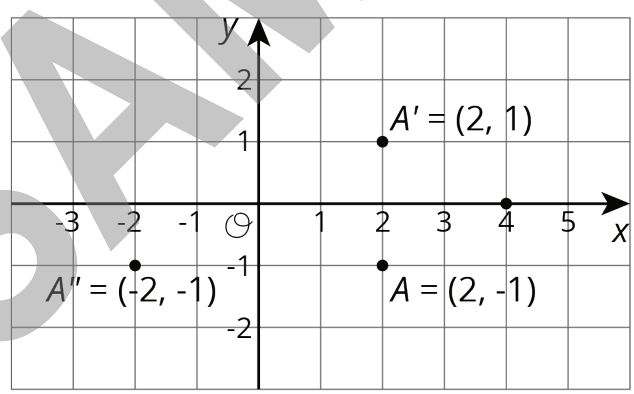
Lesson 4 Summary

We can use coordinates to describe points and find patterns in the coordinates of transformed points.

We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, segment AB is translated right 3 and down 2.

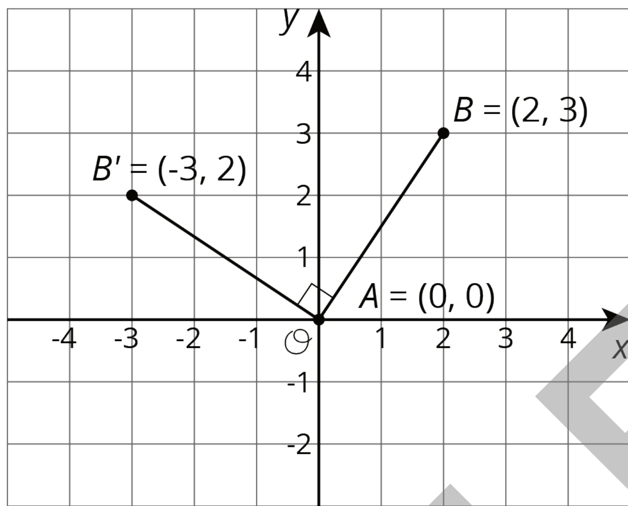


Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are $(2, -1)$ across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are $(2, 1)$. Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are $(-2, -1)$.



Reflections across other lines are more complex to describe.

We don't have the tools yet to describe rotations in terms of coordinates in general. Here is an example of a 90° rotation with center $(0, 0)$ in a counterclockwise direction.



Point A has coordinates $(0, 0)$. Segment AB was rotated 90° counterclockwise around A. Point B with coordinates $(2, 3)$ rotates to point B' whose coordinates are $(-3, 2)$.

Glossary

- coordinate plane

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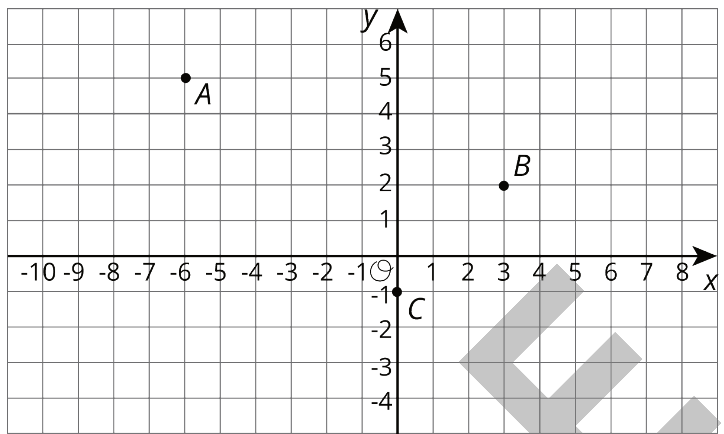
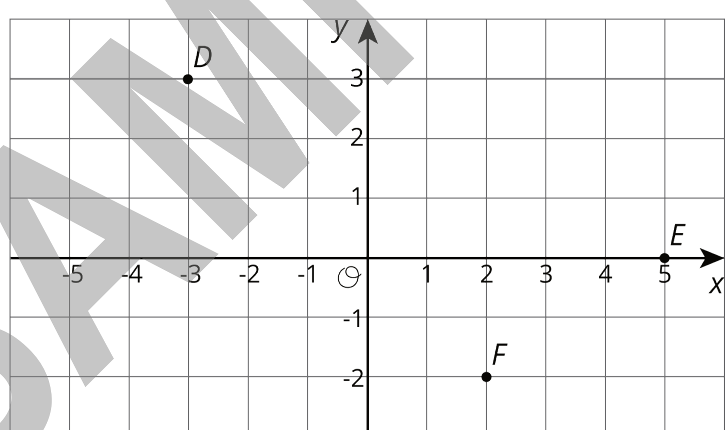
Lesson 4 Practice Problems

1. a. Here are some points.



What are the coordinates of A, B, and C after a translation to the right by 4 units and up 1 unit? Plot these points on the grid, and label them A', B' and C'.

b. Here are some points.



What are the coordinates of D, E, and F after a reflection over the y axis? Plot these points on the grid, and label them D', E' and F'.

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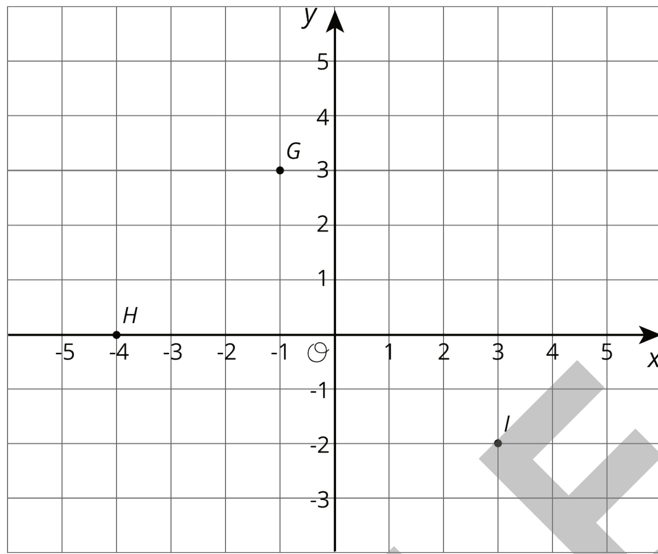
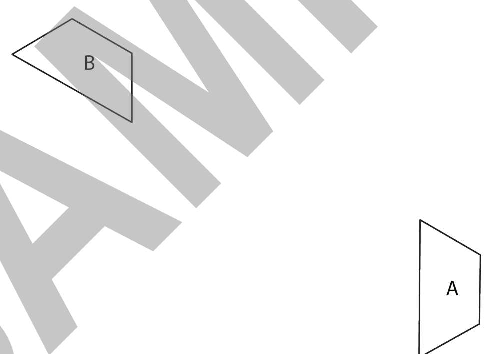
c. Here are some points.



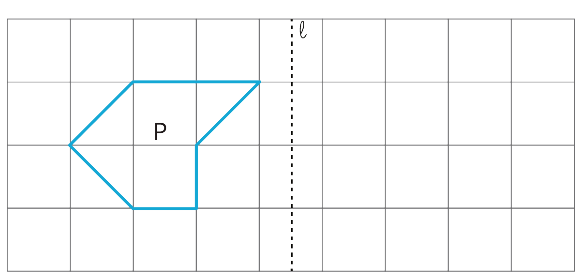
What are the coordinates of G, H, and I after a rotation about $(0, 0)$ by 90 degrees clockwise? Plot these points on the grid, and label them G', H' and I'.

2. Describe a sequence of transformations that takes trapezoid A to trapezoid B.



(From Unit 1, Lesson 3.)

3. Reflect polygon P using line ℓ.



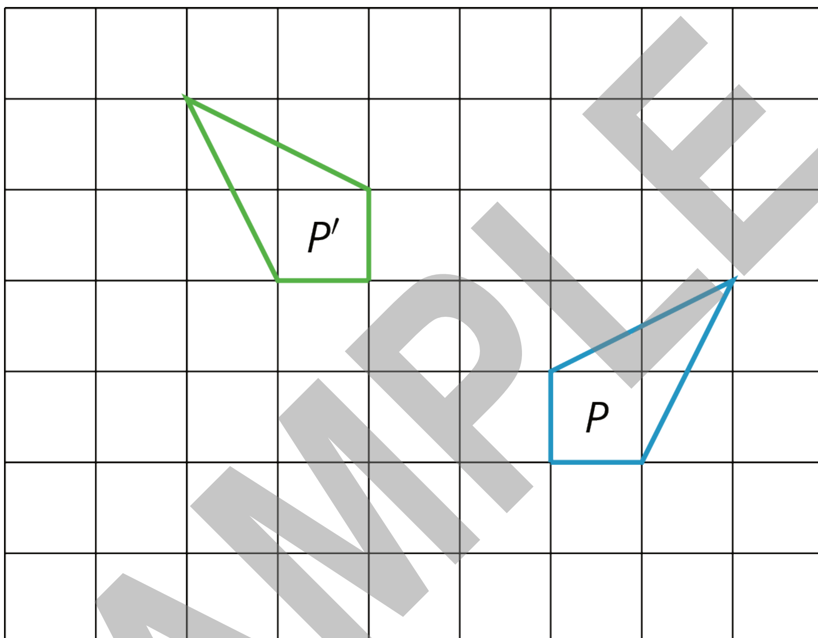
(From Unit 1, Lesson 3.)

Lesson 5: Describing Transformations

Let's transform some polygons in the coordinate plane.

5.1: Finding a Center of Rotation

Andre performs a 90-degree counterclockwise rotation of Polygon P and gets Polygon P',
but he does not say what the center of the rotation is. Can you find the center?



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5.2: Info Gap: Transformation Information

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Are you ready for more?

Sometimes two transformations, one performed after the other, have a nice description as a single transformation. For example, instead of translating 2 units up followed by translating 3 units up, we could simply translate 5 units up. Instead of rotating 20 degrees counterclockwise around the origin followed by rotating 80 degrees clockwise around the origin, we could simply rotate 60 degrees clockwise around the origin.

Can you find a simple description of reflecting across the x-axis followed by reflecting across the y-axis?

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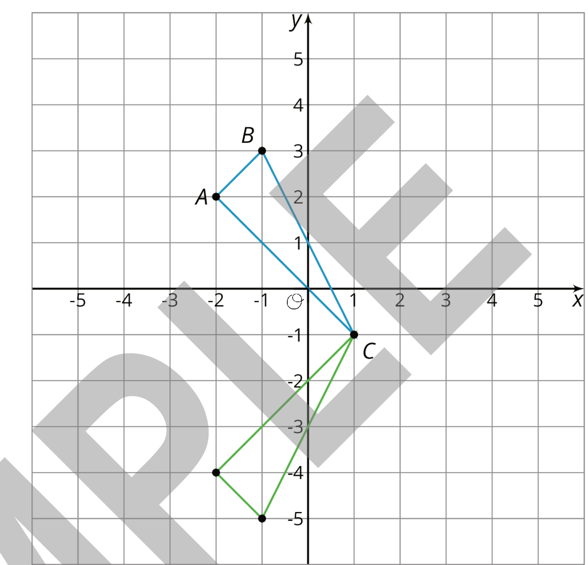
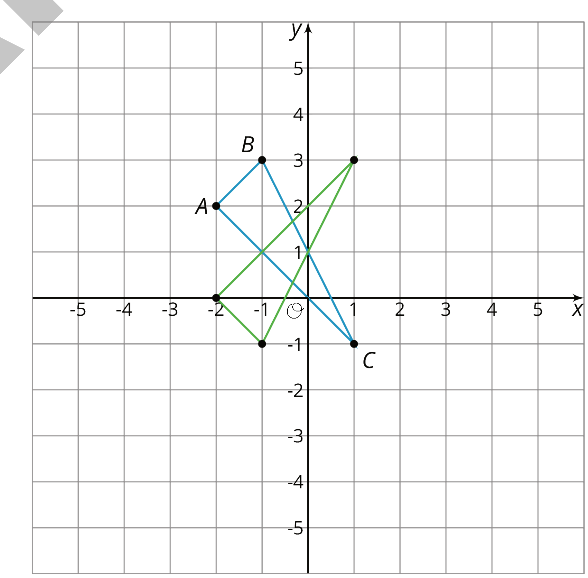
Lesson 5 Summary

The center of a rotation for a figure doesn't have to be one of the points on the figure. To find a center of rotation, look for a point that is the same distance from two corresponding points. You will probably have to do this for a couple of different pairs of corresponding points to nail it down.

When we perform a sequence of transformations, the order of the transformations can be important. Here is triangle ABC translated up two units and then reflected over the x-axis.



Here is triangle ABC reflected over the x-axis and then translated up two units.



Triangle ABC ends up in different places when the transformations are applied in the opposite order!

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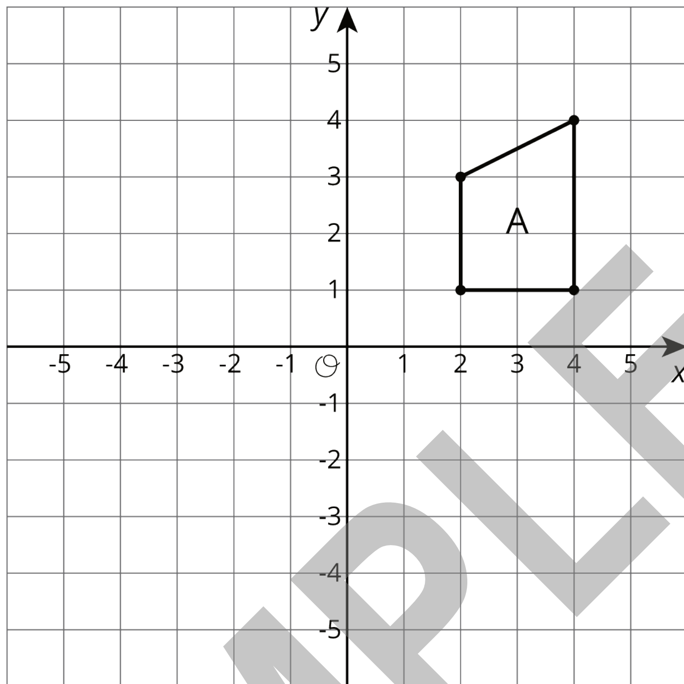
Lesson 5 Practice Problems

1. Here is Trapezoid A in the coordinate plane:



a. Draw Polygon B, the image of A, using the y-axis as the line of reflection.

b. Draw Polygon C, the image of B, using the x-axis as the line of reflection.

c. Draw Polygon D, the image of C, using the x-axis as the line of reflection.

2. The point $(-4, 1)$ is rotated 180 degrees counterclockwise using center $(-3, 0)$. What are the coordinates of the image?

A. $(-5, -2)$

B. $(-4, -1)$

C. $(-2, -1)$

D. $(4, -1)$

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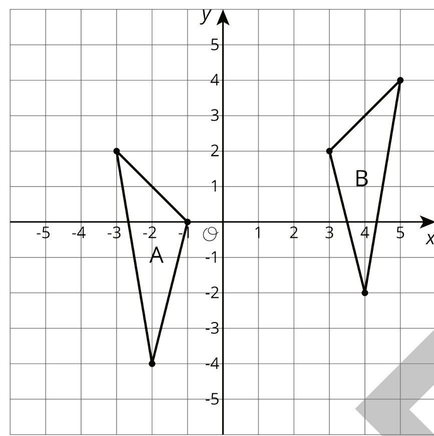
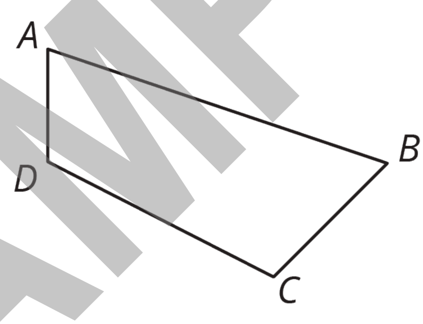
3. Describe a sequence of transformations for which Triangle B is the image of Triangle A.



4. Here is quadrilateral $ABCD$.



Draw the image of quadrilateral $ABCD$ after each transformation.

a. The translation that takes B to D.

b. The reflection over segment BC.

c. The rotation about point A by angle DAB, counterclockwise.

(From Unit 1, Lesson 2.)

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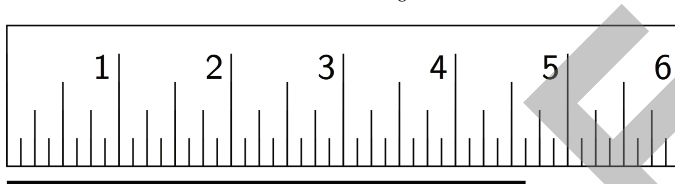
Lesson 6: No Bending or Stretching

Let's compare measurements before and after translations, rotations, and reflections.

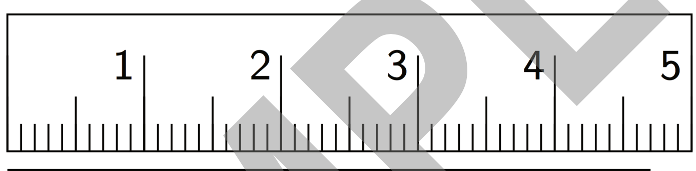
6.1: Measuring Segments

For each question, the unit is represented by the large tick marks with whole numbers.

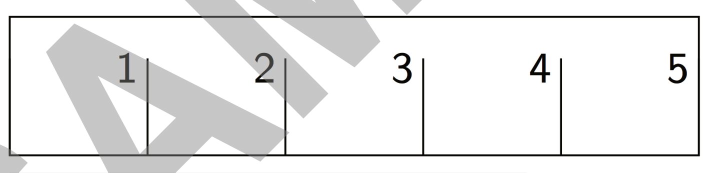
1. Find the length of this segment to the nearest $\frac{1}{8}$ of a unit.



2. Find the length of this segment to the nearest 0.1 of a unit.



3. Estimate the length of this segment to the nearest $\frac{1}{8}$ of a unit.



4. Estimate the length of the segment in the prior question to the nearest 0.1 of a unit.

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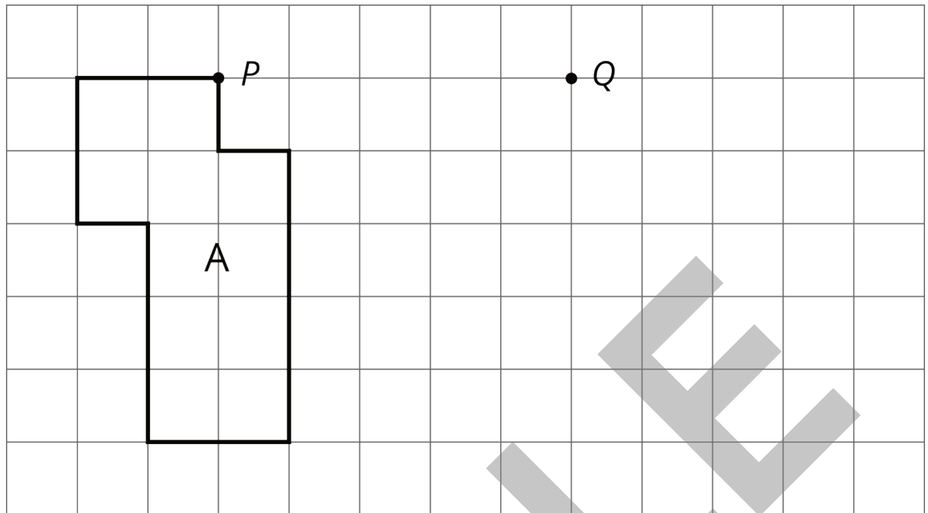
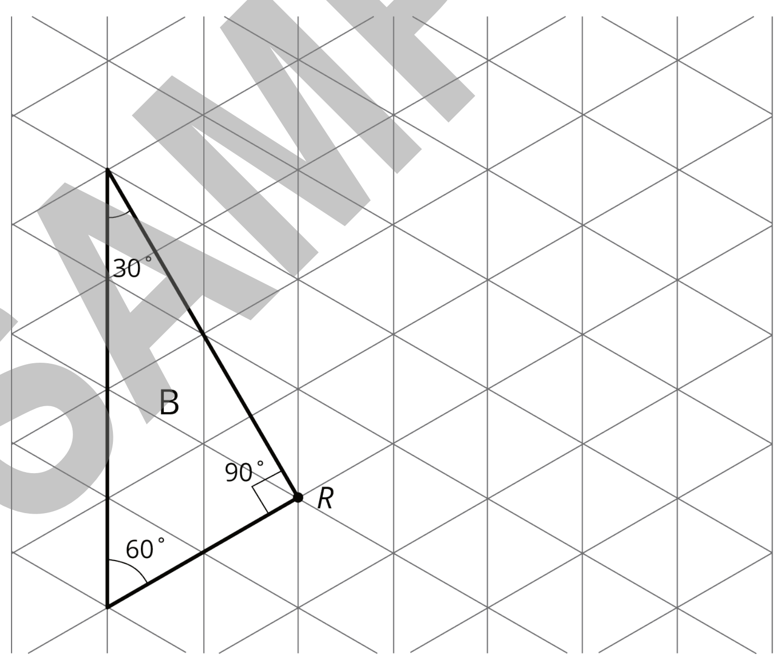
6.2: Sides and Angles

1. Translate Polygon A so point P goes to point Q. In the image, write the length of each side, in grid units, next to the side.



2. Rotate Triangle B 90 degrees clockwise using R as the center of rotation. In the image, write the measure of each angle in its interior.



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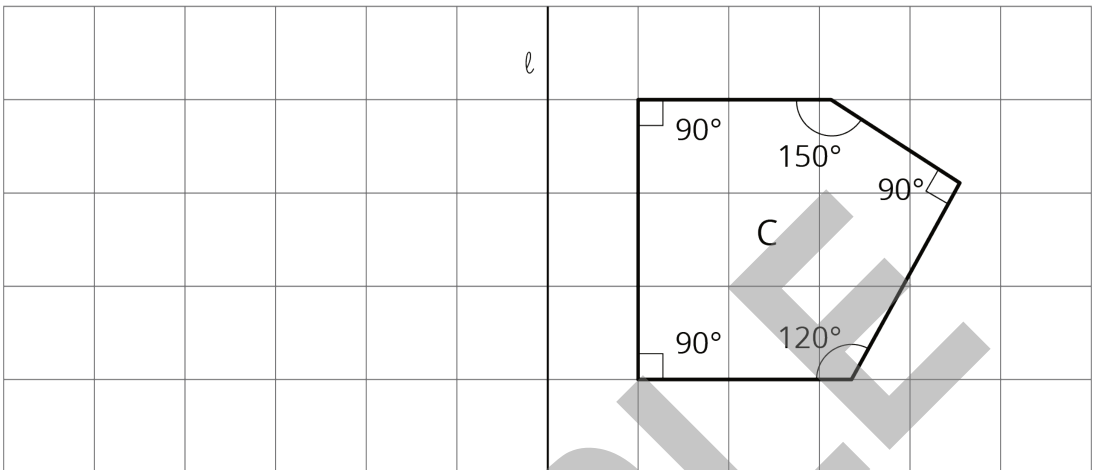
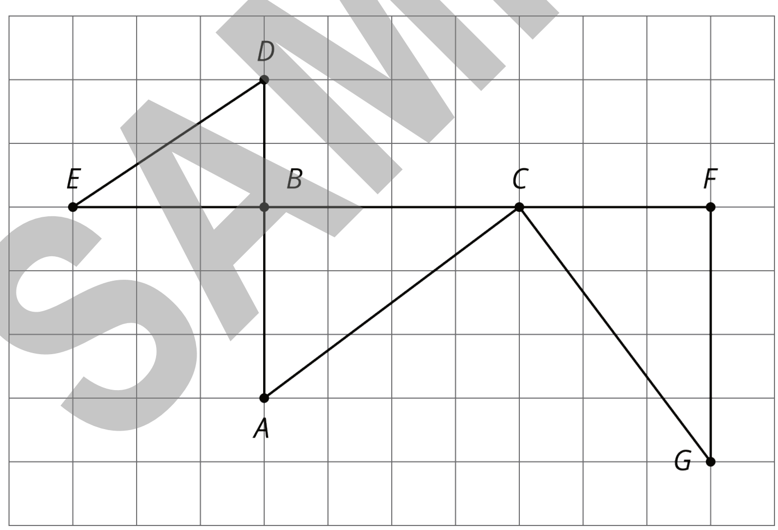
3. Reflect Pentagon C across line ℓ.
   a. In the image, write the length of each side, in grid units, next to the side. You may need to make your own ruler with tracing paper or a blank index card.

   b. In the image, write the measure of each angle in the interior.



6.3: Which One?

Here is a grid showing triangle ABC and two other triangles.



You can use a **rigid transformation** to take triangle ABC to *one* of the other triangles.

1. Which one? Explain how you know.

2. Describe a rigid transformation that takes ABC to the triangle you selected.

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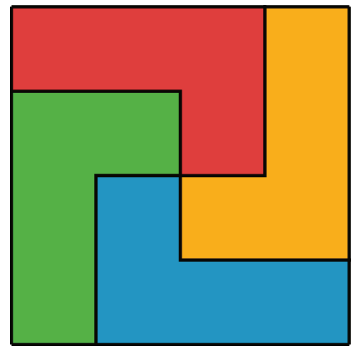
Are you ready for more?

A square is made up of an L-shaped region and three transformations of the region. If the perimeter of the square is 40 units, what is the perimeter of each L-shaped region?



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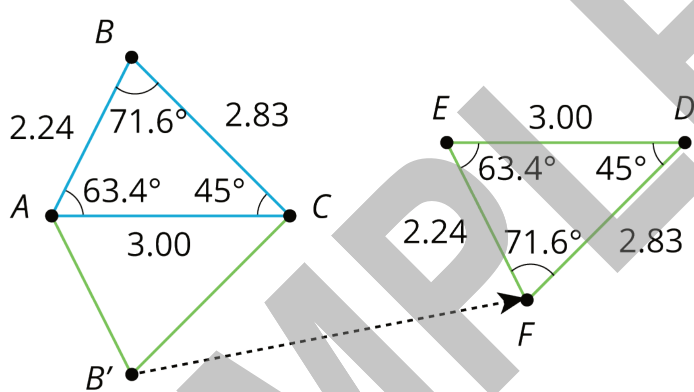
Lesson 6 Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of **rigid transformations**. A rigid transformation is a move that doesn't change measurements on any figure.

Earlier, we learned that a figure and its image have corresponding points. With a rigid transformation, figures like polygons also have **corresponding** sides and corresponding angles. These corresponding parts have the same measurements.

For example, triangle EFD was made by reflecting triangle ABC across a horizontal line, then translating. Corresponding sides have the same lengths, and corresponding angles have the same measures.



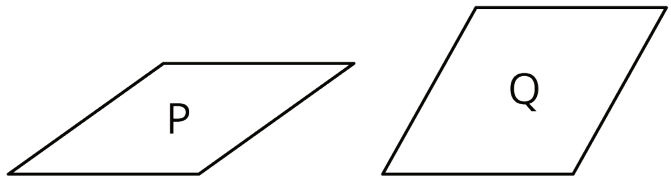
measurements in triangle ABC	corresponding measurements in image EFD
$AB = 2.24$	$EF = 2.24$
$BC = 2.83$	$FD = 2.83$
$CA = 3.00$	$DE = 3.00$
$m\angle ABC = 71.6^\circ$	$m\angle EFD = 71.6^\circ$
$m\angle BCA = 45.0^\circ$	$m\angle FDE = 45.0^\circ$
$m\angle CAB = 63.4^\circ$	$m\angle DEF = 63.4^\circ$

Glossary

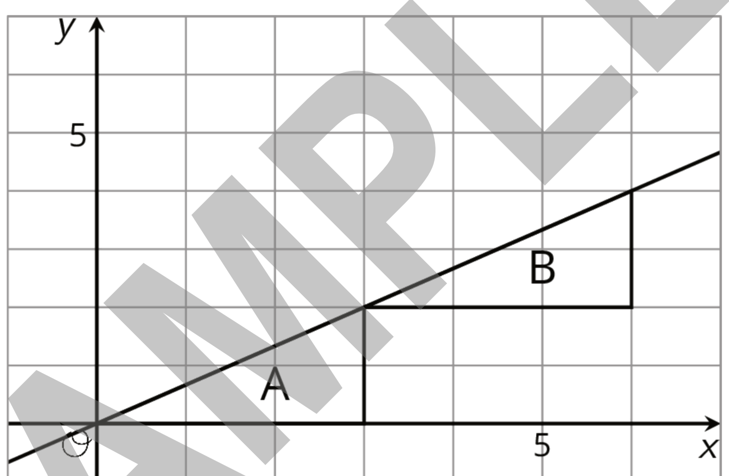
- corresponding
- rigid transformation

Lesson 6 Practice Problems

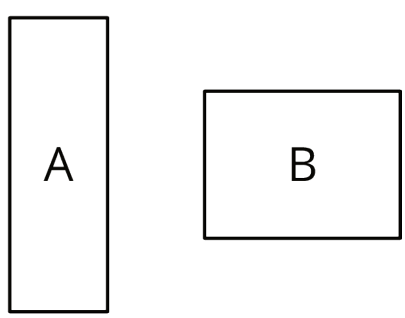
1. Is there a rigid transformation taking Rhombus P to Rhombus Q? Explain how you know.



2. Describe a rigid transformation that takes Triangle A to Triangle B.

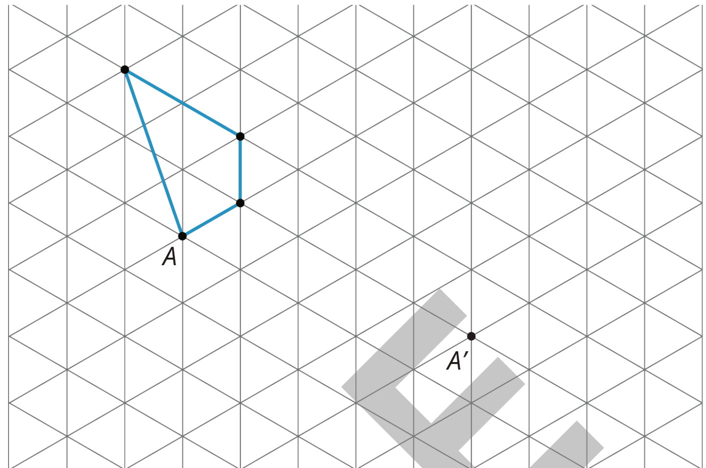


3. Is there a rigid transformation taking Rectangle A to Rectangle B? Explain how you know.

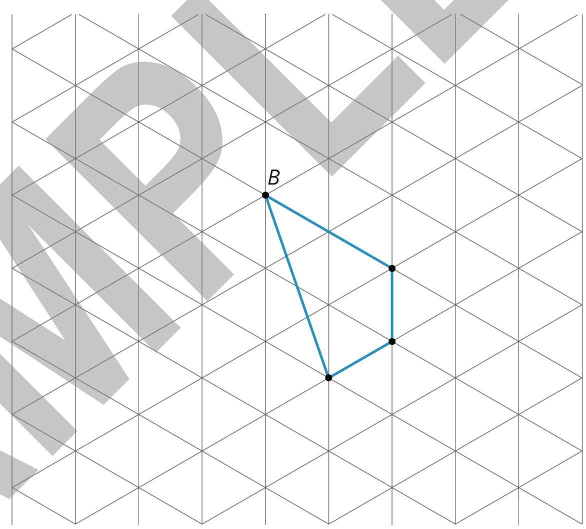


4. For each shape, draw its image after performing the transformation. If you get stuck, consider using tracing paper.

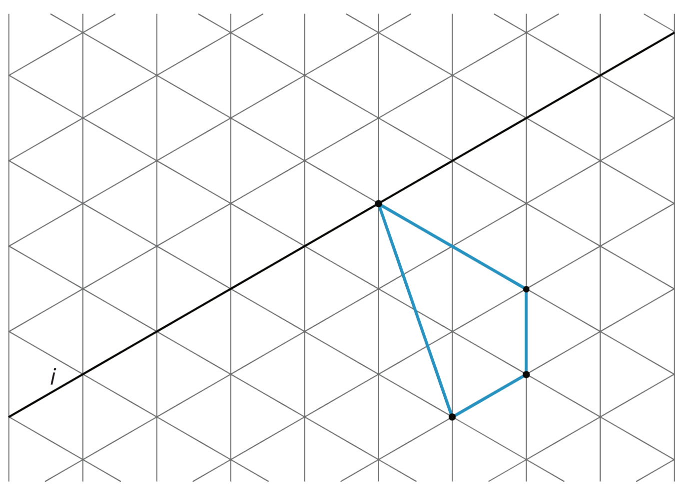
   a. Translate the shape so that A goes to A'.



   a. Rotate the shape 180 degrees counterclockwise around B.



   a. Reflect the shape over the line shown.



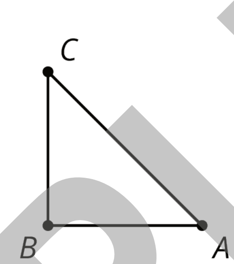
(From Unit 1, Lesson 3.)

Lesson 7: Rotation Patterns

Let's rotate figures in a plane.

7.1: Building a Quadrilateral

Here is a right isosceles triangle:



1. Rotate triangle ABC 90 degrees clockwise around B.
2. Rotate triangle ABC 180 degrees clockwise round B.
3. Rotate triangle ABC 270 degrees clockwise around B.
4. What would it look like when you rotate the four triangles 90 degrees clockwise around B? 180 degrees? 270 degrees clockwise?

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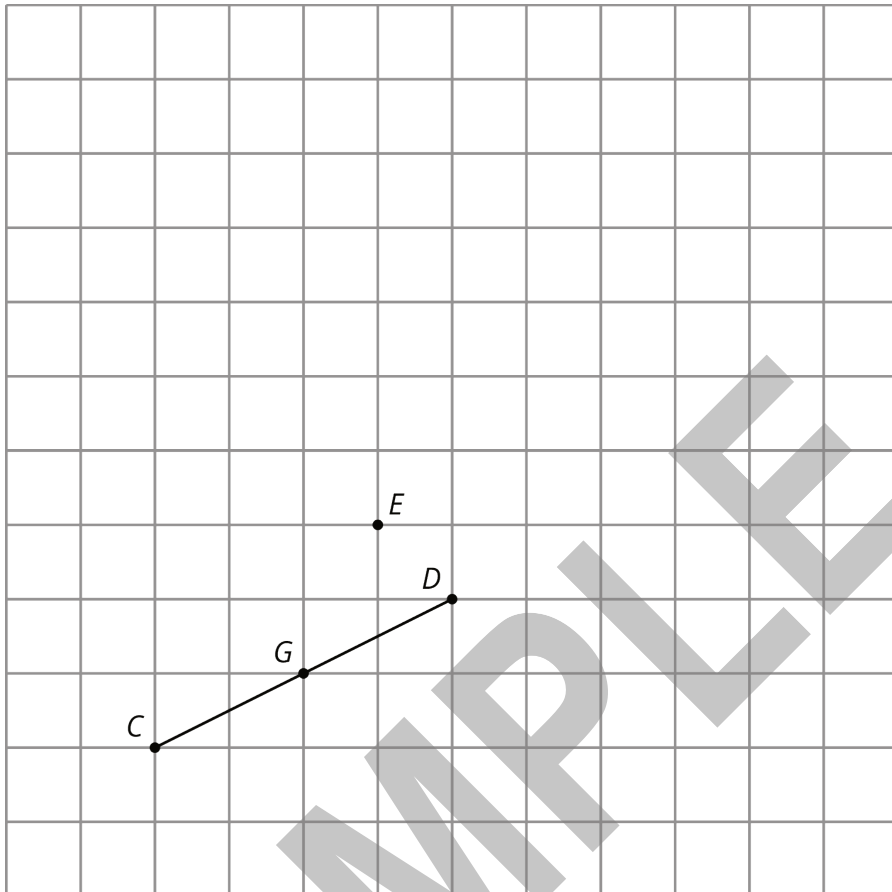
7.2: Rotating a Segment



1. Rotate segment CD 180 degrees around point D. Draw its image and label the image of C as A.

2. Rotate segment CD 180 degrees around point E. Draw its image and label the image of C as B and the image of D as F.

3. Rotate segment CD 180 degrees around its midpoint, G. What is the image of C?

4. What happens when you rotate a segment 180 degrees around a point?

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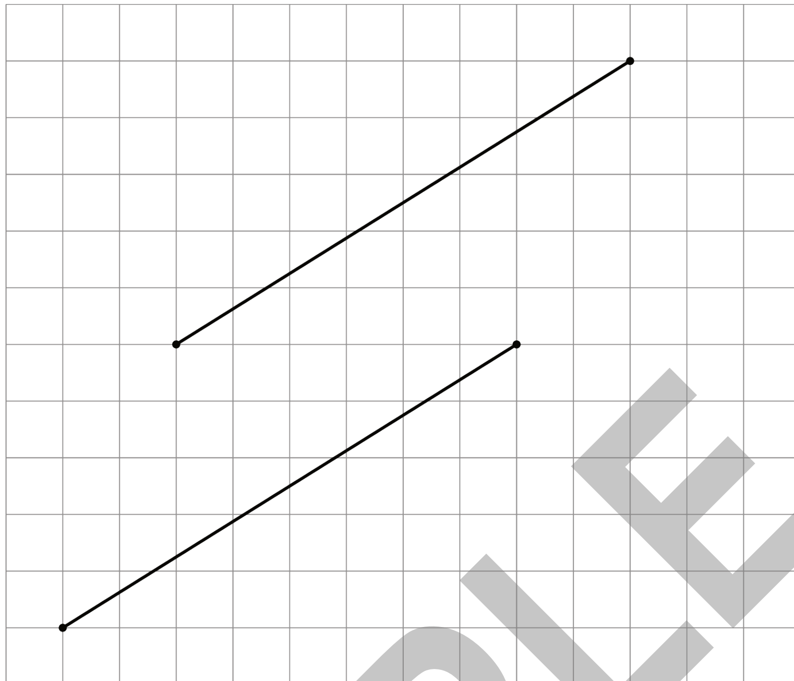
Are you ready for more?



Here are two line segments. Is it possible to rotate one line segment to the other? If so, find the center of such a rotation. If not, explain why not.

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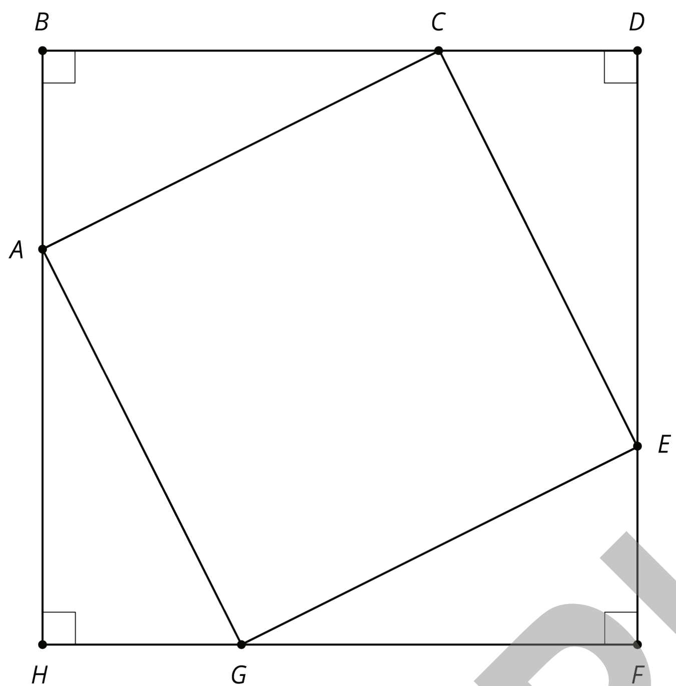
7.3: A Pattern of Four Triangles



You can use rigid transformations of a figure to make patterns. Here is a diagram built with three different transformations of triangle ABC.

1. Describe a rigid transformation that takes triangle ABC to triangle CDE.

2. Describe a rigid transformation that takes triangle ABC to triangle EFG.

3. Describe a rigid transformation that takes triangle ABC to triangle GHA.

4. Do segments AC, CE, EG, and GA all have the same length? Explain your reasoning.

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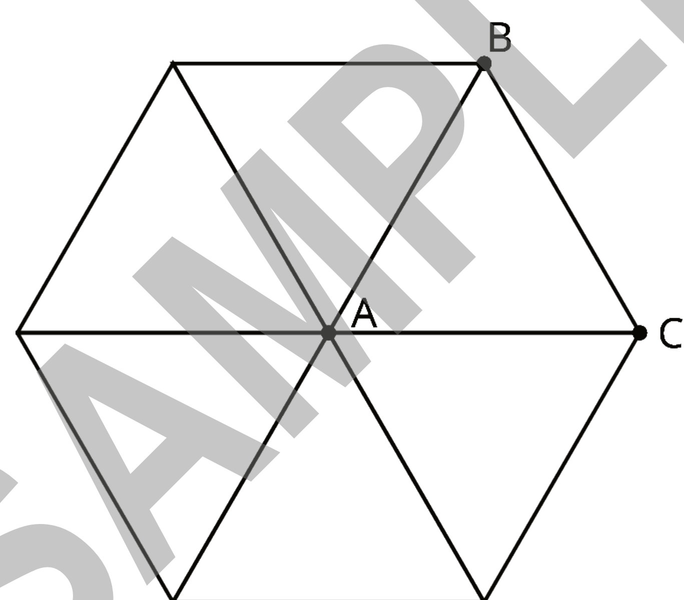
Lesson 7 Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

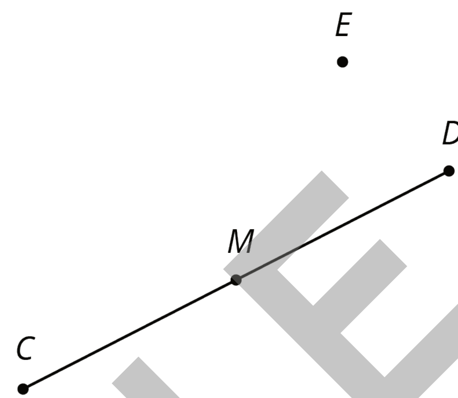
- The segment maps to itself (if the center of rotation is the midpoint of the segment).
- The image of the segment overlaps with the segment and lies on the same line (if the center of rotation is a point on the segment).
- The image of the segment does not overlap with the segment (if the center of rotation is *not* on the segment).

We can also build patterns by rotating a shape. For example, triangle ABC shown here has $m(\angle A) = 60$. If we rotate triangle ABC 60 degrees, 120 degrees, 180 degrees, 240 degrees, and 300 degrees clockwise, we can build a hexagon.



Lesson 7 Practice Problems

1. For the figure shown here,



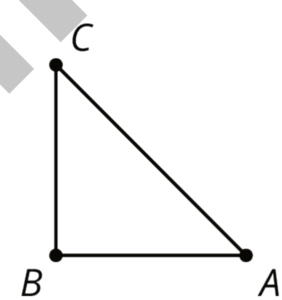
   a. Rotate segment CD 180° around point D.

   b. Rotate segment CD 180° around point E.

   c. Rotate segment CD 180° around point M.

2. Here is an isosceles right triangle:

   Draw these three rotations of triangle ABC together.



   a. Rotate triangle ABC 90 degrees clockwise around A.

   b. Rotate triangle ABC 180 degrees around A.

   c. Rotate triangle ABC 270 degrees clockwise around A.

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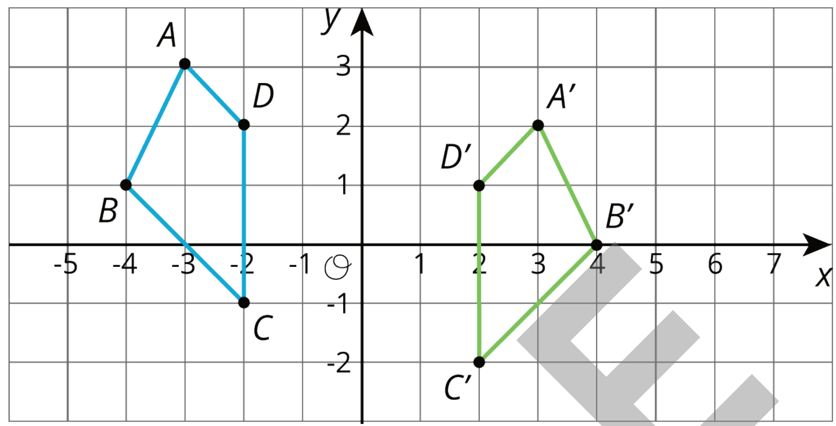
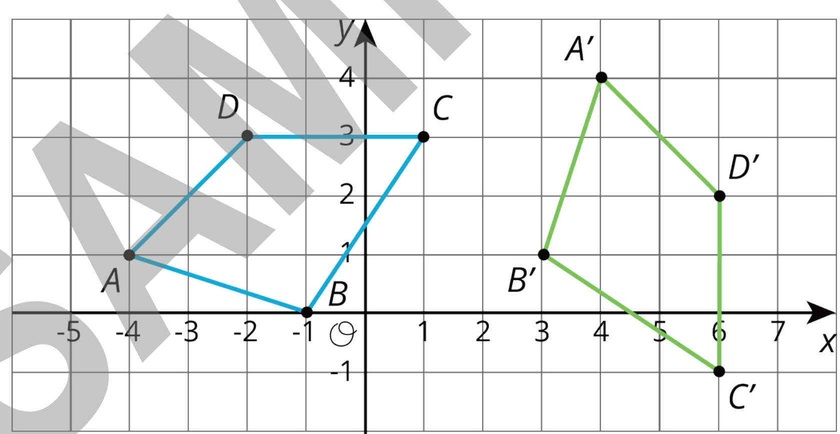
3. Each graph shows two polygons $ABCD$ and $A'B'C'D'$. In each case, describe a sequence of transformations that takes $ABCD$ to $A'B'C'D'$.

a.



b.



(From Unit 1, Lesson 4.)

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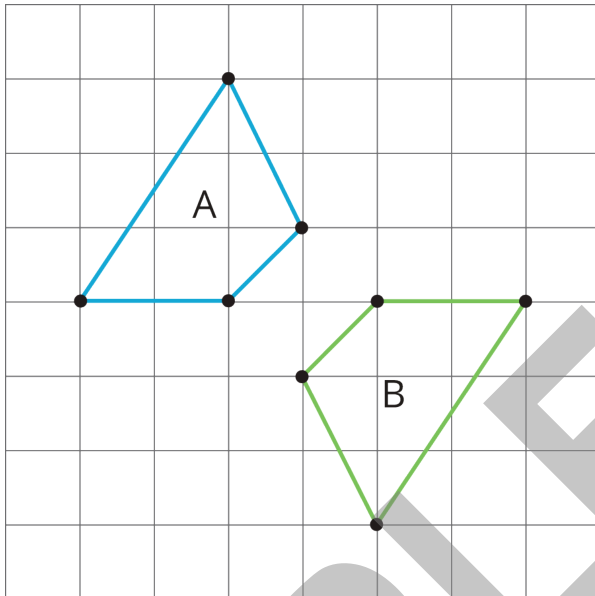
4. Lin says that she can map Polygon A to Polygon B using *only* reflections. Do you agree with Lin? Explain your reasoning.



(From Unit 1, Lesson 3.)

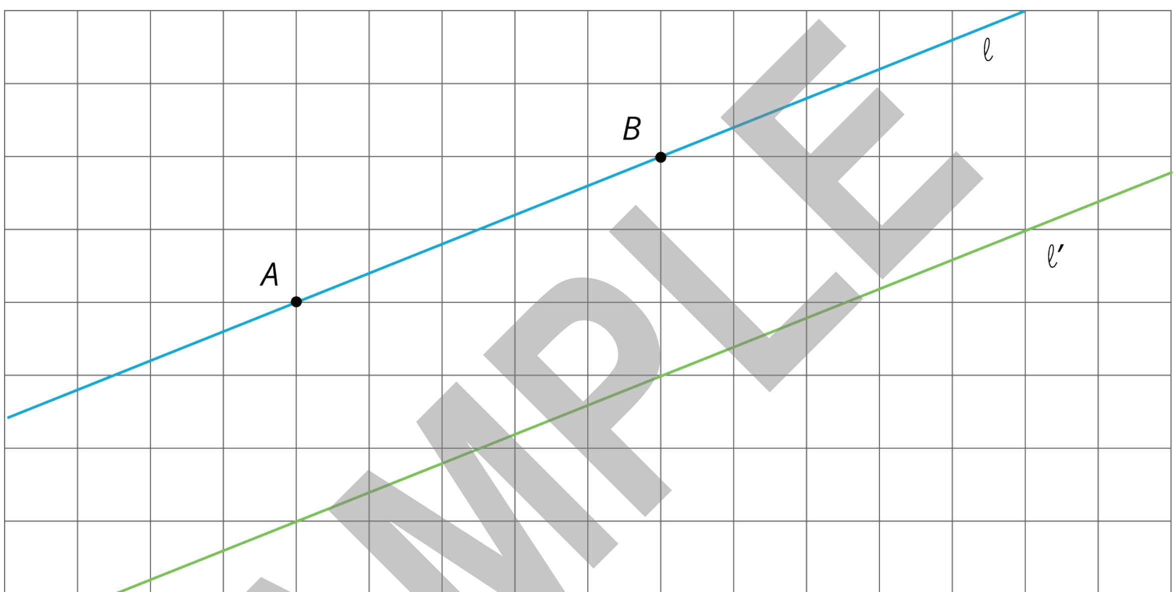
Lesson 8: Moves in Parallel

Let's transform some lines.

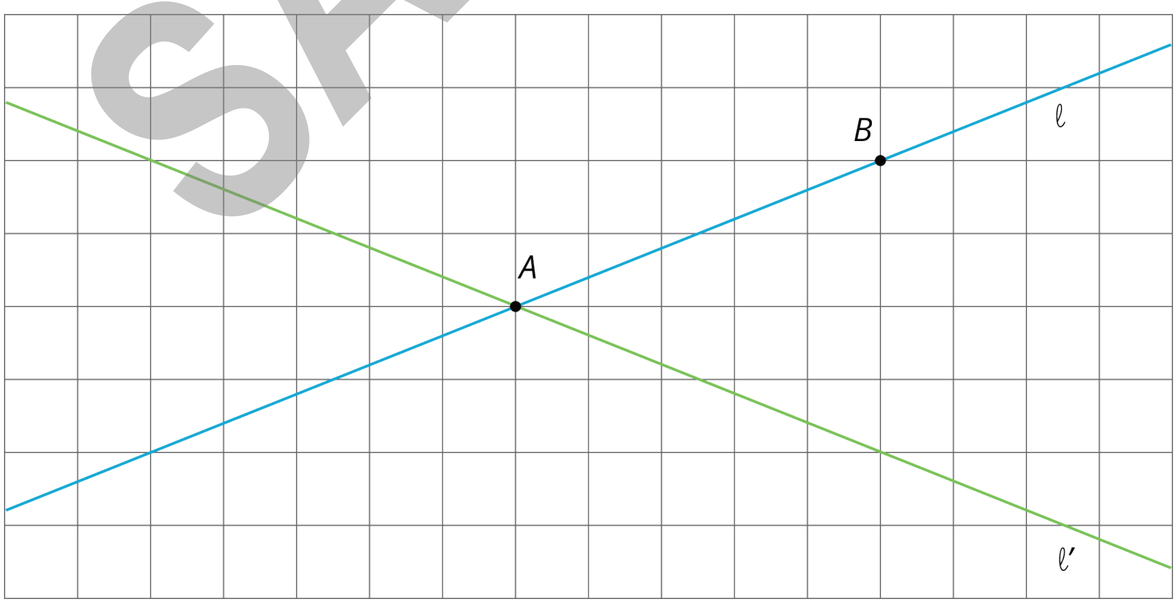
8.1: Line Moves

For each diagram, describe a translation, rotation, or reflection that takes line ℓ to line ℓ'. Then plot and label A' and B', the images of A and B.

1.



2.



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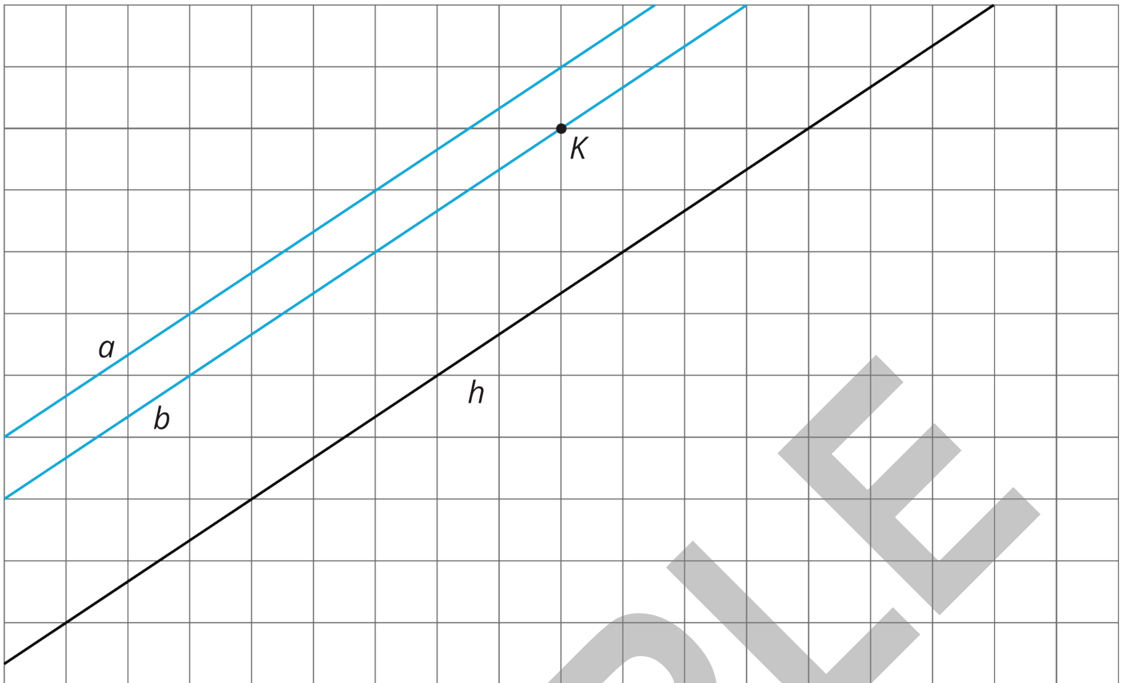
8.2: Parallel Lines



Use a piece of tracing paper to trace lines a and b and point K. Then use that tracing paper to draw the images of the lines under the three different transformations listed.

As you perform each transformation, think about the question:

What is the image of two parallel lines under a rigid transformation?

1. Translate lines a and b 3 units up and 2 units to the right.

   a. What do you notice about the changes that occur to lines a and b after the translation?

   b. What is the same in the original and the image?

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2. Rotate lines a and b counterclockwise 180 degrees using K as the center of rotation.

   a. What do you notice about the changes that occur to lines a and b after the rotation?

   b. What is the same in the original and the image?

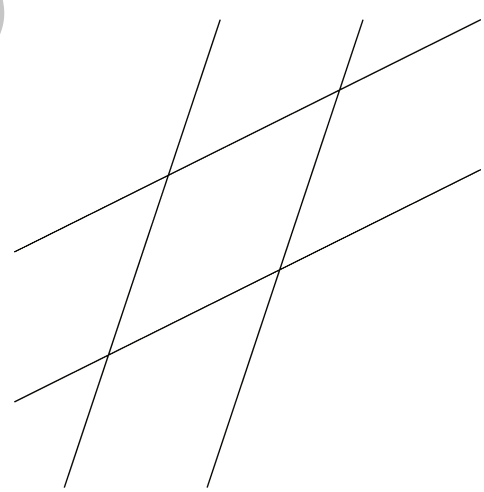
3. Reflect lines a and b across line h.

   a. What do you notice about the changes that occur to lines a and b after the reflection?

   b. What is the same in the original and the image?

Are you ready for more?

When you rotate two parallel lines, sometimes the two original lines intersect their images and form a quadrilateral. What is the most specific thing you can say about this quadrilateral? Can it be a square? A rhombus? A rectangle that isn't a square? Explain your reasoning.

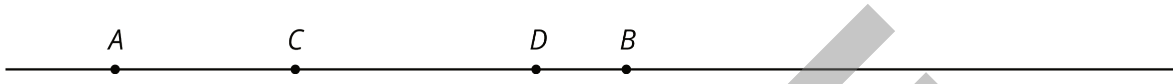


8.3: Let's Do Some 180's

1. The diagram shows a line with points labeled A, C, D, and B.
   a. On the diagram, draw the image of the line and points A, C, and B after the line has been rotated 180 degrees around point D.

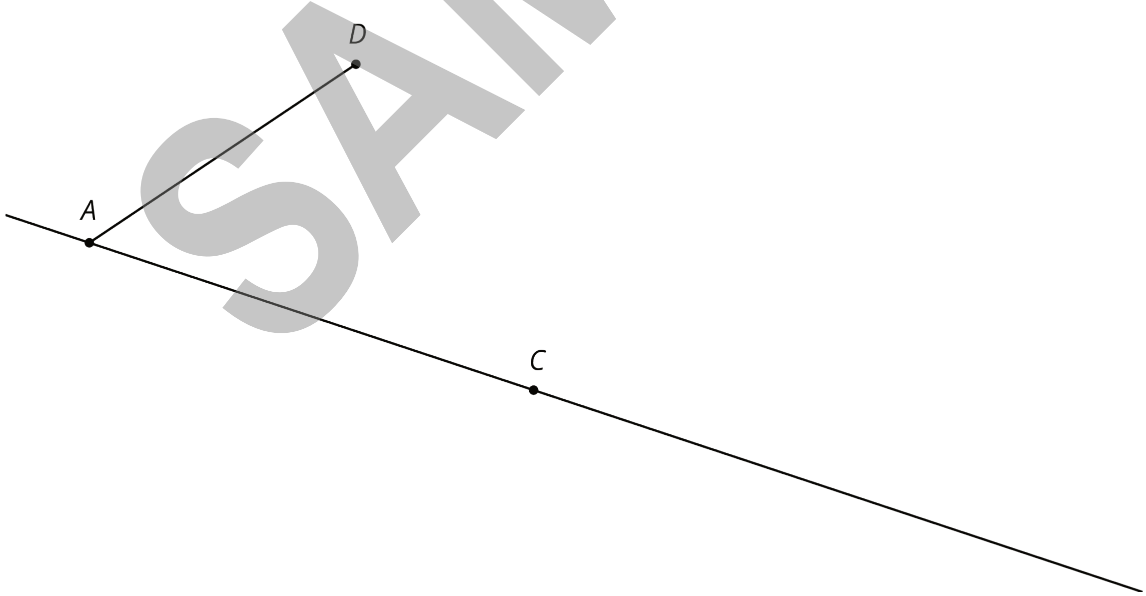
   b. Label the images of the points A', B', and C'.

   c. What is the order of all seven points? Explain or show your reasoning.



2. The diagram shows a line with points A and C on the line and a segment AD where D is not on the line.
   a. Rotate the figure 180 degrees about point C. Label the image of A as A' and the image of D as D'.

   b. What do you know about the relationship between angle CAD and angle $CA'D'$? Explain or show your reasoning.



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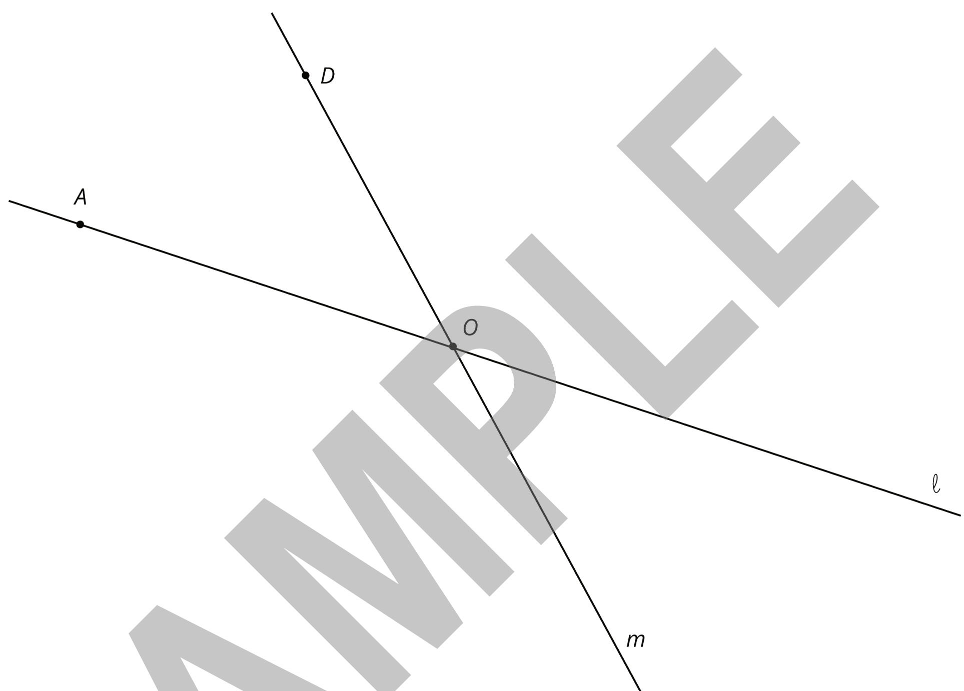
3. The diagram shows two lines ℓ and m that intersect at a point O with point A on ℓ and point D on m.
   a. Rotate the figure 180 degrees around O. Label the image of A as A' and the image of D as D'.

   b. What do you know about the relationship between the angles in the figure? Explain or show your reasoning.



Lesson 8 Summary

Rigid transformations have the following properties:

- A rigid transformation of a line is a line.
- A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.

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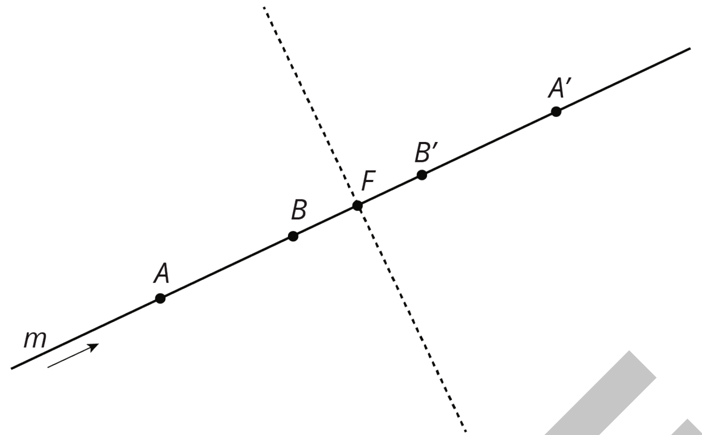
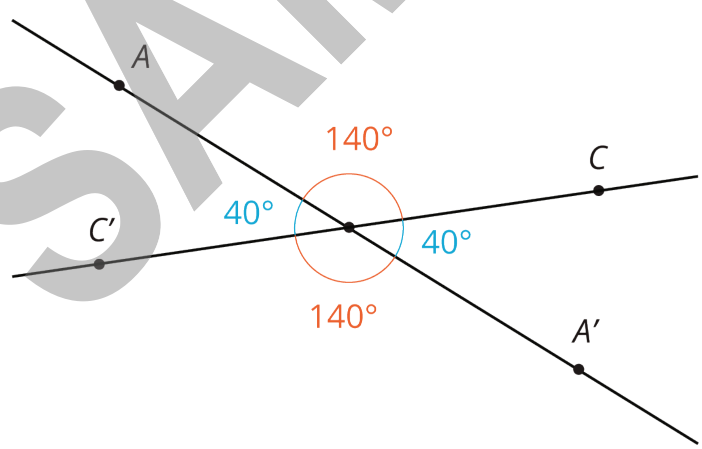
- Sometimes, a rigid transformation takes a line to itself. For example:



  - A translation parallel to the line. The arrow shows a translation of line m that will take m to itself.
  - A rotation by 180° around any point on the line. A 180° rotation of line m around point F will take m to itself.
  - A reflection across any line perpendicular to the line. A reflection of line m across the dashed line will take m to itself.

These facts let us make an important conclusion. If two lines intersect at a point, which we'll call O, then a 180° rotation of the lines with center O shows that **vertical angles** are congruent. Here is an example:



Rotating both lines by 180° around O sends angle AOC to angle $A'OC'$, proving that they have the same measure. The rotation also sends angle AOC' to angle $A'OC$.

Glossary

- vertical angles

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Lesson 8 Practice Problems

1. a. Draw parallel lines AB and CD.

   b. Pick any point E. Rotate AB 90 degrees clockwise around E.

   c. Rotate line CD 90 degrees clockwise around E.

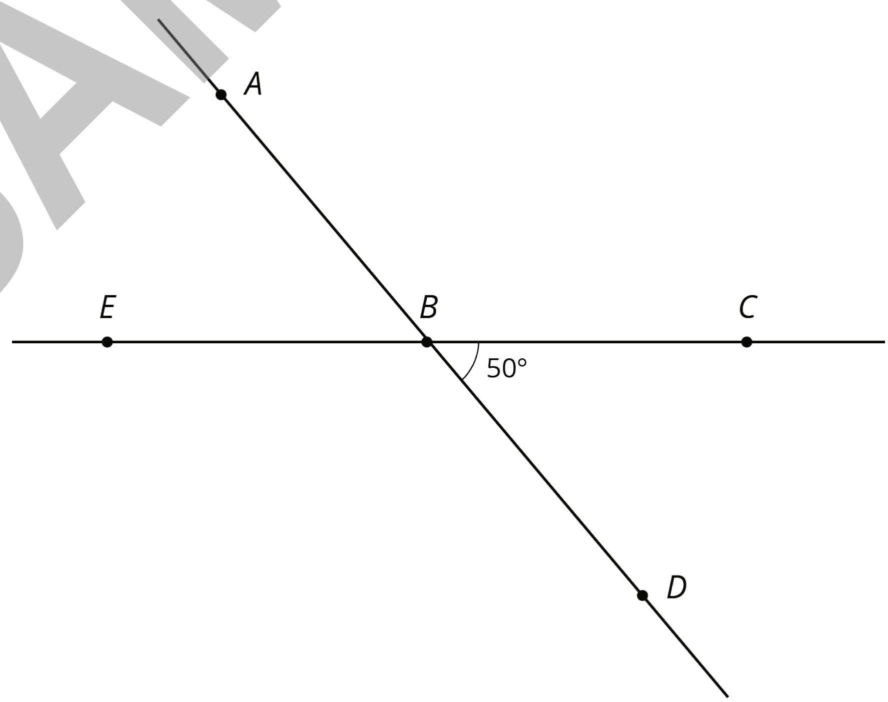
   d. What do you notice?

2. Use the diagram to find the measures of each angle. Explain your reasoning.

   a. $m\angle ABC$

   b. $m\angle EBD$

   c. $m\angle ABE$



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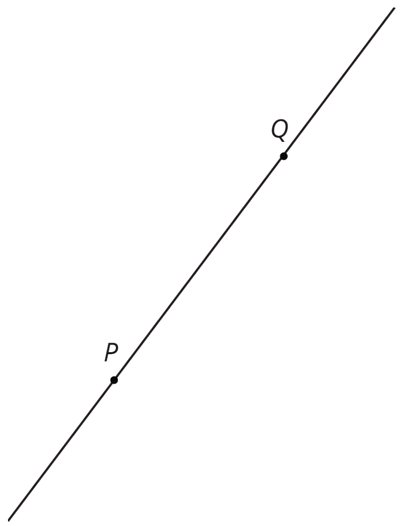
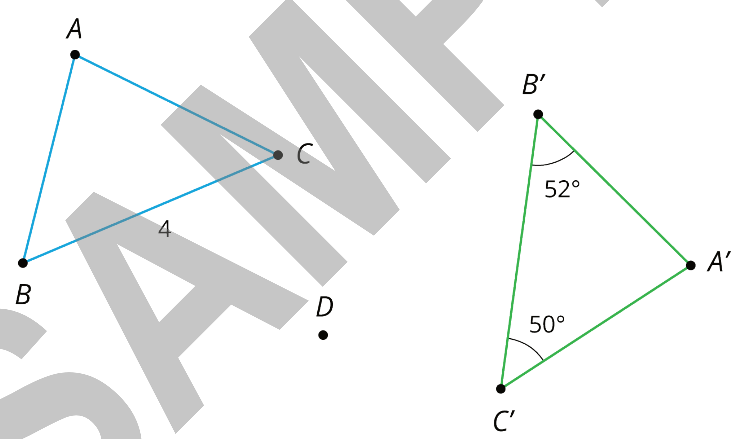
3. Points P and Q are plotted on a line.



a. Find a point R so that a 180-degree rotation with center R sends P to Q and Q to P.

b. Is there more than one point R that works for part a?

4. In the picture triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is D.



a. What is the length of side $B'C'$? Explain how you know.

b. What is the measure of angle B? Explain how you know.

c. What is the measure of angle C? Explain how you know.

(From Unit 1, Lesson 6.)

5. The point $(-4, 1)$ is rotated 180 degrees counterclockwise using center $(0, 0)$. What are the coordinates of the image?

   A. $(-1, -4)$

   B. $(-1, 4)$

   C. $(4, 1)$

   D. $(4, -1)$

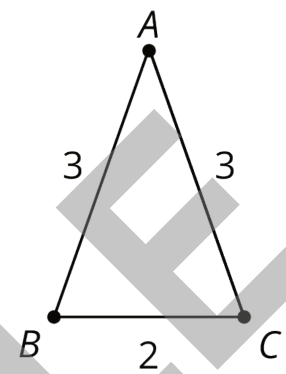
   (From Unit 1, Lesson 5.)

Lesson 9: Composing Figures

Let's use reasoning about rigid transformations to find measurements without measuring.

9.1: Angles of an Isosceles Triangle

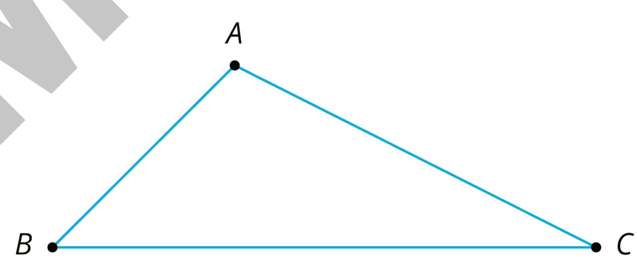
Here is a triangle.



1. Reflect triangle ABC over line AB. Label the image of C as C'.
2. Rotate triangle ABC' around A so that C' matches up with B.
3. What can you say about the measures of angles B and C?

9.2: Triangle Plus One

Here is triangle ABC.



1. Draw midpoint M of side AC.
2. Rotate triangle ABC 180 degrees using center M to form triangle CDA. Draw and label this triangle.
3. What kind of quadrilateral is $ABCD$? Explain how you know.

Are you ready for more?

In the activity, we made a parallelogram by taking a triangle and its image under a 180-degree rotation around the midpoint of a side. This picture helps you justify a well-known formula for the area of a triangle. What is the formula and how does the figure help justify it?

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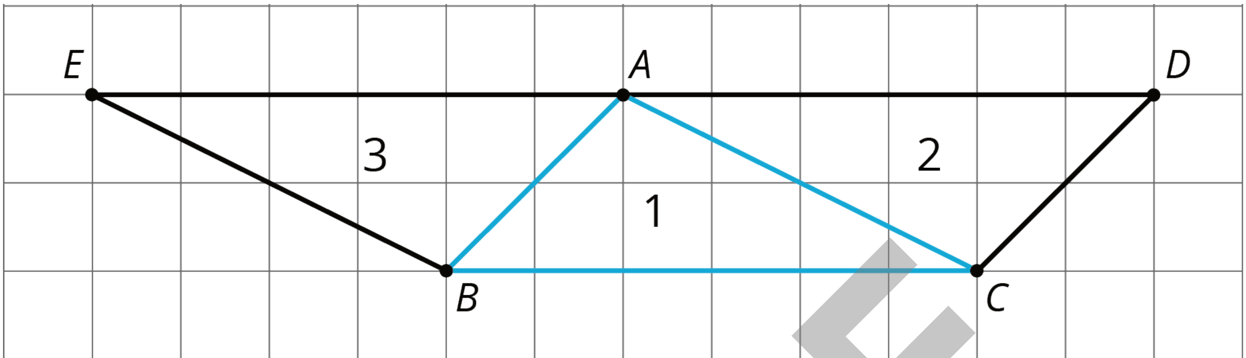
9.3: Triangle Plus Two

The picture shows 3 triangles. Triangle 2 and Triangle 3 are images of Triangle 1 under rigid transformations.



1. Describe a rigid transformation that takes Triangle 1 to Triangle 2. What points in Triangle 2 correspond to points A, B, and C in the original triangle?

2. Describe a rigid transformation that takes Triangle 1 to Triangle 3. What points in Triangle 3 correspond to points A, B, and C in the original triangle?

3. Find two pairs of line segments in the diagram that are the same length, and explain how you know they are the same length.

4. Find two pairs of angles in the diagram that have the same measure, and explain how you know they have the same measure.

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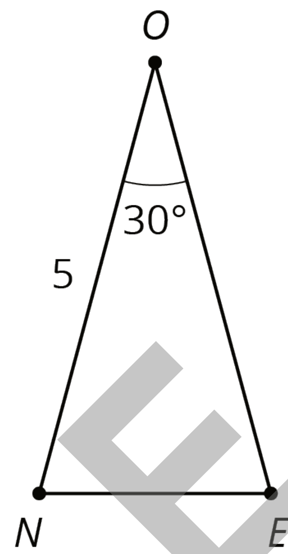
9.4: Triangle ONE Plus

Here is isosceles triangle ONE. Its sides ON and OE have equal lengths. Angle O is 30 degrees. The length of ON is 5 units.

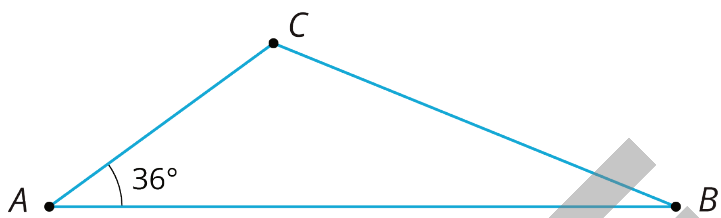


1. Reflect triangle ONE across segment ON. Label the new vertex M.
2. What is the measure of angle MON?
3. What is the measure of angle MOE?
4. Reflect triangle MON across segment OM. Label the point that corresponds to N as T.
5. How long is $\overline{OT}$? How do you know?
6. What is the measure of angle TOE?
7. If you continue to reflect each new triangle this way to make a pattern, what will the pattern look like?

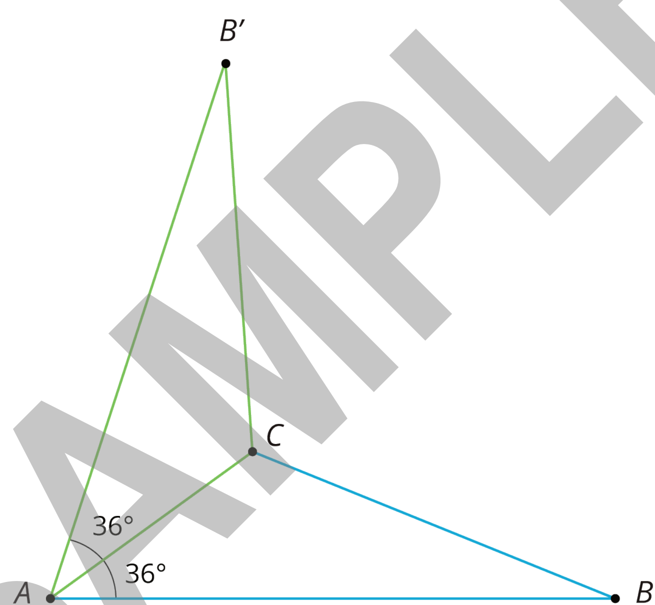
Lesson 9 Summary

Earlier, we learned that if we apply a sequence of rigid transformations to a figure, then corresponding sides have equal length and corresponding angles have equal measure. These facts let us figure out things without having to measure them!

For example, here is triangle ABC.



We can reflect triangle ABC across side AC to form a new triangle:



Because points A and C are on the line of reflection, they do not move. So the image of triangle ABC is $AB'C$. We also know that:

- Angle $B'AC$ measures 36° because it is the image of angle BAC.
- Segment AB' has the same length as segment AB.

When we construct figures using copies of a figure made with rigid transformations, we know that the measures of the images of segments and angles will be equal to the measures of the original segments and angles.

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Lesson 9 Practice Problems

1. Here is the design for the flag of Trinidad and Tobago.

   Describe a sequence of translations, rotations, and reflections that take the lower left triangle to the upper right triangle.

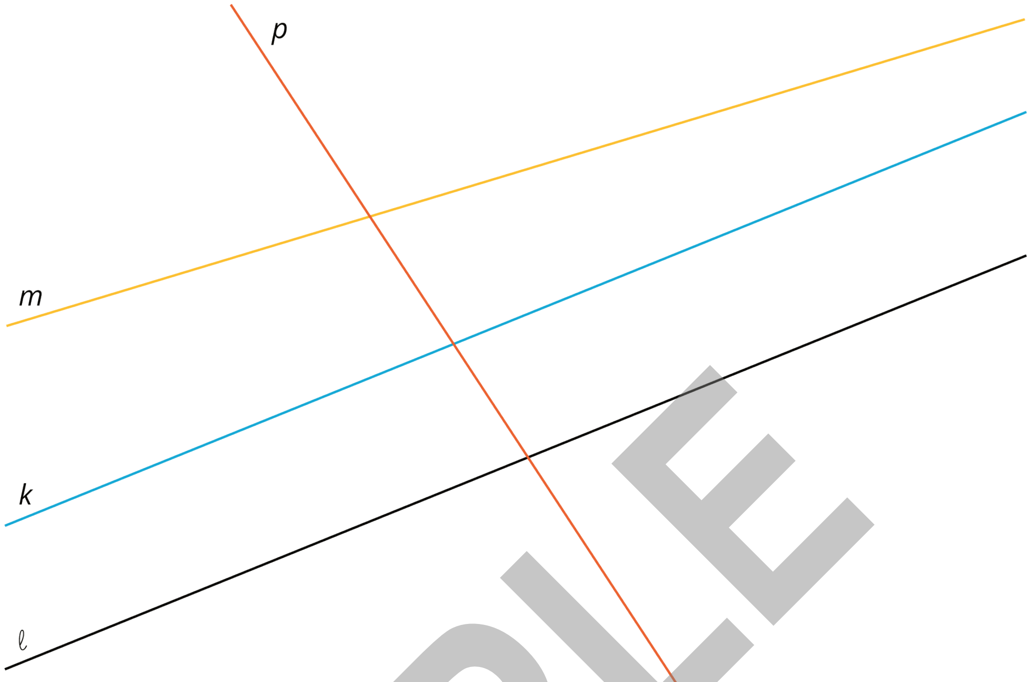
2. Here is a picture of an older version of the flag of Great Britain. There is a rigid transformation that takes Triangle 1 to Triangle 2, another that takes Triangle 1 to Triangle 3, and another that takes Triangle 1 to Triangle 4.

   a. Measure the lengths of the sides in Triangles 1 and 2. What do you notice?

   b. What are the side lengths of Triangle 3? Explain how you know.

   c. Do all eight triangles in the flag have the same area? Explain how you know.

3. a. Which of the lines in the picture is parallel to line ℓ? Explain how you know.



b. Explain how to translate, rotate or reflect line ℓ to obtain line k.

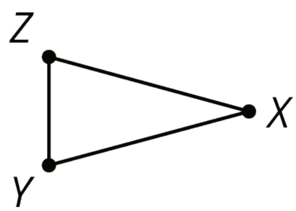
c. Explain how to translate, rotate or reflect line ℓ to obtain line p.

(From Unit 1, Lesson 8.)

4. Point A has coordinates $(3, 4)$. After a translation 4 units left, a reflection across the x-axis, and a translation 2 units down, what are the coordinates of the image?

(From Unit 1, Lesson 5.)

5. Here is triangle XYZ:



Draw these three rotations of triangle XYZ together.

a. Rotate triangle XYZ 90 degrees clockwise around Z.

b. Rotate triangle XYZ 180 degrees around Z.

c. Rotate triangle XYZ 270 degrees clockwise around Z.

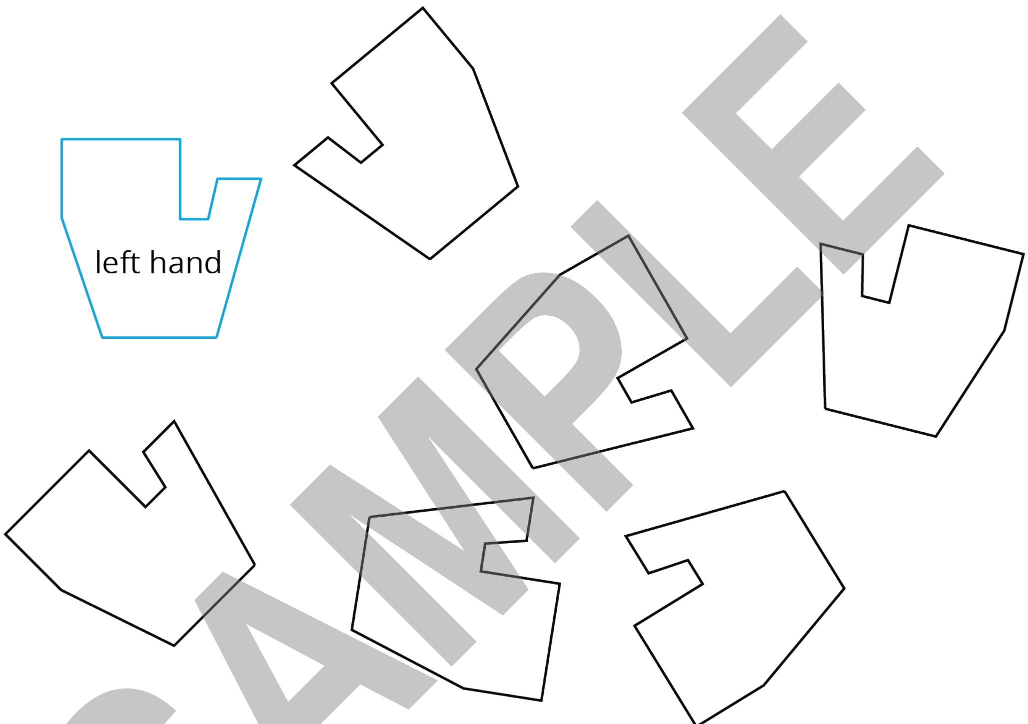
(From Unit 1, Lesson 7.)

Lesson 10: What Is the Same?

Let's decide whether shapes are the same.

10.1: Find the Right Hands

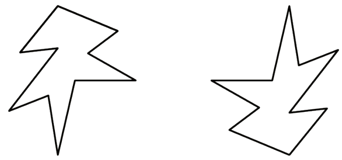
A person's hands are mirror images of each other. In the diagram, a left hand is labeled. Shade all of the right hands.



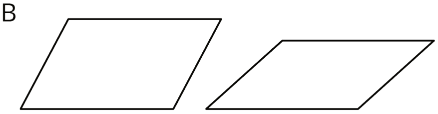
10.2: Are They the Same?

For each pair of shapes, decide whether or not they are the same.

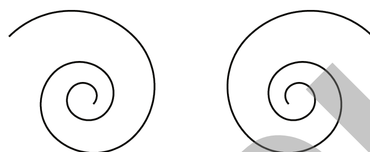
A



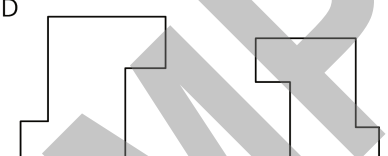
B



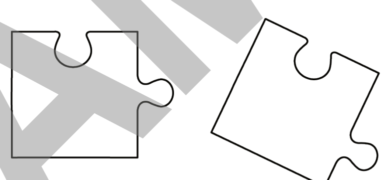
C



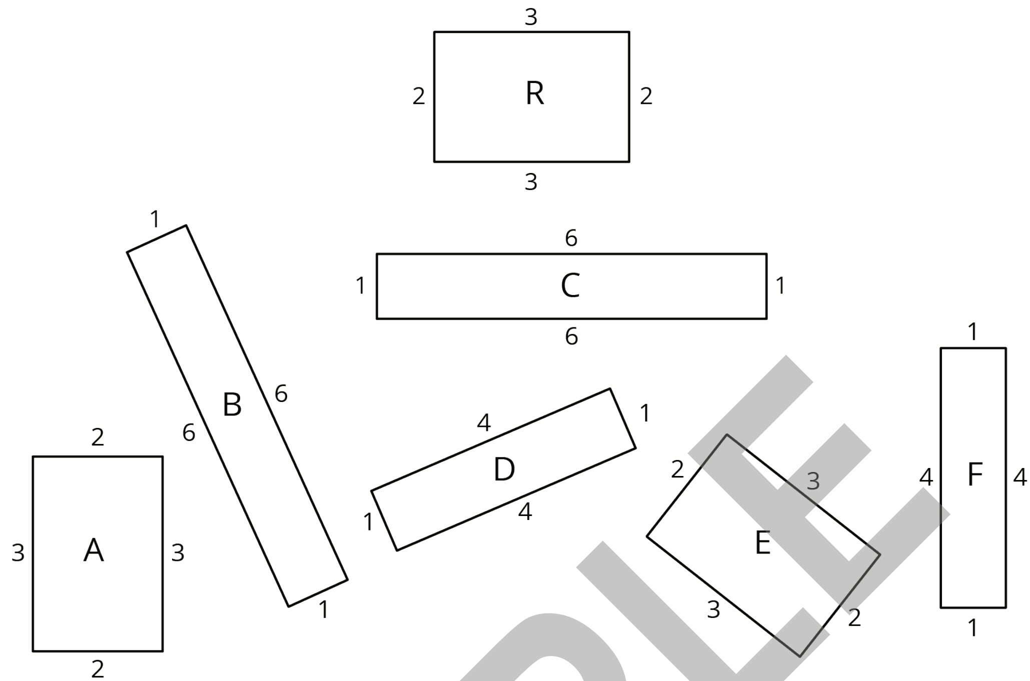
D



E



10.3: Area, Perimeter, and Congruence



1. Which of these rectangles have the same area as Rectangle R but different perimeter?

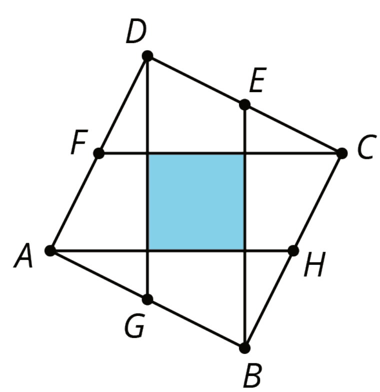
2. Which rectangles have the same perimeter but different area?

3. Which have the same area *and* the same perimeter?

4. Use materials from the geometry tool kit to decide which rectangles are **congruent**. Shade congruent rectangles with the same color.

Are you ready for more?

In square $ABCD$, points E, F, G, and H are midpoints of their respective sides. What fraction of square $ABCD$ is shaded? Explain your reasoning.



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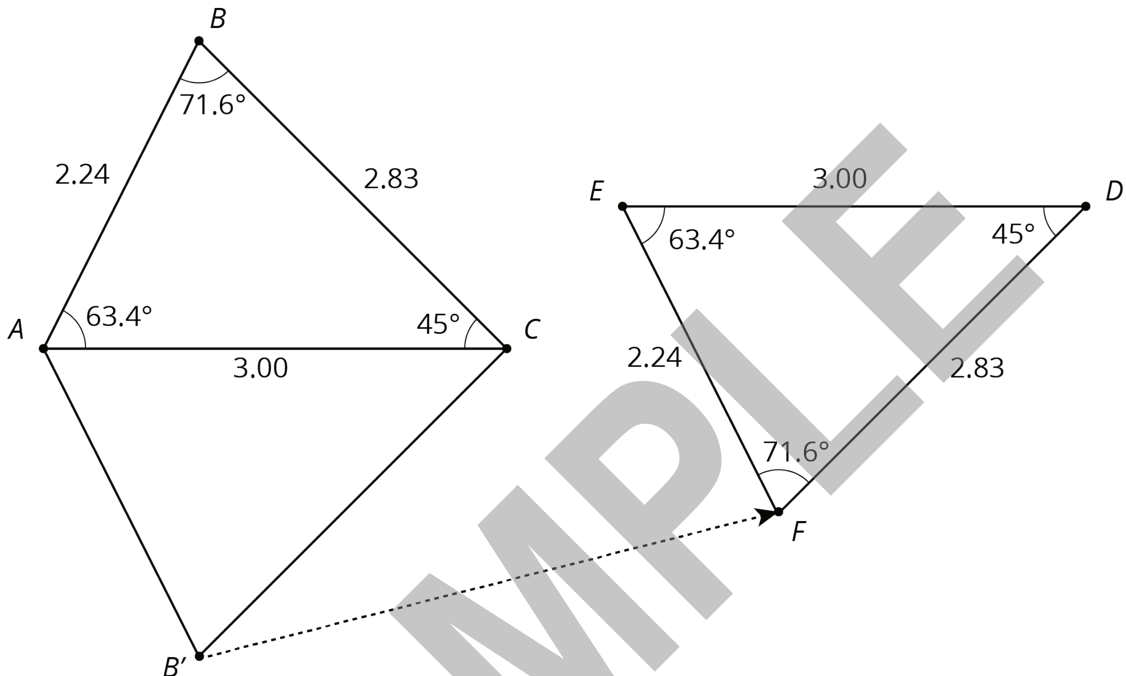
Lesson 10 Summary

Congruent is a new term for an idea we have already been using. We say that two figures are congruent if one can be lined up exactly with the other by a sequence of rigid transformations. For example, triangle EFD is congruent to triangle ABC because they can be matched up by reflecting triangle ABC across AC followed by the translation shown by the arrow. Notice that all corresponding angles and side lengths are equal.



Here are some other facts about congruent figures:

- We don't need to check all the measurements to prove two figures are congruent; we just have to find a sequence of rigid transformations that match up the figures.
- A figure that looks like a mirror image of another figure can be congruent to it. This means there must be a reflection in the sequence of transformations that matches up the figures.
- Since two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have a different perimeter or area.

Glossary

- congruent

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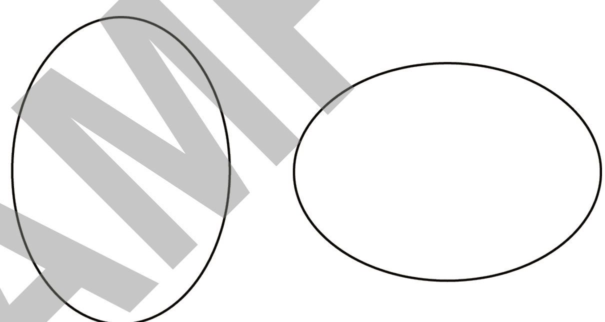
Lesson 10 Practice Problems

1. If two rectangles have the same perimeter, do they have to be congruent? Explain how you know.

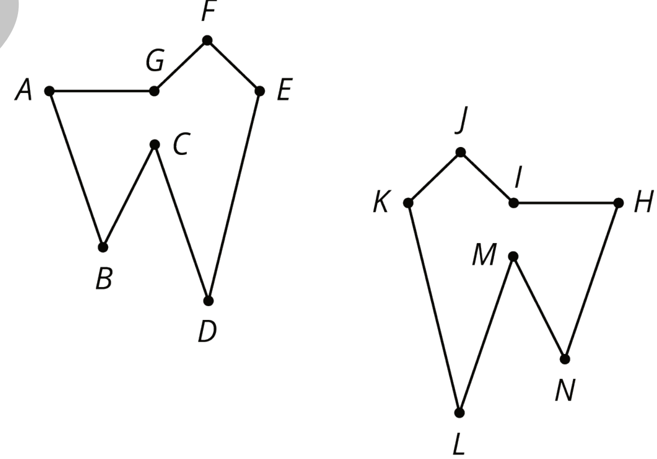
2. Draw two rectangles that have the same area, but are *not* congruent.

3. For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

   a.



   b.



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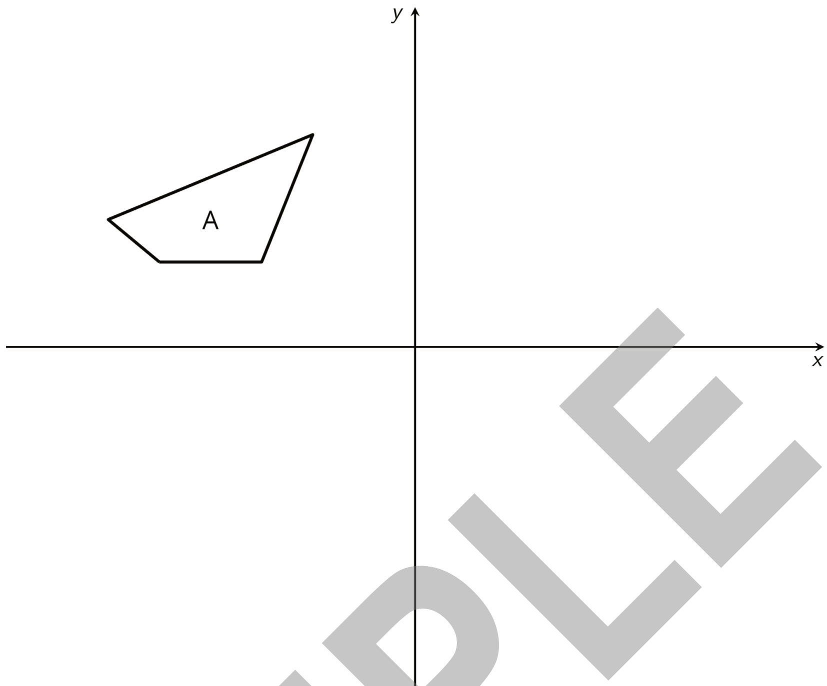
4. a. Reflect Quadrilateral A over the x-axis. Label the image quadrilateral B. Reflect Quadrilateral B over the y-axis. Label the image C.



b. Are Quadrilaterals A and C congruent? Explain how you know.

5. The point $(-2, -3)$ is rotated 90 degrees counterclockwise using center $(0, 0)$. What are the coordinates of the image?

A. $(-3, -2)$

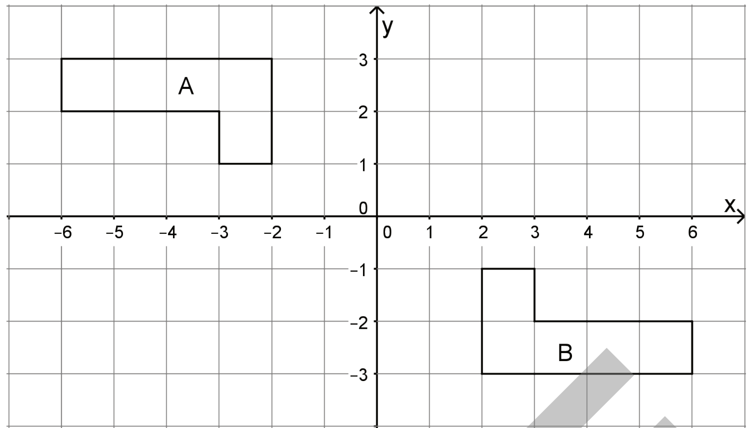
B. $(-3, 2)$

C. $(3, -2)$

D. $(3, 2)$

(From Unit 1, Lesson 5.)

6. Describe a rigid transformation that takes Polygon A to Polygon B.



(From Unit 1, Lesson 6.)

Lesson 11: Congruence

Let's decide if two figures are congruent.

11.1: Translated Images

All of these triangles are congruent. Sometimes we can take one figure to another with a translation. Shade the triangles that are images of triangle ABC under a translation.

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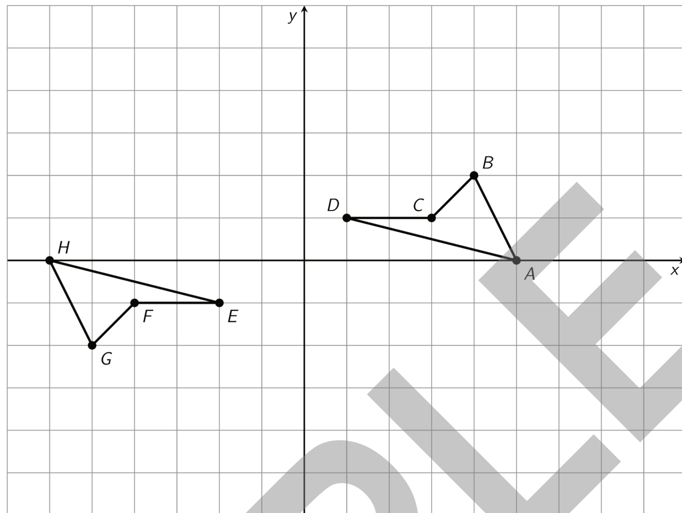
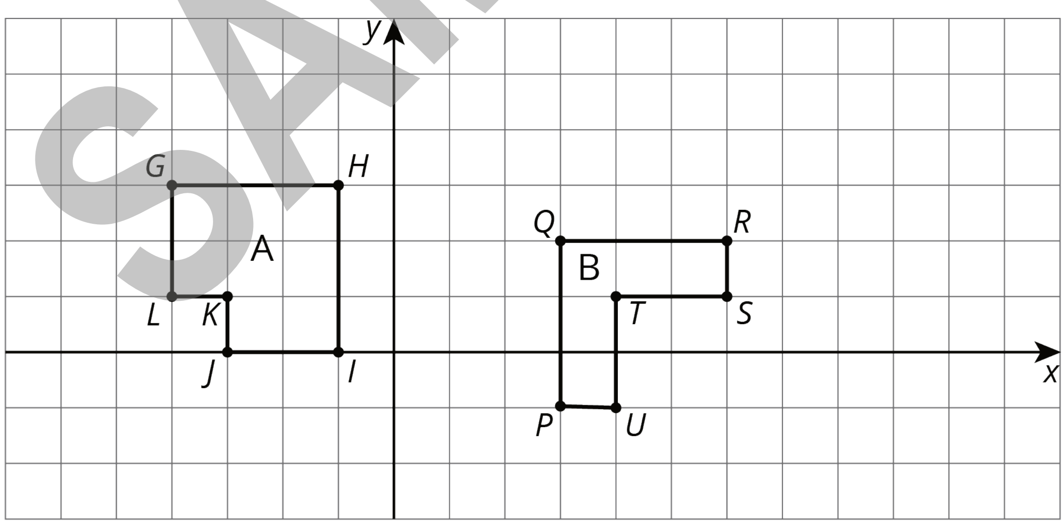
11.2: Congruent Pairs

For each of the following pairs of shapes, decide whether or not they are congruent. Explain your reasoning.

1.



2.



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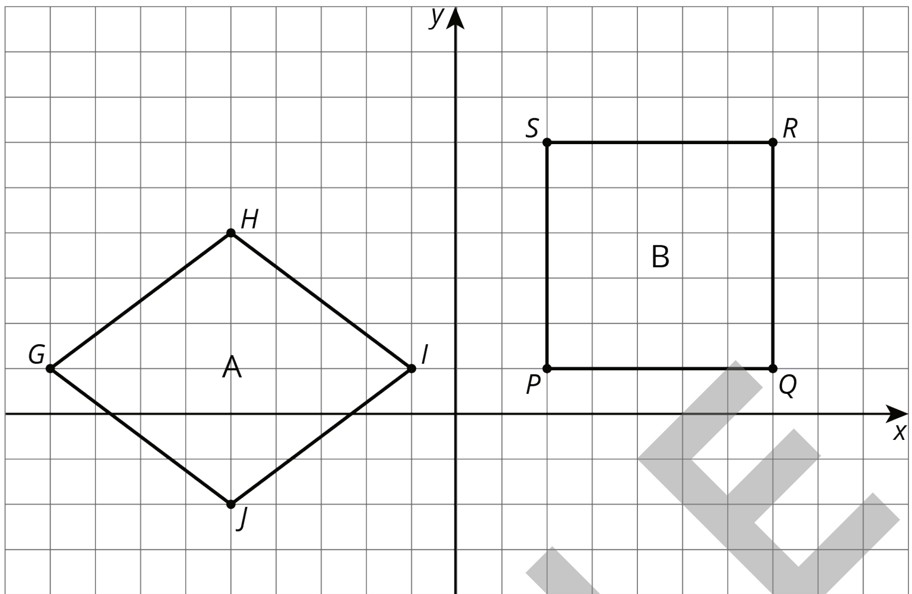
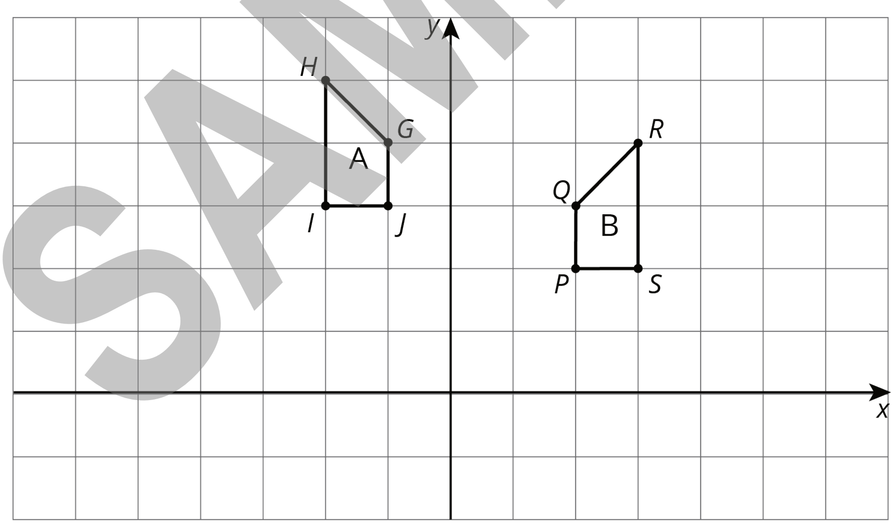
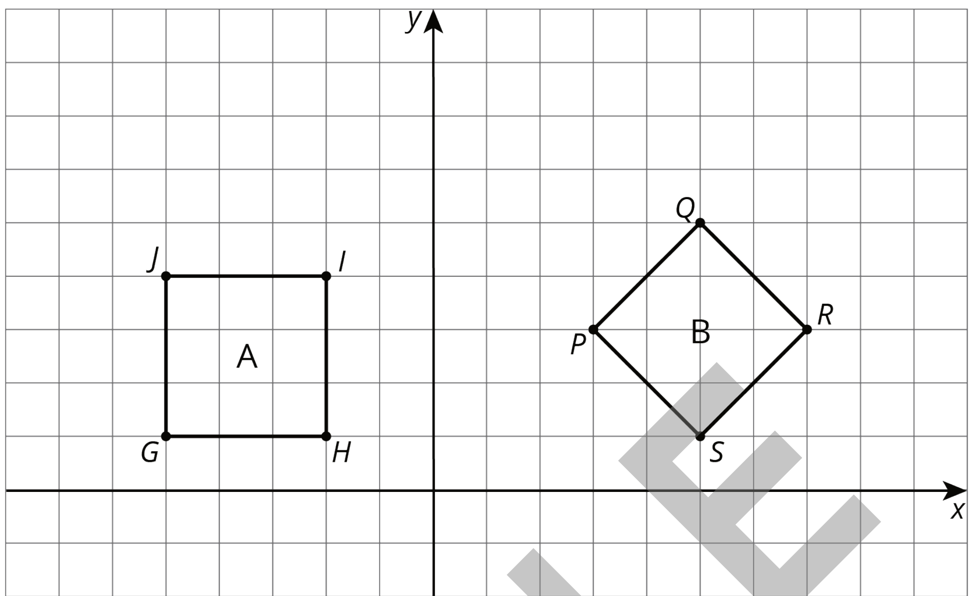
3.



4.



5.



Are you ready for more?

A polygon has 8 sides: five of length 1, two of length 2, and one of length 3. All sides lie on grid lines. (It may be helpful to use graph paper when working on this problem.)

1. Find a polygon with these properties.

2. Is there a second polygon, not congruent to your first, with these properties?

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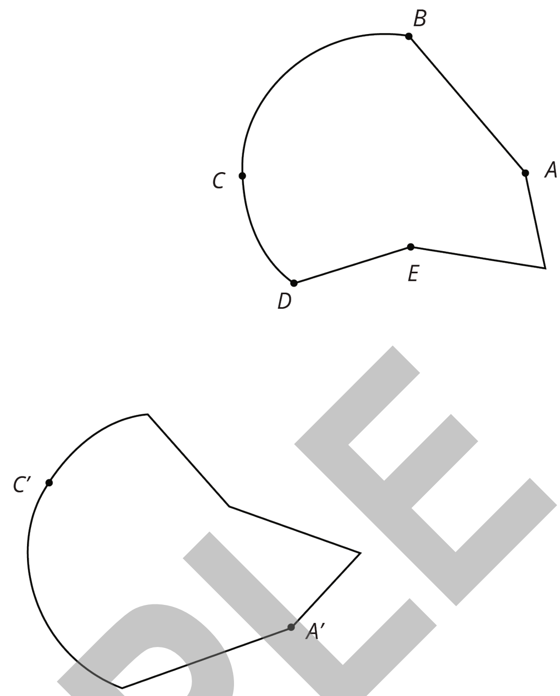
11.3: Corresponding Points in Congruent Figures

Here are two congruent shapes with some corresponding points labeled.

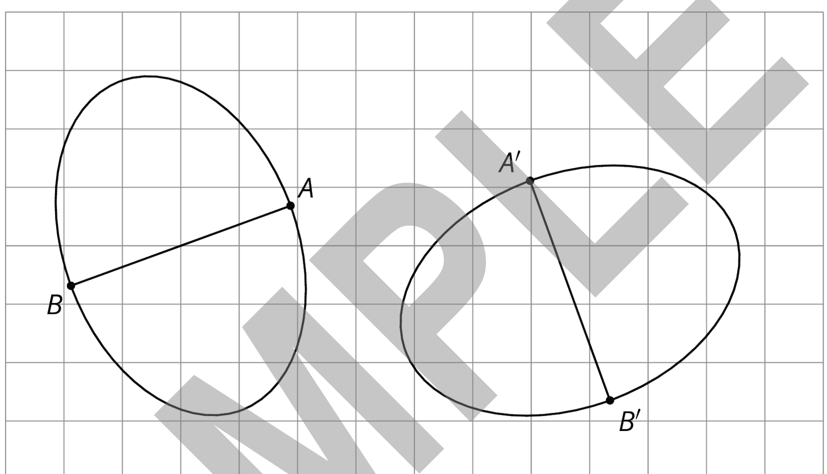


1. Draw the points corresponding to B, D, and E, and label them B', D', and E'.
2. Draw line segments AD and $A'D'$ and measure them. Do the same for segments BC and $B'C'$ and for segments AE and $A'E'$. What do you notice?
3. Do you think there could be a pair of corresponding segments with different lengths? Explain.

Lesson 11 Summary

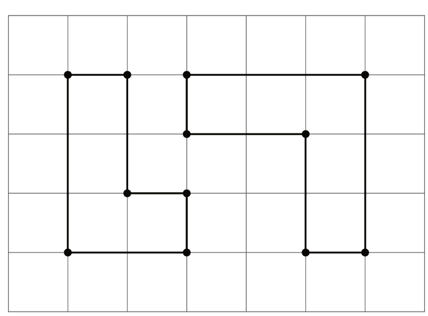
How do we know if two figures are congruent?

- If we copy 1 figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.
- We can prove that 2 figures are congruent by describing a sequence of translations, rotations, and reflections that move 1 figure onto the other so they match up exactly.
- Distances between corresponding points on congruent figures are always equal, even for curved shapes. For example, corresponding segments AB and $A'B'$ on these congruent ovals have the same length:



How do we know that 2 figures are not congruent?

- If there is no correspondence between the figures where the parts have equal measure, that proves that the two figures are not congruent. In particular,
  - If two polygons have different sets of side lengths, they can't be congruent. For example, the figure on the left has side lengths 3, 2, 1, 1, 2, 1. The figure on the right has side lengths 3, 3, 1, 2, 2, 1. There is no way to make a correspondence between them where all corresponding sides have the same length.



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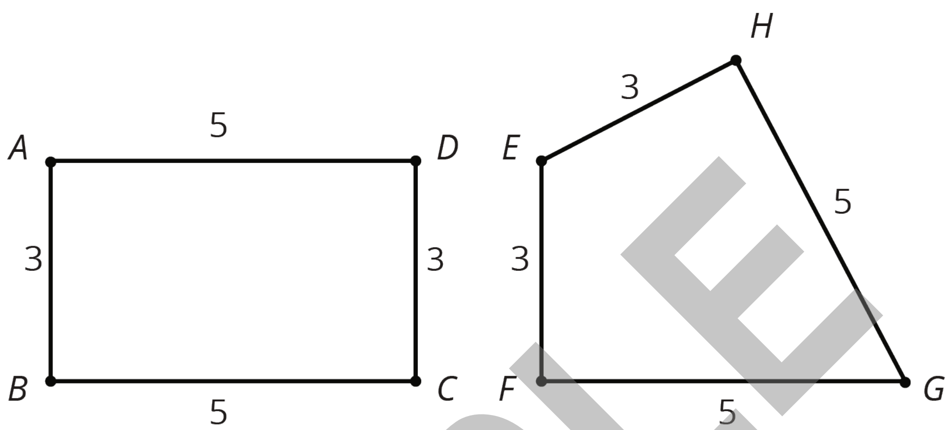
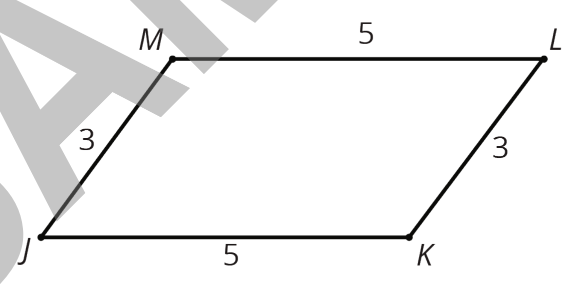
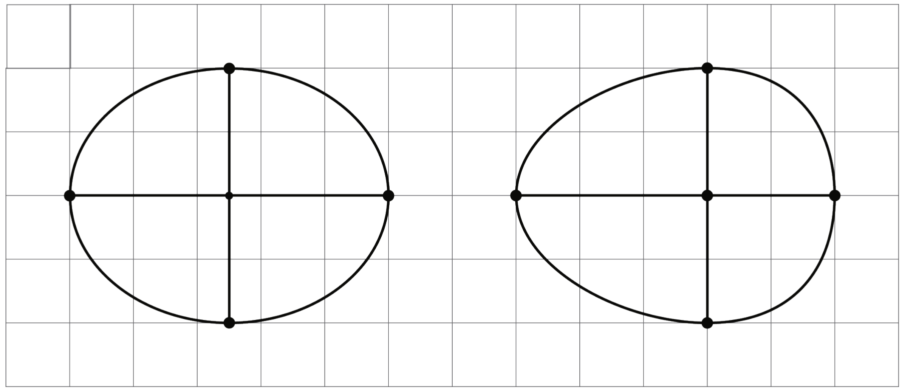
- If two polygons have the same side lengths, but their orders can't be matched as you go around each polygon, the polygons can't be congruent. For example, rectangle $ABCD$ can't be congruent to quadrilateral $EFGH$. Even though they both have two sides of length 3 and two sides of length 5, they don't correspond in the same order. In $ABCD$, the order is 3, 5, 3, 5 or 5, 3, 5, 3; in $EFGH$, the order is 3, 3, 5, 5 or 3, 5, 5, 3 or 5, 5, 3, 3.



- If two polygons have the same side lengths, in the same order, but different corresponding angles, the polygons can't be congruent. For example, parallelogram $JKLM$ can't be congruent to rectangle $ABCD$. Even though they have the same side lengths in the same order, the angles are different. All angles in $ABCD$ are **right angles**. In $JKLM$, angles J and L are less than 90 degrees and angles K and M are more than 90 degrees.



- If you have curved figures, like these 2 ovals, you can find parts of the figures that should correspond but that have different measurements. On both, the longest distance from left to right is 5 units across, and the longest distance from top to bottom is 4 units. The line segment from the highest to lowest point is in the middle of the left oval, but in the right oval, it's 2 units from the right end and 3 units from the left end. This proves they are not congruent.



Glossary

- right angle

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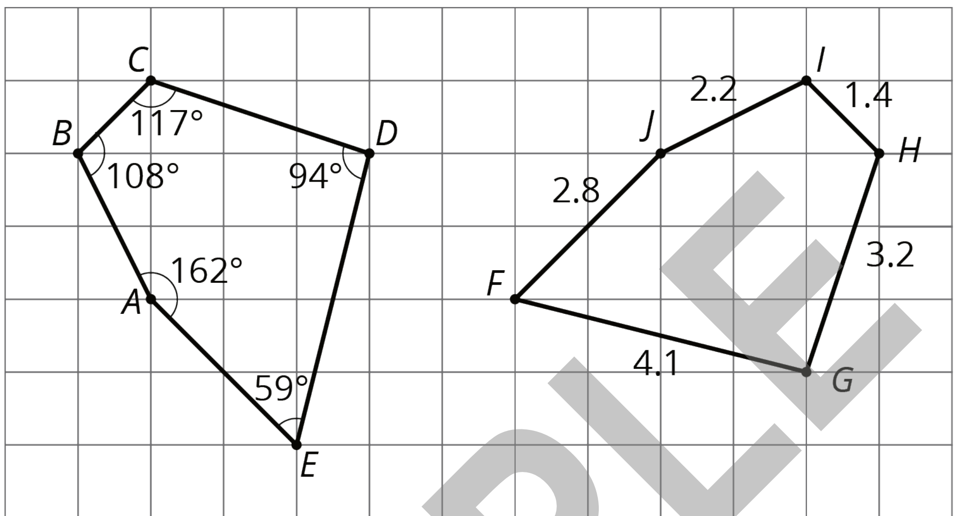
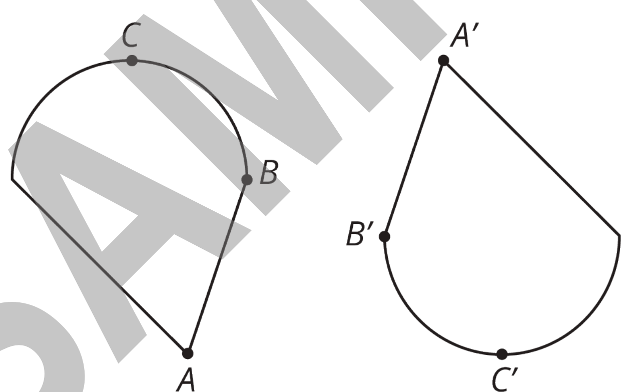
Lesson 11 Practice Problems

1. a. Show that the two pentagons are congruent.

   b. Find the side lengths of $ABCDE$ and the angle measures of $FGHIJ$.



2. These two figures are congruent, with corresponding points marked.



   a. Are angles ABC and $A'B'C'$ congruent? Explain your reasoning.

   b. Measure angles ABC and $A'B'C'$ to check your answer.

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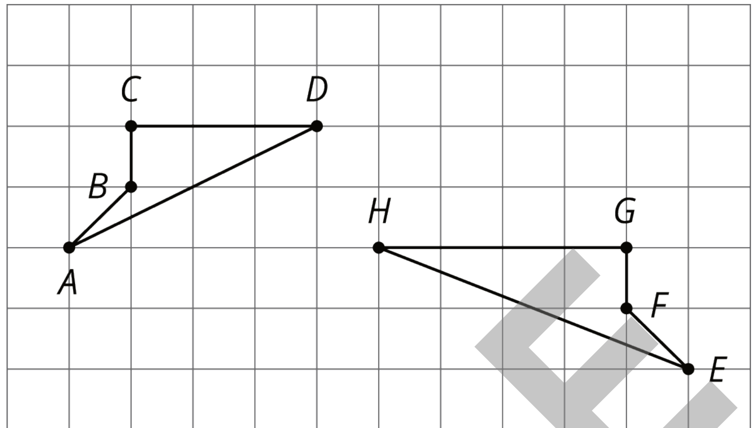
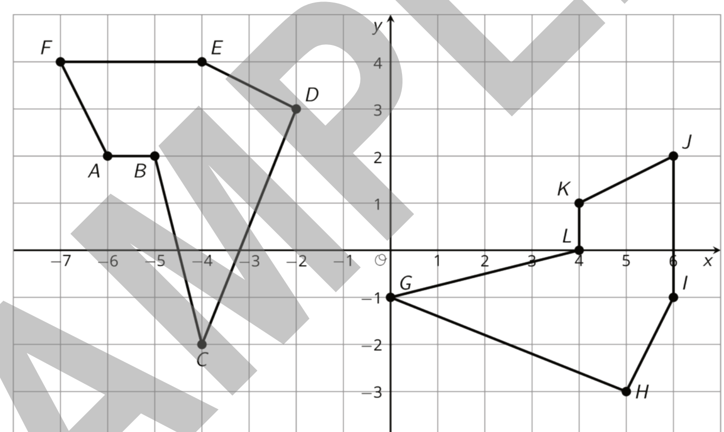
3. For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

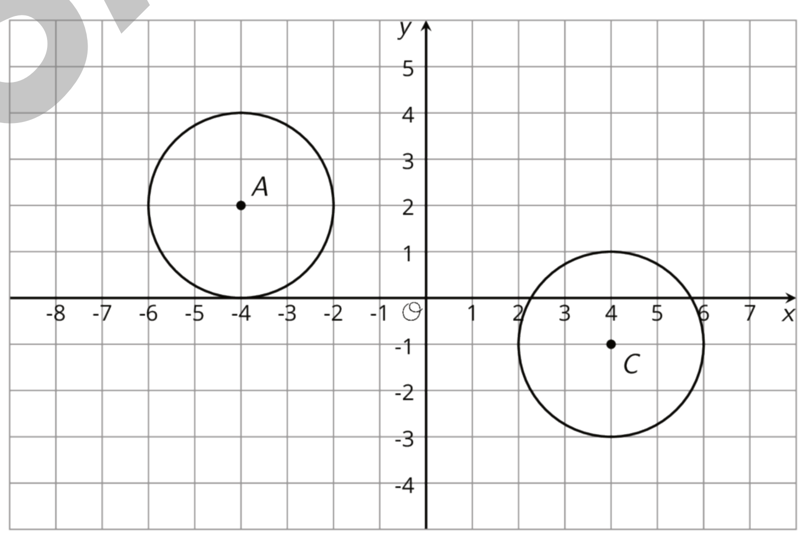
   a.



   b.



   c.



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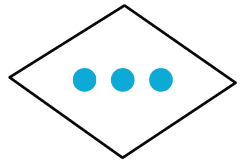
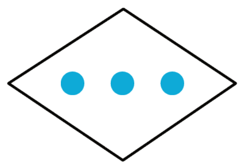
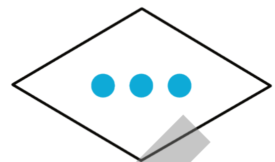
4. Which of these four figures are congruent to the top figure?



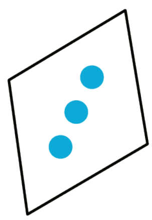
A



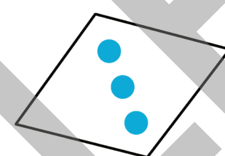
B



C



D



A. A

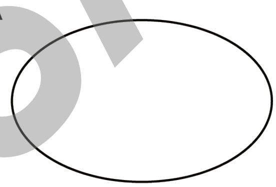
B. B

C. C

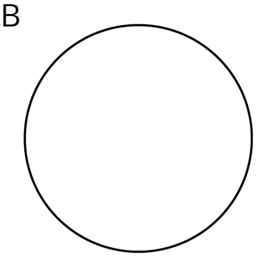
D. D

5. Here are two figures.

A



B



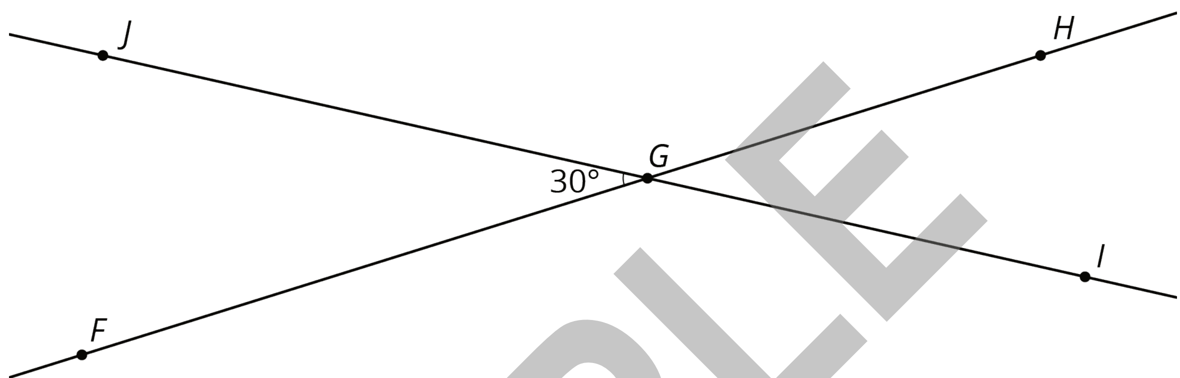
Show, using measurement, that these two figures are *not* congruent.

Lesson 12: Alternate Interior Angles

Let's explore why some angles are always equal.

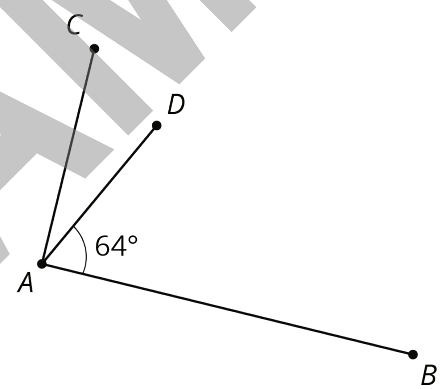
12.1: Angle Pairs

1. Find the measure of angle JGH. Explain or show your reasoning.



2. Find and label a second 30° angle in the diagram. Find and label an angle congruent to angle JGH.

3. Angle BAC is a right angle. Find the measure of angle CAD.



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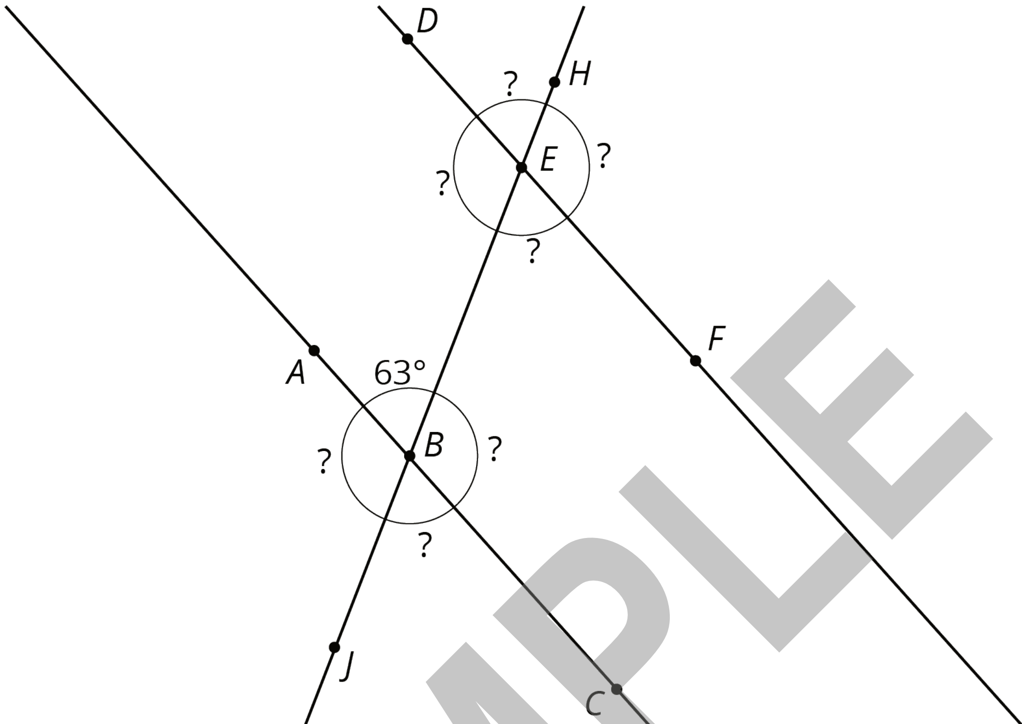
12.2: Cutting Parallel Lines with a Transversal

Lines AC and DF are parallel. They are cut by **transversal** HJ.



1. With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning.

2. What do you notice about the angles with vertex B and the angles with vertex E?

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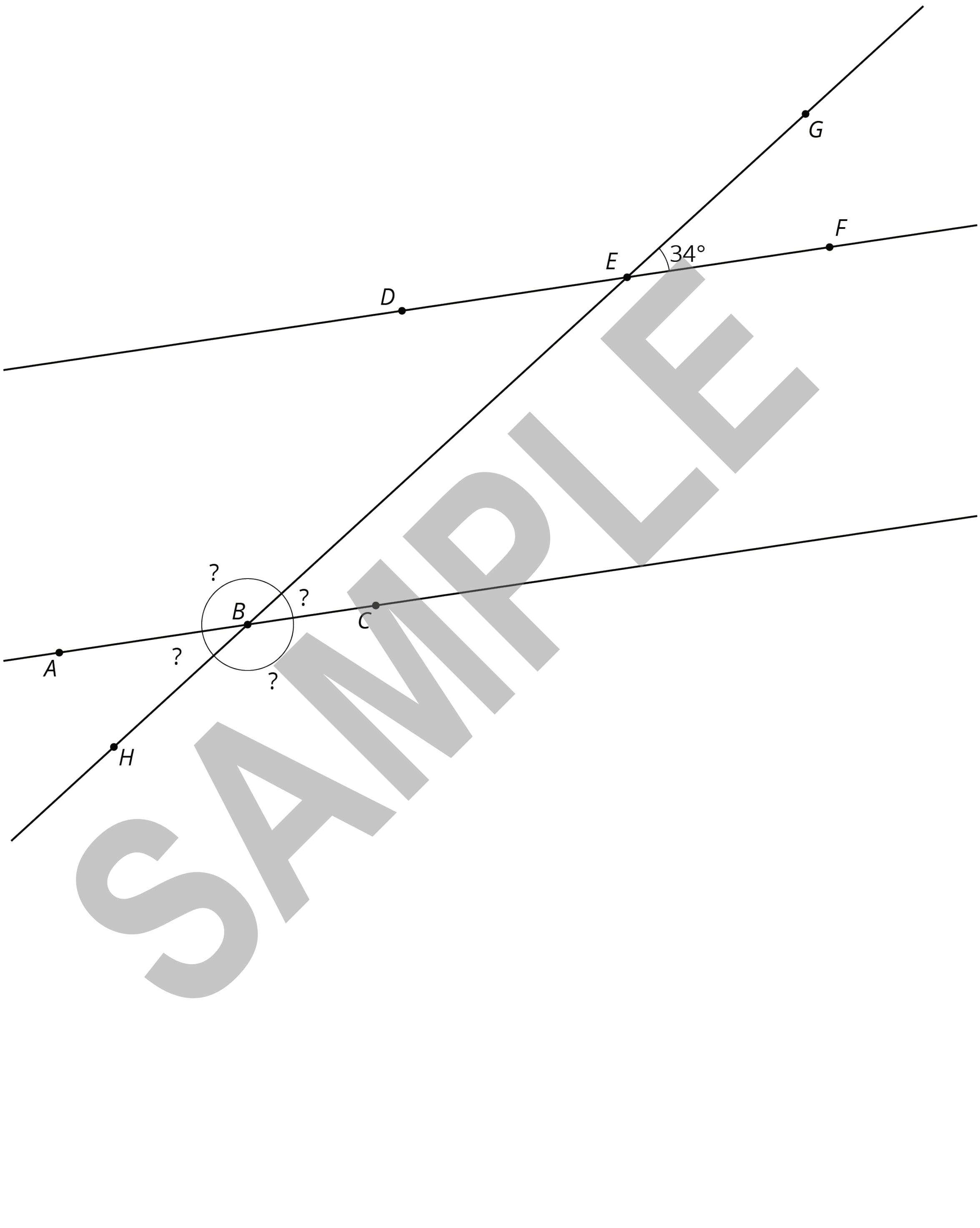
3. Using what you noticed, find the measures of the four angles at point B in the second diagram. Lines AC and DF are parallel.



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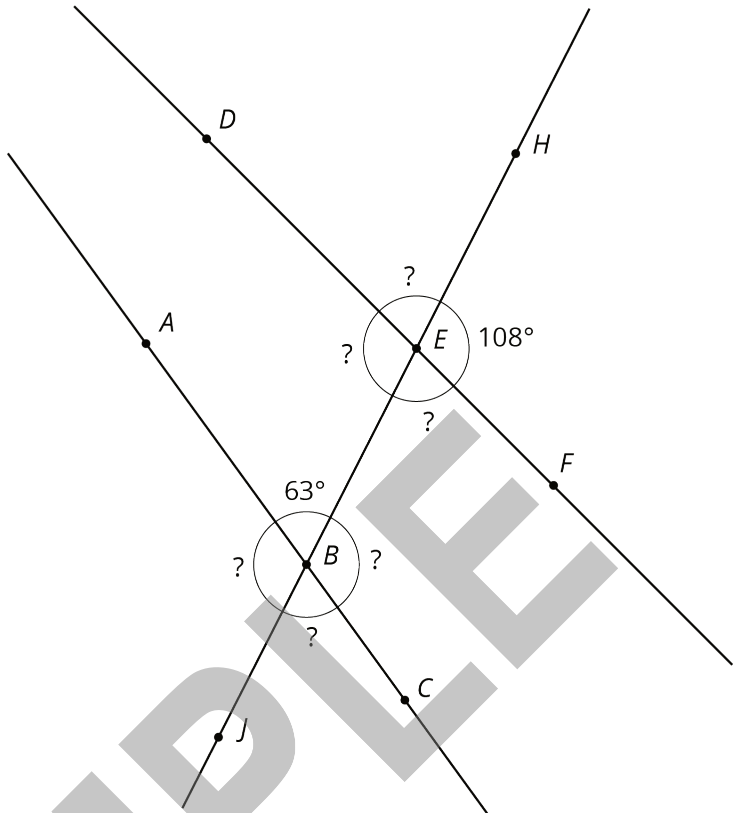
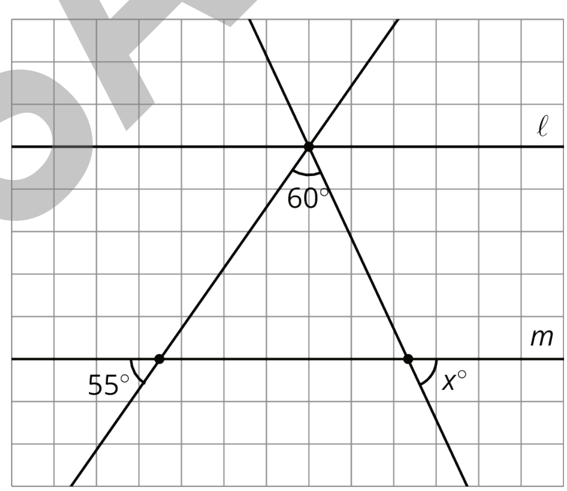
4. The next diagram resembles the first one, but the lines form slightly different angles. Work with your partner to find the six unknown angles with vertices at points B and E.



5. What do you notice about the angles in this diagram as compared to the earlier diagram? How are the two diagrams different? How are they the same?

Are you ready for more?



Parallel lines ℓ and m are cut by two transversals which intersect ℓ in the same point. Two angles are marked in the figure. Find the measure x of the third angle.

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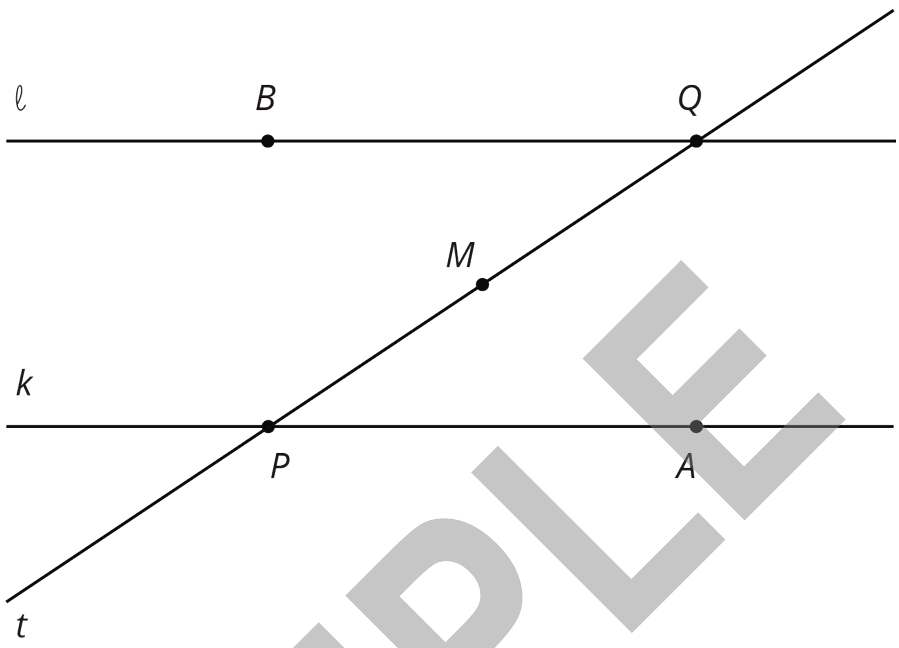
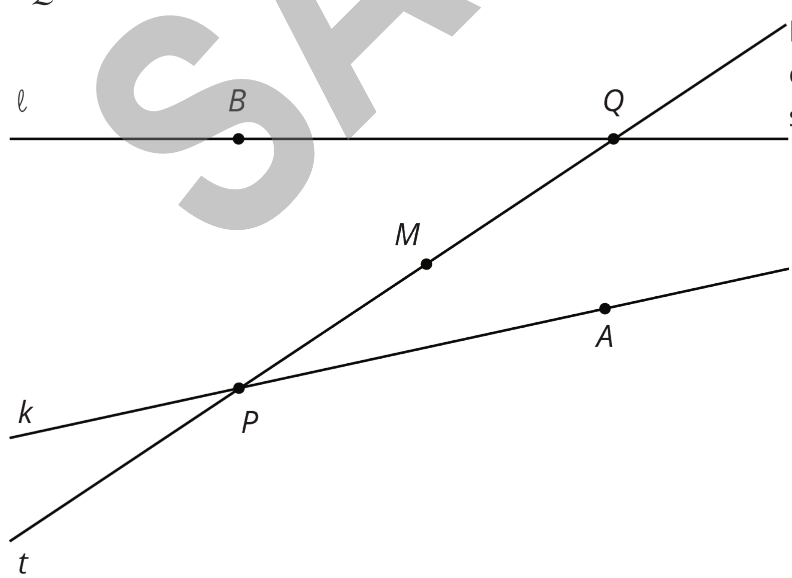
12.3: Alternate Interior Angles Are Congruent

1. Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.



Find a rigid transformation showing that angles MPA and MQB are congruent.

2. In this picture, lines ℓ and k are no longer parallel. M is still the midpoint of segment PQ.



Does your argument in the earlier problem apply in this situation? Explain.

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12.4: Info Gap: Angle Finding

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

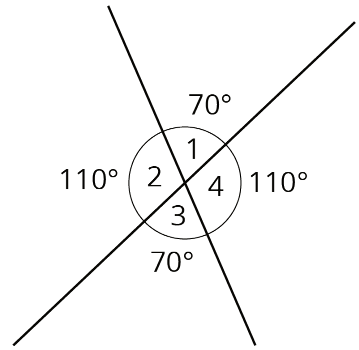
1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

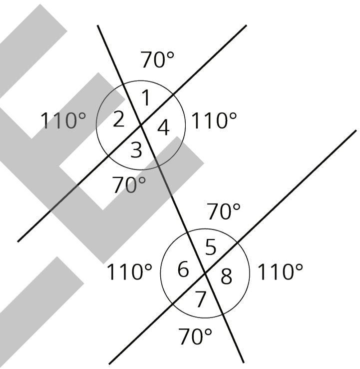
Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Lesson 12 Summary

If two angle measures add up to 90°, then we say the angles are **complementary**. If two angle measures add up to 180°, then we say the angles are **supplementary**. When two lines intersect, vertical angles are equal and adjacent angles are supplementary. For example, in this figure angles 1 and 3 are equal, angles 2 and 4 are equal, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.



When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.



Alternate interior angles are equal because a 180° rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections—can you see how rotating 180° about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is 70° we use vertical angles to see that angle 3 is 70°, then we use alternate interior angles to see that angle 5 is 70°, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is 110° since $180 - 70 = 110$. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure 70°, and angles 2, 4, 6, and 8 measure 110°.

Glossary

- alternate interior angles
- complementary
- supplementary
- transversal

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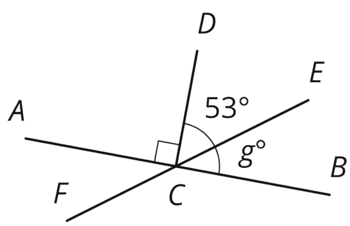
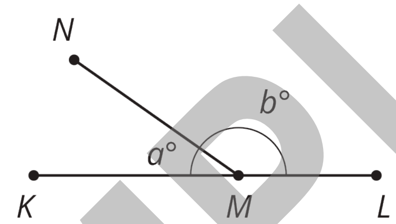
Lesson 12 Practice Problems

1. Segments AB, EF, and CD intersect at point C, and angle ACD is a right angle. Find the value of g.



2. M is a point on line segment KL. NM is a line segment. Select **all** the equations that represent the relationship between the measures of the angles in the figure.



A. $a = b$

B. $a + b = 90$

C. $b = 90 - a$

D. $a + b = 180$

E. $180 - a = b$

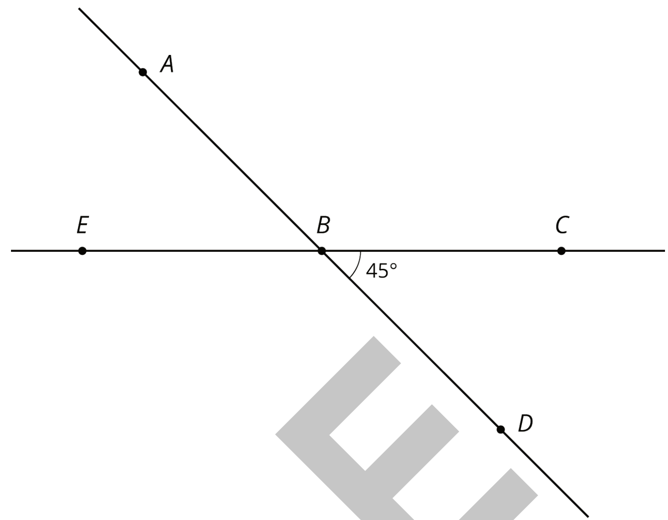
F. $180 = b - a$

3. Use the diagram to find the measure of each angle.

   a. $m\angle ABC$

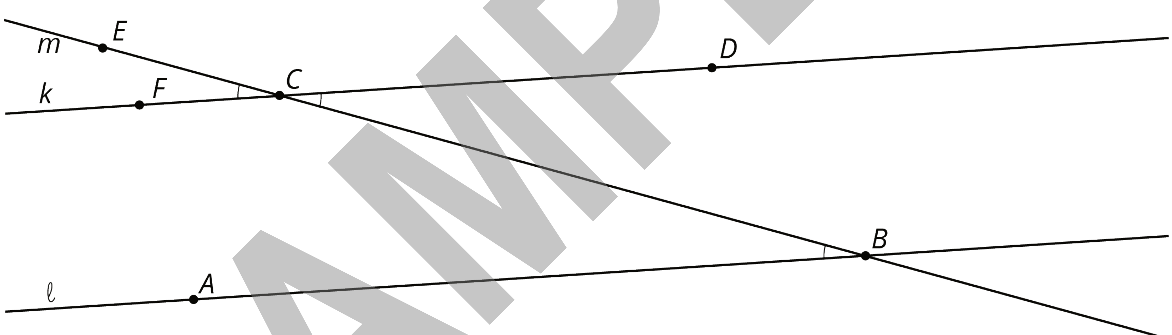
   b. $m\angle EBD$

   c. $m\angle ABE$



(From Unit 1, Lesson 8.)

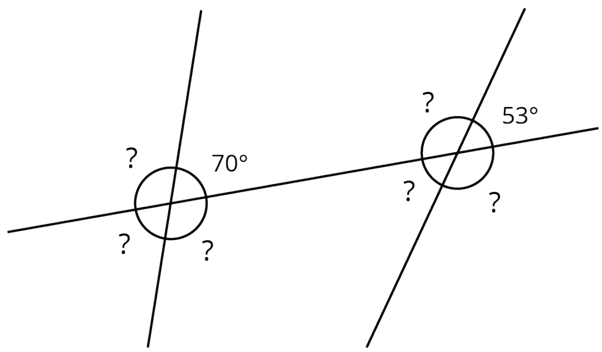
4. Lines k and ℓ are parallel, and the measure of angle ABC is 19 degrees.



   a. Explain why the measure of angle ECF is 19 degrees. If you get stuck, consider translating line ℓ by moving B to C.

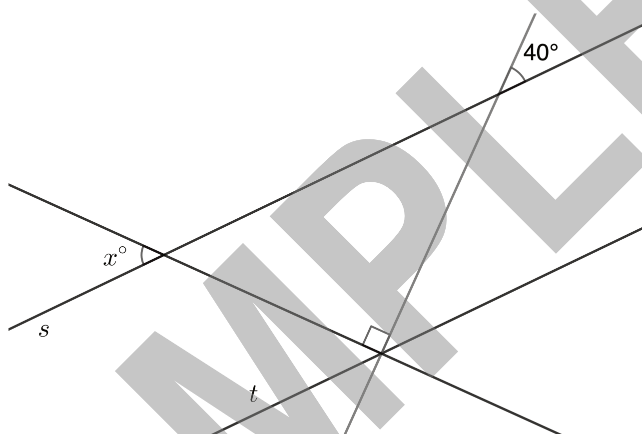
   b. What is the measure of angle BCD? Explain.

5. The diagram shows three lines with some marked angle measures.



Find the missing angle measures marked with question marks.

6. Lines s and t are parallel. Find the value of x.



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Lesson 13: Adding the Angles in a Triangle

Let's explore angles in triangles.

13.1: Can You Draw It?

1. Complete the table by drawing a triangle in each cell that has the properties listed for its column and row. If you think you cannot draw a triangle with those properties, write "impossible" in the cell.

2. Share your drawings with a partner. Discuss your thinking. If you disagree, work to reach an agreement.

	acute (all angles acute)	right (has a right angle)	obtuse (has an obtuse angle)
scalene (side lengths all different)			
isosceles (at least two side lengthsare equal)			
equilateral (three side lengths equal)			

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13.2: Find All Three

Your teacher will give you a card with a picture of a triangle.

1. The measurement of one of the angles is labeled. Mentally estimate the measures of the other two angles.

2. Find two other students with triangles congruent to yours but with a different angle labeled. Confirm that the triangles are congruent, that each card has a different angle labeled, and that the angle measures make sense.

3. Enter the three angle measures for your triangle on the table your teacher has posted.

13.3: Tear It Up

Your teacher will give you a page with three sets of angles and a blank space. Cut out each set of three angles. Can you make a triangle from each set that has these same three angles?

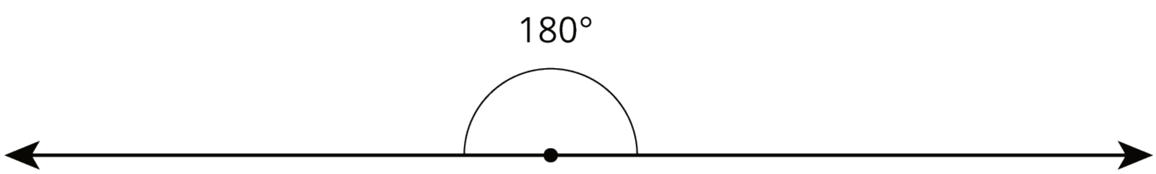
Are you ready for more?

1. Draw a quadrilateral. Cut it out, tear off its angles, and line them up. What do you notice?

2. Repeat this for several more quadrilaterals. Do you have a conjecture about the angles?

Lesson 13 Summary

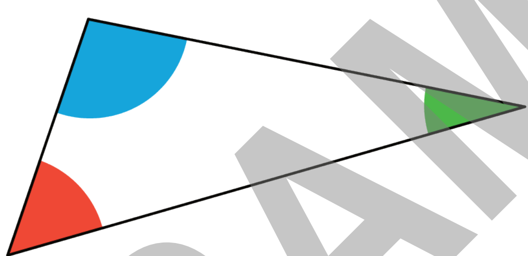
A 180° angle is called a **straight angle** because when it is made with two rays, they point in opposite directions and form a straight line.

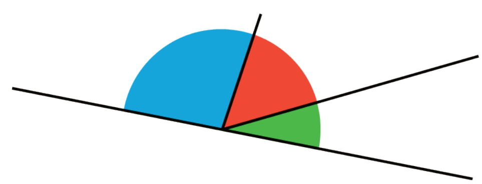


If we experiment with angles in a triangle, we find that the sum of the measures of the three angles in each triangle is 180°—the same as a straight angle!

Through experimentation we find:

- If we add the three angles of a triangle physically by cutting them off and lining up the vertices and sides, then the three angles form a straight angle.
- If we have a line and two rays that form three angles added to make a straight angle, then there is a triangle with these three angles.





Glossary

- straight angle

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Lesson 13 Practice Problems

1. In triangle ABC, the measure of angle A is 40°.

   a. Give possible measures for angles B and C if triangle ABC is isosceles.

   b. Give possible measures for angles B and C if triangle ABC is right.

2. For each set of angles, decide if there is a triangle whose angles have these measures in degrees:

   a. 60, 60, 60

   b. 90, 90, 45

   c. 30, 40, 50

   d. 90, 45, 45

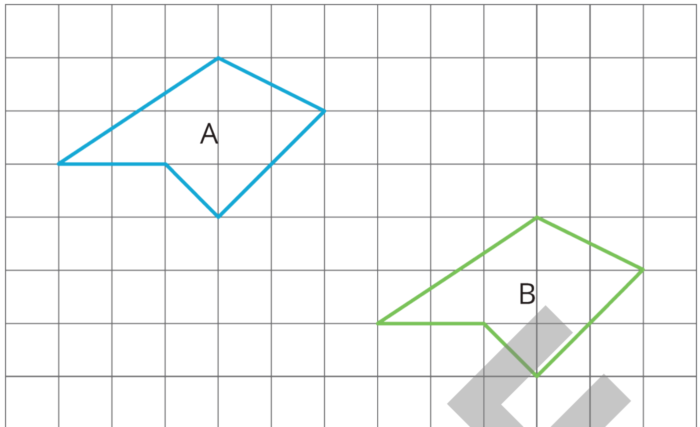
   e. 120, 30, 30

   If you get stuck, consider making a line segment. Then use a protractor to measure angles with the first two angle measures.

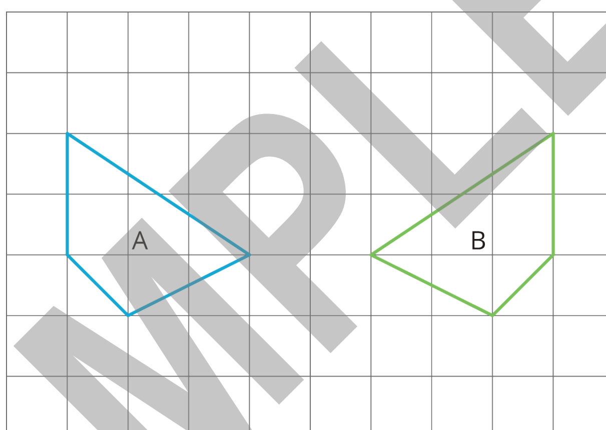
3. Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your reasoning.

4. For each pair of polygons, describe the transformation that could be applied to Polygon A to get Polygon B.

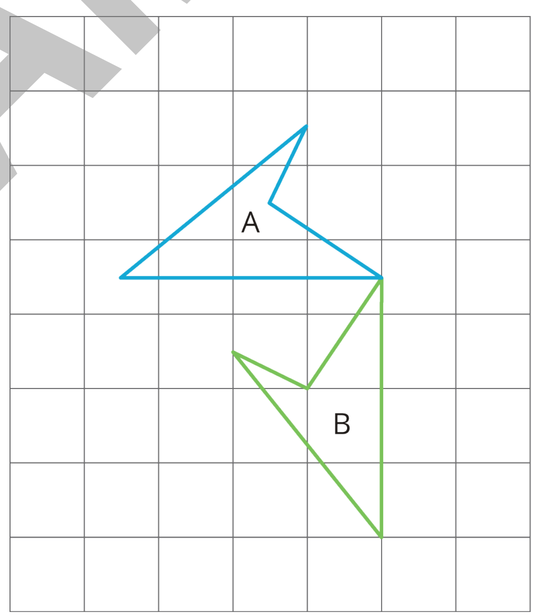
a.



b.



c.



(From Unit 1, Lesson 3.)

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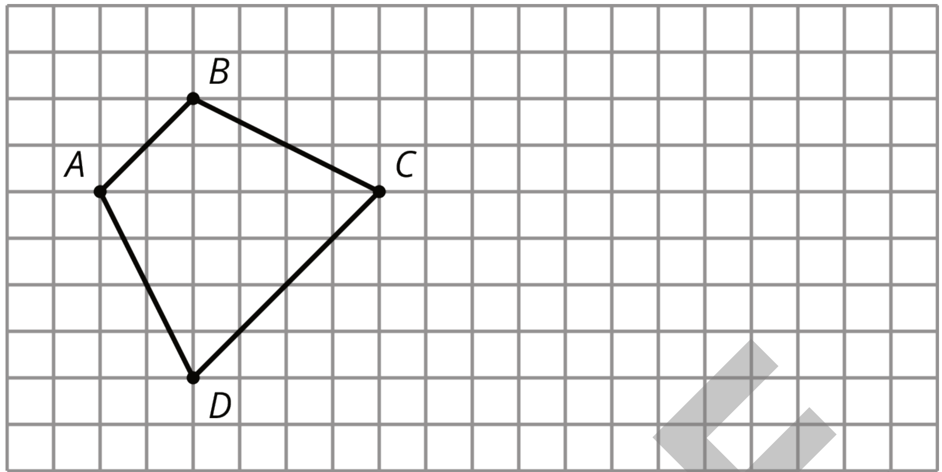
5. On the grid, draw a scaled copy of quadrilateral $ABCD$ using a scale factor of $\frac{1}{2}$.



(From Unit 1, Lesson 12.)

Lesson 14: Parallel Lines and the Angles in a Triangle

Let's see why the angles in a triangle add to 180 degrees.

14.1: True or False: Computational Relationships

Is each equation true or false?

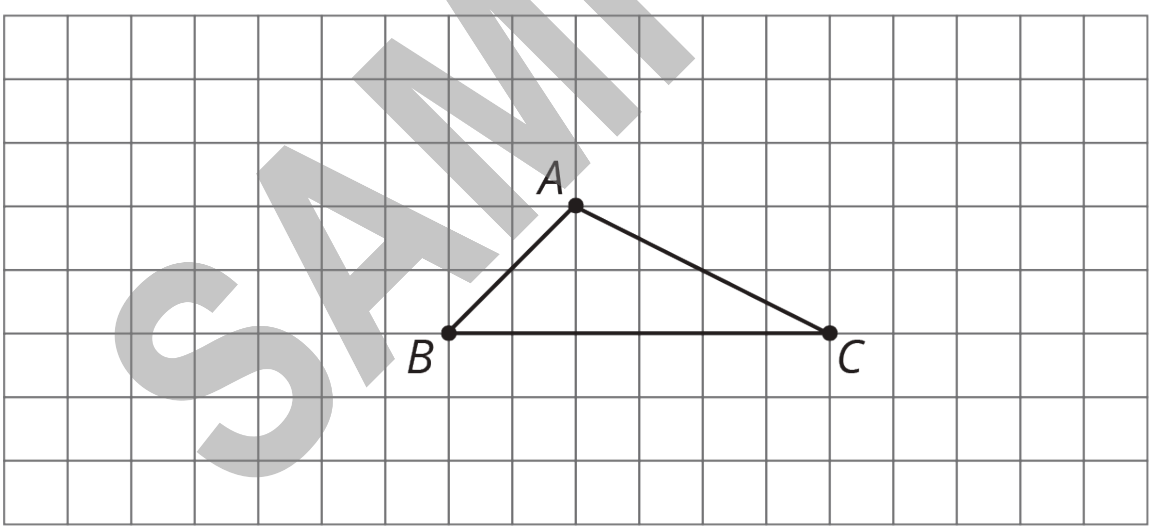
$62 - 28 = 60 - 30$

$3 \cdot \text{-}8 = (2 \cdot \text{-}8) - 8$

$\frac{16}{\text{-}2} + \frac{24}{\text{-}2} = \frac{40}{\text{-}2}$

14.2: Angle Plus Two

Here is triangle ABC.



1. Rotate triangle ABC 180° around the midpoint of side AC. Label the new vertex D.

2. Rotate triangle ABC 180° around the midpoint of side AB. Label the new vertex E.

3. Look at angles EAB, BAC, and CAD. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.

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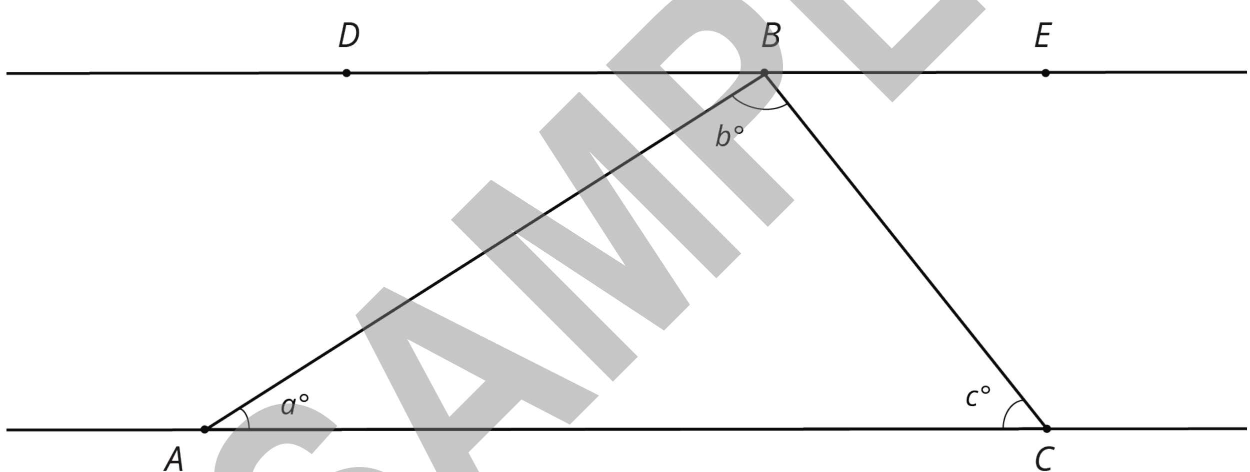
4. Is the measure of angle EAB equal to the measure of any angle in triangle ABC? If so, which one? If not, how do you know?

5. Is the measure of angle CAD equal to the measure of any angle in triangle ABC? If so, which one? If not, how do you know?

6. What is the sum of the measures of angles ABC, BAC, and ACB?

14.3: Every Triangle in the World

Here is $\triangle ABC$. Line DE is parallel to line AC.



1. What is $m\angle DBA + b + m\angle CBE$? Explain how you know.

2. Use your answer to explain why $a + b + c = 180$.

3. Explain why your argument will work for *any* triangle: that is, explain why the sum of the angle measures in *any* triangle is 180°.

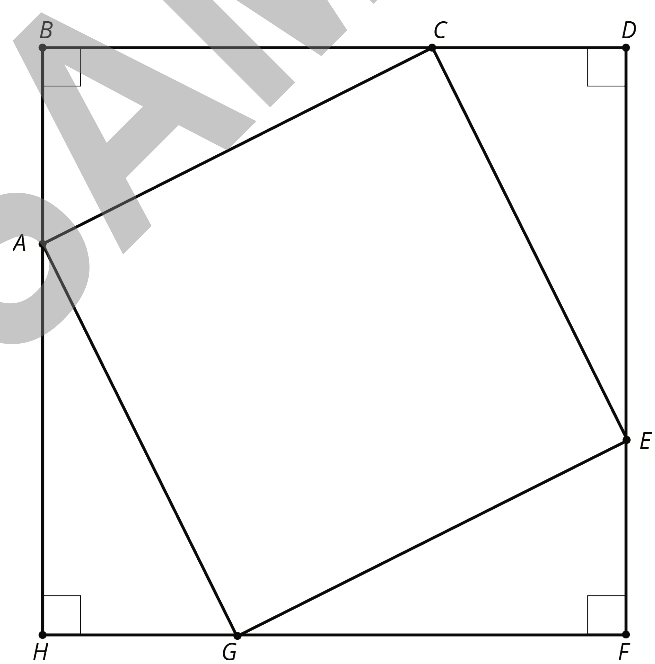
Are you ready for more?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?

2. Come up with an explanation for why anything you notice must be true (hint: draw one diagonal in each quadrilateral).

14.4: Four Triangles Revisited

This diagram shows a square $BDFH$ that has been made by images of triangle ABC under rigid transformations.



Given that angle BAC measures 53 degrees, find as many other angle measures as you can.

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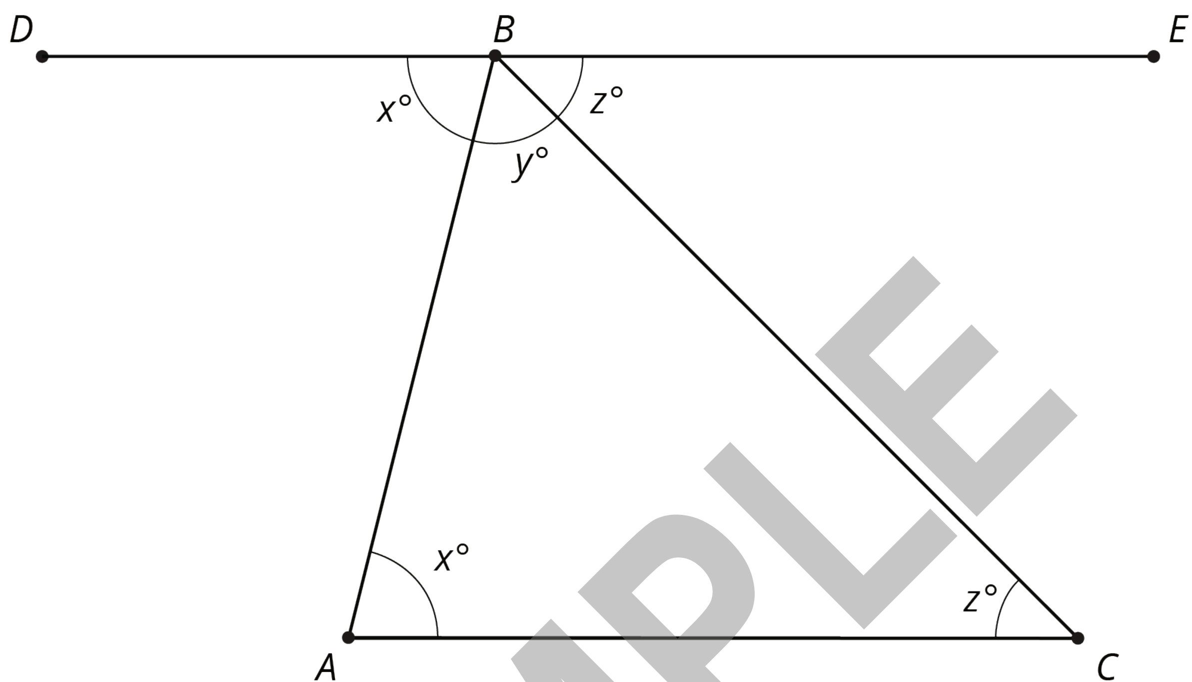
Lesson 14 Summary

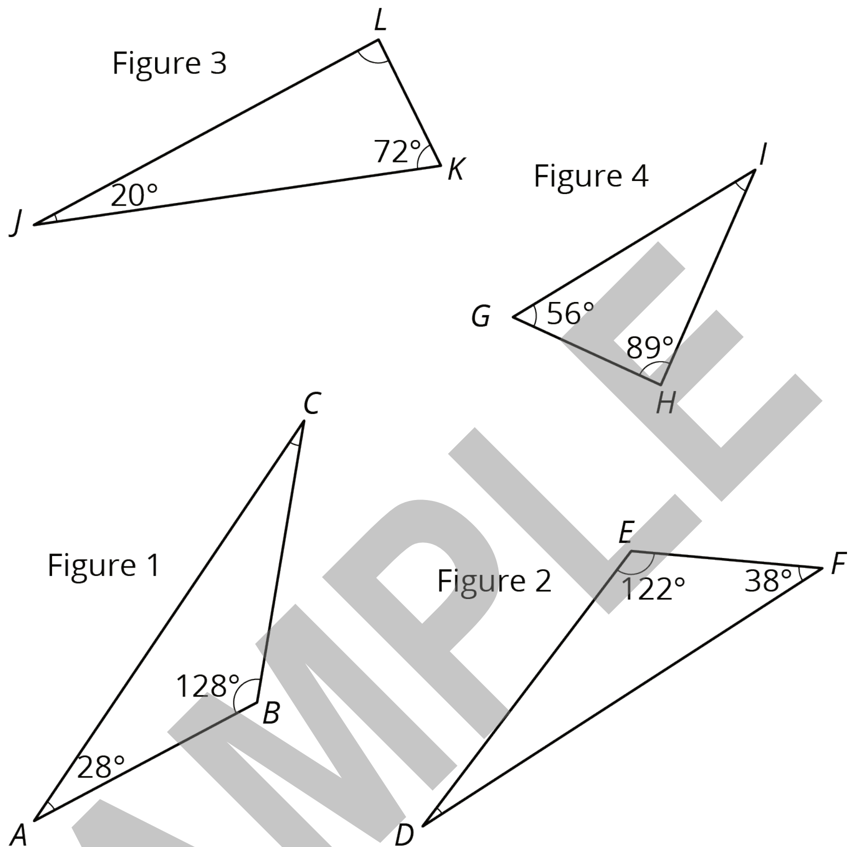
Using parallel lines and rotations, we can understand why the angles in a triangle always add to 180°. Here is triangle ABC. Line DE is parallel to AC and contains B.



A 180 degree rotation of triangle ABC around the midpoint of AB interchanges angles A and DBA so they have the same measure: in the picture these angles are marked as x°. A 180 degree rotation of triangle ABC around the midpoint of BC interchanges angles C and CBE so they have the same measure: in the picture, these angles are marked as z°. Also, DBE is a straight line because 180 degree rotations take lines to parallel lines. So the three angles with vertex B make a line and they add up to 180° ($x + y + z = 180$). But x, y, z are the measures of the three angles in $\triangle ABC$ so the sum of the angles in a triangle is always 180°!

Lesson 14 Practice Problems

1. For each triangle, find the measure of the missing angle.



2. Is there a triangle with *two* right angles? Explain your reasoning.

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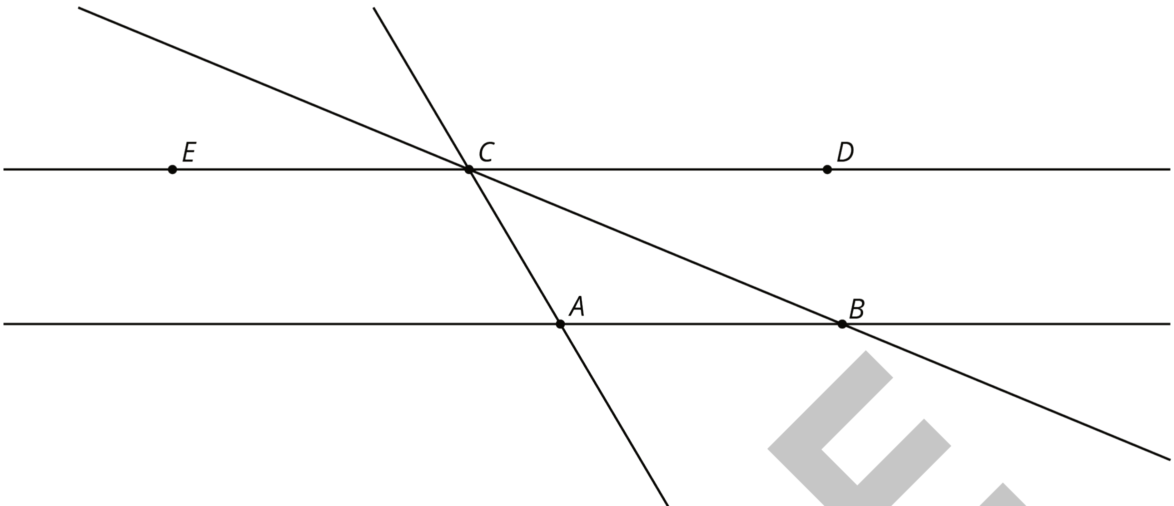
3. In this diagram, lines AB and CD are parallel.



Angle ABC measures 35° and angle BAC measures 115°.

a. What is $m\angle ACE$?

b. What is $m\angle DCB$?

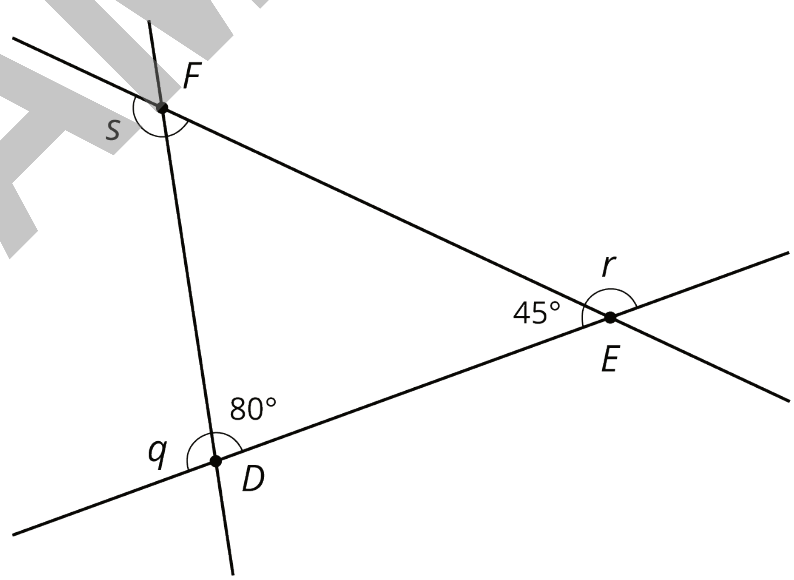
c. What is $m\angle ACB$?

4. Here is a diagram of triangle DEF.

a. Find the measures of angles q, r, and s.

b. Find the sum of the measures of angles q, r, and s.

c. What do you notice about these three angles?



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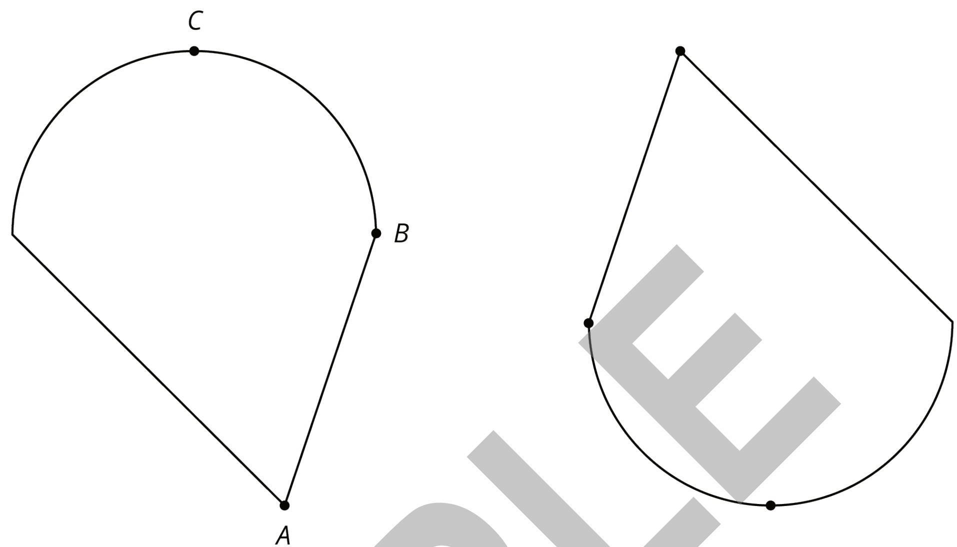
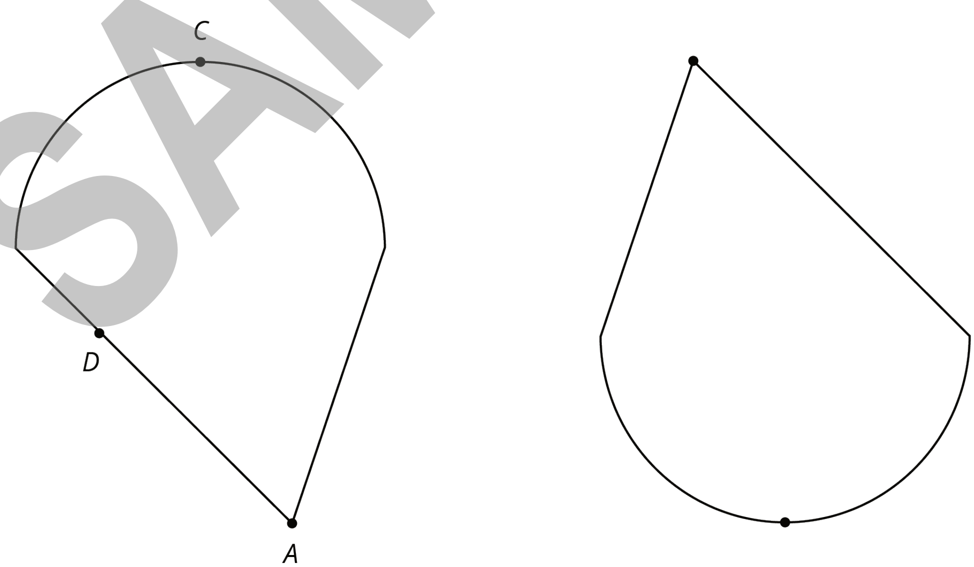
5. The two figures are congruent.

   a. Label the points A', B' and C' that correspond to A, B, and C in the figure on the right.



   b. If segment AB measures 2 cm, how long is segment $A'B'$? Explain.

   c. The point D is shown in addition to A and C. How can you find the point D' that corresponds to D? Explain your reasoning.



(From Unit 1, Lesson 11.)

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Lesson 15: Building Polygons

Let's build shapes.

15.1: Where Is Lin?

At a park, the slide is 5 meters east of the swings. Lin is standing 3 meters away from the slide.

1. Draw a diagram of the situation including a place where Lin could be.

2. How far away from the swings is Lin in your diagram?

3. Where are some other places Lin could be?

15.2: Building Diego's and Jada's Shapes

1. Diego built a quadrilateral using side lengths of 4 in, 5 in, 6 in, and 9 in.

    a. Build such a shape.

    b. Is your shape an identical copy of Diego's shape? Explain your reasoning.

2. Jada built a triangle using side lengths of 4 in, 5 in, and 8 in.

    a. Build such a shape.

    b. Is your shape an identical copy of Jada's shape? Explain your reasoning.

15.3: Swinging the Sides Around

We'll explore a method for drawing a triangle that has three specific side lengths. Your teacher will give you a piece of paper showing a 4-inch segment as well as some instructions for which strips to use and how to connect them.

1. Follow these instructions to mark the possible endpoints of one side:

    a. Put your 4-inch strip directly on top of the 4-inch segment on the piece of paper. Hold it in place.

    b. For now, ignore the 3-inch strip on the left side. Rotate it so that it is out of the way.

    c. In the 3-inch strip on the *right* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where a 3-inch side could end.

    d. Remove the connected strips from your paper.

2. What shape have you drawn while moving the 3-inch strip around? Why? Which tool in your geometry toolkit can do something similar?

3. Use your drawing to create two unique triangles, each with a base of length 4 inches and a side of length 3 inches. Use a different color to draw each triangle.

4. Reposition the strips on the paper so that the 4-inch strip is on top of the 4-inch segment again. In the 3-inch strip on the *left* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where another 3-inch side could end.

5. Using a third color, draw a point where the two marks intersect. Using this third color, draw a triangle with side lengths of 4 inches, 3 inches, and 3 inches.

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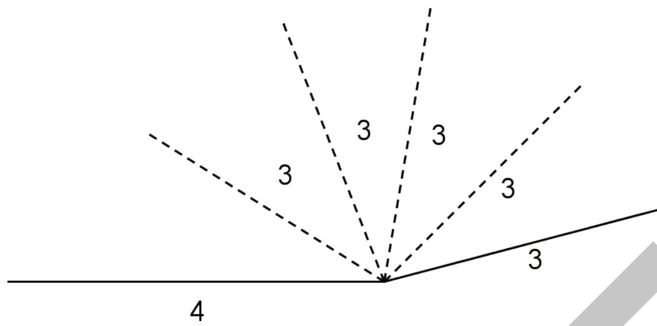
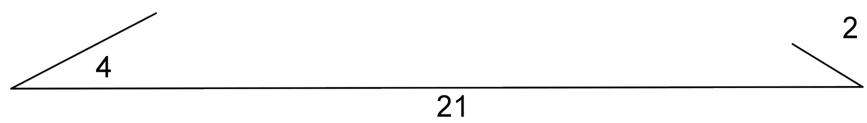
Lesson 15 Summary

If we want to build a polygon with two given side lengths that share a vertex, we can think of them as being connected by a hinge that can be opened or closed:



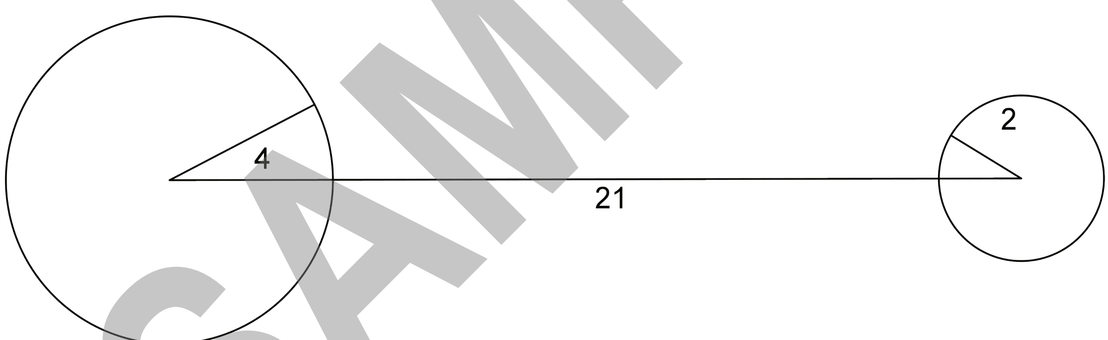
All of the possible positions of the endpoint of the moving side form a circle:

You may have noticed that sometimes it is not possible to build a polygon given a set of lengths. For example, if we have one really, really long segment and a bunch of short segments, we may not be able to connect them all up. Here's what happens if you try to make a triangle with side lengths 21, 4, and 2:



The short sides don't seem like they can meet up because they are too far away from each other.

If we draw circles of radius 4 and 2 on the endpoints of the side of length 21 to represent positions for the shorter sides, we can see that there are no places for the short sides that would allow them to meet up and form a triangle.



In general, the longest side length must be less than the sum of the other two side lengths. If not, we can't make a triangle!

If we *can* make a triangle with three given side lengths, it turns out that the measures of the corresponding angles will *always* be the same. For example, if two triangles have side lengths 3, 4, and 5, they will have the same corresponding angle measures.

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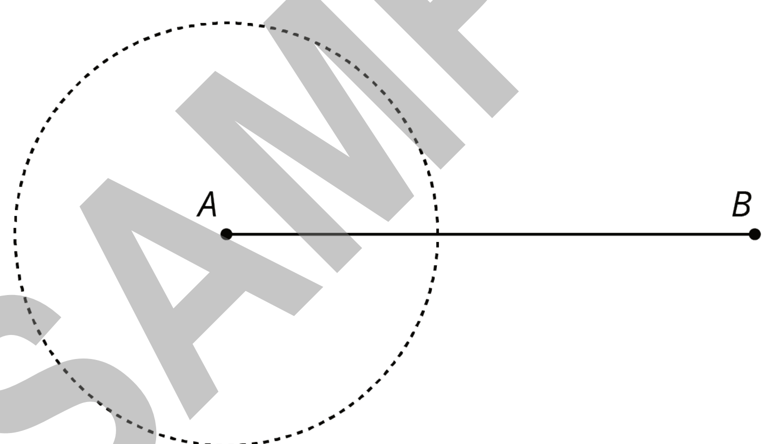
Lesson 15 Practice Problems

1. A rectangle has side lengths of 6 units and 3 units. Could you make a quadrilateral that is not identical using the same four side lengths? If so, describe it.

2. Come up with an example of three side lengths that can not possibly make a triangle, and explain how you know.

3. In the diagram, the length of segment AB is 10 units and the radius of the circle centered at A is 4 units. Use this to create two unique triangles, each with a side of length 10 and a side of length 4. Label the sides that have length 10 and 4.



4. Select **all** the sets of three side lengths that will make a triangle.

   A. 3, 4, 8

   B. 7, 6, 12

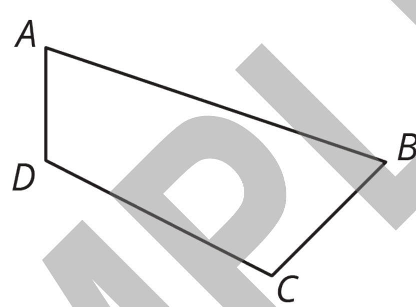
   C. 5, 11, 13

   D. 4, 6, 12

   E. 4, 6, 10

5. Based on signal strength, a person knows their lost phone is exactly 47 feet from the nearest cell tower. The person is currently standing 23 feet from the same cell tower. What is the closest the phone could be to the person? What is the furthest their phone could be from them?

6. Here is quadrilateral $ABCD$.



Draw the image of quadrilateral $ABCD$ after each rotation using B as center.

a. 90 degrees clockwise

b. 120 degrees clockwise

c. 30 degrees counterclockwise

(From Unit 1, Lesson 2.)

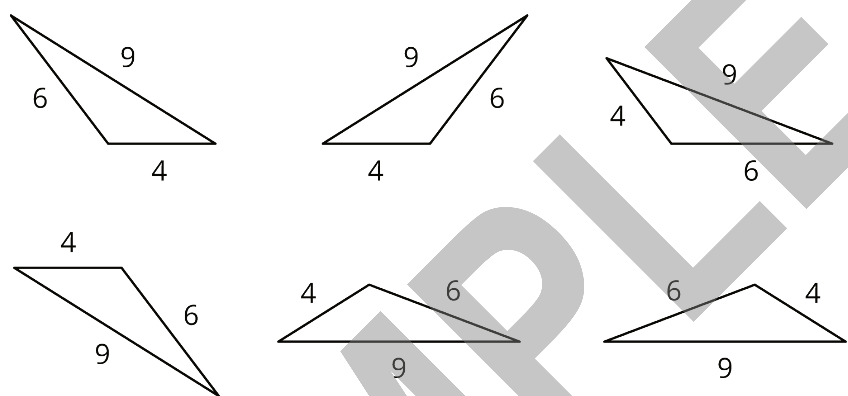
Lesson 16: Triangles with 3 Common Measures

Let's contrast triangles.

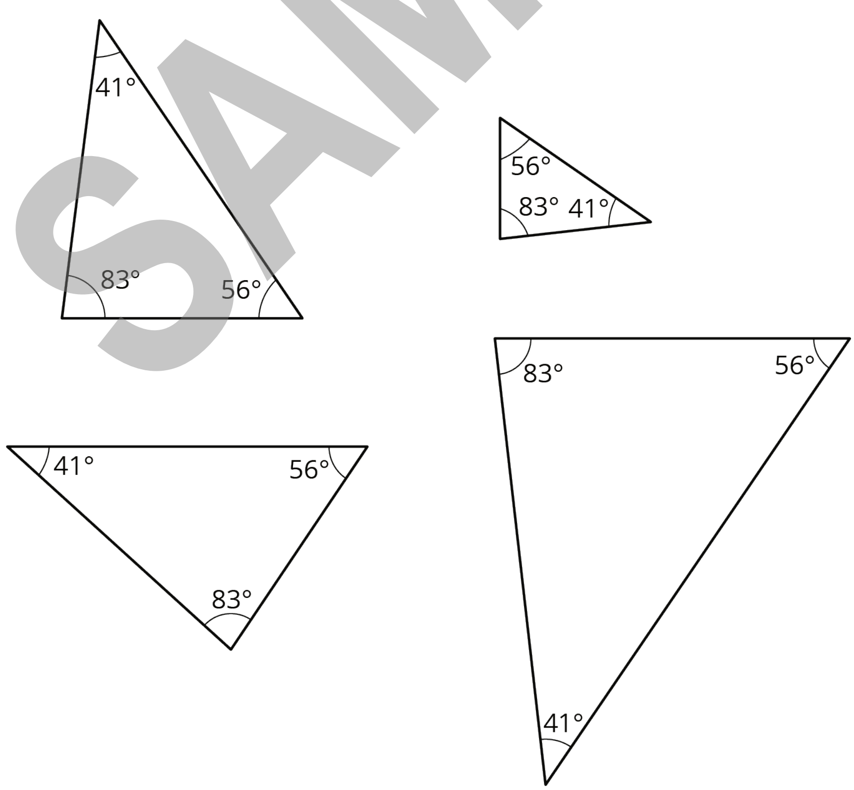
16.1: 3 Sides; 3 Angles

Examine each set of triangles. What do you notice? What is the same about the triangles in the set? What is different?

Set 1:



Set 2:



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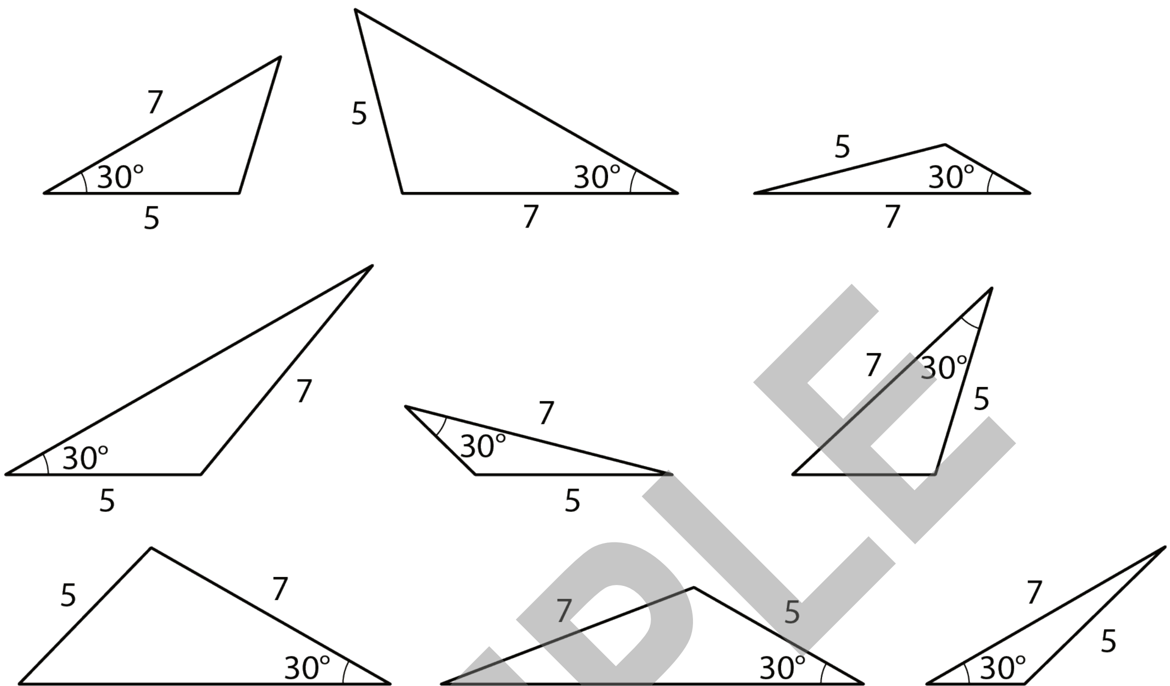
16.2: 2 Sides and 1 Angle

Examine this set of triangles.



1. What is the same about the triangles in the set? What is different?

2. How many different triangles are there? Explain or show your reasoning.

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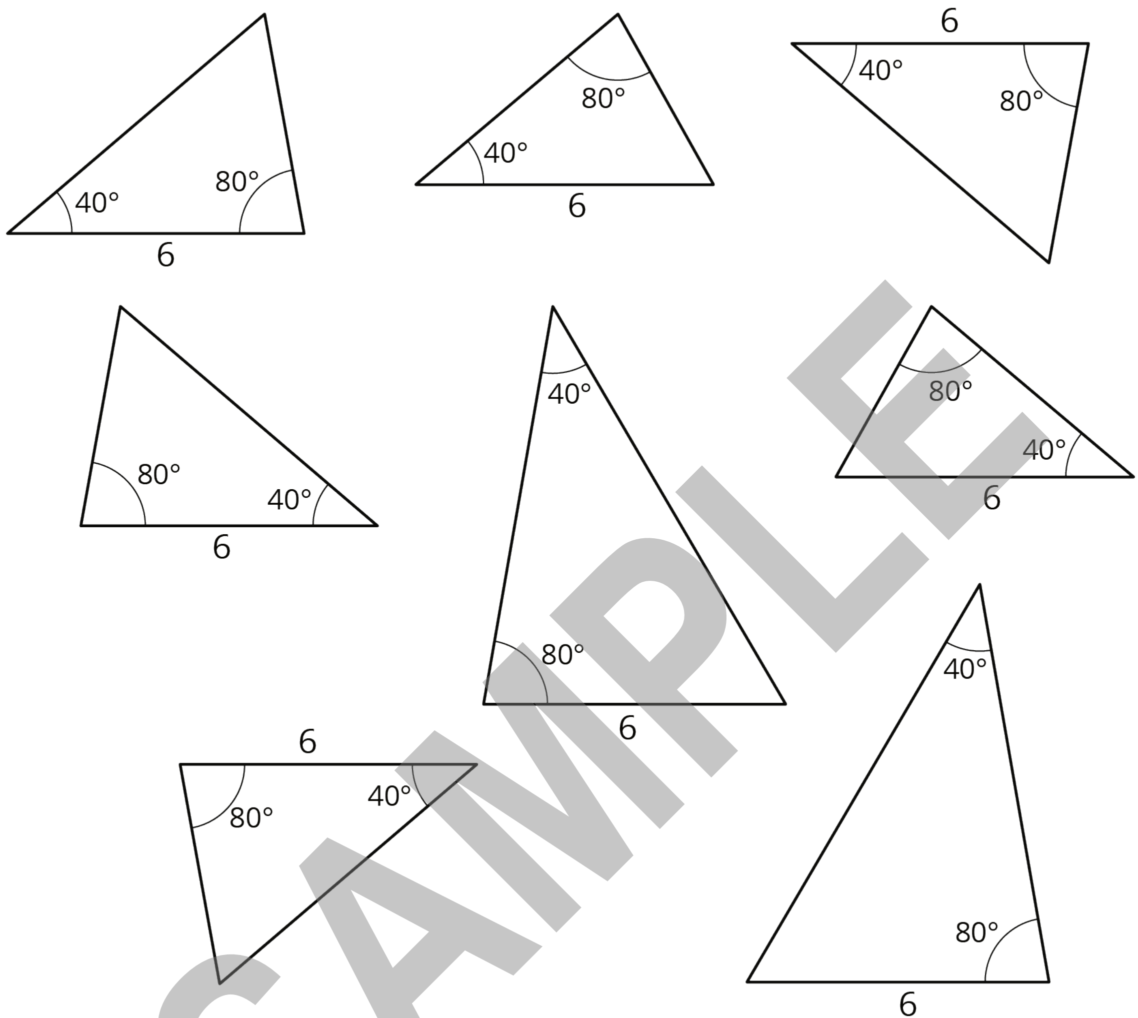
16.3: 2 Angles and 1 Side

Examine this set of triangles.



1. What is the same about the triangles in the set? What is different?

2. How many different triangles are there? Explain or show your reasoning.

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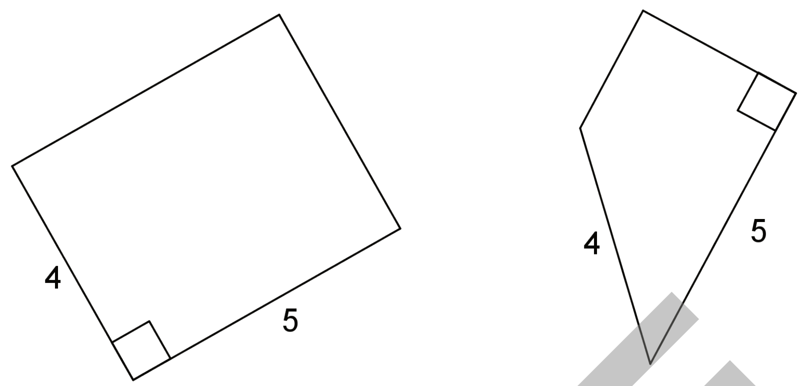
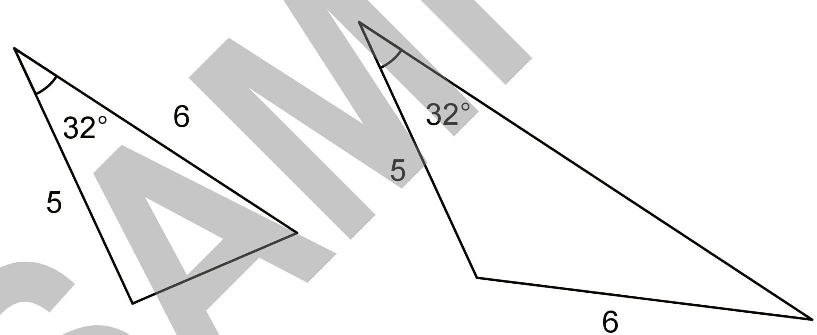
Lesson 16 Summary

Both of these quadrilaterals have a right angle and side lengths 4 and 5:



However, in one case, the right angle is *between* the two given side lengths; in the other, it is not.

If we create two triangles with three equal measures, but these measures are not next to each other in the same order, that usually means the triangles are different. Here is an example:



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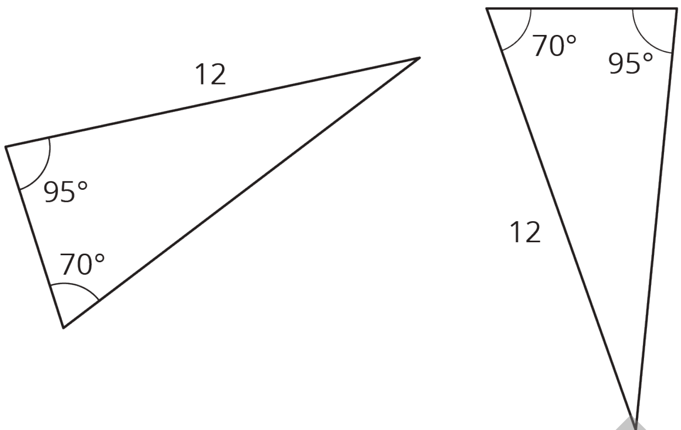
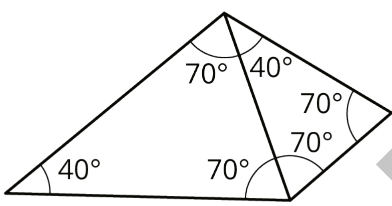
Lesson 16 Practice Problems

1. Are these two triangles identical? Explain how you know.



2. Are these triangles identical? Explain your reasoning.



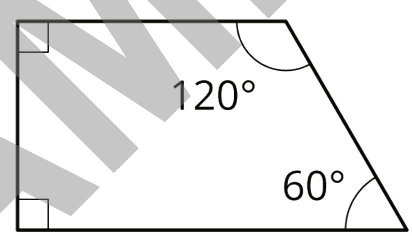
3. Tyler claims that if two triangles each have a side length of 11 units and a side length of 8 units, and also an angle measuring 100°, they must be identical to each other. Do you agree? Explain your reasoning.

4. a. Draw segment PQ.

b. When PQ is rotated 180° around point R, the resulting segment is the same as PQ. Where could point R be located?

(From Unit 1, Lesson 7.)

5. Here is trapezoid $ABCD$.



Using rigid transformations on the trapezoid, build a pattern. Describe some of the rigid transformations you used.

(From Unit 1, Lesson 9.)

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Lesson 17: Drawing Triangles

Let's see how many different triangles we can draw with certain measurements.

17.1: Using a Compass to Estimate Length

1. Draw a 40° angle.

2. Use a compass to make sure both sides of your angle have a length of 5 centimeters.

3. If you connect the ends of the sides you drew to make a triangle, is the third side longer or shorter than 5 centimeters? How can you use a compass to explain your answer?

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17.2: How Many Can You Draw?

1. Draw as many different triangles as you can with each of these sets of measurements:

   a. Two angles measure 60°, and one side measures 4 cm.

   b. Two angles measure 90°, and one side measures 4 cm.

   c. One angle measures 60°, one angle measures 90°, and one side measures 4 cm.

2. Which of these sets of measurements determine one unique triangle? Explain or show your reasoning.

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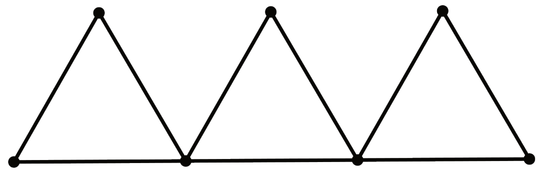
Are you ready for more?



In the diagram, 9 toothpicks are used to make three equilateral triangles. Figure out a way to move only 3 of the toothpicks so that the diagram has exactly 5 equilateral triangles.

17.3: Revisiting How Many Can You Draw?

1. Draw as many different triangles as you can with this set of measurements.
    a. One angle measures 40°, one side measures 4 cm, and one side measures 5 cm.

    b. Do these measurements determine one unique triangle? How do you know?

2. Draw as many different triangles as you can with each of these sets of angle measurements. Do either of these sets of measurements determine one unique triangle? Explain how do you know.
    a. One angle measures 50°, one measures 60°, and one measures 70°.

    b. One angle measures 50°, one measures 60°, and one measures 100°

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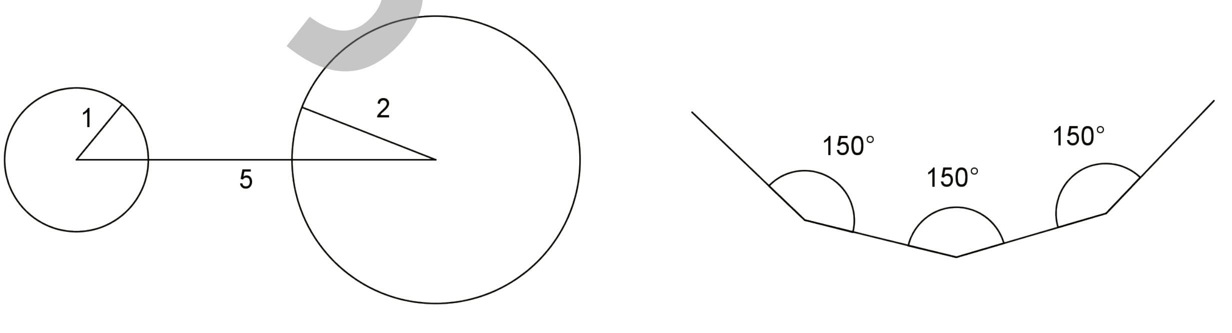
Are you ready for more?

Using *only* a compass and the edge of a blank index card, draw a perfectly equilateral triangle. (Note! The tools are part of the challenge! You may not use a protractor! You may not use a ruler!)

Lesson 17 Summary

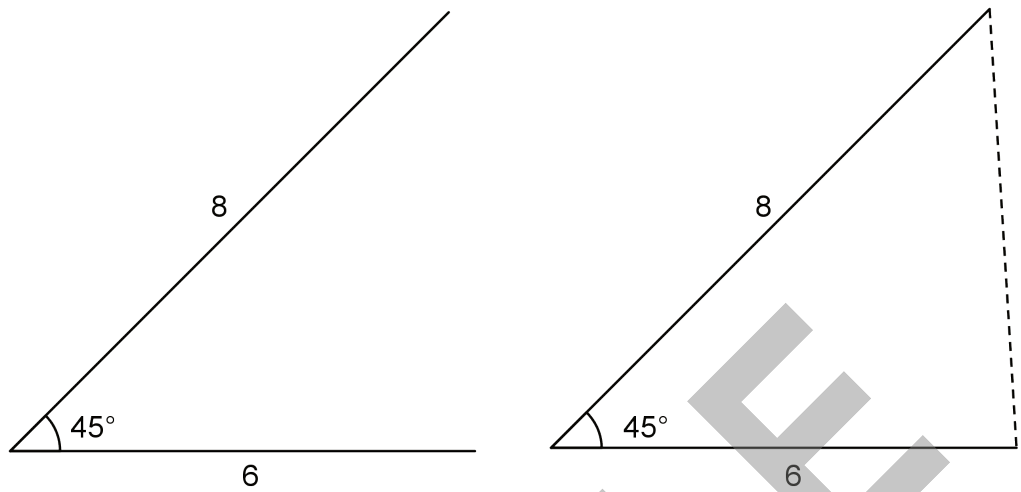
A triangle has six measures: three side lengths and three angle measures.

If we are given three measures, then sometimes, there is no triangle that can be made. For example, there is no triangle with side lengths 1, 2, 5, and there is no triangle with all three angles measuring 150°.

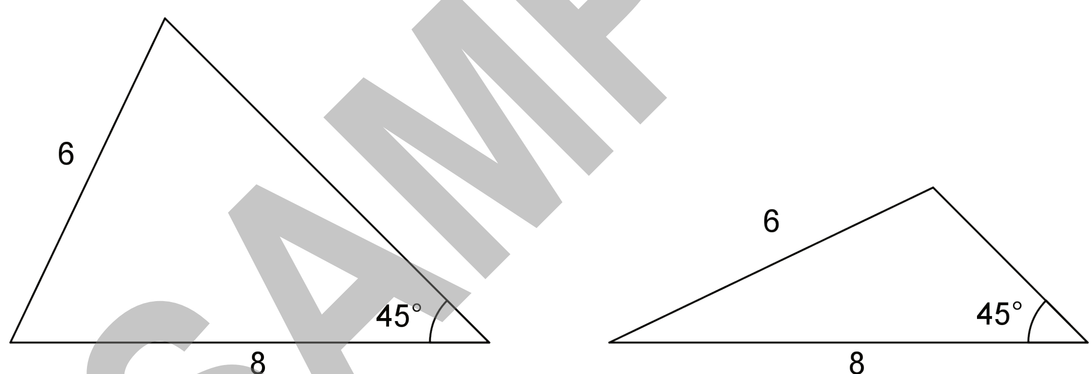


Sometimes, only one triangle can be made. By this we mean that any triangle we make will be the same, having the same six measures. For example, if a triangle can be made with three given side lengths, then the corresponding angles will have the same measures.

Another example is shown here: an angle measuring 45° between two side lengths of 6 and 8 units. With this information, one unique triangle can be made.



Sometimes, two or more different triangles can be made with three given measures. For example, here are two different triangles that can be made with an angle measuring 45° and side lengths 6 and 8. Notice the angle is not between the given sides.



Three pieces of information about a triangle's side lengths and angle measures may determine no triangles, one unique triangle, or more than one triangle. It depends on the information.

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Lesson 17 Practice Problems

1. Use a protractor to try to draw each triangle. Which of these three triangles is impossible to draw?

   a. A triangle where one angle measures 20° and another angle measures 45°

   b. A triangle where one angle measures 120° and another angle measures 50°

   c. A triangle where one angle measures 90° and another angle measures 100°

2. A triangle has an angle measuring 90°, an angle measuring 20°, and a side that is 6 units long. The 6-unit side is in between the 90° and 20° angles.

   a. Sketch this triangle and label your sketch with the given measures.

   b. How many unique triangles can you draw like this?

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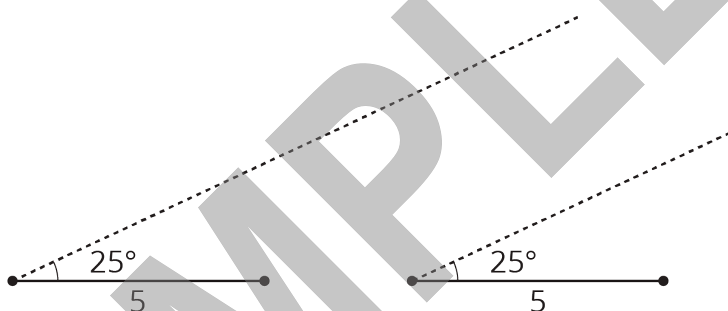
3. A triangle has sides of length 7 cm, 4 cm, and 5 cm. How many unique triangles can be drawn that fit that description? Explain or show your reasoning.

4. A triangle has one side that is 5 units long and an adjacent angle that measures 25°. The two other angles in the triangle measure 90° and 65°. Complete the two diagrams to create two *different* triangles with these measurements.



5. Is it possible to make a triangle that has angles measuring 90 degrees, 30 degrees, and 100 degrees? If so, draw an example. If not, explain your reasoning.

Lesson 18: Rotate and Tessellate

Let's make complex patterns using transformations.

18.1: Deducing Angle Measures

Your teacher will give you some shapes.

1. How many copies of the equilateral triangle can you fit together around a single vertex, so that the triangles' edges have no gaps or overlaps? What is the measure of each angle in these triangles?

2. What are the measures of the angles in the

    a. square?

    b. hexagon?

    c. parallelogram?

    d. right triangle?

    e. octagon?

    f. pentagon?

18.2: Tessellate This

1. Design your own **tessellation**. You will need to decide which shapes you want to use and make copies. Remember that a tessellation is a repeating pattern that goes on forever to fill up the entire plane.

2. Find a partner and trade pictures. Describe a transformation of your partner's picture that takes the pattern to itself. How many different transformations can you find that take the pattern to itself? Consider translations, reflections, and rotations.

3. If there's time, color and decorate your tessellation.

18.3: Rotate That

1. Make a design with rotational symmetry.

2. Find a partner who has also made a design. Exchange designs and find a transformation of your partner's design that takes it to itself. Consider rotations, reflections, and translations.

3. If there's time, color and decorate your design.

Glossary

- tessellation

Learning Targets

Lesson 1: Moving in the Plane

- I can describe how a figure moves and turns to get from one position to another.

Lesson 2: Naming the Moves

- I can identify corresponding points before and after a transformation.
- I know the difference between translations, rotations, and reflections.

Lesson 3: Making the Moves

- I can use grids to carry out transformations of figures.
- I can use the terms translation, rotation, and reflection to precisely describe transformations.

Lesson 4: Coordinate Moves

- I can apply transformations to points on a grid if I know their coordinates.

Lesson 5: Describing Transformations

- I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.

Lesson 6: No Bending or Stretching

- I can describe the effects of a rigid transformation on the lengths and angles in a polygon.

Lesson 7: Rotation Patterns

- I can describe how to move one part of a figure to another using a rigid transformation.

Lesson 8: Moves in Parallel

- I can describe the effects of a rigid transformation on a pair of parallel lines.
- If I have a pair of vertical angles and know the angle measure of one of them, I can find the angle measure of the other.

Lesson 9: Composing Figures

- I can find missing side lengths or angle measures using properties of rigid transformations.

Lesson 10: What Is the Same?

- I can decide visually whether or not two figures are congruent.

Lesson 11: Congruence

- I can decide using rigid transformations whether or not two figures are congruent.
- I can use distances between points to decide if two figures are congruent.

Lesson 12: Alternate Interior Angles

- I can find unknown angle measures by reasoning about complementary or supplementary angles.
- If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements.

Lesson 13: Adding the Angles in a Triangle

- If I know two of the angle measures in a triangle, I can find the third angle measure.

Lesson 14: Parallel Lines and the Angles in a Triangle

- I can explain using pictures why the sum of the angles in any triangle is 180 degrees.

Lesson 15: Building Polygons

- I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle.
- I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals.
- I can show whether or not 3 side lengths will make a triangle.

Lesson 16: Triangles with 3 Common Measures

- I understand that changing which sides and angles are next to each other can make different triangles.

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Lesson 17: Drawing Triangles

- Given two angle measures and one side length, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle.

Lesson 18: Rotate and Tessellate

- I can repeatedly use rigid transformations to make interesting repeating patterns of figures.

- I can use properties of angle sums to reason about how figures will fit together.

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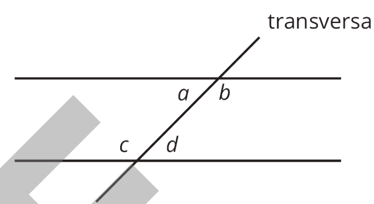
Glossary

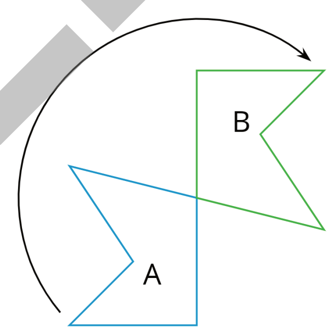
alternate interior angles
Interior angles are angles that are made by a transversal crossing two parallel lines. They are the angles that lie between the parallel lines, not outside them.

If two interior angles lie on opposite sides of the transversal they are called alternate interior angles.

In the figure, a and d are alternate interior angles, and b and c are also alternate interior angles.



clockwise
Clockwise means to turn in the same direction as the hands of a clock. The top turns to the right. This diagram shows Figure A turned clockwise to make Figure B.



complementary
Two angles are complementary to each other if their measures add up to 90°. The two acute angles in a right triangle are complementary to each other.

congruent
One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

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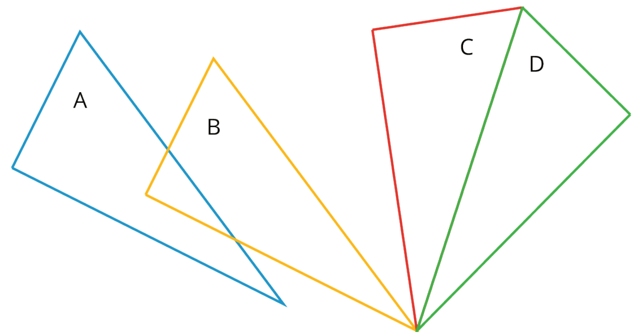
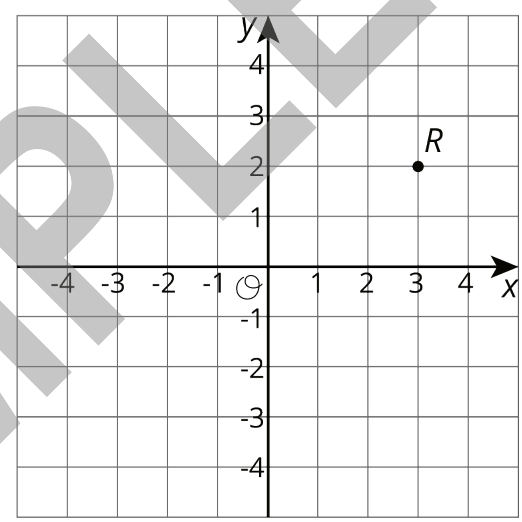
In the figure, Triangle A is congruent to Triangles B, C, and D. A translation takes Triangle A to Triangle B, a rotation takes Triangle B to Triangle C, and a reflection takes Triangle C to Triangle D.

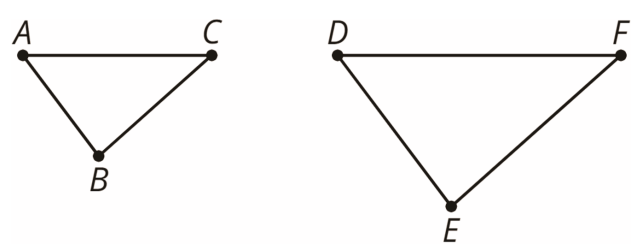


coordinate plane
The coordinate plane is a system for telling where points are. For example. point R is located at $(3, 2)$ on the coordinate plane, because it is three units to the right and two units up.



corresponding
When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

For example, point B in the first triangle corresponds to point E in the second triangle. Segment AC corresponds to segment DF.



counterclockwise
Counterclockwise means to turn opposite of the way the hands of a clock turn. The top turns to the left.

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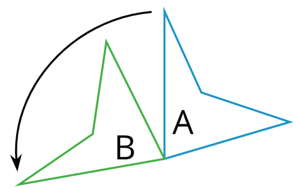
This diagram shows Figure A turned counterclockwise to make Figure B.

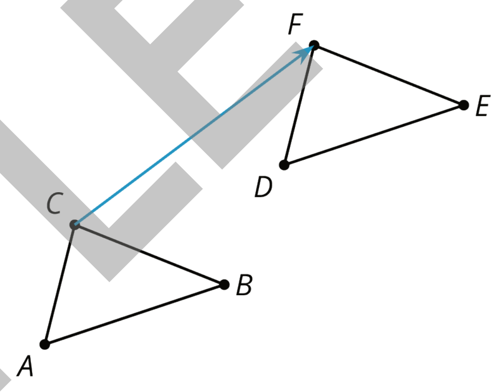


image

Translations, rotations, and reflections move objects in the plane. Points, segments, and other parts of the original all have corresponding parts on the "moved object." The moved object is called the **image**.

For example, here is triangle ABC and a translation to the right and up which is labeled DEF.

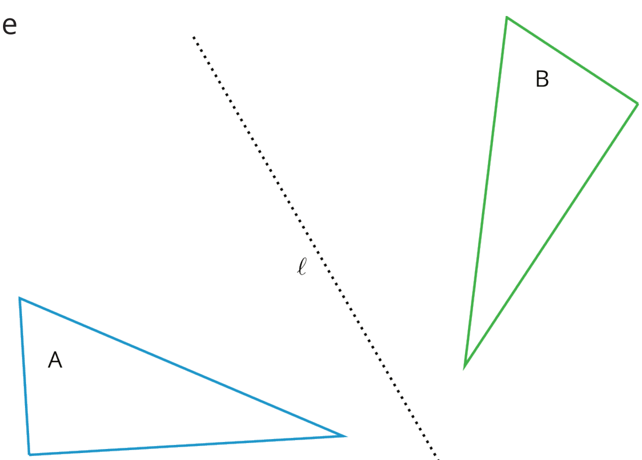
Point F in the image corresponds to point C, segment EF in the image corresponds to segment BD, and angle DEF corresponds to angle ABC.



reflection

A reflection across a line moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.

This diagram shows a reflection of A over line ℓ that makes the mirror image B.



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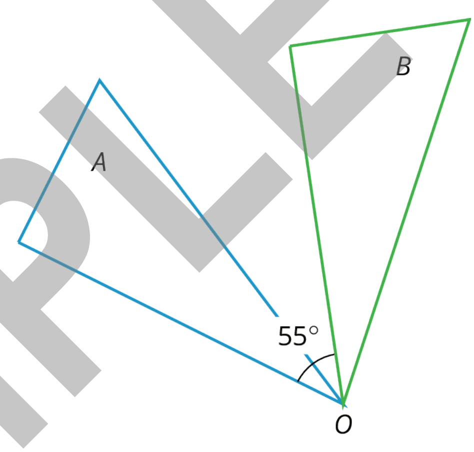
right angle
When you divide a straight angle into two angles with equal measures, each of the two angles is a right angle. For example, the four corners of a square are right angles.

rigid transformation
A rigid transformation is a move that does not change any measurements of a figure. Translations, rotations, and reflections are rigid transformations, as is any sequence of these.

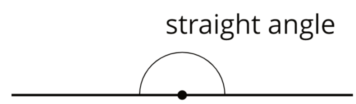
rotation
A rotation moves every point on a figure around a center by a given angle in a specific direction.

This diagram shows Triangle A rotated around center O by 55 degrees clockwise to get Triangle B.



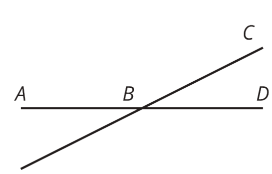
sequence of transformations

straight angle
A straight angle is an angle that forms a straight line. It measures 180 degrees.



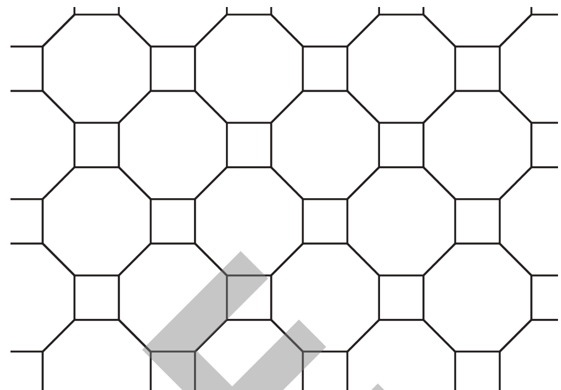
supplementary
Two angles are supplementary to each other if their measures add up to 180°.

For example, angle ABC is supplementary to angle CBD, because they add up to a straight angle, which has measure 180°.



tessellation
A tessellation is a repeating pattern of one or more shapes. The sides of the shapes fit together perfectly and do not overlap. The pattern goes on forever in all directions.

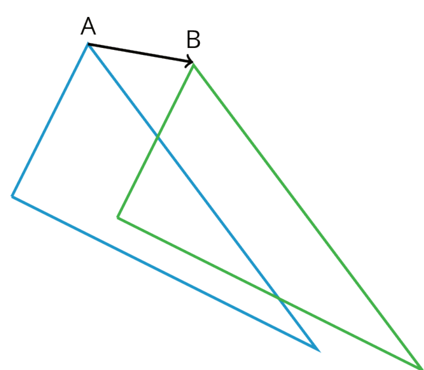
This diagram shows part of a tessellation.



transformation
A transformation is a translation, rotation, reflection, or dilation, or combination of these. There is also a more general concept of a transformation of the plane that is not discussed in grade 8.

translation
A translation moves every point in a figure a given distance in a given direction.

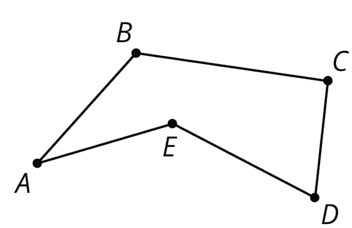
This diagram shows a translation of Figure A to Figure B using the direction and distance given by the arrow.



transversal

vertex
A vertex is a point where two or more edges meet. When we have more than one vertex, we call them vertices.

The vertices in this polygon are labeled A, B, C, D, and E.



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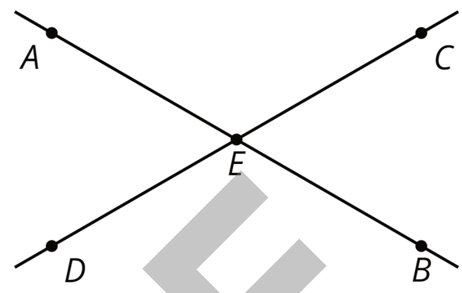
vertical angles

Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal.

For example, angles AEC and DEB are vertical angles. If angle AEC measures 120°, then angle DEB must also measure 120°.

Angles AED and BEC are another pair of vertical angles.



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